2016

Structure Constant of Twist-2 Light Ray Operators in the Regge Limit

Ian Balitsky
Old Dominion University

Vladimir Kazakov

Evgeny Sobko

Repository Citation

Balitsky, Ian; Kazakov, Vladimir; and Sobko, Evgeny, "Structure Constant of Twist-2 Light Ray Operators in the Regge Limit" (2016). *Physics Faculty Publications*. 73.
https://digitalcommons.odu.edu/physics_fac_pubs/73

Original Publication Citation

Balitsky, I., Kazakov, V., & Sobko, E. (2016). Structure constant of twist-2 light-ray operators in the Regge limit. *Physical Review D, 93*(6), 1-6. doi:10.1103/PhysRevD.93.061701

PHYSICAL REVIEW D **93,** 061701(R) (2016)

Structure constant of twist-2 light-ray operators in the Regge limit

Ian Balitsky,[1] Vladimir Kazakov,[2] and Evgeny Sobko[3]

[1]*Physics Department, Old Dominion University, Norfolk, Virginia 23529, USA and Theory Group, JLAB, 12000 Jefferson Ave, Newport News, Virginia 23606, USA*

[2]*LPT, École Normale Superieure, 24, rue Lhomond 75005 Paris, France and Université Paris-VI, Place Jussieu, 75005 Paris, France*

[3]*DESY Hamburg, Theory Group, Notkestraße 85, 22607 Hamburg, Germany*

(Received 14 October 2015; published 11 March 2016)

We compute the normalized structure constant of three twist-2 operators in $\mathcal{N} = 4$ SYM in the leading Balitsky-Fadin-Kuraev-Lipatov (BFKL) approximation at any N_c. The result is applicable to other gauge theories including QCD.

DOI: 10.1103/PhysRevD.93.061701

I. INTRODUCTION

The problem of high-energy behavior of amplitudes has a long story [1,2]. One of the most popular approaches is to reduce the gauge theory at high energies to 2+1 effective theory which can be solved exactly or by computer simulations. Unfortunately, despite the multitude of attempts, the Lagrangian for $2 + 1$ QCD at high energies is not written yet. In this context the idea to solve formally the high-energy QCD or $\mathcal{N} = 4$ SYM by the calculation of anomalous dimensions and structure constants in the Balitsky-Fadin-Kuraev-Lipatov (BFKL) limit seems to be very promising.

$\mathcal{N} = 4$ SYM is a superconformal theory, and its most important physical properties are encoded into the operator product expansion (OPE) characterized by the spectrum of anomalous dimensions and by the structure constants. While the former is now exactly and efficiently computable at large N_c due to quantum integrability [3], the calculation of the OPE structure constants is these days on a fast track, especially after the ground-breaking all-loop proposal of Ref. [4].

In this work we calculate the three-point correlator of twist-2 operators $\mathcal{O}^j(x) = \mathrm{tr}F_{+i}D_+^{j-2}F_+^i +$ fermions $+$ scalars in $\mathcal{N}=4$ SYM in the BFKL limit [5] when $\omega = j - 1 \to 0$, the 't Hooft coupling $g^2 \equiv \frac{N_c g_{\mathrm{YM}}^2}{16\pi^2} \to 0$ and $\frac{g^2}{\omega}$ fixed, for arbitrary N_c. The symbol $+$ in the field-strength tensor F_{+i} means contraction with light-ray vector n_+, and the summation over index i goes over two-dimensional space orthogonal to n_+ and n_-. Since the contribution of fermions $+$ scalars is subleading at this limit, including the internal loops, the result is valid for the pure Yang-Mills theory as well. The case of the two-point correlator was elaborated in our previous paper [6] where we defined the generalized operators with complex spin as special light-ray operators [7] (regularized as a narrow rectangular Wilson contour called a "frame") and calculated their correlator using OPE over Wilson lines [8] with a rapidity cutoff and the BFKL evolution (see Fig. 1). Here we use the same light-ray operators: one along the n_+ direction and two along n_-. In this case we should use more general Balitsky-Kovchegov (BK) evolution [9,10], and the leading BFKL contribution comes from the BK vertex.

II. LIGHT-RAY OPERATORS AND THEIR RELATION TO LOCAL OPERATORS

The generalization of local operator $\mathcal{O}^j$ for the case of complex spin j was constructed in Ref. [6]. It has a form of light-ray operator $\check{\mathcal{S}}^J$ stretched along the n_+ direction and realizing the principal series representation of $sl(2|4)$ with conformal spin $J = \frac{1}{2} + i\nu$ which is related to Lorentz spin j as $J = j + 1$. The full regularized operator reads as follows,

$$\check{\mathcal{S}}^{j+1}(x_{1\perp}) = \check{\mathcal{S}}_{gl}^{j+1}(x_{1\perp}) + \frac{i}{2}(j-1)\check{\mathcal{S}}_{f}^{j+1}(x_{1\perp}) - \frac{1}{2}(j)(j-1)\check{\mathcal{S}}_{sc}^{j+1}(x_{1\perp}), \quad (1)$$

where, for example, the regularized gluon operator is

$$\check{\mathcal{S}}_{gl}^{j+1}(x_{1\perp}) = \lim_{|x_{31\perp}|\to 0} |x_{13\perp}|^{-\gamma_j} \mathcal{S}_{gl}^{j+1}(x_{1\perp}, x_{3\perp}),$$

