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Estimating Freeway Traffic Volume Using Shockwaves and Probe Vehicle Trajectory Data

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Abstract

Probe vehicle data are increasingly becoming the primary source of traffic data. In current practice, traffic volumes and speeds are collected from inductive loop or similar devices. As probe vehicle data become more widespread, it is imperative that methods are developed so that traffic state estimators like speed, density and flow can be derived from probe vehicle data as well. In this paper, a methodology to estimate traffic flow on a freeway based on probe vehicle trajectory data combined with traffic shockwave theory is proposed. In essence, probe vehicle trajectory can indicate the free-flowing and congested regimes. By using LWR kinematic wave model, a shockwave can be identified that separates both regimes. From the formation of the shockwave, flows for each regime are estimated. To identify the shockwave, *k*-means clustering is applied to the data. When applied to simulated data, the error of the estimated flow during free-flow ranges from -9% to 1% with an average of -5%. The estimated flow during congestion has an error of 0%. Based on the results, this paper shows that the proposed method can predict traffic flow with a reasonable accuracy under congested and free-flow conditions.

Keywords: probe vehicle; traffic flow estimation; LWR shockwave;

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1. Introduction

Advancements in probe vehicle technology have made it an attractive dataset for traffic state estimations. While data collected from inductive loop or similar detectors are still in use, the emergence of probe vehicle (PV) as an alternative traffic data has garnered a significant attention. As a result, PV data have been used for traffic state estimations such as travel time, speed and density estimation.

The objective of this study is to estimate traffic flow using PV trajectory data combined with the shockwave theory. While there have been other studies on flow estimation from PV data, the approaches have been different (e.g., fundamental diagram, vehicle spacing and Kalman filter). The proposed shockwave approach fills the gap in this field of study.

Traffic state can be estimated by different variables. The most common and widely used traffic estimation from PV data is speed. For this application, it is inferred that PV speed (sample) represents the speed of general traffic (population). Travel time which is the inverse of speed can also be estimated from PV data using the same inference.

Other traffic state estimations such as density and flow cannot be directly inferred from PV data since not all vehicles in the traffic stream serve as probes. Density is the number of vehicles occupying a space. In current practice, density is not an actual measurement but instead an estimation based on the ratio of flow over speed collected at a specific point. On the other hand, flow (and speed) is a direct measurement of the traffic. Flow is the sum of all vehicles passing a point and is aggregated in time ranging from 20 seconds to 15 minutes.

Flow along with other variables such as density and travel time are important traffic state estimators. There are several applications where flow is a critical input. In transportation management and planning, flow is required to determine the number of lanes in freeway design. In traffic control, signal timing is dependent upon the flow at the intersection. In travel demand models, calibration of the model is dependent upon flow. If PV data ever to replace loop detector data, it becomes imperative that methods are developed to estimate flow from PV data. Due to these practical aspects, flow is taken as the main parameter to be estimated. However, the methods presented here are equally applicable to density estimation.

The remainder of this paper is organized as follow: Section 2 is a review of relevant work. Section 3 is the methodology used in this paper. Followed by a description of data in Section 4. The results are then presented in Section 5. Finally, Section 6 draws conclusions of this paper and future work.

2. Literature Review

Traffic state estimations are modeled using a variety of approaches, all of which require data which can come from loop detector or vehicle trajectory. Loop detector or similar devices are considered as static sensors are commonly referred to as Eulerian measurement. Vehicle trajectory or motion of a car is known as mobile sensor is traditionally referred to as Lagrangian measurement.

Modeling of traffic state estimators from loop detector data has been tackled from a variety of angles. Treiber, Kesting, and Wilson (2011) introduced an adaptive smoothing method that analyzes loop detector data to reconstruct traffic states. Sun, Muñoz, and Horowitz (2004) applied Kalman filtering to loop detector data to predict traffic state estimators. Daganzo (1994) proposed the cell transmission model CTM which divides a road segment in "cells". The model computes the flow or density of each cell according to the principle of conservation of vehicle. The CTM approach was further advanced by other studies for traffic state estimation (Sumalee, Zhong, Pan, and Szeto (2011), Tian, Yuan, Treiber, Jia, and Zhang (2012) and Celikoglu (2014)).

