May 2017

# A Novel Distributed Control Method for Economic Dispatch in Islanded Microgrid

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# A NOVEL DISTRIBUTED CONTROL METHOD FOR ECONOMIC DISPATCH IN ISLANDED MICROGRID

by

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A Thesis Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Master of Science

in Engineering

at

The University of Wisconsin-Milwaukee

May 2017

# ABSTRACT

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by

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The University of Wisconsin-Milwaukee, 2017
Under the Supervision of Professor David C. Yu



Microgrid (MG) is proposed to integrate the distributed generations and improve the energy efficiency. Economic dispatch is an important tool to obtain the economic operation in MG, which deals with the power in efficient ways while meeting the constraints of total load demand as well as the generator constraints. In research, a two-layer MG is proposed, where the top layer is a communication network for information transmission, while the bottom layer is the MG for power transmission. A fully distributed control method is proposed to obtain the minimum operation costs of the distributed generations by reaching the consensus of the incremental cost, which can address the equality and inequality constraints. Then, seven cases are designed to evaluate the performance of the methods. The simulation results demonstrate the effectiveness of proposed approach.

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# ACKNOWLEDGEMENTS

I would first like to thank my thesis advisor Professor Yu. The door to Prof. Yu's office always open whenever I ran into a trouble spot or had a question about my research or writing. He allowed this research to be my own work, but steered me in the right direction whenever he thought I needed it. My supervisor also gave me warm concern about my life, and he made me feel like stay at my home county even though I am totally new when I just arrived in US.

I would also like to thanks to Doc. Cuzner who gave us the power electronics class and will attend my defense to be one of the committee member. His patience in helping me solve my class question and his profound knowledge impress me.

I would also like to acknowledge my domestic advisor Doc. Li Qiang and Prof. Chen Minyou. They encouraged me to pursue such an extraordinary experience in UWM and gave advice on my research directions.

Finally, I must express my very profound gratitude to my parents for providing me with unfailing support and continuous encouragement throughout my years of study and throughout the process of researching and writing this thesis. This accomplishment would not have been possible without them. Thank you.

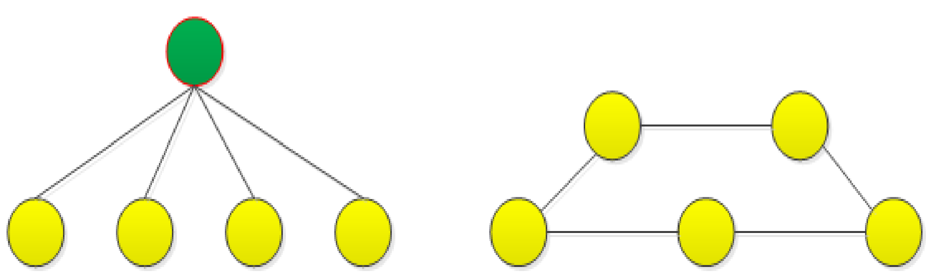
# Ⅰ. Introduction

As the energy crisis and environmental issues have become increasingly prominent, it has become the general consensus of human society to develop low carbon economy and construct ecological civilization to achieve sustainable development for all the countries in the world [1]. The development of the clean and renewable energy has become the main strategy to realize the economic and sustainable development.

Distributed generation is a technology that uses a variety of dispersed energy to generate power, including renewable energy and local fossil fuels. Two main categories can be classified, namely which require a supplied fuel and do not require a supplied fuel. For the type with supplied fuel, microturbine and the fuel cell can be typical. Microturbines are scaled down tubine engines with integrated generators and power electronics. Distributed generation has the advantage of the less pollution, higher energy efficiency, and more flexible installation, it can save transmission and distribution resources and operating costs, and reduce the line loss of concentrated transmission.  It can also reduce the total capacity of power grid, improve peak and valley performance and the reliability of power supply. It is a strong complement and effective support of power grid. With the penetration of the DG increasing, its inherent problems are apparent. It has high costs of single access and control difficulties. On the one hand, DG is an uncontrollable source, so large system tends to take ways of restriction, isolation to deal with it, in order to reduce the impact on the power grid.  On the other hand, DG has some characteristics, which make DG have to connect and operate in the form of load, this results in the structure of the distributed generation would be extremely limited.

In order to integrate the distributed generation into the grid, and maximize the advantages of the distributed generation in economy, energy and environment, the concept of MG is put forward. MG is a smaller, independent and decentralized system, which consists of loads, generators, energy storage systems. It can reduce the feeder loss, and increase the reliability of local power supply and improve energy efficiency. MG works not only in islanded mode but also the grid-connected mode through the PCC (point of common coupling) to exchange power with the host grid. In grid-connected mode, MG and the host grid are backup of each other, thus improving reliability of power supply.

Two main methods are widely used in MG control, namely centralized control and distributed control, shown in Figure 1. For the centralized control, a central controller is needed to collect all the information from units and send the information back after processing. Further, there is no information exchanging among units. When all the information flows into the central controller, it should be huge task for central controller to deal with them, and the breaking down possibility will be high for the central controller. The other type is the distributed control, which all the units will communicate with their neighbors and no central controller exists.



(a) Centralized control

(b) Distributed control

Figure 1. Different control methods

Compared with conventional power system, the manipulated variables of MG are more abundant, such as the active power output of distributed power, the interface current of current source inverters, etc. In the conditions of the system operating constraint, MG can achieve optimal operation and reasonable distribution of energy by controlling and regulating these variables, maximize the use the renewable energy, and ensure the economic operation of the entire MG.

Since various types of DG units such as wind turbines, photovoltaic (PV) panels, and fuel cells may coexist, one pivotal problem in MG management is the economic dispatch problem (EDP) which balances the power generation and loads at a minimum monetary cost [2].

Different from traditional power systems with thermal energy power generators, the economic dispatch in MGs is challenging because of the intermittent and climate-dependent nature of renewable energy sources. EDP is commonly formulated as an optimization problem in the literature [3]. Solving EDP is to find a power output combination of all generators, which give the lowest operating cost while maintaining systems constraints [4].

Traditional algorithms for solving economic dispatch problem can be classified into two categories [5]. The first category is analytical method, such as Lagrange multipliers [6], gradient search methods [7], and linear programing [8]. The second category is heuristic method such as genetic algorithm [9], evolutionary programming, and particle swarm optimization [10].

Most of such methods are centralized and deployed at a large timescale [5]. Although the centralized economic dispatch has the advantage of high efficiency, it suffers from the problem of a single point of failure and high deployment cost in terms of a powerful central controller and a communication infrastructure. Moreover, both the future power gird and the communication network are likely to have a variable topology, which further undermines the efficiency of centralized mechanisms [11]. Because of the large amounts of data to process and the significant delays with communications systems, it is difficult for a centralized method to provide optimal control setting in real time. Thus, the response speed of such solutions needs to be improved.

To overcome the disadvantage of the centralized methods, various types of decentralized methods have been proposed. The main idea comes from the distributed consensus theory multi-agent systems [12]-[14]. As pointed in [15], [16], [17] and [18], the future power grid can be considered as aggregation of controllable power electronics devices which are controlled by distributed agents though the communication network. The consensus algorithm requires only neighbor-to neighbor interaction [19]. The distributed algorithm in [20] and [21] can eventually converge to optimal allocations, however, it takes relatively long time to achieve that. Moreover, in order to drive the overall allocation towards optima, power mismatch is unavoidable for the algorithm, thus unexpected oscillation can be observed during optimization process. In [22]-[23], the authors proposed a two-level incremental cost consensus algorithm, corresponding to the leader level and follower level respectively. The algorithm requires leader to be able to access the global generation and load information, which requires extra time for the communication. In addition, the algorithm does not consider the power loss of the system which degrades the optimality of the obtained solutions. In [24], the authors proposed a population dynamics approach for the dispatch of distributed generators in a smart grid. However, an auctioneer agent acting like a centralized coordinator is still required. The work in

[25] presented a class of distributed gradient algorithms for optimal resource allocation over a network, where only generation-demand equality constraints were considered and it did not take the influence of intermittent renewable generation into consideration. In this research, a fully distributed multi agent based on optimal economic dispatch control method for islanded MG is proposed. According to the proposed solution, each generator has an associate agent that communicates with its neighboring agents only. No centralized or specialized agent is used to coordinate the operation of the autonomous agents. The intermittent renewable generations are taken into consideration. The major contributions of this research are summarized as follows.

1) A systematic method is given, which consists of two parts. One is the rules to establish a communication network, and the other is the method to derive two sets of control laws based on the communication network. It no longer needs any other agents to collect the power mismatch.
2) Both equality constraints and inequality constraints are addressed.
3) The consensus of incremental cost will be obtained in a relatively short time, even after large fluctuation of our system.
4) The intermittent renewable generations are fluctuating with environment, and the real time information can be collected by agents. Thus, the influence of renewable generation changing is taken into consideration.

The rest of this research in organized as follows. The economic dispatch problem and the equal incremental cost principle are introduced in Chapter II. In Chapter III, the MG network topology is introduced, and also the control algorithms are derived. Chapter IV introduces the structure of the MG and the parameters of DGs for simulations. Simulations are performed in Chapter V, and the results are analyzed and compared. Chapter VI presents the conclusions of this research.

# Ⅱ. Problem Formulation

This section introduces the economic dispatch problem for islanded MG, which aims at minimizing the total active power generation cost. After the MG is given, the design rules for the communication network are presented first. Further, the validations of the theorem are carried out. Then applying our proposed method to the economic dispatch problem, the consensus of the incremental cost is introduced, which to make the cost minimum.

**A. Economic Dispatch Problem**

A MG usually consists of multiple distributed power generators [29]. For a conventional diesel generators, generation cost $C_i(x_i)$ can be expressed as convex quadratic function of active power output, i.e.,:

$$C_i(x_i) = \frac{(x_i+\alpha_i)^2}{2\beta_i} + \gamma_i \tag{1}$$

Where $x_i$ denotes the active power output of generator i, and $\alpha_i \geq 0, \beta_i > 0, \text{and } \gamma_i \leq 0.$

The following represents a simple formulation of the EDP:

$$\text{Minimize} \quad C=\sum_{i=1}^{n} Ci\ (xi) \tag{2}$$

Subject to $\quad x_i^{min} \leq x_i \leq x_i^{max}$ (3)

$$\sum_{i=1}^{n} x_i = D \tag{4}$$

Where $D$ is the total demand satisfying $\sum_{i=1}^{n} x_i^{min} < D < \sum_{i=1}^{n} x_i^{max}$

The incremental cost for the generator *i* is $\mathrm{d}C_i(x_i)/dx_i = (x_i + \alpha_i)/\beta_i$. The solution to traditional EDP is the equal incremental cost criterion:

$$\begin{cases} \frac{x_i - \alpha_i}{\beta_i} = \lambda^* \quad for \; x_i^{min} < x_i < x_i^{max} \\ \frac{x_i - \alpha_i}{\beta_i} < \lambda^* \quad for \qquad x_i = x_i^{max} \\ \frac{x_i - \alpha_i}{\beta_i} > \lambda^* \quad for \qquad x_i^{min} = x_i \end{cases} \tag{5}$$

Where $\lambda^*$is the optimal incremental cost.

Note the parameter $\gamma_i$ in the cost function does not affect the incremental cost.

