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Warranty Period and Product Price Optimization for Remanufactured Products

Yuxi Liu
University of Wisconsin-Milwaukee

WARRANTY PERIOD AND PRODUCT PRICE OPTIMIZATION FOR REMANUFACTURED PRODUCTS

by

Yuxi Liu

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ABSTRACT

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Yuxi Liu

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This study considers a remanufactured electrical product under a tiered warranty policy. Warranty is key in ensuring a good manufacturer—consumer relationship. Manufacturers hope to minimize warranty costs while consumers believe that good warranty promises better product quality and reliability. This Thesis presents an optimal warranty period from the perspective of a manufacturer to maximize the total expected profits, while ensuring sustained consumer relation. We use real data from a local company with a global supply chain to provide a numerical example.

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1 Chapter 1: Introduction

1.1 Prospects of the Remanufacturing Industry

Remanufacturing is a comprehensive and rigorous industrial process by which a previously sold, worn, or non-functional product or component is returned to a “like-new” or “better-than-new” condition and warranted in performance level and quality. The Remanufacturing Industrial Council gives this definition of remanufacturing. Different from rebuilding, refurbishing, reconditioning, repaired and other similar names, remanufacturing is becoming the standard term for the process of restoring used products to a “like new” condition [1].

According to Lund, Robert T. and Hauser, William M. [2], remanufacturing industry has existed in the United States for at least a century. At start of the 20th century, remanufacturing was practiced in electric motor fields. However, the large-scale use of remanufacturing in industry is World War II. Most of companies have to produce military equipment, while there is no capability on private cars. Many used cars have to be repaired to drive [3].

After a century of developments, remanufacturing is currently an important and growing activity in many industrial sectors. Some of the most commonly remanufactured product categories are: aircraft components, automotive parts, electrical and electronic equipment, engines and components, medical equipment, office furniture, printing equipment, restaurant and food-service equipment.

Completed for the U.S. Trade Representative, a latest report, which was published by the U.S. International Trade Commission (USITC) [22], provides an overview of the U.S. and global remanufactured goods industries and markets.

In the view of global, according to the report, although the statistical data on global trade in remanufactured goods are largely unavailable, the United States and Europe account for the bulk of remanufacturing and trade in the world, while other countries are developing their own remanufacturing industries quickly. In European Union, UK is the leading remanufacturer, which the estimated remanufactured goods production is £4.9 billion and employment at 44,300 workers in 2002. The United States is the world's largest producer, consumer, and exporter of remanufactured goods. The largest U.S. remanufacturing sectors are aerospace, heavy-duty and off-road equipment, and motor vehicle parts. Other developing country like Brazil, India and China, even though size of remanufacturing is relative small and undeveloped, they have own characteristic way in remanufacturing industry.

In order to comprehend the rapid development of remanufacturing industry, we should know constituents of it. There are three main participants in remanufacturing industry. First is the manufacturer or we say remanufacturer. They can be an original equipment manufacturer (OEM) or an independently operating remanufacturer (IO). There are some existing literature researching about the difference between OEM and IO. Generally, the OEM produces new products and sells them. After the products worn out, OEM collects the used back to remanufacture them. However, IO collects used products no matter who manufactured them before, remanufactures these used ones to sell. In this paper, we consider OEM and IO as similar remanufacturing entities, which collect worn product and renews them, then sells the remanufactured product with a warranty agreement. To these manufacturers, in most situations, remanufacturing can recover the energy and value for material recovery, products manufacturing and fabrication, avoiding repeated energy consumption and environment pollution, which is not only more protective of environment but also saving a lot of raw material cost. No matter shoulder the environment duty or profit for enterprise, remanufacturing is a good choice.

Second part is consumer who will purchase the remanufactured products. Consumer can be an individual person or a company. To these consumers, remanufactured products can bring them like-new product with lower price than new one. Considering the same quality and function with lower price, some customers prefer this economical deal while not buying a new one. Furthermore, with the increasingly environmental awareness, more and more consumers will learn about the sustainable industry and accept remanufactured products.

The third main participant is the government. The USITC found that: U.S. exports of remanufactured goods total $11.7 billion in 2011, up 50 percent compared with 2009. Canada, the European Union, and Mexico are important markets for U.S. exports of remanufactured goods. During the period 2009-2011, production of remanufactured goods grew by 15 percent to at least $43 billion in U.S. and 180,000 full-time jobs are supported by this industry (USITC). That means remanufacturing is not just a concept on environmental protection any more, while it can really promote employment market and create value in country's economy.

1.2 Remanufacturing status of electrical and electronic product

We consider the electrical and electronic products, where remanufacturing activities are significant and sufficient number of transactions for both new and corresponding remanufactured products can be found [4].

The general procedure of remanufacturing electrical and electronic products should be: Collection- Disassembly- Cleaning- Inspection and sorting- Reprocess（repair, recondition, replacement）- reassembly- testing- sale.

1.3 Challenge of Remanufacturing Industry

Even though there is a huge market opportunity in the remanufacturing industry, we cannot ignore the challenge the current and future challenges, some of which will be discussed the following section.

Lack of regulations and standards

This challenge is government related, especially in emerging economies like India and China. An important barrier in the remanufacturing industry is the lack of regulations, enforceable laws, penalties and incentives to motivate manufacturers. For example, if the government can provide some economic support or preferential tax policies, we believe manufactures would ensure that they incorporate strategies for responsible manufacturing, such as remanufacturing in the business plan.

Product technology changes rapidly

The 1-year rate of change (how long consumers keep a product) for electronic products in U.S. is almost 9% [5]. This percentage is indicative of the overall rate of technology change in the electronic industry. This change means that manufacturers have to invest more money on new products production or marketing. However, for remanufacturing industry, the manufacturer has to spend money and time to collect returns, train operators to enable them work on older products and create a remanufacturing process that is flexible enough to accommodate a wide range of product returns. The more frequently upgrade products, the higher cost for them [6].