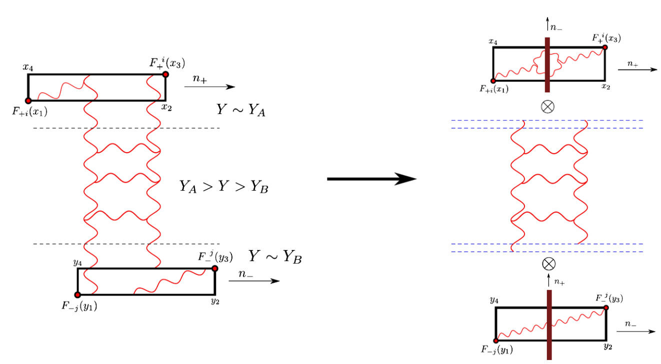

FIG. 1. Scheme of computation of the two-point correlator. In the lhs the long sides of regularizing rectangular Wilson frames are stretched along the light ray and the short sides in the orthogonal directions. In the rhs we use OPE of frames over color dipoles and compute their correlator; see Ref. [6] for details.

$S_{gl}^{j+1}(x_{1\perp}, x_{3\perp}) = \int_{-\infty}^{\infty}\int_{x_{1-}}^{\infty}\frac{dx_{1-}dx_{3-}}{x_{31-}^{j-1}}\mathrm{tr}F_+{}^i(x_1)[1,3]_\square F_{+i}(x_3)$ and $x_1 = (x_{1-}, 0, x_{1\perp})$, $x_3 = (x_{3-}, 0, x_{3\perp})$. The anomalous dimension γ_j corresponds to operator $\check{\mathcal{S}}^{j+1}(x_{1\perp})$. Here we introduced the notation $[1,3]_\square$ for a rectangular Wilson contour with coordinates x_1, x_3 of two diagonally opposite corners, as in Fig. 1. In the case of even integer Lorentz spin j, it can be rewritten as an integral of local operator $\mathcal{O}^j(x)$ with dimension $\Delta(j)$ along a light-ray direction n_+:

$$\check{\mathcal{S}}^{j+1}(x_\perp)|_{j\in\mathrm{Even}} \sim \int_{-\infty}^{\infty} dx_- \mathcal{O}^j(x). \tag{2}$$

In this case the correlator of two light-ray operators stretched along n_+ and n_- vectors, normalized as $\langle n_+ n_-\rangle = 1$, is just the double integral of two-point correlator of local operators with respect to light-ray directions $n_\pm$:

$$\langle \check{\mathcal{S}}^{j_1+1}(x_\perp)\check{\mathcal{S}}^{j_2+1}(y_\perp)\rangle = \frac{\delta(j_1-j_2)b_{j_1}}{(|x-y|_\perp^2)^{\Delta(j_1)-1}}. \tag{3}$$

In this work we calculate the correlator of three light-ray operators, restricting ourselves to a particular simple kinematics: one light-ray operator is stretched along the n_+ light-ray direction, and two others are stretched along n_-. The correlator of three light-ray operators can be obtained by integrating the correlator of three local operators along these light rays. The tensor structures of such local correlators are known from general group-theoretical considerations [11], up to a few structure constants depending on the coupling and symmetry charges. The main problem which we are addressing here is the calculation of these nontrivial constants. Remarkably, if the coordinates of all three light-ray operators in the transverse space are restricted to the same line, all these structures collapse into a single one [12], with a single overall structure constant which we are going to compute. Note that after a conformal transformation the three points in the transverse space take arbitrary positions.

However, the configuration with two collinear light-ray operators is singular, so we first consider three different light-ray directions n_1, n_2, n_3 and then take the limit $n_2 \to n_3$. The result of integration along light rays is quite simple and contains only one unknown overall constant,

$$\begin{aligned}&\langle \check{\mathcal{S}}^{j_1+1}(x_\perp)\check{\mathcal{S}}^{j_2+1}(y_\perp)\check{\mathcal{S}}^{j_3+1}(z_\perp)\rangle\\ &\quad = C_{\{n_i\}}(\{\Delta_i\},\{j_i\})\\ &\quad\cdot\frac{\langle n_1 n_2\rangle^{[j]_{1,2;3}}\langle n_1 n_3\rangle^{[j]_{1,3;2}}\langle n_2 n_3\rangle^{[j]_{2,3;1}}}{(|x-y|_\perp^2)^{[\Delta]_{1,2;3}}(|x-z|_\perp^2)^{[\Delta]_{1,3;2}}(|y-z|_\perp^2)^{[\Delta]_{2,3;1}}},\end{aligned} \tag{4}$$

where we use a short-hand notation $[a]_{i,j;k} \equiv \frac{1}{2}(a_i + a_j - a_k - 1)$ and $\{a_i\} \equiv \{a_1, a_2, a_3\}$. In what follows, we assume the existence of a good analytic continuation for $C_{\{n_i\}}(\{\Delta(j_i)\},\{j_i\})$ to noninteger $\{j_i\}$'s. We take the limit $n_1 = n_+$, $n_2 = n_-$, $n_3 \to n_2$ with the normalization $\langle n_+ n_-\rangle = 1$. In the BFKL regime $j_i = 1 + \omega_i \to 1$ we obtain

