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Photon impact factor in the next-to-leading order

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An analytic coordinate-space expression for the next-to-leading order photon impact factor for small-x deep inelastic scattering is calculated using the operator expansion in Wilson lines.

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I. INTRODUCTION

It is well-known that the small-x behavior of structure functions of deep inelastic scattering is determined by the hard pomeron contribution. In the leading order the pomeron intercept is determined by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [1], and the pomeron residue (the $\gamma^*\gamma^*$-pomeron vertex) is given by the so-called impact factor. To find the small-x structure functions in the next-to-leading order, one needs to know both the pomeron intercept and the impact factor. The next-to-leading order (NLO) pomeron intercept was found many years ago [2] but the analytic expression for the NLO impact factor is obtained for the first time in the present paper.

We calculate the NLO impact factor using the high-energy operator expansion of T product of two vector currents in Wilson lines (see e.g. the reviews [3,4]). Let us recall the general logic of an operator expansion. In order to find a certain asymptotical behavior of an amplitude by operator product expansion, one should

(i) Identify the relevant operators and factorize an amplitude into a product of coefficient functions and matrix elements of
(ii) Find the evolution equations of the operators with respect to the factorization scale
(iii) Solve these evolution equations
(iv) Convolute the solution with the initial conditions for the evolution and get the amplitude.

Since we are interested in the small-x asymptotics of deep inelastic scattering (DIS), it is natural to factorize in rapidity: we introduce the rapidity divide η which separates the "fast" gluons from the "slow" ones.

As a first step, we integrate over gluons with rapidities $Y > \eta$ and leave the integration over $Y < \eta$ for the later time. It is convenient to use the background field formalism: we integrate over gluons with $\alpha > \sigma = e^\eta$ and leave gluons with $\alpha < \sigma$ as a background field, to be integrated over later. Since the rapidities of the background gluons are very different from the rapidities of gluons in our Feynman diagrams, the background field can be taken in the form of a shock wave due to the Lorentz contraction. To derive the expression of a quark (or gluon) propagator in this shock-wave background, we represent the propagator as a path integral over various trajectories, each of them weighed with the gauge factor $\mathrm{P}\exp(ig\int dx_\mu A^\mu)$ ordered along the propagation path. Now, since the shock wave is very thin, quarks (or gluons) do not have time to deviate in transverse direction so their trajectory inside the shock wave can be approximated by a segment of the straight line. Moreover, since there is no external field outside the shock wave, the integral over the segment of straight line can be formally extended to $\pm\infty$ limits yielding the Wilson-line gauge factor

$$U_x^\eta = \mathrm{P}\exp[ig\int_{-\infty}^{\infty} du\, p_1^\mu A_\mu^\sigma(up_1 + x_\perp)],$$
$$A_\mu^\eta(x) = \int d^4k\,\theta(e^\eta - |\alpha_k|)e^{ik\cdot x}A_\mu(k), \quad (1)$$

where the Sudakov variable α_k is defined as usual, $k = \alpha_k p_1 + \beta_k p_2 + k_\perp$. (We define the lightlike vectors p_1 and p_2 such that $q = p_1 - x_B p_2$ and $p_N = p_2 + \frac{m_N^2}{s}p_1$ where p_N is the nucleon momentum). The resulting structure of the propagator in a shock-wave background is shown in Fig. 1. The explicit form of quark propagator in a shock-wave background can be taken from Ref. [5]:

$$\langle T\{\hat\psi(x)\hat{\bar\psi}(y)\}\rangle_A \overset{x_*>0>y_*}{=} \int d^4z\,\delta(z_*)\frac{(\not x - \not z)}{2\pi^2(x-z)^4}\not p_2 U_z \frac{(\not z - \not y)}{2\pi^2(x-z)^4}. \quad (2)$$

As usual, we label operators by hats and $\langle\hat{\mathcal{O}}\rangle_A$ means the vacuum average of the operator $\hat{\mathcal{O}}$ in the presence of an external field A. Hereafter, use the notations $x_* = p_2^\mu x_\mu = \frac{\sqrt{s}}{2}x^+$, $x_\bullet = p_1^\mu x_\mu = \frac{\sqrt{s}}{2}x^-$ [and our metric is $(1,-1,-1,-1)$]. Note that the Regge limit in

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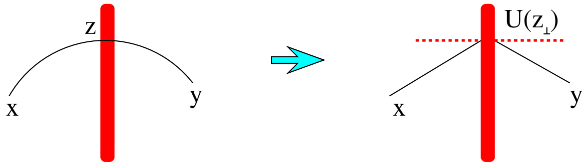

FIG. 1 (color online). Propagator in a shock-wave background

the coordinate space corresponds to $x_* \to \infty$, $y_* \to -\infty$ while $x_\perp$, $y_\perp$ are fixed; see the discussion in Refs. [6,7].