Consumer concepts

Most literature about remanufacturing focuses on the operational and organization aspects. However, this is a double-sided challenge not only to consumer but also to manufacturer [7]. On one side, consumers lack confidence in the quality of remanufacturing products, and on the other side, the manufacturer incurs more costs to ensure quality, which in turn increases the price of remanufactured products. This calls for models that are able to strike a cost-quality-warranty tradeoff.

1.4 Warranty Analysis

1.4.1 Definition of Warranty

It is not easy to give a definition for warranty, because there are many concepts and meanings are associated with it. However, despite the differences in meaning implications, it is generally agreed that a warranty is a seller's binding assurance to the respective consumer that the seller will be liable should product or service fail when used correctly and for the intended purpose. In this Thesis, we believe "the warranty is the representation of the characteristics or quality of product" [8]. It is also difficult to find out the origins of warranty exactly.

According to the English law, warranty has existed over 600 years but American courts adopted concept of warranty in the twentieth century [9]. For a more official definition of warranty, the Uniform Commercial Code (U.C.C.) §2-313 states that warranty is:

> "An affirmation of fact or promise made by the seller to the buyer, which relates to the goods and becomes part of the basis of the bargain, creates an express warranty that the goods shall conform to the affirmation or promise."

1.4.2 Warranty Policies

Generally, warranty policies can be divided into two groups based on whether or not a policy involves product development after sale (Figure 1). Policies that do not involve product development can be further divided into two subgroups: Group A, comprising of policies that are applicable for single item sales, and Group B, comprising policies that are applicable only for the sale of groups of items, such as military order. In this Thesis, we mainly focus on the Group A, i.e. single item warranty situation.

Policies in Group A can be subdivided further into two subgroups based on whether the policy is renewing or nonrenewing. In renewing policy, whenever an item fails under warranty, it is replaced by a new item with a new warranty replacing the old one. While, in the case of a nonrenewing policy, replacement of a failed item does not alter the original warrant.

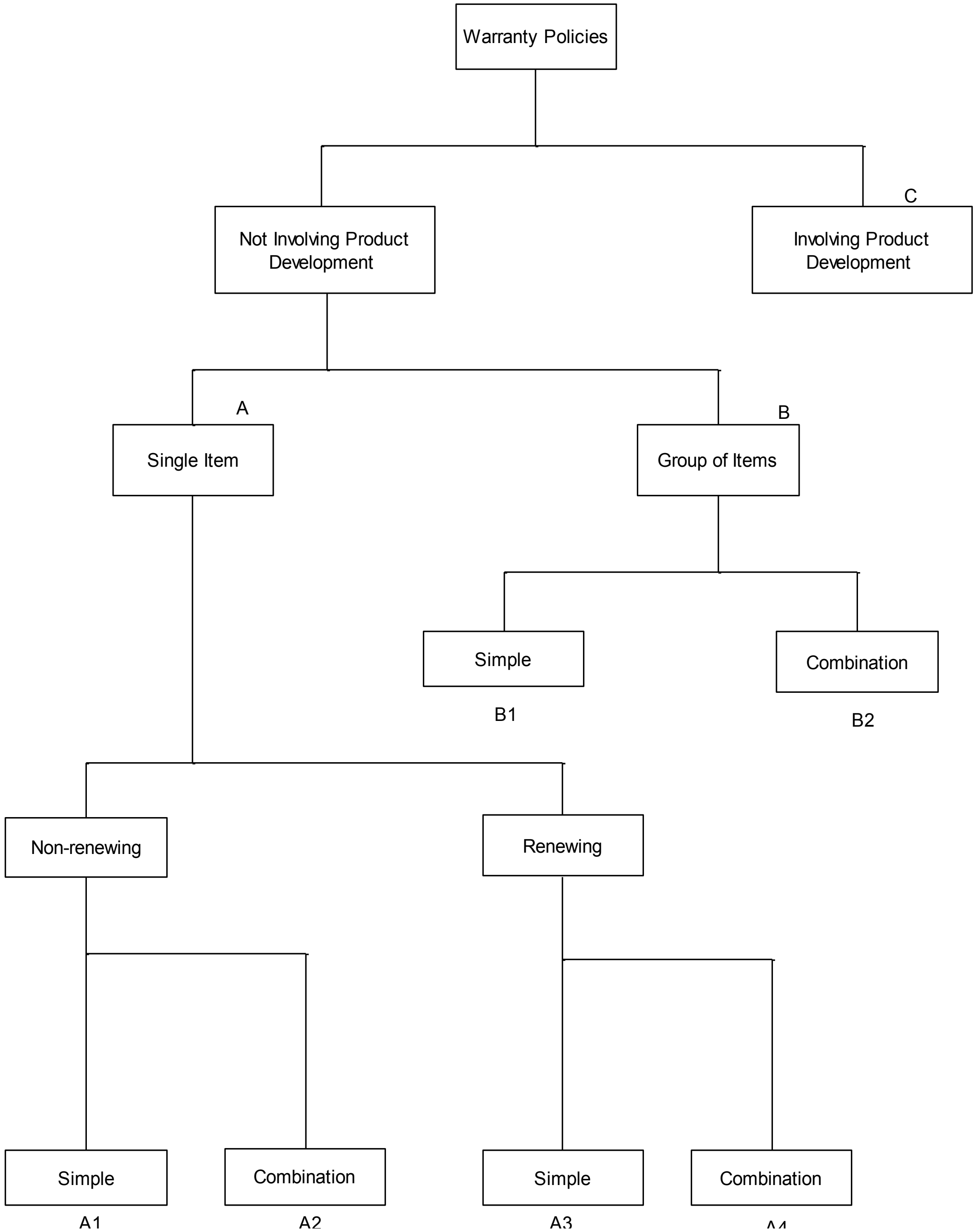

Figure 1: Type of Warranties (Source: Blischke, 1994[9])

A further subdivision comes about in that warranties may be classified as simple or combination. The free replacement and pro-rata policies are simple policies. A combination policy is a simple policy combined with some additional features or a policy is that combines the terms of two or more simple policies.

