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Cross Ratio

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Cross Ratio

Peyton Burlingame

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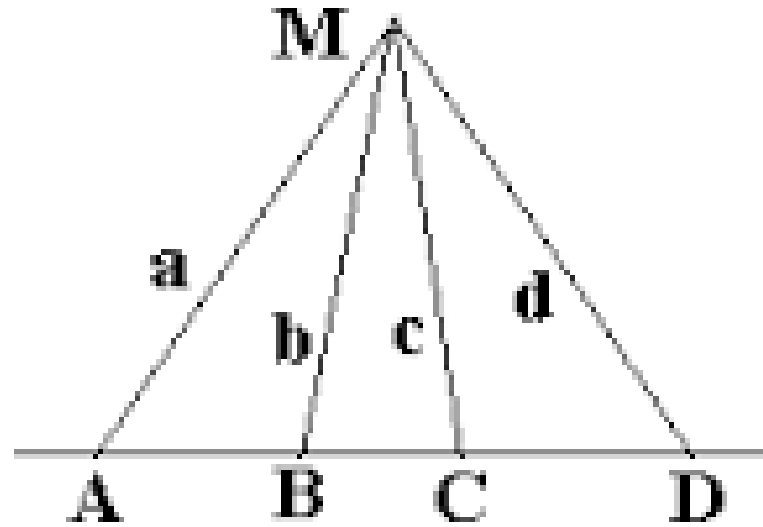
Problem

Given four points A , B , C , and D in order on a line in Euclidean space, under what conditions will there be a point P off the line such that the angles $\angle APB$, $\angle BPC$, and $\angle CPD$ have equal measure.

-MAA Monthly Problem #11915

Cross Ratio

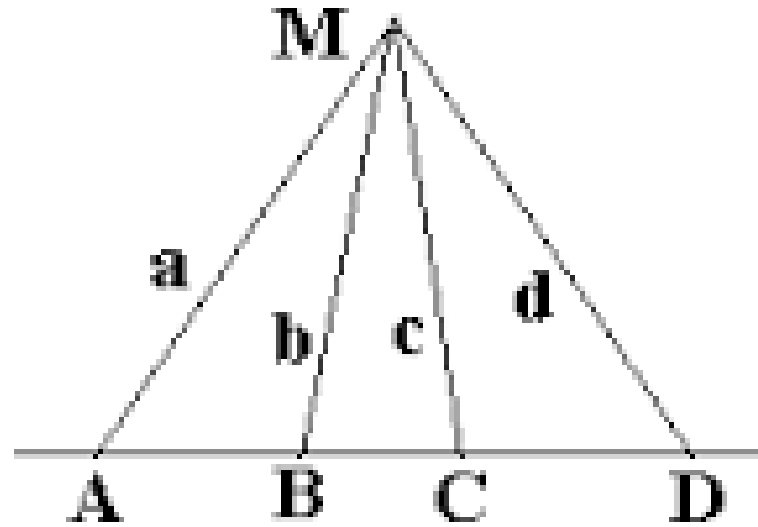
$$(ABCD) = \frac{\frac{AC}{CB}}{\frac{AD}{DB}}$$