The result of the integration over gluons with rapidities $Y > \eta$ gives the impact factor—the amplitude of the transition of virtual photon in two-Wilson-lines operators (sometimes called "color dipole"). The leading order impact factor is a product of two propagators (2) (see Fig. 2),

$$\langle T\{\bar{\hat{\psi}}(x)\gamma^\mu \hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu \hat{\psi}(y)\}\rangle_A$$
$$== \frac{s^2}{2^9\pi^6 x_*^2 y_*^2}\int d^2 z_{1\perp} d^2 z_{2\perp}\frac{\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}}{(\kappa\cdot\zeta_1)^3(\kappa\cdot\zeta_2)^3}$$
$$\times\frac{\partial^2}{\partial x^\mu \partial y^\nu}[2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)-\kappa^2(\zeta_1\cdot\zeta_2)] + O(\alpha_s). \tag{3}$$

Here we introduced the conformal vectors [8,9]

$$\kappa = \frac{\sqrt{s}}{2x_*}\left(\frac{p_1}{s} - x^2 p_2 + x_\perp\right) - \frac{\sqrt{s}}{2y_*}\left(\frac{p_1}{s} - y^2 p_2 + y_\perp\right)\zeta_i$$
$$= \left(\frac{p_1}{s} + z_{i\perp}^2 p_2 + z_{i\perp}\right), \tag{4}$$

and the notation $\mathcal{R} \equiv \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$. The above equation is explicitly Möbius invariant. In addition, it is easy to check that $\frac{\partial}{\partial x_\mu}(\text{r.h.s.}) = 0$.

Our goal is the NLO contribution to the right-hand side (r.h.s.) of Eq. (3), but first let us briefly discuss the three remaining steps of the high-energy operator product expansion. The evolution equation for color dipoles has the form [5,10]

$$\frac{d}{d\eta}\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\} = \frac{\alpha_s}{2\pi^2}\int d^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2}[\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}$$
$$\times\,\mathrm{tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c\,\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$
$$+\ \text{NLO contribution.} \tag{5}$$

(To save space, hereafter z_i stand for $z_{i\perp}$ so $z_{12}^2 \equiv z_{12\perp}^2$, etc.) The explicit form of the NLO contributions can be found in Refs. [4,11,12], while the agrument of the coupling constant in the above equation (following from the NLO calculations) is discussed in Refs. [13,14].

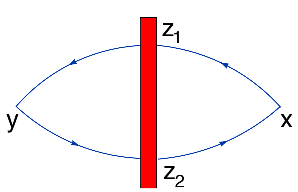

FIG. 2 (color online). Impact factor in the leading order. Solid lines represent quarks.

The next two steps, solution of the evolution in Eq. (5) with appropriate initial conditions and the eventual comparison with experimental DIS data, are discussed in many papers (see e.g. [15]). It is worth noting that, contrary to the evolution equation, the NLO correction to the impact factor has nothing to do with running of the coupling constant—it starts at the next-to-next-to-leading order level. Thus, the argument of the coupling constant at the NLO level is determined solely by the evolution equation for color dipoles. For numerical estimates involving the impact factor, one can take $\alpha_s(|x-y|)$ as the first approximation since the characteristic transverse distances in the impact factor are $\sim|x-y|$.

II. CALCULATION OF THE NLO IMPACT FACTOR

Now we would like to repeat the same steps of operator expansion at the NLO accuracy. A general form of the expansion of T product of the electromagnetic currents in color dipoles looks as follows:

$$(x-y)^4 T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}$$
$$= \int\frac{d^2z_1 d^2z_2}{z_{12}^4}\Big\{I^{\mathrm{LO}}_{\mu\nu}(z_1,z_2)\Big[1+\frac{\alpha_s}{\pi}\Big]\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$
$$+\int d^2z_3 I^{\mathrm{NLO}}_{\mu\nu}(z_1,z_2,z_3;\eta)$$
$$\times[\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}\mathrm{tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]\Big\}. \tag{6}$$

Unfortunately, in terms of Wilson-line approach there is no direct way to get the NLO impact factor for the BFKL pomeron. One needs first to find the coefficient in front of the four-Wilson-line operator (which we will also call the NLO impact factor) and then linearize it.

