

August 2015

Transient Performance Analysis of Serial Production Lines

Yang Sun

University of Wisconsin-Milwaukee

Follow this and additional works at: <https://dc.uwm.edu/etd>



Part of the [Industrial Engineering Commons](#)

Recommended Citation

Sun, Yang, "Transient Performance Analysis of Serial Production Lines" (2015). *Theses and Dissertations*. 1023.
<https://dc.uwm.edu/etd/1023>

This Thesis is brought to you for free and open access by UWM Digital Commons. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of UWM Digital Commons. For more information, please contact open-access@uwm.edu.

TRANSIENT PERFORMANCE ANALYSIS OF
SERIAL PRODUCTION LINES

by

Yang Sun

A Thesis Submitted in
Partial Fulfillment of the
Requirements for the Degree of

Master of Science
in Engineering

at

The University of Wisconsin-Milwaukee

August 2015

ABSTRACT

TRANSIENT PERFORMANCE ANALYSIS OF SERIAL PRODUCTION LINES

by
Yang Sun

The University of Wisconsin-Milwaukee, 2015
Under the Supervision of Professor Liang Zhang

Production lines with unreliable machines and finite buffers are characterized by both steady-state performance and transient behavior. The steady-state performance has been analyzed extensively for over 50 years. Transient behavior, however, is rarely studied and remains less explored. In practice, a lot of the real production systems are running partially or entirely in transient periods. Therefore, transient analysis is of significant practical importance.

Most of the past research on production systems focuses on discrete materials flow which utilizes Markov chain analysis. This dissertation is devoted to investigate the effects of system parameters on performance measures for transient serial production line with other machine reliability models. The reliability models investigated in this dissertation are exponential and no-exponential (Weibull, Gamma, Log-normal).

In a real production line system, machine reliability models are much more difficult to identify. Strictly speaking, it requires the identifications of the histograms of up- and downtime, which requires a very large number of measurements during a long period of

time. The result may be that the machines' real reliability model on the factory floor are, practically, never known. Therefore, it is of significant practical importance to investigate the general effects of system parameters on performance measures for transient serial production line with different reliability models. The system parameters include machine efficiency e , ratio of N and T_{down} (K), machines' average downtime T_{down} , and coefficient of variation CV . The performance measures include settling time of production rate (t_{sPR}), settling time of work-in-process (t_{sWIP}), total production (TP), production loss (PL). The relationship between the performance measures and system parameters reveals the fundamental principles that characterize the behavior of such systems and can be used as a guideline for product lines' management and improvement.

Most previous research studies are limited to two or three machine system due to the technical complexity. Furthermore, presently there are no analytical tools to address the problems with multiple machines and buffers during transient periods. This dissertation addresses this problem by using simulations with C++ programming to evaluate the multiple machines (up to 10) and buffers and demonstrate the transient performance at different conditions. The simulation method does not only provide quantified transient performance results for a given production line, but also provides a valuable tool to investigate the system parameter effects and how to manage and improve the existing production line.

© Copyright by Yang Sun, 2015
All Rights Reserved

To
my parents, my husband
and my son

TABLE OF CONTENTS

LIST OF FIGURES	viii
LIST OF TABLES	xi
Chapter 1 Introduction.....	1
1.1 Motivation	1
1.2 Outline.....	4
Chapter 2 System Models and Problems.....	5
2.1 Terminology	5
2.2 Machine Reliability Models	7
2.3 Systems Considered	10
2.3.1 Continuous Serial Production Lines	10
2.3.2 Systems Considered.....	12
2.3.3 System Parameters.....	13
2.4 Performance Measures	14
2.5 Problem Statement	15
Chapter 3 Exponential Systems.....	17
3.1 Transients of Exponential Lines.....	17
3.1.1 Transients of Individual Exponential Machine	17
3.1.2 Transients of Bufferless Exponential Serial Line.....	19
3.1.3 Exponential vs. Geometric	21
3.2 Transient Performance Analysis	28
3.2.1 Effects of e	28
3.2.2 Effects of K	30
3.2.3 Effects of T_{down}	31
3.3 Settling Time	32
3.3.1 Effects of e	32
3.3.2 Effects of K	34
3.3.3 Effects of T_{down}	36
3.4 Total Production.....	38
3.4.1 Effects of e	38
3.4.2 Effects of K	40
3.4.3 Effects of T_{down}	41
3.5 Production Loss.....	42
3.5.1 Effects of e	43
3.5.2 Effects of K	44
3.5.3 Effects of T_{down}	46
3.6 Summary	47
Chapter 4 Weibull, Gamma, Log-Normal Systems.....	51

4.1	Transient Performance Analysis	51
4.1.1	Effects of e	51
4.1.2	Effects of K	54
4.1.3	Effects of T_{down}	55
4.1.4	Effects of CV_{up}	57
4.1.5	Effects of CV_{down}	58
4.2	Settling Time	60
4.2.1	Effects of e	60
4.2.2	Effects of K	62
4.2.3	Effects of T_{down}	64
4.2.4	Effects of CV_{up}	66
4.2.5	Effects of CV_{down}	67
4.3	Total Production.....	69
4.3.1	Effects of e	69
4.3.2	Effects of K	71
4.3.3	Effects of T_{down}	73
4.3.4	Effects of CV_{up}	75
4.3.5	Effects of CV_{down}	77
4.4	Production Loss.....	79
4.4.1	Effects of e	79
4.4.2	Effects of K	81
4.4.3	Effects of T_{down}	83
4.4.4	Effects of CV_{up}	85
4.4.5	Effects of CV_{down}	86
4.5	Summary	88
Chapter 5 Conclusions and Future Works.....		91
Bibliography		95

LIST OF FIGURES

Figure 1 Serial production line	5
Figure 2 Exponential Reliability Model	8
Figure 3 Structural Model.....	10
Figure 4 Transients of machine state of an individual exponential machine ($T_{up} = 80$, $T_{down} = 20$).....	19
Figure 5 Transients of bufferless five-machine exponential line with identical machines ($T_{up} = 80$, $T_{down} = 20$, machines initially down)	20
Figure 6 Transients of bufferless five-machine exponential line with identical machines ($T_{up} = 80$, $T_{down} = 20$, machines initially up).....	21
Figure 7 . Approximation of PR(t) of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty).....	23
Figure 8 Approximation of CR(t) of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty).....	24
Figure 9 Approximation of WIPi(t) of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty).....	25
Figure 10 Approximation of STi(t) of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty).....	26
Figure 11 Approximation of BLi(t) of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty).....	27
Figure 12 Effects of e on Transient Performance of Exponential Lines.....	29
Figure 13 Effects of K on Transient Performance of Exponential Lines	31
Figure 14 Effects of T_{down} on Transient Performance of Exponential Lines	32
Figure 15 Effects of e on t_{sPR} and t_{sWIP} of Exponential Lines	34

Figure 16 Effects of K on t_{sPR} and t_{sWIP} of Exponential Lines	36
Figure 17 Effects of T_{down} on t_{sPR} and t_{sWIP} of Exponential Lines	37
Figure 18 Effects of e on Total Production of Exponential Lines	39
Figure 19 Effects of K on Total Production of Exponential Lines	41
Figure 20 Effects of T_{down} on Total Production of Exponential Lines	42
Figure 21 Effects of e on Production Loss of Exponential Lines	44
Figure 22 Effects of K on Production Loss of Exponential Lines	45
Figure 23 Effects of T_{down} on Production Loss of Exponential Lines	47
Figure 24 Effects of e in Transient Process of Non- Exponential Lines	53
Figure 25 Effects of K in Transient Process of Non- Exponential Lines	55
Figure 26 Effects of T_{down} in Transient Process of Non- Exponential Lines	57
Figure 27 Effects of CV_{up} in Transient Process of Non- Exponential Lines	58
Figure 28 Effects of CV_{down} in Transient Process of Non- Exponential Lines.....	60
Figure 29 Effects of e on t_{sPR} and t_{sWIP} of Non-Exponential Lines.....	62
Figure 30 Effects of K on t_{sPR} and t_{sWIP} of Non-Exponential Lines.....	64
Figure 31 Effects of T_{down} on t_{sPR} and t_{sWIP} of Non-Exponential Lines	65
Figure 32 Effects of CV_{up} on t_{sPR} and t_{sWIP} of Non-Exponential Lines.....	67
Figure 33 Effects of CV_{down} on t_{sPR} and t_{sWIP} of Non-Exponential Lines.....	68
Figure 34 Effects of e on Total Production of Non-Exponential Lines	70
Figure 35 Effects of K on Total Production of Non-Exponential Lines.....	72
Figure 36 Effects of T_{down} on Total Production of Non-Exponential Lines	74
Figure 37 Effects of CV_{up} on Total Production of Non-Exponential Lines	76
Figure 38 Effects of CV_{down} on Total Production of Non-Exponential Lines	78

Figure 39 Effects of e on Production Loss of Non-Exponential Lines.....	80
Figure 40 Effects of K on Production Loss of Non-Exponential Lines.....	82
Figure 41 Effects of T_{down} on Production Loss of Non-Exponential Lines	84
Figure 42 Effects of CV_{up} on Production Loss of Non-Exponential Lines	86
Figure 43 Effects of CV_{down} on Production Loss of Non-Exponential Lines	88

LIST OF TABLES

Table 1 Expectation, variance, and coefficients of variation of continuous random variables	10
Table 2 Exponential System Transient Performance Analysis Values.....	28
Table 3 Exponential System Parameters (Effects of e on transient performance)	29
Table 4 Exponential System Parameters (Effects of K on transient performance)	30
Table 5 Exponential System Parameters (Effects of T_{down} on transient performance).....	31
Table 6 Exponential System Parameters (Effects of e on t_{sPR} and t_{sWIP})	32
Table 7 Exponential System Parameters (Effects of K on t_{sPR} and t_{sWIP})	34
Table 8 Exponential System Parameters (Effects of T_{down} on t_{sPR} and t_{sWIP}).....	37
Table 9 Exponential System Parameters (Effects of e on TP).....	38
Table 10 Exponential System Parameters (Effects of K on TP).....	40
Table 11 Exponential System Parameters (Effects of T_{down} on TP)	41
Table 12 Exponential System Parameters (Effects of e on PL).....	43
Table 13 Exponential System Parameters (Effects of K on PL).....	44
Table 14 Exponential System Parameters (Effects of T_{down} on PL)	46
Table 15 Effects of system parameters (e , K , and T_{down}) on the t_{sPR} and t_{sWIP} , TP and PL	48
Table 16 Non-Exponential System Transient Performance Analysis Values	51
Table 17 Non-Exponential System Parameters(Effects of e on transient performance) ..	52
Table 18 Non-Exponential System Parameters (Effects of K on transient performance) .	54
Table 19 Non-Exponential System Parameters (Effects of T_{down} on transient performance)	55
Table 20 Non-Exponential System Parameters (Effects of CV_{up} on transient performance)	57

Table 21 Non-Exponential System Parameters (Effects of CV_{down} on transient performance)	58
Table 22 Non-Exponential System Parameters (Effects of e on t_{sPR} and t_{sWIP})	60
Table 23 Non-Exponential System Parameters (Effects of K on t_{sPR} and t_{sWIP})	62
Table 24 Non-Exponential System Parameters (Effects of T_{down} on t_{sPR} and t_{sWIP})	64
Table 25 Non-Exponential System Parameters (Effects of CV_{up} on t_{sPR} and t_{sWIP})	66
Table 26 Non-Exponential System Parameters (Effects of CV_{down} on t_{sPR} and t_{sWIP})	67
Table 27 Non-Exponential System Parameters (Effects of e on TP)	69
Table 28 Non-Exponential System Parameters (Effects of K on TP)	71
Table 29 Non-Exponential System Parameters (Effects of T_{down} on TP)	73
Table 30 Non-Exponential System Parameters (Effects of CV_{up} on TP)	75
Table 31 Non-Exponential System Parameters (Effects of CV_{down} on TP)	77
Table 32 Non-Exponential System Parameters (Effects of e on PL)	79
Table 33 Non-Exponential System Parameters (Effects of K on PL)	81
Table 34 Non-Exponential System Parameters (Effects of T_{down} on PL)	83
Table 35 Non-Exponential System Parameters (Effects of CV_{up} on PL)	85
Table 36 Non-Exponential System Parameters (Effects of CV_{down} on PL)	86
Table 37 Effects of system parameters (e , K , and T_{down}) on the t_{sPR} and t_{sWIP} , TP and PL	89

ACKNOWLEDGMENTS

First and foremost, I would like to express my deepest gratitude to my advisor, Dr. Liang Zhang, for his guidance and continuous support throughout my master study. Dr. Zhang not only passed me knowledge but also taught me how to think and solve problems in my research. Without his support and guidance, it would not have been possible for me to complete this dissertation. It is him who brought me into the fascinating world of production system engineering. I have benefited tremendously from his vision, technical insight and professionalism. His dedicated research altitude deeply influences my study and research.

I am grateful to all my committee members for their help and support. I appreciate Dr. Wilkistar Otieno and Dr. Xiang Fang for their valuable time and serving my committee. I am thankful to my colleague, Guorong Chen, for helping me solving lots of difficulties during my study.

Finally, I would like to dedicate this thesis to my parents, Bojun Sun and Yuhua Yang, my husband, Tiefu Zhao, and my son, Ethan Zhao. Their support and love make my master study a happy journey.

Chapter 1 Introduction

1.1 Motivation

Production lines with unreliable machines and finite buffers are characterized by both steady-state performance and transient behavior. The steady-state performance has been analyzed extensively for over 50 years [1-8]. In contrast, transient behavior is less studied in the past. Actually a lot of systems are running partially or entirely in transient period. For instance, in some systems, buffers will be purged at the end of a shift and therefore the systems will begin production under empty buffer condition. In other systems, machines can start or shut down at different time in which case systems are also running in transient period. Therefore, in a manufacturing environment, production transients, i.e., the processing time to reach steady state, are of significant practical importance. If the steady state is reached after a relatively long period of time, the system may suffer substantial production losses. For instance, it has been shown that if the cycle time of a production system is 1 minute and the plant shift is 500 minutes, the system may lose more than 10% of its production due to transients, if at the beginning of the shift all buffers were empty [9]. Therefore, transient analysis in production lines is indispensable for a practical production system.