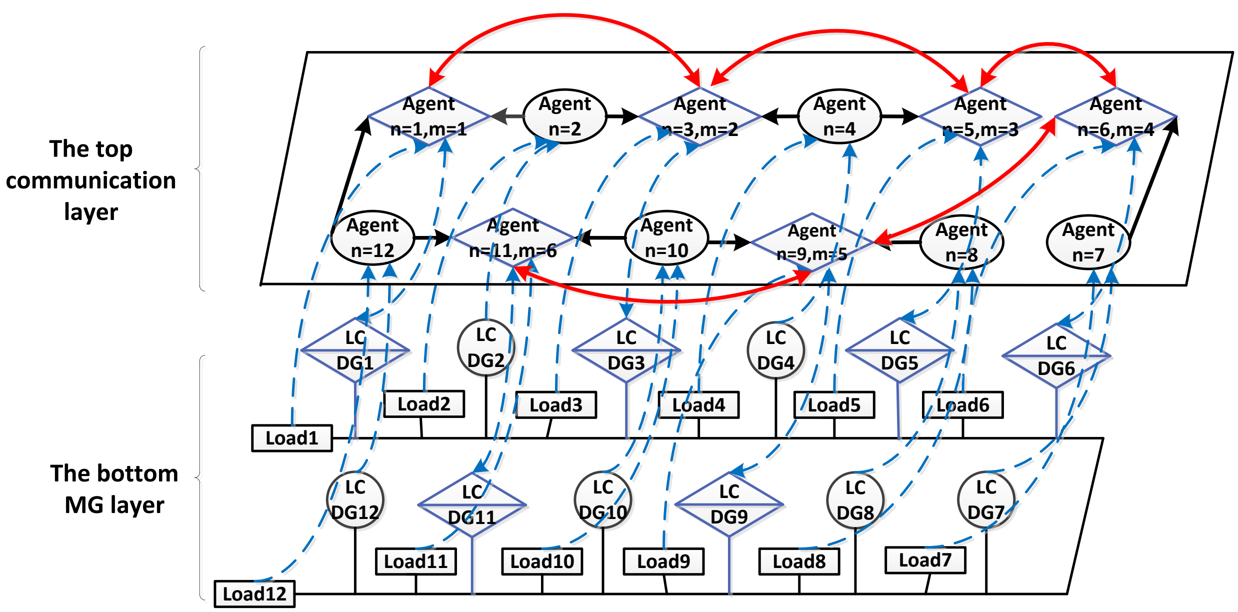
**Remark 1:** the cost function in (1) is slightly different from the one in (3), and they are essentially the same.

$$\mathrm{C}_i(x_i) = a_i x_i^2 + b_i x_i + c_i \tag{6}$$

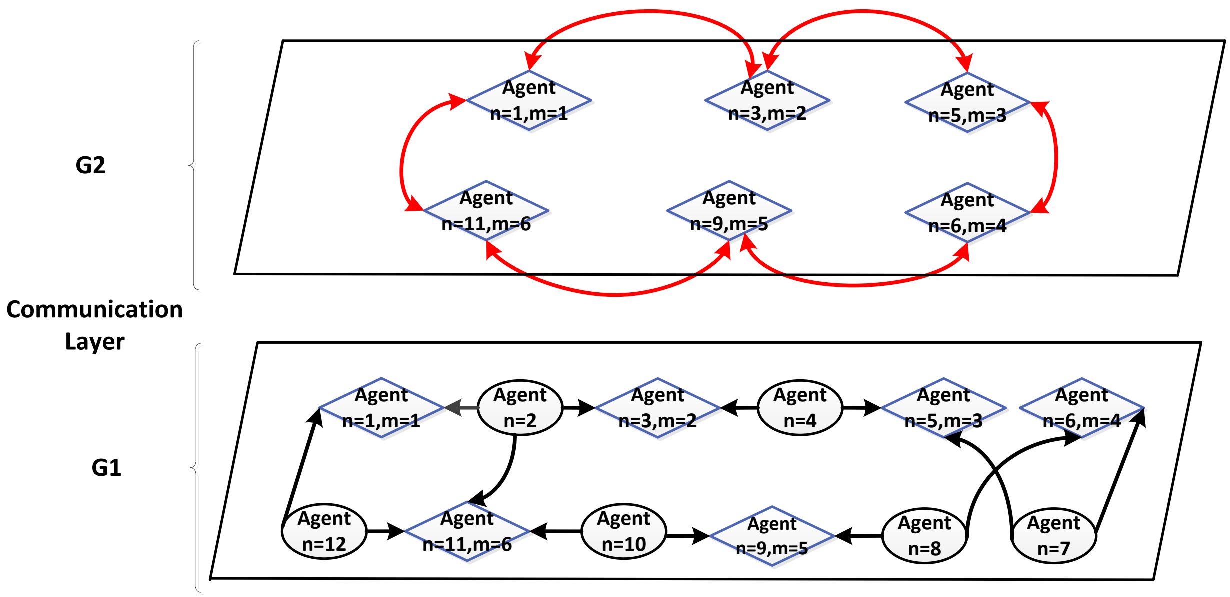
By setting$\alpha_i = b_i/2a_i\,, \beta_i = 1/2a_i\,, and\; \gamma_i = c_i - (b_i^2)/(4a_i)$, (1) and (3) are identical. The main motivation of using (1) is for notational simplicity in the following parts.

## B. Topology of Microgrid

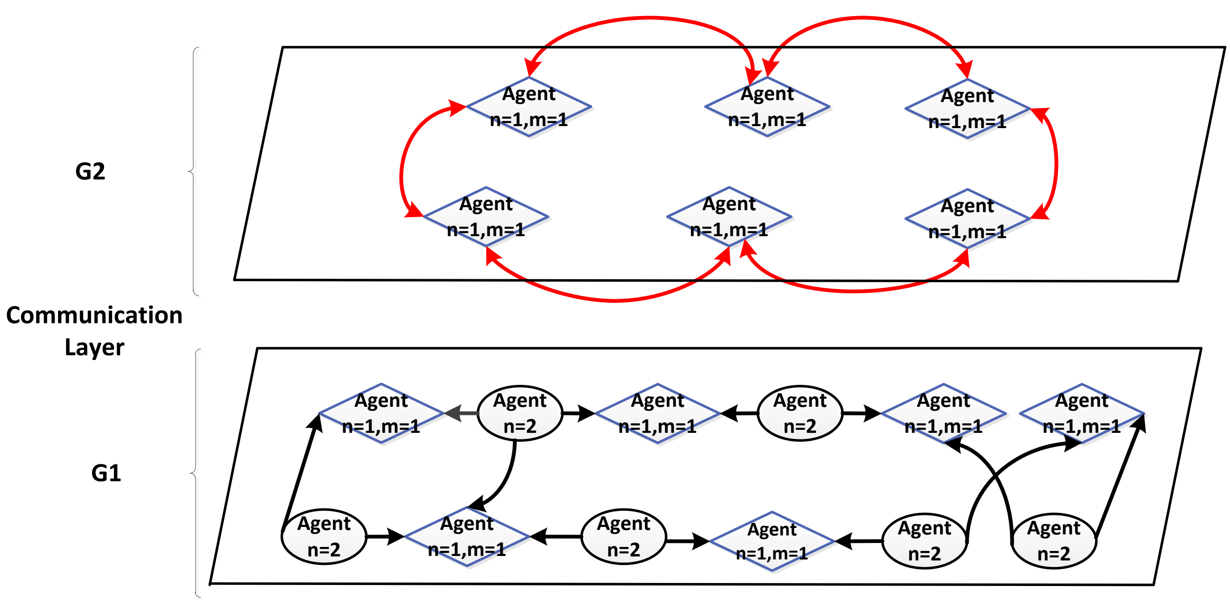
In islanded microgrid, a two-layer model is employed, as shown in figure 2, where the top layer is the communication network for information transmission, while the bottom layer is MG for power transmission. The MG layer consists of DGs and loads. The DGs in an islanded microgrid can be classified into three types. The first is the uncontrollable DGs, such as photovoltaic (PV) systems or wind turbines (WTs). These DGs rely on the renewable resources and the outputs are changing with the environment. On the contrary, the second type is the controllable DGs, such as microturbines, whose output can be controlled in terms of instructions. In islanded MG, the third is a battery energy storage system (BESS), which provide the frequency and voltage reference for the MG. Hence, it is regarded as the partially controllable DG.



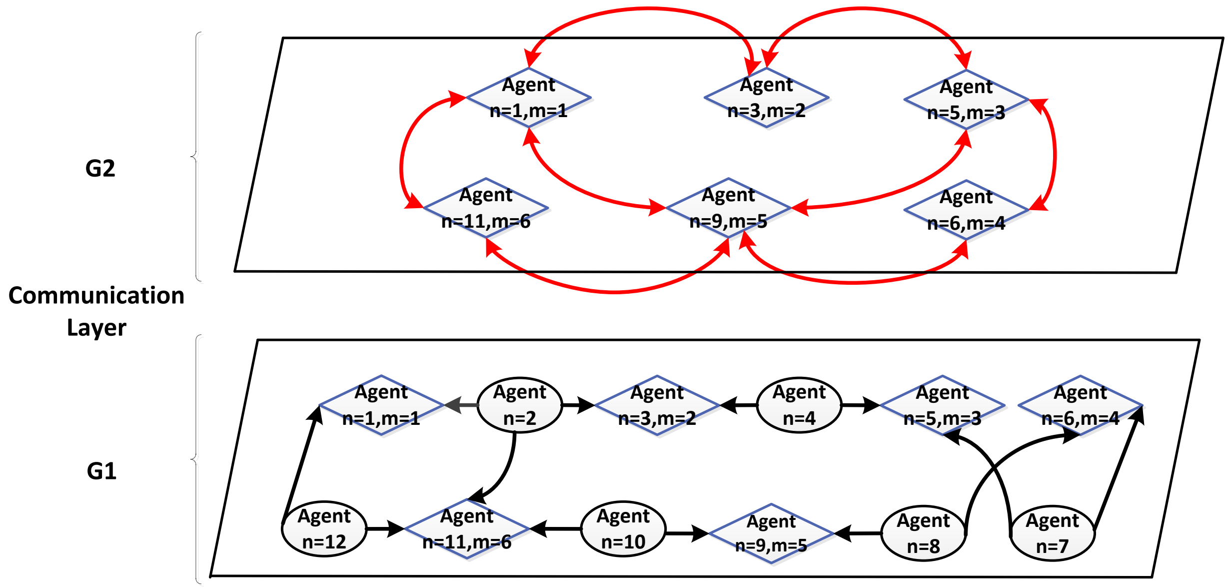

(a)


G2
Agent n=1,m=1
Agent n=3,m=2
Agent n=5,m=3
Agent n=11,m=6
Agent n=9,m=5
Agent n=6,m=4
Communication Layer
G1
Agent n=1,m=1
Agent n=2
Agent n=3,m=2
Agent n=4
Agent n=5,m=3
Agent n=6,m=4
Agent n=12
Agent n=11,m=6
Agent n=10
Agent n=9,m=5
Agent n=8
Agent n=7

(b)


G2
Agent n=1,m=1
Agent n=1,m=1
Agent n=1,m=1
Agent n=1,m=1
Agent n=1,m=1
Agent n=1,m=1
Communication Layer
G1
Agent n=1,m=1
Agent n=2
Agent n=1,m=1
Agent n=2
Agent n=1,m=1
Agent n=1,m=1
Agent n=2
Agent n=1,m=1
Agent n=2
Agent n=1,m=1
Agent n=2
Agent n=2

(c)




(d)

Figure 2. The two-layer control model for an MG. where uncontrollable and partially controllable agents are indicated by circles, while controllable agents are indicated by diamonds. (a) Network 1. power transmission and information transmission in MG a. (b) the communication network in MG a. (c) Network 2. (d) Network 3.

In the communication layer, a directed and weighted graph $A = G(V, E)$ is applied to describe the connection of the network, where $V$ is the set of agents (nodes) and $E$ is the set of edges. In our model, the type of the agents is defined according to the type of DGs which they are connected to. The agents who connect to the controllable DGs are the controllable agents; the agents who connect to the uncontrollable DGs are the uncontrollable agents; the agents who connect to partially controllable DGs are partially controllable agents. The controllable agents can not only receive the information but also send information. At the same time, the partially controllable agents and the uncontrollable agents can only send information to the controllable agents. The communication network consists of two subgraphs, namely the subgraph G1, which includes all types of agents, and the subgraph G2, which only includes the controllable agents. In subgraph G1, the controllable agents do not only have outgoing links but also have the ingoing links, while the uncontrollable and partially controllable agents can only have the outgoing links. It is worth noting that the weight on an edge from an agent *i* to agent *j* is $a_{ij}(i \neq j)$. Moreover, if the outgoing degree of ucontrollable agent is $d_i$, weight on each outgoing edge of uncontrollable agent should be $1/d_i$.For subgraph G2, each agent can have the bidirectional links.