$$\begin{aligned}&\langle \check{\mathcal{S}}^{2+\omega_1}(x_\perp)\check{\mathcal{S}}^{2+\omega_2}(y_\perp)\check{\mathcal{S}}^{2+\omega_3}(z_\perp)\rangle\\ &\quad = \lim_{n_3\to n_2 = n_-}\frac{\langle n_2 n_3\rangle^{\frac{\omega_2+\omega_3-\omega_1}{2}}}{\omega_2+\omega_3-\omega_1}\\ &\quad\times\frac{C_{+--}(\{\Delta_i\},\{1+\omega_i\})}{|x-y|_\perp^2)^{[\Delta]_{1,2;3}}(|x-z|_\perp^2)^{[\Delta]_{1,3;2}}(|y-z|_\perp^2)^{[\Delta]_{2,3;1}}},\end{aligned} \tag{5}$$

where $\Delta_i = \Delta(1+\omega_i, g^2)$ is given by the BFKL spectrum (see below). We explicitly pulled out the denominator $\frac{1}{\omega_2+\omega_3-\omega_1}$ because it will emerge in our forthcoming calculation using the BK evolution. We interpret $\lim_{\langle n_2 n_3\rangle\to 0}\frac{\langle n_2 n_3\rangle^{\frac{\omega_2+\omega_3-\omega_1}{2}}}{\omega_2+\omega_3-\omega_1}$ as a delta function $\delta(\omega_2+\omega_3-\omega_1)$ reflecting the boost invariance. In addition we keep ω_i positive through the paper.

Finally the structure constant is normalized using the corresponding two-point correlators:

$$C_{\omega_1,\omega_2,\omega_3} = \frac{C_{+--}(\{\Delta_i\},\{1+\omega_i\})}{\sqrt{b_{1+\omega_1}b_{1+\omega_2}b_{1+\omega_3}}}. \tag{6}$$

III. DECOMPOSITION OVER DIPOLES AND BK EVOLUTION

When calculating the two-point correlator [6], we used a point splitting regularization in the orthogonal direction, replacing light-ray operators by infinitely narrow Wilson frames with inserted fields in the corners (see Fig. 1). Now, for the sake of simplicity, we carry out our calculation for pure Wilson frames, related to our operators with zero R-charge in the following way:

$$\begin{aligned}&\partial_{x_{1\perp}}\cdot\partial_{x_{3\perp}}\int\int\frac{dx_{1-}dx_{3-}}{(x_{3-}-x_{1-})^{2+\omega_1}}[x_1,x_3]_\square \to\\ &\xrightarrow[x_{13\perp}\to 0,\omega_1\to 0]{} |x_{13\perp}|^{\gamma_{j_1}}c(g_{\mathrm{YM}}^2, N_c, \omega_1)\check{\mathcal{S}}^{2+\omega_1}(x_{1\perp}).\end{aligned} \tag{7}$$

The coefficient $c(g_{\mathrm{YM}}^2, N_c, \omega_i)$ [denoted below as $c(\omega_i)$] depends on the local regularization procedure, and at weak coupling it behaves as $c(\omega_i) \sim \frac{g_{\mathrm{YM}}^2}{\omega_i}$, but its explicit form is irrelevant for us because we are going to calculate the normalized structure constant where it cancels. In general, there are a few types of leading twist-2 operators which appear in this decomposition, but in the BFKL limit a single one with the smallest anomalous dimension survives. In addition, in the $\omega_i \to 0$ limit, only the term built out of gauge fields alone contributes [6].

Following the OPE method [8], the pure Wilson frames can be replaced by regularized color dipoles,

$$[x_1, x_3]_\square \to N(1 - \mathbf{U}^{\sigma_+}(x_{1\perp}, x_{3\perp})) \tag{8}$$

where

$$\mathbf{U}^{\sigma_+}(x_{1\perp}, x_{3\perp}) = 1 - \frac{1}{N}\mathrm{tr}(U^{\sigma_+}_{x_{1\perp}} U^{\sigma_+\dagger}_{x_{3\perp}}), \tag{9}$$

$$U^{\sigma_+}_{x_\perp} = P\exp\left[ig_{\mathrm{YM}}\int_{-\infty}^{\infty} dx_+ A^{\sigma_+}_-(x)\right], \tag{10}$$

$$A^{\sigma_+}_\mu(x) = \int d^4k\theta(\sigma_+ - |k_+|)e^{ikx}A_\mu(k) \tag{11}$$