Despite the importance of transient analysis, transient performance is less studied and still remain unexplored in literatures. Among the reviewed literatures, performance analysis of serial production lines with Bernoulli machines during transients have been

discussed in [9]. They investigated properties of transients of production rate and work-in-process for Bernoulli machine lines by using analytical method which is Markov chain analysis. It is shown that the transients of production rate and work-in-process are determined by the second largest eigenvalue of the transition matrix of the associate Markov chain and the pre-exponential factor. The settling time and production loss due to transients are also analyzed. To avoid the production loss during transients, it is suggested that all buffers are initially at least half full.

On the other hand, most of the past research on production systems focuses on discrete materials flow which utilizes Markov chain analysis [10, 11, 12]. There is an increasing number of research on production lines with continuous materials flow. Among them, a complementary study of continuous materials flow production systems has been conducted [13]. The throughput and bottleneck in assembly systems with non-exponential machines are also studied [14]. However, it is mostly assumed that the time to failure and the time to repair are exponentially distributed or deterministic. For instance, Baris (1998) consider a continuous materials flow production system with multiple machines in series but no intermediate buffers. However, machines' processing time is deterministic [15]. Some research focuses on other performance measures, such as production rate and due-time performance. Jacobs and Meerkov (1995) performed system theoretic analysis of due-time performance in production systems [16]. Tan and Yeralan (1997) proposes a decomposition model for continuous materials flow production systems to evaluate production rate in steady state [17]. Li and Meerkov (1995) evaluates throughput in serial production lines with non-exponential machines [18]. However, properties of settling time and production loss receive less attention.

It is important to extend the transient analysis to serial production lines with other machine reliability models, i.e. exponential and no-exponential (weibull, gamma, log-normal). However, the machine reliability model is much more difficult to identify. Strictly speaking, it requires the identifications of the histograms of up- and downtime, which require a very large number of measurements during a long period of time. The result is that the machines' real reliability model on the factory floor are, practically, never known. Realistically speaking, machines' average up- and downtime (T_{up} and T_{down}) and the coefficient variance of up- and downtime (CV_{up} and CV_{down}) may be the only characteristics of reliability models available from the factory floor. Therefore, it is critical to investigate the impacts of the machine parameters, such as T_{up} , T_{down} , CV_{up} , CV_{down} , Efficiency (e), and buffer size (N) to the production lines' transient performances, such as production rate, settling time, total production, and production loss.

Previous research mostly focus on the two or three machine system to reduce the system complexity. For instance, transient behavior of two-machine lines with Geometric reliability was studied by Meerkov et al. (2010) [19]. Baris and Stanley (2009) analyzes general Markovian continuous materials flow production systems with two machines [20]. Bruno (2001) considers a fluid system with two machines whose states are Markovian and a finite buffer between them [21]. Kim et al. (2002) provided an upper bound for carriers in a three-machine serial production line [22]. Currently there are no analytical tools to address the problems with multiple machines and buffers during transient periods, this dissertation uses simulation with C++ programming study the multiple machines and buffers (up to 10) and illustrates the transient performance by case studies.

1.2 Outline

The outline of this dissertation is as follows: Chapter 2 introduces machines reliability models and the system models considered in this thesis. Chapter 3 and Chapter 4 investigate the effects of system parameters (including machine efficiency e , ratio of N and $T_{\text{down}}(K)$, Machines' average downtime T_{down} , and coefficient of variation CV) on different performance measures, including settling time of production rate (t_{sPR}), settling time of work-in-process (t_{sWIP}), total production (TP), production loss (PL). Chapter 3 investigates the transients of serial production line with machines reliability model satisfying exponential distribution. Chapter 4 explores the transient performances for Weibull, Gamma, Log-Normal production lines, respectively. Finally, the conclusions and topics for future research are provided in Chapter 5.

Chapter 2 System Models and Problems

In order to formalize the system modeling and problems, this chapter defines a set of standard vocabulary used throughout this thesis [23].

2.1 Terminology

Serial production line: Serial production line – a group of producing units, arranged in consecutive order, and material handling devices that transport parts (or jobs) from one producing unit to the next. Figure 1 shows the block diagram of a serial production line where m_i , $i=1\dots M$, represent producing units and b_i , $i=1\dots M-1$, are material handling devices.



Figure 1 Serial production line

Cycle time (τ) : the time necessary to process a part by a machine. The cycle time may be constant, variable, or random. In large volume production systems, τ is practically always constant or close to being constant. This is the case in most production systems of the automotive, electronics, appliance, and other industries. Variable or random cycle time takes place in job-shop environments where each part may have different processing specifications. In this research, we consider only machines with a constant cycle time;

however, similar developments can be carried out for the case of random (e.g., exponentially distributed) processing time.

Machine capacity c : the number of parts produced by a machine per unit of time when the machine is up. Clearly, in the case of constant τ ,

$$c = \frac{1}{\tau}.$$

Machines in a production system may have identical or different cycle times. In the case of identical cycle time, the time axis may be considered as slotted or unslotted.

Slotted time: the time axis is slotted with the slot duration equal to the cycle time. In this case, all transitions - changes of machines' status (up or down) and changes of buffers' occupancy - are considered as taking place only at the beginning or the end of the time slots.

Unslotted time or continuous time: changes of machines' status (up or down) and changes of buffers' occupancy may occur at any time moment. If the cycle times of all machines are identical, such a system is referred to as synchronous. If the cycle times are not identical, the system is called asynchronous. Production systems with machines having different cycle times are typically considered as operating in unslotted time.

In the unslotted case, production systems can be conceptualized as typically considered as discrete event systems or as flow systems.

Discrete event system: a job (i.e. part) is transferred from the producing machine to the subsequent buffer (if it is not full) only after the processing of the whole job is complete. In this case, the buffer occupancy is a non-negative integer.

Flow system: infinitesimal parts of the job are (conceptually) transferred from the producing machine to the subsequent buffer if it is not full. Similarly, an infinitesimal part of a job is taken by a downstream machine from the buffer, if the machine is not down and the buffer is not empty. In this case, there is a continuous flow of parts into and from the buffers. Clearly, the buffer occupancy in this situation is a non-negative real number. Flow systems are sometimes easier to analyze and often lead to reasonable conclusions.

Machine reliability model: the probability mass functions (pmf's) or the probability density functions (pdf's) of the up- and downtime of the machine in the slotted or unslotted time, respectively. In addition, the expected value and coefficient of variation of up- and downtime are denoted as T_{up} , T_{down} , CV_{up} and CV_{down} , respectively.

2.2 Machine Reliability Models

In this dissertation, the following four machine reliability models are used: Exponential, Weibull, Gamma and Log-normal. In the continuous time case, each machine is denoted as $[f_{tup}(t), f_{tdown}(t)]$, where $[f_{tup}(t), f_{tdown}(t)]$, are the pdf's of up- and downtime, respectively. The expected value and coefficient of variation of up- and downtime, T_{up} , T_{down} , CV_{up} and CV_{down} , respectively are shown in Table 1.

Exponential reliability model (exp): Consider a machine in Figure 2, which is a continuous time analogue of the geometric machine. Namely, if it is up (respectively, down) at time t , it goes down (respectively, up) during an infinitesimal time δt with probability $\lambda \delta t$ (respectively, $\mu \delta t$). The parameters λ and μ are called the breakdown and repair rates, respectively.

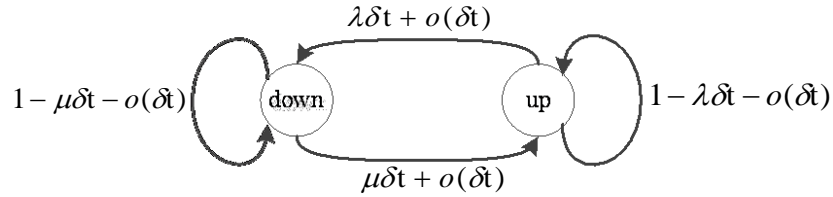


Figure 2 Exponential Reliability Model

It can be shown that the pdf's of the up- and downtime of this machine, denoted as t_{up} and t_{down} , are as follows:

$$f_{t_{up}} = \lambda e^{-\lambda t}, \quad t \geq 0,$$

$$f_{t_{down}} = \mu e^{-\mu t}, \quad t \geq 0,$$

Clearly, t_{up} and t_{down} are exponential random variables and we refer to such a machine as an exponential machine, i.e., obeying the exponential reliability model. In addition, it is easy to show that for an exponential machine

$$T_{up} = \frac{1}{\lambda}, T_{down} = \frac{1}{\mu},$$

$$CV_{up} = 1, CV_{down} = 1,$$

$$e = \frac{\mu}{\lambda + \mu}$$

Weibull reliability model (W): Weibull distribution is widely used in Reliability Theory.

For a machine obeying Weibull reliability model, its up- and downtime pdf's are given by

$$f_{t_{up}}(t) = \lambda^\Lambda e^{-(\lambda t)^\Lambda} \Lambda t^{\Lambda-1}, \quad t \geq 0,$$

$$f_{t_{down}}(t) = \mu^M e^{-(\mu t)^M} M t^{M-1}, \quad t \geq 0.$$

where μ and M are positive numbers. It can be calculated that for a Weibull machine

$$T_{\text{up}} = \frac{1}{\lambda} \Gamma\left(1 + \frac{1}{\Lambda}\right), T_{\text{down}} = \frac{1}{\mu} \Gamma\left(1 + \frac{1}{M}\right),$$

$$CV_{\text{up}} = \frac{\sqrt{\Gamma\left(1 + \frac{2}{\Lambda}\right) - \Gamma^2\left(1 + \frac{1}{\Lambda}\right)}}{\Gamma\left(1 + \frac{1}{\Lambda}\right)}, CV_{\text{down}} = \frac{\sqrt{\Gamma\left(1 + \frac{2}{M}\right) - \Gamma^2\left(1 + \frac{1}{M}\right)}}{\Gamma\left(1 + \frac{1}{M}\right)}.$$

Gamma reliability model (ga): For a machine obeying the gamma reliability model, its up- and downtime pdf's are given by gamma distribution,

$$f_{t_{\text{up}}}(t) = \lambda e^{-\lambda t} \frac{(\lambda t)^{\Lambda-1}}{\Gamma(\Lambda)}, t \geq 0,$$

$$f_{t_{\text{down}}}(t) = \mu e^{-\mu t} \frac{(\mu t)^{M-1}}{\Gamma(M)}, t \geq 0.$$

Where,

$$\Gamma(x) = \int_0^{\infty} s^{x-1} e^{-s} ds,$$

and Λ and M are positive numbers. In addition, it can be calculated that for a gamma machine

$$T_{\text{up}} = \frac{\Lambda}{\lambda}, T_{\text{down}} = \frac{M}{\mu},$$

$$CV_{\text{up}} = \frac{1}{\sqrt{\Lambda}}, CV_{\text{down}} = \frac{1}{\sqrt{M}}.$$

Log-normal reliability model (LN): For a machine obeying the log-normal reliability model, its up- and downtime pdf's are given by

$$f_{t_{\text{up}}}(t) = \frac{1}{\sqrt{2\pi\Lambda t}} e^{-\frac{(\ln t - \lambda)^2}{2\Lambda^2}}, t \geq 0,$$

$$f_{t_{\text{down}}}(t) = \frac{1}{\sqrt{2\pi M t}} e^{-\frac{(\ln t - \mu)^2}{2M^2}}, t \geq 0,$$

Where, Λ and M are positive numbers. In addition, it can be calculated that for a log-normal machine

$$T_{\text{up}} = e^{\lambda + \frac{\Lambda^2}{2}}, T_{\text{down}} = e^{\mu + \frac{M^2}{2}}$$

$$CV_{\text{up}} = \sqrt{e^{\Lambda^2} - 1}, CV_{\text{down}} = \sqrt{e^{M^2} - 1}$$

Table 1 Expectation, variance, and coefficients of variation of continuous random variables

Random variable	Expectation	Variance	CV
Exponential	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	1
Gamma	$\frac{\Lambda}{\lambda}$	$\frac{\Lambda}{\lambda^2}$	$\frac{1}{\sqrt{\Lambda}}$
Weibull	$\frac{1}{\lambda} \Gamma(1 + \frac{1}{\Lambda})$	$\frac{1}{\lambda^2} \left[\Gamma(1 + \frac{2}{\Lambda}) - \Gamma^2(1 + \frac{1}{\Lambda}) \right]$	$\frac{\sqrt{\Gamma(1 + \frac{2}{\Lambda}) - \Gamma^2(1 + \frac{1}{\Lambda})}}{\Gamma(1 + \frac{1}{\Lambda})}$
Log-normal	$e^{\lambda + \frac{\Lambda^2}{2}}$	$e^{2\lambda + \Lambda^2} (e^{\Lambda^2} - 1)$	$\sqrt{e^{\Lambda^2} - 1}$

2.3 Systems Considered

2.3.1 Continuous Serial Production Lines

Continuous serial production lines are illustrated in Figure 3 where circles represent machines and rectangles represent buffers.



Figure 3 Structural Model

Conventions:

- (a) Blocked before service.
- (b) The first machine is never starved; the last machine is never blocked.
- (c) Flow model, i.e., infinitesimal quantity of parts, produced during δt , are transferred to and from the buffers.
- (d) The state of each machine (up or down) is determined independently from all other machines.
- (e) Time-dependent failures.

