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Strategic Nurse Allocation Policies Under Dynamic Patient Demand

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STRATEGIC NURSE ALLOCATION POLICIES UNDER DYNAMIC PATIENT DEMAND

by

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ABSTRACT

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Several studies have shown a strong association between nurse staffing and patient outcomes. When a nursing unit is chronically short-staffed, nurses must maintain an intense pace in order to ensure that patients receive timely care. Over time this can result in nurse burnout, as well as dissatisfied patients and even medical errors. Improved accuracy in the allocation of nursing staff can mitigate these operational risks and improve patient outcomes. Nursing care is identified as the single biggest factor in both the cost of hospital care and patient satisfaction. Yet, there is widespread dissatisfaction with the current methods of determining nurse staffing levels, including the most common one of using minimum nurse-to-patient ratios. Nurse shortage implications go beyond healthcare quality, extending to health economics as well. In addition, implementation of mandatory nurse-to-patient ratios in some states creates a risk of under- or over-estimating required nurse resources. With this motivation, this dissertation aims to develop methodologies that generate feasible six-week nurse schedules and efficiently assign nurses from various profiles to these schedules while controlling staffing costs and understaffing ratios in the medical unit. First, we develop and test various medium-term staff allocation approaches using mixed-integer optimization and compare their performance with respect to a hypothetical full information scenario. Second, using stochastic integer programming approach, we develop a short-term staffing level adjustment model under a sizable list of patient admission scenarios. We begin by providing an overview of the organization of the dissertation.

Chapter 1 presents the problem context and we provide research questions for this dissertation.

Chapter 2 provides a review of the literature on nurse staffing and scheduling specifically from the Operations Management journals. We introduce the challenges of nursing care and nurse scheduling practices. We identify major research areas and solution approaches. This is followed by a discussion of the complexities associated with computing nursing requirements and creating rosters. Staffing requirements are the result of a complex interaction between care-unit sizes, nurse-to-patient ratios, bed census distributions, and quality-of-care requirements. Therefore, we review the literature on nursing workload measurement approaches because workloads depend highly on patient arrivals and lengths of stay, both of which can vary greatly. Thus, predicting these workloads and staffing nurses accordingly are essential to guaranteeing quality of care in a cost-effective manner. For completeness a brief review of the literature on workforce planning and scheduling that is linked to the nurse staffing and scheduling problem is also provided.

Chapter 3 develops a framework for estimating the daily number of nurses required in Intensive Care Units (ICUs). Many patient care units, including ICUs, find it difficult to accurately estimate the number of nurses needed. One factor contributing to this difficulty is not having a decision support tool to understand the distribution of admissions to healthcare facilities. We statistically evaluate the existing staff allocation system of an ICU using clinical operational data, then develop a predictive model for estimating the number of admissions to the unit. We analyze clinical operational data covering 44 months for three wards of a pediatric ICU. The existing staff allocation model does not accurately estimate the required number of nurses required. This is due in part to not understanding the pattern and frequency of admissions, particularly those which are not known 12 hours in advance. We show that these "unknown" admissions actually follow a Poisson distribution. Thus, we can more accurately estimate the number of admissions overall. Analytical predictive methods that complement intuition and experience can help to decrease unplanned requirements for nurses and recommend more efficient nurse allocations. The model developed here can be inferred to estimate admissions for other intensive care units, such as pediatric facilities.

Chapter 4 examines an integrated nurse staffing and scheduling model for a Pediatric Intensive Care Unit (PICU). This model is targeted to recommend initial staffing plans and schedules for a six-week horizon given a variety of nurse groups and nursing shift assignment types in the PICU. Nurse rostering is an NP-hard combinatorial problem, which makes it extremely difficult to efficiently solve real life problems because of their size and complexity. Usually, real-problem instances have complicated work rules related to safety and quality of service issues, as well as preferences of the personnel. In order to avoid the size and complexity limitations, we generate feasible nurse schedules for the full-time equivalent (FTE) nurses, using algorithms that will be employed in the mixed-integer programming models we develop. Pre-generated schedules eliminate the increased number of constraints, and reduce the number of decision variables of the integrated nurse staffing and scheduling model. We also include a novel methodology for estimating nurse workloads by considering the patient, and individual patients acuity, and activity in the unit. When the nursing administration prepares the medium-term nurse schedules for the next staffing cycle (six weeks in our study), one to two months before the actual patient demand realizations, it typically uses a general average staffing level for the nursing care needs in the medical units. Using our mixed-integer optimization model, we examine fixed vs. dynamic medium-term nurse staffing and scheduling policy options for the medical units. In the fixed staffing option, the medical unit is staffed by a fixed number of nurses throughout the staffing horizon. In the dynamic staffing policy we propose, historical patient demand data enables us to suggest a non-stationary staffing scheme. We compare the performance of both nurse allocation policy options, in terms of cost savings and understaffing ratios, with the optimal staffing scheme reached by the actual patient data. As a part of our experimental design, we evaluate our optimization model for the three medical units of the PICU in the "as-is" state.

In Chapter 5, we conduct two-stage short-term staffing adjustments for the upcoming nursing shift. Our proposed adjustments are first used at the beginning of each nursing shift for the upcoming 4-hour shift. Then, after observing actual patient demand for nursing at the start of the next shift, we make our final staffing adjustments to meet the patient demand for nursing. We model six different adjustment options for the two-stage stochastic programming model – five options available as first-stage decisions and one option available as the second-stage decision. Because the adjustment horizon is less than 12 hours, the current patient census, patient acuity, and the number of scheduled admissions/discharges in the current and upcoming shift are known to the unit nurse manager. We develop a two-stage stochastic integer programming model which will minimize total nurse staffing costs (and the cost of adjustments to the original schedules developed in the medium-term planning phase) while ensuring adequate coverage of nursing demand.

Chapter 6 provides conclusions from the study and identify both limitations and future research directions.

To

my parents,

my wife,

and especially my daughter and son

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Chapter 1

Problem Motivation & Statement of the Research Questions

1.1 Problem Motivation

1.1.1 Rising Healthcare Costs, Quality of Patient Care and Nursing Shortages

The enactment of the Affordable Care Act (ACA) in 2010 brought significant changes to U.S. health care policy (Altman and Frist, 2015). The legislation aimed to increase the number of individuals with health insurance, improve the quality of care, and alleviate seemingly inevitable increases in the cost of care (The Affordable Care Act, 2010). But, U.S. health care costs continue to rise, despite the advent of the Affordable Care Act (Patton, 2015). Recent estimates suggest that national health care expenditures increased between 5% and 6% in both 2014 and 2015, and are estimated at $3.2 trillion. These rates are substantially higher than inflation, and some experts suggest that similar increases will continue through 2024 (Bauchner and Fontanarosa, 2016). Nursing care is identified as the single biggest factor in both the cost of hospital care and patient satisfaction (Yankovic and Green, 2011). Several studies have shown that there exists a strong association between nurse staffing and patient outcomes. When a nursing unit is chronically short-staffed, nurses are forced to maintain an intense pace in order to ensure patients receive timely care. Over time, this can result not only in nurse burnout, but also in patient dissatisfaction and even medical errors. Improved accuracy in the allocation of nursing staff could mitigate these operational risks and improve patient outcomes.

Given the fact that wages and benefits for Registered Nurses (RNs) constitute a substantial portion of overall hospital costs, comprising approximately 25% of hospital operational costs (Maenhout and Vanhoucke, 2013b), hospital administrators have attempted to reduce nurse staffing as a means to reduce costs and increase profitability (Rivers et al., 2005). On the other hand, projections suggest that by 2020 approximately 36% of nursing positions in the United States will remain unfilled (Wright and Bretthauer, 2010). Buerhaus et al. (2009) suggest that the U.S. nursing shortage could reach half a million by 2025. Therefore, rising healthcare costs and increasing nurse shortages make cost-effective nurse staffing of vital importance (Kortbeek et al. 2015). The shortage of nurses has attracted considerable attention due to its direct impact on the quality of patient care (Punnakitikashem et al. 2013). This issue is expected to worsen, especially given the aging population of baby-boomers, who are also part of the nurse workforce. This has resulted in risk exposure for hospitals, including patient safety issues, the inability to detect complications, and potential mortality rate increases (Paul and MacDonald, 2013).

1.1.2 Complexity of Computing Nursing Requirements & Rostering

Staffing requirements are the result of a complex interaction between care-unit size, nurse-to-patient ratios, bed census distributions, and the quality-of-care requirements. The optimal configuration strongly depends on the particular characteristics of a specific case under study (Kortbeek et al., 2015a). In addition, Green et al. (2013) indicate establishing the appropriate nursing level for a particular hospital unit during a specific shift is complicated by the need to make staffing decisions well in advance (e.g., six to eight weeks) of that shift, as well as labor constraints dealing with the number of consecutive and weekend shifts worked per nurse, vacation schedules, personal days, and preferences (Miller et al. 1976, Wright et al. 2006). The management of the nursing workforce is typically seen as a multi-phase sequential planning and control process that basically consists of a staffing period, a shift scheduling effort, and an allocation phase (Maenhout and Vanhoucke, 2013). The decisions made in each phase of this hierarchical process constrain subsequent phases.

Maenhout and Vanhoucke (2013) define the staffing phase as a strategic, long-term budgeting phase that determines the quantity and mix of nursing resources. The shift scheduling phase focuses on the mid-term assignment of the budgeted nurses to workdays and/or daily work shifts (e.g., early, late or night shift). This shift assignment aims to satisfy the minimum coverage requirements while meeting time-related rules and practices (e.g., personal time requirements, contract stipulations, specific workplace conditions, national or state mandates) that define acceptable individual schedules for the nurses and the hospital. Burke et al. (2013) also indicate that creating rosters is a challenging search problem requiring the satisfaction of many constraints and the balancing of a variety of requirements. This time consuming and frustrating duty often falls to a head nurse who would rather be concentrating on their primary duty of caring for patients. Many scholars also underscore that regular rescheduling may also be required to deal with staff sickness and absences. The study suggests that computerized, automated rostering can remove the vast majority of this workload and create higher quality schedules that are fair, impartial and satisfy more preferences. Compliance with legal requirements can also be ensured, management statistics collected and monthly reports generated, all reducing paperwork (Burke et al., 2013).

Although nursing care is identified as the single biggest factor in both the cost of hospital care and patient satisfaction, there still remains widespread dissatisfaction with the current methods of determining nurse staffing levels, including the most common one of using minimum nurse-to-patient ratios (Yankovic and Green, 2011). In many hospitals, staffing levels are a result of historical development, given that hospital administrator lack the tools to base current staffing decisions on information about future patient demand (Kortbeek et al. 2015). According to Paul and MacDonald (2013) nurse shortage implications go beyond healthcare quality, extending to health economics as well. Inaccurate estimates of the nursing resources required to satisfy patient demand in a hospital environment could make this already-challenging problem worse. In addition, mandatory nurse-to-patient ratio methods implemented in some states, providing for simplification from a demand estimation perspective, create a risk of under- or overestimating required nurse resources. As a result of research demonstrating the positive impacts of higher nurse-to-patient ratios on the quality of care, patient safety, mortality, etc. (Aiken et al. 2010, Needleman et al. 2006), some states have made nurse-to-patient ratios mandatory. Even though high nurse-to-patient ratios may be a good strategy from a health quality perspective, they are not a strategy every hospital and state can possibly afford, and it is one that can also further exacerbate the nursing shortage (Paul and MacDonald, 2013). One shortcoming of this method is that it is its assumption that demand for services and the requirement for nurse resources in a hospital behaves in a linear manner, which is far from reality (Clancy, 2007).

1.2 Potential Benefits of Efficient Nurse Scheduling & Need for Decision Support Tools

Efficient, effective nurse scheduling can deliver significant benefits to healthcare environments. Burke et al. (2013) suggest that high-quality nurse rosters benefit nurses, patients and managers. Patients receive better healthcare if nurses are able to spend more time with them, and mistakes are less likely if nurses are not stressed, tired and overworked due to poorly crafted schedules. Improved rosters not only decrease nurse fatigue, but also help them to maximize the use of their leisure time and increase job satisfaction. From a management point of view, better and more flexible scheduling can improve nurse retention, aid recruitment, reduce tardiness and absenteeism, increase morale and productivity, and provide better patient service and safety. The direct result is cost reduction. Given the perspectives outlined above, decision support methods can help with efficient and effective nurse scheduling.

Many patient care units face challenges in accurately estimating the daily number of nurses needed. Analytical methods that complement intuition- and experience-based decisions on nurse staffing and workload would help decrease the unplanned last-minute scheduling requirements for nurses, and improve healthcare delivery through efficient nurse allocation. One factor making such estimates difficult is the lack of a decision support tool for understanding the distribution of admissions to healthcare facilities. We aim to statistically evaluate the existing staff allocation system of an ICU using clinical operational data, and then develop a predictive model for estimating the number of admissions to the unit. It is difficult to understand the pattern and frequency of admissions, particularly those admissions that are not known twelve hours in advance (i.e unscheduled admissions). In Chapter 3, we first show that these "unknown" admissions can be modeled via Poisson distributions. The purpose of this chapter is to provide a framework for accurately estimating the number of nurses required in Intensive Care Units (ICUs) on a given day. The model developed there is generalizable for implementation in other intensive care units.

1.3 Statement of Research Questions

1.3.1 RQ 1: Medium-Term Integrated Nurse Staffing & Scheduling

The enactment of the Affordable Care Act (ACA) resulted in more and sicker patients entering the healthcare system. This increases nursing workload, leading to higher risk of nurse burnout in an already short-staffed environment. With this motivation, we study integrated nurse staffing and scheduling in Intensive Care Units, a 7-day x 24-hour care environment facing unscheduled patient admissions with dynamic acuity levels. Our research objective is to construct staffing patterns, which specify the number of nursing personnel from various job profiles to be scheduled in the medical units and nursing shifts of a scheduling period. Our solution approach aims to reduce nurse staffing costs while balancing the under- and over- staffing risks, which will help mitigate nurse burn-out, improve patient outcomes and manage hospital staffing costs.

Nurse rostering is an NP-hard combinatorial problem that is extremely difficult to efficiently solve real-sized problems. Usually, real-problem instances face complicated work rules related to safety and quality of service issues, as well as rules about preferences of the personnel. In order to avoid the size and complexity limitations, we generate feasible nurse schedules for the full-time equivalent (FTE) nurses using algorithms. These algorithms will be used in the mixed-integer programming models developed in this work. Pre-generated schedules reduce the increasing number of constraints and the number of decision variables of the integrated model. Our optimization model recommends initial staffing plans and schedules for a six-week staffing horizon. This is based on a variety of nurse groups and nursing shift assignment types, for the medical units in the PICU. A novel methodology for estimating nurse workloads (due patient census, patient acuity, and activity in the unit) is also incorporated.

When the nursing administration prepares the medium-term nurse schedules for the next staffing cycle, one to two months before the actual patient demand realizations, target staffing levels for the upcoming nursing shifts are typically determined by a general average staffing level for the nursing care needs in the medical units. Using a mixed-integer optimization model, we evaluate fixed vs. dynamic medium-term nurse staffing and scheduling policy options for the medical units. In the fixed staffing option, the medical unit staff is fixed throughout the planning horizon. The dynamic staffing policy we propose uses historical patient demand data to suggest a non-stationary staffing scheme during the staffing horizon. We test the fixed staffing policy alternative using various staffing level options. Then, for the dynamic staffing alternative, we prepare a "heat map" of patient census, patient acuity, and admissions-discharges-transfers (ADT) in the medical units of the PICU, for example, and compare the performance of dynamic heatmap-based policy against the alternative fixed staffing policies. Chapter evaluates the performance of both nurse allocation policy options with the optimal staffing scheme reached by the actual patient data to study our first research question:

RQ 1: Do dynamic medium-term nurse staffing policies that use patient demand forecasts outperform the historically-employed fixed staffing policy for the intensive care medical units?

1.3.2 RQ 2: Controlling the Understaffing Levels in the Medical Units

As the nurse workload increases, overtime becomes more burdensome. In fact, nurses cite undesirable schedules and overtime as primary reasons for burnout (Aiken et al., 2002). Additionally, unsatisfactory working conditions and policies have contributed to higher turnover rates (Aiken et al., 2002; Cline, Reilly & Moore, 2003). Jones (2007) suggests that the cost of turnover in the United States is approximately 1.2-1.3 times the average annual salary for each vacancy. U.S. hospitals spend approximately $300,000 annually for every 1% increase in the turnover rate (Price Waterhouse Coopers, 2007). It is no surprise that some U.S. lawmakers have proposed legislation that limits the use of overtime and the number of patients to whom a nurse can be assigned. There are 21 states with restrictions on the use of overtime (American Nurses Association, 2011). When a nursing unit is chronically short-staffed, patient care is at risk. Over time, this can result not only in nurses burnout, dissatisfied patients, and even medical errors (www.americansentinel.edu).

Aiken et al. (2001) surveyed nurses in five countries and found that an increased workload causes basic nursing interventions with patients went undone during the shift. The inability to provide the required level of patient care was linked to lower job satisfaction and staff retention. High workloads and undesirable schedules are two major reasons causing job dissatisfaction (Punnakitikashem et al. 2013). Penoyer (2010) reviewed the literature on nurse staffing and patient outcomes in critical care units. The author examined the major nursing and medical literature for 1998 to 2008 articles focused on intensive care units or critical care populations. This review clearly demonstrates an association between nurse staffing in the intensive care unit with patient outcomes. Since patient safety is jeopardized when medical care units are understaffed, a scarcity of nursing capacity can lead to both costly staff sourcing from third party agencies, and to undesirable ad hoc bed closings in the ward (Kortbeek et al. 2015).

Kuntz et al. (2014) estimate the association between the occupancy levels that patients experience during a hospital stay and the probability of in-hospital survival. They suggest that when occupancy is very high, the ability to respond by exploiting staffing buffer becomes constrained. The authors suggest that the strain is passed on to employees, who are forced to ration limited resources to cope with excessive demand, and the associated stress impairs their cognitive abilities. High occupancy levels and stress lead to safety tipping points in hospitals. Neither the organization nor its clinical staff are able to absorb another increase in occupancy beyond the safety tipping point without significant deterioration of the quality of care. Empirical analysis from the Kuntz et al. (2014) article demonstrates that such tipping points exist. Mortality risk begins to increase significantly when occupancy levels exceed a tipping point of 92.5%. Burnout and the total workload experienced by nurses can usually be managed with scheduling shifts. Vericort and Jennings (2011) suggest that these shifts should limit nurse working hours, allow for enough breaks, and consider individual preferences. In fact, some hospitals offer flexible shifts with long recovery periods in order to retain nurses. The authors suggest that, in conjunction with efficient scheduling systems, hospital managers consider limiting the utilization rates experienced by nurses.

To mitigate nurse burnout and improve the appeal of ICU nursing, we incorporate "understaffing penalty" as a mechanism to control the understaffing in the medical units. We analyze how various levels of understaffing penalty affects outcomes in the medical unit. We also evaluate the impact of the number of available schedules (NAS) on understaffing ratios in the medical units. We explore whether there exists a saturation level for the NAS. To study these aspects of the medium-term nurse staffing and scheduling problem, our second research question is formulated as follows:

RQ 2: Can understaffing penalty cost be utilized as a mechanism to control the understaffing levels and possibly mitigate nurse burnout and medical errors?

1.3.3 RQ 3: Short-Term Nurse Schedule Modifications to Better Mimic the Patient Demand

Nurse schedules are constructed well ahead the occurrence of actual patient demand for nursing. In an environment where 30 to 70% of patient admissions are not known 12-hours ahead of the actual admission and where patient acuities are diverse, the nursing administration constantly face the challenge of adjusting the nurse schedules. When a medical unit is understaffed, staffing alternatives available to the administration include: (1) general nurse float pool in the hospital, (2) on-call nurses (i.e. FTE overtime and additional PRN hours) and (3) mandatory overtime. When the scheduled nursing hours exceed the hours required by current loads, the charge nurse can: (1) float the nurse to another unit, (2) reassign her to a later day in the same staffing horizon, (3) cancel the shift (and one of the following designations is used for the time off: vacation, personal day, holiday, or unpaid leave; Bard and Purnomo, 2005a). Each option listed above has its own unique cost implications. The central aspect of the short-term nurse schedule modification problem is the requirement of a very efficient solution algorithm. Practically, the charge nurse will run the solution algorithm at the beginning of each 4 to 8-hour shift and expect to have a solution in less than an hour, preferably in less than 10 minutes.

As detailed in Chapter 5, we conduct two-stage short-term staffing adjustments for the upcoming nursing shift. Our proposed adjustments are first conducted prior to each nursing shift, then following the observation of actual patient demand for nursing for the start of the next shift final staffing adjustments are made. Since the adjustment horizon is less than 12 hours, the current patient census, acuity levels of the existing patients, the number of scheduled admissions and discharges in the current and upcoming shift are known to the unit nurse manager. A two-stage stochastic integer programming model minimizes the total nurse staffing costs and cost of adjustments to the original schedules developed in the medium-term planning phase, while ensuring the coverage of nursing demand of patients in the unit.

At the start of a current shift, we assume following patient information is available to the unit charge nurse: (1) Current patient census, (2) Acuity assignments of the existing patients, (3) Scheduled and unscheduled patient admissions and discharges and their associated acuity groups during the current shift, (4) Number of scheduled patient admissions and discharges (and acuity scores of the discharged patients) in the upcoming shift. On the other hand, (1) Number of unscheduled patient admissions in the upcoming shift, and (2) Acuity assignments of patients from scheduled and unscheduled admissions in the upcoming shift are unknown to the charge nurse at the start of the current shift. A stochastic integer programming model is developed to address these shortcomings. A new expected nursing requirement is calculated and compared to the provided nursing hours after using the available schedule adjustment options. Our decision variables in both of the stages include the number of adjustment actions taken from each available adjustment type (i.e. number of cancelled shifts, number of nurses requested from the float pool etc.).

In addition to the patient information described above, the two-stage stochastic integer programming model takes as an input: (1) the number of FTE and PRN nurses scheduled for the current and upcoming shift, (2) the number of available float pool and on-call nurses in each shift, and (3) the nurse profiles and schedule of the nurses for the previous and upcoming three shifts (for potential overtime requests). We also investigate the scheduling flexibility needs of the medical units, and formulate our third research question as follows:

RQ 3: Can short-term schedule modifications that are based upon decisions attained from two-stage stochastic integer programming model lower cost and reduce understaffing levels, compared to original medium-term staffing plans?

Chapter 2

Review of Nurse Staffing and Scheduling Approaches

2.1 Introduction

This chapter provides a review of the relevant literature. Section 2.1 reviews the literature on nursing workload measurement approaches. Staffing and scheduling healthcare personnel involves determining the number of personnel of the required skills and assigning them to the predetermined shifts in order to meet predicted patient demand requirements. It is often referred to as workforce planning and scheduling in other personnel planning environments. The literature on nurse staffing and scheduling is significantly related to the workforce planning and scheduling. Section 2.2 discusses the related literature from workforce planning and scheduling. Section 2.3 provides a comprehensive review of the nurse staffing and scheduling literature found in the Operations Management and Operations Research focused journals. In particular, the review in Section 2.3 includes the areas of nurse planning stages, nurse staffing policy options, cyclic and non-cyclic scheduling of nurses, algorithmic solution approaches to the nurse staffing and scheduling problems, cross-utilization of nurses in medical units, nurse absenteeism, scheduling under demand uncertainty with stochastic solution approaches, short-term nurse staffing and nurse-to-patient assignment.

2.2 Nursing Workload Measurement

2.2.1 Dynamic Nature of Patient Demand in Hospital Environments

Workloads in nursing wards depend highly on patients arrival and their lengths of stay, both of which are inherently variable. Predicting the workloads and staffing nurses accordingly are essential for guaranteeing quality of care in a cost-effective manner (Kortbeek et al., 2015a). Measures of workload as used in the literature includes characteristics of patients (e.g. casemix) and patient turnover, as well as patient acuity/intensity (Duffield et al., 2011). Green et al. (2013) suggests that the problem of determining nurse staffing levels in a hospital environment is a complex task because of variable patient census levels and uncertain service capacity caused by nurse absenteeism. In determining staffing requirements, such factors as total census, intensity-of-care levels, and type of ward must be estimated for appropriate planning to be accomplished (Helmer et al., 1980). Hourly changes in patient census and acuity cause the demand for nursing services to depart from the planned schedule several times a day, which requires hospitals to update their staffing needs on a continuing basis (Bard and Purnomo, 2005b). Some additional factors of consideration to achieve an effective nurse staffing system would be the nurse preferences regarding work schedules, nurse absenteeism and patient acuity (Purnomo and Bard, 2007; Wang and Gupta, 2014). An acuity-based staffing system regulates the number of nurses on a shift according to the patients' needs, and not according to raw patient numbers.

Among the earlier studies, Helmer et al. (1980) developed a series of multi-variate regression models, where using ward, month, day, shift, and time as independent variables, the number of patients in each level of care are predicted. The number of patients are then used to predict nursing man-hour requirements. De Vries (1987) introduce a nursing workload measurement instrument. The study classified each patient into one of four categories, i.e. self-care, medium, high and intensive care. There are nine indicators, such as independency, need for help with bathing and/or feeding, need for observation, which determine the patient category. Using sampling and observation studies for each category, a coefficient is determined for the corresponding staff need. By classifying the daily patient mix and multiplying the number per category with the input coefficients, the workload is determined (in nursing hours, or full-time equivalents). A measure for the staff capacity utilization is then obtained by relating the assessed workload to the available staff. The ratio of these variables is called "work pressure," which is said to be 100% when the supply and demand of nursing care are balanced.

2.2.2 Nursing Workload Index, Patient Acuity and Predictability of Patient Volume

Brusco and Showalter (1993) define patient service level in terms of the number of nurse labor hours required to achieve a desired quality of patient care. They introduce a "workload index" (WI) and a "conversion factor" (CF) to compute the number of daily recommended nursing hours for the patient care unit. The WI is computed by summing up the product of relative acuity index and census of patients in that acuity category. The CF represents the number of direct care hours which should be provided for a patient in the baseline acuity category. It is used as a surrogate measure of quality of patient care. Siferd and Benton (1994) define patient acuity mathematically, for an individual patient, as the number of nurses in the unit needed by one patient during one shift. They represent the number of nurses needed in the unit during the shift as a multiplicative model of mean patient acuity, number of patients, and the mean rate of change in patient acuities. The study projects the number of nurses needed to staff the next shift as a function of the number of patients expected to be assigned to the unit at the beginning of that shift, and the level of care required by those patients. The authors show that changes in these factors interact to cause wide swings in the number of nurses needed to staff the next shift.

Harper et al. (2010) extended a hospital capacity simulation tool that determines the required size and skill mix of hospital nursing teams. Their approach incorporated discrete event simulation and stochastic programming to determine optimal nursing requirements by staff grade. Outputs from the three-phase discrete event simulation are fed into a stochastic program which recommends the optimal number of nurses to employ (full-time equivalents) by skill-mix and by shift. A novel feature of the tool is its ability to predict and compare nursing needs based on different methods of capturing patient-to-nurse ratios as currently adopted across the UK National Health Service. Yankovic and Green (2011) represent the nursing system as a variable finite-source queuing model and develop a two-dimensional model to approximate the actual interdependent dynamics of bed occupancy levels and demands for nursing. They use this model to show how unit size, nursing intensity, occupancy levels, and unit length-of-stay affect the impact of nursing levels on performance and thus how inflexible nurse-to-patient ratios frequently lead to either understaffing or overstaffing.

Paul and MacDonald (2013) develop a series of process flow-based models that consider the inherent complexity in key hospital departments and hence provide a basis for empirical models to estimate nurse demands.

Using an illustrative example of a simple intensive care unit system, they demonstrate the challenges associated with mandatory nurse-to-patient ratios to address the nurse shortage crisis when subjected to varying patient demand and hospital service quality goals. Results suggest that relying merely on mandatory nurse-to-patient ratios is not an effective strategy, especially considering the issue of nursing shortages. Kim et al. (2014) technical report evaluate the predictability of patient volume in Hospital Medicine (HM) groups using a variety of known forecasting techniques. HM groups experience fluctuations in patient volume which may be difficult to predict. Results from univariate and multivariate methods were compared with a benchmark of historical means. The mean absolute percentage error (MAPE) was used to measure the accuracy of forecast. Autocorrelations and cross-correlations of patient volume across the services were also analyzed. Results from the study indicate that the forecasting models outperformed the historical average based approach by reducing MAPE from 17.2% to 6% in one-day-ahead forecast and to 8.8% MAPE in a month-ahead forecast. The ARIMA method outperformed the other methods.

2.2.3 Advantages and Disadvantages of Mandatory Minimum Nurse-to-Patient Ratios

Nurse-to-patient ratios are commonly applied when determining staffing levels (Yankovic and Green, 2011). These ratios indicate how many patients a registered nurse can care for during a shift, taking into account both direct and indirect patient care. Staffing based on nurse-to-patient ratios can be performed in two ways. The ratios can be considered as mandatory lower bound, such as in California, or alternatively nursing administration can use these ratios as guidelines that must be satisfied for a certain proportion of time. The advantage of mandatory minimum nurse-to-patient ratios is that a consistently high level of patient safety is guaranteed (Kane et al., 2007). The disadvantage, however, is that all beds need to be continuously staffed because there is always a possibility that all beds are occupied and, as described, the nurse rosters have to be settled in advance. Therefore, overstaffing is a threat because there is little flexibility to adjust staffing levels to the predicted patient demand. Application of nurse-to-patient ratios as guidelines help overcome the overstaffing threat. In such a case, the assumption is that there is slack in the time window during which certain indirect patient care tasks can be performed, without having direct negative consequences on patient safety or work stress. Kortbeek et al. (2015a) combine the advantages of both approaches by using two nurse-to-patient ratio targets. Kortbeek et al. (2015b) present a generic analytical approach to predict bed census on nursing wards by hour, as a function of the Master Surgical Schedule and arrival patterns of emergency patients. Along these predictions, insight is gained on the impact of strategic (case mix, care unit size, care unit partitioning), tactical (allocation of operating room time, misplacement rules), and operational decisions (time of admission/discharge). Results suggest that larger facilities can operate under a higher occupancy level than smaller ones in trying to achieve a given patient service level, since randomness balances out.

2.3 Workforce Planning and Scheduling

In this section, we provide a brief review of the literature on workforce planning and scheduling that is linked to the nurse staffing and scheduling problem. Defraeye and Van Nieuwenhuyse (2016) provide a state-of-the-art literature review (1991 - 2013 time frame) on staffing and scheduling approaches that account for non-stationary demand (i.e. the number of customers fluctuates over time according to a stochastic, though to some extent predictable, pattern) for service. The authors categorize the literature according to system assumptions, performance evaluation characteristics, optimization approaches and real-life application contexts. Van den Bergh et al. (2013) also reviewed the literature on personnel scheduling problems. They identify different perspectives from which to classify the existing literature, which include: (1) Personnel characteristics, decision delineation and shifts definition, (2) Constraints, performance measures and flexibility, (3) Solution method and uncertainty incorporation, (4) Application area and applicability of research.

2.3.1 Classification of the Labor Scheduling Research

Ernst et al. (2004) presents a review of staff scheduling and rostering problems in specific application areas, and the models and algorithms that have been reported in the literature for their solution. The authors define personnel scheduling, or rostering as the process of constructing work timetables for staff so that an organization can satisfy the demand for its goods or services. The first stage of this process involves determining the number of staff, with particular skills, needed to meet the service demand. Individual staff members are allocated to shifts so as to meet the required staffing levels at different times, and duties are then assigned to individuals for each shift. All industrial regulations associated with the relevant workplace agreements must be observed during the process. Hur et al. (2004) structure workforce staffing and scheduling decisions as a three-stage hierarchical process. Stage one deals with deciding the size and composition of the workforce. Stage two focuses upon assigning the staff to work tours covering a given time interval. Stage three concerns the process of modifying the work schedule while implementing it during a day.

Bechtold et al. (1991) classify the labor scheduling research into three categories: (1) days-off, (2) shift, and (3) tour. Days-off research specifies work and non-work days for employees when the employee work week is shorter than the operating week of the service delivery system. Shift scheduling research determines a set of employee work schedules (as defined by start, finish, and rest/meal break times) across a daily planning horizon. Tour scheduling research addresses both days-off and shift scheduling over a weekly planning horizon. Bechtold et al. (1991) list objective function criteria used in the literature on labor scheduling research as: total labor hours scheduled, total number of employees, labor costs, unscheduled labor costs, customer service, over-staffing, under-staffing, number of schedules with consecutive days off, number of different work schedules used, or some combinations of the above. They also list variety of constraints that were used for labor scheduling flexibility and resource limitations that relate to: labor requirements, labor schedule duration, labor schedule start time, meal and rest breaks, consecutive/nonconsecutive days off, labor productivity, number of employees, equipment capacity, labor availability, labor location site, hours per day of operation, schedule planning horizon or some combination of the listed.

2.3.2 Algorithms for Shift Starting Times, Shift Lengths, and Break Placement

Next, we provide a brief review of the methodology articles in the workforce planning and scheduling literature used in the nurse staffing and scheduling studies. Among the earlier studies, Baker (1974) presents a simple algorithm for the problem of assigning days-off to full-time staff given a cyclic seven-day demand pattern. The formulation assumes that employees are entitled to two consecutive days off each week with the objective to find a minimum staff size capable of meeting the requirements. Baker and Magazine (1977) examine the problem of scheduling days-off in continuous (seven-day-a-week) operations under a variety of day-off policies, when demand for manpower change on weekdays and weekend days. The study consider a number of policies governing employee work assignments and in each case give a formula for the minimum workforce size and a schedule construction algorithm.

Bailey and Field (1985) present an LP model for personnel scheduling when alternative work hours are permitted. They introduce the concept of ‘Flexshifts’, which develops schedules of 6-, 8-, and 10-hour shifts against a 12- and 24-hour daily demand profile. Burns and Koop (1987) introduce a multiple-shift manpower scheduling algorithm that constructs schedules that use no more than the minimum number of workers necessary. Constraints include two off-days each week, a specified number of off-weekends in any fixed number of consecutive weekends, a maximum of six consecutive work shifts and different staffing demands for each type of shift. Bechtold and Showalter (1987) examine the problem of scheduling employees in a service delivery system subject to demand variability. The manual heuristic proposed assigns full-time employees to weekly work schedules with the objective of minimizing the total number of labor hours scheduled.

Brusco and Jacobs (1995) develop a local-search heuristic based on the simulated annealing algorithm to generate feasible integer personnel schedules in continuously operating organizations. Thompson (1995) presents an integer programming model for developing optimal shift schedules while allowing extensive flexibility in terms of alternate shift starting times, shift lengths, and break placement. The model combines the work of Moondra (1976) and Bechtold and Jacobs (1990) by implicitly matching meal breaks to implicitly represented shifts. Moreover, the model extends the work of these authors to enable the scheduling of overtime and the scheduling of rest breaks.

2.3.3 Work Tour Scheduling

Loucks and Jacobs (1991) examine the dual problem of work tour scheduling and task assignment involving workers who differ in their times of availability and task qualifications. The problem is presented in the context of a fast food restaurant. The authors indicate developing a week-long labor schedule is a nontrivial problem, in terms of complexity and importance, which a manager spends as much as a full workday solving. The primary scheduling objective (the manager's concern) is the minimization of overstaffing in the face of significant hourly and daily fluctuations in minimum staffing requirements. The secondary objective (the workers' concern) is the minimization of the sum of the squared differences between the number of work hours scheduled and the number targeted for each employee. Contributing to scheduling complexity are minimum and maximum shift lengths and a maximum number of workdays. They demonstrate that a goal programming formulation of a representative problem is too large to be solved optimally. Subsequently, they propose a computerized heuristic procedure capable of producing a labor schedule requiring at most minor refinement by a manager. Easton and Rossin (1991) indicate policies governing employee scheduling practices may permit millions of different tours in some service organizations. A common heuristic strategy is to reformulate the problem from a small working subset of the feasible tours. Solution quality depends on the number and types of schedules included in the model. They describe a working subset heuristic based on column generation. The method accommodates a mix of full- and part-time employees. Experiments revealed its formulations had similar objective values to the models using all feasible tours. They were also significantly lower than those generated by alternative working subset procedures.

Bechtold et al. (1991) evaluate the performance of four LP-based and five construction heuristic methods with respect to minimizing total labor hours scheduled. Each of the methods is applied to a tour scheduling problem, subject to a variety of labor demand requirements distributions. Statistical analysis of the results indicate that effective tour schedule solutions are generated by both LP-based and construction methods. The authors conclude that researchers should consider integrating these heuristic methods into a decision support system. Brusco and Jacobs (1993) presents the application of a simulated annealing heuristic to a cyclic staff-scheduling problem. The heuristic is designed for use in a continuously operating scheduling environment with the objective of minimizing the number of employees necessary to satisfy forecast demand. They suggest that the simulated annealing-based method tends to dominate the branch-and-bound algorithms and the other heuristics in terms of solution quality and speed of convergence to a low-cost solution. Bechtold and Brusco (1994) study working set generation methods for labor tour scheduling. Working set generation method is selection of a subset of decision variables from the set of variables specified in the complete problem, which alleviates the problem complexity. They classify previous working set generation procedures as being either structural, demand-based, or refinement. Two new working set procedures are compared with previously published generation procedures within the context of a discontinuous tour scheduling environment where the sole objective is minimization of total labor hours scheduled.

Brusco and Jacobs (1998) address the restricted starting-time tour-scheduling problem (RSTP), which involves the determination of the hours of the day (shifts) and days of the week (days on) that employees are assigned to work. RSTP is characterized by restrictions on the number of daily time periods in which employees may begin their shifts. The authors propose a two-stage heuristic solution strategy for the RSTP. The output of first stage is the set of shift starting times that yields the best LP shift scheduling solution identified for RSSP. Once this set of starting times is determined, they are used to construct tours and the problem is managed as an unrestricted starting-time tour- scheduling problem (USTP). The initial solution to the tour-scheduling problem is constructed using a variation of a common greedy heuristic used in Morris and Showalter (1983). At each iteration, the procedure begins by checking to see if a part-time employee can be added to the schedule without violating the staffing mix constraint. To evaluate the solution strategy for RSTP, study used the actual environmental conditions associated with plane-side (baggage handling, etc.) operations at unionized United Airlines airport stations. Brusco and Jacobs (2000) present an implicit tour-scheduling formulation of the "7x24" continuous tour scheduling problem that incorporates both meal-break and start-time flexibility into an integer-programming model. The integer-programming model extends Bechtold and Jacob's (1990) implicit modeling of meal breaks to the continuous tour problem and integrates Jacobs and Brusco's (1996) implicit modeling of start-time bands. The model is generalized to allow for specification of a start-time interval that indicates the number of periods between starting times. The study suggests that real-world problems containing such flexibility can be solved optimally using general purpose integer-programming software.

2.3.4 Cross-Training Employees and Schedule Adjustment Problem

Brusco and Johns (1998) present an integer linear programming model for evaluating low-cost staffing plans with appropriate cross-training configurations. The authors study a work environment where cross-trained employees have different productivity levels in multiple work activity categories. The objective of the model is to minimize workforce staffing costs subject to the satisfaction of minimum labor requirements across a planning horizon of a single work shift. Cross-training structures and the labor requirement patterns were established based on data collected from maintenance operations at a large paper mill in the United States. The authors suggest that asymmetric cross-training structures that permit chaining of employee skill classes across work activity categories are particularly useful.

Bhulai et al. (2008) present an efficient method for shift scheduling in a multiskill environment when considering a service-level constraint in each planning period. The study introduce a two-step method for shift scheduling in multiskill call centers. First, staffing levels are determined, and next, the outcomes are used as input for the scheduling problem. The scheduling problem relies on a linear programming model that is easy to implement and has short computation times. The authors suggest, short computation times potentially enable the method to be used as a part of an iterative procedure that combines shifts into rosters.

Hur et al. (2004) presents a mathematical formulation of the real-time schedule adjustment problem for settings with a heterogeneous workforce. The authors indicate that available worker capacity does not match with actual demand during a given day, which requires modifications to the planned work schedule in order to improve service and increase profitability. The study propose mathematical formulations for this type of real-time work schedule adjustment decisions and develops efficient heuristic approaches for this decision. The authors compare the effectiveness of these heuristics with the decisions of experienced service managers. The study investigates the effect of the degree of schedule adjustment on profitability, and assesses the effect of demand forecast update errors on the performance of the schedule adjustment efforts. Results indicate that the computer based heuristics achieve higher profit improvement than experienced managers. The authors also suggest that active adjustments of work schedules are beneficial as long as the direction of demand change is accurately identified.

2.4 Nurse Staffing and Scheduling in OM Literature

Over the past 30-40 years, many different approaches have been used to solve nurse rostering problems of varying forms and complexity. Among the earlier studies, Warner and Prawda (1972) defined the "Nursing Personnel Scheduling Problem" as the identification of staffing pattern which specifies the number of nursing personnel of each skill class to be scheduled among the wards and nursing shifts of a scheduling period, and minimizes a "shortage cost" of nursing care services provided for the scheduling period. Solution approaches included mathematical programming, constraint programming, goal programming, multi objective approaches, case-based reasoning and a great variety of local search and meta-heuristic approaches (Burke et al., 2013). Nurse rostering is an NP-hard combinatorial problem which makes it extremely difficult to efficiently solve real life problems (Valouxis et al., 2012). Usually real problem instances have complicated work rules related to safety and quality of service issues in addition to rules about preferences of the personnel. This section reviews the literature on nurse staffing and scheduling specifically from the Operations Management journals. We identify major research areas and also solution approaches to the problem.

Cheang et al. (2003) and Burke et al. (2004) provide a very detailed analysis of modeling approaches and methods to the nurse staffing and scheduling problems in the literature. Tables 2.1 and 2.2 below detail the articles related to nurse staffing and scheduling that appear in prominent operations management (OM) journals. Kellogg and Walczak (2007) review the nurse scheduling literature from the implementation perspective. The authors examine the models that academia has produced and the models that hospitals have actually used. The study use data from various sources, including research articles, e-mail and telephone surveys, an industry database, and a software source catalog. The authors indicate that only 30% of systems that research articles discuss are implemented, and there is very little academic involvement in systems that third-party vendors offer. Below, we provide a list of nurse staffing and scheduling literature in Operations Management journals.

Authors, Year & Journal Title	**Subject**	**Methodology**
Kim et al. (2015) - OR	Stochastic IP approach to integrated nurse staffing and scheduling	Stochastic optimization; Heuristic algorithms
Kortbeek et al. (2015a) - IJPE	Flexible nurse staffing based on hourly bed census predictions	Stochastic demand prediction
Wang & Gupta (2014) - MSOM	Nurse absenteeism and staffing strategies for hospital inpatient units	Stochastic optimization; Heuristic algorithms
Yom-Tov & Mandelbaum (2014) - MSOM	Healthcare staffing with time-varying queue with reentrant patients	Queueing networks
Wong et al. (2014) - Comp. & OR	Two-stage heuristic approach for nurse scheduling problem in ER	Heuristic algorithms
Green et al. (2013) - MS	Nurse staffing in the presence of endogenous nurse absenteeism	Econometric analysis
Burke et al. (2013) - Informs J on Comp.	A time predefined variable depth search for nurse rostering	Heuristic algorithms
Wright & Mahar (2013) - Omega	Centralized nurse scheduling to improve schedule cost and satisfaction	Multi-criteria math programming
Maenhout & Vanhoucke (2013a) - Omega	Longer term nurse allocation analysis	Integer programming; Heuristics algorithms
Maenhout & Vanhoucke (2013b) - HCMS	Analyzing the nursing organizational structure	Linear programming, Set partitioning
Punnakitikashem et al. (2013) - IIE Tran.	Integrated nurse staffing and assignment	Stochastic optimization; Heuristic algorithms
He et al. (2012) - MSOM	Timing of nurse staffing decisions with workload heterogeneity	Econometric Analysis
Valouxis et al. (2012) - EJOR	A systematic two phase approach for the nurse rostering problem	Integer programming; Heuristic algorithms
Burke et al. (2012) - Annals of OR	Pareto-based search methodology for multi-objective nurse scheduling	Multi-objective optimization; Meta-heuristics
Vericourt & Jennings (2011) - OR	Nurse staffing in medical units: A queueing perspective	Queueing Theory
Glass & Knight (2010) - EJOR	A critical appraisal of the nurse rostering problem	Mixed-integer linear programming
Burke et al. (2010) - EJOR	Variable neighborhood search for highly-constrained nurse rostering	Integer programming, Heuristic Algorithms
Wright & Bretthauer (2010) - DS	Strategies for addressing the nursing shortage	Math programming, Heuristic algorithms
Li et al. (2009) - Informs J on Comp.	Component-based heuristic search method for nurse scheduling	Heuristic Algorithms
Maenhout & Vanhoucke (2009) - JORS	Incorporating nurse-specific characteristics in cyclical scheduling	Integer programming, Heuristic algorithms
Gnanlet & Gilland (2009) - DS	Sequential decision making for optimizing nurse flexibilities	Stochastic optimization
Punnakitikashem et al. (2008) - COA	Stochastic programming for nurse assignment	Stochastic prog., Bender's decomposition
Kellogg & Walczak (2007) - Interfaces	Review of implementation for published nurse scheduling articles	Review article
Parr & Thompson (2007) - Annals of OR	Multi-objective nurse scheduling problem with weighted cost function	Meta-heuristic; Simulated annealing

Table 2.1: Nurse Staffing and Scheduling Literature in Operations Management Journals

Authors, Year & Journal Title	Subject	Methodology
Purnomo & Bard (2007) - NRL	Cyclic preference scheduling for nurses using branch and price	Integer programming; Heuristic algorithms
Bard & Purnomo (2007) - J of Sched.	Cyclic preference scheduling of nurses: A Lagrangian-based heuristic	Integer programming; Heuristic algorithms
Li & Benton (2006) - JOM	Hospital technology and nurse staffing management decisions	Hypothesis Testing
Wright et al. (2006) - DS	Nurse scheduling problem: Staffing ratios and nursing shortages	Integer programming; Heuristic algorithms
Bard & Purnomo (2005a) - EJOR	Preference scheduling for nurses using column generation	Integer programming; Heuristic algorithms
Bard & Purnomo (2005b) - HCMS	Short-term nurse scheduling with daily demand fluctuations	Integer programming
Aickelin & Dowsland (2004)-Comp. & OR	Genetic algorithm for a nurse-scheduling problem	Heuristic algorithms
Aicklein & White (2004)-Annals of OR	Building better nurse scheduling algorithms	Integer programming, Heuristic algorithms
Bellanti et al. (2004) - EJOR	Greedy neighborhood search approach to nurse rostering	Heuristic algorithms
Burke et al. (2004) - J of Sched.	The state of the art of nurse rostering	Review article
Cheang et al. (2003) - EJOR	Nurse rostering problems??a bibliographic survey	Review article
Dowsland & Thompson (2000) - JORS	Nurse scheduling with knapsacks and tabu search	Integer programming; Heuristic algorithms
Dowsland (1998) - EJOR	Nurse scheduling with tabu search and strategic oscillation	Heuristic Algorithms
Bretthauer & Cŏté (1998) - DS	Planning resource requirements in health care organizations	Math Programming, Queuing Networks
Jaumard et al. (1998) - EJOR	A generalized linear programming model for nurse scheduling	Column generation; Constrained shortest path
Millar & Kiragu (1998) - EJOR	Scheduling of 12 h shift nurses by network programming	Math Programming
Venkataraman & Brusco (1996) - Omega	An integrated analysis of nurse staffing and scheduling policies	Mixed-integer linear programming
Brusco & Showalter (1993) - Omega	Constrained nurse staffing analysis	Linear programming, Response surface
Easton et al. (1992) - POM	Analysis of alternative scheduling policies for hospital nurses	Integer programming, simulation
Trivedi (1981) - OR	A mixed-integer goal programming model for nurse budgeting	Mixed-integer goal programming
Warner (1976) - OR	Scheduling nursing personnel according to nursing preference	Mathematical programming
Miller et al. (1976) - OR	Nurse scheduling using mathematical programming	Mathematical programming
Abernathy et al. (1973) - OR	A three-stage nurse planning and scheduling model	Stochastic programming, Heuristic algorithms
Warner & Prawda (1972) - MS	A math programming model for scheduling nursing personnel	Mixed-integer quadratic programming

Table 2.2: Nurse Staffing and Scheduling Literature in Operations Management Journals (Cont'd...)

2.4.1 Stages of Nurse Planning

Warner (1976) structure three major areas of personnel planning decisions in nurse staffing and scheduling research: staffing, scheduling and reallocation of nurses. For the scheduling phase of the problem study introduces five criteria for evaluating alternative models: (1) Coverage: difference between the required and the scheduled number of nurses, (2) Quality: schedules fairness, work stretch length for a particular schedule, (3) Stability: perception of nurses in terms of consistency and predictability of days on and off and weekend work, (4) Flexibility: system's ability to adapt changes in the environment, (5) Cost: cost and number of resources consumed in making the decision (Burke et al., 2004).

Warner et. al. (1990) define several aspects of scheduling nursing personnel within the general context of nursing management and review the history of the application of operations research and computers to scheduling nurses. The study also describe what nursing administration is looking for in an automated scheduling system. The study divides the nurse management into patient-oriented issues, such as patient care philosophy, care plans, task assignment, etc., and employee-oriented issues, such as budgeting, staffing, scheduling, sick-leave tracking, productivity, etc. Venkataraman and Brusco (1996) present an integrated nurse staffing and scheduling system for analyzing nurse workforce management policies. The authors study the effects of staffing and scheduling policies on labor costs. Mixed-integer linear programming models are used to develop a nurse staffing model, which is used to determine aggregate labor requirements for a 6-month planning horizon; afterwards, another model disaggregates the nurse staffing plan into 2-week labor schedules. Results of the study suggest important interactions between staffing and scheduling policies.

Punnakitikashem et al. (2008) describe four stages of nurse planning as nurse budgeting, nurse scheduling, nurse rescheduling, and nurse assignment. Focusing on the last stage of nurse assignment, authors first present a two-stage stochastic programming model that minimizes excess nurse workload, and algorithmic approaches for solving the stochastic model. The authors solve the second-stage subproblem with a greedy algorithm. The authors suggest that nurse assignment is usually performed within 30 minutes before each shift. Consequently, the study focus is to find a good solution with the time limitation. Patient-to-nurse ratio constraints are introduced to balance the workload of nurses as well as improve the overall performance of the algorithm. Valouxis et al. (2012) use a two phase strategy where in the first phase the workload for each nurse and for each day of the week was decided while in the second phase the specific daily shifts were assigned. The study also applied local optimization techniques for searching across combinations of nurses' partial schedules.

2.4.2 Nurse Staffing Policy Options

Miller et al. (1976) formulated the nurse-scheduling problem as one of selecting a configuration of nurse schedules that minimize an objective function that balances the trade-off between staffing coverage and schedule preferences of individual nurses. Trivedi (1981) presents a mixed-integer goal programming model that incorporates cost containment and provide appropriate nursing hours for delivering quality nursing care, which considers trade-offs among full-time, part-time and overtime nurses on weekdays as well as weekends.

Easton et al. (1992) compare expected nursing expense and workforce requirements to staff medical and surgical nursing units, under alternative scheduling policies alleged to improve nurse turnover. The authors study alternative nurse scheduling patterns and present scheduling policies, reporting the number of distinct schedules or tours (i.e. scheduling patterns in use involve shifts of 8, 10, 12, or 16 hours. These shifts are combined to form tours with a variety of days-off patterns and compensation schemes.) that each policy allows, the number of monthly (28 days) paid hours and working hours for each pattern, and a ratio that reflects relative wage rates. Using simulation and an integrated staffing and scheduling methodology, the study suggests that the expected nursing wages and workforce requirements for some policies differed by as much as 33%. The study also indicates that the expected labor costs for certain policies could erode the benefits expected from improved retention. In contrast, other policies in the study allow high utilization of nursing resources, enhancing the expected benefits of reduced turnover with significant reductions in expenses for labor, recruiting, training, and fringe benefits.

Brusco and Showalter (1993) evaluates the impact of nurse staffing policy options on annual nursing labor costs. A linear programming staffing model served as the research vehicle for the study and response surface methodology was used to investigate the relationship between labor costs and the policy options. The primary nurse staffing policy options available to hospital management include: (1) staffing mix; (2) overtime; (3) flex-staff; and (4) external staff assignment. Staffing mix refers to the work force composition of registered nurses (RN), licensed practical nurses (LPN) and nurses' assistants (NA). Overtime refers to the use of nursing staff for more than 8 hr. per shift or more than 80 hr. per bi-weekly period. Flex-staffing is the use of part-time (less than 80 hr. per bi-weekly period) employees working throughout the hospital. External staffing consists of RNs signed to 13-week contracts as well as temporary nurse hires from local agencies. The impact of the nursing shortage was incorporated by assuming the currently available pool of nurses. The available nurse hours of each skill class in each patient care unit was used as a measure of nurse labor availability. The authors use a conversion factor (CF) as a surrogate measure of quality of patient care for the service level. CF is multiplied by the workload index (WI) to compute the aggregate (across all skill levels) nursing labor hour requirements for patient care units in each planning period. As their experimental research methodology authors used "Response Surface Methodology (RSM)", which employs a low-order polynomial for approximating a response variable over a specified range of design variables. In this study, the response variable is annual nursing labor costs and design variables are the policy decisions. The authors chose to use a second-order model which would capture quadratic effects if they are present. Results from the study indicate that service level, nurse labor availability, nurse staffing mix and flex-staff assignment had the most significant effects on annual nursing labor costs.

Li and Benton (2006) investigates the relationship among hospital size, location, technology, nurse management, and overall hospital performance using a comprehensive covariance structure model. The results of the study suggest that nurse management decisions have a significant effect on hospital cost and quality performance. Wright et al. (2006) develops a scheduling model to evaluate how mandatory nurse-to-patient ratios and other policies impact schedule cost and schedule desirability from the nurses' perspective. The authors adapt a three-phase workforce management framework seen in Campbell (1999). In the planning phase, the manager makes decisions concerning how many employees to hire, how many to dedicate to each unit or area of the organization, and how many employees to schedule for each shift. The authors present a "workload model" for determining these requirements. In the scheduling phase, the manager develops a schedule that shows when each employee works over the scheduling horizon using the "tour assignment model". The authors used a bi-criteria objective function approach. The first objective function minimizes total regular-time and overtime nurse wages. The second objective function minimizes the total number of undesirable shift assignments and weekends worked for nurses who do not want weekends. The authors indicate that nurse wage costs can be highly nonlinear with respect to changes in mandatory nurse-to-patient ratios.

Wright and Bretthauer (2010) present strategies to help combat the U.S. nursing shortage by providing an attractive work schedule and work environment, which help retaining existing nurses and attracting new nurses to the profession, while at the same time using the set of available nurses as effectively as possible. The authors develop a model that coordinates scheduling, schedule adjustment, and agency nurse decisions across various nurse labor pools, each of differing flexibility levels, capabilities, and costs, allowing a much more desirable schedule to be constructed. The coordinated scheduling model assigns specific nurses to each shift over the scheduling horizon (five weeks in this study). This model extends the model developed by Wright et al. (2006) by incorporating the ability to take advantage of flexible float nurses and unit nurses and the addition of agency nurses. Using the resulting schedules from the coordinated scheduling model, a second model provides a method to account for forecast error and adjust the schedule at the beginning of each shift by allocating float nurses to particular units and reassigning unit nurses. Results from the study suggest that labor costs can be reduced substantially because, without coordination, labor costs on average are 16.3% higher based on an actual hospital setting, leading to the availability of additional funds for retaining and attracting nurses.

2.4.3 Cyclic and Non-Cyclic Scheduling of Nurses

Millar and Kiragu (1998) present a mathematical model for cyclic and non-cyclic scheduling of 12-hour shift nurses. Cyclic scheduling refers to the scheduling approach, where fixed patterns of days on and days off are established and the staff is rotated continuously through them. The authors introduce a “stint”, which is a pattern characterized by a start date, a length, a ’cost’ and the shifts worked. Nurse schedules in the model are composed of alternating sequence of “work-stretch” and “off-stretch” patterns. Using the stints as nodes in a network, the authors construct an acyclic graph on which the nurse’s schedules can be defined. The resulting model is a shortest-path problem with side constraints. With a minor modification, authors use the network to define both the cyclic and non-cyclic scheduling problems.

Bard and Purnomo (2007) addresses the problem of developing cyclic schedules for nurses while taking into account the quality of individual rosters. Quality of a given schedule is determined by the absence of certain undesirable shift patterns. The study aims to offer management greater flexibility in constructing rosters by combining the principal components of cyclic and preference scheduling in a single model. The problem is formulated as an integer program (IP) and then decomposed using Lagrangian relaxation. To find solutions to the large-scale integer program (IP), authors develop a hybrid algorithm comprising both heuristic and exact procedures. Two approaches are explored, the first based on the relaxation of the preference constraints and the second based on the relaxation of the demand constraints.

Purnomo and Bard (2007) also study cyclic and preference scheduling methodology on nurse rostering with the objective of striking a balance between satisfying individual preferences and minimizing personnel costs. To find solutions, the authors develop a branch-and-price algorithm that makes use of several branching rules and an effective rounding heuristic. Maenhout and Vanhoucke (2009) investigate the benefits of integrating nurse-specific characteristics in the cyclic scheduling approach. The authors analyze to what extent these characteristics should be incorporated and compare this approach with a general and more robust cyclical scheduling approach and the flexible acyclical rostering of nursing personnel. The study suggests performance improvements in terms of robustness, scheduling effort, and solution quality by constructing new individual nurse schedules for each nurse separately incorporating nurse-specific characteristics.

2.4.4 Algorithmic Solution Approaches

Jaumard et al. (1998) presents a 0-1 column generation model with a resource constrained shortest path auxiliary problem for nurse scheduling. The master problem finds a configuration of individual schedules to satisfy the demand coverage constraints while minimizing salary costs and maximizing both employee preferences and team balance. A feasible solution of the auxiliary problem is an acceptable schedule for a given nurse, with respect to requirements such as seniority, workload, rotations and days off. Bretthauer and Cǒté (1998) present a general model and solution methodology for planning resource requirements (i.e., capacity, including nursing staff size) in health care organizations. The authors develop an optimization/queueing network model that minimizes capacity costs while controlling customer service by enforcing a set of performance constraints, such as setting an upper limit on the expected time a patient spends in the system.

Dowsland (1998) tackled the nurse staffing problem using tabu search with strategic oscillation. The objective ensures that enough nurses are on duty at all times while taking account of individual preferences and requests for days off in a way that is seen to treat all employees fairly. To achieve this goal, the author used a variant of tabu search which repeatedly oscillates between finding a feasible cover, and improving it in terms of preference costs. Dowsland and Thompson (2000) then compared integer programming models and heuristic methods in terms of providing good quality solutions to the nurse rostering problems. The authors indicate that advanced IP packages can be memory intensive, and solution times may vary considerably over different problem instances of a similar size. On the other hand, heuristics may not give solutions of consistent quality, are often criticized for being slow, and may have difficulty in converging to good feasible solutions when applied to highly constrained problems. The study illustrates how a modem heuristic and two classical integer programming models have been combined to provide a solution to a nurse rostering problem at a major UK hospital. The authors use a variant of tabu search as the core method, but applying knapsack and network flow models in pre- and post-processing phases. Bellanti et al. (2004) introduce a local search approach, which is based on a neighborhood operating on partial solutions completed by means of a greedy procedure so as to avoid the generation of infeasible solutions. Both a tabu search procedure and an iterated local search procedure are proposed for the studied nurse rostering problem.

Aicklein and White (2004) model and solve a complex nurse scheduling problem with an integer programming formulation and evolutionary algorithms. The study introduces two different algorithmic approaches: First, an encoding that follows directly from the IP formulation, which is also presented in Aickelin and Dowsland (2000). A second approach is the combination of an indirect Genetic Algorithm (GA) with a separate heuristic decoder function. The authors also propose a statistical method of comparing nurse rostering algorithms and hence build better scheduling algorithms by identifying successful algorithm modifications. The comparison method captures the results of algorithms in a single figure that can then be compared using traditional statistical techniques. Aickelin and Dowsland (2004) also describe a GA approach to the nurse scheduling problem arising at a major UK hospital. The study use an indirect coding based on permutations of the nurses, and a heuristic decoder that builds schedules from these permutations. Computational experiments are used to evaluate three different decoders with varying levels of intelligence, and four well-known crossover operators.

Parr and Thompson (2007) investigates the effectiveness of three meta-heuristic techniques based on local search in producing suitable nurse schedules. The authors consider the nurse scheduling as a constraint satisfaction problem where weights are associated with each constraint. The objective is then to minimize the sum of the weights using iterative techniques. The study also examine many different objectives to consider for the problem, each of differing importance and requires various strategies for dealing with. Combining the objectives into a linear cost function and optimizing them using simulated annealing has been compared with using the SAWing technique which places more emphasis on those constraints that are difficult to satisfy. Additionally the noising method has been used to add random variation to the weights. The noising method worked particularly well and produced schedules for a variety of real datasets that were superior to those produced manually or generated using simulated annealing.

Li et al. (2009) aim to create weekly schedules for wards of nurses by assigning each nurse one of a number of predefined shift patterns in the most efficient way. The authors report a new component-based heuristic search approach with evolutionary eliminations, which implements optimization on the components within single schedules. The main idea here is to decompose a schedule into its components (i.e., the allocated shift pattern of each nurse), and then implement two evolutionary elimination strategies mimicking natural selection and the natural mutation process on these components, respectively, to iteratively deliver better schedules. Burke et al. (2010) present a decomposition technique by combining integer programming (IP) and variable neighborhood search (VNS) to deal with complex constraints and requirements of the nurse scheduling problem. The IP is first used to solve a subproblem including all hard constraints and a subset of soft constraints. For the selection of a subset of soft constraints, more priority is given to the constraints that have low complexity (i.e. the number of variables and constraints it may add in the IP model), high importance (i.e. the degree to which the constraint is considered to be desirable by the hospital), or a trade-off between complexity and importance. Glass and Knight (2010) provide a methodology for handling rostering constraints and preferences arising from the continuity from one scheduling period to the next.

Burke et al. (2012) propose a Pareto-based search technique to solve the multi-objective nurse scheduling problem. The authors first design a generating heuristic which randomly builds a set of legal shift patterns for each nurse. The authors then employ an adaptive heuristic to quickly find a solution with the least violations on coverage demands. Next, the authors apply a coverage repairing heuristic to make the resulting solution feasible. Finally, the study proposes a simulated annealing based search method with two options to address user preferences in different ways. Burke et al. (2013) review neighborhood search methods that have been previously used to solve nurse rostering problems and present a variable depth search methodology. The algorithm works by chaining together single neighborhood swaps into more effective compound moves. It achieves this by using heuristics to decide whether to continue extending a chain, and which candidates to examine as the next potential link in the chain. The problem requires the production of non-cyclical schedules which satisfy all hard constraints and as many working preferences and requests as possible. The authors indicate that there are so many conflicting constraints and requests that if they were all hard constraints, a feasible solution would generally not exist. Instead, the authors model majority of constraints as soft constraints and given relative priorities using weights. The authors suggest that today's technology allows these larger neighborhoods to be exhaustively searched very quickly. Wong et al. (2014) employ a spreadsheet-based two-stage heuristic approach for the nurse scheduling problem in a local emergency department. First, an initial schedule satisfying all hard constraints is generated by the simple shift assignment heuristic. Second, the sequential local search algorithm is employed to improve the initial schedules by taking soft constraints (nurse preferences) into account. The proposed approach is benchmarked with the existing approach and

0-1 programming. The study focuses specifically on the emergency department, where the scheduling rules are much more restrictive due to the intense and dynamic work environment.

2.4.5 Cross-Utilization of Nurses & Nurse Absenteeism

Maenhout and Vanhoucke (2013a) discuss an integrated methodology for allocating a given workforce over multiple departments based on the hospital's nurse staffing policies, each ward's shift scheduling policies and the nurses' characteristics. The study examine the effects associated with two staffing policies, i.e., the use of cross-used (float) nurses and the employment of part-time versus full-time personnel and two shift scheduling policies, i.e., the minimum work stretch and the minimum assignment period of float nurses to a single department. The baseline roster consists of a configuration of individual nurse schedules that is generated by incorporating multiple objectives into the developed model, i.e., unit efficiency (cost), personnel job satisfaction (schedule desirability) and unit effectiveness (providing quality nursing care). This approach is endorsed by the confluence of the need to achieve a greater efficiency (due to the rising salaries of nursing personnel and the increasing pressure on hospitals to contain costs), the shortage of well-trained nursing personnel, the need to increasingly accommodate employee preferences and flexibility while maintaining high quality of care provided to patients. The results indicate that nursing efficiency, effectiveness and nurses' satisfaction can be highly variable with respect to changes in staffing and shift scheduling policies. In general, the results confirm the delicate trade-off, i.e., the higher the flexibility in scheduling, the higher the job satisfaction and the unit's efficiency and the lower the effectiveness of providing high-quality care.

Wright and Mahar (2013) investigate how centrally scheduling cross-trained nurses across multiple units in a hospital can be used to reduce costs and improve nurse satisfaction. The centralized nurse scheduling model proposed in the study is a bi-criteria integer scheduling model with objectives for schedule cost and schedule desirability. The schedule desirability objective accommodates each nurse's individual desirability (undesirability) for certain shifts (i.e. overtime, weekends). Results of the study show how centralized nurse scheduling in these hospitals improves the desirability of nurse schedules by approximately 34% and reduces overtime by approximately 80% while simultaneously reducing costs by just under 11%.

Maenhout and Vanhoucke (2013b) investigate the impact of different nurse organization structures and different organizational processes. In these organization structures nurses can typically follow a fixed staffing policy, i.e. a nurse is permanently assigned to a specific ward, or a variable staffing policy, i.e. a nurse is part of a pool of cross-trained nurses floating between different wards that require approximately the same types of skills. The latter strategy is generally recognized as to be at the expense of the nurse (dissatisfaction, stress, poor group dynamics, etc) and the patient's quality of care and requires more training and orientation. The study demonstrates how hospitals can substantially improve the nurse organization using a methodology that is based on integration and centralization over decentralized and non-integrated decision support systems. The authors suggest that the results confirm that it is best to incorporate the lower-level staffing and shift scheduling policies and the characteristics of the available nurses in the staffing decision process.

Green et al. (2013) combine an empirical investigation of the factors affecting nurse absenteeism rates with an analytical treatment of nurse staffing decisions using a novel variant of the newsvendor model. Using data from the emergency department of a large urban hospital, this study finds that nurse absenteeism is exacerbated when fewer nurses are scheduled for a particular shift. This finding highlights the need for hospital managers to use better methods to identify nurse staffing levels that are adequate to handle the anticipated workload. Wang and Gupta (2014) use data from multiple inpatient units to study which factors, including unit culture, short-term workload, and shift type, explain nurse absenteeism. The analysis highlights the importance of paying attention to heterogeneous absentee rates among individual nurses. The study develop models to investigate the impact of demand and absentee rate variability on the performance of staffing plans and obtain some structural results.

2.4.6 Scheduling Under Demand Uncertainty: Stochastic Solution Approaches

Among the earlier studies, Abernathy et al. (1973) presents a staff planning and scheduling model that has specific application in the nurse-staffing process in acute hospitals. The study formulates the planning and scheduling stages as a stochastic programming problem, suggests an iterative solution procedure using random loss functions, and develops a non-iterative solution procedure for a chance-constrained formulation that considers alternative operating procedures and service criteria. Gnanlet and Gilland (2009) consider two types of flexibility, demand upgrades and staff flexibility, which are used to coordinate patient beds and nursing staff as resources and satisfy stochastic patient demand at minimum cost. Demand upgrades refers to the flexibility of upgrading patients to a more acute unit if space is available in that unit. Under staffing flexibility, nurses cross-trained to work in more than one unit are used in addition to dedicated and contract nurses. The authors analyze four flexibility configurations (no flexibility, staffing flexibility, demand upgrades, and full flexibility) under simultaneous decision making (patient bed and staffing decisions are made at a single point in time) and sequential decision making (bed and staffing decisions are conducted at different points in time). The authors use two-stage stochastic programming to find optimal resource levels for two non-homogeneous hospital units that face stochastic demand following a continuous, general distribution. Results of the experiments suggest that benefit of using staffing flexibility on average is greater than the benefit of using demand upgrades. However, the two types of flexibilities have a positive interaction effect and they complement each other.

Vericourt and Jennings (2011) present a closed queueing model to determine efficient nurse staffing policies, where each patient alternates between requiring assistance and not. The performance of the medical unit is based on the probability of excessive delay, the relative frequency with which the delay between the onset of patient neediness and the provision of care from a nurse exceeds a given time threshold. Yom-Tov and Mandelbaum (2014) analyze a queueing model, named "Erlang-R", where the "R" stands for reentrant customers. Erlang-R accommodates customers who return to service several times during their visit within the system. The study was motivated by healthcare systems, in which offered-loads vary over time and patients often go through a repetitive service process. The authors use the developed Erlang-R model to answer questions such as how many physicians and/or nurses are required to achieve predetermined service levels.

He et al. (2012) study the problem of setting nurse staffing levels in hospital operating rooms when there is uncertainty about daily workload. The authors define the workload as the number of operating room hours used by a medical specialty on a given day to perform surgical procedures. Variable costs consist of wages at a regular (scheduled) rate and at an overtime rate when the realized workload exceeds the scheduled time. Using a newsvendor framework, study determine optimal staffing levels with different information sets available at the time of decision: no information, information on number of cases, and information on number and types of cases. Kortbeek et al. (2015a) introduces a stochastic method that uses hourly census predictions to derive efficient nurse staffing policies. The generic analytic approach minimizes staffing levels while satisfying so-called nurse-to-patient ratios. The authors explore the potential of flexible staffing policies that allow hospitals to dynamically respond to their fluctuating patient population by employing float nurses. The study evaluate the complex interaction between staffing requirements and several interrelated planning issues such as case mix, care unit partitioning and size, as well as surgical block planning.

