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Induced Mach wave–flame interactions in laminar supersonic fuel jets

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A model problem is proposed to investigate the steady response of a reacting, compressible laminar jet to Mach waves generated by wavy walls in a channel of finite width. The model consists of a two-dimensional jet of fuel emerging into a stream of oxidizer which are allowed to mix and react in the presence of the Mach waves. The governing equations are taken to be the steady parabolized Navier–Stokes equations which are solved numerically. The kinetics is assumed to be a one-step, irreversible reaction of the Arrhenius type. Two important questions on the Mach wave–flame interactions are discussed: (i) how is the flame structure altered by the presence of the Mach waves, and (ii) can the presence of the Mach waves change the efficiency of the combustion processes?

I. INTRODUCTION

For the past several years, there has been a great deal of interest in the study of supersonic free shear layers and their instabilities. This renewed interest is primarily motivated by the recent technological needs of achieving better mixing rates for supersonic flows. In the scramjet combustors currently under consideration for the propulsion system of hypersonic vehicles, for example, the proposed National Aerospace Plan (NASP), the fuel and air (the oxidizer) are introduced into the combustion chamber at very high speeds. Mixing of the two gases takes place in the shear layer forming the boundary of the two streams, and combustion occurs when there is both sufficient fuel and oxidizer present at the same point. Since the velocities of the flow are high, the residence time of the fuel and air in the combustion chamber is very short; therefore, it is extremely important that a high mixing rate of the fuel and oxidizer be achieved so that complete combustion is attained before the fuel is convected out of the engine. Compounding the problem of very short residence times in high-speed flows is that the mixing rates of supersonic shear layers have recently been shown to be considerably lower than the mixing rates of subsonic shear layers due to the effects of increased compressibility of the flow. Although the actual combustion process inside the combustor is complicated, a two-stream supersonic shear layer (jet, wake, or mixing layer) in a bounded or unbounded domain serves as an excellent physical model for many investigations. Experiments based on the two-stream shear layer model (see Clemens[1] and the references cited therein) reveal that the spreading rate, and thus the mixing rate, of high-speed free shear layers decreases rapidly as the Mach number of the flow increases; the mixing rate can be reduced by as much as 75% when the convective Mach number of the shear flow increases from subsonic to supersonic. Current research, both experimental and analytical, is now directed toward forced and unforced mixing enhancement techniques.

One obvious mixing enhancement technique is to force the shear layer at some prescribed frequency, usually computed from linear stability analysis. The reader is referred to Mankbadi[2] and the references cited therein for the case of excited jets and to Claus *et al.*[3] and the references cited therein for the case of a mixing layer. Alternative mixing enhancement schemes have also been proposed. These include the effect of unbalancing the inflow conditions on both sides of the bounded mixing layer (Farouk *et al.*,[4] and the references cited therein), modifying the trailing edge of the splitting plate to enhance the natural flow instabilities (Papamoschou[5]), and the use of an oscillating shock to interact with the supersonic shear flow and therefore enhance the downstream vorticity (Kumar *et al.*[6]).

More recently, Tam and Hu[7] and Hu and Tam[8] suggest the possibility of using a periodic Mach wave system generated by wavy walls to enhance the flow instabilities. In their studies, the rectangular channel for the fuel and air in the combustion chamber is modified to be slightly wavy at the top and bottom. One advantage of this scheme over some of the other alternative mixing augmentation schemes is that no intrusive object need be placed inside the flow. Alternatively, the wavy walls generate small-amplitude spatially periodic Mach waves that penetrate the mixing layer for supersonic flows without the need for obstacles within the flow field. Such a periodic Mach wave system can induce flow instabilities in two ways. First, two acoustic modes of the ducted shear flow are driven into resonances and instability.[7] In this way, the interaction of one acoustic wave mode with the Mach wave system produces a forcing on the other and vice versa. Second, parametric instabilities akin to the secondary instabilities of low-speed boundary layers and shear layers studied by Herbert,[9,10] Orszag and Patera,[11] and others, are computationally shown to be induced by the periodic Mach wave system.[8] The growth rates of these new instability waves vary nearly linearly with the amplitude of the Mach waves. Clearly, the supersonic shear layer is destabilized by the periodic Mach waves. As such, the wavy wall concept pro-

