High-Order Mooring Simulations for Increased Accuracy in Wave Energy Applications

Johannes Palm   Claes Eskilsson   Lars Bergdahl

Department of Shipping and Marine Technology
Chalmers University of Technology
Gothenburg, Sweden

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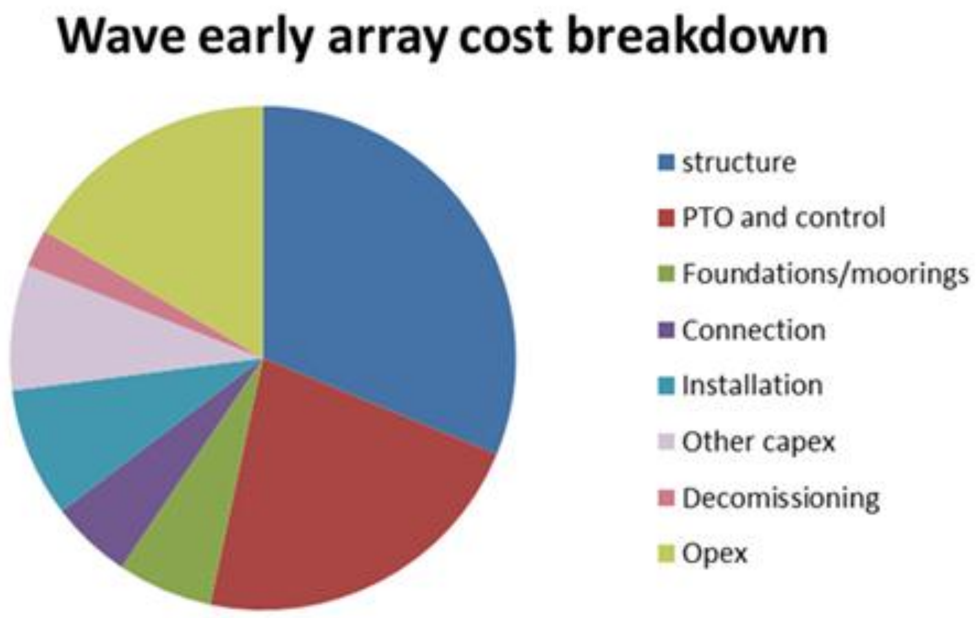
Mooring for Wave Energy Converters (WECs)

- Reduce cost of mooring (6-30%)
- Related to installation and connection
- Project started 2012
- Initiated by industry (Ocean Energy Centre)



- Moorings are costly : OK for oil and gas, (< 2% of investment)
- Reduce the cost of moorings for WECs (6 – 30 %)

In-house mooring code MooDy

- In development since 2012, now at 3rd re-write
- High-order finite elements
- Discontinous Galerkin method
- Explicit time stepping
- Coupled to OpenFOAM and recently to WEC-Sim

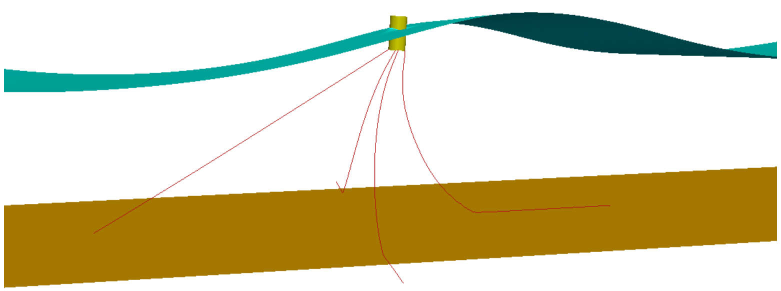


Fig. 6. The solution state after 50 s of simulation. Note the design failure due to completely lifted cable in mooring cable 1.

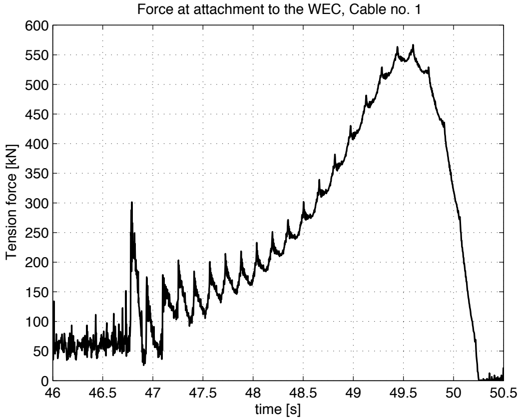
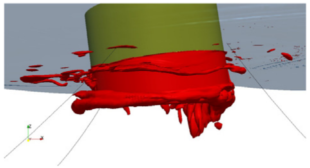
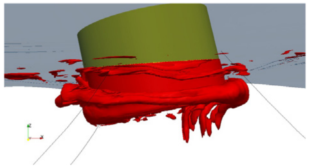


Fig. 7. Tension force in attachment point to the WEC of mooring cable 1.