The structure of the NLO contribution is clear from the topology of diagrams in the shock-wave background,; see Fig. 3 below. Also, the term $\sim 1+\frac{\alpha_s}{\pi}$ can be restored from the requirement that at $U=1$ (no shock wave) one should get the perturbative series for the polarization operator $1+\frac{\alpha_s}{\pi}+O(\alpha_s^2)$.

In our notations

$$I^{\mathrm{LO}}_{\mu\nu}(z_1,z_2) = \frac{\mathcal{R}^2}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$$
$$\times\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)-\frac{1}{2}\kappa^2(\zeta_1\cdot\zeta_2)\Big], \tag{7}$$

which corresponds to the well-known expression for the leading order impact factor in the momentum space.

The NLO impact factor is given by the diagrams shown in Fig. 3. The calculation of these diagrams is similar to the calculation of the NLO impact factor for scalar currents in $\mathcal{N}=4$ SYM carried out in our previous paper [12]. The gluon propagator in the shock-wave background at $x_* > 0 > y_*$ in the lightlike gauge $p_2^\mu A_\mu = 0$ is given by [16,17]

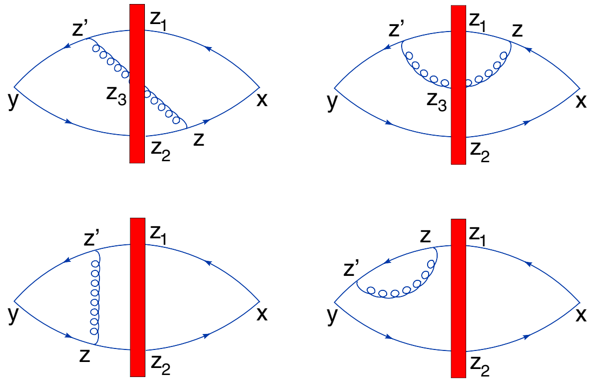

FIG. 3 (color online). Impact factor in the next-to-leading order.

$$\langle T\{\hat{A}^a_\mu(x)\hat{A}^b_\nu(y)\}\rangle \overset{x_*>0>y_*}{=} -\frac{i}{2}\int d^4z\,\delta(z_*) \times \frac{x_* g^\perp_{\mu\xi} - p_{2\mu}(x-z)^\perp_\xi}{\pi^2[(x-z)^2+i\epsilon]^2} U^{ab}_{z_\perp}\frac{1}{\partial^{(z)}_*} \times \frac{y_*\delta^{\perp\xi}_\nu - p_{2\nu}(y-z)^\xi_\perp}{\pi^2[(z-y)^2+i\epsilon]^2}, \tag{8}$$

where $\frac{1}{\partial_*}$ can be either $\frac{1}{\partial_*+i\epsilon}$ or $\frac{1}{\partial_*-i\epsilon}$ which leads to the same result. (This is obvious for the leading order and correct in NLO after subtraction of the leading-order contribution; see Eq. (15) below).

The diagrams in Fig. 3 can be calculated using the conformal integral

$$\int d^4z \frac{\not{x}-\not{z}}{(x-z)^4}\gamma_\mu \frac{\not{z}-\not{y}}{(z-y)^4}\frac{z_\nu}{z^4} - \mu\leftrightarrow\nu = \frac{\pi^2}{x^2y^2(x-y)^2}\Big[-\not{x}\gamma_\mu\not{y}\Big(\frac{x_\nu}{x^2}+\frac{y_\nu}{y^2}\Big) + \frac{1}{2}(\not{x}\gamma_\mu\gamma_\nu - \gamma_\mu\gamma_\nu\not{y}) + 2x_\mu y_\nu\frac{\not{x}-\not{y}}{(x-y)^2}\Big] - \mu\leftrightarrow\nu, \tag{9}$$

which gives the 3-point $\psi\bar{\psi}F_{\mu\nu}$ Green function in the leading order in g. Using Eqs. (2), (8), and (9), performing integrals over $z_\bullet$'s and taking traces one gets after some algebra the NLO contribution of diagrams in Fig. 3 in the form

$$I^{\text{Fig. 3}}_{\mu\nu}(z_1,z_2,z_3) = \tilde{I}^{\mu\nu}_1(z_1,z_2,z_3) + I^{\mu\nu}_2(z_1,z_2,z_3), \tag{10}$$

