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An Inquiry into the Metaphysical Foundations of Mathematics in Economics

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AN INQUIRY INTO THE METAPHYSICAL FOUNDATIONS OF MATHEMATICS IN ECONOMICS

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ABSTRACT

Economics is supposed to fall somewhere between a hard science and a social science. During the last half century, economics has become highly mathematical trying to mimic physics. The purpose of this study is to look at the metaphysical statements linked to mathematical models, specifically, Game Theory. In doing so, it will be demonstrated that Game Theory, as part of neoclassical economics, engages in analysis which can be categorized as metaphysical, with real metaphysical implications. In categorizing the metaphysical assumptions of neoclassical economists/game theorists we will see how much of their analysis is consists in a reductive, implausible metaphysical view. Problems that arise from this view are hardly taken into consideration most economists. This lack of consideration has non-trivial consequences for economics as a discipline and for its methodology.

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CONTENTS

1. Introduction

The present study of economics has become increasingly reliant on extensive use of mathematics. This use of mathematics in economics requires one to forgo the otherwise rigorous analysis that one requires when using philosophy, and its relation to other disciplines, in particular, economics. Economists tend to overlook some statements that can be deemed philosophical. The purpose of this study is to capture some of the "philosophical presuppositions" in economics that might affect its theoretical coherence. Economic ontology (the study of beings as they relate to the economy and their behavior in the world) are increasingly becoming areas of inquiry in relation to economic theory.[1] Economic Ontology seeks to uncover those "philosophical [ontological] presuppositions" that lie at the bottom of economic theory.[2] Uskali Mäki cites the Duhem-Quine thesis as an example of how mathematics and the use of empirical methods might affect the conclusions by economists and scientists. The Duhem-Quine thesis states that scientific theories are not able to be "proven" based on the results from empirical testing.[3] Thus, Mäki states, the results from empirical testing are not able to discriminate among competing theories—that is, results are not able to establish the merits, or demerits of a certain scientific (and economic) theory.[4] The purpose of philosophical considerations is not only to question the foundations on which mainstream economic

[1] *The Economic World View, Studies in the Ontology of Economics* presents a series of essays that deal with some of the main issues (or philosophical presuppositions) in various areas of economics.

[2] See Uskali Mäki, "The what, why, and how of Economic Ontology," pg. 10.

[3] See Ibid. pg. 9

[4] Ibid. pg. 9

analysis is done, but also to better our understanding of the world. This paper argues that the mere use of mathematics is not sufficient as a justification of economic arguments.[5]

The need to look outside economic analysis is imperative to have a more sound economic view of the world. Some practicing economists/applied mathematicians state that they do not make "metaphysical" statements without realizing that mathematics, as well as natural science is founded on certain "metaphysical" statements. Martin Heidegger's essay "Modern Science, Metaphysics, and Mathematics" states precisely how the history of science is founded upon seemingly "evident" truths that Isaac Newton inherits from philosophers going back to Aristotle and other Greeks.[6] Metaphysics, that which is beyond the physical (constituted of space and time) is inherently present in the study of motion; it is arguably the case that thought/thinking are also metaphysical "things." The different stages of history will affect theories of motion (that of Aristotle, or that of Newton). Heidegger's analysis of the "mathematical" starts by presenting the etymology of the word mathematics. Mathematics, he states, has to do with "number," but this is an inherently narrow definition of the mathematical. Ta mathēmata, that which can be learned, and mathēsis—that which can be taught, is at the foundation of "mathematics." What can be learned and what can be taught, for Heidegger, is a

[5] Mäki suggests that it is indeed to uncover the limitations of scientific/economic theories that lead us to a more coherent view of the world—only after we have discovered such limitations we can "justify" the merits of any given theory (pg. 10).

[6] See Martin Heidegger, "Modern Science, Metaphysics, and Mathematics," pp. 281-288.

philosophical problem which deals in deciphering the "thingness of things."[7] What is it about mathematics that helps us decipher the "thingness of things." When we answer the question about numbers, and their relation to the "thingness of things," it is possible, according to Heidegger, to do the learning. The number 3, he states, is a seemingly simple concept already at hand for us to analyze. When we see three chairs, he states, we immediately see the number three. Conversely, when we try to grasp the concept of "threeness," we are left to referring to the natural series of natural numbers.[8] What Heidegger points out using this example is that there are two senses of the "mathematical." The first sense of the mathematical is that which is learnable and comes naturally from observation. But the second sense of the "mathematical" is

> the manner of learning and the process itself[;] [t]he mathematical is that evident aspect of things within which we are always already moving and according to which we experience them as things at all, and as such things. The mathematical is this fundamental position we take toward things by which we take up things as already given to us, and as they must and should be given. The mathematical is thus the fundamental presupposition of the knowledge of things.[9]

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[7] Ibid. pg. 274, This seemingly odd formulation has to do with the world as we encounter it. The philosopher's task is to try to decipher the world. Heidegger cites five different areas in connection with ta mathēmata:

1) Ta physica: things insofar as they originate and come forth from themselves
2) Ta poioumena: things insofar as they are produced by humans and exist as such
3) Ta chrēmata: things insofar as they are in use or subsist at our disposal—these might be any object relating to ta physica or ta poioumena such as rocks, or in the case of poioumena, anything we might make
4) Ta pragmata: things insofar as we encounter them at all, whether we use them, work on them, transform them.
5) Ta mathēmata: what can be learned insofar as 1-4.

[8] Ibid. pg. 277

[9] Ibid. pp. 277-278, Heidegger cites the sign at the entrance of Plato's academy stating: "Let no one enter who has not grasped the meaning of the mathematical." This reference relates clearly to the conception of the mathematical strictly relating to number and the need to go beyond this understanding.

If we take Heidegger's formulation of the mathematical, we see clearly that the mathematical itself is not a simple set of numerical relations. What does the "knowledge of things" entail? Heidegger points out that the project of the mathematical (conceptualized in the manner of that which goes beyond number), is to project things as they first show themselves (as in the example of 3 chairs being just there). The project of the mathematical is axiomatic—that is the mathematical project sets out to make statements about the world from fundamental propositions. These fundamental propositions are set out in advance in order for the experimenter to have access to this (mathematical) axiomatic project.[10] This is the mathematical system developed by Newtonian mechanics (relating to the motion of bodies), or the infinitesimal calculus of Leibniz. The relations of objects are analyzed by a closed system that is coherent and is developed from axioms. "Knowledge of things" in the case of Newtonian mechanics, or infinitesimal calculus, *attempts* to give rise to knowledge of things generally; but in both cases, according to Heidegger, we have the "narrow" sense of mathematics at work.[11] Heidegger states that the 'calculation' which is the result from the mathematical formalism and intuitive determination of things has given modern science (economics is mentioned only in passing) its status of stature. In reality, the "burning questions" about things, and specifically beings remain unanswered and unquestioned.[12] If we are to dig deep into the foundations of

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[10]Ibid. pp. 291-292

[11] Ibid. pg. 297

[12]Ibid. pg. 296

mathematics and its relation to other sciences, it is only through metaphysics that this can be done—this is so because metaphysics reaches farthest not only to beings or things, but to beings in totality.[13]

What exactly does metaphysics mean to economics? The way in which economists construe agents and economic structures will have implications as to what predictions will come from within the specific economic presuppositions. This applies not only for economists, but also other social scientists that make statements about complex human reality. From this complex human reality, it follows that the structures in which humans exist are also complex and have an impact on how scientists and economists do science. Sections 2-3 state that mathematics *could* be the foundation of economic analysis, but it is only through an inherently metaphysical analysis that allows us to posit this mathematical foundation. Section 4 on economic methodology tries to show the lack of progression in the mainstream economics with regards to methodological issues that affect economic analysis. Section 5 deals with the current view of mainstream economics which involves rigorous "mathematical formulations," but its assumptions about individuals in an economy is far from economic reality. Section 6 challenges mainstream economic theory with respect to value among other things. This whole of this study involves a truly interdisciplinary approach utilizing psychology, economics, philosophy and history. Thus, the study of economics is not merely the study of "economic agents" all of whom can be reduced to mathematical algorithms (determined and deterministic calculations). Economics is not only about making tractable formulations, abstracted from any type

[13] Ibid. pg. 296

of worldly reality. Knowing how these formulations come to be will help us understand the how radical our conclusions might be in relation to history, science, philosophy and the development of knowledge (and the lack of it).

2. Kant and the Possibility for a Science

Kant's genius allowed him to ponder the question whether philosophy could ground itself like a science in order for knowledge to be possible. Kant's idea for philosophy does not rely on a form of metaphysics from which all positings are derived. Mathematics, for example, seems to be an important case for the philosopher of science to ponder in order to dismantle the metaphysics that go with the grounding that is given to some metaphysical forms of mathematics. Mathematics is possible formulated as idealism. That is, mathematical objects exist outside of the mind, and therefore they are independent of human positing for existence. Plato is the main influence in the history of philosophy to bring forth such a foundation to philosophical thought. And although Plato's view provided a great deal of insight on philosophical questions, it allows for a metaphysics which posits objects outside of the realm of experience; something which ultimately is (according to Kant) 'spurious metaphysics.' According to Kant, time & space are 'pure sensibilities' which must ground logic. That is, it is impossible to come to a conclusive positing about objects *outside* of space & time. For systems of mathematics, what follows from this is that mathematical objects viewed as independent entities become another form of spurious metaphysics. Kant's insight about the way in which we ground our epistemology will impact the type of philosophy and science (including mathematics and natural science). If we want to

steer away from spurious metaphysics, Kant's epistemology must be carefully examined against other forms of idealism, and this, will be the ground for philosophy, and ultimately other sciences.

In order to posit the existence of something, first, that object must lie within space & time. Plato's strong disagreement against Kant lies in the fact that, if we posit objects inside of space & time, we will not really ground anything because of the unreliability of the senses, from which we experience objects. Thus the great disagreement between Plato and Kant is about the senses. Plato requires positing objects outside of space & time in order to make these objects unchanging. Thus, Plato would view Kant's "psychologism" as unreliable because Plato views the senses as an unreliable mechanism through which we can come to "know" things.[14]

The clearest statement for the integration of mathematics as a ground for a science is in the *Prolegomena to any Future Metaphysics*. Kant's Transcendental Aesthetic reformulates the problem of dogmatism and empiricism. Kant's reformulation of the empiricist/dogmatic opposition leads to a philosophical view that allows truth with certainty without having to refer to any spurious metaphysics.

2. 1 Kant's Refutation of Idealism

According to Kant, Idealism has some general characteristics. Idealism states objects exist "in-themselves" outside of space & time, and therefore outside any possibility of experience; furthermore, it is impossible to provide any proof for the existence of such objects. The two examples provided by Kant are Descartes

[14] Plato ultimately thinks that we cannot know anything. But this follows clearly from the view that there are these immutable objects outside of space & time which can not be "known." Plato thinks that things in the world of space & time "participate" in the universal forms (See Naomi Reshotko, unpublished manuscript, "Plato's Epistemological Paradox: The Know*able* Cannot be Known").

"material idealism" and Berkeley's "dogmatic idealism. Kant observes that Berkeley's idealism is problematic in general because it assumes that space is imaginary. What follows from this assumption is that objects in space are also imaginary.[15] Thus, there are only imaginary objects in the world, and we never know what they are.[16] It is clear that Berkeley's idealism is too problematic to defend in any length. The "Material Idealism" of Descartes is not a better formulation of the problem of knowledge according to Kant. Descartes' idealism, according to Kant, is no better than Berkeley's idealism. Descartes wants to prove existence by assuming external existence (non-imaginary); proof of external existence allows for Descartes to posit the 'I am,' but nothing more, we are unable, according to Descartes to go beyond proving our own existence.[17]

Kant goes on to talk about intuition in idealism. Idealism does not allow a coherent theory of intuition in order to come to any knowledge. As stated above, any "knowledge" that we think we have is unstable. This is yet another corollary of idealism, and the positing of objects outside of space & time. Furthermore, since we can't come to know objects directly (through intuition—i.e. Plato), we need to make

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[15] Kant, Immanuel, *The Critique of Pure Reason*, trans. Norman Kemp, B275. Although Plato is not mentioned in this, we see that Plato's formulation of Idealism makes this claim as well.

[16] Kant provides his own theory on how we gather knowledge of objects in space & time through intuition (that is, how objects appear to us in their immediacy without going into any technical philosophical jargon—Plato's objection to this is that objects in space & time are too unstable, that is why he want to posit objects *outside* of space & time, in order to guarantee that not one perception is what is called "knowledge." This would be the response by Plato ultimately, for a detailed discussion see Socratic dialogue, *Protagoras.*

[17] Ibid. B275

inferences from the objects that appear to us. This inference is supposed to be stable (see Reshotko's account of Platonic epistemology mentioned above).[18]

For Kant, old metaphysics consists of just this lack of distinction between things-in-themselves, and objects of experience. Things in-themselves are objects which are unconditioned by the human mind, while objects of appearance are conditioned objects. Space & time make the dividing line between the unconditioned and the conditioned. If objects are possible objects of experience, Kant states that these objects lie in space & time; we can have knowledge about them, and make knowledge claims. If on the other hand, objects are not possible objects of experience, then they necessarily lie outside of space & time—and therefore, nothing can be known about them.[19]

2.2 Natural Science as Philosophy

The Prolegomena to Any Future Metaphysics presents what Kant's view of what a science should be, in connection to positing things *in* space and time. Before Kant, science suffers from the over-abundance of metaphysics. The main problem with this sort of metaphysics is twofold. One the one hand, there is Hume as the main proponent of the view that it is impossible to come to know anything at all because of all the flux in the world which constitutes our proximate reality. This reality is nothing but flux; anything that we come to say about the world is unfounded because

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[18] Ibid. B291-B294, Reshotko states that it is necessary to "set the bar high" with respect to knowledge, this guarantees that we have a stable and reliable epistemology. Thus, Ultimately, Kant would be categorized as a version of *Protagoras*.

[19] Platonists will obviously disagree with this but this is not meant to give an argument against Platonists. At the conclusion of this section, it will be demonstrated that Kant's view can be developed into a metaphysically consistent view with current theories of mathematics (as well as some versions of Platonism).

we do not take into consideration the complexity of a world in flux. The main consequence of this view is that we are unable to come to any knowledge a priori. That is, we are unable to come to any knowledge without experience. Furthermore, we are unable to come to know anything after we 'experience' things. Thus, we are unable to have any cognition whatever.[20] Kant's main problem for philosophy lies in asking whether it is possible to come to know things a priori, or before experience. In order to do this Kant sets himself the task to reveal the failure of philosophy before him. According to Kant, the failure of philosophy before him is due to being trapped in "spurious metaphysics." Philosophers before Kant see philosophy as the vehicle to ultimate truth—Truth as "things-in-themselves." This kind of philosophizing is what has allowed for philosophy to be stuck without advancing. Kant's view of philosophy comes from the use of the *synthetic method* as well as the *analytic method* which will bring about a "science that shall display all its articulations, as the structure of a quite peculiar faculty of cognition in its natural combination."[21] For Kant this science is nearer intuition than other sciences which in the past have attempted such endeavors, but failed because these sciences have ended up with seemingly coherent metaphysics of abstract objects.[22]

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[20] Kant, Immanuel, *Prolegomena to any Future Metaphysics*, [4:258], here Kant presents Hume's "destructive philosophy" which criticizes any type of metaphysics. Kant very poignantly rejects Hume's empiricism. Kant notes that Hume himself is puzzled about the question of metaphysics—Hume himself falls into a metaphysical trap.

[21] Ibid. [4:264]

[22] Ibid. Kant does not mention this but we could mention Plato, Aristotle, Descartes, Spinoza, Leibniz to mention a few. The rationalists definitely fall under this category. At least the rationalist want a rigorous way to derive their views, but they also end up with spurious metaphysics because they fail to make the distinction between the "things-in-themselves" and objects of possible experience.

Kant's complaint against the "old metaphysics" consists in the fact that there is no necessary link between cognition and objects. Cognition is simply what is possible to experience in this world. Before Kant, it was necessary to make objects in the world completely separate entities about which metaphysical statements are merely assumed. The proofs that follow from these metaphysical statements are merely a priori; that is, a priori definitions are given, proofs follow from definitions/axioms, then (propositions) and corollaries from propositions.[23] Kant's critique of this kind of "proof" and "proving" both are merely theorizing about "things-in-themselves," about which for Kant, nothing can be known.

Kant's response to this "old metaphysics" is to state how it is possible to come to know things through reason, and be certain that this is actual knowledge (Hume's challenge). According to Kant the foundation for anything that can be stated is in analytic or synthetic judgments. The former is merely *explicative* and the latter *ampliative*.[24] Analytic judgments are explicative as they are merely tautological/definitional statements. Analytic judgments have non-contradiction as their principle. Synthetic judgments are ampliative in that these statements *add* something to our cognition.[25] That is, a synthetic judgment adds something to our knowledge. When I say a triangle is a three sided figure, I am merely restating a fact. But, when I say that the sum of the internal angles of any triangle is equal to 180˚, I

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[23] The certainly rigorous example of this is Spinoza's proof of the existence of God. What is important to note is how Spinoza is talking about God as a "thing-in-itself." One of the striking conclusions for Spinoza is that there is part of the human mind which has an infinite attribute. Otherwise, it would be impossible to make the connection between the human and the divine.

[24] Ibid. [4:266-267]

[25] Ibid. [4:267]

am making a synthetic judgment. For Kant, it is in this way that the structure of knowledge is constructed. Although analytic judgments are always a priori, and follow the principle of contradiction, synthetic judgments can be a posteriori.[26] All judgments about experience are synthetic (a posteriori); and without exception mathematical judgments are synthetic.[27] Kant's famous example of this is in the arithmetic statement "7+5=12." Following the foundation that Kant has provided, it is necessary that we use intuition to start analyzing this problem. We might come to the realization that this statement is analytic. That is, we simply respond that the answer is 12. But this is *incorrect* because if it is the case that we are able to do this, we should have not problem doing this with larger numbers (where it is clear that using our hands to count would be quite cumbersome). Thus, this is a synthetic statement, a priori. That is, we do not need to experience this sum in the world for its truth to hold. Euclidean Geometry also has this characteristic.[28]

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[26] Ibid. [4:267-268] Analytic a posteriori judgments is a null class of judgments, that is it is impossible, according to Kant, to come to know things are they are in themselves through experience. We note that in the case of the "old metaphysics" this is not the case since we are able to somehow "know" these things.

[27] Ibid. [4:268] Mathematicians hold that the principle of contradiction is at the bottom of the reliability of mathematical knowledge. Kant keenly observes that the principle of contradiction only works if we have another judgment to accompany our analysis; it is not by itself that synthetic judgments (in this way) work—this only works by presupposing other synthetic propositions.

[28] Ibid. [4:268-269] Kant states that it is clear that this statement is synthetic. Kant's point about intuition is that we need to use our fingers to count if needed. But there is nothing in "7+5=12" which immediately gives us the number 12. Kant's point is clearer if we try 234+585=819. There is nothing in the sum that leads us to the answer, 819. We don't immediately get "819" when we think "234+585." Thus, this is a synthetic judgment, that is, it contains information about the world which I did not already know—such as the case for analytic statements. For a further discussion on this see Gottfried Martin's *Arithmetic and Combinatorics, Kant and his Contemporaries,* Ch. 6, "Synthetic Judgment in Arithmetic" (discussed below); also see Johann Schulz, Appendix in *Arithmetic and Combinatorics, Kant and his Contemporaries,* Schulz gives a detailed formulations of how mathematical proofs are constructed. He deals with the way in which theorems hold by way of either using a conceptual axiomatic approach, or Kant's approach through the forms of intuition. Kripke

The main problem Kant sees in all previous philosophy is that mathematic propositions are thought of as analytic (a priori) while metaphysical propositions are thought of as synthetic a priori. Thus we see that Hume would not have allowed for mathematical statements as synthetic a priori (given his skepticism).[29] Finally, Kant states that metaphysical statements have to be grounded in cognition. The conclusion from all this is that for any metaphysical statement to be grounded, it has to be an object of possible experience.[30] Kant argues in order to have a ground for a possibility of grounding metaphysics in possible experience by answering the following questions:

1) How is pure mathematics possible?
2) How is pure natural science possible?
3) How is metaphysics in general possible?
4) How is metaphysics as a science possible?[31]

The answer to these questions makes up the whole of Kant's argument against the "old metaphysics."

The common thread among all these questions is that judgments are to be grounded in pure intuition.[32] One might question (this was certainly Plato's thesis for

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(1972, pp. 274-275) offers a devastating objection to Kant's distinction between the analytic/synthetic distinction stating that some a priori judgments are both a priori and contingent. Kripke's example is referencing a yard stick in Paris at $time_0$ measuring one meter. A priori, the observer knows that the measurement is "one meter" long. The observer must fix the reference to this one meter stick. Fixing the reference makes it a problem of a priori judgments to be conclusive. Resolving this is a topic for another paper. The idea is that if we take Kant as the point of departure, we are able to see the minimum requirements for us to think about beliefs and judgments in general.

[29] [4:272-273] Kant once again charges Hume for holding this view—Hume rejects mathematics as synthetic a priori, and as the rest of all other philosophers and mathematicians, holds that all mathematical propositions are analytic. What Kant's sees as a grave mistake is to regard metaphysics as synthetic a priori. That is, metaphysical propositions are true regardless of our cognitions; they give us new information about the world (and even beyond).

[30] Ibid. [4:274], this is also the subject of "How is Cognition from Pure Reason Possible?" [4:276-280].