In continuous time case, serial production lines shown by Figure 3 operate according to the following assumptions:

- a) The system consists of M machines m_i , $i = 1, \dots, M$, and $M-1$ buffers, b_i , $i = 1, \dots, M - 1$.
- b) Each machine m_i , $i = 1, \dots, M$, has two states: up and down. When up, the machine is capable of producing with rate c_i (parts/unit of time); when down, no production takes place.
- c) The up- and downtime of each machine are continuous random variables, $t_{up,i}$ and $t_{down,i}$, $i = 1, \dots, M$, and are determined by its reliability model. It is assumed that these random variables are mutually independent.
- d) Each in-process buffer b_i , $i = 1, \dots, M - 1$, is characterized by its capacity, $0 < N_i < \infty$.
- e) Machine m_i , $i = 2, \dots, M$, is starved at time t if it is up at time t and buffer b_{i-1} is empty at time t .

- f) Machine m_i , $i = 1, \dots, M - 1$, is blocked at time t if it is up at time t , buffer b_i is full at time t and machine m_{i+1} fails to take any work from this buffer at time t .

2.3.2 Systems Considered

In this dissertation, continuous serial production lines are operated according to the following assumptions:

- a) The system consists of M identical machines m_i , $i = 1, \dots, M$, and $M-1$ identical buffers, b_i , $i = 1, \dots, M - 1$.
- b) Each machine m_i , $i = 1, \dots, M$, has two states: up and down. When up, the machine is capable of producing with rate c_i (parts/unit of time); when down, no production takes place. Machines are down initially.
- c) Machines m_i , $i = 1, \dots, M$, operate independently and obey continuous reliability model.
- d) Each buffer b_i , $i = 1, \dots, M - 1$ has finite capacity $0 < N_i < \infty$, and is empty initially.
- e) Machine m_i , $i = 2, \dots, M$, is starved if it is up and buffer b_{i-1} is empty. It is assumed that machine m_1 is never starved;
- f) Machine m_i , $i = 1, \dots, M-1$, is blocked if it is up, buffer b_i has N_i parts and machine m_{i+1} fails to take a part. It is assumed that m_M is never blocked.

Note that continuous serial production lines with three or ten identical machines, whose reliability model satisfies Exponential, Weibull, Gamma and Log-normal, respectively, are researched in this dissertation. Therefore, flow model is used in the research. As a result, the state of the buffer is a real number between 0 and N . Since simulation method is used in this work, machines' cycle times are set to 1 minute. In other words, each

machine's capacity c is 1 part/min. Time axis is divided into several time infinitesimal slots δt . δt is set to 0.05 minute.

2.3.3 System Parameters

- T_{up} , T_{down} : Machines' average up- and downtime.
- CV_{up} , CV_{down} : Coefficient of variation of up- and downtime.
- e : machine efficiency, which is the expected value of the number of parts produced during a cycle time. In this case, e is demonstrated by equation below:

$$e = \frac{T_{up}}{T_{up} + T_{down}}$$

- N : buffer capacity, the maximum number of parts that the buffer can store. It is assumed throughout that $N < \infty$, implying that buffers are finite. The number of parts contained in a buffer at a given time is referred to as its occupancy. Since in a production system, the occupancy of a buffer at a given time (slot or moment) depends on its occupancy at the previous time (slot or moment), buffers are dynamical systems with the occupancy being their states. If the machines are modeled as discrete event systems, the state of the buffer is an integer between 0 and N . In flow models, states are real numbers between 0 and N .
- K : Ratio of N and T_{down} .

$$K = \frac{N}{T_{down}}$$

The larger K is, the more protection to machines from starvation and blockage produced by buffers.

2.4 Performance Measures

The following performance measures are considered:

- Production rate (PR): average number of parts produced by the last machine of a production system per cycle time in the transient state of system operation.
- Total work-in-process (WIP): average number of parts contained in all the buffers of a production system in the transient state of its operation.
- Total production (TP) : average total number of parts produced by the last machine in the time duration T.

$$TP = \sum_{t=0}^T PR(t)$$

TP quantifies how much production will be gained in time T.

- Production loss (PL): the percentage of reduced production from the beginning to time T compared with total production in steady state.

$$PL = \frac{\sum_{t=0}^T [PR_{ss} - PR(t)]}{PR_{ss} \times T}$$

Where, PR_{ss} is the production rate in the steady state of system operation. PL quantifies the percentage of production loss due to transient process.

- Settling time of production rate(t_{sPR}): the expected time necessary for PR to reach and remain within $\pm 5\%$ of PR_{ss} .

t_{sPR} measures how fast the system enters steady state regarding production rate.

- Settling time of work-in-process(t_{sWIP}): the expected time necessary for WIP to reach and remain within $\pm 5\%$ of WIP_{ss} ;

t_{sWIP} measures how fast the system enters steady state regarding work-in-process.

In this work, we analyze total production and production loss during a shift of duration T cycles. We assume that $T = 500$ minutes, which is typical for automotive assembly plants where the cycle time is around 1 minute and the shift is 8 hours. T is set to 500 minutes in all the systems investigated in this thesis to simulate a typical automotive assembly plants.

2.5 Problem Statement

Previous research of transient performance are limited to some non-Exponential production lines, and the analysis are mainly focused on the effects of coefficient variance (CV) to the production rates (PR). It is shown that the production rate is monotonically decreasing function of coefficient variation [23]. It is important to extend these transient analysis to other machine models, and furthermore to investigate the general effects of system parameters on other performance measures.

1. Extend these transient analysis to other machine models. This dissertation is devoted to investigate the effects of system parameters on performance measures for transient serial production line with other machine reliability models. The reliability models investigated in this dissertation include exponential and non-exponential (Weibull, Gamma, Log-normal).
2. Investigate the general effects of system parameters on other performance measures. In a real production line system, machine reliability models are much more difficult to identify. Therefore, it is of significant practical importance to investigate the general effects of system parameters on performance measures for transient serial production line. This dissertation investigates the effects of system parameters (including e , K , T_{down} and CV) on other performance measures,

including settling time of production rate (t_{sPR}), settling time of work-in-process (t_{sWIP}), total production (TP), production loss (PL). The relationship between the performance measures and system parameters reveals the fundamental principles that characterize the behavior of such systems and can be used as a guideline for product lines' management and improvement.

Chapter 3 Exponential Systems

This chapter investigates transients of exponential serial line. First, transients of individual exponential machine and bufferless exponential serial line are analyzed. Second, production rate of exponential line is approximated by using geometric line. Third, the effects of system parameters, e , K and T_{down} , of exponential serial production line on the transient performance measure which are settling time, total production and production loss are analyzed. The exponential serial production line which is operated under the assumptions (a-f) in section 2.3.2. The parameters of machines reliability model are determined by the system parameters table in each section.

3.1 Transients of Exponential Lines

3.1.1 Transients of Individual Exponential Machine

Let $x_i(t)$, $i \in \{0 = \text{down}, 1 = \text{up}\}$ be the probability that the machine is in state i at time t .

Then, the evolution of $\mathbf{x}(t) = [x_0(t) \ x_1(t)]^T$ can be described a Markov chain:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t),$$

$$x_0(t) + x_1(t) = 1,$$

$$\mathbf{A} = \begin{bmatrix} -\mu & \lambda \\ \mu & -\lambda \end{bmatrix}.$$

The eigenvalues of matrix \mathbf{A} are 0 and $(\lambda + \mu)$ and the corresponding eigenvectors are:

$$\begin{bmatrix} \lambda \\ \mu \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Let

$$Q = \begin{bmatrix} \lambda & 1 \\ \mu & -1 \end{bmatrix}.$$

Then,

$$Q^{-1} = \frac{1}{\lambda + \mu} \begin{bmatrix} 1 & 1 \\ \mu & -\lambda \end{bmatrix},$$

the evolution of the system state can be calculated as:

$$\begin{aligned} \begin{bmatrix} x_0(t) \\ x_1(t) \end{bmatrix} &= Q \begin{bmatrix} e^{0t} & \\ & e^{-(\lambda+\mu)t} \end{bmatrix} Q^{-1} \begin{bmatrix} x_0(0) \\ x_1(0) \end{bmatrix} \\ &= \begin{bmatrix} \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} e^{-(\lambda+\mu)t} & \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t} \\ \frac{\mu}{\lambda + \mu} - \frac{\mu}{\lambda + \mu} e^{-(\lambda+\mu)t} & \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t} \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_1(0) \end{bmatrix} \\ &= \begin{bmatrix} \frac{\lambda}{\lambda + \mu} + \left(\frac{\mu}{\lambda + \mu} x_0(0) - \frac{\lambda}{\lambda + \mu} x_1(0) \right) e^{-(\lambda+\mu)t} \\ \frac{\mu}{\lambda + \mu} - \left(\frac{\mu}{\lambda + \mu} x_0(0) - \frac{\lambda}{\lambda + \mu} x_1(0) \right) e^{-(\lambda+\mu)t} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\lambda}{\lambda + \mu} + \left(x_0(0) - \frac{\lambda}{\lambda + \mu} \right) e^{-(\lambda+\mu)t} \\ \frac{\mu}{\lambda + \mu} + \left(x_1(0) - \frac{\mu}{\lambda + \mu} \right) e^{-(\lambda+\mu)t} \end{bmatrix}. \end{aligned}$$

Since λ and μ are both positive, $e^{-(\lambda+\mu)t}$ tends to 0 as t approaches infinity, and, therefore,

$$\begin{bmatrix} x_0(t) \\ x_1(t) \end{bmatrix} = \begin{bmatrix} x_0(\infty) + (x_0(0) - x_0(\infty))e^{-(\lambda+\mu)t} \\ x_1(\infty) + (x_1(0) - x_1(\infty))e^{-(\lambda+\mu)t} \end{bmatrix},$$

where

$$x_0(\infty) = \frac{\lambda}{\lambda + \mu} = \frac{T_{down}}{T_{up} + T_{down}}, x_1(\infty) = \frac{\mu}{\lambda + \mu} = \frac{T_{up}}{T_{up} + T_{down}}.$$

Clearly, the transient of an individual exponential machine is characterized by

- 1) The distance between the initial condition and the steady state; and
- 2) System mode $e^{-(\lambda+\mu)t}$.

In addition, if the machine is initially in the steady state, i.e.,

$$x_0(0) = \frac{\lambda}{\lambda + \mu}, x_1(0) = \frac{\mu}{\lambda + \mu},$$

then it remains in the steady state for all t :

$$x_0(t) = \frac{\lambda}{\lambda + \mu}, x_1(t) = \frac{\mu}{\lambda + \mu}.$$

An illustration is given in Figure 4 for an exponential machine with $T_{\text{up}} = 80$, $T_{\text{down}} = 20$ (i.e., $\lambda = 0.0125$ and $\mu = 0.05$).

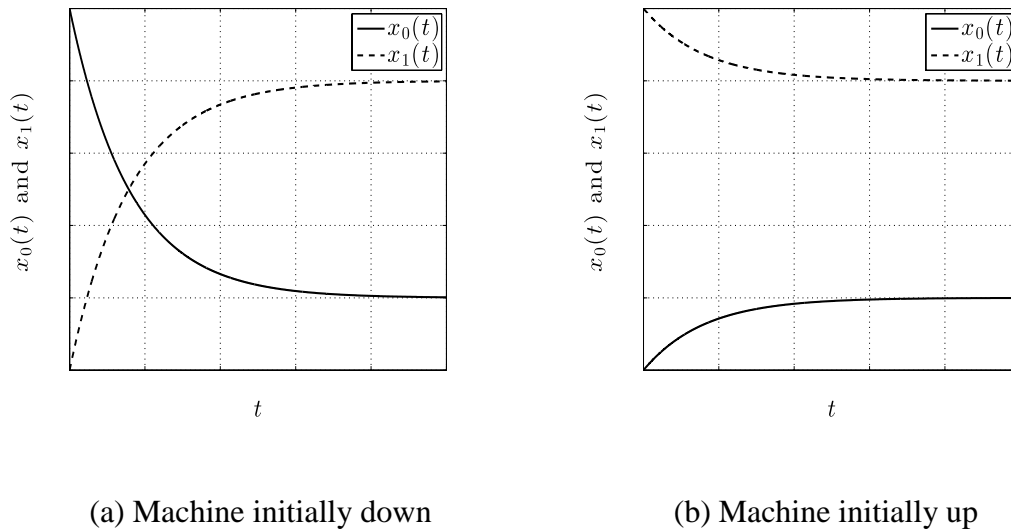


Figure 4 Transients of machine state of an individual exponential machine ($T_{\text{up}} = 80$, $T_{\text{down}} = 20$)

3.1.2 Transients of Bufferless Exponential Serial Line

Consider an exponential serial line having all buffers with zero capacity. Clearly, the production rate of this line at time t is given by:

$$PR(t) = \Pr[\text{all machines are up at time } t].$$

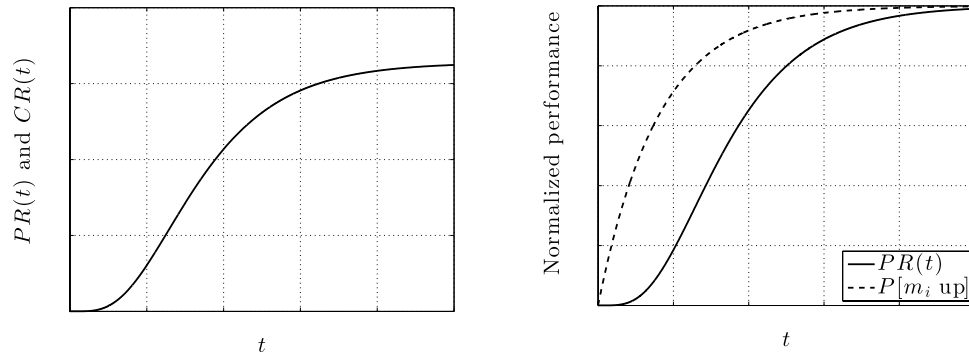
Let $x_{i,j}(t)$, $i \in \{0 = \text{down}, 1 = \text{up}\}$, $j \in \{1, 2, \dots, M\}$ be the probability that the machine m_j is in state i at time t . Then,

$$PR(t) = \prod_{i=1}^M x_{1,i}(t) = \prod_{i=1}^M \left[e_i + (x_{1,i}(0) - e_i) e^{-(\lambda_i + \mu_i)t} \right]$$

The steady state production rate is $PR(\infty) = \prod_{i=1}^M e_i$ and the transient of $PR(t)$ contains a number of modes defined by all possible combinations of the machines in the system.