(b)



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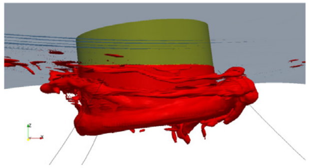

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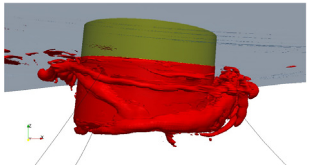
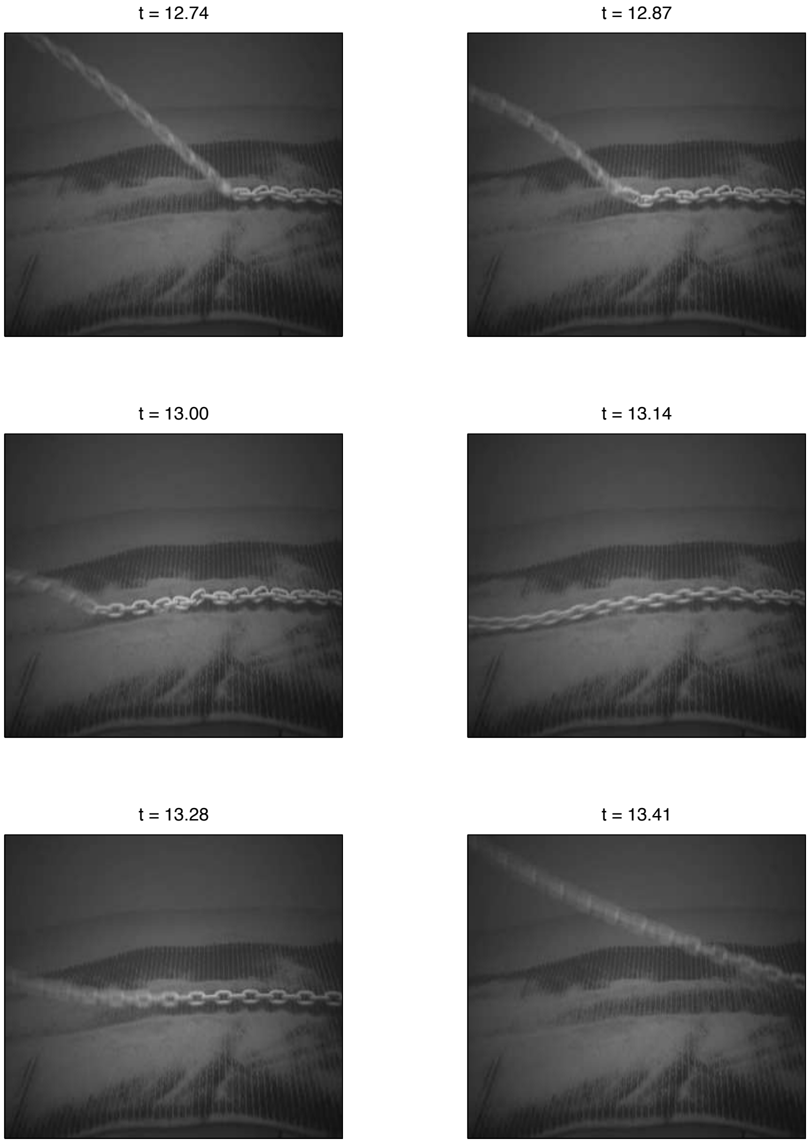


(k)