Each of the preceding four groupings (A1 to A4 in Figure 1) can be further subdivided into two subgroups based on whether the policy is one-dimensional or two-dimensional. The dimension of a policy is the number of variables specified in defining the warranty limits. A one-dimensional policy is almost always based on either time or age of the item but could instead be bases only on usage. In contrast, a typical two-dimensional policy is based on time or age and usage.

1.4.3 Warranty Type

Whether a policy is renewing or non-renewing, the seller or manufacturer must formulate specific warranty terms to apply to each policy. In this Section, we demonstrate some common warranty types to explain the nuances of policies in real life. The most common warranty forms are the free-replacement and pro-rata warranties.

Free-Replacement Warranty

Under the free-replacement warranty, the manufacturer is fully responsible for product failure. That means, the manufacturer can repair or replace for failed products free of charge up to a time say W or a usage say U from the initial purchase. The time W is called the warranty period or warranty length and U the usage limit. Free-replacement warranty is widely applicable to repairable or inexpensive non-repairable items.

Pro-Rata Warranty

Under pro-rata warranty, the manufacturer and customer will prorate sharing the maintenance or replacement cost. This means that the manufacturer can refund a fraction of the purchase price when the product fails before time W or usage limit U from the initial purchase. Here the prorated refund amount is a linear or nonlinear function of the residual warranty and the current condition of product.

Complex One-Dimensional Warranties

As explained in Section 1.4.2, the dimension of a policy is the number of variables specified in defining the warranty limits, like warranty period W or usage limit U.

Sometimes, the manufacturer will combine the free-replacement and pro-rata warranty with one dimensional limit. Under this policy, the manufacturer agrees to replace or repair freely up to time W_1 ($W_1<W$) or usage U_1 ($U_1<U$). Then, from time W_1 to W or from usage U_1 to U, the refund becomes prorated. This combined warranty is referred to as a complex one-dimensional warranty.

Two-Dimensional Warranty Policies

A two-dimensional warranty remains effective only if the product satisfies both the warranty period W and usage limit U. For example, in the automobile sector, a dealer will provide both warranty period and the mileage limit to a customer.

Reliability Improvement Warranties

This kind of warranty is an extension to one of the basic warranty types, i.e free-replacement or pro-rata warranty. It guarantees the reliability of the item for the defined length of time. According to Blischke and Murthy [9], the reliability warranty approach was first used by the airline industry especially in purchases of commercial aircrafts, and later adopted by the military airline sector. From these applications, the four main components of a reliability improvement warranty include:

1. A guaranteed mean time between failures.

2. Manufacturer supported engineering changes.

3. A guaranteed turnaround time for repaired or replaced units.

4. A supply of consignment spares for use by the buyer at no cost until the guaranteed mean time between failures is demonstrated.

1.4.4 Warranty Cost

Scholars and manufacturers alike invest on research on warranties due to the concomitant cost implications. These costs include: administrative costs, transportation cost (to the repair facility and return to consumer), repair or replacement cost (comprising material cost and labor cost) and spare parts inventory costs.

1.5 The Impact of Warranty on Remanufactured products

The remanufactured products have existed for over 100 years in the U.S., however, today, a large proportion of consumers still have difficulty accepting remanufactured goods. The main problem is that consumers have a preconceived notion "that a remanufactured product is a used product." This means they do not trust that a remanufactured product is as good as new. Even though many consumers are attracted to the relatively cheap price of remanufactured products, they still harbor the impression that these products have poor quality.

Manufacturers are therefore forced to find ways to assure and promise consumers about their remanufactured products' good quality at relatively lower prices. Warranty, one of the ways manufacturers offer assurance has a key role to keep a balance between product quality and consumers' impression.

There are two papers in literature, Pang [10] and Rubio [11] that analyze customer behavior when purchasing remanufactured products in the UK and Spain respectively.

They use different study methods but have same conclusion that consumers regard both the product's price and the reputation of the remanufacturer as of particular importance in their intention to purchase a remanufactured product. Particularly, a good warranty policy (but not necessarily the length of warranty) has a positive indication on the reputation of a remanufacturer.

These findings by Pang [10] and Rubio [11] necessary research questions: What is a good warranty policy for a remanufactured product? It is definitely not "the longer the policy better" because Pang's study results already show that the warranty length is not a significant factor. From a remanufacturer's perspective an optimal warranty should maximize the remanufacturer's profit while portraying high product quality to the consumers. This Thesis study therefore, seeks to answer this question by determining the most optimal combination of warranty and price for remanufactured products. To make the analysis as realistic as possible, real failure data of remanufactured electronic products was obtained from a reputable fortune 500 company based in the U.S.

2 Chapter 2: Literature Review

2.1 Warranty Analysis Review

Over the past decades, several review papers have been written about product and service warranty policies and models from both manufacturing and management. A rich compilation of warranty literature can be found in Shafiee et al. [12], Thomas et al. [13], Murthy et al. [14] and, Karim et al. [15] all of which are review articles. An elaborate list of close to 190 warranty articles can be found in Murthy and Djamaludin [14], in which literature on warranty, especially for new products is broadly categorized into the following areas: warranty policies, warranty cost analysis, warranty and engineering, warranty and marketing, warranty and logistics and warranty and management. In addition, the book by Blischke and Murthy [9] provides a wide coverage of warranty taxonomy, theories, policies, and models, cost analyses as well as a few case studies. The warranty model that has been used in this study is adopted from Blischke's book.

There is also a substantial amount of literature covering warranty for second hand products. Chattopadhyay and Murthy [16] are the earliest researchers to publish articles in this area. In their paper “warranty cost analysis for second-hand products”, Chattopadhyay and Murthy suggest that dealers of second-hand products should estimate the warranty cost and build this cost into the product price structure. This is because customers of second hand goods are more concerned with protection against product failure. They then provide two approaches to demonstrate how to estimate the expected warranty cost under free-replacement warranty and pro-rata warranty from a system or component level.

