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ESSAY

AT THE END OF *PALSGRAF*, THERE IS CHAOS: AN ASSESSMENT OF PROXIMATE CAUSE IN LIGHT OF CHAOS THEORY

Edward S. Adams*
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ABSTRACT

Palsgraf articulated the doctrine of proximate cause, necessary to prove the tort of negligence. *Palsgraf* needs to be reexamined in light of today's understanding of cause and effect. The case concerned a woman (Mrs. Palsgraf) standing on a train platform who was injured by a roof tile that fell as the result of the vibrations caused by the explosion of another passenger's package. Mrs. Palsgraf sued the railway for negligence and prevailed at the trial court level. The New York Court of Appeals reversed the trial court, however, holding that the railway company's actions were not the proximate cause of Mrs. Palsgraf's injuries.

Modern science recognizes that the railway station constituted a complex dynamic system. *Palsgraf* was decided in 1928 at a time when understanding of cause and effect in complex dynamics was minimal and steeped in a linear mindset. Because the understanding of cause and effect in these systems has been significantly advanced by the field of nonlinear dynamics in recent years, the case

should be reexamined in this new light. Linear systems have outputs that are proportional to their inputs and are therefore predictable. But linearity inadequately models most of the real world. Nonlinear systems, though not recognized by most as such, are more prevalent than linear systems and have outputs disproportionate to their input. A subset within the group of nonlinear systems are chaotic systems, the main focus of this essay. The title "chaotic" is misleading as these systems still follow discrete physical laws. But it is their sensitivity to initial conditions that makes them unpredictable. What appears as a random result may actually be a strong reaction to immeasurable inputs at the beginning of a sequence of events. The weather is a good example of this principle, termed deterministic chaos. There is no way to isolate and define each initial condition that goes into a weather pattern, but as will be shown, each initial condition may radically affect the resulting systems.

Chaotic systems exist alongside predictable linear systems. In *Palsgraf*, the train station had both regular, predictable systems, such as the track being able to carry the weight of the train, and some unpredictable, chaotic systems, such as the interaction of the exploding package, the roof tiles, and Mrs. Palsgraf. If an accident happened that involved a regular system, then it is more likely that the railway should have been held liable. This is because the engineers who designed the station should have known the linear parameters within which the station could be used safely. Had they neglected to act on this knowledge, the company would have been negligent. However, because the system involved was a high dimension chaotic system—many interacting degrees of freedom—the system was subject to the principle of sensitivity to initial conditions, and Mrs. Palsgraf's injuries were in no way foreseeable or direct even though chaos theory elucidates the causal link between explosion and injury. Because they were in no way foreseeable or direct, the defendant had no duty toward the plaintiff with regard to the roof tile. So, according to the doctrine of proximate cause as articulated in this case, it still holds that the company was not negligent and the court's finding is supported.

There is a second way to look at proximate cause and *Palsgraf* in light of modern scientific theories. Modern theories postulate that the apparently chaotic phenomenon which are occurring at the station are actually following rules. A deterministic pattern of behavior exists even though it is not readily discernible. All of the operative degrees of freedom define a phase space, and phase space analysis may elucidate the system's deterministic behavioral patterns. Phase space analysis is a tool used to visualize the behavior of a dynamic system over time. Many dynamic systems generally behave in a stable manner, but intrusions from outside a system can alter the stability in varying degrees. Depending on the disturbance, the system may return to its original pattern, or may be permanently disrupted, adopting a completely new pattern of behavior. Phase space analysis provides a second way to look at *Palsgraf*. In a system made of a train station and a train, there are a number of phase space portraits which demonstrate predictable patterns, such as the location of the station's platform. Other portraits, such as those that include the movement of passengers, are never stable. The railway company has a responsibility to maintain the stable system within safe parameters so that it is not permanently disturbed by outside systems. Because the roof tiles at the train station were loose, they were subject to being disturbed. The loose tiles created what scientists call a zone of danger, and the accident was therefore foreseeable. The railway station did not take the proper precautions, and liability results. Using phase space analysis, *Palsgraf* would have been resolved as Judge Andrews suggested in his dissent, and the train company would have been liable.

These same theories and scientific principles can be applied to most proximate cause tort cases. The appendix following this essay contains examples of actual cases decided using these principles.

I. *PALSGRAF*

Televisions were mounted above the coffee bar in the cybercafe, their sounds slightly muted so patrons could sit at tables, drinking coffee or surfing the Internet on the computer terminals. Casablanca played continuously on the screens.

It was Friday evening and the cafe was busy. When the young genius took his coffee from the bar, he looked around and saw that there was but a single empty chair. He approached the table and asked the young woman who sat in the other chair if he might join her. She was dressed in sensible casual clothes. The woman, also a young genius, looked up at the young man and smiled through her wire-rimmed eyeglasses.

"If you don't make too much noise," she said airily. "I'm reading the *Palsgraf*[1] case." She pronounced both "A's" in *Palsgraf* so the word sounded overtly German, in the way that certain pedantic law professors do. The young man nodded uncertainly and sat across the table from her. He quietly watched the screen-saver of flying toasters on the monitor and then touched the mouse and the initial Netscape screen appeared. The sensible young woman glanced at the screen and snickered.

The young man looked up asked her, "you don't surf the Net?"

"Oh, *please*," the young woman said. "I learn from *casebooks*, the way my ancestors did when *they* studied the law."

"You're in law school, then?" said the young man.

"I am."

"What's your name?"

"Alexis. Alexis Loci Delicti,"[2] the sensible young woman responded. "And yours?"

"Tortuffe."[3]

"Pardon me," Alexis said.

"Tortuffe," the male genius replied. "It has religious implications."

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1. Palsgraf v. Long Island R.R. Co., 162 N.E. 99 (N.Y. 1928).

2. The character of Alexis Loci Delicti is based on *lex loci delicti*: "The law of the place where the crime or wrong took place. The 'lex loci delicti', or 'place of the wrong', is the state where the last event necessary to make an actor liable for an alleged tort takes place." BLACK'S LAW DICTIONARY 911 (6th ed. 1990).

3. The character Tortuffe is based on Moliere's *Tartuffe*.

"Well, Tortuffe," Alexis said, "I need to return to *Palsgraf.*" She smoothed her blond hair and made sure none of it had strayed from its large green barrette.

"What the hell's *Palsgraf,* anyway?" Tortuffe asked, running his fingers through his long wavy hair which, it occurred to Alexis, made him look terribly disheveled.

A. Palsgraf *Is THE Tort Case*

"*Palsgraf,*" Alexis said, "is the tort case—"

"Tort?" Tortuffe interrupted.

"A tort is a private or civil wrong or injury," Alexis said, "as opposed to a criminal wrong, in which the state prosecutes the offender."[4]

"And *Palsgraf?*" Tortuffe asked.

" '*Palsgraf*' is *Palsgraf v. Long Island Railroad Co.,* the most famous of all tort cases," Alexis said.

"What happened in the case?"

"Mrs. Palsgraf, the plaintiff, was waiting for her train on the defendant's railroad platform after she bought a ticket to Rockaway Beach.[5] As another train was leaving, two men ran to catch it.[6] One of the men boarded the moving train without incident, but the second man jumped for the train and was about to fall.[7] A guard on the train pulled him onto the train, and a guard standing on the platform pushed from behind.[8] During this pushing and pulling, a package, wrapped in newspaper, fell and exploded.[9] The package contained fireworks.[10] The shock from the explosion threw down some of the scales—or roof tiles—at the far end of the platform.[11] The falling scales struck Mrs. Palsgraf, and she sued the railroad for her injuries."[12]

"That's the most famous tort case?" Tortuffe asked.

"Hmmpff," Alexis snorted. "Obviously, you don't understand the great implications of the case."

"Oh, tell me, dear Alexis," Tortuffe said, "what are the great implications of *Palsgraf?*"

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4. *See* BLACK'S LAW DICTIONARY 370-71, 1489 (6th ed. 1990).
5. *See Palsgraf,* 162 N.E. at 99.
6. *See id.*
7. *See id.*
8. *See id.*
9. *See id.*
10. *See id.*
11. *See id.*
12. *See id.*

"*Palsgraf* articulated the doctrine of proximate cause,"[13] Alexis answered.

"Why is proximate cause important?" Tortuffe asked.

B. *Proximate Cause Is Needed to Prove the Tort of Negligence*

"Because it is one of the elements needed to prove the tort of negligence," Alexis said. "To prove negligence you must show that the defendant had a duty toward the plaintiff—a duty that required the defendant to abide by certain standards.[14] Additionally, you must show that the defendant breached this duty, or was careless, that there was a link of causation between the defendant's failure to conform to the duty, and that this causal link brought about actual damage to the plaintiff."[15]

"So," Tortuffe said, catching on, "*proximate cause* is cause that is proximate, or close to, the act of negligence."

"Yes," Alexis said, almost smiling. She was enjoying this young man's interest in her knowledge of tort law. She continued, "but *proximate* implies a limitation. The doctrine is limited to situations where risks are foreseeable as likely to occur because of a negligent act, or the consequences of the negligent act are 'of the same general sort that was risked.' "[16]

Tortuffe crossed his arms and looked upward as if thinking. When he finally spoke it was deliberate and slow. "So there can't be too much distance, whether in time or in space, between cause and effect, and there can't be other causes that supersede or intervene to cancel out the impact of the original cause."[17]

"That's right," Alexis said.

"But who decides whether the cause and effect are proximately linked?" Tortuffe asked.

"The finder-of-fact," Alexis said. "In *Palsgraf*, the New York Court of Appeals reviewed the facts as found by the trial court, and applied the doctrine of proximate cause to those facts.[18] The case was decided in 1928, with the majority opinion written by Chief Justice Benjamin Cardozo."[19]

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13. *See id.* at 99-101.
14. *See* BLACK'S LAW DICTIONARY 1032 (6th ed. 1990).
15. *See id.*
16. *In re* Kinsman Transit Co., 338 F.2d 708, 724 (2d Cir. 1964).
17. *See* BLACK'S LAW DICTIONARY 1225-26 (6th ed. 1990).
18. *See Palsgraf*, 162 N.E. at 99-101.
19. *See id.* at 99.

Tortuffe and Alexis Loci looked across the table at each other for a moment, slightly puzzled, before Tortuffe continued.

"So Mrs. Palsgraf won the case, right?" Tortuffe said.

"No,"[20] Alexis said. "Think, Tortuffe, of proximate cause. Her injury was not foreseeable or direct in any sense of the word. It was a freak, random accident. Neither the railroad's employees, guards, or porters had a duty of care to Mrs. Palsgraf, to whom there was no foreseeable risk."

"The fireworks exploded and the shock of it made the scales fall and injure her," Tortuffe said, "but the defendant railroad was not liable for her injuries."[21]

"That's correct," Alexis said. "In the interests of a practical and workable doctrine the court had to draw the line somewhere. The cause was not proximate to the effect—Mrs. Palsgraf's injuries."

"It's an interesting doctrine," Tortuffe said, "but, in this case more intricacy was present than either the court realized, or, seemingly, than anyone since has realized."

"Excuse me?" Alexis sputtered.

"The case needs to be reexamined with the various aspects of it dissected away from each other; seen clearly as separate parts," Tortuffe asserted.

"How would you know?" Alexis said. "You aren't a lawyer."

"I know," Tortuffe responded smugly, "precisely *because* I am not a lawyer."

"Well then, Mr. Know-it-all," Alexis said, "what *are* you?"

"I'm a graduate student of physics," he haughtily replied.

"Hmmpff," Alexis said dismissingly. "Proximate cause and the *Palsgraf* interpretation have lasted for many years, and they're not about to change now."

"But they *should* change," Tortuffe said. "The interpretation of proximate cause in the *Palsgraf* case is oversimplified, it's outdated."

"Why is it outdated?" Alexis asked.

"Because it is based on a linear conception of how the world works," Tortuffe replied. "We can do better than that with our modern understanding of dynamic systems."

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20. *See id.* at 101.
21. *See id.*

II. LINEARITY IMPLIES OUTPUT PROPORTIONATE TO INPUT

A. *Introduction to and History of Our Understanding of Linearity*

"Linear, dynamic systems?" Alexis exclaimed. "What are you talking about?"

