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Restall's Proof-Theoretic Pluralism and Relevance Logic

Teresa Kouri

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Abstract

Restall (2014) proposes a new, proof-theoretic, logical pluralism. This is in contrast to the model-theoretic pluralism he and Beall proposed in Beall and Restall (2000) and in Beall and Restall (2006). What I will show is that Restall has not described the conditions on being admissible to the proof-theoretic logical pluralism in such a way that relevance logic is one of the admissible logics. Though relevance logic is not hard to add formally, one critical component of Restall's pluralism is that the relevance logic that gets added must have connectives which mean the same thing as the connectives in the already admitted logic. This is what I will show is not possible.

1 Introduction

Restall (2014) proposes a new, proof-theoretic, logical pluralism. It is based on a sequent calculus, where the connectives are implicitly defined (or constituted) by their left and right logical rules. Defining the logical constants by their left and right logical rules is far from new. We see Dummett, Gentzen and even Prawitz suggesting similar moves in their respective systems, though they are not logical pluralists. What is unusual is that Restall takes these rules to implicitly define the same set of connectives *across different logics*, all of which are taken to be "right". Not all pluralists hold that connective meanings are the same across all admissible logics, and so this is part of what makes his system's approach to his chosen three interesting. He holds that because the change in logics is affected by differences in things other than the left and right logical rules for a connective, the connectives mean the same thing in all three logics.

I will show that Restall's new proof-theoretic pluralism is such that it cannot include a relevance logic. For the purposes of this paper, I take a relevance logic to be one which minimally avoids the positive and negative forms of Lewis's paradox.[1] In this paper I will demonstrate why Restall's system cannot accommodate such a logic. I will suggest that this is because Restall has not provided adequate conditions on which logics are admissible, and that the reasonable ways of spelling out such conditions do not allow us to include a relevance system in this framework. I hold that this provides a basis for dismissing this framework for pluralism in general, as Restall ought to want his pluralism to be able to accommodate a relevance logic.

There are at least two reasons to think Restall should want his pluralism to include a relevance logic. First, in his work with Beall (see Beall and Restall (2006)), they include a (particular) relevance logic as a legitimate logic. This gives us at least reason to suspect that Restall thinks relevance logics are legitimate, and that at least one should be included as a legitimate logic in a pluralism. Second, a pluralism which includes only three logics – none of which are relevance logics – seems unnecessarily restrictive, especially since the logics which are admitted into the pluralism are generated by altering structural rules, and one can generate

[1] The positive form is that from a contradiction, one can infer anything. The negative for is that from a contradiction, one can infer any negated sentence.

a relevance logic by altering structural rules (though, I will show, not without changing the meanings of the connectives in Restall's system). Together, these two reasons suggest that if Restall's proof-theoretic pluralism cannot include a relevance logic, then it ought to be dismissed. [2]

2 Restall's Proof-Theoretic Pluralism

Restall (2014) develops his pluralism on the basis of a sequent calculus. This pluralism explicitly encompasses classical, intuitionistic and dual intuitionistic logic.[3]

Restall's calculus encompasses at least the following rules:

$$\text{identity}\ \frac{}{\Gamma, A \vdash A, \Delta} \qquad \wedge\text{L}\ \frac{\Gamma, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}$$

$$\text{cut}\ \frac{\Gamma \vdash A \qquad A \vdash \Delta}{\Gamma \vdash \Delta}$$

$$\text{weakening R}\ \frac{\Gamma \vdash A}{\Gamma \vdash A, B} \qquad \wedge\text{R}\ \frac{\Gamma \vdash \Delta, A \qquad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \wedge B}$$

$$\text{weakening L}\ \frac{\Gamma \vdash A}{\Gamma, B \vdash A} \qquad \vee\text{R}\ \frac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \vee B, \Delta}$$

$$\neg\text{L}\ \frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta} \qquad \vee\text{R}\ \frac{\Gamma \vdash B, \Delta}{\Gamma \vdash A \vee B, \Delta}$$

$$\neg\text{R}\ \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \Delta, \neg A}$$

$$\wedge\text{L}\ \frac{\Gamma, A \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \qquad \vee\text{L}\ \frac{\Gamma, A \vdash \Delta \qquad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}$$

For any connective, $*$, Restall claims that $*$L and $*$R constitute the meaning of, or implicitly define, the connective $*$. The $*$L and $*$R are the logical rules

[2] The lessons drawn for Restall here will certainly have an impact on the debate on the possibility of defining logical connectives inferentially. See, for example, Hjortland (2014) and Paoli (2014), who discuss the possible meaning-constituting laws of the logical connectives.

