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Sensitivity analysis method to address user disparities in the analytic hierarchy process

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ABSTRACT

Decision makers often face complex problems, which can seldom be addressed well without the use of structured analytical models. Mathematical models have been developed to streamline and facilitate decision making activities, and among these, the Analytic Hierarchy Process (AHP) constitutes one of the most utilized multi-criteria decision analysis methods. While AHP has been thoroughly researched and applied, the method still shows limitations in terms of addressing user profile disparities. A novel sensitivity analysis method based on local partial derivatives is presented here to address these limitations. This new methodology informs AHP users of which pairwise comparisons most impact the derived weights and the ranking of alternatives. The method can also be applied to decision processes that require the aggregation of results obtained by several users, as it highlights which individuals most critically impact the aggregated group results while also enabling to focus on inputs that drive the final ordering of alternatives. An aerospace design and engineering example that requires group decision making is presented to demonstrate and validate the proposed methodology.

1. Introduction

Many methods have been developed and made available to decision makers to streamline and facilitate complex decision making activities. When decision makers aim to understand the relative value of a set of alternatives, based on the ratio of gained benefits over the implementation cost, a Cost-Benefit Analysis is typically implemented. This method is well suited for decision problems that entail the optimization of a utility function but has clear limitations when there are qualitative parameters or when multiple objectives are introduced, as described by Cascetta, Carteni, Pagliara, and Montanino (2015). Multi-criteria decision analysis (MCDA) methods have been developed to support the decision making process in these more complex cases. In the 1960′s, Bernard Roy developed the ELimination Et Choix Traduisant la REalite (ELECTRE) method (Elimination and Choice Expressing Reality), based on rankings and vetoes (Figueira, Greco, Roy, & Słowiński, 2013). Thomas Saaty developed the Analytical Hierarchy Process (AHP) in the following decade, deriving priority vectors from matrices of pairwise comparisons. The method was first mentioned in 1972 and a full description of the model was provided in 1980 (Saaty, 1980). In parallel, fuzzy sets were first introduced by Bellman and Zadeh in the 1970′s in an effort to translate qualitative linguistic statements in mathematical expressions (Zoraghi, Amiri, Talebi, & Zowghi, 2013) and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) was developed by Hwang and Yoon in 1981 (Behzadian, Otaghsara, Yazdani, & Ignatius, 2012). In more recent developments, Saaty (2005) proposed the Analytic Network Process (ANP). Contrarily to AHP which assumes interdependence of the criteria, ANP accounts for the dependence that is inherent to the decision making factors. MCDA methods have also been coupled with consensus building techniques such as the Delphi method (Le Pira, Inturri, & Ignaccolo, 2016). Among these models, the Analytic Hierarchy Process constitutes one of the most studied and utilized MCDA methods. AHP has been used in a variety of fields, to include, among many others: public transportation planning (Le Pira, Inturri, Ignaccolo, & Pluchino, 2015, 2016, and Cascetta et al., 2015) marketing and portfolio management, shipping assets selection, military applications, the evaluation of the environmental impact of construction projects, marine biology and medical applications (Forman & Gass, 2001). In recent years, there has been a noticeable increase in the

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Nomenclature

a_{kl}	Pairwise comparison in the kth row and the lth column in a pairwise comparison table
$A_k^{P_j}$	Normalized row geometric mean of the kth criterion in the pairwise comparison table input by user P_j
$\bar{A}_k^{P_j}$	Row geometric mean of the kth criterion in the comparison table input by user P_j
C^{P_j}	Pairwise comparison matrix input by user P_j
ε_{ij}	Error associated with element a_{ij}
$G_k^{(i,j_{i-1})}$	Normalized group row geometric mean for the kth criterion at the ith level under the criteria j_{i-1}
$\bar{G}_k^{(i,j_{i-1})}$	Group row geometric mean for the kth criterion at the ith level under the criteria j_{i-1}
$\lambda_{\max}$	Largest eigenvalue of a pairwise comparison table
μ	Consistency Index of a pairwise comparison table
$n^{(i,j_{i-1})}$	Number of criteria in level i under the criteria j_{i-1} in the upper level
P_i	User i
S_q	Relative weight of design alternative q
$W^{I,J}$	Relative weight of the criteria J in level I
W_q	Relative weight of design alternative q

use of AHP for applications in mechanical and aerospace engineering. Lafleur, Sharma, and Apa (2007) developed a framework to down-select a vehicle for robotic space exploration. AHP was used in conjunction with a Pareto plot to rapidly outline solutions that concurrently offer high value and low cost. Cho et al. (2008) used AHP in conjunction with TOPSIS and Quality Function Deployment (QFD) in a hybrid method to evaluate the preliminary shape of a very light jet. Conrow (2011) used AHP to assess the Technology Readiness Level (TRL) scale used by the Department of Defense (DoD) and the National Aeronautics and Space Administration (NASA).

A comprehensive literature survey revealed that multi-user aggregation methods for AHP are still widely debated and researched. Solutions have been proposed to combine individual results or develop consensus frameworks. The decision making process indeed rarely resides in the hands of one decision maker, who is as defined by Cascetta et al. (2015), as the person "formally in charge of the choice" (p.28). Similarly, complex decision frameworks receive inputs from groups of individuals, who all contribute to the process. The process is seldom based on the input of a single Subject Matter Expert (SME) or a single stakeholder, individuals with an interest in the problem but with no decision power (Cascetta et al., 2015). Individuals involved in the decision making process have varying backgrounds, experience and personal goals. In recent years, a certain emphasis has been put on individualistic differences among groups of decision makers, Subject Matter Experts and stakeholders. It has been highlighted that individuals who collaborate on a common project and provide pairwise comparisons of criteria and alternatives do not share absolutely identical profiles. These individuals have different perspectives of the different parts of the system. Some have more experience relevant to certain aspects of the problem of concern than others; while other have more experience with the AHP methodology itself. The traditional AHP methodology assigns equal importance to each individual's priority vectors and ranking, and ignores the disparities in profiles. Little information is available to interpret the sensitivity of a given pairwise comparison on the obtained results. The motivation of this study is to investigate this selected limitation of AHP and to present a new methodology. The impact of profile disparities in the decision making process is addressed by deriving an analytical sensitivity analysis based on local partial derivatives. The results of this effort provide AHP users with additional information about which individuals most critically impact the aggregated group results, therefore enabling decision makers to focus on inputs that drive the final ordering of alternatives in the ex-post studies (Cascetta et al., 2015; Le Pira, et al., 2015). This method is cross-cutting as it is applicable to both public and private sectors decision problems and is relevant to evaluate profile disparities among any types of AHP users, whether they are decision makers, SMEs or stakeholders.

This paper first introduces a mathematical representation of the traditional AHP method in Section 2. Variations of the traditional AHP method, and their strategies and limitations in addressing user profile disparities are discussed in Section 3. The derivation of analytical sensitivities of the weights with respect to user input in the pairwise comparison matrices is presented in Section 4. An example of the selection of a wheel design for the Space Exploration Vehicle is then presented in Section 5 to validate the derived sensitivity equations and to demonstrate how they provide information which can be used to address user profile disparities. The conclusion of this research effort is given in Section 6.

2. The traditional analytic hierarchy process

Prior to discussing the developed methodology, an overview of the traditional mathematical steps of the Analytic Hierarchy Process is presented hereafter.

2.1. Problem modeling

Saaty (1986) describes three principles used sequentially in decision making: "They are the principles of decomposition, comparative judgment and synthesis of priorities" (p. 841). The process of decomposing the problem at hand or structuring its complexity constitutes the first step of the Analytical Hierarchy Process (Forman and Gass, 2001). A hierarchical tree of the problem requirements is derived from the problem statement in order to visualize these various requirements and their logical structure. This initial problem formulation has a great impact on the derivation of the priority vectors and final ranking of the alternatives. Careful modeling is critical to the success of the methodology. Saaty (1994) comments on this "significant effect on the outcome" (p. 22) of problem modeling, defining it as the most "creative part" of the AHP methodology. Franek and Kresta (2014) also comment on the correlation between the chosen hierarchical structure and the achieved outcome. Typically, problem modeling will generate a multi-layer tree, with top-level requirements subsequently broken down into lower level sub-requirements. Requirements are commonly referred to as criteria in the literature. The modeling of the problem should yield clusters of criteria, with a commonality of focus within clusters (Forman & Gass, 2001). The number of criteria that are under consideration should be sufficient, but limited. A total number of criteria smaller than 9 is typically recommended, as studies have shown that "a person cannot simultaneously perceive and estimate more than 7+/− 2 objects" (Tsyganok, Kadenko, & Andriichuk, 2012). In this study, the number of sub-criteria in level i, under the criteria j_{i-1} of the upper level $i-1$ is denoted as $n^{(i,j_{i-1})}$. This yields a total number of sub-criteria $n^{(i,:)}$ in a given level i as

$$n^{(i,:)} = \sum_{j_{i-1}=1}^{j_{i-1}=n^{(i-1,:)}} n^{(i,j_{i-1})} \tag{1}$$

2.2. Pairwise comparison

Once the problem modelling is completed, users proceed to comparing each criterion against every other criterion, in pairs. As these pairwise comparisons are performed, they are sorted in reciprocal matrix format. For a given criterion, a matrix C^{P_i} of size $m \times m$ stores all pairwise comparisons provided by evaluator P_i (Eq. (2)), where m is set to be $n^{(i,j_{i-1})}$. The pairwise comparison in the kth row and the lth column is denoted as a_{kl}. It should be noted that not all comparisons are independent. One has $a_{kl}=1$ for $k=l$ and $a_{kl}=1/a_{lk}$ for $k < l$. This is due to the reciprocity axiom, which implies that only pairwise comparisons in the upper triangle of the matrix above the diagonal are independent (Forman & Gass, 2001). Furthermore, the rule of transitivity (Franek & Kresta, 2014) is used to check that an acceptable level of consistency is achieved as users perform pairwise comparison. The following equality should be true for any indices k, ℓ and p: $a_{k\ell}a_{\ell p}=a_{kp}$.

$$C^{P_i} = \begin{bmatrix} 1 & a_{12} & \dots & a_{1m} \\ 1/a_{21} & 1 & \dots & a_{2m} \\ \dots & \dots & 1 & \dots \\ 1/a_{1m} & 1/a_{2m} & \dots & 1 \end{bmatrix} \tag{2}$$

The pairwise comparison values stored in the matrix are then aggregated to form a vector of relative weights for each criterion considered in the matrix. This aggregation can be performed with either the right eigenvector method or the row geometric mean method (Dijkstra, 2011; Davoodi, 2009). Both methods yield satisfactory results and are appropriate to use. In this study, the row geometric mean method is used for both its computational simplicity and its compatibility with MS Excel. One of the objectives of this study was to develop a methodology that could be easily deployable with commonly used software. MS Excel is widely used for engineering applications, and constitutes an excellent platform to implement the application of the analysis and to produce visualization aids. The ease of implementation of the row geometric mean method in MS Excel when compared to the right eigenvector method made the method more suitable for the purposes of this study and was therefore selected.