In the connection between top layer and bottom layer, each agent will handle one DG and one load, and the dashed line between the two layers instructs the information flow direction. Note that the agents can not only collect the current information from DGs but also send the command information to the DGs. In our model, the number of the agents should be equal to the number of DGs or loads. In this case, the dimension of our matrix can be equal in our control laws. Sometimes, the number of loads will be bigger than the number of agents. To handle this, some neighboring loads will be combined into one large load to connect the agent. Conversely, if the number of loads is smaller than the number of agents, some virtual loads with zero demand can be added into our network.

## C. Distributed control algorithms

### （1）Control algorithm in subgraph G1

Based on the network described above, a set of control algorithms should be derived. In control process, firstly, in subgraph G1, controllable agents and uncontrollable agents communicate with each other to obtain the power balance in the whole network. Then, in subgraph G2, controllable agents readjust the output of controllable DGs to make the operation cost minimum among all the controllable DGs. Finally, all the information is sent to the MG for execution.

In subgraph G1, there is n-node-weighted and directed graph. A $n * n$ weighted matrix is used to describe the connection of nodes and the weight of each edge. Assume there is a link from node i to node j, the weight on this link should be

$$w_{ij} = 1/d_i \tag{7}$$

Where $d_i$ is the outgoing degree of node i. Under this case, we set the information of each agent is dispatched averagely on each link. If the outgoing degree of node i is 0, $w_{ij} = 0$. It is worth noting that the sum of each row in weight matrix should be 1. For example, the sum of weight in ith row should be

$$\sum_{j=1}^{n} w_{ij} = 1 \tag{8}$$

Also $W^T$ represents the transpose of matrix W. Set a $n * n$ diagonal matrix B to denote the type of agent. If agent i is controllable, then $b_{ii} = 1$, otherwise, $b_{ii} = 0$ for the uncontrollable agent and partially controllable agent.

Additionally, we consider the system is balanced, if the sum of outputs of controllable DGs at next time step t+1 is equal to the difference between the total amounts of load demand and the sum of outputs of uncontrollable DGs at time t,

$$\begin{cases} \sum[B \cdot P(t+1)] = \sum L^P(t) - \sum(I-B) \cdot P(t) \\ \sum[B \cdot Q(t+1)] = \sum L^Q(t) - \sum(I-B) \cdot Q(t) \end{cases} \tag{9}$$

Where $P(t+1) = [P_i(t+1)]_{n\times 1}$, $Q(t+1) = [Q_i(t+1)]_{n\times 1}$, $L^P(t) = [L_i^P(t)]_{n\times 1}$, $L^Q(t) = [L_i^Q(t)]_{n\times 1}$ are the active power, reactive power, the active load demand and reactive load demand column vector, respectively, while *I* is an $n * n$ identity matrix. In islanded MG, the DG controlled in V/f mode can provide the loss of the system, which can maintain the stability of the voltage and frequency. In V/f mode, DG can adjust the outputs or inputs of active and reactive power referring to deviations between frequency, voltage and their reference, thus the deviations can be diminished and the stability of frequency and voltage can be upheld. Also, the power response returns to zero gradually after DG adjusts the outputs or inputs according to the deviations.

According to subgraph G1, the following control algorithm can be derived.

$$\begin{cases} B \cdot P(t+1) = B \cdot P(t) + W^T \cdot [L^P(t) - P(t)] \\ B \cdot Q(t+1) = B \cdot Q(t) + W^T \cdot [L^Q(t) - Q(t)] \end{cases} \quad (10)$$

Set k[th] agent to be the partially controllable agent. Then we can express one of equation (10) as follow.

$$\begin{aligned} & B \cdot P(t+1) = B \cdot P(t) + W^T \cdot [L^P(t) - P(t)] \\ & = (b_{11} \cdot P_1(t) + \cdots + b_{kk} \cdot \gamma \cdot P_k(t) + \cdots + b_{nn} \cdot P_n(t)) \\ & + (w_{11} + w_{12} + \cdots + w_{1n}) \cdot [L_1^P(t) - P_1(t)] \\ & + \cdots + (w_{k1} + w_{k2} + \cdots + w_{kn}) \cdot [L_k^P(t) - \gamma P_k(t)] \\ & + \cdots + (w_{n1} + w_{n2} + \cdots + w_{nn}) \cdot [L_n^P(t) - P_n(t)] \end{aligned}$$

Based on (8)

$$
\begin{aligned}
&B \cdot P(t+1) = B \cdot P(t) + W^T \cdot [L^P(t) - P(t)] \\
&= (b_{11} \cdot P_1(t) + \cdots + b_{kk} \cdot \gamma \cdot P_k(t) + \cdots + b_{nn} \cdot P_n(t)) \\
&+[L_1^P(t) - P_1(t)] + \cdots + [L_k^P(t) - \gamma P_k(t)] \\
&+ \cdots + [L_n^P(t) - P_n(t)] \\
&= (b_{11} - 1) \cdot P_1(t) + \cdots + (b_{kk} - 1) \cdot \gamma \cdot P_k(t) + \cdots \\
&+(b_{nn} - 1) \cdot P_n(t) + L^P(t) \\
&= L^P(t) - (I - B) \cdot P(t)
\end{aligned}
$$

So the power balance is proved.

**(2) Control algorithm in subgraph G2**

In subgraph G1, the power balance has been realized, while the minimum operation cost of controllable DGs cannot be ensured. In order to obtain the minimum cost, based on subgraph G2, the output of the controllable DGs should be readjusted.

Subgraph G2 consists of m controllable agents, now define a $m * m$ matrix *H* to denote the relationship among all the controllable agents. Assume there is a link from node i to node j, the weight on this link should be

$$h_{ij} = 1 - \frac{\beta_j/d_j}{\beta_i/d_i + \beta_j/d_j} \quad (j \in N_i, j \neq i) \tag{11}$$

$$h_{ii} = d_i - \sum \frac{\beta_j/d_j}{\beta_i/d_i + \beta_j/d_j} \quad (j \in N_i) \tag{12}$$

Where $\beta_i$, $\beta_j$ are the cost parameters of DG i and DG j respectively, and $d_i$, $d_j$ is the ingoing degree of node i and node j respectively. $N_i$ is the set of nodes which connect to node i. Thus, the updating rule for each node in network G2 should be

$$P(t+1) = H^T \cdot [(P(t) + [\alpha])./D] - [\alpha] \tag{13}$$

Where $P(t+1) = [P_i(t+1)]_{m\times 1}$, $P(t) = [P_i(t)]_{m\times 1}$ is the active power vector.$[\alpha] = [\alpha_i]_{m\times 1}$ is the cost parameter vector. $D = [d_i]_{m\times 1}$ is the ingoing degree vector.

Assume $P_i' = P_i + \alpha_i$, we can transform (10) into

$$P'(t+1) = H^T \cdot [P'(t)./D] \tag{14}$$

Let G2 to be the network with m controllable agents. If controllable agents on network G2 recalculate the set points of controllable DGs referring to the control laws (13) or (14), then the system is still balanced.

$$\begin{aligned}
&\sum_{k=1}^{m} P^{'}(t+1) = \sum_{k=1}^{m} (h_{11} \cdot \frac{P_1+\alpha_1}{d_1} + h_{21} \cdot \frac{P_2+\alpha_2}{d_2} + \cdots \\
&+ h_{m1} \cdot \frac{P_m+\alpha_m}{d_m} + \cdots + h_{1m} \cdot \frac{P_1+\alpha_1}{d_1} + h_{2m} \cdot \frac{P_2+\alpha_2}{d_2} + \cdots + h_{mm} \cdot \frac{P_m+\alpha_m}{d_m}) \\
&= (h_{11} + h_{21} + \cdots + h_{m1}) \cdot \frac{P_1+\alpha_1}{d_1} + (h_{12} + h_{22} + \cdots + h_{m2}) \cdot \frac{P_2+\alpha_2}{d_2} + \cdots \\
&+ (h_{1m} + h_{2m} + \cdots + h_{mm}) \cdot \frac{P_m+\alpha_m}{d_m} \\
&= d_1 \cdot \frac{P_1+\alpha_1}{d_1} + d_2 \cdot \frac{P_2+\alpha_2}{d_2} + \cdots + d_m \cdot \frac{P_m+\alpha_m}{d_m} \\
&= \sum_{k=1}^{m} P_k + \alpha_k
\end{aligned}$$

After the adjustment of the method in subgraph G2, we can get the outputs of all the controllable DGs to realize the minimum operation cost, while, some of the outputs may exceed their maximal capacity of the DGs, so it is necessary to readjust the outputs to make them with the capacity.

Here are the steps for us to readjust the outputs. Step 1) through the above algorithms, we can calculate the optimal outputs of the controllable DGs, and get the optimal power generation ${P_i}^*$ for each individual generator. Step 2) when the outputs violate the generator constraints (3), let $P_i = P_{imax}$ if ${P_i}^* > P_{imax}$ and $P_i = P_{imin}$ If ${P_i}^* < P_{imin}$. Let $\Omega_p$ denote the set of the generators for which the optimal $P_i = P_{imin}$ or $P_i = P_{imax}$. Step 3) then graph reconfiguration can be introduced to handle the outputs of the remaining DGs excluding those which have set their value to the boundaries.

**Graph reconfiguration:** if an agent (agent n) is newly added in, it tries to find its nearest neighbors, get permission from them and then add them in its neighbor list. If an agent (agent j) is removed, its neighboring agent i, $j \in N_i$ will delete agent j from its neighborhood list $N_i$ and also tries to setup communication with other agent k ( $k \in N_i \setminus j$), which is also the neighbor of agent $i$. If $k \in \emptyset$, i.e., no other neighbor of agent $i$ exists, the nothing is needed to be done but just deleting agent $i$.

A simple graph reconfiguration example is illustrated in Figure 2. Suppose agent 1 is removed, its neighboring agents $N_i = \{2,3,4,5\}$ monitor this situation respectively. For agent 2, it needs to setup new communication channel with it new neighbors, agent 3, 4, 5. Also it is the same with agent 3, 4, 5 to setup new communication with its neighbors.

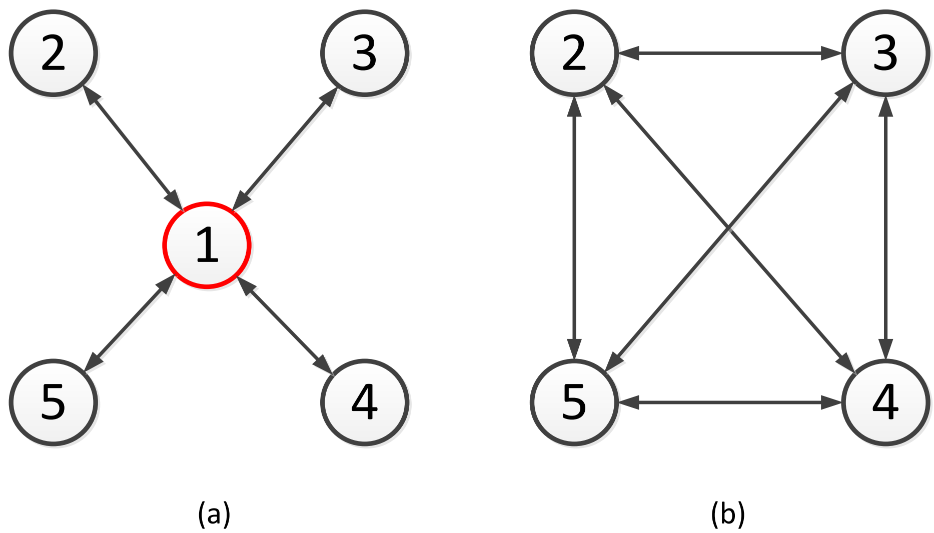



Figure 3. Graph reconfiguration illustrated example. (a) Before disconnection (b) After disconnection

Under this case, the control algorithm in subgraph G2 is modified after the agent is added or deleted in network. That is the following.

$$P(t+1) = \begin{cases} H^T \cdot [(P(t)+[\alpha])./D] - [\alpha], & if\ i \nexists\ \Omega_p \\ P_{imin}\ or\ P_{imax} & if\ i \in \Omega_p \end{cases} \tag{15}$$

# Ⅲ. Architecture of an Example Microgrid System

A test bed is established in Matlab/Simulink to evaluate the performance of our method, which consists of 12 DGs and 12 loads in a radial MG, shown in Figure 4. moreover, the communication network is realized in Matlab function module. In MG, there are 6 controllable DGs, 5 uncontrollable DGs and 1 partially controllable DG. The six controllable DGs are considered as ideal DC voltage sources $V_{dc}$ that work in PQ control mode, while the 5 uncontrollable DGs are regarded as PVs and permanent magnet synchronous generator wind turbines (PMSG-WTs), which all work in the maximum power point tracking (MPPT) control mode. Furthermore, the partially controllable DG is the BESS, which absorbs or injects the power into system to maintain the stability of the system frequency and the voltage under system fluctuation. In our simulation, the outputs of each DG cannot exceed the maximal capacity, and also they will never less than zero.