[3] Dual intuitionistic logic is a paraconsistent logic. It has the same sentential theorems as classical logic, but not the same counter-theorems. In fact, it does not disprove all contradictions. See Urbas (1996) for more details.

for the connective $*$. For example, $\neg$ is implicitly defined by $\neg$L and $\neg$R. Any rule which does not explicitly involve a connective is a structural rule. Identity, cut, weakening L and weakening R are such structural rules.[4] Though the meanings of the connectives are constituted by the rules given above, it will become important for our purposes that, according to Restall, the connective meanings are constituted by those rules *read in a particular way*. The rules must fit with reading the sequent as "every evaluation which makes everything on the left true makes something on the right true". I will discuss this requirement in more detail below.

Now we are in a position to show how Restall's pluralism works. It is well known that for this calculus, restricting the number of sentences on the right (in all sequents) to at most one results in a calculus where the law of excluded middle is no longer valid. This restriction, in fact, results in intuitionistic logic. This is how Restall, like Gentzen originally, captures intuitionistic consequence with this classical sequent calculus. The original sequent calculus proves all and only the classical truths, and if we restrict ourselves to proofs in which each sequent has at most one sentence on the right, we can prove all and only the intuitionistic consequences. Moreover, if we restrict ourselves to sequents with at most one formula on the left, we get what Restall calls dual-intuitionistic logic. Restall claims that since he has not changed the left- and right-logical rules, he has not changed the meanings of the connectives. Thus, he holds that this is a framework for a pluralism where the connectives are the same across logical consequence relations.

In a sense, what we have is the following: one object-language, one meta-language, and three "notions" of validity.[5] That is, we have a meta-language

[4]The two weakening rules are not given explicitly in Restall (2014). They have been adapted from Restall (2000).

[5]I am not quite sure what term to use here, and have settled on "notion" (with thanks to a blind reviewer for the suggestion). It is something like a reading, a sharpening, an interpretation or a

which contains three meta-relations which correspond to three "notions" of validity. I will call these relations $\vdash_C$, $\vdash_I$ and $\vdash_{DI}$, for the relations which correspond to classical validity, intuitionistic validity and dual intuitionistic validity. On "notion" one, we have classical validity, which is characterized by the meta-relation $\vdash_C$, and is given by the full complement of structural and logical rules. On "notion" two, we have intuitionistic validity, which is characterized by the meta-relation $\vdash_I$, and is given by the full complement of logical rules with a restriction on succedents in all the rules. On "notion" three, we have dual-intuitionistic validity, which is characterized by the meta-relation $\vdash_{DI}$, and is also given by the full complement of logical rules but with a different restriction, this time on the antecedents in the rules. For $i = C, I, DI$, $\Gamma \vdash_i \Delta$ can be read "every evaluation which takes every element of $[\Gamma]$ to be true takes some element of $[\Delta]$ to be true" (Restall, 2014, p 282). All three "notions" of validity have the same logical rules, and so all three associated object-languages have the same connectives.

It is of critical importance that all three "notions" result in validity relations, i.e. that they are all "notions" of the concept of validity. In their book, Beall and Restall propose that a logical consequence[6] relation must be necessary, normative and formal. A major problem with Restall (2014) is that it does not present such criteria. However, there needs to be one. We would like, for example, to rule out a calculus with the structural rules that there can only ever be exactly zero formulae on the left and exactly zero formulae on the right. This structural restriction rules out making any inferences at all. Additionally, if we chose instead to rule out the identity rule, we could not even infer that $A \vdash A$, and given that this is the only unconditional rule, we would never be able to prove anything unconditionally.

precisification. I will use the word "notion" in order to remain neutral on this this topic, as the results I present only require that any "notions"/readings/sharpenings/etc. of the same term have something in common, which all of these ways to spell out what a "notion" is.