With similar goals as Chattopadhyay and Murthy, Saidi-Mehrabad et al. [17] researched about minimizing the total mean cost of product from a seller’s perspective, by choosing the optimal reliability improvement and warranty strategy for second-hand products. They applied two methodologies—the virtual age model and screening test approach to estimate the reliability improvement cost. They then add this improvement cost to the corresponding warranty cost, under a specific warranty policy to optimize the total cost. On one hand, their results show that reliability improvement is usually costly and adds directly to the sale price of the second-hand products. On the other hand, reliability improvement cost it reduces the overall warranty cost. So the key is to find out the balance between optimal upgrade level of reliability improvement and warranty cost of second-hand products.

2.2 Warranty Analysis for Remanufactured Products

This Thesis is concerned with the analysis of warranty for remanufactured products. There are also many studies in the field of remanufacturing as a whole. One of the earliest studies of remanufacturing was by Lund in the early 1970s, and since then, more research began to emerge. However, most of the researchers are concerned with the remanufacturing-also called the closed-loop supply chain operations as well as establishing feasible models such as the End-of-life (EOL) model for a variety of products. Quite rarely do studies consider warranty analysis of remanufactured products. In this section, we will discuss the four papers found in literature that directly relate to warranty analysis of remanufactured products.

Research on the impact of quality variations on the warranty cost of remanufactured-product under FRLW policy.

Yu and Peng [18] presented the impacts of the quality variations on the remanufactured product in the process of remanufacturing and build models for reliability and the warranty cost under the Free Rectification Lifetime Warranty (FRLTW) policy. They list three possible ways that the reliability of remanufactured products can be affected, i.e. in the initial product design stages, presence of non-conforming components and errors during reassembly. They model the failure distribution for each of these causes and integrate them into the warranty cost model. Their goal is to determine how the three process variations affect the warranty cost. They summarize that reassembly error affects the reliability of the remanufactured product more than non-conforming components.

Warranty policy analysis for end-of-life product in reverse supply chain

One of the earliest research presented in the area of warranty policies in reverse supply chain was by Glickman and Berger [19]. They maximize the profit of a product sold under warranty by optimizing the price and warranty length. Alqahtani [20] evaluated the warranty cost for end-of-life products and predict an optimal warranty period for the disassembled components using sensor information embedded in a product. From the sensor information, they track the age and usage of each end-of-life product to meet the material demand while minimizing the warranty cost.

3 Chapter 3: Mathematical Model

3.1 Repairable and Remanufactured Product

Since there is no existing warranty cost model particularly established for remanufactured product, we opted to find an applicable warranty model for new product, and extend it to the remanufacturing field. In our case, we make use of the warranty cost model for repairable items. This is because we can assume that the repair process is able to renew the product—to as good as new condition. The difference therefore, between a repaired and remanufacture is that a repairable item's outcome can range from as "good as new" to "better than old" (minimal repair). Similar to remanufacturing, when the failed product is repaired to as good as new condition, it is assumed that the failure rate of repaired product is that same as the failure rate of a new product.

3.2 Basic Cost Model for Remanufactured Product

According to Blischke and Murthy [9], there is a basic cost model for supplier to replace a single repairable product under Free-Replacement warranty (FRW).

$$C_s(w) = c_s + c_r N_r(w) \quad (1)$$

Where c_s is the average cost to the seller of providing a new product without warranty and c_r is the expected total cost of supplying a repaired product under warranty. $N_r(w)$ is the expected number of repairs required during the warranty time W. So that $c_r N_r(w)$ is the expected warranty cost and $C_s(w)$ is the estimated total cost for the Original Equipment Manufacturer (OEM) to sell a new product with repairable warranty length w.

In the remanufacturing industry, normally two entities carry out the process. One is Original Equipment Manufacturer (OEM) OEM who produces both the new product and refurbished used one to new. The other is an Independent Remanufacturer (IR), who only focuses on the used product for remanufacture.

In reality, whether the process is carried out by an OEM or IR, c_s can be seen as the average cost for initial product without warranty. For OEM, the initial product is the new product, while for an IR, the initial product is a product that has been used once, and brought back for remanufacture. Hence,

$$C_R(w) = c_m + c_r N_r(w) \quad (2)$$

Where c_m is the average cost to OEM or IR of providing an initial product without warranty and C_R (w) is the total cost for OEM or IR to sell a remanufactured product with warranty W. N_r (w) and c_r retain their notation similar to Equation (1). From the basic cost model of remanufactured product, the initial cost c_m may include manufacturing costs, distribution costs and all other costs associated with providing the item to consumers. Remanufacturing cost c_r includes material costs, labor costs, shipping costs and administrative costs, among other direct and indirect remanufacturing costs. So, c_rN_r (w) is the warranty cost for remanufactured products. We can get c_r from company's accounts immediately. However, we cannot know N_r (w) in straight but calculate it out. To do this, we should know the failure rate of product first.

3.3 Failure Rate and Expected Numbers of Failure in Warranty

N_r (w) is the number of remanufacturing sessions required for a given product during the warranty period W. In reality, N_r (w) is a random variable that varies from one product to another. We therefore need to find a way to estimate N_r(w). The renewal function of a product offers a credible approach to estimate the expected value of N_r (w).

3.3.1 Failure Rate

Since we intend to investigate the warranty cost for remanufactured products, we must be able to estimate the expected number of them are claimed to have failed during the warranty period. Failure rate is a key in estimating the expected number of failures during warranty. Generally, we consider the failure rate as the frequency with which an industrial system or component fails.

Let X_1 denote the time of a product's first failure. Let F(x) denote the distribution function for the X_1. Then the probability of product failure in the interval [x, x+t), given that no failure happens before x, is

$$F(t|x) = [F(t+x) - F(x)]/\bar{F}(x) \tag{3}$$

Failure rate is therefore

$$r(x) = \lim_{t\to 0} \frac{F(t|x)}{t} = \frac{f(x)}{\bar{F}(x)} \tag{4}$$

For example, if we consider a product's first failure time is exponentially distributed, F(x)

= 1- $e^{-\lambda t}$. Then the failure rate for this product becomes

$$r(x) = \lim_{t \to 0} \frac{F(t|x)}{t} = \frac{f(x)}{\bar{F}(x)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda \qquad (5)$$

for $0 \leq x < \infty$, $\lambda > 0$.