where

$$\tilde{I}^{\mu\nu}_1(z_1,z_2,z_3) = \frac{\alpha_s}{4\pi^2} I^{\rm LO}_{\mu\nu}(z_1,z_2)\frac{(\zeta_1\cdot\zeta_2)}{(\zeta_1\cdot\zeta_3)(\zeta_1\cdot\zeta_3)} \times \int_0^\infty \frac{d\alpha}{\alpha} e^{i\alpha(s/4)\sigma\mathcal{Z}_3}, \tag{11}$$

and

$$(I_2)_{\mu\nu}(z_1,z_2,z_3) = \frac{\alpha_s}{16\pi^8}\frac{\mathcal{R}^2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big\{\frac{(\kappa\cdot\zeta_2)}{(\kappa\cdot\zeta_3)}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[-\frac{(\kappa\cdot\zeta_1)^2}{(\zeta_1\cdot\zeta_3)} + \frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)} + \frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)(\zeta_1\cdot\zeta_2)}{(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_3)} - \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}\Big] + \frac{(\kappa\cdot\zeta_2)^2}{(\kappa\cdot\zeta_3)^2}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)}{(\zeta_2\cdot\zeta_3)} - \frac{\kappa^2(\zeta_1\cdot\zeta_3)}{2(\zeta_2\cdot\zeta_3)}\Big] + (\zeta_1\leftrightarrow\zeta_2)\Big\} \tag{12}$$

[recall that $z^2_{ij\perp} = 2(\zeta_i\cdot\zeta_j)$ and $\mathcal{Z}_i = \frac{4}{\sqrt{s}}(\kappa\cdot\zeta_i)$]. We obtained this expression at $x_*>0>y_*$, but from the conformal structure of the result, it is clear that this expression holds true at $x_*<0<y_*$ as well.

The integral over α in the r.h.s. of Eq. (11) diverges. This divergence reflects the fact that the contributions of the diagrams in Fig. 3 is not exactly the NLO impact factor since we must subtract the matrix element of the leading-order contribution. Indeed, the NLO impact factor is a coefficient function defined according to Eq. (6). To find the NLO impact factor, we consider the operator Eq. (6) in the shock-wave background (in the leading order $\langle\hat{U}_{z_3}\rangle_A = U_{z_3}$):

$$\langle T\{\hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x)\hat{\bar{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}\rangle_A - \int\frac{d^2z_1d^2z_2}{z^4_{12}} I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\rangle_A = \int\frac{d^2z_1d^2z_2}{z^4_{12}} d^2z_3 I^{\rm NLO}_{\mu\nu}(x,y;z_1,z_2,z_3;\eta) \times [{\rm tr}\{U_{z_1}U^\dagger_{z_3}\}{\rm tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c\,{\rm tr}\{U_{z_1}U^\dagger_{z_2}\}]. \tag{13}$$

The NLO matrix element $\langle T\{\hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x)\hat{\bar{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}\rangle_A$ is given by Eq. (10) while the subtracted term is

$$\frac{\alpha_s}{2\pi^2}\int\frac{d^2z_1d^2z_2}{z^4_{12}} I^{\rm LO}_{\mu\nu}(z_1,z_2)\int_0^\sigma\frac{d\alpha}{\alpha}\int d^2z_3\frac{z^2_{12}}{z^2_{13}z^2_{23}} \times [{\rm tr}\{U_{z_1}U^\dagger_{z_3}\}{\rm tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c\,{\rm tr}\{U_{z_1}U^\dagger_{z_2}\}], \tag{14}$$

as follows from Eq. (5). The α integration is cut from above by $\sigma = e^\eta$ in accordance with the definition of operators $\hat{U}^\eta$; see Eq. (1). Subtracting (14) from Eq. (10), we get

$$I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3;\eta) = I^{\mu\nu}_1(z_1,z_2,z_3;\eta) + I^{\mu\nu}_2(z_1,z_2,z_3),$$