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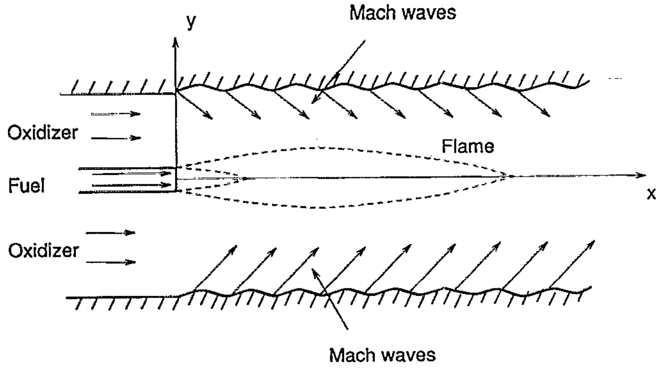

FIG. 1. Schematic showing the fuel jet interacting with Mach wave system.

vides a potentially promising scheme for mixing enhancement.

This paper explores a model problem to investigate the steady response of a *reacting*, compressible laminar jet to Mach waves generated by wavy walls in a channel of finite width. The model consists of a two-dimensional jet of fuel emerging into a stream of oxidizer. The governing equations are taken to be the steady parabolized Navier–Stokes equations which are solved numerically. The pressure gradients in the downstream and cross-stream directions are kept in the equations thus allowing for wave propagations. Parabolic equations are commonly used in the study of turbulent supersonic flames.[12,13] In the present model, instabilities of the jet are excluded; however, the steady-state solutions can serve as the basic state for future stability investigations. In this work, the value of the constant Reynolds number is chosen to compensate for turbulent diffusion effects, even though a turbulence model is not being used. The chemical kinetics is taken to be a one-step, irreversible reaction of the Arrhenius type. Two important questions on the Mach wave–flame interactions are addressed below: (i) how is the flame structure altered by the presence of the Mach waves, and (ii) can the presence of the Mach waves change the efficiency of the combustion processes? Section II contains the formulation of the model problem, results are presented in Sec. III, and the conclusions are given in Sec. IV.

II. FORMULATION

Consider a reacting flow inside the channel of a combustion chamber (Fig. 1) such that x is in the direction of the flow and y is normal to the flow. A jet of fuel with width $2\hat{d}$ is injected into a uniform ambient flow of oxidizer inside the chamber at $x=0$. Both the oxidizer and the fuel are assumed to be at supersonic velocities. The channel walls are made slightly wavy to generate Mach waves that propagate through and interact with the flames formed by the diffusion of fuel into oxidizer and oxidizer into the fuel. The wavy wall surfaces at the bottom and the top of the channel are given by the equations:

top wall: $y=H_1-A_1\cos(K_1x)$,

bottom wall: $y=-H_2-A_2\cos(K_2x)$.

The strength of the generated Mach waves depends on the product A_2K_2 for the bottom wall and A_1K_1 for the top wall (Liepmann and Roshko[14]). For convenience of discussion, define

$$\epsilon=\frac{A_2K_2}{2\pi}=\frac{\text{wavy wall amplitude}}{\text{wavelength}},\quad \alpha=\frac{A_1}{A_2},\quad l=\frac{K_1}{K_2},$$

where ϵ is typically small so that nonlinearity at the walls is negligible and the parameters α and l are taken to be of order unity. In this study we assume ϵ is no more than 0.02.

