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2010

Comment on "Superfluid Turbulence from Quantum Kelvin Wave to Classical Kolmogorov Cascades"

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Yepez et al. Reply: We agree with Krstulovic and Brachet [\[1\]](#page-1-0) that the k^{-3} power law, in the energy spectrum for a linear vortex, marks the presence of a vortex core, using the standard kinetic energy definition, $\int dx^3 \frac{1}{2} m v(x)^2 |\varphi(x)|^2$.
Yet the k^{-3} power law also marks the presence of a vortex Yet, the k^{-3} power law also marks the presence of a vortex tangle with a Kelvin wave (KW) cascade, provided it occurs with a $k^{-5/3}$ power law at small k.

Our initial vortices had winding number $n = 6$, equivalent to 6 overlapping $n = 1$ vortices, a highly unstable configuration as illustrated in Fig. [1](#page-1-1). We used ray tracing

to image surfaces around the nodal lines $\varphi = 0$.
Consider an $L^3 = 2048^3$ simulation with initial vortex Consider an $L^3 = 2048^3$ simulation with initial vortex
we number $k \in \{40\}$ and vortex-vortex separation $\ell \sim$ wave number $k_{\xi} = 40$ and vortex-vortex separation $\ell \sim \sqrt{\frac{L^3}{n}} = \frac{2048}{241} \approx 241$ using a total vortex length $f_{\text{tot}} = 12nL$. $\sqrt{\frac{L^3}{L_v}} = \frac{2048}{\sqrt{72}} \approx 241$, using a total vortex length $\mathcal{L}_v = 12nL$.
In the initial linear vortex spectrum the transitional wave In the initial linear vortex spectrum, the transitional wave number between k^{-1} and k_{line}^{-3} number between k^{-1} and k^{-3}_{linear} related to the inverse co-
herence length, k_{ξ}^{linear} , is pronounced. In contrast, in the quantum turbulence spectrum with clean $k^{-5/3}$ and k_{tan}^{-3} power laws, the transitions related to the inverse Kolmogorov scale, $k_{\text{outer}} = k_{\ell} \sim \ell^{-1}$, and an inner scale, k_{angle} are both propounced. This is seen in Fig. 2 with $k_{\text{inner}}^{\text{tangge}}$, are both pronounced. This is seen in Fig. [2](#page-1-2) with $k_{\text{inner}}^{\text{tangle}} \sim \sqrt{3} L = 40$ at $t = 0$ (no $K_{\text{WQ}}^{\text{VQ}}$) and with $k_{\text{inner}}^{\text{tangle}} \sim$ $k_{\xi}^{\text{linear}} \equiv \frac{\sqrt{3}}{2}$ $\frac{\sqrt{3}}{2}$ $\frac{L}{\xi}$ = 40 at $t = 0$ (no KWs) and with $k_{\ell}^{\text{tangle}} \approx$ 40 at $t = 20000$ in a KW cascade. Thus, we find $k_{\xi}^{\text{linear}} \approx k_{\xi}^{\text{tangent}}$ and this similarity also occurred for the 5760³ k_{ℓ}^{image} , and this similarity also occurred for the 5760³
simulation reported in our Letter [2]. We identified the simulation reported in our Letter [\[2\]](#page-1-3). We identified the classical to quantum transition region as $k_{\text{outer}} \leq k \leq$ k_{inner} , and identified the outer scale with the Kolmogorov length ($k_{\text{outer}} \approx k_{\ell}$) and the inner scale with the coherence length. When the k^{-3} spectrum is absent or significantly diminished, temporarily due to intermittency [\[3\]](#page-1-4), we do not see a vortex tangle with a KW cascade. When the k^{-3} spectrum at high $k \ge k_{\text{inner}}$ is present (along with a $k^{-5/3}$ Kolmogorov spectrum at small $k \leq k_{\text{outer}}$ marking a vortex tangle), we see distorted vortices supporting KWs undergoing kelvon-kelvon couplings, including at $k > k_{\xi}^{\text{linear}}$.

We believe there is essential dynamics at high wave numbers $k > k_{\xi}$. The $L^3 = 5760^3$ grid simulation we reported has $\sim 10^{11}$ microscopic (bit) particles, and a single
vortex can contain hundreds of thousands of grid points vortex can contain hundreds of thousands of grid points. The unitary algorithm $\Psi' = U\Psi$ employs a tensor product
state $\Psi = \psi(r) e^{i\Phi}$ separated over the L³ points of the state $\Psi = \psi(x)^{\otimes L^3}$ separated over the L^3 points of the system, where each local ket $\psi(x)$ is a 2-spinor. This gives system, where each local ket $\psi(x)$ is a 2-spinor. This gives an exact quantum simulation modulo the lattice cutoff $\ll \xi$ that accurately solves the Gross-Pitaevskii equation. A

FIG. 1 (color online). Two initially nearly intersecting rectilinear $n = 6$ vortices on a portion of a 4032³ grid (left). By $t =$ 4000, many $n = 1$ vortices are subject to the KW instability by mutual interaction (middle). By $t = 57,500$, a vortex tangle is evident (right).

FIG. 2 (color online). Incompressible kinetic energy spectra with 12 linear $n = 6$ vortices at $t = 0$ (left) and during turbulence with a KW cascade at $t = 20000$ (right) for a 2048³ grid. Low-k power-law regression fits: $k^{-1.00}$ (left) and $k^{-1.67}$ (right). High-k power-law fits: $k^{-3.16}$ (left) and $k^{-3.03}$ (right). Initially, the wave number cutoff is $k_{\xi} \approx 40$ (red vertical line). Later at $t = 20000$, we find $k_{\text{outer}} = k_{\ell} \approx 40$ (green vertical line) and $k_{\text{inner}} \approx \pi k_{\text{outer}} = 127$ (red vertical line).

fluctuating part of $\psi(x)$ are quasiparticles

$$
\delta \psi(x) \cong \varepsilon \left(\begin{array}{c} u(x)e^{-i\omega t} \\ -v^*(x)e^{i\omega t} \end{array} \right)
$$

governed by the Bogoliubov–de Gennes (BdG) equations,

$$
i\hbar\left(\begin{array}{c}\partial_t u\\-\partial_t v\end{array}\right)=\left(\begin{array}{cc}\mathcal{L} & -g\varphi_v^{*2}\\-g\varphi_v^{*2} & \mathcal{L}\end{array}\right)\left(\begin{array}{c}\nu\\v\end{array}\right),\,
$$

with a spatial operator $\mathcal{L} = -\frac{\hbar^2}{2m}\nabla^2 + 2g|\varphi_v|^2 - \mu$. High
k-space resolution especially at large *k* is vital to ensure k -space resolution, especially at large k , is vital to ensure these fluctuations are numerically represented inside the cores. Finally, high- k kelvons are known experimentally [\[4\]](#page-1-5), and such kelvons have been verified numerically at the BdG level [\[5,](#page-1-6)[6](#page-1-7)]. The cutoff $r_c < \xi$ is inside the core with a modified KW dispersion relation [\[6\]](#page-1-7).

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