The row geometric mean $\overline{A}_r^{P_j}$ provides the weight of the rth criterion in the pairwise comparison matrix C^{P_j} prepared by the jth user as shown in Eq. (3).

$$\overline{A}_r^{P_j} = \sqrt[m]{\left(\prod_{s=1}^{m} a_{rs}\right)} = \sqrt[m]{a_{r1}a_{r2}\dots a_{rm}} \tag{3}$$

Considering element dependency in the pairwise comparison matrix, the expression for the geometric mean can also be written as Eq. (4).

$$\overline{A}_r^{P_j} = \sqrt[m]{\frac{1}{a_{1r}}\dots\frac{1}{a_{k-1,r}}a_{r,k+1}\dots a_{r,m}} \tag{4}$$

The computed weights are then normalized for the jth user and the rth criterion. The normalized row geometric mean $A_r^{P_j}$ of the rth criterion, evaluated by the jth user is shown in Eq. (5).

$$A_r^{P_j} = \frac{\overline{A}_r^{P_j}}{\sum\limits_{q=1}^{m} \overline{A_q^{P_j}}} \tag{5}$$

The value of $A_r^{P_j}$ represents the weight of the rth criterion assigned by user P_j, among the m number of criteria of the ith level, which is a sub-criterion under criterion j_{i-1} one level above. Note that for conciseness, m is set to be the same as $n(i, j_{i-1})$ in the derivation. Saaty (1980, p. 180) proposed an index, the Consistency Index μ, to measure the level of inconsistency of a given pairwise comparison matrix C^{P_j} (Eq. (6)),

$$\mu = \frac{\lambda_{\max} - m}{m-1} = \sum_{i<j}\left[\frac{1}{2}\left(\varepsilon_{ij} + \frac{1}{\varepsilon_{ij}}\right) - 1\right] \div \left(\frac{1}{2}m(m-1)\right) \tag{6}$$

where $\lambda_{\max}$ is the largest eigenvalue of the pairwise comparison matrix C^{P_j} and ε_{ij} is the error for element a_{ij} in the matrix. The error is defined as $\varepsilon_{ij} = a_{ij} \times A_j^{P_j}/A_i^{P_j}$, where $A_i^{P_j}$ and $A_j^{P_j}$ are the weights associated with rows i and j. A perfectly consistent pairwise comparison matrix has an error of $\varepsilon_{ij}=1$ for any element in the matrix. Saaty defines the Random Index as the averaged consistency indices for randomly generated pairwise comparison tables. Table 1 lists the values of Random Indices for different sizes of pairwise comparison tables. Saaty defines the threshold for suitable consistency in a pairwise comparison matrix as 10% of the ratio between the Consistency Index and the Random Index. The weights used to compute the Consistency Index as defined in Eq. (6) can be calculated either by the right eigenvector method or by the row geometric mean method. Two sets of Random Indices are reported in Table 1, based upon the method used to calculate the weights.

2.3. Group aggregation

Two methods are available to perform the row geometric mean method at the group level: the Aggregation of Individual Judgements (AIJ) and the Aggregation of Individual Priorities (AIP) (Escobar, Aguarón, & Moreno-Jiménez, 2004). AIJ obtains a group judgment matrix from individual matrices and then derives the group priorities. AIP first computes individual priority vectors from the individual matrices and then derives the group priorities. Escobar et al. (2004) show that both methods yield the same alternatives priorities. Also, for both AIJ and AIP, the group inconsistency equals or outperforms the worst individual inconsistency. AIP is less computationally intensive and was therefore selected as the aggregation method in this study.

For a given number of users P, the aggregated group geometric mean $\overline{G}_r^{(i,j_{i-1})}$ for the rth criterion at the ith level under the criterion j_{i-1} of the upper level $i-1$ is given by Eq. (7).

$$\overline{G}_r^{i,j_{i-1}} = \sqrt[P]{\left(\prod_{\ell=1}^{P} A_r^{\ell}\right)} = \sqrt[P]{A_r^1 A_r^2 \dots A_r^P} \tag{7}$$

Similarly to the weight calculations for a pairwise comparison table, the priority vector given in Eq. (7) is then normalized. The normalized aggregated group geometric mean $G_r^{(i,j_{i-1})}$ for the rth criterion at the ith level under the criterion j_{i-1} of the upper level $i-1$ is given by Eq. (8),

$$G_r^{(i,j_{i-1})} = \frac{\overline{G}_r^{(i,j_{i-1})}}{\sum_{q=1}^{m} \overline{G}_q^{(i,j_{i-1})}} \tag{8}$$

2.4. Final ranking of design alternatives

Similarly to the method used to determine the relative weight of each criterion, the design alternatives available to the users are evaluated and ranked against each base criterion in the lowest level of the decision making hierarchy. In this study, a base criterion is defined as a criterion that is not connected to any subcriterion in the hierarchical table. The dimension of the pairwise comparison table used to weight available design alternatives in this case is equal to the number of design alternatives. The ranking of the design alternative q against the selected base criterion r of level i under criterion j_{i-1} one level above is denoted by $S_q^{(i,j_{i-1})}$, which can be obtained by using Eqs. (3)–(8). The overall weight of each base criterion, denoted by $W^{I,J}$, is obtained by aggregating all weights of the criteria in levels that are above the base criterion of concern, as stated by Eq. (9),

$$W^{I,J} = G_r^{(i,j_{i-1})} G_{j_{i-1}}^{(i-1,j_{i-2})} \dots G_{j_2}^{(2,j_1)} G_{j_1}^{(1,0)} \tag{9}$$

Table 1
Random indices for consistency check (Dijkstra, 2011, p.108).

	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
Geometric method	0.52	0.87	1.08	1.22	1.32	1.39	1.44	1.48
Eigenvector method	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

The pair of indices I,J indicates that the overall weight is associated with a base criterion, which is the rth criterion in level i under criteria j_{i-1} in the hierarchical table.

The final ranking of the design alternative q, aggregated from all users and denoted by $S_q^{I,J}$ can then be expressed as

$$S_q^{I,J} = W^{I,J} S_q^{(i,j_{i-1})} \tag{10}$$

It should be noted that $S_q^{(i,j_{i-1})}$ is the local ranking of design alternative q, among all design alternatives, measured solely against a ith level criterion under the upper level criterion j_{i-1}, while $S_q^{I,J}$ is the ranking that considers not only the merit of design alternative q among all alternatives but also the relative importance of the criterion I,J among all base criteria in decision making. The final ranking of design alternative q is then obtained by summing all $S_q^{I,J}$ for all base criteria in the hierarchical table.

$$W_q = \sum_{I,J} S_q^{I,J} \tag{11}$$

3. Variations of traditional AHP

Recently, great emphasis has been put on qualifying and integrating disparities in user profiles. Traditional AHP aggregates user priority vectors with the assumption that every individual contributes equally to the process. This assumption is rarely valid. User profiles vary greatly and the traditional AHP methodology lacks the ability to take this diversity into account. A discussion of some attempts to integrate these disparities in the AHP model and their limitations follows. First, the categorization of disparities among users will be discussed. Strategies that use AHP to differentiate users will then be presented, followed by qualitative strategies, and then quantitative strategies. Lastly, the group consensus approach will be discussed.

3.1. Categorization of disparities among users

If the presence of disparities among users is commonly accepted, the formulation of the exact nature of the disparities is rare. In one study, common variations among users are described as “underestimation, optimism and limited capacity for concurrent analysis of multi-factor problems” (Bulut, Duru, Keçeci, & Yoshida, 2012, p. 1911). A different interpretation is given by Aly and Vrana (2008), who use the term “importance” to qualify users. The study breaks down importance in three categories: “Knowledge: the amount of important knowledge and information each expert bears. Experience: the age and historical deepness of the expertise contained in each expert. Relevance: the degree of how much each expert has knowledge pertaining and relating to the decision problem” (p. 533). Bennour and Crestani (2007) define professional competence as the “combination of knowledge (theoretical, contextual and procedural), know-how (practical and implemented in empirical manners) and behaviors (attitudes and relational or cognitive behaviors)” (p. 5745).

An extensive literature search revealed that there is no commonly accepted definition of disparities in user profiles and abilities to use the model. The different interpretations found in the literature can be combined and organized as follows:

1. Competence: years of experience, relevance of educational and professional training, familiarity with part or the totality of the subject of the study.
2. Ability: familiarity with the formulation of judgments for multi-factors problems, familiarity with decision-making strategies, ability to formulate consistent comparisons.
3. Compliance: use of the decision-making model as intended, lack of personal interest, absence of use of power or influence, and absence of coalition between users.

The need to integrate the disparities in user profiles into the final rankings is demonstrated by Tsyganok et al. (2012), who develop two mathematical models to study the dependency of final rankings on group size. Following two different distribution laws, the authors randomly generate “expert opinions” or rankings. The number of users varies between 3 and 200 while the number of criteria varies between 3 and 9. The study finds that the minimum number of users in a group for which disparities can be ignored is 50, the threshold at which the discrepancy between the model adjusted for user competency and the unadjusted model is under 5%.

Groups involved in decision making typically do not comprise such a large number of individuals. This study therefore justifies the need to take into account user profiles into the final rankings obtained from an AHP study.

3.2. Current AHP-based strategies and limitations

The earlier strategies developed to integrate the discrepancies in user profiles involve the use of AHP. Authors use the AHP process to obtain weights for users. In a 1994 study by Ramanathan and Ganesh (1994), each user rates the other users with the AHP model, formulating pairwise comparisons. The individual weights are then aggregated and produce a priority vector for the group. Users are also required to include a rating of themselves in the comparison matrix. Other studies (Cook, Kress, & Seiford, 1996; Aly & Vrana, 2008, p.532) introduce the notion of a “supra decision-maker.” One decision maker is assumed to have knowledge of all of the other users’ profiles and performs pairwise comparisons of the other users with the AHP tool. To do so, Cook et al. (1996) use the traditional AHP methodology with a 1–9 scale of crisp numbers. Aly and Vrana (2008) introduce a Fuzzy-AHP tool to weight the knowledge of experts. Fuzzy numbers and their membership functions are used in an effort to take the qualitative aspect of the ranking into account.

Shortfalls of these methods are described hereafter. In the case of all of the users supplying rankings for the entire group, there is a high risk for conflicting personal interest or coalitions. The expertise of users is also a matter of perspective and rating expertise can be highly subjective. Also, there are cases where users do not physically interact. Criteria rankings might be obtained with no in-person meetings, rendering users unaware of the profile of the other users. In the case of the supra decision-maker, it may be difficult for one individual to have a precise knowledge of all of the user profiles. The supra decision-maker can easily obtain quantitative elements from the users, for example the number of years worked in a given field or the highest level of education achieved. It may however be difficult for the supra decision-maker to determine the familiarity of a user with the formulation of multi-factor decisions, or whether or not the user performed rankings with an ulterior motive in mind.