and σ_+ is a longitudinal cutoff in the n_+ direction. Now we can write

$$\begin{aligned}&\langle S^{2+\omega_1}(x_{1\perp}, x_{3\perp})S^{2+\omega_2}(y_{1\perp}, y_{3\perp})S^{2+\omega_3}(z_{1\perp}, z_{3\perp})\rangle \\ &= -D_\perp \int_{-\infty}^{\infty} dx_{1-}\int_{x_{1-}}^{\infty} dx_{3-}x_{31-}^{-2-\omega_1}\int_{-\infty}^{\infty} dy_{1+} \\ &\times \int_{y_{1+}}^{\infty} dy_{3+}y_{31+}^{-2-\omega_2}\int_{-\infty}^{\infty} dz_{1+}\int_{z_{1+}}^{\infty} dz_{3+}z_{31+}^{-2-\omega_3} \\ &\times \langle \mathbf{U}^{\sigma_{1-}}(x_{1\perp}, x_{3\perp})\mathbf{V}^{\sigma_{2+}}(y_{1\perp}, y_{3\perp})\mathbf{W}^{\sigma_{3+}}(z_{1\perp}, z_{3\perp})\rangle,\end{aligned} \tag{12}$$

where $\mathcal{D}_\perp = \frac{N^3(\partial_{x_{1\perp}}\cdot\partial_{x_{3\perp}})(\partial_{y_{1\perp}}\cdot\partial_{y_{3\perp}})(\partial_{z_{1\perp}}\cdot\partial_{z_{3\perp}})}{c(\omega_1)c(\omega_2)c(\omega_3)}$.

In our kinematics two dipoles $\mathbf{V}$ and $\mathbf{W}$ have zero n_+ projection, and in the BFKL approximation they form a "pancake" field configuration in the reference frame related to $\mathbf{U}$. This means that the rapidity of $\mathbf{U}$ serves as the upper limit for integrations with respect to rapidities of $\mathbf{V}$ and $\mathbf{W}$ in our logarithmic approximation. Now we use the BK evolution equation [9,10] to calculate the quantum average in (12). It gives the evolution of the dipole $\mathbf{U}^Y$ with respect to rapidity $Y = e^\sigma$, namely

$$\sigma\frac{d}{d\sigma}\mathbf{U}^\sigma(z_1, z_2) = \mathcal{K}_{\mathrm{BK}} * \mathbf{U}^\sigma(z_1, z_2), \tag{13}$$

where $\mathcal{K}_{\mathrm{BK}}$ is an integral operator having the following form in the leading-order (LO) approximation:

$$\begin{aligned}\mathcal{K}_{\mathrm{LO\,BK}} * \mathbf{U}(z_1, z_2) = \frac{2g^2}{\pi}\int d^2z_3\frac{z_{12}^2}{z_{13}^2z_{23}^2}[&\mathbf{U}(z_1, z_3) \\ &+ \mathbf{U}(z_3, z_2) - \mathbf{U}(z_1, z_2) \\ &- \mathbf{U}(z_1, z_3)\mathbf{U}(z_3, z_2)].\end{aligned} \tag{14}$$

The evolution of $\mathbf{U}^{Y_1}$ goes from Y_1 to an intermediate Y_0 with respect to the linear part of (13), and then the BK vertex acts at Y_0 and generates two dipoles which can be contracted with $\mathbf{V}^{Y_2}$ and $\mathbf{W}^{Y_3}$. Schematically, it can be written as

$$\int dY_0(\mathbf{U}^{Y_1} \to \mathbf{U}^{Y_0}) \otimes (\mathrm{BKvertex\ at}\,Y_0) \otimes \begin{pmatrix}\langle \mathbf{U}^{Y_0}\mathbf{V}^{Y_2}\rangle \\ \langle \mathbf{U}^{Y_0}\mathbf{W}^{Y_3}\rangle\end{pmatrix}.$$

The linear BFKL evolution of $\mathbf{U}^{Y_1}$ from Y_1 to Y_0 gives

$$\begin{aligned}\mathbf{U}^{Y_1}(x_1, x_3) = &\int d\nu\int d^2x_0\frac{\nu_1^2}{\pi^2}E_{\nu_1}(x_{10}, x_{30})e^{\aleph(\nu_1)Y_{10}} \\ &\cdot\frac{1}{\pi^2}\int\frac{d^2\gamma d^2\beta}{|\gamma-\beta|^4}E^*_{\nu_1}(\gamma - x_0, \beta - x_0)\mathbf{U}^{Y_0}(\gamma, \beta),\end{aligned} \tag{15}$$

where we denoted $Y_{ij} \equiv Y_i - Y_j$ and we introduced the function $E_\nu(z_{10}, z_{20}) = (\frac{|z_{12}|^2}{|z_{10}|^2|z_{20}|^2})^{1/2+i\nu}$ which projects dipoles on the eigenstates of the BFKL operator with the eigenvalues $\aleph(\nu) = 4g^2(2\psi(1) - \psi(1/2+i\nu) - \psi(1/2-i\nu))$. We take here only the sector $n = 0$, where n is the discrete quantum number of $SL(2, C)$ because it gives the leading contribution.