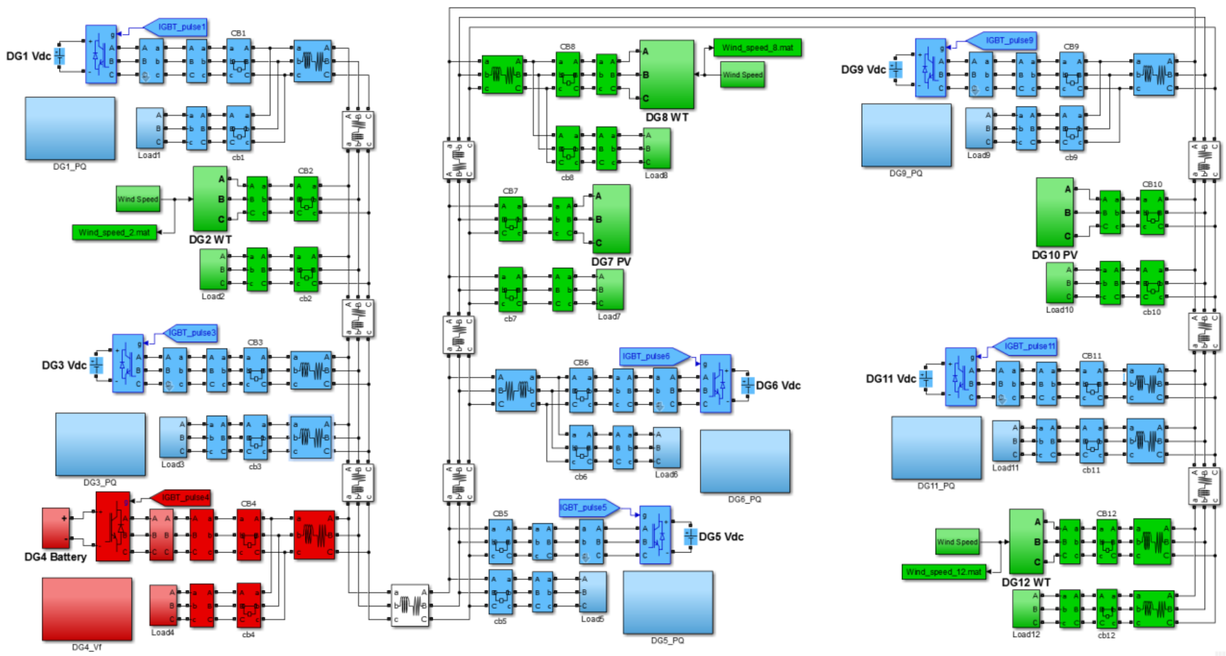



Figure 4. Microgrid test bed

Table 1. Setup and parameters of DGs and loads

| Sources | Capacities | Control | $a(\$/MW^2h)$ | $b(\$/MW^2h)$ | $c(\$/h)$ | Load | Max. Demand |
|---|---|---|---|---|---|---|---|
| $DG_1$ | 50kW,40kVar | PQ | 0.059 | 6.71 | 80 | $Load_1$ | 20kW,0kVar |
| $DG_2$ | 30kW,0kVar | MPPT | - | - | - | $Load_2$ | 35kW,0kVar |
| $DG_3$ | 60kW,25kVar | PQ | 0.047 | 7.08 | 56 | $Load_3$ | 10kW,20kVar |
| $DG_4$ | 20kW,20kVar | V/f | - | - | - | $Load_4$ | 30kW,0kVar |
| $DG_5$ | 55kW,20kVar | PQ | 0.066 | 6.29 | 43 | $Load_5$ | 20kW,20kVar |
| $DG_6$ | 65kW,30kVar | PQ | 0.031 | 7.53 | 35 | $Load_6$ | 10kW,10kVar |
| $DG_7$ | 50kW,0kVar | MPPT | - | - | - | $Load_7$ | 20kW,0kVar |
| $DG_8$ | 35kW,0kVar | MPPT | - | - | - | $Load_8$ | 30kW,15kVar |
| $DG_9$ | 45kW,38kVar | PQ | 0.069 | 4.57 | 48 | $Load_9$ | 40kW,10kVar |
| $DG_{10}$ | 45kW,0kVar | MPPT | - | - | - | $Load_{10}$ | 20kW,15kVar |
| $DG_{11}$ | 70kW,28kVar | PQ | 0.038 | 5.86 | 91 | $Load_{11}$ | 15kW,20kVar |
| $DG_{12}$ | 50kW,0kVar | MPPT | - | - | - | $Load_{12}$ | 20kW,0kVar |

The parameters of DGs and loads are listed in Table 1. It is worth noting that the large loads are located near the DGs with small capacity, while the small loads are arranged near the

DGs with large capacity. Also, we assume all the uncontrollable DGs do not produce any reactive power and set the voltage and frequency reference at 380 voltage and 50 Hz, respectively. In order to take the line loss into consideration, we set the line impedance to be $0.169 + j0.77\Omega/km$. Note that the MG system works in a balanced state initially.

Further, on the subgraph G1, uncontrollable agents send their information to the controllable agents every 45ms, and then the control laws calculate the set point of the controllable DGs in terms of (10). The information is exchanged between all the controllable agents and the set points are recalculated by (14). It is worth noting that when information is not transmitted on the subgraph G1, controllable agents on the subgraph G2 send the current value of the set point of controllable DGs to their neighbors in every 1 ms. Further, the set points of controllable DGs are recalculated and the outputs of controllable DGs are regulated. In this way, active powers dispatch in an economic way to make the operation cost of all the controllable DGs minimum.

## Ⅳ. Results

In this section, some cases are discussed to evaluate the performance of the proposed algorithms. In case 1 and 2 the effects of the load and environment fluctuations on the effectiveness of the algorithm are investigated without generator constraints and with generator constraints. In case 3, the algorithm is applied to different topology to test the feasibility. Further, in the following cases, the communication delay, package loss and broken linkage are illustrated, respectively. Moreover, the plug and play property of the algorithm is tested. Finally, all results are shown and discussed.

**Case 1: both environmental and load demand fluctuations without considering generator constraints**

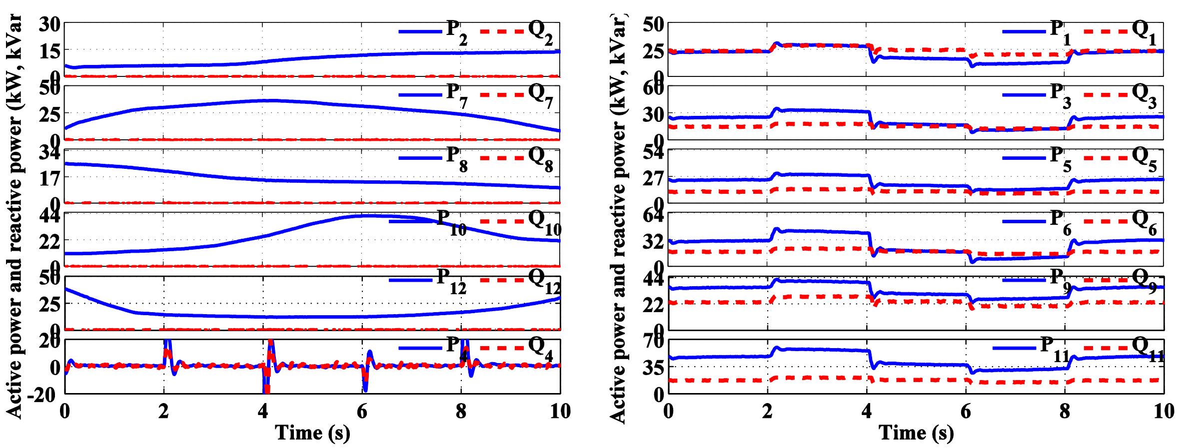
In this case, the outputs of the 3 PMSG-WTs and 2 PVs change with the environmental fluctuations, and the output curves are set, shown in Figure 3(a). Meanwhile, the load demands are scheduled as followings.

- t=2 s: all active and reactive power increase by 20%,
- t=4 s: $load_4$ and $load_8$ are taken away for the MG,
- t=6 s: all active and reactive power decrease by 20%,
- t=8 s: $load_4$ and $load_8$ are reconnected to the MG.

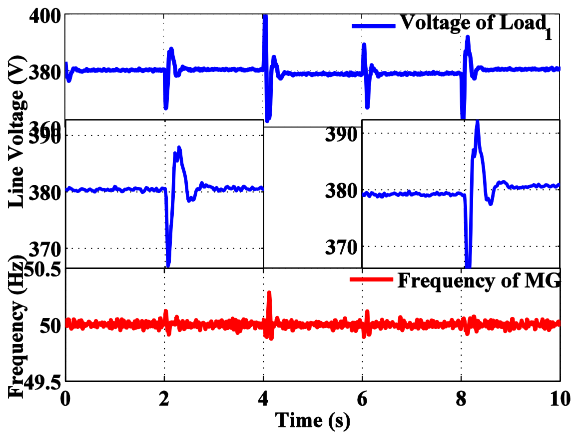
For all the controllable DGs, the generator constraints are not taken into consideration. In optimization process, the goal is to get the incremental cost of all the DGs to a common value. In time slot t=0-2s, the optimal controllable power outputs are $P_1^* = 23.6kW$, $P_3^* = 25.6kW$ , $P_5^* = 24.0kW$ , $P_6^* = 31.4kW$ , $P_9^* = 35.8kW$ , $P_{11}^* = 47.7kW$ and the incremental costs of all the controllable DGs are $\lambda^* = 9.49\ kW/\$$, shown in Fig.5(b) and Fig.5(d). At t=2 s, the total active and reactive power of load demand increase by 20%, all the reactive and active outputs of the DGs increase to fulfill the new power balance. The new optimal power outputs are $P_1^{*\prime} = 29.8kW$, $P_3^{*\prime} = 33.5kW$ , $P_5^{*\prime} = 29.6kW$ , $P_6^{*\prime} = 42.9kW$ , $P_9^{*\prime} = 40.8kW$ , $P_{11}^{*\prime} = 56.8kW$. At the same time, the new incremental cost consensus value increase to $\lambda^* = 10.23kW/\$$ with rising active power outputs of controllable DGs. Similarly, in other time slots, the incremental cost should vary according to the principal, in which, the more the active power outputs, the larger the value is.

For DG4, it plays the role of ESS and reacts first to smooth the fluctuations of the controllable DGs when the system fluctuates. The outputs of DG4 will decrease to zero gradually when the fluctuations diminish and the controllable DGs can share the outputs provided by the ESS. In Figure 5(c), the fluctuations of the line voltage and the system frequency are presented. It is clear that the line voltage and frequency fluctuations can settle down quickly regardless of how large the fluctuations are.

