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Exploring the use of known strategies for achieving fluency of multiplication and division facts in third grade.

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ABSTRACT

In order to complete higher level math tasks, fluency of multiplication and division is crucial. Based on state standards, third grade students are expected to become fluent with multiplication and division facts from 0-100 by the end of the year. Throughout history many educators have relied on timed tests to teach and assess math fact fluency. Best practices for helping students become fluent with math facts is now highly debated. The purpose of this teacher action research study was to evaluate the effectiveness of using math fact games and visual math cards as means of distributed practice in order to achieve fluency with multiplication and division facts among a group of forty-seven, third-grade students in a suburban school district in Central Ohio. A mixed-method design was framed within a cycle of inquiry used in teacher action research, including a pre-assessment, student interviews, and summative assessments for data collection. The focus of the study was to implement research-based instructional strategies for boosting fluency of math facts, then assessing, student fluency. Data from the Post-Assessment, verbal assessment, rating scales, and interviews showed that students did in fact learn to multiply and divide when taught these strategies. Students reported that learning strategies were crucial, math games were helpful as well as enjoyable, and timed tests would cause anxiety. Moreover, the assessment used in this study was a more valid measure of what fluency in mathematics really encompasses. Regardless of the sequence in which the two strategies were introduced, all students demonstrated growth in strategy usage and efficiency over the two-week period.

SECTION ONE

Introduction

Does memorizing 100 multiplication and division flashcards truly prepare an individual to be successful with everyday math? Do regular timed tests in elementary school truly teach students anything about math? Research suggests that requiring memorization of math facts in elementary school does not ensure future success with math tasks (Boaler, 2014, 2015; Kling, 2014, 2015). Lasting success with mathematical tasks is rooted in understanding, not memorization. So why do we continue to force children to engage in this task? I propose a different method to helping students boost math fact fluency.

As a practicing third grade teacher, I have never supported giving students timed tests, but I have wondered what else was out there to use. I really became interested in researching this topic when I found that the Ohio State Standards define fluency as "the ability to use efficient, accurate, and flexible methods for computing," and more explicitly, "fluency does not imply timed tests" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 92). When I read that, I noticed the state explicitly rejects the implication that fluency is about performance on timed fact test, and I began to wonder why many teachers *are* teaching and assessing fact fluency with timed tests? On several accounts in my teaching experience I have come across teachers in various districts and states who continue to use timed tests to assess fluency. This practice is not specific to my school.

At my elementary school, the third-grade team is composed of sixteen teachers with diverse backgrounds, teaching experience, and viewpoints on current issues in education. With so many different teaching styles and viewpoints there is little agreement on what is best for kids in regards to math fact fluency instruction and assessment. Like many other colleagues, I have strong opinions on how to best help my students become fluent and how I want to assess that fluency.

In a meeting last year that included all sixteen third-grade teachers, we debated extensively about how fluency should be taught, and about the existing requirement that we give timed tests to determine a report card grade. More questions arose for me. I was concerned with the level of anxiety that these timed tests provoked in my students. When I brought my concerns to the team, the curriculum director suggested that, if the tests caused too much anxiety for a particular student, then I would not have to give the test to that student.

How then do I assess fluency? Do I assume these anxious students aren't fluent or that students who do well on timed test are fluent? How do I complete the timed-test section of the report card? Additionally, I wondered about the validity of timed tests in measuring a student fluency, especially in light of the state standards definition written above. After this particular team meeting I knew that I had to look more into this topic. I knew I could make a difference by providing evidence to support my previous claims and by sharing the existing research on teaching and assessing math fact fluency in the elementary classroom.

When I think about what is the best way for my students to learn math facts and for me to assess them, I know timed tests are not the answer. As a new third grade math

teacher, I quickly, realized that my students exhibited anxiety towards math assessments, especially when being timed. After giving just one timed assessment to these students, they became anxious that every assessment would be timed. I constantly field questions from students like, “is this timed?!” or “how long do we have?” I do not want to crush students’ eagerness to explore multiplication and division by drilling facts and requiring them to simply memorize facts. Instead, I want to support their fluency by building meaning with multiplication and division. What is uncertain to our third-grade teaching team, however, is how best to do this. A study that explores a range of research-based math fact fluency methods will inform my practice and perhaps the teaching approaches of my colleagues.

In this study I plan to implement research-based instructional strategies for boosting fluency of math facts then gather data regarding the effectiveness of these strategies in measuring and increasing student fluency. The findings from this study will be shared with the third-grade team as well as the curriculum coordinator so we can examine them in order to inform our own practices. The purpose of this action research project is to evaluate the effectiveness of math fact games and visual math cards for achieving fluency with multiplication and division facts among a group of forty-seven, third-grade students in a suburban school district in Central Ohio. The research questions I would like to answer are “what happens when math fact games and visual math cards are used to support fact fluency?” “How do these two activities impact students’ feelings towards math tasks and fluency in math as defined by state standards?” “How do these activities impact students’ accuracy with multiplication and division facts?” “Are there alternative ways to collect data on a students’ fluency, that encompass the entire

definition of fluency?" And "given that we know timed tests are not a valid or complete measure of students' fluency ability, what strategies do students use when given opportunity to explain their thinking?"

SECTION TWO

Literature Review

The foundation of true math ability lies in number sense and fluency with numbers. Tikhomirova, et al. (2017) quotes Dehaene (2011) who illuminates the above claim by asserting, "individuals with high levels of mathematical fluency manage quickly and accurately to perform initial calculations, saving more cognitive resources for completing the task" (p. 96). The need for fluency is crucial as individuals age and encounter more complex mathematical problems. Higher levels of fluency allow a mathematician's cognitive power to be used for more complex work instead of basic facts. However, building this fluency can be challenging.

Research on best practices for teaching math fact fluency has pointed out how countless teachers tend to focus their math fact instruction in unproductive ways (Boaler, 2015). Instead of teachers communicating to their students the importance of understanding basic mathematics and having number sense, they focus solely on speed. Little research suggests that timed tests are crucial to help students become fluent in math facts and successful in further aspects of mathematics. Yet, an overwhelming number of teachers today are still using these timed tests due to fear of change and lack of knowledge of an alternative (Boaler, 2014; Kling, 2014).

After studying effective strategies for helping students to be successful with math facts, many researchers and math experts seem have examined the alternatives to timed tests. The findings from past research on math fluency, instruction, and assessment, as well as current research on alternatives to timed tests, are presented in this literature review. I begin by presenting the research on fluency, then more specifically, on math

fluency, then shed light on the many ways fluency is and can be assessed. Finally, I draw on the literature to outline suggestions for teaching fluency. The information presented in this Literature Review comes from scholars in the field of education and mathematics over the past ten or so years.

Defining fluency

The word fluency can mean a number of things depending on the context. Although the focus of this literature review is math fact fluency, there is value in first examining the research on fluency in a different educational setting. Moving past the outdated belief some may have that fluency is just speed, the National Institute of Child Health and Human Development (2000) adds to the definition of fluency noting that it also includes reading at an appropriate speed, as well as accurately decoding words and using expression while reading. Knowing that fluency is viewed as much more than reading quickly, reading teachers consider all of these domains when listening to a student and concluding that they read fluently.

This relates to fluency of mathematics in that is reiterates that fluency is more than answering facts quickly. Students need to understand that when assessed on fluency, they should be striving for more than speed. They should be improving their number sense and flexibility with strategies. In reading assessment, data collection would include accuracy and expression as well in order to get a complete picture of the student's reading ability. Marcell (2011) supports her claim that [math] fluency needs to mean more to students than speed to get a *good* score by adding that the students focus moves from wondering if they achieved higher than their previous score, to then excitement about their growth and true understanding of the content. Fluency for math, just like

reading, is more than speed and the score. It is about the task as a whole. It is a complex skill including understanding strategies and applying them. It is about the "lightbulb" moments when students realize they learned something. It is about understanding, not just memorization.

Math fluency for elementary age students

Math fact fluency is not as simple as memorizing math facts and answering as many questions as possible in one minute. Math fluency in elementary school includes multiple steps towards mastery. Wallace and Gurganus (2005) echo The National Council of Teachers of Mathematics when claiming that students need a balance of conceptual understanding and computation skills to achieve fluency. To be truly fluent with math facts, students need to have deeper understanding of their work than just the answer. According to the Common Core State Standards for Mathematics, fluency is "the ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 92). This definition encompasses using previously learned strategies to solve unknown problems. For example, if students know their doubles facts in addition, they can use this skill to solve multiplication problems with a multiplier of two. Adding to this skill, they can use doubles facts to solve problems with a multiplier of three. It is a doubles set plus one extra set of the other number.

Bray and Maldonado (2018), assert that students must be taught to use previously learned facts to *derive* or solve for new and unknown facts in their journey towards automaticity. Likewise, Kling and Bay-Williams (2015) explain that students start deriving their answers by using appropriate strategies and reasoning skills, before finally

mastering production of facts accurately and efficiently. Kling and Bay-Williams share three phases in building fluency; modeling to find the answer, deriving answers through use of reasoning strategies based on known facts, and finally mastery (2015). From the research above, it is clear that in order to build fluency, instructional strategies need to teach students to use previously learned facts to help them learn new facts, and subsequently become more fluent. Accuracy is of course crucial in mathematics. Being close is not enough, so rushing students with timed tests may take away from accuracy. Efficiency is also included in the definition because students need to flexibly use numbers to make the most of their time. Although there are many ways to solve problems, the most efficient strategy for any given problem should be applied.

Another common misconception is that math fluency is no more than memorization. It is true that students will start to know facts from memory or otherwise known as, "by heart," but how this is different than memorization is explained by stating, "the phrase 'by heart' implies that they have deeply internalized the knowledge, [and] that they are fully aware of its worth and are apprised of its meaning," (*The Unacceptability of Drilling and Memorization*, 2017, p. 21). Fluency implies that the knowledge has been learned with meaning, and therefore will be retained long term. This is very different from memorization in that it is using one's memory to skillfully complete the process of solving a math problem. It takes flexibility, accuracy, efficiency, and appropriate use of numbers to be fluent. Fluency requires critical thinking and is crucial in the elementary years and beyond.

Assessing fluency

How fluency should be assessed is a widely debated topic. A common practice for assessing fluency in the primary school is with timed tests. But, can timed tests provide a valid measure of student fluency? To provide a reasoned response to that question, it is necessary to next review the literature on the benefits and drawbacks of timed tests to assess fluency. Following this, I present literature on the use of teacher observations, fact interviews, journaling, and short quizzes as forms of assessment.

Research over the past two decades overwhelmingly points out drawbacks to timed math assessments, especially in regards to the correlation between math fluency and anxiety (Ashcraft, 2002; Boaler, 2014; Geist, 2010; Kling, 2014, 2015). Limited and dated research, however, suggests that timed tests can be used in practice to challenge students to strive for growth (Brookhart, Andolina, Zuza, & Furman, 2004). In a study conducted by Brookhart, Andolina, Zuza, and Furman (2004), 41 third graders were included in exploring the use of timed math assessments paired with self-assessment reflection. Over ten weeks the two classes were given several timed math fact tests after predicting their results. Once the results were predicted and the assessments were taken, students examined their results on graphs and made predictions for the next timed test. The idea presented here was that self-assessment reflection paired with the timed tests would motivate students to learn the facts in order to graph positive results.

The researchers saw an opportunity to promote student growth by suggesting that, as students monitor their own growth and are thinking about their work, they are better able to learn times tables (Brookhart et al., 2004). They gather that if students are thinking about their thinking and what they have learned so far, they will be motivated to

continue to strive and grow with their math fact retention. The graphing and reflecting on each assessment followed by predicting the results of the subsequent assessment will encourage them to memorize more facts. By the end of the ten-week intervention students in both classes did get better at predicting their scores, and students did increase scores. No data was presented as to whether students retained the content over an extended period of time of not. The researcher claimed that when students engage in self-assessment they are more successful in learning. However, even with support for self-assessment, an abundant amount of research seems to dispel the belief that timed tests should ever be used, and many scholars make a case for doing away with timed tests as they believe that memorizing the facts is not sufficient to create fluent mathematicians (Boaler, 2014, 2015; Kling, 2014, 2015).

Making an analogy between knowing family and knowing math facts, Caron (2007) asked the reader if they had to memorize the name of their loved ones. He suggests instead, "through interaction and meaningful engagement over time, they have become an undeniable part of your long- term memory" (Caron, 2007, p. 280). He continues to make his point by relating this analogy to multiplication in that, in order for students to be successful in fluency, they do not need to be endlessly tested (Caron, 2007).

A significant amount of research suggests timed tests are not the answer. Facts may be memorized for the tests, but critical math thinking is not learned and sustained (Boaler, 2014, 2015; Kling, 2014, 2015). "Teachers begin to focus on repetition and speed or 'timed tests' as important tools for improving mathematical prowess and skill which can undermine the child's natural thinking process and lead to a negative attitude

toward mathematics" (Popham, 2008; Scarpello, 2017; Thilmany, 2004; Tsui & Mazzocco, 2007 as cited in Geist, 2010, p. 25). Timing students makes them feel pressure and anxiety. This causes them to turn to rote memorization in order to be what they think is successful at the task at hand (Geist, 2010).

Some scholars are comparing assessment in math to most recent practices for assessment in reading. A drawback to the math timed tests is that it is not fully assessing all that fluency encompasses (Boaler, 2014, 2015; Kling, 2014, 2015). Kling (2014) starts by relating math fluency assessment, to reading fluency assessment by asking if teachers in this setting also listen to their students as well as observe them and ask questions to assess comprehension. Reading assessments include observing students and asking comprehension questions. It includes accuracy and self-correction. Why would math assessments not include these same things? Kling (2014) then asks her reader to take time to think about what all you would accurately learn about your students' fluency if you *only* used timed tests to assess. It is not effective and accurate. Some scholars are concerned there are lost opportunities in assessment by basing fact fluency solely on timed tests (Kling, 2014, 2015).

