A Characterization of the Solution of a Fredholm Integral Equation with L∞ Forcing Term

Hideaki Kaneko
Old Dominion University, hkaneko@odu.edu

Richard Noren
Old Dominion University, rnoren@odu.edu

Yuesheng Xu

Original Publication Citation

Kaneko, H., Noren, R., & Xu, Y. (1990). A characterization of the solution of a Fredholm integral equation with L∞ forcing term. *Journal of Integral Equations and Applications, 2*(4), 581-594. doi:10.1216/jiea/1181075587

JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 2, Number 4, Fall 1990

A CHARACTERIZATION OF THE SOLUTION OF A FREDHOLM INTEGRAL EQUATION WITH L^∞ FORCING TERM

HIDEAKI KANEKO, RICHARD NOREN AND YUESHENG XU

Dedicated to John A. Nohel on the occasion of his sixty-fifth birthday

ABSTRACT. In this paper we investigate the regularity properties of the Fredholm equation $\phi(s) - \int_a^b g_\alpha(|s-t|)k(s,t)\phi(t)dt = f(s)$, $a \le s \le b$. The kernel is the product of the smooth function k and the singular function g_α defined as $g_\alpha(|s-t|) = |s-t|^{\alpha-1}$, for $0 < \alpha < 1$, and $g_\alpha(|s-t|) = \log|s-t|$, for $\alpha = 1$. The forcing function f is in L^∞. We obtain a decomposition of the solution as the sum of two functions—one with a discontinuity reflecting that of the forcing function—and the other a regular function. Our results extend those of C. Schneider [**6**], who assumes a condition that is stronger than $f \in C[a,b] \cap C^m(a,b)$ (for some integer m).

1. Introduction. In this paper, we study the solution $\phi = \phi(s)$ of the Fredholm integral equation

$$\phi(s) - \int_a^b g_\alpha(|s-t|)k(s,t)\phi(t)\,dt = f(s), \quad a \le s \le b, \tag{1.1}$$

where g_α satisfies

$$g_\alpha(|s-t|) = \begin{cases} |s-t|^{\alpha-1}, & \text{if } 0 < \alpha < 1, \\ \log|s-t|, & \text{if } \alpha = 1, \end{cases} \tag{1.2}$$

and k and f satisfy

$$k \in C^{m+1}([a,b] \times [a,b]), \quad f \in L^\infty[a,b]. \tag{1.3}$$

In order to describe regularity results for (1.1) we need to define a class of functions and an auxiliary function. For $0 < \alpha \le 1$ and