For traffic flow estimation from vehicle trajectory, Neumann, Touko Tcheumadjeu, Bohnke, Brockfield, and Bei (2013) and Anuar, Habtemichael, and Cetin (2015) used PV data combined with the fundamental diagram (FD). Though the approach is similar, the studies differ in terms of (1) number of FD models, (2) calibration of the FD and (3) aggregation of time interval. Neumann, Touko Tcheumadjeu, and Bohnke (2013) extended their work by applying Bayesian probability to estimate flow. Using this technique, given a set of loop detector data containing traffic speed and the respective flow, estimate traffic flow for a given PV speed using Bayesian probability.

Utilizing forward facing cameras mounted on probe vehicles, Seo, Kusakabe, and Asakura (2015a) estimated flow and density by measuring the spacing between the lead and follower vehicle. They then demonstrated a data assimilation technique to estimate flow (Seo, Kusakabe, & Asakura, 2015b). By applying Kalman filtering and

Newtonian relaxation methods Juan C Herrera and Bayen (2010) predicted the flow and density of traffic. Similar approach by Work et al. (2008) and Roncoli, Bekiaris-Liberis, and Papageorgiou (2015) estimated traffic density using Kalman filter.

PV data have also been used in queue length estimation. Ban, Hao, and Sun (2011), Cetin (2012), Anderson, Ran, Jin, Qin, and Cheng (2011), and Comert and Cetin (2009) all used PV trajectory and the shockwave theory to estimate queue length. Cai, Wang, Zheng, Wu, and Wang (2014) relied on loop detector data in addition to PV trajectory to estimate queue length.

To understand the reliability of PV as a traffic data source, Bar-Gera (2007) studied the reliability of travel time and speed of traffic for data collected from PV. Meanwhile Juan C. Herrera et al. (2010) developed a traffic monitoring system based on PV data for the San Francisco bay area. Kim and Coifman (2014) performed a study on the reliability of PV data for Ohio Department of Transportation as the agency moves away from inductive loop. PV data have also been used to analyze the resiliency of a transportation network as performed by Donovan and Work (2015).

The methodology proposed in this paper which is combining PV data and shockwave theory fills the research gap by introducing a new concept in estimating flow. This methodology is fairly new and has never been applied in other studies.

3. Methodology

The methodology proposed in this paper estimates traffic flow using PV data in combination with the shockwave theory as proposed by Lighthill-Whitham-Richards (LWR) (1955; 1956). The LWR model is used to analyze traffic flow dynamics, in particular estimating the shockwave boundary and speed. Derived from a FD and the conservation law (Equation 1 below), the LWR model, also known as the kinematic wave model, describes the evolution of system state in terms of density, flow, or speed over time and space. The conservation equation and the shockwave speed can be formulated as:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{1}$$

$$w = {(q_j - q_f)}\Big/{(k_j - k_f)} \tag{2}$$

In the shockwave equation, w is the shockwave speed while q and k are flow and density, respectively. It is written for a particular interface or boundary where a free-flow condition is mixing with jammed traffic, and j and f denotes jam (or congested) and free-flow conditions, respectively. Using the fundamental relationship between the three variables, k can be represented as:

$$k = {q}/{u} \tag{3}$$

Substituting k into Equation 2 and solving for q_f:

$$w = {q_j - q_f}\Big/{\frac{q_j}{u_j} - \frac{q_f}{u_f}} \tag{4}$$

$$q_f = \frac{q_j - w\frac{q_j}{u_j}}{1 - \frac{w}{u_f}} \tag{5}$$

The objective from implementing the shockwave equation is to solve for flow q regardless of free-flow or congestion. By relying solely on PV data, three of the five variables - w, u_j and u_f - can be estimated, leaving q_j and q_f as the two unknown variables. To estimate w, u_j and u_f, a breakpoint speed has to be selected. Breakpoint speed is the point where traffic transitions from free-flow to congestion. To estimate w, fit a linear regression line through the break points. w is the slope of the linear regression line. After selecting a breakpoint speed, any PV speed observations greater than breakpoint speed is considered to be free-flowing and any observations smaller than breakpoint speed is considered under congestion. u_j and u_f are the average PV speed for the congested and free-flow regions, respectively.

After calculating w, u_j and u_f, to solve Equation 2 one of the two unknown variables (q_j or q_f) must be estimated. During free-flow period, there is fluctuation of flow and varying space between vehicles. In contrast, during congestion vehicle flow and spacing are relatively constant. Because of this characteristics, it is expected that there is less variation in $\hat{q}_j$ compared to $\hat{q}_f$. With $\hat{q}_j$ being uniform it is expected that there would be less variation in $\hat{q}_f$ when solving the shockwave equation. Hence the decision to first solve $\hat{q}_j$ instead of $\hat{q}_f$.