[31] Ibid. [4:280]

arguing against any position that relied on the senses) how it is possible to *intuit anything a priori…*Kant's answer to this is to look at things as a representation as they appear in their immediacy.[33] One might ask, how can the intuition precede the object itself? The answer arises from a reference to old metaphysics, where any kind of statement regarding objects of intuition is impossible because these statements refer to things as *they are in themselves*. It is only in this way that intuitions (that lead to representations) would not take place a priori.[34] Kant's formulation of how we are able to intuit things a priori is as follows:

> There is thus only one way in which it is possible for my intuition to precede the actuality of the object and take place as cognition a priori, *namely if it contains nothing else than the form of sensibility, which in me as subject precedes al lactual impressions through which I am affected by objects*. […] from it [that objects can be intuited in the form of sensibility] follows: that all propositions which concern merely this form of sensible intuition will be possible and valid for objects of the senses; equally the converse, that intuitions which are possible a priori can never concern any other things other than objects of our senses.[35]

The pure intuitions Kant is referring to are space & time (S&T). Adding to the critique of the old metaphysics, Kant states that prior to him, space & time were thought of as pure concepts or as "things-in-themselves". When talking about objects in the world, such as geometrical figures, Kant states that it should be the case that we should be able to completely find two exact objects. If it was the case that space & time were "things-in-themselves" and not pure forms of intuition, we would have no

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[32] Ibid. [4:281]

[33] Ibid. [4:282]

[34] Ibid. [4:282-283]

[35] Ibid. [4:282-283]

problem with this task. However, when we see that the reflection of our left hand is our right hand, there is something awkward about this.[36] Kant's point is that there is incongruence when we try to combine our left hand to the reflection in the mirror (the right hand). The reason for this is that things are not appearances of things as they are in themselves, but forms of sensible intuition.[37] Thus, Kant states, we come to our conclusion about mathematics, specifically, geometry and arithmetic; mathematical objects are grounded in sensible intuition—they are not appearance of things as they are "in-themselves"[38] One might ask, as Plato did, why we would not want to do this kind of epistemological "grounding." Kant's position could be categorized as a relativist position, because we could say that our senses or our sensibility[39] are too unstable to be able to guarantee any type of epistemological ground. This seems to be an easy way out from Kant's genius, and general insight about the limitations of philosophy and metaphysics specifically. For Kant, the geometer, as he/she sits in his desk thinking about geometric figures and propositions can see that lines, for example are part of sensible experience. They are not merely subjective illusions, but objects of ordinary experience.[40] Kant's view of Idealism thus:

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[36] Ibid. [4:286], Kant's examples also include geometrical figures, but his point is clearer when we use the hand example. If it is the case that we see things that "participate" in forms, it should be the case that we find two things that are identical at all times.

[37] Ibid. [4:286]

[38] Ibid. [4:287]

[39] This would certainly be Spinoza's critique of Kant. For Spinoza, the affects are unreliable, and thus we must do away from sensibility to make room for Reason.

[40] Kant, Immanuel, *Prolegomena to any Future Metaphysics*, [4:288] Here, once again, Kant states that if we want to call for metaphysical entities when we speak about geometric figures, we are not able to

> Idealism consists in the assertion that there are none other than thinking beings; the other things which we believe to perceive in intuition are only representations in the thinking beings, to which in fact no object outside the latter corresponds. Say on the contrary: things are given to us as objects of our senses situated outside us, but of what they may be in themselves we know nothing; we only know their appearances, i.e. the representations they bring about in us when they affect our senses.[41]

Thus it seems that the idealist position has a harder time justifying objects as appearances of "things-in-themselves."

Furthermore, geometers, and mathematicians do not merely use the senses to comet to proof the indubitability of geometric figures, or arithmetical axioms. It is necessary to use the understanding to come to judgments about the world which might be true or false.[42] Kant is clear that "illusions" can arise whether we conceive of space & time as sensible intuitions or as "things-in-themselves." The "illusion" arises from our carelessness. To this regard Kant makes the following point about mathematics:

> My doctrine of the ideality of space & time, therefore, so far from making the whole world of the senses into mere illusion, is rather the only means of securing the application to actual objects of one of the most important cognitions, namely that which mathematics expounds a priori, and of preventing it from being held to be mere illusion, because without this observation it would be quite impossible to decide whether the intuitions of space & time, which we take from no experience and which yet lie in our representation a priori, were not chimeras of the brain made by us to which no object corresponds, at least not adequately, and thus geometry itself a mere illusion; whereas on the contrary, just because all objects of the world of the senses are mere appearances, we have been able to show the indisputable validity of geometry in respect to them.[43]

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ground our concepts except in "spurious metaphysics" where we call for "pure logic" (as a thing-in-itself).

[41] Ibid. [4:289]

[42] Ibid. [4:290-292]

[43] Ibid. [4:292]

Kant's main argument here is that if we grant that we are speaking about appearances of "things-in-themselves," we cannot possibly be sure of the validity of our statements (since we can never know things as they are in themselves).

The second part of the *Prolegomena* deals with the question "how is Pure Natural Science possible?" From the foregoing discussion, we can already see that Kant has provided the ground for the possibility of natural science. For Kant, "[n]ature is the *existence* of things, insofar as the latter is determined according to universal laws."[44] Once again if by nature, we meant the existence of things-in-themselves, we would never know nature; not a priori—and certainly not a posteriori. A priori reasoning deals with analysis of concepts (from which we form analytic judgments). These analytic judgments, as mentioned earlier, are mere tautologies. We cannot possibly learn anything new by analyzing these types of concepts (about nature). According to Kant, knowing things a priori necessarily involves our understanding's conformity to these laws, not the other way around (that is, we do not go around just positing laws of nature…). Furthermore, knowledge of nature a posteriori, or through experience, is impossible because this would indicate that we could have such cognition about things-in-themselves.[45]

What then, is the ground for the possibility of science? The key is mathematics applied to the appearances.[46] Kant claim is that this formulation allows the possibility for objects of *inner* senses as well as objects of *outer* senses. How is it

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[44] Ibid. [4:294]

[45] Ibid. [4:294]

[46] Ibid. [4:295]

possible to come to such a conclusion? It is because we are concerned with the *sum total of all objects of experience*. Thus, cognition about what could not be objects of experience would be "hyperphysical." That is this cognition would only hold in thought, not in application. It is thus, that we are able to come to hold that objects of experience are a priori possible and precede all experience. [47]

After we have this possibility for science, there seems to be a lack of clarity as to what exactly these objects we have thus mentions are about. In sections §21 & §21[B][48], Kant provides tables which are subject to the universal conditions of intuition: Namely, the "Logical table of judgments", the "transcendental table of concepts of the understanding," and the "Pure physiological table of universal principles of natural science." These are supposed to be the foundational principles as the ground for the possibility of knowledge. These tables not only guarantee knowledge, but they guarantee synthetic judgments a priori. That is, this foundation Kant has provided is not only knowledge a priori (prior to any experience), but we can also build upon this knowledge (thus the synthetic element). This is the cornerstone of a natural science.[49]

Experience and reason are well connected, according to Kant, but it is not always the case that reason leads us to judgments about the world which are true. The third section of the *Prolegomena* deals with this specific question. How is it that

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[47] Ibid. [4:295-296]

[48] Ibid. [4:302-305]

[49] The last two questions about the possibility of metaphysics, and metaphysics as a science follows the same argument Kant has given thus far. In order for us to have any type of knowledge about the world we must have a ground upon which synthetic a priori knowledge is possible.

reason can make mistakes, and yet, we are to have an objective system about nature (and the world) which will hold independently of experience? This question seems to follow from the two previous questions about mathematics and science. It is not clear, Kant states, that once we have established the objectivity of mathematics and natural science, we are better because the objectivity of such things is *for its own sake*. The purpose of metaphysics, for Kant, is

> namely the occupation of reason merely with itself and the acquaintance with objects that is supposed to arise immediately from brooding over its own concepts, without needing the mediation of experience or in any way being able to reach that acquaintance through experience.[50]

Mathematics and natural science exist for themselves. Therefore, it is of no use for us to be able to see that they exist in this independent manner. What is interesting is to be able to come to know things that are objective, but that depend on the structure of reason as it exists in humans—that is Kant's project, to provide "the grounding for a science which is to contain the system of all these cognitions a priori [ideas which correspond to objects in the world], that without such a separation metaphysics is absolutely impossible, and at best random […]"[51] In order to see that Kant provides a ground for the possibility of synthetic knowledge a priori, we need to go back and see that if we thought that metaphysics consisted of describing "things-in-themselves," we would merely have analytic propositions to ground our knowledge for science (or any other knowledge for that matter).[52] One of Kant's clearest statements about the

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[50] Ibid. [4:327]

[51] Ibid. [4:329]

[52] Reshotko's response to this once again is to say that although we are not able to know the forms of things in the world, we clearly see that some people are on better epistemological grounds than others;

grounds for the possibility of knowledge was to divide our realm of inquiry into the realm of understanding (space & time), and the realm of Reason (as a things that deals with *all* objects including those that are not inside space & time).[53]

Kant furthers his explanation in sections §46-§49 of the *Prolegomena* on the psychological ideas. Previous to Kant, there was a confusion of the formulation "S is P." Descartes for example, as already mentioned, could only hold the validity of the "S;" and while others could not even do this (Hume),[54] Kant's formulation of ideas is allows for the formulation "S is P" to hold a priori. This is not merely by assuming away the validity of S, or P, but through stating that our knowledge depends on the agreement between ideas (in the understanding and in Pure Reason), and things in nature.[55]

The last part of Kant's *Prolegomena* deals with the questions about the possibility of metaphysics in general and metaphysics as a science. The answer to these questions should be clear at this point. It is only possible to know objects of possible experience. Any formulation that deals with objects outside the realm of

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this could be a criticism of Kant, but it is not necessarily clear that Kant disagrees with this—even in Kant's formulation of the grounds for a science, we are clearly in a position to make errors, and Kant does state this about the understanding [that is, reason restricted to space & time]. The way for us to disentangle mistakes of the understanding from knowledge of things as they appear to us is to have already established an ontology of objects of possible experience. This way, we are able to rely on fundamental principles (i.e. the principle of contradiction) as the ground for the possibility making objective epistemic claims.

[53]Ibid. [4:329], §41-§42 make this clear, Kant states, "all pure cognitions have this in common." That is, all pure cognitions exist a priori as possible objects of experience, in space & time. Section [4:331] also makes it clear that the task of deriving metaphysics is for us to keep a list of things that are possible to experience, but, with the help of mathematics and natural science.

[54] Although Hume does assume that there something *to whom* things in the world appear…

[55] Ibid. [4:329]

experience will deal with objects about which we can only speculate, but cannot know.[56]

[56] See sections §50-§56; section §56 deals with the Theological Idea in which Kant makes this point starkly. It should be highly stressed that the first two antimonies presented about space & time; and about the world make the whole of Kant's view on epistemology even clearer. The first antimony states that either: (thesis) space & time have a beginning, or (antithesis) space & time are infinite. In the first case, we are dealing with Kant's realm of the space & time as boundaries for experience; in the latter case, we are dealing with things-in-themselves (because it is not possible to have the infinite as an object of possible experience). Some versions of Platonism respond to Kant by showing that objects outside space and time are knowable (e.g. Full-Blooded-Platonism).

3. Ontology of Mathematics

Kant's conception of mathematics as the ground of knowledge cannot be overlooked when talking about mathematics. Kant argues that mathematics is constructed due to the capacity for experience with which we are equipped. For Kant, mathematics is the truest case of synthetic judgments a priori par excellence. Ontology of mathematics refers to what type of entities mathematics things are. For Kant, objects in nature are part of how humans are and how the world is. The Kantian/Newtonian model gives us a view of the world in which there is integration between the "pure forms of intuition" (space & time) and the physical world. The way we conceive of mathematical objects and mathematical structures affects our understanding of these objects. Kant's argument for the "construction" of mathematical objects within space and time, for him, assures that we have epistemic access for mathematical objects. The competing camp in the ontology of mathematics against "constructing" our knowledge of mathematical structures and objects goes back to Platonic Idealism. This section presents different ways that the mathematics can be conceived, furthering and challenging Kant's vision for the structure of knowledge and mathematics lying at the foundation of knowledge.

3.1 Gottfried Martin, Kant's Ontology of Mathematics[57]

Gottfried Martin's *Arithmetic and Combinatorics, Kant and his Contemporaries* gives a detailed account of the history of mathematics and its attempt to ground mathematics as a pure science. It seems that all mathematicians follow a platonic line of thought to ground mathematics. It is only through Johann Schulz that

[57] Arithmetic and geometry

we have a correct interpretation of Kant's philosophical insight about knowledge and its dependence on the forms of intuition.

This question is dealt with in "The Axiomatics and Logic of Mathematics." The main problem (already raised in the *Prolegomena*) seems to be that the axiomatization of mathematics is a conceptual/analytic form of grounding. Our epistemology depends on assumptions about concepts, from which we can derive our proofs.[58] Martin cites five different ways in which mathematics can be derived (starting with axioms):

1) Arithmetic and geometry depend on axioms and are constructive in structure. Kant
2) Arithmetic and geometry depend on axioms but are deductive in structure Jakob Friedrich Fries (1773-1843), Husserl, Hilbert, Peano, Zermelo, Johann Friedrich König (1798-1865)
3) Arithmetic and geometry can be deduced purely logically, both principles and the theorems. Leibniz, Wolff, Hermann Günter Grassmann (1809-1877), Russell, [Alfred North Whitehead (1861-1947), Wittgenstein
4) Arithmetic is logically deductive, geometry is axiomatically constructive. Practically all the great mathematicians of the nineteenth century followed Gauss in making such a distinction between arithmetic and geometry […]
5) Arithmetic is logically deductive, geometry axiomatically deductive. Frege, Vloemans.[59]

What is important to notice here is that there is a disagreement about the way in which our grounding of mathematics should take place. Virtually all 2-5 go against Kant's view. Martin's states that we could easily classify all these formulations in Kantian terms of *analytic* and *synthetic*. Any logistic or axiomatic formulation will

[58] See Martin, Gottfried, *Arithmetic and Combinatorics, Kant and his Contemporaries*, Ch. 1, pp. 3-10.

[59] Ibid. pp. 6-7

fall under analytic statements.[60] Any constructive or deductive formulation will fall under synthetic propositions.[61] Martin points out that Kant is not well known for his thinking about mathematics, but we see that Kant has great insight not only for the axiomatization of mathematics but also philosophy in general.

Kant's *The Concept of Negative Quantities* (1763) shows the relation between numbers of opposition. It is not that a "negative quantity" exists.[62] With respect to mathematicians, Kant states:

> The concept of negative quantities has long been used in mathematics and it is also of the greatest importance there. Nevertheless, the ideal which most have gotten of it and the explanation they have given is astonishing and contradictory, although no inaccuracy has arisen in application, for the particular rules replaced the definition and guaranteed the use; but what may have been mistaken in the judgment about the nature of the abstract concept has remained useless and has been without consequence[63]

Thus, even though mathematics is derived through methods 2-5 (see above), we still see a great deal of misinterpretation about the status of negative numbers.[64] Kant clearly states that negative numbers are merely the opposition in quantity. The clearest example is in wealth and debts, regarding which, the relation ship is one of

[60] Recall above our formulation of Kant's view of a science. If it is the case that we are to have a reliable ground for the possibility of a science, it is not the case that we can derive our entire epistemology merely from analytic propositions. In the case of Kant, axioms are properties of the structure with which we form our understanding, thus, axioms are not merely statements derived from logic.

[61] Ibid. pg. 7

[62] Ibid. pg. 54

[63] Ibid. pg 55, Kant's commentary from *The Concept of Negative Quantities*, reproduced in Martin (1985 [1972]).

[64] Given the deduction of "negative numbers" for example, "negative numbers" are numeric entities, not a quantity relation.

"canceling out." The interpretation that negative numbers are entities on their own comes from the lack of consideration of the transcendentality of numbers.[65]

"Combinatorics and the Idea of a Systematic Ontology" discusses the way in which we are to build an ontology through which we can describe the world as it appears to us. The main works in the history of mathematics where we find an attempt to derive an ontology of mathematics is Leibniz's *De arte Combinatoria*. This work presents the way in which our knowledge is made up of elementary concepts and complex concepts.[66] These concepts form knowledge through a system of signs (that is of the form "S is P"). Martin clearly points out that Leibniz's system depends on the existence of elementary concepts. Furthermore, it is not clear that Leibniz thinks that there are synthetic propositions (where the S is already contained in the P—i.e. the sum of the interior angles of any triangle sum to 180°).[67] Recall Kant's rigorous derivation of what a science should be. The foundation of any science is made up of analytic and synthetic propositions; these propositions make up knowledge. There are stark differences between Kant and Leibniz though: with regards to the distinction of appearances of things in space & time and "things-in-

[65] Ibid. pp. 55-58, Martin gives a detailed account of Kant's insight into irrational numbers (which includes π and e) and imaginary, or complex numbers (√-1). Mathematicians simply ignore what this could possibly mean in terms of philosophy…

[66] Ibid. pp. 60-61, Leibniz's attempt to derive all our knowledge depends on basic *definitions* once again (recall Kant's insistence on making a distinction between appearances and "things-in-themselves." Leibniz's systematic ontology depends on analytic judgments (à la Kant). The problem with Leibniz's systematic ontology is that it is supposed to describe all our knowledge of things through the various combinations of "elementary" and "complex" concepts. Leibniz states that there is a universal science (sciencia universalis) which is made up of the art of signs (ars characteristica). Thus, simple and complex concepts are in line with this coupling of the art of signs which make up a universal science (pg. 61).

[67] Ibid. pg. 61

themselves," Leibniz makes reference to "things-in-themselves," whereas Kant clearly thinks this is impossible to do.[68]

Thus we come to Kant's assertion that any ontology, if it is to have any solid grounding upon which we can make epistemic claims, must be derived from fundamental axioms, which stand on their own, but connect the human mind to the outer world of experience. In the *Critique of Pure Reason* Kant states:

> Pure *synthesis, thought of generally*, gives us the pure concept of the intellect. By this synthesis, however, I mean that which depends on a base of synthetic unity a priori. Thus our counting (above all seen in the larger numbers) is a *synthesis* according to concepts, because it is done according to a common base of unity (e.g., the decimal)[69]

Thus, synthetic judgments in arithmetic (and in general) depend on an "Allness" which provides the unity of concepts upon which knowledge stands.[70] This "Allness" can be described as Kant's "Schematism of the pure concepts of understanding." The schematism connects apperception to pure concepts. That is, to synthetic propositions a priori, in this case, arithmetic propositions.[71] Gottfried Martin notes

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[68] Leibniz's definition of a monad comes to mind; see "The Principles of Philosophy, or the Monadology" in his *Philosophical Essays*, pg. 213, a Monad—"a simple substance that enters into composites—simple, that is, without parts […] [a]nd there must be simple substances, since there are composites […]" Leibniz goes on to define the mind of the Monad as substance that can have knowledge of God (29, pg. 217). What is even more startling is that Leibniz equates God with knowledge (47, 48, pg. 219). It is clear from Leibniz's conception of knowledge dependant on God that Kant's critique of philosophy aims to surpass this type of idealist ontology. Leibniz clearly states that there is no difference between "things-in-themselves" and things as they appear to us. Leibniz goes even further and states that we as Monads participate in the "best of possible worlds" that depends on God [its architect]. Kant's objection is that we cannot know this.

[69] Kant, Immanuel, *The Critique of Pure Reason*, trans. F. Max Müller, B104.

[70] See Martin, Gottfried, *Arithmetic and Combinatorics, Kant and his Contemporaries*, pg. 86.

[71] See Kant, Immanuel, *The Critique of Pure Reason*, trans. Norman Kemp, B176-B187. For Kant, schemata represent the unity of apperception with appearances of objects in the world (space & time) (B179-B180). These schemata, Kant states, are products of imagination, but it is not imagination in the ordinary sense, it is imagination as it is connected with objects that are actually appearing in the

Kant's obscurity[72] in trying to work out the exactness of schematism with regards to mathematics, specifically, to numbers. Martin's states that we must think of the concept of number as "inseparable from the perception of time."[73] Thus, the schematism allows the senses to be connected to the outer appearances in unity. It is ultimately this unity which allows us to have synthetic judgments a priori (in particular those of arithmetic[74] and geometry[75]).

3.2 From Kant to Contemporary Views on the Ontology of Mathematics

This section deals with current views in the philosophy of mathematics. We draw extensively from Balaguer (1998) who states that there are two equally defensible philosophies of mathematics. These philosophies of mathematics are

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world (That is why we are able to have an objective reality that is stable, without having to fall under a Humean, or Berkeleyan trap).

[72] In this regard, Kant has been attacked.

[73] See Martin, Gottfried, *Arithmetic and Combinatorics, Kant and his Contemporaries*, pp. 87-88. Objections to Kant seem to be attacks about Kant's "vagueness" and "obscurity." It is true that Kant's chapter on the schemata is difficult to follow. This does not mean that we revert back to Leibnizian logicism and think of numbers as having to necessitate a divine mind, given their eternal validity (Leibniz himself refers to this formulation as divine mathematics (mathesis divina).

[74] Martin gives the example of "7+5=12" and refers to the *Prolegomena*. In addition, Kant states in the *Critique of Pure Reason* (trans. Kemp, B15-B16): "We might, indeed, at first suppose that the proposition 7+5=12 is a merely analytic proposition, and follows by the principle of contradiction from the concept of a sum of 7 and 5. But if we look more closely we find that the concept of the sum of 7 and 5 contains nothing save the union of the two numbers into one, and in this no thought is being taken as to what that single number may be which combines both. The concept of 12 is by no means already thought immediately in thinking of the union of 7 and ; and I may analyze my concept of such a possible sum as long as I please, still I shall never find the 12 in it. We have to go outside these concepts, and call in the aid of the intuition which corresponds to one of them, our five fingers, for instance as Stegler does in his *Arithmetic*, five points adding to the concept of 7, unit by unit, given the five of intuition. For starting with the number 7, and for the concept of 5 calling in the aid of the fingers of my hand as intuition, I now add on e by one to the number 7 the unites which I previously took together to form the number 5, and with the aid of that figure [the hand] see the number 12 come into being."

[75] A similar kind of analysis can be done with geometry.

essential to economics to serve as foundations for what is and what is not plausible in economic theory.