The nondimensional parabolized equations governing the steady two-dimensional flow of a compressible, reacting jet of an ideal gas in which the material properties of the fuel are assumed to be the same as the oxidizer are (Williams[15] and Anderson *et al.*[16])

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0, \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{1}{\gamma M^2}\frac{\partial p}{\partial x}+\frac{1}{\text{Re}}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{1}{\gamma M^2}\frac{\partial p}{\partial y}+\frac{1}{\text{Re}}\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right), \tag{3}$$

$$\rho u\frac{\partial T}{\partial x}+\rho v\frac{\partial T}{\partial y}-\frac{(\gamma-1)}{\gamma}\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}\right)$$
$$=\frac{1}{P_r\text{Re}}\frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right)+\frac{1}{\text{Re}}(\gamma-1)M^2\left[\mu\left(\frac{\partial u}{\partial y}\right)^2\right.$$
$$\left.+\frac{4}{3}\mu\left(\frac{\partial v}{\partial y}\right)^2\right]+\beta\rho Y_OY_FDe^{-\text{Ze}/T} \tag{4}$$

$$\rho u\frac{\partial Y_O}{\partial x}+\rho v\frac{\partial Y_O}{\partial y}=\frac{1}{\text{Sc}_O\text{Re}}\frac{\partial}{\partial y}\left(\mu\frac{\partial Y_O}{\partial y}\right)$$
$$-\nu_O\rho Y_OY_FDe^{-\text{Ze}/T}, \tag{5}$$

$$\rho u\frac{\partial Y_F}{\partial x}+\rho v\frac{\partial Y_F}{\partial y}=\frac{1}{\text{Sc}_F\text{Re}}\frac{\partial}{\partial y}\left(\mu\frac{\partial Y_F}{\partial y}\right)$$
$$-\nu_F\rho Y_OY_FDe^{-\text{Ze}/T}, \tag{6}$$

$$p=\rho T. \tag{7}$$

In these equations, u and v are the velocity components in the x and y directions, respectively; T is the temperature; ρ is the density; p is the pressure; and Y_F and Y_O are the mass fractions of the fuel and the oxidizer, respectively. The velocities, pressure, temperature, and density are nondimensionalized by their values in the oxidizer stream, and the spatial coordinates are normalized by the height of the channel. The reaction is assumed to be irreversible and of the Arrhenius type. The viscosity μ is taken to be a function of temperature only, assumed to obey a linear law. The nondimensional parameters appearing above are the Mach number M of the oxidizer stream, the specific heats ratio γ, the Damkohler number D, the Zeldovich number Ze, the

Schmidt numbers Sc_O and Sc_F, the heat release parameter β per unit mass fraction of the fuel, and finally ν_O and ν_F are the parameters involving the stoichiometry for the two species. The Reynolds number, Re, based on channel height and the velocity of the oxidizer stream is taken to be a constant whose value is chosen to compensate for turbulent diffusion effects. Numerical simulations of a turbulent planar mixing layer inside a channel indicate that a constant Reynolds number ranging from 500 to 5000 is adequate when compared with the experimental data.[17] In our calculations reported below Reynolds number is taken to be 2000.

The initial conditions at the jet inlet are

$$u(0,y)=\begin{cases}1, & d<y\leqslant h_1,\\ u_j, & -d\leqslant y\leqslant d,\\ 1, & -h_2\leqslant y<-d,\end{cases}$$

$$T(0,y)=\begin{cases}1, & d<y\leqslant h_1,\\ T_j, & -d\leqslant y\leqslant d,\\ 1, & -h_2\leqslant y<-d,\end{cases}$$

$$Y_F(0,y)=\begin{cases}0, & d<y\leqslant h_1,\\ 1, & -d\leqslant y\leqslant d,\\ 0, & -h_2\leqslant y<-d,\end{cases}$$

$$Y_O(0,y)=\frac{1}{\phi}\,[1-Y_F(0,y)], \tag{8}$$

$$p(0,y)=1,$$

$$v(0,y)=0,$$

where $d=\hat{d}/(H_1+H_2)$, $h_{1,2}=H_{1,2}/(H_1+H_2)$, and ϕ is the equivalence ratio. If $\phi=1$, the mixture is stoichiometric, if $\phi>1$ it is fuel rich, and if $\phi<1$, it is fuel lean.

