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Cascading Failures and Fundamental Uncertainty: Divergence in Financial Risk Assessment

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ABSTRACT

By applying common financial risk assessment models to the network economy formalized in Delli Gatti et al. (2006), and by contextualizing both in the broader literature on complexity in economic systems, the question of convergence in economic models is addressed. Critically, a formal state condition is identified which can contribute to the emergence of periods of extreme divergence from expected conditions even in a model characterized by restrictive assumptions regarding agent choice and market structure. The strength of the impact of this state condition, here the topology of a credit network, on the dynamics of the economic system is furthermore shown to be highly dependent upon the structure of the market. The existence of such state conditions has fundamental implications for the evaluation of risk and institutional design in economic systems

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I Complexity

Reductive Theories and Complexity

In our attempts to understand the world we live in we are often forced to reduce extreme complexity to a tractable form. This is the pursuit of the many theorists across all disciplines who have built the great abstract constructions of science and philosophy: to render a complex world understandable. In doing so it is unavoidable that we exchange some degree of precision in subjective observation for the creation of abstract frameworks. It must granted that such reduction is necessary, for without imposing order on our observations we have no way of assessing the effect of any hypothetical action, or attempting to predict future states upon which to base our decisions. Without a structured understanding of reality we are left rudderless; we cannot choose. The critical question, then, is to understand to what degree our impositions on reality blind us to the very realities we are attempting to capture. When, in reducing observed phenomena to an understandable form, do we fool ourselves into dismissing the possibility that our framework may be wrong? Are we prepared to speak accurately to how severe the deviation may be? How do we evaluate whether the framework serves as an aid to decision making and not an obstruction? The possible questions about any such framework are numerous, but resolve themselves into three general forms. The first is a

question of optimality: does the model approach reality as closely as possible given its limitations? The second is one of complexity: how can we understand the possible magnitude of the deviations from the predictions of the model, given that the true system it attempts to summarize is in reality much more complex? One might be tempted to say that this second question is really subsumed into the usual formulation of a confidence interval. In reality, it is not. A confidence interval speaks to the level of error that is possible given the stability of the system, we might say it is the inherent instability given that the model obtains. Since the reality is much more complex, we must be concerned about whether the dynamics possible in the higher order system are significant enough to confound our lower order model, *whether there is a more fundamental uncertainty in parameter choice than can be captured in a model of lower-order complexity.* Finally, as our capabilities in modeling complexity have increased, we must ascertain whether or not opportunities exist to lower the degree to which we have abstracted ourselves from reality and in so doing begin to understand in some way the fundamental uncertainty that plagues our models. While I do not propose to have complete answers to these questions, in some sense their application to our understanding of financial risk is the aim of this work. In order to do so it will be necessary to rest upon a particularly important abstraction, one which attempts to categorize complexity.

In the introduction to *Barriers and bounds to Rationality*, Peter Albin presents just such an abstraction. The system is designed to mirror Chomsky's presentation of formal language and presents a useful framework by which to understand the somewhat nebulous term, "complex dynamics", through the dynamics of the emergent behavior a

given system is capable of producing. In Albin's framework, which for want of a better term I will refer to as our "complexity scale", the lowest category describes what we might call trivial systems. Such systems lack dynamics entirely and can be defined as equilibrium states, in such a framework prediction is meaningless since current conditions will persist for all time. The second state, which might be named "Convergent Dynamics", define the family of systems producing dynamics which can be modeled perfectly by well-behaved systems, their behavior exhibits no discontinuity and collapses into a linear combination of the variables in parameter space. Within these models prediction is possible indefinitely: the dynamics support only systems with constant or absent variability. The third level is familiarly referred to as "Chaos"; systems exhibiting third-order complexity produce dynamics that are imperfectly predictable over the short term, and which can be understood as exhibiting the qualities of chaotic attractors. The dynamics may be attracted to fixed points, but their rate of convergence might be arbitrarily large or small given the position of the endowment point in parameter space. Furthermore, within systems of third-order complexity, the relationships governing the emergent dynamics of the system are variable over periods of time. The resulting shifting patterns of attraction can lead to the appearance and disappearance of predictable system behaviors, prediction is severely impeded in third-order systems, but possible within certain bounds. Finally, fourth-order complex systems produce dynamics perhaps best formalized as white noise with infinite variance. No objective model can be overlaid upon observations to produce prediction; each observation exists in its own subjective reality (Albin 1998).

It's important to note that Albin does not assert that a system sufficiently complex to produce fourth-order dynamics necessarily does so, merely that once a certain level of complexity in the relationship between agents and their respective choice matrices is achieved, such dynamics *are possible*. Whether or not dynamics of varying degrees obtain at any given time is dependent on several *state conditions*, and these dependencies may be more or less apparent in any given system. If these conditions vary over time, the dynamics presented may bifurcate from one level of complex behavior to another. An example: today we may all agree that the value of Apple stock is fairly well priced at $636, which will cause us to interact in the market in a fairly benign way with minor adjustments around that value-pricing due to our various beliefs about the long term validity of that price. However, if some major event called that price into question, for example if the patents that form the basis of Apple's revolutionary tablet and smartphone platforms were invalidated, a great many of us may begin to disagree fundamentally with that price. Worse, we all may begin to disagree with one another over the new price as well, greatly increasing volatility and reducing the convergence of the time-evolution of price to any given function of known parameters. The value of the stock begins to behave unpredictably.

In analyzing the exhibited dynamics of the above example we might note that the state condition supporting convergence was the set of factors supporting the level of certainty of the agents interacting in the market, when opinions became much less certain, a model predicting the price of the stock would have "diverged" with very wide implied variance. State conditions are then the conditions which support the foundational

relationships required for a given model to obtain, the environmental factors which support dynamics pointing to a given assumption. In the context of the financial crisis, for example, the growth of fraud, affirmation bias, leverage, and the mispricing of risk served as evidences of the collapse of the state conditions supporting the orderly valuation of mortgages. The collapse of these conditions themselves did not immediately lead to the severe uncertainty and chaotic price movements characteristic of the worst days of the crisis, instead, it rendered such a scenario possible. One could assert that the *bifurcation* from orderly market interactions to disorderly ones was made possible via these changes. Any given model is characterized by a set of such conditions, and failures to understand how those conditions might change over time constitute an assumption that a given model, which is by definition reductive, expresses a perfect reflection of reality.

The vast majority of economic models are predicated upon a theoretical framework constructed within a system capable only of producing at most second-order complexity. These are the familiar equilibrium models introduced in freshman economics courses, in which systems of supply and demand schedules interact such that any point in endowment space collapses quickly to equilibrium (the framework in general lacks even the dynamics to describe this collapse, it is assumed to take place in an arbitrarily small period of time in relation to the period of concern of the model). The ultimate expansion of these models merely extends the concept into arbitrary numbers of interacting markets. Event those economic systems concerned with dynamics are in general fairly limited in their use of the available family of dynamics systems. Neoclassical models of capital formulation generate either linear growth, or variations of cyclical behavior. Even the

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most extreme examples deal with the interaction of multiple equillibria; and the dynamics of movement between these equilibria is often quite thoroughly underdefined. In fact, in applying fairly simple stochastic elements derived from the field of thermodynamics to explain the *dynamics* of agent interaction in the familiar 2x2x2 Edgeworth box model, Duncan Foley effectively destroys a common neoclassical conclusion: that agents beginning with the same endowments arrive at the same final allocations (Smith and Foley 2008). Such a result is the consequence of applying a model which exists in static time to a world which exists in complex space.

In every case, as soon as inter-temporal considerations are allowed, second order systems begin to impose strange restrictions on the behavior they model. We are often forced to imagine the ability of economic agents to solve optimization in infinite time (as in game theory) or to assume that the full trajectory of future allocations of consumption and capital can be correctly assessed by agents (as in General Equilibrium with investment). Such paradoxes have been touted as evidences of the strength of the deductive reasoning behind them: "economist Robert Lucas has boasted that the axioms underlying classical economics are 'artificial, abstract, patently unreal'. […]Lucas insists that such unreal assumptions are 'the only scientific method of doing economics'" (Davidson 2009, p.5). In actuality, for a discipline occasionally accused of suffering due to its love affair with science, in terms of complexity, economics has been left behind by the physical sciences. While I am not personally familiar with much in the way of theoretical physics, the postulation of steady state systems appears positively Newtonian. As was mentioned before, the application of statistical dynamics can invalidate

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neoclassical conclusions, and there likely exists a wealth of additional value to be gained from the further study of complexity in economics. Similarly, meteorology has advanced beyond economics by leaps and bounds (with great results too, once our partners in fruitless prognostication, meteorologists are now able to predict the occurrence of tornadoes by several minutes in most cases, a critical improvement. We economists find it difficult even to agree that certain economic phenomena have obtained in the past, let alone that they will obtain in the future).

We are forced to conclude that a vast sample of economic models by their nature ignore, or treat in only the most perfunctory or arcane manner, the question of agent behavior in a world with a time dimension. In the realm of risk management this is highly problematic, as all risk is by nature inter-temporal, and worse, intertemporally uncertain. To the extent that economic systems do in fact exhibit third-order and fourth-order complexity, the possibility exists that we are severely mispricing risk. Such an assertion would seem to be fairly obvious in the context of the financial turmoil of the past few years. However, it might be fair to wonder what statistical artifacts would enable us to assess whether recent mispricings may have been due to the inherent complexity of financial systems (in effect whether pricing predictions were too tight and implied more foresight than may have been possible), or whether the mispricing was due to market failures that drove market pricings away from long run equilibrium (in effect, that there was no problem with the implied precision of the estimate, but rather only that significant biases existed). The answer lies in the fundamental assertion of financial economics of the validity of mean field approximation.

Challenges for Classical Convergence

As has been previously alluded, the degree of precision in prediction falls precipitously as the degree of complexity of a given system increases. Unfortunately, this lack of precision has been criticized as a weakness of complex models when assessed against the cohesive structure which serves as the foundation of mainstream economics. As a consequence the models of the mainstream often fail to implement the lessons of complexity, and practitioners are repeatedly surprised when the "impossible" obtains, when in actuality they should expect a landscape in which the state conditions supporting their models are prone to bifurcation. In finance, the efficient market hypothesis postulates a marketplace in which deviations from "true" pricing, or that implied by the correct assessment of the probability of future returns, are allowed only to obtain in the short run (see the definition of EMH given by Malkiel 1992). Long run pricing assumptions common to such equilibrium models have served to reinforce the sorts of affirmation bias that have crippled the ability of the financial system and regulatory agencies to correctly asses the likelihood and impact of systemic failure. Such paradigms do not allow for the existence of the bifurcations possible in the real world.

This mainstream framework of fundamental coherence has been challenged by significant bodies of economic inquiry. Notably, Kahneman and Traversky have submitted a theory of decision making which poses very difficult questions for the rational agent hypothesis, leading to the emergence of studies into Behavioral Economics, in which agents cannot be expected to exhibit classically rational behavior (Kahneman

2011). But, we need not even go as far as the behavioral economists have in order to find crippling breakdowns of the system's conclusions; without even relaxing the assumption that agents behave rationally, it can be shown that the conclusions of the neoclassical system fail to obtain when agents are faced with a limited subset of the available knowledge (Greenwald and Stiglitz 1988).