Kim and Mehrotra (2015) study the problem of integrated staffing and scheduling under demand uncertainty.

The study aims to reduce overall labor costs by right-sizing staff by balancing under- and overstaffing costs. Scheduling plans and staffing decisions are usually generated well ahead of time, and adjustments are made when more accurate demand information is available. The "here-and-now" decision is to find initial staffing levels and schedules. The "wait-and-see" decision is to adjust these schedules at a time closer to the actual date of demand realization. The authors formulate the problem as a two-stage stochastic integer program with mixed-integer recourse. The problem is a challenging large-scale one because the scheduling decisions introduce a large number of integer variables due to possible shift combinations. It is also a two-stage stochastic program because at a distant future adjustments to scheduling decisions are needed. At the beginning of the planning horizon, the staffing and scheduling decisions are made to minimize the sum of total staffing cost, expected adjustment cost, and expected overstaffing and understaffing cost. The authors integrate the staffing, scheduling, and adjustment decisions since an understaffed shift requires additional workers to maintain the desired quality of service, while an overstaffed shift results in lost wages because of limited salvage value of the scheduled staff. Weekly scheduling patterns and eight adjustment patterns were generated by using a recursive procedure. The results from the study suggest that compared with a deterministic model, the two-stage stochastic model leads to significant cost savings. The cost savings increase with mean absolute percentage errors in the patient volume forecast.

2.4.7 Short-Term Nurse Staffing and Nurse-to-Patient Assignment

Bard and Purnomo (2005a, 2005b) study efficient modifications to short-term nurse schedules due to dynamic nature of patient demand and nurse availability constraints. The authors present an integer programming model that takes the current set of rosters for regular and pool nurses and the expected demand for the upcoming 24 hours as input, and produces a revised schedule that makes the most efficient use of the available resources, which involve the use of overtime, outside nurses, and floaters. The model is formulated and solved at a hospital-wide level rather than for each unit separately. To improve retention, management must now take into account individual preferences and requests for days off in a way that is perceived as fair, while ensuring sufficient coverage at all times. Bard and Purnomo (2005a) solve this multi-objective problem with a column generation approach that combines integer programming and heuristics. The integer program is formulated as a set covering-type problem whose columns correspond to alternative schedules that a nurse can work over the planning horizon. The two main criteria used to judge the quality of a schedule are the number of preference violations and the number of outside nurses required. The implementation is judged in part by the amount of time spent in finding solutions. One of the weaknesses of the presented model is that it does not allow shifts to be split among time units (i.e. in 4-hour blocks). The authors indicate that the difficulty is that the size of the decision space grows exponentially with the number of periods over which the regular and pool nurse variables are defined.

Punnakitikashem et al. (2013) study short-term nurse staffing and nurse-to-patient assignment problem. The authors integrate these two problems within a stochastic programming model with an objective to minimize an expected excess workload on nurses taking patient care uncertainty into consideration subject to the hard budget constraint. The authors indicate that nurse staffing models in the literature have mainly focused on nurse scheduling and ignored nurse-to-patient assignment. Based on the nurse staffing level, a charge nurse assigns each patient to a nurse at the beginning of a shift, which is referred to as a nurse assignment. In general, a nurse assignment is performed approximately 30 minutes prior to a shift. The authors indicate the significance of taking patient information into consideration, which will enable the nursing administration to meet patients' needs while using nursing staff efficiently. The authors also suggest that most of the existing models proposed in the optimization literature are deterministic and exclude uncertainty in patient care, while patient care is stochastic in nature due to its fluctuations during the shift and its enormous variation. The authors provide three Stochastic Integrated Nurse Staffing and Assignment (SINSA) decompositions and solution methods based on the L-shaped method, which are (i) Benders' decomposition; (ii) Lagrangian relaxation with Benders' decomposition; and (iii) nested Benders' decomposition. By providing nurses several non-dominated solutions with different staffing costs and different workloads, nurses can select what they believe is the best solution from a set of quality ones. As a potential future research area, the authors suggest that incorporating dynamic patient acuity in the nurse staffing and assignment model will provide better results. As the progress of a patient's condition changes over time, the acuity level is changed. Patients with different levels of acuity require different amounts of required care from nurses.

Chapter 3

Nurse Allocation Policy Evaluation and Analysis of Admissions in an ICU

The purpose of this chapter is to provide a framework for accurately estimating the number of nurses required in Intensive Care Units (ICUs) on a given day. One factor making such estimates difficult is the lack of a decision support tool for understanding the distribution of admissions to healthcare facilities. We aim to statistically evaluate the existing staff allocation system of an ICU using clinical operational data, and then develop a predictive model for estimating the number of admissions to the unit. We analyzed clinical operational data of 3 ICU wards for a period of 44 months. The existing staff allocation models for these 3 units does not accurately estimate the required number of nurses. It is difficult to understand the pattern and frequency of admissions, particularly those admissions that are not known 12 hours in advance. We first show that these "unknown" admissions can actually be predicted fairly accurately by fitting the pattern of admissions to a Poisson distribution. Then we provide improvements in estimating the overall number of admissions. Analytical predictive methods that complement intuition and experience-based decisions on nurse staffing and workload would help decrease the unplanned last-minute scheduling requirements for nurses, and improve healthcare delivery with more efficient nurse allocation. The model developed here is generalizable for implementation in other pediatric intensive care units.

3.1 Introduction

Nurse staffing is crucial to providing quality healthcare because nurses are a critical component of a safe health care delivery system (Barton, 2009). In a landmark report, the principal finding of the Institute of Medicine's Committee on the Adequacy of Nurse Staffing in Hospitals and Nursing Homes states: "Nursing is a critical factor in determining the quality of care in hospitals and the nature of patient outcomes" (Wunderlich et al., 1996, p.92). However, many patient care units face challenges in accurately estimating the number of nurses needed on a daily basis. The Pediatric Intensive Care Unit (PICU) that is the focus of this study experiences this problem.

One of the challenges in identifying the required number of nurses stems from the fact that the subject PICU, like many patient care units, has difficulty estimating accurately the number of admissions to each ward. Some admissions are known 12 or more hours in advance and are hereafter called 'scheduled' admission. However, the remaining admissions are 'unscheduled.' admissions. PICUs have a very high rate of unscheduled admissions due to the acute and critical nature of the population served. Among the unscheduled admissions, some are known fewer than 12 hours in advance: for example, a patient that requires emergent, unscheduled surgery only 8 hours prior to needing an ICU bed. Other unscheduled admissions are not known in advance: for example, when an admitted acute care patient's health condition deteriorates. In both types of unscheduled admissions, there is not enough time to modify nursing work schedules to accommodate the unforeseen needs. The lack of an analytical decision support tool to analyze unscheduled admissions, and to assist the charge nurses in their decision-making, results in either chronically short-staffed wards (increasing the workload for nurses) or over-staffed wards (costly and inefficient).

In this manuscript we analyze the operations of a PICU by using clinical operational data extracted from two different databases. We develop a robust model for reliable estimation of the unscheduled PICU admissions, thereby estimating more accurately the number of nurses required for patient care in each work shift. This two-phase study first evaluates the accuracy of the existing nurse staffing allocation system, then uses a second data set to model and predict the unscheduled patient population for this PICU. The ability to accurately estimate admissions will prove to be a valuable input for building accurate cyclical nurse staffing schedules.

3.2 Materials and Methods

The PICU under consideration is a 72-bed unit with three 24-bed wards in a free-standing children's hospital. Wards A and B focus primarily on cardiac and non-cardiac surgery patients, respectively, and Ward C is the medical intensive care unit. Although planning for staffing is based on three 8-hour shifts (Day, Evening, Night), the administrators prefer to use six consecutive 4-hour shifts (Day 1, Day 2, Evening 1, Evening 2, Night 1, and Night 2. starting at 7:00AM, 11:00AM, 3:00PM, 7:00PM, 11:00PM and 3:00AM, respectively).

The study consists of two phases. In phase one we use data from the PICU's internally-developed staff allocation tool (called "Staff Assist") and from a national clinical database called Virtual PICU Performance System (VPS) (VPS LLC, Los Angeles, CA), to evaluate the accuracy of the existing staff allocation tool. In the second phase, we use the VPS data to develop a reliable decision support tool for estimating the number of unscheduled admissions. That is, we estimate the expected number of admissions for work shifts at each ward of the PICU, then compare these expected values with the actual number of admissions.

StaffAssist records the estimated number of admissions and recommended/desired/agreed number of nurses. VPS is dedicated to standardized data sharing/ benchmarking among pediatric ICUs, and records the actual admissions data for the PICU. Importantly, every admission in the VPS is classified as scheduled or unscheduled based on whether the admission was known 12 or more hours prior to patient arrival in the PICU. Data for the period 02/01/11-12/31/12 was collected from StaffAssist and for the period 04/01/2009-12/31/2012 from VPS. We obtained institutional Research Ethics Board approval to use the data.

3.2.1 Evaluation of StaffAssist Performance and Descriptive Statistics for Admissions

This section provides an overview of the current approach to staff planning at the medical units. StaffAssist first combines the existing census (number of patients in the unit at a given nursing shift) and the expected admission/transfer in/discharge numbers entered by charge nurses (head nurses), to compute the predicted census. It then uses the ward-specific work load factor WHPUOS (worked hours per unit of service) to recommend the number of nurses for each daily shift based on predicted census for the ward. WHPOUS is based on historical data entered into the StaffAssist system. WHPUOS is a measure of productivity, which is computed by the ratio of actual hours worked divided by the volume of service for the same period.

Each PICU ward is assigned a specific number of nurses, called the "agreed" number of nurses. For each 4-hour shift, charge nurses enter into StaffAssist the current patient census and the expected number of admissions. Expected admissions include the scheduled and unscheduled admissions that are known (in less than 12 hours) to be coming to the PICU. The StaffAssist tool then recommends the number of nurses to use for each shift at each ward (Rather StaffAssist does not incorporate the unscheduled admissions into the nurse requirement estimates). Each charge nurse, enters their "desired" number of nurses. They consider the PICU census and the expected unscheduled admissions. If an unscheduled admission becomes known, the charge nurses will account for this patient in their entry request. However, charge nurses neither use an analytical method to estimate the number of unscheduled admissions, nor address the issue of "unknown" unscheduled admissions. To obtain the "agreed" number of nurses, the "recommended" number of nurses from StaffAssist is compared with the "desired" number of nurses. Finally, the nurse staffing office review both entries to allocate an agreed number of nurses to work shifts at each ward throughout the PICU and hospital.

To achieve our phase one goals, we extracted the historical data for the recommended, desired and agreed number of nurses from StaffAssist as well as the data for the actual number of admissions from VPS. Using these data, we evaluated the accuracy of the existing staff allocation system by comparing the number of nurses recommended by StaffAssist to the number desired by charge nurses. Then we analyzed the admissions (scheduled and unscheduled) data to obtain descriptive statistics and inferences on the distribution of unscheduled admissions data. All phase one analyses were completed using Microsoft® Excel® 2010 (Microsoft, Redmond, WA).

3.2.2 Development of a Decision Support Tool to Estimate the Unscheduled Admissions

In this step, we used the unscheduled admissions data from the VPS database to develop a reliable estimation tool for the number of unscheduled admissions. This tool might improve the accuracy for nurse staffing. We classified the unscheduled admissions data abstracted from VPS by shift, day of the week and ward. Analysis of the number of unscheduled admissions on each day of the week (see Table 4 and Figure 5) indicated that the weekday admissions (Monday through Friday) are higher than those on weekends (Saturday and Sunday). Therefore, we combined all the admissions into two day groups and fit distributions that characterize the number of unscheduled admissions on six daily shifts for each ward. In other words, we fit individual distributions for each one of 6·2·3 = 36 subsets of the data (e.g., D1-weekday-WardA, N2-weekend-WardB, etc.). We fit distributions by using JMP® Pro 11.1.1 (SAS, Cary, NC).

In each data subset, Poisson distribution emerged as the best fit for characterizing the distribution of unscheduled admissions. Poisson distribution is widely used for modeling arrivals to a medical unit. A discrete random variable is said to have a Poisson distribution with parameter λ, if the probability mass function is given by:

$$f\left(k;\lambda\right)=P\left(X=k\right)=\frac{\lambda^{k}\cdot e^{-\lambda}}{k!}\qquad for\ \ k=0,1,2... \tag{1}$$

where e is the Euler's number and *k!* represents the factorial of *k*. The positive real number λ is equal to both the expected value and the variance of X, i.e λ = E[X] = Var[X]. In our context, λ is the estimator for the average number of unscheduled admissions in each subset. Finally, k is the number of unscheduled admissions.

After obtaining the value of λ for each such subset, we generate the cumulative probability distribution (CDF) for the number of unscheduled admissions in each subset. The probability mass function obtained by plugging λ into equation (1) and evaluating this function for k = 0, 1, 2 ... is used to generate its CDF for the number of unscheduled admissions in each subset. After obtaining these probability distributions, we constructed a simulation model to imitate the unscheduled admissions in all three wards of the PICU and to verify the accuracy of the estimates obtained by the probability distributions.

3.2.3 Simulation Model

This section describes the simulation model used to generate the unscheduled patient admissions in all three wards of the PICU. Let *F(X)* denote the CDF for the number of unscheduled admissions variable. For each subset, we simulated the number of unscheduled admissions by using the inverse function, i.e. for $Y \sim U(0,1)$, we generated a uniform random variate y (a realization of Y) and found the corresponding number of unscheduled arrivals for y by using $F^{-1}(y)$. For example, for shift E1 on weekdays in Ward B, the arrival rate λ is estimated as 0.7381 (i.e., on average 0.7381 admissions occur for shift E1 on weekdays in Ward B). Therefore, we obtain the probability mass function values as:

$$P(0) = \frac{e^{-0.7381} \cdot (0.7381)^0}{(0)!} = 0.4780; \qquad P(1) = \frac{e^{-0.7381} \cdot (0.7381)^1}{(1)!} = 0.3528;$$

$$P(2) = \frac{e^{-0.7381} \cdot (0.7381)^2}{(2)!} = 0.1302; \qquad P(3) = \frac{e^{-0.7381} \cdot (0.7381)^3}{(3)!} = 0.0320;$$

$$P(4) = \frac{e^{-0.7381} \cdot (0.7381)^4}{(4)!} = 0.0070.$$

In this example $P(X = k)$ denotes the probability that there are k unscheduled admissions in shift E1 on a weekday in Ward B. Using the probability mass function values, we obtain the following CDF for the number of unscheduled admissions to Ward B during shift E1 on weekdays:

$$F(x) = \begin{cases} 0.4780; x < 1 \\ 0.8308; 1 \leqslant x < 2 \\ 0.9610; 2 \leqslant x < 3 \\ 1.0000; x \leqslant 4 \end{cases}$$

where x is a realization of X and X follows Poisson distribution with rate 0.7381. Now, to generate a simulated number of arrivals for this shift, we generated a uniform random variate y and found the corresponding number of arrivals for y using $F^{-1}(y)$. For example, when y is equal to 0.9854 we estimate that two unscheduled admissions will occur.

We organized the VPS data into the "Training" and "Testing" sets. The Training set covered the period from 04/01/2009 to 12/31/2011 and the Testing set covered the period from 01/01/2012 to 12/31/2012. We simulated the unscheduled admissions in each subset for 100 times. Following this, we obtained the average number of unscheduled admissions and compared these average values with the actual number of admissions. Appendix A provides a flow chart for depicting the estimation method described above. The simulation program for estimation of admissions was developed in Microsoft® Excel® 2010 (Microsoft, Redmond, WA).

3.3 Results

This section summarizes the evaluation of the existing staff allocation system and the descriptive statistics for the admissions data. We then present the results analyzing the accuracy of our estimation tool used for predicting the number of admissions.

3.3.1 Phase I Results

Table 3.1 reports the comparison of the number of nurses recommended by StaffAssist versus that desired by charge nurses where 'n' represents the frequency of cases with the specified condition in that ward and % values represent the proportion of cases among the overall number of observations for that ward. The data show that StaffAssist overestimates the required number of nurses in Ward A, while it underestimates that in Wards B and C. We also compared the recommended and agreed upon number of nurses from StaffAssist and found that StaffAssist recommends the same number of nurses as the agreed upon number of nurses only at 26.5%, 31.0% and 32.3% of the time for Wards A, B, and C, respectively, and is thus fairly inaccurate.

Registered Nurses (RNs)	Ward A		Ward B		Ward C	
	n[1]	%[2]	n	%	n	%
Recommended > Desired	2,218	52.7%	369	8.9%	825	19.6%
Recommended = Desired	1,159	27.6%	1,306	31.3%	1,396	33.2%
Recommended < Desired	830	19.7%	2,493	59.8%	1,987	47.2%

[1] n values represent the number of cases with the specified condition in each ward.

[2] % values represent the % of cases among all observations within that ward.

Table 3.1: StaffAssist Recommendation vs. Desired Number of Nurses

The magnitude of the underestimation/overestimation is also a point of interest. On average, StaffAssist overestimates the number of nurses by 0.50 in Ward A, while it underestimates the number of nurses by 0.85 and 0.48 in Wards B and C, respectively. Also, among the cases that underestimation/overestimation of required number of nurses occurs, the average values of the underestimation/overestimation are provided in Table 3.2. Table 3.2 indicates that the average value for the underestimation in StaffAssist is higher than that for the overestimation in all wards.

		Work Shift						Total
		Day 1	Day 2	Evening 1	Evening 2	Night 1	Night 2	
Ward A	Underestimate	17.68%	22.91%	25.21%	20.51%	17.99%	13.80%	19.73%
	Equal	26.31%	28.85%	31.02%	30.41%	24.65%	23.89%	27.55%
	Overestimate	56.01%	48.23%	43.77%	49.08%	57.37%	62.31%	52.72%
Ward B	Underestimate	66.20%	59.97%	54.67%	63.93%	63.55%	49.53%	59.81%
	Equal	28.29%	27.44%	33.43%	31.40%	30.78%	37.26%	31.33%
	Overestimate	5.52%	12.59%	11.90%	4.67%	5.67%	13.21%	8.85%
Ward C	Underestimate	50.64%	44.48%	44.84%	53.18%	50.00%	39.82%	47.22%
	Equal	32.53%	32.01%	36.49%	35.08%	31.92%	30.91%	33.17%
	Overestimate	16.83%	23.51%	18.67%	11.74%	18.08%	29.27%	19.61%

Table 3.2: Accuracy of Nursing Requirements in Each Work Shift for the Wards

Figure 3.1(a)-(c) display the accuracy of StaffAssist in identifying the number of nurses in each shift for Wards A-C, respectively. In Ward A, StaffAssist overestimates the nurse requirements mostly in the night shifts, and underestimates mostly in the evening shifts (see Figure 3.1(a)). In Wards B and C, StaffAssist underestimates the nurse requirements in all the shifts and more noticeably so at D1, E2 and N1 (see Figures 3.1(b) and 1(c)).

In summary, the overestimation of nurse requirements in Ward A and the underestimation in Wards B and C cause significant difficulties in managing the PICU nursing staff. This analysis shows that improvements to the current decision support tool may offer more accurate estimations of nurse requirements.

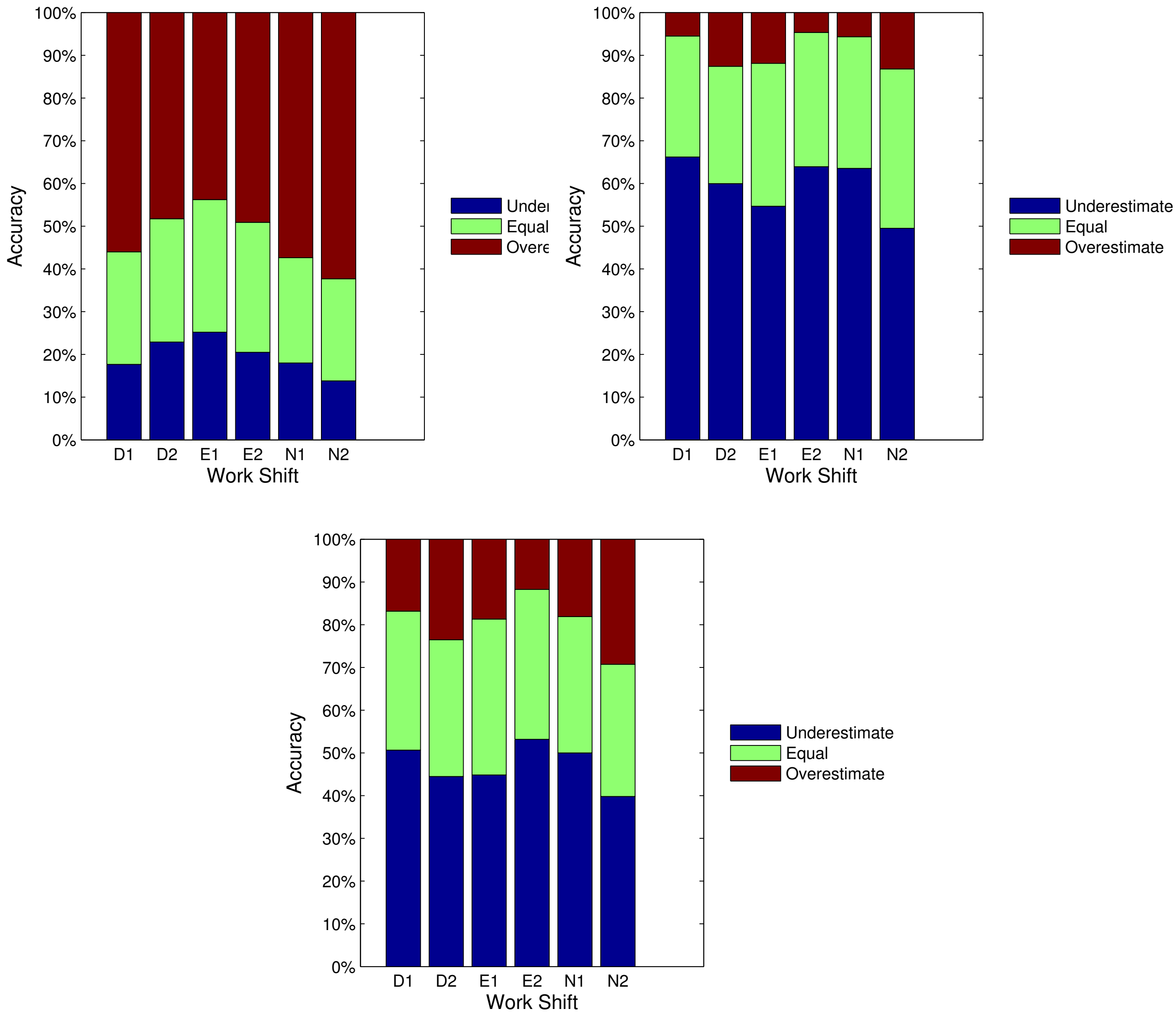
One of the inputs for StaffAssist is the number of admissions to each ward. Table 3.3 shows the number of scheduled and unscheduled admissions at each ward and as a total for the PICU. VPS data shows that about 69% of admissions to the PICU are unscheduled. Ward A has a higher percentage of scheduled admissions (61.3%) whereas Wards B and C have significantly higher percentages of unscheduled admissions (67.4 and 87.9%, respectively).

Table 3.4 shows the shift and day based number of unscheduled admissions for the PICU. Most unscheduled admissions occur in E1, D2 and E2 shifts with a peak in E1 shift, and during the weekdays (Monday-Friday)

Figure 3.1: Accuracy of Nursing Requirements in Each Shift

(a) Ward A

(b) Ward B



(c) Ward C

with a peak on Wednesday.

Figure 3.2(a) shows the breakdown of the number of unscheduled admissions by ward for each shift and Figure 3.2(b) shows that for each day of the week. The majority of the unscheduled admissions are for Wards B and C. Unscheduled admissions numbers in D1 and N2 shifts are considerably smaller than their counterparts in the other shifts. Figure 3.2(b) displays that the unscheduled admissions occur with higher

Data Category	Ward A		Ward B		Ward C		PICU Total	
	n^1	$\%^2$	n	%	n	%	n	%
Scheduled Admissions	993	61.3%	1,235	32.6%	353	12.1%	2,581	31.0%
Unscheduled Admissions	627	38.7%	2,556	67.4%	2,555	87.9%	5,738	69.0%
Total Admissions	1,620		3,791		2,908		8,319	
Total Discharges	1,596		3,764		2,889		8,249	

[1] n values represent the number of cases with the specified condition in each ward.

[2] % values represent the % of cases among all observations within that ward.

Table 3.3: Scheduled vs. Unscheduled Admissions Recorded in VPS

Shift	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Shift Total
Day1	64	83	85	58	66	63	65	484
Day2	196	195	215	212	206	124	108	1,256
Evening1	246	229	249	255	240	136	155	1,510
Evening2	182	171	170	164	159	132	161	1,139
Night1	127	141	146	107	128	132	124	905
Night2	51	68	71	59	55	73	67	444
Day Total	866	887	936	855	854	660	680	5,738

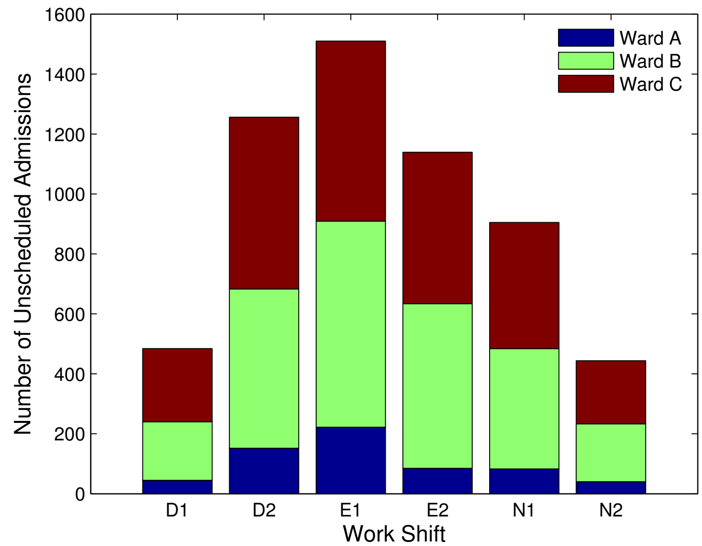
Table 3.4: Number of Unscheduled Admissions in Each Shift on Each Day at the PICU

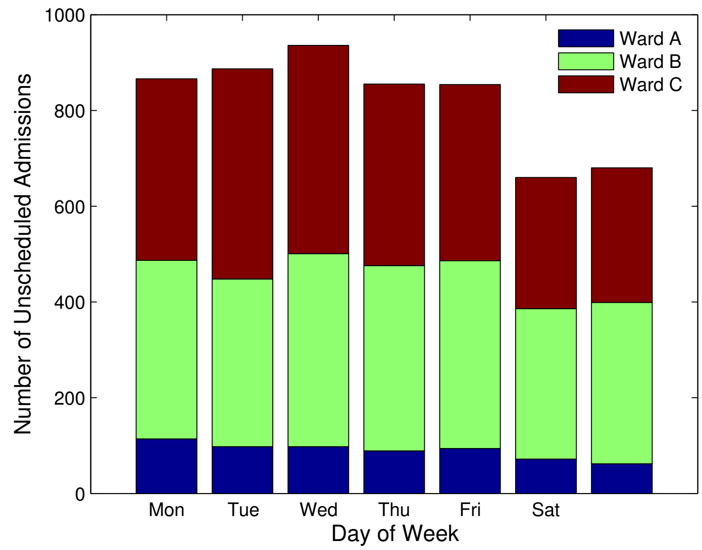
frequency during the weekdays (Monday-Friday) than they do during the weekends. Other than Wednesday, the remaining weekdays have similar unscheduled admission frequencies. Saturday and Sunday frequencies also resemble each other.

Although unscheduled admissions are hard to predict, they have a significant impact on the accuracy of the nursing requirements in StaffAssist. Figures 3.3(a)-(c) indicate the challenges the nurses have in accurately estimating the total (scheduled and unscheduled) number of admissions for each shift. In Ward A, charge nurses significantly overestimate the number of admissions in shifts D2 and E1 by 59.69% and 23.89%, respectively. In Ward B, charge nurses overestimate the number of admissions for shifts D2 and E1 (by 37.99% and 18.90%, respectively), while underestimate that in all the other shifts (on average by 35.64%). In Ward C, charge nurses underestimate the number of admissions in all shifts with a significant margin (ranging from 36.15% to 47.54%). This analysis shows that the charge nurses do not properly adjust the expected

Figure 3.2: Unscheduled Patient Admissions

(a) by Shifts in a Day



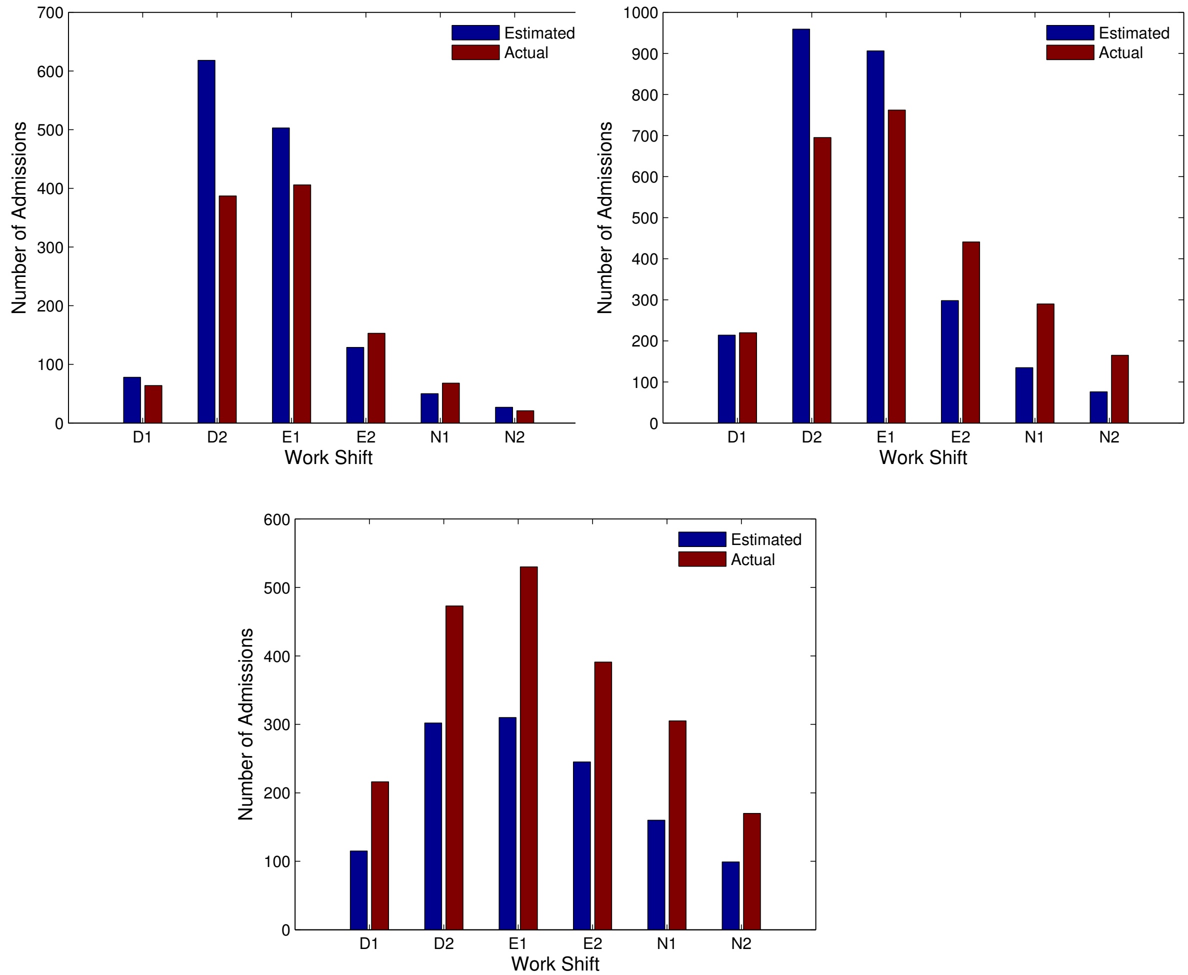


(b) by Days of a Week

admissions with the unscheduled admissions number and signals the need for a more reliable and objective decision support tool.

Figure 3.3: Accuracy in Estimating Admissions

(a) Ward A (b) Ward B



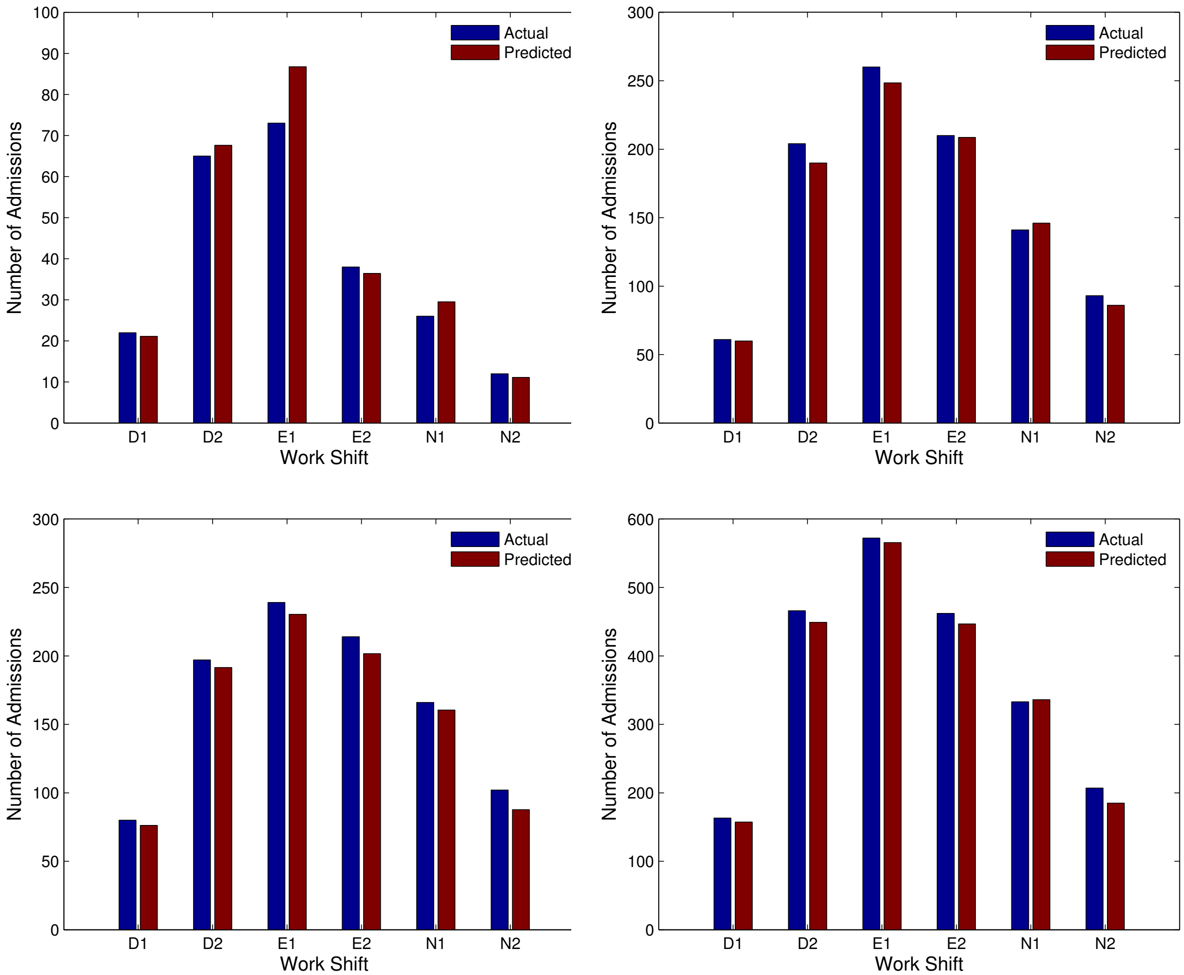
(c) Ward C

3.3.2 Phase II Results

Following the evaluation of the existing staffing system, we focused on the development of an improved tool estimating unscheduled admissions. Using the method described in Phase II of the study, we predicted the number of unscheduled admissions in each ward (and the PICU as total) and compared these estimated values with the actual number of unscheduled admissions from the VPS database. Figure 3.4(a)-(d) display these comparisons where the values listed are the aggregated values for the shifts (combining the weekday and weekend values to be able to compare with values in Figures 3.3(a)-(c)).

Figure 3.4: Actual vs. Predicted Number of Unscheduled Admissions

(a) Ward A (b) Ward B



(c) Ward C (d) PICU Total

As seen in Figures 3.4(a)-(d), the unscheduled admissions can be predicted with reasonable accuracy by characterizing their distribution with Poisson distribution. We computed the % deviation in the number of unscheduled admissions by using the following formula:

$$\frac{(Predicted\ Unscheduled\ Admissions - Actual\ Unscheduled\ Admissions)}{(Actual\ Unscheduled\ Admissions)} \cdot 100\%.$$

The % deviation in Wards A, B, and C ranged from -7.25% to 18.82% (on average 3.48%), from -7.48% to 3.55% (on average -2.94%), and from -14.06% to -2.80% (on average -5.74%), respectively.

In addition, Tables 3.5 - 3.7 show the day of week and shift based accuracy computations for Wards A-C.

When we disaggregate the data points (i.e. analyze weekdays and weekends separately), the prediction accuracy deteriorates, but still manages to provide good estimations for the number of unscheduled admissions. The high deviation values for Ward A, especially in the weekends, are due to the lack of data points to properly estimate the corresponding λ values. Prediction accuracy in weekends for Wards B and C are also slightly worse than those for weekdays, again due to the small sample size. The overall % deviation for the PICU ranges from -7.61% to 0.86% and from -17.64% to 5.51% on the weekdays and weekends, respectively.

Ward A	**Weekday**			**Weekend**		
Shift	**Actual**	**Predicted**	**% Deviation**	**Actual**	**Predicted**	**% Deviation**
Day1	14	14.41	2.93%	8	6.69	-16.38%
Day2	56	54.75	-2.23%	9	12.87	43.00%
Evening1	62	73.79	19.02%	11	12.95	17.73%
Evening2	34	30.19	-11.21%	4	6.23	55.75%
Night1	20	21.57	7.85%	6	7.94	32.33%
Night2	6	7.69	28.17%	6	3.44	-42.67%

Table 3.5: Prediction Accuracy by Work Shift and Day of Week - Ward A

Ward B	**Weekday**			**Weekend**		
Shift	**Actual**	**Predicted**	**% Deviation**	**Actual**	**Predicted**	**% Deviation**
Day1	42	40.66	-3.19%	19	19.27	1.42%
Day2	164	153.56	-6.37%	40	36.34	-9.15%
Evening1	214	195.45	-8.67%	46	53.04	15.30%
Evening2	158	155.38	-1.66%	52	53.28	2.46%
Night1	102	106.19	4.11%	39	39.81	2.08%
Night2	61	58.62	-3.90%	32	27.42	-14.31%

Table 3.6: Prediction Accuracy by Work Shift and Day of Week - Ward B

Ward C	**Weekday**			**Weekend**		
Shift	**Actual**	**Predicted**	**% Deviation**	**Actual**	**Predicted**	**% Deviation**
Day1	57	58.90	3.33%	23	17.26	-24.96%
Day2	166	162.42	-2.16%	31	29.07	-6.23%
Evening1	194	190.06	-2.03%	45	40.23	-10.60%
Evening2	168	157.13	-6.47%	46	44.45	-3.37%
Night1	128	120.61	-5.77%	38	39.82	4.79%
Night2	76	65.81	-13.41%	26	21.85	-15.96%

Table 3.7: Prediction Accuracy by Work Shift and Day of Week - Ward C

3.4 Discussion

In this study we develop a reliable tool for estimating the number of unscheduled admissions. This will, in turn, improve the accuracy of identifying the nursing needs for work shifts at the PICU. Several studies have shown that a strong association exists between nurse staffing and patient outcomes (Blegen & Vaughn, 1998; Kovner & Gergen, 1998; Aiken et al., 2002; Needleman & Buerhaus, 2003). When a nursing unit is chronically short-staffed, nurses are forced to keep up an intense pace in order to ensure patients receive timely care. Over time, this can result in nurse burn-out,patient dissatisfaction, and even medical errors (American Sentinel University - Healthcare, 2015). Improved accuracy in the allocation of nursing staff could mitigate these operational risks and improve patient outcomes.

Nursing care is identified as the single biggest factor in both the cost of hospital care and patient satisfaction (Yankovic & Green, 2011). Yet, there is widespread dissatisfaction with the current methods of determining nurse staffing levels, including the most common one of using minimum nurse-to-patient ratios (Yankovic & Green, 2011). Nurse shortage implications go beyond healthcare quality, extending to health economics as well. In addition, implementation of mandatory nurse-to-patient ratios in some states creates a risk of underestimating or overestimating required nurse resources (Paul & MacDonald, 2013).

Green et al. (2013) suggest that the task of determining nurse staffing levels in hospitals is complex because of variable patient census levels and uncertain service capacity caused by nurse absenteeism (Green et al., 2013). To determine appropriate staffing requirements, factors such as total census, care intensity levels, and ward type must be estimated (Helmer et al., 1980). Hourly changes in patient census and patient acuity levels cause frequent fluctuations in the number of nurses required vs. the initial planned levels, forcing the healthcare providers to revise the staffing needs on a continuing basis (Bard & Purnomo, 2005). Additional factors to consider for effective nurse staffing include nurse preferences regarding work schedules and nurse absenteeism (Purnomo & Bard, 2007; Wang & Gupta, 2014).

Penoyer (2010) provided an annotated review of major nursing and medical literature to demonstrate the association of nurse staffing with patient outcomes in critical care units and populations. Coro et al. (2013) employed a large multi-center PICU database to investigate the characteristics associated with mortality in unplanned and planned pediatric cardiac intensive care unit (ICU) admissions. The mortality rate in the cardiac ICU was significantly higher among the unplanned admissions than the planned admissions. This study develops a decision support tool to understand the distribution and timing of unscheduled (unplanned) admissions in the PICU to mitigate such risks. The methods used here are easy to replicate in various healthcare settings. However, the accuracy of estimations will depend on obtaining reliable and sufficient historical data.

There are significant economic implications for optimizing nurse staffing based on an improved understanding of patient volume. While the cost of overstaffing can easily be viewed as waste, there are also costs for understaffing patient care units. Understaffing has been linked to hospital-acquired infections and their significant preventable costs (Cimiotti et al., 2012). Additionally, the loss of nurses through burnout is estimated to cost $300,000 per year for each percentage of annual nurse turnover (PriceWaterhouseCoopers, 2007). Thus, applying novel models to the chronic problems may lead to significant reductions in healthcare costs.

There are limitations to our findings. First, the findings reflect data from one PICU. Until our methods are applied to other PICUs (or even acute care units, if necessary data are available), our findings are provocative at best. A second limitation is the influence of charge nurse behavior on our evaluation of predicted versus desired nurses. In the absence of a "gold standard" for the true number of nurses needed at any moment, the "desired" number is prone to gaming and may reflect other factors beyond what the true perceived needs are. A final limitation is the role of infectious disease outbreaks and disasters on staffing needs. As evidenced in outbreaks of Influenza A H1N1, Enterovirus D68 or even the international spread of Ebola, it is unlikely any model can anticipate staffing needs created by unforeseen demands on staffing. Future work might couple models such as the one described here with machine learning to allow recalibration in the face of emerging diseases.

3.5 Conclusions

Determining accurate nurse staffing levels has been a topic of interest due to healthcare quality requirements, financial constraints, limited resources, patient safety requirements, and nurse shortages. This study confirms the influence of unscheduled admissions on the accuracy of predicting PICU admissions. We show that estimating the number of unscheduled admissions by obtaining the probability distribution of historical unscheduled admissions will provide higher precision compared to using only experience and intuition to do so. We propose a convenient, objective, simulation-based statistical methodology to assist healthcare providers in estimating the number of admissions and required number of nurses. Further research should be carried out to understand the nature of scheduled admissions before StaffAssist can be refined. In addition, the investigators also identified gaps between the expected admission and discharge numbers, and additional research will focus on understanding discharge patterns to resolve this discrepancy.

The potential contribution of this study is improved nurse staffing models, which in turn will enable nurses to deliver better quality care and improve patient outcomes. To our knowledge, predictability of unscheduled admissions has yet to appear in the literature in general, and in PICU literature specifically. Analytical predictive methods that complement intuition and experience-based decisions on nurse staffing and workload would help to decrease the unplanned/last-minute scheduling requirements for nurses, and to decrease costs with more efficient nurse staffing planning. Our model is generalizable to implement in other (pediatric) intensive care units for nurse staffing and could be a valuable input for future nurse staffing models.

Chapter 4

Integrated Nurse Staffing and Scheduling: Medium-Term Staffing Strategies

4.1 Introduction

The Affordable Care Act (ACA) is the comprehensive health care reform law enacted in March, 2010). The law has three primary goals. First, it seeks to make affordable health insurance available to more people. It does this by providing consumers with subsidies, or "premium tax credits", which lower costs for households with incomes between 100% and 400% of the federal poverty level. Second, it expands the Medicaid program to cover all adults with income below 138% of the federal poverty level. Third, the ACA supports innovative medical care delivery methods designed to lower the costs of health care generally (www.healthcare.gov). ACA led to more and sicker patients entering the healthcare system. This increased the nursing workload, leading to a higher risk of nurse burnout in already short-staffed hospital medical units. Over time, this can result in dissatisfied patients and even medical errors (www.americansentinel.edu). These developments require the hospital administrations to better control understaffing in the medical units while keeping a balance of the staffing costs.

This chapter focuses on integrated nurse staffing and scheduling in Intensive Care Units (ICUs), which are

7-day x 24-hour care environments facing unscheduled patient admissions with dynamic acuity levels. Our research objective is to construct staffing patterns, which specify the number of nursing personnel from various job profiles to be scheduled in the medical units and nursing shifts of a scheduling period. Our solution approach targets reducing the nurse staffing costs while balancing the under- and over- staffing risks, which will help mitigate nurse burn-out, improve patient outcomes and manage hospital staffing costs. Nurse rostering is an NP-hard combinatorial problem which makes it extremely difficult to efficiently solve real-life problems due to their size and complexity. Usually real problem instances have complicated work rules related to safety and quality of service issues as well as rules about preferences of the personnel. In order to avoid the size and complexity limitations, we generate feasible nurse schedules for the full-time equivalent (FTE) nurses using algorithms that will be used in the mixed-integer programming (MIP) models developed in Chapter 3. Pre-generated schedules eliminate the increased number of constraints and reduce the number of decision variables of the integrated nurse staffing and scheduling model. The MIP model recommends initial staffing plans and schedules for a six-week staffing horizon for the medical units, given a variety of nurse groups and nursing shift assignment types. We also include a novel methodology for estimating nurse workloads by considering patient census, acuity and activity in the unit.

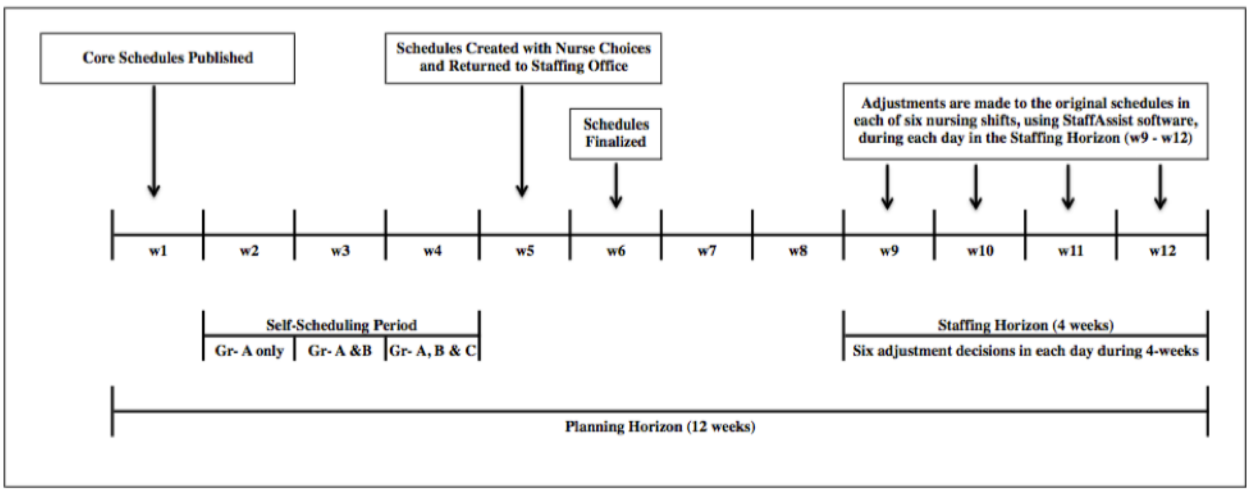
When the nursing administration prepares the medium-term nurse schedules for the next staffing cycle, one to two months prior to the actual patient demand realizations, target staffing levels for the upcoming nursing shifts are typically determined using historical average staffing levels for the nursing care needs. Using the MIP model, we examine fixed vs. dynamic medium-term nurse staffing and scheduling policy options for the medical units. In the fixed staffing option, the medical unit is targeted to be staffed at a fixed level throughout the staffing horizon. This chapter proposes a dynamic staffing policy option which uses historical patient demand data to instead suggest a non-stationary staffing scheme during the staffing horizon. We evaluate the fixed staffing policy alternative with various staffing level options (i.e. by staffing the medical unit with 11, 13 or 15 nurses throughout the staffing horizon). As an example, for the dynamic staffing alternative, we prepare a “heat map” of patient census and acuity, as well as admission-discharge-transfer (ADT) activity, in the medical units and compare the performance of dynamic heat map based policy vs. the alternative fixed staffing policies. We compare the performance of both nurse allocation policy options, in terms of cost savings and understaffing ratios, with the optimal staffing scheme reached by the actual patient data. We evaluate whether the dynamic medium-term nurse staffing policies that use patient demand forecasts outperform the historically employed fixed staffing policy for the intensive care medical units. In order to reduce nurse burnout and make the job more appealing to the new RN candidates, we introduce the concept of "understaffing penalty" as a mechanism to control the understaffing in the medical units. We analyze how various levels of understaffing penalty (the cost of understaffed hours given as a ratio to the cost to the FTE nurse staffing) affect the outcomes (in terms of staffing costs and understaffing levels) in the medical unit. We also evaluate the effect of the number of available schedules (NAS) per FTE nurse profile on the objective function costs and understaffing ratios in the medical units. We explore whether there exists a saturation level for the NAS, where increases in the NAS do not bring any additional cost savings. We use the MIP model as a mechanism to control the understaffing levels in the medical units that often trigger nurse burnout and medical errors.

Chapter 3 confirmed the influence of unscheduled admissions on the accuracy of predicting PICU admissions and demonstrates that using the probability distribution of historical unscheduled admissions improves the accuracy of estimating the number of unscheduled admissions. This in turn improves the nursing workload requirement estimations. The performance of the scheduling models discussed in the chapters that follow relies on the forecast accuracy for nursing workload estimates, which is complicated by the nature of unscheduled admissions to the PICU. Therefore, it is critical that models discussed in later sections use the enhanced forecasting methodologies developed in this chapter. Next, we discuss the motivation and significance of using the desired modeling approaches. The PICU Wards A and B mainly focus on cardiac and non-cardiac surgery patients, respectively, and Ward C is the medical intensive care unit. Each unit has a capacity of 24 beds for inpatients. Two data sources described in Chapter 3 are used for this study (i.e. VPS and StaffAssist). We use the distribution of patient acuities at the admissions in each medical unit as a proxy to estimate the acuity score of each patient in the unit.

4.2 Current Scheduling Practices at the PICU

There are multiple planning stages for nurse scheduling at the PICU. Figure 4.1 below illustrates a summary of current practices for nurse staffing and scheduling. There is a 12-week planning horizon, among which weeks 9 to 12 constitute the actual staffing horizon for the medical units. Major phases of this planning horizon are discussed next.

Figure 4.1: Illustration of Current Scheduling Implementation at the PICU



1. *Prepare Core Schedules*: The first phase of the scheduling prepares and publishes the core schedules for each medical unit. These core schedules list how many nurses are needed for each nursing shift in each day of the four-week staffing horizon, W9 - W12. These core schedules are targeted to be published during the week before the self-scheduling period begins (i.e. in week 1 as indicated in Figure 4.1), seven weeks before the actual staffing horizon. During the preparation phase, administrators attempt to maintain a predetermined level of staffing throughout the staffing horizon for each medical unit.

2. *Self-Schedule*: Following the publication of core schedules, the next three weeks are self-scheduling periods for the nurses (i.e. weeks two, three and four in Figure 4.1). The nurses are divided into three priority groups for self-scheduling. Group A gets to select the desired schedule first, then Group B, then Group C. These groups are rotated among the nurses in each scheduling cycle for fairness (i.e. a group A nurse will be in Group B in next cycle and in Group C in the following cycle). All the nurses have day and night nurse classifications (i.e. some nurses are designated to day shifts, 7:00 AM to 7:00 PM, and some are designated to night shifts, 7:00 PM to 7:00 AM) and they will pick their desired

schedule accordingly. If schedulers want the nurses to rotate for the benefit of maintaining the staffing level, they query nurses before they self-schedule, using the anticipated leaves and resignations to help guide that decision. When the schedule opens for self-scheduling, one of the unit schedulers has already “mastered” in (i.e. fixed the assignment) of that particular nurse’s weekends and any holidays that fall during the schedule period. The nurse then selects the preferred shift and enters that as a pending assignment code so that the unit schedulers know which are “masters” and which are “self-schedule”.

Each medical unit/ward in the hospital has a target (core) nurse staffing level, which is modified annually depending on the “Budgeted Average Daily Census” and “Required Care Hours.” Previous-year patient census and acuity scores are used to estimate the levels of nursing workload indicators. Once calculated, these “core” staffing levels serve as caps for nurse assignments. Occasionally, some units use caps which are one above the core staffing levels. During the self-scheduling period, the staffing caps constrain too many nurses to select a specific set of nursing shifts (i.e. once the cap is reached for a nursing shift, no additional nurses can self-schedule for that specific shift any more).

3. *Finalize Schedules with Nurse Choices*: When the self-scheduling period closes (Week 5), the unit schedulers attempt to smooth the staffing levels, following the predefined scheduling rules (i.e. none of the nurses can be scheduled for four 4-hour shifts in a row unless they choose that, or schedulers cannot put them on a Friday, Saturday, or Sunday that’s not their assigned weekend). The staffing office finalizes the schedules in week six, two and a half weeks before the staffing horizon begins. Once they are finished, it is reviewed by a unit leader and published for all to view. Figure 4.2 provides a sample chart demonstrating the first three phases of the current PICU scheduling process.

Figure 4.2: Screenshot from PICU Scheduling Office for the First Three Stages

4 Week Schedule Period	Self Schedule Groups (if used, follow unit guidelines)	Planning/Self Scheduling Period *** Remember to*** Publish Core Schedules prior to the first day of the planning/self scheduling period	This is the time period when employees will not be able to view pending schedules	1. Create schedule 2. Complete CRU orientation schedules and return to Staffing Office	Schedules are Final. ***Archive*** Complete schedules in the following format Pre mm/dd/yy unit name Post mm/dd/yy unit name
SUN - SAT	GROUPS	FRI - THUR	FRI-TUES	FRI - FRI	WED
12/20/2015-1/16/2016	All Unit	10/30/2015-11/19/2015	11/20/2015-12/1/2015	11/20/2015-11/27/2015	12/2/2015
Christmas 12/25	A	10/30/15-11/19/15			
New Years 1/1/16	B	11/6/15-11/19/15			
	C	11/13/15-11/19/15			

4. *Adjust the Schedule Phase*: During the four-week staffing horizon nurse schedulers adjust the finalized schedules, but must inform the nurses, ideally 72 hours prior the time of adjustment. Schedulers are assumed firmed and fixed 24 hours prior to the actual staffing shift.

5. *Staffing the Next Nursing Shift Using StaffAssist*: The schedulers currently use PICUs internally developed staff allocation tool StaffAssist for adjusting staffing levels for the next nursing shift. While the majority of nurses work in 12-hours shifts, planning for staffing is based on three 8-hour shifts (Day, Evening, Night). However, because of the ever-changing unit census, the schedulers prefer to use six consecutive 4-hour shifts – Day 1 (starting at 7:00AM), Day 2, Evening 1, Evening 2, Night 1, and Night 2) to identify the required number of nurses for each shift more accurately. The details of StaffAssist procedures are already discussed in Section 3.2.

   During the StaffAssist adjustments phase, when the scheduled nurses are more than the recommended and/or agreed upon nurses, the extra nurses are noted in StaffAssist and they will be floated to another unit if needed there. If not, downtime is granted based on requests and seniority. If the scheduled nurses are fewer than the recommended and/or agreed upon nurses, the resources for the required extra nurses will be the float pool (called CRU - Clinical Resource Unit) or any staff members able to float from other units. Competencies must be matched (i.e. an acute care floor RN is not able to take many patient assignments in the PICU's). If the shortage persists, the unit works short. All nursing staff are assigned to a specific unit/ward and within the PICU there are three different staffs (W3, W4, W5). If a ward is overstaffed, preference is to send extra staff to another PICU ward. However, the ultimate decision is made by the Patient Care Manager on-call for that shift, as part of the house-wide staffing assignment process.

6. *Nurse-to-Patient Assignment Phase*: The final stage is the nurse-to-patient assignment phase occurring during the shift. Nursing assignments to specific patients are made in 4-hour increments. Although every attempt is made to keep a nurse with the same assignment for the duration of his or her shift, both for patient safety as well as nurse satisfaction, assignments may need to change based on the staff level.

4.3 Nurse Classification and Job Profiles

While the majority of nurses work 12-hour shifts, staffing plans are based on three 8-hour shift formats (Day, Evening, and Night). However, because of the ever-changing unit census, schedulers prefer to use six consecutive 4-hour shift blocks of time – Day 1 (D1) starting at 7:00AM, Day 2 (D2), Evening 1 (E1), Evening 2 (E2), Night 1 (N1) and Night 2 (N2) – to more accurately identify the required number of nurses for each shift. Below we discuss various classifications for the nurses:

Employment Types: There are two main Employment Type classifications for PICU nurses working: FTE (full-time equivalent) nurses, and PRN ("pro re nata," a Latin phrase that roughly translates to "as needed" or "as the situation arises") nurses. FTE nurses are further defined in clusters. An FTE - 1.0 nurse will work 40 hours per week, an FTE - 0.9 nurse will work 36 hours per week, etc. The work hours per week and shift types for the FTE nurses are given in the table below. PRN nurses also have three tiers: Tier-I nurses will work at least 20 hours, Tier-II nurses will work at least 44 hours and Tier-III nurses will work at least 76 hours during a scheduling period (i.e. 4 weeks). Details on nurse job profiles and associated shift patterns for each job class appear in Table 4.1, and we present nurse job profile distributions for the PICU in Table 4.2:

Self-Scheduling and Weekend Assignment Groups: The nurses are divided into three priority groups for self-scheduling: A, B, and C as detailed in Section 4.2. During the scheduling cycle, each nurse group determines the self-scheduling rank and week for each nurse to select his or her shift assignments (from here forward "his or her" will be presented only as "her"). The self-scheduling priority group assignments also determine the weekends on which the nurse is assigned to work. For example, a Group A nurse will work the first Friday, Saturday and Sunday, which is the technical definition of a weekend at the hospital (D1 shift on Friday through an N2 shift on Monday). She will then be assigned to another weekend shift three weeks later.

Day and Night Shift Assignment Groups: All the nurses have day shift and night shift classifications. Some nurses are designated to day shifts, working 7:00 AM to 7:00 PM, and others to night shifts, working 7:00 PM to 7:00 AM). Nurses pick their desired schedule accordingly.

Profile	Employment Type	Shift Description	Shift Type	Weekend Gr.
1	FTE - 0.9	Three 12-hour shifts; 36 hrs/wk	Day	Group A
2	FTE - 0.9	Three 12-hour shifts; 36 hrs/wk	Day	Group B
3	FTE - 0.9	Three 12-hour shifts; 36 hrs/wk	Day	Group C
4	FTE - 0.9	Three 12-hour shifts; 36 hrs/wk	Night	Group A
5	FTE - 0.9	Three 12-hour shifts; 36 hrs/wk	Night	Group B
6	FTE - 0.9	Three 12-hour shifts; 36 hrs/wk	Night	Group C
7	FTE - 0.8	Two 12-hour, one 8-hour shifts; 32 hrs/wk	Day	Group A
8	FTE - 0.8	Two 12-hour, one 8-hour shifts; 32 hrs/wk	Day	Group B
9	FTE - 0.8	Two 12-hour, one 8-hour shifts; 32 hrs/wk	Day	Group C
10	FTE - 0.8	Two 12-hour, one 8-hour shifts; 32 hrs/wk	Night	Group A
11	FTE - 0.8	Two 12-hour, one 8-hour shifts; 32 hrs/wk	Night	Group B
12	FTE - 0.8	Two 12-hour, one 8-hour shifts; 32 hrs/wk	Night	Group C
13	FTE - 0.6	Two 12-hour shifts; 24 hrs/wk	Day	Group A
14	FTE - 0.6	Two 12-hour shifts; 24 hrs/wk	Day	Group B
15	FTE - 0.6	Two 12-hour shifts; 24 hrs/wk	Day	Group C
16	FTE - 0.6	Two 12-hour shifts; 24 hrs/wk	Night	Group A
17	FTE - 0.6	Two 12-hour shifts; 24 hrs/wk	Night	Group B
18	FTE - 0.6	Two 12-hour shifts; 24 hrs/wk	Night	Group C
19	FTE - 0.5	One 12-hour, one 8-hour shift; 20 hrs/wk	Day	Group A
20	FTE - 0.5	One 12-hour, one 8-hour shift; 20 hrs/wk	Day	Group B
21	FTE - 0.5	One 12-hour, one 8-hour shift; 20 hrs/wk	Day	Group C
22	FTE - 0.5	One 12-hour, one 8-hour shift; 20 hrs/wk	Night	Group A
23	FTE - 0.5	One 12-hour, one 8-hour shift; 20 hrs/wk	Night	Group B
24	FTE - 0.5	One 12-hour, one 8-hour shift; 20 hrs/wk	Night	Group C
25	FTE - 0.3	One 12-hour shift; 12 hrs/wk	Day	Group A
26	FTE - 0.3	One 12-hour shift; 12 hrs/wk	Day	Group B
27	FTE - 0.3	One 12-hour shift; 12 hrs/wk	Day	Group C
28	FTE - 0.3	One 12-hour shift; 12 hrs/wk	Night	Group A
29	FTE - 0.3	One 12-hour shift; 12 hrs/wk	Night	Group B
30	FTE - 0.3	One 12-hour shift; 12 hrs/wk	Night	Group C
31	PRN - Tier I	32+ hr/schedule; $\leqslant$ 40 hrs/wk		
32	PRN - Tier II	68+ hr/schedule; $\leqslant$ 40hrs/wk		
33	PRN - Tier III	116+ hr/schedule; $\leqslant$ 40hrs/wk		

Table 4.1: Job Profiles for Nurses

Job Profile	Ward A	Ward B	Ward C	PICU	% in PICU	Hours
FTE - 0.9	59	48	50	157	62.8%	36 hrs. per week
FTE - 0.8	4	0	1	5	2.0%	32 hrs. per week
FTE - 0.6	20	11	18	49	19.6%	24 hrs. per week
FTE - 0.5	2	1	1	4	1.6%	20 hrs. per week
FTE - 0.3	2	1	0	3	1.2%	12 hrs. per week
PRN - I	0	0	0	0	0%	32+ hr/schedule
PRN - II	4	2	2	8	3.2%	68+ hr/schedule
PRN - III	12	8	4	24	9.6%	116+ hr/schedule

Table 4.2: Nurse Job Profile Distributions in the PICU

4.4 Two-Phase Procedure for Optimal Nurse Assignments

Chapter 3 evaluated the existing staff allocation system of a PICU, and we developed a method to reliably estimate the number of unscheduled admissions to the PICU. The following sections of Chapter 4, now focus on developing optimization methods that will more accurately identify the PICU nursing needs. Our main objective for the optimization model we have developed is to reduce nurse staffing costs while balancing the under- and over-staffing risks. To do this we use a two-phase procedure for optimal nurse assignments. Phase 1 of our solution procedure generates feasible FTE nurse schedules for the staffing horizon of six weeks, while satisfying the constraints imposed by the nurse profile. Phase 2 assigns FTE nurses to the pre-generated feasible nurse schedules and PRN nurses to the nursing shifts, using mixed-integer optimization models. We first develop a "heat map" of patient census and ADT activity in the medical units for the dynamic staffing policy option. To develop the heat map we estimate monthly seasonality index for Patient Census, Acuity and ADT Activity. Then we estimate Patient Census, Acuity and ADT Activity averages for all Day of Week and Shift of the Day combinations. The desired heat map of patient demand is generated by multiplying the monthly seasonality factors with the historical Day-Shift averages for the medical units. Using the heat map and the mixed-integer optimization models we analyze whether dynamic staffing policies outperform the currently-used fixed staffing policy. We also compare the performance of both options with the optimal staffing scheme reached by the actual patient data.

The Dynamic Staffing Model aims to minimize the staffing costs from FTE and PRN nurses, along with understaffing penalty costs. The Fixed Staffing Model minimizes the total difference between a predetermined target staffing level and actual staffing levels resulting from nurse assignments. Figure 4.3 below summarizes the modeling approaches adopted in this chapter. The first distinction among alternative models is a fixed vs. dynamic medium-term nurse staffing target for the staffing horizon. For the alternative that uses fixed staffing targets, we use the Fixed Staffing Model and historical patient demand data. For the dynamic staffing targets,we use the optimal staffing model as a benchmark for model performance measurement. This uses actual patient demand observed during the studied staffing horizon. Our Dynamic Staffing Model uses the heat map approach outlined above for the patient demand forecast throughout the staffing horizon. We give a description of these models in Section 4.6.

The output of the optimization using our integrated nurse staffing and scheduling models will be available schedules for nurses to pick from among the suitable ones to their profiles. We then can have "open", "firm"

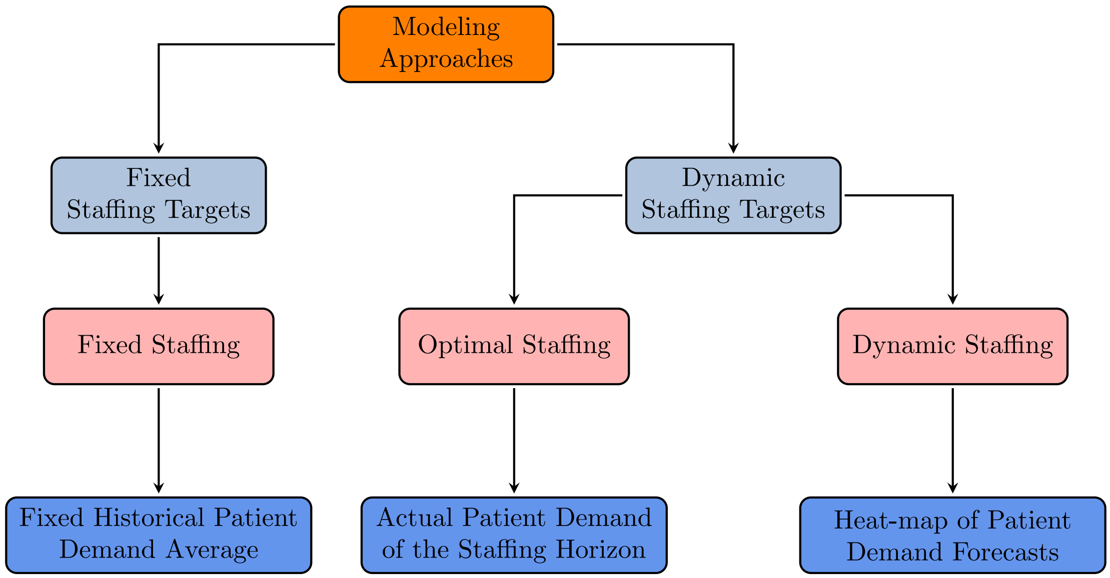


Figure 4.3: Modeling Approaches

and “frozen” phases of nurse assignments. In the “open phase”, we allow nurses to have a period similar to their current self-scheduling period, in which they will be allowed to pick a schedule that meets their job class and type. We can assume this selection of schedules process to be completed in a manner similar to the current self-scheduling process in use (i.e., certain nurse groups might be allowed to select from the available schedules depending on a rotating priority scheme). To enhance nurse satisfaction with the schedules even further, unit nurse managers can allow nurses, in the “firm phase” to switch some blocks within their initial assignments among each other. Following the firm phase, the schedules are final and no further changes are allowed in the “frozen phase”.