Math fluency used to be defined just with speed, but today's Common Core State Standards for Mathematics have expanded that definition. The standards suggest that fluency also should assess flexibility, accuracy, efficiency and appropriateness of strategies (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Timed tests are not designed to assess all of these domains because the product turned in only shows the answers written; not what the student may fully know, or the strategy they implemented to help them. Timing math tests makes

math seem more about the answer instead of the process. It improperly places the educational emphasis on performing rather than learning (Boaler, 2014). The true importance in learning math and thinking flexibly is lost due to an assessment which seemingly prevents a student from being able to fully analyze and utilize the problem-solving skills they have learned, because the time constraint takes that away from them. Instead, it is all about finding the right answer as quickly as possible when the real challenge and exercise that will accurately measure fluency must ensure that the student has the necessary time to observe, detect and solve with the skills that they have previously been taught.

Several articles claim timed tests are causing anxiety levels to rise among students. Math anxiety caused by timed tests creates an issue for valid assessment because the working memory is being compromised while processing due to disruptions from math anxiety (Ashcraft, 2002). If students cannot think clearly because they are anxious, the assessment is not testing what the students know, but rather how anxious they are or how well they cope with a blockage of the working memory.

Hunt and Sandhu (2017) conducted a study to look closer at time pressure on eighty adults solving math problems. The researchers concluded that participants' error count was significantly higher when they were asked to perform with limited time. Upon reviewing results, they found four times the amount of errors were made in math anxious individuals when timed versus untimed (Hunt, 2017). Although this study did not look at elementary aged students and timed multiplication tests, it did find a correlation between timing assessments and the anxiety response in humans leading to incorrect responses. Anxiety starts with timed tests but later becomes a learned response to all forms of math

they encounter. Geist (2010) asserts that the cause of anxiety about math does not stem from the mathematics computation, but it actually comes from the way math is presented in school. This means teachers are causing anxiety in students by their means of practice and assessment. The anxiety is not about the numbers, the anxiety is about being rushed and timed.

In attempts to more accurately assess students' fluency and provide teachers with alternatives to timed tests, Kling (2014) offers four strategies to teachers as better substitutes to timed tests to assess math fact fluency. First, observations can be useful for assessing fact fluency (Kling, 2014). This can be during everyday classroom instruction, practice opportunities, and even games. The teacher actively observes, takes anecdotal notes on what the students are doing, and records if they are applying taught strategies effectively. Games can be fun and interactive for students, but Kling (2014) also reminds that when students are playing, they need to be able to tell their partner not only the answer but also how they found it in order for this to be beneficial. Through teacher observations while students play against classmates for fluency practice, student responses can be recorded on a rubric and assessed for understanding.

Another offered strategy by Kling (2014) is interviewing students to assess their mathematical knowledge and thinking ability. She offers the idea that these interviews are quite the opportunity for students to learn from each other and hear explanations from the classmates on how they engaged in problem solving. She continues to say that teachers should also follow up these interviews with clarifying questions. In this scenario, students are given higher-level thinking opportunities to explain and clarify their work. Justifying or clarifying an answer gives proof to the teacher that the student really has the

understanding piece, not just memorization. They are no longer simply remembering, they are understanding, applying, analyzing and evaluating. This can be a one-on-one conversation with prompts to check for understanding while observing the student work, or conferencing with students about their work after they have completed it.

The third assessment strategy suggested by Kling (2014) is journaling, in that it assesses flexible thinking and understanding of strategy selection and use. Students are given opportunities to explain and defend their work in writing. They are able to draw pictures, write words, and show numbers to convey their thoughts and strategies. Kling suggests that this can be done by prompting students to explain in writing, to a friend, how they would figure out the solution to a given math problem. The teacher can then take a look at not only the students understanding, but also see how they apply their skills and knowledge to solve problems.

Fourth, Kling (2014) recommends the idea of quizzes to quickly assess fluency. This is different than timed tests because it gives students a chance to show the facts they "just know." On these quizzes, students are also asked to identify how they solved the problems. These facts assessed on the quizzes, are call fundamental facts because they are fundamental in learning all other facts. Above, Kling's four main strategies for assessing fact fluency without timed tests are presented, but next there is merit in exploring known strategies for helping students boost math fluency.

Cognitive development theories

Before exploring strategies for boosting math fluency specifically, there is merit in touching on theory about how children learn in general. Seminal research is presented on the topic of child development by Jean Piaget and Lev Vygotsky in the early 1900's.

Piaget discusses how interactions between peers in their physical environment plays a crucial role in their development.

Students learn from their peers. They hear thoughts and ideas of their classmates, and grow from their interactions with them by adjusting any previous schema they have about a topic. This process of deciding what they think is part of assimilation and accommodation where "children either modify an existing scheme to account for the new object or event or else form an entirely new scheme to deal with it," (Ormrod, 2012, p. 142). Lloyd and Fernyhough continue to explain that Piaget believed social interactions with peers help build logic in that they cause a "cognitive conflict" through disequilibrium, (1999) and the child learns from the given experience. From Piaget's perspective, interactions with peers merely help children realize there are other perspectives that they may or may not agree with.

Vygotsky however would go much further to assert that foundations of *all* cognitive development are due to social interactions, (Ormrod 2012). Lloyd and Fernyhough, (1999) explain, as students share their knowledge, they are helping others grow within their Zone of Proximal Development. This means that through interactions with peers, students can achieve beyond their individual limits, while supported by a peer (Lloyd & Fernyhough, 1999; Ormrod, 2012). Due to the support and interactions of their peers, students are making cognitive growth, and practicing previously taught skills. They are more able to engage in complex tasks than they otherwise would be if asked to work independently.

Piaget also had four stages for which he classified cognitive development. Children ages six to twelve fall into the "concrete operational" stage. At this stage they

lack ability to think abstractly, but rather think concretely. They are limited to thinking logically about concrete or real-life things (Ormrod, 2012), so teachers must account for this when planning learning experiences.

When thinking about how to best boost math fact fluency among third graders, these theories should be considered. Taking from Piaget and Vygotsky's theories, one can conclude that activities that encourage peer interactions and involve concrete tasks will best help boost student growth within any subject area.

Known strategies for boosting fluency

Experts encourage alternative methods of not only assessing fluency which was seen above, but also teaching math for fluency, which involves students using flexibility, accuracy, efficiency, and strategy, rather than using timed tests to force students to skip over understanding and just memorize facts (Boaler, 2014; Kling, 2014, 2015; Nelson, Burns, Kanive, & Ysseldyke, 2013). There are many strategies to boost number sense, in order to help all students, succeed in mathematical experiences.

One suggestion by Boaler (2014) for teaching math fluency is Number Talks, a strategy to help students develop a conceptual understanding of math principles, while also presenting creative and flexible ways to do math (Boaler, 2014). This strategy is used when a teacher poses a math problem, and encourages students to solve the problem mentally. When students are ready, they silently show a signal. This again is so students are not anxious by timing (including seeing others done before them). It is again the process, not the speed. Once all students have had adequate time, several students share their answers. Then a few students are called on to share strategies and the teacher

models their thinking for the class. The stress is on the process in reaching the answer, not the answers themselves.

Number Talks encourage students to think flexibly and understand math concepts. They take students to high level thinking skills. With practice in Number Talks, students will build number sense and be more successful in their math problem solving than memorizing math facts for a timed test (Boaler, 2014). Bray and Maldonado (2018) also propose an instructional strategy similar to Number Talks; they call it Number Strings. With this method students looks for patterns and relationships between numbers over a course of several problems. The sequence of the problems is strategically planned by the teacher to draw attention to targeted mathematics strategies and guide rich discussion (Bray & Maldonado, 2018). They reassure that this instructional strategy is effective by adding, "repeated engagement in such discussions overtime does promote student reasoning about relationships, which in turn leads to increased fact fluency," (Bray & Maldonado, 2018, p.92)

A second suggestion to boost math fluency is the use of math cards. This is a substitute for the classic flash card. A current leading math expert, and Professor of Mathematics Education at Stanford University, Jo Boaler explains how these cards also help build strong number sense in her research (2015). Although students are still answering the cards as they would in a flash card situation, they are also matching the expression on the card to another card with a modeled picture. Boaler rationalizes this strategy by clarifying that when students are matching cards, they are also explaining their thoughts on how they knew cards were equal. An activity like this one does have students rehearse math facts, but more importantly provides opportunities for students to

prove understanding. (Boaler, 2015). Students need more than memorization; they need to understand and make connections. By using the visual math cards and matching a fact to an image or visual representation, the students are understanding what the numbers and mathematical symbols in the equation mean and how they relate to each other.

A third source suggests online fact practice, due to its ability to work at the student's level and provide activities that help them progress from where they are. The researchers claim, that programs encompassing the use of technology are proven to be effective in helping students become fluent in recalling math facts (Nelson et al., 2013). Students these days constantly want to use technology, and having programs that allow students to complete interactive fact practice on the computer, would be engaging and draw students in.

Another source proposes mnemonic strategies as a tool for success with math facts because instead of depending on repetition, the mnemonic will cue students to remember facts easily, and therefore improve their computation skills and fluency (Nelson et al., 2013). Implementing mnemonic strategies leads students back to earlier instruction to guide them towards recalling a strategy. Mnemonics are catchy and take the working memory back to the time the material was taught. This strategy is used across the curriculum and Nelson asserts that it is just as effective for math as well.

Additional research suggests math games and hands-on activities are an interactive and effective means of teaching math facts (Boaler, 2015; Kling, 2015). The students are immersed in practice over a period of time to help them think strategically and use their mathematical knowledge in solving math facts. Kling (2015) asserts that these games to practice multiplication facts are not only relevant but enjoyable

experiences, that avoid the unneeded stress of timed tests, but instead involve students in several opportunities to calculate efficiently and practice math. Kiili et. al (2015) also support the use of games in the classroom saying they are effective interventions, as well as engaging activities for students to facilitate positive attitudes.

A study by Afari et al. (2012) investigated the effectiveness of games used in the mathematics classroom. Participants included 90 students in three different schools took part in this six-week intervention. Findings from the study showed students who were engaged, excited, and motivated to participate in math. Students collaborated and the teachers reported positive learning environments. One student proclaimed, "[games make] the ideas stick in my mind. In the test, I actually remembered one question that we played in the class; so it is kind of fun and important," (Afari et al., 2012, p.1386).

Summary

Compelling research creates a case for the disposal of timed math tests and replacing them with more meaningful practices to teach and assess math fluency. By first looking broadly at fluency, and then honing in on math fluency, the flaws in a popularly used fluency teaching method and assessment, timed tests, has become apparent. Based on the evidence presented in this Review of the Literature, it is clear there are many more drawbacks to timed tests than benefits. Drawbacks to timed tests include the generation of anxiety in students, limiting of critical thinking skills, creation of the idea that the process has less importance than the answer, and lacking valid assessment for all parts of fluency.

There are several other alternative strategies for teaching and assessing math fluency among students as seen in the following lists.

Teaching:

1) Instruction using Number Talks, a whole class interactive discussion and practice opportunity to boost number sense (Boaler, 2014).
2) Visual math cards to provide meaning and promote understanding of numbers and expressions versus the traditional memorization of flash cards (Boaler, 2015).
3) Online fact practice with programs created to boost math fact fluency (Nelson et al., 2013).
4) Mnemonic Strategies to cue the memory (Nelson et al., 2013).
5) Interactive games played between students to practice strategies taught (Afari et al., 2012; Boaler, 2015; Kiili et. al (2015); Kling, 2015).

Assessing:

1) Teacher observations and anecdotal notes during student activities and games.
2) Interview questions to follow up to student work, provided by the teacher individually.
3) Student journaling to give opportunities to defend and explain their thinking in a low-risk, informal way to a peer.
4) Quizzes as a simple check in to find out what facts are just known, and which facts require a strategy to solve.
5) Teaching mnemonic devices to evoke memory of strategies during later times.

Research shows that timed tests are not effective for helping students to achieve math fluency in elementary aged students. Any of the above suggested strategies would be more effective and student-friendly approaches for helping students build solid math foundational skills and positive self-concept of math competency.

SECTION THREE

Research design and method

Through a mixed-method study, I plan to answer the questions "what happens when math fact games and visual math cards are used to support fact fluency?" "How do these two activities impact students' feelings towards math tasks and fluency in math as defined by state standards?" "How do these activities impact students' accuracy with multiplication and division facts?" "Are there alternative ways to collect data on a students' fluency, that encompass the entire definition of fluency?" And "given that we know timed tests are not a valid or complete measure of students' fluency ability, what strategies do students use when given opportunity to explain their thinking?" I plan to assess the extent to which students are able to fluently answer given multiplication and division problems.

As stated earlier in this study, fluency is not implying speed in student response, but instead more complex understanding of the concept. For the purpose of this study, math fluency is defined as "the ability to use efficient, accurate, and flexible methods for computing," (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 92). I will be looking for flexibility in strategy use as well as efficiency, accuracy, and appropriate use of strategy. Drawing on the research on effective teaching and assessment practices, I have created an assessment designed to measure student fluency as it is defined through my stated definition. The procedure that I have developed to guide me through this research as well as the assessment created will be described in this section.

Setting and participants

This study takes place in a suburban school district in Central Ohio. Most families residing within this school district limits are of high socioeconomic status. In this area the poverty rate is less than 3%. The school building where this study takes place is located on a central learning campus along with all other school buildings in this district. The building where the study takes place houses students in grades one through three. There are 1,074 students within this school, including 515 female students and 559 male students. These students are distributed across the three grades as follows: 378 third graders, 334 second graders, and 360 first graders. Seven hundred and ten students identify as Caucasian, 157 Asian, 90 African American, 83 Multi-Racial, 32 Hispanic, and 2 American Indian. Of the 1,074 students, 122 qualify for services as Students with Disabilities, 82 are identified as gifted, and 74 are identified as English Language Learners.

The participants of this study all are members of the third-grade class. Of the 378 students at this grade level, 143 are female and 235 male. They are broken up into sixteen classes across the school building. The average class size for these students is twenty-four. Students range from eight to nine years old.

As the researcher of this study and teacher, I will conduct this study with two of the third-grade classes; both taught by myself. Since I do not have a self-contained elementary classroom, I see two classes or groups of students per day. Each class receives eighty minutes of math instruction per day. Class A receives math instruction in the afternoons and Class B receives math instruction in the mornings. Class A includes 25 students: 13 males and 12 females, while Class B includes 22 students: 11 males and 11

females. Class A contains a Gifted Cluster and overall higher achieving math students than in Class B. There is no specific clustering in Class B, however of the twenty-five students in this class, seven students receive reading support daily outside of the classroom. Evidence shows that the make of Class A is overall higher academically than the makeup of Class B.

