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# Precision measurement of the proton and deuteron spin structure functions $g_{2}$ and asymmetries $A_{2}{ }^{\text {m }}$ 

## E155 Collaboration

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#### Abstract

We have measured the spin structure functions $g_{2}^{p}$ and $g_{2}^{d}$ and the virtual photon asymmetries $A_{2}^{p}$ and $A_{2}^{d}$ over the kinematic range $0.02 \leqslant x \leqslant 0.8$ and $0.7 \leqslant Q^{2} \leqslant 20 \mathrm{GeV}^{2}$ by scattering 29.1 and 32.3 GeV longitudinally polarized electrons from transversely polarized $\mathrm{NH}_{3}$ and ${ }^{6} \mathrm{LiD}$ targets. Our measured $g_{2}$ approximately follows the twist- 2 Wandzura-Wilczek calculation. The twist- 3 reduced matrix elements $d_{2}^{p}$ and $d_{2}^{n}$ are less than two standard deviations from zero. The data are inconsistent with the Burkhardt-Cottingham sum rule if there is no pathological behavior as $x \rightarrow 0$. The Efremov-LeaderTeryaev integral is consistent with zero within our measured kinematic range. The absolute value of $A_{2}$ is significantly smaller than the $A_{2}<\sqrt{R\left(1+A_{1}\right) / 2}$ limit.


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The deep inelastic spin structure functions of the nucleons, $g_{1}\left(x, Q^{2}\right)$ and $g_{2}\left(x, Q^{2}\right)$, depend on the spin distribution of the partons and their correlations. The function $g_{1}$ can be primarily understood in terms of the quark parton model (QPM) and perturbative QCD with higher twist terms at low $Q^{2}$. The function $g_{2}$ is of particular interest since it has contributions from quark-gluon correlations and other higher twist terms at leading order in $Q^{2}$ which cannot be described perturbatively. By interpreting $g_{2}$ using the operator product expansion (OPE) [1,2], it is possible to study contributions to the nucleon spin structure beyond the simple QPM.

[^0]The structure function $g_{2}$ can be written [3]:
$g_{2}\left(x, Q^{2}\right)=g_{2}^{W W}\left(x, Q^{2}\right)+\overline{g_{2}}\left(x, Q^{2}\right)$
in which
$g_{2}^{W W}\left(x, Q^{2}\right)=-g_{1}\left(x, Q^{2}\right)+\int_{x}^{1} \frac{g_{1}\left(y, Q^{2}\right)}{y} d y$,
$\overline{g_{2}}\left(x, Q^{2}\right)=-\int_{x}^{1} \frac{\partial}{\partial y}\left(\frac{m}{M} h_{T}\left(y, Q^{2}\right)+\xi\left(y, Q^{2}\right)\right) \frac{d y}{y}$,
$x$ is the Bjorken scaling variable and $Q^{2}$ is the absolute value of the virtual photon four-momentum squared. The twist-2 term $g_{2}^{W W}$ was derived by Wandzura and Wilczek [4] and depends only on $g_{1}$ [5-10]. The function $h_{T}\left(x, Q^{2}\right)$ is an additional twist- 2 contribution [ 3,11 ] that depends on the transverse polarization density in the nucleon. The $h_{T}$ contribution to $\overline{g_{2}}$ is suppressed by the ratio of the quark to nucleon masses $m / M$ [11] and its effect is thus small for up and down quarks. The twist-3 part $(\xi)$ comes from quark-gluon correlations and is the main focus of our study. Lowprecision measurements of $g_{2}$ and $A_{2}$ exist for the proton and deuteron [12-14], as well as for the neutron [7,15]. In this Letter, we report new, precise measurements of $g_{2}$ and $A_{2}$ for the proton and deuteron.