"Your theory of proximate cause is based on linear dynamics," Tortuffe said. "That is, it's based on the notion that things are predictable because many of the things we work with in life *are* predictable in such ways. A linear system is predictable because its output is proportionate to input."[22] "For instance," stated Tortuffe as he unscrewed his pen and removed the spring, "pulling the spring a quarter of an inch takes just so much force while pulling the spring half an inch takes double that amount. Such linear behavior is described by Hooke's law.[23] The concept of linearity is based on Newton's laws of motion, which can be explained using Euclidean geometry.[24] Euclidean geometry is great if you're trying to get across a flat Midwestern town, but if you want to put a man, ugh, or a woman, on Mars, or sail from California to Fiji, geometry of the non-Euclidean variety is a better choice since it represents curved space more accurately."

"Newton's *Philosophiae Naturalis Principia Mathematica* was published in English in 1729, two years after Newton's death.[25] The work was the culmination of thought by Galileo, Kepler, and other great scientists.[26] It presented a mathematical way of understanding the mechanics of the world.[27] Newton's techniques allow description of dynamic systems like the planets, or billiard balls in motion.[28] The techniques are also predictive in nature because the systems examined were linear, or at least nearly so. Newton's proofs made use of Euclidean geometry, though another way to model these linear systems was via differential equations—the calculus that Newton and, independently, Leibniz invented.[29] By the late 1700's the methods of calculus were widely available and were being used with Newton's laws of motion to model more of the natural world and to further technology."[30]

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22. *See* IAN STEWART, DOES GOD PLAY DICE? THE MATHEMATICS OF CHAOS 81-84 (1989).
23. *See* DANIEL J. BOORSTIN, THE DISCOVERERS 51-52 (1983).
24. *See* STEWART, *supra* note 22, at 77-81.
25. *See* BOORSTIN, *supra* note 23, at 401, 406.
26. *See id.* at 406.
27. *See id.*
28. *See id.* at 405-07.
29. *See id.* at 413-16.
30. *See* STEWART, *supra* note 22, at 33-35.

"Newton, Leibniz, Hooke, Galileo, Kepler, and Euclid, famous men to be sure, but, what happens if you stretch your spring one inch?" asked Alexis inquisitively.

"That's exactly what I am getting at!" exclaimed Tortuffe.

B. Most of Us Think the World Works in Linear Ways

"A clicking pen, a bat hitting a ball, the Moon's orbit and countless other everyday phenomena are commonplace to us, and while most of us do not know the math used to describe them, we know that they are basically linear systems and that, because of their inherent linearity, they and their future behavior can be described and predicted through linear differential equations. Thus, we have a feel for this world-view because it is what we often experience. The bridges we drive over, the cars we drive over them, and many other devices are the products of Newton's legacy. These are systems whose dynamics are described readily and are predictable, that is, their outputs can be predetermined because they are proportionate to their inputs: no 'squared' or higher order terms, no sine or cosine functions, no exponentials or natural logarithms, and no cross multiplication between the variables. So, they are categorized as linear deterministic systems. We've discussed linearity; deterministic means they obey laws such that if one knew the laws and all the initial states of the system then the outcome could be predicted."

"Predictability of linear systems results despite errors at the beginning of the sequences. For instance, if the initial value of a variable used to predict the behavior of a linear system is slightly incorrect, the prediction will still match the behavior temporarily because if the variable in the system truly did start with that incorrect value, the behavior of the system would nonetheless be very similar. The point is that a small change in the system leads only to a slowly growing change later in the system. Similarly, if there are two very nearly identical systems, but there is a small difference between them, they will behave very similarly down the road. You could call this insensitive dependence on initial conditions. It's important to us that bridges, cars, and other systems we think about in everyday life be predictable. Otherwise they would act in unpredictable ways, usually break, and not be of much use to us," stated Tortuffe as he irreparably, and rather unexpectedly, stretched his pen's spring to several inches in length. "The point is that a large part of our experience, and therefore a large part of how we think about the world and make judgments, is based on linear dynamics. The people thinking about proximate cause in the past have been bound to this linear bias."

C. Linear Dynamics Inadequately Models Much of the Real World

"But the problem with linear dynamics is that it is inadequate to understand much of our world. Newton used his linear laws of motion to calculate positions and velocities of two mutually, gravitationally attractive material bodies.[31] Yet no one has been able to predict future positions of three material bodies interacting via gravity because groups consisting of more than two bodies interact in a nonlinear fashion. Actually, two bodies interact in a nonlinear way as well, but the interaction is simple enough that it can be modeled with linear equations. This three-body problem is evident when we send 'space probes to other planets.'[32] Scientists chart a course to where the planet will be located in its orbit when the probe arrives 'but midcourse corrections are nevertheless necessary because Newtonian [linear] physics can predict accurately only the interaction of two bodies, not three.'[33] You may question how it is that we can know the future orbits of all the planets in the solar system as there are more than two and so it would seem to be greater than a two-body problem. The answer is that the Sun is so massive and the planets small enough so that each orbit is effectively a function of the gravitational interaction between just the Sun and the planet in question—a two-body problem. The space probe, when close enough to a planet, is acted upon by both the Sun and the planet, and so is involved in a three-body problem."[34]

"Many other examples of nonlinear interactions are, literally, closer to home. The interaction between some man-made structures and wind forms chaotic resonance phenomena. In November, 1940, high winds across the Puget Sound caused an unstable resonant oscillation to develop in the Tacoma Narrows Bridge.[35] The nonlinear resonant oscillation

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31. *See* Lawrence A. Cunningham, *From Random Walks to Chaotic Crashes: The Linear Genealogy of the Efficient Capital Market Hypothesis*, 62 GEO. WASH. L. REV. 546, 582 (1994). For additional sources on chaos theory in other areas of law, see Robert C. Downs, *Law and Economics: Nexus of Science and Belief*, 27 PAC. L.J. 1 (1995) (criticizing the law and economics movement for over-generalizing human behavior in light of chaos theory); Edward J. Imwinkelried, *Evidence Law Visits Jurassic Park: The Far-Reaching Implication of the Daubert Court's Recognition of the Uncertainty of the Scientific Enterprise*, 81 IOWA L. REV. 55 (1995) (examining the scientific belief system of evidence law using chaos theory); Glenn Harlan Reynolds, *Chaos and the Court*, 91 COLUM. L. REV. 110 (1991) (discussing the jurisprudence of the Supreme Court using chaos theory).
32. Cunningham, *supra* note 31, at 582.
33. *Id.* (citing EDGAR E. PETERS, CHAOS AND ORDER IN THE CAPITAL MARKETS 135 (1991)).
34. *See* STEWART, *supra* note 22, at 32-35.
35. *See* Jan Glidewell, *The Vintage Whines of Pasco Return Series*, ST. PETERSBURG TIMES, Mar. 29, 1998, at 1; *see also* Northwestern Mut. Fire Ass'n v. Union Mut. Fire Ins. Co., 144 F.2d 274, 275 (9th Cir. 1944).

grew until the bridge collapsed—the bridge was less than a year old.[36] Perhaps you remember some of those old war movies, where soldiers were ordered to break step when crossing bridges to prevent such destructive resonant oscillations. A nonlinear phenomenon in resource management is the population dynamics of fish stocks. Yet another example of nonlinear phenomena arises in the flight dynamics of highly maneuverable aircraft like jet fighters which may be operated in 'unstable steady states' to enhance performance. These are just some examples of phenomena that directly affect peoples' lives and cannot be understood through linear dynamics—there are many others."

"Why," Alexis asked, "do we need to talk about linear dynamics not adequately modeling the world, when *Palsgraf* is so obviously a random, freak accident?"

"I'm trying to tell you that your current doctrine of proximate cause is based on linear notions. These notions do not accurately explain complex systems like those found in real life. Linear physics can't encompass the *Palsgraf* system—the interaction of many variables and many material bodies, including the explosion, the shock waves resulting from the explosion, the railroad platform, and the scales that fell on yet another variable, Mrs. Palsgraf."

"Is proximate cause, as exemplified in *Palsgraf*, valid in light of nonlinear dynamics?" Tortuffe asked rhetorically. "Perhaps," he said, "but it must be reinterpreted through one of the tools of nonlinear dynamics. After an interpretation based on a more realistic understanding of dynamic systems we'll find that *Palsgraf* was oversimplified by the court because they were thinking according to linear principles."

"*What*, O great scientist," Alexis asked, "is this tool with which we should reinterpret *Palsgraf?*"

"Chaos," Tortuffe said, leaning back in his chair.

"What do you mean—*chaos*?" Alexis asked. Her eyes squinted belligerently behind her eyeglasses. "You, who know nothing about the law, come here and tell me that a major, precedent-setting case in my field was oversimplified! Well, Mr. Scientist, what is a more realistic way to look at the facts of *Palsgraf?*"

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36. *See Northwestern Mut. Fire Ass'n*, 144 F.2d at 275.

III. NONLINEARITY IMPLIES OUTPUT DISPROPORTIONATE TO THE INPUT

A. Chaos Theory Is a Subfield of Nonlinear Dynamics

"Chaos theory," Tortuffe said, "is a sub-field of nonlinear dynamics which has developed since about the mid-1960's."

"I don't understand," Alexis said.

"Have you seen the movie *Jurassic Park*?"[37] Tortuffe asked.

"Yes, I may recall that one," Alexis wryly smirked. "Oh yes, how silly of me—one of the top grossing films of all time."

"Alright smarty," Tortuffe said, rolling his eyes, "what happens in the movie?"

"Well," Alexis said. "The dinosaurs break free of their cages and pandemonium breaks loose. Oh, and everybody cheers when the T-Rex eats the lawyer."

"Pretty much," Tortuffe said. "Mr. Hammond and his team of scientists tried to do what they thought was a very simple thing; that is, they tried to create a simple zoo in which each animal was precisely controlled."

"And it didn't work," Alexis said. "It wasn't simple at all, it was complex and as soon as the electricity that ran through the perimeter fences was shut off, things became, well, *chaotic*."

"So," Tortuffe said, "both simple and complex systems can produce complex and seemingly unpredictable behavior. This is a main tenet of chaos theory.[38] Furthermore, most engineers know that process control units become part of the total system and thus can easily complicate an otherwise simple system."

"Why do some simple systems produce complex behavior?" Alexis asked. "I thought that simple systems could be explained in a linear fashion."

B. The Butterfly Effect Is Characteristic of Chaotic Systems

"Some nonlinear systems produce complex behavior because of *The Butterfly Effect*, or *sensitivity to initial conditions*.[39] Chaotic systems comprise a subcategory of nonlinear systems." Tortuffe paused, gathering his thoughts, "look—any experiment has a margin of error for factors that cannot be controlled. Yet these factors are there and in chaotic nonlinear systems; thus, they can tremendously alter the results of the experi-

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37. JURRASIC PARK (Universal 1990).

38. *See* MICHAEL CRICHTON, JURASSIC PARK 75-76, 194-95 (1990).

39. *See* STEWART, *supra* note 22, at 139-42.

ment. This can happen quickly, even if they are very small factors. This is why the phenomenon is called sensitivity to initial conditions."

"The best way to understand the butterfly effect is to see it," Tortuffe continued. "We'll visualize it with an equation that is subject to sensitivity to initial conditions. This equation has become the paradigm of the phenomenon and is called the logistic equation. Iterate the equation $x \longrightarrow kx(1 - x)$ where k is a constant and x is either the initial chosen value or, subsequently, the previous output of the equation. For instance, if k = 3 and you've chose 0.5 for your first value of x, then you have 3(0.5)(1 - 0.5) and this equals 0.75.[40] Now, to iterate, take the output, 0.75, and make it the input for the next round. The equation is now 3(0.75)(1 - 0.75) and the new output is 0.14. If you iterate 100 times, you can make a graph of the ups and downs of the output. For certain values of k, this system is sensitive to initial conditions as follows. If you run the iteration y times with an initial "x" of 0.0897 and again y times but starting with 0.0898, you might expect the graphs to look similar. They do for a number of iterations but then begin to diverge and before too long they are in no way correlated with each other—all because of a 0.0001 difference in the initial value of x." (See Figure 1)

Figure 1: The Butterfly Effect[41]

The curves represent initial conditions differing by only 0.0001. At first they appear to coincide, but soon chaotic dynamics leads to independent, widely divergent trajectories.

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40. *See id.* at 155.
41. *See id.* at 141. Reprinted by permission of the Publisher.