Fig. 1 illustrates the relationship between trajectory and w highlighting the free-flow and congested regions. In this figure, solid lines are the PV trajectory, dashed lines are non- PV trajectory and dotted line is w.

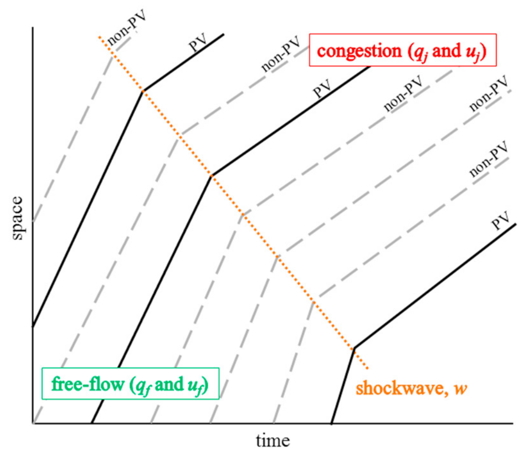

Fig. 1. Sample illustration of trajectory and shockwave

To calculate $\hat{q}_j$ the Northwestern (Drake, Schofer, & May, 1967) congested regime u - k relationship is implemented. It can be formulated as:

$$k = {u_B - u}/_{0.265} \tag{6}$$

k is density, u is observed speed and u_B is breakpoint speed (= 40). Re-writing Equation 3 in terms of $\hat{q}_j$ and substituting k into the equation:

$$\hat{q}_j = \left(\frac{u_B - u_P}{0.265}\right) u_P \tag{7}$$

$\hat{q}_j$ is the estimated flow during congestion, u_P is the average probe vehicle speed during congestion.

As stated earlier, three of the five variables - w, u_j and u_f - can be estimated from PV data. It is already established that w is a linear regression line between free-flow and congested regions. While u_j and u_f are the average PV speed which are smaller or greater than u_B, respectively.

From the relationships describe earlier it can be seen that $\hat{q}_j$, u_j and u_f are dependent upon u_B. Since these three variables are used in the Equation 2, they also affect the results for $\hat{q}_f$. To have a good estimation of $\hat{q}_j$ and $\hat{q}_f$, it is imperative that u_B is selected properly.

u_B is the speed at which traffic flow transitions from free-flow to congestion and vice versa. When this transition points are connected together, they form $\widehat{\omega}$. Note the difference between w and ω. While they both stands for shockwave, in this paper the term w is used for flow estimation while ω is used to determine u_B.

By adjusting u_B it changes the transition points which in turns affects $\widehat{\omega}$. For an assigned u_B, $\widehat{\omega}$ is estimated by fitting a linear regression line through a group of transition points. In addition to the transition points time-space coordinate, another factor that affects $\widehat{\omega}$ is the number of points contained in a group. Each group is assigned a specific number n of PV. Too small of an n per group may cause severe fluctuation of $\widehat{\omega}$ while too large of an n may result in averaging the $\widehat{\omega}$ over too large of time period. Basically after identifying the total number of PV from the dataset, they are divided into p number of groups containing n PV.

To determine the proper u_B the $\widehat{\omega}$ for each group is compared against a known ω. The objective is to adjust u_B so that the difference between $\widehat{\omega}$ and ω is minimized. The objective function can be formulated as:

$$Min\ Z = \sum_{i=1}^{p} \frac{|\widehat{\omega}_i - \omega|}{p} \tag{8}$$

where: $\widehat{w}_i$ is estimated shockwave for each group
w_i is known shockwave for each group
p is total number of groups

After selecting u_B the variables - w, u_j and u_f - can be calculated. They are then plugged into Equation 7 and 5 to calculate $\hat{q}_f$. To measure the accuracy of the results, the difference between $\hat{q}_f$ and q_f are calculated.

4. Data

A segment at a length of three miles is simulated in Vissim. The segment is a three lane freeway facility reducing to a two lane segment at the end of the roadway. No ramps exist between the start and end points of the segment. Vehicle demand is increased at the beginning of the simulation before decreasing towards the end. Fig. 2 is an illustration of the network while the simulated demand is listed on Table 1.

This segment is not intended to simulate any actual road segment. It is a hypothetical segment with the aim of producing synthetic data to be applied to the proposed methodology. To create congestion a bottleneck from three to two lanes is intentionally introduce with a high demand in the beginning and slowly decreasing towards the end. All other simulation parameters (e.g., car following, free-flow speed) are based on Vissim default values.