Electron beams with energies of 29.1 and 32.3 GeV and longitudinal polarizations of $P_{b}=(83.2 \pm 3.0) \%$ struck approximately transversely polarized $\mathrm{NH}_{3}$ [6] (average polarization $\left\langle P_{t}\right\rangle=0.70$ ) or ${ }^{6} \mathrm{LiD}[16]\left(\left\langle P_{t}\right\rangle=\right.$ $0.22)$ targets. The beam helicity was randomly chosen pulse by pulse. Scattered electrons were detected in three independent spectrometers centered at $2.75^{\circ}$, $5.5^{\circ}$ and $10.5^{\circ}$. The two small-angle spectrometers
were the same as in SLAC E155 [9], while the largeangle spectrometer had additional hodoscopes and a more efficient pre-radiator shower counter. Further information on the experimental apparatus can be found in Refs. $[6,8,9]$. The approximately equal amounts of data taken with the two beam energies and opposites signs of target polarization gave consistent results.

The measured asymmetry, $\tilde{A}_{\perp}$, differs from transverse asymmetry $A_{\perp}$ because the target polarizations were not exactly perpendicular to the beam line. It was determined using

$$
\begin{gather*}
\tilde{A}_{\perp}=\frac{1}{f_{\mathrm{RC}}}\left[\frac{C_{1}}{f P_{t}}\left(\left(\frac{N_{L}-N_{R}}{N_{L}+N_{R}}\right) \frac{1}{P_{b}}-A_{\mathrm{EW}}\right)\right. \\
\left.+C_{2} \frac{\sigma_{p}}{\sigma_{d}} \tilde{A}_{\perp}^{p}\right]+A_{\mathrm{RC}}, \tag{2}
\end{gather*}
$$

where $N_{L}$ and $N_{R}$ are the measured counting rates from the two beam helicities, including small corrections for pion and charge symmetric backgrounds, dead-time and tracking efficiency, and $A_{\text {EW }}$ is the electroweak asymmetry ( $\approx 8 \times 10^{-5} Q^{2}$ ). The target dilution factor, $f$, is the fraction of free polarizable protons $(\approx 0.13)$ or deuterons $(\approx 0.18)$ for a given spectrometer acceptance. For the proton target, the nuclear correction $C_{1} \approx 0.98$ is due to the polarization of the ${ }^{15} \mathrm{~N}$ and $C_{2}=0$. The deuteron data were extracted from the ${ }^{6} \mathrm{LiD}$ results by applying a slightly $x$-dependent nuclear correction $C_{1} \approx 0.52$ to account for the lithium and deuterium nuclear wave functions with ${ }^{6} \mathrm{Li} \sim \alpha+d$ [16]. An additional correction $C_{2}(x) \approx-0.042$ accounts for the $\sim 4 \%$ polarized ${ }^{7} \mathrm{Li}$ in the target. The quantities $f_{\mathrm{RC}}$ and $A_{\mathrm{RC}}$ are radiative corrections determined using a method similar to E143 [6]. The quantity $1-f_{\mathrm{RC}}$ was calculated as the proportion of events in a bin coming from elastic and quasielastic tails, and $A_{\mathrm{RC}}$ included polarization-dependent elastic and quasi-elastic as well as inelastic and vertex corrections. The radiative dilution factor $f_{\mathrm{RC}}$ has the effect of increasing the statistical errors at low $x$. Uncertainties in the radiative corrections were estimated by varying the input models over a range consistent with the measured data.

Because $\tilde{A}_{\perp}$ is close to zero, the relative statistical errors are always greater than $25 \%$. The uncertainties due to target and beam polarization and dilution factor combined are $5.1 \%$ (proton) and $6.2 \%$ (deuteron).

They are multiplicative and small compared to the statistical errors.