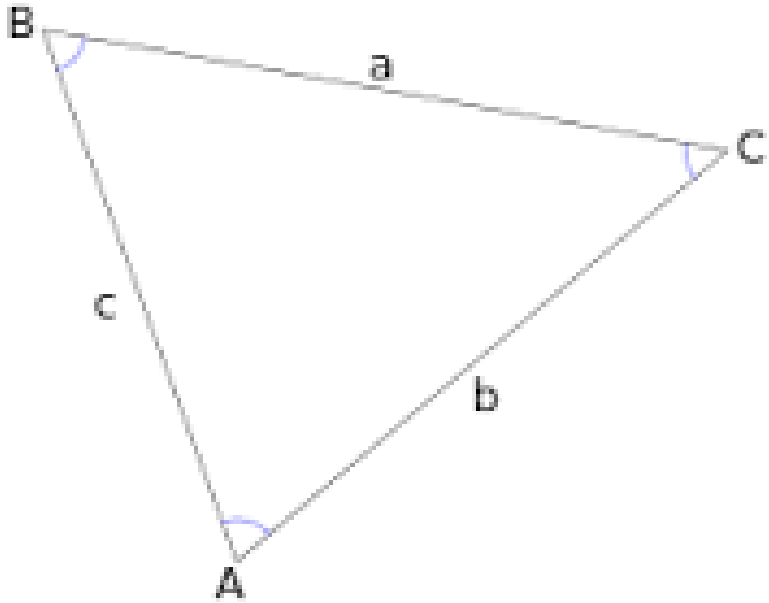
Geogebra Link

Cross Ratio Theorem

The cross ratio will be the same for any line crossing the four rays (a, b, c, d) starting at point M .



Law of Sines

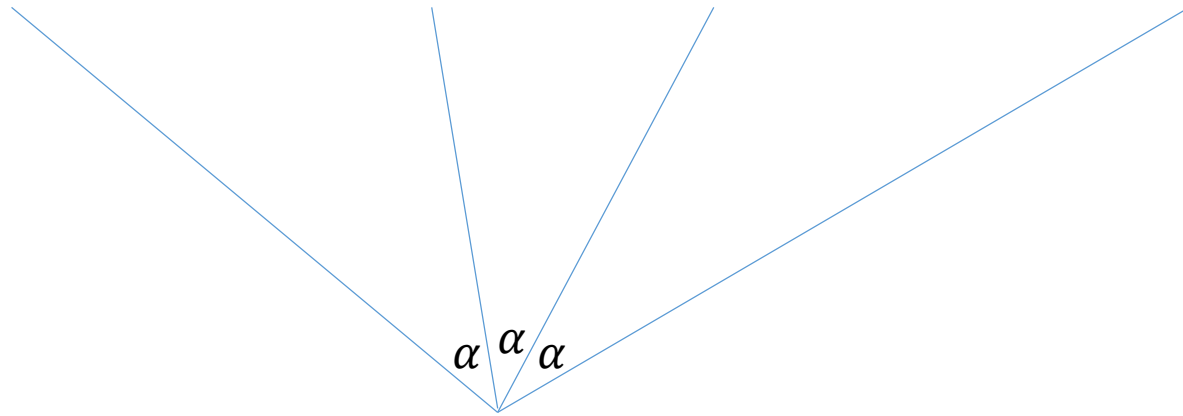


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

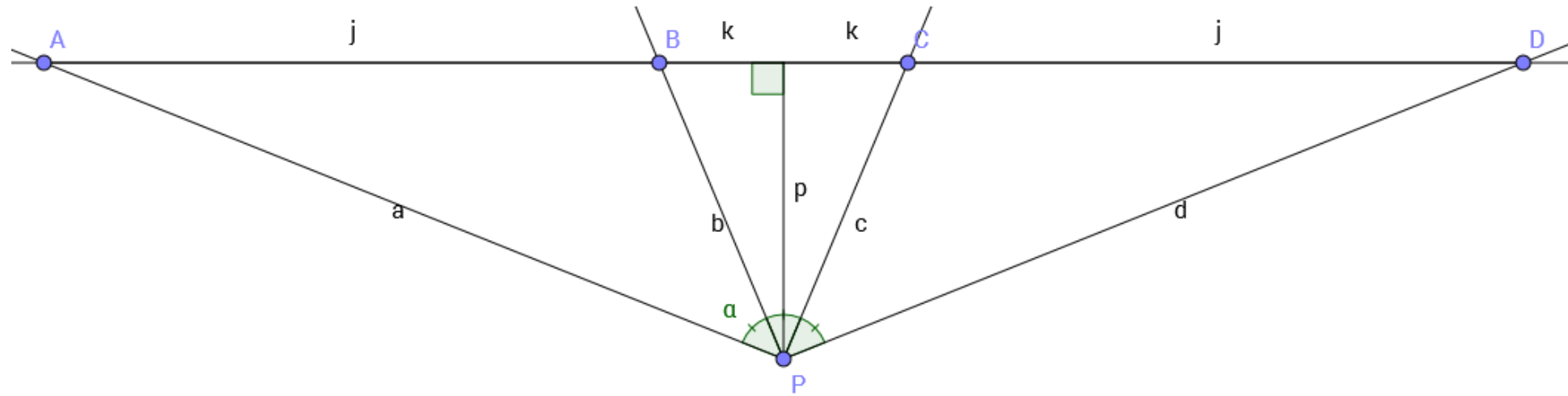
$$\Rightarrow \frac{\sin A}{\sin B} = \frac{a}{b}$$