Since our interest is in the mechanism of Mach wave–flame interaction, the boundary layers at the upper and lower walls of the channel are neglected; thus the no-slip conditions are replaced by slip boundary conditions. The appropriate boundary conditions imposed on the mean position of the channel walls for the normal velocity are

$$v=2\pi\alpha l\epsilon\sin(k_1x), \qquad \text{at} \quad y=h_1, \tag{9a}$$

$$v=2\pi\epsilon\sin(k_2x), \qquad \text{at} \quad y=-h_2, \tag{9b}$$

where $k_{1,2}=K_{1,2}(H_1+H_2)$. Boundary values of p are derived from the characteristics at the walls with corresponding Riemann invariants and the boundary conditions for T, u, and ρ are found from local isentropic relations.[14]

Since the considered flow is everywhere supersonic, no information propagates upstream. Hence a downstream marching method of solution is appropriate. We use a Crank–Nicolson-type central difference discretization for Eqs. (1)–(6). The numerical scheme for the nonlinear equations (1)–(7) employs an iterative process. In (2)–(6), the pressure derivative terms, energy dissipation terms, and the reaction source terms are evaluated as the average of the values at the current and the previous mesh points and updated after each iteration. As a result, Eqs. (2)–(6) are decoupled and each yields a tridiagonal system for u, v, T, Y_O, and Y_F, respectively. Equation (1) is then used to find ρ and the gas law (7) is used to find the pressure. The iteration at each marching step is considered complete when the relative error between two iterations are less than 10^{-6}.

III. RESULTS AND DISCUSSIONS

In this section numerical solutions of Eqs. (1)–(7) are presented. In all calculations, $h_1=h_2=\frac{1}{2}$; $d=0.05$; the Mach number is taken to be 2; and the Prandtl number, the Schmidt numbers, and the equivalence ratio are taken to be unity. In addition, the coefficients ν_F and ν_O are also assumed to be unity, and the heat release parameter β is 2. The computational mesh is taken to be 2000 grid points in the y direction, with the grid spacing in the x direction being such that grid points lie on the characteristics for the oxidizer stream, i.e., $\Delta x = \sqrt{M^2-1}\Delta y$; a test of the numerical resolution was carried out with twice the number of grid points confirming the accuracy of the reported results. All calculations were done in 64-bit precision. In all of the results presented, the Mach wave system is initiated by the wall deformations beginning at $x=0$ which is in line with the fuel jet inlet. As a result, the initial mixing of the fuel and oxidizer occurs under undisturbed conditions with the first point of interaction determined by the Mach angle. In practice, the inlet jet can be placed closer to the Mach wave field by either moving the inlet of the fuel jet downstream or, equivalently, initiating the waviness of the walls upstream of the fuel jet inlet. Since the aim of this investigation is to study the effect of the Mach waves on combustion for a model problem, no attempt is made in regards to optimizing the geometry.

The two questions raised in the Introduction concerning the Mach wave–flame interactions are (i) how is the flame structure altered by the presence of the Mach waves, and (ii) can the presence of the Mach waves change the efficiency of the combustion processes? These questions are addressed below.