Within the group of forty-seven student participants in this study there are five students who identify as African American, seven Asian students, one Hispanic student, one multiracial student and thirty-three Caucasian students. None of the students are classified as English Language Learners. Five students receive services for Speech IEP's. The class also includes ten students who qualify as Gifted. These forty-seven students will take part in the accuracy portion of the Pre-Assessment, all daily intervention, the accuracy portion of the Mid-Point Assessment, the accuracy portion of the Post-Assessment, as well as rating scales on their feelings towards various math activities.

A smaller selection of students will be chosen through stratified sampling (Ary et al., 1990) to take part in verbal assessment (Appendix A) as well as interviews to represent a focus group. To gather a stratified sample, I will select three high-performing students, three average-performing students and three low-performing students for this portion of the assessment from each class. The performance groupings for this focus group will be based on scores from a math fact assessment administered on ALEKS (Assessment and LEarning in Knowledge Spaces). ALEKS is computer-based program by McGraw Hill that has the capabilities to assess student achievement in mathematics with an online assessment. The assessment goes through each multiplication and division problem within 100 and reports data on known facts to the teacher. This group of forty-

seven students have used ALEKS throughout this school year and therefore are familiar with ALEKS. The assessment data can be analyzed and pattern of strength or areas for improvement can be identified. I will use this as a diagnostic tool as a predictor of mathematical achievement with fact fluency. Each performance group, high, average, and low will include six students; three from each class, making a total of eighteen total focus students. Knowing that Class A is more advanced overall than Class B, ranges for high, average, and low groups are different for the two classes. The ranges that were used to group the students are presented in Table 1. Class A contains a gifted cluster of students as well as other high-performing individuals. That being said, high, average, and low students for the focus group will be grouped by score as compared to their classmates not grade level peers. Students in Class A who showed mastery of fifty percent or greater qualified for the high group, whereas forty percent or greater qualified as part of the high group in Class B. Students in Class A who showed mastery of thirty percent or greater qualified for the average group, whereas twenty percent or greater qualified as part of the average group in Class B. Students in Class A who showed mastery of less than thirty percent qualified for the low group, whereas below twenty percent qualified for the low group in Class B.

Table 1. Ranges used to group students

Level	**High**	**Average**	**Low**
% Range Class A	50% or greater	30%-49%	Below 30%
% Range Class B	40% or greater	20%- 39%	Below 20%

Focus group selection

Using data from the ALEKS assessment, I determined that the focus group would include the following students (pseudonyms):

High Group: Henry, Anthony, Leon, Aaron, Mandy, and Ivan

Average Group: Nina, Will, Gretchen, Carter, Emma, and Anna

Low Group: Lynn, Addie, Nick, Makayla, Lee, and Gabby

These students include nine females, and nine males. Of the females two are gifted, none are on an IEP, none are ELL. Of the females, six identify as Caucasian, one identifies as Hispanic, and two identify as Asian. Of these males four are gifted, two have speech IEP's, zero are ELL. While none of the students have IEP's with reading goals, four of the focus students receive reading support outside of the classroom. Of these males, six identify as Caucasian, one identifies as Multiracial, and two identify as Asian. A breakdown of demographics for each student are presented in Table 2. Exact scores from the ALEKS Multiplication and Division Assessment that I used as a diagnostic to identify my eighteen focus group students can be seen in Appendix U.

Table 2. Demographics of students in focus group

Name	**Class**	**Level**	**Ethnicity**	**Gifted**	**IEP**	**ELL**	**Reading Support**
Henry	Class A	High	Asian	Yes	No	No	No
Anthony	Class A	High	Multiracial	Yes	No	No	No
Leon	Class A	High	Caucasian	Yes	Yes	No	No
Aaron	Class B	High	Asian	No	No	No	No
Mandy	Class B	High	Caucasian	No	No	No	No
Ivan	Class B	High	Caucasian	No	Yes	No	No
Nina	Class A	Average	Asian	Yes	No	No	No
Will	Class A	Average	Caucasian	No	No	No	No
Gretchen	Class A	Average	Caucasian	Yes	No	No	No
Carter	Class B	Average	Caucasian	No	No	No	Yes
Emma	Class B	Average	Caucasian	No	No	No	No
Anna	Class B	Average	Asian	No	No	No	No
Lynn	Class A	Low	Caucasian	No	No	No	Yes
Addie	Class A	Low	Caucasian	No	No	No	No
Nick	Class A	Low	Caucasian	Yes	No	No	No
Makayla	Class B	Low	Hispanic	No	No	No	Yes
Lee	Class B	Low	Caucasian	No	No	No	Yes
Gabby	Class B	Low	Caucasian	No	No	No	No

Method and procedure

I have developed a mixed-method study, framed within a cycle of inquiry used in teacher action research, including a pre-assessment, mid-point assessment, post-assessment, rating scales and interviews for data collection. A mixed-method study includes quantitative and qualitative methods in the same study to help provide the researcher with a complete picture (Mahmood, 2019). A description of interventions and

the assessment protocol for qualitative and quantitative data will be explained in this section.

Stratified sampling was used to identify several students for a focus group in this study. All students selected for this focus group will have approved consent from parents or guardians before participating in the study. Interview questions will be asked in order to gather information about the students' experience with math games and visual math cards versus feelings about timed math experiences.

First, I will administer all forty-seven student participants a Pre-Assessment. While I give the verbal version of the assessment to the eighteen focus students individually, and scribe as they answer my questions, I will evaluate the other twenty-nine students based on the accuracy of their written answers using a paper-pencil format. In addition to evaluating the accuracy of focus students' responses, I will document their problem-solving processes for efficiency, flexibility, accuracy, and appropriateness of strategy usage.

The accuracy portion of the assessment will include three multiplication and three division questions that test for accuracy in solving. All items on the assessment will have products less than one-hundred. In addition to the six accuracy questions, the focus group students will verbally answer three scenario-based questions designed to elicit deeper explanations regarding their problem-solving processes; specifically, the appropriateness, efficiency, and flexibility of their strategy use. Questions were derived from "Assessing Basic Fact Fluency (Kling, 2014) as seen in figures 4 and 5. The verbal assessment can be seen in Appendix A.

Appropriate strategy selection	Flexibility
• Explain how to use the "count on" strategy for 3 + 9. • What strategy did you use to solve 6 + 8? • A friend is having trouble with some of his times 6 facts. What strategy might you teach him? • Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Is this a good strategy? Tell why or why not.	• How can you use 7 × 10 to find the answer to 7 × 9? • Solve 6 × 7 using one strategy. Now try solving it using a different strategy. • Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Does this strategy always work?
Efficiency	**Accuracy**
• What strategy did you use to solve 9 + 3? • How can you use 7 × 7 to solve 7 × 8? • Which facts do you "just know"? For which facts do you use a strategy?	• Crystal explains that 6 + 7 is 12. Is she correct? Explain how you know. • What is the answer to 7 × 8? How do you know it is correct (how might you check it)?

Figure 4. Assessing Fact Fluency Sample Questions.

Solve these problems and tell how you solved them.

4 × 5 = _____	Check one: _____	I used this strategy: _____	
	_____	I just knew.	
10 × 6 = _____	Check one: _____	I used this strategy: _____	
	_____	I just knew.	
6 × 2 = _____	Check one: _____	I used this strategy: _____	
	_____	I just knew.	
5 × 3 = _____	Check one: _____	I used this strategy: _____	
	_____	I just knew.	
2 × 9 = _____	Check one: _____	I used this strategy: _____	
	_____	I just knew.	
3 × 10 = _____	Check one: _____	I used this strategy: _____	
	_____	I just knew.	
5 × 7 = _____	Check one: _____	I used this strategy: _____	
	_____	I just knew.	
8 × 10 = _____	Check one: _____	I used this strategy: _____	
	_____	I just knew.	

Figure 5. Assessing Fact Fluency Sample Assessment.

Once data is collected, I will then plan two different interventions (A: games and B: visual math cards) for the two groups. Each intervention will be described in detail shortly. Using the cycle of inquiry, I will implement intervention A with one class, while implementing intervention B with the other class. Once the two-week cycle is complete, I will then collect Mid-Point Assessment data to evaluate this set of interventions. The Mid-Point assessment will be a replica of the Pre-Assessment where all students complete the accuracy questions with a paper-pencil format, and the focus students answer the verbal assessment questions.

After the first cycle and the Mid-Point assessment, I will reverse the interventions for each group and implement the opposite strategy for another two weeks. Following cycle two, I will administer the Post-Assessment as a final data point to measure student growth. In the end I intend to use my findings to evaluate what the effects of these interventions are on math fact fluency in third grade.

Scoring procedures for the focus group verbal assessment are outlined in Table 3.

Table 3. Scoring procedures for the focus group verbal assessment.

Points	**Criteria**	**Number of Items**
1	Accuracy: Student accuracy finds missing product or quotient	6
1	Efficiency: Student uses and explains an efficient strategy to solve. It takes a reasonable amount of time to solve. Student does not count by ones.	6
1	Appropriate Strategy: The strategy selected makes sense and would lead to correct answer. There strategy is not counting on by ones.	6
1	Explanation: Student explains what they would do in the situation to solve. The first question by explaining an appropriate strategy, the second question by solving efficiently, and the third question by flexibly solving the same problem two ways.	3

Treatment

The two strategies I will implement as treatments are Math Games and Visual Math Cards. Although there are many research-based methods to teaching multiplication and division, I have selected these two methods to implement to supplement my daily math instruction during this research project. My daily instruction of multiplication and division already heavily focuses on teaching strategies to solve multiplication and division problems. These two treatments will be used as distributed practice of the skills they learn. The use of math games for students will provide several opportunities to apply and practice multiplication and division concepts. Games can be fun and interactive for students, (Boaler, 2015; Kling, 2015) but Kling (2014) also explains that when students are playing, they need to be able to tell their partner not only the answer but also how they found it. Having a chance to explain their thinking will help move these math concepts from working memory to long term memory for long term success in math. Knowing the benefits of using math games to support learning, I have planned for each intervention time to last thirty minutes. The math games were all games created by *Math Learning Center* and part of our district's math program, *Bridges.* During the intervention the students will play a version of each of the following games. The fact family focus of the game depends on results of the previously mentioned ALEKS assessment. Students will play with a partner to work on a fact family that they have not yet shown mastery of based on the assessment. I have briefly described the purpose of the games, and the full game directions and game sheets can be seen in the Appendices.

1) *Cover Up* (Appendix B-C)*:* Students will practice multiplication skills by spinning two spinners and multiplying the two numbers. Students model work by drawing arrays to help them find solutions.
2) *Scout Them Out* (Appendix D-L)*:* Students will practice multiplication skills by classifying given problems in appropriate fact families, then solving them using known strategies for that family.
3) *Division Capture* (Appendix M-R)*:* Students will practice division skills in a game similar to tic-tac-toe. They spin a spinner and the answer spun is the quotient to a division problem on the board. Students try and get three in a row across the board.
4) *Line 'Em Up* (Appendix S-T)*:* Students will practice division skills by splitting up different dividends into arrays. They will divide the dividend by two, three, four, five, and six and create an array to model the problem.

The second treatment includes visual math cards (Boaler, 2015). These are different than traditional flashcards in that they do not have the answers on the back. Students are not just memorizing answers to problems, but instead building understanding of equal groups. Students will create their own set of visual math cards with meaning to them. Students will be given a multiplication and division table with facts that they proved mastery of highlighted. An example of the table can be seen in Appendix V. Once students look over their given tables, they will be asked to take one fact family at a time and create visual math strategy cards for unknown facts. Rather than writing an answer on the back of the cards, students will draw a picture, create a model, or write a connection to represent what the equation met. This way they have a personal connection as well as a way to spark their memory on how to solve when they look at the

back. The card does not contain the answer, but rather a visual that the students will have made to help them solve. Students will create up to forty cards starting with their unknown facts in the following order X0, X1, X2, X10, X5, X4, X8, X3, X9, X6. This is the order that the strategies are taught in our math program, *Bridges*, so the same order will be followed.

For instance, If I were going to make a card for 3x4 because I have not yet mastered fours facts, I would draw something of meaning to me on the back. Since I have an interest in horses, I would draw three pastures with four horses in each on the back of the card. Every time I would then go back to practice with my cards I would think "three groups of four." Eventually the answers will be learned and retained because I learned them with meaning rather than just memorizing an abstract concept of a number times another number equals a third number. This process will be taken a step further, and turned into a game. As stated above, games are an excellent way for students to practice their skills over several opportunities.

Once cards are made, students will practice solving with known strategies by playing a math version of the card game "war" with a partner. In this game the student has to explain how they would solve the problem on their card, then the player with the higher number wins both cards. The person with the most cards in the end wins the game. Students have a chance to practice their own facts, but also may have a chance to review previously learned facts, in this game. If a student notices their partner did not correctly answer the problem even with a higher total, they have a chance to steal the cards if their partner was not accurate and they can explain to their partner a strategy to solve that fact. A schedule of the intervention for each class is presented in Table 4.

Table 4. Schedule of Intervention

Day	CLASS A	Topic	CLASS B	Topic
1	Game: Cover Up	Single Digit Multiplication with products under 100	Create Visual Math Cards	Multiplication with products under 100 and Division with dividends under 100
2	Game: Scout Em Out	Single Digit Multiplication with products under 10	Card Game War vs. Partner	Multiplication with products under 100 and Division with dividends under 100
3	Game: Scout Em Out and Division Capture	Single Digit Multiplication and Division with dividends under 100	Card Game War vs. Partner	Multiplication with products under 100 and Division with dividends under 100
4	Game: Division Capture	Division with dividends under 100	Card Game War vs. Partner	Multiplication with products under 100 and Division with dividends under 100
5	Game: Line Them Up	Division with dividends under 100 (some with remainders)	Card Game War vs. Partner	Multiplication with products under 100 and Division with dividends under 100
	MID-POINT ASSESSMENT		MID-POINT ASSESSMENT	
1	Create Visual Math Cards	Multiplication with products under 100 and Division with dividends under 100	Game: Cover Up	Single Digit Multiplication with products under 100
2	Card Game War vs. Partner	Multiplication with products under 100 and Division with dividends under 100	Game: Scout Em Out	Single Digit Multiplication with products under 10
3	Card Game War vs. Partner	Multiplication with products under 100 and Division with dividends under 100	Game: Scout Em Out and Division Capture	Single Digit Multiplication and Division with dividends under 100
4	Card Game War vs. Partner	Multiplication with products under 100 and Division with dividends under 100	Game: Division Capture	Division with dividends under 100
5	Card Game War vs. Partner	Multiplication with products under 100 and Division with dividends under 100	Game: Line Them Up	Division with dividends under 100 (some with remainders)

Data analysis

Accuracy data from all forty-seven participants along with data involving student strategies, and experiences learning multiplication and division will be collected and analyzed. Although I am not trying to prove students learn better from the interventions implemented in this study than they would with timed tests, I aim to study if they are still learning and even enjoying their work. If they are not harmful, then why wouldn't future teachers use these interventions? I aim to analyze data based on the Ohio State Standards definition of fluency. I want to know if students are choosing appropriate strategies, efficient in their work, and able to think flexibly when solving. I want to look at data that validly assesses what the standard is asking students to be able to do. I will take the quantitative data and look for trends among the high, average, and low groups in the class. I will create a descriptive profile to get a complete picture of what is happening with the participants accuracy. I will also use data from interviews and examine them for themes to answer the question if visual math cards and math games are effective for boosting math fact fluency.