Such assertions, that economic actors are not omniscient, or that economic agents fail to exhibit perfectly rational behavior, are in most cases granted. In fact, most economists would likely agree that the behavioralists have discovered interesting artifacts of human behavior. What is contended, however, is that while such disturbances can cause short term instabilities, the long run systemic dynamics can be defined within the bounds of the rational model, that instabilities are cancelled *in the mean*. This is of course a direct rejection of third and fourth-order complexity, under which such mean field approximations can fail as useful descriptions of systemic behaviors since state conditions are subject to change. Such losses of convergence, and to a resulting extent therefore a loss of numerical precision in the definition of the dynamics, are artifacts of real world market behavior. The qualitative story told becomes deeply cautionary: the value of precise modeling is bounded by the fluctuating state of the system.

Interestingly, while increases in the power of modern computing and the sophistication of statistical methods have made it possible to estimate the predictions of increasingly complex models, the result seems to have been ever stronger assertions of certainty and coherence rather than a understanding of the implications of divergence in higher-order complex systems. Commentary during the years leading up to the most

recent financial crises has been characterized by a common theme. *"Innovations" in the financial services sector have purported to leverage these advances to increase the efficiency of economic allocation* (Ang and Cheng 2005, Wiel and Yves 2010). In particular, the refrain has been that the increased pace of innovation in the financial markets has likewise increased the efficiency of the allocation of financial risk, those actors most willing (and possibly most able, depending on the model examined) to assume the risk do so and are accurately compensated for it (These largely flow from Arrow 1964). In the aggregate the risk is "shared" with complex interconnections between financial agents ensuring that the effects of risks when realized are diversified and that their impact is dispersed in such a way that the ability of the financial sector to facilitate real production is not unduly impacted.

Approach

It is unfortunate that this ideal diversification has not obtained in actual markets. Purely or primarily *financial* crises have caused massive damage to economic systems, arising in markets characterized by differing degrees of regulation and government intervention. The narratives proposed as backdrops to each of these financial crises have certainly varied, and it is not the aim of this work to support or refute any particular narrative of any particular crisis. Instead, by studying the nature of financial prediction (and by extension, the entirety of the general equilibrium framework under Arrow-Debreu) in light of the rich complexities possible in agent based models, it is hoped that something constructive can be expressed in the direction of answering an ontological

question: What are our abilities in the face of a financial system that is characterized by a highly variable structure?

Agent Based Modeling (ABM) is a simulation approach which models micro-level behavior (agent level) to produce systemic results. Similar to the neoclassical micro foundations of macro behavior, in ABM agents are granted endowments in the form of initial conditions, and rules defining their reactions to external stimuli (the role played by profit and utility-maximization in the neoclassical framework). However, the similarities end there. The neoclassical approach goes on to formulate assumptions about agent choice in a very specific way, with continuously defined and differentiable utility and production functions, with the express aim of arriving at an analytical solution to the problem of agents' choices in a system with many agents. Indeed, the Arrow-Debreu framework was in a sense constructed to solve exactly that problem and arrive at the General Equilibrium condition as an analytical solution to economic systems. Of course, complex systems often have no such convenient analytical solutions. In agent based modeling agent choice is not so constrained, the decision rules granted agents can allow for discontinuity and context sensitivity, allowing for the creation of systems of the highest levels of complexity. Agents arrive at choices in an algorithmic way, indeed, analytical solutions may not obtain. The interaction of these agents when placed in an economic context proceeds via a market or a set of conditions contextualizing their choices, and the resulting emergent behavior makes up the macroeconomic results of differing micro-economic decision matrices. Importantly, causation can go both ways, from agents onto macroeconomic effects, and from institutional structure onto agents.

The definition of agent choice must proceed with caution, however, due to the possibility of arriving at fourth order dynamics (infinite variance). It is therefore perhaps most useful to employ agent based modeling towards a better understanding of the conditions that drive bifurcation and uncertainty in economic systems. The extent to which such conditions exist may define the limits of economic forecasting. In this work ABM is employed in informing the extent of our ignorance, rather than purporting to improve our ability to precisely forecast future events.

Several agent based models have been proposed to explore the possible determinants of real world behavior; in particular the literature has focused on the restriction of agent's ability to garner complete information about their world, building perhaps on the introduction of heterogeneous reactions on the part of agents to observed phenomena. The problem of choice, then, becomes the complex attempt on the part of the agent to assess the best reaction to real world phenomena, given what they observe. Results garnered are centered on the problem of framing behavioral choices to lead to optimal outcomes (from the realm of behavioral economics), or upon the best policy anticipation for the results of uninformed choices (Alan 2011, Chakrabati et al. 2011). Another body of inquiry, and that studied in the few models touched upon below, centers around the effects of the structure of an economy upon the outcomes of the actions of agents with tightly defined choice sets. In a given "network" economy, the choices available to the agent are defined by the topology of the network. An agent may only be able to enter into a credit or purchase relationship with a defined set of partners within the model or it may have a set of choices regarding which partners it wishes to do business

with. Agent relationships, behavior, and economic outcomes, are consequently dependent on network which is in turn defined by behavior. The resulting feedback loops prove to lead to the sorts of chaotic returns and divergent expectations common in the real world, and certainly provides an avenue for exploring the complexity of the time path of such second order metrics as covariance/correlation.

Most importantly, when leveraging such agent based models, it becomes possible to speak in a very concrete way about the structural foundations of the emergent behavior of the system. The main thrust of this work will center on a well-known agent based model, presented in Delli Gatti et al. (2006), which constructs a framework for assessing the effects of credit topologies (the weights and layout of the various credit arrangements in the productive process) on firm profits and bankruptcy cascades. By leveraging an existing risk evaluation technique, a diversification coefficient based upon value-at-risk and defined in Perignon and Smith (2010), the model can be shown to create dynamics which "break" the foundational assumptions of a variety of risk-management techniques. Furthermore, it will be shown that the severity of this violation is dependent in a fundamental way on the degree of concentration of credit connections. This narrow assertion (focusing on the limitations of certain families of risk management techniques) sits solidly in the context of a broader ongoing argument over the fundamental limits of prediction in economic systems, as such, it will be beneficial to contextualize the work via a discussion of the question of quantifiable vs. fundamental uncertainty as it obtains in the frameworks posited by Keynes (writing on fundamental uncertainty) and Arrow and Debreu (assuming quantifiable uncertainty). Once placed in this broader context, a

description of the model and techniques used in the work will be followed by an exploration of the mechanisms that drive the complex results discovered. Finally, a presentation of the results obtained will illustrate clearly the conclusions of the work, namely that the simple model produced in Delli Gatti et al. (2006), and to some extent the 2010 reformulation, produces a bifurcating realization of variable state-conditions (in this case the structure of the correlation between bank profits). In short the fundamental question of prediction ceases to be framed in the context of correctly calibrating signal weights, and must be predicated upon the realization that real-world economies are shaped by a system capable of producing the highest of complexities, in which not all state-conditions lead to predictive models that converge.

II. Lit Review

Keynes, Minsky, Albin, and Fundamental Uncertainty

In a 2009 article Paul Davidson asserts that there exist two distinct modeling approaches which attempt to define the nature of the operation of a capital economy. He names these two approaches "Classical Economy Theory", and the "Keynesian Theory of Liquidity in an Entrepreneurial Economy" (Davidson 2009). As Davidson explains, the primary difference between the two theoretical frameworks lies in their treatment of economic agents' decision making in the context of unknown future events. In the Classical theory, agents' current period decisions are contingent upon their understanding of the likelihood of future events, which is in turn defined by their assessment of the statistical occurrence of like events in the past. In all cases, for the operating of the general equilibrium assumptions underpinning classical ideas of equilibrium (and thus the entire basis for the theoretical conclusion of efficiency), this understanding of the likelihood of future events must be complete, in that to every future possibility a probability must be defined. Davidson goes on to explain that not only must these probability distributions be defined, but that additionally "these subjective distributions must be equal to the objective probability distributions that will govern outcomes at any

particular future date" (Davidson 2009), a requirement known as the ergodic axiom (more on this later).

By contrast, Keynes asserts in the General Theory that when making investment decisions in the current period, agents are faced with a set of future conditions that while conditional on current actions, are fundamentally uncertain. Thus, when attempting to make these decisions, agents must rely on their "subjective degree of belief regarding future events" (Davidson 2009) (in effect the assertion is that agents can be *wrong*). It is interesting to note as well that Keynes' understanding of economic agents as fundamentally uncertain, as opposed to fundamentally certain, lends support to and quite obviously predates the observations of "herd" behavior and other exhibitions of human response to extreme uncertainty commented upon by the behavioralists (Davidson 2010). In such a context equilibrium conditions cannot be expected to obtain, and the conclusions of the Efficient Market Hypothesis (as derived from the equilibrium results of Arrow 1964) are called into serious question. If agents cannot be relied upon to make decisions to invest based upon the optimal coordination of all future states, and if that failure is not due to the usual descriptions of market failure, whether they obtain in the form of "government shocks" forcing the system from a long run equilibrium, or from sub-optimal equilibria due to incentive structures such as "excessive discounting" or any other relaxation of the strict neoclassical theory, but rather is due to failure of equilibrium conditions to obtain even in a free market, then the conclusions of the neoclassical synthesis are severely challenged.

From an empirical standpoint, the ergodic axiom does not have a good track record. Most immediate to the mind of the reader, perhaps, is the utter economic chaos resulting from house price projections which ignored the stability of the assumptions necessary to sustain growing prices. It's not even as though the necessary assumptions were few; in order to support the value proposition of the credits backed by the houses, sufficient (or in this case infinite) market depth and perfect information were among the conditions required to obtain. Of course, this severe empirical refutation of the Efficient Market Hypothesis has largely failed to lead to a rejection of the theory. It is still taught, and the neoclassical models are still the basis of a majority of economic thought. In general, such major empirical evidences of a non-ergodic world lead to qualifications of the EMH; economists create multiple equilibrium models in which various factors conspire to cause long run deviations from equilibrium. If the "failures" behind these deviations could be corrected, it is argued, the unfettered economy would gravitate toward efficiency (Stiglitz 1989). What is desperately needed is a discussion of whether the ergodic axiom can itself obtain in any conditions.

It shouldn't be assumed that a rejection of neoclassical modeling techniques leads necessarily to the transformation of economics from a "scientific" and quantitative pursuit to a qualitative one (although really, what use is an incorrect quantitative "science" to anyone?). With advances in the realm of complexity theory and increased understanding of human decision making, which is the real contribution of the behavioralists, it is becoming increasingly possible to speak to uncertainty in a technical

sense. Or rather, it has been possible to do so for a while, but the ground is ripe for further advance.

In the realm of investment – to narrow our scope a bit – the application of Keynes' theory of liquidity paints a fundamentally different picture than that of the classical theories. In that pursuit, Hyman Minsky's financial instability hypothesis builds upon Chp. 12 of the General Theory to produce a comprehensive discussion of the progression of financial markets from stability to fragility. In financial markets, Minsky asserts that realized profits in one period provide the rationalization for increased leverage in the next. Creditors are willing to extend credit based on past profits, but the injection of fresh capital in the form of these credits enables the sector to further increase profits, and new investment floods the profitable sector. Over time the dynamic is unsustainable. As new sources of financing are exhausted, prices cannot continue their previous meteoric rises. This has two immediate effects, first, the flow of credit is further restricted as profits fall, second, if the cycle has progressed far enough, firms will have become dependent on the flow of new financing to facilitate the servicing of past period debts. Minsky calls this state "Speculative Finance" or "Ponzi Finance" depending on the severity of the phenomenon (Minsky 2008). We might imagine the degree of leverage in the sector as a state condition determining the possible degree of correction which would result from a restriction of new finance. Early in the cycle the correction might be small, as firms are able to finance based on carried profits. As the degree of leverage in the system increases, the size of possible corrections increases, thus decreasing the degree of convergence for any predictive model which correctly assesses the loss of new financing

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as a real possibility; the range of possible next period outcomes grows. It might be possible to extend this line of reasoning to assert that under sufficiently extreme conditions, prediction becomes computationally impossible, as the severity of the chaos of a possible market meltdown grows.