[6]Though Beall and Restall (2006) speak of logical consequence relations, and I am here speaking of validities, I take these two notions to be similar enough. A logical consequence relation can, for example, be thought of as a set/class of validities satisfying certain closure properties.

Since, I take it, part of what it is to be a right logic is to be capable of licensing unconditional inferences, these types of restrictions cannot be permitted.[7] We need some way to rule them out, and giving criteria of what counts as a "notion" of validity proper is one way to do this. Moreover, it is not simply a matter of ruling out just one or two odd systems. We can generate these odd systems fairly simply: we just need to restrict the number of formulae on the left- and right-hand side of the sequent to an exact number. The calculus generated by the structural restriction of exactly three sequents on the left and exactly four on the right, for example, is also as odd, and it is not clear that it ought to be included in any calculus claiming to give us the "right" logics.

The question that we have to ask now is whether there is a fourth "notion" of validity, with a fourth meta-relation, call it $\vdash_R$, which corresponds to some type of relevance validity. The rest of this paper is devoted to exploring some possibilities for such admissibility criteria of logics into the proof-theoretic pluralism, and assessing whether they admit a relevance logic without admitting the undesirable calculi mentioned above. What I will show is that no reasonable criteria provide us with a framework which admits classical, intuitionist and relevance logics, and which preserves connective meanings across all such logics. In the next section, I will claim that the most natural way Restall has to give a criterion to rule out undesirable "logics" as above is to claim that real "notions" of validity can be read "every evaluation which takes every element of $[\Gamma]$ to be true takes some element of $[\Delta]$ to be true". This, I will show, causes problems when attempting to include a relevance logic. Following this, I will consider two other options,

[7] I take it that this requirement on a logic is inherently plausible, and hold that Beall and Restall (2006) would agree. As they note, their system does not license logics which are not reflexive; logics which do not license the inference from A to itself. They refer to such systems as "logics by courtesy and by family resemblance" (Beall and Restall, 2006, p 91). I take it that their dismissal of non-reflexive systems suggests that they would agree that the logic developed by removing the identity rule from the sequent calculus ought not to be one of the right logics, and that they hold a right logic needs to license at least some unconditional inferences.

both of which fail as well.

3 No relevance validity

Now, one might think that some relevance logics can be added to this pluralism quite simply. It is known that removing the weakening rules given above results in a sequent calculus where it is not possible to prove any irrelevancies, and can reasonably be called a relevance logic (see, for example, Restall (2000)).[8] Since the structural rule changes do not change the logical rules, Restall could reasonably claim that the connectives in this new system still mean the same thing as those in the original system, and hence that this change allows a relevance logic into his proof-theoretic pluralism. What I will show in this section is that removing the weakening rules requires a change in the sequent calculus which is not acceptable in Restall's proof-theoretic pluralism, and thus relevance logic cannot be added to his system.

The fact that Restall's 2014 pluralism cannot take relevance logics seriously stems from two simple, easily derivable and well known, equivalences between the weakening rules and the following two intuitively true sequents:

$$A \vee B \vdash A, B \tag{1}$$

and

[8]To my knowledge, this is the only method that has taken hold for this particular proof theory. At the very least, Restall (2000) claims this is a method which is natural to use. It is due to Tennant (1984, 1992).