C. The Extreme Examples of Nonlinearity Are Those Systems That Are "All or Nothing"

Tortuffe continued, "before discussing the Butterfly Effect further, let me first explain general nonlinearity. The linear systems previously discussed (springs, baseballs, two-body gravitational problems, etc.) actually are nonlinear systems of the class that, over some controlled range, can be very closely approximated by linear equations. Since there is no truly linear system, nonlinearity is reality," Tortuffe stated as he gazed at his pen's unsprung spring, "and '[n]onlinearity means that a small variation in one side of the equation may result in large results on the other side of the equation.'[42] For example, a vehicle's wind resistance is proportional to the velocity cubed. People who spend a lot of time on the highways know that gas mileage is much better at 55 mph than at 85 mph, and that driving at 70 mph will not yield miles per gallon equal to the average of that at 55 mph and 85 mph."

"I don't think I understand," Alexis said

"The most extreme type of nonlinearity is anything that is all or nothing. This is the best example of output not being proportional to input. Consider a nonlinear switch modeled by an equation that has, as input, the amount of energy being supplied to flick the switch. On the other side of the equation it has, as output, a one or a zero, depending on whether the switch is on or off. An energy of amount x is needed to flip the switch. So, assume the switch is in the off position and that $\frac{1}{4} x$ is supplied—the switch is not flipped. Now supply $\frac{1}{2} x$. The switch is still not flipped and neither is it the case that it is twice as much flipped as with $\frac{1}{4} x$—indicating that this is not a linear system. Nor is it even the case that the switch is just a little more flipped than it was with $\frac{1}{4} x$. Put $\frac{9}{10} x$ energy in and the right side of the equation still equals 'off.' "

"So only x or more energy switches it," Alexis interjected. "This is very nonlinear! The roof tile is like a switch; when the force of an explosion is applied, it either falls or it doesn't. Since this is an all or nothing outcome, you would say this interaction is a nonlinear one."

"That's right," Tortuffe said. He looked out of the corner of his eye and cleared his throat before he continued. "The light switch has a very sudden threshold—this model is called a transfer function. Light switches are a common example of this. Another common phenomenon modeled by a transfer function is the firing of nerve impulses in our nervous sys-

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42. Thomas Earl Geu, *The Tao of Jurisprudence: Chaos, Brain Science, Synchronicity, and the Law*, 61 TENN. L. REV. 933, 937 (1994) (emphasis omitted).

tems. A neuron may receive excitatory stimuli of varying levels from other neurons' dendrites, for example, but it won't fire a signal of its own toward other neurons until the sum of the incoming excitatory stimuli exceeds a certain threshold. And just like the light switch, the output is not proportional to the input. Rather it is on or off with virtually no in-between states. This is a nonlinear system."

"OK, I understand the difference between linear and nonlinear systems, now, let's get back to sensitivity to initial conditions," Alexis directed.

## D. Sensitivity to Initial Conditions Implies Various Degrees of Unpredictability

"Let me give you more examples of sensitivity to initial conditions," Tortuffe said. "Imagine a billiards table with rounded ends. From one flat side of the table you strike the ball with a pool cue and the ball rolls across the table and strikes the cushion along the other flat side of the table. One can measure the angle at which the ball strikes the far cushion, and the angle at which the ball leaves the far cushion. If there is no friction, the ball will depart from the point of impact on the cushion at the same angle that it approached the cushion. Knowing the initial position of the ball and the angle at which it was sent off, a prediction can be made of its position after x bounces. However, the initial position of the ball cannot be known exactly, the key point. An error in the initial location of the ball will amount to a minor difference between an actual and a predicted position as the ball bounces off the parallel sides. However, the ball and our predicted ball will reach the rounded ends eventually and then our predictions may quickly become completely wrong. This is because the error grows exponentially with each subsequent impact so that even within two or three bounces the prediction could be utterly wrong. This system I have posited is deterministic, but the future position of the ball cannot be accurately predicted because of the exponential increase in the error."

"How could this be? Construe each rounded end as a series of angles that approximates a half circle. If the actual initial position of the ball and the measured initial position of the ball differ by 1°, this difference will be maintained as the ball bounces in the parallel section of the billiards table. However, when the ball encounters the rounded ends it's possible that the 1° difference between actual and predicted position straddles the vertex of one of the angles comprising the approximation of a half circle. At this point the prediction may 'bounce' off the surface to

the right of the angle in which case the actual ball will bounce off the surface to the left of the angle and they will subsequently travel in very different directions and encounter more angles. This would decouple prediction and actual bounce pattern. A true half circle end is basically a high resolution version of the angled ends we just discussed. So, now you see how once the ball bounces to an end, prediction and actual pattern diverge quickly.

"If you understand that a very small error in describing the initial position of the ball can make a tremendous difference between prediction and actual future position, then you understand that two balls which started from almost the same position and were sent off at the exact same angles will, after a few bounces, follow utterly different paths. This is sensitivity to initial conditions. Note that the concept exemplified on the pool table is qualitatively equivalent to the iterated nonlinear equation we encountered previously—each has a pair of trajectories that start infinitesimally apart from each other, yet quickly diverge wildly." (See Figure 2 below)

Figure 2: A Billiards Table with Rounded Ends[43]
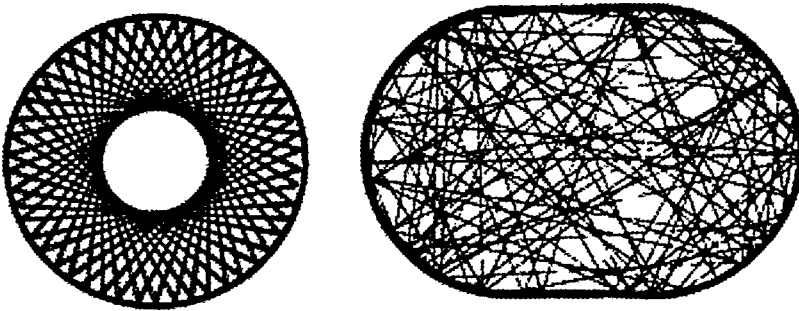
Classical billiards, a key to quantum chaos. A particle bounces off the edges of some region, like a ball on a billiards table. A circular ring (left) leads to regular behavior, one like a stadium (right) leads to chaos.

"Before giving you the next example of the sensitivity to initial conditions, let me introduce some terms important to the concept—*choice*

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43. *See* STEWART, *supra* note 22, at 294. Reprinted by permission of Professor Sir Michael Berry and the Royal Society.

points and *perturbations*. A choice point is a physical state from which a system will evolve down one of several possible paths.[44] Perturbations are phenomena that disturb the system.[45] If disturbed enough, the system may evolve away from the choice point down a different path than it would have if it had not been perturbed. In the pool table example, each angle potentially straddled by the actual and predicted positions represented a choice point—the ball could hit the side it actually hit or the side it was predicted to hit and the subsequent paths would differ greatly. Perturbations that effect significant change may be vanishingly small. A 0.01° difference between two pool balls on our table with angled ends could cause one ball to hit the right side of an angle and the other to hit the left side, in which case they may travel in very different directions. So, a very small perturbation can have a huge effect on where a ball bounces because that perturbation could cause the 0.01° difference. In the *Palsgraf* case, the explosion is a perturbation and its existence results in the choice point that has the possible outcomes of either the system relaxing to the original, pre-explosion state, or relaxing to a new state and potentially harming someone in the process."

Alexis coughed loudly as someone she knew walked by. The woman waved, but did not approach. Alexis saw no alternative but to continue to submit to the discussion.

"OK, the next example of sensitivity to initial conditions," Tortuffe continued unabashed, "is the weather. It's important to note that it's also a deterministic system. Even so, a slight change in the input of a weather system, even an extremely simplified model system created on computers, can cause the weather to change dramatically. That is one reason sensitivity to initial conditions is also called The Butterfly Effect—a butterfly flapping its wings in New York may affect the weather in Peking."[46]

"How does that happen?" Alexis asked.

"Well, the Butterfly Effect posits that a storm system in North America has a number of large scale and obvious input, but also a large number of smaller and temporally remote input. These input are not obvious, but, as their effects grow, give rise to the obvious input. With this line of thinking we can understand how the flapping of a butterfly's wings in one part of the world; an input that creates a tiny air current, could, in time, drastically change the weather in another part of the

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44. EDWARD OTT, CHAOS IN DYNAMICAL SYSTEMS 108-31 (1993).
45. *See* JOHN BRIGGS & F. DAVID PEAT, TURBULENT MIRROR 27-28 (1989).
46. *See* JAMES GLEICK, CHAOS: MAKING A NEW SCIENCE 8 (1987).

world. If we knew all the input to the weather to infinite precision and knew how to model it, then, we could predict it because it's a deterministic system. However, we don't know all the input to infinite precision, and a butterfly's wing currents are part of those things we can't know at all, let alone to infinite precision. And, because it's a chaotic system, that very small air current is a perturbation that can create a cascade of choices that lead to a completely different weather system from the unperturbed one—as changing the position of the ball on the pool table an infinitesimal amount can completely change its trajectory, relative to what it would have been if started at the first position, after just a few bounces. So, you can see that perturbations and choice points are the basis of sensitivity to initial conditions. At the social level, an analogy to the Butterfly Effect can be found in the play, *An Inspector Calls*,[47] by J.B. Priestley, in which the seemingly trivial negative comments and actions of individuals toward a young woman compound each other and put her life into a tail-spin that ends years later with her suicide.[48] Each of these comments and actions was a 'butterfly.' "

"I understand what you're saying," Alexis said to Tortuffe. "But if each small change at the beginning of the system produces infinite variety throughout the system and to the end of the system, how is there any kind of order underlying the system's chaotic, random variety?"

E. Deterministic Chaos Implies Systems That Are Nonrandom but, Even So, Have Outcomes That Are Not Predictable

"Complex systems, seemingly random and without order, conform to certain patterns," Tortuffe said. "What we have, then, is *deterministic chaos*. Deterministic chaos is the idea that a system can follow strict rules governing its behavior (this is the deterministic part) and at the same time appear random, or chaotic, to us because of sensitive dependence on initial conditions.[49] We've seen examples of the deterministic aspects of these systems—the strict mathematical regularity of the logistic equation and the physical rules that determine the way a ball bounces on a billiards table, or those that determine the outcome of two gusts of wind interacting. These systems only look random to us because they are

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47. J.B. PRIESTLEY, AN INSPECTOR CALLS: PLAY IN THREE ACTS (1972).

48. *See id.* It is not clear that the young woman actually commits suicide, or if it was a trick on the individuals who had been cruel to the young woman. The play illustrates that a number of individual and unrelated events could collectively cause someone to end their life.

49. *See* GLEICK, *supra* note 46, at 250-52.

subject to sensitivity to initial conditions. Contrast deterministic chaos to the linearly oriented world view of the finder-of-fact in 1928."

"So where does that take us?" Alexis asked. "How does deterministic chaos apply to allocating liability in *Palsgraf*?"

IV. APPLICATION OF CHAOS THEORY TO *PALSGRAF*

A. The Train Station Is a System Composed of Subsystems—Some Are Predictable and Some Are Chaotic

Tortuffe moved slowly to the counter and refilled his cup before he continued. "Think of the variables in *Palsgraf* as a system. This system is composed of the people, the trains, the train station—but also think of it as composed of smaller systems. Some of the subsystems are rather predictable in nature—the motion of the train on the tracks is very predictable and small variables in the system, like a pebble on the track, will not change the predicted course of action. Some of the subsystems are chaotic in nature. An example of the chaotic aspect is the interaction between the vibrations caused by the exploding fireworks, the scales that fell on Mrs. Palsgraf, and her placement on the platform. Small changes in the initial inputs to the nonlinear systems could have drastically changed the outcomes of the explosion. For instance, if the explosion had been just barely weaker, the tile might not have fallen at all. As we said before, anything that is all or nothing is intrinsically nonlinear, like the light switch example."

"Let's think more thoroughly about why this system would be classified as chaotic. Picture this: it is obvious that the explosion created a primary shock wave that traveled through the air and hit the first part of the roof it encountered initially, and hit consecutively further sections of the roof in sequence. However, the primary shock wave hit other, nearby structures like buildings and the floor of the station and bounced off as secondary shock waves, some of which went toward the roof again, and these all hit the roof at different times slightly after the primary shock hit the roof. Like a rock thrown into a pool of water, each part of the primary shock wave that hit the roof slightly before the next part of the wave hit the next part of the roof created a sound wave emanating out in a circle within the material of the roof. The secondary shock waves arriving just after the primary also created various sound waves within the roof. Each of these sound waves had energy. The energy of a single wave was probably not enough to knock a roof tile off."