In *Barriers and Bounds to Rationality*, Peter Albin raises just such a question in the context of computational systems. Albin suggests the modeling of economic dynamics at the level of interacting agents (which he names, somewhat cumbersomely, "Cellular Automata"). As we have seen, leveraging the ideas of Gödel, Albin asserts that under certain conditions it can be shown that systems of interacting agents can produce dynamics for which any fully specified predictive model would be "unknowable", in that in a computational sense no agent could ascertain how long a set of methods built to assess the models future time path would take to converge. Indeed, the agent would be unable to know whether the methods would converge to a conclusion in any finite period of time. The assertion is supported by a proof which rests on the work of Kurt Gödel, an Austrian/American mathematician made famous by his development of incompleteness theorems obtaining within Whitehead and Russell's Principia Mathematica, a supposedly complete formalization of all mathematical truth from basic axioms. In showing that paradoxes (the reason for the name incompleteness: completeness implies all statements can be proved either true or false; it is a lack of undecidability) existed within Whitehead and Russell's work, Gödel essentially proved paradox fundamental to mathematical logic. In order to do so, Gödel relied on a clever formalization of self-reference to create the statement "this statement is false" in formal logic. (For a complete discussion of Godel

incompleteness including its connections to economic prediction and a proof of the inconstructibility of a predictor in systems with self-reference see Barriers and Bounds to Rationality Chapter 2).

For Albin, the existence of formal undecidability is supported in economic models of Cellular Automata through the formalization of increasingly complex decision rules for automata. If an agent in endowed with the ability to formulate its own decision on that of its neighbor, and its neighbor likewise chooses at least partially based on the original agent's own observed choice, it can clearly be seen that both agents' choice matrices are to some extent self-referential (Albin 1998). Simple formulations of game theory often solve the problem of circular reference in formal games through the conceptualization of infinite time horizons, but such solutions are difficult to believe (an agent solves an infinite set?) and have been shown problematic in real world experiments (Eichberger and Kelsey 2011). Albin shows that fairly simple choice sets can lead to completely unpredictable emergent behavior at the aggregate level, and suggests that the existence of such dynamics in simple models should all but prove its existence in real markets, where agents are allowed much more complex and variable choice sets. In the context of broader economic thought, dynamics matter. It's entirely possible that equilibrium may fail to obtain, and even that any static picture could completely fail to characterize the system.

Arrow-Debreau, Copula Pricing, and Value at Risk

Traditionally, the treatment of the quantification of uncertainty (and I use the word purposefully, the majority of efforts to quantify prediction have ignored the possibility of fundamental uncertainty) has progressed according to two interconnected paradigms. The first involves the evaluation of statistical models upon historical data, complicated trend parameterizations such as GARCH/ARCH and ARIMA fall into this category. Such models necessarily require that at least on some level current conditions obtain over the time horizon of the forecast (ie. second or third-order complexity, for a survey of time-series forecasting methods applied to financial markets, see Taylor 2008). Should whatever statistical artifacts a given model is built upon be violated during the period forecast, the forecast will, for obvious reasons, be rendered incorrect. In the context of this first paradigm, prediction is facilitated by the correct estimation of the *patterns* underlying economic data, possible failures explored by the practitioner might include model misspecifications around the memory of the system, the periodicity of cyclical behavior, or bias in estimator values. The second paradigm has to do with the choice of specific statistical artifact and implies the application of a structural framework to empirical data in the form of an economic model. In the context of the earlier noted ergodic paradigm, equilibrium models require that when the correct economic frameworks are applied to statistical estimations of past conditions, the resulting conclusions must obtain over every future period (Davidson 2009) so that agents are able to correctly decide upon current period behavior such that all possible future utility is maximized.

The work of Arrow and Debreu codified the idea of general equilibrium into a comprehensive mathematical model. By assuming economic agents to possess preference sets that were both complete (all possible bundles of goods are associated with preferences), and convex (implying a decreasing marginal rate of substitution) the Arrow-Debreu model proves the existence of a unique equilibrium point at which preferences are optimally balanced with constraints. The restricted model is then expanded to show the existence of general equilibrium under an expanded set of conditions for the economy as a whole (Varian 1992).

Such a concentration on equilibrium has led to a massive foundation of mainstream thought on the concept of comparative statics. The earliest foundations of economic teaching are in general pictures of supply and demand graphs depicting nicely drawn smooth curves intersecting at ubiquitous black dots. Students are then taught the factors which move the supply and demand curves, and are told to reproduce the movements and descriptions of the format "quantity increases and price increases when demand increases" on tests and draw nice arrows describing the movements. In more complicated frameworks many economic models still often rest upon these parameterizations of equilibrium conditions. The quantification of the effects of a given policy on the welfare of the poor takes current conditions as current equilibrium, and assesses the effects on the various determinants of the welfare function and assess the cost to one group, benefit to another, and pronounce the new intersection (in this case a hyperplane separating convex sets) the result of the proposed policy. At this point it should be noted that a "dynamic" neoclassical model expressing convergence to a

constant rate of change, for example in capital accumulation (even those with multiple equilibria, see Böhm and Vachadze 2007), is qualitatively no different from a static equilibrium, the constant state merely exists in the derivative; this is *not* accounting for "dynamics" as such, and does not constitute an understanding of how equilibrium is met, it is simply integrated comparative statics. If equilibria fail to obtain as often as not, it is very problematic for such predictive modeling.

The vast majority of financial pricing models rely directly on the existence of Arrow-Debreu equilibrium. Indeed the A-D framework itself attempts to solve the problem of the intertemporal allocation of capital; in order to arrive at a general equilibrium the framework arrives at a specification of allocations which solve the system of equations for resource allocation across time (Davidson 2009). The application of this equilibrium model to financial markets is then a logical extension, which Arrow takes up in his 1964 article *The Role of Securities in the Optimal Allocation of Risk-Bearing*. An examination of Arrow's discovery of an optimal risk allocation across all future states is contingent upon the assessment in the current period of the likelihood that any given future state should occur, otherwise his utility function: $V_i = \sum_{i=1}^{S} \pi_s U_i$ describing the expected utility conditional on the likelihood that state *s* occurs in some future period, wouldn't exist since π_s would not exist (Arrow 1964, p94). If it is further the case that the optimal distribution in some future state, *s*, is itself contingent on the conditional probability of state *t* obtaining at some point after *s*, we can support Davidson's claim that Arrow-Debreu requires the assessment of the optimal distribution of goods be conditional upon not only the likelihood of any future state, (the *s* in Arrow's utility)*,* obtaining in an

immediate future, but also upon the distributions of all possible future states across all time. This makes a certain kind of sense if we realize that current investment decisions are predicated upon opinions of the future, and that future investment decisions will likely continue to be defined by similar considerations. What *is* odd is the assertion that agents can account for this infinite set of possibilities in infinite time. While Arrow's approach to the solution for optimal risk-sharing is mathematically coherent, it would appear to pose critical problems of computability. How is it possible for an economic agent to ascertain the likelihood of any given state with perfect accuracy, let alone the likelihood of any state at any arbitrary point in the future? Critically, the conclusion that market pricings are generally correct in an equilibrium sense is necessary for the current body of theory supporting models of financial risk. From Aroujo et al. (2010):

> "Since the Arrow's Role of Securities seminal paper, the financial general equilibrium models assume that the price of assets satisfies equilibrium conditions in a competitive setting where many agents demand assets profiles in accordance with their preferences and their endowments, providing the foundations for the study of financial markets by a celebrated fundamental result asserting that financial markets must not offer arbitrage opportunities" (Aroujo et al. 2010, p.2).

In particular, the pricing of complex derivative products has often required the utilization of such assumptions, since if financial general equilibrium fails to obtain, we have fundamental mispricicing in the market. The evaluation of the risk profiles of complex derivative instruments relies heavily on the assumption of perfect market pricing. In Li (2000), three separate approaches are described to estimate the present value of such a portfolio of credits (specifically he is referring to the now infamous mortgage backed securities and other CDOs). The first is the estimation of asset and default correlation based on a body of historical knowledge which we might name the statistical/historical

approach, the second uses the Merton option-theoretical approach, and the third operates via asset swap spreads (Li 2000). The second and third approaches are expressions of the Arrow-Debreu framework applied to financial markets (and the first is a tautological representation of the ergodic axiom). In all cases, convergence conditions can only be met via the assumption of equilibrium pricing under perfect competition. The swap spread approach infers default probabilities through the spread between treasuries (or securities for which the default probability is zero) and the credits in question. Such "pricing" requires an accurate assessment by the market of the default probability of the credits, and such a correct assessment could only be fully supported by a perfect market. Given the degree of fraud prevalent in the mortgage market at the time these pricing models found widespread use, such a claim is farcical (A quick look at bank litigation losses since the crisis would give a rough valuation).

All of the above approaches utilize a copula function to generate a multivariate probability distribution for the assessment of future price at the derivative level. In multivariate statistics, a copula function is any function of the form $C = \Phi(\phi^{-1}(A_i) \forall i \in \mathbb{Z}, \Sigma)$ which generates a multivariate distribution function from a set of observed marginal distributions. In the case of the CDO, the marginal distributions are the observed survival times of the assets that make up the instrument. Once the multivariate distribution is evaluated, expectations of future price, and more importantly expressions of aggregate risk at the portfolio level can be quantified. The correlation matrix Σ is given through analysis of survival time or asset value correlation and can have a critical effect on the evaluated copula (Li 2000). Thus these copula pricing models required accurate

calibration of the correlation matrix, Li's three methods attempt to do just that via either straight expressions of historical correlations or inferred relationships between constituent credits as expressed by market pricing. In the context of fundamental uncertainty, this calibration presents great difficulty. Both observed historical correlations and market assessments of risk are subject to state conditions that are prone to variability over time. At best, predictions might be expected to obtain over short time horizons (under third-order dynamics). Once again, CDO structures don't lend themselves to an optimistic assessment in this regard; their underlying credits were in many cases 30 year mortgages.