$A, B \vdash A \wedge B$

$$(2)$$

Recall that we are meant to read the sequent $\Gamma \vdash \Delta$ as "every evaluation which takes every element of $[\Gamma]$ to be true takes some element of $[\Delta]$ to be true" (Restall, 2014, p 282). Thus, the rules (1) and (2) ought to be at least intuitively valid. (1) "says" that if $A \vee B$ is true, then one of A or B is true. (2) "says" that if both A and B are true, then $A \wedge B$ is true. So far so good. The following proofs show that weakening R and weakening L are respectively inter-derivable with (1) and (2):

$$\dfrac{\dfrac{A \vdash A}{A \vdash A \vee B}\,\vee\mathrm{R} \qquad A \vee B \vdash A, B}{A \vdash A, B}\,\mathrm{CUT} \qquad\qquad \dfrac{\dfrac{A \vdash A}{A \vdash A, B}\,\mathrm{WR} \qquad \dfrac{B \vdash B}{B \vdash A, B}\,\mathrm{WR}}{A \vee B \vdash A, B}\,\vee\mathrm{L}$$

$$\dfrac{A, B \vdash A \wedge B \qquad \dfrac{A \vdash A}{A \wedge B \vdash A}\,\wedge\mathrm{L}}{A, B \vdash A}\,\mathrm{CUT} \qquad\qquad \dfrac{\dfrac{A \vdash A}{A, B \vdash A}\,\mathrm{WL} \qquad \dfrac{B \vdash B}{A, B \vdash B}\,\mathrm{WL}}{A, B \vdash A \wedge B}\,\wedge\mathrm{R}$$

This is a simple result which most people familiar with sequent calculus could prove, and Restall is no exception. The fact is, though, that Restall does not seem to have realized the impact it has on his pluralism. What causes the problem for Restall is that invalidating (1) or (2), as we would need to in order to add a relevance consequence to his system, is problematic.

The combination of weakening R and $\neg$L actually proves a form of Lewis's paradox, which violates our minimal criteria of a relevance logic:

$$\dfrac{\dfrac{A \vdash A}{A \vdash A, B}\,\mathrm{WR}}{A, \neg A \vdash B}\,\neg\mathrm{L}$$

Thus, we need to "get rid of" either weakening R or $\neg$L. In this instance, it is weakening which must go, since we need to preserve connective meanings, and to do so we must keep all of the logical rules.

To invalidate weakening L and weakening R, we must make (1) and (2) invalid.[9] If we want to claim (1) and (2) are invalid, we have to claim that there is some evaluation on which the formulae on the left hand side of the sequents are true, but no formula on the right hand side is true (this is just the negation of what it means for a sequent to be valid).[10] In the case of (1), this means we have to claim that there is some evaluation on which $A \vee B$ is true, but neither A nor B is true. In the case of (2), this means we have to claim there is some evaluation on which both A and B are true, but $A \wedge B$ is not true. Neither of these is possible; there is no evaluation which satisfies these claims. Thus, I claim, (1) and (2) must be valid if valid sequents are read as Restall suggests.

There is another way to run this argument: one can run it directly against the weakening rule itself. If the sequent, $\Gamma \vdash \Delta$ is meant to be read "every evaluation which makes everything on the left true must make something on the right true", then if an evaluation which makes all of Γ true must make at least one member of Δ true, then an evaluation which makes all of Γ true and also makes A true must still make some member of Δ true. Adding a sentence to the premise set, on this conception of the reading of the sequent, must not change the validity of the sequent.[11]

This has a drastic consequence for Restall's proof-theoretic pluralism. On this strict reading of the sequents, relevance cannot fit. That is, if the sequents must be read "every evaluation which makes everything on the left true must make something on the right true", then adding a relevance consequence obtained by removing the weakening rules will require evaluations which are not possible.

[9] There are other options if we remove the CUT rule, see Tennant (forthcoming) for a relevance logic without CUT. Restall, however, has made claims that logics must be unrestrictedly transitive (see Beall and Restall (2006, p 91)), and so I will not consider the possibility here.

[10] This will work if the meta-language is classical. It is plausible to think that Beall and Restall (2006) take it to be classical, and they are criticized for doing so by Read (2006).

[11] Thanks to Marcus Rossberg and Nathan Kellen for pointing this out to me. Restall's response to this version of the argument would presumably be to read the sequent intentionally, and thus the same reasoning as in section 4 will apply here as well.

Thus, relevance consequence cannot share the common core shared by the other "notions" of validity, and thus it cannot be a "notion" of validity in Restall's system.

