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Kaneko, Hideaki; Noren, Richard D.; and Xu, Yuesheng, "Numerical Solutions for Weakly Singular Hammerstein Equations and Their Superconvergence" (1992). *Mathematics & Statistics Faculty Publications*. 29. https://digitalcommons.odu.edu/mathstat_fac_pubs/29

Original Publication Citation

Kaneko, H., Noren, R. D., & Xu, Y. (1992). Numerical solutions for weakly singular Hammerstein equations and their superconvergence. *Journal of Integral Equations and Applications*, 4(3), 391-407. doi:10.1216/jiea/1181075699

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JOURNAL OF INTEGRAL EQUATIONS AND APPLICATIONS Volume 4, Number 3, Summer 1992

NUMERICAL SOLUTIONS FOR WEAKLY SINGULAR HAMMERSTEIN EQUATIONS AND THEIR SUPERCONVERGENCE

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ABSTRACT. In the recent paper [7], it was shown that the solutions of weakly singular Hammerstein equations satisfy certain regularity properties. Using this result, the optimal convergence rate of a standard piecewise polynomial collocation method and that of the recently proposed collocation-type method of Kumar and Sloan [10] are obtained. Superconvergence of both of these methods are also presented. In the final section, we discuss briefly a standard product-integration method for weakly singular Hammerstein equations and indicate its superconvergence property.

1. Introduction. We consider the Hammerstein equation with weakly singular kernel

(1.1)
$$\varphi(s) - \int_a^b g_\alpha(|s-t|)k(s,t)\psi(t,\varphi(t))\,dt = f(s), \quad a \le s \le b,$$

where

$$g_{\alpha}(s) = \begin{cases} s^{\alpha - 1} & \text{for } 0 < \alpha < 1\\ \log s & \text{for } \alpha = 1. \end{cases}$$

Throughout this paper, we assume that

(i) $k \in C([a, b] \times [a, b])$

(ii) $\psi \in C([a, b] \times (-\infty, \infty))$ and satisfies the Lipschitz condition $|\psi(t, y_1) - \psi(t, y_2)| \leq A|y_1 - y_2|.$

In the recent paper [7], it was shown that under assumptions (i), (ii) and

(iii) AG < 1, where $G \equiv \sup_{a \le s \le b} \int_a^b |g_\alpha(|s-t|)k(s,t)| dt$,

there is a unique solution to equation (1.1).

Generalizing the argument of C. Schneider [14], regularity properties of the solution φ were also obtained in [7]. For our present purposes, these results can be summarized as follows:

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