A second family of better known risk evaluation models, known under the common name Value-at-Risk (VaR), rest upon the assessment of the downside tail of returns. In general trading firms and major financial institutions will forecast the expected 1% or 5% one day ahead upper bound for losses across their portfolios. This Value-at-Risk effectively quantifies the short term risks facing the firm and is used in the specification of various aspects of firms' required capital cushions. It should be noted that the technical inference of VaR is often implicitly mischaracterized as the maximum value a firm can expect to lose with a 95 or 99% confidence interval. In reality, the previous statement severely understates the implied possible risk facing the firm, a correctly calibrated one-day-ahead 1% Value-at-Risk should be expected to generate VaR "Breaks" (days on which losses *at least exceed* the VaR) between three to four times a year. So, a correct statement of the interpretation of VaR would be that the firm should expect to lose *at least* the VaR three to four times a year. Additionally, in the context of such

leptokurtosis in returns as obtains in the stock market, the actual loss during a VaR break could be orders of magnitude larger than the VaR itself (one only needs to visit the list of trading losses on Wikipedia to understand how true this statement is). A multitude of methods have been proposed to calculate value at risk, both in the context of normal (Linsmeier and Pearson 2000) or non-normal returns, and in dealing with complex correlation structures within a portfolio. The technical construction can be built off of historical values or Monte Carlo simulations given current distribution structures (Linsmeier and Pearson 2000). Value-at-Risk models have even been proposed building upon copula-based multivariate distributions (Miller and Liu 2006), and Monte Carlo simulations built off of ARCH/GARCH models (Mancini and Trojani 2011) to parameterize possible conditional variance among the credits of a portfolio. Whatever the technical aspects of their construction, at their core VaRs are simply the value of the cumulative distribution function at either 5 or 1%, or by extension the area in the 1% or 5% tail of the distribution of loss.

Perignon and Smith (2010) provides a convenient method by which to assess the degree of covariance within a portfolio via Value-at-Risk by assessing the percent deviation between the observed VaR of the portfolio and the VaR of a theoretical portfolio with identical dollar weights but perfect correlation across assets. Leveraging this "diversification coefficient" will allow us to assess the time evolution of such correlations, and in so doing begin to study the possible uncertainty inherent in the state conditions necessary to support the large family of derivative Copula pricing models: extreme variance and kurtosis in the time-evolution of diversification would seriously

cripple the degree of convergence. The ultimate goal is much deeper than merely specifying an expectation of variation in parameter values, through the application of an Agent Based Model in concert with Albin's discussions of unpredictability a conclusion of periodic fundamental uncertainty in these risk-pricing models is strongly supported.

Agent Based Modeling (ABM)

While there exists a sizable body of literature, the use of non-linear dynamics in economic models is a developing work. In many important areas they remain on the periphery, secondary in importance to the more common applications of comparative statics (Zhang 2005). Less work has been done, however, in the examination of agent based models, which enable the simulation of the highest levels of complexity. Even more than non-linear dynamics, agent based modeling allows for the study of the functional determinants of convergence and divergence in dynamics. Depressingly, the application of Monte Carlo simulations to the estimation of portfolio risk is likely the most widely applied instance of simulation today. It is important to note that there exists significant difference between the simulation of a statistical expression of emergent behavior (Monte Carlo simulations of correlated returns for example) and the simulation of the dynamics which produce these emergent behaviors. Indeed, the very advantage of ABM is that it allows us to specify choice matrices for economic agents that do not necessarily lead to convergent models, but which instead allow us to study possible determinants of the conditions which are necessary for modeling convergence. As was touched upon above, if we cannot support the stability of these conditions, it is possible

that the emergent behavior of the system could be computationally "unknowable" and prediction models would diverge. The realization that such states could exist is critical, especially if the conditions driving convergence/divergence can be better understood. There is certainly a *growing* body of literature on ABM (agent based models), and network models in particular, however, the critical question of convergence vs. divergence has not generally been addressed.

In Delli Gatti et al. (2006 and later expanded in Delli Gatti et al. 2010), the authors present an economy characterized by three types of agents, downstream firms, upstream firms, and banks. The three groups interact through a series of credit relationships, both to finance the purchase of intermediate goods, and to fund their wage-bills. The firm network acts as a semi-closed system: all production is consumed, providing the energy input to the system, while bankruptcies and the wage bill extract wealth from the system. Within the model a number of factors lead to outcomes that mimic observed reality in important ways. Due to the nature of the connections between firms and banks, the normally distributed stochastic energy input (the revenue of downstream firms) results in several non-normal outcomes. Sufficient loss in a single downstream firm can spread as a cascading bankruptcy through the rest of the network, the extent of which is only constrained by the margins on which agents are operating and the depth of the connections in the network. Furthermore, the interest rate mechanism is biased towards larger firms, as growing net worth implies stability to banks, supporting lower borrowing costs. The results highlighted are kurtosis in the aggregate value of bad debt due to cascading failure, and fat tails in firm size distribution. These results arise not

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solely due to the interest rate mechanism but also due credit constraints which allow growing nodes to continue to grow but in the case of loss constrain others, reinforcing the growth tendency through a variety of factors (Delli Gatti et al. 2006).

Since the publication in 2006 of the initial model, the group has updated the model to be more refined. By introducing an endogenous choice to the formation and dissolution of credit connections, the updated model has adopted a possible explanation of the method by which the network can evolve. The change has two advantages. First, endogenous agent choice allows for a closer approximation of reality. Firms are not completely restricted in their choice of credit and real connections (although, as Stiglitz himself has pointed out, choice can be severely constrained in credit possibilities for small and medium enterprises. Often, a single bank advances credit in the light of a long term relationship with the borrower, finding a replacement can often be difficult for the SME, see Greenwald and Stiglitz 2003). Instead they have choice in respect to with whom they chose to do business. Second, the observation is made that the results of the initial model are reinforced through the introduction of an endogenous and variable network topology. Since incentives exist to encourage firms seeking new credit partners and thereby to gravitate towards established network nodes (again, size implies stability), leptokurtosis in firm size distribution is reinforced, and the excess kurtosis in the time evolution of bad debt is similarly strengthened (Delli Gatti et al. 2010).

The results of the model were already well established at the time of its publication; Axtell (2001) finds robust results detailing a zipf distribution of firms, and kurtosis in returns to investment are highlighted in a massive number of studies on

various stock markets (Mandlebrot 1963, presenting perhaps an iconic example) and is a well-recognized result. However, there is an additional result which was not highlighted in the 2006 or the 2010 formulation of the credit-network economy, but which is supported by work in Mizgier, Wagner, and Holyst (2012) and Battison et al. (2007), which is that the concentration of a network economy has drastic implications on the generation of global and local risk. Wagner et al. (2009), studied supplier default dependencies via a copula approach, and this work will leverage the Delli Gatti et al. model to connect the body of supplier default correlation/cascade models and literature on portfolio pricing in an attempt to understand the implications of fundamental uncertainty on risk assessment. The results are similar, the default correlation structure of the financial markets are dependent upon network topologies. In contrast to their work, the focus of this piece is to highlight the factors complicating the prediction of future value based on current state conditions and thus the assumption of convergence to equilibrium. The ability of such network models to produce dynamics which either experience sustained and volatile deviations from equilibrium conditions, or to fail to approximate any recognizable notion of equilibrium, introduce in a very real way the possibility of divergence in expectations of price.

In E. Nier et al. (2006), the network economy is formalized as an Erdös-Rení random graph, wherein the nodes of the network economy are connected and summarized given a few broad statements about the distribution of credit connections. The model is defined by a number of nodes, N, a probability expression p_{ij} defining the likelihood that any given bank has lent to another bank in the system, the average mix of eternal to

internal assets (within the system, the proportion of interbank to external lending), a parameter defining the net worth as a function of total assets, and a residual deposit variable (E. Nier et al. 2006). In generating the simulations, the authors choose to shock single banks and study the propagation of default chains through the system. Their isolated formal approach has the advantage of studying the propagation of default risk in networks absent the muddying influence of the dynamic systems studied in this paper. There could be a lot to be gained from applying a similar approach to the propagation of risk in the Delli Gatti network over time. Indeed, if any network could be modeled as a series of fluctuating Erdös-Rení random graphs, the conclusions of E. Neir et al. could be used to describe the time evolution of risk *uncertainty*, especially if the time-evolution of the parameters of the Erdös-Rení map could be modeled. Such an exercise would, in essence, be a return to Albin's discussion of the predictability of a system; since not all state conditions can be expected to produce predictive models which converge, it is important to study, to whatever extent possible, the determinants of these conditions.

III. The Model

A credit network model for production with an intermediate good

A version of the original model defined in Delli Gatti et al. (2006) in which three layers of agents interact with one another in reaction to a stochastic final price is adopted for analysis. A downstream industry produces goods for consumption at the final price, an upstream industry produces inputs which are consumed by downstream firms and financed by short term credit by the upstream firms. Both upstream and downstream firms turn to the banking industry to finance their wage bills. The resulting network of credit and productive relationships constitute the topology of the network model. By assessing the experience of the correlation matrix of bank profits, this work will assess the variability of observed risk under a varying topology.

The model produces a simulation of a credit market, with a stochastic final price defining the profits of downstream firms, which must enter into relationships with upstream producers and banks to secure capital goods and financing to cover the costs of financing. In each timestep all downstream firms are presented with a new stochastic price, and the experience of the network is defined over time by resulting experience of these firms. Firms and banks are replaced after bankruptcy such that the topology of the

network is unaffected. This work studies profits in the banking sector over time by iterating this simulation through several thousand timesteps.

Figure 1: Credit Network

Downstream firms produce based on their current net worth via the following relationship, along with linear labor and capital requirement functions:

1) $\quad Y_{it} = \phi A_{it}^{\beta}; N_{it} = \delta_a Y_{it}, K_{it} = \gamma Y_{it}$

Where the firm's output at time t Y_{it} is a function of its net worth A and the parameters ϕ and β. The requirement functions define the necessary labor and capital to produce Y_{it} by the coefficients δ_a and γ. Furthermore, labor unit-costs are defined as w and capital unit-costs p, such that the cost function (when all production is financed via carried profits) can be given as:

2) $\quad C_{it} = wN_{it} + pK_{it}$

All downstream production is consumed at the stochastic final price $u_{it} \sim U[0,2]$.

Upstream firms produce capital inputs as intermediate goods in the downstream production process. Upstream firms produce to fulfill downstream demand with an analogous requirement function with only labor as an input:

3) $Y_{jt} = D_{jt}; N_{jt} = \delta_u Y_{jt}.$

Upstream firms supply ½ of the demand for each of their proximate downstream partners, such that the demand for the upstream firm's production is:

4) $D_{ut} = 1/2[\gamma Y_{it} + \gamma Y_{(i+1)t}].$

(given the capital requirement function from (1)). Throughout the model the defining characteristic is the topology of credit connections. All firms finance their production through a combination of carried profits and credit relationships for the employment of resources. Once production is realized, firms will have either netted sufficient revenue to pay their creditors, or they will default. All firms rely on bank credit to finance their labor inputs, such that the labor cost of production for the firm is given by:

5) $C_n = \delta_n Y_{nt} w (1 + r_n^b).$

where $n = u$ for upstream firms and d for downstream firms, r_n^b is the interest rate charged to the firm by the bank, δ_n is the labor requirement coefficient from (1) and (3),

and w is the labor unit-cost, constant across all firms and all time steps. Additionally, downstream firms enter into a credit agreement with upstream firms for the delivery of capital inputs into the production process, such that the capital cost of the downstream firm is given by:

$$6) \quad C_d = \gamma Y_{dt} p (1 + r_d^u).$$

where r_d^u is the interest rate charged to downstream firms by upstream firms, p is the price of capital goods, constant across firms and time, and γ is the capital requirement coefficient from (1). Defaults, when they occur, are characterized by total loss on the part of the creditor. Finally, the bankrupt agent is removed from the model and replaced in the next time step with an agent with given starting net worth.