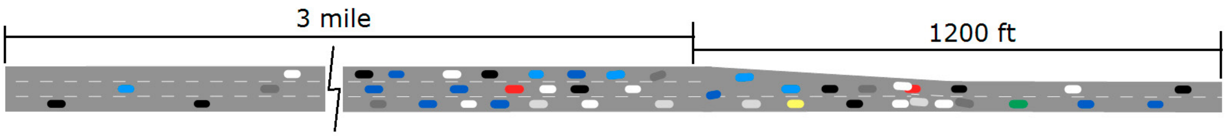

Fig. 2 Simulated Vissim network

Simulation time is two hours with a total of 6,667 vehicles. Five percent of the vehicles are randomly selected as PV and their trajectories are illustrated in Fig. 3. With five percent penetration rate, there is a total of 333 PV.

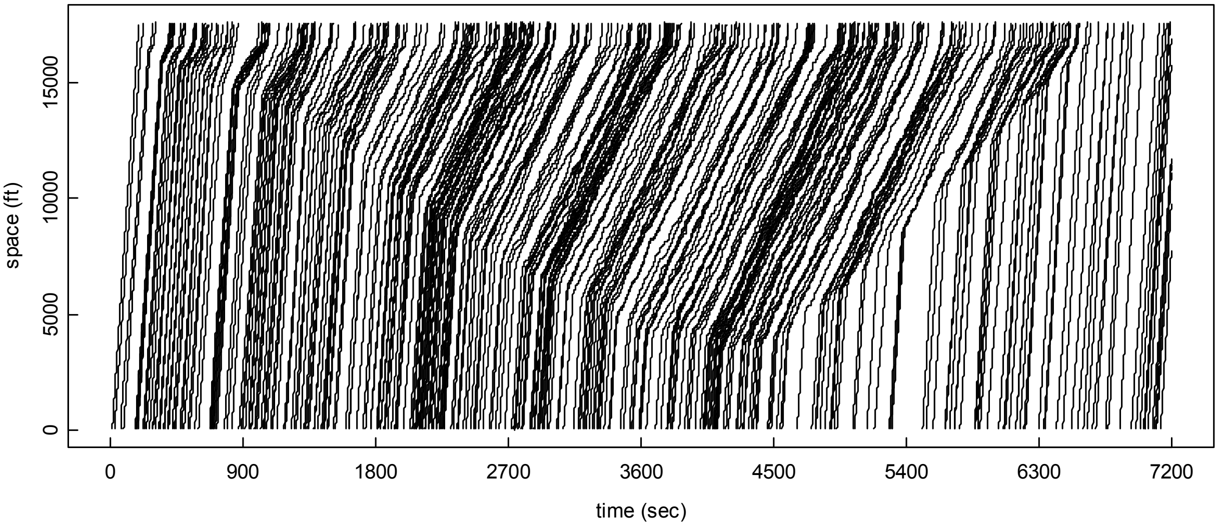

Fig. 3. Probe vehicle trajectory

In Fig. 3 each line is the trajectory for a single vehicle. Near vertical lines indicate free-flow condition while angle lines indicate congestion. From visual observation a backward moving shockwave can be observe between time zero to about 4000 second. As demand decreases, the shockwave changes to forward moving starting around 4000 second and finally disappearing around 6500 second.

Using data for all vehicles congested flow q_j and jam density k_j can be determined. To calculate q_j the number of vehicles is counted around location 16000 feet between 1800 and 5400 second time period. k_j is estimated by counting the number of vehicles at time 3600 second between locations 10000 and 15280 feet. From calculation of all vehicle data at the specified time-space, q_j = 1159 veh/hour/lane (vphpl) and k_j = 128 veh/mile/lane (vpmpl). These values are the ground truth and will be used in other calculations.

As stated earlier in this section, demand is increased at the beginning of the simulation before decreasing towards the end. In the simulation the demand is modified every 900 seconds or 15 minutes. Throughout the simulation the demand is modified eight times that can be identified as time segment i. To calculate ω_i for each time segment, the shockwave equation is used where:

$$\omega_i = {(q_j - q_i)}\Big/{(k_j - k_i)} \tag{9}$$

where: ω_i is shockwave for time segment i
q_i is flow for time segment i
k_i is density for time segment i
q_j is flow during congestion (calculated to be 1159 vphpl)
k_j is jam density (calculated to be 128 vpmpl)

q_i is the number of all vehicles entering the segment at each time segment i. k_i is the density of vehicle at mid-time for each time segment for a one half mile section during free-flow period. Multiply by two to convert k_i to vehicle per mile. With all four values of q_i, k_i, q_j and k_j known ω_i can be calculated. The simulated vehicle demand, q_i and ω_i is shown in Table 1.