The nonlinear part of the BK evolution (13) is described by the following renormalization group equation:

$$\begin{aligned}\frac{\partial}{\partial Y}\mathbf{U}^Y(\gamma, \beta)\Big|_{Y=Y_0} = &-\frac{2g^2}{\pi}\int d^2\alpha\frac{|\gamma-\beta|^2}{|\gamma-\alpha|^2|\beta-\alpha|^2} \\ &\times \mathbf{U}^{Y_0}(\gamma, \alpha)\mathbf{U}^{Y_0}(\alpha, \beta).\end{aligned} \tag{16}$$

Finally we contract the two emerging dipoles $\mathbf{U}^{Y_0}(\gamma, \alpha)$ and $\mathbf{U}^{Y_0}(\alpha, \beta)$ with $\mathbf{V}^{\sigma_{2+}}(y_{1\perp}, y_{3\perp})$ and $\mathbf{W}^{\sigma_{3+}}(z_{1\perp}, z_{3\perp})$. Thus for the planar contribution, we get

$$\begin{aligned}&\langle \mathbf{U}^{Y_1}(x_{1\perp}, x_{3\perp})\mathbf{V}^{Y_2}(y_{1\perp}, y_{3\perp})\mathbf{W}^{Y_3}(z_{1\perp}, z_{3\perp})\rangle_{pl} \\ &= -\frac{2g^2}{\pi}\int dY_0\int d\nu_1\int d^2x_0\frac{\nu_1^2}{\pi^2}E_{\nu_1}(x_{10}, x_{30})e^{\aleph(\nu_1)Y_{10}} \\ &\times\frac{1}{\pi^2}\int\frac{d^2\alpha d^2\beta d^2\gamma}{|\gamma-\beta|^2|\gamma-\alpha|^2|\beta-\alpha|^2}E^*_{\nu_1}(\gamma - x_0, \beta - x_0) \\ &\cdot(\langle \mathbf{U}^{Y_0}(\gamma, \alpha)\mathbf{V}^{Y_2}(y_{1\perp}, y_{3\perp})\rangle\langle \mathbf{U}^{Y_0}(\alpha, \beta)\mathbf{W}^{Y_3}(z_{1\perp}, z_{3\perp})\rangle \\ &+ \langle \mathbf{U}^{Y_0}(\gamma, \alpha)\mathbf{W}^{Y_3}(z_{1\perp}, z_{3\perp})\rangle\langle \mathbf{U}^{Y_0}(\alpha, \beta)\mathbf{V}^{Y_2}(y_{1\perp}, y_{3\perp})\rangle).\end{aligned} \tag{17}$$

The last two terms in (17) give the same contribution, so it is enough to know the correlators of two dipoles [6],

$$\begin{aligned}&\langle \mathbf{U}^{Y_0}(\gamma, \alpha)\mathbf{V}^{Y_2}(y_{1\perp}, y_{3\perp})\rangle \\ &= \frac{8g^4(1-N_c^2)}{N_c^4}\int d^2y_0\cdot\int\frac{d\nu_2\nu_2^2e^{Y_{02}\aleph(\nu_2)}}{(\frac{1}{4}+\nu_2^2)^2} \\ &\times E_{\nu_2}(\gamma - y_0, \alpha - y_0)E^*_{\nu_2}(y_{10}, y_{30}),\end{aligned} \tag{18}$$

and similarly for $\langle \mathbf{U}^{Y_0}(\alpha, \beta)\mathbf{W}^{Y_3}(z_{1\perp}, z_{5\perp})\rangle$. It was argued in Ref. [6] that we can make the identification for rapidities in dipole correlators, $Y_{02} = \ln\frac{L_0y_{31+}}{\Lambda^2}$, $Y_{03} = \ln\frac{L_0z_{31+}}{\Lambda^2}$, where Λ a cutoff of which the precise value is irrelevant in LO. On the other hand, the difference of rapidities of the first dipole and of the BK vertex $Y_{10} = \ln\frac{x_{31-}}{L_0}$ corresponds to BFKL evolution. The integral over $Y_0 = \ln\frac{L_0}{\Lambda}$ goes from Y_1 to $\max(Y_2, Y_3)$. If we plug (17) and (18) into (12) and do the integrals over light-ray directions, i.e. over rapidities, we obtain the following planar contribution:

$$\begin{aligned}&\langle S^{2+\omega_1}(x_{1\perp},x_{3\perp})S^{2+\omega_2}(y_{1\perp},y_{3\perp})S^{2+\omega_3}(z_{1\perp},z_{3\perp})\rangle_{pl}\\&\quad=\frac{2^8g^{10}(N_c^2-1)^2}{\pi^3N_c^8}\delta(\omega_1-\omega_2-\omega_3)D_\perp\int d\nu_1\frac{\nu_1^2}{\pi^2}\frac{1}{\omega_2+\omega_3-\aleph(\nu_1)}\int\frac{d\nu_2\nu_2^2}{(\frac14+\nu_2^2)^2}\frac{1}{\omega_2-\aleph(\nu_2)}\\&\qquad\cdot\int\frac{d\nu_3\nu_3^2}{(\frac14+\nu_3^2)^2}\frac{1}{\omega_3-\aleph(\nu_3)}\int d^2x_0d^2y_0d^2z_0E^*_{\nu_1}(x_{10},x_{30})\cdot E^*_{\nu_2}(y_{10},y_{30})E^*_{\nu_3}(z_{10},z_{30})\Upsilon_{pl}(\nu_1,\nu_2,\nu_3;x_0,y_0,z_0).\end{aligned}\tag{19}$$