3.3. Current qualitative strategies and limitations

Several studies attempt to introduce qualitative strategies. Fuhua, Hongke, and Guoqiang (2010) allocate a weight vector to the users based on the "expert's experience value" (p. 3788). The "expert's experience value" is determined from "experience and familiarity." This approach implies that the value is either allocated by a supra decision-maker or that it is self-determined by the user. The shortfalls described in the previous section on using AHP to determine the weights of users apply here also. Bulut et al. (2012) assign a coefficient to users, described as "Lambda coefficients [that] correspond to the expertise priority" (p. 1918). Bardossy, Duckstein, and Bogardi (1993) investigate combining fuzzy-AHP results from users. When others typically focus on determining which experts have the most knowledge and experience, Bardossy et al. approach the issue from a reliability standpoint. Finally, Van den Honert (1998) uses "suitable weights for the group members" (p.100). The term "suitable" however is not defined.

Srdjevic, Srdjevic, Blagojevic, and Suvocarev (2013) discuss two shortcomings that stem from qualitative strategies. The task of assigning weights can be difficult when there are many users. Also, determining weights for users prior to the decision process "may lead to a result which the participants do not feel to be their own" (p. 6671). The qualitative approach leaves room for subjectivity and bias on the part of the user who is determining the weights. This approach conflicts with the analytical method of AHP, which strives to introduce structure and objectivity in the decision-making process.

3.4. Current quantitative strategies and limitations

Three recent studies describe quantitative strategies to integrate user profiles in the final rankings. Jongsawat and Premchaiswadi (2010) base their methodology on the Euclidian distance between the priorities of a user and the group aggregated priorities. Weights are then allocated to users, which allow the authors to determine whether users contribute positively or negatively to the decision problem. Users with negative contributions are excluded. The process is iterated until only a sub-set of users with positive contributions is left. Although this method was not specifically designed to be applied with AHP, it could easily be transposed to the AHP model. Duru, Bulut, and Yoshida (2012) base the prioritization of users on the consistency they achieved in the ranking process. The authors question the traditional approach of correlating years of experience with greater importance in terms of user priority: "While experience has specific importance, it is not a robust indicator of accurate decisions at all" (p. 4955). Rather than relying on the experience level of the users, the authors choose to use the consistency achieved while producing their rankings, stating, "individual consistency is one of the objective indicators of the quality of judgment" (p. 4954) and "level of consistency is one of the unique indicators of the decision quality and robustness" (p. 4964). Other studies derive user priorities based on their consistency. Dong, Zhang, Hong, and Xu (2010) correlate consistency with soft consensus. Their iterative algorithm adjusts individual rankings to reach an acceptable collective consistency index. A 2011 study combines both Euclidian distance and consistency strategies to develop a unique method and obtain individual weights for the users (Srdjevic, Blagojevic, & Srdjevic, 2011). The study aims at minimizing "the risk of negligent, incompetent, or irresponsible decision making" (p. 531).

A shortfall of these methods is the assumption that users who should be granted greater priority will achieve greater consistency. This assumption may not be valid, and the literature review did not find any justification for that statement. Experience is not a necessary condition for consistency: a user who is familiar with decision-making methodologies can achieve an acceptable consistency ratio without an in-depth knowledge of the topic of interest. Also, AHP can be used iteratively until a desired consistency ratio is obtained. In that case, any user regardless of experience can input pairwise comparisons until a satisfactory consistency level is achieved. A limitation of the study by Dong et al. (2010) stems from the adjustment of individual priorities without any input from the users. The consensus can then appear artificial as it is not reached based on user feedback. Removing the input of users who obtain a negative rating in the method developed by Jongsawat and Premchaiswadi (2010) may also prevent an objective representation of the group diversity. Priority vectors that deviate from the average priority vector may not necessarily symbolize a lack of knowledge or a desire to skew the results; rather a user may have a different understanding of the problem at hand which may bring additional value to the decision making process. Removing outlying opinions may then be detrimental to the outcome of the process. Quantitative methods have the benefit of not relying on the subjective opinion of a decision maker. However, quantitative methods are still under investigation and few validation studies seem to have been conducted.

3.5. The group consensus approach

The strategies presented so far take user profile diversity into consideration by defining weights, priorities or importance for each user. Beyond the aggregation of individual priorities, judgments or preference structures, an alternative approach is to strive to reach group consensus (Moreno-Jiménez, Aguarón, & Escobar, 2008). Bryson (1996) proposes a measure of consensus based on the value of the sine between priority vectors. The method identifies consensus builders as well as users who impact the consensus building process negatively. Users can have two types of influence on the rest of the group. Informational influence arises when users treat information from others as "evidence about reality" (p. 30). Normative influence occurs when users have "the desire to conform to the expectation of other group members" (p. 30). During the consensus building process, users learn from each other and adjust their own priorities based on this informative feedback. They can cooperate, compromise, and/or compete. The consensus approach removes the bias inherent to allocating weights to users. This method also emphasizes the importance of the discussion among experts. This discussion increases the liability of users for the opinions they provide. The discussion also provides them with the opportunity to increase their knowledge about some aspects of the problem, and review and refine their judgment.

A major shortcoming of the consensus approach provided by Bryson is the time investment that is required. The necessary iterations lengthen the decision making process and may not always be practical. Another limitation resides in the uncertain outcome of the consensus approach: reaching consensus is not guaranteed.

Disparities among users have been identified and many methods have attempted to capture these disparities to adjust the outcome of an AHP-based decision process accordingly. These methods, however, show some limitations, and the comprehensive literature review presented here has not revealed a methodology that provides a suitable solution to the problem. Recently, researchers have placed their focus on consensus building in policy making instead of capturing disparity (Le Pira et al., 2015, 2016; Cascetta et al., 2015). To this effort, a consensus indicator was proposed by Le Pira et al. (2015), which measures the similarity or overlap between two preference decision lists. This area of research is particularly important for public policy decision making when numerous stakeholders are involved. The entire decision making process is iterative, made of three major tasks: policy making by decision-makers and planners, stakeholder engagement, and revision and reevaluation. The direct communication between stakeholders, decision-makers and planners brings about the consensus of the final decision. AHP-based methods are proved to be effective for alternative evaluation and evaluation aggregation in this broad-based decision process. It is believed that the sensitivity analysis proposed here can enhance stakeholder engagement as it can identify the critical input elements in any AHP-based process and redirect the focus of the discussion to these critical elements.

4. Sensitivity analysis method to address user disparities in AHP

A variety of methods have been reviewed in the previous section, which addressed the issue of profile disparities within a group of users tasked involved in a common decision making problem. These methods, however, show various limitations. Building on this observation, this study proposes to address user disparities without quantifying them. This novel method is based on sensitivity analysis, which indicates critical pairwise comparisons. With the sensitivity analysis based method, the decision maker gains insight in which pairwise comparisons are most critical to the decision process. The decision maker now has a tool to identify users who are most influential to the group outcome.

While this new approach provides information to decision makers on the most critical pairwise comparisons and on which users influences most the group results for a given criterion, this approach does not relieve decision makers from the concern of identifying relevant SMEs for a given decision making problem. Similarly, this new approach does not provide a validation of the obtained rankings. The scope of this sensitivity analysis based approach is to allow decision makers to identify critical inputs, and critical users who drive the aggregated group results. This sensitivity based method can be applied after users have been vetted and calibrated with other methods, such as the method developed by Cooke and Goossens (2008). Prior to using the sensitivity analysis presented here, decision makers should also ensure that SMEs are "nominated and selected via a traceable and defensible procedure" and that they "undergo a training and familiarization session" (Cooke & Kelly, 2010, p. 4).

The proposed sensitivity analysis method is based on local partial derivatives, a method used in engineering disciplines to analyze uncertainty, such as in structural analysis or in optimization problems. This section presents the analytical derivatives of the individual variables that directly affect the outcome of the hierarchy decision model. Those variables are the individual weight $A_r^{P_j}$ for criterion r, the aggregated group weight $G_r^{(i,j_{i-1})}$, the consistency index μ, and the final ranking of a design alternative $S_q^{I,J}$, with respect to an entry in the pairwise comparison table provided by a user, P_j.

4.1. Analytical derivatives of weightings

A change in value of a pairwise comparison, $a_{k\ell}$, at the ith level by user P_j will affect the criterion weights $A_k^{P_j}$ and $A_\ell^{P_j}$ since $a_{\ell k}=1/a_{k\ell}$; l, $k=1, 2, ..., m$. The input of the pairwise comparison table $a_{k\ell}$, $k<\ell$, in Eq. (2) is considered hereafter as an independent variable. The general formula for the derivatives of $A_r^{P_j}$ with respect to $a_{k\ell}$ is given by directly differentiating Eq. (5). The detailed derivation procedure is given in the Appendix. In the case when r is equal to k, one has

$$\frac{\partial A_k^{P_j}}{\partial a_{k\ell}}=\frac{a_{\ell k}}{m}\times A_k^{P_j}\times\left[1-\left(A_k^{P_j}-A_\ell^{P_j}\right)\right] \tag{12}$$

And in the case when r is equal to ℓ, one has

$$\frac{\partial A_\ell^{P_j}}{\partial a_{k\ell}}=-\frac{a_{\ell k}}{m}\times A_\ell^{P_j}\times\left[1+\left(A_k^{P_j}-A_\ell^{P_j}\right)\right]$$

Finally, in the case when $r\neq k$, ℓ, one has

$$\frac{\partial A_r^{P_j}}{\partial a_{k\ell}}=-\frac{a_{\ell k}}{m}\times A_k^{P_j}\times\left(A_k^{P_j}-A_\ell^{P_j}\right) \tag{13}$$

After aggregation of each user's weights for the rth criterion at the ith level, one obtains an aggregated group weight $G_r^{(i,j_{i-1})}$. The dimension of the aggregation table is $m\times P$, where P is the total number of users and $m=n^{(i,j_{i-1})}$ is the number of criteria or alternatives. A change in value of the pairwise comparison $a_{k\ell}$ at the ith level by user P_j will affect the aggregated weight $G_r^{(i,j_{i-1})}$ as follows:

$$\frac{\partial G_r^{(i,j_{i-1})}}{\partial a_{k\ell}}=\sum_{s=1}^{m}\left(\frac{\partial G_r^{(i,j_{i-1})}}{\partial A_s^{P_j}}\times\frac{\partial A_s^{P_j}}{\partial a_{k\ell}}\right) \tag{14}$$

On the right-hand side of Eq. (14), the derivative $\frac{\partial A_s^{P_j}}{\partial a_{k\ell}}$ has been given in Eq. (12) for $s=k$ or ℓ and Eq. (13) for $s\neq k$, ℓ. The other derivative, $\frac{\partial G_r^{(i,j_{i-1})}}{\partial A_s^{P_j}}$, is the effect on the aggregated group weight of the rth criterion due to a change of the individual weight, $A_s^{P_j}$, evaluated by user P_j on the sth criterion. It can be evaluated as follows:

$$\frac{\partial G_r^{(i,j_{i-1})}}{\partial A_s^{P_j}}=\frac{G_r^{(i,j_{i-1})}\times\left(\sum_{q=1,q\neq r}^{m}G_q^{(i,j_{i-1})}\right)}{P\times A_r^{P_j}}\quad\text{for } r=s, \tag{15}$$

and

$$\frac{\partial G_r^{(i,j_{i-1})}}{\partial A_s^{P_j}}=\frac{-G_r^{(i,j_{i-1})}\times G_s^{(i,j_{i-1})}}{P\times A_s^{P_j}}\quad\text{for } r\neq s, \tag{16}$$