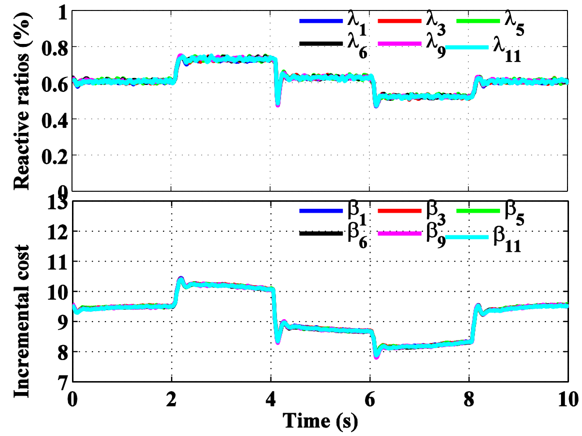
Here, it is worth noting that all the controllable DGs' outputs are within the generator ranges. In the next case study, the outputs of the DGs will be increase to verify the effectiveness the modified distributed algorithm which meets the generator constraints.




(a) (b)




(c)



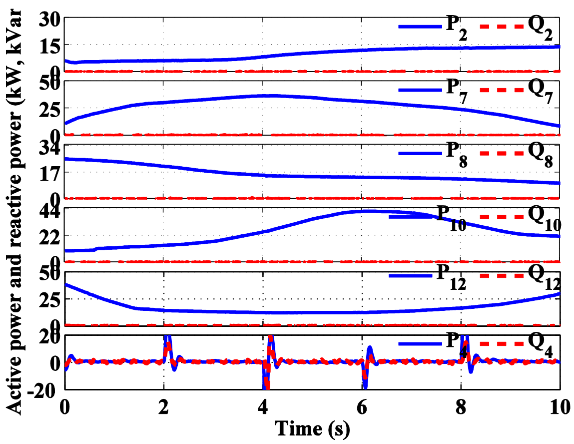

(d)

Figure 5. Simulation results without generator constraints. (a) Active and reactive power outputs of uncontrollable and partially controllable. (b) Active and reactive power outputs of controllable DGs. (c) the line voltage and frequency in the MG. (d) The incremental cost of controllable DGs. (e) The reactive power ratios of uncontrollable DGs.

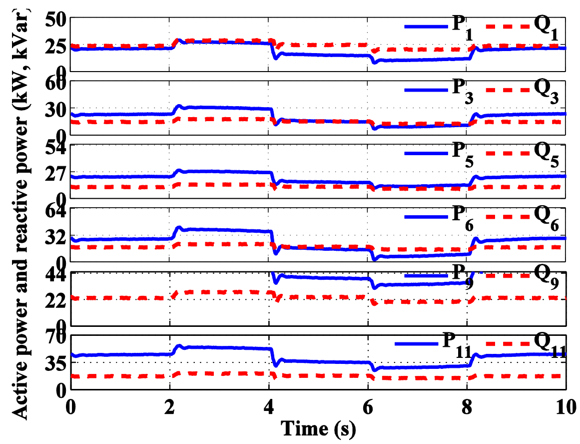
### Case 2: both environmental conditions and load demand fluctuations with considering generator constraints

In this case, the generator constraints are added. To increase the power outputs, we change the parameters a , b, c of DG9 to 0.05 $\$/MW^2h$, 4.57$\$/MWh$, 48$\$/h$, and all the other uncontrollable DGs and load demands are set the same with those in case 1. In time slot t=0-2 s, the new optimal outputs are $P_1^* = 21.9kW$, $P_3^* = 23.6kW$ , $P_5^* = 22.8kW$ , $P_6^* = 28.0kW$ , $P_9^* = 46.9kW$ , $P_{11}^* = 44.7kW$. Even though all the incremental costs reach the consensus, the outputs of DG9 exceed its maximal capacity 45kW, shown in Figure 6(b). Based on (12), the outputs of DG9 are set to 45kW, and the outputs of the other controllable DGs are readjusted to compensate the power deficiency caused by resetting the DG9. From Fig.5, the new optimal outputs after adjustment are $P_1^* = 22.2kW$, $P_3^* = 24.0kW$ , $P_5^* = 22.9kW$ , $P_6^* = 28.5kW$ ,

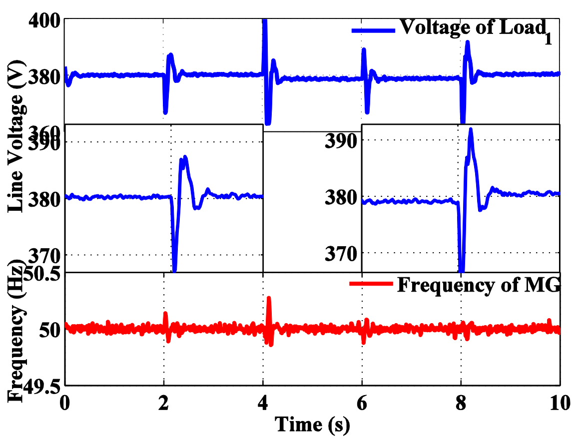
$P_9^* = 45.0kW$ , $P_{11}^* = 45.4kW$. The new incremental cost of DG9 is $\lambda_9^* = 9.07kW/\$$, while the incremental costs of the other DGs should still reach the consensus from $\lambda^* = 9.29kW/\$$ to $\lambda^* = 9.32kW/\$$.



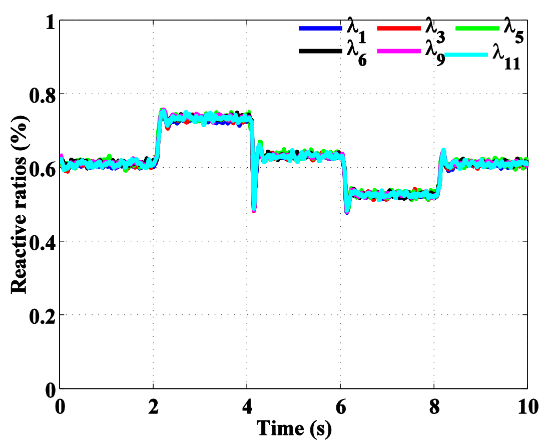

(a)




(b)




(c)




(d)

Figure 6. Simulation results before changing the parameters of DG9 (a) Active and reactive power outputs of uncontrollable and partially controllable. (b) Active and reactive power outputs of controllable DGs. (c) the line voltage and frequency in the MG. (d) The incremental cost of controllable DGs. (e) The reactive power ratios of uncontrollable DGs.

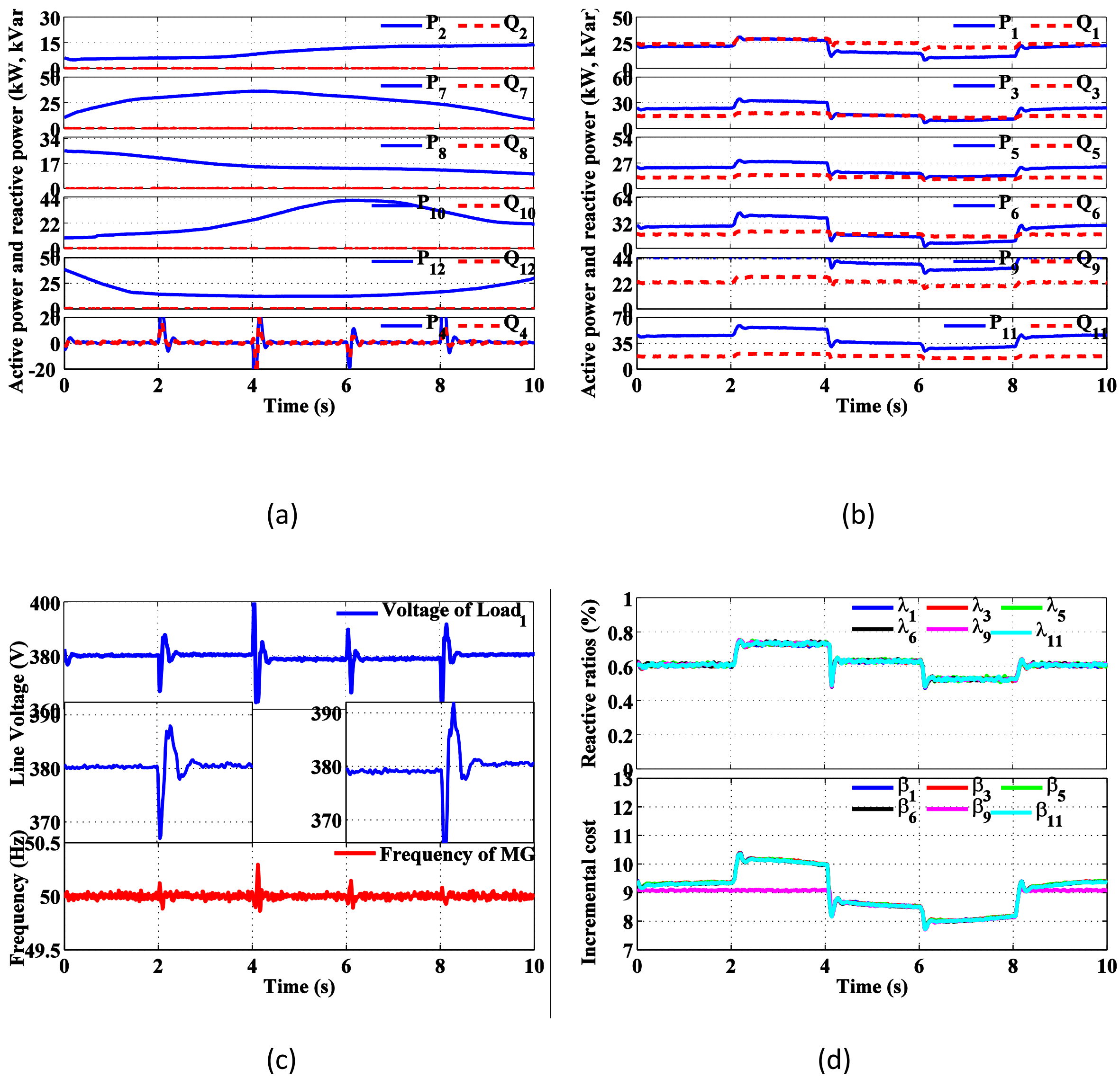
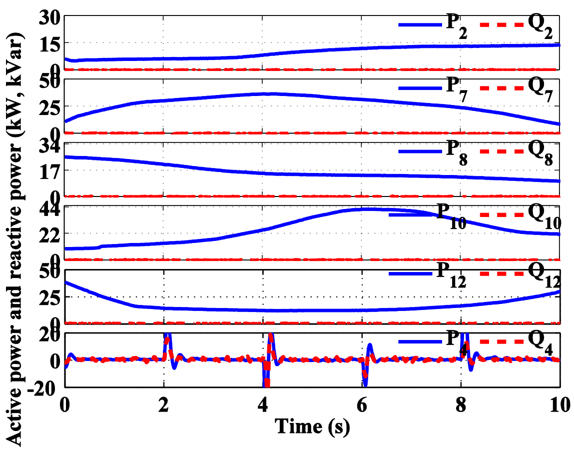


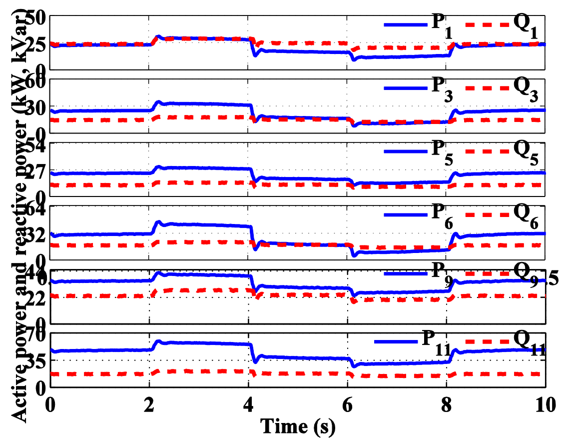

Figure 7. Simulation results after changing the parameters of DG9. (a) Active and reactive power outputs of uncontrollable and partially controllable. (b) Active and reactive power outputs of controllable DGs. (c) the line voltage and frequency in the MG. (d) The incremental cost of controllable DGs. (e) The reactive power ratios of uncontrollable DGs.