Table 1. Simulation demand, flow and shockwave for different time segments

Time Segment	1	2	3	4	5	6	7	8
Time (sec)	0	900	1800	2700	3600	4500	5400	6300
Demand (vph)	4000	4600	4400	4200	3500	3000	3000	2500
q_i (vph)	4071	4401	4323	4179	3477	2133	1980	2103
ω_i (mph)	-1.90	-2.80	-2.71	-2.17	0	3.80	4.38	3.95

From Table 1, ω_i is backward moving (negative value) in time segments 1 through 4. In time segment 5, ω_i is transitioning from backward to forward resulting in a zero value. As the simulation concludes ω_i is forward moving (positive value). ω_i shown in Table 1 will be used as ground truth to determine u_B.

5. Results

Prior to estimating $\hat{q}_j$ or $\hat{q}_f$ a proper u_B value must be selected. In this study $\widehat{\omega}_i$ is calculated for every twentieth PV with u_B search values ranges from thirty to fifty mph. The results are shown in Table 2. From this table u_B=35 resulted in the lowest Z at 0.442. Based on this results u_B=35 will be used to calculate $\hat{q}_j$ and $\hat{q}_f$.

Table 2. Results of u_B and Z

u_B	Z
30	0.470
31	0.478
32	0.476
33	0.462
34	0.479
35	0.442
36	0.466
37	0.475
38	0.517
39	0.515
40	0.502
41	0.503
42	0.487
43	0.501
44	0.533
45	0.536
46	0.549
47	0.628
48	0.588
49	0.572
50	0.577

For each u_B the transition points between free-flow and congested regions are identified for every twenty PV. Within each group of twenty PV the transition points are compared against each other. Any statistical outliers in term of space are removed from the analyses. The outliers are due to PV speed being smaller than u_B upstream of the congestion. Fig. 4 illustrates the grouping for every 20^{th} PV. In this figure the circles are the transition points (excluding the outliers), dashed lines are trajectories of the first, every 20^{th} and last probe vehicle and solid lines are the shockwaves. Note that the number of PV for the last group is less than twenty.

Table 3 summarizes the results of $\hat{q}_f$ and $\hat{q}_j$ when u_B is equal to 35. For comparison purpose, the same table summarizes $\hat{q}_f$ and $\hat{q}_j$ when u_B is equal to 40 and 45. $\hat{q}_f$ and $\hat{q}_j$ are compared to q_f and q_j with the difference calculated in terms of percent error. The formula for percent error is written in the table. Note that u_B=40 is the value proposed by the Northwestern congested regime FD.

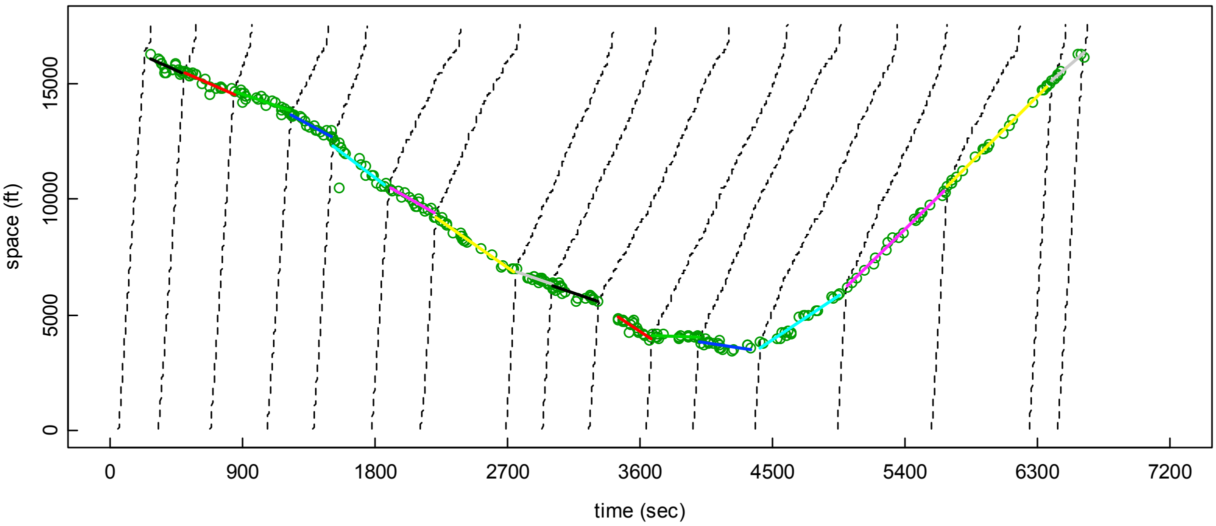

Fig. 4. Probe vehicle trajectory for every 20th vehicle, transition points and its shockwave