Banks supply credit based on a leverage limit:

$$7) \quad L_{zt}^s = A_{zt}/\alpha$$

where α is a regulatory target set by authorities which is invariant over the course of the scenario and A_{zt} is the net worth of bank z at time t (while not addressed in this work, the introduction of a government agent to set this limit based on various behavioral regimes might provide insight into unintended policy effects). In the base model each bank is connected via credit arrangements with one downstream and one upstream firm, and the

number of these connections will increase as we restrict the size of the set of banks. The assessed interest rate is given by the relationship

$$8) \qquad r_{nt}^{bu} = k/(\frac{A_{nt}}{\tilde{A}_t})\wedge k$$

where A is the net worth of the nth firm, and $\tilde{A}$ is the median wealth of firms, calculated separately among upstream and downstream banks ($n = u$ for upstream and $n = d$ for downstream). Finally, it is assumed that deposits, D_{zt}, are residual on banks' balance sheets, such that the sum of the credit supplied by the bank is covered by the value of the bank's carried profits and the value of the deposits: $L^s = D + C$ where C is carried profits and D deposits. This yields the relationship: $L^s - D = Net\ Worth$ when all carried profit is fully deployed in lending. A constant interest rate (r_d) is assessed against deposits over the duration of the simulation. Banks react to the aggregate demand of their partner firms and, in the case in which they have insufficient credit available to meet the demand, turn to the interbank market to raise funds. The interbank rate is constant over the duration of the scenario, and default among banks leads to total loss on their creditors' interbank portfolios.

The scenario progresses according to the observed beginning asset values of the downstream firms. Aggregate downstream production is calculated, and orders filter upward to upstream firms for capital inputs and on to banks to finance the payment of wages. Banks asses the incoming demand for credit against their credit supplies, attempt to resolve credit constraints through the interbank market and transmit the available credit

and derived interest rates to firms. Firms then adjust their production based on any applicable credit constraints, and place their orders.

All downstream production is consumed at the stochastic price, u_{it}. If a downstream firm fails to receive a sufficient price to prevent bankruptcy it defaults completely on its credit agreements. Thus, a downstream firm's bankruptcy increases the bankruptcy risk for both its supplier and bank. Additionally, the increased likelihood of bank default can serve to "infect" the broader banking system via borrowing on the interbank market.

Since all production is contingent upon the network of credit connections within the economy, the distribution of returns is highly dependent on the time-evolution of the interbank market. Credit constraints can pose a real restriction on the pace of growth, and since the interest rates are assessed against firms based upon their performance against the median in their industry, banks servicing "successful" firms operate on thinner margins and can garner lower profit, whereas those banks servicing frail firms might realize greater potential profits, but at a higher risk. Realized returns in the banks' portfolios up to and including default have real effects on the likelihood of any given bank leveraging the interbank market. A default on a given bank's balance sheet can depress its capital cushion and necessitate a deepening of its credit relationship with its neighbor banks. Additionally, those banks replacing bankrupt banks can require outside investment (read "borrowing on the interbank market") to service their partner firms. When default occurs, the potential for contagion runs only as deep as the credit connections that exist between banks, thus, a comprehensive characterization of the

correlation among banks will serve as a useful expression of the effects of changing network topologies.

It is interesting to note that the model essentially operates within the restrictive bounds of utility maximizing behavior. Due to the static nature of the credit network and the stochastic price element, all firms are price takers in their respective markets. The addition of competition or differentiation in agent's choice algorithms could only be expected to increase the degree of observed complexity. Interesting extensions are likely possible into the question of market power and further heterogeneity among agents.

Perignon and Smith's Diversification Coefficient

In Delli Gatti et al. (2006 and 2010) the model above (although with drastic alterations in the later paper) was run to various specifications. However, the basic identity, $N_u = N_d = N_b$ where N_x is the size of the set of each industry, is held constant across simulations. The following results, then, will be assessed against a series of simulations of the above model with a varying number of banks. Once the scenarios are run, the results are analyzed according to Perignon and Smith (2010), in which a Value at Risk (a measure of the boundary, usually 1 or 5% upper tail of the 1-day ahead expectation for loss) approach is leveraged to study diversification in various portfolios of bank credits. In this analysis the net worth of banks are formalized as a portfolio with returns given by bank profit in each period and Perignon and Smith's diversification coefficient is used to assess the time evolution of the level of correlation across sections of the economy. In the context of the Copula approach to derivative pricing we are quite

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literally examining the state conditions of the credit market (and how difficult prediction based on such models may become). Since copula functions are quite sensitive to changes in the correlation matrix, severe variation will imply that state changes in the topology of the credit network fundamentally affect the ability of such predictive models to obtain. From Perignon and Smith (2010):

1) $\qquad \delta = \dfrac{\sum_{i=1}^{n} VaR_i - DVAR}{\sum_{i=1}^{n} VaR_i}$

In the above context $\sum_{i=1}^{n} VaR_i$, the sum of the individual VaRs of credits in the portfolio, is given through the identity:

2) $\qquad VaR_i = k\sigma_i x_i$

where k is a scale parameter varying with the shape of the distribution (here assumed normal for ease of derivation: $k=2.33$), and x_i is the dollar position in the asset, in this case the standard deviation of returns is assessed against changes in net worth. Since net worth is itself taken to be the dollar position, x_i can be omitted. The sum of the individual VaRs of credits across a portfolio is equal to the VaR of the total portfolio only if the asset correlation of the portfolio's constituent credits is perfect. In all cases some imperfect level of correlation will obtain between credits, and the extent to which credits are uncorrelated will depress the Value at Risk of the portfolio as a whole. The

calculation of DVaR (or diversified value at risk, see Perignon and Smith 2010 for a complete derivation) is given by:

$$3) \qquad DVaR = \sqrt{V'RV}$$

where V is a column vector of the individual VaRs within the portfolio and R is the asset correlation matrix. The diversification coefficient δ in (1) ranges from 0 to 1 measuring the percentage deviation from a perfectly correlated portfolio due to the structure of the correlation matrix of a portfolio's constituent credits, a higher value for the diversification coefficient will imply a weaker correlation structure and lower vise-versa. Note that in all cases this analysis will be conducted against the *profits* of the various banks. Should deep credit relationships develop between banks, they will serve as unrealized avenues of correlation until default occurs. Without fully quantifying the nature and depth of the various credit relationships in the network (something we could not expect the various participants to be capable of due to informational constraints) statistical estimation of risk will be "surprised" by the strength of correlation during downside shocks. Characterization of the resulting mispricing requires a deeper discussion of both the development over time of the credit relationships among firms and banks and the determinants and effects of default.

IV. The determinants of the correlation matrix:

What follows is a technical examination of the determinants and nature of default probabilities at each level of the network, and the methods by which bankruptcy risk propagates. It will be especially necessary to study the development of two conditions, bankruptcy and credit constraints, the first being the only mechanism by which second and higher order effects (default cascades) can be felt through the model, and the second being the determining factor of the strength of those higher order effects.

Notice that the expression of the profit of the ith downstream firm at time t is given by:

1) $\pi_{it} = u_{it}(Y_{it}) - (1 + r_d^u)p\gamma Y_{it} - (1 + r_d^b)w\delta_d Y_{it}$

(from 1,2,5, and 6 in section 3.1) It is immediately apparent that the distribution of profit for an individual downstream firm is largely independent of the profit of an adjacent downstream firm (for initial analysis the effect of r_d^b, which is partially determined by the median worth of the downstream sector, is assumed to be negligible). Thus, the likelihood of default of a downstream firm, $P[\pi_{it} < -A_{it}]$, is iid $\forall$ i aside interest rate effects.

Likewise we can see that the profit of the jth upstream firm is given by:

2) $\pi_{jt} = (1 + r_d^u)p\gamma Y_{it} - (1 + r_u^b)w\delta_u(\frac{1}{2}\gamma Y_{it} + \frac{1}{2}\gamma Y_{(i+1)t}) - BD$

(from 3,4, and 5 in 3.1) where BD is the aggregate value of credit supplied to downstream firms which defaulted in time t. By extension it can be seen that:

3) $P[\pi_{ji} < -A_{jt}] \propto P[\pi_{it} < -A_{it}] \forall i \in Z, w\delta_u, r_u^b$

Where $P[\pi_{jt} < -A_{jt}]$ is the likelihood of collapse for the upstream firm j at time t, Z is the set of debtors of j, $w\delta_u$ is the implied cost multiplier for the labor cost of the upstream firm, and r_u^b is the interest rate charged by the bank to the upstream firm. It is apparent that the distribution of returns to the upstream firm's investment will approximate the copula function $C = \Phi(\phi^{-1}(A_i) \forall i \in Z, \Sigma)$ where sigma is the identity matrix, since the asset values of all of its downstream partners are independent absent interest rate effects. Considering that the implication is that the observation of bankruptcy for any element of Z has no effect on the likelihood of default for any other element, it can be deduced that:

4) $P[\pi_{jt} < -A_{jt}] \propto \prod_{i=1}^{x} P[\pi_{it} < -A_{it}]$

Where x is the number of bankruptcies in Z necessary to induce bankruptcy in u. Since the expected value of each downstream firm i at time t is identical save for credit

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constraints and interest rate disparities driven by banks' perception of solvency in firms, the expected weight of each downstream firm is identical. Thus, the likelihood that a sufficient number of firms default to force default in *j* is:

5) $$P[\pi_{jt} < -A_{jt}] \approx \prod_{i=1}^{x} P[\pi_{it} < -A_{it}]$$

and falls exponentially with *N*, the number of firms in Z, *ceteris paribus*. Concentration at the level of the upstream firm is the perfect example of diversification absent interest rates, since the defaults of *i* firms are *iid*.

It is not the case, however, that the survival times of the *j* and *j+1* upstream firms are uncorrelated. In the initial case first described by Delli Gatti et al. (2006), the ith downstream firm receives half of its required capital input from upstream firms *j* and *j+1*, and thus the incidence of default in the *i*th downstream firm appreciably increases the likelihood of default for both upstream firms. That, however, is as far as the shock can be transmitted, absent the interest rate mechanism noted above. The conditional probability of default in the jth upstream firm is contingent upon only the observation of default in the *j+1* and *j-1* upstream firms. A shock cannot be transferred further until it is significant enough to affect the median net worth in the industry or should the supplying bank face difficulty repaying its obligations on the interbank market.

Additionally, given that time evolution of profit for the upstream firm is given by:

6) $\pi_{jt} = \sum_{i \in Z}\left[(1 + r_{jt}^i) - (1 + r_{zt}^j)w\delta_u\right]\gamma\phi A_{it}^\beta + (1 + r_{zt}^j)A_{jt};$ (A_{jt} being

the value of carried profits financing production)

If we maintain that the upstream firm shares the demand of the ultimate downstream firms (such that $Z_j \cap Z_{j+1}$ contains only one downstream firm). The strength of the correlation between two adjacent upstream firms would fall if the size of Z were to increase.

When we consider the effect of the bank's interest rate decisions upon the default and asset value correlations of the up and downstream firms we notice that the interest rate decision is given by:

7) $r_{yt}^{bx} = k/(\frac{A_{yt}}{\widetilde{A_{xt}}})\wedge k$

Where A_{yt} is the net worth of a given upstream or downstream firm at time t ($y=i$, $x=d$ *for downstream and y=j, x=u for upstream)* $\tilde{A}$ is the median net worth of firms at time t, computed separately for upstream and downstream firms. The interest rate decision includes a dual force component. First, it acts on the individual firm by reinforcing its divergence from the median. Weaker firms will face more stringent borrowing costs and stronger firms vice versa. Secondly, interest rates rise as the industry's median falls further increasing systemic weaknesses after negative shocks. Within any given industry, however, the effects of the interest rate mechanism on pairwise correlations are minimal. To the extent that it does have an effect, the mechanism reinforces comovement in the

previous period, and provides minor resistance to correlation between firms whose profits did not exhibit comovement in the period prior. The division of the sectors into banks that serve as nodes of credit provision and their constituent firms likely interacts with these factors to produce long range dependence and the observed variability of the correlation matrix.