We determined $g_{2}\left(x, Q^{2}\right)$ and $A_{2}\left(x, Q^{2}\right)$ from $\tilde{A}_{\perp}$ (dominant contribution) and the previously measured $g_{1}$ (small contribution) using:

$$
\begin{align*}
g_{2}= & \frac{y F_{1}}{2 E^{\prime}(\cos \Theta-\cos \alpha)} \\
& \times\left[\tilde{A}_{\perp} \nu \frac{(1+\epsilon R)}{1-\epsilon}-\frac{g_{1}}{F_{1}}\left[E \cos \alpha+E^{\prime} \cos \Theta\right]\right] \tag{3}
\end{align*}
$$

$A_{2}=\gamma\left(g_{1}+g_{2}\right) / F_{1}$,
where $\cos \Theta=\sin \alpha \sin \theta \cos \Phi+\cos \alpha \cos \theta$, $\theta$ is the spectrometer angle, $\Phi$ is the angle between the spin plane and the scattering plane, $\alpha=92.4^{\circ}$ is the angle of the target polarization with respect to the beam direction, $y=v / E, v=E-E^{\prime}, E$ and $E^{\prime}$ are the incident and scattered electron energies, $\epsilon^{-1}=1+2\left[1+1 / \gamma^{2}\right] \tan ^{2}(\theta / 2), \gamma=\sqrt{Q^{2} / \nu^{2}}$ and $F_{1}=F_{2}\left(1+4 M^{2} x^{2} / Q^{2}\right) /[2 x(1+R)]$. We used a new $Q^{2}$-dependent parameterization of $g_{1}$ [9] world data, the NMC fit to $F_{2}\left(x, Q^{2}\right)$ [17] and the SLAC fit to $R\left(x, Q^{2}\right)=\sigma_{L} / \sigma_{T}$ [18]. The structure functions for $p, d$, and $n$ are related by $g_{2}^{d}=\left(g_{2}^{p}+g_{2}^{n}\right)(1-$ $\left.1.5 \omega_{D}\right) / 2$, where $\omega_{D}=0.05$, the fraction of D-wave in the deuteron wave function.

Results for $A_{2}$ and $x g_{2}$ for the three spectrometers and two energies are given in Table 1 with statistical errors. The systematic error on $x g_{2}$ is much smaller than the statistical error and is given approximately by $a+b x$ where $a_{p}\left(a_{d}\right)=0.0016(0.0009)$ and $b_{p}\left(b_{d}\right)=$ $-0.0012(-0.0008)$. It includes the systematic errors on $\tilde{A}_{\perp}$ as well as a $5 \%$ normalization uncertainty on $g_{1}$. The data cover the kinematic range $0.02 \leqslant x \leqslant 0.8$ and $0.7 \leqslant Q^{2} \leqslant 20 \mathrm{GeV}^{2}$ with an average $Q^{2}$ of $5 \mathrm{GeV}^{2}$. Fig. 1 shows the values of $x g_{2}$ as a function of $Q^{2}$ for several values of $x$ along with results from E143 [6] and E155 [14]. The data approximately follow the $Q^{2}$ dependence of $g_{2}^{W W}$ (solid curve), although for the proton, the data points are lower than $g_{2}^{W W}$ at low and intermediate $x$ and higher at high $x$. The predictions of Stratmann [19] are closer to the data.

To get average values at the average $Q^{2}$ for each $x$ bin we used the $Q^{2}$ dependence of $g_{2}^{W W}: g_{2}\left(Q_{\text {avg }}^{2}\right)=$ $g_{2}\left(Q_{\text {exp }}^{2}\right)-g_{2}^{W W}\left(Q_{\text {exp }}^{2}\right)+g_{2}^{W W}\left(Q_{\text {avg }}^{2}\right)$. These averaged results for $A_{2}$ and $x g_{2}$ are listed at the bottom

Table 1
Results for $A_{2}$ and $x g_{2}$ with statistical errors for proton and deuteron at the measured $x$ and $Q^{2}\left[(\mathrm{GeV} / c)^{2}\right]$. The systematic error on $x g_{2}$ is given by $a+b x$, where $a_{p}\left(a_{d}\right)=0.0016(0.0009)$ and $b_{p}\left(b_{d}\right)=-0.0012(-0.0008)$