After we know the failure rate of a product, we can determine the expected number of failures during a specific time period using the renewal function approach.

3.3.2 Ordinary and Delayed Renewal Function

According to Blischke and Murthy [9] a counting process {N (t), t ≥ 0} can be used to represent a delayed renewal process if the following conditions hold:

1. N (0) = 0.
2. X_1, the time to first event, is a nonnegative random variable with distribution function F (x).
3. X_j, j ≥ 2, the time intervals between jth and (j-1)st events, are independent and identically distributed random variables with distribution function G (x), which is different from F (x).

4. N (t) = sup {n: $S_n \leq t$}, where $S_0 = 0$ and, for $n \geq 1$, where S_n is the time instant of the nth renewal (or remanufacture in our application).

$$S_n = \sum_{i=1}^{n} Xi \quad (6)$$

When G(x) equals F(x), the process becomes an ordinary renewal process:

For a repairable product, it is usually assumed that the repair process is less than perfect and the repaired items are not good as new. An alternative model in which is recognized widely in literature is based on the assumption that the initial product has a lifetime of X_1 with distribution F(.), and the repaired items during warranty are assumed to have lifetimes X_2, X_3…, identically distributed with distribution G(.). Since the definition indicates that the remanufactured products have the same function with new one, then the failure rate of remanufactured product will not be changed. That means the initial remanufactured product has a lifetime X_1 with distribution F(.), and the recurrent remanufactures of the same item while in warranty, X_2, X_3…, are assumed to have lifetime distribution F(.) too. Therefore, we conjecture that remanufactured products exhibit an ordinary renewal process.

3.3.3 Analysis of Ordinary Renewal Processes

Blischke and Murthy already proved that the expected number of renewals in [0,t) is

$$M(t) = F(t) + \int_0^t M(t-x)f(x)dx \qquad (7)$$

The function in Equation 7 is called the renewal integral equation and M(t) is called the renewal function associated with the distribution function F(t). Considering the remanufacturing industry, M(t) means the expected number of products need to be remanufactured during warranty time [0,t). As will be seen in later case study, the renewal integral equation plays an important role in the warranty analysis. The difficulty in figuring out M(t) from Equation 9 is that M(t) appears on both sides of the equation. It is only for a small group of distribution functions F(t) that M(t) can be calculate analytically in closed form. For example, Blischke and Murthy [9] demonstrate that for exponential distribution with parameter λ, the renewal function is M(t) = λt.

That means if a kind of remanufactured product's lifetime follows exponential distribution with parameter λ. Then,

$$M(t) = 1 - e^{-\lambda t} + \int_0^t \lambda(t - x)\,\lambda e^{-\lambda x}dx = \lambda t \qquad (8)$$

3.4 Optimal Warranty Period Model

Once we know the lifetime distribution, failure rate and expected number of returns during warranty length of a product, we can start to build mathematical model to seek the optimal warranty period for remanufactured products. The warranty length is a critical factor. A longer warranty not only means greater protection to consumers, it also indicates a better quality. That means warranty length will affect marketing demand, hence it is also a factor in determining the profitability of a product.

3.4.1 The Demand Function

Generally the demand for remanufactured product depends on the marketing strategies. According to Glickman and Burger research [19], here we use the demand function $Q\left(C_p, w\right)$ as shown in Equation (9)

$$Q\,(C_p, w) = k_1 {C_p}^{-a}(w + k_2)^b \qquad (9)$$

Where w is the length of the warranty period; C_p is the sale price of product; $k_1 > 0$ is an arbitrary constant, an amplitude factor; $k_2 \geq 0$ is a constant of time displacement: it allows for non-zero demand with no warranty offered, $a>1$ is the parameter of the price elasticity; $0<b<1$ is the parameter of warranty length elasticity. In this demand function, we can see that the product demand decreases exponentially with respect to price and increases exponentially with warranty period.

3.4.2 Profit Maximization Model

As mentioned in Chapter 3, $C_R(w)$ is the total cost for OEM or IR to sell a remanufactured product with warranty w, so, $E[C_R(w)]$ denote the expected cost per unit sale.

$$E[C_R(w)] = c_m + c_r E[N_r(w)] \tag{10}$$

Then the expected profit per unit sale $\pi\left(C_p, w\right)$ is given by

$$\pi\left(C_p, w\right) = C_p - E[C_R(w)] \tag{11}$$

Multiplied by the demand function (9), we can get the total expected profit

$$\Pi\left(C_p, w\right) = Q\left(C_p, w\right)\pi\left(C_p, w\right) \tag{12}$$

Using (10) and (11) in (12), we have

$$\Pi\left(C_p, w\right) = k_1 {C_p}^{-a}(\mathrm{w} + k_2)^b\{C_p - c_m - c_r E[N_r(w)]\} \tag{13}$$

What we need is to maximize the $\Pi\left(C_p, w\right)$value and then get the optimal value of w and C_p, which denote warranty length and price per item respectively.

For example, assume the failure distribution of a product is exponential distribution with parameter λ. Then according to (), the expected value of $N_r(w)$ is given by $E[N_r(w)]$ = M(w)=λw. The total expected profit is given by

$$\Pi\left(C_p, w\right) = k_1 {C_p}^{-a}(\mathrm{w} + k_2)^b\{C_p - c_m + c_r \lambda \mathrm{w}\} \tag{14}$$

4 Chapter 4: Case Study

4.1 Back ground statement

This research was designed in collaboration with a global fortune 500 company based in the Midwest region of the U.S. The company is a world leader in the production of industrial automation and information. The company is recognized for innovation and excellence throughout the world. Its ten global ISO certified remanufacturing facilities use the same high quality parts, standards, and specifications as the original manufacturing process to restore customer's automation equipment to like new condition.