An illustration is given in Figure 5 for a bufferless five-machine exponential line with identical machines ($T_{up} = 80$, $T_{down} = 20$). The machines are assumed to be down initially. In Figure 5 (a), we plot the transients of the system throughput rate, $PR(t)$ (which is equal to $CR(t)$ for bufferless lines). To compare the transients of system performance with individual machine, we plot the transients of the probability that a machine is up in Figure 5(b) along with $PR(t)$. In addition, we normalize both terms by their corresponding steady state values.

As one can see from the figure, due to the interaction of the machines in the system, the transients of the system performance is slower than those of individual machines. Similar observation can be made when the initial condition is changed (see Figure 6).

(a) $PR(t)$ and $CR(t)$

(b) Normalized performance

Figure 5 Transients of bufferless five-machine exponential line with identical machines ($T_{up} = 80$, $T_{down} = 20$, machines initially down)

It should be noted that, when the machines are initially up, the system throughput, in fact, benefits from the transients as it never enters below its steady state level. For general initial conditions, the system throughput should be slower than the machine with the slowest transients.

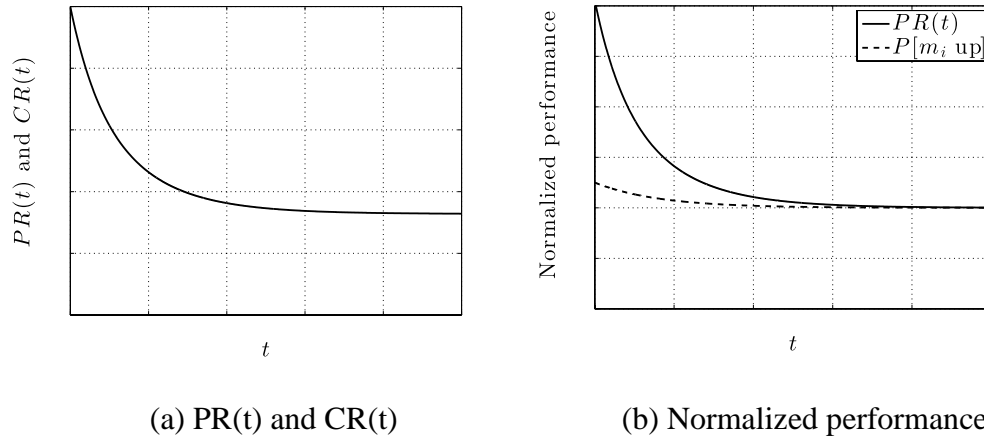


Figure 6 Transients of bufferless five-machine exponential line with identical machines ($T_{\text{up}} = 80$, $T_{\text{down}} = 20$, machines initially up)

3.1.3 Exponential vs. Geometric

It is well known that geometric distribution is the discrete counterpart to the exponential distribution. The geometric reliability model is defined as follows: Let $s(n) \in \{0 = \text{down}, 1 = \text{up}\}$ denote the state of a machine during cycle time n . Then, the transition probabilities are given by:

$$\begin{aligned} \Pr[s(n+1) = 0 \mid s(n) = 1] &= P, \Pr[s(n+1) = 1 \mid s(n) = 1] = 1 - P, \\ \Pr[s(n+1) = 1 \mid s(n) = 0] &= R, \Pr[s(n+1) = 0 \mid s(n) = 0] = 1 - R, \end{aligned}$$

where P and R are referred to as the breakdown and repair probabilities, respectively. Clearly, the up- and downtime of a machine with the reliability model above are geometric random variables with mean T_{up} and T_{down} given by:

$$T_{up} = \frac{1}{P}, T_{down} = \frac{1}{R}.$$

The main difference between the geometric reliability model and the exponential model is that, a geometric machine operates under a slotted time axis (i.e., in discrete time) with the slot duration equal to its cycle time and all events (machine breakdown/repair, transportation of parts, etc.) take place either at the beginning or at the end of a time slot, while an exponential machine operates in continuous time and an event can take place at any time instant. In addition, the flow model considered for the exponential case allows infinitesimal parts to travel within the system, while the geometric case only moves whole parts around. Despite these differences, the two models are very similar.

It can be shown that production lines with geometric machines are characterized by discrete-time discrete-space Markov chains. Analytical studies have been carried out to investigate the transient behavior of such systems (see [1]) and an analytical procedure based on recursive aggregation has been developed to approximate the transient performance of a geometric line with high accuracy. On the other hand, production lines with exponential machines are characterized by continuous-time mixed-space Markov process, which is much more difficult to study analytically. Since both systems share a number of similarities, it becomes interesting to see if it is possible to study the transients of serial lines with exponential machines by transforming the system into one with geometric machines. This transformation is, indeed, very straightforward: Consider a serial line with exponential machines defined by section 2.3.2, then its corresponding serial line with geometric machines are given by:

$$P_i = \lambda_i, R_i = \mu_i, N_i^{geo} = N_i,$$

where P_i and R_i are the breakdown and repair rates of machine m_i in the geometric line and N_i^{geo} is the capacity of buffer b_i in the geometric line. Let $PR^{geo}(n)$ denote the production rate of the geometric line during time slot n . Then, we may approximate the production rate of the original exponential line using:

$$PR^{app}(t) = PR^{geo}(\lfloor t \rfloor) + \frac{PR^{geo}(\lceil t \rceil) - PR^{geo}(\lfloor t \rfloor)}{\lceil t \rceil - \lfloor t \rfloor} \cdot (t - \lfloor t \rfloor).$$

In other words, at integer time instants (i.e., $t = 1, 2, 3, \dots$), the production rate of the exponential line is approximated using the production rate of the geometric line during the same time slot. For non-integer time instants, the production rate of the exponential line is approximated using linear interpolation of the production rates of the geometric line during the nearest two time slots. An illustration of this approximation is provided in Figure 7. As one can see, the geometric line-based formula has very good accuracy in approximating the transient production rate of an exponential line.

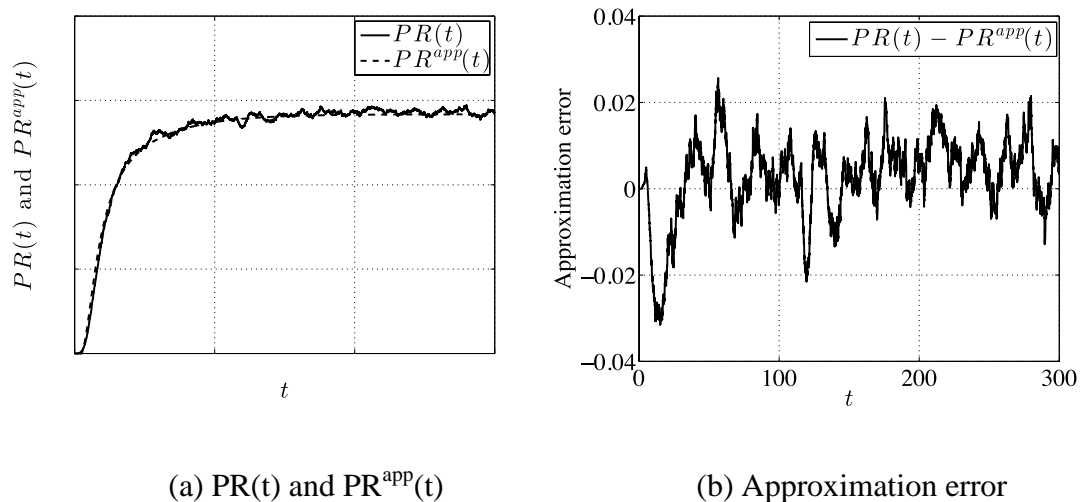


Figure 7 . Approximation of $PR(t)$ of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty)

Similarly, approximation formulas for other transient performance measures are proposed as follows:

$$CR^{app}(t) = CR^{geo}(\lfloor t \rfloor) + \frac{CR^{geo}(\lceil t \rceil) - CR^{geo}(\lfloor t \rfloor)}{\lceil t \rceil - \lfloor t \rfloor} \cdot (t - \lfloor t \rfloor),$$

$$WIP_i^{app^{app}}(t) = WIP_i^{app^{geo}}(t)(\lfloor t \rfloor) + \frac{CR^{geo}(\lceil t \rceil) - CR^{geo}(\lfloor t \rfloor)}{\lceil t \rceil - \lfloor t \rfloor} \cdot (t - \lfloor t \rfloor), i = 1, \dots, M - 1,$$

$$ST_i^{app^{app}}(t) = ST_i^{geo}(t)(\lfloor t \rfloor) + \frac{ST_i^{geo}(\lceil t \rceil) - ST_i^{geo}(\lfloor t \rfloor)}{\lceil t \rceil - \lfloor t \rfloor} \cdot (t - \lfloor t \rfloor), i = 2, \dots, M,$$

$$BL_i^{geo}(t) = BL_i^{geo}(t)(\lfloor t \rfloor) + \frac{BL_i^{geo}(\lceil t \rceil) - BL_i^{geo}(\lfloor t \rfloor)}{\lceil t \rceil - \lfloor t \rfloor} \cdot (t - \lfloor t \rfloor), i = 1, \dots, M - 1,$$

As an illustration, we study the approximation of these transient performance measures for the same five-machine line considered above. The results are summarized in Figure 8- Figure 11. As one can see the accuracy of consumption rate approximation is similar to that of the production rate estimation. The accuracy of work-in-process approximation is lower but still within 5% of the buffer capacity in most cases. The accuracy of starvation and blockage approximation is similar, typically within ± 0.02 .

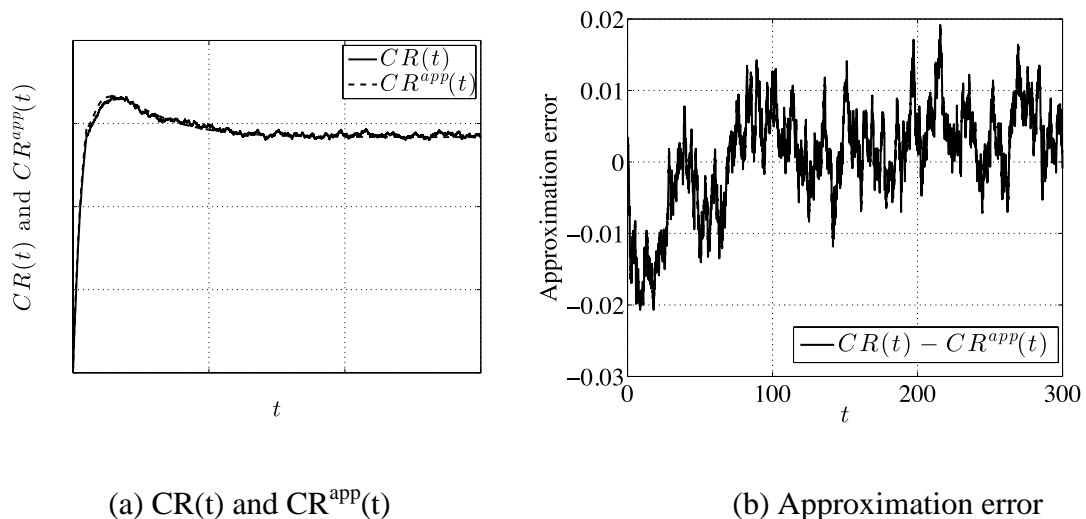


Figure 8 Approximation of $CR(t)$ of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty)

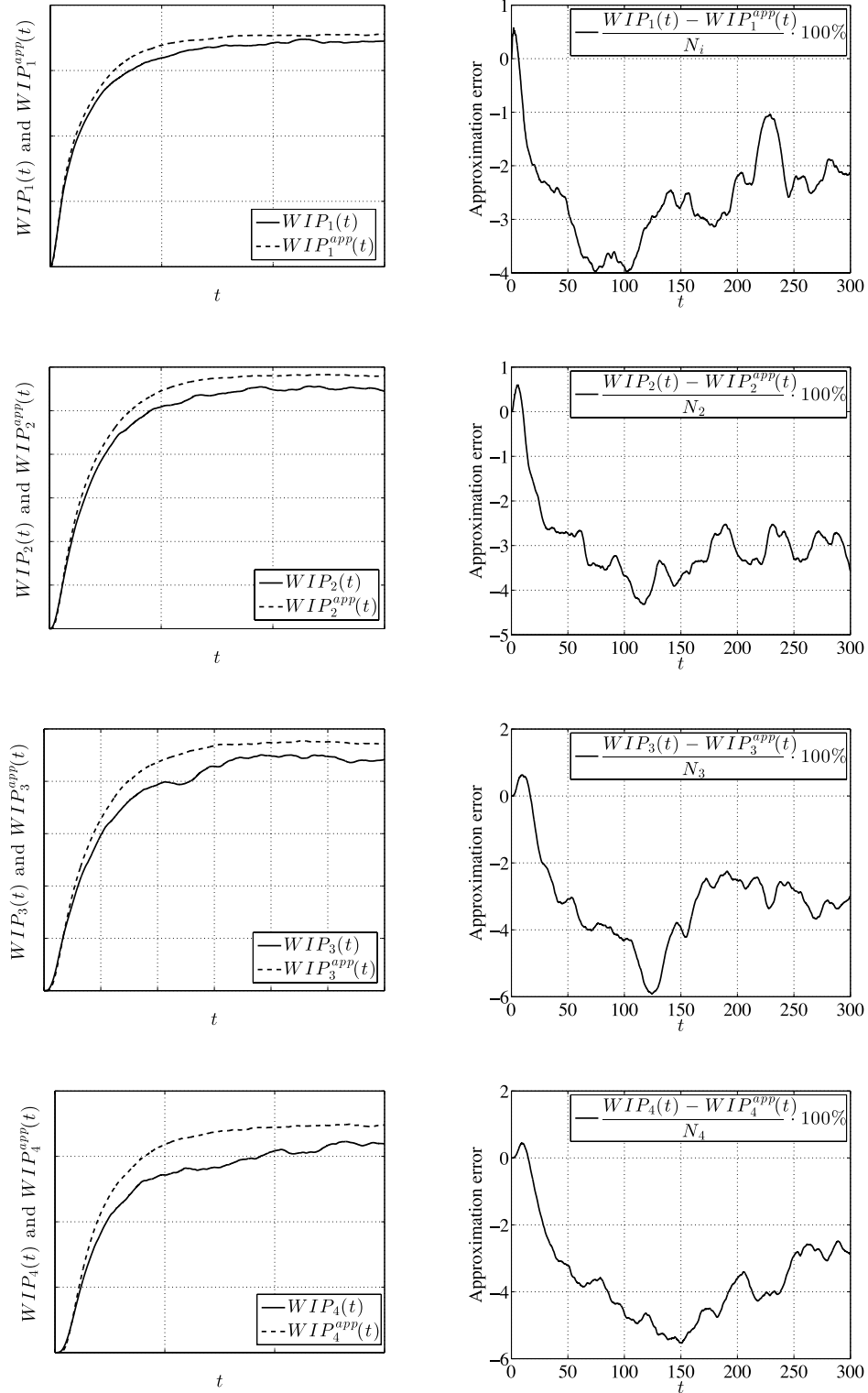


Figure 9 Approximation of $WIP_i(t)$ of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty)