"Picture the wakes of two boats on a lake. Where they meet they add to each other and become higher, resulting is what is called con-

structive interference. The resulting wave has more energy than either of the first two. It's like singing in the shower, the sound waves bouncing around the roof doing the same thing as one's voice bouncing off the tiled shower walls. Depending on how they meet, some cancel each other and some add to each others' energy. The interaction between these waves and the tiles was a chaotic resonance phenomenon similar to Maria Calas breaking a crystal glass with her voice. A perfectly normal, healthy tile could have been subjected to the added force of a number of waves at one time, and thus been dislodged although it wasn't loose in the first place. Perhaps the addition of two waves wasn't enough to dislodge the tile—if it is dislodged it falls, if it is not dislodged, it does not fall. But, maybe the energy in three coinciding waves was enough—this could be modeled by the transfer function so we know the system was at least strongly nonlinear!"

"Furthermore, if one sound wave would have been ten centimeters to the right because the structural beam it bounced off had been ten centimeters to the right, the three waves would not have coincided and so would not have dislodged the roof tile. This would be sensitivity to initial conditions, an indicator that chaos was present! Finally, it's obvious that all the bouncing waves appeared to be random but each was the result of specific physical laws being obeyed—this was deterministic chaos! So, we conclude the system composed of the explosion, the roof and its tiles, and Mrs. Palsgraf constituted a chaotic subsystem within the larger system that was the whole train station."

"So you're saying that there are two different types of systems, predictable and chaotic, that interact in the *Palsgraf* scenario," Alexis said, "and that they are interrelated?"

"Yes," Tortuffe said.

"So there must be two different types of accidents that might occur, depending on which system is affected?"

B. Regular Mishaps May Occur in the Predictable Systems

"Exactly," he said. "The first type of accident we will call a *regular mishap*. These affect the predictable systems and could be called predictable mishaps. But I will use the term "regular" because it is the term physicists use when they differentiate between regular and chaotic systems. The engineers who designed the railroad in *Palsgraf* had access to equations and tables which told them which metals would support the loads the rails had to hold. Assume, for sake of example, that the explosion had not happened and Mrs. Palsgraf had boarded the train. If there

had been a bend in the track a few miles after she had boarded, and the metal of the track was weak enough that it broke under the pressure applied by the turning train, the train would have crashed and Mrs. Palsgraf would have been injured. With that metal and that physical load, *injury was almost certain.* The rules applying to this crash were *regular and known and thus could have been prevented* by the design engineers."

C. Chaotic Mishaps May Occur in the Chaotic Systems

"And the second type of accident is a *chaotic mishap,*" Alexis suggested.

"Yes," Tortuffe said, smiling at her quickness. "Assume," he said, "that the explosion in *Palsgraf* was due to another type and amount of explosive. Now the initial conditions have changed: although there is still an explosion, perhaps the outcome will be different. How could this be? The system composed of the explosion, the resulting vibrations and the scales was, as we found earlier, chaotic. Therefore, a set of different vibrations from even an infinitesimally smaller explosion or a slightly differently placed structural beam *might have resulted in no tile falling and nothing injurious happening.* In what ways might injury not have been sustained? The initial choice point was whether or not the fireworks would explode upon impact. Once this choice was made, injury depended on two more choice points. The first was whether or not scales would fall from anywhere on the roof due to the perturbation that was the explosion. The second choice was whether or not the scales 'chosen' to fall would have the correct trajectory to strike Mrs. Palsgraf. Thus, there were three choice points that had to coincide for injury to occur. First, the explosion had to happen; second, scales had to fall; third, the scales that fell had to follow the correct trajectory to injure Mrs. Palsgraf. The physical rules applying to this complex system were unknown, and *even if they were,* because the interaction is chaotic, the results of an explosion could not be predicted other than very coarsely—a big explosion will cause structural and personal harm, a small explosion may not have negative effects."

"I simplified that a bit. Actually we need to divide systems into a number of categories based on the predictability of their outcomes. First, as we've discussed, linear systems, or nonlinear systems that closely approximate linearity, can be predicted. Then we divide chaotic systems into two classes. The first class is called low dimensional chaos[50] and the

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50. *See id.* at 271.

second is called high dimensional chaos.[51] The outcomes of neither type of chaotic system can be predicted, but those of systems exhibiting low dimensional chaos can be assigned probabilities. You can't even assign probabilities to the outcomes of systems exhibiting high level chaos. The train system was a high dimensional chaotic system. So, the only way to prevent unknown and unforeseeable mishaps is to build everything as solidly as is humanly possible which would be an economically infeasible solution."

"I think I can see the difference between the regular and the chaotic mishaps," Alexis said. "In the case of the regular mishap, someone or something (the engineering company that designed the railroad) was close enough to the cause of the injury to have been able to prevent it. They had the information necessary to foresee the problem and to prevent it."

"Yes," Tortuffe said, *"then there would have been proximate cause."*

"Liability could be assigned because of the direct and proximate connection between the faulty design and the injury."

"That's right," Tortuffe said. "And in the second case, the case of the *chaotic mishap*, we know there was a link of causation between the explosion and the injury—part of what's necessary to prove proximate cause. However, the injury was not foreseeable or direct in any sense of the word because it was the outcome of a high dimensional chaotic system; therefore, the defendant railroad had no duty toward plaintiff and so breached no such duty—the other part of proving proximate cause. So, *there was no proximate cause."*

D. Application of the Concept of Sensitivity to Initial Conditions to Palsgraf

"I see in all of this, then," Alexis said, *"a workable rule* for tort law that can be derived from our discussion: if cause and injurious effect are linked by a regular system or a series of regular systems with very little opportunity for unstable nonlinear behavior to intrude, then, as long as the principles governing these linear behaviors are generally known, the defendant can be found negligent for not applying that knowledge to the system that resulted in injury. If, however, the cause and injurious effect constitute a high dimensional chaotic system or are linked by a series of systems, some of which are high dimensional chaotic systems,

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51. *See id.*

then the defendant cannot be held liable because the specific consequence could not be foreseen as likely to occur."

"The rule you've articulated seems largely to support the result the majority reached in *Palsgraf*," Tortuffe said.

"It seems so, yes," Alexis agreed, "it's the same conclusion but based on a more thorough analysis made possible by a sophisticated understanding of dynamic systems."[52]

V. AN ANALYSIS OF THE CASE BASED ON CLASSICAL STATISTICAL MECHANICS RESULTS IN THE RAILROAD COMPANY BEING FOUND LIABLE

"But," Tortuffe drove on, "*there is a different way*, a second way, of looking at what happened in *Palsgraf*, a way that might assign liability to the railroad, and result in a different outcome from the majority in *Palsgraf*."

"Using the concept of deterministic chaos, that a system can appear random, but in fact be following strict rules governing its behavior?" she asked.

"No," he said. "Now we will assume the station behaves according to classical statistical mechanics instead of chaotic dynamics."

"This sounds complicated," she said. "You know my legal background does not encompass these scientific theories."

Tortuffe asked, "Remember the Butterfly Effect? Non-random phenomena may appear to be random because of it. But also remember that the point was that chaotic phenomena actually are following rules, so there should be some pattern to the behavior, though it may not be readily discerned. So, it is observed that, '[n]ature [is] *constrained*. Disorder [is] channeled . . . into patterns with some common underlying theme.'[53] That underlying theme, that underlying order, is described as the movement of the system in phase space.[54] Phase space analysis also can be used to describe the behavior of non-chaotic systems, as we'll discuss in this second way of looking at *Palsgraf*."

"Well," Alexis said, "I don't know what phase space is. I wouldn't care to admit that I don't know, but how could one be expected to know something so arcane?"

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52. In his dissenting opinion in *Palsgraf*, Judge Andrews alluded to a sophisticated and interrelated dynamic system. *See* Palsgraf v. Long Island R.R. Co., 162 N.E. 99, 103 (N.Y. 1928) (Andrews, J., dissenting). This shows that at least one judge in the early twentieth century did not have a linear mindset.

53. GLEICK, *supra* note 46, at 152.

54. *See id.* at 49-50, 136.

"Let me explain it to you," Tortuffe said. "And I wouldn't expect you to understand phase space. Indeed, until today, I had never heard of proximate cause."

"But proximate cause is a basic concept," Alexis said.

"And so is phase space," Tortuffe said. "It is basic to understanding the complex systems in which we live, like the system of which Mrs. Palsgraf was a part."

"All right," Alexis sighed. "Tell me about phase space. But then you'd better tell me how it relates to proximate cause."

A.   The Behavior of a System Can Be Reduced to the Motion of a Point Moving in Multidimensional Phase Space

"Phase space analysis is a tool used to visualize the behavior of a dynamic system over time.[55] The numbers representing all of the information from a system at one instant can be reduced to a single point in multidimensional phase space. This is very similar to reducing the coordinates 'x, y, and z' to a single point in a three-axis graph, except that describing the state of a dynamic system at any one instant often requires many more than three dimensions. As the system changes from one instant in time to the next, a new point represents the new variation of the system. Tracing a curve through the temporally consecutive points creates a *phase space portrait*. This portrait, or *trajectory*, shows the behavior of the system."[56]

"I will give you an example of a phase space portrait in a spring-driven pendulum—a simple, nonlinear system," Tortuffe said. "For the spring-driven pendulum, the phase space is a plane and a single dot in the plane represents the condition (the state of all variables) of the pendulum at time X. The position of the pendulum is plotted on one axis and its velocity is plotted on the other axis. These two variables encompass all possible behavior of the pendulum. When the pendulum has swung as far to the left as it will, for an instant it stops, or, as Newton may have said, 'the kinetic energy has been transferred to potential energy.' Also note that any kinetic energy lost due to friction is replaced by the spring drive, which we shall assume is wound periodically. If the position is on the x-axis and velocity is on the y-axis, then at this point the dot representing the state of the system is on the x-axis (because there is no velocity, the y value is 0) as far to the left (negative) as it will be.
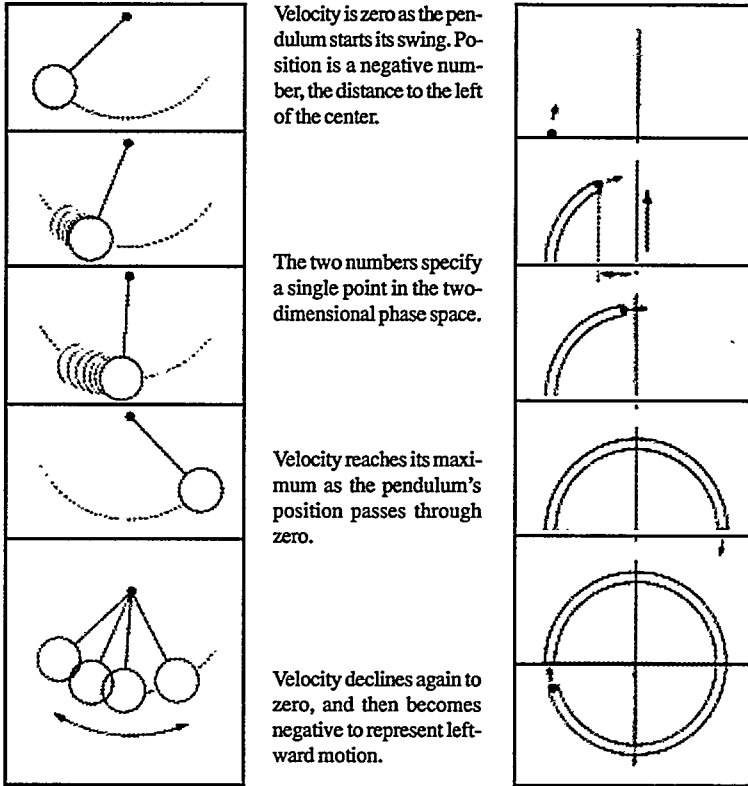
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55.   *See id.* at 134-39.
56.   *See id.* at 134-35.