Leaving aside concentration in the upstream markets for the moment, we can see that for the bank, then, a shift in network topology towards increased concentration doesn't necessarily decrease the likelihood of default. Each upstream credit in the bank's portfolio has a positive default correlation with the credits of adjacent firms, and the downstream firms associated with them. Furthermore, banks will readily employ idle balances on the interbank market, both to fill unmatched demand for credit, and to supply short term liquidity to their neighbors in the case of liquidity crises. As the depth of these interbank credits increase, *possible* correlation among banks becomes increasingly variable. Through the interbank market, then, there exists an unbounded transmission mechanism. In essence a seemingly stable "equilibrium" may bifurcate from a strong attractor in the dynamic sense (the sector quickly recovers from a single bank's default) to a weak one (a single bank's default leads to systemic problems). Thus, the observation of default of a single downstream firm can increase the likelihood of the observation of default for the entire *banking sector* (with a decreasing effect further out along the network). Furthermore, negative shocks increase the fragility of the system in two ways. First, negative shocks decrease the net worth of firms and banks. Second, to the extent that the banks which experienced negative shocks turn to the interbank market to either

match demand for credit or to cover liability crises, negative shocks deepen the liability structure. As is seen from the time evolution of profits in the zth bank:

$$8) \qquad \pi_{zt} = \sum_{i \in Z_j} r_d^b w \delta_d A_{it} + \sum_{j \in \Lambda_z} r_u^b w \delta_u A_{jt} + r^{bb} IB - BD - r_d DD$$

where IB is the position of bank z on the interbank market, BD the value of bad debt on the bank's balance sheet, Λ_z is the set of upstream firms with credit connections to the zth bank, and Z_j is the set of downstream firms with credit connections to the jth upstream firm. It can be noted that the risk profile of the bank is determined by the size of the positions the bank has taken in the various markets. When the bank's balance sheet is weighed towards the up and downstream markets, the bank's expected value is characterized by the correlations among its own credits. By contrast, when the bank's balance sheet is weighed toward interbank lending, default can occur even in the presence of strong performance in the rest of its portfolio. The experience of the concentrated network is not trivially an expression of the aggregated experience of the banks in the diffuse network, much larger portions of the banking sector can be effected negatively following an identical downstream experience. When such negative impacts occur, the fragility of the entire system is increased, raising the likelihood of a departure from a convergent expression of the network's expected "equilibrium".

V. Results

The model was run first against a network with 100 members of each set of firms: upstream, downstream and banks. The simulations covered periods of 2500 timesteps, and results are examined against time steps beyond 500. Subsequent model specifications decrease the number of banks first to 50, and then 25, such that in the base model each bank has credit connections with a single upstream and downstream firm, in the second case each bank has credit connections with four downstream-upstream firm pairs and finally eight in the concentrated case.
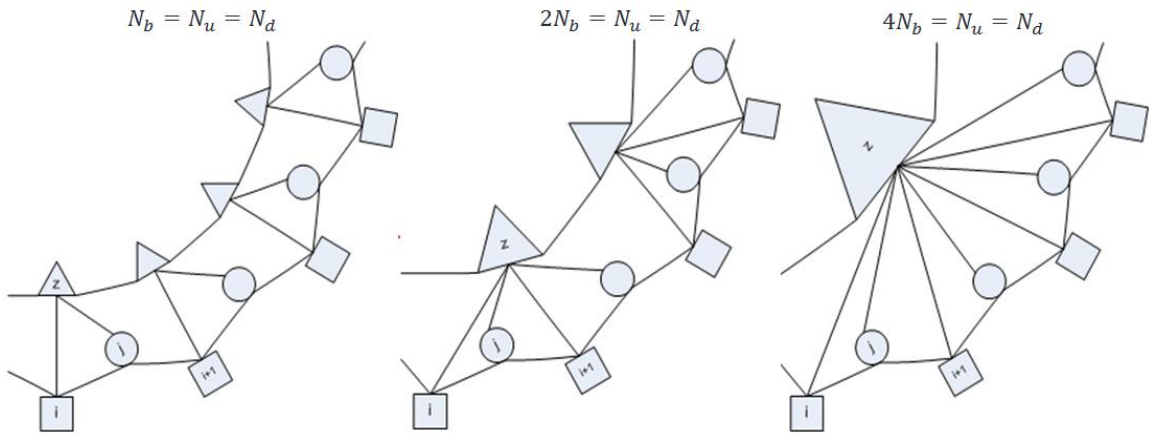

Figure 2: Increasing concentration at the bank level

A theoretical portfolio is then constructed from the resulting net worth of the banks given by the iterations of the model. By examining the exhibited evolution of the correlation matrix among these banks via Perignon and Smith's diversification coefficient, the

predictability of the theoretical portfolio can be formalized. It is shown that the variable depth of credit connections imposes an increasing degree of uncertainty on this portfolio as the banking sector is constricted, as the effect of unrealized avenues of correlation on the interbank market are generalized to ever larger portions of the portfolio. The structure of the correlation matrix can exhibit strong bifurcation. The following three specific statistical artifacts support this conclusion.

Kurtosis in single period returns:

single period returns:
25 banks, Kurtosis = 6.5

single period returns:
50 Banks, Kurtosis = 3.0

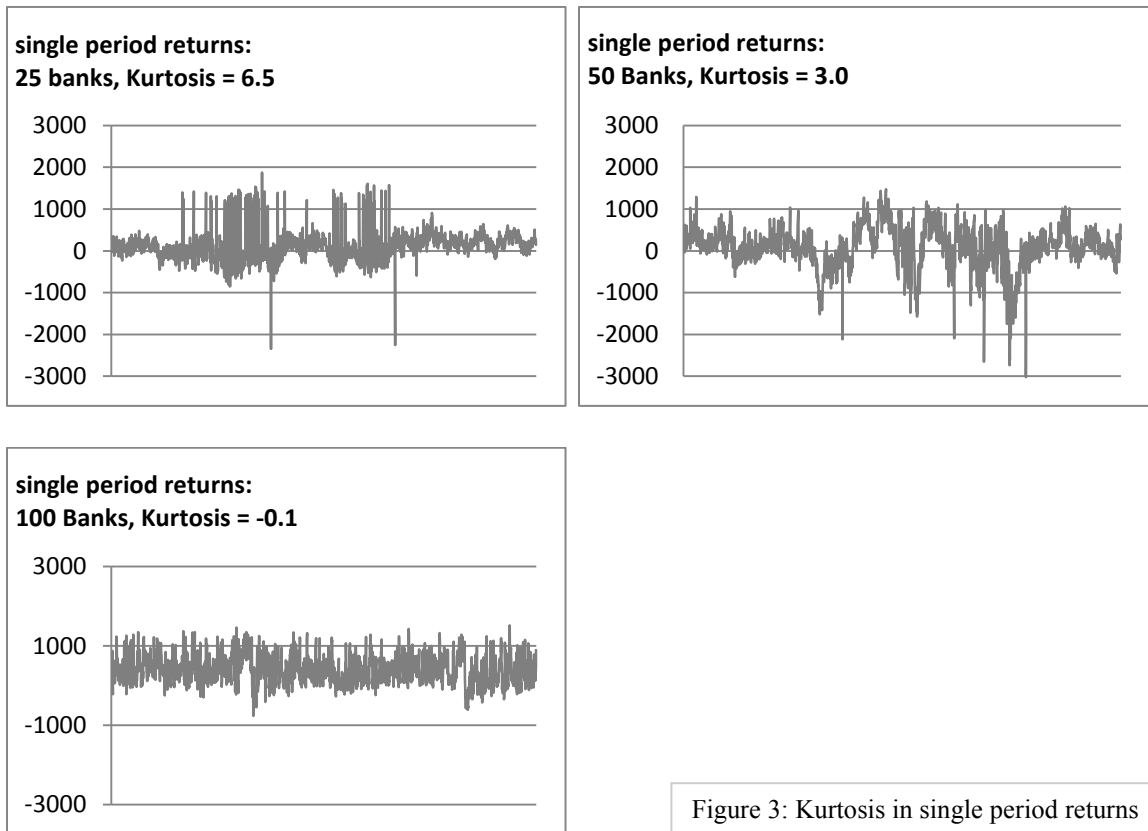single period returns:
100 Banks, Kurtosis = -0.1

Figure 3: Kurtosis in single period returns

A marked increase in the kurtosis on single period profits to the banking sector is observed to correlate with increasing concentration among banks. Visual inspection

shows a nearly Brownian time evolution of profit degrading into a signal that bifurcates between periods of more or less normal returns and periods of abnormal profit, punctuated by a large loss before returning to normal profits. Importantly, the somewhat noisy emergent profit in the 50 bank case collapses to a stable signal punctuated by large breaks in the 25 bank case. Complex interactions between the ability of increasingly large nodes to "cover" occasional failures in the productive sectors and the effect of systemic risk due to increasingly deep credit connections can, to some extent, explain the qualitative change. As the credit network becomes concentrated, the true level of risk in downstream and upstream firms is masked by greater bank assets. As these lower level failures remain unobserved, the likelihood of failures large enough to be transmitted along the interbank market diverges from observed frequency of normal failure. Furthermore, the effect of "hidden" avenues of correlation realized under default increases in the aggregate as larger portions of total assets are tied up in close proximity to any bank default. (Remember that the likely effect of cascades on the interbank market dissipates with distance. A concentrated interbank market provides much less space for dissipation and a much less varied experience.)