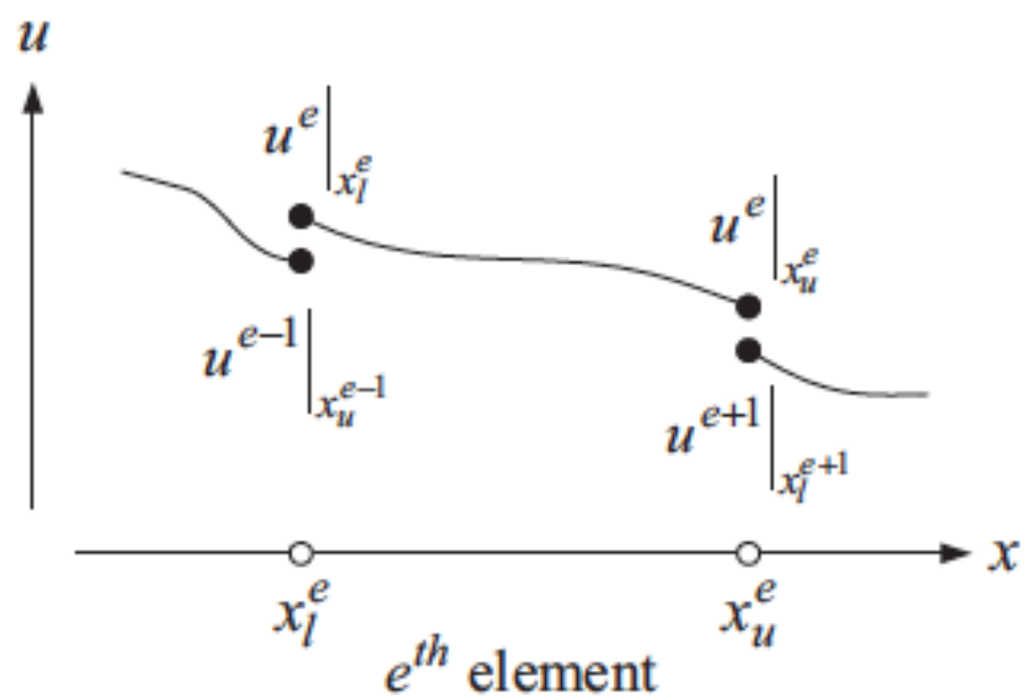
Snap loads

- ”When the touchdown point speed exceeds the transverse wave speed of the mooring line a shock forms in the tension. […] Shocks during upward motion of the mooring lead to a snap load in the tension record. Shocks during downward motion lead to slack tension at the touchdown point"

Gobat and Grosenbaugh, (2001)



Capture snap loads by Discontinuous Galerkin (DG)



Model equation: Nonlinear hyperbolic equation

Formulation in conservative form: $\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = \mathbf{S}$

$$\gamma_0 \frac{\partial^2 r}{\partial t^2} - \frac{\partial}{\partial s}\left(T \frac{\frac{\partial r}{\partial s}}{\left| \frac{\partial r}{\partial s} \right|} \right) = f$$

r : cable position vector

$T = EA_0\epsilon$: tension force magnitude (tangential to cable, no bending stiffness included)

f : external forces (added mass, buoyancy, ground model and drag)

γ_0 : cable mass per unit length

s : unstretched cable coordinate

Modelling of nonlinear hyperbolic equations

- *Lax-Wendroff's theorem* (Lax and Wendroff, 1960)
  If a conservation law is solved with a conservative method the solution converge to a unique and correct solution

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = \mathbf{S}$$

- *Hue-LeFloch's theorem* (Hue and LeFloch, 1994)
  If a non-conservative method is used – well then the solution will simply not be correct..

- *Godunov's theorem* (Godunov, 1959)
  There is no second- or higher order scheme with constant coefficients that do not produce non-physical maximum/minimum

Snap loads demands on numerical scheme

- Shock waves → Be able to handle discontinuities (DG)
- Peak load can be very high and important → Accurate peak captures, i.e. no over-undershoots (limiter)
- High celerity → High temporal resolution (regardless explicit or implicit)
- Many load cycles → Low numerical damping (high spatial resolution, high p)
- (Snap load generation → Accurate ground model)

Key step 1: Equation in conservation form

$$\gamma_0 \frac{\partial^2 r}{\partial t^2} - \frac{\partial}{\partial s}\left(T \frac{\frac{\partial r}{\partial s}}{\left|\frac{\partial r}{\partial s}\right|} \right) = f$$

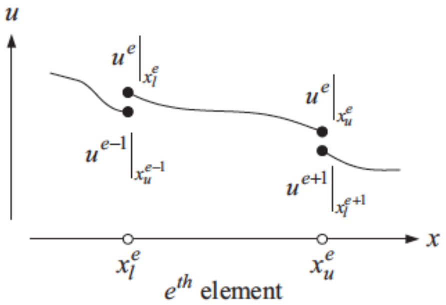
$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = \mathbf{S}$$

$$\mathbf{U} = \begin{bmatrix} r \\ q \\ L \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} 0 \\ \frac{L}{\gamma_0} + \beta r \\ T\frac{q}{|q|} \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} \frac{L}{\gamma_0} \\ -\beta q \\ f\frac{q}{|q|} \end{bmatrix}$$

where $q = \partial r/\partial s$ and $L = \gamma_0 \partial r/\partial t$ is the momentum of the cable and β is a penalty parameter

Key step 2: Conservative numerical method
Discontinuous Galerkin method

Partition the cable domain Ω of unstretched coordinate $s \in [0/L_c]$ in to N_{el} elements regions Ω_e. Inside the elemental region a function y is approximated with a pth order Legendre polynomial:



$$y(s,t) \approx y^e_\delta(s,t) = \sum_{k=0}^{p} \phi_k(s)\tilde{y}^e_k(t)$$

where $\tilde{y}_k$ is the kth modal expansion coefficient.