3.2a Mathematical Platonism

We have seen that in Kant's formulation of science depends on the senses as a vehicle for understanding. Mathematics, for Kant, allows us to "know" things independently of our ability to grasp what is at hand—but Kant's analysis can only take us so far. Kant's "refutation" of idealism is not a refutation of Platonism altogether. Furthermore, our analysis of the interrelation between natural science and mathematics has only been carried in terms of geometry and arithmetic. Although, for the purposes of epistemology this is fine, we need to further analyze the consequences for the ontology of mathematics of the later developments in mathematics—namely set theory. Mark Balaguer's *Platonism & anti-Platonism in Mathematics* gives us a nice way to bring both Kantian and Platonic adherents in the philosophy of mathematics under an equally defensible ontology[76] (and epistemology) of mathematics. Balaguer states that there is a "fictionalist" account of mathematics which holds that statements like "3 is a prime number" is a fictional statement—the entity of the number 3 need no necessarily exist. The Platonist variant of this is that the number 3 does in fact exist. Balaguer's analysis allows us to view two ways in which a mathematical ontology (the nature of mathematical "entities")

[76] The Kantian version ultimately gets called "fictionalism" and the Platonist version FBP (full-blooded-Platonism). Balaguer ultimately thinks that in terms of ontology and epistemology, these are the only defensible views—and ultimately what he calls "isomorphic" (or equivalent).

and a mathematical epistemology (the acquisition of knowledge of these entities) can be carried out in a non-question-begging manner.[77]

The first half of *Platonism & anti-Platonism in Mathematics* deals with defending Full-Blooded-Platonism (FBP). FBP is the theory that mathematical objects exist only insofar as they are logically possible.[78] For this task, Balaguer has to answer to typical objections to Platonism in Mathematics (as mathematical objects). The main argument against Platonism is the epistemological argument which states that:

1) Human beings exist entirely within space-time.
2) If there exist any abstract mathematical objects, then they exist outside of space and time

Therefore by CTK [Causal Theory of Knowledge][79],

3) If there exist any abstract mathematical objects, then human beings could not have knowledge of them.

Therefore,

4) If mathematical Platonism is correct, then human beings could not attain mathematical knowledge.
5) Human beings have mathematical knowledge.

Therefore,

6) Mathematical Platonism is not correct.[80]

Balaguer proceeds to give a taxonomy of strategies to call into question the validity of the epistemic argument against Platonism. Ultimately, what Balaguer want to do is defeat all versions of Platonism except the FBP version. The first strategy is to call into question the validity of the CTK. It is not clear that the transition from (2) to (3)

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[77] For a further breakdown of Platonism in mathematics not covered below See Mark Balaguer, *Platonism and Anti-Platonism in Mathematics*, Introduction, pp. 5-11.

[78] See Mark Balaguer, *Platonism and Anti-Platonism in Mathematics*, pg. 5.

[79] The causal theory of knowledge states that "in order for a person S to know p, it is necessary that S be causally related to the fact that p in an appropriate way (ibid. pg. 22).

[80] Ibid. pg. 22

follows smoothly. One possible view is that we can attain some knowledge of these non-spatiotemporal objects via *mathematical intuition*. The question now becomes how this is possible. We have to argue for the immateriality of the mind. This view, however, is unintelligible, because the "transfer" of knowledge takes place between non-spatiotemporal objects (which are causally inert) and the minds in space and time. What this amounts to is focusing on the immateriality of the mind—as separate from the brain, which could allow for a type of cross-realm communication. This is a simple argument to block (1) but of course not enough. What ultimately needs to be developed from this is an ontological thesis which states that there exist real mental states which are irreducible to merely physical states.[81] If it is the case that the mind is spatiotemporal, Balaguer argues, statements and thoughts are not only reducible to physical states, but to Turing Machines—that is causally connected statements by reductive algorithms. If our minds are physical, then we can reduce minds to machines. It is arguably the case that our minds are irreducible to machines (for this has consequences in all realms of philosophy and other sciences). Balaguer here is presenting Kurt Gödel's view on what mathematical objects might be, and how it is possible to have cross-realm contact with such objects.

Gödel's view was that the purpose of the mind is through intuition to come to know what "reality" was like in forming thoughts, through the senses, about the appearances of reality. Thus, for Gödel, it seems that reality is connected through the

[81] Ibid. pp. 25-26 Balaguer rightly points out that this could take another book on refuting Cartesian-dualism which we are not to do in this setting anyway.

spatiotemporal "reality" in some way.[82] We have called into question (1) but we must move onto the rest of the argument.

The next move for Balaguer is to try to reject (2). Although this takes quite a bit of effort, we shall only provide the mode of ("naturalistic") Platonism represented by Penelope Maddy. This view states that it is possible to have knowledge of abstract objects through sense perception.[83] Maddy's view amounts to correlating sense perception with the notion of *sets*.[84] Balaguer has a discussion about the way in which sets are defined in the philosophy of mathematics—one can define sets in the traditional abstract sense where one can state the categorical relationship among objects inside a set. The second definition of a set deals with a "naturalized" variant which deals with defining sets in terms of physical objects (further divided into singular and aggregate). Thus the set of 1 egg, or many eggs represents a singular set, and the latter an aggregate (indefinite) set. For the purposes of this paper, Balaguer concludes that it is not possible to correlate singular sets or aggregate sets to mental states (which make reference to the way in which we perceive each set differently). For Maddy, it seems that she wants to overcome the epistemic problem of knowledge by asserting a perceptual mechanism by which we are able to come in contact with mathematical objects (namely sets). The problem raised by Balaguer is the way in

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[82] Ibid. pg. 28

[83] Ibid. pg. 28

[84] In the philosophy of mathematics, the notion of the set, and how a set is defined has non-trivial consequences for the kind of the set theory at hand. For example, the definition of set's given by Gödel gives rise to a "naive set-theory." Naive set theory is "groundless" because it relies on "axioms of logic" which give rise to inconsistencies in this "naïve" conception of set-theory. On the other hand, the Iterative conception of the set corrects the "groundlessness" because it is non-reductive metaphysically (See George Boolos' "The Iterative Conception of the Set," pp. 486-488).

which we divide the singular set from the aggregate set is problematic when we start talking about mental states (and mental stuff in general).[85] We have failed to reject (2) at this point, Balaguer calls on other forms of Platonism to try to do this.

The whole of these other arguments, as Balaguer suggests, are different ways to say that there can be knowledge without contact. The first is a truly innocuous position which states that we do not need an explanation for how we can acquire knowledge of non-spatiotemporal objects. This view of course tries to avoid the problem by not addressing it.[86]

The No-Contact Theory of Intuition (NCTI) which states that we can get knowledge of mathematical objects via "intuition."[87] This is an internal view that is problematic because it "merely restates the problem."[88]

Next on the list we have holism and empirical confirmation of Willard Quine among others. Quine's view states that we have good reason to believe our mathematical theories are true because they are central to our worldview. This includes having those mathematical theories being confirmed empirically (by using them in our scientific theories). Balaguer clearly states that the problem with confirmation holism is that it is outright false. Empirical findings do not *prove*

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[85] See Mark Balaguer, *Platonism and Anti-Platonism in Mathematics*, pg. 34.

[86] Ibid. pg. 35-36

[87] "Intuition," we have already seen, as used in Kant, deals with connecting the whole of the cognitive apparatus (beyond one person) to be able to come to knowledge which is in space and time. The notion here is slightly different, not worked out much more than to refer to a cognitive apparatus—but, the sense in which NCTI Platonists use this term leaves a gap between the abstracts objects and the "apparatus." This is exactly the problem that Kant foresees in the *Critique of Pure Reason*, although there is obviously debate as to how well Kant formulates his "solution."

[88] Ibid. pp. 38-40

theories. With respect to our worldview, confirmation holism does not speak to the "nominalistic content" of our scientific theories. "Nominalistic content" refers to the use of non-mathematical language in order to express the theory at hand—even if the ontology is deemed "fictional" (mathematical objects are constructed). Balaguer cites the "nominalization of quantum mechanics" as and example of this way of a "nominalization" which challenges that mathematics behind quantum mechanics can be expressed simply by logical relations. Balaguer's point is that the use of highly technical mathematical language in quantum mechanics does not prove that the mathematical theories behind the mathematical language are true. The question here is the expression of the theory in terms of language; the 'truth' of the theory clearly does not depend on the method used (as in FBP).[89]

The "necessity" view bears similar problems to the conformational view. The necessity view states that we can acquire knowledge about mathematical objects through our senses, and this knowledge is necessarily true. Taking the example of adding two numbers in arithmetic we can construct an entire "knowledge" of numbers according to this necessity view. It seems that this view is an attempt at restating (although badly) what Kant points to, and Gottfried Martin further develops. The only necessity in connection to arithmetic is dealing with what Kant called analytic statements (from which we cannot learn anything). Analytic statements must hold independent of how we construct our theory of arithmetic. Balaguer's objection is very much Kant's, we cannot have knowledge of things merely by positing a

[89] Ibid. pp. 40-41 more on the nominalization view with respect to "fictionalism."

definition and deriving a system of "logic" from these definitions (this amounts to a tautological account of mathematics).[90]

All these versions of Platonism are what Balaguer calls non-plenitudinous forms of Platonism (that is, these forms of Platonism want to commit to some forms of mathematical objects and not others) which fail to satisfy all the epistemic challenges posed mainly by Paul Benacerraf which deal with the objection against Platonists that knowledge of mathematical objects is not possible without contact. This attack does defeat the various versions of Platonism that we have been briefly talking about. The next step in the argument is to defend FBP against the Benacerrafian epistemic challenge.

As defined above, Full-Blooded Platonism commits itself to the existence not only of mathematical objects, but all objects which are logically possible. In order to get epistemic access to this non-spatiotemporal realm, Balaguer claims that all we need to do is acquire knowledge of a purely mathematical theory which is consistent, acquiring knowledge of this theory is acquiring knowledge of the mathematical realm. This is a seemingly simple response to the Benacerrafian challenge. If we ponder about Nepalese villages that exist, we can say that there exist Nepalese villages. Now, if we can take all possible Nepalese villages that could exist, Balaguer points out that one cannot have knowledge of these villages, but they do exist. "Dreaming up" the Nepalese villages might be another objection that can be raised to

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[90] Ibid. pg. 42, Balaguer comments that the "necessity" based view is an uninteresting view which deals with the knowledge problem trivially—that is by stating certain axioms which might or might not help the building of knowledge. Kant clearly deals with this problem by separating the types of propositions that are made (synthetic a priori, analytic a priori, and synthetic a posteriori). See note 74 above for a concise refutation of such a view.

the FBPist. According to Balaguer, the FBPists simply can respond by asserting that this "dreaming up" because of FBP, all possible Nepalese villages *do* exist. That is, the FBPist commits to the existence of *all* possible Nepalese villages—this is the position by FBPists—every Nepalese village that has the logical possibly of existence, actually exists. We saw earlier the other forms of Platonism cannot make this claim (i.e. naturalistic forms of Platonism can only claim certain types of objects but not other types). Thus, a similar move can be done for mathematical objects. The next set of objections comes obviously from the fact that we can "dream up" anything; what about beliefs and reference, it clearly is the case that we can't just "dream up" any old story. To block this worry, we need to do more work on FBP.

The "logically possible and consistent" assertion by FBPists has to be worked out more in detail in order to be able to claim a defensible stance. Balaguer points out the two different ways in which statements can be 'about' something. These statements 'about' something can be "metaphysically thick" or "metaphysically thin." "Metaphysically thick" statements are 'about' something in which the subject has to be "connected" to the object "in an appropriate way."[91] What this amounts to is that the statement has to be mapped in the space/time grid. Furthermore, there are "metaphysically thin" statements 'about' something. These statements deal with subjects and objects which need not be necessarily real—Balaguer's example is a little girl's statement that "Santa Claus is fat." This is a consistent, coherent, statement which does not violate any 'logic;' but the fact that this statement is a "metaphysically thin" statement, is of value for the FBPists—reference about

[91] Ibid. pg. 49

mathematical objects (and others as well) need not be "metaphysically thick." In fact, reference to mathematical object can be used in the "metaphysically thin" sense to ease some worries about the "dreaming up" of objects in general. If we are to defend FBP, Balaguer states, it has to come about through this route. Through metaphysically thin statements, Balaguer, states, we are able to come to defend FBP. The full argument for FBP is given as follows:

> (i) FBP-ists can account for the fact that human beings can—without coming into contact with the mathematical realm—formulate purely mathematical theories
> (ii) FBP-ists can account for the fact that human beings can—without coming into contact with the mathematical realm—know of many of these purely mathematical theories that they are consistent
> (iii) If (ii) is true, then FBP-ists can account for the fact that (as a general rule) if mathematicians accept a purely mathematical theory T, then T is consistent
> Therefore,
> (iv) FBP-ists can account for the fact that (as a general rule) if mathematicians accept a purely mathematical theory T, then T is consistent.
> (v) If FBP is true, then every consistent purely mathematical theory truly describes the mathematical realm, that is, truly describes some collection of mathematical objects
> Therefore,
> (vi) FBP-ists can account for the fact that (as a general rule) if mathematicians accept a purely mathematical theory T, then T truly describes part of the mathematical realm.[92]

The fact that mathematicians believe a theory does not mean that the theory will be consistent, and the fact that a theory is consistent is not enough to believe it. The argument given here for FBP is to show that as a theory of mathematical objects, FBP can account for things that other versions of Platonism are not able to account for. In order to defend FBP we need to look at the main objections leveled against FBP.

[92] Ibid. pp. 51-52, the proof for (i) is given in the last few pages (31-33), if (ii) and (iii) are true, the rest of the argument follows. Thus, we need to fully defend FBP against possible objections.

The first thing to note about FBP is how it serves as a metaphysical foundation to mathematical theories. One of the main set theories in mathematics is that developed by Zermelo and Fraenkel (denoted ZF set-theory)[93]. Balaguer states that FBP services ZF set-theory much better than other forms of Platonism. Balaguer cites two types of ZF Set-Theory, one where the continuum hypothesis[94] is true (ZF + CH) and one where the continuum hypothesis is false (ZF – CH). Other forms of Platonism are limited in talking about the mathematical realm based on how those forms of Platonism are formulated (naturalistic Platonism is restricted to what we can say about the natural world). Since both set theories are consistent (that is, whether we take the continuum hypothesis to be true or false) we have two distinct theories which describe *different* universes of sets. FBP is able to account for this, whereas other forms of Platonism scramble to reformulate their commitments.[95]

The next worry is to about the term "consistency." We define consistency as relating to a theory that is logically possible and non-contradictory in its description of the mathematical realm. Thus, there is a worry that FBP entails a shift in the term "consistent" to mean "true" tautologically. That is, FBP commits itself to a theory that is true, immediately after it is deemed consistent. The response to this worry deals with providing an explanation on how we are able to restrict our domain of our mathematical theory to truly describe the mathematical realm. This is done by seeing

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[93] ZF set theory is a mathematical theory about universes of sets founded on axioms. Set theory is the foundation of how calculus is possible (among other forms of mathematics).

[94] The continuum hypothesis states that there is an unlimited continuum of universes, thus, there exists an infinite number of spaces that lie side by side.

[95] Ibid. pp. 58-59

how mathematical theories are grounded (in natural numbers, for example). By seeing how mathematical theories are grounded in a natural way, we can then give a more comprehensive definition of consistency. Consistency is that which is logically possible and also partially grounded in a natural way.[96] This type of consistency allows us to posit type-token relationships metaphysically in mathematics (although this also extends to other areas of study where metaphysics is called for). FBP allows for multiple responses to "open questions."[97] FBP allows multiple isomorphic[98] (equivalent) theories. This, of course is subject to rigorous research and discovery.[99]

If FBP does so well being the house of multiple equivalent theories, there might be a problem establishing how to weed out the "bad" theories from the good ones. Balaguer here refers us to the use of "standard models" of mathematics. These standard models are axiomatically grounded models that have been developed based

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[96] Ibid. pg. 61, Balaguer speaks of a Language L which is grounded in a natural way that relates to mathematical theories that actually describe the mathematical realm. What should be noted here is that FBP covers more mathematical ground than other forms of Platonism. Furthermore, FBP can better account for distinctions between actual and fictional statements which are logically consistent. Other forms of Platonism are not able to do this because they are foundationally committed—thus, the only way to incorporate a logical fictional account (a novel) vs. an actual logical account (set theory) is to keep referring to the axioms of logic upon which *each* of these areas is founded.

[97] Ibid. pg. 62, Open questions are items in the philosophy of mathematics for which no singular truth can be ascribed. The example here is once again ZF set theory where the continuum hypothesis is true or false. Balaguer states that traditional Platonist accounts of mathematics "dictate" open questions (pg. 63).

[98] In the case that mathematical theories are not equivalent, we might have to do more work to determine if those theories describe different parts of the mathematical realm (as in ZF+CH and ZF-CH).

[99] A more detailed discussion of "consistency" in general, with links between logic and mathematics is given in Ibid. pp. 69-75. Balaguer describes the foundation of mathematics as relying in intuitive, non-contradictory logical observations on which the more complex mathematical statements are formed. A theory is "consistent if it is semantically and syntactically consistent." That is, if our notion of consistency is intuitively valid (we have good reasons to rule out skeptical arguments), then we have a *possibility* to develop a sound mathematical theory.

on our limited contact with the mathematical realm. This limited contact with the mathematical realm, Balaguer states, is enough ground mathematical theories. Thus, the research that mathematicians do requires them to build upon existing "standard models." This "building upon" allows mathematicians to extend other theories by extending "standard models" or by building new models based on "our intuitions, notions and conceptions."[100]

If we rely on our intuitions, notions and conceptions, one might say, as Balaguer states, that "there is no number 7." According to FBP, this is truly describes the mathematical realm. The question arises when we speak of the *entire* mathematical realm. A sentence like "there is no number 7" could amount to a mathematical theory with a whole in the sequence of natural numbers which is *part* of the mathematical realm, but it could not amount to a theory about the entire mathematical realm, since it is contradictory with our current notion of mathematics, and in particular, natural numbers.[101] Thus, FBP is truly committed to logical statements which truly describe the mathematical realm. Statements like "there is no number seven" might sound challenging to FBP but they are not interesting metaphysically.[102]

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[100] Ibid. pg. 65

[101] Ibid. pg. 67, one can object that since we are dependent on the notion of our current mathematical theories "being right" we might never be able to counter such sentences. Thanks to Professor Urquhart for this observation.

[102] Another interpretations of this sentence might be non-cognitivism about the whole of mathematics—in which case *all* theories are in the same position of indeterminacy with respect to being able to build upon axioms which depend on our notions of mathematics.

Another objection similar to the last one is to say that FBPists concede that sentences like "there is no number 7 and "2+2=5" truly describe part of the mathematical realm, yet, these sentences seem plain false. We notice that the objection is trying to attack FBPists inability to distinguish between theories. The problem here, Balaguer, states, is that, there might be a way to formulate a model in a "non-standard" way where these sentences truly describe part of the mathematical realm. But, when we use these sentences based on our current standard mathematical theories, these statements are false (in some absolute sense, and FBP is able to account for this by allowing the possibility to expand on open questions in mathematics). Balaguer states that we might be able to formulate a theorem where "2+2=5" or that '5' is equivalent to our current usage of '4.' This might be a confusing way to proceed with any argument to try to build a new mathematical theory.[103]

The last important objection to FBP is to say that FBP does not account for the uniqueness of mathematical theories. The non-uniqueness objection states that since we don't have unique definitions of numbers, we cannot possibly come to any conclusive description of the mathematical realm.[104] Since there is a contradiction in

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[103] Ibid. pg. 67; pg. 53 provides a brief explanation on the relationship between mathematics and physics. This relationship allows us to have "internalist" and "externalist" versions of mathematics. Thus, one way to counter the current objection is to say that our current use of mathematical language is also grounded on our observation of the physical world—the next step of the refutation would be to provide compelling reasons why we think mathematics truly describes the physical world. This would not be hard to do (e.g. Newtonian mechanics).

[104] See Benacerraff's "What numbers Could not be" in *Philosophy of Mathematics, Selected Readings*.

these this instance, FBP must be false. This is called the uniqueness objection to FBP.[105]

In "What Numbers Could not be," Benacerraf deals with the non-uniqueness of mathematical theories which states that mathematical theories cannot be reduced to a unique theory. That is, if it is the case that there are mathematical objects, and we are able to access the mathematical realm, why is it that mathematical objects can be described in the same manner? To make this objection clearer, let's look back at Kant's example of "7+5." This objection deals with breaking down the 7 and the 5 into their respective sub-components—that is stating 7 as "1+1+1+1+1+1+1" and 5 as "1+1+1+1+1." Another way to break down either 7 or 5 is into other sums. Thus, 5 can be expressed as "3+2" or "4+1" or "3+1+1" or "2+1+1+1" etc.[106] Thus, there is nothing in "7+5" that immediately leads to "12." The problem here is epistemic. According to Benacerraf, there is no way to come to any conclusive statement about the mathematical realm because of this problematic. The conclusion drawn by Benacerraf is a form of mathematical fictionalism (to be defended below) where our mathematical theories are "fictions," but these "fictions" truly describe the mathematical realm. Balaguer's response to the non-uniqueness objection to FBP is to assert that non-uniqueness is not really a threat to FBP. Looking at our Full Conception of Natural Numbers (FCNN), Balaguer states that the problem of uniqueness disappears when we look at our mathematical theories from a structural stance. Viewing our mathematical theory structurally, uniqueness is overshadowed.

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[105] See Mark Balaguer, *Platonism and Anti-Platonism in Mathematics*, Ch. 4.

[106] The set-theoretical formulation of the objection is given in Benacerraff (1983 [1965]).

This is not merely a shift of focus. If it was the case that a theory needs to have a unique reference for every object, then theories in other areas might be threatened.[107] Structuralism allows us to discern the main patterns at work in our mathematical theories. When talking about abstract objects, Balaguer observes an intimate connection between numbers in general, abstract objects, reference and language. It is not the case that we make reference to a chair in a singular way. The chair might have accidental properties which affect the way in which the chair is presented in the mind—there is not one unique way to conceive a chair. FBP can embrace this simply by seeing that this non-uniqueness truly describes the way we experience the world and reference concrete *and* abstract objects.[108] In the case of abstract objects, such as numbers, we can see that the number "7" for example, if referred to as "5+2" or any other combination instead of "7" can lead to skepticism. There might be an endless questioning of what "7" really means. In order to deflect skeptical arguments against our conception of numbers, Balaguer makes reference to our "full conception of natural numbers." This conception of natural numbers grounds our ability to come to knowledge, at least of numbers.

Thus far we have considered some of the main worries that we might have in holding the FBP view. Given the various objections to FBP, according to Balaguer, FBP is able to answer these objections and is the best version of Platonism. The main worry against Platonism, according to Balaguer, is that Platonism does not survive the

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[107] Metaethics, consumer theory, and action theory come to mind.

[108] Ibid. pp. 80-87, this conception of abstract objects (non-uniqueness of reference) allows us to account for complexities that might not be obvious in the world.

epistemic objections brought against it. But, as we saw, knowledge without contact is possible as long as the knowledge to which we make reference is "consistent." Other versions of Platonism do not survive the epistemic challenge because the do not commit themselves to *all* logically possible objects. The next section deals with fictionalism. Fictionalism will have to answer to the questioning of FBP, and if it succeeds, we will have two defensible metaphysical views on the philosophy of mathematics.