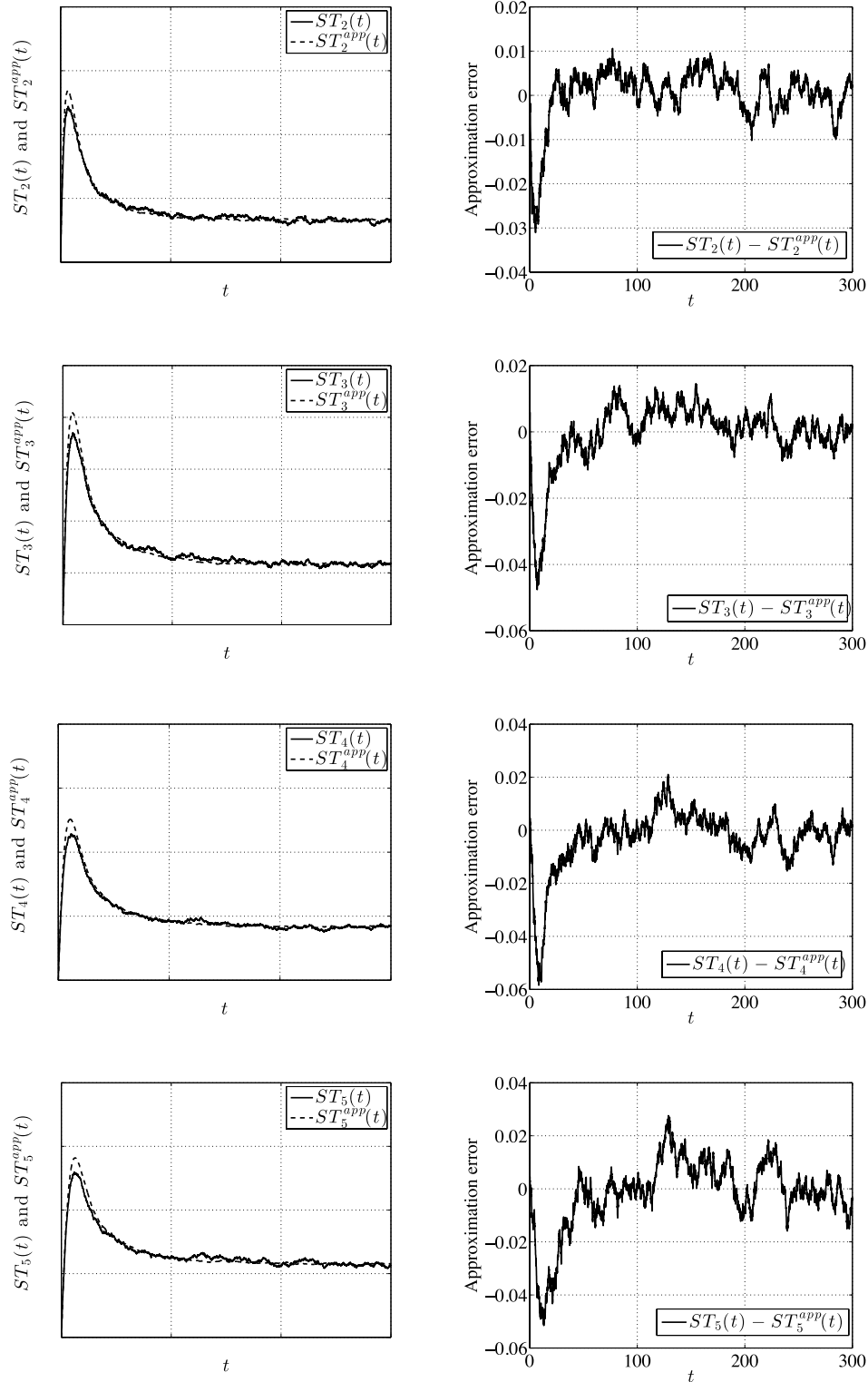


Figure 10 Approximation of $ST_i(t)$ of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty)

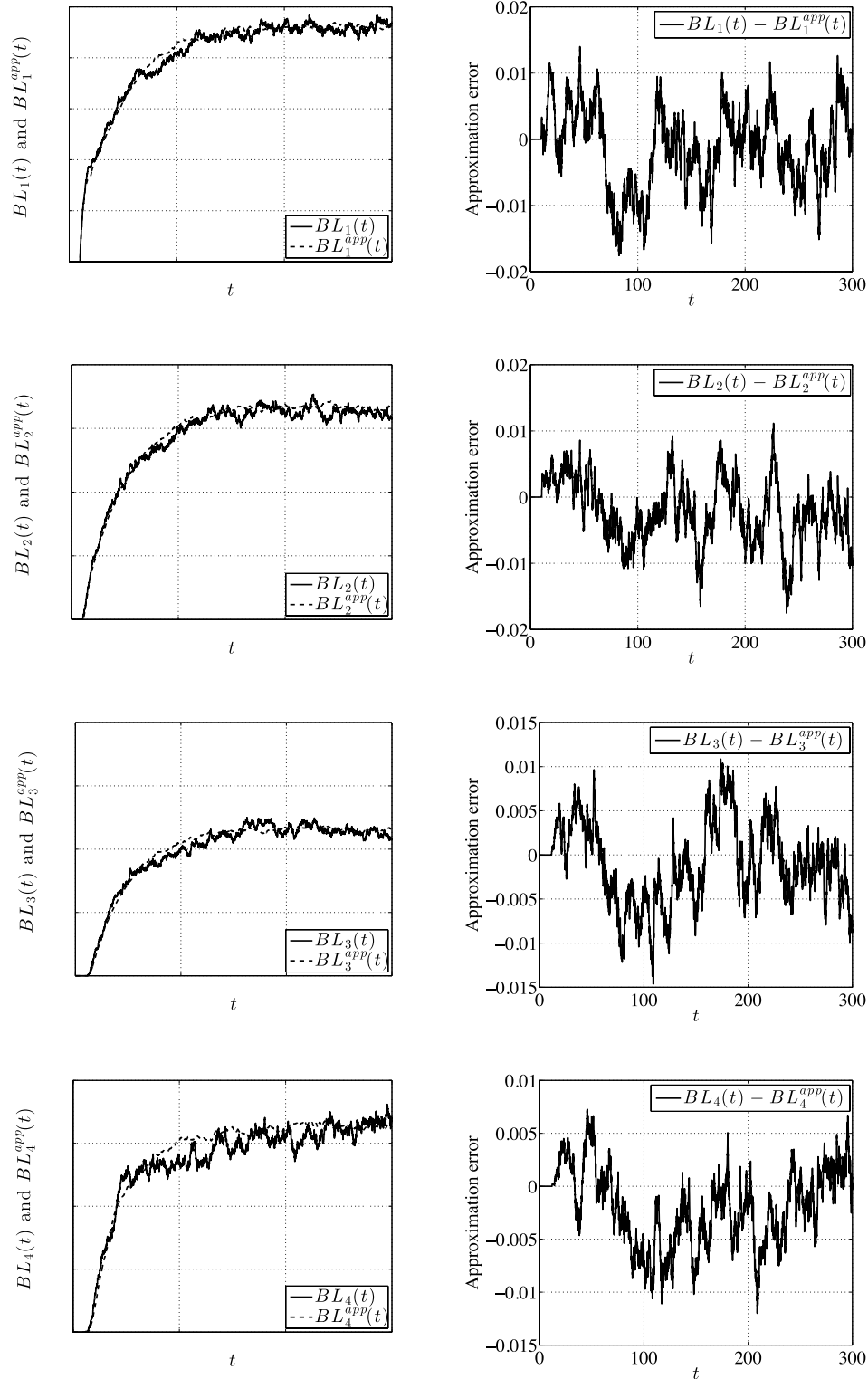


Figure 11 Approximation of $BL_i(t)$ of five-machine exponential line with identical machines and identical buffers ($T_{up} = 40$, $T_{down} = 10$, $N = 10$, all machines initially down, all buffers initially empty)

3.2 Transient Performance Analysis

To analyze the system's transient performance, settling times, t_{sPR} and t_{sWIP} , one has to know the behavior of PR and WIP as a function of t . Therefore, in this section, we first analyze the trajectories of $PR(t)$ and $WIP(t)$ and then utilize them to evaluate the settling time.

The system parameters in the simulation are shown in Table 2.

Table 2 Exponential System Transient Performance Analysis Values

Parameters	e	M	K	T_{down}
Range	[0.7,0.9]	[3,10]	[1,5]	[3,9]
Default Value	0.9	[3,10]	3	5

Based on system characteristics and simulation results in the following subsections, we have the following conjectures:

- 1) PR approaches steady state value faster than WIP in the same system;
- 2) PR approaches steady state value faster when e increases. In contrast, WIP needs longer time to reach the steady state as e increases.
- 3) PR and WIP approaches steady state slower when there are more machines in the system;
- 4) The difference of the transient times between PR and WIP also becomes more significant as the machine number increases.
- 5) PR and WIP approaches steady state value slower when K increases;
- 6) PR and WIP approaches steady state value slower when T_{down} increases;

3.2.1 Effects of e

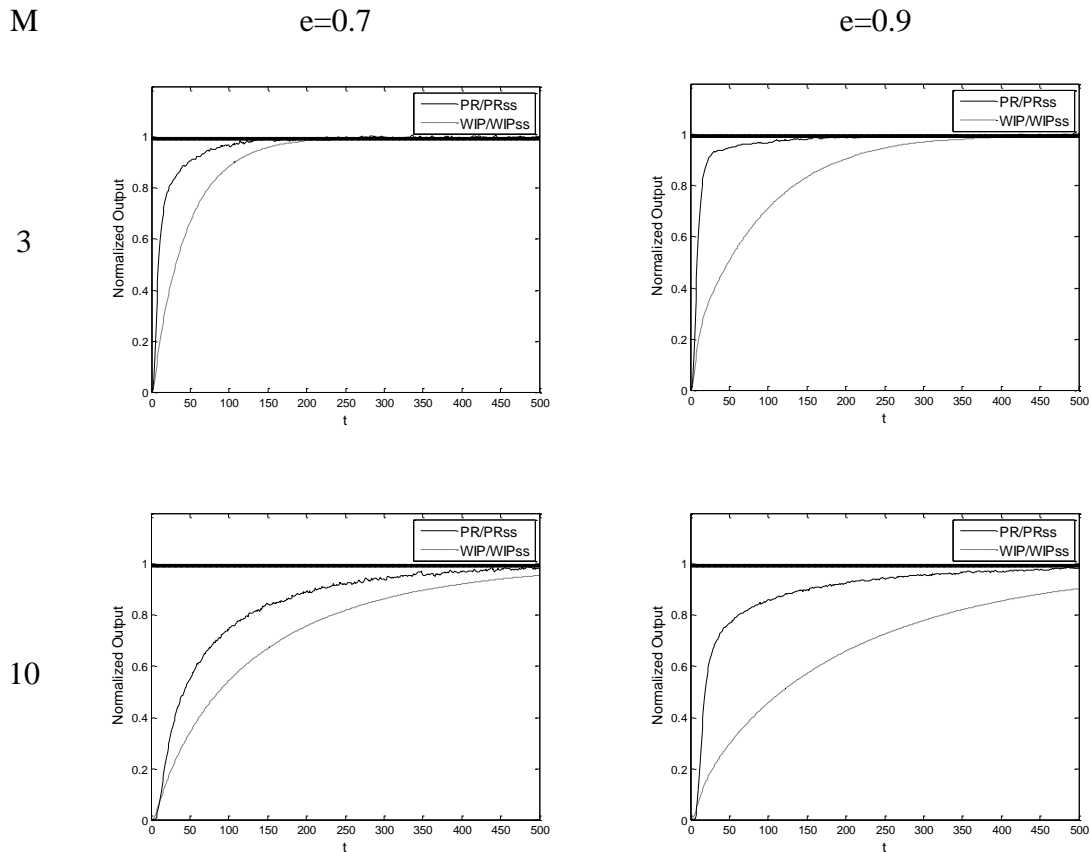
In order to analyze the effects of efficiency (e) on the transient performance. PR and WIP are simulated with the following system parameters:

Table 3 Exponential System Parameters (Effects of e on transient performance)

Parameters	e	M	K	T_{down}
Value	[0.7, 0.9]	[3, 10]	3	5

Analysis of PR and WIP: To compare the transients of PR and WIP, Figure 12 shows the graphs of PR/PR_{ss} and WIP/WIP_{ss} for various e and M and the following conjectures are observed:

- 1) The production rate (PR) approaches steady state value faster than the work-in-process (WIP) in the same system. As e becomes larger, the difference becomes more pronounced. For examples, at $M=3$, $e=0.7$, the setting time of PR and WIP are around 80 and 150, respectively. At $M=3$, $e=0.9$, the setting time of PR and WIP are around 50 and 250, respectively.