Now it starts swinging back (with a positive velocity because it is swinging to the right) as potential energy is transferred to kinetic energy and the point moves to the right as the position of the pendulum swings toward center. The point continues moving higher and higher on the y-axis because the pendulum's velocity increases under the acceleration of earth's gravity. The velocity reaches a maximum at dead center where all of the potential energy is converted to kinetic energy. Past center it is still moving toward its most extreme position on the right, but its velocity is decreasing, so the y-component of the dot plotted at x,y decreases until it is at 0, at which time the pendulum is again motionless and at its furthest right point. This behavior has plotted a semi-circle above the x-axis in the two dimensional phase plane (each variable is a dimension). Now the process begins again, but in the opposite direction, so we plot these velocities as negative and the result is a trajectory tracing a semi-circle below the x-axis. (See Figure 3) Remember, the trajectory is the curve in the phase plane that defines a system's behavior. Phase space portraits, in this case a phase plane, are a way of comparing the behavior of one system to another."

Figure 3: The Swinging Pendulum[57]

Velocity is zero as the pendulum starts its swing. Position is a negative number, the distance to the left of the center.

The two numbers specify a single point in the two-dimensional phase space.

Velocity reaches its maximum as the pendulum's position passes through zero.

Velocity declines again to zero, and then becomes negative to represent leftward motion.

Another way to see a pendulum. One point in phase space (right) contains all the information about the state of a dynamical system at any instant (left). For a simple pendulum, two numbers—velocity and position—are all you need to know.
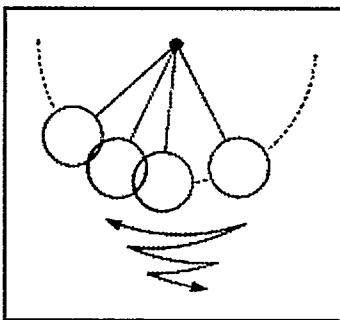
The points trace a trajectory that provides a way of visualizing the continuous long-term behavior of a dynamical system. A repeating loop represents a system that repeats itself at regular intervals forever.

If the repeating behavior is stable, as in a pendulum clock, then the system returns to this orbit after small perturbations. In phase space, trajectories near the orbit are drawn into it: the orbit is an attractor.

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57. *See id.* at 136. Reprinted by permission of the author.

B. *Attractors Are Shapes in Phase Space That Attract Nearby Trajectories*

"Some systems settle down to what's called an attractor[58] in their corner of phase space. An attractor is the phase portrait of a stable system. If the swinging of the pendulum were stable (like the spring-driven pendulum on a clock), its phase portrait would be a circle. Furthermore, if someone bumps the clock, the spring-driven pendulum may swing further to the right than it usually does, thus creating a series of points in the phase portrait that deviate from the circle, but because the system is stable (nature's kinetic energy sapping frictional forces being balanced by the energy restoring force of the unwinding spring), after a few deviant swings, the pendulum would swing with the same behavior it had before the bump, and so trace the same prior phase portrait. The circle for the spring-driven pendulum is called an attractor because any deviations from it, within certain parameters, do not have an effect on the long term trajectory. Sure, for a few literal swings or cycles of the phase plane portrait the behavior/shape might be different, but then the behavior returns to its normal shape. However, the timing of the cycle is different than if the pendulum had not been temporarily perturbed. Same attractor, different timing, indicates that a phase shift occurred. The take home point is that attractors attract behaviors that start at points nearby, and are therefore stable since forces within the system are in a sort of balance. In the pendulum example I just gave you, then, the circle formed by the pendulum is an attractor because in a stable system, despite small perturbations which cause slightly differing trajectories, all nearby trajectories are drawn to the circle."[59]

"But I assumed that the clock in the above example had a perfect, energy-balanced, spring-driven pendulum that never slowed down," Tortuffe continued. "A real pendulum that is not pumped to keep it constantly swinging will have a different attractor describing its long term behavior—a single point! The initial time of its phase portrait would be a spiral because nothing balances the frictional force. Thus, the pendulum loses velocity and distance on its swing on each cycle. Eventually it swings no more, nothing changes, and the point that represents this is the center of the spiral: $x = 0$ because there is no left to right and back again change of position, and $y = 0$ because the velocity is 0. We can see that without the spring drive, the attractor changed. In this case, the attractor

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58. *See id.* at 134, 269; *see also* STEWART, *supra* note 22, at 108-10.
59. *See* GLEICK, *supra* note 46, at 136.

is a point in phase space that draws nearby points to it, and again it is stable in the long run."

"Note that though the attractors attract points near to them and will recover from small perturbations, the attractor can be changed by an intrusion from outside the system. For instance, if a wrecking ball strikes the clock, the pendulum will fly with uncharacteristic velocity and position to a point it has never been before (both literally and in phase space), perhaps roll slightly, then stop. We see that the greatly perturbed system sends the pendulum to another, distant, point attractor."

"What do you mean, 'settles down' and 'attractor'?" Alexis asked.

"I mean that a dynamic, complex system that is mapped in phase space is attracted to one point or to a few distinct points in the phase space. For instance, a periodic behavior, by that I mean a set of motions that repeats itself exactly again and again over time—like the spring-driven pendulum—forms what is called a *limit cycle*[60] when mapped in phase space. The limit cycle is an attractor because if it is perturbed such that points in phase space tracing the action of the pendulum deviate from it, the pendulum will be attracted back to the same limit cycle thus swinging with its characteristic motion again—but perhaps phase-shifted relative to where it would have been in its behavior cycle had it not been perturbed. Remember, the spring-driven pendulum derived its stability from a balance struck between the energy gained by the spring drive and the energy lost by friction. When forces do not strike some sort of balance, instability is a likely result. For instance, a compass needle can point north or south. Pointing the needle north is a steady state or equilibrium point, and the needle returns to north after most any perturbation from north. A steady state also exists for a south-pointing needle, but this state is unstable and any perturbation will dislodge the needle, which will then swing back to north. Thus, the stable steady state attracts nearby points while the unstable state repels nearby points."

C. Separatrices Separate Attractors' Volumes of Influence from Each Other

Tortuffe paused and thought of an example, "imagine two limit cycles near to each other, one on the left and one on the right. You know that each one attracts nearby trajectories. As they are near each other, there must be a divider that dictates if an incoming trajectory falls into the attractor on the right, or that on the left. There is and this divider is

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60. *See* STEWART, *supra* note 22, at 101-03.

called a *separatrix*.[61] (See Figure 4). You can think of a separatrix as a thin island in the middle of a river viewed from above. Two flow lines right next to each other can be pulled apart to flow on opposite sides of the island as it acts to divide the river. Remember though, with separatrices and attractors we're discussing trajectories in phase spaces that describe the behavior of systems, not water. The point is that in our example the separatrix, or island, is a line in two dimensions whereas separatrices for systems of greater than two dimensions are actually surfaces."

Figure 4: A Limit Cycle, a Point Attractor, and a Separatrix

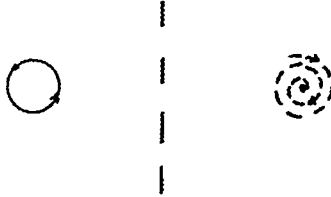Limit Cycle   Point Attractor
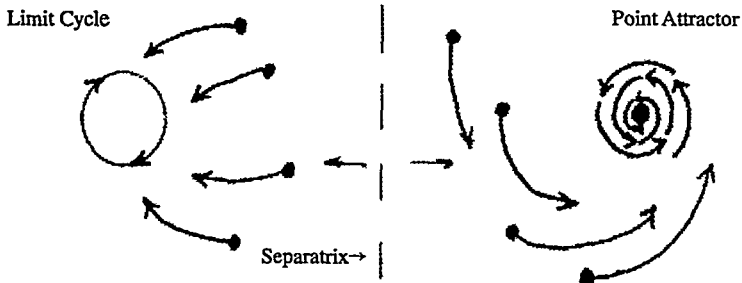
Separatrix→

Pairs of trajectories with starting points side by side but on opposite sides of a separatrix move toward different attractors.

"The pendulum example showed that a momentary energy influx will create noise in the attractor, basically increasing its volume during the period the extra energy is applied, and for a time after until the behavior settles down in the attractor again. If enough extra energy is put into the system, the behavior can stray far enough from the attractor that it is no longer under the attractor's influence. Now, if it is under a new attractor's influence, the system will fall into the behavior described by the new attractor. (See Figure 5, which illustrates this point). The influence dividing line—or surface if the system is at least tri-dimensional—is the separatrix. So, for instance, if enough energy was infused at the right moment into the system whose attractor is on the left, its behavior would be stretched away from the attractor, across the separatrix, and then fall into the attractor on the right!"

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61. *See id.* at 100.

Figure 5: An Attractor Being Stretched Enough That Its Trajectory
Crosses a Separatrix and Falls into a Different Attractor

A trajectory will settle into a new attractor if it is stretched far enough away from its first attractor.

The nonperturbed system is on the left and it remains on its limit cycle attractor. On the right is the point attractor the system will fall into if its trajectory crosses the separatrix.

If the system is perturbed slightly, the trajectory changes some but stays on the original attractor.
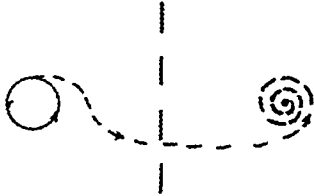
If the system is perturbed greatly, it jumps to a new attractor because the trajectory crossed the separatrix.

D.   The Train Station Constituted a Stable System Whose Behavior Could Be Modeled by an Attractor in Phase Space of Many Dimensions

"Consider a system," Tortuffe said, "made of the train station and the train. This system is closed off from the outside world, and there is a phase portrait of many, many dimensions that describes its behavior throughout the day. The behavior of the system follows a general pattern and each of the players, or variables, has its own 'dimension' that gives input to where the point representing the system as a whole is at any one time. But the system will never be in exactly the same state. Some of the variables never change: the platform does not move, the scales do not

move, the train makes a circuit and stays on its track. However, other variables *do* change: the train starts and stops, and has velocity. Also, there is the noise. For example, the train never stops in exactly the same spot. For this reason, among other noise, the line tracing the behavior of the system in phase space will never exactly repeat itself, but the line will stay within some shape in phase space. This is the attractor. The wider the variation in the variables (like the more noise present) the larger the volume of the attractor. Or, if you can't see volume because you are working with paper, you can say the width of the attractor is an indication of the amount of noise present in the system."

"Remember," Tortuffe continued, "any attractor is stable. So, the station's behavior is stable: train arrives, train departs, station superstructure remains the same. Outside influences like people boarding and the wind blowing are minor perturbations and thus create minimal noise in the system. So, the shape of the station's attractor will remain the same in spite of minor perturbations intruding from the outside and in spite of minor changes in the system—like where the train stops each time it comes around."

"The responsibility of the railroad and of the engineers was to design and maintain a stable system so that it could be depended on to be safe. *As we'll see, their responsibility was to maintain the system within safe parameters—that is, to keep the noise in the system low enough that the attractor's volume did not increase to such a point that people would be endangered.*"

E.   When the Attractor Volume Increases Enough Due to Noise, It Endangers People

"If we consider the system to be made up of subsystems, each with their own attractors, then we see that the superstructure of the station, including the scales, should be settled at point attractors: they should not be moving! This rule that they cannot move is part of the shape of the overall attractor of the system. Particularly, the tiles that struck Mrs. Palsgraf only had two possible steady states—each represented by a point attractor. One was the original steady state on the roof, and the other was the steady state of lying on the platform which would result from being dislodged from the roof. Essentially, the tile is similar to the light switch mentioned earlier, it's either 'on' or 'off' the ceiling."

"*What could cause an element of the overall attractor to 'jump' out of its usual behavior—in this case for the scale to leave its point attractor on the roof and relocate to a new point attractor?*" Tortuffe swal-

lowed and caught his breath before continuing in excitement. "Remember, the phase space that the train station's attractor encompasses is representative of all the possible combinations of the variables in the station system. One of these combinations is the one where the tile is at the same position as Mrs. Palsgraf! Having tiles at the same coordinates as peoples' heads is obviously not part of the original, safe, attractor—somehow it changed to encompass this dangerous situation, so what happened? The best way to visualize how this danger zone and the 'safe' attractor relate to each other is to extract the dimension describing the behavior of the offending roof tile from the phase space. This allows us to place a point in the new phase space that represents the tile being at the same position as Mrs. Palsgraf's head. If the trajectory describing the station's behavior passes through this point, then Mrs. Palsgraf has a problem! The easier it is to reach this condition, the closer this danger point is to the attractor. This means it takes less energy to knock the behavior of the station into such a state that most of the variables behave normally, but the tile comes to be in the same place as Mrs. Palsgraf's head. Effectively, the point represents a separatrix for the tile's behavior. If enough energy is put into the station that its attractor's volume is stretched across this point, then the tile slides to a new point attractor—the ground—and passes through her head on the way! The salient point is that it passed through the seven feet of space above the platform that should only be for patrons to occupy." (See Figure 6).