The partner company provides all customers remanufacturing service with 12-month warranty on the entire units, not just the replaced components. Unlike other repair services, the company has a comprehensive seven-step remanufacturing process for restoring used or failed equipment to their original operating condition and to make sure that they will function reliably. This seven-step remanufacturing process includes:

1. Receipt and Verification of Unit for warranty; a bar code is assigned for easy

tracking of repair history and order status.

2. Revisions and Enhancement performed to properly clean and update equipment to latest applicable hardware and copyrighted firmware.
3. Component Verification/Replacement of suspected faulty components.
4. Dynamic Functional Testing against current OEM specifications. Specialists determine operational status using dedicated test equipment including parametric testing.
5. Environmental testing to highlight intermittent problems not readily apparent, which helps prevent premature failures.
6. Final Quality Inspection is performed by Quality Control Inspectors to ensure compliance to company standards.
7. The products are the securely shipped in anti-static bags and containers, to help protect the remanufactured unit against static discharge.

4.2 Data Pre-Processing and Analysis

We received raw data received from the partner company cannot be displayed in this Thesis due to proprietary agreements. In total, there were 5192 product-return records from April 2012 to April 2015 in the database. The data included a family of automation controls, which were identifiable using their catalogue identities (CatID). Each individual product had an exclusive serial number, which was used to track the number of times a product was remanufactured. In addition, the serial numbers were used to track and estimate the time intervals between failures (returns) of each product. Once the product failed, the customer claimed the warranty or placed a remanufacturing order (RO). In the database, RO creation date is the time when a return order was created. Since RO is the closest the actual failure time, we used RO date to represent the failure time.

After pre-processing the raw data, we found out 1168 products were returned more than once for remanufacture. These 1168 data points were most useful in this study, from which we estimated the time between failures. Among the 1168 returned products, 28 of them were returned 4 times, 156 of them were returned 3 times and the rest 984 products were returned twice during April 2012 to April 2015. Next step was to calculate interval days between each return for every product. There was a total of 613 interval data. These were used as the actual time between failure data.

We sought to fit the most appropriate distribution to the failure data using the R statistical data. The Weibull distribution was the most fitful with the highest P-value of 0.4851. The second most fitting distribution was the Gamma distribution with a P-value of 0.2568. Hence we resolved use the Weibull distribution with the suggested shape parameter $\beta = 1.1794$ and scale parameter of λ of 269.93, thus $1/\lambda \approx 0.0037$. Figure 2 is the frequency curve of the actual data.

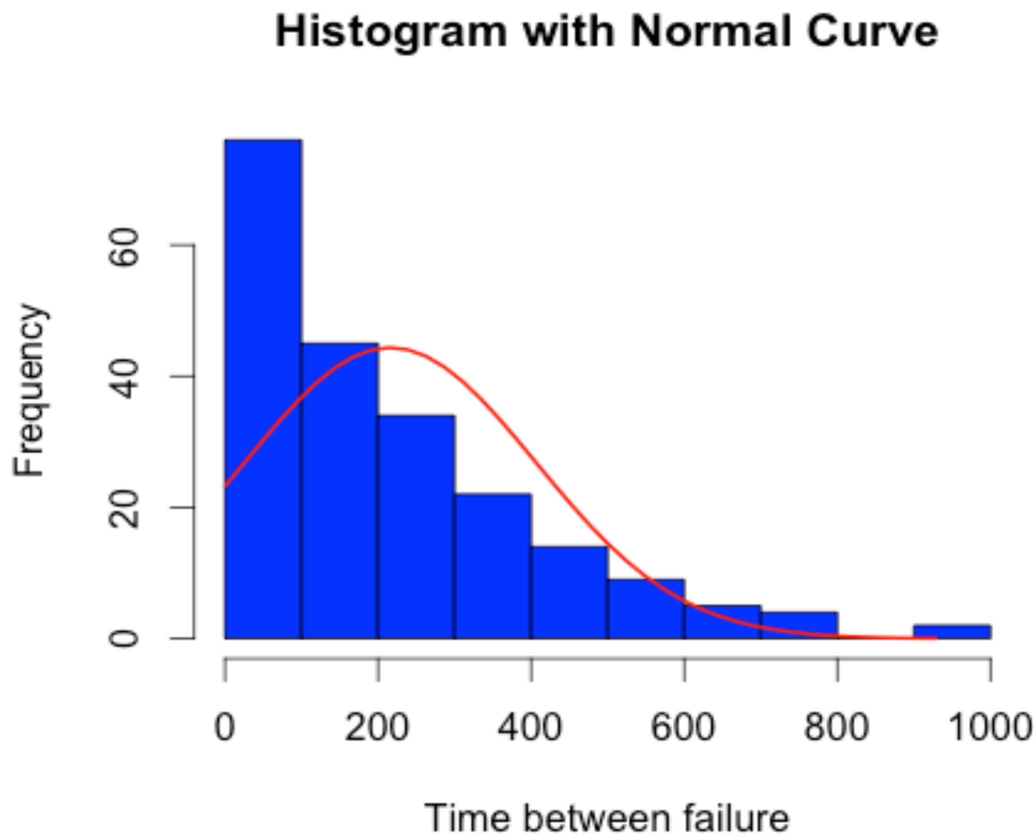

Figure 2: Product Returns Frequency Graph of Time Between Failures (in days)

Figure 3 and 4 are the probability distribution function (pdf) and the cumulative distribution function (cdf) of the time between failures respectively.