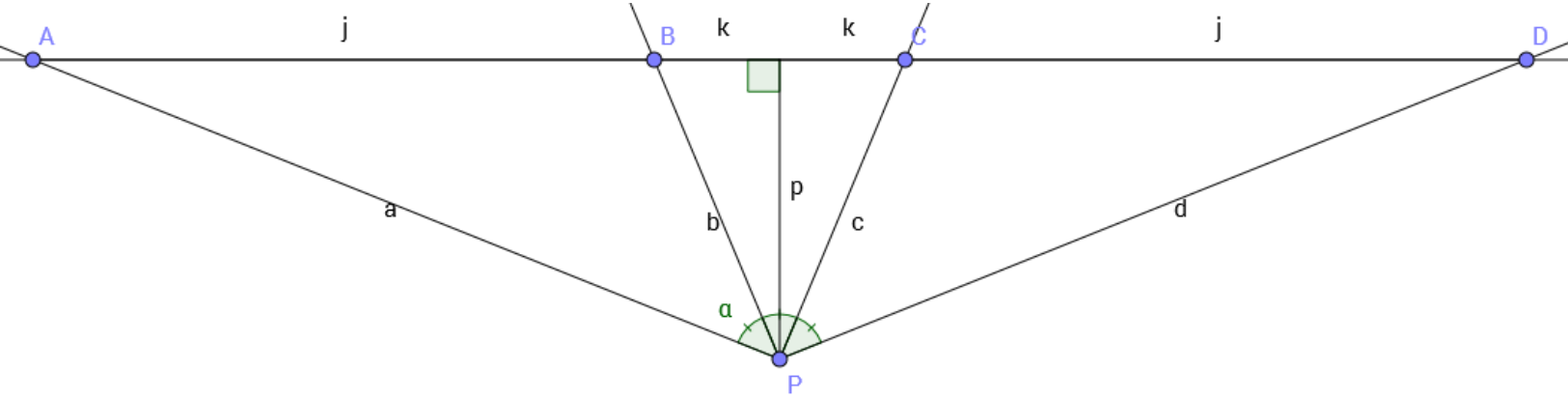
Cross Ratio Property

If the three angles are congruent, the cross ratio must be greater than $4/3$.