4.2. Analytical derivatives of inconsistency index

The partial derivative of the inconsistency index μ of Eq. (6) with respect to a given pairwise comparison $a_{k\ell}$ by user P_j is calculated as follows:

$$\frac{\partial\mu}{\partial a_{k\ell}}=\frac{1}{(m(m-1))}\sum_{i<j}\left(\left(1-\frac{1}{\varepsilon_{ij}^2}\right)\frac{\partial\varepsilon_{ij}}{\partial a_{k\ell}}\right) \tag{17}$$

where the derivative of the error term ε_{ij}, $\varepsilon_{ij}=a_{ij}\times A_j^{P_j}/A_i^{P_j}$, is given by Eq. (18), for $j>i$

$$\frac{\partial\varepsilon_{ij}}{\partial a_{k\ell}}=\frac{\partial a_{ij}}{\partial a_{k\ell}}\times\frac{A_j^{P_j}}{A_i^{P_j}}+\frac{a_{ij}}{\left(A_i^{P_j}\right)^2}\times\left(\frac{\partial A_j^{P_j}}{\partial a_{k\ell}}\times A_i^{P_j}-\frac{\partial A_i^{P_j}}{\partial a_{k\ell}}\times A_j^{P_j}\right) \tag{18}$$

It should be noted that the term $\frac{\partial a_{ij}}{\partial a_{k\ell}}$ in Eq. (18) is null most of time, except for the following two cases:

$$\frac{\partial a_{ij}}{\partial a_{k\ell}}=\begin{cases}1 & \text{for}\quad i=k, j=\ell\\ -a_{ij}^2 & \text{for}\quad i=\ell, j=k\end{cases} \tag{19}$$

The individual weight derivatives $\frac{\partial A_j^{P_j}}{\partial a_{k\ell}}$ and $\frac{\partial A_i^{P_j}}{\partial a_{k\ell}}$ can be computed by Eqs. (12) and (13). The derivative of the Consistency Index can also be computed for the right eigenvector method. In this case, the derivatives of concern are directly derived by differentiating Eq. (6) as:

$$\frac{\partial\mu}{\partial a_{k\ell}}=\frac{\left(\frac{\partial\lambda_{\max}}{\partial a_{k\ell}}\right)-m}{m-1} \tag{20}$$

where the derivative of the maximal eigenvalue can be computed as suggested by Saaty (2005, p. 30),

$$\frac{\partial \lambda_{\max}}{\partial a_{k\ell}} = \frac{\left(v_k \times w_\ell - a_{\ell k}^2 \times v_\ell \times w_k\right)}{\sum_{i=1}^{m} (v_i \times w_i)} \tag{21}$$

where v_i and w_i are the ortho-normalized left and right-eigenvectors of the pairwise comparison matrix C^{P_j}. It is evident that the derivative of the Consistency Index is much easier to compute with Eq. (17) than with Eq. (21), which is based on the eigenvalue method.

4.3. Analytical derivatives of final ranking of design alternatives

Set $G_r^{(i,j_{i-1})}$ to be the aggregated weight based on the pairwise comparison tables submitted by all users for the rth criterion in the ith level and $a_{k\ell}$ is the input value of one pairwise comparison submitted by one user. The effect of a change in the value of $a_{k\ell}$ on the overall total weight can then be obtained by differentiating the total weight equation, Eq. (9), as

$$\frac{\partial W^{IJ}}{\partial a_{k\ell}} = \frac{\partial G_r^{(i,j_{i-1})}}{\partial a_{k\ell}} G_{j_{I-1}}^{(I-1,j_{I-2})} \dots G_{j_2}^{(2,j_1)} G_{j_1}^{(1,0)} \tag{22}$$

where the derivative $\frac{\partial G_r^{(i,j_{i-1})}}{\partial a_{k\ell}}$ can be computed by Eqs. (14)–(16). Consequently, the effect of a change in $a_{k\ell}$ of a criteria comparison table on the final ranking of design alternative q, can be obtained by differentiating Eq. (10) as:

$$\frac{\partial S_q^{IJ}}{\partial a_{k\ell}} = \frac{\partial W^{IJ}}{\partial a_{k\ell}} S_q^{(i,j_{i-1})} \tag{23}$$

Numerical validation of the analytical derivation of the sensitivity analysis presented here can be found in Ivanco (2015).

This methodology can be easily implemented using commonly used software such as MS Excel. An application is shown in the following section, which showcases numerical examples and visualization of the results through graphic displays.

5. An aerospace application: down-selection of a wheel design for the space exploration vehicle

The novel sensitivity analysis-based method presented here is now applied to an aerospace engineering decision making problem. The Space Exploration Vehicle (SEV) is a modular vehicle that provides roving capability to astronauts, and enables lunar and Martian exploration. A picture of the SEV is shown in Fig. 1.

The unique mission of the SEV generates many design challenges. Whereas it may sound trivial at first in comparison to more sophisticated elements of the rover, the design and selection of the wheels is a key design point for the vehicle. The specificities of the terrain and the space environment make these wheels especially challenging to design. The National Aeronautics and Space Administration (NASA) and the National Institute of Aerospace (NIA) initiated the RASC-AL Lunar Wheel student competition in 2013 to foster innovation and propose new potential designs for the SEV wheels. An Old Dominion University (ODU) team participated and won first place in the design competition. Students had five months to design, manufacture and test a wheel concept. The manufactured wheel would be mounted on a Gator RSX to compete against other wheel designs in a roll-off competition. Budget was limited to \$10,000 to cover all aspects of the competition, to include a week-long travel to Johnson Space Center for four students.

At the time, none of the students were familiar with MCDA methods. The design challenge however involved multiple criteria that had to be taken into consideration to satisfy the competition requirements. The team also formulated several design alternatives

Fig. 1. The Space Exploration Vehicle.
Source: www.nasa.gov

and a consensus had to be reached to down-select the alternative that would be fabricated and used during the roll-off. This down-selection of an engineering design is well-suited to be analyzed with AHP and to showcase the capabilities of the new sensitivity analysis based method. In this light, several users were requested to perform pairwise comparisons to rank the proposed alternatives. Users were chosen based on their disparity in profiles, with a total number of six users, $P = 6$. User 1 and User 2 are students who were on the team and have first-hand knowledge of the competition requirements and the design process. User 3 and User 4 are aerospace engineers who are familiar with the design of components for space applications while not being as familiar with the competition requirements and the team's decision process. User 5 and User 6 have no engineering background and no prior knowledge of engineering design and manufacturing for space applications.

5.1. AHP application problem modeling

Fig. 2 shows the hierarchical tree used to model the wheel selection problem. The top level objective is to select the best wheel design among four alternatives, evaluated against four first-level and ten second-level criteria. Table 2 provides definitions for the various criteria.

In recent years, new concepts for tires and wheels have emerged in the automotive and bicycle industry, growing the pool of available alternatives. In the design phase of the project, the team developed several concepts and had to reach a consensus to down-select one design. The alternatives that were considered are an All-aluminum wheel, an All-steel wheel, an Aluminum wheel with rubber tread and an All-composite wheel (Fig. 3).

Since the team ruled the All-Composite alternative out prior to producing a finalized 3D model, Fig. 3 shows the Michelin Tweel, which is an existing design that shares similarities with the team concept. Each user was required to provide pairwise comparisons to rank the criteria at all levels. There is one group of four criteria in Level 1, for which users provided pairwise comparisons in a $n^{(1,0)} \times n^{(1,0)} = 4 \times 4$ matrix. In Level 2, there are three criteria under the Level 1 Terrain Performance criterion, with $n^{(2,1)} = 3$ (Regolith, Boulders and Rocks, Craters and Slopes). There are four criteria under the Level 1 Compatibility with Space Applications criterion, with $n^{(2,2)} = 4$ (Ability to withstand radiation, Loss Mass, Low Maintenance, Low Volume). There are no Level 2 criteria

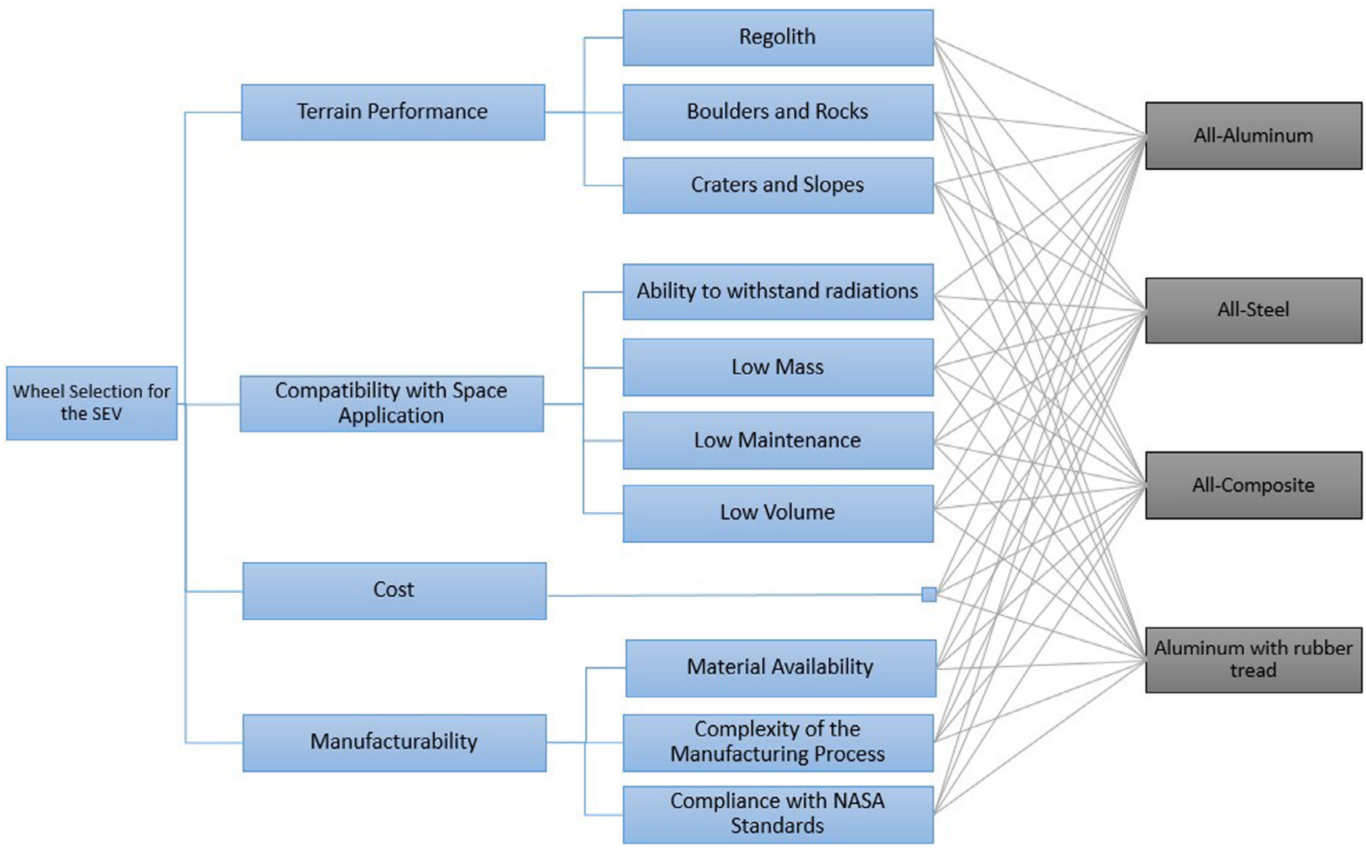

Fig. 2. Problem modeling for the selection of a lunar wheel design.