The usual delta-function $\delta(\omega_1-\omega_2-\omega_3)$ (see e.g. Ref. [14]) is a consequence of boost invariance as in the formula (5). Υ_{pl} represents the planar contribution of the BK vertex,

$$\begin{aligned}\Upsilon_{pl}(\nu_1,\nu_2,\nu_3;x_0,y_0,z_0)&=\int\frac{d^2\alpha d^2\beta d^2\gamma}{|\gamma-\beta|^2|\gamma-\alpha|^2|\beta-\alpha|^2}E_{\nu_1}(\beta-x_0,\gamma-x_0)\cdot E_{\nu_2}(\alpha-y_0,\gamma-y_0)E_{\nu_3}(\alpha-z_0,\beta-z_0)\\&=\frac{\Omega(h_1,h_2,h_3)}{|x_0-y_0|^{4[h]_{1,2;3}+2}|x_0-z_0|^{4[h]_{1,3;2}+2}|y_0-z_0|^{4[h]_{2,3;1}+2}},\end{aligned}\tag{20}$$

where $h_1=\frac12+i\nu_1$, $h_2=\frac12+i\nu_2$, $h_3=\frac12+i\nu_3$ and the function $\Omega(h_1,h_2,h_3)$ was presented in Ref. [15].

Remarkably we can also take into account the nonplanar contribution [15,16], thus providing the finite N_c answer for the BFKL structure constant. It appears as a single extra term Υ_{npl},

$$\begin{aligned}\Upsilon_{npl}(\nu_1,\nu_2,\nu_3;x_0,y_0,z_0)&=\int\frac{d^2\beta d^2\gamma}{|\gamma-\beta|^4}E_{\nu_1}(\beta-x_0,\gamma-x_0)E_{\nu_2}(\beta-y_0,\gamma-y_0)E_{\nu_3}(\beta-z_0,\gamma-z_0)\\&=\frac{\Lambda(h_1,h_2,h_3)}{|x_0-y_0|^{4[h]_{1,2;3}+2}|x_0-z_0|^{4[h]_{1,3;2}+2}|y_0-z_0|^{4[h]_{2,3;1}+2}},\end{aligned}\tag{21}$$

where $\Lambda(h_1,h_2,h_3)$ was also presented in Ref. [15], and the full answer can be obtained from (19) by replacing Υ_{pl} with Υ (see in Fig. 2):

$$\Upsilon=\Upsilon_{pl}-\frac{2\pi}{N^2}\Upsilon_{npl}\mathrm{Re}\left[\psi(1)+\psi\left(\frac12+i\nu_1\right)-\psi\left(\frac12+i\nu_2\right)-\psi\left(\frac12+i\nu_3\right)\right].\tag{22}$$

The integrals over x_0, y_0, z_0 are easily computable, e.g.

$$\begin{aligned}\int d^2x_0E_{\nu_1}(\beta-x_0,\gamma-x_0)E^*_{\nu_1}(x_{10},x_{30})&=(\tau^2)^{\frac12+i\nu_1}{}_2F_1\left(\frac12+i\nu,\frac12+i\nu,1+2i\nu,\tau\right){}_2F_1\left(\frac12+i\nu,\frac12+i\nu,1+2i\nu,\bar\tau\right)\\&\quad\times\frac{(\frac14+\nu^2)^2}{\nu^2}G(\nu)+(\nu\leftrightarrow-\nu),\end{aligned}\tag{23}$$

$$G(\nu)=\frac{\nu^2}{(\frac14+\nu^2)^2}\frac{\pi\Gamma^2(\frac12+i\nu)\Gamma(-2i\nu)}{\Gamma^2(\frac12-i\nu)\Gamma(1+2i\nu)},\tag{24}$$

where $\tau=\frac{|x_1-x_3||\beta-\gamma|}{|x_1-\beta||x_3-\gamma|}$. In the limit $x_1,x_3\to x$, we can replace $\frac{|x_1-x_3||\beta-\gamma|}{|x_1-\beta||x_3-\gamma|}\to\frac{|x_1-x_3||\beta-\gamma|}{|x-\beta||x-\gamma|}\to0$. For small τ we close the ν_1 contour in the lower (upper) half-plane for first (second) term, respectively, both of them giving the same contribution. Integrals over α,β,γ in (19) can be reduced to Υ_{pl} represented in Ref. [15] in terms of hypergeometric and Meijer G functions and Υ_{npl} in terms of Γ functions. Integrals over ν_i can be done by picking up the BFKL poles $\omega_i=\aleph(\nu_i^*)$.