SECTION FOUR

Results

Descriptive data for whole class based on accuracy:

All 47 participants completed the accuracy portion of the Pre-Assessment. The Assessment can be found in Appendix A. Students completed three multiplication and three division problems to check for accuracy**.** Due to prior learning experiences earlier this year, some students have already shown mastery with accuracy on the given multiplication and division problems. Numbers of students in the high, average, and low groups scoring 100% accuracy on the Pre-Assessment are presented in Table 5. Five of the six high students in Class A achieved 100% accuracy on the Pre-Assessment. All three high students in Class B achieved 100% accuracy on the Pre-Assessment. However, since *fluency* encompasses much more than just accuracy or speed, students continued to be a part of the intervention. Two of the seven average students in Class A started with a 100% accuracy score on the Pre-Assessment. Two of the seven average students in Class B started with a 100% accuracy score on the Pre-Assessment. None of the nine low students in Class A achieved 100% accuracy on the Pre-Assessment. Likewise, none of the fifteen low students in Class B achieved 100% accuracy on the Pre-Assessment. Accuracy means for all groups on Pre-Assessment, Mid-Point Assessment, and Post-Assessment are presented in Table 7.

Table 5. Pre-Assessment Accuracy Scores of 100%.

Pre-Assessment 100% Accuracy		
	Class A	**Class B**
High	5/6	3/3
Average	2/7	2/7
Low	0/9	0/15

Once Pre-Assessment results were collected and the data were analyzed, the first intervention took place. This first intervention period was supposed to last seven school days however with three consecutive snow days, as well as teacher absence for professional development, the first cycle was shortened to five intervention days. That being said, in order to keep the interventions consistent, the second intervention cycle was also shortened to five intervention days. After the five intervention days Mid-Point Assessment data were collected. Data from the Mid-Point assessment can be seen in Appendix X. The mean growth in accuracy of Class A was 23% while the mean growth in accuracy of Class B was 30%.

At the Mid-Point assessment 77% of students (17/22) in Class A and 56% of students (14/25) in Class B achieved accuracy scores of 100%. That is ten additional students in Class A and nine additional students in Class B. A breakdown of those students by academic level are presented in Table 6. Whether it be due to the intervention or not, they are still showing growth in accuracy measures across the board.

Table 6. Mid-Point Assessment Accuracy Scores of 100%.

Mid-Point Assessment 100% Accuracy		
	Class A	**Class B**
High	6/6	2/3
Average	5/7	3/7
Low	5/9	8/15

Post-Assessment data revealed that 95% of all students (21/22) in Class A achieved mastery with accuracy of multiplication and division on the given problems. In Class B, 88% of all students (22/25) achieved mastery with accuracy of multiplication and division on the given problems. A breakdown of those students by academic level are presented in Table 7.

Table 7. Post-Assessment Accuracy Scores of 100%.

Post-Assessment 100% Accuracy		
	Class A	**Class B**
High	6/6	3/3
Average	7/7	6/7
Low	8/9	14/15

Of the 47 participants, 44 had 100% mastery of accuracy at the time of the post-assessment. Also, one of the three students who did not achieve 100% accuracy, experienced a traumatic life event at the time of this study so it is expected that she would not have performed to her full potential.

Class A and Class B qualitative data based on survey:

Once I had accuracy data from all students in Class A and Class B, I was interested in their feelings towards this learning process. What was most helpful to them? What activities were most enjoyable to them? I had students complete a survey with rating scales to give feedback on their experience with different tasks that they did participate in, as well as activities that were excluded from this study such as memorizing traditional flashcards and timed tests (Fig. 1, 2, and 3).

Students were asked to rate the following activities as no (not fun), kind of fun, yes fun, or my favorite (very fun):

1) Scout Them Out (Work Place Game)
2) Cover Up (Work Place Game)
3) Timed Tests (not completed during this study)
4) Division Capture (Work Place Game)
5) Line 'Em Up (Work Place Game)
6) Other Work Place Games (not completed during this study)
7) Visual Math Card Game "War"
8) Memorizing Traditional Flash Cards
9) Using learned strategies when solving (example: Double Doubles for X2)
10) Building arrays with tiles

Figure 1 illustrates spikes in the blue (no) line when it came to timed tests and memorizing traditional flash cards as being fun. However, majority students reported all math games that we played as fun. Additionally, a majority of students thought the strategies they learned to help them multiply and divide along with building arrays with tiles were fun as well. The most popular game among all forty-seven participants was War with the visual math cards. The data provides strong evidence that these students have enjoyed learning strategies to solve math problems and practicing their strategies through math games. Students who are engaged in their learning experiences are going to

take more away from them in the long run than disengaged students. As reported by students the games were successful in having students enjoy their learning.

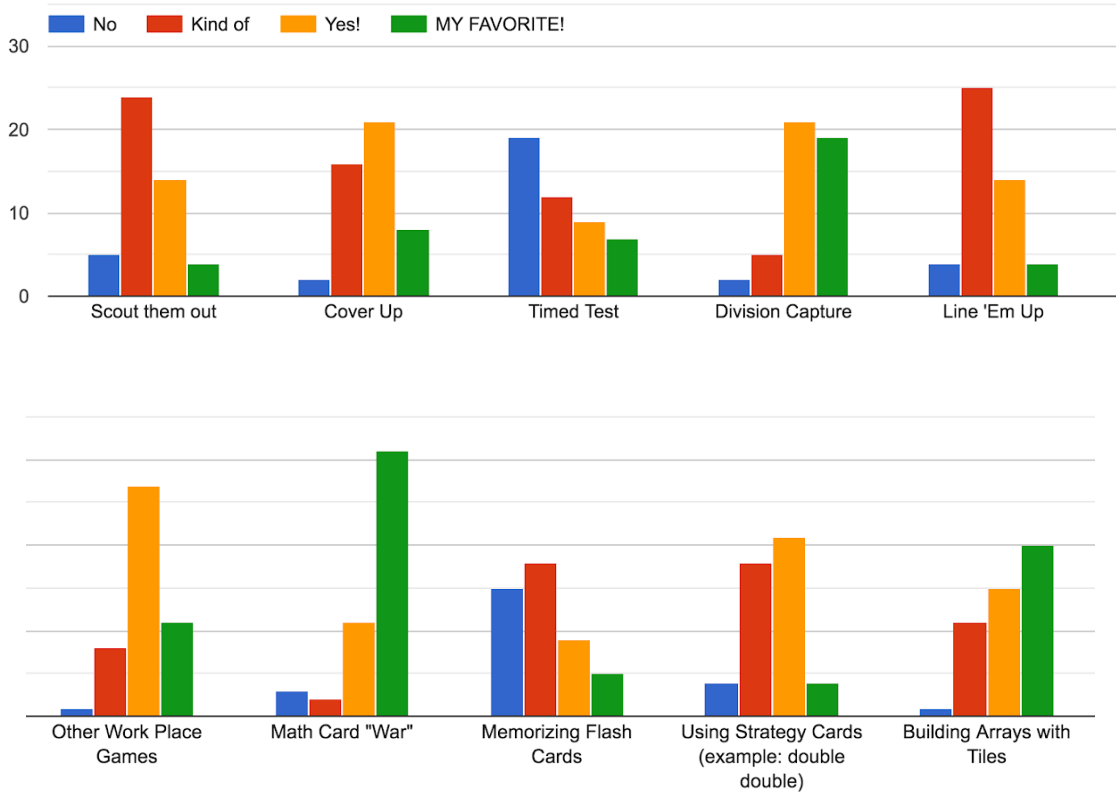

Figure 1. Rating scale indicating how fun the math activities are.

Next, students were asked to rate the same activities based on how helpful they were in learning multiplication and division (Fig. 2). Students responded with strongly disagree, disagree, neutral, agree, and strongly agree. Again, the data table represents a spike in *strongly disagree* to timed test and memorizing traditional flashcards. Students felt that games such as Scout Them Out, Division Capture, Other Work Place games, Visual Math Card War, as well as using strategies in solving are most helpful to learning to multiply and divide. Not only do students enjoy the games and strategies over timed tests and memorizing facts, but they also feel they are more helpful than the latter.

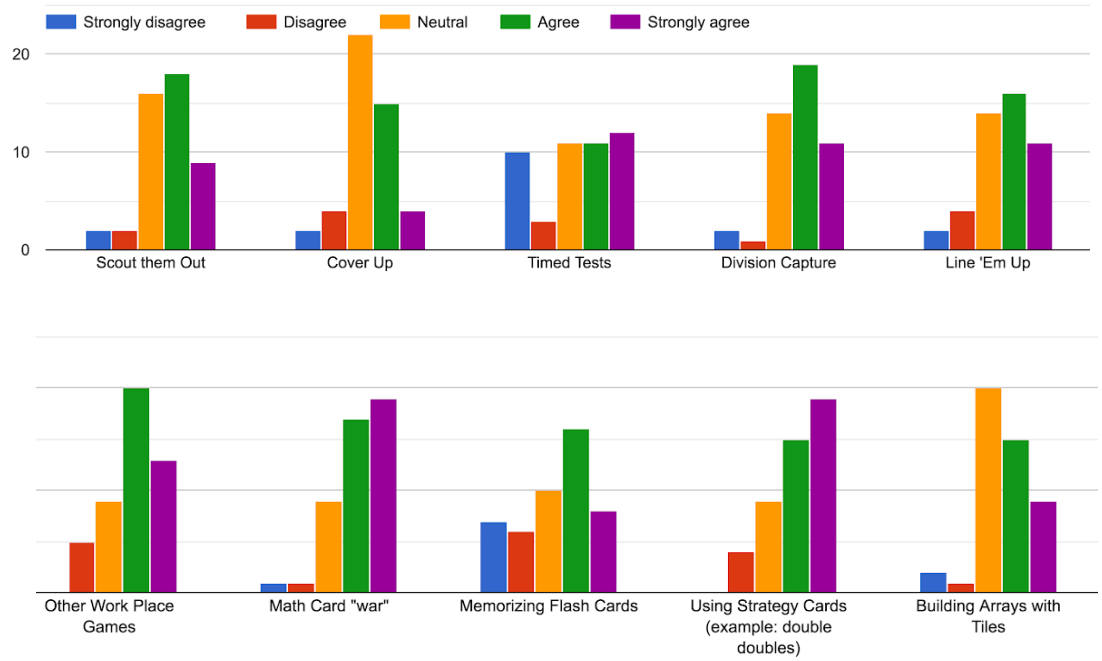

Figure 2. Rating scale indicating how helpful the math activities are.

Finally, I asked students to select methods that may help them or someone else become good at solving math problems (Fig. 3). I was interested in what they felt was important. Low percentages of students felt rushing, being quizzed daily, practicing traditional flashcards, and memorizing facts would help in being *good* at solving math problems. Instead, most students felt taking their time, thinking about their work, using known strategies to help, practicing with games, building with arrays, and Visual Math Cards may help them be more successful. Aligning with the data from the previous two questions, as well as interview questions answered by focus students, these participants felt strongly about what they need to be successful with math. Time, strategies, and opportunities to practice manipulating numbers is what stands out.

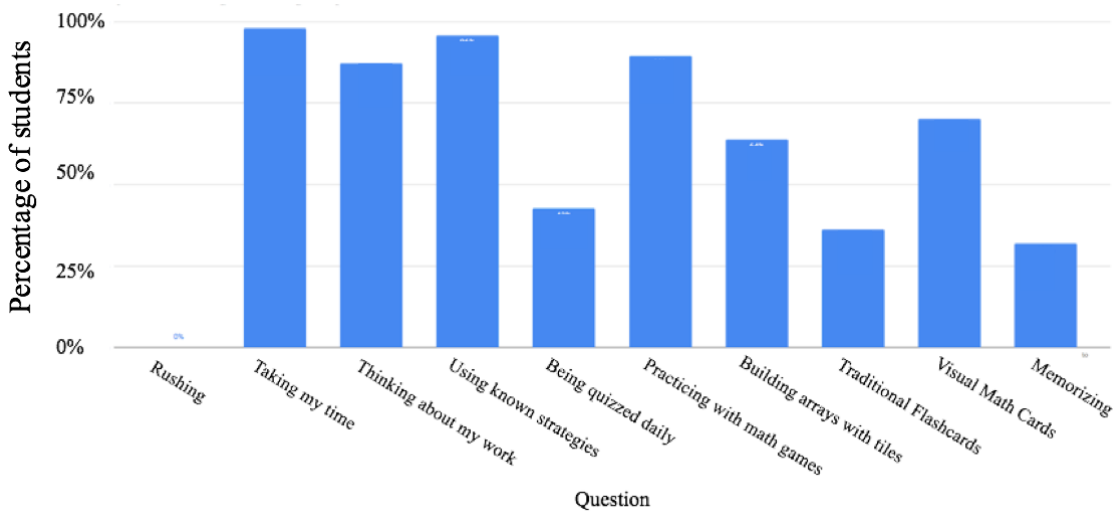

Figure 3. Survey results on how to best become good at solving math problems.