Table 2
Definitions of criteria for the selection of a wheel design for the Space Exploration Vehicle.

	Terrain Performance
Regolith	Wheel should not sink nor clog.
Boulders and Rocks	Wheel should climb over boulders and rocks without incurring significant damage.
Craters and Slopes	Vehicle should be able to ascent and descent up to 15° slopes without sliding. SEV should be able to navigate crater rims.
	Compatibility with Space Application
Ability to withstand radiations	Wheel should be able to withstand space radiations, with no or little degradation of the material properties and little impact on the fatigue cycle.
Low Mass	Mass should be minimized.
Low Maintenance	Required maintenance should be minimized. Human and/or robotic in-situ repairs or replacements should be possible.
Low Volume	Footprint should be minimized.
	Cost
Cost	Encompasses fabrication, testing and shipping costs.
	Manufacturability
Material Availability	Material should be available within the timeframe of the competition ramp up.
Complexity of the manufacturing process	Encompasses simplicity of the design, machinability of the chosen material, and complexity of welding, fastening or extrusion techniques.
Compliance with NASA standards	Wheel design must have the ability to comply with NASA fabrication standards for space applications.

under Level 1 criterion Cost, $n^{(2,3)} = 0$. There are three Level 2 criteria under Level 1 criterion Manufacturability, with $n^{(2,4)} = 3$ (Material Availability, Complexity of the Manufacturing Process, Compliance with NASA Standards). Each evaluator subsequently performed pairwise comparisons in three comparison tables for Level 2 criteria, with respective dimensions $n^{(2,1)} \times n^{(2,1)} = 3 \times 3$, $n^{(2,2)} \times n^{(2,2)} = 4 \times 4$ and $n^{(2,4)} \times n^{(2,4)} = 3 \times 3$. Each user then had to evaluate the design alternatives against each Level 2 criteria, where $n^{(2,:)} = 11$. This yielded eleven 4×4 tables, as there were four design alternatives considered.

5.2. Results

The weights associated with the pairwise comparisons submitted by all users to rank Level 1 and Level 2 criteria are calculated with Eq. (5). The aggregated results among all users are then calculated based upon Eq. (8) (Table 3). The group ranked Terrain Performance as the most significant criterion, followed by Cost, Space Application and lastly Manufacturability.

Table 4 shows the weights associated with the pairwise comparisons provided by each user for the four design alternatives, for

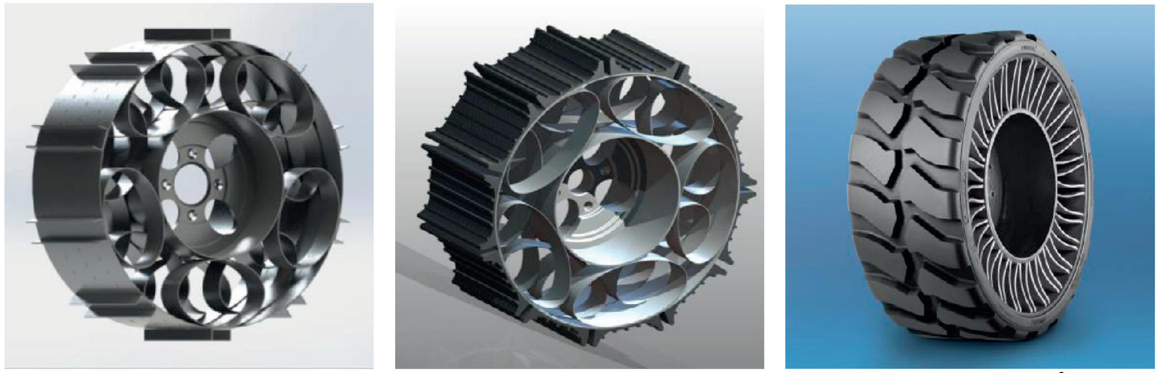

Fig. 3. Proposed design alternatives (Michelin Tweel shown here. By TweelTech (Own work) [CC BY-SA 3.0 (http://creativecommons.org/licenses/by-sa/3.0)], via Wikimedia Commons).

Table 3
Criteria weights summary.

	User 1	User 2	User 3	User 4	User 5	User 6	Group
Terrain Performance	**0.151**	**0.402**	**0.100**	**0.627**	**0.445**	**0.439**	**0.379**
Regolith	0.540	0.750	0.699	0.688	0.537	0.500	0.651
Boulders and Rocks	0.163	0.171	0.064	0.064	0.364	0.250	0.157
Craters and Slopes	0.297	0.078	0.237	0.248	0.099	0.250	0.191
Space Application	**0.067**	**0.232**	**0.050**	**0.191**	**0.315**	**0.439**	**0.208**
Radiation Resistance	0.108	0.325	0.263	0.045	0.432	0.706	0.297
Low Mass	0.430	0.193	0.089	0.245	0.314	0.106	0.262
Low Maintenance	0.077	0.359	0.610	0.636	0.116	0.155	0.319
Low Volume	0.385	0.123	0.038	0.074	0.138	0.033	0.122
Cost	**0.391**	**0.232**	**0.565**	**0.150**	**0.141**	**0.088**	**0.269**
Manufacturability	**0.391**	**0.134**	**0.284**	**0.043**	**0.099**	**0.035**	**0.144**
Material Availability	0.586	0.460	0.582	0.062	0.493	0.084	0.356
Fabrication Process	0.353	0.221	0.348	0.285	0.311	0.147	0.348
Compliance with NASA standards	0.061	0.319	0.069	0.653	0.196	0.769	0.296

each Level 2 criterion under consideration. These weights are calculated with Eq. (5).

The weights are aggregated for the group using Eq. (8) and are denoted by $S_q^{(i,j_{i-1})}$. As an example, the weight $S_q^{(i,j_{i-1})}$ of the All-Steel wheel design alternative is 0.284 indicated in Table 5 for the Level 2 criterion Regolith under the Level 1 criterion Terrain Performance.

Fig. 4 provides a visual representation of the relative importance of the alternatives for each criterion. There are eleven columns in the figure, to reflect the eleven Level 2 criteria. Each alternative is represented by its own color, so that each column is composed of four colors representing the four alternatives. The bar height corresponds to the aggregated group weight listed in the last column of Table 4 multiplied by the Level 1 aggregated group weight listed in bold in Table 3. The Regolith and the Cost criteria are given the most significant importance in the down-selection of the leading design for the SEV wheel design.

Table 5 provides a summary of the aggregated group results for all criteria and alternatives. Final rankings of the alternatives for the group are shown in Table 6. As stated by Eqs. (10) and (11), the final ranking of each design alternative is the inner product between the overall weights of the Level 2 criteria and the weight of the design alternative of concern. Specifically, the final rankings given in Table 6 are the result of the inner product between column 2 and one of columns from 3 to 6 in Table 5.

The group selected the All-Aluminum wheel design as its leading alternative. This selection was due to the high score given to the All-Aluminum design for cost, which has a high priority weight among the criteria. This design also ranked high for its performance on regolith, another criterion with a significant weight. The All-Steel alternative was ranked second. The All-Composite and the Aluminum wheel with rubber tread were allocated much lower scores in the final rankings than the first two alternatives. The leading alternative obtained with the AHP method coincides with the alternative selected by the student team, who elected to manufacture and compete with the All-Aluminum design.

5.3. Sensitivity analysis

The sensitivity analysis method is now applied to demonstrate its use in determining which pairwise comparisons have the greatest impact on the aggregated group results. First, the method is applied to the group weights obtained for the four Level 1 criteria. As indicated in Table 3, the group weighting coefficients are:

$$\boldsymbol{G}_{\cdot}^{(1,0)} = \begin{pmatrix}0.379 & 0.208 & 0.269 & 0.144\end{pmatrix}^T$$

The contributions from User 2 and User 6 are, respectively $\boldsymbol{A}_{\cdot}^{2} = (0.402 \quad 0.232 \quad 0.232 \quad 0.134)^T$ and $\boldsymbol{A}_{\cdot}^{6} = (0.439 \quad 0.439 \quad 0.088 \quad 0.035)^T$. To evaluate the influence of a user's weight on the aggregated group weight, the derivatives of $\boldsymbol{G}_{\cdot}^{(1,0)}$ with respect to $\boldsymbol{A^2}$ and $\boldsymbol{A^6}$ are calculated based upon Eqs. (15) and (16), as follows:

$$\left[\frac{\partial G_s^{(1,0)}}{\partial A_r^2}\right] = \begin{bmatrix}0.0976 & -0.0567 & -0.0732 & -0.0677\\ -0.0327 & 0.1185 & -0.0403 & -0.0372\\ -0.0423 & -0.0403 & 0.1413 & -0.0481\\ -0.0226 & -0.0215 & -0.0278 & 0.1531\end{bmatrix}$$

$$\left[\frac{\partial G_s^{(1,0)}}{\partial A_r^6}\right] = \begin{bmatrix}0.0893 & -0.0300 & -0.1931 & -0.2594\\ -0.0300 & 0.0626 & -0.1061 & -0.1426\\ -0.0387 & -0.0213 & 0.3725 & -0.1842\\ -0.0207 & -0.0114 & -0.0733 & 0.5861\end{bmatrix}$$

Table 4
Alternative weights summary.