3.2b Mathematical Fictionalist (Anti-Platonism)

Fictionalism is a form of anti-realism. That is, it states that all mathematical objects that are referred to in our mathematical theories are "fictions." Furthermore, there is no really a "number 3." When we speak of the "number 3" we use this term only as a manner of speaking. The term "number 3" is a vacuous term. When we use it, it is simply used at *face value*.[109] Thus, fictionalism states that there aren't *any* mathematical objects, and the terms like "3" are vacuous terms.[110] Fictionalism states that statements like "3 is a prime number" can be accounted for in the 'story of mathematics' *only*. Like FBP, fictionalism relies on the development of mathematics to provide it with a framework in which the "fictions" hold. Like FBP, Fictionalists

[109] Ibid. Introduction, pg. 12.

[110] Fictionalism, like Full-Blooded-Platonism is the best version of anti-realist Anti-Platonism. Other forms of Anti-Platonism include 'psychologism' which states that mathematical objects are mental objects only (ideas); conventionalism which states that mathematical truth are true by convention, and mathematical If-then-ism which states that mathematical truths require axioms from which all mathematical truths are derived (from *necessity* of If-then statements). The objection to all these theories is that they are only internally consistent. When reference is sought to justify statements externally, these theories break down (ibid. pp. 11-12). These versions anti-realist anti-Platonism are rejected because they are based on controversial claims about the ontology of mathematical objects, unlike fictionalism, which takes the meaning of mathematical objects at "face value" (pg. 102).

must account for all the metaphysical worries about epistemology, and ontology of mathematics.

The main difference between anti-realists and realists (Platonists) is that anti-realists agree that 'facts' about mathematical objects are "true" by *invention*—that is mathematicians are free to "construct" theories within certain limitations; Platonists agree that 'facts' about mathematical objects are true in some non-vacuous way, and those 'facts' are *discovered*.

Our next task is to see why fictionalism is the best version of Anti-Platonism. One of the main views in philosophy of mind is referred to as psychologism, where mental objects (ideas) are true as they are constructed by us. These objects are mind dependent, so, if we all died, sentences like "2+2=4" would cease to exist (and become false) due to our deaths.[111] This is the main problem with psychologism, because statements like "2+2=4" are mind dependent, we are only able to construct truth. This is problematic if we want truth statements to be objective, independent of ideas created in the mind.

Another important view in the philosophy of mathematics that needs to be considered is realistic Anti-Platonism. This type of Anti-Platonism is different than other forms like psychologism, and conventionalism in that these other theories are anti-realist. According to Balaguer, realist anti-Platonism is famously attributed to John Stuart Mill. This view states that mathematics is the most general of the natural sciences. The truth of statements like "2+2=4" is accounted by us observing physical objects and intuitively seeing how when objects are lumped together, this gives us an

[111] Ibid. pg. 105

aggregate of objects—the ontology of mathematics under this view is strictly tied to physical objects. Thus, 2 eggs, coupled with another 2 eggs, would give us 4 eggs. This type of Realist anti-Platonism ties the truths of mathematics to physical objects. One of the main and fatal objections to this theory is its inability to account for different configurations of sets. Thus, this theory lacks the ability to account for the "stuffness" of objects. Adding eggs to chairs is the exact same thing as adding eggs to eggs. Thus, egg-stuff is commensurate with chair-stuff. According to Balaguer, hierarchical mathematical theories like ZF set-theory are able to account for these differences, simply unavailable to John Stuart Mill's form of realism.[112]

Truth's like "2+2=4" according to Mill are arrived by our study of physical objects. When we consider such sentences, it is objectionable that these sentences depend in any way to the relation of physical objects. For Mill, it is possible to derive an entire system of axioms just from observing physical objects. This is a bit of a stretch, because it requires the falsification of mathematical facts like "2+2=4" as a form of an empirical science. That is, facts like "2+2=4" would depend on our observation, and empirical methods (however sophisticated or antiquated they might be). Mill's project concerns itself about physical objects in the world, leaving the "metaphysical objects" out. Thus Mill thinks that we can arrive at a full description of the mathematical realm strictly from the physical world. This places enormous pressure on empirical methods, their predictions, and how that affects the "truth" of scientific theories. According to Mill, we are able come to truths based solely on our observation, and modeling of the physical world. In the case of simply accounting for

[112] Ibid. pg. 106

physical objects, Mill's project breaks down when dealing with large quantities (or extremely small ones as it is the case of quantum physics). If we take the statement that "2+2=4," we can derive the "truthness" of the statement by observing objects that indeed adding 2+2 we get 4. When dealing with large quantities, we have a harder time gathering the objects, counting, and deriving the truth of the outcome. For Mill, mathematics is a strictly empirical science.

Given all the problems that we have encountered, fictionalism can account for these because all that is said about our mathematical theories is not true. All other theories, not only state that our mathematical theories are true, but these "truths" depend on mind (psychologism) or on physical objects (Mill's scientific realism).

If we accept that fictionalism is the best form of anti-realist anti-platonism, our next worry is whether fictionalism can answer to the Frege's indispensability argument.[113] Thus, fictionalism would be false if it is not able to account for how our mathematical theories are indispensable to empirical science.

The main argument against fictionalism is Gottob Frege's indispensability argument which states that:

> (i) the only way to account for the truth of our mathematical theories is to adopt Platonism
> (ii) The only way to account for the fact that our mathematical theories are applicable is and/or indispensable to empirical science is to admit that these theories are true
> Therefore,
> (iii) Platonism is true and anti-Platonism is false.[114]

[113] Frege's indispensability argument states that mathematical theories are true because they are indispensable to our empirical theories.

[114] Ibid. pg. 95 Balaguer's move is to proceed to attack (ii) by showing that mathematics is not indispensable to empirical science; furthermore, the fact that mathematics is applicable in an 'indispensable' way can be accounted for without abandoning fictionalism (ibid pg. 96); Colyvan

It seems that Frege's argument relies heavily on (ii) to try to refute all versions of anti-Platonism. Balaguer shows that (ii) is false by stating that fictionalists can account for the 'indispensability' of mathematical theories in a similar manner to the FBPist. The fictionalist can state that the 'truth' of our mathematical theories, in sentences like "2+2=4" is part of the *standard story* of mathematics whereas other sentences, like "2+2=5" are *not* part of the standard story of mathematics.[115] There is another problem that arises for Platonists, as well for fictionalists. Platonists are in trouble because they are unable to point to the *relevant* applications of mathematics to empirical science. Balaguer's example is that if mathematics provides us information about Mars and Charles Manson, it doesn't provide us with what information is relevant about that particular relationship. Platonists can account for relevant applications of mathematics to empirical sciences by using a representational account of mathematics in empirical science. For, example, the way in which we use temperature depends on the real number line. The relations between numbers (degrees) are the same as the relations between numbers in arithmetic.[116] This is called a "representational account" of mathematics. Now, although this is a simple example, we can talk about other examples in physics, chemistry, and biology. The problem for Platonists is that this "representational account" will not account for *all* the applications in the empirical sciences. Fictionalists are able to do the same

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(2001) presents a more detailed argument for the indispensability of mathematics, but comes to the same conclusion.

[115] Ibid. pg. 99

[116] Ibid. pg. 111

because they are able to call on a "representation" to refer to the characteristics of temperature. This is called a representational mapping that is *homomorphic* to the relations of the empirical structure in question.[117] Balaguer simply shows that there are only some indispensable mathematical theories for our empirical sciences, and that these indispensable mathematical theories can be *nominalized*[118].

[117] Ibid. pg. 112, a homomorphism refers to a structure that replaces another structure and is equivalent to that other structure in its function.

[118] Ibid. Ch. 6, this chapter deals with the nominalization of Quantum Mechanics from a fictionalist point of view. Because QM can be nominalized, it is possible to nominalize other empirical structures which require the aid of mathematics, the only thing that we say differently under fictionalism is that these structures are fictions.

4. Economic Methodology

Section 3 dealt with providing a basic metaphysical framework of analysis in which mathematics is grounded. This basic metaphysical framework serves as the foundation upon which "economics" can occur—specifically, mathematical economics. With the rise of the mathematization of economics, it has been assumed that mathematics in general inherently has an indubitable logical foundation at its core. At first glance, it seems that this is the case, mathematics, does seem to provide a foundation upon which the acquisition of knowledge occurs irrefutably. The purpose of this section is to draw a correlation between the metaphysical foundations of mathematics, and its methodological implications in economics.

Kevin Hoover's review of four seminal works in the methodology of economics rightly points out *why methodology matters in economics*. This is precisely the title of his review. In it, Hoover provides a realist defense of the new classical programme of research for economics. This realist defense can be construed obviously in its platonic correlate. The four main works reviewed in Hoover's "Why does methodology matter for Economics" are Mark Blaug's *The Methodology of Economics: Or How Economists explain*; Daniel Hausman's *The Inexact and Separate Science of Economics*; Alexander Rosenberg's *Economics: Mathematical Politics or Science of Diminishing Returns?*; and Lawrence Boland's *The Principles of Economics: Some Lies my Teachers told me*. Blaug is characterized as a "falsificationalist" following the Popperian/Lakatosian tradition.[119] Blaug's definition of methodology (as quoted in Hoover) is:

> …a study of the relationship between theoretical concepts and warranted conclusions about the real world; in particular, methodology is that branch of economics where we examine the ways in which economists justify their theories and the reasons they offer for preferring one theory over another; methodology is both a descriptive discipline - "this is what most economists do" - and a prescriptive one - "this is what economists should do to advance economics"…[120]

There is a good reason to start with Blaug's definition of methodology. We get a rough idea of why it might be good to understand the "behind the scenes" working to uphold economic theorizing and applications.

Hoover's criticism of Rosenberg lies in portraying Rosenberg as merely philosophizing about economics, without generating a general critique about economics from particulars.[121]

According to Rosenberg, as rightly pointed out by Hoover's review of Rosenberg (1992), economics belongs as a branch of applied mathematics.[122] Rosenberg's critique of the current state of economics is its lack of progress between theorizing and predictability (two important components of a research programme – i.e. as seen in Newtonian mechanics). Rosenberg's attack on the current state of the economics enterprise[123] is precisely directed at economics-as-mathematical-politics. According to Rosenberg, economists use mathematics as a tool to justify statements

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[119] See Kevin Hoover, "Review: Why Does Methodology Matter for Economics?" Pg. 716

[120] Ibid.

[121] Ibid. pg. 726

[122] Ibid.

[123] This might be a controversial way of bundling up neoclassical economics and its correlates. Kevin Hoover demands that we do away with the term "neoclassical economics" and we rename it "New Classical Economics" in light of the "rational expectations" models in economics having to do with macroeconomic developments in the last century (for a detailed discussion see Ibid. pg. 729, and Hoover 1988, pg. 13).

about the economy which might not have very much merit. Thus, Rosenberg states that the "intentions" of economists aren't academic or scientific, but political. In Rosenberg's analysis, the "political" of economics can be characterized as an oligarchical dictatorship, where a set of economists have the only means (mathematics) by which economics is "properly done."

Rosenberg compares economics to meteorology. Hoover rightly points out that the state of technology in economics and meteorology have *both* improved, and therefore, Rosenberg's critique and comparison do not hold—that is, it is *not* the case, as Rosenberg claims, that the economics enterprise fails completely to predict economic phenomena. Economics doesn't fail anymore than meteorology to predict the weather. Hoover's "refutation" of Rosenberg's critique seems to lie in the fact that we *have* improved our method of collecting and predicting economic behavior.[124]

Rosenberg's critique goes much deeper than meteorology. A better way to critique Rosenberg's critique of the economics enterprise would be the fact that Rosenberg states that the theory of supply & demand is analogous to Euclidean geometry. Rosenberg's argument starts by stating that supply & demand are assertions about the economic space we observe. Rosenberg then compares supply & demand to Euclidean geometry. The last step is to show that Euclidean geometry is shown to be false given the developments in non-Euclidean geometry (mainly Einstein's work on relativity which makes use of non-Euclidean geometry to show that the Euclidean view is actually false).[125] According to Rosenberg, supply &

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[124] Ibid. pp 726-729

demand are "vacuous" as theoretical terms because they rely on something that has long been done away with in physics. Given that Euclidean geometry is false, why is it that we use a false structure to describe the economic realm? Our analysis in section 2 deals with arithmetic because it is the simplest (yet complicated) way to uncover the philosophical problems at hand. Kevin Hoover skips over this part of Rosenberg's critique, which is extremely valuable methodologically. Hoover's defense of the current economics enterprise is summarized with the following metaphor: it is better to have a house with a leaking roof, than no house at all.[126]

Hoover fails to capture Rosenberg's deeper insight and its effect on economic methodology. Rosenberg asserts that microeconomics (in which ultimately, game theory takes the front seat) is a 'maximal theory.' A 'maximal theory' seeks to "maximize" or "minimize" something within the theoretical framework. In the case of neoclassical economics, analogously to Newtonian mechanics, Rosenberg states that the model of neoclassical economics behaves in a 'maximal' way—the goal of the theoretical model is to "maximize" or "minimize" *something*. It is in this spirit that the individual in neoclassical economics microeconomics is built.[127]

Evolutionary biology presents an interesting case of comparison, according to Rosenberg. Evolutionary biology uses differential calculus to state that organisms

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[125] See Alexander Rosenberg, *Economics: Mathematical Politics or Science of Diminishing Returns*, pp. 239-241.

[126] We shall come back to this claim in later sections (this connection will be made explicit with regards to competing theories which do not take mathematics unqualifyingly (i.e. "innocuous falsificationism") as a foundational methodological device).

[127] See Alexander Rosenberg, *Economics: Mathematical Politics or Science of Diminishing Returns*, pp. 230-232, Neoclassical economics seeks to explain individual behavior this dual simplistic manner.

maximize "fitness." It is arguably the case that the use of a "maximal" theory fits evolutionary biology better than economics given that evolutionary biology does not make any claims about individual organisms behaving in a certain manner linked to mathematical formulations.[128] In this light, Rosenberg's critique can bear some fruitful conversation about economics. Hoover's simplistic "refutation" of Rosenberg seems to miss this point.[129] Thus, Rosenberg's insight about neoclassical economics, specifically game theory, is that it might not be as fruitful to apply principles of differential calculus to economics as these methods are applied in natural systems (physics, or biology).[130] Rosenberg points out that the success of evolutionary biology and physics do not give methodological credence to economics.[131]

Kevin Hoover's review proceeds with Mark Blaug's book *The Methodology of Economics, or how Economists Explain*. Hoover characterizes Blaug's methodology as belonging to the Lakatosian/Popperian tradition of "falsificationism." Falsificationism sets out to test hypotheses about scientific phenomena (and of course, economic phenomena as well).[132] Thus, Blaug's concern as a

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[128] Ibid. pg. 234

[129] It seems that Rosenberg points in the direction that propels this project. Rosenberg's seems to look comparatively at other systems in the sciences methodologically analogous (b/c of the use of differential calculus) in order to critique how economics as a discipline might hold up against these other theories with the use of the same tools—differential calculus in mathematics.

[130] In Neoclassical economics, then people, are captured at the same level of mathematical algorithms, which could describe perfectly well a system of nature. This point will become important when defining "individuality" with respect to intentionality. As we have seen, the game-theoretical assertion is that a general-equilibrium-evolutionary-biology-like maximal behavior exists in economic agents.

[131] See Alexander Rosenberg, *Economics: Mathematical Politics or Science of Diminishing Returns*, pg. 234.

[132] See Kevin Hoover, "Review: Why Does Methodology Matter for Economics?" Pg. 724.

"falsificationist" is that the economics enterprise does not "properly" test its theories—Blaug calls current economic theorizing "innocuous falsificationism." According to Hoover, Blaug's worry as a "falsificationist" is misplaced because one can only hope for "innocuous falsificationism."[133]

Blaug's Ch. 3 presents verificationism as prehistory of methodology of economics. Verificationism deals with a priori truths and how to verify them. What is interesting about verificationism is that there is inherently no attempt to work out a "methodology" in order to formulate hypotheses.[134] Verificationism assumes that theory of choice is true (by assumption), and the task of the researcher is to "verify" that the theory is true. According to Blaug, Adam Smith's case is of most importance for economics. Adam Smith attempts to give a version of verificationism especially in *The Wealth of Nations*.[135]

Another important figure in the history of economics is John Stuart Mill. John Stuart Mill's 1836 essay *On the Definition of Political Economy* which begins with a definition of economic man. According to John Stuart Mill, referring to the economic man as a whole would require a lot, and basically, economic man is too complicated.

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[133] This is in connection with the leaking roof metaphor provided by Hoover.

[134] See Mark Blaug, *The Methodology of Economics, or how Economists Explain*, pg. 52, among these early political economists are Adam Smith, David Ricardo, and Thomas Malthus. Blaug's point is that a priori truths were taken to be obvious enough not to have to state them explicitly. This is certainly the case in Adam Smith's *The Wealth of Nations*. Smith is a particularly interesting case because he does have an essay that is methodological on the philosophy of science—*The Principles which Lead and Direct Philosophical Inquiries; Illustrated by the History of Astronomy* which deals Smith's epistemology through Newton's *Principia*: 'certain principles, primary or proved, in the beginning, from whence we account for the several phenomena, connecting all together by the same chain' (as quoted in Blaug, ibid. pg. 52).

[135] Ibid. it is not coincidental that Smith has ties to Newton, and ultimately to the Kantian tradition. What is important to notice is the call of the a priori as a precondition for the methodology of economics by Smith.

Thus, John Stuart Mill suggests creating a "fictional" economic man—where the purpose of economic analysis (then political economy proper) is to get at the basics of this "fictional" economic man. Mill's methodological move to construct a "fictional" economic man allows us to analyze the way in which the "fictional" economic man's goals are to maximize wealth with as little sacrifice as possible.[136] Mill thus calls for an a priori method tied to verificationism. Maximizing wealth is not a "fact" we can verify. Insofar as the *science* of political economy goes, it fails to prove what we hypothesize as the behavior of the "fictional" economic man if we try to apply it to real world situations. The solution to this problem is that we should seek to verify our theories about our "fictional" economic man. Mill calls for a priori statements, which are then corroborated by a posteriori statements which are supposed to be the result of "philosophical investigation"—this very process of investigation is deemed verificationism. That is, we shouldn't try to *falsify* our economic theories. It is rather a method of verification that we should seek to verify our assertions made before any investigation is done (a priori).[137] Mark Blaug points out that this type of verificationism leads Mill to limit his analysis to "tendency laws." These tendency laws are what are at the bottom of the phenomena we observe in the world (whether in physics or economics). The importance in mentioning physics or other sciences for Mill, is that it is clear for Mill that there exist "Laws" for which we can apply the a

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[136] Ibid. pp. 54-56

[137] Ibid. pg. 57 Mill's example (in a passage quoted by Blaug) is friction in mechanics. Mill sees that we observe friction in the world. Our next step is to assert an a priori statement, we then observe something in the world that verifies our assertion (neither Mill nor Blaug state this but this is Mill's version of acquiring scientific knowledge).

priori/a posteriori formulation. This formulation 'comes into its own, not as a way of discovering truth, but as a way of verifying it'.[138]

What is important about this is the lineage in methodology that is clearly represented by *ceteris paribus* statements widely used in economics today. As Blaug states:

> [a] *tendency statement* in economics can be regarded, therefore, as a promissory note that is only redeemed when the *ceteris paribus* clause has been spelled out and taken into account, preferably in quantitative terms[139]

Furthermore, Blaug states that ceteris paribus statements are a "sort of catchall" statements for all that is unknown. In comparison to the physical sciences (physics, chemistry, biology), ceteris paribus statements are used in a restrictive sense which does not encompass the whole of the theory at hand.[140] Blaug further states that unless there is a restriction placed on these ceteris paribus clauses (the restriction amounts to "disturbances" on the system being analyzed), they are useless as attempts to blanket other "causes" in the analysis that is being carried out.

John Stuart Mill was aware of some of these issues which he tried to address in his S*ystem of Logic* (1843). Mill's system of logic tries to "demolish" Kant's foundational epistemology (that is how synthetic a priori statements can come to be).[141] Blaug's analysis of Mill's *Logic* renders Mill as an important figure in the philosophy of science, but rather than to label Mill's *Logic* as a theoretical account of

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[138] Ibid. pg. 58, Mill as quoted in Blaug (1992), Blaug clearly states the importance for Mill to relate natural laws to economic laws.

[139] Ibid. pg. 58

[140] Ibid. pg. 60

[141] Ibid. pg. 62

logic, Blaug wants to label it "an analysis of the scientific method relating to the evaluation of evidence." Mill's contributions to economics, in his *Logic* deal with the way in which logical induction is made. Mill's canons of induction are "a set of non-demonstrable rules of confirmation." Mill tries to solve Hume's problem of induction using his *four methods of agreement*.[142] Blaug further points out that Mill's indiscriminate use of the term induction "largely spoils" the rest of the *Logic*.[143]

Ultimately, Mill's "logic" neither solves Hume's induction problem, nor even attempts to "defeat" Kant's epistemology. Mill neither succeeds in stating how it is possible that we acquire knowledge following Kant's project; or how we go from impressions to knowledge following Hume's challenge to Kant. Instead Mill takes over Jeremy Bentham's utilitarianism.[144] Bentham's utilitarianism is founded on methodological individualism (that is, economic agents are atomic units in the economy). Mill's ultimate goal was to provide the methodological foundation for the social sciences in general. Mill did not call for a "descriptive" type of methodological individualism which attempts to describe how economic agents truly are in their "atomic" state. Mill attempts to develop a normative account the economy, and economic agents (in terms of methodological individualism). Blaug

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[142] Ibid. pg. 63, what Mill's system comes down to, according to Blaug, is an analysis of economic phenomena by presupposing a system of nature from which we can draw causal connections. On Mill's four methods of agreement, see Ibid. note 10.