Focus group data based on verbal assessments:

Focus Group students not only completed the accuracy portion of the assessment, but they also responded to verbal assessment questions to check to flexibility, efficiency, and appropriate strategy usage (Appendix A). The scoring guide for this assessment was presented in Table 3 and results from their verbal assessments are seen in Appendix W. On the Pre-Assessment, three of the eighteen focus students were efficient when solving on the Pre-Assessment; two of which were in the high group, and one of which classified as gifted. Five of the six students in the high group achieved 100% on their strategy score. This means they had an appropriate strategy to solve the given problems. None of the other twelve students had strategies for all given problems. Five of the six low students scored fifty percent or lower on the strategy portion. The sixth student, who achieved ⅚, is classified as gifted. Finally, all high students were able to explain their thinking in the verbal assessment whereas a majority of the other two groups did not have 100% mastery on this skill. Accuracy scores of focus students across all three levels

closely reflected accuracy scores of participants across the whole group (within six or less percentage points), therefore they are an accurate reflection of the whole group. Both class means for each academic level can be seen as compared to the same level focus group are presented in Table 8 and 9.

Table 8. Class A accuracy mean for all assessments.

Class A Accuracy						
	Pre-Assessment		Mid-Point Assessment		Post-Assessment	
	Entire Group Class A	Focus Group Class A	Entire Group Class A	Focus Group Class A	Entire Group Class A	Focus Group Class A
High	98%	100%	100%	100%	100%	100%
Average	79%	78%	93%	100%	100%	100%
Low	50%	44%	93%	98%	98%	100%

Table 9. Class B accuracy mean for all assessments

Class B Accuracy						
	Pre-Assessment		Mid-Point Assessment		Post-Assessment	
	Entire Group Class B	Focus Group Class B	Entire Group Class B	Focus Group Class B	Entire Group Class B	Focus Group Class B
High	100%	100%	94%	94%	100%	100%
Average	75%	78%	86%	94%	98%	100%
Low	43%	44%	93%	89%	97%	100%

Knowing the focus group students were a good representation of their academic level peers, I looked deeper into the focus student's strategy usage to get a more complete picture of their fluency ability. Although the high group did achieve 100% accuracy on the Pre-Assessment, they were not completely fluent. With a mean of eighty-nine percent in Class A and a mean of eighty-seven percent in Class B on the complete Verbal Pre-Assessment, they validated the notion that they still need further instruction and practice to become fluent with multiplication and division along with the rest of the participants in

the average and low group. Through the duration of the study accuracy scores for the high, average, and low focus groups aligned with accuracy scores of their academic level peers. Therefore, they continued to be reliable representations of their class's fluency ability. The mean for each focus group of Class A and Class B at the time of the Pre-Assessment, Mid-Point Assessment and Post-Assessment are presented in Table 10.

Table 10. Focus group mean for verbal assessment

Focus Group Complete Verbal Assessment						
	Pre-Assessment		Mid-Point Assessment		Post-Assessment	
	Focus Group Class A	Focus Group Class B	Focus Group Class A	Focus Group Class B	Focus Group Class A	Focus Group Class B
High	89%	87%	97%	92%	100%	100%
Average	58%	57%	94%	89%	100%	98%
Low	37%	33%	78%	65%	91%	86%

Once again, the focus group took part in the accuracy measures during the Mid-Point Assessment, but also the verbal portion of the assessment (Appendix A). Results from the Focus Group on the Mid-Point Verbal Assessment are presented in Appendix X, and the scoring guide can be viewed in Table 3. Students were graded on appropriate and efficient strategy, accuracy, and flexibility. The mean for accuracy measures from Pre-Assessment to Mid-Point Assessment increased across the board in both Class A and Class B, as well as among the focus groups, with the exception of one student in the high focus group of Class B. Ivan had one inaccurate answer to a given division problem. By the Mid-Point Assessment, the high, average, and low focus group had all increased their mean score for the complete verbal assessment (Table 11). The high focus group for Class A increased by eight percent and the high focus group for Class B increased by five percent. The average focus group for Class A increased by thirty-six percent and the

average focus group for Class B increased by thirty-two percent. The low focus group for Class A increased by forty-one percent and the low focus group for Class B increased by thirty-two percent. All focus groups made significant gains during their first week of treatment. The largest gains were seen in the low and average group. The treatments in this study appear to have a positive effect on low achieving students in increasing fluency. The order in which the two treatments were administered did not appear to have an effect on student growth. They have become more efficient, able to use appropriate strategies, and explain their thinking.

Table 11. Focus Group growth since Pre-Assessment

Focus Group growth since Pre-Assessment		
	Focus Group Class A	**Focus Group Class B**
High	8%	5%
Average	36%	32%
Low	41%	32%

The mean for accuracy measures from Mid-Point Assessment to Post-Assessment increased across the board in both Class A and Class B as well as among the focus groups, with the exception of the high students Class A, the high focus group in Class A, and the average focus group in Class A because they already had achieved 100% accuracy at the Mid-Point Assessment (Tables 8 and 9).

By the Post-Assessment the high, average, and low focus group had all increased their mean score for the complete verbal assessment since the Mid-Point Assessment (Table 12). The high focus group for Class A increased by three percent and the high

focus group for Class B increased by eight percent. The average focus group for Class A increased by six percent and the average focus group for Class B increased by nine percent. The low focus group for Class A increased by thirteen percent and the low focus group for Class B increased by twenty-one percent. All focus groups made additional gains during their second week of treatment. Again, the largest gains were seen in the low group, which makes sense because they had the furthest to go. All students appeared to benefit from the treatment in this study, but the low achieving group made the most gains.

Table 12. Focus Group growth since Mid-Point Assessment

Focus Group growth since Mid-Point Assessment		
	Focus Group Class A	**Focus Group Class B**
High	3%	8%
Average	6%	9%
Low	13%	21%

Table 13 shows the mean growth over the course of the treatment period for the Focus Group students. From the time of Pre-Assessment to Post-Assessment, the mean scores for fluency of the high focus group for Class A increased by eleven percent and the mean of the high focus group for Class B increased by thirteen percent. The mean of the average focus group for Class A increased by forty-two percent and the mean of the average focus group for Class B increased by forty-one percent. The mean of the low focus group for Class A increased by fifty-four percent and the mean of the low focus group for Class B increased by fifty-three percent. Students were more able to choose appropriate and efficient strategies, as well as solve problems accurately while thinking

about numbers flexibly. Again, there was no drastic difference in any group as to whether they had the Visual Math Cards first or the Work Place Games.

Table 13. Focus Group growth Pre-Assessment to Post-Assessment

Focus Group growth Pre-Assessment to Post-Assessment		
	Focus Group Class A	**Focus Group Class B**
High	11%	13%
Average	42%	41%
Low	54%	53%

Mastery of all fluency measures was calculated for the Focus Group and results are presented in Table 12. All students in the high focus group achieved mastery of all criteria by the Post-Assessment. Five of the six students in the average group achieved mastery of all criterion by the Post-Assessment. The final student in that group missed one point for efficiency. Of the low group only one student achieved mastery of all criterion by the post-assessment, however the other five were close behind with scores of 86%. The students in the low group who lost points were in the area of efficiency. In the end the students in the low group made growth anywhere from 33% to 67% across the criterion.

Table 12. Mastery of all Criteria on Verbal Post-Assessment

Mastery of all Criteria		
	Focus Group Class A	**Focus Group Class B**
High	3/3	3/3
Average	3/3	2/3
Low	1/3	0/3

Focus group interview data:

Given that this is a mixed method study, I next look at qualitative data to complete the picture. The purpose of gathering qualitative data was to find out more about students' feelings towards the best way to become fluent with multiplication and division as well as what most helped them in this learning experience. I began by examining student responses to interviews about their attitudes toward traditionally implemented math, and toward the treatment strategies. When examining their responses about their experience learning multiplication and division, four themes stood out: The data indicates that students know that having as well as using strategies leads to success with multiplication and division, they enjoy playing math games, strongly believe the games help them learn, and feel that timed tests provoke anxiety.

Theme 1: Strategies

The first theme that stood out from the focus students was that students expressed that having strategies and using them to solve enables you to be successful in all multiplication and division scenarios. When discussing what makes someone *good* at multiplication or division, the word or theme of *strategy* was repeatedly used by students. For example, Leon, a member of the high group, said that good mathematicians "don't need to memorize facts, they need to remember strategies." Anthony and Henry (also members of the high group?) added that not only *knowing* the strategies are important but it is about *using* them. Anthony continues, "eventually you just end up remembering some." This aligns with research claims of Caron (2007) when he makes an analogy between knowing family and knowing math facts, he asked the reader if they had to memorize the name of their loved ones. He suggests instead, "through interaction and

meaningful engagement over time, they have become an undeniable part of your long-term memory" (Caron, 2007, p. 280). These students are sharing that through meaningful practice with strategies, they do end up remembering some answers to given problems, taking these "derived facts" and then achieving mastery (Kling & Bay-Williams, 2015). The others in the high group had agreeing answers sharing that you need to know the strategies and practice them to become good at multiplication and division. Aaron is proud of his use of strategies sharing, "my mom was impressed with me because I had a quick strategy to solve 15X5. I did 15X10= 150 so half of that is 75. I know X5 are half ten facts."

The average focus group had similar responses. While Carter stated that you need to use what you know to help you solve a new problem, Will included in his response that you have to have different ways to solve. Emma shared that efficiency in strategy usage was important. She explained, "having a variety of known strategies will help you choose an efficient strategy for a problem." Anna shared, "you need to be able to not just memorize the answers but also prove you answers and explain your thinking." Gretchen emphasized the word 'how' when saying "you actually need to know *HOW* to get the answer." These students agreed that just knowing a select few answers to given problems is not enough. You need to have several strategies, remember to use them, and understand how they work. Later on in life this will be useful because they will be equipped with strategies to solve any given problem, and think critically rather than having to pull out a calculator when they are posed with a challenging problem.

Once again, the low group talked about strategies being important to be good at multiplying and dividing. Their answers really focused on using what you know to help

you. Nick, Gabby, and Addie were all in agreement when sharing that we need to use what we know to help solve harder problems. Makayla said "we need to think of what other math problems we can do to help us get to the answer." Makayla's response supports the idea of Number Talks or Number Strings which were mentioned in Section Two. Our daily math program often calls for these meaningful conversations about a string of problems that have a pattern. We call it Problem Strings in our classroom setting. This conversation helps students recognize patterns and build strategies for solving. (Bray & Maldonado, 2018) share that "repeated engagement in such discussions overtime does promote student reasoning about relationships, which in turn leads to increased fact fluency," (p.92). What Makayla is describing in thinking back to those whole group Number Talks, and being able to that independently in her head. Finally, Lynn shared that once you use these strategies over and over, you really do start to remember some of the answers. This approach to learning multiplication and division equips students with strategies to confidently use on any given problem, while still allowing them to learn some facts by heart. The high, average, and low group were all in agreement that strategies are important. Most of the focus students actually used the word *strategy* in their discussion with me, but some even added that having strategies includes using what you know in order to help you solve the unknown. They are not forced to memorize a given set of facts, that may later be lost from memory. Instead they are using meaningful strategies and practicing manipulating numbers to better encode the content in their brains. This theme presented by the participants in this study indicated that using strategies was crucial to their thinking and mastery of facts, which aligns with research found in Kling and Bay-Williams *Three Steps to Mastering Multiplication Facts.* Kling

and Bay-Williams claim "students make more rapid gains in fact mastery when emphasis is placed on strategic thinking" (National Research Council, 2001; Cook & Dossey, 1982; Heege, 1985; Thornton, 1978 as cited in Kling & Bay-Williams, 2015, p. 551).

Theme 2 and 3: Enjoyable and helpful

The second and third theme that stood out went hand-in-hand. The students truly enjoyed playing the math games and felt they were helpful in learning multiplication and division. Repeatedly I heard from students these games were so fun *and* it helped them learn! Whether it be the game of "War" with the Visual Math Cards, or the Workplace Games, students always talked about how enjoyable these activities were. While observing students I also saw engagement and on task behavior, which tells me they like what they were involved in that the time.

The high group started off by saying how fun the games were. Mandy shared that she was able to practice important strategies though the games. The games were a great time to practice what she was learning and that helped her become better at multiplication and division. Leon described it as "fun and creative. They helped me learn not to count on my fingers. Also, the games were fun because you take math and turn it into a game. They could even be played outside of school!" Students wanting to continue to play the game even when not required to is a good indication that it was an enjoyable experience for students. Anthony shared, "the games were fun and good practice. I learned more strategies and got faster at solving." Aaron expressed that the games were helpful in making him feel more confident.

Likewise, the average group reported positive attitudes towards the games. Anna shared, "It was AWESOME because it's a *game* and you're *learning,* so that's fun!"

Kling (2015) asserts that using games to practice multiplication facts are not only relevant but enjoyable experiences, that avoid the unneeded stress of timed tests, but instead involve students in several opportunities to calculate efficiently and practice math. All six of the students in the average group used the words *fun* and *helpful* as part of their response in describing their experience with the games. All four games were well liked. Nina said, "I really like the games because they make math easy and I get more efficient by practicing and playing the games." Gretchen stated "I use multiplication to help me with division. In Scout them Out the multiplication was helpful to understanding fact families and [therefore] the division part of the game. Also, using multiplication in Division Capture and Line Em Up helped me get better at division." Will added on that having games with arrays (Cover Up and Line Em Up) was helpful "because the arrays were a good visual to helping me understand multiplying and dividing."

When asking the low group about their experience playing the games, Addie reported "they helped me think out problems and the activities were fun." Lynn also shared, "they were enjoyable and helped me learn even though some were challenging. I used strategies to help solve the harder ones." Lee talked about the use of strategies in the game when saying, "the games were fun because if you aren't good at the multiplication fact, you can learn more about the strategy, and with the Visual Math Cards, the pictures help you too. Gabby shares a bit on confidence when saying, "I would have never thought I could solve any of these problems without the strategies. I thought about those strategies when playing the games. That was helpful." All students in the low group reported liking the games, however they appeared to enjoy the multiplication games more than the division games. After observing these students' work, and interacting with them

during the games, I can infer that the multiplication games were more enjoyable than division games because they were in the students' Zone of Proximal Development. The division games may have been easier, more enjoyable, and a more effective method of distributed practice if they had been implemented later on for this group of students. Yet, even when the math was challenging, students enjoyed the games. For example, Addie shared, "Scout Them Out was especially fun even though it was challenging, because it was repetitive. It was a fun way to practice strategies."