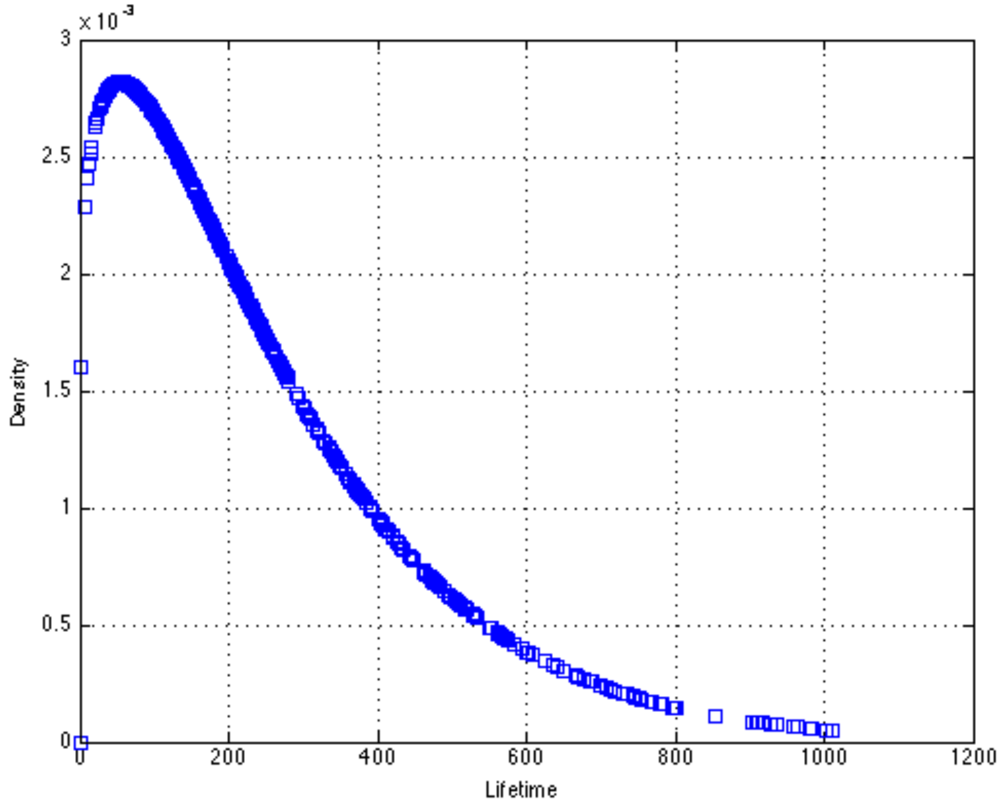

Figure 3: Plot of Probability Density Function of Product's Time Between Failures (in days)

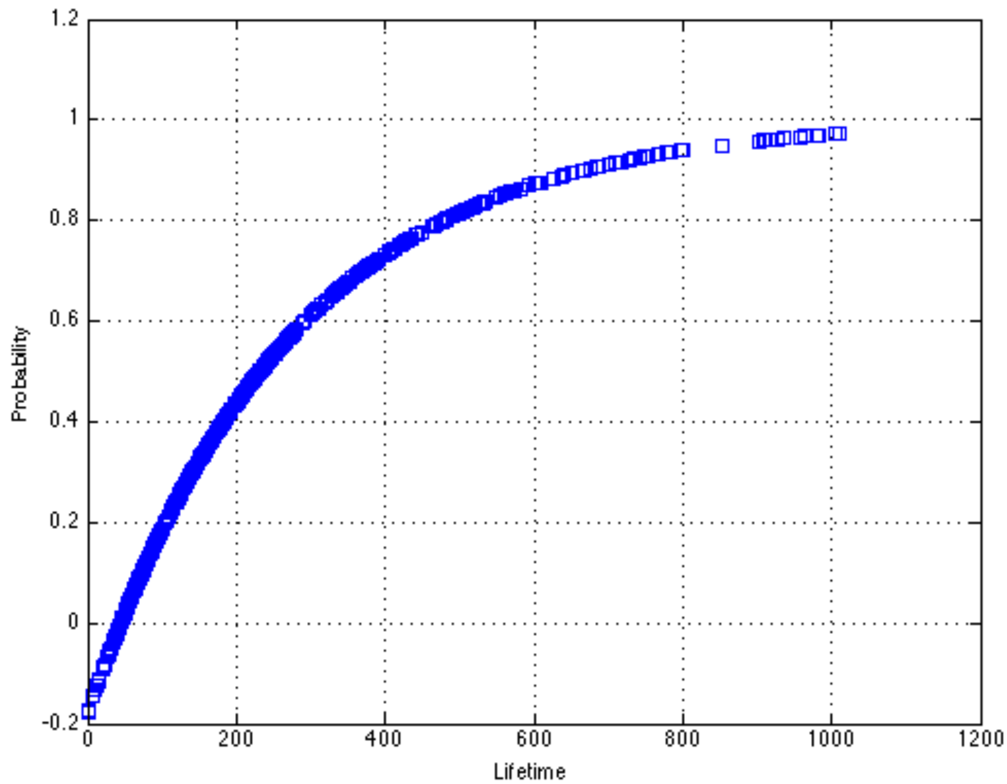

Figure 4: Plot of Cumulative Distribution Function of Product's Time Between Failures (in days)

Once we know the probability distribution function, the next step is to calculate the expected number of returns during warranty by using the renewal function M(.). However, just like we discussed in Section 3.3.3, only a small group of distribution's renewal function can be solved analytically in closed form. Unfortunately, the Weibull distribution is not one of them. We have to therefore seek another approach.