Combining (19), (22) and (23), we come to the final expression for the three-point correlation function,

$$\begin{aligned}&\langle S^{2+\omega_1}(x_{1\perp},x_{3\perp})S^{2+\omega_2}(y_{1\perp},y_{3\perp})S^{2+\omega_3}(z_{1\perp},z_{3\perp})\rangle\\&\quad=-ig^{10}\frac{\delta(\omega_1-\omega_2-\omega_3)}{c(\omega_1)c(\omega_2)c(\omega_3)}H\cdot\frac{\Psi(\nu_1^*,\nu_2^*,\nu_3^*)|x_{13}|^{\gamma_1}|y_{13}|^{\gamma_2}|z_{13}|^{\gamma_3}}{|x-y|^{2+\gamma_1+\gamma_2-\gamma_3}|x-z|^{2+\gamma_1+\gamma_3-\gamma_2}|y-z|^{2+\gamma_2+\gamma_3-\gamma_1}}\end{aligned}\tag{25}$$

RAPID COMMUNICATIONS

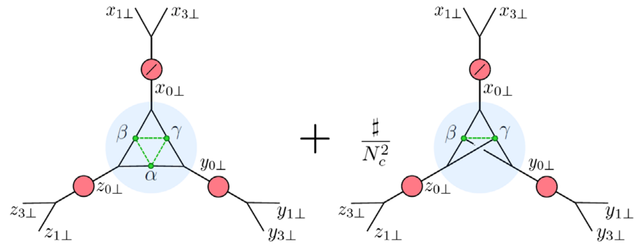

FIG. 2. The structure of the three-point correlator. Red circles correspond to BFKL propagators [the crossed one has an extra multiplier $(\frac{1}{4}+\nu_1^2)^2$]. The blue blob corresponds to the three-point functions of two-dimensional BFKL conformal field theory. The triple "Y"-veritces correspond to E functions. For example, a vertex with ends labeled as $z_{1\perp}$, $z_{3\perp}$, $z_{0\perp}$ corresponds to $E_\nu(z_{10\perp}, z_{30\perp}) = (\frac{|z_{13\perp}|^2}{|z_{10\perp}|^2|z_{30\perp}|^2})^{1/2+i\nu_3}$. The $\alpha\beta\gamma$-triangle in the first, planar term and $\beta\gamma$-link in the second, nonplanar term correspond to the triple pomeron vertex.

$$\text{where } H = \frac{2^{10}(N_c^2-1)^2}{\pi^2 N_c^5}\gamma_1^2(2+\gamma_1)^4(2+\gamma_2)^2 \times (2+\gamma_3)^2\frac{G(\nu_1^*)}{\aleph'(\nu_1^*)}\frac{G(\nu_2^*)}{\aleph'(\nu_2^*)}\frac{G(\nu_3^*)}{\aleph'(\nu_3^*)}, \tag{26}$$

$\gamma_i = \gamma(1+\omega_i)$ are anomalous dimensions and the coefficient $\Psi(\nu_1^*,\nu_2^*,\nu_3^*)$ has the form

$$\Psi(\nu_1^*,\nu_2^*,\nu_3^*) = \Omega(h_1^*,h_2^*,h_3^*) - \frac{2\pi}{N_c^2}\Lambda(h_1^*,h_2^*,h_3^*)\cdot \mathrm{Re}(\psi(1) - \psi(h_1^*) - \psi(h_2^*) - \psi(h_3^*)),$$
$$h_i^* = \frac{1}{2} + i\nu_i^* = 1 + \frac{\gamma_i}{2}. \tag{27}$$

The functions $\Omega(h_1,h_2,h_3)$ and $\Lambda(h_1,h_2,h_3)$ (defined in (20) and (21)) and calculated in [15].

Our final result for the normalized structure constant is

$$C_{\omega_1,\omega_2,\omega_3} = -i^{1/2}g^4\frac{2}{\pi^5}\frac{\sqrt{N_c^2-1}}{N_c^2}\gamma_1^2(2+\gamma_1)^2 \cdot\sqrt{\frac{G(\nu_1^*)}{\aleph'(\nu_1^*)}\frac{G(\nu_2^*)}{\aleph'(\nu_2^*)}\frac{G(\nu_3^*)}{\aleph'(\nu_3^*)}}\Psi(\nu_1^*,\nu_2^*,\nu_3^*). \tag{28}$$

Specifying the dependence on parameters $\{\frac{g^2}{\omega_i}\}$, g^2 and N_c, we can write $C_{\omega_1,\omega_2,\omega_3} = g\frac{\sqrt{N_c^2-1}}{N_c^2}f(\frac{g^2}{\omega_1},\frac{g^2}{\omega_2},\frac{g^2}{\omega_3})$, where f is a function which depends only on the ratios $\{\frac{g^2}{\omega_i}\}$. In the limit $\frac{g^2}{\omega_i}\to 0$, we get the asymptotics:

$$\Omega(h_1^*,h_2^*,h_3^*) \to -\frac{16\pi^3}{\gamma_1^2\gamma_2^2\gamma_3^2}\cdot[\gamma_1^2(\gamma_2+\gamma_3)+\gamma_2^2(\gamma_1+\gamma_3) + +\gamma_3^2(\gamma_1+\gamma_2)+\gamma_1\gamma_2\gamma_3)(1+O(g^2/\omega_i))$$
$$\times\Lambda(h_1^*,h_2^*,h_3^*) \to \frac{8\pi^2(\gamma_1+\gamma_2+\gamma_3)}{\gamma_1\gamma_2\gamma_3}(1+O(g^2/\omega_i)). \tag{29}$$