Kurtosis in diversification over time:

Of course, multiple factors could conspire to lead to kurtosis in bank profits. Delli Gatti et al (2006 and 2010) show the development of kurtosis in firms' experiences due to the nature of credit constraints and supplier connections in the productive sectors. In order to study the development of asset correlation among banks in particular, Perignon

and Smith's diversification coefficient is assessed against a rolling window of 200 observations. Over time the dynamics of this diversification coefficient should accurately present the nature of the diversification of the correlation matrix, and by extension model the variability of the credit-network topology and thus the viability of predictive modeling. More precisely, the diversification coefficient captures the dynamics of the correlation matrix as current risk evaluation methods would, observed kurtosis and variability in the experience of the diversification coefficient will imply the sudden realization of potential correlation (via credit-network topology) which would result in severe mispricing of risk:
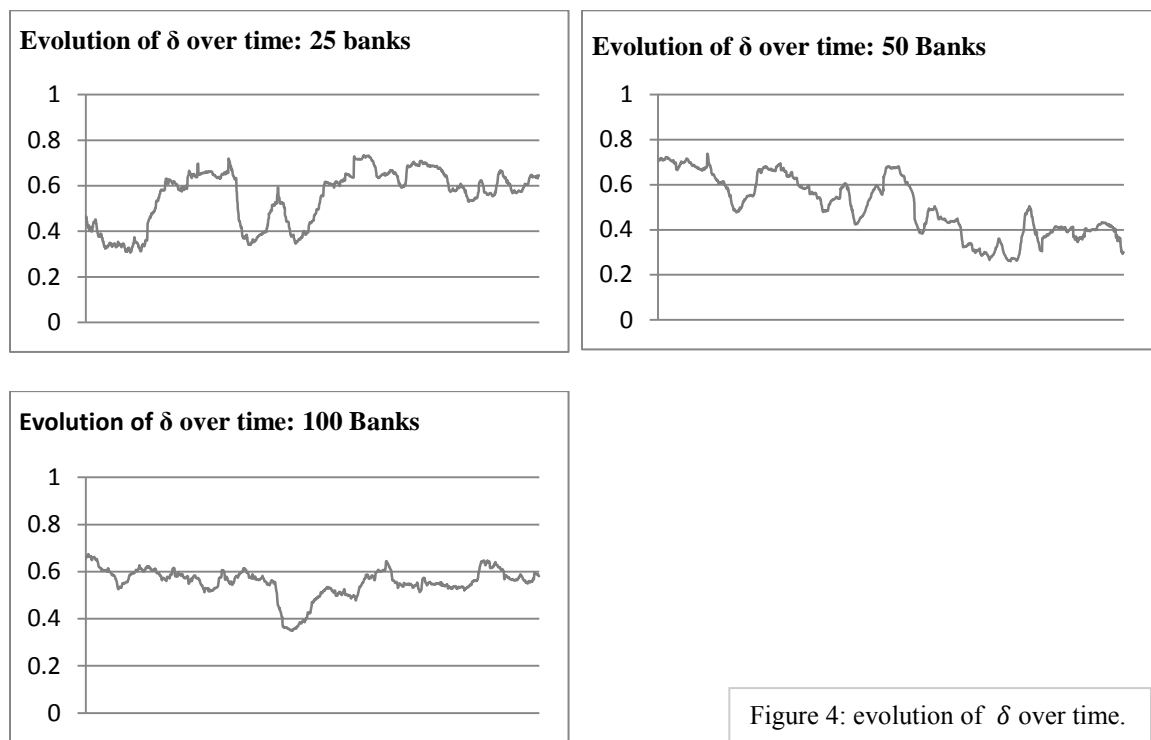
Evolution of δ over time: 25 banks

Evolution of δ over time: 50 Banks

Evolution of δ over time: 100 Banks

Figure 4: evolution of δ over time.

By visual inspection δ exhibits an increasing degree of variability with the concentration of the network. In the context of this credit network, the variability of the correlation matrix speaks to the strength of the *realized* connections among the banks

constituting this "portfolio" and the resulting correlations in profits over time. There are two avenues by which comovement in profits can arise in the model. First, if firms grow more quickly than their partner banks, the demand for credit can outpace the supply available from their supplying bank, in which case other banks either view the bank as a profit opportunity and invest their excess profits in it (through the extension of credits as described above) or growth is constrained. Such dependencies can persist over multiple periods, leading to possible long run dependence in the time evolution of the diversification coefficient. Additionally, such dependencies effectively generate elevated downside risk, as any bankruptcy in a single bank is more likely to cause a bankruptcy cascade that "infects" other parts of the network. Second, single period losses can lead to increased fragility in the system by causing the redistribution of excess profits to cover the troubled bank. In either case a statistical assessment would be blind to the magnitude of the resulting increase in risk. Should a node with "deep" network connections experience failure, the resulting bankruptcy cascade could be severe, cascading to multiple nodes along the chain. The two interact in a complex manner and the emergent behavior exhibited in the panel above gives evidence to the increasing importance of these inter-node effects in the concentrated model.

The above results show that the diversification coefficient, as an expression of the correlation matrix, exhibits qualitatively different time evolution dependent on the level of concentration in the model. However, a statistical expression of this variability is complicated by difficulties in accurately assessing long run dependence, see CITATION for a discussion. Examining the first difference of the diversification coefficient yields

convincing concrete results in support of the hypothesis that increasing concentration (and the resulting "masking" of effective risk) yields divergence in risk assessment:
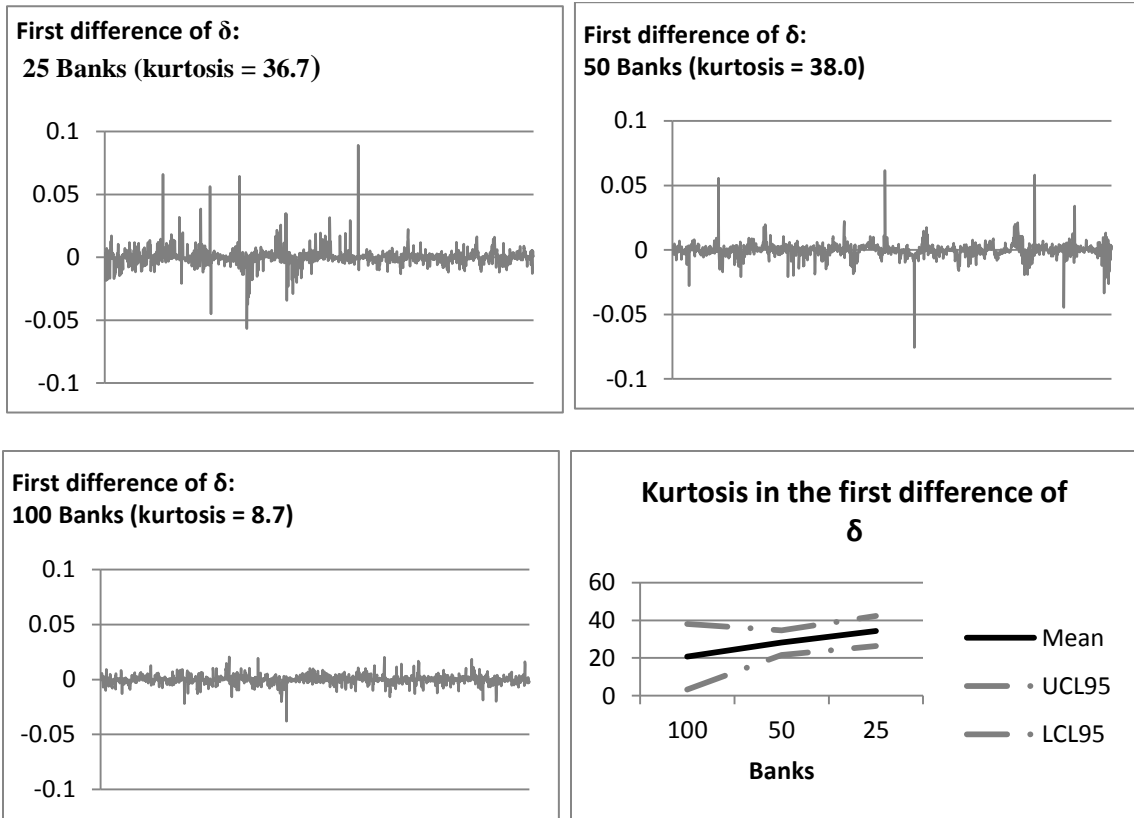

Figure 5: Increasing concentration at the bank level

Visual inspection again shows a strong increase in the variability of the implied correlation structure of the banking sector; critically, kurtosis in the above first differences of the evaluated Value-at-Risk increases in the restricted models. This observed increase in kurtosis is due to the unobserved risk inherent to the concentrated case; the concentration of constituent credits into ever larger "portfolios" (ie. the banks' balance sheets) induces imperfect diversification within the bank's balance sheets, and masks the development of risky structures until they are "realized" through loss. The occurrence of a multi-period loss in any given bank remains constant or declines as

concentration increases. However, the resulting effect on the magnitude of loss is drastically increased as larger portions of the banking sector are "tied up" in the fortunes of a single bank. The results were additionally found to be robust as an expression of the mean experience of a test of 50 iterations of the simulation under each model specification.

Net Worth

Finally, the number of banks operating in the market appears to have an effect on the time evolution of net worth in the economy. In the following three scenarios the aggregate net worth of the banking sector in initial conditions remains unchanged. As in the previous exploration, the number of banks is restricted:
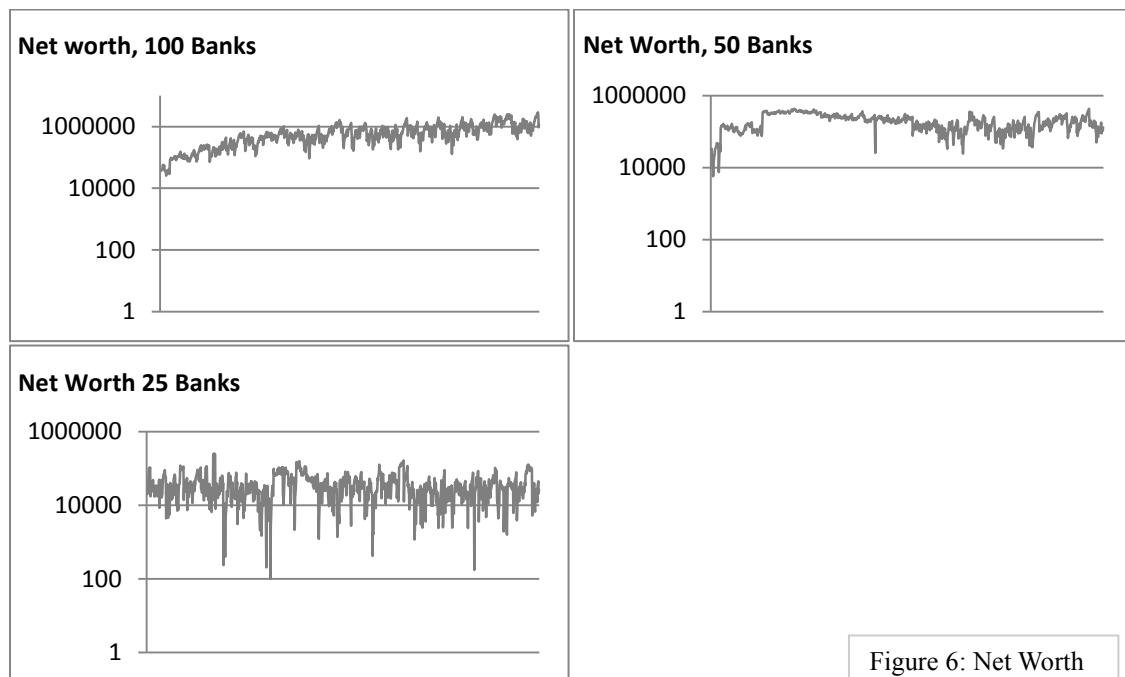

Figure 6: Net Worth

As the number of banks declines growth in the economy collapses. Stationarity forms the lower bound due to the construction of the model, as firms are not allowed to hold negative net worth and furthermore all bankruptcies are replaced by a firm with a set "initial" net worth.

Remember that the supply of credit is determined by the prudential lending target, $L_{zt} = A_{zt}/\alpha$, where A_{zt} is the net worth of bank z at time t, α is a statutory capital ratio, and L_{zt} is the supply of credit from bank z at time t. If defaults occur too frequently, or are too severe, banks remain unable to grow net worth to support the growing economy and the economy becomes credit constrained. In fact, from the results above, it would appear that the trade off created by concentrated financial markets – less frequent but more sever failures – does not lead to a long run increase in the total welfare of the economy. The severity of the downside risk during cascades is sufficient to more than overcome the improved performance during normal periods.