4.4.1 Determination of Required Staffing Levels

While research has established that staffing is associated with patient safety, few studies have examined ways to measure nurse workload and its impact on patient safety (Laschinger & Leiter, 2006). Various methodologies and staffing management tools are used at different hospitals to calculate the nursing workload and good staffing levels for clinical units. Average Daily Census (ADC), counted as the number of patients at midnight, has been used by hospitals to determine capacity needs, budgeting, and staffing, but it is not clear that this measure captures the full extent of demand for beds or its dynamic nature (Kosnik, 2006). Midnight census (MC) undercounts the workload for high-occupancy hospitals that have the most beds occupied at any given time, but also often discharge a patient in the morning, then admit a different patient later in the day in the same bed (Baernholdt et al., 2010). Although there is debate on how much time nurses spend on admitting, transferring, or discharging patients, previous studies agree that services required for ADT activities is a major component of the nursing workload (Baernholdt et al., 2010). Significant improvements have been made in capturing patient acuity, but staffing management systems still often underestimate workload in terms of dynamic patient flows (Wagner, 2005), and nurse managers lack the tools to reliably measure nursing workload (Lee & Cummings, 2008). Once based solely on volume-driven ratios, the number of nurses scheduled for each shift was dependent upon the number of patients occupying the unit during MC. This approach proved to be imprecise, and over time more factors such as patient acuity, admissions, discharges and transfers were taken into account (Harper, 2012).

Nurse staffing requirements in a medical unit are the result of a complex interaction between care unit sizes, nurse-to-patient ratios, bed census distributions, and quality-of-care requirements. The optimal configuration strongly depends on the particular characteristics of a specific case under study (Kortbeek et al., 2015a). In addition, Green et al. (2013) indicate that establishing the appropriate staffing level for a particular hospital unit during a specific shift is complicated by the need to make staffing decisions well in advance (e.g., six to eight weeks) of that shift, and labor constraints. These limits include the number of consecutive and weekend shifts worked per nurse, vacation schedules, personal days and preferences (Miller et al. 1976, Wright et al. 2006). Furthermore, hospital location (urban vs. rural), population density and hospital type (trauma, general rehab, children's) also influence decisions. Management of the nursing workforce is typically seen as a multi-phase, sequential planning and control process consisting of staffing, shift scheduling and allocation phases (Maenhout and Vanhoucke, 2013). The decisions made in each phase of this hierarchical process constrain subsequent phases. Workloads in nursing wards depend highly on patient arrivals and lengths of stay, both of which are inherently variable. Predicting these workloads and staffing nurses accordingly are essential to guaranteeing quality of care in a cost-effective manner (Kortbeek et al., 2015a). Measures of workload as used in the literature includes characteristics of patients (e.g., Case Mix), patient turnover, and patient acuity/intensity (Duffield et al., 2011). In many hospitals, staffing levels are a result of historical development because hospital managers lack the tools to base current staffing decisions on information about future patient demand (Kortbeek et al., 2015a).

We use a nursing requirement computation which takes into account the patient census, acuity mix, and total ADT activity in the unit for a given shift. PICUs in our focal children's hospital use a six-class categorization for patient acuity levels, say A to F, with F the category of the most nursing-workload-intense group. For Critical Care, the rough guidelines for nursing time requirement for each acuity group per 8 hour shift are:

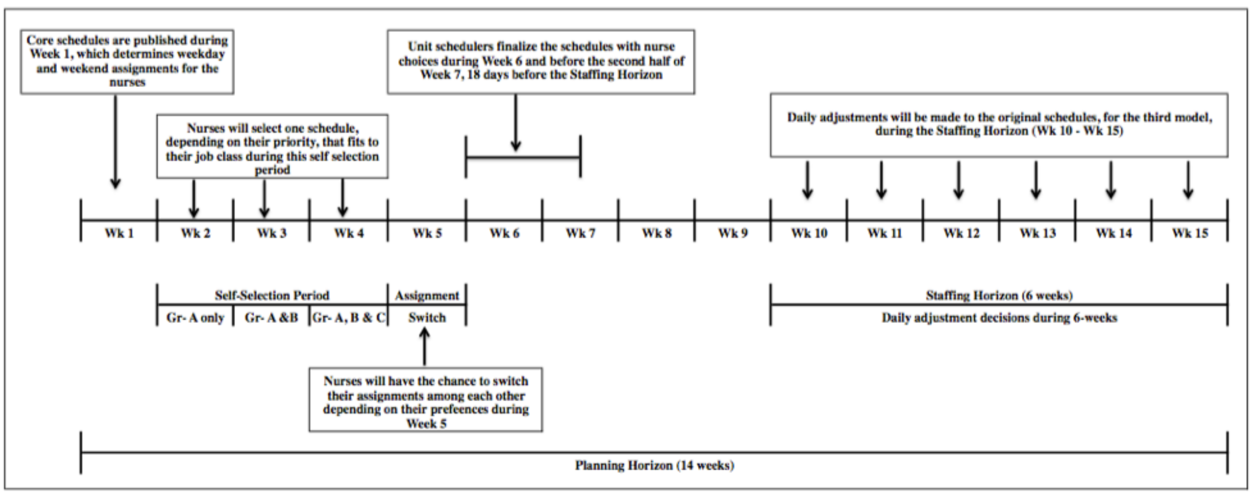
A=1 hour, B=2 hours, C=3 hours, D=4-5 hours, E=8 hours, F=16 hours. (For 16 hours, 2 RNs are assigned for 1 patient.) PICUs generally don't admit patients with acuity levels A and B , and only occasionally admit C patients. For the purpose of this study, we assume the nursing time required for the ADT activities occur during a given shift. Studies in the literature suggest roughly one-half hour nursing time for each ADT activity. Using patient census, patient acuity, and ADT activity occurring during a specific shift, allows us to compute the required total nursing workload for the unit for a specific nursing shift.

4.4.2 Incorporating Unscheduled Admissions & Extending Staffing Horizon

Another improvement target for the intended model is to incorporate unexpected admissions in the expected census estimations using historical patient data. Currently, StaffAssist does not incorporate the unexpected admissions number in the nurse requirement estimates. Charge nurses, using their own intuition and experience, enter the "desired" number of nurses in the StaffAssist system based on the current census and their estimate of unscheduled admissions. If an unscheduled admission is known, such as the patient going to the operating room 8 hours prior to needing a bed, the charge nurse will account for this patient in her request. However, in both these steps, the charge nurses use no analytical method to estimate the number of unscheduled admissions, nor do they address the issue of unscheduled admissions.

Current nurse weekend shift definitions also create complexities in terms of nurse job classes. As mentioned earlier, technically a weekend shift starts with a D1 shift on Friday and ends with an N2 shift on the following Monday. However, for job class definition, Fridays and Saturdays belong to one week and the following Sunday and Monday shifts belong to the following week. We propose starting a week with Monday D1 shift and ending the week with a Monday N2 shift. This scheme allows all the weekend assignments to belong to the last 18 shifts of a certain week. The PICU currently uses a staffing horizon of four weeks, but there are three self-scheduling and weekend assignment groups for nurses: A, B and C. Using a four-week staffing horizon complicates the tracking of self-scheduling priority among the three groups. We propose extending the staffing horizon to six weeks, which will allow for two full cycles of rotation among the three nurse groups. Then every new staffing horizon will begin with a group A nurse priority and weekend assignment. Figure 4.4 below presents an illustration of the timeline of our proposed scheduling approach:

Figure 4.4: Illustration of Proposed Scheduling Approach



4.5 Phase 1: Pre-Generation of Feasible Nurse Schedules

The nurse scheduling problem in this study involves many requirements depending on the nurse job profile, employment type, shift type and weekend assignment groups. Ensuring that any candidate solution satisfies these requirements is no trivial task. Here we name those requirements:

1. 12-hour break between two successive shift assignments for each nurse,
2. No nurse can work more than 3 consecutive 4-hour shifts at a time,
3. No nurse can work more than 40 hours/week,
4. All FTE nurses will be scheduled at least for two consecutive shifts (i.e., they cannot be scheduled only for a single 4-hour shift),
5. No FTE nurse can be scheduled more than 4 work days per week,
6. Minimum and maximum work hours allowances per week and per staffing horizon for different classes of PRN and FTE nurses,
7. Day shift nurse, Night shift nurse assignment limitations,
8. Holiday and/or weekend shifts assignment rotations,
9. All FTE nurses must be scheduled in compliance with their shift structure defined in their job profile.

Any life-sized tour assignment model that addresses these requirements will suffer from too many constraints caused by the growing size of these requirements. In addition, the larger the unit, the larger the problem dimensionality. This decreases the odds of solving the problem. For that reason, we use an algorithm that generates alternative nurse schedules for each FTE nurse group, while making sure the schedules are feasible. The schedule generation algorithm (used in C++ environment for one sample nurse job profile, job profile #1 as defined in Table 4.1: FTE - 0.9 nurses with "Day" shift assignments and weekend group of "Group A") appears in Figure 4.5. A step-by-step description for the developed algorithm follows below. Samples from the code appear in Appendices D.

Step 1 - Defining functions to be used in the program: First we define a function "comb" that generates all combinations of a set of numbers given an array of numbers. Array "b" is the input array that has "n" elements. Using the "comb" function we generate all possible combinations of size "r"; and output them into

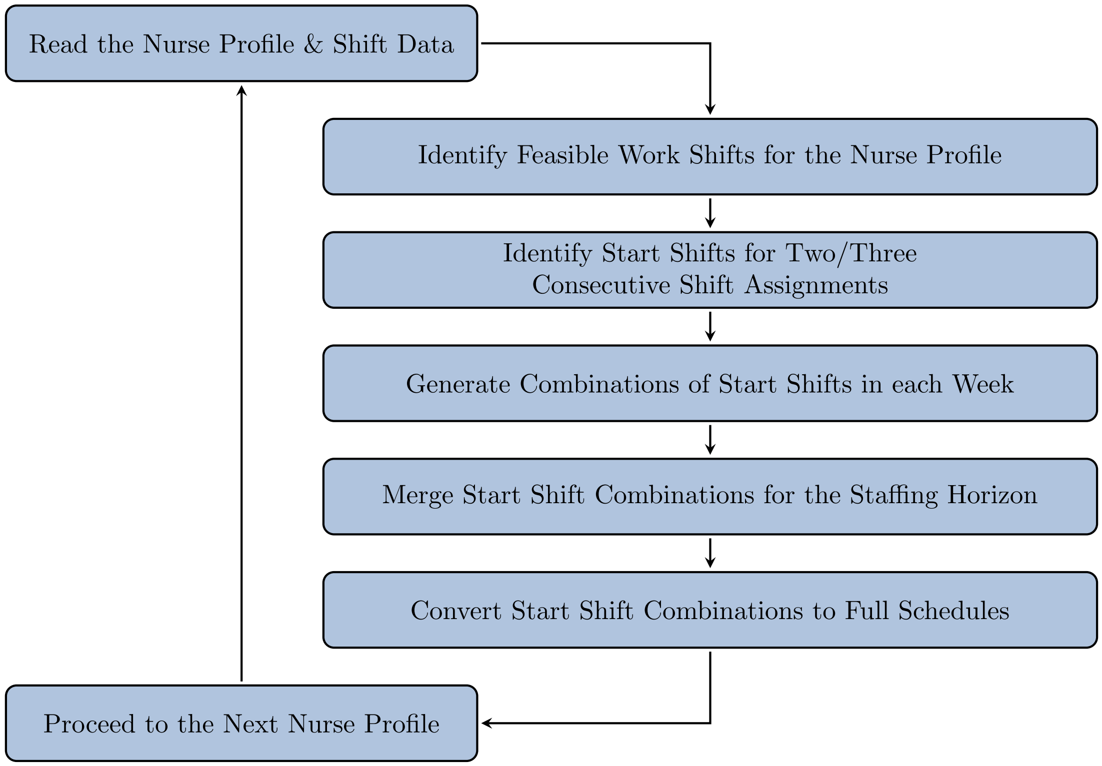


Figure 4.5: Schedule Generation Steps for FTE Nurses

an array. We use this function to generate all possible shift start time combinations, given alternative shift start times. We also define the "factorial" function that computes the factorial of a given integer. Finally, we define function "combination", which computes the number of subsets size "m" of a set with size "n". We use these functions to generate a schedule for the staffing horizon of six weeks given available start times for each week individually. Appendix Figure D.1 presents the partial code for this step.

Step 2 - Defining variables and parameters; reading the shift data: In step 2, we start the main program and define our variables, parameters and arrays. Then we read the shift data from a text file: "Shift Data.txt", which contains the information on: "Week #", "Shift #", "Shift Name", "Shift Type", "Day Type" for a given shift (i.e. For any shift from 0 to 251, this file contains the week number for the shift; shift number in the week, shift 1 to shift 42; shift name as D1, D2, E1, E2, N1 and N2; type of the shift as a "day" or "night" shift and weekend or weekday classification for the shift. This information is used when evaluating available shifts for a specific nurse job profile. Appendix Figure D.2 presents the partial code for this step.

Step 3 - Reading the nurse job profiles from the data: Step 3 reads the nurse job profile data from a text file: "Nurse Types.txt". This file has the information on the "Job Class" (i.e. ID number for each unique nurse profile), "Employment Type" (i.e. FTE - 0.9 or PRN-Tier III), "Shift Category" (i.e. "Day" shift nurses) and "Weekend Group" (i.e. weekend assignment group A, works during the weekends in weeks 1 and 4 only throughout the staffing horizon) for each nurse job profile. Appendix Figure D.3 presents the partial code for this step.

Step 4 - Identifying available work shifts for the given nurse profile: Step 4 assigns a "0" to all shifts that are classified as night shifts, as this code generates schedules for nurses that work the day shifts. We assign zeros to weekday shifts in weeks 1 and 4 because nurses from the given profile can only work weekend shifts for those weeks. We assign a "1" to all other shifts, since these are available work shifts for this nurse job profile. Figure 4.6 below present the identification of feasible work shifts for the studied nurse profile. Appendix Figure D.4 presents the partial code for this step.

G_SHIFT	Week #	Shift #	Shift Name	Shift Type	Day Type	Schedule
21	1	22	E2	NIGHT	WEEKDAY	0
22	1	23	N1	NIGHT	WEEKDAY	0
23	1	24	N2	NIGHT	WEEKDAY	0
24	1	25	D1	DAY	WEEKEND	1
25	1	26	D2	DAY	WEEKEND	1
26	1	27	E1	DAY	WEEKEND	1
27	1	28	E2	NIGHT	WEEKEND	0
28	1	29	N1	NIGHT	WEEKEND	0
29	1	30	N2	NIGHT	WEEKEND	0
30	1	31	D1	DAY	WEEKEND	1
31	1	32	D2	DAY	WEEKEND	1
32	1	33	E1	DAY	WEEKEND	1
33	1	34	E2	NIGHT	WEEKEND	0
34	1	35	N1	NIGHT	WEEKEND	0
35	1	36	N2	NIGHT	WEEKEND	0
36	1	37	D1	DAY	WEEKEND	1
37	1	38	D2	DAY	WEEKEND	1
38	1	39	E1	DAY	WEEKEND	1
39	1	40	E2	NIGHT	WEEKEND	0
40	1	41	N1	NIGHT	WEEKEND	0
41	1	42	N2	NIGHT	WEEKEND	0
42	2	1	D1	DAY	WEEKDAY	1
43	2	2	D2	DAY	WEEKDAY	1
44	2	3	E1	DAY	WEEKDAY	1
45	2	4	E2	NIGHT	WEEKDAY	0
46	2	5	N1	NIGHT	WEEKDAY	0
47	2	6	N2	NIGHT	WEEKDAY	0
48	2	7	D1	DAY	WEEKDAY	1
49	2	8	D2	DAY	WEEKDAY	1
50	2	9	E1	DAY	WEEKDAY	1
51	2	10	E2	NIGHT	WEEKDAY	0
52	2	11	N1	NIGHT	WEEKDAY	0
53	2	12	N2	NIGHT	WEEKDAY	0
54	2	13	D1	DAY	WEEKDAY	1
55	2	14	D2	DAY	WEEKDAY	1
56	2	15	E1	DAY	WEEKDAY	1
57	2	16	E2	NIGHT	WEEKDAY	0
58	2	17	N1	NIGHT	WEEKDAY	0
59	2	18	N2	NIGHT	WEEKDAY	0

Figure 4.6: Identifying Feasible Work Shifts for the Given Nurse Profile

Step 5 - Identifying the shifts that are available for three consecutive assignments: Step 5 identifies the shifts which are available to start a three consecutive four-hour shift assignments. The reason for this is, the nurse job profile we are studying in this example will only assigned to work three times for three consecutive four-hour shifts. Then we label those shifts "start times" and list the IDs for those shifts. Figure 4.6 marks examples of these shifts with a star. Appendix Figure D.5 presents the partial code for this step.

Step 6 - Build sets of available start times for the individual weeks: In step 6, we separate the potential start times in each week and generate a set of available start times for each week. This is needed to generate combinations of those start times for alternative schedules. Appendix Figures 3.6 (a) and (b) present the

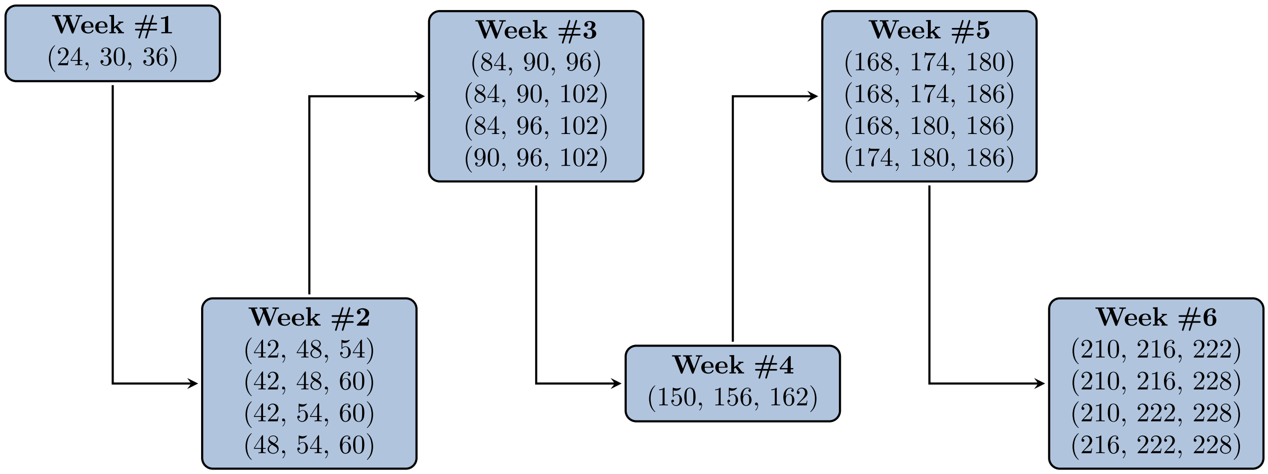


Figure 4.7: Generating Combinations of Start Shifts

pieces of codes used for this step. Appendix Figure D.6 and D.7 presents the partial code for this step.

Step 7 - Generate potential start time combinations for each week: Step 7 generates potential start time combinations for each week using the pre-defined "comb" function. The nurse job profile we study will be assigned three shift start times for each generated combination. Resulting bundles of three start times will mark the starting shifts for each of three consecutive four-hour shift assignments. This nurse profile will be assigned three of those assignments in each week. Figure 4.7 present an example of the process described in this step. Appendix Figure D.8 presents the partial code for this step.

Step 8 - Combining weekly start time combinations to a complete schedule: Step 8 generates combinations of potential start times for the entire planning horizon. We combine the weekly start time combinations to a complete schedule. The output from this step will list (3*6=) 18 start times among the set of shifts. This results in a complete enumeration of possible schedules for the nurse job profile. Appendix Figure D.9 presents the partial code for this step. Figure 4.8 present sample start time combinations for some FTE-0.9 nurse profiles.

Step 9 - Converting the potential start time combination arrays to the full set of schedules: The final step converts the potential start time combination arrays into the full set of schedules containing assignments for each shift using a boolean variable (i.e. the code generated 256 different schedules for the presented nurse profile. Appendix Figure D.9 presents the partial code for this step.

Figure 4.8: Sample start time combinations for some FTE-0.9 nurse profiles

Given are the start times for three consecutive four-hour shift assignments for the nurse type

First shift is shift "0" and last shift is shift "251"; 6 weeks of scheduling period, each week has 42 four-hour shifts.

FTE_0.9 - Day Shifts - Group A

Week 1			Week 2			Week 3			Week 4			Week 5			Week 6		
3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S
24	30	36	42	48	54	84	90	96	150	156	162	168	174	180	210	216	222
			42	48	60	84	90	102				168	174	186	210	216	228
			42	54	60	84	96	102				168	180	186	210	222	228
			48	54	60	90	96	102				174	180	186	216	222	228

FTE_0.9 - Day Shifts - Group B

Week 1			Week 2			Week 3			Week 4			Week 5			Week 6		
3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S
0	6	12	66	72	78	84	90	96	126	132	138	192	198	204	210	216	222
0	6	18				84	90	102	126	132	144				210	216	228
0	12	18				84	96	102	126	138	144				210	222	228
6	12	18				90	96	102	132	138	144				216	222	228

FTE_0.9 - Day Shifts - Group C

Week 1			Week 2			Week 3			Week 4			Week 5			Week 6		
3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S
0	6	12	42	48	54	108	114	120	126	132	138	168	174	180	234	240	246
0	6	18	42	48	60				126	132	144	168	174	186			
0	12	18	42	54	60				126	138	144	168	180	186			
6	12	18	48	54	60				132	138	144	174	180	186			

FTE_0.9 - Night Shifts - Group A

Week 1			Week 2			Week 3			Week 4			Week 5			Week 6		
3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S	3S
27	33	39	45	51	57	87	93	99	153	159	165	171	177	183	213	219	225
			45	51	63	87	93	105				171	177	189	213	219	231
			45	57	63	87	99	105				171	183	189	213	225	231
			51	57	63	93	99	105				177	183	189	219	225	231

We generated codes for 30 different nurse job profiles and identified the total number of available schedules for each nurse job profile. Figure 4.9 lists total number of start time alternatives in each week and total number of resulting schedules for each nurse job profile. As we can observe from the results in the figure, total available schedules for FTE classes 0.8 and 0.5 generate millions of available schedules, which negates their use in an optimization model. Next section presents the optimization model.

Figure 4.9: Total Available Schedule Alternatives for each Nurse Job Profile

			Shift Assignment Alternatives in Each Week for FTE Nurses						
EMPLOYMENT_TYPE	SHIFT_CATEGORY	WEEKEND_GROUP	Wk #1	Wk #2	Wk #3	Wk #4	Wk #5	Wk #6	TOTAL_SCHEDULES
FTE_0.9 (36 hr/week)	Day	A (Weeks 1 & 4)	1	4	4	1	4	4	256
FTE_0.9 (36 hr/week)	Day	B (Weeks 2 & 5)	4	1	4	4	1	4	256
FTE_0.9 (36 hr/week)	Day	C (Weeks 3 & 6)	4	4	1	4	4	1	256
FTE_0.9 (36 hr/week)	Night	A (Weeks 1 & 4)	1	4	4	1	4	4	256
FTE_0.9 (36 hr/week)	Night	B (Weeks 2 & 5)	4	1	4	4	1	4	256
FTE_0.9 (36 hr/week)	Night	C (Weeks 3 & 6)	4	4	1	4	4	1	256
FTE_0.8 (32 hr/week)	Day	A (Weeks 1 & 4)	6	24	24	6	24	24	11,943,936
FTE_0.8 (32 hr/week)	Day	B (Weeks 2 & 5)	24	6	24	24	6	24	11,943,936
FTE_0.8 (32 hr/week)	Day	C (Weeks 3 & 6)	24	24	6	24	24	6	11,943,936
FTE_0.8 (32 hr/week)	Night	A (Weeks 1 & 4)	6	24	24	6	24	24	11,943,936
FTE_0.8 (32 hr/week)	Night	B (Weeks 2 & 5)	24	6	24	24	6	24	11,943,936
FTE_0.8 (32 hr/week)	Night	C (Weeks 3 & 6)	24	24	6	24	24	6	11,943,936
FTE_0.6 (24 hr/week)	Day	A (Weeks 1 & 4)	3	6	6	3	6	6	11,664
FTE_0.6 (24 hr/week)	Day	B (Weeks 2 & 5)	6	3	6	6	3	6	11,664
FTE_0.6 (24 hr/week)	Day	C (Weeks 3 & 6)	6	6	3	6	6	3	11,664
FTE_0.6 (24 hr/week)	Night	A (Weeks 1 & 4)	3	6	6	3	6	6	11,664
FTE_0.6 (24 hr/week)	Night	B (Weeks 2 & 5)	6	3	6	6	3	6	11,664
FTE_0.6 (24 hr/week)	Night	C (Weeks 3 & 6)	6	6	3	6	6	3	11,664
FTE_0.5 (20 hr/week)	Day	A (Weeks 1 & 4)	12	24	24	12	24	24	47,775,744
FTE_0.5 (20 hr/week)	Day	B (Weeks 2 & 5)	24	12	24	24	12	24	47,775,744
FTE_0.5 (20 hr/week)	Day	C (Weeks 3 & 6)	24	24	12	24	24	12	47,775,744
FTE_0.5 (20 hr/week)	Night	A (Weeks 1 & 4)	12	24	24	12	24	24	47,775,744
FTE_0.5 (20 hr/week)	Night	B (Weeks 2 & 5)	24	12	24	24	12	24	47,775,744
FTE_0.5 (20 hr/week)	Night	C (Weeks 3 & 6)	24	24	12	24	24	12	47,775,744
FTE_0.3 (12 hr/week)	Day	A (Weeks 1 & 4)	3	4	4	3	4	4	2,304
FTE_0.3 (12 hr/week)	Day	B (Weeks 2 & 5)	4	3	4	4	3	4	2,304
FTE_0.3 (12 hr/week)	Day	C (Weeks 3 & 6)	4	4	3	4	4	3	2,304
FTE_0.3 (12 hr/week)	Night	A (Weeks 1 & 4)	3	4	4	3	4	4	2,304
FTE_0.3 (12 hr/week)	Night	B (Weeks 2 & 5)	4	3	4	4	3	4	2,304
FTE_0.3 (12 hr/week)	Night	C (Weeks 3 & 6)	4	4	3	4	4	3	2,304

4.6 Phase 2: Optimal Nurse Assignments to Pre-Generated Schedules - Model Description

This section provides a detailed description of the model used for optimal nurse assignments to pre-generated feasible schedules. The model integrates staffing with scheduling by assigning nurses to pre-generated full nurse schedules covering the six-week horizon. We also describe two alternative models for the integrated nurse staffing and scheduling in Appendices B and C. The initial model in Appendix B falls into the class of tour-assignment models described in Chapter 2. The tour-assignment model contains a significant number of constraints and a very large number of decision variables, which makes it inefficient in terms of computational complexity. The second model uses assignment of nurses to pre-generated schedules, and assigns PRN nurses from three tiers to the nursing shifts. This model enjoys the reduced constraint and decision variable sizes, but suffers from the problem of ensuring the requirement of not assigning any specific PRN nurse to four consecutive nursing shifts. Details of the model and the problem regarding the PRN nurse assignments are discussed in Appendix C.

For the reasons described above, we use pre-generated FTE nurse schedules as an input for the optimization model. This eliminates the increased number of constraints and reduces the number of decision variables. We use PRN nurses to reduce the risk of understaffing. We present a detailed nurse job profile portfolio which offers great flexibility for the nurses working at the PICU. Offering flexible nurse schedules is a crucial enhancement for high nurse retention and avoidance of burnout.

Appendix E presents the AMPL modeling code to be used in our optimization experiments for the developed medium-term nurse assignment model. Appendix F presents a step-by-step description of a small problem instance of the developed medium-term optimization model in AMPL environment. The small problem instance presented involves 120 alternative schedules for nurses from 30 different job profiles (i.e., four schedule alternatives for each FTE nurse profile). Schedules are generated using the C++ codes developed and selected among 16 randomly selected schedules for the given nurse job profiles using the presented AMPL maximally different schedule selection model described in next section of this chapter. Next, we present the mixed-integer programming models we use for medium-term integrated nurse staffing and scheduling.

4.6.1 Sets and Nurse Job Profiles

J : Set of alternative FTE nurse job profiles for the medical unit; (i.e. $J = \{1, 2, 3, ..., 30\}$)

S_j : Set of all available schedules for nurses from job profile j

P: Set of all PRN nurses.

We assume PRN nurses $\{1...PT_1\}$ are PRN Tier-1 nurses, nurses $\{(PT_1{+}1)...(PT_1{+}PT_2)\}$ are PRN Tier-2 nurses, nurses $\{(PT_1{+}PT_2{+}1)...(PT_1{+}PT_2{+}PT_3)\}$ are PRN Tier-3 nurses.

T : Set of four-hour nursing shifts during the scheduling period of six week $T = \{0, 1, 2, 3, ..., 251\}$ (i.e. 42 shifts a week, six weeks in a schedule; 252 four-hour shifts in total).

i.e. A typical week starts with the nursing shift $l = 1$, which is a Monday D1 shift and ends with shift $l =$ 42, which is a Monday N2 shift.

$w \in \{1, 2, ..., 6\}$, is the index of weeks during the staffing horizon and T_w is the subset of shifts during week w.

G : Set of patient acuity categories $G = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$

i.e. For $g \in G$ acuity category g =1 indicates that patient belongs to the acuity designation A in hospital terminology, similarly g =2 indicates acuity group B, g=3 indicates acuity group C, g=4 indicates acuity group D, g=5 indicates acuity group E, g=6 indicates acuity group F.

4.6.2 Model Parameters

$a_{s,t}$: 1 if for schedule $s \in S_j$ can be assigned to work at shift t ; 0 otherwise.

$\vartheta_{g,t}$: Vector keeping the number of patients in acuity group $g \in G$ at the unit for shift $t \in T$.

h_g: Nursing hours required for patient care for acuity group g in a four-hour nursing shift (i.e. $h =$ [0.5, 1, 1.5, 2.5, 4, 8] ; a patient with acuity 'F', g=6, will require eight hours of nursing care in a four-hour shift).

α_t: Number of admission and transfer-in activities to a unit for shift t

β_t: Number of discharge and transfer-out activities to a unit for shift t

c_j : Staffing cost per four-hour shift for the FTE nurses from job profile j

b_p : Staffing cost per four-hour shift for PRN nurse $p \in P$

γ : Nursing hours required for one patient admission / transfer-in activity

δ : Nursing hours required for one patient discharge / transfer-out activity

c_u^p : Penalty cost of one hour understaffed nursing care

n_j : Number of FTE nurses from job profile $j \in J$

4.6.3 Decision Variables

x_s: number of FTE nurses from that are assigned to work for schedule $s \in S_j$; $x_s \in \mathbb{Z}$.

$y_{p,t}$: 1 if PRN nurse $p \in P$ is assigned to work for shift $t \in T$; 0 otherwise.

z_p: 1 if PRN nurse $p \in P$ is assigned to work for any shift $t \in T$ during the staffing horizon of six weeks; 0 otherwise.

U_t: Total understaffing for shift $t \in T$; $U_t \in \mathbb{R}$.

We are introducing the binary decision variable $y_{p,t}$ for each individual PRN nurse for each nursing shift. We also introduce decision variable z_p just to gain control over the PRN assignment hours during the staffing horizon. We want to make sure if a PRN nurse is assigned to any shift during the staffing horizon, then that same nurse is assigned at least the required minimum number of nursing hours depending upon the PRN tier.

4.6.4 Dynamic Staffing Model

Objective Function Cost Components

Minimize {FTE Staffing Costs + PRN Staffing Costs + Total Understaffing Penalty Costs }:

$$\left[\underbrace{\sum_{j \in J} \sum_{s \in S_j} \sum_{t \in T} c_j \cdot x_s \cdot a_{s,t}}_{\text{FTE Staffing Cost}} + \underbrace{\sum_{p \in P} \sum_{t \in T} b_p \cdot y_{p,t}}_{\text{PRN Staffing Cost}} + \underbrace{\sum_{t \in T} c_u^p \cdot U_t}_{\text{Understaffing Penalty}} \right]$$

Our objective in this optimization problem is to minimize total costs of FTE and PRN nurse staffing and total penalty costs associated with the understaffing levels in the unit throughout the staffing horizon.

Model Constraints

- Understaffing Constraint:

We set the lower bound for the understaffing variable (U_t):

$$\left[\overbrace{\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g \in G} \vartheta_{g,t} \cdot h_g}^{\text{Required Nursing Hrs.}} - \overbrace{\sum_{j \in J} \sum_{s \in S_j} 4 \cdot x_s \cdot a_{s,t}}^{\text{Sch. FTE Hrs.}} - \overbrace{\sum_{p \in P} 4 \cdot y_{p,t}}^{\text{Sch. PRN Hrs.}} \right] \leqslant U_t$$

and $U_t \geqslant 0 \qquad \forall \quad t \in T.$

Required nursing hours minus the provided nursing hours from FTE and PRN nurses should define the lower bound for the understaffing variable U_t. As mentioned earlier, we measure the nursing requirement in a shift by multiplying the number of patients in each acuity group by the associated nursing hours and aggregate the nursing hours required due to the admissions and discharge, ADT, activities. Provided nursing hours come from the FTE Staffing which are assignments to pre-generated schedules and assignment of individual PRN nurses to specific nursing shift depending on the foreseen patient demand.

- Constraints related to the number of available FTE nurses from each job profile $j \in J$:

$$\sum_{s \in S_j} x_s \leqslant n_j \qquad \forall \;\; j \in J;$$

We cannot assign more than available number of FTE nurses from each job profile $j \in J$.

- Constraints related to the rule of avoiding four consecutive four-hour shift assignments for the PRN nurses:

$$(y_{p,t} + y_{p,(t+1)} + y_{p,(t+2)} + y_{p,(t+3)}) \leqslant 3 \qquad \forall \;\; p \in P, t \in \{1 \ldots (T-3)\};$$

- Constraints related to avoiding single four-hour shift breaks between two consecutive assignments of PRN nurses:

$$(y_{p,t} - y_{p,(t+1)}) + (y_{p,(t+2)} - y_{p,(t+1)}) \leqslant 1 \qquad \forall \;\; p \in P, t \in \{1 \ldots (T-2)\};$$

- Constraints related to avoiding two four-hour shift breaks between two consecutive assignments of PRN nurses:

$$(y_{p,t} - y_{p,(t+1)}) + (y_{p,(t+3)} - y_{p,(t+2)}) \leqslant 1 \qquad \forall \;\; p \in P, t \in \{1 \ldots (T-3)\};$$

- Available maximum PRN hours for a given week during staffing horizon:

$$\sum_{t \in T_w} y_{p,t} \leqslant 10 \qquad \forall \;\; p \in P, w \in \{1, 2, ..., 6\}.$$

  For any PRN nurse $p \in P$ shift assignments should be smaller than 10 four-hour shifts for a given week during the staffing horizon of six weeks, where w is the index for the weeks and T_w is the subset of shifts during week w.

- Constraint related to the minimum work hours requirement of PRN nurses:

  Assignments for the PRN nurses in a tier shouldn't be less than the minimum work hours required for that PRN tier.

  For Tier-1:

$$8 \cdot z_p \leqslant \sum_{t \in T} y_{p,t} \qquad \forall \;\; p \in \{1 ... PT_1\}$$

  Tier-1 PRN nurses should be assigned to a minimum of eight four-hour shifts during the staffing horizon of six weeks. Or no assignments.

  For Tier-2:

$$17 \cdot z_p \leqslant \sum_{t \in T} y_{p,t} \qquad \forall \;\; p \in \{(PT_1 + 1) ... (PT_1 + PT_2)\}$$

  Tier-2 PRN nurses should be assigned to a minimum of 17 four-hour shifts during the staffing horizon of six weeks. Or no assignments.

  For Tier-3:

$$29 \cdot z_p \leqslant \sum_{t \in T} y_{p,t} \qquad \forall \;\; p \in \{(PT_1 + PT_2 + 1) ... (PT_1 + PT_2 + PT_3)\}$$

  Tier-3 PRN nurses should be assigned to a minimum of 29 four-hour shifts during the staffing horizon of six weeks. Or no assignments.

- We leave the option of no assignment for specific PRN nurses open during the staffing horizon.

$$\sum_{t \in T} y_{p,t} \leqslant z_p \cdot M \qquad \forall \;\; p \in \{1 ... (PT_1 + PT_2 + PT_3)\}$$

  where, M is a large enough positive integer. The constraint ensures that if any PRN nurse $p \in$

$\{1...(PT_1 + PT_2 + PT_3)\}$ was not assigned to work during the staffing horizon, all shift assignments $y_{p,t}$ are forced to be zero.

We can present the developed Dynamic Staffing Model, with PRN nurses modeled individually, as follows:

Minimize {FTE Staffing Costs + PRN Staffing Costs + Total Understaffing Penalty Costs }:

$$\Bigg[\underbrace{\sum_{j\in J}\sum_{s\in S_j}\sum_{t\in T} c_j \cdot x_s \cdot a_{s,t}}_{\text{FTE Staffing Cost}} + \underbrace{\sum_{p\in P}\sum_{t\in T} b_p \cdot y_{p,t}}_{\text{PRN Staffing Cost}} + \underbrace{\sum_{t\in T} c_u^p \cdot U_t}_{\text{Understaffing Penalty}} \Bigg]$$

subject to

$$\Bigg[\overbrace{\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g\in G} \vartheta_{g,t} \cdot h_g}^{\text{Required Nursing Hrs.}} - \overbrace{\sum_{j\in J}\sum_{s\in S_j} 4 \cdot x_s \cdot a_{s,t}}^{\text{Sch. FTE Hrs.}} - \overbrace{\sum_{p\in P} 4 \cdot y_{p,t}}^{\text{Sch. PRN Hrs.}} \Bigg] \leqslant U_t$$

$$\sum_{s\in S_j} x_s \leqslant n_j \qquad \forall\ j \in J;$$

$$\left(y_{p,t} + y_{p,(t+1)} + y_{p,(t+2)} + y_{p,(t+3)}\right) \leqslant 3 \qquad \forall\ p \in P, t \in \{1 \ldots (T-3)\};$$

$$\left(y_{p,t} - y_{p,(t+1)}\right) + \left(y_{p,(t+2)} - y_{p,(t+1)}\right) \leqslant 1 \qquad \forall\ p \in P, t \in \{1 \ldots (T-2)\};$$

$$\left(y_{p,t} - y_{p,(t+1)}\right) + \left(y_{p,(t+3)} - y_{p,(t+2)}\right) \leqslant 1 \qquad \forall\ p \in P, t \in \{1 \ldots (T-3)\};$$

$$\sum_{t\in T_w} y_{p,t} \leqslant 10 \qquad \forall\ p \in P, w \in \{1, 2, \ldots, 6\}.$$

$$8 \cdot z_p \leqslant \sum_{t\in T} y_{p,t} \qquad \forall\ p \in \{1, \ldots, PT_1\}$$

$$17 \cdot z_p \leqslant \sum_{t\in T} y_{p,t} \qquad \forall\ p \in \{(PT_1+1), \ldots, (PT_1+PT_2)\}$$

$$29 \cdot z_p \leqslant \sum_{t\in T} y_{p,t} \qquad \forall\ p \in \{(PT_1+PT_2+1), \ldots, (PT_1+PT_2+PT_3)\}$$

$$\sum_{t\in T} y_{p,t} \leqslant z_p \cdot M \qquad \forall\ p \in \{1, \ldots, (PT_1+PT_2+PT_3)\}$$

$$U_t \in \mathbb{R} \text{ and } U_t \geqslant 0 \qquad \forall \quad t \in T;$$

$$O_t \in \mathbb{R} \text{ and } O_t \geqslant 0 \qquad \forall \quad t \in T;$$

$$x_s \in \mathbb{Z} \text{ and } x_s \geqslant 0 \qquad \forall \quad s \in S_j;$$

$$z_p,\ y_{p,t} \in \{0, 1\} \qquad \forall \quad p \in P,\ t \in T$$

4.6.5 Fixed Staffing Model

For the fixed staffing model our objective is to minimize the total difference between FTE and PRN staffing levels in each nursing shift and a pre-determined, targeted fixed staffing level throughout the staffing horizon. In addition to the original constraints presented in the dynamic model above, we add another constraint which limits the total staffing level in each shift with the target staffing level. We present the objective function and the additional constraints as follows:

- Objective Function:

  Minimize: Total Staffing Difference w.r.t. Target Staffing Level

$$\left[(n_{target} \cdot T) - (\sum_{t \in T} \underbrace{\overbrace{\sum_{j \in J} \sum_{s \in S_j} x_s \cdot a_{s,t}}^{\text{FTE Staffing}} + \overbrace{\sum_{p \in P} y_{p,t}}^{\text{PRN Staffing}}}_{\text{Staffing Level at Shift } t}) \right];$$

- Additional Constraints:

$$\sum_{j \in J} \sum_{s \in S_j} x_s \cdot a_{s,t} + \sum_{p \in P} y_{p,t} \leqslant n_{target} \qquad \forall \;\; t \in \{1 \ldots T\};$$

  , where 'n_{target}' is the target fixed staffing level for all shifts throughout the staffing horizon. The additional constraint is in the form of an inequality, since in many cases staffing all nursing shifts with the fixed staffing level may not be a feasible option. As a result of this optimization, we might observe some shifts having less than target staffing level, especially in the medical units with less PRN nurses.

We test the performance of both static and dynamic models against the perfect information scenario, where assumed that the patient demand is perfectly known at the time of scheduling. Patient demand pattern for nursing for a typical 6-week staffing horizon consists of census data for each patient acuity group and ADT activity during each nursing shift for the staffing horizon.

The Fixed Staffing Model, with PRN nurses modeled individually, is as follows:

Minimize: Total Staffing Difference w.r.t. Target Staffing Level

$$\left[(n_{target} \cdot T) - (\sum_{t \in T} \underbrace{\overbrace{\sum_{j \in J} \sum_{s \in S_j} x_s \cdot a_{s,t}}^{\text{FTE Staffing}} + \overbrace{\sum_{p \in P} y_{p,t}}^{\text{PRN Staffing}} \quad)}_{\text{Staffing Level at Shift } t} \right];$$

subject to

$$\sum_{j \in J} \sum_{s \in S_j} x_s \cdot a_{s,t} + \sum_{p \in P} y_{p,t} \leqslant n_{target} \qquad \forall \; t \in \{1 \ldots T\};$$

$$\left[\overbrace{\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g \in G} \vartheta_{g,t} \cdot h_g}^{\text{Required Nursing Hrs.}} - \overbrace{\sum_{j \in J} \sum_{s \in S_j} 4 \cdot x_s \cdot a_{s,t}}^{\text{Sch. FTE Hrs.}} - \overbrace{\sum_{p \in P} 4 \cdot y_{p,t}}^{\text{Sch. PRN Hrs.}} \right] \leqslant U_t$$

$$\sum_{s \in S_j} x_s \leqslant n_j \qquad \forall \; j \in J;$$

$$(y_{p,t} + y_{p,(t+1)} + y_{p,(t+2)} + y_{p,(t+3)}) \leqslant 3 \qquad \forall \; p \in P, t \in \{1 \ldots (T-3)\};$$

$$(y_{p,t} - y_{p,(t+1)}) + (y_{p,(t+2)} - y_{p,(t+1)}) \leqslant 1 \qquad \forall \; p \in P, t \in \{1 \ldots (T-2)\};$$

$$(y_{p,t} - y_{p,(t+1)}) + (y_{p,(t+3)} - y_{p,(t+2)}) \leqslant 1 \qquad \forall \; p \in P, t \in \{1 \ldots (T-3)\};$$

$$\sum_{t \in T_w} y_{p,t} \leqslant 10 \qquad \forall \; p \in P, w \in \{1, 2, \ldots, 6\}.$$

$$8 \cdot z_p \leqslant \sum_{t \in T} y_{p,t} \qquad \forall \; p \in \{1, \ldots, PT_1\}$$

$$17 \cdot z_p \leqslant \sum_{t \in T} y_{p,t} \qquad \forall \; p \in \{(PT_1 + 1), \ldots, (PT_1 + PT_2)\}$$

$$29 \cdot z_p \leqslant \sum_{t \in T} y_{p,t} \qquad \forall \; p \in \{(PT_1 + PT_2 + 1), \ldots, (PT_1 + PT_2 + PT_3)\}$$

$$\sum_{t \in T} y_{p,t} \leqslant z_p \cdot M \qquad \forall \; p \in \{1, \ldots, (PT_1 + PT_2 + PT_3)\}$$

$$U_t \in \mathbb{R} \text{ and } U_t \geqslant 0 \qquad \forall \quad t \in T;$$

$$O_t \in \mathbb{R} \text{ and } O_t \geqslant 0 \qquad \forall \quad t \in T;$$

$$x_s \in \mathbb{Z} \text{ and } x_s \geqslant 0 \qquad \forall \quad s \in S_j;$$

$$z_p, \; y_{p,t} \in \{0, 1\} \qquad \forall \quad p \in P, \; t \in T$$

$$n_{target} \in \mathbb{Z}$$

4.7 Schedule Selection for the Optimization Experiments

In order to select a subset of available schedules for each FTE nurse profile to feed into the optimization model, we create two sub models: the Maximum Difference Model and the Random Schedule Model. For the Maximal Difference Model we select a subset of schedules of maximally different given size from the pool of schedules. These schedules are mutually compared for each nursing shift, and a difference score is computed for each comparison. The sum of all difference scores becomes the objective value of the maximization problem. The idea is that, as the schedules become more diverse, the optimization model will generate better results in terms of cost minimization and nursing demand coverage. Below we present the mathematical representation of the Maximum Difference Model.

(1) Maximum Difference Model:

- *Parameters*:
  - S: Total number of available schedules to the nurses
  - T: Number of four-hour shifts in the scheduling period
  - N: Size of desired subset of schedules to feed in optimization model
  - $a_{s,t}$: 1 if a nurse for schedule $s \in S$ can be assigned to work at shift $t \in T$; 0 otherwise.
- *Decision Variables*:
  - X_s binary, $s \in$ {1...S}; 1 if schedule $s \in S$ is selected within the subset, 0 otherwise.
  - $D_{s,k,t}$ binary, $s \in$ {1...(S-1)}, $k \in$ {2...S)}, $t \in \{1..T\}$; 1 if assignment of two compared schedules, s and k, in shift t different from each other, 0 otherwise.
  - $W_{s,k}$ binary, $s \in$ {1...(S-1)}, $k \in$ {2...S)}; 1 if schedules s and k are selected within the subset, 0 otherwise.
  - $\mathit{\Delta}_{s,k,t}$, binary $s \in$ {1...(S-1)}, $k \in$ {2...S)}, $t \in \{1..T\}$; 1 if assignment of two compared schedules, s and k, in shift t different from each other and schedules s and k are selected within the subset , 0 otherwise.
- Objective Function: Maximize Total Mutual Difference of Schedules within the Selected Subset of Schedules:

Max

$$\sum_{s,k,t} \Delta_{s,k,t}, \qquad \forall \quad s \in \{1...(S-1)\}, k \in \{(s+1)...S\}, t \in \{1...T\}$$

- *Constraints:*

$$\sum_{s \in \{1...S\}} X_s = N;$$

$\forall \;\; s \in \{1...(S-1)\}, k \in \{(s+1)...S\}, t \in \{1...T\}$:

$$[a_{s,t} - a_{k,t}]^2 = D_{s,k,t};$$

$$W_{s,k} \leqslant X_s;$$

$$W_{s,k} \leqslant X_k;$$

$$\Delta_{s,k,t} \leqslant D_{s,k,t};$$

$$\Delta_{s,k,t} \leqslant W_{s,k};$$

We solve the optimization problem using CPLEX solver in the AMPL environment. The optimization model gives a desired size of subset of schedules that are maximally different among each other. Figure 4.10 below presents the AMPL model of the optimization model that selects maximally different schedules. We also present a sample data file in Figure 4.11 that shows an instance where we select four schedules among the available 16.

(2) Random Schedule Selection Model:

The Maximum Difference Model is not efficient for large size schedule selections (e.g., selecting 256 schedules from 10,000 available schedules). Multi-index decision variables and a large set of constraints make it difficult to obtain a solution in a reasonable period of time. So we use a second technique, the Random Schedule Model, in which we select a given size of random schedules from the pool of all available schedules. We model the random selection routine in C++ using the Mersenne-Twister random-number engine.

Figure 4.12 presents a sample selection code for FTE nurses from $FTE-0.6$ employment working in day

Figure 4.10: Selecting Maximally Different Schedules - Model Description

```
###  DATA  ###
param S > 0; # total number of available schedules to the nurses
param T > 0; # number of four-hour shifts in the scheduling period
param N > 0; # size of desired subset of schedules
param a{1..S, 1..T} >=0;
# 1 if for schedule s in S_{j} can be assigned to work at shift t; 0 otherwise.

###  VARIABLES  ###

var X {1..S} >=0  binary;
var D {1..(S-1), 2..S, 1..T} >=0 binary;
var W {1..(S-1), 2..S} >=0  binary;
var Diff {1..(S-1), 2..S, 1..T} >=0 binary;

###  OBJECTIVE  ###
maximize Total_Difference:

    sum {s in 1..(S-1), k in (s+1)..S, t in 1..T} Diff[s,k,t];

###  CONSTRAINTS  ###

subject to Difference {s in 1..(S-1), k in (s+1)..S, t in 1..T}:

    (a[s,t] - a[k,t])*(a[s,t] - a[k,t]) = D[s,k,t];

################################################################################

subject to Total_Schedules:

    sum {s in 1..S} X[s] = N;

################################################################################
subject to Product1 {s in 1..(S-1), k in (s+1)..S}:

W[s,k] <= X[s];

################################################################################

subject to Product2 {s in 1..(S-1), k in (s+1)..S}:
W[s,k] <= X[k];

################################################################################

subject to Diff1 {s in 1..(S-1), k in (s+1)..S, t in 1..T}:
Diff[s,k,t] <= D[s,k,t];

subject to Diff2 {s in 1..(S-1), k in (s+1)..S, t in 1..T}:
Diff[s,k,t] <= W[s,k];

#################################################################################
```

shifts and have weekend assignment Group A. Among the available 11,664 schedules for this nurse profile, we are randomly selecting 256 schedules that will feed into our optimization model. The experimental results presented in the next section, show that randomly selecting 256 schedules for each nurse profile (i.e. 256 schedules/nurse profile * 30 nurse profiles = 7680 schedules for each optimization model) provides a sufficiently large selection of schedules for the optimization experiments. Increasing the number of available schedules further would yield minimal benefits in terms of incremental cost savings, but would increase the

Figure 4.11: Selecting Maximally Different Schedules - Data File

```
data;

param S := 16; # total number of available schedules to the nurses for the job profile

param T := 252; # number of four-hour shifts in the scheduling period

param N := 4; # size of desired subset of schedules


########### SCHEDULES ################
# a[s,t] is 1 if for schedule s.

# t = {1,2,...,252} nursing shifts in the staffing horizon of six weeks

param a (tr): # this is the transpose of matrix a[s,t]
    1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16 :=
1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
2   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
3   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
4   0   0   0   0   0   0   0   0   1   1   0   0   0   0   1   1
5   0   0   0   0   0   0   0   0   1   1   0   0   0   0   1   1
6   0   0   0   0   0   0   0   0   1   1   0   0   0   0   1   1
7   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
8   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
9   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
10  1   0   0   1   0   0   0   0   0   0   0   1   0   1   0   0
11  1   0   0   1   0   0   0   0   0   0   0   1   0   1   0   0
12  1   0   0   1   0   0   0   0   0   0   0   1   0   1   0   0
13  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
14  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
15  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
16  0   0   0   0   1   0   1   1   0   0   1   0   1   0   0   0
17  0   0   0   0   1   0   1   1   0   0   1   0   1   0   0   0
18  0   0   0   0   1   0   1   1   0   0   1   0   1   0   0   0
19  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
20  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
21  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
22  0   1   1   0   0   1   0   0   0   0   0   0   0   0   0   0
```

computational complexity and solution time dramatically.

Figure 4.12: Random Schedule Selection Using C++

```
////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////
//Here we output all schedule alternatives for this nurse type to a txt file.

ofstream myfile ("/Users/OTA/Desktop/Optimization with Cpp and Cplex/schedule_fte06_day_weekendA_256.txt");
if (myfile.is_open())
{
    for (int t=0; t<256; t++)


    {
        std::random_device rd;     // only used once to initialise (seed) engine
        std::mt19937 rng(rd());    // random-number engine used (Mersenne-Twister in this case)
        std::uniform_int_distribution<int> uni(0,11664); // guaranteed unbiased

        auto random_integer = uni(rng);
        myfile << "Schedule "<< random_integer <<"\t\t";
        for (int g=0; g<G_SHIFTS; g++)
        {
            myfile << schedule_fte06_day_weekendA[random_integer][g] << "\t";
        }
        myfile << "\n";
    }
    myfile.close();
}
else
{
    cout << "Unable to open file";
}


////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////
```

4.8 Experimental Design for RQ1 and RQ2

The efficient and effective management of nursing personnel is of critical importance in a hospital's environment, comprising a vast share of operational costs. Burke et al. (2013) suggest that high-quality nurse rosters benefit nurses, patients and managers. From a management point of view, better and more flexible scheduling can help retain nurses and aid recruitment, reduce tardiness and absenteeism, increase morale and productivity, and provide better patient service and safety. Costs can be reduced by needing to hire fewer agency nurses and by lowering staff turnover. A lack of methodologies and decision support tools to improve scheduling is still a strategic problem to the hospital administrations. The adopted nurse workforce practices and policies highly affect nurses' working conditions and quality of care (Maenhout & Vanhoucke, 2013). Healthcare managers are seriously challenged as all these issues converge. One way to ease this pressure is to develop better decision support systems that provide insight into the consequences and outcomes of various nurse staffing and shift scheduling policies. All these elements affect personnel management. Managing a proper personnel policy has a positive impact on nurses' working conditions, which are strongly related to quality of care (Wright et al., 2006).

We study the interaction between various factors affecting nurse staffing and scheduling process, as well as alternative performance measures, using the optimization models presented earlier. We propose an experimental design to identify penalty costs, which will be used in the optimization model for understaffing reduction in the medical units. We also determine how robust the nurse staffing and scheduling models are, using different nurse mix and patient demand in the medical units. Our experimental design presented in this section is intended to explore answers for our first two research questions in this study:

- RQ 1: Do dynamic medium-term nurse staffing policies that use patient demand forecasts outperform the historically-employed fixed staffing policy for the intensive care medical units?
- RQ 2: Can understaffing penalty cost be utilized as a mechanism to control the understaffing levels which possibly mitigate nurse burnout and medical errors?

Next, we identify the significant factors and build an experimental design using those factors, which enables us study their impact on desired performance measures.

4.8.1 Identifying Significant Design Factors

In order to evaluate the impact of various design factors on nurse staff scheduling approaches and performance measures, we develop an experimental design that is based upon the following significant factors:

1. Nurse Profile Mix (NMIX): Nurse mix for the studied three PICU Wards (i.e. distribution of nurses over FTE and PRN groups in the medical units). Cases used in the experimental design are presented in Nurse Profile Mix table, Table 4.4, below.

2. Number of available schedules for each FTE nurse profile (NAS): (i.e. Number of schedules provided as an input for the optimization model).

3. Understaffing Penalty Cost (UPC): Penalty cost for one nursing hour understaffing at the medical unit. FTE nurse hourly rate is normalized to one unit. Base level of UPC is determined as 1.5, due to the fact that mandatory overtime cost of a nurse is typically 50% higher than regular hourly rate.

4. Staffing Policy (SPO): Fixed versus dynamic staffing policy, compared against the perfect information optimal staffing scenario.

5. Patient Demand (PD): Patient demand pattern used in the optimization models for nursing for a typical 6-week staffing horizon. Consists of census data for each patient acuity group and ADT activity during each nursing shift for the staffing horizon.

Table 4.3 below present the experimental design factors and various levels of these factors used in this study. Next, we discuss the significance of these design factors.

Nurse Mix (NMIX)	**# of Schedules (NAS)**	**Understaffing Penalty (UPC)**	**Staffing Policy (SPO)**	**Patient Demand PD**
Ward A	4	1.5	Optimal Staffing	Actual Demand
Ward B	16	2.0	Fixed Staffing - L1	Fixed Demand - L1
Ward C	64	3.0	Fixed Staffing - L2	Fixed Demand - L2
	256	10.0	Dynamic Staffing	Heat-Map Demand

Table 4.3: Experimental Design Factors

Implications of Understaffing in Medical Units

There are significant economic implications for optimizing nurse staffing based on an improved understanding of patient volume. When a nursing unit is chronically short-staffed, nurses are forced to maintain an intense pace in order to ensure patients receive timely care. According to Paul and MacDonald (2013) nurse shortage implications go beyond healthcare quality, also extending to health economics. While the cost of overstaffing can easily be viewed as waste, there are also costs for understaffing patient care units. Understaffing has been linked to hospital-acquired infections and their significant preventable costs (Cimiotti et al., 2012). Additionally, the loss of nurses through burnout is estimated to cost $300,000 per year for each percentage of annual nurse turnover (PriceWaterhouseCoopers, 2007). Thus, applying novel models to the chronic problem of understaffing in medical units may lead to significant reductions in healthcare costs, nurse job satisfaction and patient safety.

Nursing Shortage, Nurse Turnover, Fatigue and Burn-out:

Burnout and the total workload experienced by nurses are usually managed by adequately scheduling shifts. Cline et al. (2003) examine the results of a qualitative study designed to enhance the understanding of RNs' perceptions of the factors prompting them to leave employment in acute care settings. The authors sought to identify any disparity between what RNs gave employers as their reasons for leaving and what they would reveal to a neutral third party. Two major themes emerged from the discussion: management and staffing concerns. Because of the staffing shortages, nurses felt patient care was compromised and their licenses were at risk due to their inability to provide appropriate, necessary care. The sheer cost of turnover - $64,000 for an ICU nurse and $42,000 for a medical, surgical nurse coupled with low morale and potentially dangerous situations caused by inadequate staffing, compels managers to examine ways to decrease turnover (Kerfoot, 2000). Unattractive schedules, poor practice environments and high workloads are identified as important factors leading to discontentment and a high nursing turnover. This initiate hospitals to adopt policies that increasingly accommodate preferences and requests of their nursing staff while ensuring suitably qualified staff on duty at the right time (Maenhout & Vanhoucke, 2013).

Patient Safety and Outcomes:

The shortage of nurses has attracted considerable attention due to its direct impact on the quality of patient care (Punnakitikashem et al. 2013). This issue is expected to worsen, especially given the aging population of baby-boomers, which includes those that are part of the nurse workforce. This has resulted in a wide variety of problems, including patient safety issues, inability to detect complications, and potential patient mortality rate increases (Paul and MacDonald, 2013). Penoyer (2010) reviewed the literature evaluating the association of nurse staffing with patient outcomes in critical care units and populations. An annotated review of major nursing and medical literature from 1998 to 2008 was performed to find research studies conducted in intensive care units or critical care populations where nurse staffing and patient outcomes were addressed. Findings from this review clearly demonstrate an association of nurse staffing in the intensive care unit with patient outcomes. Patients receive better healthcare if nurses are able to spend more time with them and mistakes are less likely if nurses are not stressed, tired and overworked due to poor scheduling and understaffing (Burke et al., 2013). Since patient safety is jeopardized when medical care units are understaffed, a scarcity of nursing capacity can lead to expensive hiring of nurses from external agencies and to undesirable ad hoc bed closings (Kortbeek et al. 2015a).

Nurse Job Satisfaction and Absenteeism:

Improved rosters not only decrease nurse fatigue but also help maximize the use of their leisure time and satisfy more of their personal requests (Burke et al., 2013). Aiken et al. (2001) surveyed nurses in five countries and found that one result of increased workload was that basic nursing interventions were left undone. Being unable to provide the required level of patient care was linked to lower job satisfaction and staff retention. High workloads and undesirable schedules are two major reasons for nurses to report job dissatisfaction (Punnakitikashem et al. 2013). Green et al. (2013) combine an empirical investigation of the factors affecting nurse absenteeism rates with an analytical treatment of nurse staffing decisions using a novel variant of the newsvendor model. Using data from the emergency department of a large urban hospital, this study finds that absenteeism rates are consistent with nurses exhibiting an aversion to higher levels of anticipated workload. Kuntz et al. (2014) argue that safety tipping points occur when managerial escalation policies are exhausted and workload variability buffers are depleted. Front-line clinical staff is forced to ration resources and, at the same time, becomes more error prone as a result of elevated stress hormone levels.

As our design factor related to the understaffing levels, we use the Understaffing Penalty Cost (UPC). Penalty cost for one nursing hour understaffing at the medical unit. We first normalize all the cost parameters by the FTE nurse hourly rate (i.e. FTE nurse hourly cost is assumed to be one unit). Base level of UPC is determined as 1.5, due to the fact that mandatory overtime cost of a nurse is typically 50% higher than regular hourly rate and mandatory overtime is the frequently used method to cover understaffing in the medical unit. Cost of understaffing cannot be limited with the nursing cost of mandatory overtime. The impact of understaffing on the nursing staff, nurse turnover and patient outcomes needs to be addressed as a part of the penalty cost. For that reason, as demonstrated in the experimental design factors table above (i.e Table 4.3), we study four different levels for the UPC: 1.5, 2, 3 and 10. Understaffing cost (or penalty) is not an observable factor in the day-to-day operations of the PICU medical units. The sensitivity of the nursing administration for avoiding potential understaffing will be reflected to our experiments as various levels of Understaffing Penalty Cost. We analyze how the performance measures (mean, median, max of understaffing percentage in each nursing shift) are impacted by various levels of understaffing penalty cost. We seek to minimize potential understaffing risk under 5% level, given that medical units under consideration for this specific study are part of a Pediatric Intensive Care Unit. We evaluate which penalty costs keep the understaffing under desired levels given various patient demand patterns (PD) and nurse mix (NMIX), while keeping the total staffing costs at a reasonable level.

Nursing Care is Costly:

U.S. health care costs continue to rise, despite the advent of the Affordable Care Act (Patton, 2015). Nursing care is identified as the single biggest factor in both the cost of hospital care and patient satisfaction (Yankovic and Green, 2011). Recent estimates suggest that national health care expenditures increased between 5% and 6% in 2014 and 2015 and are estimated at $3.2 trillion. Given the fact that registered nurse wages and benefits constitute a substantial portion of overall hospital costs, comprising approximately 25% of the hospital's operational costs (Maenhout and Vanhoucke, 2013b), hospitals have attempted to reduce nurse staffing as a means to reduce costs and increase profitability (Rivers et al., 2004). On the other hand, projections suggest that by 2020 approximately 36% of nursing positions in the United States will remain unfilled (Wright and Bretthauer, 2010). Rising healthcare costs and increasing nurse shortages make cost-effective nurse staffing of vital importance (Kortbeek et al. 2015a).

Mandatory Nurse-to-Patient Ratios Create a Risk of Overstaffing:

There is widespread dissatisfaction with the current methods of determining nurse staffing levels, including the most common one of using minimum nurse-to-patient ratios (Needleman and Buerhaus, 2003). Mandatory nurse-to-patient ratios implemented in some states also create a risk of underestimating or overestimating required nurse resources. Yankovic and Green (2011) represent the nursing system as a queuing model and develop a two-dimensional model to approximate the actual interdependent dynamics of bed occupancy levels and demands for nursing. The authors use this model to show how unit size, nursing intensity, occupancy levels, and unit length-of-stay affect the impact of nursing levels on performance and thus how inflexible nurse-to-patient ratios can lead to either understaffing or overstaffing. Paul and MacDonald (2013) demonstrate the issues with mandatory nurse-to-patient ratios in addressing the nurse shortage crisis when subject to varying patient demand and hospital service quality goals. Results from the study suggest that relying merely on mandatory nurse-to-patient ratios is not an effective strategy, especially considering the issue of nursing shortages. Even though high nurse-to-patient ratios may be a good strategy from a health quality perspective, it is not a strategy every hospital and state can possibly afford, and it is one that can also further exacerbate the nursing shortage (Paul and MacDonald, 2013). One shortcoming of this strategy is the assumption that demand for services and the requirement for nurse resources in a hospital behave in a linear manner, which is far from reality (Clancy, 2007).

In summary, due to the budget constraints faced by the hospital administrations and costly nature of nursing care, avoiding overstaffing by better matching patient demand has a crucial importance. We analyze how the performance measures like mean, median, max of overstaffing percentage in each nursing shift is impacted by various levels of UPC and other design factors. Note that overstaffing cost is imposed in the objective function cost components in the form of additional staffing cost from the extra nurses used.

Nurse Mix - Ward Size and Structure

There are more than sixty-five hundred hospitals in the United States that are described as short-stay or long-term, depending on the length of patient stay. Short-stay facilities include community, teaching, and public hospitals. Sometimes short-stay hospitals are referred to as acute care facilities because the services provided within them aim to help resolve pressing problems or medical conditions, such as a heart attack, rather than long-term chronic conditions such as the need for rehabilitation following a head injury. There are various nurse profiles working within these different types of hospitals. Next is a short list of main categories for nurse profiles: (1) Registered Nurses (RNs): RNs are nurses with an associate or bachelor's degree in nursing. They assist physicians in hospitals and a variety of medical settings and help in treating patients with illnesses, injuries and medical conditions. RNs constitute the largest population of employment in most U.S. hospitals. (2) PRN (Pro-Re-Nata) or Per Diem (per day) Nurses: PRN nurses carry out the same essential duties of an RN but on a part-time or temporary basis. Some per diem nurses work in this capacity to gain additional hours, some to have shorter working hours and some may just want to gain a variety of experiences to explore their opportunities. (3) Travel (or agency) Nurses: Travel nurses work temporary jobs nationally and internationally, sometimes for weeks at a time and sometimes for a few years. Travel nurses perform many of the same duties as standard RN, often working for an agency that supplements staff to facilities in need. (4) Licensed Practical Nurses (LPNs): LPNs perform a variety of tasks under the supervision of an RN. They administer medicine, check vital signs and give injections.

Brusco and Showalter (1993) suggest that staffing mix and the assignment of flex-pool nursing hours were the two most important nurse staffing policy options for affecting annual nursing labor costs. Nurse staffing policy options are defined in terms of the flexibility they provide to hospital management to match nursing staff to demand requirements overtime. The primary nurse staffing policy options included in the study and available to hospital management include: (1) Staffing Mix; (2) Over-Time; (3) Flex-Staff; and (4) External Staff Assignment (Brusco and Showalter, 1993). Staffing mix refers to the workforce composition of registered nurses (RN), licensed practical nurses (LPN) and nurses' assistants (NA). Overtime refers to the use of nursing staff for more than 40 hrs per week and more than 12 hrs. per day. Flex-staffing is the use of part-time employees working throughout the hospital. External staffing consists of the RNs signed to 13-weeks contracts as well as temporary nurse hires from local agencies. Staffing patterns differ across nursing care units in hospitals. This affects nursing intensity and the direct costs of nursing care. For example, patients admitted to an intensive care unit typically have patient-to-RN staffing ratios of 2:1 or 1:1. An adult medical/surgical ward may have ratios between 4:1 and 8:1. The patient-to-nurse ratio determines the mean hours of care delivered on the unit, yet individual patients may require more or less care than the mean (Welton et al., 2006).

As revealed from the discussion above, there are various types of hospitals with a variety of nurse job profiles. Any nurse staffing and scheduling model should address the variety in the nature of this problem. Our model includes 30 different FTE nurse job profiles, which consider the day-night shift assignment of nurses, weekly work hour limits, unique shift structures and weekend job assignment rotations. We also consider PRN nurses with three different job tiers, depending on total lowest work hours limit per schedule, which help the nursing administration fill in the gaps of their schedules. As a part of our experimental design, we evaluate our optimization model for the three medical units of the PICU in the as-is state. These three different wards will enable us to test our model with respect to three different instances of nurse mix (i.e. three different sets of FTE and PRN nurse combinations, which will help us validate the reliability of the developed model. We observe how variations in the nurse mix impact the schedule cost, computational time and number of iterations for an optimal or near-optimal solution. From the nursing administration perspective, testing the model with three different nurse mix instances will demonstrate how the range of under and overstaffing percentages is impacted by the changing size and combination of FTE and PRN nurses, as the administration would like to keep these ratios within a certain range. We also evaluate how the size of PRN nurse pool impact the performance measures and problem complexity. Results will provide insights with respect to the role PRN nurses play in the medical units. Table 4.4 below summarize the current nurse mix in the three PICU wards that are used in our experiments.

Nurse Profile	Employment Type	Shift Type	Weekend Group	Number of RNs		
				Ward A	Ward B	Ward C
1	FTE - 0.9	Day	Group A	10	8	8
2	FTE - 0.9	Day	Group B	10	8	8
3	FTE - 0.9	Day	Group C	10	8	9
4	FTE - 0.9	Night	Group A	10	8	9
5	FTE - 0.9	Night	Group B	9	8	8
6	FTE - 0.9	Night	Group C	10	8	8
7	FTE - 0.8	Day	Group A	1	0	0
8	FTE - 0.8	Day	Group B	1	0	1
9	FTE - 0.8	Day	Group C	0	0	0
10	FTE - 0.8	Night	Group A	0	0	0
11	FTE - 0.8	Night	Group B	1	0	0
12	FTE - 0.8	Night	Group C	1	0	0
13	FTE - 0.6	Day	Group A	3	2	3
14	FTE - 0.6	Day	Group B	3	2	3
15	FTE - 0.6	Day	Group C	4	2	3
16	FTE - 0.6	Night	Group A	4	2	3
17	FTE - 0.6	Night	Group B	3	2	3
18	FTE - 0.6	Night	Group C	3	1	3
19	FTE - 0.5	Day	Group A	0	0	0
20	FTE - 0.5	Day	Group B	0	0	0
21	FTE - 0.5	Day	Group C	1	0	0
22	FTE - 0.5	Night	Group A	0	0	0
23	FTE - 0.5	Night	Group B	1	0	0
24	FTE - 0.5	Night	Group C	0	1	1
25	FTE - 0.3	Day	Group A	1	0	0
26	FTE - 0.3	Day	Group B	0	0	0
27	FTE - 0.3	Day	Group C	0	1	0
28	FTE - 0.3	Night	Group A	0	0	0
29	FTE - 0.3	Night	Group B	1	0	0
30	FTE - 0.3	Night	Group C	0	0	0
31	PRN - Tier I			0	0	0
32	PRN - Tier II			4	2	2
33	PRN - Tier III			12	8	4
Total # of RNs				**103**	**71**	**76**

Table 4.4: Nurse Mix at the PICU Wards

Dynamic Patient Demand for Nursing

Workloads in nursing wards depend highly on patient arrivals and lengths of stay, both of which are inherently variable. Predicting these workloads and staffing nurses accordingly are essential for guaranteeing quality of care in a cost-effective manner (Kortbeek et al., 2015a). Measures of workload as used in the literature includes characteristics of patients (e.g. casemix) and patient turnover, as well as patient acuity/intensity

(Duffield et al., 2011). Green et al. (2013) suggests that the problem of determining nurse staffing levels in a hospital environment is a complex task because of variable patient census levels and uncertain service capacity caused by nurse absenteeism. In determining staffing requirements, such factors as total census, intensity-of-care levels, and type of ward must be estimated for appropriate planning to be accomplished (Helmer et al., 1980). Hourly changes in patient census and acuity cause the demand for nursing services to depart from the planned schedule several times a day, This requires hospitals to update their staffing needs on a continuing basis (Bard and Purnomo, 2005b). Some additional factors of consideration to achieve an effective nurse staffing system would be the nurse preferences regarding work schedules, nurse absenteeism and patient acuity (Purnomo and Bard, 2007; Wang and Gupta, 2014).