|  | $\langle x\rangle$ | $\left\langle Q^{2}\right\rangle$ | $A_{2}^{p}$ | $x g_{2}^{p}$ | $A_{2}^{d}$ | $x g_{2}^{d}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\theta \approx 2.75^{\circ} ; E=29.1 \mathrm{GeV}$ | 0.021 | 0.80 | $-0.015 \pm 0.012$ | $-0.037 \pm 0.026$ | $0.003 \pm 0.017$ | $0.009 \pm 0.036$ |
|  | 0.026 | 0.90 | $-0.009 \pm 0.008$ | $-0.026 \pm 0.015$ | $0.010 \pm 0.011$ | $0.020 \pm 0.021$ |
|  | 0.038 | 1.10 | $0.016 \pm 0.006$ | $0.020 \pm 0.010$ | $-0.013 \pm 0.009$ | $-0.021 \pm 0.014$ |
|  | 0.061 | 1.30 | $0.026 \pm 0.008$ | $0.017 \pm 0.009$ | $-0.017 \pm 0.011$ | $-0.024 \pm 0.013$ |
|  | 0.098 | 1.60 | $0.014 \pm 0.010$ | $-0.011 \pm 0.009$ | $0.025 \pm 0.015$ | $0.016 \pm 0.013$ |
|  | 0.155 | 1.80 | $0.061 \pm 0.015$ | $0.005 \pm 0.010$ | $0.008 \pm 0.024$ | $-0.005 \pm 0.013$ |
|  | 0.245 | 2.00 | $0.098 \pm 0.024$ | $-0.005 \pm 0.010$ | $0.058 \pm 0.038$ | $0.002 \pm 0.014$ |
|  | 0.380 | 2.10 | $0.258 \pm 0.064$ | $0.007 \pm 0.018$ | $-0.008 \pm 0.105$ | $-0.031 \pm 0.024$ |
| $\theta \approx 5.5^{\circ} ; E=29.1 \mathrm{GeV}$ | 0.061 | 2.70 | $0.033 \pm 0.036$ | $0.045 \pm 0.061$ | $0.059 \pm 0.052$ | $0.094 \pm 0.084$ |
|  | 0.098 | 3.50 | $0.029 \pm 0.009$ | $0.019 \pm 0.013$ | $0.000 \pm 0.014$ | $-0.009 \pm 0.018$ |
|  | 0.155 | 4.40 | $0.020 \pm 0.008$ | $-0.017 \pm 0.009$ | $0.024 \pm 0.012$ | $0.012 \pm 0.012$ |
|  | 0.245 | 5.30 | $0.042 \pm 0.011$ | $-0.021 \pm 0.008$ | $0.037 \pm 0.017$ | $0.000 \pm 0.011$ |
|  | 0.380 | 6.10 | $0.035 \pm 0.019$ | $-0.043 \pm 0.007$ | $0.086 \pm 0.032$ | $0.002 \pm 0.010$ |
|  | 0.580 | 6.70 | $0.107 \pm 0.045$ | $-0.020 \pm 0.006$ | $0.137 \pm 0.082$ | $-0.004 \pm 0.008$ |
|  | 0.780 | 7.00 | $-0.131 \pm 0.130$ | $-0.012 \pm 0.003$ | $0.444 \pm 0.232$ | $0.003 \pm 0.004$ |
| $\theta \approx 10.5^{\circ} ; E=29.1 \mathrm{GeV}$ | 0.155 | 7.10 | $0.030 \pm 0.018$ | $-0.001 \pm 0.024$ | $-0.023 \pm 0.032$ | $-0.042 \pm 0.039$ |
|  | 0.245 | 9.90 | $0.018 \pm 0.016$ | $-0.036 \pm 0.016$ | $0.029 \pm 0.031$ | $0.006 \pm 0.025$ |
|  | 0.380 | 13.10 | $0.054 \pm 0.025$ | $-0.026 \pm 0.013$ | $0.035 \pm 0.052$ | $-0.006 \pm 0.021$ |
|  | 0.580 | 16.30 | $0.090 \pm 0.068$ | $-0.010 \pm 0.010$ | $0.031 \pm 0.156$ | $-0.009 \pm 0.017$ |
|  | 0.780 | 18.40 | $-0.182 \pm 0.259$ | $-0.008 \pm 0.005$ | $0.795 \pm 0.625$ | $0.010 \pm 0.009$ |
| $\theta \approx 2.75^{\circ}, E=32.3 \mathrm{GeV}$ | 0.021 | 0.80 | $-0.001 \pm 0.008$ | $-0.007 \pm 0.020$ | $0.003 \pm 0.014$ | $0.006 \pm 0.031$ |
|  | 0.026 | 0.90 | $0.002 \pm 0.006$ | $-0.004 \pm 0.014$ | $-0.006 \pm 0.011$ | $-0.010 \pm 0.022$ |
|  | 0.038 | 1.10 | $0.007 \pm 0.005$ | $0.001 \pm 0.009$ | $0.003 \pm 0.008$ | $0.006 \pm 0.014$ |
|  | 0.061 | 1.30 | $0.019 \pm 0.006$ | $0.009 \pm 0.008$ | $0.015 \pm 0.010$ | $0.015 \pm 0.013$ |