[143] Ibid. pg. 62, note 9 makes reference to Medawar (1967, pg. 137) which states: 'Unfortunately, we in England have been brought up to believe that scientific discovery turns upon the use of a method analogous to, and with the same logical stature as deduction, namely the method of *Induction*—a logically mechanized process of thought which, starting from simple declarations of fact arising out of the evidence of the senses, can lead us with certainty to the truth of general laws. This would be an intellectually disabling belief if anyone actually believed it, and it is one for which John Stuart Mill's methodology of science must take most of the blame…'

[144] Ibid. pp. 63-65

states, "[e]ven those who are most sympathetic to Mill's economics agree that he was at best a lukewarm verificationist."[145]

In the section on verificationism, Blaug has one last segment on modern Austrians, among whom the notable are Ludwig von Mises and Friedrich Hayek. The case of Mises is of particular interest because of the poignant assertion that he is on Kantian ground. Ludwig von Mises was against any form of verificationism, or empiricism which looked anything like scientific research which had been taking place in the tradition of Mill among others. Von Mises central claim was that all human intentions were based on an underlying purpose (which is formulated in a radical mode of a priorism).[146] What is important to notice about the modern Austrians, especially von Mises is his reluctance to give in to the temptation of indiscriminate use of mathematics in economics.

Von Mises wants to follow Kant in his "synthetic a priori" method. Von Mises, however, falls short in his critique of the use of mathematics in economics. Von Mises thinks that mathematics is at the root of the problem in economic theory. Mathematics, as argued by Kant (section 2 above) is supposed to be the foundation of knowledge, and science. Thus, economists will agree that mathematics makes the analysis clearer by putting all the assumptions about the specific model on the table (whether economists think about the implications about the use of mathematics is a different story). Von Mises misses Kant's point in the *Prolegomena* with respect to the intimate connection between science, mathematics, and epistemology. The whole

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[145] Ibid. pg. 67

[146] Ibid. pg. 80

of Kant's project is due to this connection. Just referring to terms such as "synthetic a priori" and the like without any regard to Kant's original insight is misguided.[147]

Following the prehistory of methodology, Blaug gives an account of the methodology of economics in the 20th century. Many of the developments seem to be a continuation of Mill's project stemming from his *Logic* and his seminal essay *On the Definition of Political Economy*. As one of my professors used to say when teaching us advanced micro-economics: "it's the same story, I just ramp-up the math!" The same story goes for the methodology of economics, as mathematical techniques in economics become more sophisticated, the importance to maintain an eye towards ontological problems get's even more hushed up.

The main methodological techniques of the 20th century take several forms of falsificationism (this is to be differentiated to the rigorous falsificationism for which Blaug calls).

Paul Samuelson's 1949 *The Operational Significance of Economic Theory* brings the research programme of "Operationalism." Operationalism, according to Samuelson, deals with deriving 'operationally meaningful theorems.' A meaningful theorem, according to Samuelson, is 'simply a hypothesis about empirical data which could conceivably be refuted if under ideal conditions.'[148] Blaug analysis of

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[147] We shall not dwell on this point here, but notice that Kant's insight will be useful to address problems not only in science, but in mathematics and economics (as these are considered separate but intimately related areas).

[148] Ibid. pg. 87 Blaug notes that this formulation differs from the Operationalism in the Philosophy of Science laid down by Percy Bridgeman's *The Nature of Physical Theory* (1936) which deals with connecting "abstract concepts physical theory to the experimental operations of concrete physical measurement (ibid. pg. 88)." For another important statement similarly made in economics is Alan Gibbard and Hal Varian's "Economic Models" which states that capturing reality whole is an

Samuelson's "Operationalism" in particular is that it is undermined by the lack of clarity to specify a more fundamental mode of falsification—Blaug's characterization of Operationalism (at least Samuelson's brand) is to say that it is merely empirical.[149]

Samuelson's attempt to formulate a clear project for economists seems to fail in clarifying exactly the relationship between the theory and the falsification that is to take place given the theory at hand.[150] The whole of Samuelson's project is to study economic phenomena as they appear in the world by way of positing the economic phenomena's dynamic stability, then via "causal empiricism," provide proof of the economic phenomena as it actually is through the model that is posited. Samuelson's project aims at formulating a theory in a "realistic" manner. That is, a theory which deals with connecting the "real world" to the economic theories at issue.[151]

Milton Friedman's seminal "Essay on the Methodology of Positive Economics" (1953) comes precisely to the aid of aspiring economists. Although it is difficult to discern what Friedman "really" meant in this essay with regards to "positive economics" Blaug's statement about the essay will do for now.[152] The main

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impossibility, thus, we must use "caricatures" (models) that are essentially (instrumental) to our economic theories (pp. 665-666).

[149] Ibid. pg. 89

[150] Ibid. pg. 88 Blaug's example in capturing Samuelson's methodology is the Keynesian marginal propensity to consume. Samuelson's first premise is that we can establish the "algebraic sign" of the function—that is we can uncover a well defined negatively sloped demand curve. The second premise is that the marginal propensity to consume is dynamically stable. The proof, as Blaug states it is via an axiomatic framework (which allows us to use premises 1 & 2).

[151] Ibid. pg. 90

[152] Uskali Mäki's (forthcoming) "Reading *the* methodological essay in twentieth century economics: Map of multiple perspectives" makes this point rather clearly stating that unrealistic assumptions need not be a detriment to a scientific theory. Mäki's stance, however, is at the margin in terms of its interpretation of Friedman's essay. That is, Mäki is interested in seeing how Friedman's essay could

statement for Friedman's 1953 essay is the "irrelevance-of-assumptions thesis." This thesis states that the assumptions on which a theory is grounded (that is, the set of axioms which lie in support of the theory at hand) do not matter as long as the theory has explanatory power. A theory's explanatory power is how well the theory is "confirmed" by experience.[153] The term "Unrealistic" is to be stripped of any metaphysical connotation. "Unrealistic" simply means the way in which the hypothesis, and its assumptions are able to incorporate (and explain) the economic variables that the hypothesis is trying to explain in the world. Friedman rightly points out that the whole of scientific analysis is full of such "unrealistic assumptions." This point is worth exploring in a bit of detail. The example that comes under analysis is the perfect competition/monopolistic competition framework, and maximization of profit in economics. According to Friedman, when we assume that there is perfect competition in the (actual) market place, and that businessmen seek to maximize profits, we are merely stating "abstracting from reality." Abstracting from reality refers to the use of economic models in order to analyze what truly is going in the economy. According to Friedman, whether these assumptions are "realistic" has no bearing on our theory (that economic agents seek to maximize profit, and that the structure under which this maximization takes place is perfect competition). The Darwinian process of rivalry will assure that the agents who actually maximize will

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fit in a framework of scientific realism (how scientific phenomena behave under an objective structure). Mäki's analysis allows us to move beyond the segmented and, thus far unclear account of economic methodology.

[153] Ibid. pg. 91, this formulation is what comes to be called the F-twist. The extreme form of the F-twist is expressed in Friedman's essay as: 'to be important…a hypothesis must be descriptively false in its assumptions.'

survive.[154] Friedman's "methodology" even goes further than this. Blaug calls into question the way in which assumptions might be used to support (formulate) a hypothesis:

> 1) statements of motivation such as utility and profit maximization; 2) statements of overt behavior of economic agents; 3) statements of functionality and stability of certain functional relationships; 4) restrictions on the range of variables taken into account; and 5) boundary conditions under which the theory is held to apply.[155]

The methodological way in specifying each of these assumptions is not straightforward. One has to really look at the issues at hand (for which assumption 3 is useful). The purpose of these "assumptions," according to Friedman, is to allow economists a ground upon which they conduct their research without having to make any methodological claims. Milton Friedman's "irrelevance-of-assumptions thesis" merely ignores any need to deal with such questions—ultimately what this leads to treating business firms as "black boxes" from which all theorizing and hypothesizing takes place. This framework, allows for the theories about maximizing agents and perfect competition to be "unfalsifiable;" unfalsifiable theories are not subject to questioning, or disproof, they are simply working frameworks that can never be debunked.[156]

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[154] Ibid. pg. 92 This is an interesting point which definitely has been of much use. Minkler (1999) provides a detailed analysis of what "Darwinism" might entail in the atomist/individualistic economic agent. Minkler concludes that individualistic-type competition would have in fact killed off humans long ago. The "Darwinism" that has more potential is the group-type. Groups are essentially more able to carry out social plans (reproduction, governance etc.)…What is interesting is that Adam Smith himself sees this group link, although it works in a metaphysical way (as an "invisible hand.") Thus, it is not at all clear that the individualistic/atomistic "Darwinism" to which Friedman makes reference is substantial enough to allow Friedman's hypothesis of maximizing returns to hold.

[155] Ibid. pg. 94

[156] Ibid. pg. 96

Kevin Hoover's "Why does methodology matter for Economics," seems to agree with Mark Blaug's characterization of present day mainstream economics as a field that practices "innocuous falsificationism."[157] As stated above, "falsificationism" deals with empirical "testing" regarding economics (or any other type) theories in order to determine which theory is better. By "better," we mean to say that the theory explains the behavior of the phenomena which the theory intends to describe. Hoover's assertion about economics is that "innocuous falsificationism" is the best thing for which we can hope. If we are to be "innocuous" about debunking our economic theories, our intent regarding economic theories in general is not to debunk those theories but gather data which "fits" those theories (e.g. Rosengberg's charge on economics being mere "curve fitting.") Hoover, however, does not take into account Rosenberg's critique of mathematical economics as it relates to connecting theory with the world—a story clearly corroborated by Blaug's account of the methodology of economics culminating in Friedman's 1953 essay.

[157] Ibid. pg. 244

5. NEO-CLASSICAL ECONOMICS (OR SOME DERIVATIVE THEREOF)

Section 4 dealt with giving a brief account of the methodology of economics, it seems that one can draw a well defined (almost unchanged) line from the time of Mill up until the time of Friedman's 1953 essay. This section deals with the question about mainstream economics from a historical perspective, its lack of change, and its implementation of mathematics as a theoretical grounding.

5.1 Neoclassical Economics Defined

Neoclassical economics comes directly from Jeremy Bentham's Utilitarianism found in the *Principles of Morals and Legislation*. Secondly, Neoclassical economics is also directly linked to Locke's *An Essay on Human Understanding*. Although the nomenclature changes, the basic framework for economic methodology slightly changes to accommodate a strand of utility theory advanced by Bentham. The study of economics remains, in neoclassical economics, as the study of a natural system.[158] The evolution of economic theorizing has led us to rely on the mathematization of economic theory. As we saw in section 2, Kant's project sets out to ground knowledge in general, and particularly in science via mathematics. Thus, we have a good argument why this mathematization occurs. Almost two centuries later, Milton Friedman's *Essays in Positive Economics* sets out to derive a "value-free" economic methodology, and what a better way to do this than through mathematics.[159] This section revisits neoclassical economics to its past. In doing so, we see that this has specific implications for economic methodology.

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[158] This thought is due to Prof. Robert Urquhart and his course on history of economic thought.

[159] See Milton Freedman, *Essays in Positive Economics*, Essay 1.

Locke provides the perfect human psychology on which neoclassical economists are able to build models. The Lockean view deals with the inner subjectivity of individuals. This inner subjectivity is disentangled from the world.[160] Adam Smith's *Wealth of Nations* synthesizes the chaotic view of the individual (as in Hobbes) with the Lockean (more coherent) view. The *Wealth of Nations* does this by calling upon an invisible hand. Individual subjectivities come together in the market place, and through the aid of the invisible hand of the market, form a continuous social whole. Smith simply continues Locke's idea about individual subjectivity through his formulation of competitive individuals in the market.[161] Furthermore, for Smith, the interaction of competitive individuals in the market reveals an underlying psychology about individuals. Smith did not directly engage in trying to investigate behavior by individuals in specific settings (rather, the market arises as a natural system from the behavior of individuals and individuals' self-interest).[162] For Smith, self-interest creates social cohesion by creating interdependent economic relations among individuals.

John Davis characterization of Adam Smith is what Davis calls Scottish Enlightenment Dualism. Dualism refers to "mental states," and their correspondence with "activity" and objects in the world. Dualism has an underlying type of

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[160] See John Davis, *Theory of the Individual in Economics, Identity and Value*, pg. 24; Davis makes a clear distinction between the Lockean subjectivism from that of Hobbes (as chaotic), or even Newton's formulation, in which individuals might not even take part in the universe.

[161] Ibid. pp. 24-25 Davis, notes, that even though Smith provides a framework where the whole of society is affected by the invisible hand, Smith does not provide a framework as to how society as a whole might influence the market.

[162] Once again we notice the causal one-directional link.

metaphysics which seeks to point to every "correspondence" between what goes on in the brain, and phenomena in the world. That is, every act, and every object is linked to thoughts. Dualism does not provide a way to distinguish imaginary objects from "real" ones. This Locke/Smith formulation of the individual is taken up by *neo*classical economists.[163]

Neoclassical economists ignore any type of reference to philosophy, which would not only help economics deal with these types of problems, but also construct better theoretical models.

Neoclassical economics takes after Adam Smith but rejects everything but the way in which the market mechanism works. John Davis summarizes the neoclassical argument as follows:

1) Individuals are unique among all possible units of analysis in economic life because their behavior alone can be understood in terms of choice.
2) Choice can only be explained in reference to individuals' inner states, that is, their private tastes and desires, because this is the basis on which individuals discriminate among their options.
3) Individuals discriminate among their options because they apply marginalist principles.
4) Marginalist principles explain markets because they account for the determination of prices[164]

Any further development of mainstream economic theory resembles this formulation. Davis' characterization of neoclassical economics allows us to make explicit two points which are carried through the rest of our analysis of game theory (defined below). First, individuals are independent of their environment. Second, everything

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[163] Among them are in the 19th century - Jevons, Menger, Walras, and Marshall.

[164] Ibid. pp. 25-26

is explained by choices made by individuals in the economy. Using choice as a mode of analysis for the economy, neoclassical economists "solve" any problems dealing with the psychology of individuals. Neoclassical economics is "rigorously" defended with the rise of game theory. With the development of game theory, the attempt to solve problems about the economy given the complexity of individuals is merely reworded more sophisticated jargon, but the same neoclassical statements about individuals and the economy.

5.2 Evolution in Terminology, not in Economic Theory

One of the main platforms for neoclassical economics is the reduction of the individual to a maximizing agent. The main assumption about individuals in general is that they are "rational" with respect to their actions. This means that they follow the best set of actions which will best satisfy the agent's ends.[165] Individuals have 'objective functions' which they maximize.[166]

The nineteenth century saw the rise of neoclassical marginalism. Among the distinguished economists are Gossens, Jevons, Menger, Walras and Marshall. These economists held that the "individual's objectives are associated with the psychology of wants and desires."[167] John Davis states that the formulation of the individual does not change from Jeremy Bentham's formulation of the individual basically ruled by pain and pleasure. Neoclassical marginalism sought to take the individual's

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[165] Ibid. pg. 27, Davis notes that the theory of choice is agnostic about the ends of choices (we will see that this is a direct consequence of the incommensurability of value under a purely subjectivist internal theory of the individual).

[166] Ibid. pg. 27, Here Davis points out that the 'objective function' formulation is supposed to signal a departure away from the Lockean subjectivist view to the Newtonian object world.

[167] Ibid. pg. 27

"psychology" further by attributing a measurement—this theory receives the name of "cardinal utility theory." Cardinal utility theory states that everything can be measured quantitatively. The primary influence of this idea is Jeremy Bentham's *Principles of Morals and Legislation*.

For Bentham, human psychology was subject to the sovereign masters of pain and pleasure.[168] Individuals are under the rule of these "masters," and perform a calculus which seeks pleasure and avoids pain.[169] The calculus of pain and pleasure was an actual calculation, according to Bentham, to decide what course of action would be better depending on the amount of pleasure produced. For Bentham, as well for the early marginalists in the nineteenth century, this calculus was cardinal—that is, possible to measure as a quantity, such as weight and height.[170] Thus, neoclassical economics, in its early development resembles Bentham's cardinal utilitarianism.[171] The 20th century saw a different development in utility theory. The focus now was shifted to the ordinality of preferences—ordinal utility theory. Economists very quickly realized that utility was in fact not like weight or height.

According to Davis, Pareto (1979 [1909])[172] made the first step towards distancing economic theorizing from human psychology. Pareto's formulation is in

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[168] See Jeremy Bentham, *Principles of Morals, and Legislation*, Ch. 1, pg. 1.

[169] Ibid.

[170] See John Davis, *Theory of the Individual in Economics, Identity and Value*, pg. 27.

[171] Davis makes reference to (Maas 2001), "Mechanical Reasoning: William Stanley Jevons and the making of modern economics," PHD dissertation, University of Amsterdam.

[172] Pareto, V. *Manual of Political Economy*, Trans. A. Schweier, New York: Augustus M. Kelley.

terms of individuals' preference[173] combinations. For Pareto, and this development of neoclassical economics, rank ordering has nothing in particular in reference to individual psychology. The key to this type of analysis arrives at preferences from individuals that are "revealed" from individuals' choices.[174]

Hicks and Allen (1934)[175] develop Pareto's ordinal utility framework into indifference curves.[176] Robins (1935 [1932], 1938) goes as far as to make interpersonal comparisons of utility among economic agents (which lead to policy recommendations).[177] Davis points out that these particular statements of utility and the individual are part of what Davis calls "emotivist ethics." Emotivist ethics sets out to describe ethical judgments in terms of individuals' emotions or attitudes.[178] If we look at the case of either the Benthamite formulation of "human nature" with respect to what ought to be done ethically, we see that Bentham emphasizes quite clearly that ethical judgments are derived from "objective" concepts—pleasure and pain. This is not the case with the later formulations of utility. The naturalistic framework with which Bentham develops his utilitarianism seeks to arrive at ethical judgments from objective natural states (pleasure and pain). The nineteenth century

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[173] "Preference" means that goods can be rank ordered.

[174] See John Davis, *Theory of the Individual in Economics, Identity and Value*, pg. 27.

[175] See Hicks, J. and Allen, R. "A reconsideration of the theory of value," in Davis (2003) pg. 28.

[176] See John Davis, *Theory of the Individual in Economics, Identity and Value*, pg. 28. In consumer theory, indifference curves are supposed deal with consumer behavior in consumption of goods. This is the fully developed way in which economic agents are maximizing utility subject to a budget constraint—all this takes place of course in the ordinal utility framework.

[177] Ibid. the works by Lionel Robins are *An Essay on the Nature and Significance of Economic Science* (1935) and "Interpersonal comparisons of utility: a comment" (1938).

[178] Ibid. pg. 29

development of ordinal utility complicates the Benthamite formulation by asserting that ethical judgments come from individuals' emotions or attitudes. Thus, there is a shift from "nature," in Bentham, and the subjective emotions and attitudes of individuals.[179] If we are to derive ethical judgments from individuals' attitudes or emotions, we are only left with individuals' judgments about ethics and the economy. If we are to derive any objective framework which does not depend on the subjective attitudes of individuals, it is only through reference to that objective framework that we can arrive at doing so (see section 6.3 below). However, if we only state that individuals have subjective attitudes towards the ethical or economic realm, we are left with just those subjective attitudes towards the economic or ethical realms, without attempting to discern what the actual economic or ethical realm. [180]

The 20th century is the story of Paul Samuelson and "revealed preferences." Paul Samuelson undermines utility theory and moves away to revealed and axiomatic preferences. Samuelson's project is to have a theory of choice that does away with any reference to psychology and individual subjectivity (Samuelson 1938). Preferences do not dictate individual choices. Instead, choices "reveal" an individual's preferences (Samuelson 1948).[181] Paul Samuelson's project was to model economic theory to make it as "scientific" as possible by using models from

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[179] Ibid. pg. 29

[180] For a more detailed argument against "naturalistic" arguments in general, see Russ Shafer-Landau, *Moral Realism*: *a Defense*, Ch. 1.

[181] See John Davis, *Theory of the Individual in Economics, Identity and Value*, pg. 30, Samuelson's 1938 paper is "A note on the theory of consumer behavior" and his 1948 paper is "Consumption theory in terms of revealed preference."

thermodynamics.[182] Samuelson's influenced economics by restating the utility in terms of choice formally—that is mathematically. In "Some Psychological Aspects of Mathematics and Economics**"** Samuelson provides an argument for the complete mathematization of economics. He divides the "literal" economists from the mathematical economists. This division, he admits, is an arbitrary one. What Samuelson states is that it is possible to reduce economics to formal mathematics.[183] Samuelson's call for the non-mathematician is to become one.[184]

As Davis clearly points out, the critique of neoclassical economics does not stick because they are "not trying to explain human psychology." Instead their aim is to explain human choice. But in order to do this neoclassical economists resort need to start with economic agents and their exogenous preferences.[185]

Since neoclassical economics must be purely formal, they resort to "methodological individualism" to accomplish this. This, according to Davis, is the "last ditch defense" for neoclassical economics. The mathematization of economics gives rise to "methodological individualism" (where economic agents are homogeneous atomic agents).[186] What follows from this is that reductionism takes

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[182] Ibid. pg. 30, specifically equilibrium thermodynamics, according to Davis, Samuelson goal in formulating individuals interactions in the economy in terms of "equilibrium" resolve the problem of having to deal with the subjective preferences of individuals.

[183] Pg. 383

[184] The entire argument follows from the fact that mathematics is thought of as a pure science. As we saw in section one, it is not clear that mathematicians themselves know that they might be engaging in spurious metaphysics, so to say that just because one is a mathematician, one is a pure scientist is merely asserted.

[185] Ibid. pg. 31

place. Reductionism, in philosophy of science, states that if a theory can be explained by a simpler (more fundamental) theory, than the larger theory can be reduced to the simpler theory.[187] The criterion for reductionism is to translate the less fundamental theory into the simpler theory (via "bridge laws"). Secondly, all explanations in the less fundamental theory must be translated to the simpler, thus eliminating the unnecessary elements from the less fundamental theory, in order to express everything in terms of the simpler theory.[188]

New Classical Economics[189] (differentiated from neoclassical) is an attempt to be the ultimate defense of neoclassical economics by resorting to general equilibrium models. New Classical Economics stems is based on modeling general equilibrium in the economy based on individual preferences, endowments and technology. The well known Debreu-Arrow-McKensie general equilibrium model attempts to explain individual preferences, endowments, and technology (in the aggregate) via fluctuations in unemployment.[190] Davis concludes that these models make "global"

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[186] Ibid. pg. 35, "methodological individualism" comes from Joseph Schumpeter's 1954 *History of Economic Analysis*. Schumpeter's original term "sociological individualism" regarded individuals own actions as they translated in social phenomena. The term "methodological individualism" is derived from the latter—although Schumpeter thought that that it was incoherent to speak of a social whole as divided in to a totality of individuals.

[187] Ibid. pg 36, the example in philosophy of science is how Kepler's laws were a special case of Newton's laws, and Newton's laws were a special case of Einstein's laws.