Several researchers have proposed several methods to solve the renewal function of the Weibull distribution. We chose to consider the approximation method by Jiang [21] because his approximation methodology is not only appropriate for our assumptions, but also sufficiently accurate and relatively simple to execute. In Jiang's method, the renewal function of Weibull distribution can be approximated as below equation:

$$M(t) = F^{p}(t)H^{q}(t) \qquad (15)$$

where F(t) is the cumulative Weibull distribution given by

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\lambda}\right)^{\beta}\right] \qquad (16)$$

H(t) is the cumulative hazard function which is given by

$$\mathrm{H(t)} = \left(\frac{t}{\lambda}\right)^{\beta} \qquad (17)$$

Here p, q > 0 and they are parameters to be determined by form:

$$P(\beta) = 1 - \exp\left[-\left(\frac{\beta-1}{A}\right)^{B}\right] \qquad (18)$$

Where a and b are hyper-parameters of the statistical distribution of β, which is the shape parameter of Weibull distribution. In our case study the values are a = 1.0571, b = 1.0518. Then q = 1- p(β). Applying Jiang's approximation methodology in our case we get,

$$M(t) = \{\, 1 - \exp\left[-\left(\frac{w}{\lambda}\right)^{\beta}\right]\}^{p} * [\left(\frac{w}{\lambda}\right)^{\beta}]^{q} \quad (19)$$

Where w is the variable of warranty length, then put λ = 269.9341, β =1.1794, p = 0.1434, q = 0.8566 into it.

$$M(t) = \{\, 1 - \exp\left[-\left(\frac{w}{269.9341}\right)^{1.1794}\right]\}^{0.1434} * [\left(\frac{w}{269.9341}\right)^{1.1794}]^{0.8566} \quad (20)$$

After simplified, the form is:

$$M(t) = \{\, 1 - \exp[-0.0014w^{1.1794}]\}^{0.1434} * [0.0037w^{1.1794}]^{0.8566} \quad (21)$$

4.3 Solution Model

We built the optimization model for our case by applying the total profit maximization function in Equation 13. For company in this case study the initial sale cost c_m=c_r=$737.17. The expected number of returns $E\ [N_r(w)]$=M(t). When t is considered as the warranty period w, then is $E\ [N_r(w)]$=M(w).

$$\Pi\left(C_p, w\right) = k_1 {C_p}^{-a}(\mathrm{w} + k_2)^{b}\{C_p - c_r - c_r\mathrm{M(w)}\} \quad (22)$$

Substituting Equation 21 into 22, we have

$\Pi\left(C_p, w\right) =$

$$k_1 C_p^{-a}(\mathrm{w} + k_2)^b\{C_p - 737.17 - 737.17\{1 - \exp[-0.0014w^{1.1794}]\}^{0.1434} *$$

$$[0.0037w^{1.1794}]^{0.8566}\} \quad (23)$$

We need to maximize the total expected profit $\Pi\left(C_p, w\right)$ by obtaining the values of C_p, w, for at which their respective partially derivate are equated to zero.

Thus the partial derivatives of Equation 23 were solved using Mathematica and the optimal value of C_p was obtained as follows:

$$C_p^* = \frac{737.17a + 2.5715\,a\,(1 - e^{-0.0014w^{1.1794}})^{0.1434} w^{1.0103}}{a-1} \quad (24)$$

Substituting the optimal C_p from Equation 24 back into Equation 23, we can get optimal warranty period w under different assumption of other parameters.

4.4 Numerical Expression and Sensitive Analysis

Let us assume that the sale price C_p range includes the actual remanufacturing cost of $737.17 (i.e. zero profit) to an arbitrary upper limit of $30000. Let the warranty period also vary from 180 days (half year) to 1080 days (3 years)—this range subsumes the actual warranty periods offered by the company. Let k_1= 80000, k_2= 2, a ε {1, 1.5, 2 } and b ε { 0.8, 0.85, 0.90}, then the maximum value of total profits are listed in the sensitive analysis Table 1.

From Table 1 we can see that when the warranty period is w=360 days, a=1.5 or 2, the maximum total profit is the highest $7,015,200 and the optimal C_p is $29,000.

Table 1 also indicates that if the sale price ranges from $737.17 to $30000, then 1-year warranty is the optimal choice for the remanufactured products of the company.

		w=180 days		w=360 days		w=540 days	
		Total Profit	Cp	Total Profit	Cp	Total Profit	Cp
a=1	b=0.80	$160,200	$29,000	$3,892,000	$29,000	$3,255,100	$29,000
	b=0.85	$207,900	$29,000	$5,225,200	$29,000	$4,459,300	$29,000
	b=0.90	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
a=1.5	b=0.80	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
	b=0.85	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
	b=0.90	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
a=2	b=0.80	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
	b=0.85	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
	b=0.90	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
		w=720 days		w=900 days		w=1080 days	
		Total Profit	Cp	Total Profit	Cp	Total Profit	Cp
a=1	b=0.80	$1,443,300	$29,000	-$142,180.00	$29,000	-$5,268,900	$29,000
	b=0.85	$2,005,800	$29,000	-$142,180.00	$29,000	-$5,268,900	$29,000
	b=0.90	$2,787,500	$29,000	-$142,180.00	$29,000	-$5,268,900	$29,000
a=1.5	b=0.80	$2,787,500	$29,000	-$26,000.00	$29,000	-$96,400	$29,000
	b=0.85	$2,787,500	$29,000	-$26,000.00	$29,000	-$96,400	$29,000
	b=0.90	$2,787,500	$29,000	-$26,000.00	$29,000	-$96,400	$29,000
a=2	b=0.80	$2,787,500	$29,000	-$500.00	$29,000	-$1,800	$29,000
	b=0.85	$2,787,500	$29,000	-$500.00	$29,000	-$1,800	$29,000
	b=0.90	$2,787,500	$29,000	-$500.00	$29,000	-$1,800	$29,000

Table 1: Sensitivity to Elasticity's Parameters

5 Chapter 5: Conclusion and Future Research

In this Thesis, we developed a stochastic process model, based on the renewal function from manufacturers' point of view, to optimize the warranty period and price of remanufactured products under free-replacement warranty policy. The objective function is the manufacturer' total expected profit. For calculation purposes, the total cost function is assumed to only depend on the demand function, sale price and warranty period. In the quest to determine the optimal solution of case study, we applied a simple and accurate approach to approximate the renewal function for the Weibull distribution.

This is a new exploratory study that attempts to find optimal warranty period for remanufactured products under free-replacement policy. There are also some uncertain factors in our model, such as amplitude parameter, price-demand elasticity and warranty-demand elasticity factors all of which are attributes of the demand function, hence they change with the real market conditions. In the future study, we will continue to improve and optimize this model to serve the remanufacturing industry better, by testing it to real products of other remanufacturing companies.

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