Kim et al.'s (2014) technical report evaluated the predictability of patient volume in Hospital Medicine (HM) groups using a variety of known forecasting techniques. HM groups experience fluctuations in patient volume which may be difficult to predict. Results from univariate and multivariate methods were compared with a benchmark of historical means. The mean absolute percentage error (MAPE) was used to measure forecast accuracy. Autocorrelations and cross-correlations of patient volume across the services were also analyzed. Results from the study indicate that the forecasting models outperformed the historical average based approach by reducing MAPE from 17.2% to 6% in one day ahead forecast and to 8.8% MAPE in a month ahead forecast. The ARIMA method outperformed the other methods a day (or beyond) ahead forecast.

Given the dynamic nature of patient demand for nursing in medical units, we consider patient demand generated by various periods of six-week staffing horizon data for patient census, mix and ADT activity in the medical units. As outlined by the previous literature above, an acuity-based staffing system regulates the number of nurses on a shift according to the patients' needs, and not according to raw patient numbers. We study different six-week time periods for each medical unit in the PICU. While Fixed Staffing models use a fixed level of patient demand throughout the staffing horizon, Dynamic Staffing models use heat map approach for better mimicking the dynamic nature of patient demand. All results are compared with the outcomes of the Optimal Staffing model, which use actual patient demand data. We evaluate how the performance measures of schedule costs, computational time and under- and over-staffing percentages are impacted with different instances of patient demand data.

Number of Available Schedules for FTE RNs (NAS)

Appropriate staffing and shift scheduling of the healthcare workforce are central components and are essential for the delivery of care to patients. Moreover, as labor costs typically represent more than 40% of a hospital's total budget, hospitals are under increasing pressure to manage their nursing workforce efficiently (Maenhout and Vanhoucke, 2013). Vericort and Jennings (2011) suggest that these shifts should limit nurse working hours, allow for enough breaks, and consider individual preferences. In fact, some hospitals offer flexible shifts with long recovery periods in order to retain nurses. The authors suggest in conjunction with efficient scheduling systems, hospital managers might also want to limit the utilization rates experienced by nurses.

Given the pressures hospitals face to manage their nursing workforce efficiently, offering a sizable number of alternative schedules to each nurse profile could help nursing administration better match patient demand. In this respect, we also investigate the impact of number of available schedules (NAS) on the schedule performance. In this context, it is possible to then observe how increasing the number of schedules impact our performance measures. We explore answers for questions as follows: Does increasing the number of available schedules bring significant objective function cost savings? How does problem complexity and solution time is impacted by various levels of available schedules per nurse group? How does the understaffing and overstaffing levels are impacted by the number of available schedules in the optimization model? Four distinct levels on the NAS factor are used to evaluate schedule performance; 4 schedules, 16 schedules, 64 schedules, 256 schedules per FTE nurse group. We use the random selection routine in C++ using the Mersenne-Twister random-number engine to select 16, 64 and 256 schedules among the available schedule pool. Selecting 4 schedules out of randomly selected 16 schedules is conducted by the maximally different selection using the optimization model in AMPL environment.

4.8.2 Description of Performance Measures

Below is a summary of the performance measures to be evaluated from the experimental design:

Description of the Performance Measures for the Optimization Model Experiments

- Obj. Value (Total Cost): Resulting objective function cost of the optimization model. It is the total cost of FTE and PRN staffing costs and understaffing penalty costs.
- Optimality Gap (%): The percentage gap between the best integer solution achieved and the objective value of the relaxed LP model.

- FTE Staffing Cost: Total staffing cost for the FTE nurses, in the medium-term, during the scheduling horizon of 6 weeks.
- PRN Staffing Cost: Total staffing cost for the PRN nurses, in the medium-term, during the scheduling horizon of 6 weeks.
- Understaffing Penalty Cost: Total penalty cost for understaffing during the scheduling horizon of 6 weeks.
- Median U_t: Median percentage understaffing during the scheduling horizon of 6 weeks.
- Average U_t: Average percentage understaffing during the scheduling horizon of 6 weeks.
- Max U_t: Maximum level of percentage understaffing observed during the scheduling horizon of 6 weeks.
- Median O_t: Median percentage overstaffing during the scheduling horizon of 6 weeks.
- Average O_t: Average percentage overstaffing during the scheduling horizon of 6 weeks.
- Max O_t: Maximum level of percentage overstaffing observed during the scheduling horizon of 6 weeks.
- # FTEs: Total number of FTE nurses assigned to work in the unit (among the total available FTE nurse pool) during the scheduling horizon of 6 weeks.
- FTE % Utilization: Percentage of FTE nurses assigned to work in the unit (among the total available FTE nurse pool) during the scheduling horizon of 6 weeks.
- # PRNs: Total number of PRN nurses assigned to work in the unit (among the total available PRN nurse pool) during the scheduling horizon of 6 weeks.
- Avg. PRN Hours per Week: Average hours of work assignment for the PRN nurses per week during the scheduling horizon of 6 weeks.
- PRN % Utilization: Average percentage utilization of PRN nurses per week during the scheduling horizon of 6 weeks, compared to a 40 hours work week.
- Minimum Staffing per Shift: Minimum number of total nursing personnel in one shift, including both FTE and PRN nurses, during the scheduling horizon of 6 weeks.
- Average Staffing per Shift: Average number of total nursing personnel in one shift, including both FTE and PRN nurses, during the scheduling horizon of 6 weeks.
- Maximum Staffing per Shift: Maximum number of total nursing personnel in one shift, including both FTE and PRN nurses, during the scheduling horizon of 6 weeks.

4.8.3 Preparation Steps of Heat-Map for Patient Demand

In this section, we provide a description of patient demand data used in the optimization experiments in this chapter. Then, we provide a description of the steps we used to develop the heat map approach used for forecasting input data for the dynamic staffing model. Table 4.5 present the description of the sample and full dataset of the three PICU Wards used in this study. We use three different six-week time period for each medical unit. Mean, median, minimum, maximum, standard deviation and coefficient of variation of the patient demand for nursing hours (i.e. the nursing requirement based on our computation approach that use patient acuities and ADT activity) are presented for the used sample and the full dataset in Table 4.5. As can be observed from the summary statistics, all three samples used in the study closely mimic the characteristics of the full dataset spanning more than four years of time period.

Next, we describe the steps used to develop the heat map for the studied three PICU wards as an example. A similar approach can be followed to produce a unit specific heat map for any medical unit.

Step 1: Search for Monthly Seasonality

As a first step of preparing the patient demand heat map, we search for monthly seasonality in patient data. Figure 4.13 below presents the monthly seasonality in patient census data for the three PICU wards. The averages are computed using the full dataset. The figure clearly suggests a monthly seasonality for average patient census, which has to be addressed in the heat map preparation. We also check average monthly ADT activity as a part of nursing requirements. Figure 4.14 below presents the monthly seasonality in patient ADT activity for the three PICU wards. The ADT averages are also computed using the full dataset. Figure 4.14 also suggests a monthly seasonality for average ADT activity, more significant for Ward C.

Step 2: Computing the Monthly Seasonality Index for Patient Census and ADT activity

In order to calculate the monthly seasonality index, we first count the nursing shifts that matches the searched month of the year, then we sum the census data for all those shifts. Dividing the total census with total count of the shifts give us average census for the month. We repeat the same process for all months to compute the general census average. Then, we divide the average monthly census data to the overall average census to find the monthly seasonality index. Table 4.6 below demonstrates the discussed computations for Ward A. We repeat the same procedure to calculate the seasonality index for patient ADT activity.

Figure 4.13: Average Patient Census by Months

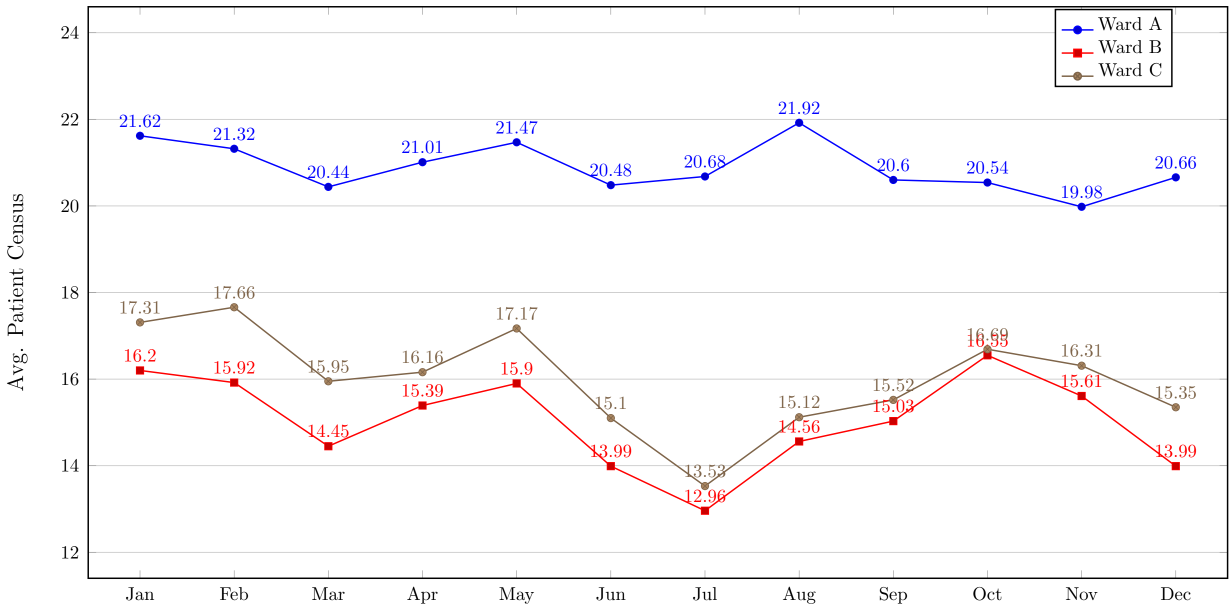
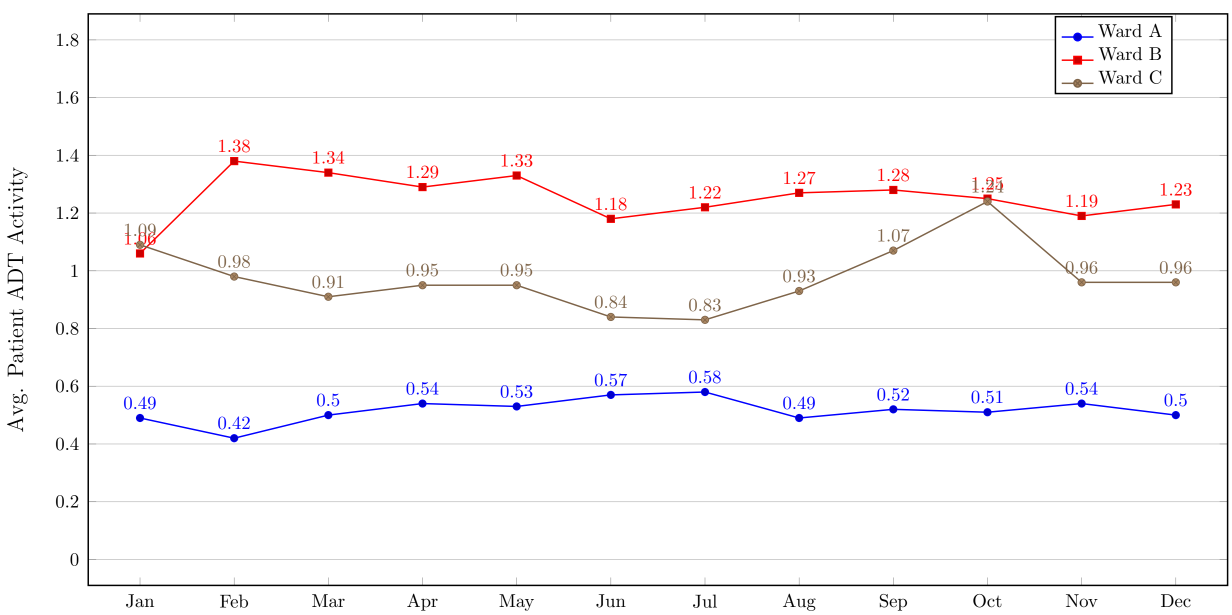


Figure 4.14: Average Patient ADT Activity by Months



Sample Data Period	Ward A	Ward B	Ward C
Start Date	11/19/12	4/9/12	10/19/13
End Date	12/31/12	5/21/12	11/30/13
Patient Demand per Shift (hrs.)			
Mean (hrs.)	72.98	51.54	46.09
Median (hrs.)	74.50	51.50	47.50
(Min, Max) (hrs.)	(51.0, 90.0)	(27.0, 74.0)	(25.0, 71.5)
Std. Deviation (hrs.)	8.88	9.29	8.59
Coefficient of Var. (CV)	12.17%	18.02%	18.64%
Full Dataset Period	**Ward A**	**Ward B**	**Ward C**
Start Date	4/7/09	4/7/09	4/7/09
End Date	12/22/13	12/22/13	12/22/13
Patient Demand per Shift (hrs.)			
Mean (hrs.)	77.31	45.55	46.77
Median (hrs.)	78.49	45.37	46.99
(Min, Max) (hrs.)	(37.4, 92.1)	(15.5, 75.3)	(17.4, 71.7)
Std. Deviation (hrs.)	8.70	9.49	9.77
Coefficient of Var. (CV)	11.26%	20.84%	20.88%

Table 4.5: Data Summary & Patient Demand per Shift (hrs.) - Wards A, B & C

Step 3: Search for Seasonality in Days of the Week and Shifts in a Day

In step 3, we search for seasonality in days of the week and shifts in a day. Figure 4.15 presents the average census data with respect to days of a week. Figure indicates a gradual increase in patient census as the we move from Monday to Thursday, followed by a decrease in the census as we move towards the weekend. The pattern is pretty similar for all three PICU wards. Figure 4.16 presents the average patient ADT activity by days of the week. The ADT activities appear to be fairly stable during the weekdays, followed by a significant drop in the weekends. Both of these characteristics has to be addressed in the heat map development.

Next, we check the seasonality of patient census and ADT with respect to the six nursing shifts in a day. Figure 4.17 presents the average patient census by nurse shifts in a day. The average census decrease with

Figure 4.15: Average Patient Census by Days of Week

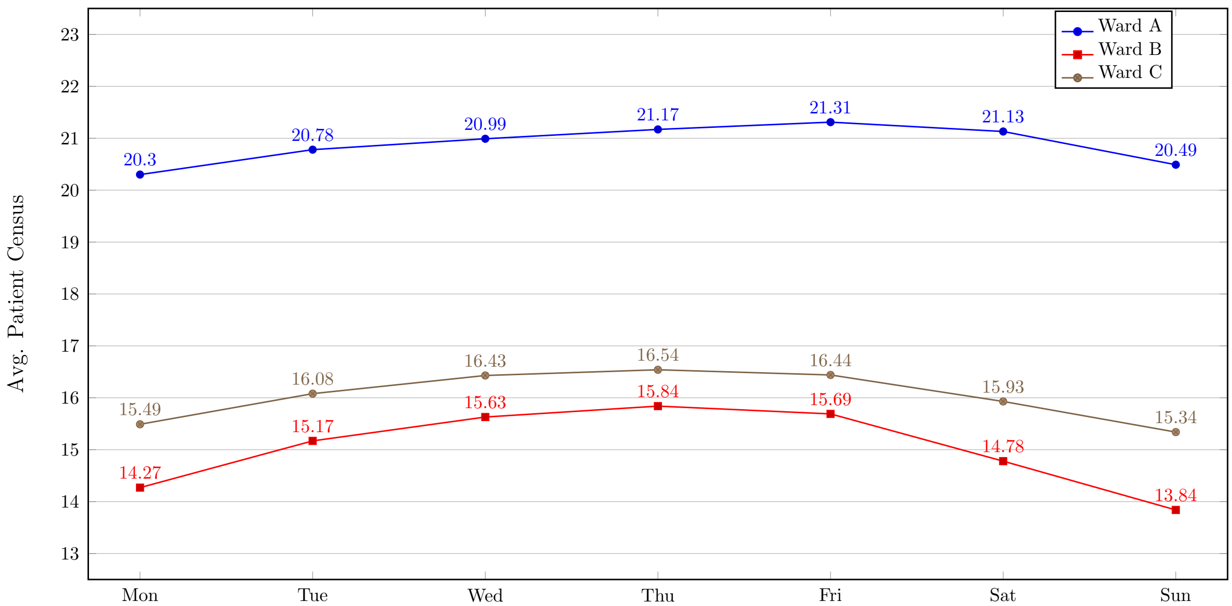
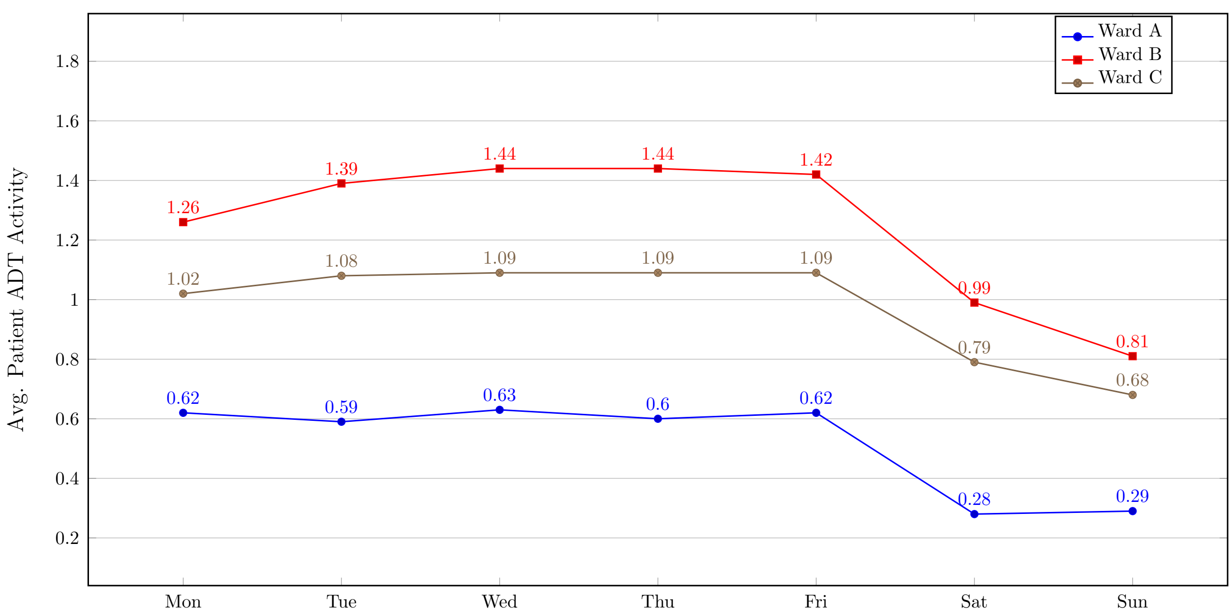


Figure 4.16: Average Patient ADT Activity by Days of Week



MONTH	**CENSUS TOTAL**	**# of SHIFTS**	**AVG. CENSUS**	**S. INDEX**
Jan	16,084	744	21.618	1.035
Feb	14,454	678	21.319	1.021
Mar	15,209	744	20.442	0.979
Apr	18,129	863	21.007	1.006
May	19,969	930	21.472	1.028
Jun	18,434	900	20.482	0.981
Jul	19,228	930	20.675	0.990
Aug	20,382	930	21.916	1.050
Sep	18,541	900	20.601	0.987
Oct	19,099	930	20.537	0.983
Nov	17,982	900	19.980	0.957
Dec	18,116	877	20.657	0.989
Grand Avg.	215,627	10,326	20.882	

Table 4.6: Computing Monthly Seasonality Index for Patient Census - Ward A

the discharges in shifts E1 and E2, then start to increase again with the unscheduled admissions starting from shift E2. Patient ADT in nurse shifts is presented in Figure 4.18. ADT level starts with a low level at shift D1, reaches it's peak at shift D2, and starts to decrease beginning from shift E1, an expected pattern since majority of the scheduled admissions and discharges occur during the daytime. We decide to compute the average census and ADT for each day-shift combination (i.e. 7 days a week, 6 shifts in a day), resulting in 42 different options. Next, we discuss how to compute the day-shift averages for patient census and ADT.

Step 4: Computing Average Census and ADT activity for each Day-Shift Combination

In step 4, we compute the average census and ADT for each day and shift combination. We search the data for a specific day and shift combination and add to the sum and count if the shift matches. Then, we divide the summation by the count to find the average census and ADT for each option. Table 4.7 demonstrates an example of average census computation for each day-shift combination for Ward A. We repeat the same procedure for other medical units and for computing the average ADT values.

Step 5: Adjusting Day-Shift Averages by Monthly Seasonality Index

In Step 5, we adjust each average census and ADT value for day-shift combinations via multiplying with

Figure 4.17: Average Patient Census by Nurse Shifts

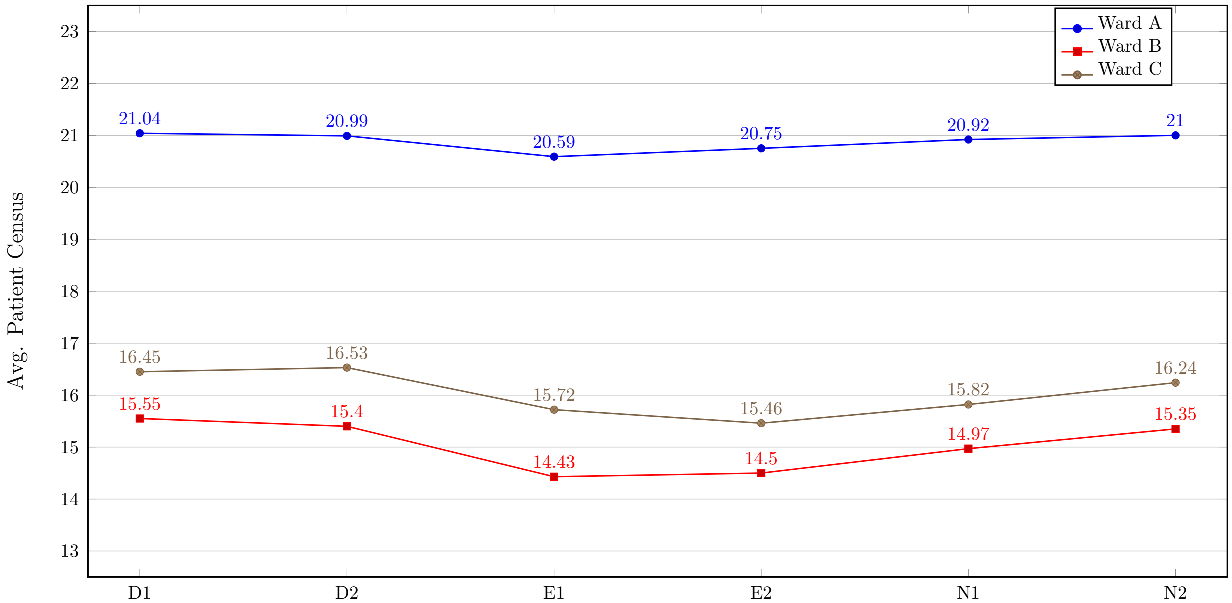
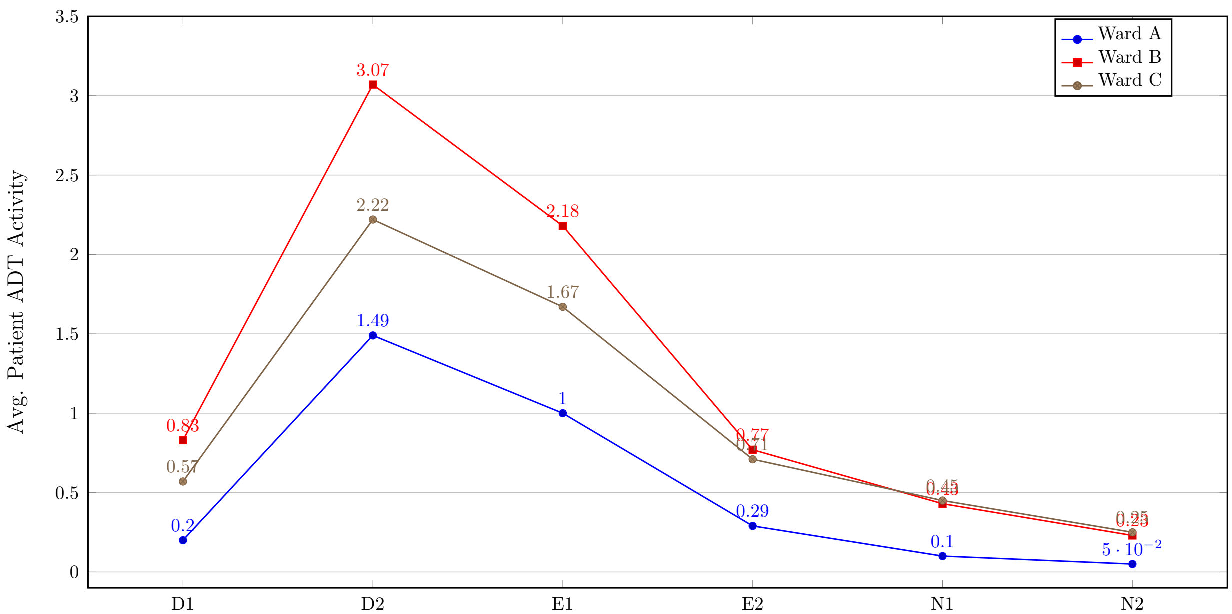


Figure 4.18: Average Patient ADT by Nurse Shifts



DAY	SHIFT	CENSUS TOTAL	# of SHIFTS	AVG. CENSUS
1- Monday	1- D1 - 7:00 AM to 11:00 AM	4,955	245	20.224
1- Monday	2- D2 - 11:00 AM to 3:00 PM	4,952	245	20.212
1- Monday	3- E1 - 3:00 PM to 7:00 PM	4,911	245	20.045
1- Monday	4- E2 - 7:00 PM to 11:00 PM	5,010	245	20.449
1- Monday	5- N1 - 11:00 PM to 3:00 PM	5,074	245	20.710
1- Monday	6- N2 - 3:00 PM to 7:00 PM	4,965	246	20.183
2- Tuesday	1- D1 - 7:00 AM to 11:00 AM	5,123	246	20.825
2- Tuesday	2- D2 - 11:00 AM to 3:00 PM	5,110	246	20.772
2- Tuesday	3- E1 - 3:00 PM to 7:00 PM	5,045	246	20.508
2- Tuesday	4- E2 - 7:00 PM to 11:00 PM	5,120	246	20.813
2- Tuesday	5- N1 - 11:00 PM to 3:00 PM	5,162	246	20.984
2- Tuesday	6- N2 - 3:00 PM to 7:00 PM	5,094	245	20.792
3- Wednesday	1- D1 - 7:00 AM to 11:00 AM	5,197	246	21.126
3- Wednesday	2- D2 - 11:00 AM to 3:00 PM	5,184	246	21.073
3- Wednesday	3- E1 - 3:00 PM to 7:00 PM	5,077	246	20.638
3- Wednesday	4- E2 - 7:00 PM to 11:00 PM	5,130	246	20.854
3- Wednesday	5- N1 - 11:00 PM to 3:00 PM	5,202	246	21.146
3- Wednesday	6- N2 - 3:00 PM to 7:00 PM	5,186	246	21.081
4- Thursday	1- D1 - 7:00 AM to 11:00 AM	5,228	246	21.252
4- Thursday	2- D2 - 11:00 AM to 3:00 PM	5,213	246	21.191
4- Thursday	3- E1 - 3:00 PM to 7:00 PM	5,128	246	20.846
4- Thursday	4- E2 - 7:00 PM to 11:00 PM	5,200	246	21.138
4- Thursday	5- N1 - 11:00 PM to 3:00 PM	5,255	246	21.362
4- Thursday	6- N2 - 3:00 PM to 7:00 PM	5,223	246	21.232
5 - Friday	1- D1 - 7:00 AM to 11:00 AM	5,280	246	21.463
5 - Friday	2- D2 - 11:00 AM to 3:00 PM	5,261	246	21.386
5 - Friday	3- E1 - 3:00 PM to 7:00 PM	5,173	246	21.028
5 - Friday	4- E2 - 7:00 PM to 11:00 PM	5,211	246	21.183
5 - Friday	5- N1 - 11:00 PM to 3:00 PM	5,260	246	21.382
5 - Friday	6- N2 - 3:00 PM to 7:00 PM	5,271	246	21.427
6- Saturday	1- D1 - 7:00 AM to 11:00 AM	5,289	246	21.500
6- Saturday	2- D2 - 11:00 AM to 3:00 PM	5,283	246	21.476
6- Saturday	3- E1 - 3:00 PM to 7:00 PM	5,132	246	20.862
6- Saturday	4- E2 - 7:00 PM to 11:00 PM	5,096	246	20.715
6- Saturday	5- N1 - 11:00 PM to 3:00 PM	5,107	246	20.760
6- Saturday	6- N2 - 3:00 PM to 7:00 PM	5,280	246	21.463
7 - Sunday	1- D1 - 7:00 AM to 11:00 AM	5,133	246	20.866
7 - Sunday	2- D2 - 11:00 AM to 3:00 PM	5,124	246	20.829
7 - Sunday	3- E1 - 3:00 PM to 7:00 PM	4,963	246	20.175
7 - Sunday	4- E2 - 7:00 PM to 11:00 PM	4,944	246	20.098
7 - Sunday	5- N1 - 11:00 PM to 3:00 PM	4,951	246	20.126
7 - Sunday	6- N2 - 3:00 PM to 7:00 PM	5,125	246	20.833

Table 4.7: Computing Average Census for each Day-Shift Combination - Ward A

the associated monthly seasonality index generated in Step 2. Now, we have a monthly adjusted census and ADT average for each day of a week and shift in a day combination.

Step 6: Distributing Average Census over Individual Acuity Groups

In Step 6, we distribute average census values into individual acuity categories using historical acuity distribution of admits at the medical units. Table 4.8 below presents the distribution of patient admissions over the acuity groups throughout the full data period. We use the computed percentages to forecast number of patients in each acuity group in the medical unit. Seasonally adjusted census of day-shift averages are multiplied by associated percentages to forecast the number of patients in each acuity group in the unit.

	Ward A		**Ward B**		**Ward C**	
Acuity	**# Admits**	**%**	**# Admits**	**%**	**# Admits**	**%**
A	0	0.00%	0	0.00%	0	0.00%
B	1	0.04%	12	0.16%	2	0.04%
C	7	0.25%	73	1.04%	131	2.51%
D	1,184	42.53%	4,592	68.21%	3,709	71.18%
E	1,375	49.39%	1,991	29.48%	1,320	25.34%
F	217	7.79%	81	1.12%	48	0.93%
Total Admits	2,784		6,749		5,211	

Table 4.8: Historical Acuity Distribution in PICU Wards

Step 7: Combining Acuity Distribution and ADT Activity to Calculate Patient Demand

In Step 7, we combine the forecasted number of patients in each acuity group with the seasonally adjusted ADT activity forecasts to compute an estimated patient demand (i.e. nursing requirement) for each nursing shift throughout the targeted staffing horizon. Figure 4.19 below, summarizes the preparation steps of heat-map for patient demand.

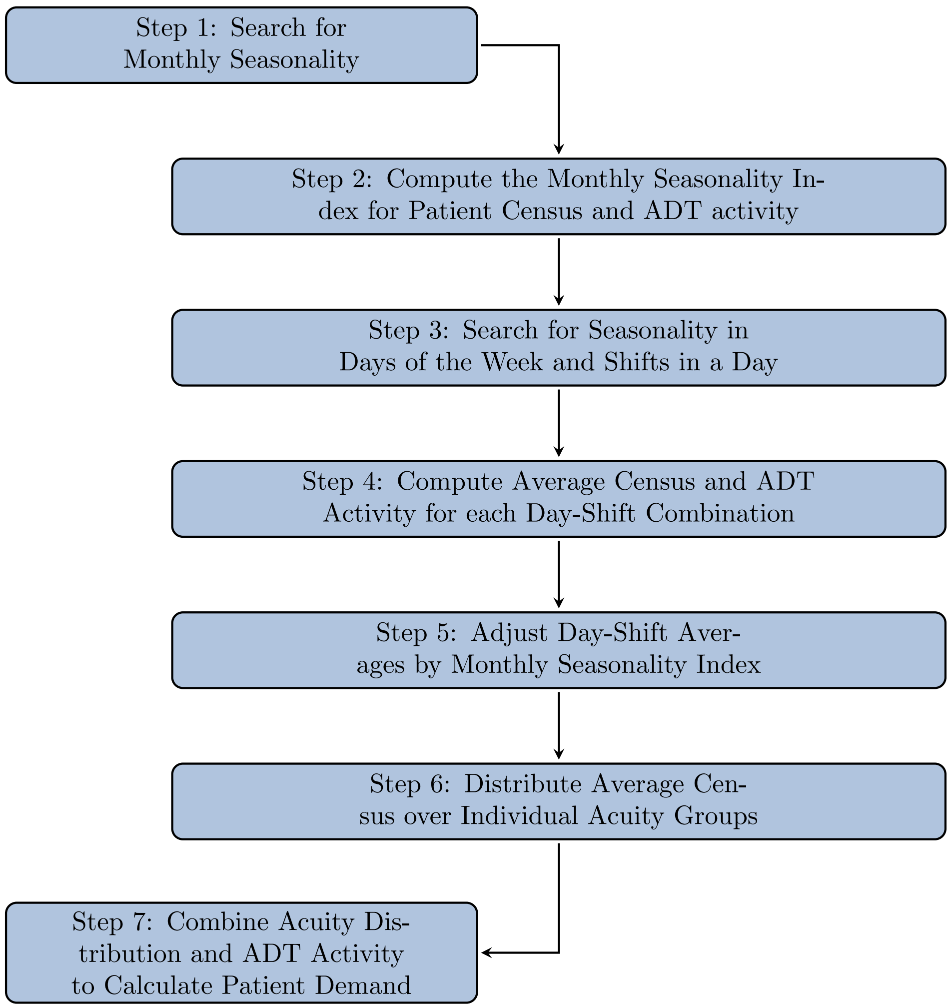


Figure 4.19: Preparation Steps of Heat-Map for Patient Demand

4.9 Results & Discussion

We present and discuss the results of our experiments in this section. We used AMPL programming interface to develop our optimization models. Our preferred solver for the optimization problems is IBM's CPLEX v12.6.3 Solver and the developed models in this chapter are in the class of mixed-integer programming models. We use three alternative stopping criteria for the optimization experiments: (1) Optimality gap of 0.1% is reached; (2) Time limit of 6 hours is reached; (3) Tree memory size limit of 100 GB is reached. Next, we discuss the results of the experiments with respect to each design factor and evaluate some performance measures.

4.9.1 Impact of Number of Available Schedules (NAS)

To test the impact of an increase in NAS on performance measures and objective value, we used the Optimal Staffing model for Ward A, where we used the actual patient data for the staffing horizon. We tested the model for three different levels of understaffing penalty (UPC) (or C_u^p as presented in the table below). Base level of UPC is determined as 1.5, due to the fact that mandatory overtime cost of a nurse is typically 50% higher than regular hourly rate. We also tested for UPC = 2 and UPC = 3 cases. We test for 4, 16, 64 and 256 different schedules per nurse profile in each experiment. Total number of schedules range from 120 schedules (i.e for the 4 schedule per nurse profile case) to 7680 schedules (i.e for the 256 schedule per nurse profile case) since we model for 30 different nurse profiles. We use random schedule selection technique for selecting 16, 64 and 256 schedules from the available feasible schedule pool; we select 4 schedules using the maximum difference model presented in the schedule selection section. Since we use the maximum difference model, selected 4 schedules are a subset of the 16 schedule set; which is not the case for other scenarios (i.e. randomly selected 16 or 64 schedules may not be a subset of randomly selected 256 schedules).

As presented in Table 4.9 below, objective value (i.e. the total cost for the objective function) is either slightly reduced or kept stable as we increase the NAS from 4 to 256. We conclude that feeding the alternative staffing models with 256 schedules per FTE profile (i.e. 7,680 total different schedules for the optimization model) is sufficiently large for providing schedule diversity. Even suggested maximally different four schedules per nurse profile approach seems to be providing efficient solutions. Further increases in the NAS, above 256 schedules per nurse profile, would not bring any cost savings but will increase the problem complexity, hurting the solution performance of the developed models. As a result, the rest of the experiments in this chapter, use standard 256 schedules per FTE profile in all optimization problems. In addition, observe that in all experiments cost savings are realized due to a reduced level of overstaffing in the unit. All experiments presented in Table 4.9 are solved to 0.1% or less optimality gap before reaching the time and memory size limit.

Design Factors		Cost Elements				Staffing Summary		
NAS	C_u^p	Objective Value	FTE Staffing	PRN Staffing	Understaffing Penalty	% Under. Avg.	% Over. Avg.	Staffing Avg.
4	1.5	19,338.15	14,712.00	3,524.40	1,101.75	3.67%	2.20%	17.77
16	1.5	19,338.15	14,688.00	3,524.40	1,125.75	3.79%	2.12%	17.75
64	1.5	19,338.15	14,712.00	3,524.40	1,101.75	3.67%	2.18%	17.77
256	1.5	19,338.15	14,688.00	3,524.40	1,125.75	3.77%	2.10%	17.75
4	2.0	19,579.60	15,312.00	3,471.60	796.00	1.98%	3.75%	18.32
16	2.0	19,573.60	15,264.00	3,493.60	816.00	2.02%	3.62%	18.29
64	2.0	19,573.40	15,240.00	3,502.40	831.00	2.04%	3.58%	18.28
256	2.0	19,574.40	15,240.00	3,502.40	832.00	2.05%	3.58%	18.28
4	3.0	19,808.40	16,296.00	3,128.40	384.00	0.64%	6.42%	18.99
16	3.0	19,807.70	16,272.00	3,159.20	376.50	0.63%	6.41%	18.99
64	3.0	19,807.70	16,272.00	3,159.20	376.50	0.63%	6.32%	18.99
256	3.0	19,810.70	16,272.00	3,159.20	379.50	0.63%	6.40%	18.99

Table 4.9: Impact of NAS: Optimal Staffing, Ward A

4.9.2 Staffing Policy Evaluation for PICU Wards

Our first research question in this study was: "Does dynamic medium-term nurse staffing policies that use patient demand forecasts outperform the historically employed fixed staffing policy for the intensive care medical units?" In this section, we evaluate the results of our experiments involving a comparison of alternative staffing policies with the aim of finding answers to our first research question. Table 4.10 present the results of experiments for Ward A under various understaffing penalty costs. For the base case of UPC = 1.5, Dynamic Staffing policy provided better outcomes in terms of objective value compared to two Fixed Staffing policy alternatives with 19 and 20 nurses. The objective value under Dynamic Staffing policy is only 3.97% more than the objective value under Optimal Staffing. Median understaffing under the Dynamic Policy is 0%, with an average understaffing of 3.01%. Median and average understaffing percentages are lower for the Fixed Staffing policies at a cost of average overstaffing percentages of 8.45% and 12.23%. As can be observed from the "Staffing Summary" section of the table, Dynamic Staffing policy achieved the cost savings via lowering the average staffing level throughout the staffing horizon. Also note that, even under Fixed Staffing policies achieving a stable level of staffing throughout the staffing horizon may not be feasible given the unit specific nurse mix and profiles. That is why we observe a staffing average of 18.99 under Fixed Staffing policy with a staff size target of 19 and a staffing average of 19.98 with a staff size target of 20. As the UPC is increased to 2.0, average staff size under Dynamic Staffing policy is slightly increased to 18.88 and the objective value is still less than Fixed Staffing alternatives (i.e. cost savings are 2.11% and 3.70%, respectively). Median understaffing is kept stable at 0% and average understaffing ratio is decreased to 2.67%, well below the acceptable 5% level, via increased staff size to alleviate the increased UPC. For the scenario of UPC = 3.0, Dynamic Staffing policy continue to provide cost savings at 1.01% and 0.90% levels compared to the two Fixed Staffing policy options. Now, the average staff size is increased to 19.21, which brings the average understaffing to 2% level. As the UPC is further increased to 10.0, the Fixed Staffing policy with 20 nurses provides slightly better objective values compared to the Dynamic Policy. Note that, we only use Optimal Staffing as a means of measuring the performance of alternative models. Optimal Staffing assumes perfectly known patient acuities and ADT activity for the upcoming staffing horizon, which makes it a hypothetical alternative.

Table 4.11 present the results of experiments for Ward B under various understaffing penalty costs. For Ward B, we test two alternative Fixed Staffing alternatives with staff sizes 11 and 12 against the Dynamic Staffing. For all UPC alternatives Dynamic Staffing policy provided cost savings compared to the Fixed Staffing alternatives. Savings range from 0.62% to a staggering 32.25% for UPC = 10.0 scenario and with respect to the Fixed Staffing with 11 nurses alternative. Objective values attained using the Dynamic Staffing model compared to the Optimal Staffing was comparable for lower UPC values (i.e. 3.61% and 5.95% for UPC = 1.5 and 2.0, respectively). As the UPC is further increased to 3.0 and 10.0 the gap between Optimal and Dynamic Staffing alternatives become more significant. Average staff size remained between 11.69 and 12.29 under the Dynamic Staffing policy. Understaffing percentages are higher for this Ward, compared to Ward A, for both Dynamic and Fixed Staffing policy alternatives. This observation indicates a higher than usual patient demand level for the studied specific staffing horizon. While median understaffing percentages under the Dynamic Policy ranged between 3.16% to 8.01%, the average understaffing percentages realized between 8% and 10.74%. Dynamic Staffing policy understaffing percentages are lower compared to the Fixed Staffing alternatives under same scenarios. Overall, we can conclude that Dynamic Staffing policy demonstrated superior performance compared to Fixed Staffing models for both cost savings and stable understaffing per-

Design Factors		Objective		Understaffing		Overstaffing		Staffing Summary		
Staffing Policy	C_u^p	**Value**	**Savings**	**Med.**	**Avg.**	**Med.**	**Avg.**	**Min.**	**Avg.**	**Max.**
Optimal Staffing	1.5	19,338.15	-3.97%	1.45%	3.77%	0.00%	2.10%	12.00	17.75	22.00
Dynamic Staffing	1.5	20,137.00	0.00%	0.00%	3.01%	2.01%	7.59%	17.00	18.78	20.00
Fixed Staffing - 19 nurse	1.5	20,288.50	0.75%	0.00%	2.64%	2.01%	8.45%	18.00	18.99	19.00
Fixed Staffing - 20 nurse	1.5	20,781.75	3.20%	0.00%	0.97%	7.38%	12.23%	19.00	19.98	20.00
Optimal Staffing	2.0	19,574.40	-2.80%	1.23%	2.05%	0.00%	3.58%	14.00	18.28	22.00
Dynamic Staffing	2.0	20,139.00	0.00%	0.00%	2.67%	2.01%	7.75%	18.00	18.88	20.00
Fixed Staffing - 19 nurse	2.0	20,564.00	2.11%	0.00%	2.64%	2.01%	8.45%	18.00	18.99	19.00
Fixed Staffing - 20 nurse	2.0	20,885.00	3.70%	0.00%	0.97%	7.38%	12.23%	19.00	19.98	20.00
Optimal Staffing	3.0	19,810.70	-5.23%	0.00%	0.63%	2.56%	6.40%	14.00	18.99	22.00
Dynamic Staffing	3.0	20,903.00	0.00%	0.00%	2.00%	4.11%	8.88%	18.00	19.21	21.00
Fixed Staffing - 19 nurse	3.0	21,115.00	1.01%	0.00%	2.64%	2.01%	8.45%	18.00	18.99	19.00
Fixed Staffing - 20 nurse	3.0	21,091.50	0.90%	0.00%	0.97%	7.38%	12.23%	19.00	19.98	20.00
Optimal Staffing	10.0	20,164.80	-13.49%	0.00%	0.15%	3.70%	8.42%	14.00	19.43	24.00
Dynamic Staffing	10.0	23,310.00	0.00%	0.00%	1.67%	5.26%	9.28%	18.00	19.35	20.00
Fixed Staffing - 19 nurse	10.0	24,972.00	7.13%	0.00%	2.64%	2.01%	8.45%	18.00	18.99	19.00
Fixed Staffing - 20 nurse	10.0	22,537.00	-3.32%	0.00%	0.97%	7.38%	12.23%	19.00	19.98	20.00

Table 4.10: Staffing Policy Evaluation: Ward A

centages for Ward B.

Next, we evaluate the performance of staffing policy alternatives using the results of experiments for Ward C under various understaffing penalty costs in Table 4.12. For Ward C, we test the performance of the Dynamic Staffing model with respect to two alternative levels of Fixed Staffing, with 12 and 13 nurses throughout the staffing horizon. For the case of UPC = 1.5, objective values of Dynamic staffing policy and Fixed Staffing policy with 12 nurses is similar. Dynamic Staffing provided cost savings of 2.94% compared to the Fixed Staffing policy with 13 nurses. When UPC is increased to 2.0, the performance of Dynamic Staffing policy differentiates from both alternative Fixed Staffing policies. Associated cost savings are 2.93% and 4.70% for the 12 and 13 nurse alternatives, respectively. For the highest level of UPC = 10.0, cost savings with the Dynamic Staffing policy is significantly different than both alternative Fixed Staffing policies (i.e. 18.69% and 5.42%). Objective values realized under the Dynamic Policy get as close as 1.88% to the Optimal Staffing objective levels. Observe that average staffing level for the Fixed Staffing policies are not in line with the intended staff size targets (i.e. 11.57 average staff size instead of 12 and 12.43 average staff size instead of

Design Factors		Objective		Understaffing		Overstaffing		Staffing Summary		
Staffing Policy	C_u^p	**Value**	**Savings**	**Med.**	**Avg.**	**Med.**	**Avg.**	**Min.**	**Avg.**	**Max.**
Optimal Staffing	1.5	13,845.70	-3.61%	2.22%	7.19%	0.00%	3.05%	2.00	11.97	17.00
Dynamic Staffing	1.5	14,363.85	0.00%	8.01%	10.74%	0.00%	4.22%	10.00	11.69	13.00
Fixed Staffing - 11 nurse	1.5	14,482.80	0.83%	14.56%	14.49%	0.00%	2.88%	11.00	11.00	11.00
Fixed Staffing - 12 nurse	1.5	14,482.35	0.82%	6.80%	9.56%	0.00%	5.98%	12.00	12.00	12.00
Optimal Staffing	2.0	14,219.80	-5.95%	0.00%	3.54%	2.36%	7.00%	8.00	12.87	17.00
Dynamic Staffing	2.0	15,120.20	0.00%	6.80%	10.08%	0.00%	4.58%	10.00	11.82	13.00
Fixed Staffing - 11 nurse	2.0	15,560.80	2.91%	14.56%	14.49%	0.00%	2.88%	11.00	11.00	11.00
Fixed Staffing - 12 nurse	2.0	15,214.60	0.62%	6.80%	9.56%	0.00%	5.98%	12.00	12.00	12.00
Optimal Staffing	3.0	14,658.40	-11.23%	0.00%	2.00%	4.71%	10.65%	8.00	13.45	18.00
Dynamic Staffing	3.0	16,512.10	0.00%	5.42%	9.41%	0.00%	5.05%	10.00	11.96	13.00
Fixed Staffing - 11 nurse	3.0	17,716.80	7.30%	14.56%	14.49%	0.00%	2.88%	11.00	11.00	11.00
Fixed Staffing - 12 nurse	3.0	16,679.10	1.01%	6.80%	9.56%	0.00%	5.98%	12.00	12.00	12.00
Optimal Staffing	10.0	16,686.20	-32.74%	0.00%	1.74%	6.12%	11.50%	8.00	13.59	17.00
Dynamic Staffing	10.0	24,808.00	0.00%	3.16%	8.00%	0.00%	6.28%	11.00	12.29	14.00
Fixed Staffing - 11 nurse	10.0	32,808.80	32.25%	14.56%	14.49%	0.00%	2.88%	11.00	11.00	11.00
Fixed Staffing - 12 nurse	10.0	26,930.60	8.56%	6.80%	9.56%	0.00%	5.98%	12.00	12.00	12.00

Table 4.11: Staffing Policy Evaluation: Ward B

13). It seems Ward C employs the minimum number of PRN nurses compared to the number of total nurses employed in the unit (i.e. 6 out of 76 nurses) and having fewer PRN nurses limits the scheduling flexibility for the unit. Realized average understaffing percentages for the Dynamic Staffing policy are in the range of 2.3% and 6.95%, median understaffing being kept stable at 0%. Overstaffing percentages are also lower under the Dynamic Staffing policy compared to both Fixed Staffing policy for the base UPC level. As the UPC level is increased, Dynamic Staffing average staff size is increased to alleviate the understaffing penalty levels, which comes at the cost of increased overstaffing percentages.

In summary, for the experiments we conducted with the three PICU wards, the performance of Dynamic Staffing policy was either superior to the Fixed Staffing alternatives or similar. The power of Dynamic Staffing policy lies in the accuracy of forecasted heat map. As the forecasting performance in preparing the unit specific heat map is improved, the more cost savings and alleviated understaffing percentages will be observed under Dynamic Staffing policy. Regarding the Fixed Staffing policy, we need to first note that perfectly stable staff size may not be feasible in many cases, especially with a limited number of PRN nurse

Design Factors		Objective		Understaffing		Overstaffing		Staffing Summary		
Staffing Policy	$\mathbf{C_u^p}$	**Value**	**Savings**	**Med.**	**Avg.**	**Med.**	**Avg.**	**Min.**	**Avg.**	**Max.**
Optimal Staffing	1.5	12,429.70	-5.14%	3.61%	8.40%	0.00%	2.60%	5.00	10.54	16.00
Dynamic Staffing	1.5	13,103.35	0.00%	0.00%	6.95%	0.00%	9.80%	8.00	11.42	13.00
Fixed Staffing - 12 nurse	1.5	13,097.85	-0.04%	0.00%	6.11%	1.05%	10.29%	10.00	11.57	12.00
Fixed Staffing - 13 nurse	1.5	13,488.60	2.94%	0.00%	3.76%	9.47%	15.59%	11.00	12.43	13.00
Optimal Staffing	2.0	12,900.40	-1.88%	0.41%	6.02%	2.11%	5.04%	5.00	11.05	17.00
Dynamic Staffing	2.0	13,147.05	0.00%	0.00%	5.88%	1.15%	10.85%	9.00	11.68	13.00
Fixed Staffing - 12 nurse	2.0	13,532.60	2.93%	0.00%	6.11%	1.05%	10.29%	10.00	11.57	12.00
Fixed Staffing - 13 nurse	2.0	13,765.60	4.70%	0.00%	3.76%	9.47%	15.59%	11.00	12.43	13.00
Optimal Staffing	3.0	13,543.80	-5.42%	0.00%	3.36%	5.80%	10.00%	6.00	11.87	17.00
Dynamic Staffing	3.0	14,319.60	0.00%	0.00%	4.07%	5.76%	14.76%	10.00	12.31	16.00
Fixed Staffing - 12 nurse	3.0	14,402.10	0.58%	0.00%	6.11%	1.05%	10.29%	10.00	11.57	12.00
Fixed Staffing - 13 nurse	3.0	14,319.60	0.00%	0.00%	3.76%	9.47%	15.59%	11.00	12.43	13.00
Optimal Staffing	10.0	14,853.80	-13.95%	0.00%	0.69%	10.53%	23.94%	8.00	13.59	21.00
Dynamic Staffing	10.0	17,262.20	0.00%	0.00%	2.30%	15.66%	23.79%	12.00	13.52	18.00
Fixed Staffing - 12 nurse	10.0	20,488.60	18.69%	0.00%	6.11%	1.05%	10.29%	10.00	11.57	12.00
Fixed Staffing - 13 nurse	10.0	18,197.60	5.42%	0.00%	3.76%	9.47%	15.59%	11.00	12.43	13.00

Table 4.12: Staffing Policy Evaluation: Ward C

body. But eventually, our proposed Fixed Staffing modeling approach provides a reliable and efficient way of scheduling the nursing workforce. Medical units with higher variation in patient demand levels would benefit the most using the Dynamic Staffing policy proposed in this study.

4.9.3 Controlling for Understaffing Levels in the Medical Units

This section presents our experimental results regarding the use of UPC as a mechanism for controlling the understaffing levels in the medical units. Our second research question in this chapter was: "Can understaffing penalty cost be utilized as a mechanism to control the understaffing levels which possibly mitigate nurse burnout and medical errors?". We evaluate the results of our experiments in this section to gain insights to better control the understaffing levels to alleviate nurse burnout, overtime and last minute schedule adjustments. We test the impact of understaffing penalty cost (UPC), primarily on understaffing ratios in the medical units, under alternative staffing policies. We test the use of understaffing penalty cost as a mechanism to control the understaffing levels under the Dynamic Staffing policy. Next, we evaluate the impact of UPC on understaffing percentages in Ward A.

Table 4.13 presents the results of experiments under alternative staffing policies and UPC for Ward A. Under the Optimal Staffing, increasing the UPC from the base level of 1.5 to 10.0 reduce the average understaffing from 3.77% to 0.15%. This is achieved via an increase in the average staff size from 17.75 to 19.43. Average overstaffing in the unit is realized between 2.10% to 8.42% for various levels of UPC. Under Dynamic Staffing, average understaffing percentages occurred between 1.67% and 3.01%, with a median understaffing level of 0% for all UPC levels in Ward A. For the base level UPC of 1.5, Dynamic Staffing policy resulted in 3.01% average understaffing in the nursing shifts throughout the staffing horizon, which is less than the average understaffing attained under the Optimal Staffing. As the UPC is increased to 2.0, 3.0 and 10.0, the average understaffing levels slightly decreased to 2.67%, 2.00% and 1.67% for the Dynamic Staffing policy. Average staff size remained between 18.78 and 19.35, a slightly smaller variation in staff size compared to the Optimal Staffing outcomes. Average overstaffing percentages remained between 7.59% and 9.28%. The most cost savings are achieved via Dynamic Staffing policy for the cases of UPC = 1.5, 2.0 and 3.0, when compared with the Fixed Staffing alternatives. All experiments are solved to an optimality gap less than 0.33%.

For the Fixed Staffing alternatives with 19 and 20 nurses for Ward A, we observe an average understaffing of 2.64% and 0.97%. Objective values for the two alternatives are relatively close to the Dynamic Staffing policy outcomes. This might be due to the relatively stable nature of patient demand for this unit (i.e. Coefficient of Variation for this unit is 12.17% for the sample and 11.26% for the full dataset, see Table 4.5). Overstaffing ratios for the Fixed Staffing alternatives are higher compared to the Dynamic Staffing for UPC = 1.5 and 2.0. We can conclude using a UPC of 1.5 suffices to expect less than 5% average understaffing ratios for this medical unit under the Dynamic Staffing policy for the studied staffing horizon. Due to the static nature of nursing staff size under the Fixed Staffing policy alternatives, the under and overstaffing ratios are kept stable with respect to the UPC levels. As a result, under the Fixed Staffing policy nursing administration can only change the level of targeted fixed staff size, which create the risk of excess overstaffing levels in case of an increased staff size.

Table 4.14 presents the results of experiments for Ward B. All experiments are solved to an optimality gap less than 0.09%. Under Optimal Staffing policy, where we test our models with actual patient data, increasing the UPC increased the average staffing level (i.e. average staffing level in a nursing shift throughout the staffing horizon of six weeks), decreasing the average understaffing ratios in the medical unit. For the base

Design Factors		Opt.	Objective	Staffing	Understaffing			Overstaffing	
Staffing Policy	C_u^p	Gap	Value	Avg.	Penalty	Med.	Avg.	Med	Avg
Optimal Staffing	1.5	0.13%	19,338.15	17.75	1,125.75	1.45%	3.77%	0.00%	2.10%
Optimal Staffing	2.0	0.00%	19,574.40	18.28	832.00	1.23%	2.05%	0.00%	3.58%
Optimal Staffing	3.0	0.00%	19,810.70	18.99	379.50	0.00%	0.63%	2.56%	6.40%
Optimal Staffing	10.0	0.00%	20,164.80	19.43	300.00	0.00%	0.15%	3.70%	8.42%
Dynamic Staffing	1.5	0.29%	20,137.00	18.78	939.00	0.00%	3.01%	2.01%	7.59%
Dynamic Staffing	2.0	0.33%	20,139.00	18.88	831.00	0.00%	2.67%	2.01%	7.75%
Dynamic Staffing	3.0	0.18%	20,903.00	19.21	1,251.00	0.00%	2.00%	4.11%	8.88%
Dynamic Staffing	10.0	0.01%	23,310.00	19.35	3,490.00	0.00%	1.67%	5.26%	9.28%
Fixed Staffing - 19 nurse	1.5	0.10%	20,288.50	18.99	826.50	0.00%	2.64%	2.01%	8.45%
Fixed Staffing - 19 nurse	2.0	0.10%	20,564.00	18.99	1,102.00	0.00%	2.64%	2.01%	8.45%
Fixed Staffing - 19 nurse	3.0	0.10%	21,115.00	18.99	1,653.00	0.00%	2.64%	2.01%	8.45%
Fixed Staffing - 19 nurse	10.0	0.10%	24,972.00	18.99	5,510.00	0.00%	2.64%	2.01%	8.45%
Fixed Staffing - 20 nurse	1.5	0.01%	20,781.75	19.98	309.75	0.00%	0.97%	7.38%	12.23%
Fixed Staffing - 20 nurse	2.0	0.01%	20,885.00	19.98	413.00	0.00%	0.97%	7.38%	12.23%
Fixed Staffing - 20 nurse	3.0	0.01%	21,091.50	19.98	619.50	0.00%	0.97%	7.38%	12.23%
Fixed Staffing - 20 nurse	10.0	0.01%	22,537.00	19.98	2,065.00	0.00%	0.97%	7.38%	12.23%

Table 4.13: Controlling for Understaffing in Ward A

case of UPC = 1.5, under Optimal Staffing, the average understaffing in the medical unit is 7.19%, which is reduced to 1.74% for the extreme case of UPC = 10.0. The gains in observed understaffing levels come at a cost of over 20% increase in the objective value. For the base case of UPC = 1.5 the median understaffing is observed as 2.22%, which is an acceptable level for medium-term planning. Average staff size, under the Optimal Staffing, changed between 11.97 and 13.59.

Under Dynamic Staffing policy, increasing UPC from the base level of 1.5 to 10.0 causes 72.7% increase in objective value in Table 4.14. The median understaffing percentage is realized as 8.01% for the base case, with an average understaffing level of 10.74%. The average understaffing drops to 8.00% when UPC is increased to 10.0. The gain in average understaffing (i.e. from 10.74% to 8.00%) when increasing UPC from 1.5 to 10.0 is realized at the cost of significant increase in the total understaffing penalty cost (i.e. from 2,414.25 for UPC=1.5 to 12,230 for UPC=10). Average staff size is kept at a stable level with a minimum of 11.69 and maximum of 12.29 nurses under the Dynamic Staffing policy for Ward B.

Design Factors		Opt.	Objective	Staffing	Understaffing			Overstaffing	
Staffing Policy	$\mathbf{C_u^p}$	**Gap**	**Value**	**Avg.**	**Penalty**	**Med.**	**Avg.**	**Med**	**Avg**
Optimal Staffing	1.5	0.09%	13,845.70	11.97	1,612.50	2.22%	7.19%	0.00%	3.05%
Optimal Staffing	2.0	0.01%	14,219.80	12.87	1,093.00	0.00%	3.54%	2.36%	7.00%
Optimal Staffing	3.0	0.05%	14,658.40	13.45	954.00	0.00%	2.00%	4.71%	10.65%
Optimal Staffing	10.0	0.09%	16,686.20	13.59	2,835.00	0.00%	1.74%	6.12%	11.50%
Dynamic Staffing	1.5	0.03%	14,363.85	11.69	2,414.25	8.01%	10.74%	0.00%	4.22%
Dynamic Staffing	2.0	0.03%	15,120.20	11.82	3,037.00	6.80%	10.08%	0.00%	4.58%
Dynamic Staffing	3.0	0.05%	16,512.10	11.96	4,270.50	5.42%	9.41%	0.00%	5.05%
Dynamic Staffing	10.0	0.04%	24,808.00	12.29	12,230.00	3.16%	8.00%	0.00%	6.28%
Fixed Staffing - 11 nurse	1.5	0.00%	14,482.80	11.00	3,234.00	14.56%	14.49%	0.00%	2.88%
Fixed Staffing - 11 nurse	2.0	0.00%	15,560.80	11.00	4,312.00	14.56%	14.49%	0.00%	2.88%
Fixed Staffing - 11 nurse	3.0	0.00%	17,716.80	11.00	6,468.00	14.56%	14.49%	0.00%	2.88%
Fixed Staffing - 11 nurse	10.0	0.00%	32,808.80	11.00	21,560.00	14.56%	14.49%	0.00%	2.88%
Fixed Staffing - 12 nurse	1.5	0.00%	14,482.35	12.00	2,196.75	6.80%	9.56%	0.00%	5.98%
Fixed Staffing - 12 nurse	2.0	0.00%	15,214.60	12.00	2,929.00	6.80%	9.56%	0.00%	5.98%
Fixed Staffing - 12 nurse	3.0	0.00%	16,679.10	12.00	4,393.50	6.80%	9.56%	0.00%	5.98%
Fixed Staffing - 12 nurse	10.0	0.00%	26,930.60	12.00	14,645.00	6.80%	9.56%	0.00%	5.98%

Table 4.14: Controlling for Understaffing in Ward B

We test Fixed Staffing policy with two alternative staff size for Ward B, results are presented in Table 4.14. Under the eleven nurse alternative objective value is realized at 14,482.80 for the base case of UPC = 1.5. Increasing the UPC to 10.0 triggers an over 128% increase in the objective value to 32,808.80. Median understaffing is observed as 14.56% with an average understaffing level of 14.49%. When the staff size is increased to twelve nurse for the Fixed Staffing Model, median understaffing percentages drop to 6.80%, with an average understaffing percentage of 9.56%. Among the two alternatives of Fixed Staffing policy, staffing unit with twelve nurse provided better outcomes for this staffing horizon. Comparing alternative staffing policies with the Optimal Staffing outcomes, we can conclude that Dynamic Staffing policy provided better results in terms of both objective value and observed median and average understaffing percentages in the medical unit. Higher than desired understaffing percentages are observed for both Dynamic and Fixed Staffing models for the studied staffing horizon in Ward B. The results reflect the "higher than usual" patient demand for the medical unit during the studied staffing horizon. Depending on the observed levels of understaffing percentage, nursing administration can determine an appropriate level of UPC, which will help reduce nurse burnout.

Table 4.15 presents the results of experiments for Ward C. Under Optimal Staffing, observed average understaffing percentages ranged between 0.69% and 8.40%. Average staff size started at a low of 10.54 for the base UPC case and go up to 13.59 for the maximum UPC of 10. The range of staff size under Optimal Staffing reflect the need for a dynamic staff size to control understaffing levels in the medical units. Under the Dynamic Staffing policy, average staff size range between 11.42 and 13.52. Average understaffing percentages ranged between 2.30% and 6.95%. While Fixed Staffing with 12 nurses provide better performance compared to the Fixed Staffing alternative with 13 nurses for lower UPC levels, Fixed Staffing with 13 nurses appears to be a better option for higher UPC levels. Observe that, given the listed stopping criteria for the optimization experiments, best optimality gaps achieved for the Fixed Staffing levels remain above 25% for both alternatives, which demonstrates the achieving perfectly stable staff size may not be even feasible for specific medical units. Overall, we can conclude, Dynamic Staffing policy provided cost savings and lower understaffing levels for the unit compared to the Fixed Staffing alternatives. Difference in objective value drastically increase when UPC is at it's highest level. Dynamic Staffing policy successfully retained the average understaffing levels between the 2.30% and 6.95% range for the medical unit, which closely follows the Optimal Staffing outcomes.

In summary, Dynamic Staffing provide a more reliable, in terms of acceptable understaffing ratios, and flexible staffing policy which also bring cost savings for the medical units. Depending on the tolerable levels of understaffing for the medical unit, the nursing administration can determine the appropriate UPC level to be used. For unexpectedly high patient demand periods, higher understaffing levels might be observed, as the Dynamic Staffing policy is based on historical patient demand based heat maps. Heat maps can be dynamically adjusted as new demand patterns are observed for the upcoming staffing horizons. As the accuracy of patient demand forecasts are enhanced, the better performance outcomes will be achieved using Dynamic Staffing policy. Historically employed Fixed Staffing policies do not provide the required staff size flexibility to alleviate understaffing in the medical units which trigger nurse burnout. Fixed Staffing policies would also increase the need for short term schedule adjustment costs to better match the patient demand due to the static nature of staff size. In addition, for medical units with limited PRN nurse body, achieving perfectly stable staff size may not be even feasible as demonstrated in our experiments.

Design Factors		Opt.	Objective	Staffing	Understaffing			Overstaffing	
Staffing Policy	C_u^p	Gap	Value	Avg.	Penalty	Med.	Avg.	Med	Avg
Optimal Staffing	1.5	0.02%	12,429.70	10.54	1,684.50	3.61%	8.40%	0.00%	2.60%
Optimal Staffing	2.0	0.02%	12,900.40	11.05	1,654.00	0.41%	6.02%	2.11%	5.04%
Optimal Staffing	3.0	0.01%	13,543.80	11.87	1,473.00	0.00%	3.36%	5.80%	10.00%
Optimal Staffing	10.0	0.11%	14,853.80	13.59	1,075.00	0.00%	0.69%	10.53%	23.94%
Dynamic Staffing	1.5	0.26%	13,103.35	11.42	1,455.75	0.00%	6.95%	0.00%	9.80%
Dynamic Staffing	2.0	0.35%	13,147.05	11.68	1,244.25	0.00%	5.88%	1.15%	10.85%
Dynamic Staffing	3.0	2.02%	14,319.60	12.31	1,782.00	0.00%	4.07%	5.76%	14.76%
Dynamic Staffing	10.0	0.19%	17,262.20	13.52	3,505.00	0.00%	2.30%	15.66%	23.79%
Fixed Staffing - 12 nurse	1.5	33.30%	13,097.85	11.57	1,304.25	0.00%	6.11%	1.05%	10.29%
Fixed Staffing - 12 nurse	2.0	33.30%	13,532.60	11.57	1,739.00	0.00%	6.11%	1.05%	10.29%
Fixed Staffing - 12 nurse	3.0	33.30%	14,402.10	11.57	2,608.50	0.00%	6.11%	1.05%	10.29%
Fixed Staffing - 12 nurse	10.0	33.30%	20,488.60	11.57	8,695.00	0.00%	6.11%	1.05%	10.29%
Fixed Staffing - 13 nurse	1.5	25.00%	13,488.60	12.43	831.00	0.00%	3.76%	9.47%	15.59%
Fixed Staffing - 13 nurse	2.0	25.00%	13,765.60	12.43	1,108.00	0.00%	3.76%	9.47%	15.59%
Fixed Staffing - 13 nurse	3.0	25.00%	14,319.60	12.43	1,662.00	0.00%	3.76%	9.47%	15.59%
Fixed Staffing - 13 nurse	10.0	25.00%	18,197.60	12.43	5,540.00	0.00%	3.76%	9.47%	15.59%

Table 4.15: Controlling for Understaffing in Ward C

4.9.4 Analysis of Nurse Utilization

In this section we review the utilization of nurses in medical units. Table 4.16 presents the results of experiments for nurse utilization in Ward A. Ward A has 87 FTE and 16 PRN nurses (i.e. 18.4% PRN to FTE ratio). We test the number of nurses used under different scenarios and staffing policies because our optimization modeling approach does not force the assignment of all available nurses in the medical unit. In the tables that follow we present the percentage of FTE nurses used among the available pool and the percentage of average weekly PRN nurse assignment hours compared to a 40 hour per work week. All PRN nurses are used in all staffing policies under all UPC levels. Our results indicate that the PRN nurse pool, even with a 10% higher cost compared to the FTE nurses, provides crucial flexibility for the nursing administration to match patient demand with staffing levels. PRN utilizations in terms of average hours of assignment per week is also kept at higher levels, from a low of 27.71 hours per week to 35 hours per week, indicating PRN utilizations higher than 70% for all experiments. Minimum FTE nurses used during the staffing horizon is 81. The average staff size in the unit ranged between 17.75 and 19.98 nurses per shift. Nursing administration can use these observations for long-term planning (i.e., planning for hiring additional nurses) to determine whether the current nurse pool is large enough to satisfy patient demand. Under Optimal Staffing, as the UPC is increased from the base level of 1.5 to 10.0, nurse utilization from the available FTE pool starts at 86.2% and reaches 100.0%. Under the Dynamic Staffing policy FTE utilization remains stable at 96.6%, with 84 of 87 available nurses used. Under the Fixed Staffing policy, 81 nurses are used for the 19-nurse alternative and all 87 nurses are used for the 20-nurse alternative, which means that an additional 6 FTE nurses are needed to increase the fixed staff size by one nurse.