$$I^{\mu\nu}_1(x,y;z_1,z_2,z_3;\eta) = \frac{\alpha_s}{2\pi^2} I^{\rm LO}_{\mu\nu}(z_1,z_2)\frac{z^2_{12}}{z^2_{13}z^2_{23}} \times \Big[\int_0^\infty\frac{d\alpha}{\alpha}e^{i\alpha(s/4)\mathcal{Z}_3} - \int_0^\sigma\frac{d\alpha}{\alpha}\Big] = -\frac{\alpha_s}{2\pi^2} I^{\rm LO}_{\mu\nu}\frac{z^2_{12}}{z^2_{13}z^2_{23}} \times \Big[\ln\frac{\sigma s}{4}\mathcal{Z}_3 - \frac{i\pi}{2} + C\Big], \tag{15}$$

where C is the Euler constant. Note that one should expect the NLO impact factor to be conformally invariant since it is determined by tree diagrams in Fig. 3. However, as discussed in Refs. [4,7,11], formally the lightlike Wilson lines are conformally (Möbius) invariant, but the longitudinal cutoff $\alpha < \sigma$ in Eq. (1) violates this property so the term $\sim \ln\sigma \mathcal{Z}_3$ in the r.h.s. of Eq. (15) is not invariant. As demonstrated in these papers, one can define a composite operator in the form

$$[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_a = \mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\} + \frac{\alpha_s}{2\pi^2}\int d^2z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \times [\mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_3}\}\mathrm{tr}\{\hat{U}^\sigma_{z_3}\hat{U}^{\dagger\sigma}_{z_2}\} - N_c\,\mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\}]\ln\frac{4az_{12}^2}{\sigma^2 s z_{13}^2 z_{23}^2} + O(\alpha_s^2), \tag{16}$$

where a is an arbitrary constant. It is convenient to choose the rapidity-dependent constant $a \to ae^{-2\eta}$ so that the $[\mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\}]^{\mathrm{conf}}_a$ does not depend on $\eta = \ln\sigma$ and all the rapidity dependence is encoded into a dependence. Indeed, it is easy to see that $\frac{d}{d\eta}[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]^{\mathrm{conf}}_a = 0$ and $\frac{d}{da}[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]^{\mathrm{conf}}_a$ is determined by the NLO Balitsky-Kovchegov kernel which is a sum of the conformal part and the running-coupling part with our $O(\alpha_s^2)$ accuracy [4,12].

Rewritten in terms of composite dipoles (16), the operator expansion (6) takes the form:

$$T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \int\frac{d^2z_1d^2z_2}{z_{12}^4}\Big\{I^{\mathrm{LO}}_{\mu\nu}(z_1,z_2)\Big[1+\frac{\alpha_s}{\pi}\Big][\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_a + \int d^2z_3 I^{\mathrm{NLO}}_{\mu\nu}(z_1,z_2,z_3;a) \times[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - N_c\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_a\Big\}. \tag{17}$$

We need to choose the "new rapidity cutoff" a in such a way that all the energy dependence is included in the matrix element(s) of Wilson-line operators so the impact factor should not depend on energy. A suitable choice of a is given by $a_0 = -\kappa^{-2} + i\epsilon = -\frac{4x_*y_*}{s(x-y)^2} + i\epsilon$, so we obtain

$$(x-y)^4 T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \int\frac{d^2z_1d^2z_2}{z_{12}^4}\Big\{I^{\mu\nu}_{\mathrm{LO}}(z_1,z_2)\Big[1+\frac{\alpha_s}{\pi}\Big][\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0} + \int d^2z_3\Big[\frac{\alpha_s}{4\pi^2}\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big(\ln\frac{\kappa^2(\zeta_1\cdot\zeta_3)(\zeta_1\cdot\zeta_3)}{2(\kappa\cdot\zeta_3)^2(\zeta_1\cdot\zeta_2)} - 2C\Big)I^{\mu\nu}_{\mathrm{LO}} + I^{\mu\nu}_2\Big][\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - N_c\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}\Big\}. \tag{18}$$

Here the composite dipole $[\mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\}]_{a_0}$ is given by Eq. (16) with $a_0 = -\frac{4x_*y_*}{s(x-y)^2} + i\epsilon$ while $I^{\mu\nu}_{\mathrm{LO}}(z_1,z_2)$ and $I^{\mu\nu}_2(z_1,z_2,z_3)$ are given by Eqs. (7) and (12), respectively.