[188] Ibid.

[189] See Hoover (1989) for a more detailed discussion, and the importance of this distinction. For the purposes of this paper, the difference in name does not really matter.

[190] See John Davis, *Theory of the Individual in Economics, Identity and Value*, pg. 37.

assumptions that are critical for the models, which cannot be derived from individual behavior.[191]

Davis' critique of the various derivations of neoclassical economics seems to be a core critique in philosophy of science. Davis presents an array of economic views which deal with the main line of economic thought to date. Davis concludes that the way in which economics is done by neoclassical economics overlooks questions about the nature of the individual and the economy. However harsh Davis' critique might seem to neoclassical economists, Roy Weintraub thinks that economists in general do not need to philosophy of science. Philosophy of science deals with the philosophical commitments and statements of scientific theories. Weintraub thinks that this type of analysis is at a level too high to have impact in economics' practice.[192] That is, economists should worry about the practical aspects of economics. "Philosophical questions" should be left to philosophers of science and methodologists. For Weintraub takes methodologists to have little to do with the "inside" economic practice. Weintraub's main targets are Alexander Rosenberg and Donald McCloskey. Rosenberg is seen as a mere "Methodologist." Weintraub takes Rosenberg's critique of Neoclassical economics (among other brands of economic theory)[193] to be mere "metatheorizing." Rosenberg's critique of neoclassical

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[191] Ibid. Arrow's 1968 work is "Economic Equilibrium," Rizvi's 1991 work is "Specialization and the existence problem in general equilibrium theory," Janssen's 1993 work is *Microfoundations, A Critical Inquiry*. Making "global" assumptions in principle cannot be done because one has to assume that individual behavior is "atomic." See Davis' discussion of general equilibrium models (ibid. pp. 35-38).

[192] See E. Roy Weintraub, "Methodology Doesn't Matter, but the History of Thought Might" pp. 481-482, and "Controversy: Axiomatisches Mißverständnis" pp. 1837-1847.

economics starts from the fact that neoclassical economics has not produced the same empirical results as biology or physics. These "results" are in terms of predictability of the phenomena that each of these is trying to explain. Since neoclassical economics fails to bring about the same results empirically, Rosenberg questions the credibility of neoclassical economics.[194]

McCloskey on the other hand, in *The Rhetoric of Economics*, argues that economics need not concern itself with epistemic problems (problems about how we acquire knowledge in general). McCloskey's argument is that all that matters is the "art of argument," and discussion which might arise from economics as a discipline. Weintraub states that both Rosenberg and McCloskey are simply "wrong." Weintraub comes to the defense of everyday economists (economists in practice) and states that any criticism by a philosopher of science lies "outside" of economics.[195] Thus, according to Weintraub, the work of philosophers of science is redundant and has no practical impact for everyday economists. Weintraub's defense of neoclassical economists (among the rest) lies precisely here. The outsider's perspective cannot

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[193] See Alexander Rosenberg, *Economics, Mathematical Politics or Science of Diminishing Returns?*

[194] Ibid. pg. 17 – Rosenberg states the goals of neoclassical economics –to use predictive power to asses the power of the theory & Ch. 3 discusses the predictive successes of neoclassical economics. Rosenberg's discussion of the predictive success of neoclassical economics is a rather nuanced one. Milton Friedman defends neoclassical economics in "The Methodology of Positive Economics." Friedman seems complacent about the lack of predictive success in neoclassical economic theory. On the other hand, noted nobel economist Wassily Leontief is too harsh on the same subject (Ibid. pg. 63). Rosenberg's view is that predictive success is not directly caused by the unrealistic assumptions by neoclassical economists—he sees unrealistic assumptions as unproblematic in general (there are cases in physics where this happens, but the predictive success is higher). Rosenberg sees that predictive failure due partly to the high value that is placed on econometric modeling and the resilience to increase data gathering methods which might increase the predictive success of the current econometric models (pg. 64).

[195] See E. Roy Weintraub, "Methodology Doesn't Matter, but the History of Thought Might" pg. 481 From Section II on the methodology of economics, we saw that Rosenberg's critique is a bit more deep than this.

permit philosophers of science to impact the methodology of everyday economists. It is clear that Weintraub thinks that economists stick pretty close to the ground (in terms of metaphysics), thus they are "safe" from any "metatheorizing." But, it is arguably the case that economists in general, including neoclassical economists make metaphysical claims without knowing. Even when these metaphysical claims are known, the metaphysical claims are narrow and at times implausible. Metaphysical claims by everyday economists take the form of "models" and these models should be analyzed thoroughly.[196]

5.3 Game Theory, Intentionality, & Welfare

Game theory has its inception in von Neumann and Morgenstern's 1944 *Theory of Games and Economic Behavior*. Von Neumann and Morgenstern stem their analysis of the individual agent from better wanting to understand the behavior of oligopolistic and monopolistic firms. From their analysis of these types of firms, von Neumann and Morgenstern state that individual economic agents are "bearers of strategies." Treating individuals as bearers of strategies gave way to another explanation of individual behavior, "supraindividual" institutions, convention and social rules.[197] In

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[196] Some economists, such as well known economist Gary Becker uses the term "metautility" in "Altruism, egoism, and genetic Fitness: Economics and Sociobiology." Furthermore, John C. Harsanyi takes notice of the problem of G.E. Moore's "ideal utilitarianism" as it relates to his "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility" (pg. 310). Harsanyi sees the complications of metaphysical claims and thus sticks to the familiar game theoretic formulation which follows the tradition of von Neumann and Morgenstern's game theoretical formulation of economic agents see below). For a further discussion on the role of general equilibrium models in economics see Rosenberg (1992), chapters 7 & 8. For a further critique of mathematical proofs, and their relationship to economic theorizing see Rosenberg (1986).

[197] See John Davis, *Theory of the Individual in Economics, Identity and Value*, pg. 38; supraindividual refers to institutions that arise with the aggregation of the atomic individual (Davis' terminology).

the tradition of Walrasian equilibrium, the Nash equilibrium is the outcome of the aggregate of strategies by individuals in the economy.[198]

John B. Davis duly asks if individuals considered as "bearers of strategies" do any better in explaining the individual, and whether avoids the problems in neoclassical economics. The first problem we encounter is with respect to "being a bearer of strategies." Davis points out that this characterization of the individual allows for a non-exclusive relation between the individual and the strategies. That is, rather than to specify a particular type of rational individual who will act in the economy in a certain psychological way, the "bearer of strategy" definition of the individual is not to be tied to any particular type of psychological behavior.[199] Thus, Davis concludes that this game theoretic characterization of the individual rejects Locke's conception of the individual as possessing a subjective inwardness.[200] A consequence of the game theoretic formulation of strategies allows game theorists not to give explanations as to why a player might pursue one strategy over another—

> Game theorists do not say *because* players have such-and-such type strategies, games will have certain outcomes. They ask, *should* players have such-and-such types of strategy, what outcomes might games have?[201]

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[198] Ibid.

[199] Ibid. pg. 39

[200] Davis simply passes through this point but, in rejecting the Lockean inwardly subjective individual and the problems that arise from this, game theorists adapt the strategy view which leads to a kind of mathematical idealism (individuals obey a set of strategies contained in mathematical formulations).

[201] Ibid. pg 39

This game theoretic formulation ultimately shows the ongoing agnosticism by neoclassical economists about any normative claim—and thus, by remaining in the "descriptive" realm, they "avoid" value-laden claims.

The problem of reductionism, according to Davis, remains in game theory. Reductionism refers to how theories might be "reduced" to simpler, more compact theories. The "bearer of strategy" formulation is an atomist-holist equilibrium analysis. Rather than to present an account of economic agents in connection to social institutions, conventions, and social rules, the game theoretic formulation assumes this connection. Game theorists depend on "consistent alignment of beliefs" (CAB) and common knowledge rationality (CKN).[202] What follows from CAB and CKN is that individuals with the same information prior to any game will come to the same conclusion.[203] In order to simplify the problem, consistent alignment of beliefs and common knowledge rationality must be asserted. Common alignment of beliefs states that agents will not have conflicting beliefs about anything; whereas common knowledge rationality states that when it comes to knowledge that is imposed on economic agents, the reaction of agents in relation to that knowledge would be the same. If agents interact with social institutions for example, these social institutions will have the same impact on economic agents because they all have "common knowledge" about these social institutions. Common knowledge is regarded as something that is known to all the economic agents. If economic agents act against

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[202] Ibid.

[203] Ibid. pg. 40; this seems to be an interesting claim by game theorists that individuals have the exact same epistemic access. It would seem that their unwillingness to postulate a direct epistemological theory would lead them to state otherwise, because there are inherent biological, social, institutional differences among individuals, they might not reach the same conclusions independently.

this "common knowledge," then the particular action of the agent is considered "irrational." Davis points out that the common knowledge rationality would follow nicely from having to deal with social institutions if individuals were actually as they are described in game theory.[204] Common alignment of beliefs is harder to imagine without a social foundation of some sort. Rather, game theorists stick to their individualistic "strategy bearing" agent models.[205]

John B. Davis provides an account of game theory at its general level, and although we get a general idea of what problems might arise, Davis' project has taken us far enough. Alexander Rosenberg[206] provides a more detailed account of game theory and rational choice. Prediction makes a science exact, and increase in prediction increases the accuracy of a science. In the case of economics, since it is dealing with human choice, it is harder to come to prediction as it is the case in physics and biology. Rational choice theory is the main founding block of game theory (and arguably neoclassical economics). Rational choice theory (in addition to what has been discussed above) as it is formalized in game theory deals with the choices of economic agents. Rosenberg, (as many others) points out that the idea that agents are "rational" is suspect. Rosenberg's conclusion is that economic agents are

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[204] Ibid. pg. 40

[205] Aristotle's *Politics*, Book I, Ch. 2, 1152a1; 1152a29 which states that man is a political animal, resonates with Davis' point about the need to see that the individual is viewed in terms of the whole.

[206] Ch. 5, "Economics and Intentional Psychology," in *Economics, Mathematical Politics or Science of Diminishing Returns?*

in fact irrational.[207] This is but the start of the line of problems that plague rational choice theory.

Daniel Hausman's *The Inexact and Separate Science of Economics* presents the rational choice theory and its basic foundations—utility maximization, the weak axiom of revealed preference (WARP), and rationality and uncertainty: expected utility theory.[208]

It is said that economic agents maximize utility (an economic agent's index of preferences). As Hausman, states, to state that agents "maximize utility" is to state *nothing* about the agent's preferences. All that this means is that we can connect an agent's preferences with choices—"Rational individuals rank available alternatives and *choose* what they most *prefer*."[209] Thus, the working framework for rational choice theory is:

> [agent] A's **preferences** are **rational** if and only if:
> (1) A's preferences are complete,
> (2) A's preferences are transitive, and
> (3) A's preferences are continuous.
> An agent's A's **choices** are **rational** if and only if:
> (1) A's preferences are rational and
> (2) A prefers no option to the one A chooses[210]

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[207] The definition of rational hinges upon an agent A desiring x to y and always being consistent in that desire, unless there is a individual's general change in preferences, thus this would change the ranking of preferences...A more detailed discussion to follow in the last section on value.

[208] See Daniel Hausman, *The Inexact and Separate Science of Economics*, pp. 13-27, these items are given after assumptions are taken for granted about completeness and transitivity (which deals with incommensurability of ranking options) (pg. 15); also taken, these items assume Debreu's proven theorem that the ordinal representation theorem which states that given an individual's preferences which are complete, transitive, and continuous, it is possible to derive a real valued utility function (pg. 18 in Hausman, Debreu's *Theory of Value*, pp. 54-59).

[209] Ibid. pg. 18, italics in the original.

[210] Ibid. bold in the original.

Hausman states that rational choice theory is a normative theory (despite attempts by some to unqualifyingly deem it descriptive because of its mathematical rigor)—"[t]o define what rational choice and preference are, is *ipso facto* to say how one ought rationally to prefer and to choose."[211] Hausman further states that rational choice theory may be taken as "positive." Even when rational choice theory is "positive," one still has the limitation of *rationality* as defined by rational choice theory. The level of generality of these preferences not only speaks to economic activity, but psychology as well.

Samuelson's 1938 "A Note on the Pure Theory of Consumer's Choice" attempts to formulate a completely positive account of rational choice theory, but instead of rationality, *revealed preferences* take center stage. In order not to rely on preferences and choices, revealed-preference theory comes in the picture (as already mentioned above, this takes our attention away from the actual agent and shifts it to his preferences that are "revealed." The weak axiom of revealed preference (WARP) serves as the foundation for revealed-preference theory.

The WARP indicates that if it is revealed that agent A prefers x to y, then it will always be revealed that x is preferred to y. Thus, the conclusion to be drawn from WARP is that if x is revealed to be preferred to y, then agent A must always choose x over y.[212] The obvious problems for this are pointed out by (Sen 1977)

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[211] Some examples include Samuelson's "Economic Theory and Mathematics—An Appraisal," (1952), and Friedman's *Essays in Positive Economics*, (1953).

[212] See Daniel Hausman, *The Inexact and Separate Science of Economics*, pg. 22.

where an agent might choose something that the agent does not prefer; or the agent's preferences might have changed altogether (Hirschman 1985).[213]

As if these problems aren't enough, Hausman points out that revealed-preference theory faces four further serious objections. (Harsanyi 1977) shows that if agent A chooses x when y is available, then y when x is available, then agent A is not irrational—agent A is indifferent between x and y. Hausman states that if agent A is indifferent between x and y, then it would take a series of trials before we figure out what the agent's preferences are, which violates the WARP.[214]

The second objection deals with considering agents as "bearers of strategies." In the language of game theory, economic agents are "players" which are found in "games." Thus, players make decisions in terms of the rules of game theory depending on preferences. Players might do something other than their preference to mislead other players.[215] Hausman example deals with the prisoner's dilemma game. The prisoner's dilemma is a game about two prisoners which are captured, then questioned in separate rooms. If they both "cooperate," meaning if they "tell" on the other prisoner, this is considered to be the best outcome because they will both get shorter sentences for cooperating. The game theory "prediction" states that the players will always want to "not cooperate" and incriminate on the other player. The

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[213] Ibid. pg. 22, Hausman's reference is Sen's 1977 "Rational Fool's," specially sections VI-VIII, and Hirschman's "Against Parsimony, Three Ways of Complicating Some Categories of Economic Discourse."

[214] Ibid. pg. 22

[215] This objection does not fully stick because, then the game theorist states that in this case, the expected value of the game must be calculated for the player to "randomize." That is, for the player to play something which is not a high preference, the player's expectations for payoffs must outweigh the cost of loosing if the player's "bluff is called."

player who tells on the other player then gets none punishment whereas the other player gets all of it. Hausman states that this game leads to "suboptimal outcomes." The conclusion that game theorists draw from the prisoner's dilemma is that prisoners do not cooperate (that is they stick to their story instead of turning themselves in), which is a worse outcome than if both cooperated (which is the case many times). Thus, Hausman argues, players' preferences are misconstrued. Hausman states that the analogy to be drawn from this game is about market failures. If there is a market failure, then the rational thing to do is to *cooperate*.[216]

The third objection deals with completeness. The completeness assumption deals with agent's rankings—*all of them*. That is, an economic agent is required to have a full rank of all the preferences available—and to be had (in the future)…as already stated, this complicates the already mentioned preference changes, and choices for objects which are not preferred.

Revealed-preference theory supposedly gets rid of the subjective economic agent. Revealed-preference theory fails to rid itself of the subject because economic agents have desires, and these desires lead to choices. The common thread among game theorists is to try to come up with a rational version of the individual linked to mathematical formulations. But as we have seen, this is done at the expense of a coherent, more plausible account of the individual.

The last thing to say is that economic welfare is strictly linked to the construal of economic agents as either "bearers of strategies," or economic agents as they are "revealed" by their choices.

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[216] Ibid. pg. 22

The next section deals with a different yet coherent theory of agents faced with choice in general. Game Theory is highly mathematized, and this serves as the only way to justify the conclusions drawn in economic theory.[217] Because there is much resistance in economics to moving away from mathematically intensive theories like game theory, our next task will be to show that other theories might be able to accommodate the problems that game theory as rational choice theory try to overcome by abstracting from agents, or by trying to axiomatize a system of choice.

[217] Ibid. pg. 26 A more recent development of game theory deals with risk and uncertainty – Expected Utility Theory. Hausman's diagnosis about expected utility theory is that practical agent decisions under risk and uncertainty are in principle too difficult to predict. Even when we are dealing with objective probabilities (which are controversial to Bayesian statisticians who state that all probability is subjective), the problem of risk and uncertainty has severe damaging effects to predict agent behavior. Hausman's point is that expected utility theory, as an extension of rational choice theory is practical in application.

6. CHALLENGING NEO-CLASSICAL ECONOMICS, VALUE & INTENTIONALITY, RECONSTRUCTING SOCIAL ONTOLOGY

One of the main attractions of game theory is its mathematical eloquence and rigor. Game theorists claim to be able to analyze problems about economic agents formally through mathematics and derive conclusions about economic agents. Game-theorists seek to explain universal economic behavior. The foundation of this "universal" economic behavior dates back to the principle of utility, as defined by Jeremy Bentham, and taken up unchanged by economists today (in its various forms); the principle of utility states that humans seek pleasure and avoid pain. The language of game theory is that economic agents seek to maximize outcomes in economic situations. This is *the* economic problem that each agent encounters. The task of each economic agent maximizes utility to *his/her* end.[218] In the history of economic thought, this maximization starts as quantitative, but because cardinal utility is internal to the economic agent, the problem is to figure out the whole of economic agents' preferences. If we try to figure out every economic agent's preferences, assuming that we are able to do this, then we are faced with a problem figuring out whose preferences will be "the best." Ordinal utility theory replaces cardinal utility theory. Ordinal utility theory relies on *orderings* of things. Game theory makes substantial use of ordinality to claim that individuals will choose what they prefer, that their preferences are well defined, and consistent (see above, pg. 78), and finally, the choices made by economic agents are rational choices. Rationality is strictly defined as optimization and maximization. In thinking about economic agents, game

[218] Adam Smith's *Wealth of Nations*, Book I, demonstrates that individuals act in their own self-interest, and in doing so, create a social cohesion which allows society to work in an orderly form. It is the work of the "invisible hand" which allows this social cohesion to work.

theorists neglect important aspects of individuals, social theory, and value. This section will show that there are other *relevant* non-mathematical theories about economic agents, society, and *intentions* that give rise to choices. Game theory as a theory of the individual, as a social theory, and as a theory of value is untenable.

6.1 **The Philosophy of Game Theory**

Game theory seeks to explain the whole of human economic interactions. It focuses on the economic agents' choices. These choices create a reality that can be analyzed through "games." The most famous example of a game is supposed to help us think about human behavior is the prisoner's dilemma (see pg. 80 above). The economic agent's task is to maximize positive consequences, as in classical utilitarianism.[219] The rise of game theory in the 1940's with Von Neumann and Morgenstern sought to "repair" economic theory. The "repair" was to be made by requiring "rigorous" strictly mathematical formulations. These mathematical formulations, as (Mirowski 2002) argues, are simply elaborate formulations without the least bearing on economic reality. Mirowski's argument is that game-theorists seek to relate economics to a game of automata[220]—that is, a computerized/mechanized mode of economic decision. Mirowski states that game-

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[219] The main statement on the subject is John Stuart Mill's essay on Bentham's utilitarianism, and his brand of utilitarianism. Bentham's utilitarianism is a naturalized utilitarianism where pleasure arises from natural properties. Mill's utilitarianism sets out to draw from Aristotle's virtue ethics. See Mill's *Utilitarianism and Other Essays*, and Bentham's *Principles of Morals and Legislation*, especially chapters I-X. James Griffins' "Incommensurability, What's the Problem" distinguishes further between different forms of utilitarianism vs. consequentialism—consequentialism being a broader class. In terms of what is 'good,' game theory could be classified in terms of one or the other.

[220] See Philip Mirowski, *Machine Dreams, how Economics becomes a Cyborg Science*, pp. 503-516, this section is entitled "Send in the Clone." The highly controversial claim that human behavior is reducible to automata, basically meaning that we can map human behavior with mathematical algorithms is the ultimate goal of game theory.

theorists seek to make economics a "cyborg" science-where a cyborg is an organism that is part machine—the connection to game theory is that game theory relies on mathematics to make statements about "human nature." Mirowski's argument is similar to the argument in here—simply taking mathematics and mixing mathematics into economic theory does not render economic theory true. In principle, economic theory should see to develop the integration of powerful computations methods backed by abstract mathematical statements.[221] In doing so, one also much consider what it means to have such a coupling—the "rigorous mathematics" with the abstract mathematical statements. In neoclassical economics, the coupling of mathematics to economic theory has become the standard way of theorizing. Mathematics becomes the source of knowledge and sole apparatus by which justification about economic theories becomes possible (and true). Mirowski insists that the only possible proper justification for game theory is for game theorists to be not only concerned with philosophical assertions within economic theory. The game theorist, then should be concerned about history and philosophy of science (as a way to make sure tenable ontological views are succinctly analyzed).[222]

When considering economic agents, game theorists are only concerned about the preference structure of agents. As we saw earlier, the preference structure of an economic agent refers to the types of things among which the economic agent makes choices. The preference structure to which game theorists make reference is always

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[221] Ibid., Ch. 6, pp. 341-349.

[222] Ibid. 472-478, Mirowski's example in this passage is regarding game theory and its emphasis on fixed-point theorems. That is, a mathematical proof for a "solution concept." A solution concept indicates where the whole of economic individuals' interactions comes to equilibrium.

tractable, and consistent.[223] Once the theory is justified with eloquent mathematical formulations, it is then that game theorists are able to make concrete statements about the welfare of economic agents. The economic welfare of agents (which is essentially *all* the welfare of agents) is characterized by the individual agent's preference structure.

The seminal paper by Kenneth Arrow shows that collective decisions are not easily derivable. Even if it is the case that agents have well ordered preferences, the whole of economic agents (society) is unable to come to a choice that is definitive.[224] Arrow's impossibility theorem shows it is impossible to come to a situation where *all* preferences by all of the members of society are satisfied. Game theory's statement on welfare is then to assume that economic agents are accepting of choices that the whole of economic agents face. Theoretically, if we accept game theory as an economic theory, we are granting the commensuration of all preferences—that is, we grant that our preferences are measurable, even if the measure is a set of orderings, the commensuration of all these preferences give rise to a question of value and preference. Game theorists, alongside with consequentialists, are able to claim that through maximization, and the coherent structure of preferences, economic agents are able to come to a definitive statement about the maximization of economic welfare for the whole of economic agents. What follows from Arrow's impossibility theorem

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[223] A study by Sippel (1997) in Mirowski (2002) presents people with real time choices about market decisions. The study shows that the axioms of revealed preference (see pg. 72 above) are constantly violated. This comes as no surprise since, the tractability (that is the way the problem is intended to be solved mathematically) depends on such axioms.