Figure 2 shows the contours of the reaction rate $\Omega=\rho Y_F Y_O De^{-Ze/T}$ with and without Mach waves. In these calculations $u_j=2$ and $T_j=1$, the Zeldovich number is 20 and the Damkohler number is 4×10^7. As seen in Fig. 2(a), ignition of the fuel jet in the absence of the Mach waves occurs at the two mixing layers between the fuel jet and the oxidizer stream at a downstream position of about $x=2.5$. The reaction rate is the highest at the ignition points and the ignition points are followed by two active reaction zones along the jet boundaries. In the calculations of Figs. 2(b) and 2(c), a Mach wave system is generated from the bottom wavy wall for $k_2=4\pi$ with $\epsilon=0.01$ and 0.02, respectively. From the reaction rate contours, we observe that the ignition points are moved toward the jet inlet as the Mach wave strength is increased. This decrease in the ignition distance owing to the passage of the Mach wave system is a result of two separate thermal effects. First, the diffusion of fuel and oxidizer is enhanced by the temperature increases associated with the Mach waves

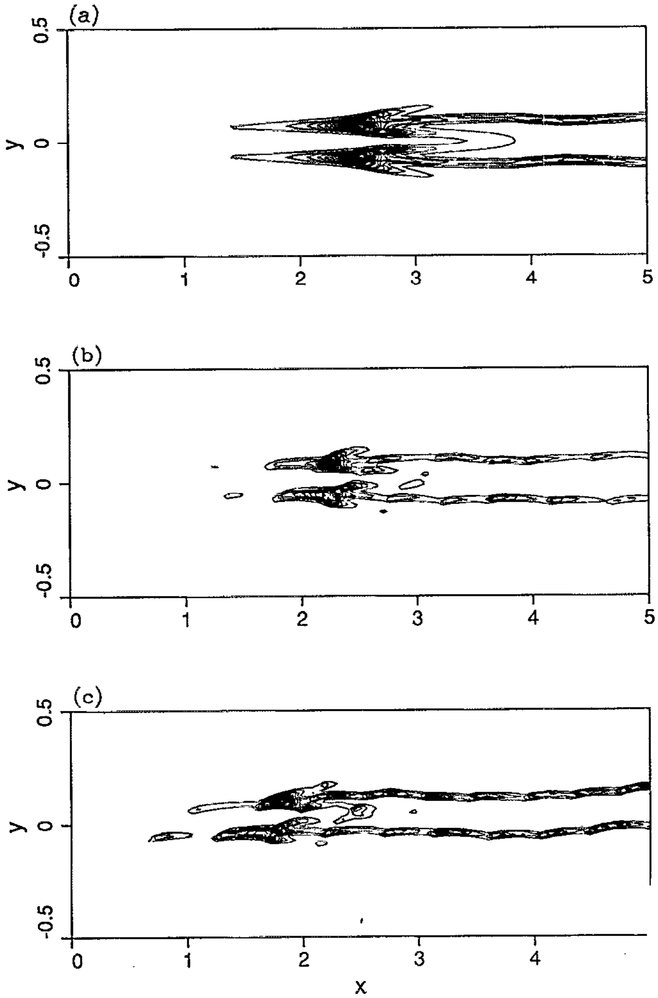

FIG. 2. Contours of the reaction rate $\Omega = \rho Y_F Y_O D e^{-Ze/T}$ with $M=2$, $u_j=2$, $T_j=1$, Ze=20, $D=4\times 10^7$, and $\beta=2$. (a) $\epsilon=0$, contour levels from 0.038 to 1.55, increment 0.038; (b) $\epsilon=0.01$, $k_2=4\pi$, contour levels from 0.11 to 2.25, increment 0.11; (c) $\epsilon=0.02$, $k_2=4\pi$, contour levels from 0.17 to 3.55, increment 0.17.