|  | 0.098 | 1.60 | $0.021 \pm 0.009$ | $-0.004 \pm 0.008$ | $-0.004 \pm 0.014$ | $-0.011 \pm 0.013$ |
|  | 0.155 | 1.80 | $0.045 \pm 0.013$ | $-0.008 \pm 0.009$ | $0.045 \pm 0.021$ | $0.015 \pm 0.013$ |
|  | 0.245 | 2.00 | $0.076 \pm 0.020$ | $-0.018 \pm 0.009$ | $0.063 \pm 0.034$ | $0.003 \pm 0.014$ |
|  | 0.380 | 2.10 | $0.209 \pm 0.053$ | $-0.004 \pm 0.017$ | $0.076 \pm 0.095$ | $-0.011 \pm 0.025$ |
| $\theta \approx 5.5^{\circ} ; E=32.3 \mathrm{GeV}$ | 0.061 | 2.70 | $-0.015 \pm 0.023$ | $-0.041 \pm 0.042$ | $0.046 \pm 0.035$ | $0.077 \pm 0.061$ |
|  | 0.098 | 3.50 | $0.017 \pm 0.007$ | $0.000 \pm 0.011$ | $0.004 \pm 0.011$ | $-0.003 \pm 0.016$ |
|  | 0.155 | 4.40 | $0.033 \pm 0.007$ | $-0.002 \pm 0.008$ | $0.028 \pm 0.010$ | $0.015 \pm 0.011$ |
|  | 0.245 | 5.30 | $0.041 \pm 0.009$ | $-0.023 \pm 0.007$ | $0.034 \pm 0.015$ | $0.003 \pm 0.010$ |
|  | 0.380 | 6.10 | $0.069 \pm 0.016$ | $-0.029 \pm 0.007$ | $0.000 \pm 0.028$ | $-0.024 \pm 0.009$ |
|  | 0.580 | 6.70 | $0.126 \pm 0.038$ | $-0.016 \pm 0.005$ | $0.078 \pm 0.074$ | $-0.008 \pm 0.007$ |
|  | 0.780 | 7.00 | $0.177 \pm 0.110$ | $-0.004 \pm 0.003$ | $0.170 \pm 0.210$ | $-0.002 \pm 0.004$ |
| $\theta \approx 10.5^{\circ} ; E=32.3 \mathrm{GeV}$ | 0.155 | 7.10 | $0.027 \pm 0.013$ | $0.001 \pm 0.019$ | $0.025 \pm 0.022$ | $0.019 \pm 0.029$ |
|  | 0.245 | 9.90 | $0.026 \pm 0.012$ | $-0.029 \pm 0.013$ | $0.006 \pm 0.021$ | $-0.016 \pm 0.018$ |
|  | 0.380 | 13.10 | $0.033 \pm 0.019$ | $-0.034 \pm 0.010$ | $-0.010 \pm 0.035$ | $-0.028 \pm 0.015$ |
|  | 0.580 | 16.30 | $0.000 \pm 0.048$ | $-0.024 \pm 0.008$ | $0.215 \pm 0.105$ | $0.013 \pm 0.012$ |
|  | 0.780 | 18.40 | $-0.146 \pm 0.191$ | $-0.008 \pm 0.004$ | $-0.527 \pm 0.424$ | $-0.011 \pm 0.007$ |
| AVERAGE | 0.021 | 0.80 | $-0.005 \pm 0.007$ | $-0.018 \pm 0.016$ | $0.003 \pm 0.011$ | $0.008 \pm 0.023$ |
|  | 0.026 | 0.90 | $-0.003 \pm 0.005$ | $-0.014 \pm 0.010$ | $0.002 \pm 0.008$ | $0.006 \pm 0.015$ |
|  | 0.038 | 1.10 | $0.011 \pm 0.004$ | $0.010 \pm 0.007$ | $-0.004 \pm 0.006$ | $-0.007 \pm 0.010$ |
|  | 0.061 | 1.40 | $0.020 \pm 0.005$ | $0.011 \pm 0.006$ | $0.003 \pm 0.007$ | $-0.001 \pm 0.009$ |
|  | 0.098 | 2.30 | $0.023 \pm 0.004$ | $-0.003 \pm 0.005$ | $0.006 \pm 0.007$ | $-0.001 \pm 0.007$ |
|  | 0.155 | 3.70 | $0.036 \pm 0.004$ | $-0.007 \pm 0.004$ | $0.026 \pm 0.007$ | $0.009 \pm 0.006$ |
|  | 0.245 | 5.00 | $0.048 \pm 0.005$ | $-0.022 \pm 0.004$ | $0.036 \pm 0.009$ | $0.000 \pm 0.005$ |
|  | 0.380 | 7.10 | $0.064 \pm 0.009$ | $-0.031 \pm 0.004$ | $0.029 \pm 0.017$ | $-0.015 \pm 0.005$ |
|  | 0.580 | 8.40 | $0.092 \pm 0.023$ | $-0.018 \pm 0.003$ | $0.122 \pm 0.047$ | $-0.004 \pm 0.005$ |
|  | 0.780 | 8.20 | $0.004 \pm 0.074$ | $-0.007 \pm 0.002$ | $0.228 \pm 0.142$ | $0.000 \pm 0.002$ |