Theme 4: Anxiety in timed tests

Finally, based on student responses as well as observations I note that most students have exhibited feelings of stress and anxiety with timed tests. The high group still expressed some feelings of nervousness about timed tests, but displayed more confidence in their abilities. For example, when asked how he felt about time tests, Anthony reported, "I feel okay. I don't like tests but I like math." Aaron claimed, "sometimes I panic and try to calm myself down. When I hear how much time there is, I rush." Mandy explained, "some are easy and some are hard. I just do the easy ones first and sometimes the hard ones are more stressful."

Henry and Leon, the highest students in the focus group, differed slightly in their opinion of timed tests, reporting that they may be more focused on answering the given problems if they were timed. Leon, however, added that timed tests do make him feel rushed anyway.

Students in the average group also expressed feeling stressed when timed especially with division, however some acknowledged that they also have to try their best if they really were to be asked to complete a timed test. Carter replied, "that's a lot of

math… it would stress me out. Even though I would still try my best, I don't like them." Emma added, "I would do better if I am not timed, so I know I don't have to rush. Then I could take my time on each problem and really think about it. Timed tests are stressful but I know if I have one I just have to focus on math."

When asked how they felt about timed tests and their ability to complete an assessment if it is timed versus not timed, all six students in the low focus group shared experiences such as feeling rushed, stressed, and not enough time to think. Addie shared, "If I have a long time to complete the problems I can be relaxed. If I am timed I would rush and may misread the problems." Each of the others agreed sharing very similar responses. Makayla added on some interesting information when responding to this question. She claimed, not only would she "do worse if I am timed because I can't focus and think," but also, "I would feel sad because I wouldn't get to play math games we have played."

Regardless of their ability level, there seems to be some degree of stress and anxiety associated with timed tests. Lower students have strong opinions, while average students seem to accept what they are asked to do and make the most of it. The higher students know they have the ability, so try to stay calm, and give it their best shot to show their achievement.

SECTION FIVE

Conclusion

Research suggests that timed tests are not the answer for boosting multiplication and division fluency in third grade. Ohio State Standards for mathematics also explain that fluency does not imply timed tests. Instead, we define fluency as, "the ability to use efficient, accurate, and flexible methods for computing," (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 92). Fluency in this study was assessed with validity, measuring more than just accuracy in a short time. It considered a student's ability to work efficiently, while choosing and explaining an appropriate strategy and flexibly thinking about numbers when computing.

Given that we know timed tests are not a valid or complete measure of students' fluency ability, students had the ability to turn to learned strategies to help solve given problems and explain their thinking. Furthermore, participants report to be much less stressed when not timed, enjoy their learning, and honestly want to interact with multiplication and division tasks. When analyzing student strategies during instructional time, I observed students implementing additional efficient strategies and moved away from counting by ones in their work. Being that these interventions did not have an adverse effect on student's accuracy measures, and the definition of fluency encompasses more than just accuracy, I do not see an issue with removing timed tests from the curriculum and replacing them with the activities and focus group assessments tried in this study.

Fluency was intended to be boosted with the use of math games and visual math cards. Students engaged in enjoyable distributed practice of the strategies taught in class

to improve on their fluency ability. Trends appeared proving that students often get overwhelmed and anxious when it comes to timed tests. Instead, they prefer distributed practice with game-like activities when learning math. They explained they need time to think and process their answers. Rushing will not help them succeed. They feel that they have learned and become better at multiplication and division over the year; some have even reported that they do start to remember the facts by heart through these experiences without ever having to memorize them.

Clearly avoiding timed tests has not negatively affected mastery of multiplication accuracy, efficiency, appropriate strategy usage, and flexibility with number in this group of students. Students also had the luxury of being much less stressed, and enjoying their learning. They wanted to engage in multiplication and tasks.

Data from the Post-Assessment, verbal assessment, rating scales, and interviews shows that students did in fact learn to multiply and divide. Moreover, the assessment used was a more valid measure of what fluency really means. It did not matter if the Visual Math Cards or Work Place Games occurred first, however all students did make great growth with strategy usage and efficiency over the two-week period. Both activities did seem to support fact fluency. These two activities lead to students having positive feelings towards math tasks and boosted fluency ability, including accuracy, in math as defined by state standards.

Implications/ Limitations

Over the course of this research several events occurred that may have affected results. There were three consecutive inclement weather days at our school which cut the intervention time short. Additionally, one of our students went through a traumatic life

event, which in turn affected all participants. All participants lost math instruction for one day. Also, this child, in Class B was not as focused for the duration of the study. Classmates in either class could have been affected more long term as well with the news affecting their classmate and friend. Each intervention was intended to last ten school days. However, which these events out of our control, the interventions each lasted only five school days. If this study were to be done again, additional time for the interventions would provide students with additional distributed practice and more incubation time for the learning to settle in.

If a follow-up study were to be completed, I wonder if these student participants would do better on problems above third grade standards. Students who have memorized the one hundred facts they need to be *fluent* with in third grade may only know those given facts. I wonder if these participants would be further equipped to complete multiplication and division problems in their future that go beyond the third-grade standards.

Dissemination

Results will be delivered to the committee of capstone reviewers through a presentation on my findings. Also, it will be submitted for electronic publication in the Otterbein Digital Commons. My findings will also be shared with my grade level team, the curriculum director and administrators of my building. It is my intention to share these findings and make an impact on the instructional practice and assessment plans of other teachers. Making sure students are learning to multiply and divide fluently today and in the future is my ultimate goal. In order for students to become proficient, appropriate instruction needs to be planned and valid assessment needs to be created.

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Appendix A

Name:

Accuracy:		
20/4=		I just knew
		I used this strategy:_____
7X3=		I just knew
		I used this strategy:_____
18/2=		I just knew
		I used this strategy:_____
6X8=		I just knew
		I used this strategy:_____
18/3=		I just knew
		I used this strategy:_____
4X8=		I just knew
		I used this strategy:_____
Appropriate strategy: A friend is having trouble solving 54/9. What would you tell him/ her to do in order to solve?		
Efficiency: How can you use the answer to 6X6 to help you find the answer to 7X6?		
Flexibility: Solve 3X4 one way. Now try solving it a different way.		

Appendix B

Work Place Guide 2C Cover Up

Summary

Players take turns spinning two numbers, drawing an array with those dimensions on a 10-by-10 grid, and finding the product represented by the array (total area of the array). Each player takes four turns and then finds the total of their four products. The player whose total is closest to 100 wins.

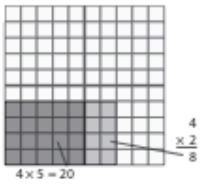
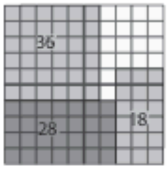
Skills & Concepts

- Interpret products of whole numbers (3.OA.1)
- Apply properties of operations as strategies to multiply (3.OA.5)
- Fluently multiply with products to 100 using strategies (3.OA.7)
- Use strategies based on place value, properties of operations, or the relationship between addition and subtraction to add fluently with sums to 1,000 (3.NBT.2)
- Represent the product of two numbers as the area of a rectangle with side lengths equal to those two numbers, and find the area of the rectangle by multiplying the side lengths (3.MD.7b)
- Use the area model for multiplication to illustrate the distributive property (3.MD.7c)
- Use and explain additive strategies (e.g., repeated addition and skip-counting) to demonstrate an understanding of multiplication (supports 3.OA)

Materials

Copies	Kit Materials	Classroom Materials
TMT8 Work Place Guide 2C Cover Up **TMT9** 2C Cover Up Record Sheet **SB 55** Work Place Instructions 2C Cover Up	• 3 spinner overlays	• colored pencils or markers in 2 different colors

Assessment & Differentiation

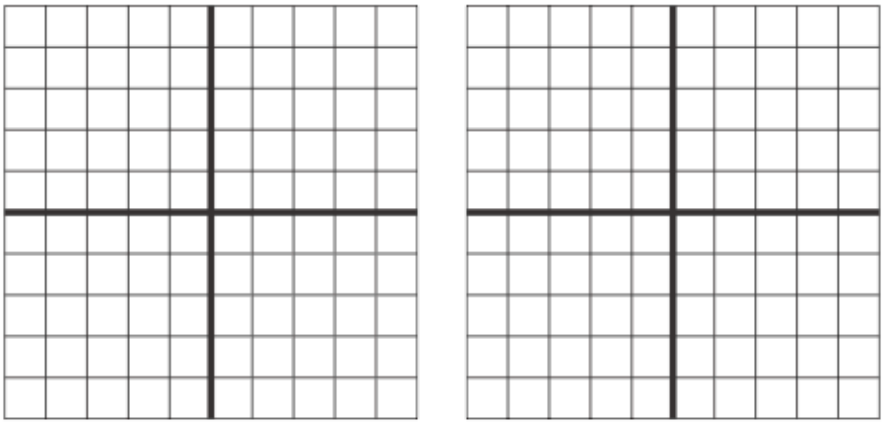
If you see that...	Differentiate	Example
Students are struggling to find the products.	**SUPPORT** Helps students use the structure of the arrays to help find the products. Can they locate a fact they do know in the array and use it to find the total product? You won't want to encourage students to count the squares in the arrays by 1s, but using the facts they do know and the arrays they do recognize can help them develop mental strategies that contribute to fluency with the multiplication facts.	 You would not want to encourage students to count the 28 squares in this array one-by-one. However, breaking the array into a 4-by-5 and a 4-by-2 array results in two easier products, the sum of which is the product of 4 × 7: 28.
Students are struggling to add many numbers.	**SUPPORT** Review addition strategies with these students. Encourage them to use combinations of 10 or friendly numbers. For example, when adding 12 + 16 + 15 + 24, they could add 16 + 24 to get 40. Then 12 + 16 = 28. Then 28 + 40 = 68.	
Students can efficiently multiply single-digit numbers.	**CHALLENGE** Invite students to draw either the array with the exact dimensions they spun, or an array with the same product by different dimensions. This can help students fit an array on their grid that otherwise might not fit and gives them the opportunity to consider a variety of multiplication and division facts.	 At this point in the game, if a student spun a 6 and a 2, they would not be able to fit a 6-by-2 array in the space available on the grid. They could, however, fill in a 3-by-4 or 4-by-3 array to represent that product of 12.
English-Language Learners Use the following adaptations to support the ELL students in your classroom.		
Encourage ELL students to play with same-language peers in their own language.		

Appendix C

NAME | DATE

2C Cover Up Record Sheet

Player 1 _____ Player 2 _____



First Array _____ First Array _____

Second Array _____ Second Array _____

Third Array _____ Third Array _____

Fourth Array _____ Fourth Array _____

Total Total

Appendix D

Unit 5 Module 2 | **Session 2** *1 copy kept in a clear plastic sleeve and stored in the Work Place bin*

Work Place Guide 5B Scout Them Out

Summary

In this activity for individuals, the student chooses one of the eight Scout Them Out Sheets to complete. Each sheet features two of the multiplication strategies that were introduced in Unit 2. The student finds all the multiplication facts on the sheet that match the first strategy, circles them in blue, and writes the answers to those facts in regular pencil. Then she identifies all of the multiplication facts on the sheet that match the second strategy, circles them in red, and writes the answers. Finally, she solves the division problems on the lower part of the sheet, using the answers to the multiplication problems above to help.

Skills & Concepts

- Solve division problems by finding an unknown factor (e.g., solve 32 ÷ 8 by finding the number that makes 32 when multiplied by 8) (3.OA.6)
- Fluently multiply and divide with products and dividends to 100 using strategies (3.OA.7)
- Recall from memory all products of two 1-digit numbers (3.OA.7)

Materials

Copies	Kit Materials	Classroom Materials
TM T3 Work Place Guide 5B Scout Them Out **TM T4–11** 5B Scout Them Out Multiplication & Division A-H **SB 154** Work Place Instructions 5B Scout Them Out	• colored tiles • red linear units	

Assessment & Differentiation

Here are some quick observational assessments you can make as students begin to do this activity on their own. Use the results to differentiate as needed.

If you see that…	Differentiate	Example
Students are struggling to apply the multiplication strategies.	SUPPORT Consider having these students practice the same sheet more than once, and invite them to skip the division practice on the lower part of the sheet at this time. SUPPORT Meet with these students in small groups, and review the strategies. Have them refer back to the Multiplication Table Student Book page they filled in during Unit 2, and consider revisiting some of the activities from Unit 2, Module 3, Sessions 3 and 4 by way of review.	
Students are making lots of mistakes on the sheets.	SUPPORT Ask these students to check their own work using the Multiplication Table Student Book page they completed during Unit 2, Module 3. Or, have students trade sheets and check each others' work. If and when students or their classmates discover errors on their sheets, ask that they correct them on the spot.	
Students are able to identify the fact strategies and apply them easily.	CHALLENGE Ask these students to complete more than 1 of the 8 sheets each time they visit the Work Place so they will have finished all of them before this activity is replaced by another.	
English-Language Learners Use the following adaptations to support the ELL students in your classroom.		
• Refer students back to the Multiplication Table Student Book page they completed during Unit 2, Module 3, to provide a visual reference for the strategy names.		