III. NLO IMPACT FACTOR FOR THE BFKL POMERON

For the studies of DIS with the linear NLO BFKL equation (up to two-gluon accuracy) we need the linearized version of Eq. (18). If we define

$$\hat{\mathcal{U}}_a(z_1,z_2) = 1 - \frac{1}{N_c}[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_a \tag{19}$$

and consider the linearization

$$\frac{1}{N_c^2}\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - \frac{1}{N_c}\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0} \simeq \hat{\mathcal{U}}(z_1,z_2) - \hat{\mathcal{U}}(z_1,z_3) - \hat{\mathcal{U}}(z_2,z_3),$$

one of the integrals over z_i in the r.h.s. of Eq. (18) can be performed. The result is

$$\frac{1}{N_c}(x-y)^4 T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\int\frac{dz_1dz_2}{z_{12}^4}\hat{\mathcal{U}}_{a_0}(z_1,z_2) \times\Big[I^{\mathrm{LO}}_{\alpha\beta}\Big(1+\frac{\alpha_s}{\pi}\Big) + I^{\mathrm{NLO}}_{\alpha\beta}\Big], \tag{20}$$

where

$$I^{\alpha\beta}_{\mathrm{LO}}(x,y;z_1,z_2) = \mathcal{R}^2\frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2) - \zeta_1^\alpha\zeta_2^\beta - \zeta_2^\alpha\zeta_1^\beta}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)} \tag{21}$$

(see Eq. (7) and

$$
\begin{aligned}
I^{\alpha\beta}_{\rm NLO}(x,y;z_1,z_2) = \frac{\alpha_s N_c}{4\pi^7}\mathcal{R}^2\bigg\{&\frac{\zeta_1^\alpha\zeta_2^\beta + \zeta_1\leftrightarrow\zeta_2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\bigg[4\,{\rm Li}_2(1-\mathcal{R}) - \frac{2\pi^2}{3} + \frac{2\ln\mathcal{R}}{1-\mathcal{R}} + \frac{\ln\mathcal{R}}{\mathcal{R}} - 4\ln\mathcal{R} + \frac{1}{2\mathcal{R}}\\
&- 2 + 2\Big(\ln\frac{1}{\mathcal{R}} + \frac{1}{\mathcal{R}} - 2\Big)\Big(\ln\frac{1}{\mathcal{R}} + 2C\Big) - 4C - \frac{2C}{\mathcal{R}}\bigg]\\
&+\Big(\frac{\zeta_1^\alpha\zeta_1^\beta}{(\kappa\cdot\zeta_1)^2} + \zeta_1\leftrightarrow\zeta_2\Big)\Big[\frac{\ln\mathcal{R}}{\mathcal{R}} - \frac{2C}{\mathcal{R}} + 2\frac{\ln\mathcal{R}}{1-\mathcal{R}} - \frac{1}{2\mathcal{R}}\Big]\\
&+\Big[\frac{\zeta_1^\alpha\kappa^\beta + \zeta_1^\beta\kappa^\alpha}{(\kappa\cdot\zeta_1)\kappa^2} + \zeta_1\leftrightarrow\zeta_2\Big]\Big[-2\frac{\ln\mathcal{R}}{1-\mathcal{R}} - \frac{\ln\mathcal{R}}{\mathcal{R}} + \ln\mathcal{R} - \frac{3}{2\mathcal{R}} + \frac{5}{2} + 2C + \frac{2C}{\mathcal{R}}\Big]\\
&-\frac{2}{\kappa^2}\Big(g^{\alpha\beta} - 2\frac{\kappa^\alpha\kappa^\beta}{\kappa^2}\Big) + \frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2)}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\bigg[\frac{2\pi^2}{3} - 4\,{\rm Li}_2(1-\mathcal{R}) - 2\Big(\ln\frac{1}{\mathcal{R}} + \frac{1}{\mathcal{R}} + \frac{1}{2\mathcal{R}^2} - 3\Big)\\
&\times\Big(\ln\frac{1}{\mathcal{R}} + 2C\Big) + 6\ln\mathcal{R} - \frac{2}{\mathcal{R}} + 2 + \frac{3}{2\mathcal{R}^2}\bigg]\bigg\},
\end{aligned}
\tag{22}
$$

where ${\rm Li}_2(z)$ is the dilogarithm. Here one easily recognizes five conformal tensor structures discussed in Ref. [18].

IV. CONCLUSIONS AND OUTLOOK

We have calculated the NLO impact factor for the virtual photons both in the nonlinear form (18) and with the linear (two-gluon) accuracy (20). Our results are obtained in the coordinate representation so the next step should be the Fourier transformation of Eq. (22) which would give the momentum-space impact factor convenient for phenomenological applications (and available at present only as a combination of numerical and analytical expressions [19]). The study is in progress.

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