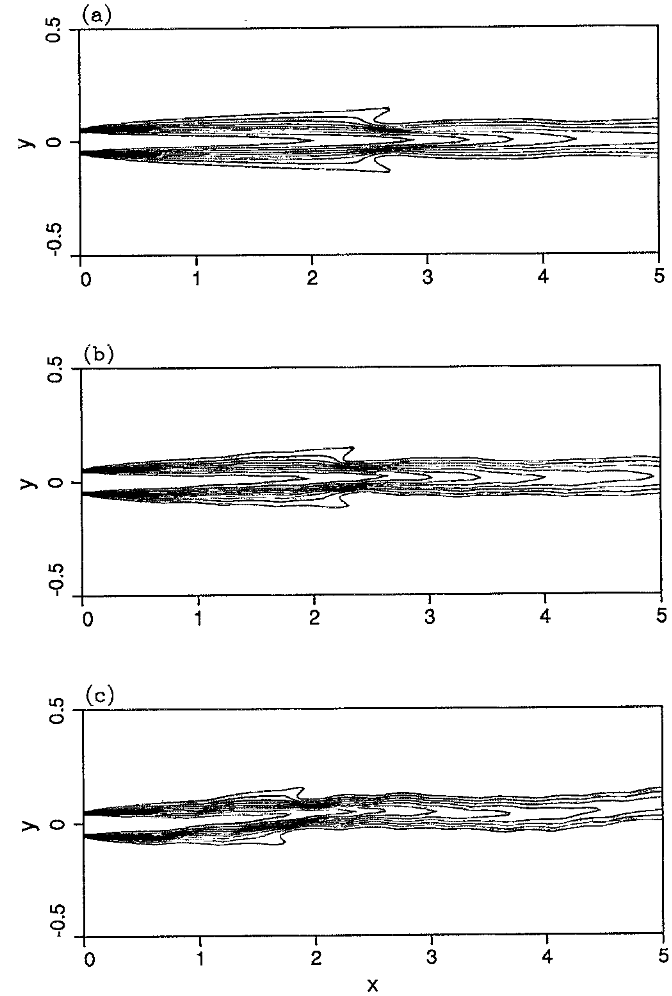

FIG. 3. Contours of fuel distribution Y_F with $M=2$, $u_j=2$, $T_j=1$, Ze $=20$, $D=4\times 10^7$, and $\beta=2$. Contour levels from 0.1 to 0.9, increment 0.1. (a) $\epsilon=0$; (b) $\epsilon=0.01$, $k_2=4\pi$; (c) $\epsilon=0.02$, $k_2=4\pi$.

through the temperature dependence of the diffusion coefficients; and second, the reaction rate which is very temperature sensitive is also greatly enhanced by the temperature increase. Also noticeable from these contours is a shift of the reaction upwards in the vertical direction and a streamwise skewness of the ignition points which follows the Mach angle of the flow. The corresponding fuel distribution contours are given in Fig. 3 showing that as the strength of the Mach wave system increases, the fuel jet is displaced and the structure becomes wrinkled.

Figure 4(a) contains contours of the pressure for the case of $\epsilon=0.02$. As the supersonic flow passes over the wavy wall, compression/expansion Mach waves are generated where the slope of the wall is positive/negative. This causes a moderate amplitude of 0.4 for the pressure fluctuations near the wavy wall. Away from the wall, the amplitude of the disturbances does not increase. However, owing to the nonlinearity of the fluctuations, the compression waves form weak shocks inside the channel toward the upper wall, much like an N wave.[18] Figure 4(b) is a profile of the pressure distribution versus the downstream position x at a fixed vertical position $y=-0.05$, together with a profile of the reaction rate Ω. The oblique shock jump conditions[14] are found to be met at the sharp rises of the pressure profile. The increase of the pressure level in the downstream direction results from the accumulated effects of the Mach wave system generated by the bottom wall waviness and the reflected waves off the top channel wall. It is seen that the reaction rate increases whenever a compression is present. This particular calculation shows ignition occurring after the fuel jet is heated by three consecutive compression waves.