	User 1	User 2	User 3	User 4	User 5	User 6	Group	
Regolith	Steel	0.167	0.415	0.167	0.122	0.276	0.569	0.284
	Aluminum	0.167	0.315	0.167	0.510	0.390	0.298	0.327
	Composite	0.333	0.229	0.333	0.267	0.195	0.065	0.242
	Al. w. rubber tread	0.333	0.041	0.333	0.101	0.138	0.068	0.147
Boulders and Rocks	Steel	0.268	0.635	0.268	0.533	0.426	0.394	0.425
	Aluminum	0.092	0.099	0.092	0.206	0.301	0.356	0.173
	Composite	0.499	0.195	0.499	0.105	0.213	0.131	0.243
	Al. w. rubber tread	0.140	0.070	0.140	0.156	0.060	0.890	0.160
Craters and Slopes	Steel	0.168	0.219	0.168	0.144	0.426	0.119	0.224
	Aluminum	0.156	0.327	0.156	0.499	0.301	0.146	0.281
	Composite	0.186	0.368	0.186	0.297	0.213	0.192	0.275
	Al. w. rubber tread	0.490	0.087	0.490	0.060	0.060	0.543	0.220
Radiation Resistance	Steel	0.441	0.393	0.533	0.671	0.501	0.448	0.513
	Aluminum	0.441	0.442	0.351	0.128	0.354	0.412	0.345
	Composite	0.040	0.126	0.039	0.073	0.036	0.049	0.057
	Al. w. rubber tread	0.078	0.039	0.077	0.128	0.109	0.091	0.085
Low Mass	Steel	0.0037	0.216	0.130	0.033	0.038	0.073	0.054
	Aluminum	0.197	0.478	0.237	0.227	0.418	0.491	0.372
	Composite	0.553	0.201	0.580	0.588	0.318	0.123	0.395
	Al. w. rubber tread	0.212	0.105	0.053	0.152	0.227	0.313	0.179
Low Maintenance	Steel	0.368	0.284	0.533	0.509	0.391	0.366	0.421
	Aluminum	0.368	0.444	0.317	0.175	0.391	0.393	0.352
	Composite	0.070	0.212	0.049	0.062	0.067	0.145	0.092
	Al. w. rubber tread	0.193	0.061	0.101	0.255	0.151	0.095	0.135
Low Volume	Steel	0.250	0.227	0.255	0.167	0.238	0.250	0.244
	Aluminum	0.250	0.423	0.098	0.262	0.313	0.250	0.261
	Composite	0.250	0.227	0.590	0.453	0.313	0.250	0.347
	Al. w. rubber tread	0.250	0.122	0.057	0.118	0.137	0.250	0.148
Cost	Steel	0.400	0.620	0.535	0.129	0.188	0.245	0.353
	Aluminum	0.456	0.216	0.321	0.295	0.654	0.245	0.392
	Composite	0.044	0.058	0.045	0.417	0.040	0.186	0.098
	Al. w. rubber tread	0.100	0.105	0.099	0.158	0.118	0.323	0.157
Material Availability	Steel	0.446	0.302	0.446	0.619	0.447	0.373	0.452
	Aluminum	0.446	0.347	0.446	0.205	0.316	0.424	0.371
	Composite	0.043	0.281	0.043	0.059	0.056	0.100	0.079
	Al. w. rubber tread	0.064	0.070	0.064	0.117	0.181	0.102	0.097
Fabrication Process	Steel	0.497	0.521	0.497	0.684	0.501	0.376	0.522
	Aluminum	0.368	0.284	0.368	0.193	0.354	0.427	0.333
	Composite	0.038	0.132	0.038	0.076	0.036	0.060	0.058
	Al. w. rubber tread	0.097	0.063	0.097	0.042	0.109	0.137	0.088
Compliance w/ Standards	Steel	0.400	0.308	0.400	0.175	0.420	0.417	0.358
	Aluminum	0.423	0.341	0.423	0.361	0.420	0.417	0.419
	Composite	0.071	0.274	0.071	0.326	0.044	0.083	0.115
	Al. w. rubber tread	0.106	0.078	0.106	0.137	0.116	0.083	0.108

Table 5
Summary of group weights.

		Steel	Aluminum	Composite	Aluminum w/rubber
Terrain Performance (0.379)	Regolith (0.247)	0.284	0.327	0.242	0.147
	Boulders and Rocks (0.060)	0.425	0.173	0.243	0.160
	Craters and Slopes (0.072)	0.224	0.281	0.275	0.220
Space Application (0.208)	Resistance to Radiation (0.062)	0.513	0.345	0.057	0.085
	Low Mass (0.054)	0.054	0.372	0.395	0.179
	Maintenance (0.066)	0.421	0.352	0.092	0.135
	Low Volume (0.025)	0.244	0.261	0.347	0.148
Cost (0.269)	Cost	0.353	0.392	0.098	0.157
Manufacturability (0.144)	Material Availability (0.051)	0.418	0.399	0.095	0.088
	Complexity of Fabrication (0.050)	0.314	0.539	0.060	0.087
	Compliance w/NASA standards (0.043)	0.403	0.382	0.109	0.106

The magnitudes of the terms reveal that $\boldsymbol{G}^{(1,0)}_{\cdot}$ is more sensitive to the weights in $\boldsymbol{A}^{6}_{\cdot}$ than in $\boldsymbol{A}^{2}_{\cdot}$ as the former has the largest component. The sensitivity of the group weights with respect to a change in one pairwise comparison supplied by one user is now calculated with Eq. (14) and the row geometric means are calculated for the matrix. For a given weighting vector, $\{A_r^{P_j}\}$ of a comparison table prepared by user P_j, for the rth criteria, where r runs from 1 to $n^{(i,j_{i-1})}$ the sensitivity of $\{A_r^{P_j}\}$ to a given pair comparison entry, a_{kl} is given by a vector

$$\frac{\partial \left\{A_r^{P_j}\right\}}{\partial a_{k\ell}} \tag{23}$$

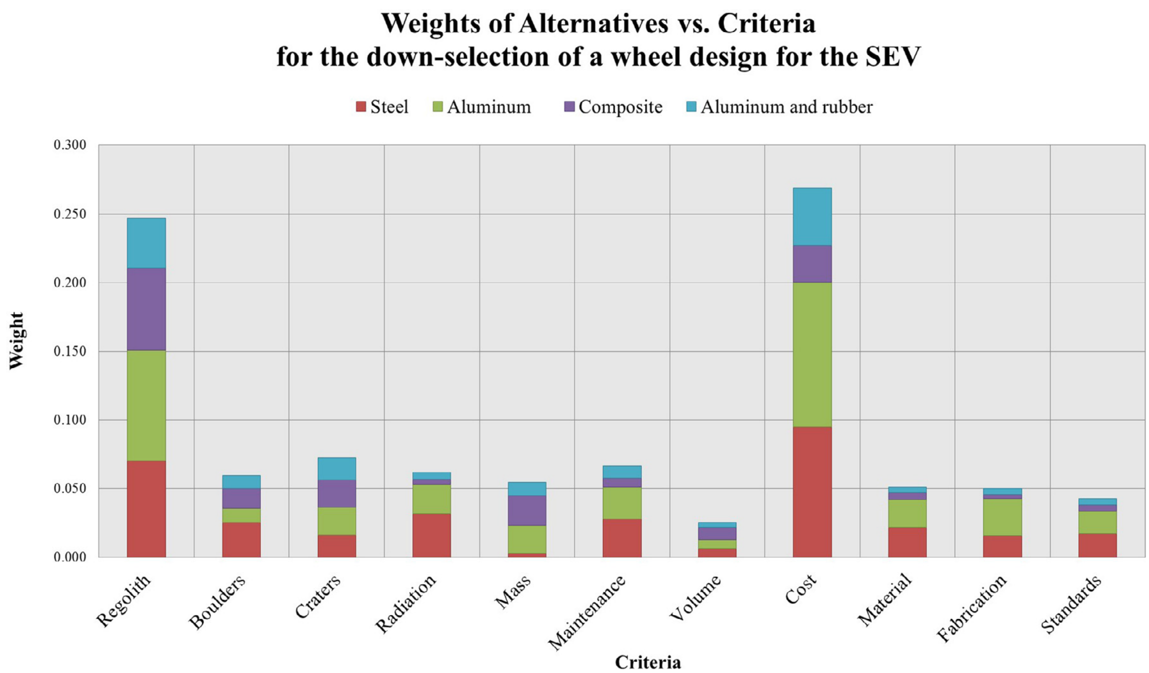

Fig. 4. Aggregated group results for the down-selection of a wheel design for the SEV.

Table 6
Aggregated group results.

Steel	Aluminum	Composite	Aluminum w/rubber
0.330	0.352	0.173	0.145

Table 7
Pairwise comparison matrix for User 6 for the top-level criteria.

	Terrain Performance	Space Application	Cost	Manufacturability
Terrain Performance	1	1	7	9
Space Application	1	1	7	9
Cost	1/7	1/7	1	5
Manufacturability	1/9	1/9	1/5	1

Note that the size of the pairwise comparison table is $n^{(i,j_{i-1})} \times n^{(i,j_{i-1})}$. Now, sum the absolute value of the vector components together to obtain a single scalar quantity representative of the magnitude of the sensitivity, called the sensitivity index, as

$$S_{k\ell}^{P_j} = \sum_{r=1}^{n^{(i,j_{i-1})}} \left| \frac{\partial A_r^{P_j}}{\partial a_{k\ell}} \right| \tag{24}$$

The $n^{(i,j_{i-1})} \times n^{(i,j_{i-1})}$ matrix, $[S_{k\ell}^{P_j}]$ is called sensitivity matrix. A surface plot of the sensitivity matrix is provided to the decision maker. Peaks on the plots provide a visualization of the pairwise comparisons that have the most critical impact on the aggregated group results.

Fig. 5 shows the sensitivity matrix plots for each user for the Level 1 criteria of the SEV case study. The x and y axes represent the rows and columns of the initial pairwise comparison matrix, and integers 1 to 4 along the axes represent the four criteria under consideration, namely Terrain Performance, Compatibility with Space Application, Cost and Manufacturing. The numerical values along the z-axis indicate the sensitivity index for each pairwise comparison, or the sum of the absolute values of the vector components obtained with Eq. (14) for $\frac{\partial G_r^{(i,j_{i-1})}}{\partial a_{k\ell}}$. It can be observed that User 6, with a maximum sensitivity index above 0.22, provided the pairwise comparison that has the greatest magnitude for sensitivity. This specific pairwise comparison influences the aggregated group results the most. Greater details on the application of the sensitivity analysis method can be found in Ivanco (2015).

Table 7 provides the pairwise comparison matrix provided by User 6 for the four Level 1 criteria. The sensitivity plots shows that the most critical pairwise comparison is on the fourth row, in the first column, which is for the comparison of Manufacturability and Terrain Performance and has an input value of 1/9.

Table 8 displays the original group weights obtained for the Level 1 criteria and what the group weights would be for these same criteria if User 6 was excluded from the results. It can be observed that the criteria weights would differ by a percent difference ranging between 10.82% and 24.55%. The sensitivity plots shown in Fig. 5 show that User 2 has the least influence on the group results. Indeed, the most critical pairwise comparison provided by User 2 has a sensitivity index of 0.07, the smallest magnitude for all maxima shown in the six plots of Fig. 5. The effect of User 2 on the group results is also investigated and shown in Table 8. It can be observed that the percent difference between the group weights for all users and the group weights adjusted with the exclusion of User 2 ranges between 1.34% and 2.89%. The decision maker can now gain insight into which user has the most influence on the group results from inspection of the sensitivity plots, as confirmed by the percent difference computed and displayed in Table 8.

Given the background of User 6, who has no experience or training in the engineering field, the impact of User 6 on the aggregated group results may be of concern. Such a result should trigger a close inspection of User 6 inputs for this specific matrix and possibly a group discussion, in an effort to determine if the group decision is in accordance with the opinion of User 6.