## Case 3: Impacts of different topologies of communication networks

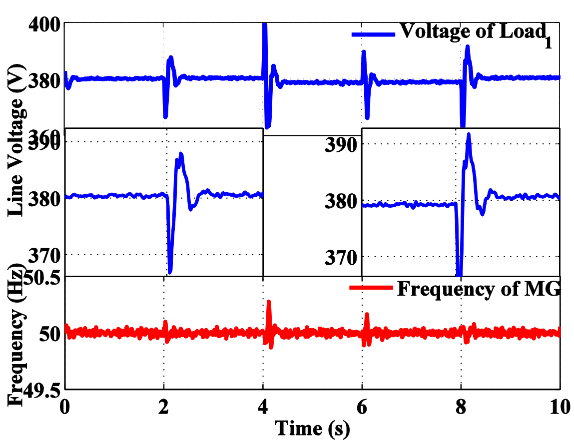
As shown in network b and network c, different network topologies are investigated to test the performance of the control algorithms. Compared with network a, two more links are added between agent 1 and agent 4, agent 2 an agent 6 respectively in subgraph G2; while in network b, one more edge is added between agent 2 and agent 11 in the subgraph G1 of network c. Here, all settings of environmental and load demand fluctuations follow those in case 1.



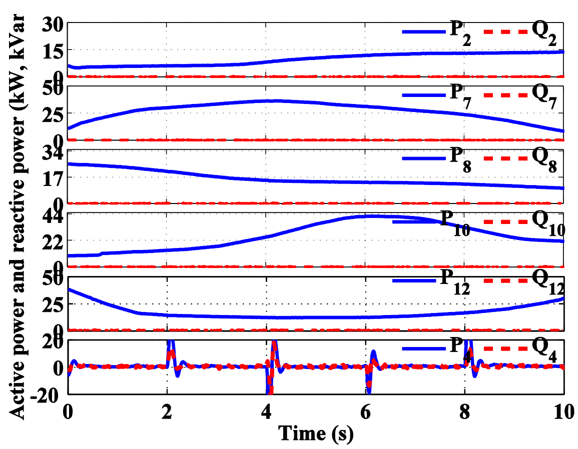

(a)



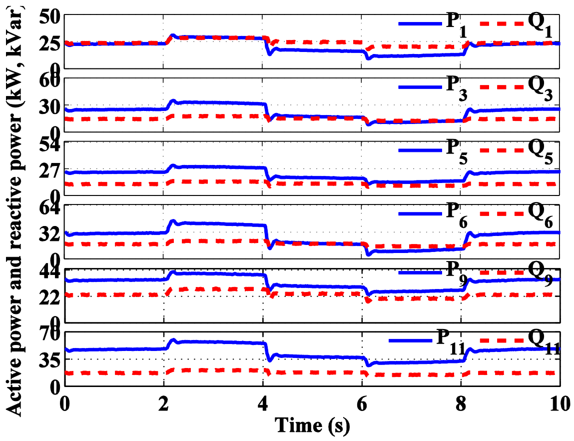

(b)



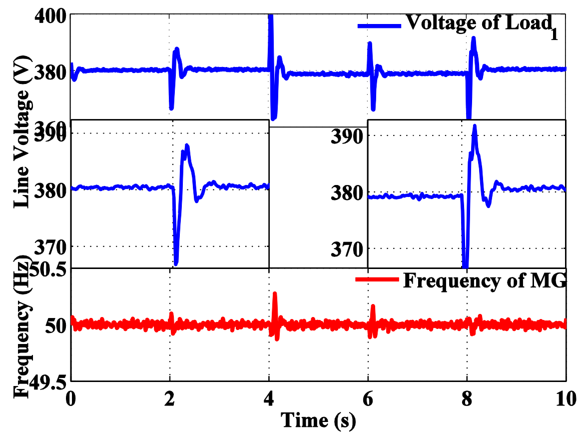

(c)




(d)




(e)



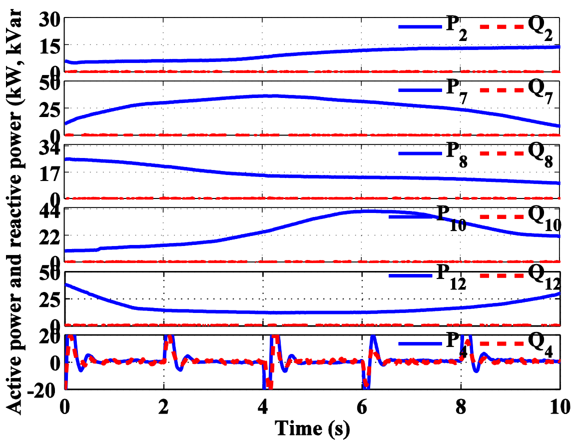

(f)

Figure 8. Simulation results under different topologies. (a), (d) Active and reactive power outputs of uncontrollable and partially controllable. (b), (e) Active and reactive power outputs of controllable DGs. (c), (f) the line voltages and frequency in the MG.(a)-(c) on network 2. (d)-(f) on network 3.

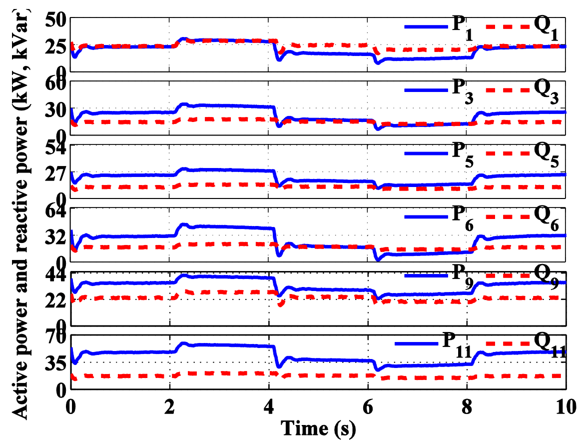
The simulation results obtained on two networks are shown in Figure 8. From the results, the outputs and incremental cost of controllable DGs, the voltage and frequency are almost the same in different topology. It is shown that the performances of our control method are not associated with the network topology, thus this laws can be applied for other network easily.

**Case 4: Impacts of time delays**

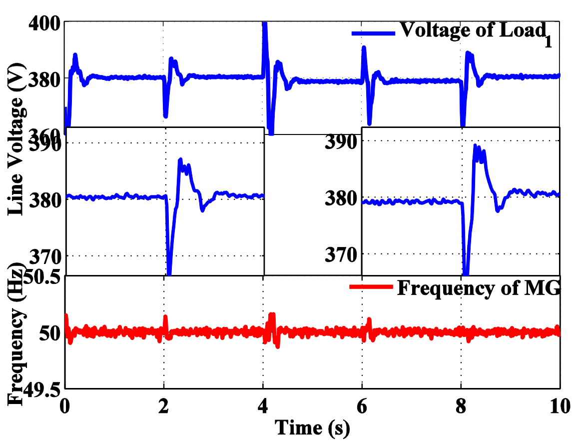
In real communication network, time delay will be unavoidable under some circumstance when the information is transmitted among the network. In my method, the communication network plays an important role in realizing the economic dispatch of all the controllable DGs. It is essential to investigate the effect of time delay on the performance of our methods. When both environmental conditions and load demands fluctuate, where the setting follows those in case 3, a fixed time delay $\alpha$ occurs. Simulations are carried out based on the time delayα =50ms, 100ms, 150ms. The simulation results are shown in Figure 4. It can be seen that the control law is robust with the time delay α =0.1s, 0.12s. However, the system dynamic situation become worse under time delay t=0.15s, compared with the former two time delays. It is clear that the longer the time delay, the more severe the fluctuations will be. While after the system settles down, the outputs of the all the controllable DGs and the responses of voltage and frequency are almost the same.



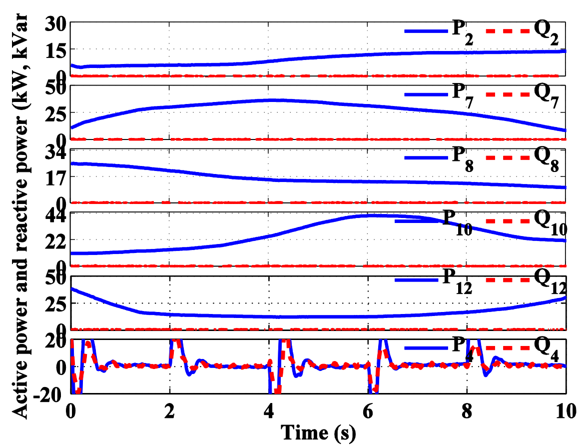

(a)



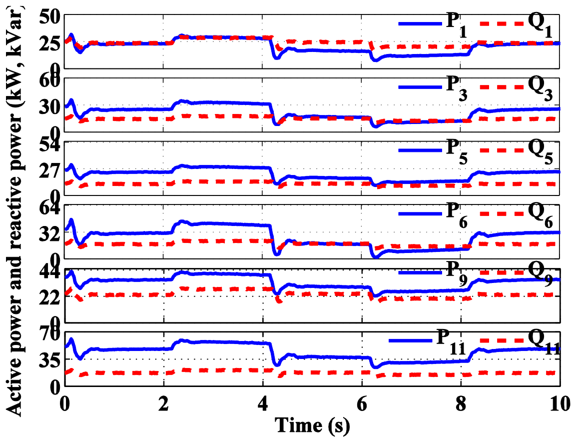

(b)



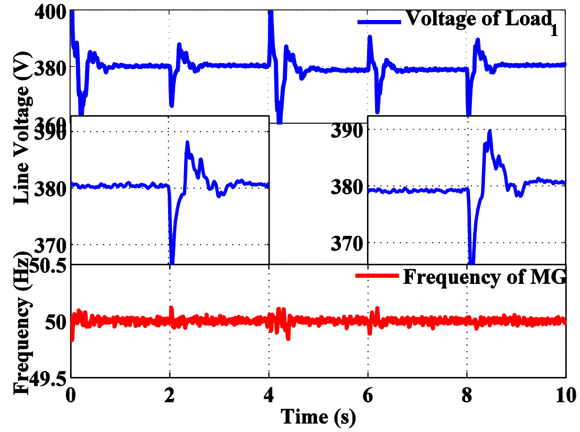

(c)



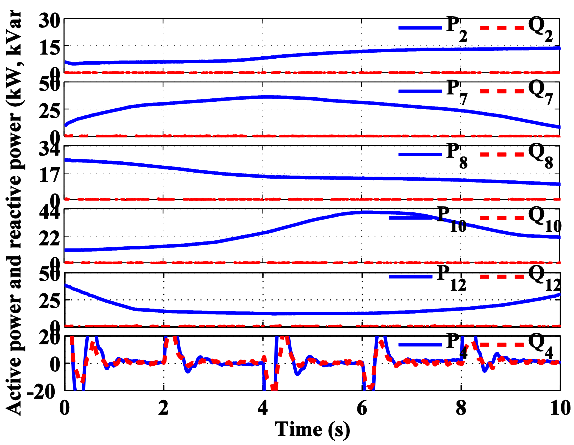

(d)


Active power and reactive power (kW, kVar)
Time (s)
P1 Q1
P3 Q3
P5 Q5
P6 Q6
P9 Q9
P11 Q11

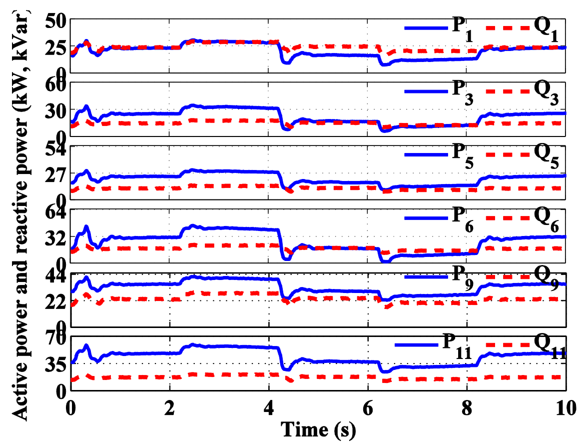
(e)