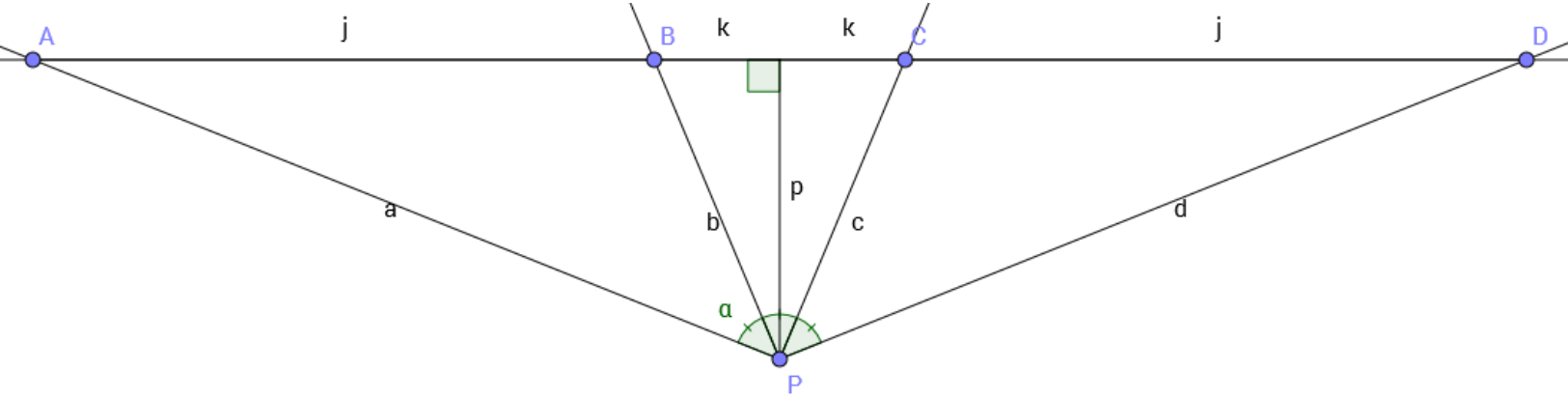


Best situation





$$(ABCD) = \frac{\frac{AC}{CB}}{\frac{AD}{DB}} = \frac{\frac{j+2k}{2k}}{\frac{2j+2k}{j+2k}} = \frac{(j+2k)^2}{2k(2j+2k)}$$



$$(ABCD) = \frac{(j + 2k)^2}{2k(2j + 2k)} = \frac{\left[\tan\left(\frac{3}{2}\alpha\right) + \tan\left(\frac{1}{2}\alpha\right) \right]^2}{4 \tan\left(\frac{1}{2}\alpha\right) \times \tan\left(\frac{3}{2}\alpha\right)}$$

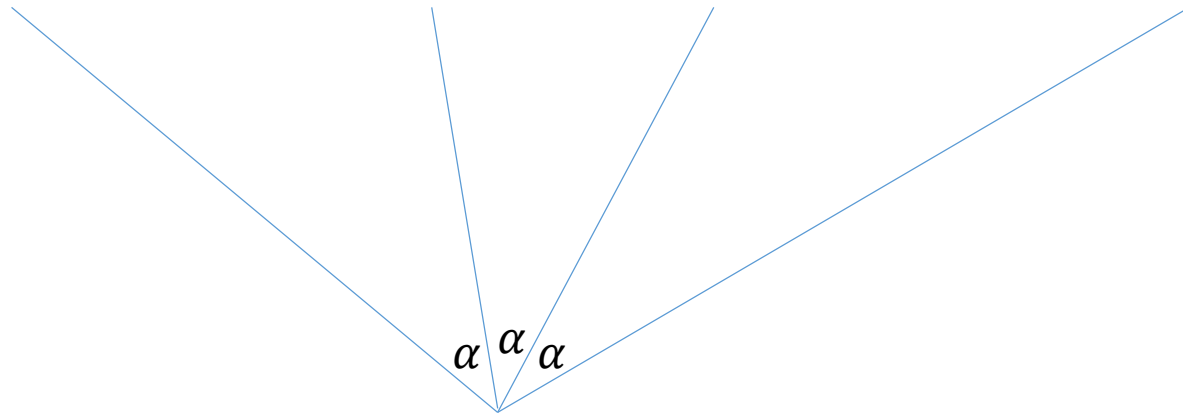
Cross Ratio is Increasing

On the interval $\left(0, \frac{\pi}{3}\right)$,

$$\frac{d}{d\alpha} \left(\frac{1}{2 \cos(2\alpha)+1} + 1 \right) = \frac{4 \sin(2\alpha)}{(2 \cos(2\alpha)+1)^2} > 0$$

Cross Ratio Property

If the three angles are congruent, the cross ratio must be greater than $4/3$.



Problem

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Partial Solution

If there exists a P then the cross ratio is greater than $\frac{4}{3}$.

Areas of further work are necessary

Acknowledgments

- Dr. Leah Childers
- Appreciation to the Honors College for encouraging us to do this project

References

- Geogebra
- Wolfram Alpha
- Milne, John J. *An Elementary Treatise on Cross-ratio Geometry, with Historical Notes*. Cambridge: U, 1911. Print.