Figure 12 Effects of e on Transient Performance of Exponential Lines

- 2) PR approaches steady state value faster when e increases. In contrast, WIP needs longer time to reach the steady state as e increases.
- 3) PR and WIP both approach steady state slower when there are more machines in the system. For instance, when there are three machines in the systems, machines' efficiency is 0.7, the settling time of PR is close to 200. In contrast, the settling time of PR is more than 500 when there are ten machines in the system.
- 4) The difference of the transient times between PR and WIP also becomes more significant as the machine number increases.

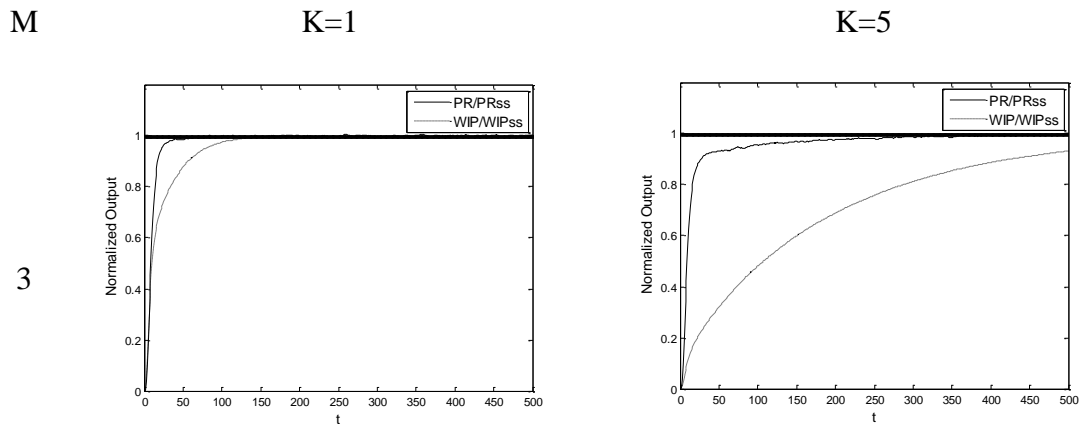
3.2.2 Effects of K

In order to analyze the effects of K on the transient performance. PR and WIP are simulated with the following system parameters:

Table 4 Exponential System Parameters (Effects of K on transient performance)

Parameters	e	M	K	T_{down}
Value	0.9	[3, 10]	[1,5]	5

Analysis of PR and WIP: To compare the transients of PR and WIP, Figure 13 shows the



10

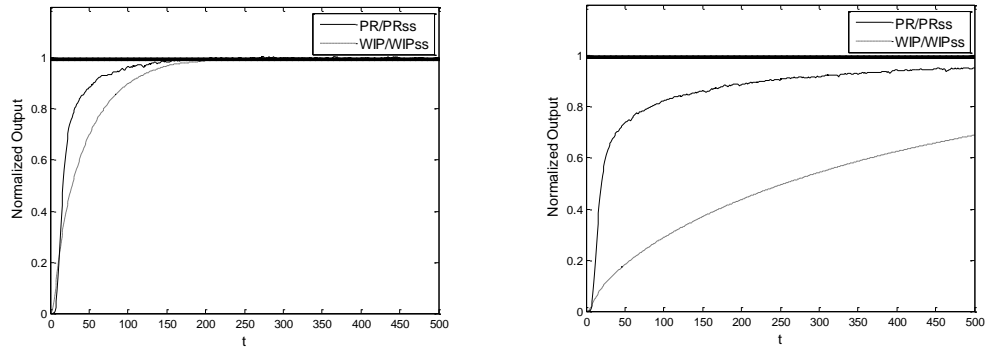


Figure 13 Effects of K on Transient Performance of Exponential Lines

graphs of PR/PR_{ss} and WIP/WIP_{ss} for various K and M, and the following conjectures is observed:

- 1) PR and WIP both approach steady state value slower when K increases.

3.2.3 Effects of T_{down}

In order to analyze the effects of T_{down} on the transient performance. PR and WIP are simulated with the following system parameters:

Table 5 Exponential System Parameters (Effects of T_{down} on transient performance)

Parameters	e	M	K	T _{down}
Value	0.9	[3, 10]	3	[3, 9]

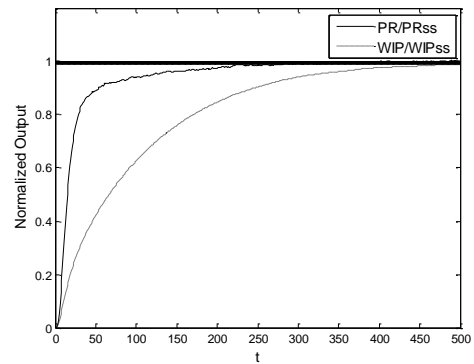
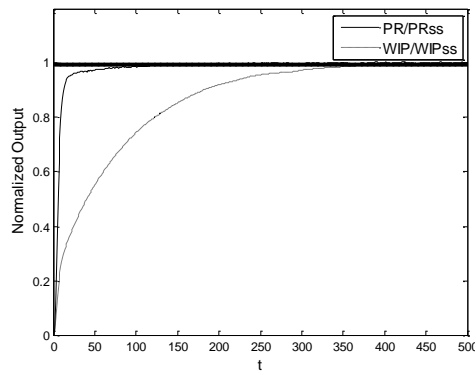
Analysis of PR and WIP: To compare the transients of PR and WIP, Figure 14 shows the

M

T_{down}=3

T_{down}=9

3



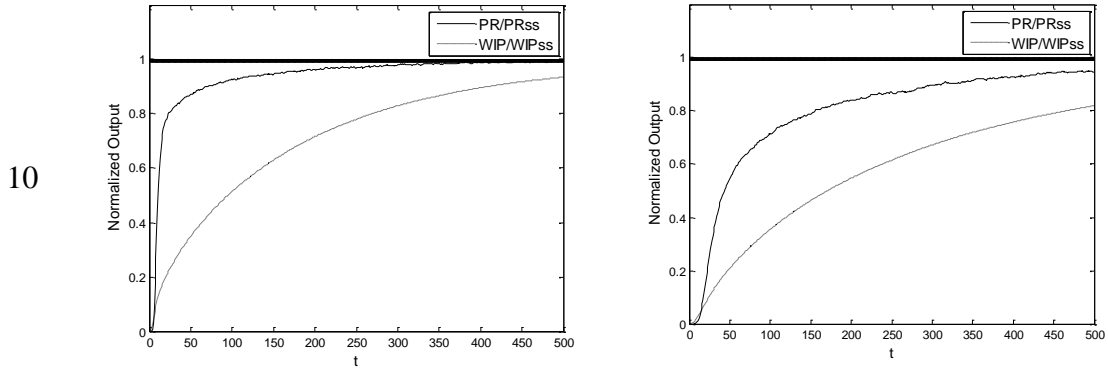


Figure 14 Effects of T_{down} on Transient Performance of Exponential Lines

graphs of PR/PR_{ss} and WIP/WIP_{ss} for various K and M , the following conjecture is observed:

- 1) PR and WIP both approach steady state value slower when T_{down} increases.

3.3 Settling Time

Settling time of PR or WIP measures how fast the system enters steady state in terms of PR or

WIP . The shorter settling time, the faster the system approaches steady state. To analyze the effects of e , K , and T_{down} on the settling time of PR (t_{sPR}) and WIP (t_{sWIP}), simulations are implemented and the system parameters are shown in each subsection.

3.3.1 Effects of e

In order to analyze the effects of efficiency (e) on the settling time t_{sPR} and t_{sWIP} , simulations are implemented with the following system parameters:

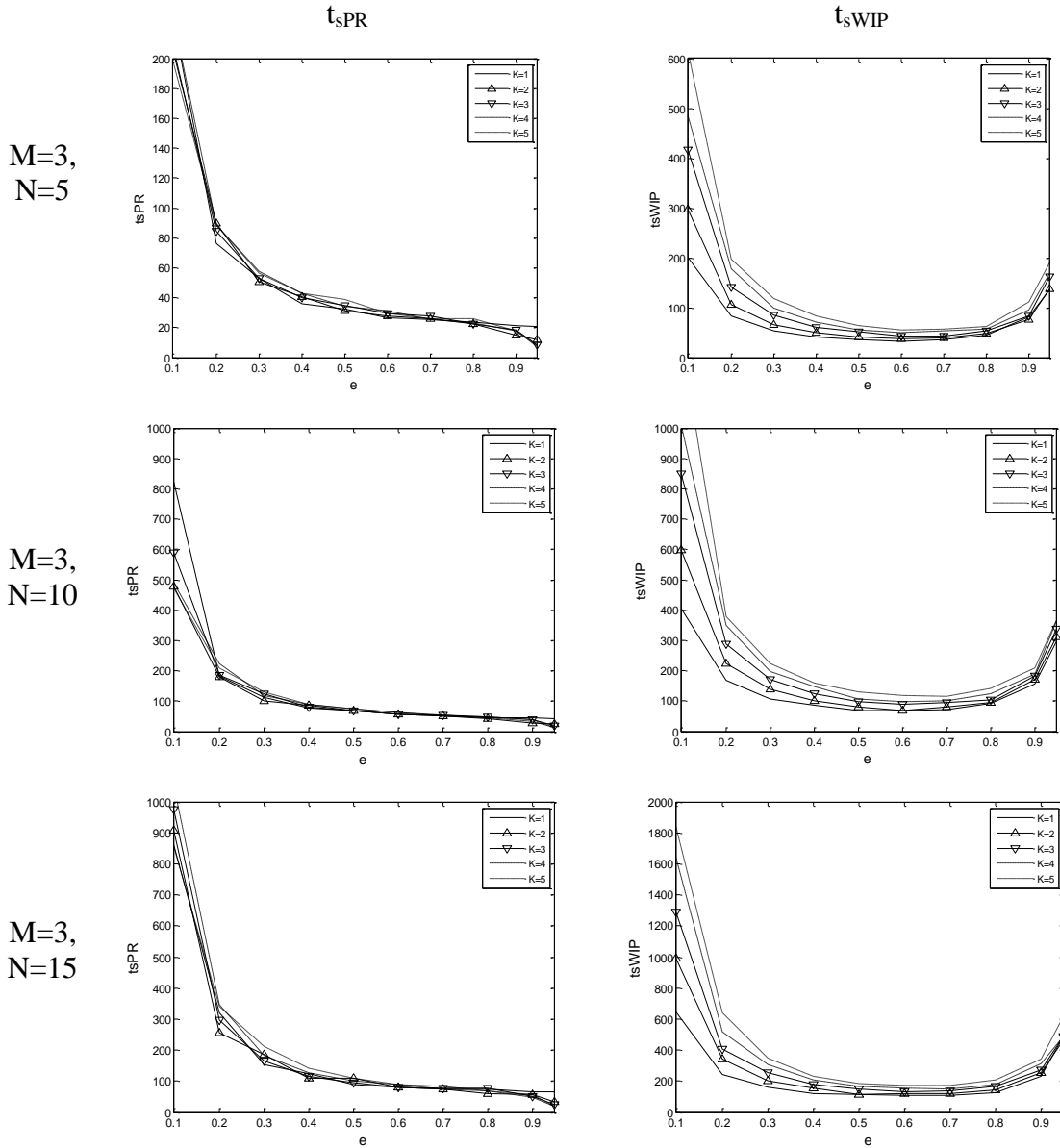
Table 6 Exponential System Parameters (Effects of e on t_{sPR} and t_{sWIP})

Parameters	e	M	K	N
Value	[0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,0.95]	[3, 10]	3	[5, 10, 15]

Analysis of t_{sPR} and t_{sWIP} :

Figure 15 shows the graphs of t_{sPR} and t_{sWIP} vs. e for various N and M .

- 1) PR has shorter settling time than WIP in the same system ($t_{sPR} < t_{sWIP}$).
- 2) t_{sPR} becomes shorter as e increases and the slope becomes larger as N increases. In contrast, t_{sWIP} is a convex function of e .
- 3) t_{sPR} and t_{sWIP} both increase if there are more machines in the system.