Design Factors		**FTE Utilization**		**PRN Utilization**			**Staffing Levels**		
Staffing Policy	$\mathbf{C_u^p}$	**# FTEs**	**% Util.**	**# PRNs**	**hrs/wk**	**% Util.**	**Min.**	**Avg.**	**Max.**
Optimal Staffing	1.5	75	86.2%	16	33.38	83.4%	12.00	17.75	22.00
Optimal Staffing	2.0	79	90.8%	16	33.17	82.9%	14.00	18.28	22.00
Optimal Staffing	3.0	84	96.6%	16	29.92	74.8%	14.00	18.99	22.00
Optimal Staffing	10.0	87	100.0%	16	29.25	73.1%	14.00	19.43	24.00
Dynamic Staffing	1.5	84	96.6%	16	27.71	69.3%	17.00	18.78	20.00
Dynamic Staffing	2.0	84	96.6%	16	28.75	71.9%	18.00	18.88	20.00
Dynamic Staffing	3.0	84	96.6%	16	30.42	76.0%	18.00	19.21	21.00
Dynamic Staffing	10.0	84	96.6%	16	32.92	82.3%	18.00	19.35	20.00
Fixed Staffing - 19 nurse	1.5	81	93.1%	16	32.71	81.8%	18.00	18.99	19.00
Fixed Staffing - 19 nurse	2.0	81	93.1%	16	32.71	81.8%	18.00	18.99	19.00
Fixed Staffing - 19 nurse	3.0	81	93.1%	16	32.71	81.8%	18.00	18.99	19.00
Fixed Staffing - 19 nurse	10.0	81	93.1%	16	32.71	81.8%	18.00	18.99	19.00
Fixed Staffing - 20 nurse	1.5	87	100.0%	16	35.00	87.5%	19.00	19.98	20.00
Fixed Staffing - 20 nurse	2.0	87	100.0%	16	35.00	87.5%	19.00	19.98	20.00
Fixed Staffing - 20 nurse	3.0	87	100.0%	16	35.00	87.5%	19.00	19.98	20.00
Fixed Staffing - 20 nurse	10.0	87	100.0%	16	35.00	87.5%	19.00	19.98	20.00

Table 4.16: Nurse Utilization, Ward A

Table 4.17 presents the results of experiments for nurse utilization in Ward B. Ward B has 61 FTE and 10 PRN nurses (i.e. 16.4% PRN to FTE ratio). Similar to Ward A, all PRN nurses are used in all staffing policies under all UPC levels for Ward B, as well. FTE nurse utilization starts from 85.2 % and reaches 100% under the Optimal Staffing for various levels of UPC. For the Dynamic Staffing policy, percentage of used FTE nurses stay stable in the 82% and 85.2% range. The difference in utilizations between the two staffing policies indicate that patient demand for this specific staffing horizon is "higher than usual", which indicates 61 FTE nurses offer a significant size capacity buffer for this medical unit. Under the Fixed Staffing alternatives, 48 and 49 nurses are used for 11 and 12 nurse alternatives, respectively. PRN to FTE ratio of 16.4% allowed the Fixed Staffing models to perfectly match their associated target staff size. PRN weekly utilization remained in the 61.8% to 79% for the tested staffing models.

Design Factors		**FTE Utilization**		**PRN Utilization**			**Staffing Levels**		
Staffing Policy	**C_u^p**	**# FTEs**	**% Util.**	**# PRNs**	**hrs/wk**	**% Util.**	**Min.**	**Avg.**	**Max.**
Optimal Staffing	1.5	52	85.2%	10	27.53	68.8%	2.00	11.97	17.00
Optimal Staffing	2.0	56	91.8%	10	25.80	64.5%	8.00	12.87	17.00
Optimal Staffing	3.0	60	98.4%	10	24.73	61.8%	8.00	13.45	18.00
Optimal Staffing	10.0	61	100.0%	10	25.87	64.7%	8.00	13.59	17.00
Dynamic Staffing	1.5	50	82.0%	10	27.60	69.0%	10.00	11.69	13.00
Dynamic Staffing	2.0	50	82.0%	10	28.53	71.3%	10.00	11.82	13.00
Dynamic Staffing	3.0	50	82.0%	10	30.93	77.3%	10.00	11.96	13.00
Dynamic Staffing	10.0	52	85.2%	10	31.67	79.2%	11.00	12.29	14.00
Fixed Staffing - 11 nurse	1.5	48	78.7%	10	26.80	67.0%	11.00	11.00	11.00
Fixed Staffing - 11 nurse	2.0	48	78.7%	10	26.80	67.0%	11.00	11.00	11.00
Fixed Staffing - 11 nurse	3.0	48	78.7%	10	26.80	67.0%	11.00	11.00	11.00
Fixed Staffing - 11 nurse	10.0	48	78.7%	10	26.80	67.0%	11.00	11.00	11.00
Fixed Staffing - 12 nurse	1.5	49	80.3%	10	31.60	79.0%	12.00	12.00	12.00
Fixed Staffing - 12 nurse	2.0	49	80.3%	10	31.60	79.0%	12.00	12.00	12.00
Fixed Staffing - 12 nurse	3.0	49	80.3%	10	31.60	79.0%	12.00	12.00	12.00
Fixed Staffing - 12 nurse	10.0	49	80.3%	10	31.60	79.0%	12.00	12.00	12.00

Table 4.17: Nurse Utilization, Ward B

Table 4.18 presents the results of experiments for nurse utilization in Ward C. Ward C has 70 FTE and 6 PRN nurses (i.e. 8.57% PRN to FTE ratio). Similar to Wards A and B, all PRN nurses are used in all staffing policies under all UPC levels for Ward C, as well. Due to the lower PRN to FTE ratio of 8.57%, the Fixed Staffing models couldn't perfectly match their associated target staff size (i.e. average staff size realized as 11.57 and 12.43 for 12 nurse and 13 nurse alternatives, respectively). FTE nurse utilization starts from 70% and reaches 94.3% under the Optimal Staffing for various levels of UPC. For the Dynamic Staffing policy, percentage of used FTE nurses realized in the range of 74.3% and 90%. For both Fixed Staffing alternatives, 54 FTE nurses were used (i.e. 77.1% of the available FTE nurse pool). PRN weekly assignment utilization observed in 90% range for both Dynamic Staffing and Fixed Staffing alternatives.

Under the Optimal Staffing, PRN utilization started at 81.4% for the base level and gradually reduced to 54.7% for the scenario of UPC=10.0. Average staff size changed between 10.54 and 13.59, under the Optimal Staffing, which demonstrates that fixed staff size doesn't really reflect the staff size needs in the medical unit.

Design Factors		FTE Utilization		PRN Utilization			Staffing Levels		
Staffing Policy	**C_u^p**	**# FTEs**	**% Util.**	**# PRNs**	**hrs/wk**	**% Util.**	**Min.**	**Avg.**	**Max.**
Optimal Staffing	1.5	49	70.0%	6	32.56	81.4%	5.00	10.54	16.00
Optimal Staffing	2.0	51	72.9%	6	30.67	76.7%	5.00	11.05	17.00
Optimal Staffing	3.0	57	81.4%	6	29.67	74.2%	6.00	11.87	17.00
Optimal Staffing	10.0	66	94.3%	6	21.89	54.7%	8.00	13.59	21.00
Dynamic Staffing	1.5	52	74.3%	6	36.56	91.4%	8.00	11.42	13.00
Dynamic Staffing	2.0	53	75.7%	6	36.33	90.8%	9.00	11.68	13.00
Dynamic Staffing	3.0	57	81.4%	6	36.00	90.0%	10.00	12.31	16.00
Dynamic Staffing	10.0	63	90.0%	6	35.89	89.7%	12.00	13.52	18.00
Fixed Staffing - 12 nurse	1.5	54	77.1%	6	36.00	90.0%	10.00	11.57	12.00
Fixed Staffing - 12 nurse	2.0	54	77.1%	6	36.00	90.0%	10.00	11.57	12.00
Fixed Staffing - 12 nurse	3.0	54	77.1%	6	36.00	90.0%	10.00	11.57	12.00
Fixed Staffing - 12 nurse	10.0	54	77.1%	6	36.00	90.0%	10.00	11.57	12.00
Fixed Staffing - 13 nurse	1.5	54	77.1%	6	36.00	90.0%	11.00	12.43	13.00
Fixed Staffing - 13 nurse	2.0	54	77.1%	6	36.00	90.0%	11.00	12.43	13.00
Fixed Staffing - 13 nurse	3.0	54	77.1%	6	36.00	90.0%	11.00	12.43	13.00
Fixed Staffing - 13 nurse	10.0	54	77.1%	6	36.00	90.0%	11.00	12.43	13.00

Table 4.18: Nurse Utilization, Ward C

4.9.5 Objective Function Cost Elements

We evaluate the cost elements that form the objective function in our optimization models in this section. Table 4.19 presents the cost elements that contribute to the objective value for Ward A. Under Optimal Staffing, when the UPC is increased from 1.5 to 10.0, first reaction is to increase available FTE staffing level, which is reflected in the increased FTE staffing cost in the table. Understaffing penalty cost is kept between 300.00 and 1,125.75 using the staff size flexibility. The results suggest why a Dynamic Staffing policy would help reduce staffing costs in a medical unit while balancing for understaffing risks. Dynamic Staffing policy follows a similar path to the Optimal Staffing. FTE staff size is increased to alleviate the increased understaffing penalty cost. For UPC = 1.5, 2.0 and 3.0 Dynamic Staffing provide cost savings compared to both alternatives of the Fixed Staffing model. As the UPC is increased to 10, Fixed Staffing with 20 nurses become less costly due to the increased staff size. We can conclude, Dynamic Staffing policy provide the required staff size flexibility in the medical units to reduce staffing costs while balancing understaffing risks.

Design Factors		Cost Elements				
Staffing Policy	**C_u^p**	**Optimality Gap %**	**Objective Value**	**FTE Staffing Cost**	**PRN Staffing Cost**	**Understaffing Penalty**
Optimal Staffing	1.5	0.13%	19,338.15	14,688.00	3,524.40	1,125.75
Optimal Staffing	2.0	0.00%	19,574.40	15,240.00	3,502.40	832.00
Optimal Staffing	3.0	0.00%	19,810.70	16,272.00	3,159.20	379.50
Optimal Staffing	10.0	0.00%	20,164.80	16,776.00	3,088.80	300.00
Dynamic Staffing	1.5	0.29%	20,137.00	16,272.00	2,926.00	939.00
Dynamic Staffing	2.0	0.33%	20,139.00	16,272.00	3,036.00	831.00
Dynamic Staffing	3.0	0.18%	20,903.00	16,440.00	3,212.00	1,251.00
Dynamic Staffing	10.0	0.01%	23,310.00	16,344.00	3,476.00	3,490.00
Fixed Staffing - 19 nurse	1.5	0.10%	20,288.50	16,008.00	3,454.00	826.50
Fixed Staffing - 19 nurse	2.0	0.10%	20,564.00	16,008.00	3,454.00	1,102.00
Fixed Staffing - 19 nurse	3.0	0.10%	21,115.00	16,008.00	3,454.00	1,653.00
Fixed Staffing - 19 nurse	10.0	0.10%	24,972.00	16,008.00	3,454.00	5,510.00
Fixed Staffing - 20 nurse	1.5	0.01%	20,781.75	16,776.00	3,696.00	309.75
Fixed Staffing - 20 nurse	2.0	0.01%	20,885.00	16,776.00	3,696.00	413.00
Fixed Staffing - 20 nurse	3.0	0.01%	21,091.50	16,776.00	3,696.00	619.50
Fixed Staffing - 20 nurse	10.0	0.01%	22,537.00	16,776.00	3,696.00	2,065.00

Table 4.19: Cost Elements, Ward A

Table 4.20 presents the cost elements that contribute to the objective value for Ward B. Under Optimal Staffing, increased UPC triggers additional FTE nurse assignments, reflected in the increased FTE staffing cost levels. PRN staff size is kept stable under the Optimal Policy. Dynamic Staffing policy provides cost savings for all UPC levels for this medical unit, compared to both Fixed Staffing alternatives. Under the Dynamic Policy, FTE and PRN staffing costs are gradually elevated as the UPC is increased from the base level of 1.5 to 10.0. All experiments are solved to less than 0.1% optimality gap for the medical unit, reflecting the capacity cushion the unit has in terms of the available nurse pool. Differences in staffing policy objective values are mostly determined by the level of understaffing penalty.

Design Factors		Cost Elements				
Staffing Policy	$\mathbf{C_u^p}$	**Optimality Gap %**	**Objective Value**	**FTE Staffing Cost**	**PRN Staffing Cost**	**Understaffing Penalty**
Optimal Staffing	1.5	0.09%	13,845.70	10,416.00	1,817.20	1,612.50
Optimal Staffing	2.0	0.01%	14,219.80	11,424.00	1,702.80	1,093.00
Optimal Staffing	3.0	0.05%	14,658.40	12,072.00	1,632.40	954.00
Optimal Staffing	10.0	0.09%	16,686.20	12,144.00	1,707.20	2,835.00
Dynamic Staffing	1.5	0.03%	14,363.85	10,128.00	1,821.60	2,414.25
Dynamic Staffing	2.0	0.03%	15,120.20	10,200.00	1,883.20	3,037.00
Dynamic Staffing	3.0	0.05%	16,512.10	10,200.00	2,041.60	4,270.50
Dynamic Staffing	10.0	0.04%	24,808.00	10,488.00	2,090.00	12,230.00
Fixed Staffing - 11 nurse	1.5	0.00%	14,482.80	9,480.00	1,768.80	3,234.00
Fixed Staffing - 11 nurse	2.0	0.00%	15,560.80	9,480.00	1,768.80	4,312.00
Fixed Staffing - 11 nurse	3.0	0.00%	17,716.80	9,480.00	1,768.80	6,468.00
Fixed Staffing - 11 nurse	10.0	0.00%	32,808.80	9,480.00	1,768.80	21,560.00
Fixed Staffing - 12 nurse	1.5	0.00%	14,482.35	10,200.00	2,085.60	2,196.75
Fixed Staffing - 12 nurse	2.0	0.00%	15,214.60	10,200.00	2,085.60	2,929.00
Fixed Staffing - 12 nurse	3.0	0.00%	16,679.10	10,200.00	2,085.60	4,393.50
Fixed Staffing - 12 nurse	10.0	0.00%	26,930.60	10,200.00	2,085.60	14,645.00

Table 4.20: Cost Elements, Ward B

Table 4.21 presents the cost elements that contribute to the objective value for Ward C. Similar to Wards A and B, Dynamic Staffing policy provides cost savings for all UPC levels for this medical unit or demonstrates similar levels, compared to both Fixed Staffing alternatives. Again, similar to Ward B, under Optimal Staffing, increased UPC triggers additional FTE nurse assignments, reflected in the increased FTE staffing cost levels. PRN staff size is gradually reduced under the Optimal Policy, in order to achieve some gains from the 10% difference in staffing costs. Under Dynamic Staffing policy, increased UPC triggers additional FTE nurse assignments, reflected in the increased FTE staffing cost levels. PRN staff size is kept stable under the Dynamic Staffing policy. Understaffing penalty is kept below 1,800 level for UPC = 1.5, 2.0 and 3.0 for the Dynamic Staffing. For UPC = 10.0, Dynamic Staffing provide significant cost savings compared to both Fixed Staffing alternatives.

Design Factors		Cost Elements				
Staffing Policy	$\mathbf{C_u^p}$	**Optimality Gap %**	**Objective Value**	**FTE Staffing Cost**	**PRN Staffing Cost**	**Understaffing Penalty**
Optimal Staffing	1.5	0.02%	12,429.70	9,456.00	1,289.20	1,684.50
Optimal Staffing	2.0	0.02%	12,900.40	10,032.00	1,214.40	1,654.00
Optimal Staffing	3.0	0.01%	13,543.80	10,896.00	1,174.80	1,473.00
Optimal Staffing	10.0	0.11%	14,853.80	12,912.00	866.80	1,075.00
Dynamic Staffing	1.5	0.26%	13,103.35	10,200.00	1,447.60	1,455.75
Dynamic Staffing	2.0	0.35%	13,147.05	10,464.00	1,438.80	1,244.25
Dynamic Staffing	3.0	2.02%	14,319.60	11,112.00	1,425.60	1,782.00
Dynamic Staffing	10.0	0.19%	17,262.20	12,336.00	1,421.20	3,505.00
Fixed Staffing - 12 nurse	1.5	33.30%	13,097.85	10,368.00	1,425.60	1,304.25
Fixed Staffing - 12 nurse	2.0	33.30%	13,532.60	10,368.00	1,425.60	1,739.00
Fixed Staffing - 12 nurse	3.0	33.30%	14,402.10	10,368.00	1,425.60	2,608.50
Fixed Staffing - 12 nurse	10.0	33.30%	20,488.60	10,368.00	1,425.60	8,695.00
Fixed Staffing - 13 nurse	1.5	25.00%	13,488.60	11,232.00	1,425.60	831.00
Fixed Staffing - 13 nurse	2.0	25.00%	13,765.60	11,232.00	1,425.60	1,108.00
Fixed Staffing - 13 nurse	3.0	25.00%	14,319.60	11,232.00	1,425.60	1,662.00
Fixed Staffing - 13 nurse	10.0	25.00%	18,197.60	11,232.00	1,425.60	5,540.00

Table 4.21: Cost Elements, Ward C

4.10 Conclusions

This chapter studied medium-term integrated nurse staffing and scheduling in Intensive Care Units, a 7-day x 24-hour care environment facing unscheduled patient admissions with dynamic acuity levels. We use a two-phase procedure to determine optimal nurse assignments. In Phase 1 we generate feasible FTE nurse schedules for the staffing horizon of six weeks, while satisfying the constraints imposed by the nurse profile. In Phase 2 we assign FTE nurses to pre-generated feasible nurse schedules, and PRN nurses to the nursing shifts, using mixed-integer optimization models. Pre-generated schedules eliminate the increased number of constraints and reduces the number of decision variables of the integrated nurse staffing and scheduling model. The optimization model we developed recommends initial staffing plans and schedules for a six-week staffing horizon for the medical units, given the variety of nurse groups and nursing shift assignment types. Our solution aims to reduce nurse staffing costs while balancing the under- and over-staffing risks. This helps mitigate nurse burn-out, improve patient outcomes, and manage hospital staffing costs. We also develop an optimization model to generate Fixed Staffing policy schedules for nurses that will help increase the scheduling efficiency of the hospital administration.

Target staffing levels for nursing shifts are typically determined by a retrospective average staffing level for the nursing care needs in medium-term scheduling. Using the mixed-integer optimization model in this chapter, we examined fixed vs. dynamic medium-term nurse staffing and scheduling policy options for the medical units. Under the Fixed Staffing policy, the medical unit is targeted to be staffed a fixed number of nurses throughout the staffing horizon. We propose a Dynamic Staffing policy, which uses historical patient demand to suggest a non-stationary staffing scheme for the staffing horizon. We test the Fixed Staffing policy with various staffing level options. For the Dynamic Staffing alternative, we prepare a "heat map" of patient census and acuity, as well as admission, discharge and transfer (ADT) activity, using Pediatric Intensive Care Units as an example. We compare the performance of the dynamic heat map-based policy vs. the alternative fixed staffing policies. In order to develop the heat map we estimate a monthly seasonality index for patient census, patient acuity, and ADT Activity. Then we estimate patient census, patient acuity, and ADT activity averages for all Day of Week and Shift of the Day combinations. The desired heat map of patient demand is generated by multiplying the monthly seasonality factors with the historical "Day-Shift" averages for the medical units. Using the heat map and the mixed-integer optimization models, we analyze whether proposed Dynamic Staffing policies outperform the currently-used Fixed Staffing policy. We also compare the performance of Dynamic and Fixed Staffing policy options with the Optimal Staffing scheme reached by the actual patient data. We introduced the concept of "understaffing penalty" as a mechanism to control the understaffing in the medical units. We analyze how various levels of understaffing penalty cost affect staffing costs and understaffing levels in the medical unit. We also evaluate, the impact of number of available schedules (NAS) per FTE nurse profile on the objective function costs and understaffing ratios in the medical units.

Our results suggest that the total objective function cost for the optimization experiments is either slightly reduced or kept stable as we increase the number of available schedules for each nurse profile from 4 to 256. We conclude that feeding the alternative staffing models with 256 schedules per FTE profile (7,680 total different schedules for the optimization model) is large enough to provide schedule diversity. Even the suggested, maximally different four schedules per nurse profile approach seems to be providing efficient solutions. Further increases in the NAS, above 256 schedules per nurse profile, would not bring any cost savings but would increase the problem complexity, hurting the solution performance of the developed models.

Regarding the staffing policy evaluation, our results for the experiments we conducted with the three PICU wards suggest that the performance of Dynamic Staffing policy was mostly superior to the Fixed Staffing alternatives (or similar for a few problem instances) in terms of understaffing percentages and total costs. The power of Dynamic Staffing policy lies in the accuracy of the forecasted heat map. As the forecasting performance using the specific heat map is improved, more cost savings and alleviated understaffing percentages will be observed. Regarding the Fixed Staffing policy, we must first note that perfectly stable staff size may not be feasible in many cases, especially with a limited number of PRN nurse body. In addition, our proposed Fixed Staffing modeling approach provides a reliable and efficient way of scheduling the nursing workforce. Medical units with higher variations in patient demand levels would benefit the most by using the Dynamic Staffing policy proposed in this study.

The results of experiments using the Dynamic Staffing policy suggest that understaffing penalty cost (UPC) be used as a reliable mechanism for controlling understaffing ratios in medical units. Depending on how much understaffing can be tolerated, the nursing administration can determine the appropriate UPC level. For unexpectedly-high patient demand periods, higher understaffing levels might be observed because the Dynamic Staffing policy is based on historical patient demand-based heat maps. Heat maps can be dynamically adjusted as new demand patterns are observed for new staffing horizons. As the accuracy of patient demand forecasts improves, the better performance outcomes will be achieved using Dynamic Staffing policy. Historically-used Fixed Staffing policies do not provide the required staff size flexibility to alleviate understaffing in the medical units, triggering nurse burnout. Because of the static nature of staff size, Fixed Staffing policies would also increase the need for short-term schedule adjustment costs to more closely match patient demand.

All PRN nurses were assigned to work in all experiments, demonstrating that PRN nurses are critical for cost savings because of the flexibility they provide for minimizing under- and over-staffing in the nursing shifts. We conclude that having a sizable PRN nurse group will alleviate nursing shortages and provide the flexibility required for the nursing administration in the medical units. Nursing administration can use the results of medium-term staffing experiments for long-term planning to determine whether the current nurse pool is large enough to satisfy patient demand. Our analysis of objective function cost elements also suggests that the Dynamic Staffing policy provides the staff size flexibility required in the medical units to reduce staffing costs while balancing understaffing risks.

Chapter 5

A Two-Stage Stochastic Mixed-Integer Programming Approach for Short-Term Nurse Schedule Adjustments

5.1 Introduction

Nurse schedules are constructed well before the actual patient demand for nursing is observed. In intensive care environments, 30 to 70% of patient admissions are not known 12 hours before the actual admission time and patient acuity is diverse. Because patient demand fluctuates, nursing administration must constantly adjust existing nurse schedules in the short-term. In Chapter 4 we developed alternative medium-term integrated staffing and scheduling policies. Our results suggest that matching patient demand with medium-term planning in a dynamic intensive care environment is not an easy task. Hospitals use short-term schedule and staff allocation adjustments to better match patient demand for nursing.

Kim and Mehrotra (2015) also studied the short-term nurse schedule adjustments problem using two-stage stochastic programming model with mixed-integer recourse. The first-stage "here-and-now" decision is to find medium-term initial staffing levels and schedules for a 12-week staffing horizon. The authors consider an 18-week planning horizon and assume that the scheduling patterns repeat from week to week during the 12-week staffing horizon. The staffing and scheduling decisions are made 6 weeks in advance of this 12-week horizon. The second- stage "wait-and-see" decision is to adjust these schedules at a time closer to the actual date of demand, at the beginning of each week. Weekly scheduling patterns and eight adjustment patterns are generated by using a recursive procedure. In many healthcare settings, being 7 to 14 days away from the actual demand realization doesn't provide a close enough time window for an accurate demand estimate. Especially, in an intensive care setting, where over 30% to 70% of patients are categorized as "unscheduled admissions" (i.e. unexpected 12-hours prior), patient demand forecasts for short-term nurse schedules a week in advance are not reliable.

As an alternative solution approach to the problem, we study the medium-term integrated nurse scheduling and staffing as a separate problem, as presented in Chapter 4; then in the short-term we make adjustment decisions for the upcoming 4-hour nursing shift when we are 4 to 8 hours away from actual patient demand realizations. As described in Chapters 3 and 4, the PICU we study uses a fixed staffing level for medium-term staffing and scheduling. Our approach is to make short-term adjustments every four hours for the upcoming nursing shift. The short-term schedule adjustment tool currently in use considers only the scheduled patient admissions. This does not exploit the historical forecasts of unscheduled patient admissions. To the best of our knowledge, this is the first study to apply a two-stage stochastic programming approach to short-term schedule adjustments, where the adjustments are conducted for the upcoming 4-hour nursing shift. Chapter 5 now extends the work on medium-term nurse staffing and scheduling to address short-term adjustments. We conduct two-stage short-term staffing adjustments to schedules developed at the medium-term planning phase. After observing actual patient demand for nursing at the start of the next shift, we consider final staffing adjustments for nurse requirements.

At the start of any current shift, we assume the following patient information is available to the nursing administration: (1) current patient census (total patients staying in the unit); (2) acuity assignments of the existing patients (total patients in each acuity group); (3) the number of scheduled and unscheduled patient admissions for the current nursing shift; (4) the number of patient discharges and their associated acuity groups during the current shift and in the upcoming shift; and (5) total scheduled patient admissions for the upcoming shift. However, the following information is not known by the administration at the start of the current shift: (1) the acuity assignments of scheduled and unscheduled patients for the current shift (i.e., we know the number of scheduled and unscheduled patients to be admitted to the unit for the current shift, but we don't know their acuity assignments since the patients have not arrived yet), and (2) the number of unscheduled patient admissions for the upcoming shift.

We develop a two-stage stochastic integer programming model that minimizes the total nurse staffing costs and the cost of adjustments to the original medium-term schedules while ensuring coverage of nursing demand. A stochastic integer programming model is attractive because the number of the unscheduled patient admissions and acuity assignments in the upcoming shift are unknown at the time of adjustments. The unscheduled patient admission and acuity distributions analyzed in Chapter 3 are used to determine the expected nursing requirement. This value is compared to the provided nursing hours after conducting the short-term schedule adjustments. We model the current 4-hour nursing shift as the first stage of adjustments, when the actual patient demand is not revealed. We model the upcoming nursing shift as the second stage of adjustments, when the actual patient demand is known. In the second stage, we make corrective actions (i.e., requesting mandatory nurse overtime) to cover the required patient demand. Using the two-stage stochastic short-term staffing adjustment model, we study our third research question:

RQ 3: *Can short-term schedule modifications that are based upon decisions attained from two-stage stochastic integer programming model lower cost and reduce understaffing levels, compared to original medium-term staffing plans?*

There are several available first-stage adjustment options available to hospital nursing administration for short-term adjustments. These include: (1) requesting nurses from the general float pool of the hospital, (2) using on-call nurses from FTE overtime and (3) requesting additional on-call PRN nurses. When the scheduled nursing hours are expected to exceed the hours demanded by the existing patient levels in the upcoming shift, the charge nurse can: (1) float some of the nurses to other units, (2) reassign some nurses to a later day in the same staffing horizon or (3) cancel the shift and use one of the following designations for the time off: vacation, personal day, holiday, or unpaid leave (Bard and Purnomo, 2005a). We combine the three alternative adjustment types into one adjustment alternative available separately to the FTE and PRN nurses. So, the charge nurse can either (1) float, reassign or cancel some FTE nurses for the upcoming shift or (2) float, reassign or cancel some PRN nurse for the upcoming shift, as two separate available options. Each adjustment option has a different cost implication, which we discuss later in this chapter.

After observing actual patient demand at the beginning of the nursing shift (i.e. stage two), the charge nurse can request that a nurse on the previous shift, who is not originally scheduled for the current shift, stay for the current shift as a mandatory overtime, in the case of observing a nursing shortage. We leave the option of excessive staffing open in the second-stage of the problem, since cancelling shifts at this stage will not yield cost savings to the hospital. The mandatory overtime adjustment option is implemented as a second-stage corrective action after the observance of actual patient demand for nursing. Thus, we model six different adjustment options for the two-stage stochastic programming model, five options available as first-stage decisions and one option available as the second-stage decision. We note that, the model ensures sufficient coverage of patient demand for nursing hours after the two-stage adjustments are complete. It is assumed that mandatory overtime hours are unlimited for ensuring the coverage of patient demand. Table 5.1 below presents the available adjustment options available in the first and second stages of the model.

First-Stage Adjustments - Increase the Staffing Level:
(1) Request nurses from the general float pool of the hospital
(2) Request on-call nurses from FTE pool as overtime
(3) Request additional on-call PRN nurses
First-Stage Adjustments - Reduce the Staffing Level:
(4) Float, reassign or cancel some FTE nurses
(5) Float, reassign or cancel some PRN nurses
Second-Stage Adjustment - Increase the Staffing Level:
(6) Request mandatory overtime from existing nurses in previous shift

Table 5.1: Available Short-Term Schedule Adjustment Options

One important aspect of the short-term nurse schedule modification is the need for a very efficient solution algorithm. Practically, the charge nurse will run the solution algorithm 21 - 42 times/week, at the beginning of each 4 to 8-hour shift depending on nursing shift structure, and expect to have a solution in less than 10 minutes. The decision variables introduced here apply to both stages and relate to the number of adjustment actions taken for each available adjustment type (i.e. number of cancelled shifts, number of nurses requested from the float pool etc.). We ensure that the nursing constraints implied by the nurse profile and employment type are maintained by the stochastic integer programming model. In addition to the patient information listed above, the two-stage stochastic integer programming model takes as an input: (1) number of FTE and PRN nurses scheduled for the current and upcoming shift, (2) number of available float pool and on-call nurses in each shift, (3) nurse profiles and schedule of the nurses for the previous and upcoming three shifts (for potential overtime requests).

5.2 Model Description

In this section, we provide a description of the two-stage stochastic programming model used in this study. A detailed description of the decision variables, parameters, objective function and constraints are provided. At the end of the section, the full representation of the mathematical model is presented.

5.2.1 Sets, Parameters, Probability Spaces and Random Variables

J : Set of alternative FTE nurse job profiles for the medical unit; (i.e. $J = \{1, 2, 3, ..., 30\}$)

S_j : Set of all available schedules for nurses from job profile j

P: Set of all PRN nurses.

We assume PRN nurses $\{1...PT_1\}$ are PRN Tier-1 nurses, nurses $\{(PT_1{+}1)...(PT_1{+}PT_2)\}$ are PRN Tier-2 nurses, nurses $\{(PT_1{+}PT_2{+}1)...(PT_1{+}PT_2{+}PT_3)\}$ are PRN Tier-3 nurses.

T : Set of four-hour nursing shifts during the scheduling period of six week $T = \{0, 1, 2, 3, ..., 251\}$ (i.e. 42 shifts a week, six weeks in a schedule; 252 four-hour shifts in total).

i.e. A typical week starts with the nursing shift $l = 1$, which is a Monday D1 shift and ends with shift $l =$ 42, which is a Monday N2 shift.

$w \in \{1, 2, ..., 6\}$, is the index of weeks during the staffing horizon and T_w is the subset of shifts during week w.

G : Set of patient acuity categories $G = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$

i.e. For $g \in G$ acuity category g=1 indicates that patient belongs to the acuity designation A in hospital terminology, similarly g=2 indicates acuity group B, g=3 indicates acuity group C, g=4 indicates acuity group D, g=5 indicates acuity group E, g=6 indicates acuity group F.

Parameters, Probability Spaces and Random Variables

$\vartheta_{g,t}$: the vector keeping the number of patients in acuity group $g \in G$ at the unit for shift $t \in T$.

h_g: nursing hours required for patient care for acuity group g in a four-hour nursing shift (i.e. $h =$ [0.5, 1, 1.5, 2.5, 4, 8] ; a patient with acuity F, g=6, will require eight hours of nursing care in a four-hour shift).

α_t^S: number of scheduled patient admission and transfer-in activities to a unit in shift t

$\alpha^S_{g,t}$: number of scheduled patient admission and transfer-in activities from acuity group g to a unit in shift t

$\alpha^U_{g,t}$: number of unscheduled patient admission and transfer-in activities from acuity group g to a unit in shift t

$\beta^S_{g,t}$: number of scheduled patient discharges and transfer-out activities from acuity group g from a unit in shift t

c_j : staffing cost per four-hour shift for the FTE nurses from job profile j

b_p : staffing cost per four-hour shift for PRN nurse $p \in P$

x_s: number of FTE nurses that are assigned to work for schedule $s \in S_j$; $x_s \in \mathbb{Z}$. Note that x_s is fed into this model as a parameter from the medium-term staffing decisions.

$y_{p,t}$: 1 if PRN nurse $p \in P$ is assigned to work for shift $t \in T$; 0 otherwise. $y_{p,t}$ is also fed into this model as a parameter from the medium-term staffing decisions.

$a_{s,t}$: 1 if for schedule $s \in S_j$ can be assigned to work at shift t ; 0 otherwise.

Unit cost of each schedule modification type:

Cost parameters related to short-term schedule modifications available to the nursing administration in the first-stage:

Cost parameters related to generating extra nursing hours for the upcoming shift:
c^+_h: cost of additional nurses requested from the general float pool of the hospital for one shift
c^+_f: cost of additional FTE nurses requested from available on-call list for one shift
c^+_p: cost of additional PRN nurses requested from available on-call list for one shift

Cost parameters related to eliminating excess nursing hours available to the nursing administration:
c^-_f: savings incurred by floating, reassigning or cancelling one FTE nurse for the upcoming shift
c^-_p: savings incurred by floating, reassigning or cancelling one PRN nurse for the upcoming shift

Cost parameters related to the second-stage decisions:
q^+_m: cost of mandatory overtime for nurses on the current shift to stay for the next shift who were not originally scheduled for the next shift.

Parameters defining the upper bound for total number of adjustments:

$n^+_{h,(t+1)}$: total number of available nurses in the general float pool of the hospital that can be assigned to work for the medical unit for shift $(t + 1)$, requested at shift t

$n^{+}_{f,(t+1)}$: maximum number of additional FTE nurses that can be requested from available on-call list for shift $(t + 1)$ at shift t

$n^{+}_{p,(t+1)}$: maximum number of additional PRN nurses that can be requested from available on-call list for shift $(t + 1)$ at shift t

Probability Spaces and Random Variables

Uncertainty is represented in terms of random experiments with outcomes denoted by 'ω' (i.e. state of the world).

The set of all outcomes is represented by 'Ω': $\omega \in \Omega$.

As usual, the particular values the various random variables will take are only known after the random experiment, i.e. , the vector $\xi = \xi(\omega)$ is only known after the experiments.

Here in our case, the random vector $\xi(\omega)$ has 13 elements, six from number of scheduled patient admissions in six acuity groups for the current shift; another six from the number of unscheduled patient admissions in six acuity groups in the current shift; and an additional 13th element for the number of unscheduled patient admissions for the upcoming shift.

For a given realization ω, the second-stage problem data become known and combining the stochastic components of the second-stage data, we obtain the vector $\xi(\omega)$. The random event ω influences all components of $\xi(\omega)$.

In this study, second-stage decisions are represented by $y(\omega)$ in order to stress that these decisions differ as functions of the outcome of the random experiment, and of course first-stage decisions.

5.2.2 Decision Variables

First-stage decision variables for generating additional nursing hours:

$x^{+}_{h,(t+1)}$: number of additional nurses requested from the general float pool of the hospital for shift $(t + 1)$ at shift t

$x^{+}_{f,(t+1)}$: number of additional FTE nurses requested from available on-call list for shift $(t + 1)$ at shift t

$x^{+}_{p,(t+1)}$: number of additional PRN nurses requested from available on-call list for shift $(t + 1)$ at shift t

First-stage decision variables for eliminating excess nursing hours:

$x^{-}_{f,(t+1)}$: number of FTE nurses floated to another unit, reassigned to a later day or cancelled for the shift $(t+1)$ at shift t

$x^{-}_{p,(t+1)}$: number of PRN nurses floated to another unit, reassigned to a later day or cancelled for the shift $(t+1)$ at shift t

Second-stage decision variables for generating additional nursing hours: $y^{+}_{m,(t+1)}(\omega)$: number of nurses from shift t, who are not originally scheduled for the shift $(t+1)$, that stay for shift $(t+1)$ as a mandatory overtime.

5.2.3 Computation of Understaffing Penalty Cost

We assume the patient demand is realized by the total of nursing demand generated by patient mix present at the unit right at the start time point of the shift and the ADT activity that occurs during that shift.

We also assume that, by definition, the total number of scheduled patient admissions are know to the unit charge nurses within a 12-hour window, but the acuity assignments of those patients are not clear, since they didn't arrive at the unit yet.

Expected number of patients in acuity group g in a unit at the upcoming shift (i.e. shift (t+1)):

$$\mathbf{E}[\vartheta_{g,(t+1)}] = \vartheta_{g,t} + \mathbf{E}[\alpha^S_{g,t}] + \mathbf{E}[\alpha^U_{g,t}] - \beta^S_{g,t}$$

Expected nursing requirement during the upcoming shift:

$$\left[\gamma \cdot (\alpha^S_{t+1} + \sum_{g\in G} \mathbf{E}[\alpha^U_{g,(t+1)}]) + \delta \cdot \beta^S_{t+1} + \sum_{g\in G} \mathbf{E}[\vartheta_{g,(t+1)}] \cdot h_g\right]$$

, plugging in the expression for $\mathbf{E}[\vartheta_{g,(t+1)}]$ from the equation above we get:

$$\left[\gamma \cdot (\alpha^S_{t+1} + \sum_{g\in G} \mathbf{E}[\alpha^U_{g,(t+1)}]) + \delta \cdot \beta^S_{t+1} + \sum_{g\in G} (\vartheta_{g,t} + \mathbf{E}[\alpha^S_{g,t}] + \mathbf{E}[\alpha^U_{g,t}] - \beta^S_{g,t}) \cdot h_g\right]$$

Total supply of nursing hours for the upcoming shift $(t+1)$ after the adjustments will be then:

$$\left[\Big(\sum_{j\in J}\sum_{s\in S_j} x_s \cdot a_{s,(t+1)} + \sum_{p\in P} y_{p,(t+1)}\Big) + \Big(x^{+}_{h,(t+1)} + x^{+}_{f,(t+1)} + x^{+}_{p,(t+1)}\Big) - \Big(x^{-}_{f,(t+1)} + x^{-}_{p,(t+1)}\Big) + \Big(y^{+}_{m,(t+1)}(\omega)\Big)\right] \cdot 4$$

, since every nurse is scheduled for the 4-hour block shift.

We can calculate the total costs incurred by the first-stage adjustments for shift $(t+1)$ at shift t as follows:

$$\left[\left(c_h^+ \cdot x_{h,(t+1)}^+\right) + \left(c_f^+ \cdot x_{f,(t+1)}^+\right) + \left(c_p^+ \cdot x_{p,(t+1)}^+\right)\right]$$

We can calculate the total savings achieved by the first-stage adjustments for shift $(t+1)$ at shift t as follows:

$$\left[\left(c_f^- \cdot x_{f,(t+1)}^-\right) + \left(c_p^- \cdot x_{p,(t+1)}^-\right)\right]$$

We can also calculate the expected total costs incurred by the second-stage adjustments for shift $(t+1)$ at shift $(t+1)$ as follows:

$$\mathbf{E}_\xi\left[\left(q_m^+ \cdot y_{m,(t+1)}^+(\omega)\right)\right]$$

Then total costs of adjustments in the first and second stages can be calculates as follows:

$$\left[\left(c_h^+ \cdot x_{h,(t+1)}^+\right) + \left(c_f^+ \cdot x_{f,(t+1)}^+\right) + \left(c_p^+ \cdot x_{p,(t+1)}^+\right)\right] - \left[\left(c_f^- \cdot x_{f,(t+1)}^-\right) + \left(c_p^- \cdot x_{p,(t+1)}^-\right)\right] + \mathbf{E}_\xi\left[\left(q_m^+ \cdot y_{m,(t+1)}^+(\omega)\right)\right]$$

5.2.4 Objective Function & Model Constraints

Our objective is then minimizing the total costs of nurse staffing and schedule adjustments (both from the first and second stages), while satisfying the nursing demand coverage at the second stage of the model when the actual patient demand is realized:

Minimize:

$$\left[\left(\sum_{j\in J}\sum_{s\in S_j} c_j \cdot x_s \cdot a_{s,(t+1)} + \sum_{p\in P} b_p \cdot y_{p,(t+1)}\right)\right] + \left[\left(c_h^+ \cdot x_{h,(t+1)}^+\right) + \left(c_f^+ \cdot x_{f,(t+1)}^+\right) + \left(c_p^+ \cdot x_{p,(t+1)}^+\right)\right]$$

$$- \left[\left(c_f^- \cdot x_{f,(t+1)}^-\right) + \left(c_p^- \cdot x_{p,(t+1)}^-\right)\right] + \mathbf{E}_\xi\left[min\left(q_m^+ \cdot y_{m,(t+1)}^+(\omega)\right)\right]$$

Note that the objective function contains several deterministic terms and the expectation of the second-stage objective $\left(q_m^+ \cdot y_{m,(t+1)}^+(\omega)\right)$ taken over all realizations of the random event ω.

x_s and $y_{p,t}$, FTE and PRN nurse schedules assignments, are fed into this model as a parameter from the medium-term staffing decisions. The feasible schedule sets, i.e. $a_{s,(t+1)}$, are also input model parameters.

Model Constraints

Our first set of constraints are related to the limits on number of adjustments.

- Total number of nurses that can be requested at shift t, to work in shift $t + 1$, from the general float pool of the hospital is limited by $n^{+}_{h,(t+1)}$:

$$x^{+}_{h,(t+1)} \leqslant n^{+}_{h,(t+1)}$$

- Total number of FTE nurses that can be requested at shift t, to work in shift $t + 1$, from the on-call list of the unit is limited by $n^{+}_{f,(t+1)}$:

$$x^{+}_{f,(t+1)} \leqslant n^{+}_{f,(t+1)}$$

- Total number of PRN nurses that can be requested at shift t, to work in shift $t + 1$, from the on-call list of the unit is limited by $n^{+}_{p,(t+1)}$:

$$x^{+}_{p,(t+1)} \leqslant n^{+}_{p,(t+1)}$$

- Total number of FTE nurses floated to another unit, reassigned to a later day or cancelled for the shift $(t + 1)$ at shift t is limited by the medium-term total FTE nurse assignments for shift $(t + 1)$:

$$x^{-}_{f,(t+1)} \leqslant \sum_{j \in J} \sum_{s \in S_j} x_s \cdot a_{s,(t+1)}$$

- Total number of PRN nurses floated to another unit, reassigned to a later day or cancelled for the shift $(t + 1)$ at shift t is limited by the medium-term total PRN nurse assignments for shift $(t + 1)$:

$$x^{-}_{p,(t+1)} \leqslant \sum_{p \in P} y_{p,(t+1)}$$

- Second-stage sufficient coverage constraint: As a constraint at the second stage of the model, we require that the nursing hours supply after second stage adjustments will be large enough to cover the nursing requirement realized after observing the actual patient demand:

Total supply of nursing hours for the upcoming shift $(t + 1)$ after the adjustments:

$$\left[\left(\sum_{j\in J}\sum_{s\in S_j} x_s \cdot a_{s,(t+1)} + \sum_{p\in P} y_{p,(t+1)}\right) + \left(x^+_{h,(t+1)} + x^+_{f,(t+1)} + x^+_{p,(t+1)}\right) - \left(x^-_{f,(t+1)} + x^-_{p,(t+1)}\right) + \left(y^+_{m,(t+1)}(\omega)\right)\right] \cdot 4$$

, since every nurse is scheduled for the 4-hour block shifts.

Expected nursing requirement during the upcoming shift $(t + 1)$:

$$\left[\gamma \cdot (\alpha^S_{t+1} + \sum_{g\in G} \mathbf{E}[\alpha^U_{g,(t+1)}]) + \delta \cdot \beta^S_{t+1} + \sum_{g\in G} (\vartheta_{g,t} + \mathbf{E}[\alpha^S_{g,t}] + \mathbf{E}[\alpha^U_{g,t}] - \beta^S_{g,t}) \cdot h_g\right]$$

Then our constraint is listed as:

$$\left[\left(\sum_{j\in J}\sum_{s\in S_j} x_s \cdot a_{s,(t+1)} + \sum_{p\in P} y_{p,(t+1)}\right) + \left(x^+_{h,(t+1)} + x^+_{f,(t+1)} + x^+_{p,(t+1)}\right) - \left(x^-_{f,(t+1)} + x^-_{p,(t+1)}\right) + \left(y^+_{m,(t+1)}(\omega)\right)\right] \cdot 4$$

$$\geqslant \left[\gamma \cdot (\alpha^S_{t+1} + \sum_{g\in G} \mathbf{E}[\alpha^U_{g,(t+1)}]) + \delta \cdot \beta^S_{t+1} + \sum_{g\in G} (\vartheta_{g,t} + \mathbf{E}[\alpha^S_{g,t}] + \mathbf{E}[\alpha^U_{g,t}] - \beta^S_{g,t}) \cdot h_g\right]$$

- Total number of second-stage mandatory overtime adjustments at shift $(t + 1)$ is limited by total nurse assignments from previous stage, at shift t:

$$y^+_{m,(t+1)}(\omega) \leqslant \sum_{j\in J}\sum_{s\in S_j} x_s \cdot a_{s,(t+1)} + \sum_{p\in P} y_{p,(t+1)}$$

5.2.5 Two-Stage Stochastic Programming Model

Merging the individual items presented in the previous subsection, we can present the full two-stage stochastic programming model as follows:

Minimize:

$$\left[\Big(\sum_{j\in J}\sum_{s\in S_j} c_j \cdot x_s \cdot a_{s,(t+1)} + \sum_{p\in P} b_p \cdot y_{p,(t+1)}\Big)\right] + \left[\Big(c_h^+ \cdot x_{h,(t+1)}^+\Big) + \Big(c_f^+ \cdot x_{f,(t+1)}^+\Big) + \Big(c_p^+ \cdot x_{p,(t+1)}^+\Big)\right]$$

$$-\left[\Big(c_f^- \cdot x_{f,(t+1)}^-\Big) + \Big(c_p^- \cdot x_{p,(t+1)}^-\Big)\right] + \mathbf{E}_\xi\left[min\Big(q_m^+ \cdot y_{m,(t+1)}^+(\omega)\Big)\right]$$

subject to

$$x_{h,(t+1)}^+ \leqslant n_{h,(t+1)}^+$$

$$x_{f,(t+1)}^+ \leqslant n_{f,(t+1)}^+$$

$$x_{p,(t+1)}^+ \leqslant n_{p,(t+1)}^+$$

$$x_{f,(t+1)}^- \leqslant \sum_{j\in J}\sum_{s\in S_j} x_s \cdot a_{s,(t+1)}$$

$$x_{p,(t+1)}^- \leqslant \sum_{p\in P} y_{p,(t+1)}$$

$$\left[\Big(\sum_{j\in J}\sum_{s\in S_j} x_s \cdot a_{s,(t+1)} + \sum_{p\in P} y_{p,(t+1)}\Big) + \Big(x_{h,(t+1)}^+ + x_{f,(t+1)}^+ + x_{p,(t+1)}^+\Big) - \Big(x_{f,(t+1)}^- + x_{p,(t+1)}^-\Big) + \Big(y_{m,(t+1)}^+(\omega)\Big)\right] \cdot 4$$

$$\geqslant \left[\gamma \cdot (\alpha_{t+1}^S + \sum_{g\in G} \mathbf{E}[\alpha_{g,(t+1)}^U]) + \delta \cdot \beta_{t+1}^S + \sum_{g\in G} (\vartheta_{g,t} + \mathbf{E}[\alpha_{g,t}^S] + \mathbf{E}[\alpha_{g,t}^U] - \beta_{g,t}^S) \cdot h_g\right]$$

$$y_{m,(t+1)}^+(\omega) \leqslant \sum_{j\in J}\sum_{s\in S_j} x_s \cdot a_{s,(t+1)} + \sum_{p\in P} y_{p,(t+1)}$$

$$x_{h,(t+1)}^+, x_{f,(t+1)}^+, x_{p,(t+1)}^+, x_{f,(t+1)}^-, x_{p,(t+1)}^-, y_{m,(t+1)}^+(\omega) \in \mathbb{Z}$$

$$x_{h,(t+1)}^+, x_{f,(t+1)}^+, x_{p,(t+1)}^+, x_{f,(t+1)}^-, x_{p,(t+1)}^-, y_{m,(t+1)}^+(\omega) \geqslant 0$$

Next we discuss the solution algorithm for the two-stage stochastic model presented.

5.3 Solution Algorithm for Two-Stage Stochastic Programming Models with Fixed Recourse: The L-Shaped Method

In this section we present the solution algorithm for two-stage stochastic programs with fixed and finite number of realizations described in Birge & Louveaux (2011). Birge & Louveaux (2011) define "*Stochastic linear programs*" as linear programs in which some problem data may be considered uncertain. "Recourse programs" are those linear programs in which some decisions or recourse actions can be taken after uncertainty is disclosed. In a recourse problem, the decision maker has one question before the uncertainty is revealed and one after it. The decision taken after uncertainty is revealed is the decision maker's recourse. The term "Data uncertainty" implies that some of the problem data can be represented as random variables. As presented in the previous section, let the random vector $\xi(\omega)$ represent the particular values the random variables take, where ω denote the outcomes, realizations, of the random vector ξ. The set of all outcomes is represented by Ω. The random vector $\xi(\omega)$ is revealed, known, only after the random experiment.

The set of decisions is then divided into two main stages. The period before the uncertainty is revealed is called the *first stage*. During this stage decisions must be made before the realizations of the random experiments are referred to as first-stage decisions. The period after the experiment is called the *second stage*, and the decisions that are made during this stage (after realization of the experiments, are referred to as second-stage decisions. First-stage decisions are represented by the vector x, second-stage recourse decisions are represented by the vector y, or $y(\omega)$ or even $y(\omega, x)$, to indicate that second-stage decisions depend on the outcome of the random experiment and the first-stage decisions. Note that these definitions of first and second stages are only related to the realization of random experiments, and each stage may contain sequences of time periods, decisions and events. The sequence of events and decision processes can then be summarized as follows:

$$x \longrightarrow \xi(\omega) \longrightarrow y(\omega, x).$$

Two-stage stochastic linear programs with fixed recourse, that are originated by Dantzig(1955) and Beale(1955), can be represented as follows:

$$
\begin{aligned}
min\ z = c^T \cdot x + E_\xi[min\, q(\omega)^T \cdot y(\omega)] \\
s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad \\
A \cdot x = b\ , \\
T(\omega) \cdot x + W \cdot y(\omega) = h(\omega)\ , \\
x \geqslant 0,\ y(\omega) \geqslant 0\ .
\end{aligned}
$$

Here in the presented stochastic linear program program above, first-stage decisions are represented by the $n_1 \times 1$ vector x. For a given realization ω, the second-stage problem data $q(\omega)$, $h(\omega)$ and $T(\omega)$ become known, where $q(\omega)$ is $n_2 \times 1$, $h(\omega)$ is $m_2 \times 1$, and $T(\omega)$ is $m_2 \times n_1$. In this stochastic linear program, "*Fixed Recourse*" occurs when the constraint matrix W has fixed, non-random, coefficients. The objective function of the stochastic linear program program above contains a deterministic term $c^T \cdot x$, which is the cost of first-stage decisions, and the expectation of the second-stage recourse problem objective $q(\omega)^T \cdot y(\omega)$ taken over all realizations of the random event ω. Note that, for each random event ω, the value of second-stage decision variable $y(\omega)$ is determined by the solution of a separate deterministic linear program.

The solution algorithm for the two-stage stochastic linear programs with fixed recourse involves making some initial decisions that minimize current costs plus the expected value of future recourse actions. One can always form a full deterministic equivalent linear program, which is called the *extensive form*, of the original stochastic model under finite number of second stage realizations. With a large set of second stage realizations, the extensive form of the problem gets quite large, which prevents achieving an efficient solution. The frequently used solution technique, the *L-shaped method*, is a family of algorithms that are based on developing an outer linearization of the recourse function. This method is a cutting plane method in that linear cuts, supporting hyperplanes, are generated to create the linearization of the recourse function. The algorithm is primarily based on generating an outer linearization of the recourse cost function and finding a solution of the first-stage problem plus this linearization. This method is a direct application of Bender's Decomposition of the stochastic program primal, or equivalently a Dantzig-Wolfe decomposition of the dual. The block structure of the extensive form has given rise to the name "L-Shaped" for the algorithm. The method has been developed by Van Slyke & Wets (1969) in stochastic programming to take care of the feasibility questions. The main principle in the L-shaped method is to approximate the nonlinear term in the objective of the stochastic programs. Since the nonlinear objective term, the recourse function, involves a solution of all second-stage recourse linear programs, we avoid numerous function evaluations by assuming an initial fixed value, θ, for it. Using the fixed term θ, we build a master problem that involves first-stage decision variables, x, only. Then, we evaluate the recourse function in the exact original form as a subproblem for each realization of the random event ω.

We use the extensive form (EF) of the two-stage stochastic model to present the L-shaped method. Let $k =$ *1,...,K* be the index for possible realizations of the random vector ξ and let p_k be the associated probabilities. We create the extensive form by assigning one set of second-stage decision, y_k, for each realization of ξ, where each realization is associated with a specific value for q_k, h_k and T_k. Below we present the described extensive form of the large scale stochastic model:

$$
\begin{aligned}
(EF) \qquad & min\ c^T \cdot x + \sum_{k=1}^{K} p_k \cdot q_k^T \cdot y_k \\
& s.t. \\
& A \cdot x = b\ , \\
& T_k \cdot x + W \cdot y_k = h_k\ , \quad k = 1, ..., K\ ; \\
& x \geqslant 0,\ \ y_k \geqslant 0\ , \quad k = 1, ..., K\ .
\end{aligned}
$$

Solution Algorithm of the L-Shaped Method

Step 0. Initialization

Set $r = s = v = 0$.

Step 1. Define and Solve the Master Program

Set v = v+1 and solve the following linear program called the "Master Program" :

(Master Program)

$$
\begin{aligned}
& min\ z = c^T \cdot x + \theta \\
& s.t. \\
& A \cdot x = b\ , \\
& D_\ell \cdot x \geqslant d_\ell\ , \qquad \ell = 1, ..., r\ ; \\
& E_\ell \cdot x + \theta \geqslant e_\ell\ , \qquad \ell = 1, ..., s\ ; \\
& x \geqslant 0\ , \qquad \theta \in \mathbb{R}.
\end{aligned}
$$

Master program is used to figure out a proposal first-stage decision variable x, to be sent to the second stage. Let $(x^v,\, \theta^v)$ be an optimal solution. The following constraint in the master problem defines a new optimality cut constraint at each iteration:

$$E_\ell \cdot x + \theta \geqslant e_\ell \ , \qquad \ell = 1, ..., s \ ;$$

Note that, while solving the very first master problem, since there is no optimality cut constraint present, θ^v is set equal to -∞ and is not considered in the computation of x^v.

Step 2. Feasibility Cuts

The following constraint in the master program above introduces a feasibility cut for the problem:

$$D_\ell \cdot x \geqslant d_\ell \ , \qquad \ell = 1, ..., r \ ;$$

In order to generate the feasibility cut, for each realization of the random vector k = *1,...,K*, we solve the following linear program:

$$Min \qquad w^{'} = e^T \cdot v^+ + e^T \cdot v^-$$

$$s.t.$$

$$W \cdot y + I \cdot v^+ - I \cdot v^- = h_k - T_k \cdot x^v$$

$$y \geqslant 0, \ v^+ \geqslant 0, \ v^- \geqslant 0,$$

where e^T = (1,...,1) until for some *k*, the optimal value $w^{'} > 0$. In this case let σ^v be the associated simplex multiplier (i.e. the simplex multiplier or shadow price of a constraint is the difference between the optimized value of the objective function and the value of the objective function, evaluated at the optional basis, when the right hand side (RHS) of a constraint is increased by one unit), then define:

$$D_{r+1} = (\sigma^v)^T \cdot T_k$$

$$d_{r+1} = (\sigma^v)^T \cdot h_k$$

The defined values are used to generate the introduced feasibility cut constraints. Set r = *r+1*, add the generated feasibility cut constraint to the master program and return to Step 1. If for all *k*, $w^{'} > 0$ then proceed to Step 3. Observe that, in our two-stage stochastic problem introduced in the previous section, the second-stage is always feasible since there is no limit on the second-stage recourse decision variable y (i.e. any nursing shortage will be covered by the mandatory nurse overtime in the second-stage). Thus, Step 2 is omitted from the solution algorithm used in our problem.

Step 3. Subproblem / Optimality Cuts

For each realization of the random vector $k = 1,\ldots,K$, we solve the following linear program:

$$Min \qquad w = q_k^T \cdot y$$

$$s.t.$$

$$W \cdot y = h_k - T_k \cdot x^v \ ,$$

$$y \geqslant 0,$$

Let π_k^v be the simplex multipliers associated with the optimal solution of subproblem for realization k of the random vector. Define,

$$E_{s+1} = \sum_{k=1}^{K} p_k \cdot (\pi_k^v)^T \cdot T_k$$

and,

$$e_{s+1} = \sum_{k=1}^{K} p_k \cdot (\pi_k^v)^T \cdot h_k.$$

Let $w^v = e_{s+1} - E_{s+1} \cdot x^v$. If $\theta^v \geqslant w^v$, stop; x^v is an optimal solution. Otherwise, set $s = s+1$, add to the master program constraint set an optimality cut using the latest computed values of e_{s+1} and E_{s+1}, and return to Step 1. As presented in the L-shaped algorithm description above, two types of constraints are sequentially added to the master program: (i) feasibility cuts and (ii) optimality cuts, until an optimal solution is reached. Next, in 5.1 below, we provide a summary of the algorithmic steps for the L-Shaped method.

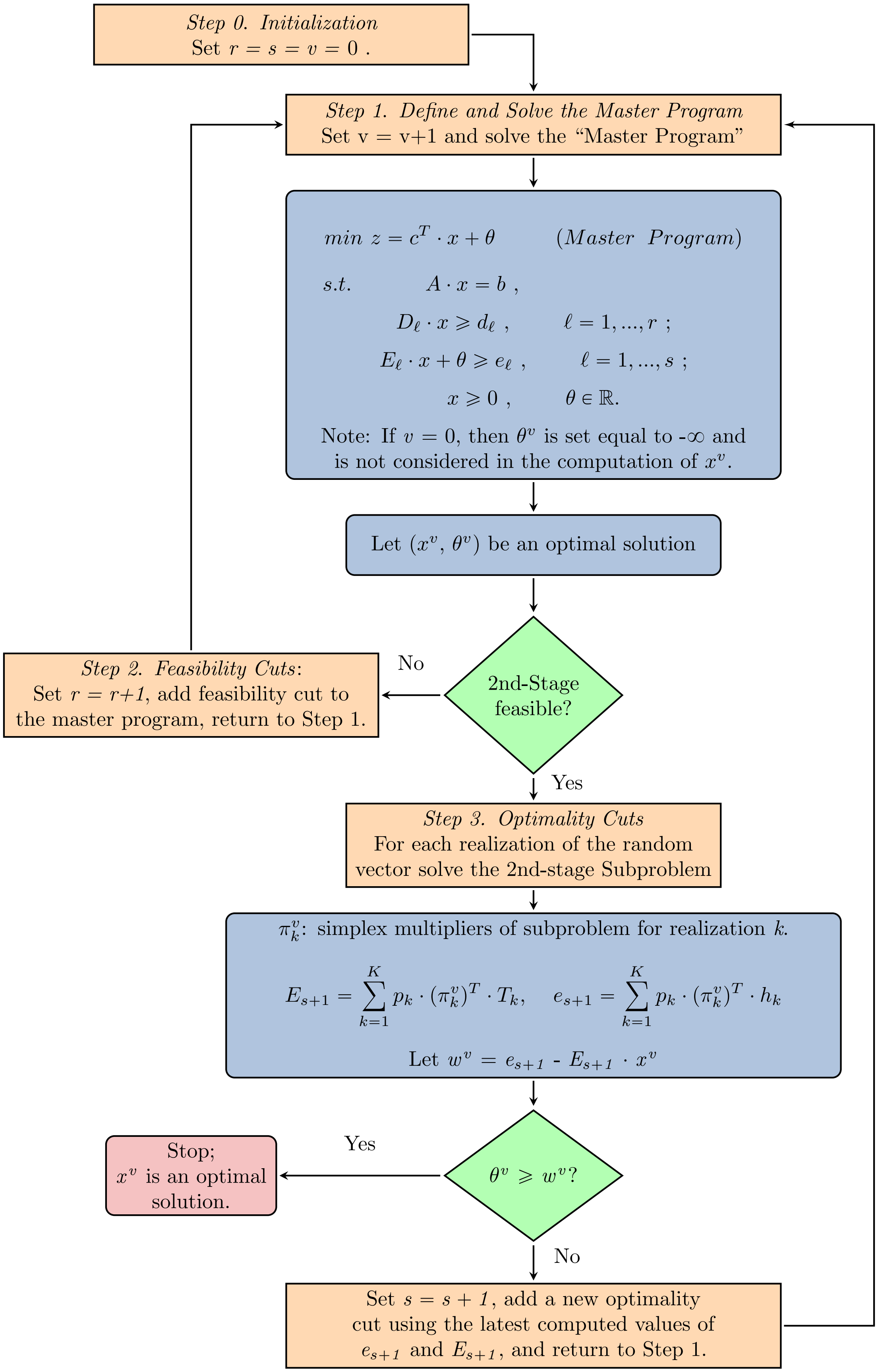


Figure 5.1: Solution Algorithm for the L-Shaped Method

5.4 Description of the Probability Matrix Generation Algorithm

As we explained in the introductory section of this chapter, the following patient data is assumed to be available to the nursing administration at the start of current shift: (1) current patient census (total patients staying in the unit); (2) acuity assignments of the existing patients (total patients in each acuity group); (3) the number of scheduled and unscheduled patient admissions for the current nursing shift; (4) the number of patient discharges and their associated acuity groups during the current shift and in the upcoming shift; and (5) total scheduled patient admissions for the upcoming shift. Howeveer, the following information is not known by the administration at the start of the current shift: (1) the acuity assignments of scheduled and unscheduled patients for the current shift (i.e., we know the number of scheduled and unscheduled patients to be admitted to the unit for the current shift, but we don't know their acuity assignments since the patients have not arrived yet), and (2) the number of unscheduled patient admissions for the upcoming shift.

In Chapter 4 we used a nursing requirement computation that takes into account the patient census, acuity mix and total Admission/Discharge/Transfer (ADT) activity in the unit for a given shift. We use a six-class categorization for patient acuities, from acuity levels A to F, F being the category of the most nursing workload-intense group. For Critical Care, the rough guidelines for nursing time requirement for each acuity group per 4-hour shift are: A=0.5 hour, B=1 hour, C=1.5 hours, D=2.5 hours, E=4 hours, F=8 hours (for patients associated with acuity F, 2 RNs are assigned for 1 patient). The PICU, like many intensive care units, generally does not admit patients with acuity levels A and B (and occasionally a C). Therefore, we expect patients only from acuity groups D, E, and F. We also consider the nursing time required for the ADT activities occurring in a given shift. Studies published in literature roughly suggest one-half hour of nursing time for each ADT activity. Given patient census, acuity assignments of the patients in the unit, and ADT activity in a specific shift, we will be able to compute the required workload for the unit for a specific nursing shift. As an example, assume that the patient mix in a medical unit at the start of a shift is as follows: 5 patients from acuity group D; 6 patients from acuity group E; 2 patients from acuity group F; and no patients from acuity groups A, B and C. Also assume that there will be 3 admissions to the unit (from scheduled or unscheduled patients) and 2 discharge. The required nursing hours for the unit will be calculated as follows:

$$(2.5\ hrs. \cdot 5) + (4\ hrs. \cdot 6) + (8\ hrs. \cdot 2) + (0.5\ hrs. \cdot 3) + (0.5\ hrs. \cdot 2) = 55\ hours$$

In order to calculate the nursing requirement for the upcoming shift, since the stochastic adjustments model is used for matching the patient demand in the upcoming shift, we need to know the patient census and mix at the start of the upcoming shift and the total number of patient admissions (i.e. scheduled and unscheduled) and discharges (i.e. all patient discharges are scheduled). In order to know the patient census and mix at the start of the upcoming shift, we need to start with the patient census and mix at the beginning of the current shift and add or subtract the patient admissions and discharges from each acuity group to be realized in the current shift. We know the current patient census and mix, we know the admissions and discharges that will occur at the current shift, but we don't know the associated acuities of these patients. For the upcoming shift, we know the number of patient discharges, but we don't know the number of unscheduled patient admissions. As a result, stochastic data in our model can be summarized as: (1) Acuity assignments of scheduled admissions in the current shift, (2) Acuity assignments of unscheduled admissions in the current shift and (3) Number of unscheduled patient admissions for the upcoming shift. We keep the available six acuity groups (A, B, C and D) for the probability matrix generation algorithm, not to loose the broader use of the methodology in non-intensive care medical units (i.e. we keep the option of admitting a patient associated with acuity group A, B and C open, but we will assume a zero probability for the event). Uncertainty is represented in terms of random experiments with outcomes denoted by 'ω' (i.e. state of the world). The set of all outcomes is represented by 'Ω': $\omega \in \Omega$. The particular values the various random variables will take are only known after the random experiment, i.e. , the vector $\xi = \xi(\omega)$ is only known after the experiments. Then, in our model, the random vector $\xi(\omega)$ has 13 elements, six from number of scheduled patient admissions in six acuity groups for the current shift; another six from the number of unscheduled patient admissions in six acuity groups in the current shift; and an additional 13th element for the number of unscheduled patient admissions for the upcoming shift. For a given realization ω, the second-stage problem data become known and second-stage recourse decisions are conducted accordingly. Table 5.2 below summarizes the factors contributing to the generation of full set of scenarios Ω in our model:

	Scheduled Admits Current Shift			**Unscheduled Admits Current Shift**			**Unscheduled Admits Upcoming Shift**
Acuity Groups	D	E	F	D	E	F	
Set of Alternatives	{0, 1, 2}	{0, 1, 2}	{0, 1}	{0, 1, 2}	{0, 1, 2}	{0, 1}	{0, 1, 2, 3, 4, 5}
# of Alternatives	3	3	2	3	3	2	6

Table 5.2: Stochastic Elements of the Random Vector

As discussed above, we have three main categories for the stochastic elements in the random vector: (1)

scheduled admits to the current shift (number of admits to each acuity group) , (2) unscheduled admits to the current shift (number of admits to each acuity group) and (3) unscheduled admits to the upcoming shift (total number of unscheduled admits from all acuity groups). We assume there is no scheduled and unscheduled admissions to the current shift for acuity groups A, B and C. So, the associated elements in the random vector always assume value 0. We also assume, scheduled and unscheduled admissions to acuity groups D and E for the current shift are limited with maximum two patients (i.e. there will be 0, 1 or 2 scheduled patient admissions to the unit for acuity groups D and E at the current shift, same holds true for unscheduled admissions as well). For the acuity group F, scheduled and unscheduled admissions for the current shift are limited with maximum one patient (i.e. there will be either 0 or 1 scheduled patient admissions to the unit for acuity group F at the current shift, same holds true for unscheduled admissions as well). Total unscheduled admits to any shift is limited by 5 admissions in total, which results in 6 different alternatives for the unscheduled patient admissions for the upcoming shift. Table 5.2 lists the number of alternatives generated by each stochastic element. The presented design results in 1944 different scenarios for the two-stage stochastic model. Presented alternatives and assumptions are inline with the historical patient admission patterns at the studied PICU. Any other patient admission pattern, for a different medical unit/hospital, can be formulated using the same algorithm that is presented in this section.

Since we know the exact number of scheduled and unscheduled admissions to the unit for the current shift at the start of the shift, the probability of each scenario 'ω' (i.e. a specific realization of the random vector) is conditional on that information. We refer to each scheduled and unscheduled admission alternative as "*Case ID*". Since both the number of scheduled and unscheduled admits to the unit can assume values from the set {0, 1, 2, 3, 4, 5}, there are 36 different Case ID options. Table 5.3 presents the Case ID assignment for each scheduled and unscheduled admission option (i.e. Case ID 13 refers to the alternative where we are expecting 2 scheduled admits and 1 unscheduled admits to the unit within the current shift). Table 5.3 also present the total number of unique scenarios that are feasible under each Case ID. Below we present how we attain number of different alternatives for each stochastic factor listed in Table 5.3.