Line Voltage (V)
Voltage of Load1
Frequency (Hz)
Frequency of MG
Time (s)

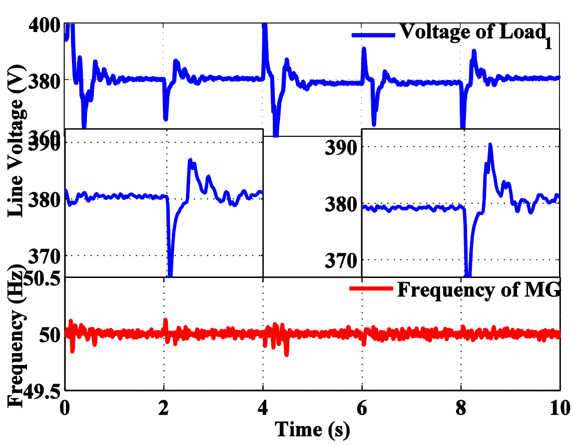
(f)


Active power and reactive power (kW, kVar
Time (s)
P2 Q2
P4 Q4

(h)


Active power and reactive power (kW, kVar)
Time (s)
P1 Q1
P3 Q3
P5 Q5
P6 Q6
P9 Q9
P11 Q11

(j)

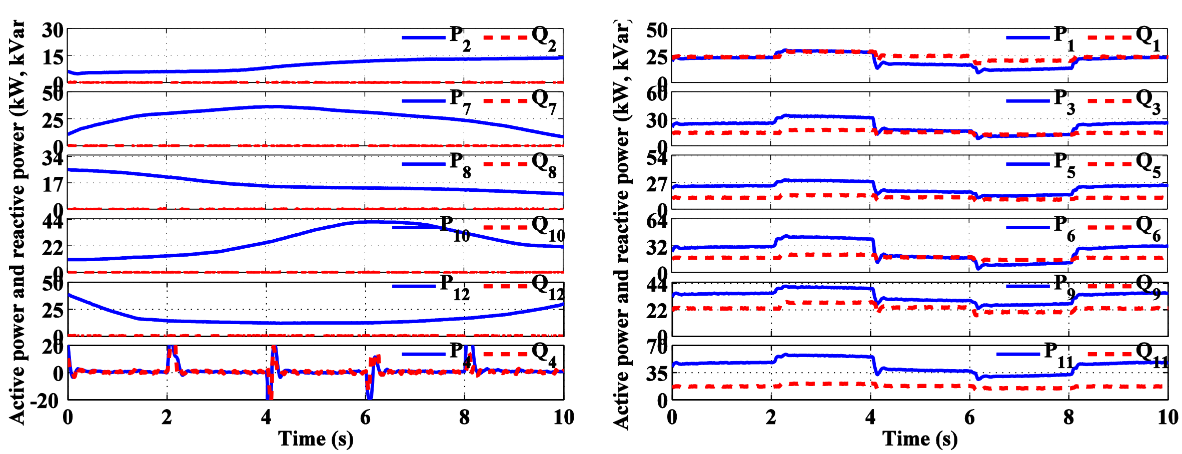

Line Voltage (V)
Voltage of Load1
Frequency (Hz)
Frequency of MG
Time (s)

(k)

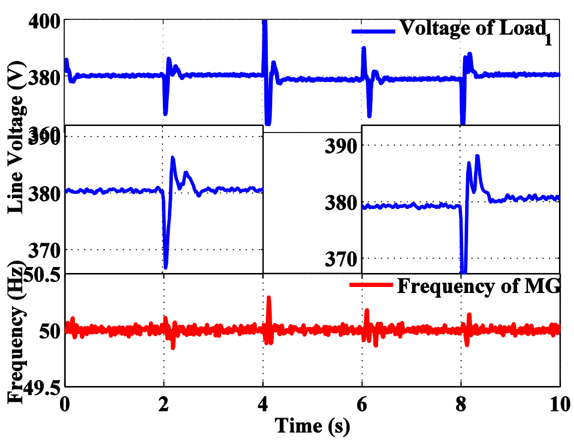
Figure 9.Simulation results under different time delay. (a),(d),(h) are the active and reactive power outputs of uncontrollable and partially controllable. (b),(e),(j) are active and reactive power outputs of controllable DGs.(c),(f),(k) the line voltages and frequency in the MG.(a)-(c) on time delay $\alpha = 50\text{ms}$.(d)-(f) on time delay $\alpha = 100\text{ms}$. (g)-(k) on time delay $\alpha = 150\text{ms}$.

**Case 5: Impacts of package loss**

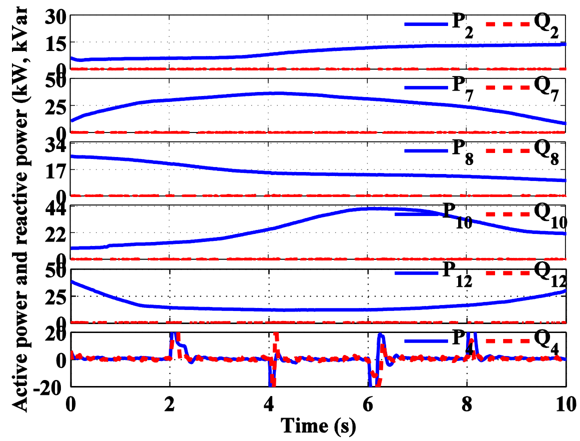
During information transmission, the unstable system will cause package loss under different situation, such as congestion. The influences of package loss on the network are tested. When the environmental conditions and load demands fluctuate, where the setting follows those in case 1, a fixed package loss rate occurs in information transferring. In network G2, we test the package loss rate β =0.3, 0.4, 0.5 happen when all the controllable agents communicate with their neighbors. It is shown that the performance of our method is associated with the package loss rate. Also, the less the information is lost, the more accurate our methods will be. While even though the package loss reaches 0.5, the system can still work in a fine state, except the large fluctuations at some specific time.



(a) (b)

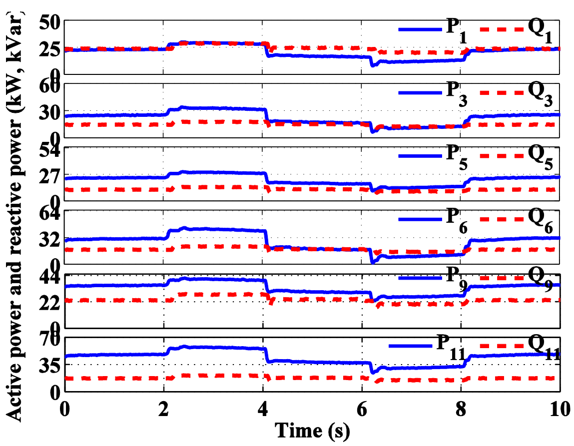


Voltage of Load1
Line Voltage (V)
Frequency (Hz)
Frequency of MG
Time (s)

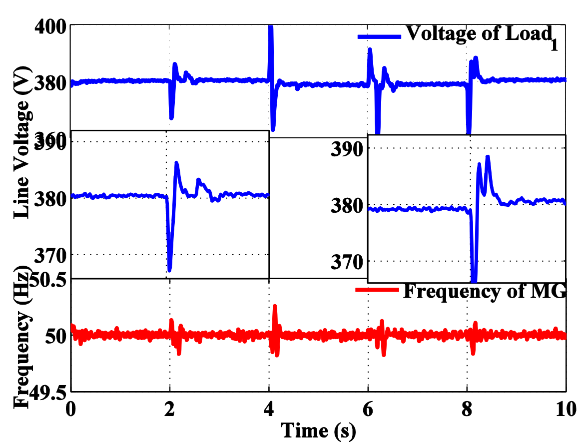



Active power and reactive power (kW, kVar
P2 Q2
P7 Q7
P8 Q8
P10 Q10
P12 Q12
P4 Q4
Time (s)


(c) (d)

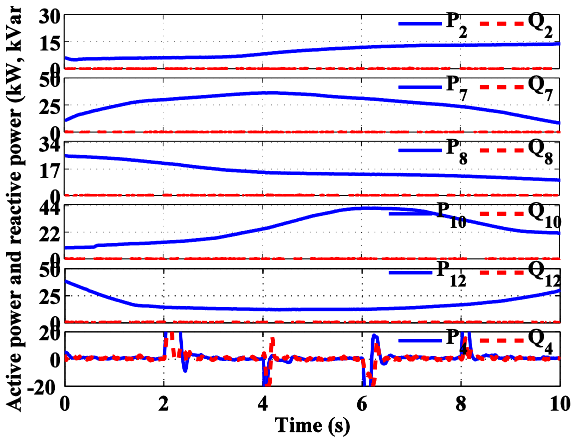


Active power and reactive power (kW, kVar)
P1 Q1
P3 Q3
P5 Q5
P6 Q6
P9 Q9
P11 Q11
Time (s)

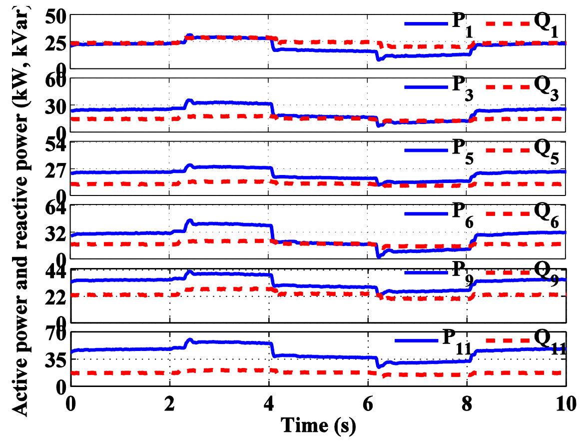



Voltage of Load1
Line Voltage (V)
Frequency (Hz)
Frequency of MG
Time (s)

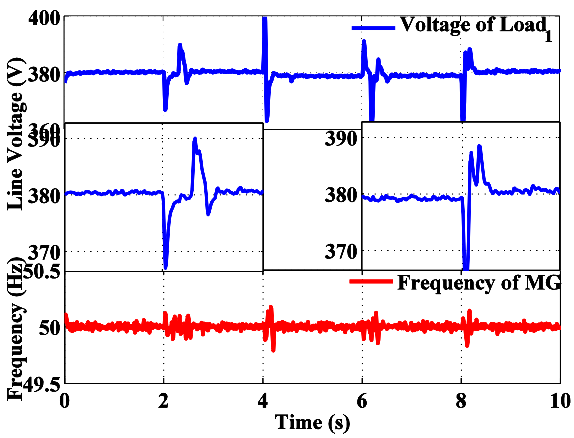

(e) (f)




(g)




(h)

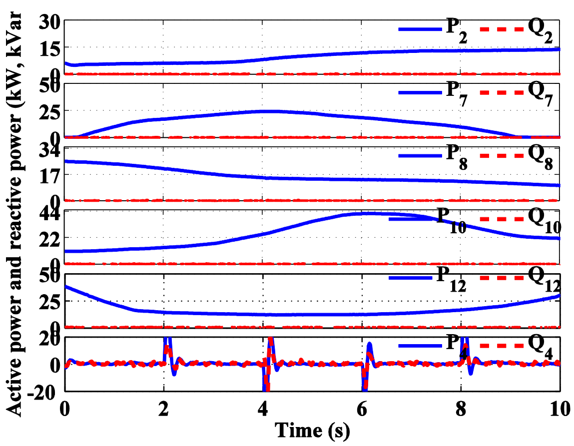



(i)

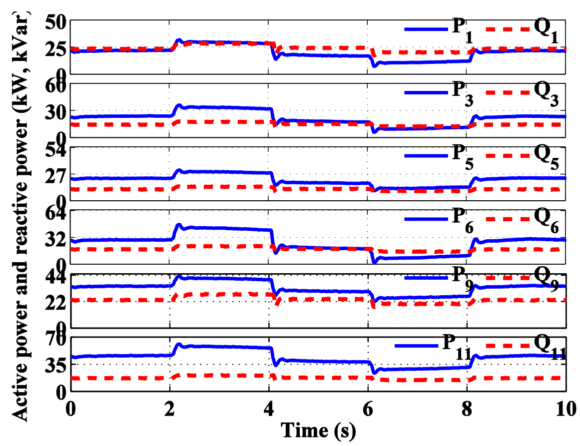
Figure 10. Simulation results under different package loss rate. (a) –(c) for simulation results under $\beta = 0.3$, (d)-(f) for simulation results under $\beta = 0.4$, (g)-(i) for simulation results under $\beta = 0.5$.