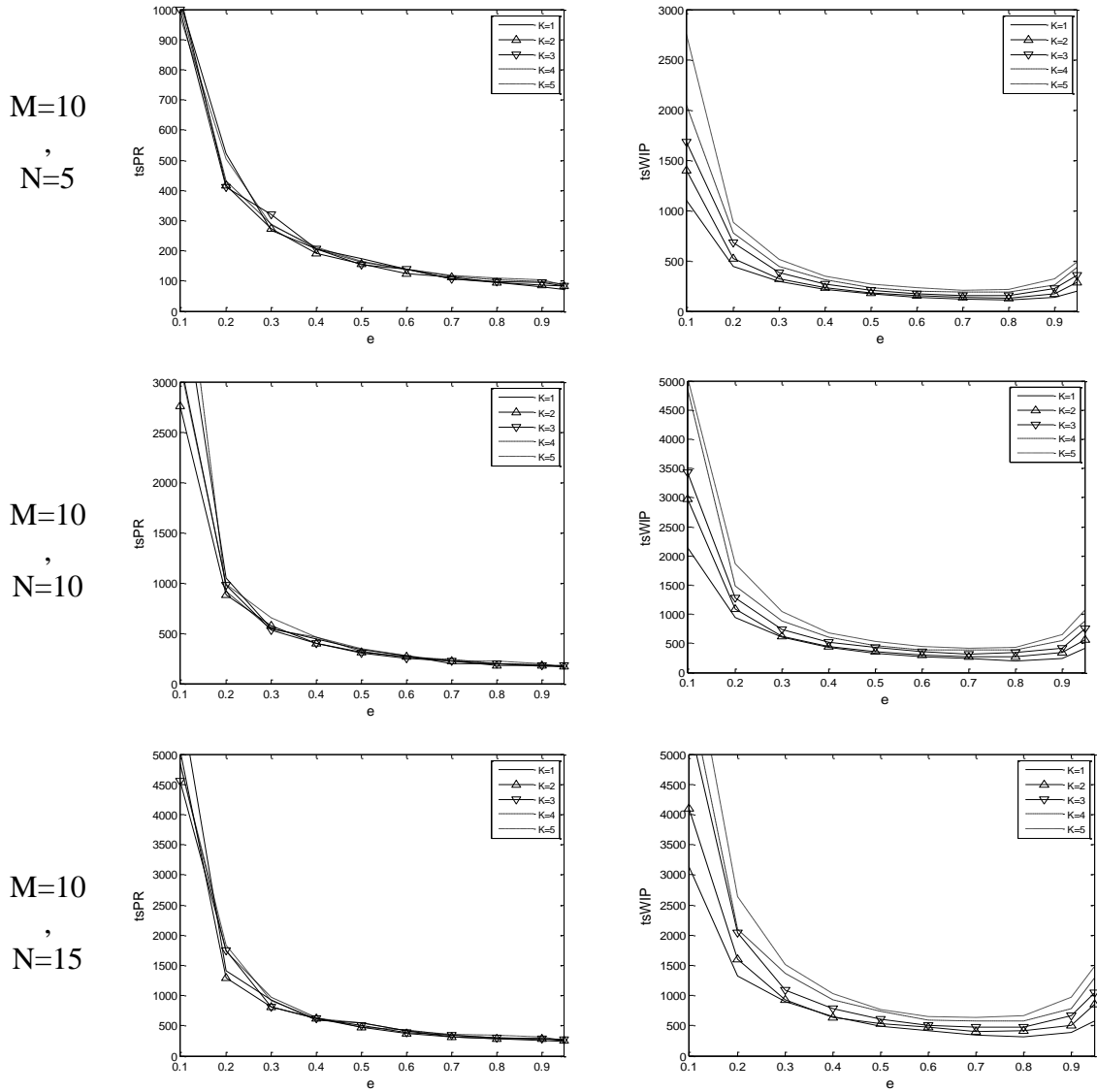


Figure 15 Effects of e on t_{sPR} and t_{sWIP} of Exponential Lines

3.3.2 Effects of K

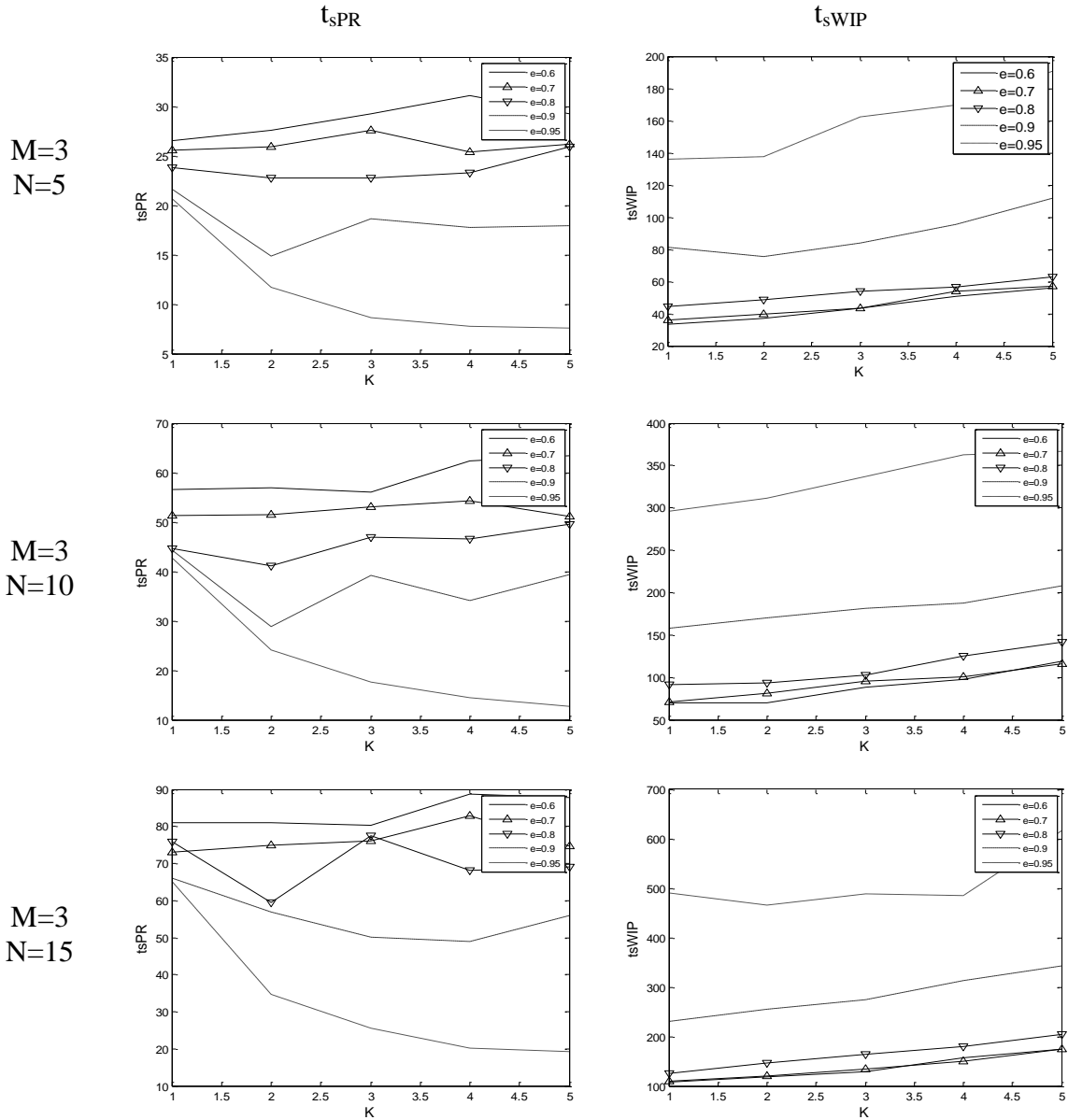
In order to analyze the effects of K on the settling time t_{sPR} and t_{sWIP} , simulations are implemented with the following system parameters:

Table 7 Exponential System Parameters (Effects of K on t_{sPR} and t_{sWIP})

Parameters	e	M	K	N
Value	[0.6, 0.7, 0.8, 0.9, 0.95]	[3, 10]	[1, 2, 3, 4, 5]	[5, 10, 15]

Analysis of t_{sPR} and t_{sWIP} : Figure 10 shows the graphs of t_{sPR} and t_{sWIP} vs. K for various N and M .

- 1) K does not have significant impact on t_{sPR} .
- 2) K has positive impact on t_{sWIP} , larger K leads to a longer t_{sWIP} . For larger M , K has more significant linear impact on t_{sWIP} .



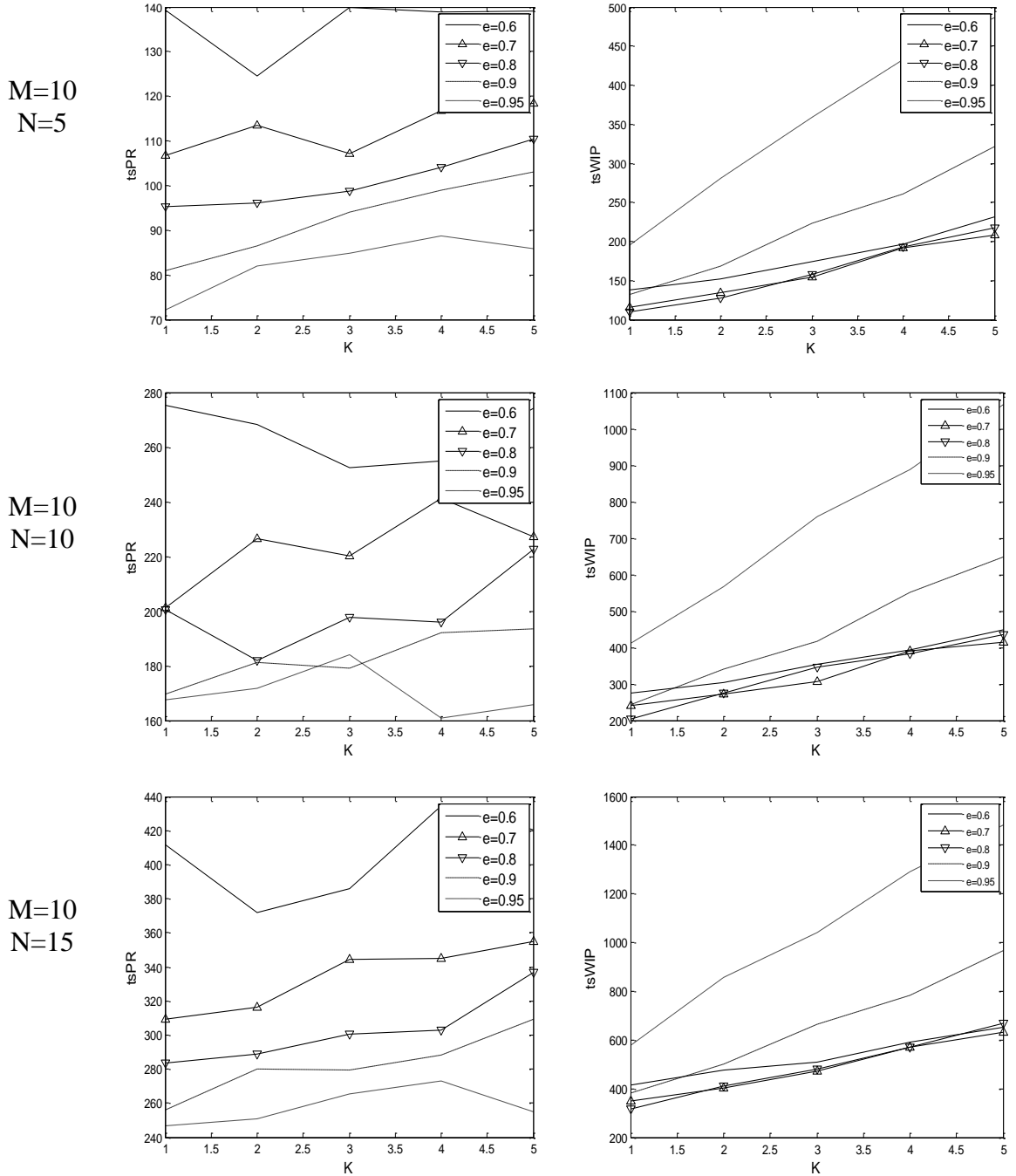


Figure 16 Effects of K on t_{sPR} and t_{sWIP} of Exponential Lines

3.3.3 Effects of T_{down}

In order to analyze the effects of T_{down} on the settling time t_{sPR} and t_{sWIP} , simulations are implemented with the following system parameters:

Table 8 Exponential System Parameters (Effects of T_{down} on t_{sPR} and t_{sWIP})

Parameters	e	M	K	T_{down}
Value	[0.6, 0.7, 0.8, 0.9, 0.95]	3	[1, 3, 5]	[5, 7, 9, 11, 13 15]

Analysis of t_{sPR} and t_{sWIP} : Figure 17 shows the graphs of t_{sPR} and t_{sWIP} vs. T_{down} for various K and M.

- 1) T_{down} has positive impact on both t_{sPR} and t_{sWIP} , larger K leads to a longer t_{sPR} and t_{sWIP} . The relationship is close to linear.