Given the number of patient admissions and set of admissions options to each acuity group, we present feasible patient admission patterns for both scheduled and unscheduled admissions. We also present the associated probabilities for each alternative for Ward A, as an example. Presented probability scores for each feasible alternative is estimated from the historical patient acuity and admissions data of the studied PICU.

Case ID	# Sch Adm Current Shf	# Unsch Adm Current Shf	Case Info	# Alt. Sch Adm	# Alt. Unsch Adm	# Unsch Adm Upcoming Shf	# of Scenarios
0	0	0	(0,0)	1	1	6	6
1	0	1	(0,1)	1	3	6	18
2	0	2	(0,2)	1	5	6	30
3	0	3	(0,3)	1	5	6	30
4	0	4	(0,4)	1	3	6	18
5	0	5	(0,5)	1	1	6	6
6	1	0	(1,0)	3	1	6	18
7	1	1	(1,1)	3	3	6	54
8	1	2	(1,2)	3	5	6	90
9	1	3	(1,3)	3	5	6	90
10	1	4	(1,4)	3	3	6	54
11	1	5	(1,5)	3	1	6	18
12	2	0	(2,0)	5	1	6	30
13	2	1	(2,1)	5	3	6	90
14	2	2	(2,2)	5	5	6	150
15	2	3	(2,3)	5	5	6	150
16	2	4	(2,4)	5	3	6	90
17	2	5	(2,5)	5	1	6	30
18	3	0	(3,0)	5	1	6	30
19	3	1	(3,1)	5	3	6	90
20	3	2	(3,2)	5	5	6	150
21	3	3	(3,3)	5	5	6	150
22	3	4	(3,4)	5	3	6	90
23	3	5	(3,5)	5	1	6	30
24	4	0	(4,0)	3	1	6	18
25	4	1	(4,1)	3	3	6	54
26	4	2	(4,2)	3	5	6	90
27	4	3	(4,3)	3	5	6	90
28	4	4	(4,4)	3	3	6	54
29	4	5	(4,5)	3	1	6	18
30	5	0	(5,0)	1	1	6	6
31	5	1	(5,1)	1	3	6	18
32	5	2	(5,2)	1	5	6	30
33	5	3	(5,3)	1	5	6	30
34	5	4	(5,4)	1	3	6	18
35	5	5	(5,5)	1	1	6	6

Table 5.3: Definition of Case IDs and Number of Associated Scenarios

- **0 Admissions Case**: If the number of scheduled or unscheduled admissions to the unit for the current shift is known to be zero, then there is only one alternative for acuity combinations under our assumptions listed in Table 5.2. Table 5.4 presents the feasible alternative and associated probability for Ward A.

	0 Admission Case						
Acuity Groups	**A**	**B**	**C**	**D**	**E**	**F**	**Prob.**
Alt # 1	0	0	0	0	0	0	1.00

Table 5.4: Feasible Patient Acuity Assignment Alternatives Under 0 Admissions - Ward A

- **1 Admission Case**: If the number of scheduled or unscheduled admissions to the unit for the current shift is known to be one, then there are three alternatives for acuity combinations under the assumptions listed in Table 5.2. Table 5.5 presents these alternatives and associated probabilities for Ward A.

	1 Admission Case						
Acuity Groups	**A**	**B**	**C**	**D**	**E**	**F**	**Prob.**
Alt # 1	0	0	0	1	0	0	0.42
Alt # 2	0	0	0	0	1	0	0.50
Alt # 3	0	0	0	0	0	1	0.08

Table 5.5: Feasible Patient Acuity Assignment Alternatives Under 1 Admission - Ward A

- **2 Admissions Case**: If the number of scheduled or unscheduled admissions to the unit for the current shift is known to be two, then there are five alternatives for acuity combinations under the assumptions listed in Table 5.2. Table 5.6 presents these alternatives and associated probabilities for Ward A.

	2 Admissions Case						
Acuity Groups	**A**	**B**	**C**	**D**	**E**	**F**	**Prob.**
Alt # 1	0	0	0	1	1	0	0.24
Alt # 2	0	0	0	1	0	1	0.13
Alt # 3	0	0	0	0	1	1	0.15
Alt # 4	0	0	0	2	0	0	0.22
Alt # 5	0	0	0	0	2	0	0.26

Table 5.6: Feasible Patient Acuity Assignment Alternatives Under 2 Admissions - Ward A

- **3 Admissions Case**: If the number of scheduled or unscheduled admissions to the unit for the current shift is known to be three, then there are five alternatives for acuity combinations under the assumptions listed in Table 5.2. Table 5.7 presents these alternatives and associated probabilities for Ward A.

	3 Admissions Case						
Acuity Groups	**A**	**B**	**C**	**D**	**E**	**F**	**Prob.**
Alt # 1	0	0	0	2	1	0	0.28
Alt # 2	0	0	0	2	0	1	0.13
Alt # 3	0	0	0	1	2	0	0.29
Alt # 4	0	0	0	1	1	1	0.14
Alt # 5	0	0	0	0	2	1	0.16

Table 5.7: Feasible Patient Acuity Assignment Alternatives Under 3 Admissions - Ward A

- **4 Admissions Case**: If the number of scheduled or unscheduled admissions to the unit for the current shift is known to be four, then there are three alternatives for acuity combinations under the assumptions listed in Table 5.2. Table 5.8 presents these alternatives and associated probabilities for Ward A.

	4 Admissions Case						
Acuity Groups	**A**	**B**	**C**	**D**	**E**	**F**	**Prob.**
Alt # 1	0	0	0	2	2	0	0.42
Alt # 2	0	0	0	2	1	1	0.28
Alt # 3	0	0	0	1	2	1	0.30

Table 5.8: Feasible Patient Acuity Assignment Alternatives Under 4 Admissions - Ward A

- **5 Admissions Case**: If the number of scheduled or unscheduled admissions to the unit for the current shift is known to be five, then there is only one single alternative for acuity combinations under the assumptions listed in Table 5.2. Table 5.9 presents the single alternative and associated probability for the Ward A.

	5 Admissions Case						
Acuity Groups	**A**	**B**	**C**	**D**	**E**	**F**	**Prob.**
Alt # 1	0	0	0	2	2	1	1.00

Table 5.9: Feasible Patient Acuity Assignment Alternatives Under 5 Admissions - Ward A

As an example, let's assume we have two scheduled and four unscheduled admissions to the medical unit for the current shift (i.e. Case ID 16). The two scheduled admissions can have five alternative acuity assignment combinations as presented in Table 5.6, and the four unscheduled admissions can have three alternative acuity assignment combinations as presented in Table 5.8. The unscheduled admissions to the upcoming shift can assume one of six feasible values (i.e. {0,1, 2, 3, 4, 5}) as presented in Table 5.2. Combining all combinations from the three stochastic elements we get 5 x 3 x 6 = 90 different scenarios under Case ID 16. The total number of scenarios in Table 5.3 are computed using this logic.

Our ultimate goal in section is to present the probability matrix generation algorithm, where the probability matrix lists the probability of each scenario under a given Case ID. The resulting probability matrix has 36 rows (i.e. one row for each Case ID) and 1944 columns (i.e. one column for each scenario). Given (1) the probability information presented in Table 5.4 through Table 5.9, (2) the probability of scheduled vs. unscheduled admissions presented in Table 5.10 and (3) the probability of having 0 to 5 unscheduled admissions for the upcoming shift presented in Table 5.11, we generate the desired unique probability matrix for each medical unit to be used in the two-stage stochastic optimization model using a C++ code.

Data Category	**Ward A**		**Ward B**		**Ward C**		**PICU Total**	
	n[1]	%[2]	n	%	n	%	n	%
Scheduled Admissions	993	61.3%	1,235	32.6%	353	12.1%	2,581	31.0%
Unscheduled Admissions	627	38.7%	2,556	67.4%	2,555	87.9%	5,738	69.0%
Total Admissions	1,620		3,791		2,908		8,319	
Total Discharges	1,596		3,764		2,889		8,249	

[1] n values represent the number of cases with the specified condition in each ward.

[2] % values represent the % of cases among all observations within that ward.

Table 5.10: Scheduled vs. Unscheduled Admissions in PICU

Table 5.10 shows the number of scheduled and unscheduled admissions at each ward and as a total for the PICU. Presented data shows that about 69% of admissions to the PICU are unscheduled. Ward A has a higher percentage of scheduled admissions (61.3%) whereas Wards B and C have significantly higher percentages of unscheduled admissions (67.4 and 87.9%, respectively). Below we also present the probability of unscheduled patient admissions for the upcoming shift for Ward A, as an example, in Table 5.11.

Below, we present the algorithmic steps of the developed probability matrix generating code. We include parts of a sample code developed for Ward A in the studied PICU in the Appendices G for reference.

Unscheduled Patient Admissions	
Admits in the Upcoming Shift	**Probability**
0 Unscheduled Admit	0.786
1 Unscheduled Admit	0.171
2 Unscheduled Admits	0.035
3 Unscheduled Admits	0.006
4 Unscheduled Admits	0.001
5 Unscheduled Admits	0.001

Table 5.11: Probability of Unscheduled Patient Admissions for the Upcoming Shift - Ward A

Step 1: Define the Cardinality of the Sets, Variables for Scheduled and Unscheduled Admissions and Associated Probabilities

In the first step of the algorithm, we define the cardinality (i.e. size) of the sets for scheduled and unscheduled admissions alternatives as presented in Table 5.2 (i.e. const int $card - Sch - Admits - t = 6$: Cardinality (size) of the scheduled admissions set (current shift), $Sch - Admits - t = \{0,1,2,3,4,5\}$). We also introduce the variables representing the number of scheduled and unscheduled admissions to the current shift, number of unscheduled admissions to the upcoming shift (stochastic data), number of scheduled admissions to the current shift for acuity groups A to F (stochastic data) and number of unscheduled admissions to the current shift for acuity groups A to F (stochastic data). We define the integer variables representing the Case ID (i.e. Case ID = {0,1,2,...,35}) and the Scenario ID (i.e. Scenario ID = {0,1,2,...,1943}). We also define the probability matrix listing the probability of each scenario given the Case ID, probability of a specific acuity distribution for scheduled and unscheduled patient admissions for the current shift and probability of having {0,1,...5} unscheduled admissions in the upcoming shift. Appendix Figure G.1 presents the code for Step 1.

Step 2: Provide the Probability Estimates of Each Scheduled and Unscheduled Admission Combination

In the second step of the algorithm, given the total number of scheduled and unscheduled admissions to the current shift, we provide the estimates of each scheduled and unscheduled admission combination in terms of the patient acuities (i.e. given that there will be two scheduled patient admissions to the unit in the current shift, what is the probability of having one patient in acuity group D and second one in acuity group F). Since each medical unit has its own patient characteristics in terms of scheduled and unscheduled admissions and patient acuity patterns, we estimate these probabilities separately for each medical unit using the historical patient data. Appendix Figure G.2 presents parts of the code for Step 2.

Step 3: Generate Case ID and Scenario ID for Each Scheduled and Unscheduled Admission Combination and Acuity Assignment

In the third step of the algorithm, we first introduce the probabilities of unscheduled patient admissions for the upcoming shift as presented in Table 5.11. Then we generate the Case ID for each scheduled and unscheduled admission combination to the current shift (i.e. Case ID 13 refers to the alternative where the medical unit expects 2 scheduled admits and 1 unscheduled admits to the unit within the current shift) as presented in Table 5.3. We then generate a "*Scenario ID*" for each acuity combination for the scheduled and unscheduled admissions to the current shift and number of unscheduled admissions for the upcoming shift. Table 5.3 also presents the total number of unique scenarios that are feasible under each Case ID. Appendix Figure G.3 presents a sample part of the code used for Step 3.

Step 4: Print the Scheduled and Unscheduled Admission Numbers for Each Acuity Group, Under Each Scenario, in the Current Shift

In the fourth step of the algorithm, we print the scheduled and unscheduled admission numbers for each acuity group, under each scenario, in the current shift. We need this data to be fed into the two-stage stochastic optimization model, since nursing requirement in the second-stage of the problem is computed accordingly. Our both first and second-stage decisions in the stochastic model use this scenario dependent data. Appendix Figure G.4 presents a sample part of the code used for Step 4.

Step 5: Print the Number of Unscheduled Admissions, Under Each Scenario, for the Upcoming Shift

In the fifth step of the algorithm, we print the number of unscheduled admissions, under each scenario, for the upcoming shift. Similar to Step 4, this stochastic data also constitute part of our nursing requirement computations and is used for decision making in the two-stage stochastic model as presented earlier in this section. Appendix Figure G.5 presents a sample part of the code used for Step 5.

Step 6: Compute the Probability of Each Scenario Given the Probability Estimates of Each Scheduled and Unscheduled Admission Combination and Case ID

In the sixth step of the algorithm, given the probability estimates of each scheduled and unscheduled admission combination in Step 2 and the Case ID and Scenario ID information generated in Step 3, we compute the probability of each scenario. In order to do so, we multiply the probability of the specific scheduled admissions acuity pattern for the current shift with the probability of the specific unscheduled admissions acuity pattern for the current shift and the probability of the specific number of unscheduled admissions in the upcoming shift, associated with each Scenario ID. Appendix Figure G.6 presents a sample part of the code used for Step 6.

Step 7: Generate the Probability Matrix for Each Scenario Given the Case ID

In the seventh and final step, we put the computed probabilities in Step 6 into a matrix, which has 36 rows (i.e. one row for each Case ID) and 1944 columns (i.e. one column for each scenario). We print the generated matrix to be used in the developed two-stage stochastic optimization model. Appendix Figure G.7 presents a sample part of the code used for Step 7. As an example, for Case ID = 13 (i.e. 2 scheduled and 1 unscheduled admits to the unit within the current shift), Table 5.12 below lists the scenarios with probabilities greater than 2%. Next, we present our experimental design that is used to test the developed stochastic model and evaluate the results.

	Sch Admits - Shift t						**Unsch Admits - Shift t**						**Unsch Adm**	
Scenario	**A**	**B**	**C**	**D**	**E**	**F**	**A**	**B**	**C**	**D**	**E**	**F**	**Shift (t+1)**	**Prob**
336	0	0	0	0	1	1	0	0	0	0	1	0	0	0.0590
360	0	0	0	0	1	1	0	0	0	1	0	0	0	0.0495
444	0	0	0	0	2	0	0	0	0	0	1	0	0	0.1022
445	0	0	0	0	2	0	0	0	0	0	1	0	1	0.0222
468	0	0	0	0	2	0	0	0	0	1	0	0	0	0.0858
768	0	0	0	1	0	1	0	0	0	0	1	0	0	0.0511
792	0	0	0	1	0	1	0	0	0	1	0	0	0	0.0429
876	0	0	0	1	1	0	0	0	0	0	1	0	0	0.0943
877	0	0	0	1	1	0	0	0	0	0	1	0	1	0.0205
900	0	0	0	1	1	0	0	0	0	1	0	0	0	0.0792
1308	0	0	0	2	0	0	0	0	0	0	1	0	0	0.0865
1332	0	0	0	2	0	0	0	0	0	1	0	0	0	0.0726

Table 5.12: Sample Probability Matrix, Ward A

5.5 Experimental Design for RQ3

To evaluate the impact of various design factors on short-term schedule adjustment decisions and performance measures, we develop an experimental design, based on the following factors, for the two-stage stochastic short-term nurse schedule adjustment problem:

1. **Nurse Profile Mix (NMIX):** Nurse mix for the three PICU wards we studied (i.e., distribution of nurses over FTE and PRN groups in the medical units). The cases used in the experimental design are presented in the Nurse Profile Mix table, Table 4.4. We study three different time periods, listed in the data description provided in Table 4.5, for the three PICU wards.

2. **Mandatory Nurse Overtime / Understaffing Penalty Cost (UPC):** As outlined in the stochastic model description, we make first-stage adjustment decisions at the start of a new shift for the upcoming shift. Any nursing shortage in the upcoming shift is then satisfied using mandatory nurse overtime. UPC is designed as a penalty cost for one nursing hour of understaffing at the medical unit. The FTE nurse hourly rate is normalized to one unit. The base level of UPC is determined as 1.5 because mandatory overtime cost of a nurse is typically 50% higher than at the regular hourly rate. We use two additional levels for the UPC, since implications of understaffing in the second stage go farther than the nurse overtime cost.

3. **Staffing Policy (SPO):** We test the performance of the developed two-stage stochastic short-term adjustments model with respect to the medium-term staffing models developed in Chapter 4. Performance measures of Fixed, Dynamic and Optimal Staffing policies with no short-term adjustments are used for comparison. The short-term adjustment model is based on the schedules developed in the medium term using the Dynamic Staffing policy.

4. **Patient Demand (PD):** The patient demand pattern used in the optimization models for nursing for a typical 6-week staffing horizon. It consists of census data for each patient acuity group and ADT activity during each nursing shift for the staffing horizon. The Optimal Staffing model assumes that actual patient data is known at the time of building medium-term schedules (i.e., hypothetical option for performance comparison), The Fixed Staffing models use a pre-determined fixed level of patient demand for nursing. The Dynamic Staffing model with no adjustments uses the "heat map" data presented in Chapter 4. The two-stage stochastic adjustments model uses the heat map data for developing medium-term schedules, then uses the patient data available in the short term at the start time of a shift (i.e., available patient data in the short-term is described in the introduction section of

this chapter).

Table 5.13 below presents the experimental design factors and various levels of these factors used in this study.

Nurse Mix (NMIX)	**Understaffing Penalty (UPC)**	**Staffing Policy (SPO)**	**Patient Demand PD**
Ward A	1.5	Optimal Staffing - No Adjustments	Actual Demand
Ward B	2.0	Fixed Staffing - L1 - No Adjustments	Fixed Demand - L1
Ward C	3.0	Fixed Staffing - L2 - No Adjustments	Fixed Demand - L2
		Dynamic Staffing - No Adjustments	Heat-Map Demand
		Dynamic Staffing with Short-Term Adjustments	Heat-Map Demand + Short-Term Data

Table 5.13: Experimental Design Factors for RQ3

Using the developed two-stage stochastic short-term staffing adjustment model and the presented experimental design in Table 5.13 we study our third research question:

RQ 3: "*Can short-term schedule modifications that are based upon decisions attained from two-stage stochastic integer programming model bring cost savings and reduction in understaffing levels, compared to keeping original medium-term staffing plans, during the nursing shifts for the medical units*?"

In addition to the cost savings and reduction in understaffing levels, using the performance measures presented below, we also evaluate the scheduling flexibility needs of the medical unit using the developed two-stage stochastic adjustments model. Number of first-stage adjustments, the ratio of shifts with any adjustments provide insights regarding the flexibility needed when building and adjusting nurse schedules. We run the developed two-stage Stochastic Adjustments model for each nursing shift throughout the staffing horizon (i.e. 252 shifts in a staffing horizon of 6 weeks). Each experimental run consists of multiple optimality cuts and for each cut the second-stage subproblem is solved for each scenario option given the Case ID information. For computational efficiency, we combined 21 nursing shifts in one single run (i.e. a single staffing horizon of 252 shifts requires 12 separate runs). Each experimental run, for the 21 shifts, is completed within two hours of run time. The whole experimental design required 9 x 12 = 108 individual runs. Next, we present the measures we use to test the performance of presented models.

5.5.1 Description of Performance Measures

Below is a summary of the performance measures to be evaluated for the results of our experimental design:

- Obj. Value (Total Cost): Resulting objective function cost of the optimization model. It is the total cost of FTE and PRN staffing costs and understaffing penalty costs. In the case of stochastic model, the objective also includes costs from first-stage adjustment decisions.
- Objective Value (O.V.) Comparison (%): Percentage difference of alternative model objective function costs with respect to the Stochastic Adjustments model objective value.
- FTE Staffing Cost: Total staffing cost for the FTE nurses, in the medium-term, during the scheduling horizon of 6 weeks.
- PRN Staffing Cost: Total staffing cost for the PRN nurses, in the medium-term, during the scheduling horizon of 6 weeks.
- Understaffing Penalty / Mandatory Overtime Cost: Total penalty cost for understaffing during the scheduling horizon of 6 weeks. In the stochastic adjustments model, the understaffing penalty is applied in the form of a mandatory overtime cost for nurses. The expected penalty is estimated by the stochastic model (i.e. sum of second-stage recourse action cost, given by θ, for the staffing horizon). The realized understaffing penalty is computed using the actual patient data during the staffing horizon and provided staffing levels, after the first-stage adjustment decisions.
- 1st Stage Staff Addition Cost: Cost of Float Pool, FTE-On-Call and PRN-On-Call nurse additions to the staff size in the first stage of stochastic adjustments model.
- 1st Stage Staff Reduction Savings: Savings from schedule cancellations for the FTE and PRN nurses to the staff size in the first stage of stochastic adjustments model.
- Median U_t: Median percentage understaffing during the scheduling horizon of 6 weeks.
- Average U_t: Average percentage understaffing during the scheduling horizon of 6 weeks.
- Max U_t: Maximum level of percentage understaffing observed during the scheduling horizon of 6 weeks.
- Median O_t: Median percentage overstaffing during the scheduling horizon of 6 weeks.
- Average O_t: Average percentage overstaffing during the scheduling horizon of 6 weeks.
- Max O_t: Maximum level of percentage overstaffing observed during the scheduling horizon of 6 weeks.

Performance measures below are used to analyze the scheduling flexibility needs of the medical units:

- Average Float Pool Request per Shift (Average X_h^+)
- Average Float Pool Request per Shift (Average X_h^+) per Average Staff Size
- Ratio of Shifts with Float Pool Request
- Average FTE-On-Call Request per Shift (Average X_f^+)
- Average FTE-On-Call Request per Shift (Average X_f^+) per Average Staff Size
- Ratio of Shifts with FTE-On-Call Request
- Average PRN-On-Call Request per Shift (Average X_p^+)
- Average PRN-On-Call Request per Shift (Average X_p^+) per Average Staff Size
- Ratio of Shifts with PRN-On-Call Request
- Average FTE Shift Cancel per Shift (Average X_f^-)
- Average FTE Shift Cancel per Shift (Average X_f^-) per Average Staff Size
- Ratio of Shifts with FTE Shift Cancel
- Average PRN Shift Cancel per Shift (Average X_p^-)
- Average PRN Shift Cancel per Shift (Average X_p^-) per Average Staff Size
- Ratio of Shifts with PRN Shift Cancel
- Ratio of Shifts with First-Stage Adjustments
- Average Nursing Hours Satisfied with Mandatory Overtime in 2nd Stage
- Average Number of Nurses Needed for the Mandatory Overtime (Average Nurse Request per Shift for Mandatory Overtime)
- Ratio of Shifts that Utilized Mandatory Overtime
- Average Mandatory Overtime Cost Per Shift

5.5.2 Parameters Utilized in the Experiments

Below, we present the fixed parameter values used in the experiments:

- *param h* := [1 0.5, 2 1.0, 3 1.5, 4 2.5, 5 4.0, 6 8.0]; nursing hours required for patient care for each acuity group in a four-hour nursing shift.
- *param c* := 4 ; staffing cost per four-hour shift for the FTE nurses.
- *param b* := 4.4 ; staffing cost per four-hour shift for the PRN nurses.
- *param gamma* := 0.5; nursing hours required for one patient admission or transfer-in activity.
- *param delta*:= 0.5; nursing hours required for one patient discharge or transfer-out activity.
- *param S* := 7680; total number of available schedules to the nurses from all job profiles.

Below, we present the cost parameters related to short-term schedule modifications available to the nursing administration in the first-stage.

Cost parameters related to generating extra nursing hours for the upcoming shift:

- c_h^+ := 4.8; cost of additional nurses requested from the general float pool of the hospital for one shift.
- c_f^+ := 4.8; cost of additional FTE nurses requested from available on-call list for one shift.
- c_p^+ := 5.2; cost of additional PRN nurses requested from available on-call list for one shift.

Cost parameters related to eliminating excess nursing hours available to the nursing administration:

- c_f^- := 3.2; savings incurred by floating, reassigning or cancelling one FTE nurse for the upcoming shift.
- c_p^- := 3.6; savings incurred by floating, reassigning or cancelling one PRN nurse for the upcoming shift.

Cost parameters related to the second-stage decisions:

- q_m^+ := 6.0, 8.0, 12.0; cost of mandatory overtime for nurses on the current shift to stay for the next shift who were not originally scheduled for the next shift. We used three levels as a part of the experimental design. Listed values represent per shift cost of mandatory overtime (i.e. \$1.5/hour $\cdot$ 4 hours/shift = \$6/shift).

Parameters defining the upper bound for total number of adjustments:

- $n_{h,(t+1)}^+$:= 3; total number of available nurses in the general float pool of the hospital that can be assigned to work for the medical unit for shift $(t + 1)$, requested at shift t.
- $n_{f,(t+1)}^+$:= 2; maximum number of additional FTE nurses that can be requested from available on-call list for shift $(t + 1)$ at shift t.

- $n^{+}_{p,(t+1)} := 2$; maximum number of additional PRN nurses that can be requested from available on-call list for shift $(t+1)$ at shift t.

Next, we present the results of our experiments.

5.6 Results & Discussion

In this section we present and discuss the results of our experiments. First of all, we need to remind that one important aspect of the short-term nurse schedule modification problem is the requirement of a very efficient solution algorithm. Practically, the charge nurse will run the solution algorithm at the beginning of each 4 to 8-hour shift and expect to have a solution in less than an hour, preferably in less than 10 minutes. All experiments conducted in this chapter reached a near-optimal solution, with an optimality gap less than 0.1%, in less than 10 minutes for one nursing shift. Table 5.14 presents the results of experiments for Ward A. We compare the results of experiments for the two-stage stochastic short-term schedule adjustments model (Stochastic Adjustments Model) with the medium-term staffing policies presented in Chapter 4. The stochastic adjustments model is based on Dynamic Staffing medium-term schedules, and introduces short-term staffing adjustments based on forecasts of the stochastic patient demand for nursing. Regarding the Objective Value, we observe that the Stochastic Adjustments model provides the least costly solution under all UPC options when compared to medium-term no-adjustment models. Even when compared to the Optimal Staffing model, which assumes patient demand data is perfectly known to the administration, the Stochastic Adjustments model brings cost savings. Savings are in the range of 0.90% to 2.09% for various UPC levels when compared with the hypothetical Optimal Staffing option. The savings increase as the UPC level is increased from 1.5 to 3.0. When compared with the Dynamic Staffing policy results, the Stochastic Adjustments model cost savings increase to a range of 5.07% to 7.72%. We compare the Stochastic Adjustment model performance with respect to two Fixed Staffing alternative policies with 19 and 20 nurses for Ward A. Our results indicate cost savings of approximately 8% for the Stochastic Adjustments model for various levels of UPC. Since staffing costs make up a considerable portion of hospital operational budgets, the savings obtained using the short-term Stochastic Adjustments model in the range of 5% to 9% seems promising.

Expected Understaffing Penalty listed in Table 5.14 is the expected total second-stage sub-problem objective function cost (i.e. Given as θ in the presented model) for the staffing horizon of six weeks. Realized Understaffing Penalty is computed using the difference between the nursing levels after the first-stage adjustments and the patient demand resulting from the actual patient data. Objective Value listed in the table is computed using the realized Understaffing Penalty levels, and is the sum of presented FTE and PRN Staffing Costs, first-stage staff addition and reductions (i.e. savings in the case of reductions, presented as negative numbers in the table) and the realized understaffing penalty level. Understaffing levels are reduced as the

UPC level is increased, for the Stochastic Adjustments model. Median and average understaffing levels observed in the shifts throughout the staffing horizon are less than 1.14% for all UPC levels, significantly lower than all medium-term staffing model alternatives. Average overstaffing level is also reduced to less than 1.54% for all UPC levels. Difference in overstaffing levels, comparing the Stochastic Adjustments and medium-term no adjustment models, is even more significant compared to the difference in understaffing levels. We can conclude that Objective Value cost savings are achieved both reducing the understaffing penalty and excessive staff size for the Stochastic Adjustments model. While first-stage staff addition costs constitute less than 3% of the objective value, first-stage staff reduction savings are realized around 5% in the Stochastic Adjustments model. Realized understaffing penalty costs, on the other hand, constitute roughly 1% of the total Objective Value of the Stochastic Adjustments model.

Table 5.15 presents the results of our experiments with the Stochastic Adjustments model for Ward B. For UPC = 1.5 cost savings of the Stochastic Adjustments model are similar to the savings observed in Ward A (i.e. 0.83% compared to Optimal Staffing, 4.60% for the Dynamic Staffing and 5.47% for the two Fixed Staffing alternatives). As the UPC level is increased to 2.0 and 3.0, cost savings achieved through the Stochastic Adjustments model drastically increase (i.e. For UPC = 2.0 observed savings for Ward B increase to 2.87% for the Optimal Staffing model, 9.39% for the Dynamic Staffing model, 10.07% for the Fixed Staffing model with 12 nurses and 12.58% for the Fixed Staffing model with 11 nurses; for UPC = 3.0 savings for Ward B even further increase to 5.05% for the Optimal Staffing model, 18.33% for the Dynamic Staffing model, 19.53% for the Fixed Staffing model with 12 nurses and 26.96% for the Fixed Staffing model with 11 nurses). We explain the drastic increase in cost savings for Ward B due to the higher levels of coefficient of variation in patient demand data for the unit, see Table 5.17, compared to the Ward A levels. Average understaffing levels are observed in the range of 0.66% to 1.99%, decreasing with an increasing UPC value. Average overstaffing levels are also observed in the range of 0.93% to 2.11%, increasing with an increasing UPC value. Similar to Ward A results, Ward B results under Stochastic Adjustments model indicate very reasonable levels of under and overstaffing, compared to all other medium-term no adjustments models. For Ward B, both staff addition and reduction adjustments costs (savings) are observed in the less than 2.5% level. Expected and realized understaffing penalty values are observed in less than 3% range for the Stochastic Adjustments model.

Table 5.16 presents the results of our experiments with the Stochastic Adjustments model for Ward C. The objective value of the Stochastic Adjustments model is significantly less than the objective value of any other medium-term staffing model, including the hypothetical Optimal Staffing model, for all levels of UPC. For UPC = 1.5, when compared to the Optimal Staffing model, Stochastic Adjustments model results indicate cost savings of 1.04%. The difference between the objective value of the two models drastically increase, for the benefit of Stochastic Adjustments model, as the UPC is increased to 2.0 and 3.0 (i.e. cost savings increase to 4.14% for UPC = 2.0 and to 7.97% for UPC = 3.0). Similar to Ward B patient data, the sample patient demand data used for Ward C experiments demonstrate higher levels of coefficient of variation (i.e. 18.64% for the sample and 20.88% for the full dataset) compared to Ward A. As the patient demand data demonstrate higher levels of variation, matching patient demand becomes more problematic and cost savings attained via the short-term Stochastic Adjustments model become more significant. Average understaffing ratios are in the range of 0.57% to 1.29% for the Stochastic Adjustments model. Average overstaffing ratios are in the range of 1.29% to 3.01% for the Stochastic Adjustments model. Slightly larger levels of overstaffing are observed for Ward C, compared to Ward A and Ward B. We explain this observation again due to the higher variation levels in patient demand data, but in addition to that the more limited PRN pool in Ward C limits the scheduling flexibility of the unit in the medium-term, causing larger differences between the nursing demand and supply (i.e. observe the PRN nurse ratio for Ward C in Table 5.17).

For the sake of generalizability of our results, we present some underlying factors that make short-term Stochastic Adjustments models more attractive to the nursing administration of any medical unit in Table 5.17. In addition, Table 5.18 presents a comparison of performance measures for the evaluated alternative models. Next, we provide insights on the impact of listed significant factors on the short-term stochastic adjustments model performance:

1. **Ratio of Unscheduled Patient Admissions to the Unit:** The more unscheduled patients admissions observed in a medical unit, the higher is the need for scheduling flexibility for the medium-term scheduling models and also the more frequently short-term schedule adjustments needed in order to better mimic patient demand for nursing. As presented in Table 5.17, Ward A observes mostly scheduled patient admissions, 61.3%, a significantly higher level compared to the scheduled admissions ratio of 32.6% for Ward B and 12.1% for Ward C. As observed in the results presented in Table 5.14, lower level of cost savings are observed compared to the values for Ward B in Table 5.15 and values for Ward C in Table 5.16. The higher level of scheduled admissions create a smoother patient demand data associated with lower levels of coefficient of variation, making it easier to match patient demand for nursing.

2. **PRN Nurse Ratio in the General Nurse Pool of the Unit:** PRN nurses are critical for cost savings due to the flexibility they provide for the minimization of under and overstaffing in the nursing shifts. Having a sizable PRN nurse body will alleviate the nursing shortages and provide the required flexibility for the nursing administration in the medical units with their scheduling process. As an example from our experiments, Ward C has the most limited PRN nurse body with 7.89% PRN nurse ratio and the short-term Stochastic Adjustments model brings significant cost savings to the unit. Due to the limited medium-term scheduling flexibility of the unit, more room for efficiency gains are left for the short-term adjustments for the unit. We conclude, the smaller PRN nurse ratio in a medical unit, the larger cost savings will be realized using the short-term Stochastic Adjustments model.

3. **Nurse Pool Size Compared to the Average Staff Size Utilized in a Shift:** The size of the available nurse pool is also an important factor, providing a capacity cushion for the desired nurse staffing levels. In order to get a non-unit specific measure, we divide the available nurse pool size for scheduling to the unit with the average staff size used in a shift throughout the staffing horizon. Smaller the ratio of used nurses to available nurses, the more scheduling flexibility the unit will observe due to the extra nurse availability for scheduling. For the PICU Wards, the used nurse ratios are in the range of 16 to 18%, very close to each other.

4. **Coefficient of Variation in the Patient Demand Data:** Coefficient of variation in patient data determines the required level of scheduling flexibility for the medical unit. We are using the coefficient of variation measure as a normalized measure with respect to the mean of patient demand data. Units observing higher levels of variation in patient demand will benefit the most using the Dynamic Staffing approach in the medium-term and the Stochastic Adjustments model in the short-term. The smoother the patient data for a medical unit, the lower cost savings will be observed using the two-stage Stochastic Adjustments model. In our case, Ward A, demonstrates lower levels of coefficient of variation compared to Wards B and C, as presented in Table 5.17, and due to that reason cost savings in terms of objective value are smaller for this unit.

In order to better understand the scheduling flexibility needs of the medical units, we evaluate some performance measures related to the frequency and size of the first and second-stage adjustment decisions and associated costs. Table 5.19 presents our experimental results related to these measures. We first provide the average medium-term no adjustment staff size for each experimental setting. We use this value to normalize some of the measures we present, since the unit nurse size would impact the magnitude of the adjustments. First measure we analyze is the average float pool request per shift. Ward A used around 0.5 requests per shift on average, a lower level compared to the 1.2 average requests per shift of Ward B. Ward C average float pool requests also realized around 0.6, similar to the Ward A patterns. The float pool requests are limited with three requests per shift and cost 20% higher than regular time FTE rate. In order to normalize with the unit size, we divide the average requests with the staff size used in the associated shift. The normalized results indicate higher levels of float pool request for Wards B and C, where we observe higher levels of patient demand variation. We also check the ratio of shifts with any amount of float pool request. Ward B observed the most requests around 52% to 55%, Ward C follows with 28.5% to 37.70% and Ward A average requests realized between 30.5% to 31.5%.

WARD A UPC = 1.5	**Stochastic Adjustments**	**Dynamic Staffing**	**Optimal Staffing**	**Fixed Staffing 19 Nurses**	**Fixed Staffing 20 Nurses**
FTE Staffing Cost	16,272.00	16,272.00	14,688.00	16,008.00	16,776.00
PRN Staffing Cost	2,926.00	2,926.00	3,524.40	3,454.00	3,696.00
1st Stage Staff Addition Cost	556.80				
1st Stage Staff Reduction Savings	-923.20				
Understaffing Penalty (Expected)	374.46				
Understaffing Penalty (Realized)	334.50	939.00	1,125.75	826.50	309.75
Objective Value (O.V.)	19,166.10	20,137.00	19,338.15	20,288.50	20,781.75
O.V. Comparison (%)	0.00%	5.07%	0.90%	5.86%	8.43%
Median Understaffing %	0.88%	0.00%	1.45%	0.00%	0.00%
Average Understaffing %	1.14%	3.01%	3.77%	2.64%	0.97%
Max Understaffing %	8.11%	15.79%	35.57%	15.56%	11.11%
Median Overstaffing %	0.00%	2.01%	0.00%	2.01%	7.38%
Average Overstaffing %	0.64%	7.59%	2.10%	8.45%	12.23%
Max Overstaffing %	38.18%	47.57%	24.39%	49.02%	56.86%

WARD A UPC = 2.0	**Stochastic Adjustments**	**Dynamic Staffing**	**Optimal Staffing**	**Fixed Staffing 19 Nurses**	**Fixed Staffing 20 Nurses**
FTE Staffing Cost	16,272.00	16,272.00	15,240.00	16,008.00	16,776.00
PRN Staffing Cost	3,036.00	3,036.00	3,502.40	3,454.00	3,696.00
1st Stage Staff Addition Cost	643.20				
1st Stage Staff Reduction Savings	-910.40				
Understaffing Penalty (Expected)	262.35				
Understaffing Penalty (Realized)	223.00	1,108.00	832.00	1,102.00	413.00
Objective Value (O.V.)	19,263.80	20,416.00	19,574.40	20,564.00	20,885.00
O.V. Comparison (%)	0.00%	5.98%	1.61%	6.75%	8.42%
Median Understaffing %	0.00%	0.00%	1.23%	0.00%	0.00%
Average Understaffing %	0.58%	2.67%	2.05%	2.64%	0.97%
Max Understaffing %	7.51%	15.29%	22.42%	15.56%	11.11%
Median Overstaffing %	0.00%	2.01%	0.00%	2.01%	7.38%
Average Overstaffing %	1.04%	7.75%	3.58%	8.45%	12.23%
Max Overstaffing %	30.91%	49.02%	60.00%	49.02%	56.86%

WARD A UPC = 3.0	**Stochastic Adjustments**	**Dynamic Staffing**	**Optimal Staffing**	**Fixed Staffing 19 Nurses**	**Fixed Staffing 20 Nurses**
FTE Staffing Cost	16,440.00	16,440.00	16,272.00	16,008.00	16,776.00
PRN Staffing Cost	3,212.00	3,212.00	3,159.20	3,454.00	3,696.00
1st Stage Staff Addition Cost	566.40				
1st Stage Staff Reduction Savings	-1,022.00				
Understaffing Penalty (Expected)	243.21				
Understaffing Penalty (Realized)	208.50	1,251.00	379.50	1,653.00	619.50
Objective Value (O.V.)	19,404.90	20,903.00	19,810.70	21,115.00	21,091.50
O.V. Comparison (%)	0.00%	7.72%	2.09%	8.81%	8.69%
Median Understaffing %	0.00%	0.00%	0.00%	0.00%	0.00%
Average Understaffing %	0.36%	2.00%	0.63%	2.64%	0.97%
Max Understaffing %	6.75%	12.73%	4.48%	15.56%	11.11%
Median Overstaffing %	1.27%	4.11%	2.56%	2.01%	7.38%
Average Overstaffing %	1.54%	8.88%	6.40%	8.45%	12.23%
Max Overstaffing %	30.91%	56.86%	56.86%	49.02%	56.86%

Table 5.14: Comparison of Two-Stage Stochastic Short Term Schedule Adjustments Model w.r.t. Medium-Term Schedules with No Adjustments - Ward A

WARD B **UPC = 1.5**	**Stochastic Adjustments**	**Dynamic Staffing**	**Optimal Staffing**	**Fixed Staffing 11 Nurses**	**Fixed Staffing 12 Nurses**
FTE Staffing Cost	10,128.00	10,128.00	10,416.00	9,480.00	10,200.00
PRN Staffing Cost	1,821.60	1,821.60	1,817.20	1,768.80	2,085.60
1st Stage Staff Addition Cost	1,694.80				
1st Stage Staff Reduction Savings	-310.00				
Understaffing Penalty (Expected)	417.69				
Understaffing Penalty (Realized)	397.50	2,414.25	1,612.50	3,234.00	2,196.75
Objective Value (O.V.)	13,731.90	14,363.85	13,845.70	14,482.80	14,482.35
O.V. Comparison (%)	0.00%	4.60%	0.83%	5.47%	5.47%
Median Understaffing %	1.49%	8.01%	2.22%	14.56%	6.80%
Average Understaffing %	1.99%	10.74%	7.19%	14.49%	9.56%
Max Understaffing %	13.73%	35.14%	88.06%	40.54%	35.14%
Median Overstaffing %	0.00%	0.00%	0.00%	0.00%	0.00%
Average Overstaffing %	0.93%	4.22%	3.05%	2.88%	5.98%
Max Overstaffing %	8.47%	57.14%	40.00%	62.96%	77.78%

WARD B **UPC = 2.0**	**Stochastic Adjustments**	**Dynamic Staffing**	**Optimal Staffing**	**Fixed Staffing 11 Nurses**	**Fixed Staffing 12 Nurses**
FTE Staffing Cost	10,200.00	10,200.00	11,424.00	9,480.00	10,200.00
PRN Staffing Cost	1,883.20	1,883.20	1,702.80	1,768.80	2,085.60
1st Stage Staff Addition Cost	1,728.80				
1st Stage Staff Reduction Savings	-294.40				
Understaffing Penalty (Expected)	334.91				
Understaffing Penalty (Realized)	305.00	3,037.00	1,093.00	4,312.00	2,929.00
Objective Value (O.V.)	13,822.60	15,120.20	14,219.80	15,560.80	15,214.60
O.V. Comparison (%)	0.00%	9.39%	2.87%	12.58%	10.07%
Median Understaffing %	0.00%	6.80%	0.00%	14.56%	6.80%
Average Understaffing %	1.16%	10.08%	3.54%	14.49%	9.56%
Max Understaffing %	13.73%	35.14%	45.95%	40.54%	35.14%
Median Overstaffing %	0.00%	0.00%	2.36%	0.00%	0.00%
Average Overstaffing %	1.47%	4.58%	7.00%	2.88%	5.98%
Max Overstaffing %	18.52%	62.96%	64.62%	62.96%	77.78%

WARD B **UPC = 3.0**	**Stochastic Adjustments**	**Dynamic Staffing**	**Optimal Staffing**	**Fixed Staffing 11 Nurses**	**Fixed Staffing 12 Nurses**
FTE Staffing Cost	10,200.00	10,200.00	12,072.00	9,480.00	10,200.00
PRN Staffing Cost	2,041.60	2,041.60	1,632.40	1,768.80	2,085.60
1st Stage Staff Addition Cost	1,776.80				
1st Stage Staff Reduction Savings	-322.00				
Understaffing Penalty (Expected)	275.25				
Understaffing Penalty (Realized)	258.00	4,270.50	954.00	6,468.00	4,393.50
Objective Value (O.V.)	13,954.40	16,512.10	14,658.40	17,716.80	16,679.10
O.V. Comparison (%)	0.00%	18.33%	5.05%	26.96%	19.53%
Median Understaffing %	0.00%	5.42%	0.00%	14.56%	6.80%
Average Understaffing %	0.66%	9.41%	2.00%	14.49%	9.56%
Max Understaffing %	13.73%	35.14%	40.50%	40.54%	35.14%
Median Overstaffing %	1.05%	0.00%	4.71%	0.00%	0.00%
Average Overstaffing %	2.11%	5.05%	10.65%	2.88%	5.98%
Max Overstaffing %	18.52%	62.96%	88.14%	62.96%	77.78%

Table 5.15: Comparison of Two-Stage Stochastic Short Term Schedule Adjustments Model w.r.t. Medium-Term Schedules with No Adjustments - Ward B

WARD C UPC = 1.5	Stochastic Adjustments	Dynamic Staffing	Optimal Staffing	Fixed Staffing 12 Nurses	Fixed Staffing 13 Nurses
FTE Staffing Cost	10,200.00	10,200.00	9,456.00	10,368.00	11,232.00
PRN Staffing Cost	1,447.60	1,447.60	1,289.20	1,425.60	1,425.60
1st Stage Staff Addition Cost	996.40				
1st Stage Staff Reduction Savings	-676.40				
Understaffing Penalty (Expected)	321.03				
Understaffing Penalty (Realized)	334.50	1,455.75	1,684.50	1,304.25	831.00
Objective Value (O.V.)	12,302.10	13,103.35	12,429.70	13,097.85	13,488.60
O.V. Comparison (%)	0.00%	6.51%	1.04%	6.47%	9.64%
Median Understaffing %	0.86%	0.00%	3.61%	0.00%	0.00%
Average Understaffing %	1.83%	6.95%	8.40%	6.11%	3.76%
Max Understaffing %	18.11%	51.15%	69.47%	44.06%	32.87%
Median Overstaffing %	0.00%	0.00%	0.00%	1.05%	9.47%
Average Overstaffing %	1.29%	9.80%	2.60%	10.29%	15.59%
Max Overstaffing %	28.00%	76.00%	43.08%	74.55%	89.09%
WARD C UPC = 2.0	**Stochastic Adjustments**	**Dynamic Staffing**	**Optimal Staffing**	**Fixed Staffing 12 Nurses**	**Fixed Staffing 13 Nurses**
FTE Staffing Cost	10,464.00	10,464.00	10,032.00	10,368.00	11,232.00
PRN Staffing Cost	1,438.80	1,438.80	1,214.40	1,425.60	1,425.60
1st Stage Staff Addition Cost	914.00				
1st Stage Staff Reduction Savings	-703.60				
Understaffing Penalty (Expected)	252.22				
Understaffing Penalty (Realized)	274.00	1,659.00	1,654.00	1,739.00	1,108.00
Objective Value (O.V.)	12,387.20	13,561.80	12,900.40	13,532.60	13,765.60
O.V. Comparison (%)	0.00%	9.48%	4.14%	9.25%	11.13%
Median Understaffing %	0.00%	0.00%	0.41%	0.00%	0.00%
Average Understaffing %	1.08%	5.88%	6.02%	6.11%	3.76%
Max Understaffing %	18.11%	45.04%	66.39%	44.06%	32.87%
Median Overstaffing %	1.05%	1.15%	2.11%	1.05%	9.47%
Average Overstaffing %	1.92%	10.85%	5.04%	10.29%	15.59%
Max Overstaffing %	28.00%	76.00%	80.00%	74.55%	89.09%
WARD C UPC = 3.0	**Stochastic Adjustments**	**Dynamic Staffing**	**Optimal Staffing**	**Fixed Staffing 12 Nurses**	**Fixed Staffing 13 Nurses**
FTE Staffing Cost	11,112.00	11,112.00	10,896.00	10,368.00	11,232.00
PRN Staffing Cost	1,425.60	1,425.60	1,174.80	1,425.60	1,425.60
1st Stage Staff Addition Cost	725.60				
1st Stage Staff Reduction Savings	-944.00				
Understaffing Penalty (Expected)	184.07				
Understaffing Penalty (Realized)	225.00	1,782.00	1,473.00	2,608.50	1,662.00
Objective Value (O.V.)	12,544.20	14,319.60	13,543.80	14,402.10	14,319.60
O.V. Comparison (%)	0.00%	14.15%	7.97%	14.81%	14.15%
Median Understaffing %	0.00%	0.00%	0.00%	0.00%	0.00%
Average Understaffing %	0.57%	4.07%	3.36%	6.11%	3.76%
Max Understaffing %	18.11%	35.77%	63.36%	44.06%	32.87%
Median Overstaffing %	1.82%	5.76%	5.80%	1.05%	9.47%
Average Overstaffing %	3.01%	14.76%	10.00%	10.29%	15.59%
Max Overstaffing %	30.91%	89.09%	104.62%	74.55%	89.09%

Table 5.16: Comparison of Two-Stage Stochastic Short Term Schedule Adjustments Model w.r.t. Medium-Term Schedules with No Adjustments - Ward C

Patient Admission Type			
	Ward A	**Ward B**	**Ward C**
Scheduled Admissions	61.3%	32.6%	12.1%
Unscheduled Admissions	38.7%	67.4%	87.9%

PRN Nurse Ratio			
	Ward A	**Ward B**	**Ward C**
# PRN Nurses	16	10	6
Total Nurse Pool	103	71	76
PRN Ratio	15.53%	14.08%	7.89%

Nurse Pool Flexibility			
	Ward A	**Ward B**	**Ward C**
Avg. Staff Size	18.88	11.82	12.31
Staff Pool	103	71	76
Ratio	18.33%	16.65%	16.20%

Coefficient of Variation in Patient Data			
	Ward A	**Ward B**	**Ward C**
Full Dataset	11.26%	20.84%	20.88%
Sample	12.17%	18.02%	18.64%

Table 5.17: Significant Factors Impacting Short-Term Stochastic Adjustments Model Performance

Unit	UPC	Unsch. Adm. %	PRN Ratio	Pool Flex.	C.V. Demand	Optimal Staffing	Dynamic Staffing	Fixed Lower	Fixed Higher	Avg. Savings
Ward A	1.5	38.7%	15.53%	18.33%	12.17%	0.90%	5.07%	5.86%	8.43%	5.07%
Ward B	1.5	67.4%	14.08%	16.65%	18.02%	0.83%	4.60%	5.47%	5.47%	4.09%
Ward C	1.5	87.9%	7.89%	16.20%	18.64%	1.04%	6.51%	6.47%	9.64%	5.92%
Ward A	2.0	38.7%	15.53%	18.33%	12.17%	1.61%	5.98%	6.75%	8.42%	5.69%
Ward B	2.0	67.4%	14.08%	16.65%	18.02%	2.87%	9.39%	12.58%	10.07%	8.73%
Ward C	2.0	87.9%	7.89%	16.20%	18.64%	4.14%	9.48%	9.25%	11.13%	8.50%
Ward A	3.0	38.7%	15.53%	18.33%	12.17%	2.09%	7.72%	8.81%	8.69%	6.83%
Ward B	3.0	67.4%	14.08%	16.65%	18.02%	5.05%	18.33%	26.96%	19.53%	17.47%
Ward C	3.0	87.9%	7.89%	16.20%	18.64%	7.97%	14.15%	14.81%	14.15%	12.77%

Table 5.18: Significant Factors Impacting Short-Term Stochastic Adjustments

	Ward A			Ward B			Ward C		
	UPC Levels			**UPC Levels**			**UPC Levels**		
Performance Measures	**1.5**	**2.0**	**3.0**	**1.5**	**2.0**	**3.0**	**1.5**	**2.0**	**3.0**
Avg. Medium-Term No Adjustment Staff Size	18.78	18.88	19.21	11.69	11.82	11.96	11.42	11.68	12.31
Avg. Float Pool Request per Shift	0.460	0.532	0.468	1.194	1.206	1.242	0.690	0.643	0.532
Avg. Float Pool Request per Shift per Nurse	0.025	0.028	0.024	0.103	0.103	0.104	0.064	0.058	0.045
Ratio of Shifts with Float Pool Request	31.35%	31.35%	30.56%	52.38%	51.98%	55.56%	37.70%	32.94%	28.57%
Avg. FTE-On-Call Request per Shift	0.000	0.000	0.000	0.202	0.214	0.218	0.103	0.091	0.060
Avg. FTE-On-Call Request per Shift per Nurse	0.000	0.000	0.000	0.018	0.018	0.018	0.010	0.009	0.005
Ratio of Shifts with FTE-On-Call Request	0.00%	0.00%	0.00%	15.48%	16.67%	17.86%	6.35%	5.95%	4.76%
Avg. PRN-On-Call Request per Shift	0.000	0.000	0.000	0.004	0.008	0.008	0.028	0.020	0.008
Avg. PRN-On-Call Request per Shift per Nurse	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.002	0.001
Ratio of Shifts with PRN-On-Call Request	0.00%	0.00%	0.00%	0.40%	0.40%	0.40%	1.98%	1.59%	0.79%
Avg. FTE Shift Cancel per Shift	0.877	0.861	0.933	0.282	0.258	0.270	0.718	0.734	1.063
Avg. FTE Shift Cancel per Shift per Nurse	0.046	0.045	0.049	0.024	0.022	0.022	0.061	0.062	0.083
Ratio of Shifts with FTE Shift Cancel	32.14%	31.35%	33.33%	17.46%	16.67%	16.67%	35.71%	38.89%	46.03%
Avg. PRN Shift Cancel per Shift	0.238	0.238	0.298	0.091	0.095	0.115	0.107	0.123	0.095
Avg. PRN Shift Cancel per Shift per Nurse	0.013	0.013	0.015	0.008	0.008	0.010	0.009	0.011	0.008
Ratio of Shifts with PRN Shift Cancel	17.86%	18.25%	21.83%	9.13%	9.52%	10.71%	10.71%	12.30%	9.52%
Ratio of Shifts with First-Stage Adjustments	76.98%	76.59%	80.95%	74.21%	73.81%	78.97%	78.57%	77.38%	77.78%
Avg. Mandatory Overtime in 2nd Stage (hrs.)	2.484	0.442	0.276	1.052	0.605	0.341	0.885	0.544	0.298
Avg. Mandatory Overtime Nurse Request	0.621	0.111	0.069	0.263	0.151	0.085	0.221	0.136	0.074
Ratio of Shifts with Mandatory Overtime	42.46%	36.11%	25.00%	57.94%	41.67%	27.38%	50.79%	35.32%	20.24%
Avg. Mandatory Overtime Cost Per Shift	3.726	0.885	0.827	1.577	1.210	1.024	1.327	1.087	0.893

Table 5.19: Evaluation of Performance Measures for the Two-Stage Stochastic Short-Term Schedule Adjustments Model

Second adjustment mechanism, available to the nursing administration, is the FTE-On-Call requests. Our results indicate that, Ward A didn't require additional nurses from FTE-On-Call for any of the shifts. Ward B used the FTE-On-Call requests between 15.48% to 17.86% of the shifts, with an average request of 0.2 nurses in a shift. Ward C FTE-On-Call requests are below 0.1 nurses per shift on average for the three UPC levels, and the unit used the available option in 4.76% to 6.35% of the shifts throughout the staffing horizon. Cost of additional FTE nurses requested from available on-call list for one shift is 20% more costly than the regular FTE rate, and limited with two requests in one shift. The third adjustment mechanism available to the unit nursing administration is the PRN-On-Call requests. Additional PRN nurses requested from available on-call list for one shift is 30% more costly than the regular FTE rate, and limited with two requests in one shift. Being the most costly staff addition option in the first stage adjustments, PRN-On-Call requests are the last option the optimization model uses. Similar to FTE-On-Call observation, Ward A didn't require any additional nurses from the PRN-On-Call list for any shift. For Ward B, the adjustment option is used for 0.4% of all shifts. Ward C used the PRN-On-Call requests in 0.79% to 1.98% of the shifts.

There are two additional adjustment mechanism, available to the nursing administration, that are used to reduce the staff size in the unit. These mechanisms are used to reduce observed overstaffing levels in the medical unit in order to better matching the patient demand via FTE and PRN shift cancellations. Savings incurred by floating, reassigning or cancelling one nurse for the upcoming shift is assumed to be 80% for the FTE nurses and and 82% for the PRN nurses. We do not introduce any upper limit on shift cancellations in our model. Average number of cancelled FTE shifts for Ward A is realized in the range of 0.86 to 0.93 for various UPC levels. Ward B average FTE shift cancellations occurred in smaller amounts, compared to Ward A, in the range of 0.25 to 0.28 cancellations per shift. Ward B observed larger staff additions and fewer shift cancellations, which can be attributed to the higher than usual sample patient demand level that is reflected within the summary statistics in Table 4.5 (i.e. mean patient demand for Ward B for the full dataset 45.55 hrs., sample data used in the optimization experiments 51.54 hrs.). Ward C FTE shift cancellations occurred in the range of 0.7 to 1.0. When we analyze the ratio of shifts with FTE shift cancellations, Ward A observed ratios in the range of 31% to 33%, Ward B in the range of 16% to 17% and Ward C in the range of 35% to 46%. The larger ratio of shift cancellations for Ward C can be attributed to the smaller PRN nurse pool for the unit (i.e. see Table 5.17, Ward C has a PRN Nurse ratio of 7.89%, much lower than the 15.53% of Ward A and 14.08% of Ward B), which limited the scheduling flexibility of the unit at the medium-term scheduling phase. The less a medical unit has scheduling flexibility in the medium-term, the more frequent short-term adjustments will be needed as demonstrated for the Ward C example. Ratio of shifts with PRN cancellations occurred at the highest rate for Ward A, 17.8% to 21.8%. Ward B and Ward C followed Ward A with similar ratios between 9.13% to 12.30%. FTE shift cancellations occurred more frequently compared to PRN nurses due to the larger nurse body in this profile and higher savings ratio.

Combining all types of first-stage schedule adjustments, Table 5.19 suggests a ratio of 75% to 80% for the shifts with any sort of short-term modifications for all PICU wards under all UPC levels. Our results indicate the nurse schedules developed in the medium-term will need some sort of adjustment in the short-term for more than 75% of the shifts throughout the staffing horizon. This observation demonstrates the crucial need for short-term schedule adjustment models in order to satisfy the most needed scheduling flexibility in the medical units. Our results also indicate, the models that use historical patient data to estimate stochastic patient demand can bring cost savings of 5% to 20% for the medical units. Table 5.19 also present the usage of mandatory nurse overtime in the second-stage as recourse decisions for the developed two-stage Stochastic Adjustments model. Our results indicate significantly lower levels of mandatory nurse overtime usage in the second-stage, less than 0.3 nurses per shift for most of the experiments. Ratio of shifts with some amount of mandatory overtime realized within the 20% to 58% range. Lower levels of second-stage mandatory overtime usage reflects the successful implementation of first-stage adjustments within the developed two-stage Stochastic Adjustments model. The developed probability matrices for the medical units reflected an accurate representation of the stochastic nature in the patient demand data, which bring the presented cost savings and lower understaffing levels. We conclude that, our experimental results demonstrate the potential benefits of using short-term Stochastic Adjustment model for the medical units reflected in the presented performance measures including lower objective values and understaffing levels. We also identified significant factors that impact the scheduling flexibility needs of the medical units, such as the ratio of unscheduled patient admissions to the unit, PRN nurse ratio in the general nurse pool of the unit, nurse pool size compared to the average staff size used in a shift and coefficient of variation in the patient demand data. These factors also relate to the frequency of short-term adjustments as presented in our experimental results.

5.7 Conclusions

In Chapter 5 we used nursing schedules from the medium-term planning phase to determine two-stage short-term staffing adjustments in the medical units for the upcoming nursing shift. Our proposed adjustments are first determined fort the beginning of each nursing shift for the upcoming 4-hour shift. After observing actual patient demand for nursing at the start of the next shift, we make our final staffing adjustments to meet the patient demand for nursing. We model six different adjustment options for the two-stage stochastic programming model: five available as first-stage decisions and one available as the second-stage recourse decision. In intensive care environments 30 to 70% of patient admissions are unscheduled (unknown 12 hours ahead of the actual admission time) and have diverse patient acuities. Because of the fluctuating patient demand, nursing administrations constantly face the challenge of adjusting previously-created nurse schedules. In Chapter 4 we developed alternative medium-term integrated staffing and scheduling policies. As our results suggest, matching perfectly the patient demand with medium-term planning in a dynamic intensive care environment is not an easy task. We develop a two-stage stochastic integer programming model which minimizes total nurse staffing costs and the cost of adjustments to the original medium-term schedules, while ensuring coverage of nursing demand.

A stochastic integer programming model is attractive because the number of unscheduled patient admissions, as well as acuity assignments, in the upcoming shift is unknown at the time of adjustments. Historical unscheduled patient admissions and acuity distributions are used to calculate an expected nursing requirement. The calculated nursing requirement is compared to the provided nursing hours after the short-term schedule adjustments. We model the current 4-hour nursing shift as the first stage of adjustments, when the actual patient demand is not revealed. The upcoming nursing shift is the second stage of adjustments, when the actual patient demand has been fully revealed. In the second stage we make corrections (i.e., requesting mandatory nurse overtime) to cover patient demand. Using two-stage stochastic short-term staffing adjustment model, we study this esearch question: "*Can short-term schedule modifications that are based on decisions obtained from the two-stage stochastic integer programming model bring cost savings and reduction in understaffing levels, compared to existing medium-term staffing plans, during the nursing shifts*?"

We also evaluate the scheduling-flexibility needs of the medical unit using the developed two-stage stochastic adjustments model. The number of first-stage adjustments, the ratio of shifts with any adjustments, provides insight regarding the flexibility needed when building and adjusting nurse schedules. We run the developed two-stage Stochastic Adjustments model for each nursing shift throughout the staffing horizon. Each experimental run consists of multiple optimality cuts, and for each cut the second-stage subproblem is solved for each scenario given the Case ID information. Our results indicate that the Stochastic Adjustments model provides the least costly solution under all UPC options when compared to medium-term no-adjustment models. Even when compared to the Optimal Staffing model, which assumes patient demand data is perfectly known to the administration, the Stochastic Adjustments model reduces costs. Cost savings are in the range of 1% to 8% for various UPC levels when compared with the hypothetical Optimal Staffing option. Our results also indicate that the models that use historical patient data to estimate stochastic patient demand can reduce costs 5% to 20%. Considering the significant portion of hospital operation budgets represented by staffing costs, the 5% to 20% savings obtained using the short-term Stochastic Adjustments model appear promising.

We present some underlying factors that make short-term Stochastic Adjustments models more attractive to the general nursing administration community. According to our results, the more unscheduled patients admissions there are, the greater the need is for scheduling flexibility for the medium-term scheduling models, and the more often short-term schedule adjustments are needed to better mimic patient demand. Higher levels of scheduled admissions create smoother patient demand data associated with lower levels of coefficient of variation, making it easier to match patient demand for nursing. PRN nurses are critical for cost savings due to the flexibility they provide for the minimization of under- and over-staffing in the nursing shifts. Having a sizable PRN nurse body will alleviate nursing shortages and provide the required flexibility for the nursing administration in the scheduling process. Because of the limited medium-term scheduling flexibility of the unit, more room is left for efficiency gains for short-term adjustments. We conclude that the smaller the PRN nurse ratio is, the greater the cost savings will be using the short-term Stochastic Adjustments model. The size of the available nurse pool is also an important factor, providing a capacity cushion for the desired nurse-staffing levels. To determine a non-unit-specific measure, we divide the nurse pool size available for scheduling to the unit by the average staff size used in a shift throughout the staffing horizon. The smaller the ratio of used nurses to available nurses, the more scheduling flexibility will be observed because of the extra nurse availability for scheduling. The coefficient of variation in patient data also affects the required level of scheduling flexibility for the medical unit. Units observing higher levels of variation in patient demand will benefit the most in the medium term using the Dynamic Staffing approach and in the short term using the Stochastic Adjustments model. The smoother the patient data for a medical unit, the lower cost savings that will be observed using the two-stage Stochastic Adjustments model.

To understand better the scheduling flexibility needs of the medical units, we evaluate some performance measures related to the frequency and size of the first- and second-stage adjustment decisions and their associated costs. Combining all types of first-stage schedule adjustments, our results indicate the nurse schedules developed in the medium term will need some sort of adjustment in the short term in more than 75% of the shifts for the staffing horizon. This observation demonstrates the crucial need for short-term schedule adjustment models in order to satisfy the most-needed scheduling flexibility. The probability matrices we developed reflect an accurate representation of the stochastic nature in the patient demand data, bringing the cost savings and lower understaffing levels we present here. Our results also indicate that the models using historical patient data to estimate stochastic patient demand can bring cost savings of 5% to 20% for the medical units. Our results indicate significantly lower levels of mandatory nurse overtime to be needed in the second stage using the Stochastic Adjustments model: fewer than 0.3 nurses per shift in most of the experiments. Lower levels of second-stage mandatory overtime usage reflects the successful implementation of first-stage adjustments within the two-stage Stochastic Adjustments model.

Chapter 6

Conclusions & Future Work

Recent estimates suggest that national health care expenditures increased between 5% and 6% in 2014 and 2015, and are estimated at $3.2 trillion. These increases are substantially higher than inflation, and some estimates suggest that similar increases will continue through 2024 (Bauchner and Fontanarosa, 2016). Nursing care is identified as the single biggest factor in both the cost of hospital care and patient satisfaction (Yankovic and Green, 2011). Several studies have shown that there exists a strong association between nurse staffing levels and patient outcomes. When a nursing unit is chronically short-staffed, nurses are forced to keep up an intense pace in order to ensure that patients receive timely care. Over time this can result not only in nurse burnout, patient dissatisfaction, and even medical errors. Improved accuracy in the allocation of nursing staff can mitigate these operational risks and improve patient outcomes. Because registered nurse wages and benefits constitute approximately 25% of all hospital costs (Maenhout and Vanhoucke, 2013b), hospitals have tried to reduce nurse staffing in order to reduce costs and increase profitability (Rivers et al., 2005). However, projections suggest that by 2020 approximately 36% of nursing positions in the United States will remain unfilled (Wright and Bretthauer, 2010). Buerhaus et al. (2009) suggests that the U.S. nursing shortage could reach one-half million by 2025. Rising healthcare costs and increasing nurse shortages make cost-effective nurse staffing vital (Kortbeek et al. 2015). This shortage of nurses has attracted considerable attention due to its direct impact on the quality of patient care (Punnakitikashem et al. 2013).

Although nursing care is identified as the single biggest factor in both the cost of hospital care and in patient satisfaction, there is still widespread dissatisfaction with the current methods of determining nurse staffing levels, including the most common one of using minimum nurse-to-patient ratios (Yankovic and Green, 2011).

In many hospitals, staffing levels are a result of historical development, given that hospital managers lack the tools to base current staffing decisions on information about future patient demand (Kortbeek et al. 2015). According to Paul and MacDonald (2013), nurse shortage implications extend beyond healthcare quality to health economicsl. Inaccurate estimates of the nursing resources required to satisfy patient demand in a hospital environment can make this already-challenging problem worse. Mandatory nurse-to-patient ratios implemented in some states, while simplifying the estimation of demand, also create a risk of under- or over-estimating nurse resource requirements. For management, better and more flexible scheduling can help retain nurses and aid in their recruitment, reduce tardiness and absenteeism, increase morale and productivity, and provide better patient service and safety. For all these reasons, the development of methodologies and decision support tools to improve nurse scheduling is still a strategic problem for hospital administrations.

Staffing requirements are the result of a complex interaction between care unit size, nurse-to-patient ratios, bed census distributions, and quality-of-care requirements. An optimal configuration depends strongly on the characteristics of a specific case study (Kortbeek et al., 2015a). Green et al. (2013) indicates that establishing the appropriate nursing level for a specific hospital unit during a specific shift is complicated by the need to make staffing decisions well in advance (e.g., six to eight weeks) of that shift. Also, labor constraints concerning the number of consecutive and weekend shifts worked per nurse, vacation schedules, personal days, and preferences further complicate matters (Miller et al. 1976, Wright et al. 2006). Management of the nursing workforce is typically seen as a multi-phase sequential planning and control process that basically consists of staffing, shift scheduling, and allocation phases (Maenhout and Vanhoucke, 2013). The decisions made in each phase of this hierarchical process constrain subsequent phases. Burke et al. (2013) also indicates that creating rosters is a challenging search problem which requires the satisfaction of many constraints and the balancing of a variety of requirements. This time-consuming and frustrating duty often falls to a head nurse who would rather be concentrating on his or her primary duty of caring for patients.

Enactment of the Affordable Care Act (ACA) resulted in more and sicker patients entering the healthcare system. The subsequent increases in nursing workload has led to a higher risk of nurse burnout in already short-staffed environments. These developments force hospital administrations to gain better control of understaffing levels in medical units while balancing staffing costs. In this dissertation we study strategic nurse allocation policies under dynamic patient demand. In Chapter 1, we present our problem and provide our research questions. In Chapter 2 we review literature on the research topics of this dissertation. In the first section of this chapter we review literature on nursing workload measurement approaches. The staffing and scheduling of healthcare personnel involves determining the number of nurses with the required skills and assigning them to the predetermined shifts to meet predicted patient demand. This process is also called workforce planning and scheduling in other personnel planning environments. In the second section we discusses related literature. In the third section we provide a comprehensive review of the nurse staffing and scheduling literature in journals that focus on Operations Management and Operations Research. This review covers research areas related to stages of nurse planning, nurse staffing policy options, cyclic and non-cyclic scheduling of nurses, algorithmic solution approaches to the nurse staffing and scheduling problems, cross-utilization of nurses in medical units, nurse absenteeism, scheduling under uncertain demand using stochastic solution approaches, short-term nurse staffing, and nurse-to-patient assignment.

Many patient care units face challenges in trying to accurately estimating the number of nurses needed on a daily basis. Analytical predictive methods, which complement intuition and experience-based decisions on nurse staffing and workload, would help decrease unplanned last-minute scheduling for nurses, and would improve healthcare delivery by providing more efficient nurse allocation. One factor making such estimates difficult is the lack of a decision support tool for understanding the distribution of admissions to healthcare facilities. We aim to statistically evaluate the existing staff allocation system of a Pediatric Intensive Care Unit (PICU) using clinical operational data, and then to develop a predictive model for estimating the number of admissions. We analyzed clinical operational data of three PICU wards for a period of 44 months. The existing staff allocation models for these three units do not accurately estimate the required number of nurses. It is difficult to understand the pattern and frequency of admissions, particularly those admissions that are not known 12 hours in advance. It is also difficult to understand the pattern and frequency of admissions, expecially those admissions that are not known twelve hours in advance. In Chapter 3 we first show that these "unknown" admissions can actually be predicted fairly accurately by fitting the pattern of admissions to a Poisson distribution. The purpose of Chapter 3 is to provide a framework for accurately estimating the number of nurses required in Intensive Care Units (ICUs) on a given day.