Take the inner product with respect to the basis function and integrate the flux vector by parts. Exchange the boundary flux with a numerical flux, and eventually integrate by parts once more yield:

$$\int_{\Omega_e} \phi \mathbf{U}_t ds + \int_{\Omega_e} \phi \mathbf{F}(\mathbf{U})_x ds + \int_{\partial\Omega_e} \phi \left(\hat{\mathbf{F}}(\mathbf{U}) - \mathbf{F}(\mathbf{U})\right) \mathbf{n} dl = \int_{\Omega_e} \phi \mathbf{S} \mathbf{ds}$$

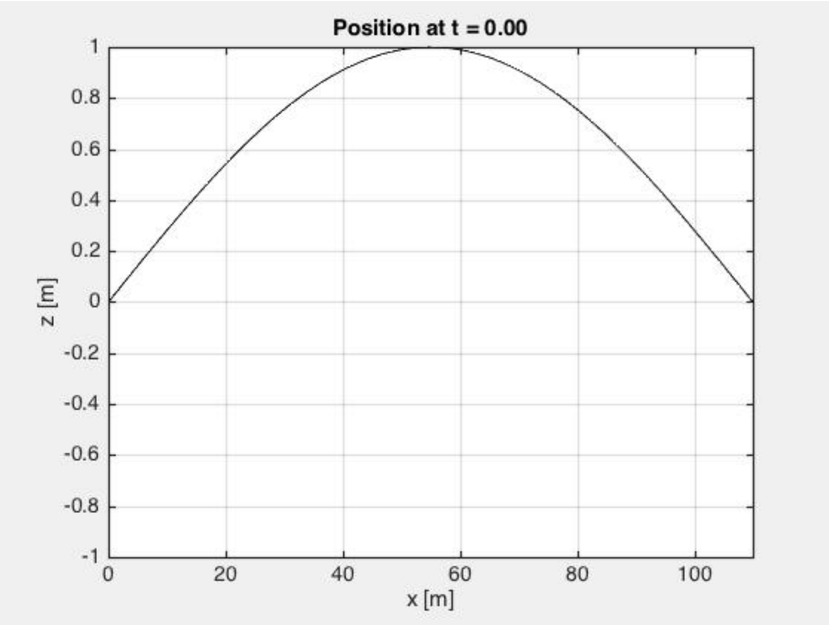
$$\hat{\mathbf{F}} = \{\mathbf{F}\} + \frac{|\lambda_{\max}|}{2} [[\mathbf{U}]]$$

Key step 3: Monotone solutions

- Application of slope limiter (Generalized MinMod). Only works for low-order elements so uses hp-adaptivity for keeping the accuracy in shock regions
- Use strong stabilisation preserving explicit time-stepping schemes (SSP-RK3)

Verification of conservative formulation

- Vibrating string test
  - Linearised tension force
  - No gravity
- Convergence plot
  - L2 norm
  - Exponential convergence (p+1/2)



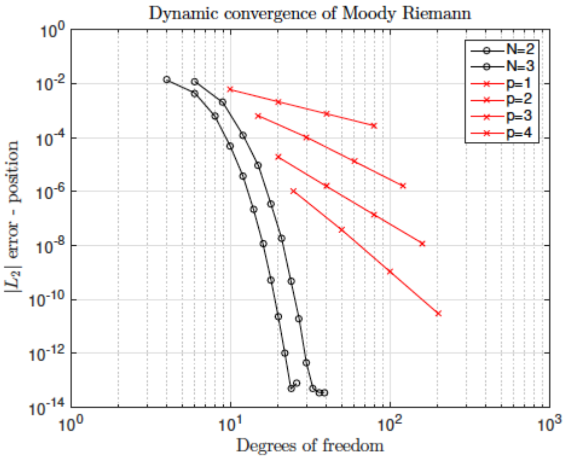


Figure 1: The error in the L_2 norm of the cable position after 1 full period of oscillation as a function of degrees of freedom in the discretisation. Shown in logarithmic scale.