Similarly, the sensitivity analysis method can be applied to the inconsistency index, to understand how a pairwise comparison impacts the inconsistency index obtained by a user. For example, using Eq. (17), the sensitivity of the inconsistency index with respect

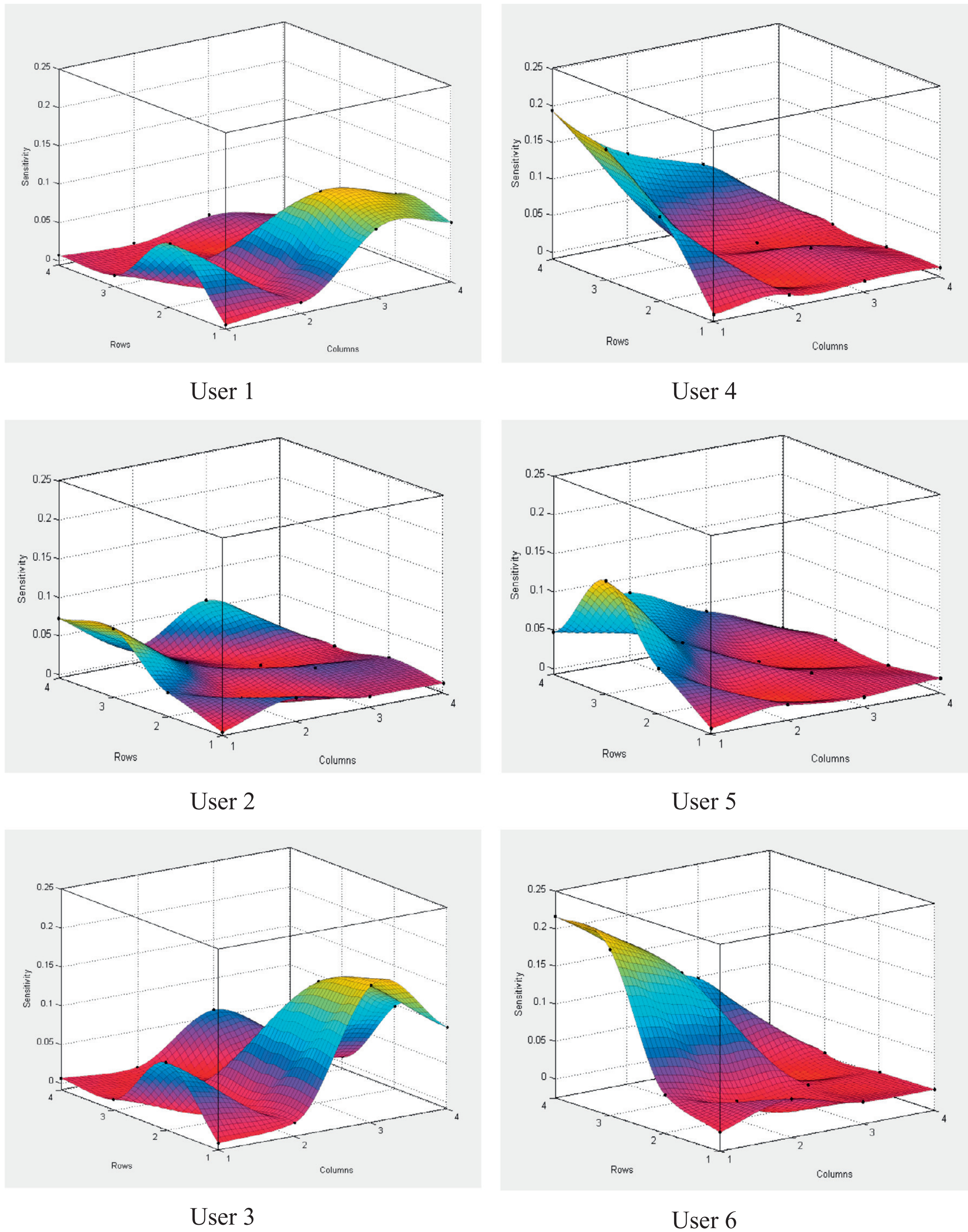

Fig. 5. Sensitivity plots by user for Level 1 criteria.

Table 8
Aggregated group weights for Level 1 criteria after removal of least and most sensitive users.

	Original group weight	Group weight, User 6 excluded	Percent difference	Group weight, User 2 excluded	Percent difference
Terrain Performance	0.379	0.342	10.82%	0.374	1.34%
Space Application	0.208	0.167	24.55%	0.204	1.96%
Cost	0.269	0.313	14.06%	0.277	2.89%
Manufacturability	0.144	0.177	18.64%	0.146	1.37%

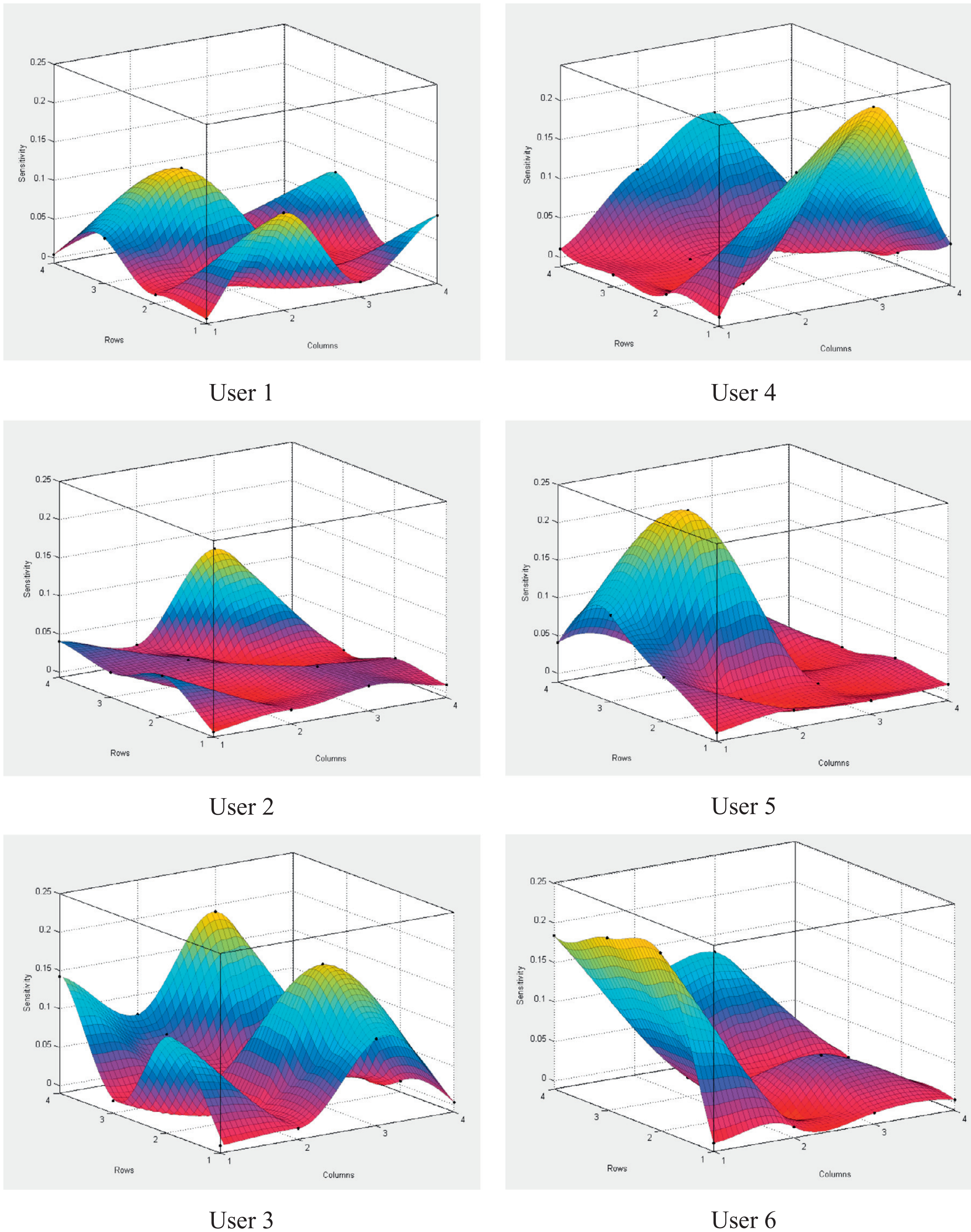

Fig. 6. Sensitivity plots by user for the Level 2 criteria under the Space Application criterion.

to the comparison of radiation resistance and low volume for the space application criterion for User 2 has a magnitude of 0.0241. Such sensitivities can be calculated for all entries in the pairwise comparison matrix of concern.

The sensitivity analysis method can be applied at any level of the hierarchy tree that models the decision problem. For example, one can now evaluate the impact of a user's pairwise comparisons to the four Level 2 criteria under the Level 1 criterion "Compatibility with Space Application". Using the same method as previously, the sensitivity indices are calculated and plotted as surface plots in Fig. 6. It can be noticed that User 4, who is an aerospace engineer who was not involved with the team, provided the pairwise comparison that has the most impact on the aggregated group results. The pairwise comparison provided in Row 1, Column 3 by User 4 indeed obtained the maximum sensitivity index in the group for the Space Application criterion. It can also be noticed that User 5 and User 6, who have no engineering experience, have more influence on the group results than User 1 and User 2 who were very familiar with the decision problem. These plots inform the deci-