Figure 6: The Idealized Station Attractor, One with Some Noise and the Tile Separatrix Point, and One in Which the Noise Has Stretched the Volume Across the Tile Separatrix Point and the Scale Falls

If the station's attractor is stretched far enough by noise, the trajectory may encounter a danger point.
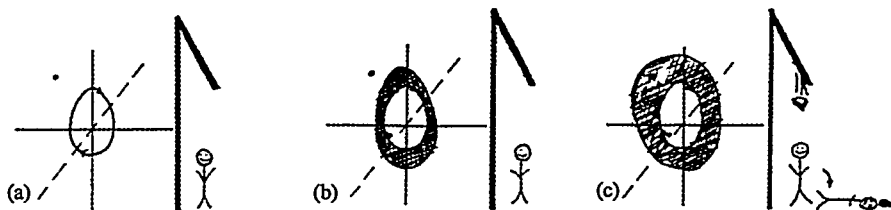
Part (a) shows a non-realistic (we don't know what the real attractor would look like) station attractor with no noise. Part (b) shows the same attractor with an everyday, safe amount of noise broadening the attractor with the tile separatrix point nearby in phase space. Mrs. Palsgraf is happy in part (b). Part (c) shows the attractor after a perturbation has caused the attractor to widen so much that it stretched across the tile separatrix point. The scale fell and passed through Mrs. Palsgraf's head!

"In Palsgraf the outside perturbation was explosives and because of the explosion, the scales were knocked from their point attractors which had previously held them in place. Thus, *the system was de-stabilized."*

F. Because the Railroad Let the Attractor Stray to the Tile Separatrix Point or Other Danger Points and Because it Could Have Prevented the Straying or Minimized its Liability but Did Not, it Should Have Been Held Liable

"So," Tortuffe concluded, "the way we have visualized the station as a system shows there are two things the railroad company could have done to protect its patrons from physical harm and to protect itself against liability. The first is to have shaken the roof (and the rest of the station) with random noise. This would simulate all small amplitude perturbations and thus would cause the attractor's volume to expand enough to encounter any nearby danger points. If instabilities like falling tiles had shown up during small amplitude random noise tests, the offending structures would have been strengthened before the station opened. The second point is that there are danger points far beyond the station's attractor and these can't be tested for. If a strong force is applied to the station, the behavior will probably change significantly, thus altering for some period of time the trajectory of the line that has traced the previously safe attractor. Many of the changes that can happen to the station are harmless, and are not danger points. However, some are. If the trajectory hits these, then injury is probable. This translates into the idea that the station should have tried to prevent large amplitude perturbations such as those caused by flammables, explosives, tigers, etc. Or they should have secured the station in such a way as to withstand such perturbations."

"A good analogy to help visualize the danger points is to think of a game played on a field littered with holes in which the objective is *not* to hit a golf ball into a hole. It is night, you have a flashlight and a wheelbarrow full of dirt, but you are not allowed to take the flashlight out of a three foot diameter circle in the center of the field. This circle has no holes in it and is analogous to a smoothly running station. Noise is represented by hitting the ball with a club. If hit lightly, the ball stays in the circle and never drops into a 'danger' hole. If hit with medium force, the ball stays in the range of the flashlight beam. If hit with heavy force, the ball leaves the range of the beam. Either way, if the flashlight is not used, the ball could fall into a hole. If the flashlight is shone from the perimeter of the inner circle, it will reveal the holes within its beam

range and they can be plugged with the dirt—then the ball can be hit with medium force and not fall into any 'danger' holes. This is like testing a structure with random, low amplitude noise to see if any parts fall down or otherwise become dangerous, and then addressing these problems prior to opening the station. The railroad company should have tested the station with low amplitude noise. If the ball is hit with heavy force, it leaves the range of the plugged holes and could fall into any of the unseen holes. This is analogous to subjecting the system to a level of noise that is too high to test. Because tests weren't run, the 'danger' points weren't found and strengthened and therefore the station could 'fall' into them. The railroad company should not have allowed classes of objects that could generate heavy noise onto the train platform."

Alexis appeared puzzled. "Is there a workable rule of tort law that can be derived from the ideas of phase space and attractors?"

"I think so," Tortuffe said. "I propose that those assigning liability based on causation consider the components involved in a case as a dynamic system that has a set of usual behaviors. They should then make a list of the classes of objects, products, behaviors, etc., that would destabilize the system. Finally, *ask if it would be reasonable—without the benefit of hindsight—to have foreseen problems* with the classes of things and so not to have allowed them into the system, or to have taken preemptive steps to prepare for them. If the answer is 'yes,' then there is proximate cause, and liability should be assigned."

"Haven't we determined," Alexis said, "that in light of chaos theory and phase space, the result in *Palsgraf* could have come out either way, with the railroad liable or the railroad not liable."

"Yes," Tortuffe stated.

"But how do we decide which way to assign liability?" Alexis said.

"It would be helpful," Tortuffe said, "to consider the important aspects of chaos theory and how they relate to the fact pattern in *Palsgraf*."

"But I am not a scientist!" Alexis protested.

"And I am not a lawyer," retorted Tortuffe.

"All right," Alexis said, not wanting to let the young scientist have center stage all to himself. "I'll take the easiest tenet first."

"Which one is the easy one?" Tortuffe asked.

"Nonlinearity is the easiest to explain and apply," Alexis said, "because it is so obviously correct."

"Spoken like a true lawyer!" Tortuffe said.

"Well," Alexis said, crossing her arms. "If you're going to make fun of me, I won't say anything."

"I wasn't making fun of you. Please continue." Tortuffe leaned back in his chair, looking at Alexis with dual interest.

"In a world with a very large number of variables, it would be a mistake to attempt to understand how all of these variables interact using linear notions of philosophy and mathematics. Proximate cause, which attempts to establish a causal nexus or link between one variable and other variables, doesn't account for the limits of the linear notions on which it is based. Linear mathematics cannot accurately predict the interaction of more than two variables. Thus, proximate cause, which attempts to link the behavior of many variables, is an outdated concept when used to achieve this aim."

"Yes," Tortuffe interrupted. "That's what I was trying to say. It's comparable to the superstition of witchcraft that was widely held in the seventeenth century in the American northeast. At that time many believed it made perfect sense, but today most people see the flaws in the witchcraft superstition and find it inadequate to explain actual events. It is the same with linear dynamics."

"We are laboring under an incorrect notion," Alexis said.

"We *were* laboring," Tortuffe said.

"Yes," Alexis agreed. "And nonlinearity also means that in the context of causation, the element of foreseeability is drastically changed."

"Exactly," Tortuffe said. "And why is foreseeability so changed?"

"What is foreseeable cannot be predicted in a linear sense," Alexis said. "Foreseeability implies linear predictions that are based on viewing one variable as acting on another, and this other variable on yet another, and in turn another. But reality, as understood in light of chaos theory, is not so simple. We know that linear notions of the predicted interaction of these variables are handicapped by an inability to accurately predict the behavior of many variables. It becomes a guessing game for the finder-of-fact to determine what was foreseeable, because in many cases there is no linear way for the tortfeasor to understand the consequences of his or her acts. As we discussed, linear dynamics is inadequate to understand the interaction of three bodies, let alone the numerous bodies that are relevant to the causation inquiry in many tort cases."

"Nonlinearity also applies to the other limitation of proximate cause that you mentioned," Tortuffe said.

"Which limitation is that?" Alexis asked.

"You said that proximate cause is limited to risks that are foresee-able, but also to consequences of a negligent act that is 'of the same general sort that was risked.' "[62]

"Yes," Alexis said. "That is another limitation which is an alternative to the foreseeability limitation."

"So," Tortuffe said, "how do you know 'the same general sort that was risked' if you don't adequately understand cause and effect and how one variable interacts with other variables?"

"I admit," Alexis said, "that 'the same general sort that was risked' is a rather amorphous concept in light of nonlinear dynamics."

"You see," Tortuffe said, "the law is trying to understand a non-linear world in a linear way. It is applying outdated concepts and it is arriving at wrong answers."

"Yes," Alexis said. "I see what you mean."

"But more on that later," Tortuffe said. "We also need to discuss how sensitivity to initial conditions and attractors apply to the doctrine of proximate cause."

"Alright," Alexis said. "Sensitivity to initial conditions applies to complex systems like the weather."

"It also applies to complex systems like reality," Tortuffe offered. "Sensitivity to initial conditions applies to any real situation in which there are more than two material bodies acting on one another."

"Yes," Alexis said. "The so-called Butterfly Effect tells us that small changes in initial variables can cause great changes throughout the system, and that those changes can grow geometrically, given a sufficient number of variables and sufficient interaction between the variables. A state of variables which seems to have a stable pattern or a perceived pattern can be upset and dramatically changed by a small change in one of the variables in the system, or from the circumstances in which that variable interacts with the others."

"Yes," Tortuffe said. "So a causation inquiry must consider that a certain system can be upset by a change in an initial condition, and that change is very much the cause of later changes within the system. Thus, the causation inquiry must include an analysis of sensitivity to initial conditions and also take into account that in the real world, changes, though not capable of being understood in a linear way, can be under-stood and, indeed predicted within the limits of chaos theory, or complexity theory."

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62. *In re* Kinsman Transit Co., 338 F.2d 708, 724 (2d Cir. 1964).

"And attractors are present in the system," Alexis said, "so that, over time, by plotting and examining a phase space portrait of the system, one could see that certain changes in initial conditions would cause great change throughout the remainder of the system. This change could be mapped and understood in a general sense. There would, then, be a way of seeing how a slight change in an initial condition would affect the behavior of the system—displaying cause and effect. Therefore, cause and effect could be mapped and seen as a pattern settled down among certain attractors which are discoverable if we could observe the system over time, and also observe the way that altering initial conditions would affect the system. The challenge is that the situation in most torts, and hence the variables to which a causation analysis must be applied, is unique in each case, and, in a practical sense, non-repeatable for experiment and determination of pattern in phase space. Hence, though we know that cause and effect are linked in a nonlinear sense, we cannot formulate the patterned behavior of the system over time."

"A causation inquiry must consider these factors," Tortuffe continued. "We can see that even in complex systems there are patterns and slight changes in initial conditions that can produce many different effects. In a linear sense these effects might be considered stochastic; that is, random or freak occurrences. Yet, when understood in light of complexity, or chaos theory, these effects which at first appear so random, are in reality part of a pattern with a limited range that may even be mapped as fractals as is the case with 'strange' attractors."

"I can see how chaos theory applies to the doctrine of proximate cause," Alexis said. "The components in the train station make up a dynamic system. Some components have a set of usual behaviors. Other components may enter the system and make it unstable. A judge should ask if it would be reasonable to expect the defendant to have foreseen the particular problem caused by the destabilizing component. If the problem was foreseeable, steps should have been taken to prevent the destabilizing component from entering the system, or to prepare for the destabilizing component. If the problem should have been prevented, proximate cause exists, and liability should be assigned. Now I think of *Palsgraf* in an entirely different way."

"Yes," Tortuffe said, "you now see *Palsgraf* as a set of many variables in a nonlinear and complex world."

"A world," Alexis said, "which proximate cause does not accurately or adequately describe. There are many variables in the *Palsgraf* system: the physical structure of the train station as it existed at the time

of the accident, the train, the porters, and the man who dropped the fireworks while attempting to board the train. There are countless other variables; the weather conditions, for example. And, last but not least, Mrs. Palsgraf herself, and her location vis-à-vis the other systems."

"Proximate cause," Tortuffe said, "and the linear understanding of the world on which it is based, cannot accurately describe what happened."