[224] This is Arrow's Impossibility Theorem, from "Values and Collective Decision making," pp. 121-125, which takes the axioms of game theory plus another few axioms which deal with collective choice to state that a situation where everybody's individual desire cannot be satisfied.

is that only by a voting system can preferences be satisfied. Arrow assumes a situation of "non-dictatorship" for the entire social system of preferences to work. This simply says that no single agent's preferences influence the preferences of all other agents in the economy. Thus, all the agents must be in agreement. This is the best situation of economic welfare. I.M.D. Little argues that even within a voting system, we are not guaranteed that the outcome be non-arbitrary.[225]

One further complication that arises about the impossibility of game theory (hinted at in section 5.1) is the constitution of the individual in game theory. Even though it seems strange to connect economic agents to automated mathematical algorithms, this is essentially the aim of game theorists regarding the individual.[226] The task of game theory is to explain individuals and the complexity of interaction among those individuals, coupled with the multiplicity in the world, but ends up being a "bricolage of algorithms."[227] What game theory actually does is reduce human behavior to maximization of "value." This maximization is easily formulated mathematically. This maximization is related to the rational choices by agents. These rational choices coincide with choosing a higher 'value.' Thus, it is irrational for an economic agent to choose a value that is less than another available 'value.' This is the extent to which game theorists treat value. It is instrumental to agents' decisions, but it remains abstract, and unanalyzed. The next section shows that

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[225] See I.M.D Little, "Social Choice and Individual Value" pp. 422-432.

[226] Philip Mirowski's *Machine Dreams, Economics Becomes a Cyborg Science*, captures the exact genealogy strictly starting from simple machines, a la Alan Turing, and the Turing Machine to the modern conception of Artificial Intelligence (See Ch. 2 & 7). The Turing Machine sought calculate simple finite algorithms (much like a simple calculator today).

[227] Ibid., pg. 516

further analysis of value leads to severe complications for maximization and choices based on optimization and maximization of value.

6.2 Value, Choice and Economics

What about value is so important to game theory, or even economics? Game theory depends on value as a quantitative scale upon which *all* things are measured as preferences. Value, in its most general sense, is far from constituting merely a quantitative scale. Value is generally related to some question about "goodness" with regard to an object or some set of events as in classical utilitarianism. The reason why value is important in a theoretical analysis in economics is that there are competing views on value which might affect what is said about economic agents (or ethical agents—whatever type of agents we speak about). Value theory seeks to answer some basic questions about value. First, the question of intrinsic value, something that has its "worth" based on its unique properties. Those properties are independent of any other properties outside of the object in question. Knowledge, for example, might be considered an intrinsic good.[228] Questions arise on whether there is one intrinsic good, or many intrinsic good—this is the debate between monists and pluralists about value. Jeremy Bentham, was a proponent of value monism.[229] John Stuart Mill on the other hand was a proponent of value pluralism. We have seen that both of these characters are important in the history of economic thought—both of them have impacted the neoclassical economics paradigm.

[228] See Mark Schroeder, "Value Theory" entry in the Stanford Encyclopedia of Philosophy, section, 2.2.

[229] Ibid.

Mark Schroeder explains that there at least three things that need to be accounted for in order to pursue any theory of value. The first one is ontology of value. Value pluralists argue that there are different types of values (knowledge and pleasure might be two distinct intrinsic values). Value monists argue that there is one and only one value.[230]

Whether one argues for value pluralism, or value monism one will have to deal with the what Schroeder calls "revisionary commitments." Given that monism is true, how might we explain that there are seemingly different values at work when, for example we talk about varieties of goodness?[231] A knife's goodness is different than the goodness of friendship. The monist might have to argue that the value displayed is no different in kind, but still remains the case how this non-difference in kind supervenes on two different cases of goodness. Pluralists on the other hand, have a different problem in that they state that there are different values. So they would no have a problem explaining why the knife is good vs. why friendship is good. Their problem arises when pluralists might have to add to their list of "values."

Thirdly, Schroeder talks about incommensurability as another obstacle for any theory of value to be enlightening. Incommensurability deals with values that are in conflict with each other. If monism is true, then incommensurability never happens. But, as Schroeder states, the fact that we live in a complex world, and the complexity of situations gives rise to doubts to how much monism holds. If we are to take a

[230] Ibid. section 2.2.1

[231] Ibid. section 1, and section 2.2.2

pluralist view about value, then there must be an explanation as to the status of conflicting values.[232]

Game theorists believe (and require) that things are measurable on a single quantitative scale. This includes the versions of game theory that construe value in terms of cardinal or ordinal utility theory. Cardinal utility theory states that tastes can be measured by actual numbers. Ordinal utility theory states that tastes need not be measured by numbers necessarily, but by rankings—we then refer to the array of possibilities and naming through which these two types of utility theory as interchanged with "revealed preferences," or with "rational choice." Game theorists attribute value (used extremely loosely without any philosophical connotations) merely to be represented by rankings, or numerically. We never get an actual theory of value except by assumption that given that the choices made by economic agents give rise to something which "pleases" the agent—going back to different forms of utilitarianism.[233]

Recall the definition of value of Jeremy Bentham places man at the mercy of two sovereign masters: pleasure and pain.[234] Bentham then provides a dichotomous, simple scale of value, that which is pleasurable is good, and that which is painful is bad. The purpose of humans is to maximize pleasure—and this comes about

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[232] Ibid. section 2.2.3

[233] We note that plausible forms of utilitarianism which analyze "psychological states" important to philosophy of mind are out of the question. These types of questions are left for the philosopher, psychologist, and sociologist to grapple with.

[234] See Jeremy Bentham, *Principles of Morals, and Legislation*, Ch. 1, pg. 1.

naturally since humans are already under the command of the two sovereign masters. This is simple construal of value which exists at the core of game theory.

If game theory is to have any credibility with respect to its value claims, it would need to take on the basic issues of comparability between at least two conflicting objects, bearers of value. The task of the economic agent in game theory is prescribed and reduced to the "preferences" of the agent already by design. We are told that agents choose among the best options available for them. This is clearly a version of utilitarianism. As argued in section 5.2 above, there is a clear evolution in the way the economic theory is presented by renaming old concepts—utility becomes preference, preference becomes "revealed preference."

Elizabeth Anderson's *Value in Ethics and Economics* gives a pragmatic account of value that will help us with our critique of game theory. Anderson's theory consists in an "expressive theory of value." People have favorable attitude towards what they deem valuable. Value, for Anderson, is intrinsic, as well as extrinsic. Extrinsic value deals with the character of something that is valued based on the characteristics of the object which depend on the person with the favorable attitude towards the object. Intrinsic value is a bit more complicated to deal with. Intrinsic value is a type of value that is independent of the person's attitude towards the object.[235]

Concrete examples of intrinsic value, according to Anderson, are people, animals, and communities. These, she states, we immediately care about and they are ends for which our actions are guided. Our attitude towards them is immediately

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[235] See Elizabeth Anderson, *Value in Ethics and Economics*, pp. 4-5.

known to be favorable. This makes people, animals, and communities intrinsic goods. We value intrinsic goods in themselves. What is intrinsically value is the object of rational favorable attitude.[236] Extrinsic goods are primarily means, and are tied primarily with states of affairs. As stated earlier, extrinsic value is dependent on some agent's attitude towards it. [237] States of affairs involve what are called instrumental goods, that is, these goods are supporting the ultimate end for which all action is being done. This distinction is important because states of affairs could be construed as intrinsic values. But it is easy to see that a view like this needs to collapse the difference between intrinsic and extrinsic value.

Elizabeth Anderson takes on versions of consequentialism, a variant of utilitarianism which states that people ought to do what brings about the best consequences for each individual. The best version of consequentialism, according to Anderson, depends on the use of extrinsic value as the main form of value. That is, the consequentialist view states that agents maximize a state of affairs that is [intrinsically] valuable for agents, thus, agents act in ways to bring about this state of affairs.[238] Under this framework, agents are also able to take into account the value of people, though only instrumentally.[239]

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[236] Ibid. pg. 21

[237] Ibid. pg. 19

[238] Ibid. pg. 22

[239] Ibid. pg. 28, a response to this is that there is no intrinsic good, that all there is extrinsic view. That is, value is derived merely from the favorable attitudes of agents. Furthermore, there is an omniscient, benevolent agent who maximizes welfare.

A more general description of consequentialism is as follows: 1) agents maximize intrinsic value, 2) intrinsic value is ascribed to a state of affairs, whereby the value is independent of peoples desires, intentions, actions, motives, 3) all these actions, intentions, motives are solely evaluated in terms of their consequences—how these consequences "embody the best states of affairs," 4) all values are agent-neutral—that is, if it gives everybody a reason to value it.[240] In game theory, as previously discussed, the goal of individuals is to maximize value—the form in which takes place varies in terminology.

Consequentialist theories justify action merely by showing that agents ought to do any action maximizes value.[241] Elizabeth Anderson looks at her own expressive theory of rationality to contrast actions under consequentialism vs. expressive theory of value. Under consequentialism, there is no direction to the intentions of the agent, except to maximize value via consequences. If we are evaluating the maximization of value, then the only thing that matters as an evaluative criterion is the consequence of the action and how well it relates to the best possible states of affairs. Thus, the agent's intentions about their actions are not directly linked—and are *not* a problem generally. Anderson's expressive theory of value states that agents have "rational favorable attitudes" towards intrinsic values. These attitudes translate into action, but the action is guided by ends. Under consequentialism, the agents are guided by consequences, which are intentionally or unintentionally maximizing welfare.

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[240] Ibid. pp. 30-31 an "agent-neutral" value maximization consequentialist theory is proposed by (Parfit 1984) where value is intrinsic and independent of agents.

[241] Ibid. pg. 33

Consequentialism requires that agents have one aim and one aim only: the production of consequences.[242] Consequentialist theories in general make intrinsic value immediately normative for desire, preference, choice, rules or action. This means that given a set of intrinsic values, agents are immediately drawn to initiate consequences that will bring about the maximization of intrinsic values tied preferences, desires, action, or rules all of which are derived from norms.[243] An important distinction between achieving consequences and intending ends (Anderson's theory) lies in the fact that those agents that maximize consequences will have not way to control the consequences. The consequences are either intended or unintended, for consequentialists, this will have no importance. Anderson's point is that when we try to evaluate an agent's intentions about a particular end, we will be in at odd determining what actually the agent's intentions were. Furthermore, if we are to maximize consequences only, it is not guaranteed that the consequences will always be the best state of affairs possible. Anderson argues primarily against this type of theorizing because of the lack of scrutiny of agent's intentions. Consequentialist theories are committed to having even the unintended actions which are actually bad, to coincide with the agent's "intention for the best consequences."

To give a clearer example borrowed from (Reshotko 2006), when we see an action, it is possible to trace what the ultimate intention of agents. For Reshotko, Shakespeare's *Romeo and Juliet* illustrates that all desire is for the "actual good."[244]

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[242] Ibid. pg 32

[243] Ibid.

In the case of Romeo, we see that he desires to be with Juliet. Given that he thinks that Juliet is dead, Romeo decides to drink the poison *thinking that Juliet is dead*. As the audience, we see that Romeo does *not* wish to drink the poison while Juliet is dead, thus he only drinks the poison thinking that Juliet is dead, therefore, Romeo only does what seems best, *and not actually what is best*. In retrospect, Reshotko argues that theories which ignore or omit desire commit themselves to stating that agents desire what seems best, and not actually what *is* best. In the case of consequentialist theories, cases like these are problematic because the best explanation we are able to give regarding human motivation is that we intended what is best without any mechanism with which to retrospectively analyze agents' intentions.[245] Anderson and Reshotko concur in investigating agents' intentionality to steer away from any abstract version of intentionality. Reshotko and Anderson agree that an agent's intentions must be accounted for with every action and choice that an agent takes. Urquhart (2005) further states that the point of having agents have real subjective tastes, preferences, and choices is part of being an individual, and no longer merely an atomic agent.[246]

Consequentialism's "advantage" over other theories, according to Anderson is its acontextuality—consequentialists can state that their "maximizing of value can

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[244] See Naomi Reshotko, *Socratic Virtue, Making the Best of the Neither-good-nor-bad*, pp 49-56

[245] Another important point that Reshotko makes about her reading of Socrates is that, with respect to consequences, intentions and ends, it is hard to have a serious evaluative attitude towards either of the three independently of each other. That is, these are connected in interesting, and complex ways, and by isolating consequences, means, or ends, results in an incoherent theory, and a misunderstanding of human behavior (ibid. pg. 64).

[246] See Robert Urquhart, *Ordinary Choices, Individuals, incommensurability, and Democracy*, Ch. 4 – "Taking things seriously."

take place in any context independent of history, thus acontextuality is ahistoricality. It is in this form that consequentialists are able to state their claims about maximization of value. The main problems, as stated by Anderson include:

> that [practical] reason can settle all questions about what to choose; that it requires the global maximization of value; and that the grounds for rational choice must be fully and decisively articulable, leaving no room for judgment and hence none for dispute.[247]

The first sentence states that everything is measurable under one scale, meaning, there are no instances of incommensurability. The second statement states that not only everything is measurable under one scale, *this is the only scale of measurement*—that is, it is a monistic theory of value. Consequentialists and game theorists[248] would dismiss Moore's non-natural monism as unattractive because it is overtly metaphysical. Another option for game theorists is to side with classical hedonists who take value to be reducible to pleasures—pleasures could also be construed as mental states where the levels of endorphin are measured, thus, the more endorphins, the more pleasure is experienced by the agent. [249] Reducing value to a mental state is a radical statement. It single handedly reduces all activity as being directed by pleasure—as the ultimate end. If all human intentionality is reducible to acts for the maximization of pleasure, then acts which cause pain, but which are only means to other ends, like running a marathon, are unexplainable, in this view.

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[247] See Elizabeth Anderson, *Value in Ethics and Economics*, pg. 45.

[248] Game theorists can be construed as consequentialists of a sort, and the same critique applies to both since both make value about what agents ought to do, or how agents actually make choices.

[249] Ibid. pg. 46, Anderson utilizes chapter 6, on monist theories of value, like Moore's aesthetic monism, and classical Hedonism (ibid. pg. 124).

While global maximization of value is untenable, Anderson states that it is still possible to defend a pragmatic theory of comparable value. A pragmatic theory of value deals with trying to commensurate things in terms of extrinsic value, and without assuming that agents maximize value globally. What ultimately must be claimed under this pragmatic theory of value is the need for a universal measure, only an objective standard which deals with comparing value. Some offer "brute preferences" as the justification of having universal scales which are objective.[250] Others justify the need to choose between two goods like money and life, "preferably in terms of money."[251] The seeming "advantage" of having such theory would eliminate the problems of incommensurability generally.[252]

The extent to which the comparative theory of value can be used is for athletic activities, which require "goodness-of-a-kind" judgments. Goodness-of-a-kind judgments are statements about of value of a certain activity, a triathlon. The properties of the athlete's performance are judged objectively by the set of rules that exist to do so. Because there are a myriad of athletic activities, Anderson states that the comparative theory of value necessitates a *plural* interpretation of value. That is, a theory of value that does not reduce value to a specific thing, such as pleasure, or an action or a consequence.[253] The second outcome of a comparative theory of value is that we are able to make sense of personal judgments of what option is best. Personal

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[250] Ibid. pg. 46, the example is Griffin (1986).

[251] Ibid. pg. 46, the example is Arrow (1967).

[252] Ibid. pg. 47

[253] Ibid. pg. 49

judgments about what is best are construed with a "multi-criterion evaluation" from impersonal goodness-of-a-kind judgments. These personal judgments constitute, but are not limited to athletic activities, but also incommensurable goods, or goods that cannot be deemed to be better than the other when compared.[254]

Elizabeth Anderson pragmatic theory of value does not go far enough in dealing with the problems at hand with value. She does start with a distinction of intrinsic vs. extrinsic form of value. She allows for incommensurability but, once again, her critique of consequentialism only goes so far. Anderson only asserts that when we have incommensurability, the objects in question might be in a sense "equal" to each other. But this "equality" is not able to tell us whether one choice was warranted whereas other choice might have not been. Elizabeth Anderson's objections to rational choice theorists do not go far enough for either rational choice theorists, or consequentialists to be worried. If we are to have a theory of value and choice, with regard to that value, what else to we have to account for?

Ruth Chang posits the problem in a different way very close to the problem of incommensurability. Incommensurability, as we have already said with a situation in which to objects are in conflict with respect to value. For Chang, incommensurability does not entail incomparability. In fact, Chang states that there is no "incomparability" generally.[255] That is, even if two objects aren't comparable in the traditional sense of "worse than," "equal," and "better than," those objects might still

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[254] Ibid. pg. 53

[255] See Ruth Chang (1997), "Introduction," pg. 4; pp. 10-13, the full defense of this claim is found in "The Possibility of Parity," "All Things Considered," and "Parity, Interval Value, and Choice."

have a value relation. Chang is extremely helpful in understanding that our critique of value in game theory needs to go further than just objecting that game theory only posits the maximization of value. Justification of choice, according to Chang, is ultimately what needs to occur in order for us to judge if our choices are "sound." Even in simple cases such as choosing between coffee and tea, Chang states, we might have difficulty relating our individual tastes for either coffee, or tea. This difficulty arises if we begin comparing a hot cup of tea with the hot cup of coffee. Chang state that, initially, the hot tea and the hot coffee are incomparable. One might go around in circles citing the texture of either, the taste of either, the quality of either in order to make a choice.[256] What ends up happening, according to Chang, is that when faced with such choices, agents start to deliberate about the qualities of each objects until those qualities are "justify" the choice of coffee over tea, or vice-versa. Chang's example is that the cup of hot tea that initially was there might now be just warm tea. How does warm tea compare to a hot cup of coffee. The process making either coffee or tea less attractive is called a "merit pump." That is, the initial characteristics of the cup of tea are devalued such that we make it less attractive to choose coffee over tea (or vice versa). Chang states that the "merit pump" problem calls for justification of choice; obviously this justification of choice cannot be ad-hoc.[257]

Chang cites seven main arguments for incomparability (of which we will only cover 4). The first type are arguments from the diversity of value—that is given that

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[256] See Ruth Chang (1997), "Introduction," pg. 11

[257] Ibid.

we have a multiplicity of values, when two of those values come in conflict, then we have a situation of incomparability. A fatal objection to these types of arguments deals with stating what Chang calls 'nominally notable' comparisons. When we compare Michelangelo and Mozart, it is true that there is no way to compare these two with respect to creativity. But when we have a third object of comparison—"Talentlessi," it is clear that talentlessi is less talented with respect to creativity to both Michelangelo and Mozart. If we have continuum of these less talented painters, Chang states, then we can compare Mozart to these less talented painters.[258]

The second type of argument for incomparability is the argument from calculation. These types of arguments state that value is construed quantitatively, or qualitatively. Value is assigned a type of scale dealing with magnitudes of some sort. Value judgments depend on "adding" or "subtracting" value from an object. Arguments from calculation assume that deliberation about values requires a type of calculation. Then, the objection raised against calculations of value simply asserts that the whole of value relations among objects cannot rest merely on calculation, therefore we have incomparability.[259] According to Chang, the fact that value judgments are required to be calculative is misinterpreting, and reducing the structure of value. The language of calculation is prevalent especially with consequentialist theories. The obvious objection is that value judgments need not be formulated with calculation in mind. That is, there are other value relations among objects besides

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[258] Ibid. pp. 15-16

[259] Ibid. pp. 18-19, Elizabeth Anderson's critique of consequentialism is an argument of this type (pg. 18).

more, less, or equal than—deliberation is "calculative in form." There might be situations where there is vagueness with respect to what type of value comparison might be made.[260] Another type of comparison that might be made with respect to two objects is a "nameless value" relation. If we take the case of comparing giving money to a charity or putting that same money in a retirement account, one has moral merits whereas the other one has practical merits. When we try to analyze the value relation among these two things we see that even though were will have to make a choice, we have will have not way to assign the type of value upon which we are calling for the comparison—this is a nameless value.[261] Arguments from calculation, then assume that deliberation about value is merely calculative, and because this calculation is not possible in some cases, there is incomparability.

Arguments from multiple rankings state that there is incomparability when there is conflict with respect to comparisons that deal with several components. Chang's example deals with hiring a philosophy professor. The candidates are Eunice and Janice. The evaluation for the job will be decided on the candidates' ability on clarity, creativeness, and competence. Eunice is a clearer writer than Janice, but Janice is slightly more competent writer. If we take these characteristics, and rearrange the importance, then we have a conflict between the different ways among which the selection of the candidates could be made. Given that we have this conflict of rankings, this type of argument state that we have incomparability. Chang

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[260] See John Broome, "Is Incommensurability Vagueness," pp. 67-70.

[261] See Ruth Chang (1997), "Introduction," pg. 30-31, this objection dates to Aristotle's *Nicomachean Ethics* (1125b), in Chang (2004) note 14.

posits a possibility, where we might have Eunice*, who might be slightly better than Eunice by being clearer. Or we might have Eunice+ who is slightly more competent than Eunice. Yet, it cannot be said that either Eunice* or Eunice+ is better overall than Janice. Thus, there might be a continuum of possibilities to construe Eunice that might render the situation comparable.[262]

Lastly, arguments from small improvements, according to Chang, are the most powerful types of arguments with respect to trying to demonstrate incomparability. These arguments state that given two objects, say (Chang's example taken from Joseph Raz) a career as a clarinetist and a career as a lawyer; take either career and improve it by an amount of money; if we improve the clarinetist career by $10, this makes the career better than the previous version of the clarinetist career, yet we cannot say that the improved career as a clarinetist is better than the career as a lawyer, therefore we have incomparability.[263] The general form of the argument is the following: (1) If A is neither better nor worse than B, (2) A+ is better than A, (3) A+ is not better than B, then (4) A and B are incomparable. The main force of the argument, states Chang, is between 1 and 3. Even though we might judge that a series of clarinetists might be better than our initial clarinetist A, it still does not follow that the series of clarinetists that are better than A are just incomparable to the career as a lawyer.[264] The reason why this argument is powerful relies on what Chang call the trichotomy thesis—that is, the relation of value among objects or

[262] Ibid. pg. 22-23

[263] Ibid. pp. 23-24

[264] Ibid. pg. 24

things is captured by "better than," "equal," or "worse than."[265] Problems arise when we have a relation that might be outside of these three value relations. Chang argues that there is a fourth relation with respect to value judgments that allows us to incorporate seeming incommensurability.