Table 3. Summary of results

PV Group	Actual flow q_f (vphpl)	$\hat{q}_j = 912\ (u_B = 35)$ PE$_j$ = -21%		$\hat{q}_j = 1105\ (u_B = 40)$ PE$_j$ = 0%		$\hat{q}_j = 1303\ (u_B = 45)$ PE$_j$ = 12%	
		$\hat{q}_f$ (vphpl)	PE$_f$ ($\frac{\hat{q}_f - q_f}{q_f} \times 100\%$)	$\hat{q}_f$ (vphpl)	PE$_f$ ($\frac{\hat{q}_f - q_f}{q_f} \times 100\%$)	$\hat{q}_f$ (vphpl)	PE$_f$ ($\frac{\hat{q}_f - q_f}{q_f} \times 100\%$)
1	1385	1074	-22	1298	-6	1526	10
2	1337	1066	-20	1288	-4	1515	13
3	1337	1030	-23	1246	-7	1466	10
4	1382	1094	-21	1322	-4	1554	12
5	1550	1176	-24	1419	-8	1667	8
6	1451	1106	-24	1335	-8	1569	8
7	1453	1145	-21	1382	-5	1624	12
8	1353	1039	-23	1256	-7	1478	9
9	1328	1023	-23	1236	-7	1455	10
10	1438	1123	-22	1356	-6	1593	11
11	1149	918	-20	1112	-3	1311	14
12	1203	966	-20	1169	-3	1377	14
13	873	652	-25	797	-9	947	8
14	687	546	-21	670	-2	800	16
15	647	530	-18	651	1	779	20
16	702	577	-18	708	1	844	20

Even though the objective function Z indicates that u_B=35 is the optimum solution, the resulting percent error in flow estimation for both free-flow and congestion is too high. During free-flow, the smallest percent error PE_f is -18%. In comparison, when u_B=40 and u_B=45, the lowest PE_f is 1% and 8% respectively. The average of the percent error is calculated to be -22% (u_B=35), -5% (u_B=40) and 12% (u_B=45).

In the congested period when u_B=35, $\hat{q}_j$ is calculated to be 912 vphpl. Comparing that to q_j which is 1159 vphpl, the PE_j is calculated to be -21%. When u_B=40 and u_B=45, the PE_j is 0% and 12%, respectively. The results indicate that the u_B value of 40 which was proposed by the Northwestern congested regime FD gave the best flow estimation.

6. Conclusions

Probe vehicle trajectory gives a useful insight on the free-flow and congested regime of a traffic flow. There is a distinct different in trajectory as probe vehicle travels through both traffic regimes. By using the LWR kinematic wave model, the separation or shockwave between the two regimes can be identified from the trajectory. Using the information provided from the shockwave supplemented with a Northwestern congested fundamental diagram, flow and speed for each regime can be estimated.

This paper explains in detail the methodology to estimate traffic flow from probe vehicle trajectory combined with the shockwave theory. By identifying the shockwave which is the boundary between free-flow and congestion, flow and speed for each condition can be predicted. As discussed earlier in this paper, the speed change points affects the final outcome of the flow estimation. Speed change points are the points where the probe vehicle speed transitions from free-flow to congestion and vice versa. In this paper a method developed to detect the speed change points did not improve the results of the flow estimation. It turns out that the speed change points proposed by the Northwestern congested fundamental diagram performed fairly well. In the end, after selecting a speed change point the results for flow estimation is acceptable with an average percent error of -5% during free-flow and 0% during congestion.

While the results look promising, there are some drawbacks to this paper. First, the simulated segment is fairly straightforward. Field data with GPS error and facility with ramps will make the analyses more complicated. Secondly with simulated data, all types of information is known and was used for analyses. Such is not the case with field data. Finally, without congestion a shockwave do not exist which renders this methodology inappropriate.

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