Shock propagation case

- 1D Test case
  - 100 m cable
  - 1000 N jump at midpoint

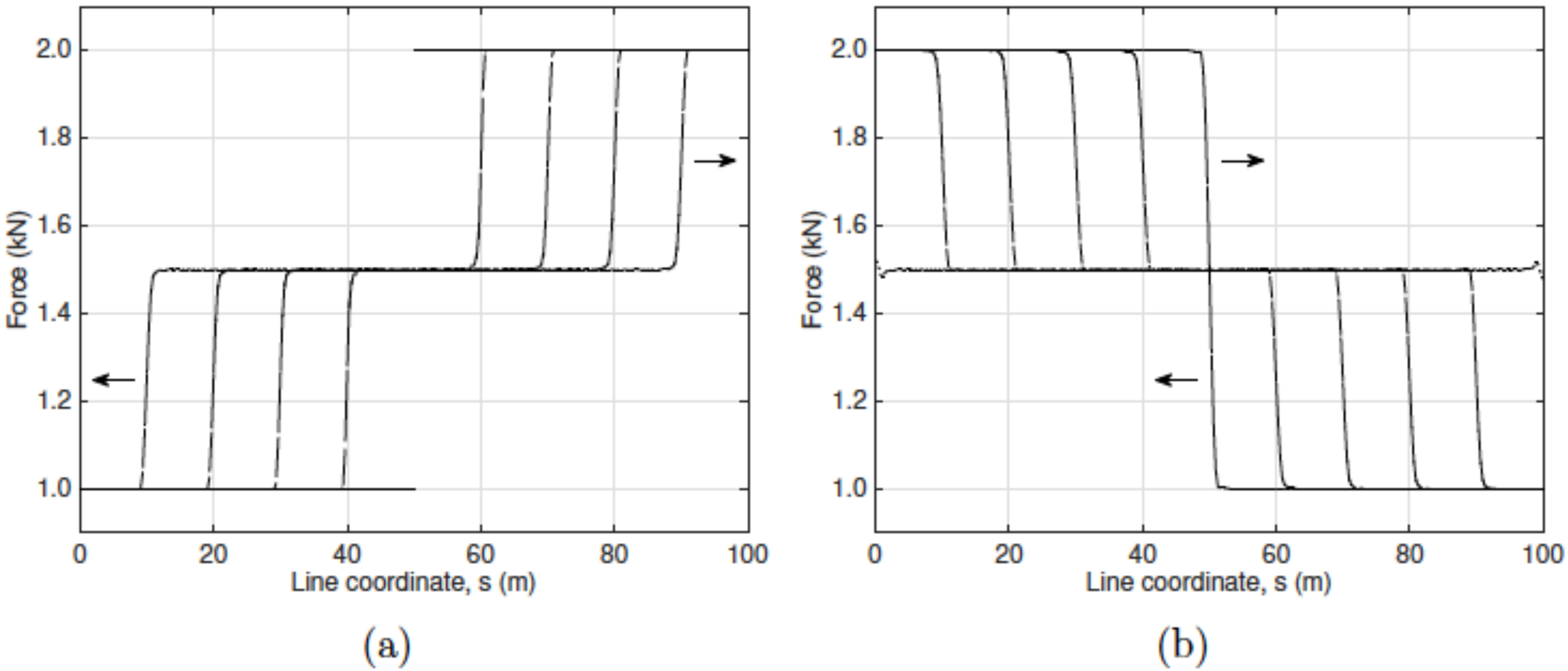
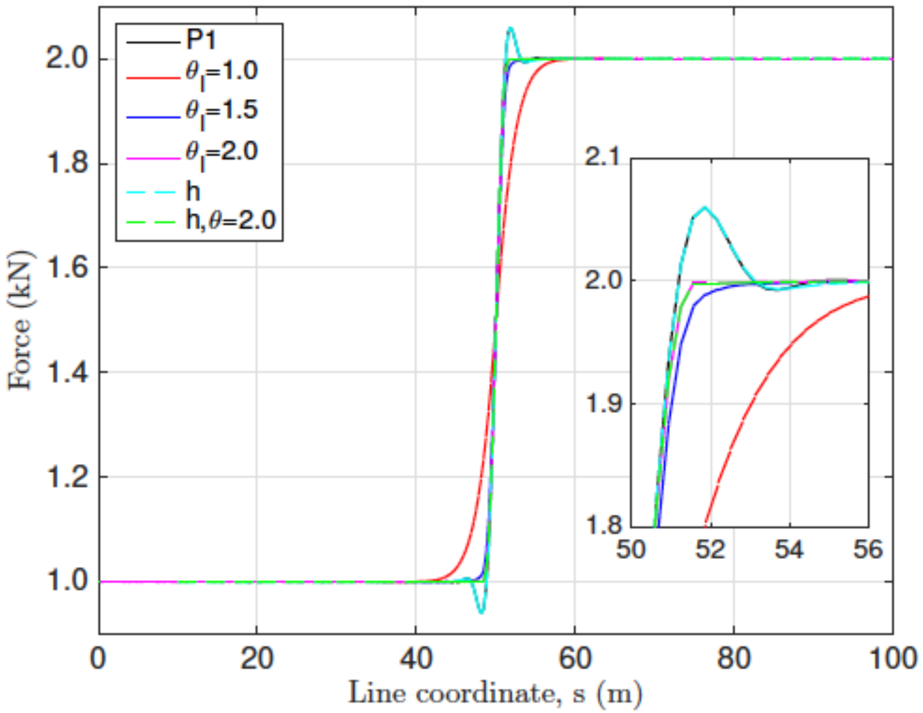