4 Other Admissibility Conditions

We might be able to make some changes to Restall's program to make relevance fit. We might say that the common core of validity is that all validity relations are given by a set of sequents generated by a particular set of structural and logical rules, rather than the "reading" given by Restall. This, though, is not satisfactory. It seems that this lets more things in as validity relations than we think there are. What, for example, about the calculus mentioned earlier, with the structural rule that there cannot be any formulae on the left or right hand side of the sequent? This is not something we would like to call a logic, since we cannot make inferences in the system. Thus, the structural restriction "exactly one on the left and none on the right" should not result in a proper notion of validity. We need some way to weed out relations which are not validities, and this manner of defining validity does not provide one. This type of change will still not allow Restall to include relevance logic, on pain of including too much, in his pluralism.

There are more drastic changes we might make to include relevance logic in an extension of Restall's proof-theoretic pluralism. Perhaps if we do not literally read the sequents as Restall suggests, then the addition of a relevance logic would not be problematic. In order to add a relevance consequence to his 2014 pluralism, then, Restall will need to provide us with a new way to read the sequent. This – though, strictly speaking, it is not his original proof-theoretic pluralism – might make relevance logic admissible.

One option would be to consider the intensional sequent as presented in Dunn and Restall (1992) (see p 100). One important difference with this sequent compared to the sequents discussed in Restall (2014) is that, rather than using quantifiers (i.e. rather than "every" and "some" in "every evaluation which takes every element of [Γ] to be true takes some element of [Δ] to be true"), they use "and" and "or". As Restall (2014) claims that the sets of sentences on the right and left hand sides of the sequents are finite (see p 282 of Restall (2014)), this difference is easy enough to overlook.[12]

The definition of the intensional sequent is constructed as follows. The intensional conjunction of A and B can be defined as $\neg(A \rightarrow \neg B)$ in Anderson and Belnap's R, but can be taken as primitive in many relevance logics as well. Intensional disjunction is similar (though not discussed in Dunn and Restall (1992)), and can either be primitive or defined as $\neg A \rightarrow B$. I will follow Dunn and Restall and use $\circ$ for intensional conjunction, and follow Read (1982) and use $+$ for intensional disjunction. The intensional sequent, $A; B \vdash C$, given by Dunn and Restall (1992) is then read as "if $A \circ B$ then C". We can expand this notion to the sequents we are currently considering. Now, rather than reading the sequent "every evaluation which takes every element of [Γ] to be true takes some element of [Δ] to be true,", we might read it "every evaluation which makes the intensional conjunction of everything in Γ true makes the intensional disjunction of everything in Δ true."

It is shown by Dunn and Restall (1992) that this sequent calculus, when we restrict Δ to at most one member, is a relevance calculus, and similar to Anderson and Belnap's R. There are then two ways to proceed with this intensional approach. Either the sequents were meant to be read intensionally all along, or we have sequents which are meant to be read "every evaluation which takes every

[12] The move to intensional connectives actually precludes the meta-language being classical. It would require a much broader change to Restall's position than I will address here.

element of [Γ] to be true takes some element of [Δ] to be true", with different "notions" of "every" and "some" as we had with validity, so that sometimes "every" and "some" mean the usual quantifiers, and sometimes they mean "intensional conjunction" and "intensional disjunction".

The problem with the first option is that there is nothing we can add to this purely intensional sequent calculus to make it classical and preserve connective meaning. It is known that our $\vee$R and $\vee$L rules fail for intensional disjunction (or, rather, our +R and +L rules). For, if we let either +R or +L be the same as $\vee$R and $\vee$L, we could derive irrelevancies:

$$\dfrac{\dfrac{\dfrac{\vdash A}{\vdash A + B}\,{+}\mathrm{R}}{\vdash \neg A \to B}\,\text{Def of } + \qquad \vdash \neg A}{\vdash B}\,\mathrm{MP}$$

This means that the meaning of + can never be constituted by the left- and right-logical rules for $\vee$ and thus that that Restall will need two disjunctions to make this strategy work: + and $\vee$ (the latter of which is constituted by the usual $\vee$R and $\vee$L rules). This, though, implies that the logical connectives will not be the same across logics, which Restall cannot allow. If we went down this path, we would need to have two disjunctions in play, rather than just one defined by the logical rules. Thus, even reinterpreting what Restall says in his original presentation as intensional rather than extensional sequents will not allow us to add a relevance logic to the pluralistic framework while preserving connective meanings.