Fig. 1. $x g_{2}$ for the proton and deuteron as a function of $Q^{2}$ for selected values of $x$. Data are for this experiment (solid), E143 [6] (open diamond) and E155 [14] (open square). The errors are statistical; the systematic errors are small. The curves show $x g_{2}^{W W}$ (solid) and the bag model calculation of Stratmann [19] (dash-dot).
of Table 1. Fig. 2 shows the averaged $x g_{2}$ of this experiment along with $x g_{2}^{W W}$ calculated using our parameterization of $g_{1}$. The combined new data for $p$ disagree with $g_{2}^{W W}$ with a $\chi^{2} /$ dof of 3.1 for 10 degrees of freedom. For $d$ the new data agree with $g_{2}^{W W}$ with a $\chi^{2} /$ dof of 1.2 for 10 dof. The data for $g_{2}^{p}$ are also inconsistent with zero ( $\chi^{2} /$ dof $=15.5$ ) while $g_{2}^{d}$ differs from zero only at $x \sim 0.4$. Also shown in Fig. 2 is the Bag Model calculation of Stratmann [19] which is in good agreement with the data, chiral soliton model calculations [20,21] which are too negative at $x \sim 0.4$ and the Bag Model calculation of Song [11] which is in clear disagreement with the data.

The average values of $A_{2}(x)$, shown in Fig. 3, are consistent with zero at low $x$, increasing to about 0.1 at the highest $x$, significantly different than zero. $A_{2}^{p}$ is many standard deviations lower than the Soffer limit [22] of $\left|A_{2}\right|<\sqrt{R\left(1+A_{1}\right) / 2}$ for all values of $x$. The same is true for $A_{2}^{d}$, except at the highest $x$ value, where the error is large.