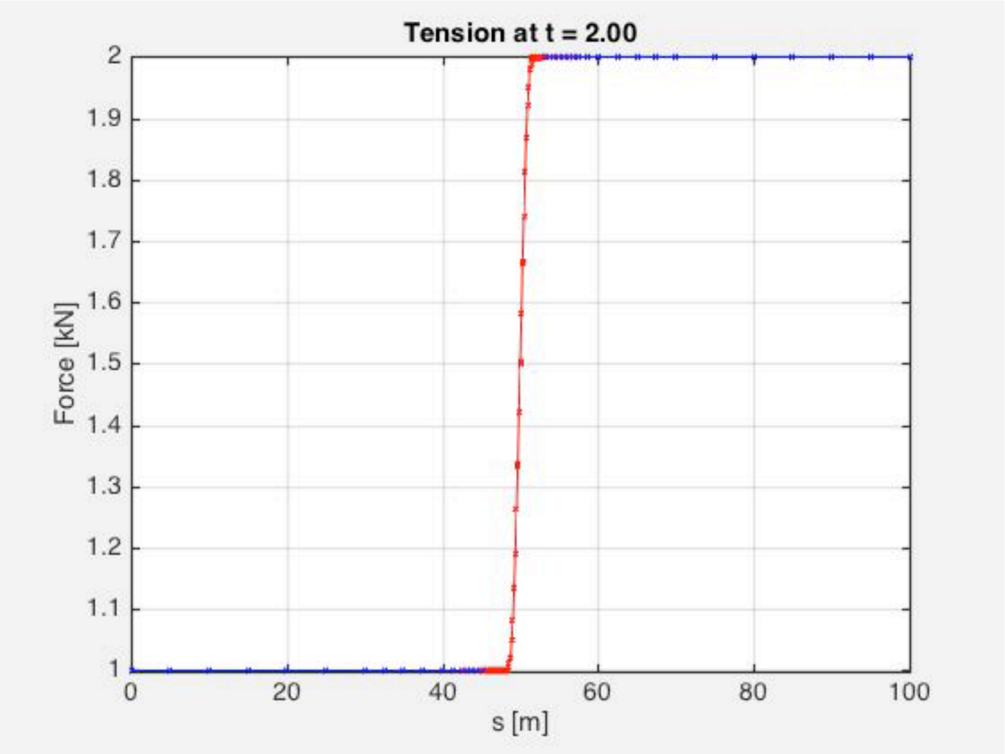
Figure 2: Shock appearance during the first second of simulation, sampled every 0.1 s from the $N = 320$ case. (a) shows $t \in [0, 0.4]$ s and (b) shows $t \in [0.5, 1.0]$ s. The arrows indicate the propagation direction of the front.