Appendix E

NAME | DATE

5B Scout Them Out Multiplication & Division A

Multiplication

1 Circle all the Doubles facts (×2) in blue. Then go back and do them.

2 Circle all the Half-Tens facts (×5) in red. Then go back and do them.

2 × 10	5 × 2	2 × 2	5 × 7	3 × 5	5 × 0
9 × 5	8 × 2	1 × 2	8 × 5	5 × 10	2 × 7
5 × 4	4 × 2	6 × 5	2 × 6	2 × 4	9 × 2

Division

3 Now solve the division problems below. Use the multiplication facts above to help.

18 ÷ 2 = _____	10 ÷ 2 = _____	6 ÷ 2 = _____	30 ÷ 5 = _____
14 ÷ 2 = _____	15 ÷ 5 = _____	20 ÷ 5 = _____	16 ÷ 2 = _____
2 ÷ 2 = _____	40 ÷ 5 = _____	50 ÷ 5 = _____	8 ÷ 2 = _____

Appendix F

NAME | DATE

5B Scout Them Out Multiplication & Division B

Multiplication

1 Circle all the Half-Tens facts (×5) in blue. Then go back and do them.

2 Circle all the Tens facts (×10) in red. Then go back and do them.

9 × 10	7 × 5	10 × 4	9 × 5	5 × 5	6 × 10
8 × 5	5 × 3	10 × 10	10 × 8	4 × 5	6 × 10
3 × 10	5 × 7	4 × 10	5 × 10	8 × 10	7 × 10

Division

3 Now solve the division problems below. Use the multiplication facts above to help.

80 ÷ 10 = _____	40 ÷ 10 = _____	70 ÷ 10 = _____	30 ÷ 5 = _____
45 ÷ 5 = _____	100 ÷ 10 = _____	40 ÷ 5 = _____	30 ÷ 10 = _____
25 ÷ 5 = _____	80 ÷ 10 = _____	10 ÷ 5 = _____	60 ÷ 10 = _____

Appendix G

NAME | DATE

5B Scout Them Out Multiplication & Division C

Multiplication

1 Circle all the Doubles facts (×2) in blue. Then go back and do them.

2 Circle all the Double-Doubles facts (×4) in red. Then go back and do them.

2 × 6	10 × 4	2 × 3	8 × 4	2 × 8	4 × 7
6 × 4	4 × 3	2 × 7	4 × 4	2 × 0	9 × 4
2 × 9	5 × 2	10 × 2	5 × 4	4 × 2	9 × 2

Division

3 Now solve the division problems below. Use the multiplication facts above to help.

8 ÷ 2 = _____	18 ÷ 2 = _____	20 ÷ 4 = _____	4 ÷ 4 = _____
40 ÷ 4 = _____	12 ÷ 2 = _____	16 ÷ 4 = _____	6 ÷ 2 = _____
32 ÷ 4 = _____	12 ÷ 4 = _____	20 ÷ 2 = _____	36 ÷ 4 = _____

Appendix H

NAME | DATE

5B Scout Them Out Multiplication & Division D

Multiplication

1 Circle all the Double-Doubles facts (×4) in blue. Then go back and do them.

2 Circle all the Double-Double-Doubles facts (×8) in red. Then go back and do them.

9 × 4	8 × 2	4 × 3	8 × 4	8 × 9	8 × 3
8 × 6	10 × 4	6 × 4	0 × 4	8 × 8	10 × 8
9 × 8	5 × 8	4 × 7	5 × 4	8 × 7	4 × 4

Division

3 Now solve the division problems below. Use the multiplication facts above to help.

56 ÷ 8 = _____	40 ÷ 4 = _____	8 ÷ 8 = _____	64 ÷ 8 = _____
36 ÷ 4 = _____	24 ÷ 4 = _____	24 ÷ 8 = _____	32 ÷ 4 = _____
16 ÷ 4 = _____	8 ÷ 4 = _____	16 ÷ 8 = _____	12 ÷ 4 = _____

Appendix I

NAME | DATE

5B Scout Them Out Multiplication & Division E

Multiplication

1 Circle all the Half-Tens facts (×5) in blue. Then go back and do them.

2 Circle all the Half-Tens Plus One Set facts (×6) in red. Then go back and do them.

5 × 3	2 × 5	6 × 3	10 × 6	6 × 6	6 × 9
7 × 6	5 × 8	5 × 5	6 × 2	7 × 5	6 × 5
6 × 0	10 × 5	9 × 5	5 × 4	6 × 8	6 × 4

Division

3 Now solve the division problems below. Use the multiplication facts above to help.

45 ÷ 5 = _____	15 ÷ 5 = _____	18 ÷ 6 = _____	50 ÷ 5 = _____
10 ÷ 5 = _____	6 ÷ 6 = _____	25 ÷ 5 = _____	40 ÷ 5 = _____
42 ÷ 6 = _____	12 ÷ 6 = _____	24 ÷ 6 = _____	35 ÷ 5 = _____

Appendix J

Unit 5 Module 2 | **Session 2** *half-class set, plus more as needed, stored in the Work Place bin*

NAME | DATE

5B Scout Them Out Multiplication & Division F

Multiplication

1 Circle all the Doubles facts (×2) in blue. Then go back and do them.

2 Circle all the Doubles Plus One Set facts (×3) in red. Then go back and do them.

2 × 4	3 × 5	3 × 3	9 × 2	3 × 8	2 × 8
7 × 2	3 × 0	3 × 4	7 × 3	6 × 2	2 × 9
3 × 2	2 × 10	1 × 3	6 × 3	2 × 5	9 × 3

Division

3 Now solve the division problems below. Use the multiplication facts above to help.

9 ÷ 3 = _____	27 ÷ 3 = _____	6 ÷ 3 = _____	18 ÷ 3 = _____
12 ÷ 3 = _____	2 ÷ 2 = _____	20 ÷ 2 = _____	12 ÷ 2 = _____
21 ÷ 3 = _____	14 ÷ 2 = _____	18 ÷ 2 = _____	4 ÷ 2 = _____

Appendix K

Unit 5 Module 2 | **Session 2** *half-class set, plus more as needed, stored in the Work Place bin*

NAME | DATE

5B Scout Them Out Multiplication & Division G

Multiplication

1 Circle all the Tens facts (×10) in blue. Then go back and do them.

2 Circle all the Tens Minus One Set facts (×9) in red. Then go back and do them.

10 × 8	10 × 10	9 × 4	9 × 9	10 × 6	9 × 6
9 × 3	7 × 10	9 × 7	9 × 2	9 × 5	9 × 10
4 × 10	10 × 1	5 × 10	10 × 2	9 × 8	3 × 10

Division

3 Solve the following division problems if you like. Can you use what you know about multiplication to help?

100 ÷ 10 = _____	45 ÷ 9 = _____	63 ÷ 9 = _____	80 ÷ 10 = _____
9 ÷ 9 = _____	18 ÷ 9 = _____	72 ÷ 9 = _____	70 ÷ 10 = _____
50 ÷ 10 = _____	20 ÷ 10 = _____	27 ÷ 9 = _____	54 ÷ 9 = _____

Appendix L

NAME | DATE

5B Scout Them Out Multiplication & Division H

Multiplication

1 Circle all the Double-Double-Doubles facts (×8) in blue. Then go back and do them.

2 Circle all the square facts (like 6 × 6 and 4 × 4) in red. Then go back and do them.

10 × 10	3 × 3	8 × 3	8 × 6	8 × 8	10 × 8
8 × 2	9 × 9	1 × 1	6 × 6	8 × 4	5 × 5
4 × 4	7 × 8	5 × 8	7 × 7	8 × 9	2 × 2

Division

3 Solve the following division problems if you like. Can you use what you know about multiplication to help?

24 ÷ 8 = _____	48 ÷ 8 = _____	49 ÷ 7 = _____	40 ÷ 8 = _____
72 ÷ 8 = _____	9 ÷ 3 = _____	36 ÷ 6 = _____	100 ÷ 10 = _____
16 ÷ 4 = _____	56 ÷ 8 = _____	81 ÷ 9 = _____	16 ÷ 8 = _____

Appendix M

Work Place Guide 5D Division Capture

Summary

Players spin to determine who goes first as well as whether to play for red or blue. Then players take turns spinning for a number they can use to complete one of 20 division combinations on a grid. Each partner uses a different color to write their numbers on the grid. Once all the equations are completed, players look for and circle in their own color any equations they completed that fall in a row, either vertically, horizontally, or diagonally. Each player earns a point for any three equations in a row they completed and 2 points for any four equations in a row they completed. The player with the higher score wins the game.

Skills & Concepts

- Solve division problems by finding an unknown factor (3.OA.6)
- Fluently divide with dividends to 100 using strategies (3.OA.7)

Materials

Copies	Kit Materials	Classroom Materials
TMT7 Work Place Guide 5D Division Capture **TMT8–12** 5D Division Capture Record Sheets A–S **SB 172–173** Work Place Instructions 5D Division Capture	• 3 clear spinner overlays • colored tiles • red linear units	• students' completed Multiplication Tables Student Book pages (from Unit 2, Module 3, Session 1) • 6 colored pencils, 3 in red and 3 in blue

Assessment & Differentiation

If you see that…	Differentiate	Example
Students are struggling with basic division facts, either because they don't understand the operation or because they haven't developed enough fluency with multiplication to be able to deal with the related division facts.	SUPPORT Encourage students to use colored tiles and red linear pieces to model and solve the combinations. This may be quite tedious, and you might consider having these students spend more time with Work Places 5B Scout Them Out, or 5C Line 'Em Up before they do too much with this Work Place. SUPPORT Allow students working at roughly the same level to use the same record sheet for several times running before they move on to the next. The 5 record sheets go in order from easiest to most challenging, so encourage these students to work them in order rather than skipping around. SUPPORT Invite all your students, not just those who are struggling, to use the Multiplication Table Student Book page they completed during Unit 2, Module 3, for help in solving division combinations.	Some pairs of students may benefit from playing the game several times with Record Sheet 1, which features division facts for 2s and 10s, before moving on to Record Sheet 2. ***Teacher*** *Looks like you're stuck on 48 ÷ 6. Can you find a fact on your Multiplication Table that would help you solve that problem?* ***Student*** *Umm…well, I see the answer 48 right here, and it says 6 × 8 is 48.* ***Teacher*** *How can you use that information to help with your division problem?* ***Student*** *If 6 × 8 is 48, then if I divide 48 by 6, the answer must be 8.*
Students are struggling to find the division combinations they need on the grid.	SUPPORT Pair students who are working at roughly the same level, and invite them to use Game Variation B. Players fill in the answers to all of the division combinations on the grid before they start playing the game. This simplifies the game because they don't have to search for the two combinations on the sheet that can be solved with any number they spin; they have only to find and circle the numbers they've already entered on the grid.	Encourage these students to use the Multiplication Table Student Book page they completed during Unit 2, Module 3, for help in finding the answers to the division combinations as they work together to fill in the grid. This will help reinforce the connection between multiplication and division facts.
Students are working easily with division facts.	CHALLENGE Pair students who are working at roughly the same level. Invite them to choose the order in which they use the sheets and possibly skip those sheets that feature facts with which they're already fluent.	

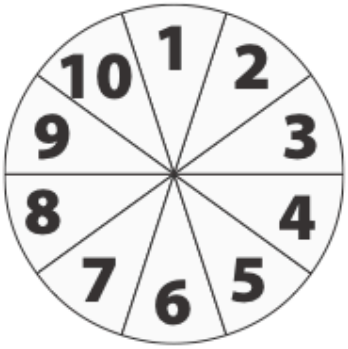
English-Language Learners Use the following adaptations to support the ELL students in your classroom.

- Play this game with small groups of ELL students, modeling how to play and what to do. Take the opportunity to reinforce the terms *divide*, *division*, and *quotient* as you work with the students.
- Encourage ELL students to play with same-language peers in their own language.

Appendix N

Unit 5 Module 3 | **Session 4** *half-class set, plus 1 copy for display*

5D Division Capture Record Sheet A, Twos & Tens

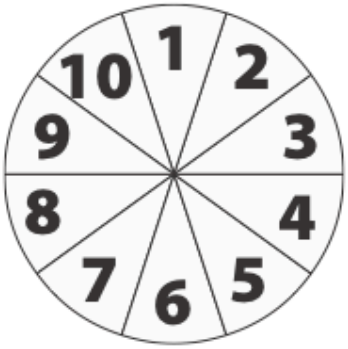


Red _____ Blue _____

100 ÷ 10	60 ÷ 10	80 ÷ 10	12 ÷ 2	50 ÷ 10	**Scoring** 3 in a row = 1 point 4 in a row = 2 points	
16 ÷ 2	10 ÷ 10	6 ÷ 2	70 ÷ 10	18 ÷ 2		
4 ÷ 2	20 ÷ 10	90 ÷ 10	40 ÷ 10	8 ÷ 2	**Red**	**Blue**
14 ÷ 2	30 ÷ 10	20 ÷ 2	10 ÷ 2	2 ÷ 2		

Appendix O

5D Division Capture Record Sheet B, Threes & Twos



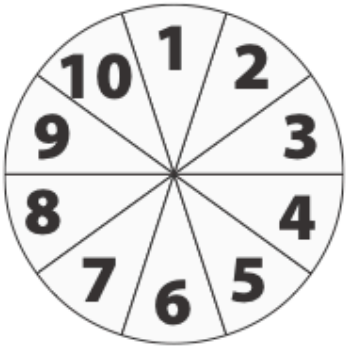
Red _____ Blue _____

21 ÷ 3	8 ÷ 2	3 ÷ 3	18 ÷ 2	12 ÷ 3	**Scoring** 3 in a row = 1 point 4 in a row = 2 points	
16 ÷ 2	24 ÷ 3	6 ÷ 2	9 ÷ 3	20 ÷ 2		
18 ÷ 3	4 ÷ 2	6 ÷ 3	10 ÷ 2	30 ÷ 3	**Red**	**Blue**
14 ÷ 2	15 ÷ 3	2 ÷ 2	27 ÷ 3	12 ÷ 2		

Appendix P

Unit 5 Module 3 | **Session 4** *half-class set, plus more as needed, stored in the Work Place bin*

5D Division Capture Record Sheet C, Fours & Twos

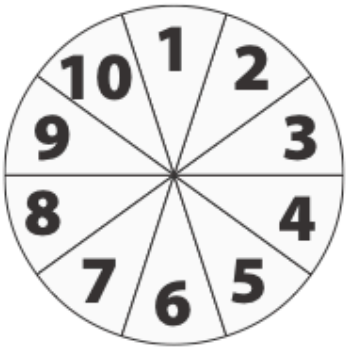


Red _____ Blue _____

12 ÷ 4	36 ÷ 4	8 ÷ 4	12 ÷ 2	28 ÷ 4	**Scoring** 3 in a row = 1 point 4 in a row = 2 points	
16 ÷ 2	16 ÷ 4	6 ÷ 2	32 ÷ 4	18 ÷ 2		
4 ÷ 2	40 ÷ 4	20 ÷ 2	20 ÷ 4	8 ÷ 2	**Red**	**Blue**
14 ÷ 2	24 ÷ 4	4 ÷ 4	10 ÷ 2	2 ÷ 2		

Appendix Q

5D Division Capture Record Sheet D, Fives & Tens



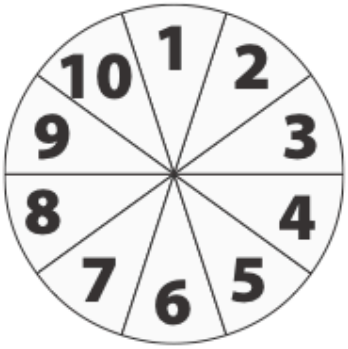
Red _____ Blue _____

15 ÷ 5	50 ÷ 10	10 ÷ 5	60 ÷ 10	35 ÷ 5	**Scoring** 3 in a row = 1 point 4 in a row = 2 points	
80 ÷ 10	20 ÷ 5	30 ÷ 10	5 ÷ 5	90 ÷ 10		
20 ÷ 10	40 ÷ 5	100 ÷ 10	30 ÷ 5	40 ÷ 10	**Red**	**Blue**
70 ÷ 10	25 ÷ 5	45 ÷ 5	50 ÷ 5	10 ÷ 10		

Appendix R

Unit 5 Module 3 | **Session 4** *half-class set, plus more as needed, stored in the Work Place bin*

5D Division Capture Record Sheet E, Sixes & Fives



Red _____ Blue _____

15 ÷ 5	50 ÷ 5	10 ÷ 5	36 ÷ 6	35 ÷ 5	**Scoring** 3 in a row = 1 point 4 in a row = 2 points	
48 ÷ 6	20 ÷ 5	18 ÷ 6	5 ÷ 5	54 ÷ 6		
12 ÷ 6	40 ÷ 5	60 ÷ 6	30 ÷ 5	24 ÷ 6	**Red**	**Blue**
42 ÷ 6	25 ÷ 5	45 ÷ 5	30 ÷ 6	6 ÷ 6		

Appendix S

Work Place Guide 5C Line 'Em Up

Summary

The game Line 'Em Up is based on the story *One Hundred Hungry Ants* (which some students may remember from Grade 2 Bridges). Each player rolls two dice, multiplies the two numbers, and then divides that number of bugs (modeled with tiles) into 2, 3, 4, 5, and finally 6 rows. Players record the results of each division, including any remainders. At the end of the game, both players add up their remainders and the player with the greater sum wins.