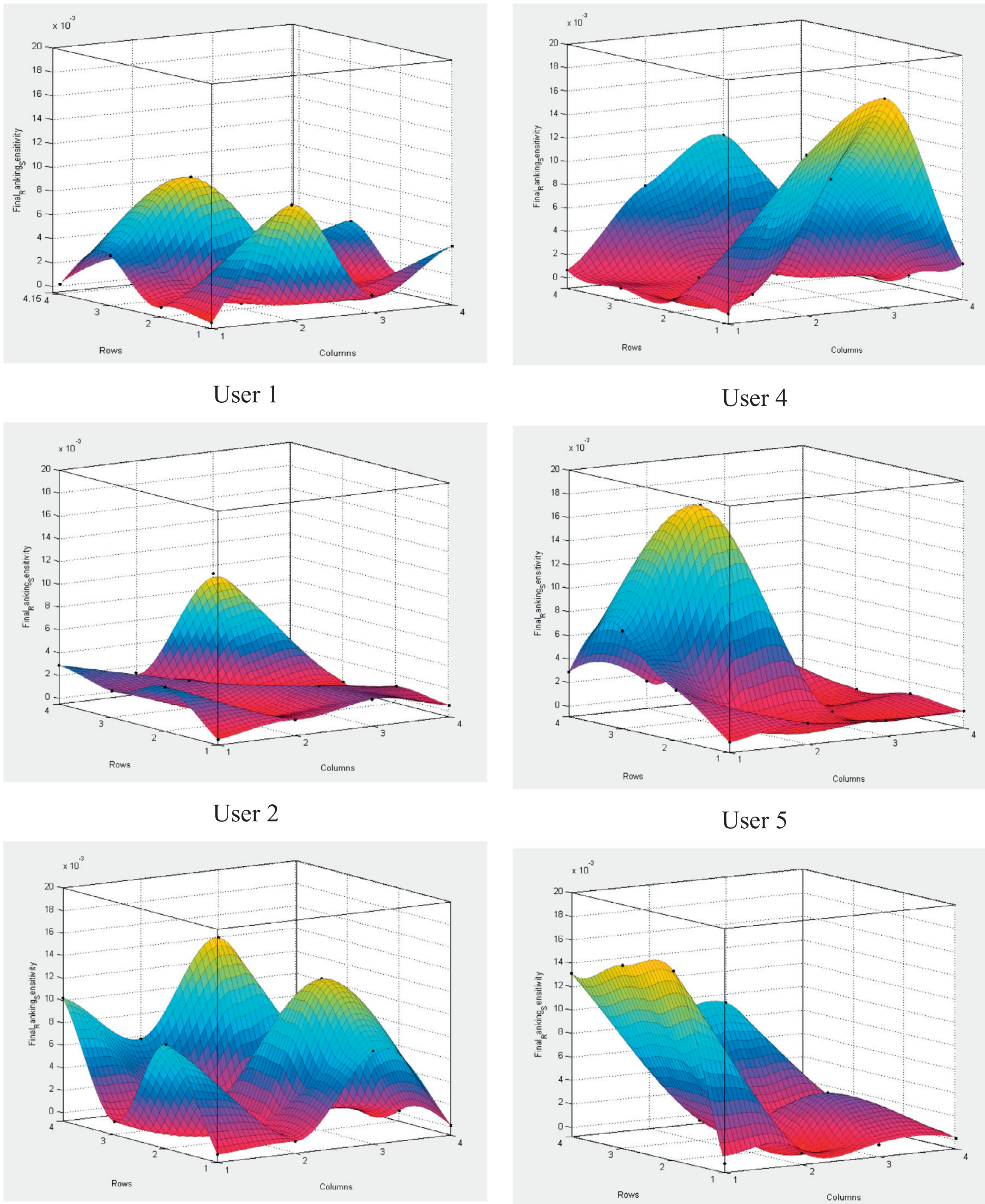

Fig. 7. Sensitivity plots by user for the overall ranking of the All-Aluminum wheel for the Space Application Criterion.

sion maker that User 4, and to a lesser extend User 5 and User 6, influence the group results the most.

The sensitivity of the overall total weight of a criterion can be calculated using Eq. (22) and the sensitivity of the final ranking of an alternative can be calculated using Eq. (23). For example, in the case of the Level 2 criteria under the Level 1 Space Application criterion, one obtains the sensitivity plots shown in Fig. 7 for the All-Aluminum wheel. It can be noticed that User 4 and User 5 drive the final ranking of the All-Aluminum design alternative when evaluated against the Space Application criterion. The decision maker can now focus on the inputs provided by these users, highlight critical pairwise comparisons as discussion points for the group and confirm that these inputs are in line with the intent of the users.

6. Conclusion

Since its development, AHP has been thoroughly studied, implemented and improved upon. Several shortfalls to AHP have been discovered and corrected over the years, with the development of improved AHP algorithms. A limitation however still remains: traditional AHP algorithms do not take the disparities of individual profiles into account. Traditional AHP grants the same importance to all individuals regardless of their experience and familiarity with the AHP method. Several qualitative and quantitative methods have been proposed to address the issue of disparities within a user group, but several shortcomings can be identified. In an effort to develop the current state of the art with regard to addressing these disparities, a new methodology was developed and presented in this paper. Rather than trying to quantify the disparities in profiles, this new approach uses an analytical sensitivity analysis to identify which AHP users have the most impact on the aggregated group results. This sensitivity analysis based method informs the decision maker of which pairwise comparisons are most critical to the final rankings so as to enable the decision maker by focusing the group effort on the most significant data points.

This new approach does not relieve decision makers from the concern of identifying relevant SMEs for a given decision making problem. Neither does this approach validates the obtained rankings. The scope of this sensitivity analysis based approach is to allow decision makers to identify critical inputs that drive the aggregated group results. The sensitivity based method can be applied after SMEs have been vetted and calibrated with other methods, such as the method developed by Cooke and Goossens (2008).

In order to assist decision makers with the implementation of AHP, an AHP tool was developed in MS Excel. Two visualization capabilities were also developed. A bar chart displays the relative importance of the criteria, and provides information on the alternatives scores for each criterion. A surface plot of the sensitivity indices allows the decision maker to gain insight in which individuals, and specifically which pairwise comparison input, impacts the group results the most. The example of the down-selection of a wheel design for the Space Exploration Vehicle was presented to illustrate how the sensitivity analysis visualization plots can be used by the decision maker to determine which individuals drive the group results. After the most influential user was identified, original group weights and group weights after the exclusion of the most influential user were compared. The sensitivity analysis method presented here allows to focus time and resources for post-AHP evaluation on the elements of the process that are most critical. This new development to the state of the art allows decision makers to gain more insight into the participation of each user to the aggregated group results and provides a methodology to address the limitation of AHP in terms of disparities in user profiles. However, this research used an academic test case with large profile disparities to demonstrate how the method could be used. Future work should investigate the applicability of the method to real world problems in which decision makers, SMEs and stakeholders may display a narrower or wider range of disparities. In addition, future work may also consider coupling this sensitivity analysis method with models that investigate the influence of users in the system, such as Social Network Analysis (Garcia Melón, Estruch Guitart, Aragones Beltrán, & Monterde Roca, 2013).

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Appendix

The sensitivity equation, Eq. (12) is derived in this appendix. Due to the reciprocity axiom, an evaluator is required only to prepare the input of elements in the upper triangle of the pairwise comparison matrix C^{P_j}. Since the goal of sensitivity analysis is to investigate the effect of variation in the user input on the weighting coefficients, only $a_{k\ell}$ with $\ell > k$ is considered as independent variable in this derivation. This is done by directly differentiating the weighting coefficient $A_r^{P_j}$ of criteria r in Eq. (5) with respect to a specified $a_{k\ell}$ in the upper triangle of the pairwise comparison matrix, C^{P_j}, provided by evaluator, P_j. It results in the following equation

$$\frac{\partial A_r^{P_j}}{\partial a_{k\ell}} = \frac{\frac{\partial \overline{A}_r^{P_j}}{\partial a_{k\ell}} \times \left(\sum_{q=1}^m \overline{A_q^{P_j}}\right) - \overline{A}_r^{P_j} \times \frac{\partial\left(\sum_{q=1}^m \overline{A_q^{P_j}}\right)}{\partial a_{k\ell}}}{\left(\sum_{q=1}^m \overline{A_q^{P_j}}\right)^2} \tag{A.1}$$

Note that $\overline{A}_r^{P_j} = e^{\ln \overline{A}_r^{P_j}}$ and $\ln \overline{A}_r^{P_j} = \ln\left[\left(\prod_{s=1}^m a_{rs}\right)^{(1/m)}\right] = \frac{\sum_{s=1}^m \ln(a_{rs})}{m}$. It leads to

$$\frac{\partial \overline{A}_r^{P_j}}{\partial a_{k\ell}} = \frac{\partial}{\partial a_{k\ell}}\left(e^{\ln \overline{A}_r^{P_j}}\right) = e^{\ln \overline{A}_r^{P_j}} \times \frac{\partial\left(\ln \overline{A}_r^{P_j}\right)}{\partial a_{k\ell}} = \frac{\overline{A}_r^{P_j}}{m} \times \left(\sum_{s=1}^m \frac{\partial(\ln a_{rs})}{\partial a_{k\ell}}\right)$$

Three cases are considered separately here for the relations between the pair of indices, (r, s) and (k, ℓ) in the last term in the above equation; $r \neq k, \ell$, $r = k$ and $r = \ell$.

Case 1: $r \neq k, \ell$

In this case, $\frac{\partial \overline{A}_r^{P_j}}{\partial a_{k\ell}} = 0$, as $\sum_{s=1}^m \frac{\partial(\ln a_{rs})}{\partial a_{k\ell}} = 0$. Thus, the variation in $a_{k\ell}$ will affect only $A_k^{P_j}$ and $A_\ell^{P_j}$.

Case 2: $r = k$

One has $\sum_{s=1}^m \frac{\partial(\ln a_{ks})}{\partial a_{k\ell}} = \frac{\partial(\ln a_{k\ell})}{\partial a_{k\ell}} = \frac{1}{a_{k\ell}} = a_{\ell k}$, which results in $\frac{\partial \overline{A}_k^{P_j}}{\partial a_{k\ell}} = \frac{a_{\ell k} \times \overline{A}_k^{P_j}}{m}$

Case 3: $r = \ell$

Since $\sum_{s=1}^m \frac{\partial(\ln a_{ks})}{\partial a_{k\ell}} = \frac{\partial(\ln a_{\ell k})}{\partial a_{k\ell}} = -\frac{\partial(\ln a_{k\ell})}{\partial a_{k\ell}} = -\frac{1}{a_{k\ell}} = -a_{\ell k}$, one then has $\frac{\partial \overline{A}_\ell^{P_j}}{\partial a_{k\ell}} = \frac{-a_{\ell k} \times \overline{A}_\ell^{P_j}}{m}$.

The above three cases lead to the following equation

$$\frac{\partial\left(\sum_{q=1}^m \overline{A_q^{P_j}}\right)}{\partial a_{k\ell}} = \frac{\partial \overline{A}_k^{P_j}}{\partial a_{k\ell}} + \frac{\partial \overline{A}_\ell^{P_j}}{\partial a_{k\ell}} = \frac{a_{\ell k}}{m} \times \left(\overline{A}_k^{P_j} - \overline{A}_\ell^{P_j}\right) \tag{A.2}$$

Finally, one can conclude by combining Eqs. (A.1) and (A.2) to obtain Eq. (12) that

$$\frac{\partial A_k^{P_j}}{\partial a_{k\ell}} = \frac{a_{\ell k}}{m} \times \frac{\overline{A}_k^{P_j}}{\left(\sum_{q=1}^m \overline{A_q^{P_j}}\right)} \times \left[1 - \frac{\left(\overline{A}_k^{P_j} - \overline{A}_\ell^{P_j}\right)}{\left(\sum_{q=1}^m \overline{A_q^{P_j}}\right)}\right]$$
$$= \frac{a_{\ell k}}{m} \times A_k^{P_j} \times \left[1 - \left(A_k^{P_j} - A_\ell^{P_j}\right)\right]$$

Similarly, one has for $r = \ell$

$$\frac{\partial A_\ell^{P_j}}{\partial a_{k\ell}} = -\frac{a_{\ell k}}{m} \times A_\ell^{P_j} \times \left[1 + \left(A_k^{P_j} - A_\ell^{P_j}\right)\right]$$

and for $r \neq k, \ell$

$$\frac{\partial A_r^{P_j}}{\partial a_{k\ell}} = -\frac{a_{\ell k}}{m} \times A_k^{P_j} \times \left(A_k^{P_j} - A_\ell^{P_j}\right) \tag{A.3}$$

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