Determining accurate nurse staffing levels has been a topic of great interest because of healthcare quality requirements, financial constraints, limited resources, patient safety requirements, and nurse shortages. In Chapter 3 we confirm the influence of unscheduled admissions on the accuracy of predicting PICU admissions. We show that estimating the number of unscheduled admissions by obtaining the probability distribution of historical unscheduled admissions will provide higher precision than using only experience and intuition. We propose a convenient, objective, simulation-based statistical methodology to assist healthcare providers in estimating the number of admissions and required number of nurses. Additional research should be conducted to understand the nature of scheduled admissions before StaffAssist can be refined. Investigators identified gaps between the expected admission and discharge numbers, and additional research will focus on understanding discharge patterns to resolve this discrepancy. This chapter aims to improve nurse staffing models, which will enable nurses to deliver better quality care and to improve patient outcomes. In general, we have not found literature, especially PICU literature, explaining how to predict unscheduled admissions. The model we developed in this chapter is generalizable for implementation in other intensive care units.

Nurse rostering is an NP-hard combinatorial problem. This makes it extremely difficult to efficiently solve real life problems because of their size and complexity. Usually real problem instances face complicated work rules related to safety and quality of service issues, as well as rules about preferences of the personnel. In order to avoid the size and complexity limitations, we use a two-phase solution procedure in Chapter 4. In Phase 1 of the procedure we generate feasible FTE nurse schedules for a staffing horizon of six weeks while satisfying constraints imposed by the nurse profile. Pre-generated schedules eliminate the increased number of constraints and reduce the number of decision variables of the integrated nurse staffing and scheduling model. In Phase 2 we assign FTE nurses to the pre-generated feasible nurse schedules and PRN nurses to the nursing shifts using mixed-integer optimization models. When the nursing administration prepares the medium-term nurse schedules for the next staffing cycle (six weeks in our case) one to two months before the actual patient demand is realized, target staffing levels for the upcoming nursing shifts are typically determined by a general average staffing level for the nursing care needs in the medical units. The optimization model in this chapter recommends initial staffing plans and schedules for a six-week staffing horizon, given a variety of nurse groups and nursing shift assignment types, in the PICU medical units.

We first prepare a “heat map” of patient census and ADT activity in the medical units for the dynamic staffing policy option. To do so we estimate a monthly seasonality index for Patient Census, Acuity, and ADT Activity. Then, we estimate Patient Census, Acuity and ADT Activity averages for all “Day of Week” and “Shift of the Day” combinations. This heat map of patient demand is generated by multiplying the monthly seasonality factors with the historical “Day-Shift” averages for the medical units. We used the heat map and the mixed-integer optimization models to analyze whether dynamic staffing policies outperform the currently-used fixed staffing policy. We also compare the performance of both options with the optimal staffing scheme reached by the actual patient data. We also include a novel methodology for estimating nurse workloads by considering patient census, acuity and activity in the unit. The dynamic staffing policy we propose uses historical patient demand data to suggest a non-stationary staffing scheme during the staffing horizon. We test the fixed staffing policy alternative with various staffing level options (i.e, the staffing of 11, 13 or 15 nurses). For the dynamic staffing alternative we prepare a heat map of patient census and acuity, as well as admission/discharge/ (ADT) activity in the PICUs (as an example) and compare the performance of the dynamic heat map based policy against the alternative fixed staffing policies. We compare the performance of both nurse allocation policy options (in terms of cost savings and understaffing ratios) to the optimal staffing scheme reached by the actual patient data . This allows us to study our first research question in Chapter 4: "*Do dynamic medium-term nurse staffing policies that use patient demand forecasts outperform the historically-used fixed staffing policy for the intensive care medical units?*"

Our results suggest that the total objective function cost for the optimization experiments is either slightly reduced or kept stable as we increase the number of available schedules for each nurse profile from 4 to 256. We conclude that feeding the alternative staffing models with 256 schedules per FTE profile (i.e. 7,680 total different schedules for the optimization model) is sufficiently large for providing schedule diversity. Even four maximally different schedules per nurse profile approach seems to provide efficient solutions. Further increases in the NAS, above 256 schedules per nurse profile, will not bring any cost savings but will increase the problem complexity, hurting the performance of the developed models. With regard to the staffing policy evaluation, our results for the experiments we conducted with the three PICU wards suggest that the performance of Dynamic Staffing policy is mostly superior to the Fixed Staffing alternatives. The performances are similar fpr a few problem instances in terms of understaffing percentages and total costs. The power of the Dynamic Staffing policy lies in the accuracy of the forecasted heat map. As the forecasting performance in preparing the unit-specific heat map is improved, more cost savings and reduced understaffing percentages will be observed. For the Fixed Staffing policy we must note that a perfectly stable staff size may not be feasible in many cases, especially thos with a limited number of PRN nurses. Also, our Fixed Staffing modeling approach provides a reliable and efficient way of scheduling the nursing workforce. Medical units with higher variation in patient demand levels would benefit the most by using the Dynamic Staffing policy proposed in this study.

As nurse workload increases because of the nursing shortage issues, overtime is becoming more of a burden on nursing staff. Nurses cite undesirable schedules and overtime as primary reasons for burnout (Aiken et al., 2002). Unsatisfactory working conditions and policies also contribute to higher turnover rates (Aiken et al., 2002; Cline, Reilly & Moore, 2003). Jones (2007) suggest that the cost of turnover in the United States is approximately 1.2-1.3 times the average annual salary for each vacancy. U.S. hospitals spend approximately \$300,000 annually for each 1% increase in the turnover rate (Price Waterhouse Coopers, 2007). Some U.S. lawmakers proposed legislation that limits the use of overtime and the number of patients that each nurse is assigned to. There are 21 states with restrictions on the use of overtime (American Nurses Association, 2011). In Chapter 4 we introduce the concept of an "understaffing penalty" as a mechanism to control the understaffing in the medical units, avoid nurse burnout, and make the job more appealing to new RN candidates. We analyze how various levels of understaffing penalty (i.e., the cost of understaffed hours given as a ratio to the cost to the FTE nurse staffing) affect outcomes (i.e., costs and understaffing percentages) . We also evaluate the impact of the number of available schedules (NAS) per FTE nurse profile on the objective function costs and understaffing ratios in the medical units. We explore whether there exists a saturation level for the NAS, at which increases in the NAS do not bring any additional cost savings. To study these aspects of the medium-term nurse staffing and scheduling problem, we ask our second research question: "*How do we control the understaffing levels in the medical units which often trigger nurse burnout and medical errors?*"

The results of experiments using the Dynamic Staffing policy suggest using understaffing penalty cost (UPC) as a reliable mechanism for controlling understaffing ratios. Depending on the tolerance levels of understaffing for the medical unit, the nursing administration can determine the UPC level to use. For unexpectedly-high patient demand periods, higher understaffing levels may be observed, as the Dynamic Staffing policy is based on historical patient -demand-based heat maps. Heat maps can be adjusted as new demand patterns are observed for new staffing horizons. As the accuracy of patient demand forecasts is enhanced, better performance outcomes will be achieved using Dynamic Staffing. Historically-employed Fixed Staffing policies do not provide the required staff size flexibility to alleviate understaffing, triggering nurse burnout. Fixed Staffing policies will increase the need for short-term schedule adjustment costs in order to better match the patient demand due to the static nature of staff size. All PRN nurses were assigned to work in all experiments, demonstrating that PRN nurses are critical for cost savings because of the flexibility they provide for the minimization of under- and over-staffing in the nursing shifts. We conclude that having a sizable PRN nurse body will alleviate the nursing shortages and provide the required flexibility for the nursing administrations in their scheduling process. Nursing administration can use the results of medium-term staffing experiments for long-term planning to determine whether the current nurse pool is large enough to satisfy patient demand. Our analysis regarding objective function cost elements also suggests that Dynamic Staffing provides the required staff-size flexibility that reduces staffing costs while balancing understaffing risks.

Nurse schedules are constructed well before actual patient demand for nursing is observed. In an environment where 30 to 70% of patient admissions are unscheduled, are unknown 12 hours before the actual admission time, and feature diverse patient acuities, the nursing administration constantly faced the challenge of adjusting the pre-developed nurse schedules in the short term. When a medical unit is understaffed, staffing alternatives available to the administration include: (1) requesting nurses from the general float pool of the hospital; (2) using on-call nurses (i.e., FTE overtime and additional PRN hours); and (3) asking nurses on the current shift to stay for the next shift (requiring that overtime be paid. When the scheduled nursing hours are greater than the hours demanded by the existing patients, the nurse manager (or the charge nurse) can: (1) float the nurse to another unit; (2) reassign her to a later day in the same staffing horizon or (3) cancel the shift for a nurse who is not willing float or be reassigned and use vacation, personal day, holiday, or unpaid leavefor the time off: (Bard and Purnomo, 2005a). Each option listed above has its own unique cost implications. One important aspect of the short-term nurse schedule modification problem is the existence of a very efficient solution algorithm. The charge nurse will usually run the algorithm at the beginning of each 4 to 8-hour shift and expect to have a solution in less than an hour, preferably in less than 10 minutes.

As an alternative approach to the problem we study the medium-term integrated nurse scheduling and staffing as a separate problem as presented in Chapter 4. Then we make short-term adjustments for the upcoming 4-hour nursing shift 4-8 hours before the actual patient demand is realized. As described in Chapters 3 and 4, the PICU we study uses a fixed staffing level for the medium-term staffing and scheduling of the upcoming shift, followed by adjustments every four hours. The short-term schedule adjustment tool usually used at the PICU considers only the scheduled patient admissions, which do not exploit the forecasts of historical unscheduled patient admissions. We belive this is the first study to apply the two-stage stochastic programming approach to make short-term schedule adjustments for the upcoming 4-hour nursing shift. This chapter extends the work on medium-term nurse staffing and scheduling to address short-term adjustments. For nurse schedules developed at the medium-term planning phase, we conduct two-stage short-term staffing adjustments for the upcoming nursing shift. Our proposed adjustments are made at the beginning of each nursing shift for the upcoming 4-hour period. Then, after the actual patient demand for the start of the next shift is realixed, we make our final staffing adjustments. We model six different adjustment options for the two- stage stochastic programming model, five of them available as first-stage decisions and one option as the second-stage decision. Because the adjustment horizon is less than 12 hours, unit nurse manager knows the current patient census, acuity levels of the patients, and the number of scheduled admissions and discharges in the current and upcoming shifts. We develop a two-stage stochastic integer programming model which will minimize the total nurse staffing costs and cost of adjustments to the original schedules developed in the medium-term planning phase, and which ensures coverage of the nursing demand in the unit. We also investigate the scheduling flexibility needs of the medical units. Thus we formulate our third research question: "*Can short-term schedule modifications based upon decisions attained from two-stage stochastic integer programming model bring cost savings and reduction in understaffing levels, compared to keeping the original medium-term staffing plans?*"

The solution algorithm for the two-stage stochastic linear programs with fixed recourse incorporates some initial decisions that minimize current costs, plus the expected value of future recourse actions. One can always form a full deterministic equivalent linear program, called the extensive form, of the original stochastic model under a finite number of second-stage realizations. Higher numbers of these second stage realizations make the extent of the problem greater, making it harder to achieve an efficient solution. The frequently-used solution technique, the "L-shaped Method," is a family of algorithms that are based on developing an outer linearization of the recourse function. This is a cutting plane method in that linear cuts, supporting hyperplanes, are generated to create the linearization of the recourse function. The algorithm is primarily based on generating an outer linearization of the recourse cost function and finding a solution of the first-stage problem plus this linearization. This method is a direct application of Bender's Decomposition of the stochastic program primal, or, equivalently, a Dantzig-Wolfe decomposition of the dual. The block structure of the extensive form has given rise to the name "L-Shaped" for the algorithm. The method has been developed by Van Slyke & Wets (1969) in stochastic programming to take care of the feasibility questions. The main principle in the L-shaped method is to approximate the nonlinear term in the objective of the stochastic programs.

Our research shows that the Stochastic Adjustments model provides the least expensive solution, under all UPC options, when compared to medium-term no-adjustment models. Even when compared to the Optimal Staffing model, which assumes patient demand data is perfectly known to the administration, the Stochastic Adjustments model delivers cost savings. These savings are in the range of 1% to 8% for various UPC levels when compared to the hypothetical Optimal Staffing option. Our research also indicates that the models using historical patient data to estimate stochastic patient demand can deliver cost savings of 5% to 20% for the medical units. Because nurse staffing costs account for a significant portion of hospital operating budgets, savings in the range of 5% to 20% seem possible using the short-term Stochastic Adjustments model. We present underlying factors that make short-term Stochastic Adjustment models more attractive to the general nursing administration community. As the number of unscheduled patient admissions rises there is a greater need for scheduling flexibility for the medium-term scheduling models. More short-term schedule adjustments are required in order to better mimic patient demand for nursing. Higher levels of scheduled admissions create smoother patient demand data associated with lower levels of coefficient of variation, making it easier to satisfy patient demand. PRN nurses are critical for cost savings because their flexibility helps minimize under- and over-staffing in the nursing shifts.

A sizable PRN body will alleviate nursing shortages and provide flexibility for the nursing administration in the scheduling process. There is more room for efficiency gains in short-term adjustments because of the limited medium-term flexibility of the unit. We conclude that, by using the short-term Stochastic Adjustment model, the smaller the PRN nurse ratio is, the greater that cost savings will be. The size of the available nurse pool is also an important factor, as it provides a capacity cushion for the desired nurse staffing levels. To obtain a non-unit specific measure, we divide the available nurse pool size for scheduling to the unit by the average staff size used in a shift throughout the staffing horizon. Lower ratios will allow more scheduling flexibility in the unit because more nurses are available to be scheduled. The coefficient of variation in patient data also affecs the level of scheduling flexibility required for the medical unit. Units with higher levels of variation in patient demand will benefit the most from the Dynamic Staffing approach in the medium term and the Stochastic Adjustments model in the short-term. The smoother the patient data is, the lower the cost savings that will be observed using the two-stage Stochastic Adjustments model.

When we combine all types of first-stage schedule adjustments, we show that nurse schedules developed in the medium term will need some sort of adjustment in the short term in more than 75% of all shifts. This observation demonstrates the crucial need for short-term schedule adjustment models to satisfy the most-needed scheduling flexibility. The probability matrices we developed accurately reflect the stochastic nature of the patient demand data and provide costs savings as well as lower levels of understaffing. We also demonstrate that models which use historical patient data to estimate stochastic patient demand can reduce costs of 5% to 20%. We show significantly lower levels of mandatory nurse overtime usage in the second stage under the Stochastic Adjustments model, less than 0.3 nurses per shift for most of the experiments. Lower levels of second-stage mandatory overtime usage reflect the successful implementation of first-stage adjustments within the developed two-stage Stochastic Adjustments model.

In future research, the multi-stage version of the two-stage Stochastic Adjustment model would allow the nursing administration to evaluate alternative adjustment models by constructing a specific scenario tree defining the time evolution of the adjustment process. Development of a Nurse Burnout Index (NBI) based on scheduling requirements, preferences, and adjustments frequency is also of interest. Obtaining access to patient demand and nurse data from various types of hospitals would enable us to compare various forms of the integrated nurse staffing and scheduling problem. Shift-based nurse adjustments data would also enable us to compare the performance of the our short-term adjustments model to what what presently exists. All of these aspects of the nurse allocation problem can be studied in future research..

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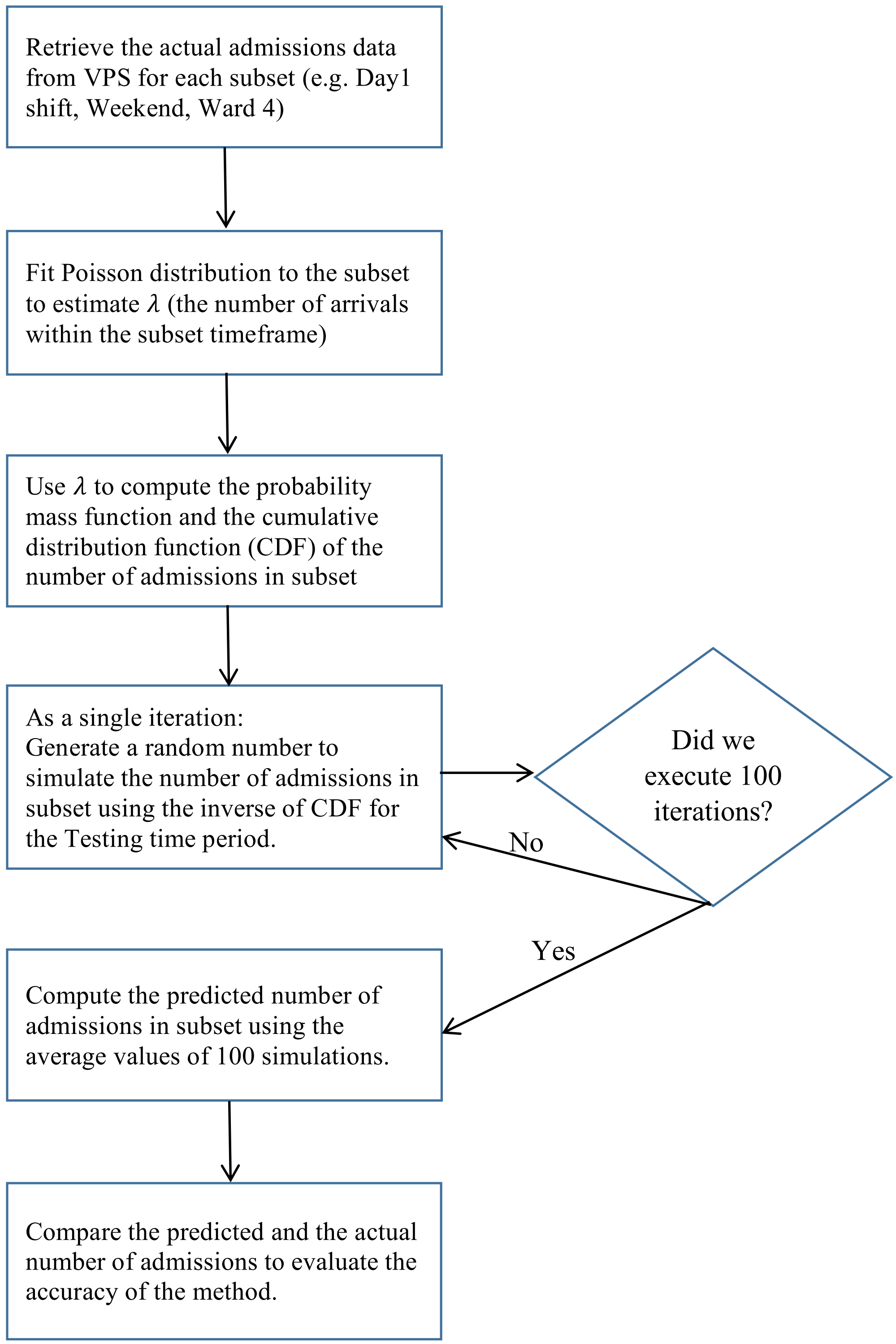
Appendices

Appendix A

Simulation Flowchart for Estimating the Number of Unscheduled Admissions

Appendix A: Simulation Flow Chart for Estimating the Number of Unscheduled Admissions



Appendix B

A Tour Assignment Model of Integrated Nurse Staffing & Scheduling

Model Description

Our decision variables in the model define when a nurse from a specific job class is assigned for a shift, and when that shift starts and ends. The objective function for the model will minimize the costs from main scheduling phase. The cost components associated with the model include: (1) Staffing cost from FTE nurses, (2) Staffing cost from PRN nurses, (3) Understaffing costs (i.e. percentage understaffing multiplied by a unit cost) and (4) Overstaffing costs.

Constraints will ensure satisfying requirements like: (1) 12-hour break between two successive shift assignments for each nurse, (2) No nurse can work more than 3 consecutive 4-hour shifts at a time, (3) No nurse can work more than 40 hours/week, (4) All FTE nurses will be scheduled at least for two consecutive shifts (i.e. they cannot be scheduled only for a single 4-hour shift), (5) No FTE nurse can be scheduled more than 4 work days per week, (6) Minimum and maximum work hours allowances per week and per staffing horizon for different classes of PRN and FTE nurses, (7) Day shift nurse, Night shift nurse limitations, (8) Holiday and/or weekend shifts assignment constraints.

Sets

I: Set of all nurses working for the medical ward / unit

Define j, w and s as follows:

j: index for alternative nurse job profiles for the medical unit; (i.e. $J = \{1, 2, 3, \ldots, 9\}$)

w: index for weekend assignment group for nurses in the medical unit; (i.e. $W = \{\text{A, B, C}\}$)

s: index for day or night shift assignment classification for nurses in the medical unit; (i.e. $S = \{\text{D, N}\}$)

Then, define $I_j^{w,s}$: Set of all nurses with job class j, weekend assignment group w and assigned to shift type s for the medical unit.

i.e. $I_2^{A,D}$ will be the set of nurses from job class 2, FTE - 0.9 nurse, assigned to weekend assignment group A and will work in day shifts.

Let $\Gamma = \{1, 2, \ldots, 24\}$ be the set of non-weekend shifts.

k : Shift assignment types for the nurses $k \in K$, $K = \{1, 2, 3\}$

i.e. Assignment type 1, k=1, assigned nurse will be assigned to work for a single four-hr shift Assignment type 2, k=2, assigned nurse will be assigned to work for two consecutive four-hr shifts Assignment type 3, k=3, assigned nurse will be assigned to work for three consecutive four-hr shifts

L : Set of four-hr nursing shifts within a week $L = \{1, 2, 3, \ldots, 42\}$

i.e. A typical week starts with the nursing shift l = 1, which is a Monday D1 shift and ends with shift l = 42, which is a Monday N2 shift.

M: Set of weeks within the staffing horizon $M = \{1, 2, 3, 4, 5, 6\}$

Θ : Set of patient acuity categories $\Theta = \{1, 2, 3, 4, 5, 6\}$

i.e. For $\theta \in \Theta$ acuity category θ =1 indicates that patient belongs to the acuity designation A in hospital terminology, similarly θ=2 indicates acuity group B, θ=3 indicates acuity group C, θ=4 indicates acuity group D, θ=5 indicates acuity group E, θ=6 indicates acuity group F.

Model Parameters

$\vartheta_{\mathbf{l,m}}$: the vector keeping the number of patients in each acuity group at the unit for shift l of week m.

i.e. $\vartheta_{21,3} = [0, 2, 5, 8, 4, 1]$ will indicate that, in shift 21 of week 3, there are 0 patients with acuity A, 2 patients with acuity B, 5 patients with acuity C, 8 patients with acuity D, 4 patients with acuity E and 1 patients with acuity F are staying in the unit.

Define $\|\vartheta_{\mathbf{l},\mathbf{m}}\| := \sum_{i=1}^{n} |\vartheta_{l,m}(i)|$, where $\vartheta_{l,m}(i)$ represent the i_{th} column of the vector $\vartheta_{l,m}$ (i.e. distance norm of the vector).

Note that $\|\vartheta_{\mathbf{l},\mathbf{m}}\|$ provides the patient census in the unit. (i.e. For shift 21 of week 3 patient census is $\|\vartheta_{21,3}\|$ $:= \sum_{i=1}^{6} |\vartheta_{21,3}(i)| = 0{+}2{+}5{+}8{+}4{+}1 = 20$

$\boldsymbol{h}$: the vector storing nursing hours required for patient care for acuity groups in a four-hour nursing shift (i.e. $\boldsymbol{h}$ = [0.5, 1, 1.5, 2.5, 4, 8] ; a patient with acuity F, θ=6, will require eight hours of nursing care in a four-hour shift).

$\alpha_{l,m}$: number of admission and transfer-in activities to a unit for shift l of week m

$\beta_{l,m}$: number of discharge and transfer-out activities to a unit for shift l of week m

c_j : stafing cost per four-hour shift for the nurse from job profile j

γ : nursing hours required for one patient admission / transfer-in activity

δ : nursing hours required for one patient discharge / transfer-out activity

q^- : Penalty cost of percentage understaffing per four-hour shift

q^+ : Penalty cost of percentage overstaffing per four-hour shift

Decision Variables

$x_{i,k}^{l,m}$ = 1 if nurse i for an assignment type k in shift l of week m is assigned to work; 0 otherwise.

$y_{i,k}^{l,m}$ = 1 if nurse i for an assignment type k starts working for a new assignment in shift l of week m ; 0 otherwise.

Objective Function Cost Components

Staffing Costs:

For all nurses in a specific job class $j \in J$ the staffing costs will be:

$$\sum_{i}\sum_{k}\sum_{l}\sum_{m} c_j . x_{i,k}^{l,m} \qquad \forall \;\; i \in I_j$$

We can then compute the total staffing costs by adding up these costs for all values of $j \in J$:

$$\sum_j c_j \left[\sum_i \sum_k \sum_l \sum_m x_{i,k}^{l,m} \qquad \forall \;\; i \in I_j \right]$$

Understaffing / Overstaffing Costs:

Total nurse hours provided for shift l in week m:

$$\sum_i \sum_k 4.x_{i,k}^{l,m}$$

Total nursing hours required for shift l in week m:

$$\vartheta_{l,m}.h^T + \gamma.\alpha_{l,m} + \delta.\beta_{l,m}$$

Understaffing Penalty Cost for shift l in week m:

$$q^-.\frac{\left[\sum_i \sum_k 4.x_{i,k}^{l,m} - \vartheta_{\mathbf{l,m}}.h^{\mathbf{T}} - \gamma.\alpha_{\mathbf{l,m}} - \delta.\beta_{\mathbf{l,m}} \right]^-}{\vartheta_{\mathbf{l,m}}.h^{\mathbf{T}} + \gamma.\alpha_{\mathbf{l,m}} + \delta.\beta_{\mathbf{l,m}}}$$

Overstaffing Penalty Cost for shift l in week m:

$$q^+.\frac{\left[\sum_i \sum_k 4.x_{i,k}^{l,m} - \vartheta_{\mathbf{l,m}}.h^{\mathbf{T}} - \gamma.\alpha_{\mathbf{l,m}} - \delta.\beta_{\mathbf{l,m}} \right]^+}{\vartheta_{\mathbf{l,m}}.h^{\mathbf{T}} + \gamma.\alpha_{\mathbf{l,m}} + \delta.\beta_{\mathbf{l,m}}}$$

Objective Function: Staffing Costs + Under/Over Staffing Penalty Costs for the Entire Staffing Horizon

$$\sum_j c_j \left[\sum_i \sum_k \sum_l \sum_m x_{i,k}^{l,m} \qquad \forall \;\; i \in I_j \right]$$

$$+ \sum_m \sum_l q^-.\frac{\left[\sum_i \sum_k 4.x_{i,k}^{l,m} - \vartheta_{\mathbf{l,m}}.h^{\mathbf{T}} - \gamma.\alpha_{\mathbf{l,m}} - \delta.\beta_{\mathbf{l,m}} \right]^-}{\vartheta_{\mathbf{l,m}}.h^{\mathbf{T}} + \gamma.\alpha_{\mathbf{l,m}} + \delta.\beta_{\mathbf{l,m}}}$$

$$+\sum_m \sum_l q^+ . \frac{\left[\sum_i \sum_k 4 . x_{i,k}^{l,m} - \vartheta_{\mathbf{l,m}} . h^{\mathbf{T}} - \gamma . \alpha_{\mathbf{l,m}} - \delta . \beta_{\mathbf{l,m}}\right]^+}{\vartheta_{\mathbf{l,m}} . h^{\mathbf{T}} + \gamma . \alpha_{\mathbf{l,m}} + \delta . \beta_{\mathbf{l,m}}}$$

Model Constraints

- Constraints related to the consistency of shift assignment types with their original definitions

  If a nurse started working on a single four-hour shift assignment then:

  $y_{i,1}^{l,m} = x_{i,1}^{l,m} \; \forall \; i, \; l, \; m$

  $(1 - y_{i,1}^{l,m}) \; . \; \kappa \geqslant x_{i,1}^{(l-1),m} + x_{i,1}^{(l+1),m} \; \forall \; i, \; l, \; m$

  If a nurse started working on a two consecutive four-hour shift assignment then:

  $y_{i,2}^{l,m} = x_{i,2}^{l,m} \; \forall \; i, \; l, \; m$

  $y_{i,2}^{l,m} \leqslant x_{i,2}^{(l+1),m} \; \forall \; i, \; l, \; m$

  If a nurse started working on a three consecutive four-hour shift assignment then:

  $y_{i,3}^{l,m} = x_{i,3}^{l,m} \; \forall \; i, \; l, \; m$

  $y_{i,3}^{l,m} \leqslant x_{i,3}^{(l+1),m} \; \forall \; i, \; l, \; m$

  $y_{i,3}^{l,m} \leqslant x_{i,3}^{(l+2),m} \; \forall \; i, \; l, \; m$

- Constraints related to the consistency of total weekly work hours with the job class definitions for FTE nurses

  (FTE - 1.0 nurses): For j=1 and $\forall \; i \in I_j$ and m : $\sum_l \sum_k x_{i,k}^{l,m} = 10$

  (FTE - 0.9 nurses): For j=2 and $\forall \; i \in I_j$ and m : $\sum_l \sum_k x_{i,k}^{l,m} = 9$

  (FTE - 0.8 nurses): For j=3 and $\forall \; i \in I_j$ and m : $\sum_l \sum_k x_{i,k}^{l,m} = 8$

  (FTE - 0.6 nurses): For j=4 and $\forall \; i \in I_j$ and m : $\sum_l \sum_k x_{i,k}^{l,m} = 6$

  (FTE - 0.5 nurses): For j=5 and $\forall \; i \in I_j$ and m : $\sum_l \sum_k x_{i,k}^{l,m} = 5$

  (FTE - 0.3 nurses): For j=6 and $\forall \; i \in I_j$ and m : $\sum_l \sum_k x_{i,k}^{l,m} = 3$

- Constraints related to the consistency of total available assignment types with the job profile definitions for FTE nurses

(FTE - 1.0 nurses): $\sum_{l} y_{i,1}^{l,m} = 0$ For j=1 and $\forall$ i, m
$\sum_{l} y_{i,2}^{l,m} = 2$ For j=1 and $\forall$ i, m
$\sum_{l} y_{i,3}^{l,m} = 2$ For j=1 and $\forall$ i, m

(FTE - 0.9 nurses): $\sum_{l} y_{i,1}^{l,m} = 0$ For j=2 and $\forall$ i, m
$\sum_{l} y_{i,2}^{l,m} = 0$ For j=2 and $\forall$ i, m
$\sum_{l} y_{i,3}^{l,m} = 3$ For j=2 and $\forall$ i, m

(FTE - 0.8 nurses): $\sum_{l} y_{i,1}^{l,m} = 0$ For j=3 and $\forall$ i, m
$\sum_{l} y_{i,2}^{l,m} = 1$ For j=3 and $\forall$ i, m
$\sum_{l} y_{i,3}^{l,m} = 2$ For j=3 and $\forall$ i, m

(FTE - 0.6 nurses): $\sum_{l} y_{i,1}^{l,m} = 0$ For j=4 and $\forall$ i, m
$\sum_{l} y_{i,2}^{l,m} = 0$ For j=4 and $\forall$ i, m
$\sum_{l} y_{i,3}^{l,m} = 2$ For j=4 and $\forall$ i, m

(FTE - 0.5 nurses): $\sum_{l} y_{i,1}^{l,m} = 0$ For j=5 and $\forall$ i, m
$\sum_{l} y_{i,2}^{l,m} = 1$ For j=5 and $\forall$ i, m
$\sum_{l} y_{i,3}^{l,m} = 1$ For j=5 and $\forall$ i, m

(FTE - 0.3 nurses): $\sum_{l} y_{i,1}^{l,m} = 0$ For j=6 and $\forall$ i, m
$\sum_{l} y_{i,2}^{l,m} = 0$ For j=6 and $\forall$ i, m
$\sum_{l} y_{i,3}^{l,m} = 1$ For j=6 and $\forall$ i, m

- Constraints related to the consistency of minimum work hours per schedule with the job class definitions for PRN nurses

  (PRN - Tier 1 nurses): $\sum_{m} \sum_{l} \sum_{k} x_{i,k}^{l,m} \geqslant 8$ For j=7 and $\forall$ i, m

  (PRN - Tier 2 nurses): $\sum_{m} \sum_{l} \sum_{k} x_{i,k}^{l,m} \geqslant 17$ For j=8 and $\forall$ i, m

  (PRN - Tier 3 nurses): $\sum_{m} \sum_{l} \sum_{k} x_{i,k}^{l,m} \geqslant 29$ For j=9 and $\forall$ i, m

- Constraint limiting the total weekly work shifts for all nurses to ten four-hour shifts per week

$\sum_{l}\sum_{k} x_{i,k}^{l,m} \leqslant 10 \ \forall \ i, \ m$

- Day shift and Night shift nurses (i.e. specific nurses will be assigned only to daytime shifts or nighttime shifts.)

  $\sum_{m}\sum_{l \in N}\sum_{k} x_{i,k}^{l,m} \leqslant 0 \ \forall \ i \in I_{j}^{w,D}$ (i.e. $s = D$)

  ,where $N \subset L$ represent nighttime shifts (i.e. $N = \{4, 5, 6, 10, 11, 12, 16, 17, 18, 22, 23, 24, 28, 29, 30, 34, 35, 36, 40, 41, 42\}$) and $I_{j}^{w,D}$ represent daytime nurses (i.e. the nurses that only get assignments in D1, D2 and E1 shifts).

  $\sum_{m}\sum_{l \in D}\sum_{k} x_{i,k}^{l,m} \leqslant 0 \ \forall \ i \in I_{j}^{w,N}$ (i.e. $s = N$)

  ,where $D \subset L$ represent daytime shifts (i.e. $D = \{1, 2, 3, 7, 8, 9, 13, 14, 15, 19, 20, 21, 25, 26, 27, 31, 32, 33, 37, 38, 39\}$) and $I_{j}^{w,N}$ represent nighttime nurses (i.e. the nurses that only get assignments in E2, N1 and N2 shifts).

- Constraints related to weekend assignments for nurses from various types (i.e. nurse groups A, B and C)

  Let $\Gamma = \{1, 2, \ldots, 24\}$ be the set of non-weekend shifts and let $\bar{\Gamma} = \{25, 26, \ldots, 42\}$ be the set of weekend shifts:

  For weekend assignment group A nurses (i.e. $w = A$):

  For $i \in I_{j}^{A,s}$, $j \notin \{1\}$, $l \in \Gamma$ and $m \in M_{A}$=$\{1, 4\}$:

  $\sum_{i}\sum_{m}\sum_{l}\sum_{k} x_{i,k}^{l,m} \leqslant 0$

  For $i \in I_{j}^{A,s}$, $l \in \bar{\Gamma}$ and $m \in M_{\bar{A}}$=$\{2, 3, 5, 6\}$:

  $\sum_{i}\sum_{m}\sum_{l}\sum_{k} x_{i,k}^{l,m} \leqslant 0$

  For weekend assignment group B nurses (i.e. $w = B$):

  For $i \in I_{j}^{B,s}$, $j \notin \{1\}$, $l \in \Gamma$ and $m \in M_{B}$=$\{2, 5\}$:

  $\sum_{i}\sum_{m}\sum_{l}\sum_{k} x_{i,k}^{l,m} \leqslant 0$

  For $i \in I_{j}^{B,s}$, $l \in \bar{\Gamma}$ and $m \in M_{\bar{B}}$=$\{1, 3, 4, 6\}$:

  $\sum_{i}\sum_{m}\sum_{l}\sum_{k} x_{i,k}^{l,m} \leqslant 0$

  For weekend assignment group C nurses (i.e. $w = C$):

  For $i \in I_{j}^{C,s}$, $j \notin \{1\}$, $l \in \Gamma$ and $m \in M_{C}$=$\{3, 6\}$:

  $\sum_{i}\sum_{m}\sum_{l}\sum_{k} x_{i,k}^{l,m} \leqslant 0$

For $i \in I_j^{C,s}$, $l \in \bar{\Gamma}$ and $m \in M_{\bar{C}}$={1, 2, 4, 5} :

$\sum_i \sum_m \sum_l \sum_k x_{i,k}^{l,m} \leqslant 0$

Note: For nurses from job class j = 1 (i.e. FTE - 1.0 nurses that work 40 hours/week), we need a relaxation for some of these constraints, because the weeks they will be assigned to work during the weekend shifts, available number of hours to work add up to 36 hr maximum, which is less than their 40 hr/week work requirement and will cause an infeasibility problem. We will let nurses from job class $j = 1$ to be assigned to one weekday Type-2 assignment during their weekend assignment weeks. Math formulation is as follows:

For weekend assignment group A nurses (i.e. $w = A$):

For $i \in I_j^{A,s}$, $j \in \{1\}$, $l \in \Gamma$ and $m \in M_A$={1, 4} :

$\sum_i \sum_m \sum_l \sum_k x_{i,k}^{l,m} \leqslant 2$

For weekend assignment group B nurses (i.e. $w = B$):

For $i \in I_j^{B,s}$, $j \in \{1\}$, $l \in \Gamma$ and $m \in M_B$={2, 5} :

$\sum_i \sum_m \sum_l \sum_k x_{i,k}^{l,m} \leqslant 2$

For weekend assignment group C nurses (i.e. $w = C$):

For $i \in I_j^{C,s}$, $j \in \{1\}$, $l \in \Gamma$ and $m \in M_C$={3, 6} :

$\sum_i \sum_m \sum_l \sum_k x_{i,k}^{l,m} \leqslant 2$

- Twelve-hour break requirement between two successive assignments, between the current and upcoming shifts, for nurse assignment type 1 (i.e. Single four-hour shift assignment):

  For $l \in$ {1,2, ..., 39}, $\forall\ i,\ k,\ m$:

$$(1 - y_{i,1}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{(l+2),m} + y_{i,k}^{(l+3),m}\right]$$

  For l=40, $\forall\ i,\ k,\ m \in \{1, 2, 3, 4, 5\}$:

$$(1 - y_{i,1}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{(l+2),m} + y_{i,k}^{1,(m+1)}\right]$$

For l=41, $\forall$ i, k, $m \in \{1, 2, 3, 4, 5\}$:

$$(1 - y_{i,1}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{1,(m+1)} + y_{i,k}^{2,(m+1)}\right]$$

For l=42, $\forall$ i, k, $m \in \{1, 2, 3, 4, 5\}$:

$$(1 - y_{i,1}^{l,m}).\kappa \geqslant \left[y_{i,k}^{1,(m+1)} + y_{i,k}^{2,(m+1)} + y_{i,k}^{3,(m+1)}\right]$$

- Twelve-hour break requirement between two successive assignments, between the current and upcoming shifts, for nurse assignment type 2 (i.e. Two consecutive four-hour shift assignment):

  For $l \in \{1,2, ..., 38\}$, $\forall$ i, k, m :

$$(1 - y_{i,2}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{(l+2),m} + y_{i,k}^{(l+3),m} + y_{i,k}^{(l+4),m}\right]$$

  For l=39, $\forall$ i, k, $m \in \{1, 2, 3, 4, 5\}$:

$$(1 - y_{i,2}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{(l+2),m} + y_{i,k}^{(l+3),m} + y_{i,k}^{1,(m+1)}\right]$$

  For l=40, $\forall$ i, k, $m \in \{1, 2, 3, 4, 5\}$:

$$(1 - y_{i,2}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{(l+2),m} + y_{i,k}^{1,(m+1)} + y_{i,k}^{2,(m+1)}\right]$$

  For l=41, $\forall$ i, k, $m \in \{1, 2, 3, 4, 5\}$:

$$(1 - y_{i,2}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{1,(m+1)} + y_{i,k}^{2,(m+1)} + y_{i,k}^{3,(m+1)}\right]$$

- Twelve-hour break requirement between two successive assignments, between the current and upcoming shifts, for nurse assignment type 3 (i.e. Three consecutive four-hour shift assignment):

  For $l \in \{1,2, ..., 37\}$ and $\forall$ i, k, m :

$$(1 - y_{i,3}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{(l+2),m} + y_{i,k}^{(l+3),m} + y_{i,k}^{(l+4),m} + y_{i,k}^{(l+5),m}\right]$$

For l=38, $\forall$ i, k, $m \in \{1, 2, 3, 4, 5\}$:

$$(1 - y_{i,3}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{(l+2),m} + y_{i,k}^{(l+3),m} + y_{i,k}^{(l+4),m} + y_{i,k}^{1,(m+1)}\right]$$

For l=39, $\forall$ i, k, $m \in \{1, 2, 3, 4, 5\}$:

$$(1 - y_{i,3}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{(l+2),m} + y_{i,k}^{(l+3),m} + y_{i,k}^{1,(m+1)} + y_{i,k}^{2,(m+1)}\right]$$

For l=40, $\forall$ i, k, $m \in \{1, 2, 3, 4, 5\}$:

$$(1 - y_{i,3}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l+1),m} + y_{i,k}^{(l+2),m} + y_{i,k}^{1,(m+1)} + y_{i,k}^{2,(m+1)} + y_{i,k}^{3,(m+1)}\right]$$

- Twelve-hour break requirement between two successive assignments, between the current and previous shifts, for each nurse:

  For $l \in \{6,7, ..., 42\}$ and $\forall$ i, k, m:

$$(1 - y_{i,k}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l-1),m} + y_{i,k}^{(l-2),m} + y_{i,k}^{(l-3),m} + y_{i,2}^{(l-4),m} + y_{i,3}^{(l-4),m} + y_{i,3}^{(l-5),m}\right]$$

  For l=5 and $\forall$ i, k, m:

$$(1 - y_{i,k}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l-1),m} + y_{i,k}^{(l-2),m} + y_{i,k}^{(l-3),m} + y_{i,2}^{(l-4),m} + y_{i,3}^{(l-4),m}\right]$$

  For l=4 and $\forall$ i, k, m:

$$(1 - y_{i,k}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l-1),m} + y_{i,k}^{(l-2),m} + y_{i,k}^{(l-3),m}\right]$$

  For l=3 and $\forall$ i, k, m:

$$(1 - y_{i,k}^{l,m}).\kappa \geqslant \left[y_{i,k}^{(l-1),m} + y_{i,k}^{(l-2),m}\right]$$

  For l=2 and $\forall$ i, k, m:

$$(1 - y_{i,k}^{l,m}).\kappa \geqslant y_{i,k}^{(l-1),m}$$

- Every nurse assignment has to start and finish within a specific week (i.e. no week overlapping assignments for nurses):

For l=42, $\forall$ i, k, m:

$$y^{l,m}_{i,2} + y^{l,m}_{i,3} = 0$$

For l=41, $\forall$ i, k, m:

$$y^{l,m}_{i,3} = 0$$

Appendix C

Nurse Staffing Model with PRN Nurses Modeled in Tiers

Sets and Nurse Job Profiles

J : Set of alternative FTE nurse job profiles for the medical unit; (i.e. $J = \{1, 2, 3, \ldots , 30\}$)

S_j : Set of all available schedules for nurses from job profile j

P : Set of PRN Tiers; (i.e. $P = \{1, 2, 3\}$)

T : Set of four-hr nursing shifts during the scheduling period of six week $T = \{0, 1, 2, 3, \ldots ,251\}$ (i.e. 42 shifts a week, six weeks in a schedule; 252 four-hour shifts in total).

i.e. A typical week starts with the nursing shift $l = 1$, which is a Monday D1 shift and ends with shift $l =$ 42, which is a Monday N2 shift.

G : Set of patient acuity categories $G = \{1, 2, 3, 4, 5, 6\}$

i.e. For $g \in G$ acuity category g =1 indicates that patient belongs to the acuity designation A in hospital terminology, similarly g =2 indicates acuity group B, g=3 indicates acuity group C, g=4 indicates acuity group D, g=5 indicates acuity group E, g=6 indicates acuity group F.

Model Parameters

a_{st} : 1 if for schedule $s \in S_j$ can be assigned to work at shift t ; 0 otherwise.

ϑ_{gt} : the vector keeping the number of patients in acuity group $g \in G$ at the unit for shift $t \in T$.

h_g: nursing hours required for patient care for acuity group g in a four-hour nursing shift (i.e. $h = [0.5, 1, 1.5, 2.5, 4, 8]$; a patient with acuity F, g=6, will require eight hours of nursing care in a four-hour shift).

α_t: number of admission and transfer-in activities to a unit for shift t

β_t: number of discharge and transfer-out activities to a unit for shift t

c_j : staffing cost per four-hour shift for the FTE nurse from job profile j

b_p : staffing cost per four-hour shift for the PRN nurse from tier p

γ : nursing hours required for one patient admission / transfer-in activity

δ : nursing hours required for one patient discharge / transfer-out activity

c_u : Penalty cost of percentage understaffing per four-hour shift

c_o : Penalty cost of percentage overstaffing per four-hour shift

n_j : Number of FTE nurses from job profile $j \in J$

k_p : Number of PRN nurses from tier $p \in \{1, 2, 3\}$

Decision Variables

x_s: number of FTE nurses from that are assigned to work for schedule $s \in S_j$; $x_s \in \mathbb{Z}$.

$y_{p,t}$: number of PRN nurses from tier $p \in P$ that are assigned to work for shift $t \in T$; $y_{p,t} \in \mathbb{Z}$.

U_t: Percentage understaffing for shift $t \in T$; $U_t \in \mathbb{R}$.

(U_t= max$\Big[$(Required Nursing hrs. in t - Provided Nursing hrs. in t)/(Required Nursing hrs. in t); 0 $\Big]$)

O_t: Percentage overstaffing for shift $t \in T$; $O_t \in \mathbb{R}$.

(O_t= max$\Big[$(Provided Nursing hrs. in t - Required Nursing hrs. in t)/(Required Nursing hrs. in t); 0 $\Big]$)

Objective Function Cost Components

Staffing Costs:

For FTE nurses the staffing costs will be computed as:

$$\sum_{j \in J} \sum_{s \in S_j} c_j \cdot x_s$$

For PRN nurses the staffing costs will be computed as:

$$\sum_{p \in P} \sum_{t \in T} b_p \cdot y_{pt}$$

We can then compute the total staffing costs for the entire scheduling horizon by adding up these costs:

$$\left[\sum_{j \in J} \sum_{s \in S_j} c_j \cdot x_s + \sum_{p \in P} \sum_{t \in T} b_p \cdot y_{pt} \right]$$

Understaffing Costs:

$$\sum_{t \in T} c_u \cdot U_t$$

Overstaffing Costs:

$$\sum_{t \in T} c_o \cdot O_t$$

Objective Function:

Minimize {Total Staffing Costs + Total Understaffing Penalty Costs + Total Overstaffing Penalty Costs}:

$$Min \left[\sum_{j \in J} \sum_{s \in S_j} c_j \cdot x_s + \sum_{p \in P} \sum_{t \in T} b_p \cdot y_{pt} + \sum_{t \in T} c_u \cdot U_t + \sum_{t \in T} c_o \cdot O_t \right]$$

Model Constraints

- Understaffing and Overstaffing Constraints:

  Total nurse hours provided by the FTE nurses for the four-shift t is:

$$\sum_{j \in J} \sum_{s \in S_j} 4 \cdot x_s \cdot a_{st}$$

  Total nurse hours provided by the PRN nurses for the four-shift t is:

$$\sum_{p \in P} 4 \cdot y_{pt}$$

Then, total nurse hours provided by all the nurses in the medical unit for the four-shift $t \in T$ is:

$$\sum_{j \in J} \sum_{s \in S_j} 4 \cdot x_s \cdot a_{st} + \sum_{p \in P} 4 \cdot y_{pt}$$

Total nursing hours required for the four-shift $t \in T$ is:

$$\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g \in G} \vartheta_{gt} \cdot h_g$$

Using the given provided and required nursing hours expressions, we can introduce the constraints, which will set the lower bound for our percentage understaffing variable (U_t) as follows:

$$\left[\frac{\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g \in G} \vartheta_{gt} \cdot h_g - \sum_{j \in J} \sum_{s \in S_j} 4 \cdot x_s \cdot a_{st} - \sum_{p \in P} 4 \cdot y_{pt}}{\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g \in G} \vartheta_{gt} \cdot h_g}\right] \leqslant U_t \qquad \forall \ \ t \in T;$$

where $U_t \geqslant 0 \qquad \forall \quad t \in T.$

Similarly, using the given provided and required nursing hours expressions, we can introduce the constraints, which will set the lower bound for our percentage overstaffing variable (O_t) as follows:

$$\left[\frac{\sum_{j \in J} \sum_{s \in S_j} 4 \cdot x_s \cdot a_{st} - \sum_{p \in P} 4 \cdot y_{pt} - \gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g \in G} \vartheta_{gt} \cdot h_g}{\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g \in G} \vartheta_{gt} \cdot h_g}\right] \leqslant O_t \qquad \forall \ \ t \in T;$$

where $O_t \geqslant 0 \qquad \forall \quad t \in T.$

- Constraints related to the number of available FTE nurses from each job profile $j \in J$:

$$\sum_{s \in S_j} x_s \leqslant n_j \qquad \qquad \forall \ \ j \in J;$$

We cannot assign more than available number of FTE nurses from each job profile $j \in J$.

- Constraints related to the number of available PRN nurses:

$$y_{pt} \leqslant k_p; \qquad \qquad \forall \ \ p \in P, \ \ t \in T$$

We cannot assign more than available number of PRN nurses from each tier $p \in P$ in any nursing shift

$t \in T$.

- Constraints related to available maximum total PRN hours for a schedule:

$$\sum_{t \in T} y_{pt} \leqslant 60 \cdot k_p \qquad \forall \;\; p \in P$$

Total PRN nurse assignments from any tier $p \in P$ for the whole staffing horizon should be smaller than 60 four-hour shifts per schedule, multiplied by the number of available PRN nurses from that tier.

- Constraints related to minimum break between two consecutive assignments rule:

$$y_{p,(t+2)} \leqslant k_p - (k_p - y_{p,t+1}) - (k_p - y_{p,t}) \qquad \forall \;\; p \in P, \;\; t \in \{1...(T-2)\}$$

Here (k_p - $y_{p,t+1}$) is the number of PRNs from tier-p who were not assigned to work at shift (t+1). Similarly, (k_p - $y_{p,t}$) is the number of PRNs from tier-p who were not assigned to work at shift t. The difference between the two expressions give the number of PRNs from tier-p who give a break to their assignments after shift t, starting from shift (t+1). Those who took a break starting from shift (t+1) cannot get any assignment in shift (t+2). That is why the difference between k_p and this expression define an upper limit for the number of PRNs to get an assignment in shift (t+2). When (k_p - $y_{p,t+1}$) - (k_p - $y_{p,t}$) is negative, constraint becomes irrelevant. We also require:

$$y_{p,(t+3)} \leqslant (k_p - y_{p,t}) \qquad \forall \;\; p \in P, \;\; t \in \{1...(T-3)\};$$

where (k_p - $y_{p,t}$) is the number of PRNs from tier-p who were not assigned to work at shift t. PRN nurses who will be assigned to work in shift (t+3) should be less than or equal to this number.

- Constraint related to the minimum work hours requirement of PRN nurses:

As an additional option, we can impose minimum work hour requirements for each PRN tier: For Tier-1:

$$8 \cdot k_1 \leqslant \sum_{t \in T} y_{1,t};$$

For Tier-2:

$$17 \cdot k_2 \leqslant \sum_{t \in T} y_{2,t};$$

For Tier-3:

$$29 \cdot k_3 \leqslant \sum_{t \in T} y_{3,t};$$

Total assignments for the PRN nurses in a tier shouldn't be less than the cumulative minimum work hour total for that PRN tier.

Notes:

1. PRN nurses can self-schedule using the output schedules generated by the optimization model, while satisfying the rules of:

    (a) Twelve-hour break requirement between two successive assignments for each nurse

    (b) No more than twelve-hour (three four-hour shifts) consecutive assignments at a time

2. Pre-generated FTE nurse schedules already satisfy the requirements given in notes 1.(a) and 1.(b)

We can present the whole model, which models PRN nurses in tiers as follows:

$$Min\left[\sum_{j\in J}\sum_{s\in S_j} c_j \cdot x_s + \sum_{p\in P}\sum_{t\in T} b_p \cdot y_{pt} + \sum_{t\in T} c_u \cdot U_t + \sum_{t\in T} c_o \cdot O_t\right]$$

subject to

$$\left[\frac{\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g\in G} \vartheta_{gt} \cdot h_g - \sum_{j\in J}\sum_{s\in S_j} 4 \cdot x_s \cdot a_{st} - \sum_{p\in P} 4 \cdot y_{pt}}{\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g\in G} \vartheta_{gt} \cdot h_g}\right] \leqslant U_t \qquad \forall \;\; t \in T;$$

$$\left[\frac{\sum_{j\in J}\sum_{s\in S_j} 4 \cdot x_s \cdot a_{st} - \sum_{p\in P} 4 \cdot y_{pt} - \gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g\in G} \vartheta_{gt} \cdot h_g}{\gamma \cdot \alpha_t + \delta \cdot \beta_t + \sum_{g\in G} \vartheta_{gt} \cdot h_g}\right] \leqslant O_t \qquad \forall \;\; t \in T;$$

$$\sum_{s\in S_j} x_s \leqslant n_j \qquad \forall \;\; j \in J;$$

$$y_{pt} \leqslant k_p; \qquad \forall \;\; p \in P, \;\; t \in T$$

$$\sum_{t\in T} y_{pt} \leqslant 60 \cdot k_p \qquad \forall \;\; p \in P$$

$$y_{p,(t+2)} \leqslant k_p - (k_p - y_{p,t+1}) - (k_p - y_{p,t}) \qquad \forall \;\; p \in P, \;\; t \in \{1 ... (T-2)\}$$

$$y_{p,(t+3)} \leqslant (k_p - y_{p,t}) \qquad \forall \;\; p \in P, \;\; t \in \{1 ... (T-3)\};$$

$U_t \in \mathbb{R}$ and $U_t \geqslant 0 \qquad \forall \quad t \in T;$

$O_t \in \mathbb{R}$ and $O_t \geqslant 0 \qquad \forall \quad t \in T;$

$x_s \in \mathbb{Z}$ and $x_s \geqslant 0 \qquad \forall \quad s \in S_j;$

$y_{p,t} \in \mathbb{Z}$ and $y_{p,t} \geqslant 0 \qquad \forall \quad p \in P, \; t \in T$

AMPL code for the Nurse Staffing and Scheduling Model - PRN Nurses Modeled in Tiers We coded the discussed models into the AMPL environment. Figures below present model parameters, decision variables, objective function and model constraints for the model that uses PRN tiers in the AMPL code.

Figure C.1: AMPL Model Parameters, Decision Variables and Objective Function - PRN Tiers

AMPL Model (.mod file)

```
###  DATA  ###

param J > 0; # number of job profiles for the FTE nurses
param S > 0; # total number of available schedules to the nurses from all job profiles
set SCHEDULES {1..J}; # set of available schedules for nurses from job profile j
param T > 0; # number of four-hour shifts in the scheduling period
param G > 0; # number of acuity groups for the patients
param P > 0; # number of PRN nurse tiers

param a {1..S, 1..T} >=0;  # 1 if for schedule s in S_{j} can be assigned to work at shift t; 0
otherwise.
param v {1..G, 1..T} >=0; # number of patients in each acuity group staying at the unit in shift t.
param h {1..G} >=0; # nursing hours required for patient care for each acuity group in a four-hour
nursing shift.
param alpha {1..T} >=0; # number of admission and transfer-in activities to a unit for shift t.
param beta {1..T} >=0; # number of discharge and transfer-out activities to a unit for shift t.
param c > 0; # staffing cost per four-hour shift for the FTE nurses (can be varied for each job profile
j).
param b {1..P} > 0; # staffing cost per four-hour shift for the PRN nurse from tier p.
param gamma > 0; # nursing hours required for one patient admission or transfer-in activity.
param delta > 0; # nursing hours required for one patient discharge or transfer-out activity.
param c_u >0; # penalty cost of percentage understaffing per four-hour shift.
param c_o >0; # penalty cost of percentage overstaffing per four-hour shift.
param n {1..J} >=0; # number of FTE nurses from job profile j in J.
param k {1..P} >=0; # number of PRN nurses from tier p in P.


###  VARIABLES  ###

var X {1..S} >=0 integer; # number of FTE nurses that are assigned to work for schedule s in S_{j}.
var Y {1..P, 1..T} >=0 integer; # number of PRN nurses from tier p that are assigned to work for shift
t in T.
var U {1..T} >=0; # percentage understaffing for shift t in T.
var O {1..T} >=0; # percentage overstaffing for shift t in T.

###  OBJECTIVE  ###

minimize Total_Cost:
        sum {j in 1..J, s in SCHEDULES[j], t in 1..T} (c*X[s]*a[s,t]) +
        sum {p in 1..P, t in 1..T} (b[p]*Y[p,t]) +
        sum {t in 1..T} (c_u*U[t] + c_o*O[t]);

        # Objective is to minimize the total costs of staffing costs from FTE and PRN nurses,
        # and also total under and over-staffing costs throughout the scheduling horizon.
```

Figure C.2: AMPL Model Constraints - PRN Tiers

```
###  CONSTRAINTS  ###

subject to Understaffing {t in 1..T}:
100*(sum {g in 1..G} (v[g,t]*h[g]) + (gamma*alpha[t]) + (delta*beta[t]) -
sum {j in 1..J, s in SCHEDULES[j]} (4*X[s]*a[s,t]) - sum {p in 1..P} (4*Y[p,t])) /
(sum {g in 1..G} (v[g,t]*h[g]) + (gamma*alpha[t]) + (delta*beta[t]))
<= U[t] ;
        # Required nursing hours minus the Provided nursing hours from FTE and PRN nurses
        # should define the lower bound for the understaffing variable.
        # Variable U_{t} is already defined as a non-negative real number.

subject to Overstaffing {t in 1..T}:
        100*(sum {j in 1..J, s in SCHEDULES[j]} (4*X[s]*a[s,t]) +
        sum {p in 1..P} (4*Y[p,t]) - sum {g in 1..G} (v[g,t]*h[g]) + (gamma*alpha[t]) +
(delta*beta[t]))
        / (sum {g in 1..G} (v[g,t]*h[g]) + (gamma*alpha[t]) + (delta*beta[t])) <= O[t] ;

        # Provided nursing hours from FTE and PRN nurses minus the Required nursing hours
        # should define the lower bound for the overstaffing variable.
        # Variable O_{t} is already defined as a non-negative real number.

subject to Available_FTE {j in 1..J}:
        sum {s in SCHEDULES[j]}(X[s]) <= n[j] ;
        # We can not assign more than available number of FTE nurses from each job profile j.

subject to Available_PRNs {p in 1..P, t in 1..T}:
        Y[p,t] <= k[p] ;
# We can not assign more than available number of PRN nurses from each tier p.

subject to Maximum_PRN_Hours {p in 1..P}:
        sum {t in 1..T} (Y[p,t]) <=  60 * k[p];
# Total PRN nurse assignments from any tier cannot be more than 60 shifts per schedule of six weeks.

subject to PRN_12Hr_Rule {p in 1..P, t in 1..(T-2)} :
                Y[p,t+2] <= k[p] - ((k[p] - Y[p,t+1]) - (k[p] - Y[p,t])) ;

# Here (k[p]-Y[p,t+1]) is the number of PRNs from Tier-p who were not assigned to work at shift (t+1).
# Similarly,(k[p] - Y[p,t]) is the number of PRNs from Tier-p who were not assigned to work at shift t.
# The difference between the two expressions give the number of PRNs from Tier-p who give a break to
their assignments after shift t, starting from shift t+1.
# Those who took a break starting from shift t+1 cannot get any assignment in shift (t+2).
# That is why the difference between k[p] and this expression define an upper limit for the number of
PRNs to get an assignment in period (t+2).
# Note: When ((k[p] - Y[p,t+1]) - (k[p] - Y[p,t])) is negative, constraint becomes irrelevant.

subject to PRN_16Hr_Rule {p in 1..P, t in 1..(T-3)} :
                Y[p,t+3] <= (k[p] - Y[p,t]);

# Here, (k[p] - Y[p,t]) is the number of PRNs from Tier-p who were not assigned to work at shift t.
# PRN nurses who will be assigned to work in shift (t+3) should be less than or equal to this number.

# Total assignments for the PRN nurses shouldn't be less than the cumulative minimum work hour total.
#subject to Minimum_PRN_Hours_Tier1:
#       8*k[1] <= sum {t in 1..T} (Y[1,t]) ;
#subject to Minimum_PRN_Hours_Tier2:
#       17*k[2] <= sum {t in 1..T} (Y[2,t]) ;
#subject to Minimum_PRN_Hours_Tier3:
#       29*k[3] <= sum {t in 1..T} (Y[3,t]) ;
```

Appendix D

A Sample Schedule Generation Code in C++

Defining functions to be used in the program

Figure D.1: Step 1 - Defining functions to be used in the program

```
/////////////////////////////////////////////////////////////////////////////////////
//  (*) Generates Nurse Schedules for "FTE_0.9" nurses working in "DAY" shifts and  //
//        has a weekend assignment "GROUP A".                                       //
//  (*) Schedule_FTE09_DAY_Weekend_A.cpp                                            //
//  (*) Created by Osman Aydas on 3/9/16.                                           //
//  (*) Copyright © 2016 Osman Aydas. All rights reserved.                          //
//                                                                                  //
/////////////////////////////////////////////////////////////////////////////////////

#include <iostream>
#include <fstream>
#include <vector>
using namespace std;
int glb_a=0;

//////////////////////////////////////////////////////////////////////////////////////
// (*) This function generates all combinations of numbers given an array of numbers. //
// (*) Array "b" is the input array that has "n" elements.                             //
// (*) Using "comb" function we are generating all possible combinations of size "r"; //
//      and output them into an array.                                                 //
// (*) We use this function to generate all possible shift start time combinations,   //
//      given alternative shift start times.                                           //
//////////////////////////////////////////////////////////////////////////////////////
void comb(int n, int r, int *arr,int **array,int *b ,int sz) {
    for (int i = n; i >= r; i --) {
        // choose the first element
        arr[r - 1] = b[i-1];
        if (r > 1) { // if still needs to choose
            // recursive into smaller problem
            comb(i - 1, r - 1, arr, array,b,sz);
        } else {
            // print out one solution
            for (int i = 0; i < sz; i ++) {
                array[glb_a][i]=arr[i];
            }
            glb_a++;
        }
    }
}
//////////////////////////////////////////////////////////////////////////////////////
//This is the factorial function that computes the factorial of a given integer.      //
//////////////////////////////////////////////////////////////////////////////////////
int factorial(int n)
{
    return (n == 1 || n == 0) ? 1 : factorial(n - 1) * n;
}

//////////////////////////////////////////////////////////////////////////////////////
//This function computes the number of subsets size "m" of a set with size "n"        //
//////////////////////////////////////////////////////////////////////////////////////
int combination(int n, int m)
{
    return (factorial(n)/(factorial(m)*factorial(n-m)));
}

//////////////////////////////////////////////////////////////////////////////////////
```

Defining variables and parameters; reading the shift data

Figure D.2: Step 2 - Defining variables and parameters; reading the shift data

```
/////////////////////////////////////////////////////////////////////////////////////////
//Here we are starting the main program and defining our variables and arrays.        //
/////////////////////////////////////////////////////////////////////////////////////////

int main(int argc, const char * argv[]) {

    const int G_SHIFTS = 252;
    const int NURSE_TYPES = 36;
    int i;
    int t;
    int start_time=0;
    int week[G_SHIFTS];
    int shift[G_SHIFTS];
    string shift_name[G_SHIFTS];
    string shift_type[G_SHIFTS];
    string day_type[G_SHIFTS];
    int job_class[NURSE_TYPES];
    string employment_type[NURSE_TYPES];
    char shift_category[NURSE_TYPES];
    char weekend_group[NURSE_TYPES];
    bool a[G_SHIFTS];
///////////////////////////////////////////////////////////////////////////////////////////
// (*) Here we are reading the data from file "Shift Data.txt".                          //
// (*) This file has the information on:                                                 //
//     "Week #", "Shift #", "Shift Name", "Shift Type", "Day Type" for a given shift.   //
///////////////////////////////////////////////////////////////////////////////////////////

    cout << "All indices begin at zero!!!" << "\n" << endl;
//Opening the .txt data file for shift data.
    const char*     filename1  = "/Users/OTA/Desktop/Optimization with Cpp and Cplex/Shift Data.txt";
    ifstream file1(filename1);
    if ( !file1 )
    {
        cerr << "ERROR: could not open file '" << filename1 << "' for reading" << endl;
        throw (-1);
    }
//Transferring shift data in .txt file to C++
    for (i=0; i < G_SHIFTS; i++)
    {
        file1 >> week[i] >> shift[i] >> shift_name[i] >> shift_type[i] >> day_type[i];
    }
//Outputting data from C++ arrays just to make sure it has been read correctly.
cout<<"| G_SHIFT "<<"| Week # "<<" | Shift #  "<<"| Shift Name"<<" | Shift Type  "<<"| Day Type | "<<'

    for (i=0; i < G_SHIFTS; i++)
    {
        cout << "\t" << i << "\t\t";
        cout << "  "<< week[i] << "\t\t\t";
        cout << shift[i] << "\t\t\t";
        cout << shift_name[i] << "\t\t\t";
        cout << shift_type[i]<< "  " << "\t\t";
        cout << day_type[i] << "\t\t\t" << endl;
    }
    cout << endl << endl;
    file1.close();
////////////////////////////////////////////////////////////////////////////////////////////////////
```

Reading the nurse job profiles from the data

Figure D.3: Step 3 - Reading the nurse job profiles from the data

```
//////////////////////////////////////////////////////////////////////////////////////////////////////
// (*) Here we are reading the data from file "Nurse Types.txt". This file has the information on:     //
//"| NURSE TYPE " << "| JOB CLASS " << "| EMPLOYMENT TYPE "<< "| SHIFT CATEGORY" << "| WEEKEND GROUP|" //
//////////////////////////////////////////////////////////////////////////////////////////////////////

//Opening the .txt data file for nurse types.

const char*     filename2  = "/Users/OTA/Desktop/Optimization with Cpp and Cplex/Nurse Types.txt";
    ifstream file2(filename2);
    if ( !file2 )
    {
        cerr << "ERROR: could not open file '" << filename2 << "' for reading" << endl;
        throw (-1);
    }

//Transferring shift data in .txt file to C++

    for (i=0; i < NURSE_TYPES; i++)
    {

        file2 >> job_class[i] >> employment_type[i] >> shift_category[i] >> weekend_group[i];
    }

//Outputting data from C++ arrays just to make sure it has been read correctly.

cout<<"| NURSE TYPE "<<"| JOB CLASS "<<"| EMPLOYMENT TYPE "<<"| SHIFT CATEGORY"<<"| WEEKEND GROUP |"<<"\n'


    for (i=0; i < NURSE_TYPES; i++)
    {
        cout << "\t\t" << i << "\t\t";
        cout << "   "<< job_class[i] << "\t\t\t";
        cout << "   " << employment_type[i] << "\t\t\t";
        cout << "\t" << shift_category[i] << "\t\t\t";
        cout << "   "<< weekend_group[i]<< "   " << "\t\t" << endl;

    }
    cout << endl << endl;

    file2.close();

//////////////////////////////////////////////////////////////////////////////////////////////////////
```

Identifying available work shifts for the given nurse profile

Figure D.4: Step 4 - Identifying available work shifts for the given nurse profile

```
//////////////////////////////////////////////////////////////////////////////////////////////////////////////////
//Here we are putting a "0" to all shifts that are classified as night shifts, since this program is for nurses //
//that work in day shifts. We are also putting zeros to weekday shifts in weeks 1 and 4 because this nurse can  //
//only work in weekend shifts for those weeks.Other than these conditions we are putting a "1" to all other     //
//shifts, since these are available work shifts for this nurse class. We output these arrays for observation.   //
//////////////////////////////////////////////////////////////////////////////////////////////////////////////////

    for (t=0; t < G_SHIFTS; t++)
    {
        if (shift_type[t]== "NIGHT")

        {
            a[t]=0;
        }
        else
        {
            if (day_type[t]== "WEEKDAY")

            {
                if (week[t]== 1 || week[t]== 4)

                {
                    a[t]=0;
                }
                else
                {
                    a[t]=1;
                }
            }
            else
            {
                if (week[t]== 1 || week[t]== 4)

                {
                    a[t]=1;
                }
                else
                {
                    a[t]=0;
                }
            }
        }
    }
cout<<"| G_SHIFT"<<"| Week # "<<"|Shift #"<<"|Shift Name"<<"|Shift Type"<<"|Day Type"<<"| Schedule |"<<"\n\n";
    for (t=0; t < G_SHIFTS; t++)
    {
        cout << "\t" << t << "\t\t";
        cout << "  "<< week[t] << "\t\t\t";
        cout << shift[t] << "\t\t\t";
        cout << shift_name[t] << "\t\t\t";
        cout << shift_type[t]<< "  " << "\t\t";
        cout << day_type[t] << "\t\t\t";
        cout << a[t] << "\t\t" << endl;
    }
    cout << endl << endl;

//////////////////////////////////////////////////////////////////////////////////////////////////////////////
```

Identifying the shifts that are available for three consecutive shift assignments

Figure D.5: Step 5 - Identifying the shifts that are available for three consecutive shift assignments

```
//////////////////////////////////////////////////////////////////////////////////////////////////////
//Here we are identifing the shifts which are available to start a three four-hour shift assignment.//
//Because, this nurse class will work three four-hour shifts each week. Then we list those shifts.  //
//////////////////////////////////////////////////////////////////////////////////////////////////////

    for (t=0 ; t < G_SHIFTS ; t++)

    {
        if (a[t]==1 && a[t+1]==1 && a[t+2]==1)

        {
            start_time=start_time+1;
        }

    }

    int *s3;
    s3 = new int[start_time];

    int cnt=0;
    for (t=0 ; t < G_SHIFTS ; t++)

    {
        if (a[t]==1 && a[t+1]==1 && a[t+2]==1)

        {
            s3[cnt] = t;
            cnt++;
        }
    }

cout<< "Potential start times for three 4-hour shifts for this nurse type are:" << "\n" << endl;

cout<< "| Week # |" << "\t" << "| Start Shift |" << "\n" << endl;

    for (int e=0; e < start_time ; e++)
    {
        cout<< "\t" << week[s3[e]] << "\t\t\t" << s3[e] << "\t\t" << endl;
    }
    cout << endl << endl;
```

Build sets of available start times for the individual weeks

Figure D.6: Step 6(a) - Build sets of available start times for the individual weeks

```
///////////////////////////////////////////////////////////////////////////////////////////
//Here we are seperating the potential start times  in each week.                        //
//This is needed to generate combinations of those start times for alternative schedules. //
///////////////////////////////////////////////////////////////////////////////////////////

    int aa=0;
    int bb=0;
    int cc=0;
    int dd=0;
    int ee=0;
    int ff=0;

    int count1=0;
    int count2=0;
    int count3=0;
    int count4=0;
    int count5=0;
    int count6=0;

    int *w1;
    for(int xx=0;xx<start_time;xx++){
        if(week[s3[xx]] == 1){
            count1++;

        }
    }

    w1 = new int[count1];


    int *w2;
    for(int xx=0;xx<start_time;xx++){
        if(week[s3[xx]] == 2){
            count2++;

        }
    }

    w2 = new int[count2];
```

Figure D.7: Step 6(b) - Build sets of available start times for the individual weeks

```
for (int d=0; d<start_time ; d++)
{
    if (s3[d]<42)
    {
        w1[aa]=s3[d];
        aa++;
    }

    if (s3[d]>41 && s3[d]<84)
    {
        w2[bb]=s3[d];
        bb++;
    }

    if (s3[d]>83 && s3[d]<126)
    {
        w3[cc]=s3[d];
        cc++;
    }
    if (s3[d]>125 && s3[d]<168)
    {
        w4[dd]=s3[d];
        dd++;
    }

    if (s3[d]>167 && s3[d]<210)
    {
        w5[ee]=s3[d];
        ee++;
    }

    if (s3[d]>209 && s3[d]<252)
    {
        w6[ff]=s3[d];
        ff++;
    }
}
cout << "Potential start times in each week are as follows:"<< "\n"<< endl;

cout << "Week #1:" << endl;

for(int x=0; x < count1; x++)
{
    cout << w1[x] << endl;
}
cout << endl << endl;

cout << "Week #2:" << endl;
for(int x=0; x < count2; x++)
{
    cout << w2[x] << endl;
}
cout << endl << endl;
```

Generate potential start time combinations for each week

Figure D.8: Step 7 - Generate potential start time combinations for each week

```
////////////////////////////////////////////////////////////////////////////////////////////////////
//Here we generate potential start time combinations in Week #1 using the pre-defined "comb" function

    cout << "Potential start time combinations in each week are as follows:"<< "\n"<< endl;
    cout << "Week #1:";
    cout << endl;

    int N1 = count1;
    int M1 = 3;
    int *arr1 = new int[M1];
    int cmb1 = combination(N1,M1);
    int** week1_comb = new int*[cmb1];

    for(int i = 0; i < cmb1; i++)
    {
        week1_comb[i] = new int[M1];
    }

    comb(N1, M1, arr1, week1_comb, w1 ,M1);

    for(int i=0;i<cmb1;i++)
    {
        for(int j=0;j<M1;j++)
        {
            cout<<week1_comb[i][j]<<" ";
        }
        cout<< endl<< endl;
    }
////////////////////////////////////////////////////////////////////////////////////////////////////
//Here we generate potential start time combinations in Week #2 using the pre-defined "comb" function

    cout << "\n"<< "Week #2:";
    cout << endl << endl;

    int N2 = count2;
    int M2 = 3;
    glb_a=0;
    int *arr2 = new int[M2];
    int cmb2 = combination(N2,M2);
    int** week2_comb = new int*[cmb2];

    for(int i = 0; i < cmb2; i++)
    {
        week2_comb[i] = new int[M2];
    }

    comb(N2, M2, arr2, week2_comb, w2 ,M2);

    for(int i=0;i<cmb2;i++)
    {
        for(int j=0;j<M2;j++)
        {
            cout<<week2_comb[i][j]<<" ";
        }
        cout<< endl<<endl;
    }
////////////////////////////////////////////////////////////////////////////////////////////////////
```

Combining weekly start time combinations to a complete schedule

Figure D.9: Step 8 - Combining weekly start time combinations to a complete schedule

```
/////////////////////////////////////////////////////////////////////////////////////////////////////////
//Here we generate combinations of potential start times for the whole scheduling period

int total_sch;
total_sch = cmb1*cmb2*cmb3*cmb4*cmb5*cmb6;
int shift_starts[total_sch][6][3];

for(int i=0;i<cmb1;i++) {
for(int j=0;j<cmb2;j++) {
for(int k=0;k<cmb3;k++) {
for(int l=0;l<cmb4;l++) {
for(int m=0;m<cmb5;m++) {
for(int n=0;n<cmb6;n++) {
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][0][0]= week1_comb[i][0];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][0][1]= week1_comb[i][1];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][0][2]= week1_comb[i][2];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][1][0]= week2_comb[j][0];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][1][1]= week2_comb[j][1];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][1][2]= week2_comb[j][2];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][2][0]= week3_comb[k][0];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][2][1]= week3_comb[k][1];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][2][2]= week3_comb[k][2];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][3][0]= week4_comb[l][0];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][3][1]= week4_comb[l][1];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][3][2]= week4_comb[l][2];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][4][0]= week5_comb[m][0];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][4][1]= week5_comb[m][1];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][4][2]= week5_comb[m][2];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][5][0]= week6_comb[n][0];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][5][1]= week6_comb[n][1];
shift_starts[(cmb1*cmb2*cmb3*cmb4*cmb5*i)+(cmb2*cmb3*cmb4*cmb5*j)+(cmb3*cmb4*cmb5*k)+(cmb4*cmb5*l)+(cmb5*m)+n][5][2]= week6_comb[n][2];

                        }
                    }
                }
            }
        }
    }
//t is the schedule rank (0 to 255); w is the week number (0 to 5); s is the assignment number (0, 1, 2) (3 assignments per week).

    cout<<"Schedule start times for each schedule are as follows: " <<"\n"<<endl;
    for (int t=0; t<total_sch; t++)
    {    cout<< "Schedule "<< t <<"\t\t";

        for (int w=0; w<6; w++)
        {
            for (int s=0; s<3; s++)
            {
                cout <<"\t"<< shift_starts[t][w][s] << "\t";
            }

        }
        cout<<endl;
    }
/////////////////////////////////////////////////////////////////////////////////////////////////////////
```

Converting the potential start time combination arrays to the full set of schedules

Figure D.10: Step 9 - Converting the potential start time combination arrays to the full set of schedules

```
///////////////////////////////////////////////////////////////////////////////////////////////////
//Here, we convert the potential start time schedules to the full set of schedules that shows assignments //
//for each shift using a boolean variable. This generates a total of 256 schedules for this nurse types.  //

    bool schedule_fte09_day_weekendA[total_sch][G_SHIFTS];

    for (int t=0; t<total_sch; t++)
    {
        for (int g=0; g<G_SHIFTS; g++)
        {
            schedule_fte09_day_weekendA[t][g] = 0 ;
        }
    }
    for (int t=0; t<total_sch; t++)
    {
        for (int w=0; w<6; w++)
        {
            for (int s=0; s<3; s++)
            {
                schedule_fte09_day_weekendA[t][shift_starts[t][w][s]] =1 ;
                schedule_fte09_day_weekendA[t][(shift_starts[t][w][s]+1)] =1 ;
                schedule_fte09_day_weekendA[t][(shift_starts[t][w][s]+2)] =1 ;
            }
        }
    }

    for (int t=0; t<total_sch; t++)
    {
        cout<< "Schedule "<< t <<"\t\t";
        for (int g=0; g<G_SHIFTS; g++)
        {
            cout << "\t" << schedule_fte09_day_weekendA[t][g] << "\t";
        }
        cout<<endl;
    }
///////////////////////////////////////////////////////////////////////////////////////////////////
//Here we output all schedule alternatives for this nurse type to a txt file.

    ofstream myfile ("/Users/OTA/Desktop/Optimization with Cpp and Cplex/schedule_fte09_day_weekendA.txt");
    if (myfile.is_open())
    {
        for (int t=0; t<total_sch; t++)
        {myfile << "Schedule "<< t <<"\t\t";
            for (int g=0; g<G_SHIFTS; g++)
            {
                myfile << schedule_fte09_day_weekendA[t][g] << "\t";
            }
            myfile << "\n";
        }
        myfile.close();
    }
    else
    {
        cout << "Unable to open file";
    }
///////////////////////////////////////////////////////////////////////////////////////////////////
}
```

Appendix E

Coding the Medium-Term Staffing Optimization Model in AMPL Environment

In this section, we present the AMPL modeling code to be used in our optimization experiments. Figure E.1 present the model parameters, decision variables, objective function for the model that assigns FTE nurses to pre-generated schedules and PRN nurses to nursing shifts individually in the AMPL code. We also present comments for explaining the parameters, variables and constraints in the model.