The OPE allows us to write the hadronic matrix element in deep inelastic scattering in terms of a series of renormalized operators of increasing twist [1,2].


Fig. 2. The $Q^{2}$-averaged structure function $x \mathrm{~g}_{2}$ from this experiment (solid circle), E143 [13] (open diamond) and E155 [14] (open square). The errors are statistical; systematic errors are shown as the width of the bar at the bottom. Also shown is our twist-2 $\mathrm{g}_{2}^{W W}$ at the average $Q^{2}$ of this experiment at each value of $x$ (solid line), the bag model calculations of Stratmann [19] (dash-dot-dot) and Song [11] (dot) and the chiral soliton models of Weigel and Gamberg [20] (dash dot) and Wakamatsu [21] (dash).

The moments of $g_{1}$ and $g_{2}$ for even $n \geqslant 2$ at fixed $Q^{2}$ can be related to twist-3 reduced matrix element, $d_{n}$, and higher twist terms which are suppressed by powers of $1 / Q$. Neglecting quark mass terms we find that:

$$
\begin{align*}
d_{n} & =2 \int_{0}^{1} d x x^{n}\left[\frac{n+1}{n} g_{2}\left(x, Q^{2}\right)+g_{1}\left(x, Q^{2}\right)\right]  \tag{5}\\
& =2 \frac{n+1}{n} \int_{0}^{1} d x x^{n} \overline{g_{2}}\left(x, Q^{2}\right) .
\end{align*}
$$

The matrix element $d_{n}$ measures deviations of $g_{2}$ from the twist-2 $g_{2}^{W W}$ term. Note that some authors $[2,23]$ define $d_{n}$ with an additional factor of two. We calculated $d_{n}$ using $\overline{g_{2}}\left(x, Q^{2}\right)$ (see Eq. (5)) with the assumption that $\overline{g_{2}}(x)$ is independent of $Q^{2}$ in the measured region. This is not unreasonable since $d_{n}$ depends only logarithmically on $Q^{2}$ [1]. The part of the integral for $x$ below the measured region was assumed to be zero because of the $x^{2}$ suppression. For


Fig. 3. The asymmetry $A_{2}$ for all spectrometers combined (solid circle) and data from E143 [13] (open diamond), E155 [14] (open square), and SMC [12] (open circles). The errors are statistical; the systematic errors are negligible. Also shown is $A_{2}^{W W}$ calculated from the twist- $2 \mathrm{~g}_{2}^{W W}$ at the average $Q^{2}$ of this experiment at each value of $x$ (solid line). The upper Soffer limit [22] is the dashed curve at the upper right.
$x \geqslant 0.8$ we used $\overline{g_{2}} \propto(1-x)^{m}$ where $m=2$ or 3 , normalized to the data for $x \geqslant 0.5$. Because $\overline{g_{2}}$ is small at high $x$, the contribution was negligible for both cases. We obtained values of $d_{2}^{p}=0.0025 \pm 0.0016 \pm$ 0.0010 and $d_{2}^{d}=0.0054 \pm 0.0023 \pm 0.0005$ at an average $Q^{2}$ of $5 \mathrm{GeV}^{2}$. We combined these results with those from SLAC experiments on the neutron (E142 [7] and E154 [15]) and proton and deuteron (E143 [6] and E155 [14]) to obtained average values $d_{2}^{p}=0.0032 \pm 0.0017$ and $d_{2}^{n}=0.0079 \pm 0.0048$. These are consistent with zero (no twist-3) to within 2 standard deviations. The values of the 2 nd moments alone are: $\int_{0}^{1} d x x^{2} g_{2}\left(x, Q^{2}\right)=-0.0072 \pm 0.0005 \pm$ $0.0003(p)$ and $-0.0019 \pm 0.0007 \pm 0.0001(d)$.