"That's right," Alexis said. "And the foreseeability test, where one is negligent when one fails to perform a duty to another to whom there is foreseeable risk, is of very limited use in a complex system. It's quite true that the man's inadvertent dropping of the fireworks onto the railroad tracks while he was pushed and pulled onto the train by the porters, did in fact cause the injury to Mrs. Palsgraf. *But for* the dropping of the fireworks, the injury would not have occurred. Yet the court in *Palsgraf* concluded that the chain of cause and effect was too long and tenuous; therefore, the railroad's porters could not have foreseen, nor predicted, the consequences of the dropping of the fireworks. But isn't it their *prediction* that was flawed? Wasn't the prediction a nonlinear one, one that didn't take into account the web-like structure of a complex system? After all, the cause and effect *did* occur."

"I agree with you," Tortuffe said. "Your notion of the web-like structure of a complex system is instructive because it implies the Butterfly Effect, or sensitivity to initial conditions. The system in *Palsgraf*, including the train station, and the human actors within the station, changes when one of the beginning variables changes. So, when the man was pushed onto the train and the fireworks fell and exploded, the system was changed throughout and was changed geometrically. And because of feedback mechanisms, the initial conditions would change as a matter of course. In light of chaos theory, it could be said that it is foreseeable that the system could change dramatically if one of the initial conditions changed even slightly."

"But," Alexis said, "the exact contours of that change could not be understood even with a grasp of chaos theory, unless the variables could be experimented with over time. The experimenter would change initial conditions and the resulting changes in the behavior of the system, if any, could be mapped in phase space. In this way, the underlying pattern that would congeal around the system's attractor could be viewed, and at least some of the initial conditions that could cause the system to deviate from its attractor would be found."

"Yes," Tortuffe affirmed. "If we could examine the system and the interaction of the variables over time, we could determine some of the ways in which Mrs. Palsgraf could have been injured when she was part of the train station system. And remember that the behavior of a complex system is constrained and limited, so the injury to Mrs. Palsgraf was not freak or random at all. Rather, her injury was within the possibilities of the system and, if the system as a whole could be examined in this rigorous fashion and the problems found fixed, the probability of her being injured could have been reduced. Obviously this depth of testing is untenable."

"The problem," Alexis said, "is that the courts must provide a workable rule, a rule that delineates exactly what kind of conduct will make a person liable for its results."

"But have courts been able to formulate such a rule?" Tortuffe asked.

"No, they haven't," she said. "Some courts use the foreseeability test articulated by the majority in *Palsgraf*.[63] That is, you are liable for the foreseeable consequences to a person to whom you owe a duty of care.[64] But you can only identify such a person through a foreseeability test. The majority opinion stated that a duty is owed to someone within the zone of danger."[65]

"And we've seen," Tortuffe said, "that in light of chaos theory such foreseeability—at least in a linear sense—is inaccurate."

"That's correct," she said, flipping the pages of her casebook. "But Judge Andrews' dissenting opinion in *Palsgraf* advocates a different test. He said that duty 'does involve a relationship between man and his fellows, but not merely a relationship between him and those whom he might reasonably expect his act would injure; rather, a relationship between him and those whom he does in fact injure.'[66] This test has been called the hindsight, or direct causation approach, because even though there may not have been a foreseeable plaintiff who was injured because of the defendant's conduct, there was in fact an injury that was caused by the conduct."[67]

"That sounds like a more realistic test," Tortuffe said.

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63. Palsgraf v. Long Island R.R. Co., 162 N.E. 99, 99-100 (N.Y. 1928).
64. *See id.* at 99.
65. *See id.* at 100.
66. *Id.* at 102 (Andrews, J., dissenting).
67. *See* WILLIAM L. PROSSER ET AL., CASES AND MATERIALS ON TORTS 338 (8th ed. 1988).

"It's true that the hindsight approach of that part of his dissenting opinion would enlarge the ambit of liability," Alexis said. "However, he continued to use linear notions when he wrote that the question of proximate cause was whether 'there was a natural and continuous sequence between cause and effect.'[68] And, of course, by *natural* he meant a *causal chain* that is linked in a way commonly understood by the common person."

"So," Tortuffe said, "if the common person could see the links in the causal chain, then it was a natural and continuous sequence?"

"Yes," she said.

"Andrews, then, because he wrote the dissenting opinion, disagreed with the result the majority reached," Tortuffe said.

"Yes," she said. "Judge Andrews would have found the defendant railroad liable for Mrs. Palsgraf's injury."[69]

"Bravo, Judge Andrews!" Tortuffe said. "Although, a causal chain will not always be natural and continuous in a linear sense."

"You couldn't expect him to be entirely accurate, Tortuffe," Alexis said. "The case took place a long time ago."

"I suppose so," Tortuffe said. "But today? Do the judges today use Andrews' approach?"

"As I said before," she said. "Some use the foreseeability approach, some use the hindsight or direct causation approach."[70]

"Then there isn't a rule that is 'workable,' as you put it," he said.

"No," she said, "there isn't. In fact there are numerous problems today with proximate cause in more contexts than the one we've discussed in *Palsgraf*. For example, some have complained that proximate cause is inadequate to assign tort liability because of its capricious nature, and because the two real questions in a tort case are: 'Who did it?' and 'Who should pay?'[71] Proximate cause has also proved problematic in the toxic and latent tort context where causation is circumstantial and based largely on statistical probability."[72]

"But," Tortuffe said, "despite the defects of proximate cause, it is still a commonly used tool in tort law."

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68. *Palsgraf*, 162 N.E. at 104 (Andrews, J., dissenting).

69. *See id.* at 105 (Andrews, J., dissenting).

70. *See supra* notes 63-67 and accompanying text.

71. *See, e.g.,* Kenneth Vinson, *Disentangling Law and Fact: Echoes of Proximate Cause in the Workers' Compensation Coverage Formula,* 47 ALA. L. REV. 723, 739-40 (1996).

72. *See, e.g.,* John M. Kobayashi, *Too Little, Too Late: Use and Abuse of Innocuous Yet Dangerous Evidentiary Doctrines,* C607 ALI-ABA 1127, 1155-56 (1991).

"Yes," Alexis said, "I'm afraid so. But at least we've shown its weaknesses. Now that we have applied chaos theory to the doctrine of proximate cause as articulated in *Palsgraf*, we can see the doctrine's limitations and why a new understanding of causation is necessary to arrive at a better understanding of events and to formulate a more just tort system. We have seen that, depending on how the court decided to draw the line of proximate cause—using traditional linear thinking, using an understanding of chaotic dynamics, or by using an understanding of classical statistical mechanics and phase space analysis—*Palsgraf* could have come out either way; that is, with the railroad liable or the railroad not liable. I think that such a significant part of tort law should not be based on an inadequate understanding of events—on such a whimsical rule."

"Certainly not," agreed Tortuffe. "What the legal doctrine *should* influence is the defendant's choice of parameters."

"That's right," Alexis said. "If the defendant's parameters create an attractor that poses an unacceptable risk of harm—or at least a harm the defendant should internalize—then the law should make the defendant liable even if the particular outcome was not foreseeable."

"Let's try to apply this theory we have developed to some of the cases that rely on *Palsgraf*," Alexis continued. "My professor gave us some examples for discussion in class."

"Sounds great," Tortuffe replied. Alexis shuffled through her three-ring binder, opened the rings, and pulled out several sheets of paper.

"Three-hole punched, I see," quipped Tortuffe. "How nice."

"I have to," Alexis replied. "I am too disorganized otherwise." She was silent for a moment as she read quickly through a case. "Here we go—*Derdiarian v. Felix Contracting Corp.*[73] The case says that the plaintiff, Derdiarian, was an employee of the defendant subcontractor.[74] He was working in an excavation site where a gas main was being installed beneath the street surface.[75] When he was working, an epileptic motorist who had failed to take his medication suffered a seizure.[76] The motorist lost control of his automobile and careened into the excavation site.[77] The motorist's automobile stuck the plaintiff and also caused a kettle of hot enamel to spill on to the plaintiff.[78] The plaintiff was burned severely.[79] It

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73. 414 N.E.2d 666 (N.Y. 1980).
74. *See id.* at 669.
75. *See id.*
76. *See id.*
77. *See id.*
78. *See id.*
79. *See id.*

was also established that the plaintiff had placed the kettle of hot enamel on the oncoming traffic side of the excavation site."[80]

"Did the jury award damages to Derdiarian?" Tortuffe asked.

"Yes, it did,"[81] Alexis replied.

"And what did the appellate court decide?" Tortuffe asked.

"I haven't looked yet," Alexis stated. "I think we should see what the result would be under our theory first."

"Alright," Tortuffe said. "It would seem to me that the contracting company created a system, or at least allowed a system to be created, that posed an unacceptable risk even under the linear approach in *Palsgraf*. The contractor could have reasonably foreseen that a car would enter the excavation site and cause injury to the workers performing their duties in the site. The contractor may not have been able to predict that the driver would be an epileptic suffering a seizure, but easily could have anticipated an out-of-control vehicle. Liability should therefore be imposed."

"But that raises two further questions," Alexis said. "First, if the contractor could have foreseen an automobile entering the trench, should not the plaintiff have foreseen the risk as well? If so, he must be at least partly at fault for his injuries. Indeed, he both entered the excavation site and placed the hot enamel on the oncoming traffic side of the trench. Second, if the linear analysis of *Palsgraf* clearly suggests liability, does that necessarily suggest liability under the approach we have developed today?"

Tortuffe thought quietly for a brief moment as he took a sip of his coffee. "My coffee has become cold," he said. He walked slowly to the bar and bought another cup. Returning to the table, he proclaimed, "I will answer your second question first. It appears to me that if a result can be predicted linearly, it necessarily can be predicted under our theory of proximate cause based upon nonlinear dynamics."

"And why is that?" Alexis asked. She thought that Tortuffe looked a bit smug and conceited at this point.

Tortuffe responded professorially, "When a person creates an attractor that poses an unacceptable risk of harm, that person is liable under our theory of proximate cause even if the particular harm is not foreseeable. If harm is foreseeable under linear dynamics, it is necessarily fore-

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80. *See id.*

81. *See id.* at 668.

seeable under nonlinear dynamics as well. The only difference is that the process is easier in the former theory than in the latter."

"I guess I can accept that," Alexis said grudgingly. "But what about the first question?"

"Well," Tortuffe said. "The first issue relates back to the elements of negligence that you mentioned when we met. I believe you said that they were a duty, a breach of that duty, a causal link between the breach and damage or injury to the plaintiff."

"That's right," Alexis said.

"Our discussion has been limited to proximate cause," Tortuffe said. "Whether Derdiarian should be held responsible for his failure to foresee the accident is a question of duty rather than of proximate cause. I assume that a person has a duty to refrain from causing him- or herself harm."

"Yes," Alexis said. "That is the basis for contributory and comparative negligence. Contributory negligence is '[c]onduct for which plaintiff is responsible amounting to a breach of duty which [the] law imposes on persons to protect themselves from injury, and which, concurring and co-operating with actionable negligence for which [the] defendant is responsible, contributes to injury complained of as a proximate cause.'[82] Contributory negligence formerly was a complete bar to recovery.[83] The defense of contributory negligence, however, has been replaced by comparative negligence in many states.[84] Under comparative negligence, negligence is measured in terms of percentage, and recovery is diminished in proportion to the amount of negligence attributable to the person for whose injury, damage or death recovery is sought.[85] Of course, there are various intricacies and exceptions, but comparative negligence is now the majority rule among the states."[86]

"Are the elements the same as in normal negligence?" Tortuffe asked.

"Yes," Alexis replied. "The defendant must affirmatively prove that the plaintiff had a duty to herself, that she breached the duty, and that the breach caused all or part of her injuries."[87]

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82. BLACK'S LAW DICTIONARY 1033 (6th ed. 1990).
83. *See id.* at 282.
84. *See id.*
85. *See id.*
86. *See id.*
87. *See id.*

"Derdiarian had no duty here," Tortuffe said. "The contractor had a duty to provide a safe workplace. Further, the contractor was the only party that had the means necessary to act upon the duty. A court could not expect Derdiarian to provide barriers or other means of protecting the excavation site."

"I understand," Alexis said. "Derdiarian had no duty to secure the excavation site, so whether the accident was foreseeable as to Derdiarian is irrelevant."

"Right," Tortuffe said. "So what did the appellate court do?"