Shock propagation case – Monotone solution



(b) $t = 2$ s

Shock propagation case - Adaptivity

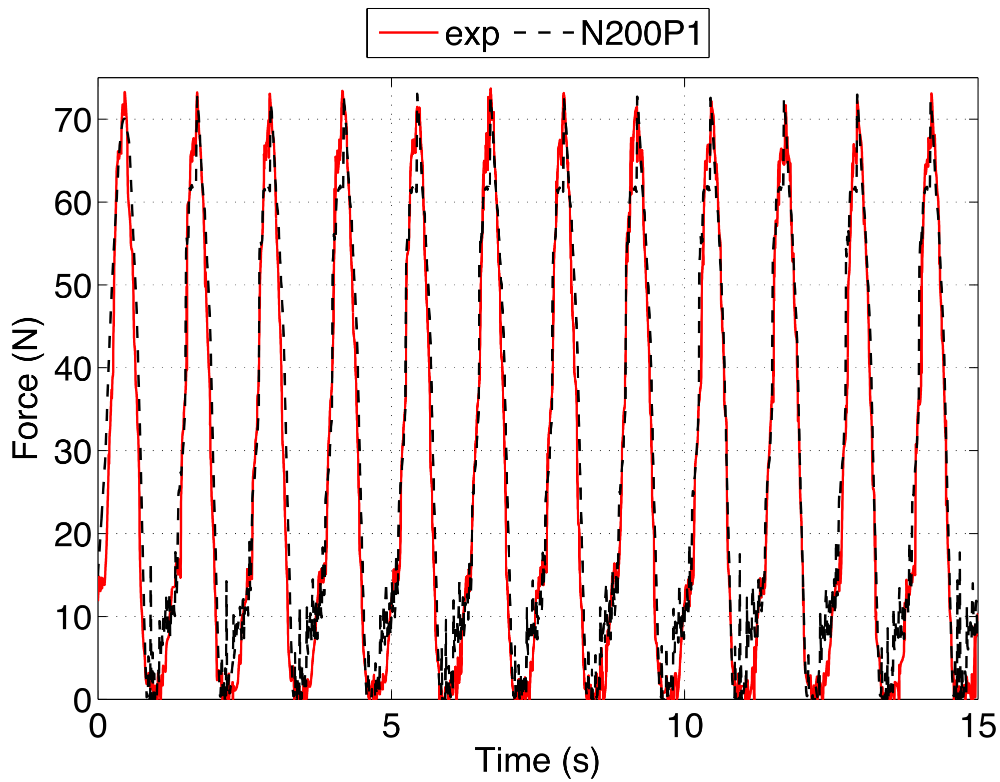


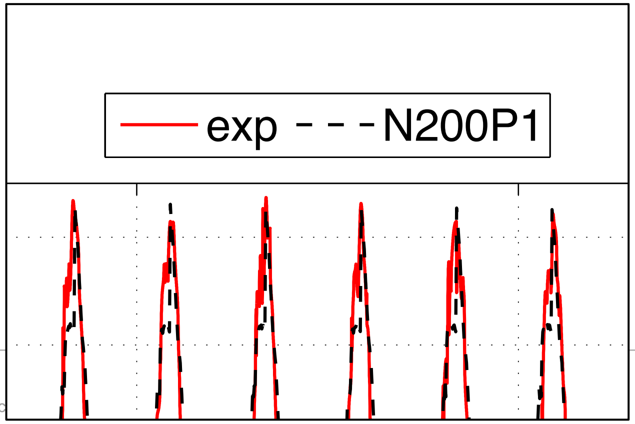
type	N	p	DoF	θ_l	relative time
static	160	1	320	NA	0.174
static	160	1	320	2.0	0.254
static	320	1	640	NA	0.675
static	320	1	640	2.0	1.000
h-adapting	48	1	96	NA	0.192
h-adapting	43	1	86	2.0	0.208

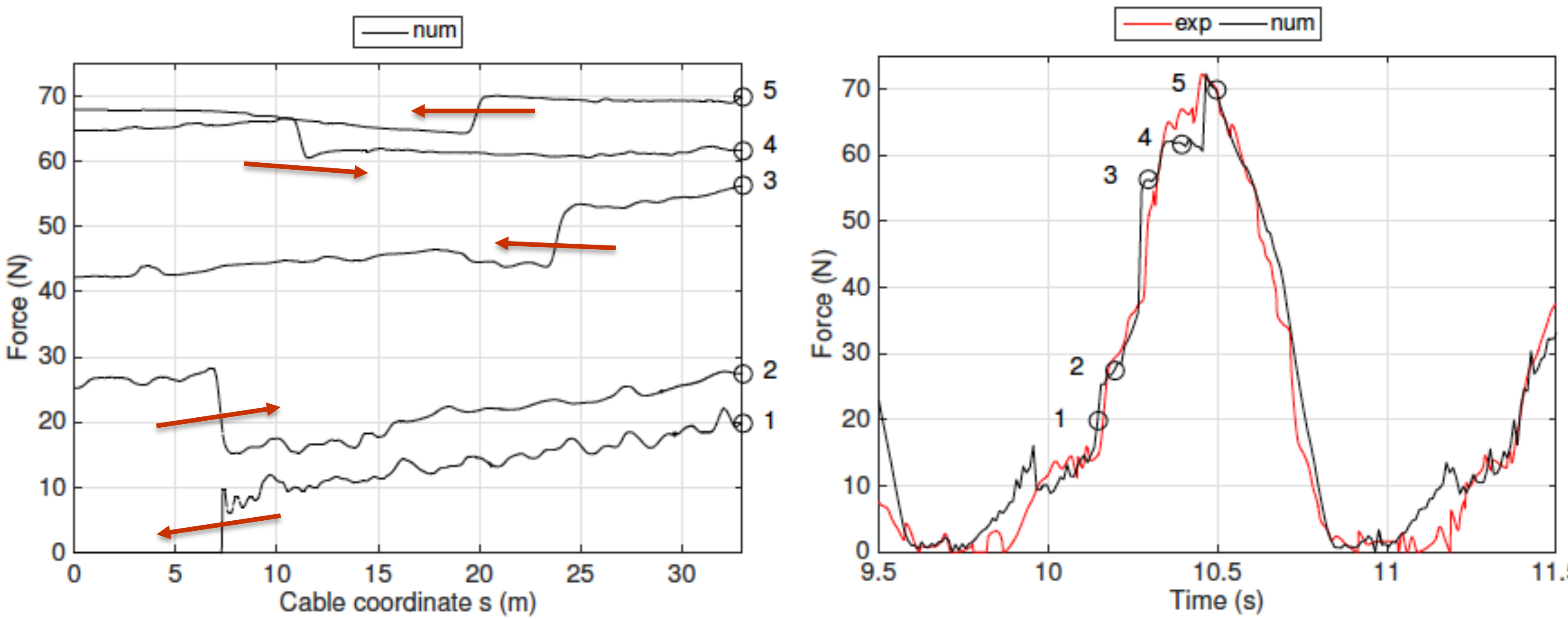
Validation case

- 33m chain on concrete floor in 3m water.
- Fixed anchor and circular motion of fair lead
- Radius 0.2m and period time 1.25s