Fig. 4 shows the experimental values of $d_{2}$ for proton and neutron with their error, plotted along with theoretical models from left to right: Bag Models (Song [11], Stratmann [19], and Ji [24]); sum rules (Stein [25], BBK [26], Ehrnsperger [27]); chiral soliton models [20,21]; and lattice QCD calculations ( $Q^{2}=5 \mathrm{GeV}^{2}, \beta=6.4$ ) [23]. The lattice and chi-


Fig. 4. The twist-3 matrix element $d_{2}$ for the proton and neutron from the combined data from this and other SLAC experiments (E142 [7], E143 [6], E154 [15] and E155 [14] (DATA). The region between the dashed lines indicates the experimental errors. Also shown are theoretical model values from left to right: bag models [11,19,24], QCD Sum Rules [25-27], Lattice QCD [23] and chiral soliton models [20,21].
ral calculations are in good agreement with the proton data and two standard deviations below the neutron data. The sum rule calculations are significantly lower than the data. The non-singlet $=3 \cdot\left(d_{2}^{p}-d_{2}^{n}\right)=$ $-0.0141 \pm 0.0170$ is consistent with an instanton vacuum calculation of $\sim 0.001$ [28].

The Burkhardt-Cottingham sum rule [29] for $g_{2}$ at large $Q^{2}, \int_{0}^{1} g_{2}(x) d x=0$, was derived from virtual Compton scattering dispersion relations. It does not follow from the OPE since $n=0$. Its validity depends on the lack of singularities for $g_{2}$ at $x=0$, and a dramatic rise of $g_{2}$ at low $x$ could invalidate the sum rule [30]. We evaluated the Burkhardt-Cottingham integral in the measured region of $0.02 \leqslant x \leqslant 0.8$ at $Q^{2}=5 \mathrm{GeV}^{2}$. The results for the proton and deuteron are $-0.044 \pm 0.008 \pm 0.003$ and $-0.008 \pm 0.012 \pm$ 0.002 , respectively. Averaging with the E143 and E155 results which cover a slightly more restrictive $x$ range gives $-0.042 \pm 0.008$ and $-0.006 \pm 0.011$. This does not represent a conclusive test of the sum rule because the behavior of $g_{2}$ as $x \rightarrow 0$ is unknown. However, if we assume that $g_{2}=g_{2}^{W W}$ for $x<0.02$,
and use the relation $\int_{0}^{x} g_{2}^{W W}(y) d y=x\left[g_{2}^{W W}(x)+\right.$ $\left.g_{1}(x)\right]$, there is an additional contribution of 0.020 (0.004) for the proton (deuteron).

The Efremov-Leader-Teryaev (ELT) sum rule [31] involves the valence quark contributions to $g_{1}$ and $g_{2}: \int_{0}^{1} x\left[g_{1}^{V}(x)+2 g_{2}^{V}(x)\right] d x=0$. Assuming that the sea quarks are the same in protons and neutrons, the sum rule takes a form $\int_{0}^{1} x\left[g_{1}^{p}(x)+2 g_{2}^{p}(x)-g_{1}^{n}(x)-\right.$ $\left.2 g_{2}^{n}(x)\right] d x=0$. We evaluated this ELT integral in the measured region using our $g_{2}$ data and the fit to $g_{1}$. The result at $Q^{2}=5 \mathrm{GeV}^{2}$ is $-0.013 \pm 0.008 \pm 0.002$, consistent with the expected value of zero. Including the data of E143 [6] and E155 [14] leads to $-0.011 \pm$ 0.008 . The extrapolation to $x=0$ is unknown, but is suppressed by a factor of $x$.

In summary, our results for $g_{2}$ follow approximately the twist- $2 g_{2}^{W W}$ shape, but deviate significantly at some values of $x$. The values obtained for the twist-3 matrix element $d_{2}$ from this measurement and the SLAC average are less than two standard deviations from zero. The data over the measured range are inconsistent with the Burkhardt-Cottingham sum rule if there is no pathological behavior as $x \rightarrow 0$. The ELT integral is consistent with zero within our measured kinematic range.

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