Alexis looked back to the papers in her hand. "The New York Court of Appeals affirmed the jury, which had apportioned liability between the contractor, the contracting party, and the driver.[88] The court said the fact '[t]hat [the] defendant could not anticipate the precise manner of the accident or the exact extent of the injuries . . . does not preclude liability as a matter of law where the general risk and character of the injuries are foreseeable.' "[89]

"That result seems to be consistent with our theory," Tortuffe said. "Do you have any more cases?"

"Let's see," said Alexis looking at her papers. "My professor also gave us *Peevey v. Burgess*.[90] She has a pretty good sense of humor and must have found this case to be a bit funny."

"What happened in the case?" Tortuffe asked.

"The court states the facts as follows," Alexis said, and began reading from the case.

On the morning of November 12, 1987, defendant drove his pickup truck to Sharrow Ford, Inc. ("Sharrow") for service. Defendant, a tobacco chewer, had attached a homemade spittoon to the emergency brake release handle under the dashboard of the truck. That morning the spittoon contained about six ounces of spit. After Sharrow mechanic Robert Shaff completed his work on the truck's alignment, he opened the driver's door to get a better view as he backed the truck off the service ramp. Shaff shifted the truck into reverse and bent to find the emergency brake release. When he pulled the handle and released the brake, the brake pedal popped up and struck the spittoon, spraying its contents into Shaff's face. As a result, Shaff's eyes burned and he became disoriented, lost control of the truck and fell out. Defendant's truck continued down the ramp and struck a vehicle being repaired by plaintiff Johnnie W. Peevey, a Sharrow employee, causing serious injury. Defendant acknowledged that, when the emergency brake was released, the brake

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88. *See* Derdiarian v. Felix Contracting Corp., 414 N.E.2d 666, 670 (N.Y. 1980). For a number of cases that may be analyzed through the framework presented in this essay, see *infra* the Table of Cases in the Appendix.

89. *Id.* at 671.

90. 596 N.Y.S.2d 250 (N.Y. App. Div. 1993).

pedal would spring up and strike the spittoon, ejecting its contents. Defendant stated that, because that occurred, he "paid attention" when he released the emergency brake.[91]

The trial court entered summary judgment in the defendant's favor, and the plaintiff appealed.[92]

"What should the appellate court have done under our theory of proximate cause?" Tortuffe asked.

"First, the court should consider the components of the relevant system," Alexis said. "In *Peevey*, the relevant dynamic system includes the defendant's truck, the defendant as its driver, the spittoon full of spit and the parking brake mechanism. As long as no part of this system is changed, there was no cause to believe that injury would result. However, the defendant should have foreseen that inviting another driver to take his place in the system would pose an unacceptable risk because of the unique nature of the interplay between the system's components. The defendant should have seen that the mechanic operating the truck was among the class of object, products, behaviors, etc. that would destabilize the system."

"The defendant should have foreseen the need to maintain the system's stability. He could have either removed the spittoon or warned the mechanic of the danger it posed," Alexis said. "The defendant had created an attractor that posed an unacceptable risk, and therefore had a duty to eliminate or minimize the risk associated with that attractor."

"Does this analysis change the result reached by the court?" Tortuffe asked.

"No. The court held that it could not say, as matter of law, that the events were unforeseeable,"[93] Alexis replied. "But even though we came to the same result, our theory provides more acceptable reasoning for the result, and the reasoning is the most important component of a legal decision."

"And so it is with physics," Tortuffe said. "The better one's reasoning in coming to a result, the better the predictive value when that reasoning is applied to another set of conditions."

"So maybe law and physics aren't all that different after all," Alexis summarily stated. Alexis gathered her papers and returned them to her binder.

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91. *Id.* at 251.
92. *See id.*
93. *See id.*

"Wonderful!" Tortuffe smiled. "I can see that you understand a bit of chaos theory and some classical statistical mechanics."

"And you understand proximate cause," Alexis said, "such as it is."

"Yes," he said, and was quiet for a moment. "So what do you do when you're not studying law?"

"Oh," she said evasively, "it's my first year, so I do nothing but study."

"Nothing?" he asked, his eyes watching hers. She would not look at him, snapping shut her knapsack, taking her coat from the chair and sliding her arms through the sleeves.

"I'm very busy," she said.

"Too bad," he said.

"Not that I want to be, you understand," she said. "I actually like to have fun."

"What do you like to do for fun?" he asked.

"My good Tortuffe," Alexis said disapprovingly, changing the subject, "I am beginning to get the feeling that you have other motivations besides explaining your precious physics and Socratic dialogue."

"Well, I would rather be Socratic than Platonic, if that is what you mean," Tortuffe smirked.

She parried quickly, "you know now that I think of it, I seem to recall a figure in French drama, with a name quite similar to yours . . . Tartuffe it was. He also was a wolf in sheep's clothing, a Trojan horse, just like you, with sneaky motives."

There was tension in the cybercafe. Both shifted uncomfortably.

Tortuffe pointed up at the television above the bar. "Do you like *Dirty Harry*?[94] I mean, for the camp of it?"

"Oh yeah," Alexis responded sarcastically, "as far as fascists go, he's simply dreamy."

They suspended their discussion and watched as the movie climaxed.

"Ah-ah, I know what you're thinking," Inspector Harry Callahan squinted as he spoke deliberately, sneering down the sight of his pistol. "You're thinking, 'did he fire six shots? Or only five?' Well, to tell you the truth, in all this excitement, I've kinda lost track myself. But being that this is a .44 Magnum, the most powerful handgun in the world and it would blow your head clean off, you've got to ask yourself one ques-

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94. DIRTY HARRY (Warner Bros. 1971).

tion. 'Do I feel lucky?' Well, do ya . . . punk?"[95]

Well, there was one more bullet. Moments later as the bad guy floated lifelessly on screen, Alexis stood up and met Tortuffe's gaze. "I bet that you thought by virtue of our cerebral exchange and your heroic display of physics you were going to endear yourself to me. But you were employing that darn linear mind-set, Tortuffe. You of all people should know that life is a lot more like Dirty Harry's handgun—deterministic chaos. Sometimes there just isn't any way to know if things are going to come out like you anticipate."

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95. *Id.*

APPENDIX: TABLE OF CASES

Case	Result	Defendant's Attractor	Plaintiff's Attractor	Chaos Result
Palka v. Servicemaster Management Servs. Corp., 634 N.E.2d 189 (N.Y. 1994)	Verdict for Plaintiff affirmed	Unstabilized—fan detached from wall	Unstabilized by Defendant, hit by fan	Verdict for Plaintiff
Derdiarian v. Felix Contracting Corp., 414 N.E.2d 666 (N.Y. 1980)	Verdict for Plaintiff affirmed	Unstabilized—car entered worksite, kettle of hot enamel	Unstabilized by Defendant, burned by split enamel	Verdict for Plaintiff
Ventricelli v. Kinney System Rent-A-Car, Inc., 383 N.E.2d 1149 (N.Y. 1978)	Verdict for Plaintiff reversed	Unstabilized—car trunk lid opened periodically, latch failure	Unstabilized by Defendant, hit by another car while closing trunk lid	Verdict for Plaintiff
Johnson v. State, 334 N.E.2d 590 (N.Y. 1975)	Verdict for Plaintiff affirmed	Unstabilized—telephoned wrong party regarding death of patient	Unstabilized by Defendant, suffered "anxiety neurosis" thought mother died	Verdict for Plaintiff
Dunn v. State, 277 N.E.2d 647 (N.Y. 1971)	Verdict for Plaintiff reversed	Unstabilized—Plaintiff harmed when inmate allowed to escape from high security mental institution	Unstabilized by Defendant, died in car crash caused by escapee	Verdict for Plaintiff
Williams v. State, 127 N.E.2d 545 (N.Y. 1955)	Verdict for Plaintiff reversed	Unstabilized—Plaintiff harmed when inmate allowed to escape from low security prison farm	Unstabilized by escapee's threats, died of brain hemorrhage	Verdict for Defendant, low security not a threat
Rosen v. Bronx Hosp., 127 N.E.2d 82 (N.Y. 1955)	Verdict for Plaintiff reversed	Stable—door operated normally (but Plaintiff attempted to introduce evidence of improper maintenance)	Unstabilized by Plaintiff's lack of hand-eye coordination, fingers detached in door	Verdict for Defendant
Flaherty v. State, 73 N.E.2d 543 (N.Y. 1947)	Summary Judgment for Defendant affirmed	Unstabilized—allowed inmate access to dangerous chemicals	Unstabilized by Defendant, acid poured over Plaintiff's head	Verdict for Plaintiff
Benenson v. National Sur. Co., 183 N.E. 505 (N.Y. 1932)	Dismissed	Unstabilized—allowed theft of documents	Unstabilized by Defendant, fraudulent bond issued in Defendant's name	Verdict for Plaintiff
Babcock v. County of Oswego, 644 N.Y.S.2d 958 (N.Y. Sup. Ct. 1996)	Summary Judgment for Defendant	Stable—broadcast tower remained operational	Unstabilized by Plaintiff's climbing and falling from tower, quadriplegic	Summary Judgment for Defendant
Rodriguez v. Oak Point Management, Inc., 618 N.Y.S.2d 772 (App. Div. 1994), rev'd, 640 N.Y.S.2d 868 (N.Y. 1996)	Denial of Summary Judgment	Unstabilized—uncontrolled dangerous activity in apartment building	Unstabilized by Defendant, hit by bullet	Denial of Summary Judgment
Cunillera v. Randall, 608 N.Y.S.2d 441 (App. Div. 1994)	Summary Judgment affirmed	Stable—drove car through hydrant's water spray	Unstabilized by Plaintiff's interaction with spray, rendered unseen by car, hit by car	Summary Judgment affirmed

Case	Court Action	Description	Outcome	Result
Batencourt v. Manhattan Ford Lincoln Mercury, Inc., 607 N.Y.S.2d 924 (App. Div. 1994)	Summary Judgment reversed	Unstabilized—car periodically leaked fluid and overheated	Unstabilized by Defendant, killed while inspecting problem, hit by truck	Summary Judgment reversed
Peavy v. Burgess, 596 N.Y.S.2d 250 (App. Div. 1993)	Summary Judgment reversed	Unstabilized—allowed uninformed operator to control "rigged" vehicle	Unstabilized by Defendant, hit by splitoon contents, chain reaction followed	Summary Judgment reversed
Amica Mut. Ins. Co. v. Town of Vestal, 594 N.Y.S.2d 918 (App. Div. 1993)	Summary Judgment for Defendants affirmed	Unstabilized—external gas pipe leaked after hit by lightning discharge	Unstabilized by Defendant, house filled with gas and exploded	Denial of Summary Judgment
Gerdowsky v. Crain's New York Business, 593 N.Y.S.2d 514 (App. Div. 1993)	Summary Judgment Denied in part	Unstabilized—vending machine toppled by 3rd party	Unstabilized by 3rd Party, vending machine fell on foot	Summary Judgment affirmed
Wagner v. Keniffic, 557 N.Y.S.2d 650 (App. Div. 1990)	Verdict affirmed	Unstabilized—hot water in urinal's cold water lines	Unstabilized by Defendant, burned by hot water after flush	Verdict affirmed
Trivino v. Jamesway Corp., 539 N.Y.S.2d 123 (App. Div. 1989)	Denial of Summary Judgment for Defendants	Stable—cosmetic puffs known to burn when exposed to heat	Unstabilized by Plaintiff wearing puffs, burned when ignited by stove	Summary Judgment for Defendants
O'Leary v. American Airlines, 475 N.Y.S.2d 285 (App. Div. 1984)	Denial Defendant's motion to Dismiss	Unstabilized—allowed passenger to become intoxicated	Unstabilized by Defendant, drunk passenger died by choking on airline food	Denial of Summary Judgment
Santiago v. New York City Hous. Auth., 475 N.Y.S.2d 50 (App. Div. 1984)	Summary Judgment for Defendant	Unstabilized—exterior door malfunctioned, difficult to open	Unstabilized by Defendant, unable to open door, hit by bullet	Denial of Summary Judgment
Bahan v. Green Bus Lines, Inc., 465 N.Y.S.2d 784 (App. Div. 1983)	Verdict affirmed	Stable—bus ran over a white half-gallon bottle, no damage to bus	Unstabilized by standing near bleach bottle, bottle hit and contents sprayed	Verdict for Plaintiff reversed