**Case 6: impacts of broken linkage**

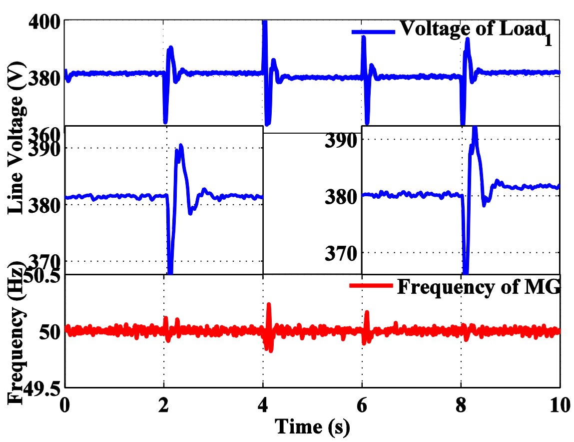
Though our method applies for different network well, it is important to state how the broken linkage affects the performance of our methods. All the following settings are in network a. In subgraph G2, the link between agent 2 and agent 3 breaks down at t=4 s and reconnect at t=8 s. In subgraph G1, we set a broken linkage between agent 8 and agent 9 at t=4 s and restore it at t=8 s. the simulation results obtained are shown in Figure 11. From the simulation, the system still keeps a good operation state even though broken linkage happens in subagraph G1 or subgraph G2. Even though, there are broken links between two agents, while each agent still has one link which connect to the whole system, and make it collect or send information from or to other agents. It is worth noting that in our system, there should not be isolated agents, which will cause the malfunction of our system.



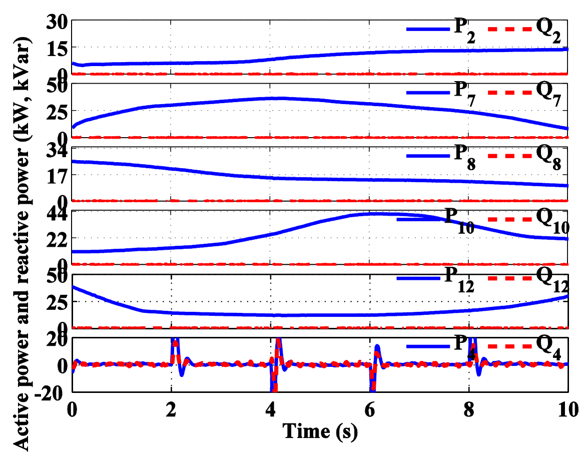

(a)



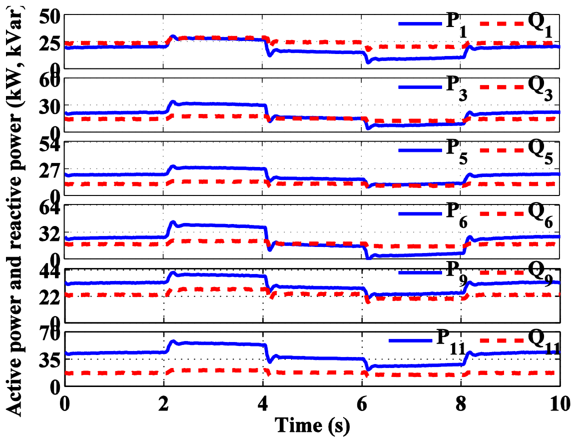

(b)



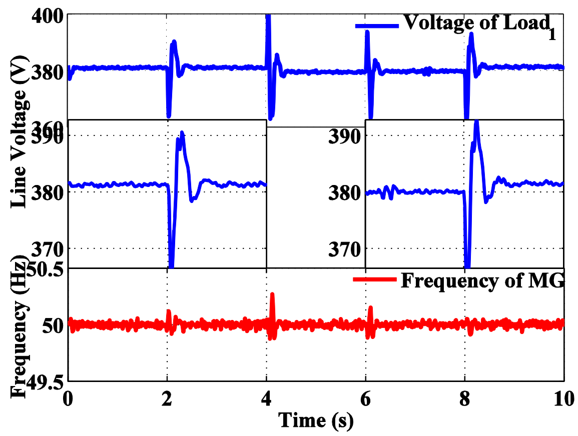

(c)




(d)




(e)




(f)

Figure 11. Simulation results if broken linkage happens. (a), (d) Active and reactive power outputs of uncontrollable and partially controllable. (b), (e) Active and reactive power outputs of controllable DGs. (c), (f) the line voltages and frequency in the MG. (a)-(c) The broken linkage between agent 2 and agent 3 in subgraph G2. (d)-(f) the broken linkage between agent 8 and agent 9 in subgraph G1.

**Case 6: Plug and play**

This case is to test the flexibility of our methods when DGs plug out or plug in at some time. Before DGs plug out or in, the incremental costs of the DGs have reached the consensus and the system works in a steady state. Here, DG1 plug out at t=4s and plug in at t=8s, and the other DGs keep connected in the system all the time. When DG1 plug out at t=4 s, the other DGs should produce more active and reactive power to compensate the power produced by DG1 to keep the power balance. Meanwhile, the incremental costs and the reactive power ratios of other DG increase as illustrated in Figure 12 (d). On the contrary, when DG1 plug in, the incremental of the other DGs decrease with the decreasing of the outputs. From Figure 12 (c), the plug out and in do not degrade the performance of the system. At same time, it keeps fast convergence speed.

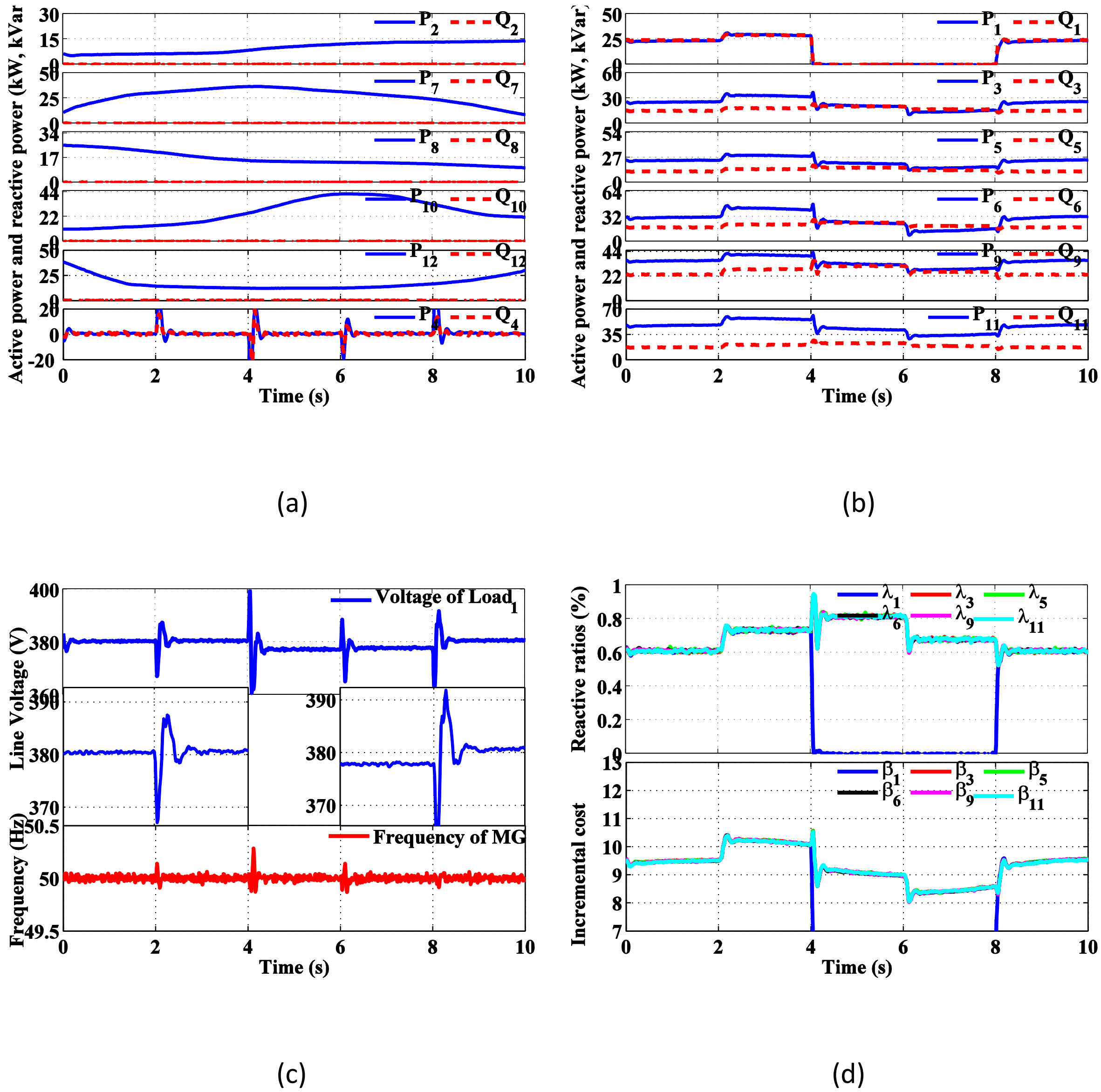



Figure 12. Simulation results of plug and play. (a), (b) Active and reactive power outputs of uncontrollable and partially controllable. (c) the line voltages and frequency in the MG. (d)-(f) the broken linkage between agent 8 and agent 9 in subgraph G1.

# Ⅴ. Conclusion

In this research, a two-layer distributed control model is proposed to realize the economic dispatch problem, where the bottom layer is the MG, while the top layer is the communication network. The communication consists of two sub-graphs, namely the one includes all types of agents, network G1 and the one only includes controllable agents, network G2.

Based on the communication networks, fully distributed control methods are derived, which no longer need the central controller to collect the global information (like power mismatch) to realize minimum operation cost. According to the equal incremental cost principal, if the outputs are satisfied, the economic dispatch is obtained.

In network G1, all the agents communicate with their neighbors, and the controllable agent will set its outputs to satisfy generation-demand balance. Then, in network G2, only the controllable agents will communicate with their neighbors to readjust the set point of all the controllable agents, in order to make the operation cost of all controllable DGs minimum. Meanwhile, all the information sent from agents will be executed by DGs.

Several simulation cases are designed to investigate the performance of control method. In simulation part, firstly, we test the performance of control methods when both environmental conditions and load demands fluctuate without and with generator constraints. In the following parts, different topology, time delay and package loss are taken into consideration. Moreover, the plug and play property is tested. From the simulation results we have obtained, the longer the communication delay, the longer time it will take for the system to get stable, also, the larger the package loss of the system, the longer time it will take to get stable. In general, the system always operates well. In other words, the voltage and the frequency can keep around the prescribed value in the steady state and also the fluctuations can be restricted in normal range even if the environmental conditions and load demands vary drastically. Furthermore, the generation cost of all the controllable DGs is minimized by our method. Meantime, the incremental cost will converge to the common constant value fast.

For future work, the algorithm should be tested in large network. Further the transmission loss here is just set at a fixed value, the reconsideration should be combined with the power flow.  Also the demand response can be investigated in further research

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