Contour plots of the reaction rate distribution are shown in Fig. 5(a) with $\epsilon=0.0$ and in Fig. 5(b) with $\epsilon=0.02$ for a cold jet. In this case, the fuel temperature at the jet inlet T_j is reduced to 0.5 but the initial velocity of the jet is the same as the previous calculations. The Damkohler number is taken to be 8×10^7. Since the jet temperature is lower, the Mach number of the jet flow is higher thus decreasing the Mach wave angle and thereby increasing the streamwise skewness. As in the initially uni-

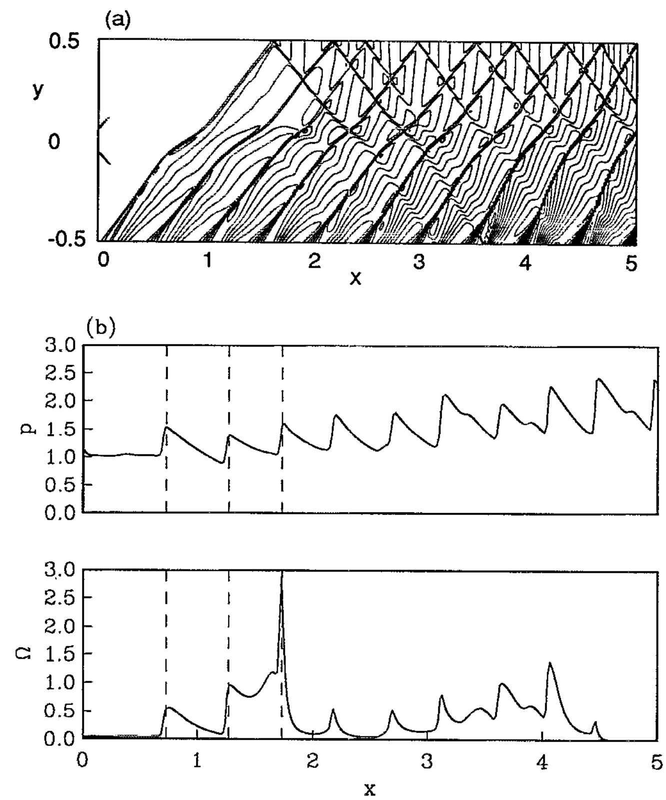

FIG. 4. (a) Contours of the pressure with $M=2$, $u_j=2$, $T_j=1$, $\mathrm{Ze}=20$, $D=4\times10^7$, $\beta=2$, $\epsilon=0.02$, and $k_2=4\pi$; (b) pressure and reaction rate profiles versus x at $y=-0.05$.

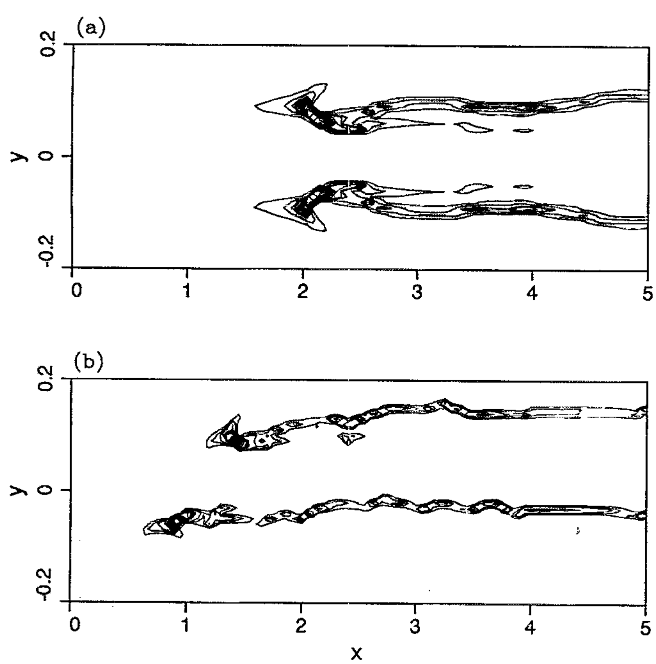

FIG. 5. Contours of the reaction rate. $\Omega = \rho Y_F Y_O D e^{-\mathrm{Ze}/T}$ with $M=2$, $u_j=2$, $T_j=0.5$, $\mathrm{Ze}=20$, $D=8\times10^7$, and $\beta=2$. (a) $\epsilon=0$, contour levels from 0.11 to 6.55, increment 0.11; (b) $\epsilon=0.02$, $k_2=4\pi$, contour levels from 0.13 to 8.25, increment 0.13.