In this limit $\gamma_i = -\frac{8g^2}{\omega_i}+o(\frac{g^2}{\omega_i})$, and the main contribution to the three-point correlator (28) comes from the planar $\mathcal{O}(g^2)$ term

$$C_{\omega_1,\omega_2,\omega_3} = -ig^2\frac{\sqrt{N_c^2-1}}{\sqrt{2\pi}N_c^2}\frac{1}{\omega_1^{\frac{5}{2}}\omega_2^{\frac{1}{2}}\omega_3^{\frac{1}{2}}}(\omega_1^2(\omega_2+\omega_3) + \omega_2^2(\omega_1+\omega_3)+\omega_3^2(\omega_1+\omega_2) + \omega_1\omega_2\omega_3)(1+O(g^2)), \tag{30}$$

whereas the nonplanar one is $\mathcal{O}(g^6)$. It might seem strange that the planar contribution does not start from $\mathcal{O}(g^4)$ terms given by the leading Feynman graphs, e.g. with four gluon vertices. However, in the BFKL approximation, we should keep $\frac{g^2}{\omega}\gg\omega$. In addition when making the point-splitting regularization, we have to keep $\frac{g^2}{\omega}|\ln(x_{31\perp}/(x-y))^2|\gg 1$. The limit $|x_{13\perp}|$ has to be taken first, which makes the value $g^2=0$ exceptional. This order of limits leads to the $\mathcal{O}(g^2)$ behavior of (30).

IV. DISCUSSION

Our result, Eq. (28), based on the BFKL approximation is a rare example of computation of a structure constant of three unprotected operators receiving contributions from all orders in a coupling constant, including infinitely many "wrapping" corrections. Moreover, our result is valid at any N_c. Since in the LO BFKL the contributions of all fields but gluons in $\mathcal{N}=4$ Supersymmetric Yang-Mills (SYM) disappear from both the definition of operators and internal loops, the result is applicable to pure Yang-Mills theory at any N_c, including $N_c=3$. It would be interesting to apply our structure constants to the OPE at hard scattering in real QCD and to work out the full "dictionary," relating them to the OPE in the two-dimensional $SL(2,C)$ conformal field theory—the basis of our BFKL computation. It is also not hopeless, though challenging, to compute these structure constants in the next-to-leading-order approximation in $\mathcal{N}=4$ SYM. Our present result may serve as an important, all-wrappings test for the future computations of similar quantities in the integrability approaches to planar AdS_5/CFT_4, such as Ref. [4] and the BFKL limit of the quantum spectral curve [17].

ACKNOWLEDGMENTS

We thank J. Bartels, S. Caron-Huot, L. Lipatov, and V. Schomerus for discussions. Our special thanks to G. Korchemsky who participated in the initial stage of this work. The work of E. S. and V. K. was supported by the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme FP7/2007-2013/

under REA Grant Agreement No. 317089 (GATIS). The work of V. K. has received funding from the European Research Council (Programme Ideas Grant No. ERC-2012-AdG 320769 AdS-CFT-solvable), from the ANR grant StrongInt (Grant No. BLANC- SIMI- 4-2011) and from ESF Grant No. HOLOGRAV-09- RNP- 092. The work of I. B. was supported by DOE Contract No. DE-AC05-06OR23177 and by Grant No. DE-FG02-97ER41028.

[1] B. L. Ioffe, V. S. Fadin, and L. N. Lipatov, *Quantum Chromodynamics: Perturbative and Nonperturbative Aspects* (Cambridge University Press, Cambridge, 2014).
[2] Y. V. Kovchegov and E. Levin, Quantum Chromodynamics at High Energy (Cambridge University Press, Cambridge, 2012).
[3] N. Beisert *et al.*, Lett. Math. Phys. **99**, 3 (2012); N. Gromov, V. Kazakov, S. Leurent, and D. Volin, J. High Energy Phys. 09 (2015) 187.
[4] B. Basso, S. Komatsu, and P. Vieira, arXiv:1505.06745.
[5] V. S. Fadin, E. A. Kuraev, and L. N. Lipatov, Phys. Lett. B **60** (1975) 50; I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978).
[6] I. Balitsky, V. Kazakov, and E. Sobko, arXiv:1310.3752.
[7] I. Balitsky and V. M. Braun, Nucl. Phys. **B311**, 541 (1989).
[8] I. Balitsky, Nucl. Phys. **B463**, 99 (1996).
[9] I. Balitsky, AIP Conf. Proc. **407**, 953 (1997).
[10] Y. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999); **61**, 074018 (2000).
[11] M. S. Costa, J. Penedones, D. Poland, and S. Rychkov, J. High Energy Phys. 11 (2011) 071.
[12] The similar phenomenon was observed in Ref. [13].
[13] V. Kazakov and E. Sobko, J. High Energy Phys. 06 (2013) 061.
[14] J. Bartels and M. Wusthoff, Z. Phys. C **66**, 157 (1995).
[15] G. P. Korchemsky, Nucl. Phys. **B550**, 397 (1999).
[16] G. A. Chirilli, L. Szymanowski, and S. Wallon, Phys. Rev. D **83**, 014020 (2011).
[17] M. Alfimov, N. Gromov, and V. Kazakov, J. High Energy Phys. 07 (2015) 164.