Skills & Concepts

- Interpret quotients of whole numbers (3.OA.2)
- Write story problems or describe problem situations to match a division expression or equation (3.OA.2)
- Fluently multiply and divide with products and dividends to 100 using strategies (3.OA.7)

Materials

Copies	Kit Materials	Classroom Materials
TM T4 Work Place Guide 5C Line 'Em Up **TM T5** 5C Line 'Em Up Record Sheet **SB 169–170** Work Place Instructions 5C Line 'Em Up	• 300 colored tiles • 36 red linear units • three 1–6 dice • three 4–9 dice • base 10 area and linear pieces for Game Variation B	

Assessment & Differentiation

If you see that...	Differentiate	Example
Students are struggling to accurately multiply the two numbers they roll, or struggling to carry out the division.	SUPPORT Have two students who are working at about the same level use two dice numbered 1–6 instead of one numbered 1–6 and one numbered 4–9. Have them build an array to model and solve the multiplication combination rolled, and then divide the resulting collection of tiles into 2, 3, 4, 5, and 6 lines. SUPPORT Insist that students support each other in taking their turns, rather than working separately (see note below).	*Teacher* *I see you rolled a 4 and a 5. Can you build a tile array to model and solve 4 × 5?* *Student* *I don't really know what that means.* *Teacher* *Let's frame it with the red linear pieces., 4 going up the side, and 5 across the top. When you fill that in, how many tiles do you think you'll have in all?* *Student* *I don't know, but it's going to have 4 rows, and 5 in every row.* *Teacher* *Why don't you build that and call me back to see your work when you're done?* 
Students are quite proficient with multiplication facts, and understand the operation of division well.	CHALLENGE Invite these students to try Game Variation B, in which they'll use 2 dice numbered 4–9. At this level, they'll be working with division facts as high as 81 ÷ 2 and should have access to base 10 area and linear pieces to model and solve the larger combinations.	
English-Language Learners Use the following adaptations to support the ELL students in your classroom.		
• Play this game with small groups of ELL students, modeling how to play and what to do. Take the opportunity to reinforce the terms *multiply, divide, division, dividend, divisor,* and *quotient* as you play. • Encourage ELL students to play with same-language peers in their own language.		

Note One of the most important features of this game is the cooperation required between partners, no matter what their skill level. While it might seem more efficient to have both students make their rolls and do their divisions at the same time, it is very valuable, both in terms of insuring accuracy and promoting dialog, to have pairs work together to make sure the tiles are counted out and divided correctly.

Appendix T

NAME | DATE

5C Line 'Em Up Record Sheet

Player 1 _____ Player 2 _____

Round 1							
	÷ 2 =		R		÷ 2 =		R
	÷ 3 =		R		÷ 3 =		R
	÷ 4 =		R		÷ 4 =		R
	÷ 5 =		R		÷ 5 =		R
	÷ 6 =		R		÷ 6 =		R

Round 2							
	÷ 2 =		R		÷ 2 =		R
	÷ 3 =		R		÷ 3 =		R
	÷ 4 =		R		÷ 4 =		R
	÷ 5 =		R		÷ 5 =		R
	÷ 6 =		R		÷ 6 =		R

After you have played both rounds, add all of your remainders. The player with the higher total wins.

Total remainders for player 1 ☐ Total remainders for player 2 ☐

Appendix U

ALEKS Diagnostic Assessment Scores for Focus Group

Name	Class	Level	Percentage of Multiplication and Division Facts within 100 known
Henry	Class A	High	67%
Anthony	Class A	High	82%
Leon	Class A	High	57%
Aaron	Class B	High	62%
Mandy	Class B	High	40%
Ivan	Class B	High	49%
Nina	Class A	Average	31%
Will	Class A	Average	47%
Gretchen	Class A	Average	30%
Carter	Class B	Average	23%
Emma	Class B	Average	33%
Anna	Class B	Average	38%
Lynn	Class A	Low	17%
Addie	Class A	Low	22%
Nick	Class A	Low	18%
Makayla	Class B	Low	4%
Lee	Class B	Low	10%
Gabby	Class B	Low	9%

Appendix V

Multiplication Table

×	0	1	2	3	4	5	6	7	8	9	10
0	0 × 0 0	0 × 1 0	0 × 2 0	0 × 3 0	0 × 4 0	0 × 5 0	0 × 6 0	0 × 7 0	0 × 8 0	0 × 9 0	0 × 10 0
1	1 × 0 0	1 × 1 1	1 × 2 2	1 × 3 3	1 × 4 4	1 × 5 5	1 × 6 6	1 × 7 7	1 × 8 8	1 × 9 9	1 × 10 10
2	2 × 0 0	2 × 1 2	2 × 2 4	2 × 3 6	2 × 4 8	2 × 5 10	2 × 6 12	2 × 7 14	2 × 8 16	2 × 9 18	2 × 10 20
3	3 × 0 0	3 × 1 3	3 × 2 6	3 × 3 9	3 × 4 12	3 × 5 15	3 × 6 18	3 × 7 21	3 × 8 24	3 × 9 27	3 × 10 30
4	4 × 0 0	4 × 1 4	4 × 2 8	4 × 3 12	4 × 4 16	4 × 5 20	4 × 6 24	4 × 7 28	4 × 8 32	4 × 9 36	4 × 10 40
5	5 × 0 0	5 × 1 5	5 × 2 10	5 × 3 15	5 × 4 20	5 × 5 25	5 × 6 30	5 × 7 35	5 × 8 40	5 × 9 45	5 × 10 50
6	6 × 0 0	6 × 1 6	6 × 2 12	6 × 3 18	6 × 4 24	6 × 5 30	6 × 6 36	6 × 7 42	6 × 8 48	6 × 9 54	6 × 10 60
7	7 × 0 0	7 × 1 7	7 × 2 14	7 × 3 21	7 × 4 28	7 × 5 35	7 × 6 42	7 × 7 49	7 × 8 56	7 × 9 63	7 × 10 70
8	8 × 0 0	8 × 1 8	8 × 2 16	8 × 3 24	8 × 4 32	8 × 5 40	8 × 6 48	8 × 7 56	8 × 8 64	8 × 9 72	8 × 10 80
9	9 × 0 0	9 × 1 9	9 × 2 18	9 × 3 27	9 × 4 36	9 × 5 45	9 × 6 54	9 × 7 63	9 × 8 72	9 × 9 81	9 × 10 90
10	10 × 0 0	10 × 1 10	10 × 2 20	10 × 3 30	10 × 4 40	10 × 5 50	10 × 6 60	10 × 7 70	10 × 8 80	10 × 9 90	10 × 10 100

- ☐ Zero facts (× 0)
- ☐ Ones facts (× 1)
- ☐ Doubles facts (× 2)
- ☐ Doubles Plus One Set facts (× 3)
- ☐ Double-Doubles facts (× 4)
- ☐ Half-Tens facts (× 5)
- ☐ Half-Tens Plus One Set facts (× 6)
- ☐ Double-Double-Doubles facts (× 8)
- ☐ Tens Minus One Set facts (× 9)
- ☐ Tens facts (× 10)

Appendix W

Complete Verbal Pre-Assessment Scores for Focus Group

Name	Class	Level	Accuracy Score	Efficiency	Strategy Score	Verbal Assessment Score	Total Score
Henry	Class A	High	6/6	0/6	6/6	3/3	15/21= 71%
Anthony	Class A	High	6/6	6/6	6/6	3/3	21/21= 100%
Leon	Class A	High	6/6	5/6	6/6	3/3	20/21= 95%
Aaron	Class B	High	6/6	6/6	6/6	3/3	21/21= 100%
Mandy	Class B	High	6/6	0/6	6/6	3/3	15/21= 71%
Ivan	Class B	High	6/6	5/6	5/6	3/3	19/21= 90%
Nina	Class A	Average	5/6	2/6	5/6	2/3	14/21= 67%
Will	Class A	Average	4/6	0/6	4/6	2/3	10/21= 48%
Gretchen	Class A	Average	5/6	0/6	5/6	2/3	12/21= 58%
Carter	Class B	Average	3/6	0/6	3/6	0/3	6/21= 29%
Emma	Class B	Average	5/6	2/6	5/6	3/3	15/21= 71%
Anna	Class B	Average	6/6	6/6	0/6	3/3	15/21= 71%
Lynn	Class A	Low	1/6	0/6	1/6	2/6	4/21= 19%
Addie	Class A	Low	2/6	0/6	2/6	1/3	5/21= 24%
Nick	Class A	Low	5/6	2/6	5/6	2/6	14/21= 67%
Makayla	Class B	Low	2/6	0/6	1/6	1/3	4/21= 19%
Lee	Class B	Low	2/6	1/6	1/6	2/3	6/21= 29%
Gabby	Class B	Low	4/6	1/6	3/6	3/3	11/21=52%

Appendix X

Complete Mid-Point Assessment for Focus Group

Name	Class	Level	Accuracy Score	Efficiency	Strategy Score	Verbal Assessment Score	Total Score	Percent Growth Since Pre-Assessment
Henry	Class A	High	6/6	5/6	6/6	3/3	20/21= 95%	24%
Anthony	Class A	High	6/6	6/6	6/6	3/3	21/21= 100%	0%
Leon	Class A	High	6/6	5/6	6/6	3/3	20/21= 95%	0%
Aaron	Class B	High	6/6	6/6	6/6	3/3	21/21= 100%	0%
Mandy	Class B	High	6/6	4/6	6/6	3/3	19/21= 90%	19%
Ivan	Class B	High	5/6	5/6	5/6	3/3	18/21= 86%	4%
Nina	Class A	Average	6/6	6/6	6/6	3/3	21/21= 100%	33%
Will	Class A	Average	6/6	6/6	6/6	3/3	21/21= 100%	52%
Gretchen	Class A	Average	6/6	2/6	6/6	3/3	17/21= 81%	23%
Carter	Class B	Average	5/6	4/6	5/6	2/3	17/21= 81%	52%
Emma	Class B	Average	6/6	3/6	6/6	3/3	18/21= 86%	15%
Anna	Class B	Average	6/6	6/6	6/6	3/3	21/21= 100%	29%
Lynn	Class A	Low	6/6	1/6	6/6	3/3	16/21= 76%	57%
Addie	Class A	Low	5/6	2/6	5/6	3/3	15/21= 71%	47%
Nick	Class A	Low	6/6	5/6	5/6	2/3	18/21= 86%	19%
Makayla	Class B	Low	5/6	0/6	5/6	2/3	12/21= 57%	38%
Lee	Class B	Low	6/6	1/6	6/6	3/3	16/21= 76%	47%
Gabby	Class B	Low	5/6	0/6	6/6	2/3	13/21= 62%	10%

Appendix Y

Complete Post-Assessment for Focus Group

Name	Class	Level	Accuracy Score	Efficiency	Strategy Score	Verbal Assessment Score	Total Score	Percent Growth Since Pre-Assessment
Henry	Class A	High	6/6	6/6	6/6	3/3	21/21=100%	29%
Anthony	Class A	High	6/6	6/6	6/6	3/3	21/21=100%	0%
Leon	Class A	High	6/6	6/6	6/6	3/3	21/21=100%	5%
Aaron	Class B	High	6/6	6/6	6/6	3/3	21/21=100%	0%
Mandy	Class B	High	6/6	6/6	6/6	3/3	21/21=100%	29%
Ivan	Class B	High	6/6	6/6	6/6	3/3	21/21=100%	10%
Nina	Class A	Average	6/6	6/6	6/6	3/3	21/21=100%	33%
Will	Class A	Average	6/6	6/6	6/6	3/3	21/21=100%	52%
Gretchen	Class A	Average	6/6	6/6	6/6	3/3	21/21= 100%	48%
Carter	Class B	Average	6/6	5/6	6/6	3/3	20/21=95%	56%
Emma	Class B	Average	6/6	6/6	6/6	3/3	21/21=100%	29%
Anna	Class B	Average	6/6	6/6	6/6	3/3	21/21=100%	29%
Lynn	Class A	Low	6/6	3/6	6/6	3/3	18/21= 86%	67%
Addie	Class A	Low	6/6	3/6	6/6	3/3	18/21= 86%	62%
Nick	Class A	Low	6/6	6/6	6/6	3/3	21/21= 100%	33%
Makayla	Class B	Low	6/6	3/6	6/6	3/3	18/21=86%	67%
Lee	Class B	Low	6/6	3/6	6/6	3/3	18/21=86%	57%
Gabby	Class B	Low	6/6	3/6	6/6	3/3	18/21=86%	34%