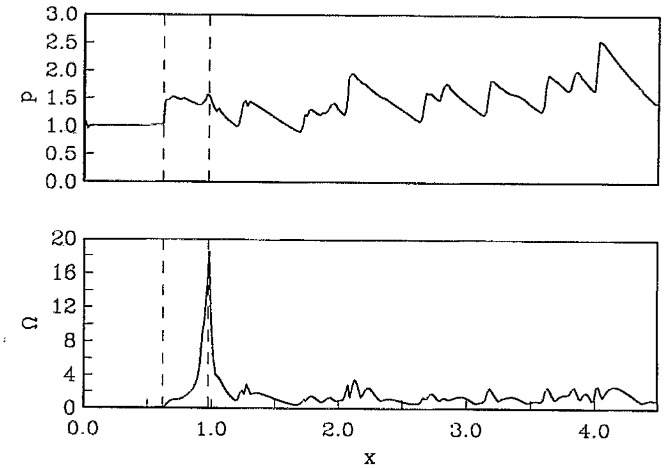

FIG. 6. Pressure and reaction rate profiles along the locus of maximum reaction rate for the lower reaction zone in Fig. 5(b). $M=2$, $u_j=2$, $T_j=0.5$, $\mathrm{Ze}=20$, $D=8\times10^7$, and $\beta=2$.

form temperature jet, the effect of increasing the Mach wave strength is to shorten the ignition distance while displacing the reaction upwards and skewing the ignition points in the streamwise direction. However, these effects are more pronounced for the cold jet indicating that the cold jet is more susceptible to enhancement by the Mach wave system. Furthermore, from Fig. 5(b) the reaction rate is seen to be significantly increased at the points where the Mach wave has induced a wrinkling of the structure. Again, this effect is present in Fig. 2(c) for the initially uniform temperature jet but is not as pronounced. Figure 6 gives the corresponding pressure and reaction rate along the locus of maximum reaction in the downstream direction for the lower reaction zone. It is seen that ignition is achieved as the first Mach wave enters the jet stream.

Finally, we have carried out calculations for the same conditions as in Fig. 5(b) but with the top wall also made wavy. This allows the investigation of the combined effects of Mach waves generated off both walls on the ignition and structure. In particular, for the case of two symmetric walls ($A_1=-A_2, k_1=k_2$) the contours of the reaction rate become symmetric about the centerline of the jet and the two ignition points are aligned in the vertical direction at about $x\approx1$.

IV. CONCLUSIONS

A model problem is considered which investigates the steady response of a reacting, compressible laminar jet to Mach waves generated by wavy walls in a channel of finite width. The model consists of a two-dimensional jet of fuel emerging into a stream of oxidizer. The governing equations are taken to be the steady parabolized Navier–Stokes equations which are solved numerically. The effects of a Mach wave system on a reacting fuel jet are demonstrated via direct comparisons of numerical results between the reactions with and without wavy walls. Effects of initial jet temperatures are also discussed. It is shown that the introduction of a Mach wave system can substantially improve combustion efficiency. In particular, we demonstrate the mechanism responsible for the decrease in the extinction distance and the wrinkling of the flame structure owing to the presence of the Mach waves. Therefore, induced Mach wave–flame interactions provides one more possible alternative augmentation scheme for reacting supersonic shear flows.

Now that the underlying physics of the Mach wave–flame interactions are understood, the effects of a time-dependent and/or turbulent flow can be investigated in an attempt to answer the question of flame stabilization, control, and vorticity enhancement. In particular, the wrinkling of the flame structure could be important in the initiation of large-scale structures. We have begun a full numerical study of this problem and will report the results at a later date.

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