The problem with the second option is that it also results in different connective meanings. As with the first option, this technique will still require two disjunctions. Our usual $\vee$ will be required for when we are using the usual "notions" of "every" and "some" in the reading of validity. However, in order to remain consistent, when using the "intensional connective" "notions" of "every" and "some" in the reading of validity, we will need +, which again cannot be defined by the left- and right-logic rules for $\vee$.

I strongly suspect that this type of result will be common with any non-*ad hoc* reinterpretation of how to read the sequents Restall uses. There will be no way to read the sequent in such a way that classical, intuitionistic *and* relevance logics are included in the proof-theoretic pluralism. It will require some sort of reinterpretation of the proof-theoretic pluralism, and I think this interpretation will be very hard to come by.

This difficulty seems to come about because the types of concerns presented above about the difficulty of adding a relevance logic to the sequent calculus given in Restall (2014) arise in part because that sequent calculus does not allow us to track premise use in any reasonable way.[13] Most proof theories for relevance logic (which include negation) either have two conjunctions and/or disjunction (e.g. $+$ and $\vee$ as above, or Belnap (1982)) or have a tool which explicitly tracks premise use (see the natural deduction system for R in Anderson and Belnap (1975)). The system provided by Restall provides us with neither of these tools. The first is explicitly prohibited by his pluralism, since it would require more than one conjunction or disjunction, thereby preventing connectives from meaning the same thing across all admissible logics. The second is a more viable option, but again is not available in Restall's system without changing the meaning of the sequent: if we have to track premises, we will need a more robust meaning than "every evaluation which takes every element of $[\Gamma]$ to be true takes some element of $[\Delta]$ to be true". More generally, I expect this inability to combine all three of classical, intuitionistic and relevance negation has something to do with the fact that intuitionistic and relevant consequence are both restrictions of classical consequence *in different ways*. Intuitionistic logic allows explosion but not double negation elimination, while relevance logic allows double negation elimination but

[13]Thanks to a blind reviewer for suggesting this issue.

not explosion. I hope to pursue this in further work.

5 Conclusion

Restall (2014) presents a proof-theoretic pluralism which cannot sanction a relevance consequence relation. This is because Restall does not provide clear enough conditions on what counts as an admissible logic in this framework (as opposed to the Beall and Restall (2006) framework, which comes with detailed admissibility conditions). Further, there seems to be no straightforward way of providing such conditions so that both a relevance consequence relation is admissible, and connective meanings are preserved across all admissible logics. I have suggested that this points to a problem more generally with finding a proof theory which accommodates exactly the logics we think are correct, which I have argued should include a relevance logic if they include both classical and intuitionistic logics.

One might also ask the following question: just what is "validity"? It seems that there are many ways to spell out what, exactly, "validity" means. Restall has happened upon at least two: the one presented with Beall in their model-theoretic pluralism, and the one he gives in his more recent proof-theoretic pluralism. Though these two types of validity do not account for the same logics, they at least give us two options to choose from when we are discussing what validity really is. The issues of what validity amounts to, and how the connective meanings influence validity are essential here. [14]

[14] I owe a debt of gratitude to Stewart Shapiro very carefully read and commented on several earlier drafts. I am grateful to comments from several people, including three blind reviewers, Scott Brown, Nathan Kellen, Chris Pincock, Marcus Rossberg, Kevin Scharp, Matt Souba and Neil Tennant. I am also grateful to audiences at the Dubrovnik Conference on Pluralism (2015), the Pacific APA (2015), PhiloSTEM (2014), the Society for Exact Philosophy (2013) and The University of Western Ontario Philosophy of Logic, Math and Physics Graduate Conference (2013).

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