Figure E.1: AMPL Model Parameters, Decision Variables and Objective Function - Individual PRNs

```
###  DATA  ###

param J > 0; # number of job profiles for the FTE nurses
param S > 0; # total number of available schedules to the nurses from all FTE job profiles
set SCHEDULES {1..J}; # set of available schedules for nurses from job profile j
param T > 0; # number of four-hour shifts in the scheduling period
param G > 0; # number of acuity groups for the patients

param PT1 >= 0; # number of PRN_Tier1 nurses
param PT2 >= 0; # number of PRN_Tier2 nurses
param PT3 >= 0; # number of PRN_Tier3 nurses
param P >=0; # number of PRN nurses. ({1..PT1} Tier 1 nurses; {(PT1+1)..(PT1+PT2)} Tier 2 nurses;
             # {(PT1+PT2+1)..(PT1+PT2+PT3} Tier 3 nurses
param a {1..S, 1..T} >=0;  # 1 if for schedule s in S_{j} can be assigned to work at shift t; 0 otherwise.
param v {1..G, 1..T} >=0; # number of patients in each acuity group staying at the unit in shift t.
param h {1..G} >=0; # nursing hours required for patient care for each acuity group in a four-hour nursing shift.
param alpha {1..T} >=0; # number of admission and transfer-in activities to a unit for shift t.
param beta {1..T} >=0; # number of discharge and transfer-out activities to a unit for shift t.
param c > 0; # staffing cost per four-hour shift for the FTE nurses (can be varied for each job profile j).
param b > 0; # staffing cost per four-hour shift for the PRN nurses.
param gamma > 0; # nursing hours required for one patient admission or transfer-in activity.
param delta > 0; # nursing hours required for one patient discharge or transfer-out activity.
param c_u >0; # penalty cost of percentage understaffing per four-hour shift.
param c_o >0; # penalty cost of percentage overstaffing per four-hour shift.
param n {1..J} >=0; # number of FTE nurses from job profile j in J.
param M >=0;

###  VARIABLES  ###

var X {1..S} >=0 integer; # number of FTE nurses that are assigned to work for schedule s in S_{j}.
var Y {1..P, 1..T} >=0 binary; # whether PRN nurse p in P is assigned to work for shift t in T.
var Z {1..P} >=0 binary; # whether PRN nurse p in P is assigned to work for any shift during the staffing horizon.
var U {1..T} >=0; # percentage understaffing for shift t in T.
var O {1..T} >=0; # percentage overstaffing for shift t in T.
var FTE_Staffing_Cost >=0 integer;
var PRN_Staffing_Cost >=0 integer;
var Under_Staffing_Cost >=0 integer;
var Over_Staffing_Cost >=0 integer;

###  OBJECTIVE  ###

minimize Total_Cost:
    sum {j in 1..J, s in SCHEDULES[j], t in 1..T} (c*X[s]*a[s,t]) +
    sum {p in 1..P, t in 1..T} (b*Y[p,t]) +
    sum {t in 1..T} (c_u*U[t] + c_o*O[t]);

    # Objective is to minimize the total costs of staffing costs from FTE and PRN nurses,
    # and also total under and over-staffing costs throughout the scheduling horizon.
```

Figures E.2, E.3 and E.4 present the model constraints for the developed optimization model in the AMPL environment, as three separate sets.

Figure E.2: AMPL Model Constraints - Individual PRNs (1)

```
###  CONSTRAINTS  ###

subject to FTE_Staffing:

FTE_Staffing_Cost = sum {j in 1..J, s in SCHEDULES[j], t in 1..T} (c*X[s]*a[s,t]);
##############################################################################################

subject to PRN_Staffing:

PRN_Staffing_Cost = sum {p in 1..P, t in 1..T} (b*Y[p,t]);
##############################################################################################

subject to Under_Staffing:

Under_Staffing_Cost = sum {t in 1..T} (c_u*U[t]);
##############################################################################################

subject to Over_Staffing:

Over_Staffing_Cost = sum {t in 1..T} (c_o*O[t]);
##############################################################################################

subject to Understaffing {t in 1..T}:
100*(sum {g in 1..G} (v[g,t]*h[g]) + (gamma*alpha[t]) + (delta*beta[t]) -
sum {j in 1..J, s in SCHEDULES[j]} (4*X[s]*a[s,t]) - sum {p in 1..P} (4*Y[p,t])) /
(sum {g in 1..G} (v[g,t]*h[g]) + (gamma*alpha[t]) + (delta*beta[t]))
<= U[t] ;

    # Required nursing hours minus the Provided nursing hours from FTE and PRN nurses
    # should define the lower bound for the understaffing variable.
    # Variable U_{t} is already defined as a non-negative real number.

##############################################################################################

subject to Overstaffing {t in 1..T}:
    100*(sum {j in 1..J, s in SCHEDULES[j]} (4*X[s]*a[s,t]) +
    sum {p in 1..P} (4*Y[p,t]) - sum {g in 1..G} (v[g,t]*h[g]) + (gamma*alpha[t]) + (delta*beta[t]))
    / (sum {g in 1..G} (v[g,t]*h[g]) + (gamma*alpha[t]) + (delta*beta[t])) <= O[t] ;

# Provided nursing hours from FTE and PRN nurses minus the Required nursing hours
# should define the lower bound for the overstaffing variable.
# Variable O_{t} is already defined as a non-negative real number.

##############################################################################################

subject to Available_FTE {j in 1..J}:
    sum {s in SCHEDULES[j]}(X[s]) <= n[j] ;

# We can not assign more than available number of FTE nurses from each job profile j.

##############################################################################################
```

Figure E.3: AMPL Model Constraints - Individual PRNs (2)

```
###  CONSTRAINTS (2)  ###

###########################################################################################

subject to PRN_Less16 {p in 1..P, t in 1..(T-3)} :
        ( Y[p,t] + Y[p,t+1] + Y[p,t+2] + Y[p,t+3] ) <= 3;

# PRN nurses cannot be assigned to four consecutive four-hour shifts.

###########################################################################################

subject to PRN_Avoid_1_0_1 {p in 1..P, t in 1..(T-2)} :
        ( Y[p,t] - Y[p,t+1]) + (Y[p,t+2] - Y[p,t+1] ) <= 1;

# Break time between two assignments cannot be a single four hour shift

###########################################################################################

subject to PRN_Avoid_1_0_0_1 {p in 1..P, t in 1..(T-3)} :
        ( Y[p,t] - Y[p,t+1]) + (Y[p,t+3] - Y[p,t+2] ) <= 1;

# Break time between two assignments cannot be a two consecutive four hour shift

###########################################################################################


subject to Maximum_PRN_Hours_Wk1 {p in 1..P}:
    sum {t in 1..42} (Y[p,t]) <=  10;

subject to Maximum_PRN_Hours_Wk2 {p in 1..P}:
    sum {t in 43..84} (Y[p,t]) <=  10;

subject to Maximum_PRN_Hours_Wk3 {p in 1..P}:
    sum {t in 85..126} (Y[p,t]) <=  10;

subject to Maximum_PRN_Hours_Wk4 {p in 1..P}:
    sum {t in 127..168} (Y[p,t]) <=  10;

subject to Maximum_PRN_Hours_Wk5 {p in 1..P}:
    sum {t in 169..210} (Y[p,t]) <=  10;

subject to Maximum_PRN_Hours_Wk6 {p in 1..P}:
    sum {t in 211..252} (Y[p,t]) <=  10;

# No PRN nurse should be assigned to more than 10 four-hour shifts in a week
# during the staffing horizon of six weeks.
###########################################################################################
```

Figure E.4: AMPL Model Constraints - Individual PRNs (3)

```
###  CONSTRAINTS (3)  ###

###############################################################################################

subject to PRN_Tier1_Min_Hours {p in 1..PT1} :
    sum {t in 1..T} (Y[p,t]) >= 8 * Z[p];

# Tier 1 PRN nurses should be assigned to a minimum of eight four-hour shifts during
# the staffing horizon of six weeks. Or no assignments.

###############################################################################################

subject to PRN_Tier2_Min_Hours {p in (PT1+1)..(PT1+PT2)} :
    sum {t in 1..T} (Y[p,t]) >= 17 * Z[p];


# Tier 2 PRN nurses should be assigned to a minimum of
# 17 four-hour shifts during the staffing horizon of six weeks. Or no assignments.

###############################################################################################

subject to PRN_Tier3_Min_Hours {p in (PT1+PT2+1)..(PT1+PT2+PT3)} :
    sum {t in 1..T} (Y[p,t]) >= 29 * Z[p];

# Tier 3 PRN nurses should be assigned to a minimum of
# 29 four-hour shifts during the staffing horizon of six weeks. Or no assignments.

###############################################################################################

subject to PRN_Tier1_Assignment {p in 1..(PT1+PT2+PT3)} :
        Z[p] * M >= sum {t in 1..T} (Y[p,t]);

# We leave the option of no assignment for PRN nurses open during the staffing horizon.

###############################################################################################
```

Appendix F

Solution of A Small Problem Instance of the Medium-Term Staffing Model in AMPL

To test the developed AMPL model, in this section we present a small problem instance which involves 120 alternative schedules for nurses from 30 different job profiles (i.e. four schedule alternatives for each FTE nurse profile). Schedules are generated using the C++ codes developed and selected among 16 randomly selected schedules for the given nurse job profiles using the presented AMPL maximally different schedule selection model. Figures F.1 to F.6 below present the screenshots from the data file of the developed AMPL model:

Description of the Problem Instance

Figure F.1: Model Parameters - Data for PRN Tiers Model

```
#####   DATA (1)   #####

data;

param J := 30;# number of job profiles for the FTE nurses

param T := 252;   # number of four-hour shifts in the scheduling period
param G := 6;     # number of acuity groups for the patients
param M := 253;   # a large enough number for PRN assignment constraints.

param PT1 := 0;     # number of PRN_Tier1 nurses
param PT2 := 4;     # number of PRN_Tier2 nurses
param PT3 := 12;     # number of PRN_Tier3 nurses
param P := 16;     # number of PRN nurses

# nursing hours required for patient care for each acuity group in a four-hour nursing shift.
param h:= 1 0.5  2 1.0  3 1.5  4 2.5  5 4.0  6 8.0;

param c:= 120 ; # staffing cost per four-hour shift for the FTE nurse from job profile j.

param b:= 132 ; # staffing cost per four-hour shift for the PRN nurses.

param gamma:= 0.5; # nursing hours required for one patient admission or transfer-in activity.

param delta:= 0.5; # nursing hours required for one patient discharge or transfer-out activity.

param c_u:= 1200; # penalty cost of percentage understaffing per four-hour shift.

param c_o:= 1200; # penalty cost of percentage overstaffing per four-hour shift.

# number of FTE nurses from job profile j in J. (87 FTE nurses in Ward 3)
param n:= 1 10  2 10  3 10  4 10  5 9  6 10  7 1  8 1  9 0  10 0  11 1  12 1  13 3  14 3  15 4
         16 4  17 3  18 3  19 0  20 0  21 1  22 0  23 1  24 0  25 1  26 0  27 0  28 0  29 1  30 0;

param S := 120;# total number of available schedules to the nurses from all job profiles
```

Results from the Small Problem Instance

We use the “run file” code in Figure F.7 below to implement the optimization model in AMPL. As can be observed from the code, we are using “CPLEX” as our solver choice. We also use several “Cplex Options” :

(1) We would like the time statistics for the optimization be reported,

(2) We would like to see mixed-integer programming steps to be displayed,

(3) We choose an optimality gap based stopping criteria of 0.1%,

(4) We set an upper time limit for the optimization experiment as 6 hours (i.e. 21,600 seconds),

(5) We also set a solution tree size limit of 100GBs,

(6) We save the compressed node file on disk.

The run file code requests AMPL to report some cost statistics including:

(1) Total Cost (i.e. Objective Value),

Figure F.2: Set of All Schedules - Data for PRN Tiers Model

```
#####   DATA (2)   #####

# set of available schedules for nurses from job profile j

set SCHEDULES[1]:=  1   2   3   4;
set SCHEDULES[2]:=  5   6   7   8;
set SCHEDULES[3]:=  9   10  11  12;
set SCHEDULES[4]:=  13  14  15  16;
set SCHEDULES[5]:=  17  18  19  20;
set SCHEDULES[6]:=  21  22  23  24;
set SCHEDULES[7]:=  25  26  27  28;
set SCHEDULES[8]:=  29  30  31  32;
set SCHEDULES[9]:=  33  34  35  36;
set SCHEDULES[10]:= 37  38  39  40;
set SCHEDULES[11]:= 41  42  43  44;
set SCHEDULES[12]:= 45  46  47  48;
set SCHEDULES[13]:= 49  50  51  52;
set SCHEDULES[14]:= 53  54  55  56;
set SCHEDULES[15]:= 57  58  59  60;
set SCHEDULES[16]:= 61  62  63  64;
set SCHEDULES[17]:= 65  66  67  68;
set SCHEDULES[18]:= 69  70  71  72;
set SCHEDULES[19]:= 73  74  75  76;
set SCHEDULES[20]:= 77  78  79  80;
set SCHEDULES[21]:= 81  82  83  84;
set SCHEDULES[22]:= 85  86  87  88;
set SCHEDULES[23]:= 89  90  91  92;
set SCHEDULES[24]:= 93  94  95  96;
set SCHEDULES[25]:= 97  98  99  100;
set SCHEDULES[26]:= 101 102 103 104;
set SCHEDULES[27]:= 105 106 107 108;
set SCHEDULES[28]:= 109 110 111 112;
set SCHEDULES[29]:= 113 114 115 116;
set SCHEDULES[30]:= 117 118 119 120;
```

(2) Total FTE Staffing Cost,

(3) Total PRN Staffing Cost,

(4) Total Under Staffing Penalty Cost,

(5) Total Over Staffing Penalty Cost.

The run file code also requests AMPL to report the resulting decision variables matrices (i.e. X, Y, Z, U and O).

Figure F.3: Available Schedules for Nurse Job Profiles - Data for PRN Tiers Model

```
#####   DATA (3)   #####

########### SCHEDULES ################
# a[s,t] is 1 if for schedule s in S_{j} can be assigned to work at shift t; 0 otherwise.

# s = {1,2,...120} this instance has 480 alternative FTE nurse schedules,

# t = {1,2,...,252} nursing shifts in the staffing horizon of six weeks


param a (tr): # this is the transpose of matrix a[s,t]

    1  2  3  4  5  6  7  8  9  10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1   0  0  0  0  1  1  1  0  0  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0
2   0  0  0  0  1  1  1  0  0  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0
3   0  0  0  0  1  1  1  0  0  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0
4   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  0  1  1  0  1  1  0
5   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  0  1  1  0  1  1  0
6   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  0  1  1  0  1  1  0
7   0  0  0  0  1  1  0  1  1  1  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
8   0  0  0  0  1  1  0  1  1  1  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
9   0  0  0  0  1  1  0  1  1  1  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
10  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  0  0  1  0  1  0
11  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  0  0  1  0  1  0
12  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  0  0  1  0  1  0
13  0  0  0  0  1  0  1  1  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
14  0  0  0  0  1  0  1  1  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
15  0  0  0  0  1  0  1  1  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
16  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  1  1  1  1  1  0  0
17  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  1  1  1  1  1  0  0
18  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  1  1  1  1  1  0  0
19  0  0  0  0  0  1  1  1  1  0  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0
20  0  0  0  0  0  1  1  1  1  0  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0
21  0  0  0  0  0  1  1  1  1  0  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0
22  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  0
23  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  0
24  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  0
25  1  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
26  1  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
27  1  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
28  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  0  0  0  0  0  0  0  0  0
29  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  0  0  0  0  0  0  0  0  0
30  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  0  0  0  0  0  0  0  0  0
31  1  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
32  1  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
33  1  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
34  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  0  0  0  0  0  0  0  0  0
35  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  0  0  0  0  0  0  0  0  0
36  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  0  0  0  0  0  0  0  0  0
```

Below in Figures F.8 to F.11 we present the resulting optimal solution with the associated objective function value, solution time and optimality gap:

Figure F.4: Patient Mix and Acuity Distribution - Data for PRN Tiers Model

```
#####   DATA (4)   #####

# Number of patients in each acuity group staying at the unit in shift t.
# We have six acuity groups {1,2,...,6} and 252 nursing shifts

param v (tr):
    1   2   3   4   5   6   :=
1   0   0   0   6   8   1
2   0   0   0   5   9   1
3   0   0   0   4   10  1
4   0   0   0   4   10  1
5   0   0   0   4   10  1
6   0   0   0   4   10  1
7   0   0   0   6   8   1
8   0   0   0   6   7   2
9   0   0   0   7   5   2
10  0   0   0   7   5   2
11  0   0   0   7   5   2
12  0   0   0   7   5   2
13  0   0   0   7   7   0
14  0   0   0   3   10  1
15  0   0   0   6   9   1
16  0   0   0   6   9   1
17  0   0   1   7   7   2
18  0   0   1   7   7   2
19  0   0   1   7   7   1
20  0   0   0   6   6   3
21  0   0   0   6   7   2
22  0   0   0   6   7   2
23  0   0   0   6   7   2
24  0   0   0   6   7   2
25  0   0   0   6   7   2
26  0   0   0   3   11  2
27  0   0   0   9   9   1
28  0   0   0   10  9   1
29  0   0   0   10  9   1
30  0   0   0   9   10  1
31  0   0   0   7   12  1
32  0   0   0   14  5   1
33  0   0   0   10  9   1
34  0   0   0   10  9   1
```

Below are the understaffing percentages associated with the near-optimal solution presented in Figure F.12.

Figure F.5: Patient Admissions & Transfer-in Activities - Data for PRN Tiers Model

```
#####   DATA (5)   #####

# Number of admission and transfer-in activities to a unit for shift t.

param alpha:=
1   0
2   0
3   1
4   0
5   0
6   0
7   0
8   1
9   0
10  0
11  0
12  0
13  0
14  0
15  3
16  0
17  1
18  0
19  0
20  0
21  0
22  0
23  0
24  0
25  0
26  1
27  3
28  1
29  0
30  0
31  0
32  0
33  0
34  0
```

Below are the overstaffing percentages associated with the near-optimal solution in Figure F.13.

AMPL Model, using the IBM's CPLEX v12.6.3 Solver was able to provide near-optimal solutions to the problem in a reasonable time. Next, we develop an experimental design based on the described optimization model and run some preliminary experiments.

Figure F.6: Patient Discharges & Transfer-out Activities - Data for PRN Tiers Model

```
#####   DATA (6)   #####

# Number of patient discharge and transfer-out activities to a unit for shift t.

param beta:=
1   0
2   0
3   1
4   0
5   0
6   0
7   0
8   1
9   1
10  0
11  0
12  0
13  0
14  0
15  1
16  0
17  0
18  0
19  1
20  1
21  0
22  0
23  0
24  0
25  0
26  0
27  0
28  0
29  0
30  0
31  0
32  0
33  0
34  0
```

Figure F.7: AMPL Run Code

```
Exp_1.mod   Exp_1.run   Exp_4S_Ward3_A.dat
model Exp_1.mod;
data Exp_4S_Ward3_A.dat;
option solver cplex;
option cplex_options 'timing=1 mipdisplay=2 mipgap=0.00099 timelimit=21600 treememlim=100000 nodefile=3';
solve;
display X;
display Total_Cost;
display FTE_Staffing_Cost;
display PRN_Staffing_Cost;
display Under_Staffing_Cost;
display Over_Staffing_Cost;
display Y;
display Z;
display U;
display O;
display O;
```

Figure F.8: AMPL Model Results - Objective Value, Solution Time & Optimality Gap

```
Elapsed time = 10971.58 sec. (1075084.36 ticks, tree = 599.74 MB)
*211042+ 6015                    2247393.0000  2244318.6493             0.14%
*216644+10545                    2245274.0000  2244359.4013             0.04%

Cover cuts applied:  7
Flow cuts applied:  10
Mixed integer rounding cuts applied:  710
Zero-half cuts applied:  1706
Lift and project cuts applied:  18
Gomory fractional cuts applied:  103

Root node processing (before b&c):
  Real time             =   10.40 sec. (7088.02 ticks)
Parallel b&c, 8 threads:
  Real time             = 10976.75 sec. (1076982.64 ticks)
  Sync time (average)   =  702.72 sec.
  Wait time (average)   =  736.44 sec.
                          ------------
Total (root+branch&cut) = 10987.14 sec. (1084070.66 ticks)

Times (seconds):
Input =  0.017829
Solve =  15689
Output = 0.262513
CPLEX 12.6.3.0: optimal integer solution within mipgap or absmipgap; objective 2245274
21647877 MIP simplex iterations
217675 branch-and-bound nodes
absmipgap = 914.599, relmipgap = 0.000407344
No basis.
```

Figure F.9: AMPL Model Results - Cost Distribution & FTE Schedule Assignments

```
ampl: display X;
display Total_Cost;
display FTE_Staffing_Cost;
display PRN_Staffing_Cost;
display Under_Staffing_Cost;
display Over_Staffing_Cost;
display Y;
display Z;
display U;
display O;

Total_Cost = 2,245,270

FTE_Staffing_Cost = 475,200

PRN_Staffing_Cost = 107,976

Under_Staffing_Cost = 674,285

Over_Staffing_Cost = 987,813


X [*] :=
  1 2    13 3    25 0    37 0    49 0    61 2    73 0    85 0    97 0   109 0
  2 1    14 5    26 0    38 0    50 0    62 2    74 0    86 0    98 1   110 0
  3 5    15 2    27 0    39 0    51 1    63 0    75 0    87 0    99 0   111 0
  4 2    16 0    28 1    40 0    52 2    64 0    76 0    88 0   100 0   112 0
  5 5    17 2    29 0    41 0    53 1    65 0    77 0    89 0   101 0   113 0
  6 0    18 1    30 1    42 0    54 0    66 0    78 0    90 0   102 0   114 0
  7 2    19 2    31 0    43 0    55 0    67 0    79 0    91 0   103 0   115 0
  8 3    20 4    32 0    44 0    56 0    68 3    80 0    92 0   104 0   116 1
  9 0    21 4    33 0    45 0    57 2    69 0    81 0    93 0   105 0   117 0
 10 2    22 4    34 0    46 0    58 0    70 0    82 0    94 0   106 0   118 0
 11 4    23 2    35 0    47 1    59 0    71 1    83 0    95 0   107 0   119 0
 12 3    24 0    36 0    48 0    60 2    72 2    84 0    96 0   108 0   120 0
;
```

Figure F.10: AMPL Model Results - PRN Shift Assignments

```
Y [*,*] (tr)
:    1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16    :=
1    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
2    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
3    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
4    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
5    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
6    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
7    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
8    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
9    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
10   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
11   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
12   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
13   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
14   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
15   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
16   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
17   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
18   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
19   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
20   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
21   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
22   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
23   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
24   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
25   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0
26   0   0   0   1   0   0   0   0   0   0   0   1   0   0   0   1
27   0   0   0   0   0   1   1   0   0   0   1   0   0   0   0   0
28   0   0   0   0   0   1   1   0   0   0   0   0   0   1   0   0
29   0   0   1   0   0   1   0   0   0   0   0   0   1   0   0   0
30   0   0   0   0   0   0   0   1   1   0   0   1   0   0   0   1
31   1   1   0   0   0   0   0   1   1   1   0   0   0   0   1   0
32   1   1   0   0   0   0   0   0   0   0   0   0   0   1   1   0
33   0   0   1   0   1   1   0   0   0   0   0   0   1   1   0   0
34   0   0   1   0   1   1   0   0   0   0   0   1   0   1   0   0
35   0   0   1   1   0   1   0   0   0   1   1   0   0   0   0   0
36   1   0   0   1   0   0   0   0   0   1   1   0   0   0   0   1
```

Figure F.11: AMPL Model Results - PRN Nurse Assignments for Staffing Horizon

```
Z [*] :=
 1  1
 2  1
 3  1
 4  1
 5  1
 6  1
 7  1
 8  1
 9  1
10  1
11  1
12  1
13  1
14  1
15  1
16  1
;
```

Figure F.12: AMPL Model Results - Understaffing Percentages (%)

```
U [*] :=
  1  0           52  0.621118   103  2.04082    154  0.653595   205  0
  2  0           53  0.621118   104  3.79747    155  2.5641     206  1.36986
  3  0           54  0.621118   105  4.86486    156  4          207  3.0303
  4  0           55  0          106  3.0303     157 33.3333     208  2.85714
  5  0           56  0          107  0          158  6.17284    209  0
  6  0           57  2.5641     108  2.5641     159  2.89017    210  4.54545
  7  0           58  1.93548    109  2.7027     160  0          211  0
  8  0           59  1.93548    110  1.60428    161  1.36986    212  0
  9  0           60  1.93548    111  2.43902    162  1.84049    213  0
 10  0           61  0          112  0          163  4.7619     214  0
 11  0           62  0          113  9.09091    164  3.0303     215  0
 12  0           63  2.43902    114  1.2987     165  4.54545    216  0
 13  0           64  1.84049    115  0.653595   166  1.2987     217  1.84049
 14  0           65  1.84049    116  0          167  4.40252    218  0
 15  0           66  1.84049    117  4          168  3.18471    219  0
 16  0           67  6.97674    118  1.75439    169  0          220  3.44828
 17  0           68  0          119 14.6067     170  1.17647    221  0
 18  0           69  1.93548    120  0          171  1.2987     222  0
 19  0           70  6.74847    121  2.85714    172  0          223  0
 20  0           71  6.17284    122  6.17284    173  2.53807    224  0
 21  0           72  1.23457    123  3.5461     174  0          225  0.653595
 22  0           73  6.14525    124  0.591716   175  1.75439    226  0.689655
 23  0           74  2.89017    125  3.5461     176  2.15827    227  0
 24  0           75  6.74847    126  0          177  2.6455     228  0
 25  0           76  1.23457    127  1.53846    178  0          229  0
 26  0           77  1.23457    128  0.621118   179  0          230  1.36986
 27  0           78  1.23457    129  1.2987     180  1.2987     231  0
 28  2.15827     79  0          130  3.44828    181  2.89017    232  1.53846
 29  1.44928     80  0.689655   131  0          182  1.84049    233  0
 30  0           81  0          132  4.63576    183  0          234  0
 31  2.04082     82  4.63576    133 30.2564     184  1.75439    235  2.29008
 32  0           83  0          134  1.75439    185  1.2987     236  3.18471
 33  1.44928     84  0          135  3.2967     186  0          237  0
 34  1.44928     85  3.44828    136  1.23457    187  0          238  0
 35  1.44928     86  3.2967     137  0          188  1.07527    239  0.518135
 36  1.44928     87  0          138  2.89017    189  0          240  0
 37  0           88  4          139 35.545      190  0          241  2.7027
 38  0           89  4          140  4.34783    191  0          242  2.43902
```

Figure F.13: AMPL Model Results - Overstaffing Percentages (%)

```
O [*] :=
  1 52.7273       64  0           127  0            190  1.33333
  2 48.6726       65  0           128  0.621118     191  4.10959
  3 38.9831       66  0           129  0            192  0
  4  3.44828      67  0           130  0            193  1.91083
  5  3.44828      68  0           131  0.00024976   194  2.8169
  6  3.44828      69  0           132  0            195  0
  7 23.6364       70  0           133  0            196  0
  8 16.6667       71  0           134  0            197  0
  9 27.7778       72  0           135  0            198  0
 10 19.6262       73  0           136  0            199  0
 11 19.6262       74  0           137  0            200  0
 12 19.6262       75  0           138  0            201 -1.8339e-12
 13 40.6593       76  0           139  0            202  0
 14 15.3153       77  0           140  0            203  0
 15 11.4754       78  0           141  0            204  0
 16 49.1525       79  0.662252    142  0            205  0.699301
 17 46.4567       80  0           143  0            206  0
 18 46.0317       81  1.40845     144  0            207  0
 19 17.1171       82  0           145  0            208  0
 20  2.3622       83  1.33333     146  3.75         209  0.662252
 21  8.47458      84  1.33333     147  0            210  0
 22 28.8136       85  0           148  0            211 11.1111
 23 28.8136       86  0           149  0            212  3.84615
 24 28.8136       87  1.33333     150  0            213  0
 25  1.69492      88  0           151  0            214  0
 26  1.47059      89  0           152  0            215  0
 27  4.41176      90  0           153  0            216  1.40845
 28  0            91  0           154  0            217  0
 29  0            92  0           155  0            218  2.7027
 30  2.12766      93  0           156  0            219  7.69231
 31  0            94  0           157  0            220  0
 32  1.5873       95  0           158  0            221 25
 33  0            96  0           159  0            222 13.4752
 34  0            97  0           160  0            223  3.59712
 35  0            98  0           161  0            224 14.9606
```

Appendix G

Probability Matrix and Scenario Generating Code for the Two-Stage Stochastic Programming Model

In this chapter of the Appendices, we present parts of a sample probability matrix and scenario generating code, developed in Xcode interface using C++, for Ward A in the studied PICU for the developed two-stage stochastic programming model presented in Chapter 5.

Step 1: Define the Cardinality of the Sets, Variables for Scheduled and Unscheduled Admissions and Associated Probabilities

Figure G.1: Define the Cardinality of the Sets, Variables for Scheduled and Unscheduled Admissions and Associated Probabilities

```
//  Probability Matrix and Scenario Generating Code for Ward A.
//  Created by Osman Aydas on 4/9/17. Copyright © 2017 Osman Aydas. All rights reserved.
/////////////////////////////////////////////////////////////////////////////////////
#include <iostream>
#include <fstream>
#include <vector>
#include <random>
using namespace std;
/////////////////////////////////////////////////////////////////////////////////////
int main(int argc, const char * argv[]) {
    const int card_Sch_Admits_t=6; // Cardinality (size) of the scheduled admissions set (current shift), Sch_Admits_t = {0,1,2,3,4,5}
    const int card_Unsch_Admits_t=6; // Cardinality of the unscheduled admissions set(current shift), Unsch_Admits_t = {0,1,2,3,4,5}
    const int card_Unsch_Admits_tp1=6; // Cardinality of unscheduled admissions set(upcoming shift), Unsch_Admits_tp1 = {0,1,2,3,4,5}
    const int card_Sch_A_t=1; // Cardinality of the scheduled admissions set for acuity group A (current shift), Sch_A_t = {0}
    const int card_Sch_B_t=1; // Cardinality of the scheduled admissions set for acuity group B (current shift), Sch_B_t = {0}
    const int card_Sch_C_t=1; // Cardinality of the scheduled admissions set for acuity group C (current shift), Sch_C_t = {0}
    const int card_Sch_D_t=3; // Cardinality of the scheduled admissions set for acuity group D (current shift), Sch_D_t = {0,1,2}
    const int card_Sch_E_t=3; // Cardinality of the scheduled admissions set for acuity group E (current shift), Sch_E_t = {0,1,2}
    const int card_Sch_F_t=2; // Cardinality of the scheduled admissions set for acuity group F (current shift), Sch_F_t = {0,1}
    const int card_Unsch_A_t=1; // Cardinality of the unscheduled admissions set for acuity group A (current shift), Unsch_A_t = {0}
    const int card_Unsch_B_t=1; // Cardinality of the unscheduled admissions set for acuity group B (current shift), Unsch_B_t = {0}
    const int card_Unsch_C_t=1; // Cardinality of the unscheduled admissions set for acuity group C (current shift), Unsch_C_t = {0}
    const int card_Unsch_D_t=3; // Cardinality of the unscheduled admissions set for acuity group D (current shift), Unsch_D_t = {0,1,2}
    const int card_Unsch_E_t=3; // Cardinality of the unscheduled admissions set for acuity group E (current shift), Unsch_E_t = {0,1,2}
    const int card_Unsch_F_t=2; // Cardinality of the unscheduled admissions set for acuity group F (current shift), Unsch_F_t = {0,1}

    int Sch_Admits_t; // Number of scheduled admissions to the current shift
    int Unsch_Admits_t; // Number of unscheduled admissions to the current shift
    int Unsch_Admits_tp1; // Number of unscheduled admissions to the upcoming shift - Stochastic Data
    int Sch_A_t; // Number of scheduled admissions to the current shift for acuity group A - Stochastic Data
    int Sch_B_t; // Number of scheduled admissions to the current shift for acuity group B - Stochastic Data
    int Sch_C_t; // Number of scheduled admissions to the current shift for acuity group C - Stochastic Data
    int Sch_D_t; // Number of scheduled admissions to the current shift for acuity group D - Stochastic Data
    int Sch_E_t; // Number of scheduled admissions to the current shift for acuity group E - Stochastic Data
    int Sch_F_t; // Number of scheduled admissions to the current shift for acuity group F - Stochastic Data
    int Unsch_A_t; // Number of unscheduled admissions to the current shift for acuity group A - Stochastic Data
    int Unsch_B_t; // Number of unscheduled admissions to the current shift for acuity group B - Stochastic Data
    int Unsch_C_t; // Number of unscheduled admissions to the current shift for acuity group C - Stochastic Data
    int Unsch_D_t; // Number of unscheduled admissions to the current shift for acuity group D - Stochastic Data
    int Unsch_E_t; // Number of unscheduled admissions to the current shift for acuity group E - Stochastic Data
    int Unsch_F_t; // Number of unscheduled admissions to the current shift for acuity group F - Stochastic Data

    int CaseID [card_Sch_Admits_t] [card_Unsch_Admits_t]; // integer variable for the Case ID = {0,1,2,...,35}

    int ScenID [card_Sch_A_t] [card_Sch_B_t] [card_Sch_C_t] [card_Sch_D_t] [card_Sch_E_t] [card_Sch_F_t] [card_Unsch_A_t]
        [card_Unsch_B_t] [card_Unsch_C_t] [card_Unsch_D_t] [card_Unsch_E_t] [card_Unsch_F_t] [card_Unsch_Admits_tp1]; // Scenario ID =
        {0,1,2,...,1943}

    long double Prob [card_Sch_Admits_t * card_Unsch_Admits_t] [card_Sch_A_t * card_Sch_B_t * card_Sch_C_t * card_Sch_D_t * card_Sch_E_t
        * card_Sch_F_t * card_Unsch_A_t * card_Unsch_B_t * card_Unsch_C_t * card_Unsch_D_t * card_Unsch_E_t * card_Unsch_F_t *
        card_Unsch_Admits_tp1]; // Probability matrix listing the probability of each scenario given the Case ID

    double P_Sch_t[card_Sch_Admits_t][card_Sch_A_t][card_Sch_B_t][card_Sch_C_t][card_Sch_D_t][card_Sch_E_t][card_Sch_F_t]; //
        Probability of a specific acuity distribution for scheduled patient admissions for the current shift
    double P_Unsch_t[card_Unsch_Admits_t][card_Unsch_A_t][card_Unsch_B_t][card_Unsch_C_t][card_Unsch_D_t][card_Unsch_E_t][card_Unsch_F_t
        ]; // Probability of a specific acuity distribution for unscheduled patient admissions for the current shift
    double P_Unsch_Admits_tp1[card_Unsch_Admits_tp1]; // Probability of {0,1,...5} unscheduled admissions in the upcoming shift.
```

Step 2: Provide the Probability Estimates of Each Scheduled and Unscheduled Admission Combination

Figure G.2: Provide the Probability Estimates of Each Scheduled and Unscheduled Admission Combination

```
for(Sch_Admits_t = 0; Sch_Admits_t < card_Sch_Admits_t ; Sch_Admits_t++)
{
    for (Sch_A_t=0; Sch_A_t < card_Sch_A_t; Sch_A_t ++)
    {
        for (Sch_B_t=0; Sch_B_t < card_Sch_B_t; Sch_B_t ++)
        {
            for (Sch_C_t=0; Sch_C_t < card_Sch_C_t; Sch_C_t ++)
            {
                for (Sch_D_t=0; Sch_D_t < card_Sch_D_t; Sch_D_t ++)
                {
                    for (Sch_E_t=0; Sch_E_t < card_Sch_E_t; Sch_E_t ++)
                    {
                        for (Sch_F_t=0; Sch_F_t < card_Sch_F_t; Sch_F_t ++)
                        {
                            if (Sch_Admits_t==0)
                            {
                                if((Sch_A_t+Sch_B_t+Sch_C_t+Sch_D_t+Sch_E_t+Sch_F_t)==0)
                                {
                                    P_Sch_t[Sch_Admits_t][Sch_A_t][Sch_B_t][Sch_C_t][Sch_D_t][Sch_E_t][Sch_F_t]=1;
                                }
                                else
                                {
                                    P_Sch_t[Sch_Admits_t][Sch_A_t][Sch_B_t][Sch_C_t][Sch_D_t][Sch_E_t][Sch_F_t]=0;
                                }

                            }
                            else if (Sch_Admits_t==1)
                            {
                                if(Sch_D_t==1 && (Sch_A_t+Sch_B_t+Sch_C_t+Sch_E_t+Sch_F_t)==0)
                                {
                                    P_Sch_t[Sch_Admits_t][Sch_A_t][Sch_B_t][Sch_C_t][Sch_D_t][Sch_E_t][Sch_F_t]=0.42;
                                }
                                else if(Sch_E_t==1 && (Sch_A_t+Sch_B_t+Sch_C_t+Sch_D_t+Sch_F_t)==0)
                                {
                                    P_Sch_t[Sch_Admits_t][Sch_A_t][Sch_B_t][Sch_C_t][Sch_D_t][Sch_E_t][Sch_F_t]=0.5;
                                }
                                else if(Sch_F_t==1 && (Sch_A_t+Sch_B_t+Sch_C_t+Sch_D_t+Sch_E_t)==0)
                                {
                                    P_Sch_t[Sch_Admits_t][Sch_A_t][Sch_B_t][Sch_C_t][Sch_D_t][Sch_E_t][Sch_F_t]=0.08;
                                }
                                else
                                {
                                    P_Sch_t[Sch_Admits_t][Sch_A_t][Sch_B_t][Sch_C_t][Sch_D_t][Sch_E_t][Sch_F_t]=0;
                                }
                            }
                            else if (Sch_Admits_t==2)
                            {
                                if(Sch_D_t==1 && Sch_E_t==1 && (Sch_A_t+Sch_B_t+Sch_C_t+Sch_F_t)==0)
                                {
                                    P_Sch_t[Sch_Admits_t][Sch_A_t][Sch_B_t][Sch_C_t][Sch_D_t][Sch_E_t][Sch_F_t]=0.24;
                                }
                                else if(Sch_D_t==1 && Sch_F_t==1 && (Sch_A_t+Sch_B_t+Sch_C_t+Sch_E_t)==0)
                                {
                                    P_Sch_t[Sch_Admits_t][Sch_A_t][Sch_B_t][Sch_C_t][Sch_D_t][Sch_E_t][Sch_F_t]=0.13;
                                }
                                else if(Sch_E_t==1 && Sch_F_t==1 && (Sch_A_t+Sch_B_t+Sch_C_t+Sch_D_t)==0)
                                {
```

Step 3: Generate Case ID and Scenario ID for Each Scheduled and Unscheduled Admission Combination and Acuity Assignment

Figure G.3: Generate Case ID and Scenario ID for Each Scheduled and Unscheduled Admission Combination and Acuity Assignment

```
P_Unsch_Admits_tp1[0]= 0.786;  // Probability of 0 unscheduled admits in the upcoming period
P_Unsch_Admits_tp1[1]= 0.171;  // Probability of 1 unscheduled admits in the upcoming period
P_Unsch_Admits_tp1[2]= 0.035;  // Probability of 2 unscheduled admits in the upcoming period
P_Unsch_Admits_tp1[3]= 0.006;  // Probability of 3 unscheduled admits in the upcoming period
P_Unsch_Admits_tp1[4]= 0.001;  // Probability of 4 unscheduled admits in the upcoming period
P_Unsch_Admits_tp1[5]= 0.001;  // Probability of 5 unscheduled admits in the upcoming period

cout << "CaseID" << "\t\t" << "Sch_Admits_t" << "\t" << "Unsch_Admits_t" <<endl  ;

for (Sch_Admits_t=0; Sch_Admits_t < card_Sch_Admits_t; Sch_Admits_t ++)
{
    for (Unsch_Admits_t=0; Unsch_Admits_t < card_Unsch_Admits_t; Unsch_Admits_t ++)
    {
        CaseID [Sch_Admits_t] [Unsch_Admits_t] = (Sch_Admits_t*card_Unsch_Admits_t) + (Unsch_Admits_t) ;

        cout<< "\t" << CaseID [Sch_Admits_t] [Unsch_Admits_t] << "\t\t\t" << Sch_Admits_t << "\t\t\t\t" << Unsch_Admits_t <<endl;
    }
}
cout << endl << endl;


for (Sch_A_t=0; Sch_A_t < card_Sch_A_t; Sch_A_t ++)
{
    for (Sch_B_t=0; Sch_B_t < card_Sch_B_t; Sch_B_t ++)
    {
        for (Sch_C_t=0; Sch_C_t < card_Sch_C_t; Sch_C_t ++)
        {
            for (Sch_D_t=0; Sch_D_t < card_Sch_D_t; Sch_D_t ++)
            {
                for (Sch_E_t=0; Sch_E_t < card_Sch_E_t; Sch_E_t ++)
                {
                    for (Sch_F_t=0; Sch_F_t < card_Sch_F_t; Sch_F_t ++)
                    {
                        for (Unsch_A_t=0; Unsch_A_t < card_Unsch_A_t; Unsch_A_t ++)
                        {
                            for (Unsch_B_t=0; Unsch_B_t < card_Unsch_B_t; Unsch_B_t ++)
                            {
                                for (Unsch_C_t=0; Unsch_C_t < card_Unsch_C_t; Unsch_C_t ++)
                                {
                                    for (Unsch_D_t=0; Unsch_D_t < card_Unsch_D_t; Unsch_D_t ++)
                                    {
                                        for (Unsch_E_t=0; Unsch_E_t < card_Unsch_E_t; Unsch_E_t ++)
                                        {
                                            for (Unsch_F_t=0; Unsch_F_t < card_Unsch_F_t; Unsch_F_t ++)
                                            {
                                                for (Unsch_Admits_tp1=0; Unsch_Admits_tp1 < card_Unsch_Admits_tp1; Unsch_Admits_tp1
                                                    ++)
                                                {
                                                    ScenID [Sch_A_t] [Sch_B_t] [Sch_C_t] [Sch_D_t] [Sch_E_t] [Sch_F_t] [Unsch_A_t]
                                                        [Unsch_B_t] [Unsch_C_t] [Unsch_D_t] [Unsch_E_t] [Unsch_F_t]
                                                        [Unsch_Admits_tp1] = (Sch_A_t * card_Sch_B_t * card_Sch_C_t * card_Sch_D_t *
                                                        card_Sch_E_t * card_Sch_F_t * card_Unsch_A_t * card_Unsch_B_t *
                                                        card_Unsch_C_t * card_Unsch_D_t * card_Unsch_E_t * card_Unsch_F_t *
                                                        card_Unsch_Admits_tp1) + (Sch_B_t * card_Sch_C_t * card_Sch_D_t *
                                                        card_Sch_E_t * card_Sch_F_t * card_Unsch_A_t * card_Unsch_B_t *
                                                        card_Unsch_C_t * card_Unsch_D_t * card_Unsch_E_t * card_Unsch_F_t *
                                                        card_Unsch_Admits_tp1)  + (Sch_C_t * card_Sch_D_t * card_Sch_E_t *
```

Step 4: Print the Scheduled and Unscheduled Admission Numbers for Each Acuity Group, Under Each Scenario, in the Current Shift

Figure G.4: Print the Scheduled and Unscheduled Admission Numbers for Each Acuity Group, Under Each Scenario, in the Current Shift

```
// printing the unscheduled admission numbers for each acuity group, under each scenario, in the current shift

cout<< "\t\t\t" << "1" << "\t\t" << "2" << "\t\t" <<  "3" << "\t\t" <<  "4" << "\t\t" << "5" << "\t\t" << "6" << "\t\t" << ":=" <<
    endl;

for (Sch_A_t=0; Sch_A_t < card_Sch_A_t; Sch_A_t ++)
{
    for (Sch_B_t=0; Sch_B_t < card_Sch_B_t; Sch_B_t ++)
    {
        for (Sch_C_t=0; Sch_C_t < card_Sch_C_t; Sch_C_t ++)
        {
            for (Sch_D_t=0; Sch_D_t < card_Sch_D_t; Sch_D_t ++)
            {
                for (Sch_E_t=0; Sch_E_t < card_Sch_E_t; Sch_E_t ++)
                {
                    for (Sch_F_t=0; Sch_F_t < card_Sch_F_t; Sch_F_t ++)
                    {
                        for (Unsch_A_t=0; Unsch_A_t < card_Unsch_A_t; Unsch_A_t ++)
                        {
                            for (Unsch_B_t=0; Unsch_B_t < card_Unsch_B_t; Unsch_B_t ++)
                            {
                                for (Unsch_C_t=0; Unsch_C_t < card_Unsch_C_t; Unsch_C_t ++)
                                {
                                    for (Unsch_D_t=0; Unsch_D_t < card_Unsch_D_t; Unsch_D_t ++)
                                    {
                                        for (Unsch_E_t=0; Unsch_E_t < card_Unsch_E_t; Unsch_E_t ++)
                                        {
                                            for (Unsch_F_t=0; Unsch_F_t < card_Unsch_F_t; Unsch_F_t ++)
                                            {

                                                for (Unsch_Admits_tp1=0; Unsch_Admits_tp1 < card_Unsch_Admits_tp1; Unsch_Admits_tp1
                                                    ++)
                                                {
                                                    cout<< "\t" << ScenID [Sch_A_t] [Sch_B_t] [Sch_C_t] [Sch_D_t] [Sch_E_t] [Sch_F_t
                                                        ] [Unsch_A_t] [Unsch_B_t] [Unsch_C_t] [Unsch_D_t] [Unsch_E_t] [Unsch_F_t]
                                                        [Unsch_Admits_tp1] << "\t\t" << Unsch_A_t << "\t\t" << Unsch_B_t << "\t\t"<<
                                                        Unsch_C_t << "\t\t"<< Unsch_D_t << "\t\t"<< Unsch_E_t << "\t\t"<< Unsch_F_t
                                                        <<endl;
                                                }
                                            }
                                        }
                                    }
                                }
                            }
                        }
                    }
                }
            }
        }
    }
}
cout << endl << endl;

// printing the number of unscheduled admission, under each scenario, for the upcoming shift

for (Sch_A_t=0; Sch_A_t < card_Sch_A_t; Sch_A_t ++)
{
```

Step 5: Print the Number of Unscheduled Admissions, Under Each Scenario, for the Upcoming Shift

Figure G.5: Print the Number of Unscheduled Admissions, Under Each Scenario, for the Upcoming Shift

```
// printing the number of unscheduled admission, under each scenario, for the upcoming shift

for (Sch_A_t=0; Sch_A_t < card_Sch_A_t; Sch_A_t ++)
{
    for (Sch_B_t=0; Sch_B_t < card_Sch_B_t; Sch_B_t ++)
    {
        for (Sch_C_t=0; Sch_C_t < card_Sch_C_t; Sch_C_t ++)
        {
            for (Sch_D_t=0; Sch_D_t < card_Sch_D_t; Sch_D_t ++)
            {
                for (Sch_E_t=0; Sch_E_t < card_Sch_E_t; Sch_E_t ++)
                {
                    for (Sch_F_t=0; Sch_F_t < card_Sch_F_t; Sch_F_t ++)
                    {
                        for (Unsch_A_t=0; Unsch_A_t < card_Unsch_A_t; Unsch_A_t ++)
                        {
                            for (Unsch_B_t=0; Unsch_B_t < card_Unsch_B_t; Unsch_B_t ++)
                            {
                                for (Unsch_C_t=0; Unsch_C_t < card_Unsch_C_t; Unsch_C_t ++)
                                {
                                    for (Unsch_D_t=0; Unsch_D_t < card_Unsch_D_t; Unsch_D_t ++)
                                    {
                                        for (Unsch_E_t=0; Unsch_E_t < card_Unsch_E_t; Unsch_E_t ++)
                                        {
                                            for (Unsch_F_t=0; Unsch_F_t < card_Unsch_F_t; Unsch_F_t ++)
                                            {

                                                for (Unsch_Admits_tp1=0; Unsch_Admits_tp1 < card_Unsch_Admits_tp1; Unsch_Admits_tp1
                                                    ++)
                                                {
                                                    cout<< "\t" << ScenID [Sch_A_t] [Sch_B_t] [Sch_C_t] [Sch_D_t] [Sch_E_t] [Sch_F_t
                                                        ] [Unsch_A_t] [Unsch_B_t] [Unsch_C_t] [Unsch_D_t] [Unsch_E_t] [Unsch_F_t]
                                                        [Unsch_Admits_tp1] << "\t" << Unsch_Admits_tp1 ;
                                                }
                                            }
                                        }
                                    }
                                }
                            }
                        }
                    }
                }
            }
        }
    }
}
```

Step 6: Compute the Probability of Each Scenario Given the Probability Estimates of Each Scheduled and Unscheduled Admission Combination and Case ID

Figure G.6: Compute the Probability of Each Scenario Given the Probability Estimates of Each Scheduled and Unscheduled Admission Combination and Case ID

```
for (Sch_A_t=0; Sch_A_t < card_Sch_A_t; Sch_A_t ++)
{
    for (Sch_B_t=0; Sch_B_t < card_Sch_B_t; Sch_B_t ++)
    {
        for (Sch_C_t=0; Sch_C_t < card_Sch_C_t; Sch_C_t ++)
        {
            for (Sch_D_t=0; Sch_D_t < card_Sch_D_t; Sch_D_t ++)
            {
                for (Sch_E_t=0; Sch_E_t < card_Sch_E_t; Sch_E_t ++)
                {
                    for (Sch_F_t=0; Sch_F_t < card_Sch_F_t; Sch_F_t ++)
                    {
                        for (Unsch_A_t=0; Unsch_A_t < card_Unsch_A_t; Unsch_A_t ++)
                        {
                            for (Unsch_B_t=0; Unsch_B_t < card_Unsch_B_t; Unsch_B_t ++)
                            {
                                for (Unsch_C_t=0; Unsch_C_t < card_Unsch_C_t; Unsch_C_t ++)
                                {
                                    for (Unsch_D_t=0; Unsch_D_t < card_Unsch_D_t; Unsch_D_t ++)
                                    {
                                        for (Unsch_E_t=0; Unsch_E_t < card_Unsch_E_t; Unsch_E_t ++)
                                        {
                                            for (Unsch_F_t=0; Unsch_F_t < card_Unsch_F_t; Unsch_F_t ++)
                                            {
                                                for (Unsch_Admits_tp1=0; Unsch_Admits_tp1 < card_Unsch_Admits_tp1; Unsch_Admits_tp1
                                                    ++)
                                                {
                                                    Prob [((Sch_A_t+Sch_B_t+Sch_C_t+Sch_D_t+Sch_E_t+Sch_F_t)*card_Unsch_Admits_t) +
                                                        (Unsch_A_t+Unsch_B_t+Unsch_C_t+Unsch_D_t+Unsch_E_t+Unsch_F_t)] [(Sch_A_t *
                                                        card_Sch_B_t * card_Sch_C_t * card_Sch_D_t * card_Sch_E_t * card_Sch_F_t *
                                                        card_Unsch_A_t * card_Unsch_B_t * card_Unsch_C_t * card_Unsch_D_t *
                                                        card_Unsch_E_t * card_Unsch_F_t * card_Unsch_Admits_tp1) + (Sch_B_t *
                                                        card_Sch_C_t * card_Sch_D_t * card_Sch_E_t * card_Sch_F_t * card_Unsch_A_t *
                                                        card_Unsch_B_t * card_Unsch_C_t * card_Unsch_D_t * card_Unsch_E_t *
                                                        card_Unsch_F_t * card_Unsch_Admits_tp1)  + (Sch_C_t * card_Sch_D_t *
                                                        card_Sch_E_t * card_Sch_F_t * card_Unsch_A_t * card_Unsch_B_t *
                                                        card_Unsch_C_t * card_Unsch_D_t * card_Unsch_E_t * card_Unsch_F_t *
                                                        card_Unsch_Admits_tp1)  + (Sch_D_t * card_Sch_E_t * card_Sch_F_t *
                                                        card_Unsch_A_t * card_Unsch_B_t * card_Unsch_C_t * card_Unsch_D_t *
                                                        card_Unsch_E_t * card_Unsch_F_t * card_Unsch_Admits_tp1) + (Sch_E_t *
                                                        card_Sch_F_t * card_Unsch_A_t * card_Unsch_B_t * card_Unsch_C_t *
                                                        card_Unsch_D_t * card_Unsch_E_t * card_Unsch_F_t * card_Unsch_Admits_tp1) +
                                                        (Sch_F_t * card_Unsch_A_t * card_Unsch_B_t * card_Unsch_C_t * card_Unsch_D_t
                                                        * card_Unsch_E_t * card_Unsch_F_t * card_Unsch_Admits_tp1) + (Unsch_A_t *
                                                        card_Unsch_B_t * card_Unsch_C_t * card_Unsch_D_t * card_Unsch_E_t *
                                                        card_Unsch_F_t * card_Unsch_Admits_tp1) + (Unsch_B_t * card_Unsch_C_t *
                                                        card_Unsch_D_t * card_Unsch_E_t * card_Unsch_F_t * card_Unsch_Admits_tp1) +
                                                        (Unsch_C_t * card_Unsch_D_t * card_Unsch_E_t * card_Unsch_F_t *
                                                        card_Unsch_Admits_tp1) + (Unsch_D_t * card_Unsch_E_t * card_Unsch_F_t *
                                                        card_Unsch_Admits_tp1) + (Unsch_E_t * card_Unsch_F_t * card_Unsch_Admits_tp1
                                                        ) + (Unsch_F_t * card_Unsch_Admits_tp1) + Unsch_Admits_tp1 ] = P_Sch_t
                                                        [Sch_A_t+Sch_B_t+Sch_C_t+Sch_D_t+Sch_E_t+Sch_F_t][Sch_A_t][Sch_B_t][Sch_C_t]
                                                        [Sch_D_t][Sch_E_t][Sch_F_t] * P_Unsch_t[Unsch_A_t+Unsch_B_t+Unsch_C_t+
                                                        Unsch_D_t+Unsch_E_t+Unsch_F_t][Unsch_A_t][Unsch_B_t][Unsch_C_t][Unsch_D_t]
                                                        [Unsch_E_t][Unsch_F_t] * P_Unsch_Admits_tp1[Unsch_Admits_tp1] ;
                                                }
                                            }
```

Step 7: Generate the Probability Matrix for Each Scenario Given the Case ID

Figure G.7: Generate the Probability Matrix for Each Scenario Given the Case ID

```
    for (int i=0; i < card_Sch_Admits_t * card_Unsch_Admits_t; i ++)
    {
        cout << "\t" << i ;
    }
    cout << ":=" << endl << endl;

    for (int j=0; j < card_Sch_Admits_t * card_Unsch_Admits_t; j++)
    {
        cout<< "Case " << j;

        for (int a=0; a < (card_Sch_A_t * card_Sch_B_t * card_Sch_C_t * card_Sch_D_t * card_Sch_E_t * card_Sch_F_t * card_Unsch_A_t *
            card_Unsch_B_t * card_Unsch_C_t * card_Unsch_D_t * card_Unsch_E_t * card_Unsch_F_t * card_Unsch_Admits_tp1); a++)
        {
            cout<< "\t" << Prob[j][a];
        }
        cout << endl << endl;
    }


    return 0;
}
```

CURRICULUM VITAE

OSMAN T. AYDAS

EDUCATIONAL HISTORY

- 2012 - 2017: Ph.D. in Management Science with a major in Supply Chain & Operations Management and minor in Information Technology Management, Lubar School of Business, University of Wisconsin - Milwaukee.
- 2000 - 2002, M.A. in Economics, Bilkent University, Ankara, Turkey.
- 1996 - 2000, B.S. in Industrial Engineering, Bilkent University, Ankara, Turkey.

PUBLICATIONS

- Aydas, O. T. (2017). "*Strategic Nurse Allocation Policies Under Dynamic Patient Demand*" (Unpublished doctoral dissertation). University of Wisconsin-Milwaukee, Milwaukee, WI.
- Ross A. D., Khajeneze M., Otieno W., Aydas O.T. (2017) "Integrated Location- Inventory Modeling under Forward and Reverse Product Flows in the Used Merchandise Retail Sector: A Multi-Echelon Formulation" *European Journal of Operational Research* 259(2), 664-676.
- Petering, M. E. H., Aydas, O.T., Kuzu, K., Ross, A. D. (2015) "Simulation Analysis of Hospital Intensive Care Unit Reimbursement Policies from the Triple Bottom Line Perspective" *Journal of Simulation*, 9(2), 86-98.
- Otieno W., Ross A. D., Aydas O. T., Omwando T. (2015) "Labor Capacity Assignment Model for Remanufacturing Environments" *International Conference on Remanufacturing Proceedings*, Amsterdam, Netherlands.
- Aydas, O.T., Metin-Ozcan, K., Neyapti, B. (2005) "Determinants of Workers' Remittances: The Case of Turkey" *Emerging Markets Finance and Trade*, Vol. 41(3), pp. 53-69.

CONFERENCE PRESENTATIONS

- 11/2016 - INFORMS Annual Meeting 2016, Nashville, TN. “Strategic Nurse Allocation Policies for Pediatric Intensive Care Units” with Ross A.D. & Kuzu K.
- 11/2016 - DSI Annual Meeting 2016, Austin, TX. “Strategic Nurse Allocation Policies for Pediatric Intensive Care Units” with Ross A.D. & Kuzu K.
- 11/2015 - INFORMS Annual Meeting 2015, Philadelphia, PA. “Nurse Allocation Policy Evaluation and Analysis of Admissions in an Intensive Care Unit” with Kuzu K., Scanlon M. & Ross A.D.
- 11/2014 - INFORMS Annual Meeting 2014, San Francisco, CA. “Labor Capacity Assignment Model for Remanufacturing Environments” with Ross A.D. & Otieno W.

TEACHING EXPERIENCE

- Spring 2016 - BUSMGMT 807 - *Supply Chain Operations*: This graduate level course covers fundamental concepts of operations and strategic supply chain management, functions within an organization, flow of goods and information, interface with supply chain partners across firms.
- Spring 2016 - BUSADM 475 - *Operations Planning & Control*: The course focuses on planning and controlling the operations of a service or manufacturing organization. Topics include forecasting, capacity planning, requirements planning, scheduling and project management.
- WinterIM 2016 - BUSADM 370 - *Introduction to Supply Chain Management*: The course includes a broad survey of the foundational concepts and principles of managing supply chain operations. Topics include customer management, purchasing, logistics, project management, demand planning.
- Summer 2015 - BUSADM 370 - *Introduction to Supply Chain Management*
- Spring 2015 - BUSADM 475 - *Operations Planning & Control*
- Fall 2015 - BUSADM 475 - *Operations Planning & Control*

PROGRAMMING SKILLS

- Programming Language
  - C++
- Statistical Computing & Data Mining

- R (R Foundation for Statistical Computing, Vienna, Austria)
- SAS®(SAS Institute Inc., Cary, NC)
- JMP®(SAS Institute Inc., Cary, NC)
- WEKA (http://www.cs.waikato.ac.nz/ml/weka/)

• Optimization and Modelling
  - IBM®ILOG®CPLEX®Optimization Studio (IBM Corporation, Armonk, NY)
  - AMPL®(AMPL Optimization, Inc., Albuquerque, NM)
  - GAMS®(GAMS Development Corporation, Washington, DC)

• Simulation Modelling
  - ProModel®(ProModel Corporation, Allentown, PA)
  - Arena®(Rockwell Automation Inc., Pittsburg, PA)
  - Simio®(Simio LLC., Sewickley, PA)

HONORS & AWARDS

- 2016, 2015 - Roger L. Fitzsimonds Doctoral Scholarship, Lubar School of Business, University of Wisconsin-Milwaukee. (Awarded to only two students among all Ph.D. students with dissertator status at the Lubar School)
- 2012 - Awarded with full scholarship plus stipend at University of Wisconsin–Milwaukee, Lubar School of Business for the Ph.D. program in the Department of Supply Chain and Operations Management.
- 2000 - Ranked 32nd out of 40,000 in the Graduate School Admission Examination in Mathematical Sciences in Turkey.
- 2000 - Awarded full scholarship plus stipend for M.A. in Economics, Bilkent University, Ankara, Turkey.
- 1996 - Ranked 210th out of more than 1.5 million attendants in the nationwide university placement examination, awarded full scholarship plus stipend for B.S. degree in the Department of Industrial Engineering, Bilkent University, Ankara, Turkey.