The fourth relation with which we might compare two things is parity. In "The Possibility of Parity" and "Parity, Interval Value, and Choice," argues that because of the complexity of value judgments and situations that call upon us to deliberate on value, acceptance of the trichotomy thesis is not enough to capture cases in which no direct comparison can be made. Such cases include the vagueness of value with respect to two or more objects being compared.[266] If we take the examples that we have been using thus far, coffee vs. tea, Michelangelo vs. Mozart with respect to creativity, the hiring of Eunice vs. Janice for a professorship—it should be clear that although these cases are hard, we are better off declaring that these are good cases for parity. If we take the tools of rational choice theory, or adhere to the trichotomy thesis of value relations, we are pushed to admit that these cases are cases of incomparable. It is arguably the case that we are on safer ground stating that these cases are cases of parity. Parity allows us to have the most complex situations of

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[265] Ibid. pg. 25

[266] See Chang (2002), "The Possibility of Parity" pp. 659-688, Chang (2005), "Parity, Interval Value, and Choice" pp. 331-350, this article defends parity not by "explaining away" but by arguing that "hard cases" where there is clearly a problem about the value relation among two or more objects. Some, Chang states, argue that these cases are cases of ignorance; this she states, is mistaken because no analysis is done, parity is merely asserted. Also Chang (2004) "All Things considered" pp. 1-22, deals with what Chang calls "all-things-considered" judgments. "All-things-considered" judgments capture every relation possible, including those of vagueness, which might entail parity. Once again, the case is made against trichotomy thesis adherents. The trichotomy thesis fails to capture these "hard cases," and when agents face difficult choices in these cases, the trichotomy thesis is only able to give ad-hoc reasons why one alternative is better than the other.

comparison without having to resort to admitting incomparability or merely asserting parity. If we take the model of consequentialism, or the model of game theory, vs. Chang's model, we see that Chang's model captures every possible case, theoretically or in real life.

The last caveat for talking about value relations is about incomparability vs. non-comparability. Incomparability is a substantive claim with respect to the composition of value of a certain object. Chang's example for non-comparability is "gustatory pleasure" with respect to chalk. When we compare the taste of chalk to the taste of food, we don't have a case of incomparability, but non-comparability. Non-comparability relates to the absence of "covering-value." The covering value for food is "gustatory pleasure." But even in cases where we might not have a named covering value, we will have to make comparisons and choices. This seemingly trivial caveat is important because even in cases of non-comparability, since no relation can be established, we will end up in the territory of vagueness one way or another, and as we have argued, judgments about vagueness with respect to value are better understood by using parity as Chang has suggested.

Up until this point, we haven't talked about the consequences for maximization of value (or even optimization). Given that we have hard cases of comparison, where we might declare that the relation of comparison is parity, how are we to maximize, or even optimize? This puts heavy doubts on the game theoretical project of "agent rationality" and maximization of value. Hsieh (2007) argues that we might not have to give up maximization that easily as a result of Chang's argument for parity. Hsieh observes that Chang's analysis requires rejecting the axiom of

transitivity.[267] Hsieh states that this rejection is to high a cost to pay.[268] Hsieh's view states that

> Optimization requires the choice of an alternative that is at least as good as other alternatives and rules out the possibility of justified choice between incomparable alternatives. In contrast, maximization only requires the choice of an alternative that is not worse than other alternatives. Because incomparable alternatives qualify as not worse than one another, maximization allows for justified choice between them[269]

Hsieh argues that when comparing to options, we need not incur the cost of rejecting the axiom of transitivity. Hsieh think that maximization is a viable option even in hard cases.[270] Although Hsieh is in disagreement with Chang about the maximization, in cases of vagueness, it is hard to see how the axiom of transitivity cannot be rejected. Take any of the cases. This is an obvious objection to Hsieh (2007). This is surprising because Hsieh refers to John Broome (on vagueness). According to Hsieh, choosing a less paying musical career (taken from Chang), might make sense, and yet, we might have a case for maximization. But the whole point of calling upon parity for such cases is that there can be an array of formulations that for which we must account. If we take our musical career we might argue that there are several configurations about the musical career that are on a par. If we take one aspect of the musical career, say a continuum of pay, it is clear that as Hsieh state, we will be able to decipher those that are "not worse," therefore, maximization holds there. But when we have aspects of talent, creativity and the like, adding continua

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[267] Hsieh (2007), pg. 66

[268] Ibid. pg. 71

[269] Ibid. pg. 71

[270] Ibid. pg. 78, "According to maximization, an alternative's not being worse with respect to the choice value is sufficient for its choice to be justified."

with respect to these will make it a lot harder for "maximization" to hold without invoking parity.

This section dealt with the justification of choice with respect to value. It is clear that an array of choices among seemingly incomparable things present a problem for the "agent rationality" of game theory. Elizabeth Anderson's critique only brought us so far against this "agent rationality" and maximization of value. Hsieh (2007) provided a way that we might construe "maximization" without referring to parity in order to justify choice. The conclusion to be drawn is that if we are able to account of all possible cases of comparison, we are left with incommensurability and vagueness if we accept the project advanced by classical utilitarianism, game theory, and neoclassical economics.

6.3 Intersubjectivity in Economics, Structures and Agents

Over the course of this paper, we have seen that that the main attraction of neoclassical theory has been its simplicity, and seeming coherence. This simplicity has allowed for interesting, yet radical statements about human interactions, and intentions. Neoclassical economic theory presents to us the atomic individual. This atomic individual's sole purpose is to maximize utility, in whatever formulation, or context in economic theory. Reality is radically different from mere atomic individuals acting in isolation. Neoclassical theory does not account for the fact that individuals do interact. Instead of having a harmonious whole, as is the case in neoclassical economic theory, we have a complex, interacting, and dynamic whole about which much more needs to be learned.

When it comes to individual agents, (Davis 2002) proposes that agents are embedded in the complex social whole. Agent's choices, intentions, and interactions depend on a social structure that predates any type of completely independent, atomic individual. The social structure, in turn, is affected by the decisions individuals make—thus, there is a reciprocal dynamic relationship between individuals and society.[271] Davis states that individuals are able to have "we-intentions" which are the foundation of reciprocal behavior of economic agents.[272] (Fehr and Falk 2002) further Davis' view by stating that atomic individuals that act on their own behalf, as maximizers of value, do *not* thrive in a competitive environment. Rather, reciprocal fairness places a halt to neoclassical theory and its claim of only self-interested individual maximizing value.[273]

In "Flaws in the Foundation, Consumer Behavior and General Equilibrium Theory," Frank Ackerman critiques what lies at the core of neoclassical theory (which also applies to game theoretical formulations). Agents are asocial, they have insatiable or unlimited wants, and consumer choices are informed by well-ordered, well-informed desires about the commodities in general. All three assumptions are easily refuted by: intersubjectivity—people are complex individuals hardly reducible to atomic agents; advertising and group peer pressure, consumption patterns are affected by these in non-trivial ways; and finally, when people consume, they stop

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[271] See John B. Davis, "Collective Intentionality, and Individual Behavior," pp. 17-18

[272] Ibid. pp. 24-26, this relates to Davis' claim about individuals being socially embedded, as opposed to atomic, isolated individuals. See also Thévenot (2002) on cooperation and convention affecting decision of individuals; and Dupuy (2002) on "imitation" in markets.

[273] See E. Fehr and A. Falk, "Reciprocal fairness, Cooperation, and Limits to Competition," pg. 40.

after the particular wants or needs are satisfied. Consumers don't consume all the food until the fridge is empty.[274] In terms of general equilibrium theory (briefly discussed above), neoclassical theory states that economies *tend* to equilibrium via the competitiveness of economic agents. This competitiveness allows for a market clearing situation where prices and quantities produced in the economy are determined.[275] Furthermore, this equilibrium situation is stable. The reality is different from this. As Ackerman states, there are two complications in economic theory which do not permit economic stability such as posited in neoclassical theory: "one involves aggregation, and the other concerns the behavioral model of the individual. Both are causes of instability of general equilibrium."[276] Ackerman suggests repairing the flaws at the foundation of neoclassical economic theory. What this amounts to changing the three assumptions that arise in consumer theory which plague the whole of neoclassical theory:

> While it [the new theory] might still involve some mathematical analysis, the variables would be different, representing human needs, desires, and experiences rather than commodities. Abstract proof of optimality of any particular market outcome would be unlikely. Instead, evaluation of economic systems would involve history, politics, and recognition of the intersubjective nature of human behavior. It would involve ethical judgments about what our society has achieved, and what else it could have done. […] in short, many contemporary economists would conclude that such a theory was not really economics.[277]

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[274] See Frank Ackerman, "Flaws in the Foundations, Consumer Behavior and General Equilibrium Theory," pg. 58.

[275] Ibid. pg. 63, mathematical economists seek to emulate physics in the modeling efforts, but, as Ackerman states, Physicists are more modest in their modeling efforts—there is an inherent difference between people, and particles, for example.

[276] Ibid. pg. 62, aggregate demand is not as well behaved as individual demand.

(Pfouts 2002) suggests that neoclassical consumer theory, which lies at the foundation of current mainstream economic analysis be discarded for a new "ontology [that] can always be stated in epistemological terms."[278]

The word ontology refers to the study of existence and to the study of the being of beings. In the context of economics, ontology refers to how economic agents behave, and how institutions might affect that agent behavior.[279] The purpose of having an explicit ontology regulates the way in which statements about beings are made. Having an explicit ontology of the consumer, according to Pfouts, "dissolves the mystery" of statements about beings about which the theory speaks.[280] Pfouts concludes:

> Too often economic theorists have not adopted logical methods appropriate for investigating the economic world, but instead have assumed an imaginary economic world that submits to the logical methods they want to use. They have preferred to abandon the world in favor of their preferences in mathematics rather than using mathematical and other methods that are effective in analyzing the real world.[281]

Herein lays the core of this whole discussion. Neoclassical methodology in whatever form, makes radical claims that go against the common occurrences of the "real world." That is not to say that their theory is entirely untrue. The lack of consideration of ontology (among others) makes mainstream economic theory suspect as a theory that pertains to the "real world." As it is stated by Pfouts, mathematical

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[277] Ibid. pg. 68

[278] See R. W. Pfouts, "On the Need for a More Complete Ontology of the Consumer," pg. 71.

[279] Ibid. pg. 71

[280] Ibid. pg. 80

[281] Ibid. pg. 83

rigor is not a substitute for ontology. Arguably, mathematicians have their own issues dealing with the ontology of mathematics on their own. Adding persons to the picture, not only makes the problem of ontology more complicated—the problem *necessitates* philosophical considerations beyond the use of mathematics to justify economic statements.

7. Conclusion

Throughout the whole of this discussion, we saw that mathematics is used as the main source of justification for mainstream economic theory. Mathematics is supposed to provide a secure foundation for economists, but as we saw in sections 2 and 3, mathematics is inherently metaphysical. The type of metaphysical system will affect the types of statements that economists/scientists will be able to make in articulating their theories; in economics, the way in which we construe the individual and the economy will drive the research and the results derived from that research. Stating clearly the metaphysical system, in whatever context, allows to clarify and to anticipate problems that lie at the foundation of economic, as well as other forms of analysis. Without such considerations, we are left in hands of obscure, drastically simplified metaphysical claims. The neoclassical theory of the individual, as well as general equilibrium are examples of neoclassical economic theory where neoclassical economists, in trying to escape metaphysics, adhere to outdated and improbable metaphysical statements about the individual and the economy as a whole. If one is to escape metaphysics, one has to realize that one is already using metaphysical statements the moment mathematics and mathematical economics are invoked.

References

Ackerman, Frank (2002). "Flaws in the Foundation, Consumer Behavior and General Equilibrium Theory," in *Intersubjectivity in Economics, Agents, and Structures*, Edward Fullbrook (ed.). New York: Routledge.

Anderson, Elizabeth (1997). "Practical Reason and Incommensurable Goods" in *Incommensurability, Incomparability, and Practical Reason*, Ruth Chang (ed.), Cambridge (MA): Harvard University Press.

Anderson, Elizabeth (1993). *Value in Ethics and Economics*. Cambridge (MA): Harvard University Press.

Aristotle (2001). Politics in *Aristotle's Basic Works*, Trans. Jowett, B, ed. McKeon R, 1127-1325. NY: Random House.

Arrow, Kenneth (1979). "Values and Collective Decision-Making" in *Philosophy and Economic Theory* F. Hahn and M Hollis (eds.), Oxford: Oxford University Press.

Arrow, Kenneth (1967). "Public and Private Values," in *Human Values and EconomicPolicy*, Sydney Hook (ed.). New York: New York University Press.

Balaguer, Mark (1998). *Platonism and Anti-Platonism in Mathematics*. NY: Oxford University Press.

Becker, Gary (1993). "A simple theory of advertising as a Good or Bad." *The Quarterly Journal of Economics*, 108 (4): 941-964.

Benacerraff, Paul (1983). "What Numbers Could not be" in *Philosophy of Mathematics, Selected Readings*, Benacerraf, P; and Putnam, H. (eds.) 2nd ed., Cambridge University Press, Cambridge.

Bentham, Jeremy (1823). *Principles of Morals and Legislation*. Oxford: Clarendon Press.

Blaug, Mark (1992). *The Methodology of Economics, Or how Economists Explain*. 2nd ed. Cambridge: Cambridge University Press.

Boolos, G. (1983). "The Iterative Conception of the Set" in *Philosophy of Mathematics, Selected Readings*, Benacerraf, P; and Putnam, H. (eds.) 2nd ed., Cambridge: Cambridge University Press.

Broome, John (1997). "Is Incommensurability Vagueness" in *Incommensurability, Incomparability, and Practical Reason*, Ruth Chang (ed.), Cambridge (MA): Harvard University Press.

Chang, Ruth (2005). "Parity, Interval Value, and Choice." 115: 331-350.

Chang, Ruth (2004). "All Things Considered." *Ethics*, 18: 1-22.

Chang, Ruth (2002). "The Possibility of Parity." *Ethics*, 112: 659-688.

Chang, Ruth (1997). "Introduction" in *Incommensurability, Incomparability, and Practical Reason*, Ruth Chang (ed.), Cambridge (MA): Harvard University Press.

Colyvan, Mark (2001). *The Indispensability of Mathematics*. New York: Oxford University Press.

Davis, John B (2003). *The Theory of the Individual in Economics, Identity and Value*. New York: Routledge.

Davis, John B (2002). "Collective Intentionality and Individual Behavior," in *Intersubjectivity in Economics, Agents, and Structures*, Edward Fullbrook (ed.). New York: Routledge.

Dupuy, Jean-Pierre (2002). "Market, Imitation, and Tradition, Hayek vs. Keynes." in *Intersubjectivity in Economics, Agents, and Structures*, Edward Fullbrook (ed.). New York: Routledge.

E. Fehr, and A. Falk (2002). "Reciprocal Fairness, Cooperation, and Limits to Competition," in *Intersubjectivity in Economics, Agents, and Structures*, Edward Fullbrook (ed.). New York: Routledge.

Friedman, Milton (1953). "Essay on the Methodology of Positive Economics," in *Essays in Positive Economics*. Chicago: University of Chicago Press.

Gibbard, A., Varian H (1991). "Economic Models." *The Journal of Philosophy*, 75 (11): 664-677

Griffin, James (1997). "Incommensurability, What's the Problem" in *Incommensurability, Incomparability, and Practical Reason*, Ruth Chang (ed.), Cambridge (MA): Harvard University Press.

Griffin, James (1986). *Well Being*. Oxford: Clarendon Press.

Harsanyi, John (1955). "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility." *The Journal of Political Economy* 63 (4): 309-321

Hausman, Daniel (1992). *The Separate and Inexact Science of Economics*. Cambridge: Cambridge University Press.

Heidegger, Martin (1993 [1967]). "Modern Science, Mathematics, and Metaphysics" in *Basic Writings*, D. F. Krell (ed.). NY: HarperCollins.

Hoover, Kevin (1995). "Review: Why Does Methodology Matter for Economics?" *The Economic Journal* 105 (430): 715-734.

Hoover, Kevin (1988). *The New Classical Economics, a Sceptical Inquiry*. Cambridge: Cambridge University Press.

Hsieh, Nien-Hê (2007). "Is Incomparability a Problem for Anyone." *Economics and Philosophy*, 23: 65-80.

Kant, Immanuel (2004). *Prolegomena to any Future Metaphysics That Will be able to Present itself as a Science*. Ed. Günter Zöller. Oxford: Oxford University Press.

Kant, Immanuel (2004). *The Metaphysical Foundations of Natural Science*. Ed. and trans. Michael Friedman, Cambridge: Cambridge University Press.

Kant, Immanuel (2003 [1929]). *The Critique of Pure Reason*. Trans. Norman Kemp Smith. New York: Palgrave McMillan.

Kripke, Saul (1972). *Naming and Necessity*. Cambridge (MA): Harvard University Press.

Leibniz, Gottfried (1989). "The Monadology" in *Philosophical Essays*, trans. Ariew, R, and Garber, D. Indianapolis: Hackett Publishing Company 1989.

Little, I.M.D (1952). "Social Choice and Individual Values." *The Journal of Political Economy*, 60 (5): 422-432

Lukes, Steven (1997). "Comparing the Incomparable, Trade-offs and Sacrifices" in *Incommensurability, Incomparability, and Practical Reason*, Ruth Chang (ed.), Cambridge (MA): Harvard University Press.

Mäki, Uskali (forthcoming). "Reading *the* methodological essay in twentieth century economics: Map of multiple perspectives" in *The Methodology of Positive*

Economics: Milton Friedman's Essay After Half a Century, Uskali Mäki (ed.), Cambridge: Cambridge University Press.

Mäki**,** Uskali (2002). "The what, why, and how of Economic Ontology," in *The Economic World View, Studies in the Ontology of Economics* U. Mäki (ed.). Cambridge: Cambridge University Press.

Martin, Gottfried (1985 [1972]). *Arithmetic and Combinatorics, Kant and His Contemporaries*. Trans. Wubnig, J. Carbondale (IL): Southern Illinois University Press.

Mill, John Stuart & Bentham, Jeremy (2004 [1987]). *Utilitarianism and Other Essays*, Alan Ryan (ed.) NY: Penguin Group.

Minkler, L. (1999). "The Problem with Utility: Toward a Non-Consequentialist/Utility Theory Synthesis." *Review of Social Economy* 42 (1): 10-13.

Mirowski, Philip (1989). *More Heat than Light*. Cambridge: Cambridge University Press.

Mirowski, Philip (2002). *Machine Dreams Economics Becomes a Cyborg Science*. Cambridge: Cambridge University Press.

Pfouts, Ralph W. (2002). "On the Need for a More Complete Ontology of the Consumer" in *Intersubjectivity in Economics, Agents, and Structures*, Edward Fullbrook (ed.). New York: Routledge.

Raz, Joseph (1997). "Incommensurability and Agency," in *Incommensurability, Incomparability, and Practical Reason*, Ruth Chang (ed.), Cambridge (MA): Harvard University Press.

Regan, Donald (1997). "Value, Comparability, and Choice" in *Incommensurability, Incomparability, and Practical Reason*, Ruth Chang (ed.), Cambridge (MA): Harvard University Press.

Reshotko, Naomi (2006). *Socratic Virtue, Making the Best of the Neither-Good-nor-Bad*. Cambridge: Cambridge University Press.

Reshotko, Naomi, "Plato's Epistemological Paradox: The Know*able* Cannot be Known," unpublished manuscript.

Robbins, Lionel (1938). "Interpersonal Comparisons of Utility: A Comment." *The Economic Journal*, 48 (192): 635-641.

Rosenberg, Alexander (1992). *Economics-Mathematical Politics, or Science of Diminishing Returns*. Chicago: University of Chicago Press.

Rosenberg, Alexander (1986). "The Explanatory Role of Existence Proofs." *Ethics*, 97 (1): 177-186.

Samuelson, Paul (1977). "A Modern Theorist's Vindication of Adam Smith." *The American Economic Review*, 67 (1): 42-49.

Samuelson, Paul (1954). "Some Psychological Aspects of Mathematics and Economics." *The Review of Economics and Statistics*, 36 (4): 380-386.

Samuelson Paul (1952). "Economic Theory and Mathematics--An Appraisal." *The American Economic Review*, 42 (2): 56-66.

Samuelson, Paul (1938). "A note on the Pure Theory of Consumer Behavior." *Economica* Vol. 5, pp. 61-71.

Schroeder, Mark (2008). "Value Theory," *The Stanford Encyclopedia of Philosophy, (Fall 2008 Edition)*, Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2008/entries/value-theory/>.

Sen, Amartya (1979 [1977]). "Rational Fools" in *Philosophy and Economic Theory* F. Hahn and M Hollis (eds.), Oxford: Oxford University Press.

Shafer-Landau, Russ (2003). *Moral Realism: A Defense*. Oxford: Clarendon Press.

Sippel, R. (1997). "An Experiment on the Pure Theory of Consumer Behavior." *Economic Journal* 107: 1431-1444

Smith, Adam (2003 [1776]). *The Wealth of Nations*. New York: Bantam Dell, Random House Inc.

Taylor, Charles (1997). "Leading a Life," in *Incommensurability, Incomparability, and Practical Reason*, Ruth Chang (ed.), Cambridge (MA): Harvard University Press.

Thévenot, Laurent (2002). "Conventions of Co-ordination and the Framing of Uncertainty" in *Intersubjectivity in Economics, Agents, and Structures*, Edward Fullbrook (ed.). New York: Routledge.

Urquhart, Robert (2005). *Ordinaryl Choices, Individuals, Incommensurability, and Democracy*. New York: Routledge.

Weintraub, E. R. (1989). “Methodology Doesn't Matter, but the History of Thought Might” *The Scandinavian Journal of Economics*, 91 (2): 477-493

Weintraub, E. R. (1998). “Controversy: Axiomatisches Mißverständnis.” *The Economic Journal*, 108 (451): 1837-1847