In the progression from a disperse network with $N_d = N_u = N_b$ to the concentrated case $N_d = N_u = 4N_b$, we can see that the magnitude of downside risk in the economy grows substantially. This supports the earlier observation that dependencies in the correlation matrix on the level of connection and concentration in the model could cause severe mispricing in models based on the assumption of convergence. In the unconcentrated case, the law of large numbers pushes correlations in asset values to cancel, and the structure approaches perfect independence in losses. Although, it should be noted that even in the unconcentrated case profits still exhibit some kurtosis due to the connections on the interbank and product markets. Furthermore, it can be seen that the

bifurcation of the network into periods of variable correlation is dependent on the depth of interbank connections, which intensify with weaknesses in the banking sector. Intuitively, the destruction of these avenues of correlation would yield actual perfect independence in the case of a non-connected network (ie. If there were no interbank market and if the credit connections between firms interacted in isolation and the assets of no two banks were in any way structurally connected). It would be perhaps a valuable contribution to extend the framework used to model an Erdös-Rení map of a network economy (Nier et al. 2009) to the limit case of absolute connection and absolute fragmentation to arrive at these same results. Intuitively, failure in the "absolute" connection case would treat the banking industry as a single bank, which would obscure all failures save those extensive enough to overcome profits from elsewhere, yielding either complete catastrophe or the appearance of stability. The case of absolute fragmentation may, by contrast, yield a perfectly Brownian structure. Critically, both the increase of long run dependence and kurtosis evidence the variability of the state conditions upon which an assessment of risk in the sector would necessarily be based.

VI. Conclusions

Minsky's Financial Instability in the emergent behavior of the model

Having discussed the nature of default and asset correlations in the Delli Gatti et al. (2006) network economy it is now possible to address the fundamental determinants of the shape of the correlation matrix and their possible applications in a broader family of interconnected systems. The genesis of extreme moments of correlation among significant proportions of the network is an act of the strength of interfirm connections. "Grouping" these connection points into larger elements then has the effect of tying together the effects of failure in portions of the market. The greater share of assets dedicated to providing a form of collateral for failure indeed lessens the likelihood of low level failure. However, it should be noted that credit constraints enter the picture as a constraint on growth, when failure effects large sectors of the banking industry, the other industries' growth can be constrained in the long term. There is a dual effect here. First, the concentrated financial system produces increased stability punctuated by periods of extreme asset correlation merely through the variable depth of credit connections. These extreme periods of correlation exacerbate downside risk and lead to a disorderly breakdown of the usual "equilibrium" dynamics, fundamentally frustrating risk valuation. The variance in the depth of those corrections is dependent on the experience of the

network, reinforcing the strength of correlation as banks increase their credit connections due to difficulty or opportunity. Second, when failure occurs, recovery is rendered more difficult even when real conditions support robust recovery due to credit constraints. (Is the crux of this argument the effect of a reactive reinforcement of credit connections as occurred in the CDS arrangements in 2007?)

Both the time evolution of banking sector profits and the variation over time of the correlation matrix of bank returns evidence the dynamic development of interdependencies in the banking sector. As differences in growth rates between firms and banks are aggravated by the interest rate mechanism, conditions can develop allowing portions of the banking sector to deploy their excess balances elsewhere in the sector, possibly developing subsidizing dependencies that can last over multiple periods. Since the potential downside risk is not realized until bankruptcy occurs, an unrealized risk can develop through avenues of credit dependency. If a bank with a higher risk profile requires credit through the interbank (or any other) market, creditors in the Delli Gatti model view the requirement as an investment opportunity, and not as risk. Indeed, if information about credit connections is proprietary, it becomes impossible for a lender to ascertain the true risk profile of another bank until bankruptcy actually occurs.

To the extent that these dependency structures exist, they can be understood as emergent behavior loosely modeling Minsky's Financial Instability Hypothesis. Minsky asserts that two separate but interconnected phenomena drive the development of financial structures which rely increasingly upon external financing. In Minsky's model three types of financing schema exist, Hedge, Speculation and Ponzi. In hedge finance

units are able to operate completely on their income account, all investment is financed via carried profits, and debts coming due are paid entirely out of profit. Speculative finance requires that debts coming due be paid at least partially by new debts, the balance sheet is rolled forward. Finally, Ponzi finance requires that debts be financed entirely by new debt, and represents an increase without bound of the indebtedness of the agent. Since all debts are liquid at the end of every timestep, this exact progression is rendered impossible by the structure of the model. However, the development and collapse over time of external finance dependent structures serves as an analogy for Minsky's hypothesis. Banks bifurcate between self-sufficient and dependent states (essentially, although not precisely, between hedge and speculative finance), as in the Financial Instability hypothesis, a dependent bank serves to increase systemic risk. Indeed it should be striking that we observe the extreme effects we do even in such a restricted model. It could easily be expected that an accurate agent based reproduction allowing for the development of Ponzi finance would exhibit more extreme behavior. Moreover, a government agent is not included in the above model, as in Minsky's formulation, and the introduction of such an agent could as easily reinforce the effects highlighted throughout this paper as mitigate them. The exact nature of Government policy, and its implications for the financial accelerator in Delli Gatti et al. (2010) could be the subject of future research.

At this stage, the strength of the connection between Minsky and the literature on Network effects in financial markets is nascent. The endogenous instability of the financial markets tie the two together and it could be asserted that Minsky described

qualitatively the quantitative expressions of network effects. The increasing depth of financial connections across a financial system serves as a *potential* conductor of financial risk. In general, these channels of possible contagion go unrecognized until their potential is realized, at which point, of course, it is too late. Indeed, it should be recognized that even in the above model, in which we know the exact credit topology of the network, we would be hard pressed to predict the behavior accurately. The reality is likely much more complex. In such a context, Minsky describes the mechanisms by which both the number of these channels is augmented, and the likelihood that failure occurs at any given point. As an economy progresses toward Ponzi finance, the relative value (indebtedness) and number (interconnectedness) of credit relationships augments, increasing the likelihood of failures with systemic effects. The types of parameters defined in an Erdos-Reni map, then, could serve as work in the direction of defining a measure of *Systemic Risk,* a concept which has received much attention in the years since the financial crisis but for which there is very little definite understanding.

The difficulties of defining systemic risk are real, due to the fundamental nature of interconnected systems, there exist patterns of reactive behavior that defy prediction. Additionally, even under such restrictive assumptions of agent choice as defined above (in fact, in Delli Gatti et al. 2006 agents are not imbued with choice, however, in the subsequent 2010 paper an agent choice matrix is presented which fails to mitigate the effects and in fact reinforces them), unpredictable patterns of behavior arise fairly rapidly. While it is true that prediction may be possible over some finite time horizon, or under some assumed conditions (such as those assumed in the formulation of copula and matrix

decomposition methods of portfolio risk evaluation), it should by now be apparent that to the extent that knowledge about network topology is unavailable to agents, the set of conditions under which future behavior is predictable is finite and smaller than the set of possible future conditions. Should *potential* systemic risk as characterized by potential avenues of contagion be realized, *predictable* behavior can rapidly degenerate into a future time path of behavior that is *unknowable*.

To what extent, exactly, is future behavior *unknowable*? It may be, again echoing the sentiments of Minsky and Albin, that the formulation of the research project approached by financial economists is in some way incomplete. While there is likely value in predicting more precisely the time evolution of profits within the economy under those state conditions which allow prediction, the pursuit of this research project in isolation or without the simultaneous pursuit of a body of study attempting to define those state conditions which might render such predictions invalid and the likelihood that these state conditions might obtain, can lead to the extremely damaging development of irrational exuberance (which, in the context of this paper could be seen as further reinforcing the likelihood of just such a state condition bifurcation). A careful examination of the financial crisis quickly shows that unrealistic expectations obtained across large portions of the financial sector, with the complex CDO and CDS instruments at their core. A large body of writing exists on this subject, see Crotty, Wolfe or any number of others for further review (Crotty 2009). An area that is greatly lacking at this moment, then, is a program of study attempting to define the probable causes of systemic bifurcations.

Real World applications

One could consider the financial accelerator to be one element of a superset of network effects. Any risk sharing scheme, should there be the possibility of network effects affecting default and asset correlations, will develop into Ponzi finance, as the structure exhibits concentration about the mean punctuated by extreme departure. If timescales fail to extend to compensate for structural changes, the increase in kurtosis will remain unobserved by actors in the economy. Worse, the structure will appear to the endogenous actor to be in a progression towards a state of greater stability. The effects of the model are agnostic to the method by which concentrations are introduced into the model, in Delli Gatti et al. (2010) the concentration is the result of an endogenous choice mechanism, other structures emphasize confidence effects, effects of moral hazards, government policy, or any number of incentive schemes. It's likely that all of the above have obtained in the past 40 years; the exhibited path over the decades leading up to the financial crisis has been a determinant of multiple factors, including Federal Reserve strategy in the case of bank failure, deregulation in financial markets, and "innovation" leading to the creation and pricing of new financial instruments. All of these developments have resulted in a more concentrated financial system, and there is ample evidence that the incentive structure of financial markets have tended to reinforce the effects.

The result, as illustrated by the agent based approach taken in the model studied here (among others) is that any reliance on mean field approximations to support

predictions of future behavior made from aggregate expressions of system position is fundamentally flawed, especially as the network connections become increasingly dense. "Network Effects" dominate in the limit. Past and current expressions of systemic risk have tended to rely casually on the net position of a financial market: observers noted in the early days of the credit crisis that the net exposure of the financial industry to subprime securities was something on the order of 200 billion, not a large figure in the context of the size of the market. However, through largely unobserved liability structures such as the now infamous (although ubiquitous) CDS, the effects of the small net aggregate exposure ballooned into likely greater than $2 trillion in destroyed wealth. A variety of factors, of course, were present in the expressed patterns of loss in the financial crisis, including liquidity problems deriving from a lack of confidence in institutions, sometimes in cases in which the institution had very little exposure. However, it was the complex nature of the credit topology of the financial network that led to risk becoming *unknowable* (which is quite obviously connected to the crisis of confidence that occurred). A single firm, or small subset of firms, which fail to cover their liabilities can lead to large effects in the emergent behavior of the system, *aggregate effects are not expressed by the aggregation of individual effects*. In short, Network effects matter.

The existing literature on ABM has focused largely on the explanation of results considered anomalous in the context of the neoclassical synthesis. In their recognition of the destabilizing effects of the complex interactions possible in networks characterized by heterogeneous interacting agents, agent based models have a clear advantage over models expressing the trajectories of economic systems in terms of aggregate statistics. A good

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application of the results of such models to the development of economic forecasting is yet to be produced. It is my hope that this work will help to inform that body of study. Perhaps the clearest conclusion of the above is that the question of forecasting must be necessarily broadened, especially in the context of the estimation of uncertainty. Following from the work of Albin, it is recognized that not all state conditions produce models whose future trajectories are predictable in a computational sense. If state conditions are variable, and studies in financial psychology should lead us to believe that they are in fact subject to bifurcation, more research is needed to study the conditions for this bifurcation. This study has technically proceeded entirely outside of the framework of choice and yet such state condition bifurcations are shown to be possible, future work would benefit from an increased understanding of just such conditions. If possible, descriptions of expectation for state conditions are critical, including as well increased effort in discovering how non-salient conditions might come to dominate system dynamics. The question of the degree to which any of the above is possible is of critical importance, and requires more attention than has been paid to it in current works.

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