- Excellent match in force time history at fair lead
- Some numerical noise in low tension region
- Pronounced extra peak, indicating snap load

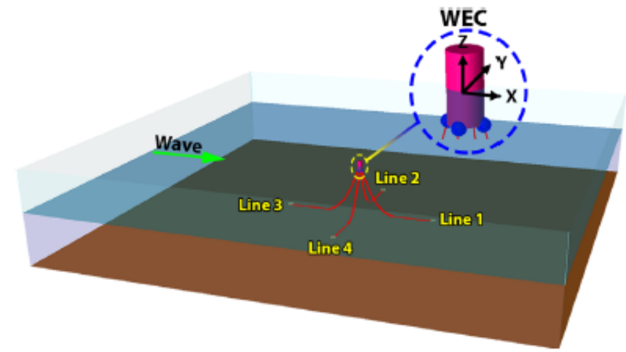






(a) Cable tension snap-shots (b) Fair-lead tension

Influence of High-Order on Fatigue Estimation



DeepC
N=100, *p*=1

MooDy
N=10, *p*=7

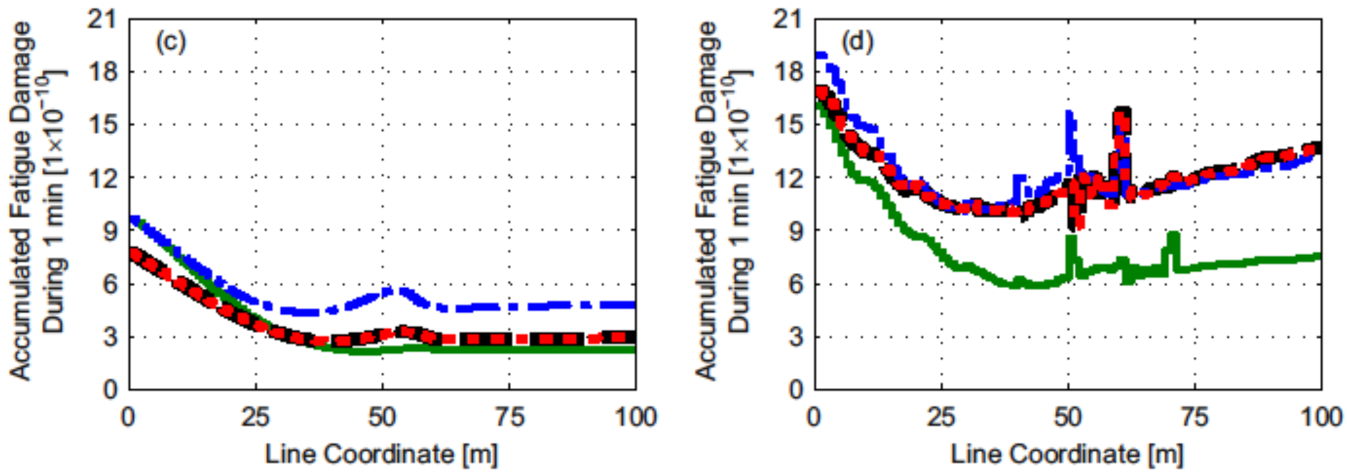


Fig. 7. Presentation of the results from the fatigue damage calculations for the four types of simulation procedures along the whole mooring line from the fairlead to the anchor point (line coordinate from 0 to 100 m). Mooring lines 1–4 are represented by a green thin line, a black dash line, a blue dash-dotted line and a red dotted line, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Yang et al., (2016)

Concluding remarks

- ***Snap load important for WECs***
  - MooDy uses a conservative method (DG) for solving the equations casted in conservation form
  - The numerical fluxes are upwinded based on an approximative Riemann solver
  - Limiters avoid over-undershoots
  - Ground model important for the generation of snap - needs a closer look
- ***hp*-adaptivity**
  - MooDy is designed as a high-order code
  - Potentially a large speedup of computations without loss of accuracy
  - Low numerical diffusion have influence on fatigue estimates
- ***Slack cables without bending stiffness***
  - An ill-posed problem
  - Inclusion of bending stiffness needed to avoid numerical noise also important for the generation of snap
- ***MooDy is intended for use as a mooring module***
  - Coupled to WEC-SIM for standard irregular waves events
  - Coupled to CFD (OpenFOAM) for extreme events

CHALMERS
UNIVERSITY OF TECHNOLOGY

johannes.palm@chalmers.se

claes.eskilsson@chalmers.se

lars.bergdahl@chalmers.se