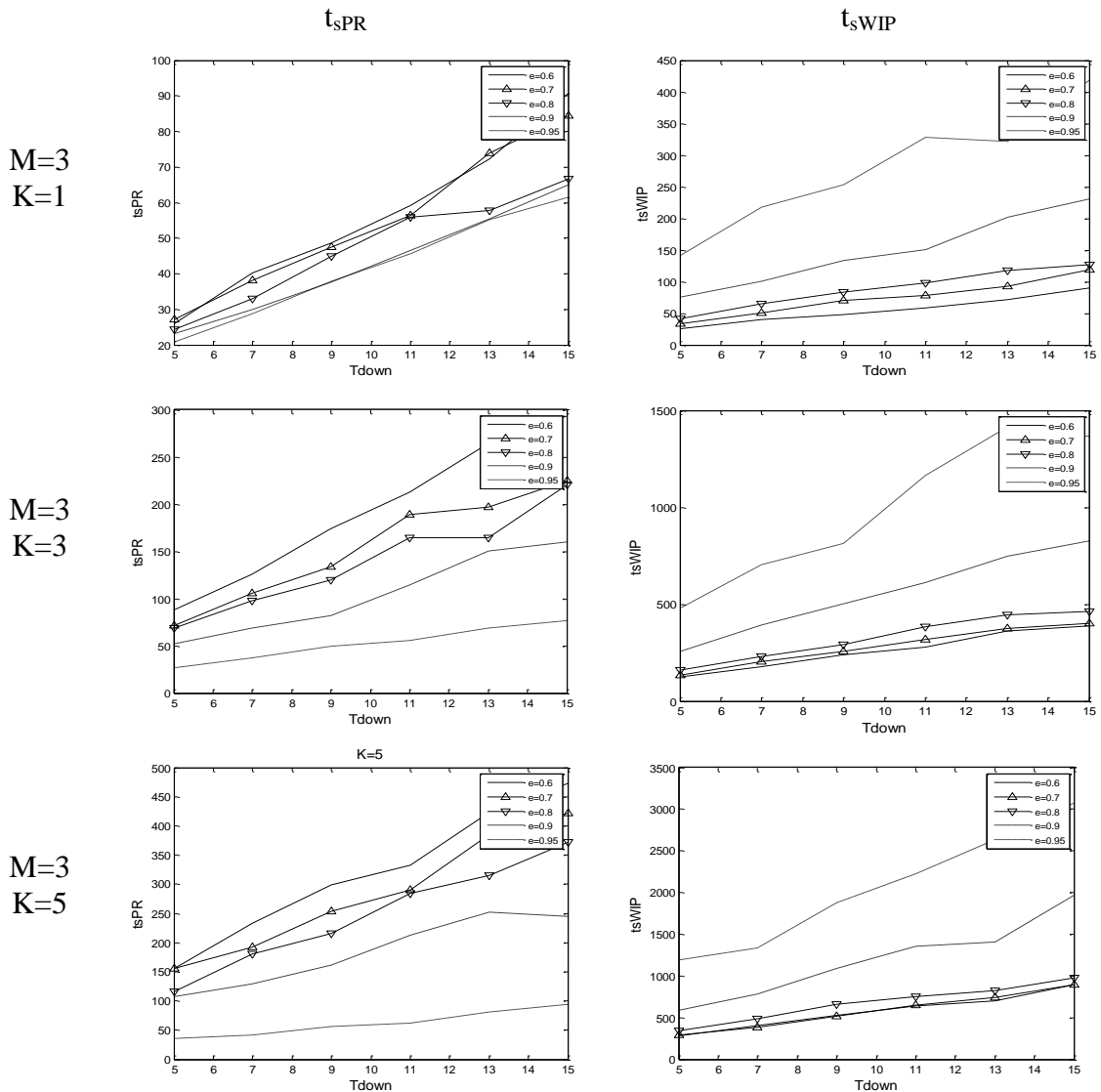


Figure 17 Effects of T_{down} on t_{sPR} and t_{sWIP} of Exponential Lines

3.4 Total Production

Total production (TP) describes how many products can be produced in the production duration. TP is one of the most important indices in system performance evaluation. Although PR in transient period can be higher than PR_{SS} occasionally, TP during transient will be smaller than total production under the same system in steady states. The analysis in this section will investigate the impact on TP due to system parameters. According to section 4.2.1, most research systems reach steady state during time slot T which is equal to 500 if e is larger than 0.5, therefore, we analyze the systems with machine efficiency larger than 0.5.

3.4.1 Effects of e

In order to analyze the effects of efficiency (e) on TP, simulations are implemented with the following system parameters:

Table 9 Exponential System Parameters (Effects of e on TP)

Parameters	e	M	K	T_{down}
Value	[0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95]	[3, 10]	3	[5, 10, 15]

Analysis of TP:

Figure 18 shows the graphs of TP vs. e for various T_{down} and M.

- 1) e has positive impact on TP, larger e leads to a larger TP, the relationship is close to linear.
- 2) The larger e, the less impact of K on TP. If machines efficiency is 0.95, for instance, there is no significant difference when K increases from 1 to 5.

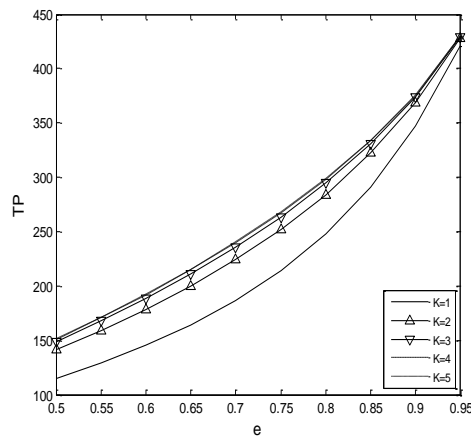
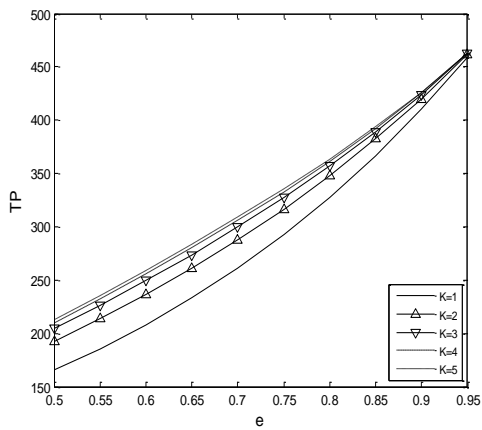
T_{down}

M

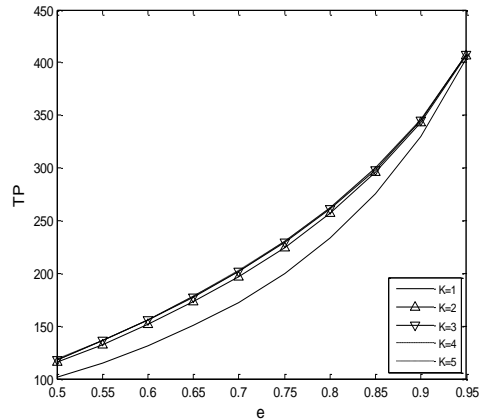
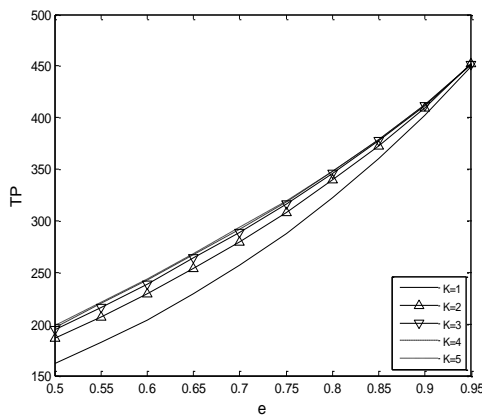
5

3

10



10



15

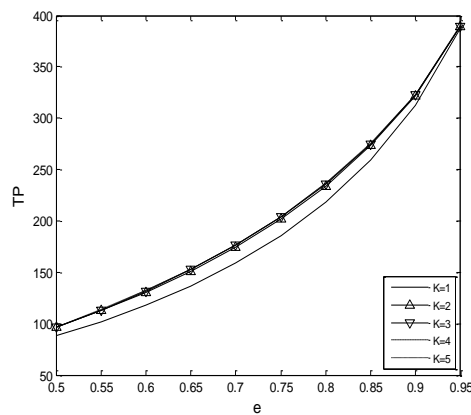
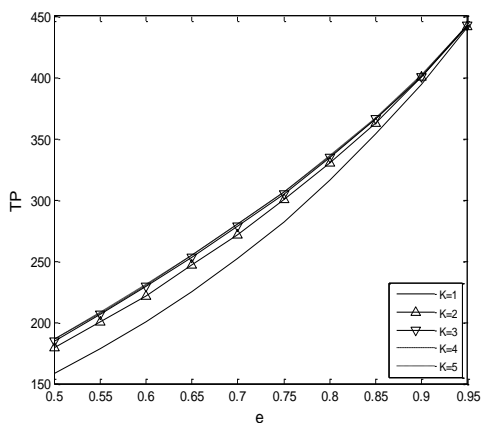


Figure 18 Effects of e on Total Production of Exponential Lines

3) TP decreases if there are more machines in the system.

3.4.2 Effects of K

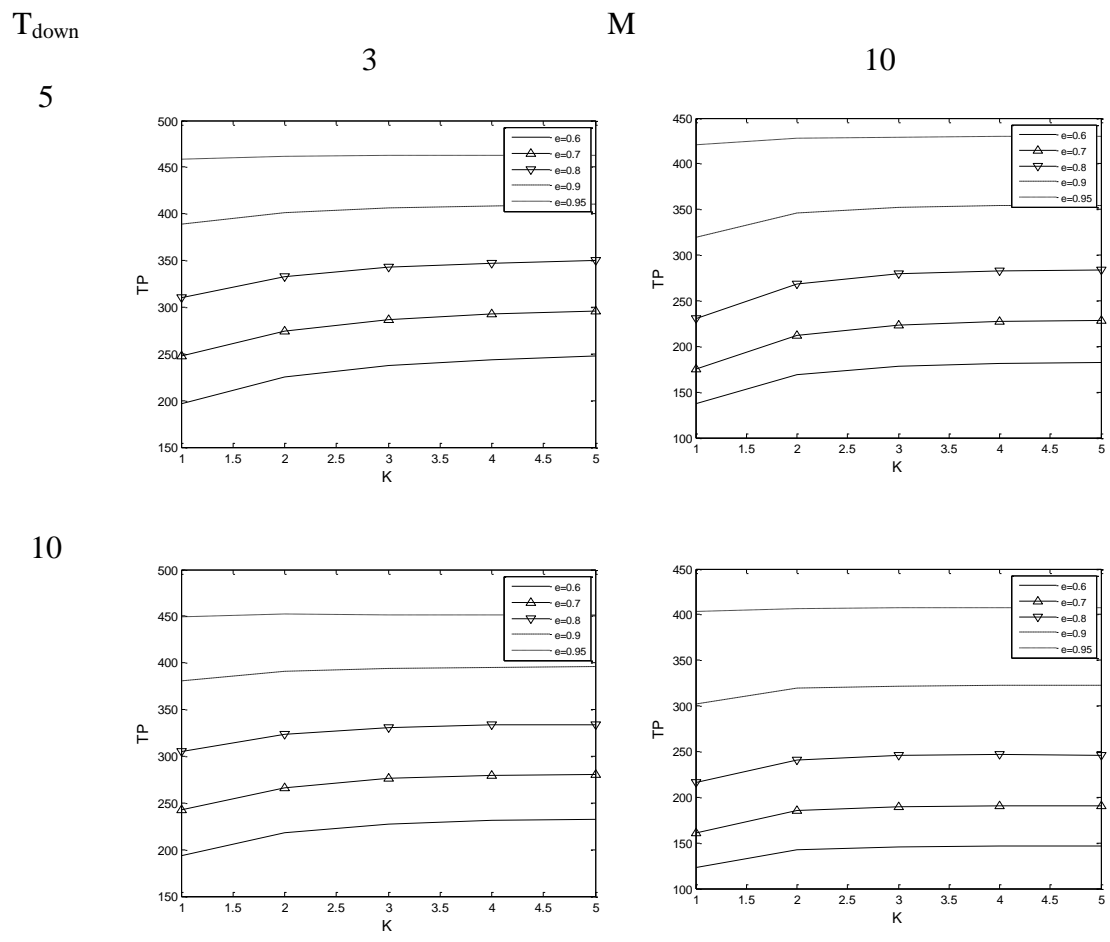
In order to analyze the effects of K on TP, simulations are implemented with the following system parameters:

Table 10 Exponential System Parameters (Effects of K on TP)

Parameters	e	M	K	T _{down}
Value	[0.6, 0.7, 0.8, 0.9, 0.95]	[3, 10]	[1, 2, 3, 4, 5]	[5, 10, 15]

Analysis of TP: Figure 19 shows the graphs of TP vs. K for various T_{down} and M.

1) K has positive impact on TP, larger K leads to a larger TP. However, TP saturates around K=2.



15

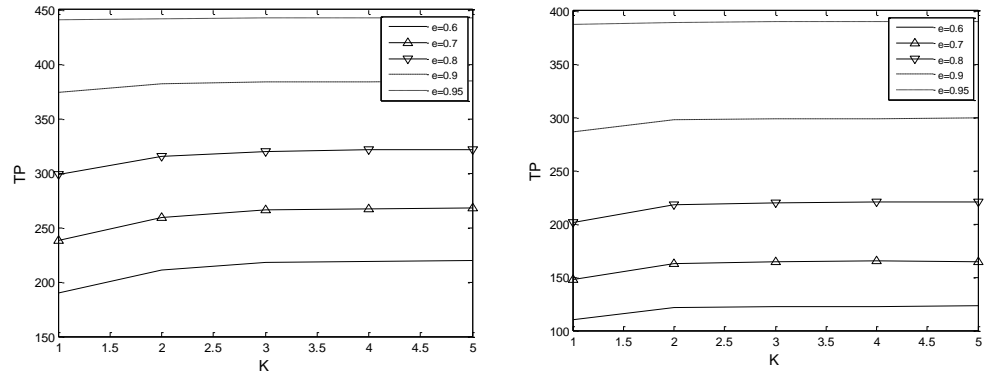


Figure 19 Effects of K on Total Production of Exponential Lines

In other words, the increase of TP from K = 1 to K = 2 is significant; the impact on TP for K > 2 is negligible.

3.4.3 Effects of Tdown

In order to analyze the effects of T_{down} on the TP, simulations are implemented with the following system parameters:

Table 11 Exponential System Parameters (Effects of T_{down} on TP)

Parameters	e	M	K	T _{down}
Value	[0.6, 0.7, 0.8, 0.9, 0.95]	[3, 10]	[1, 3, 5]	[5, 7, 9, 11, 13 15]

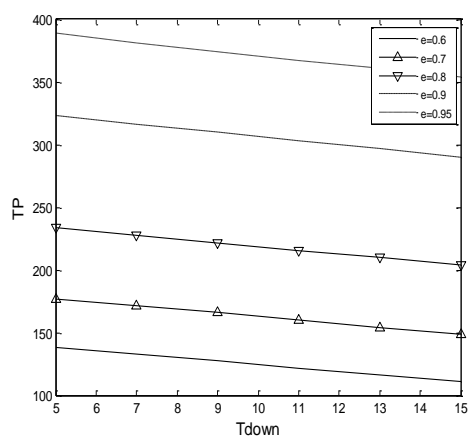
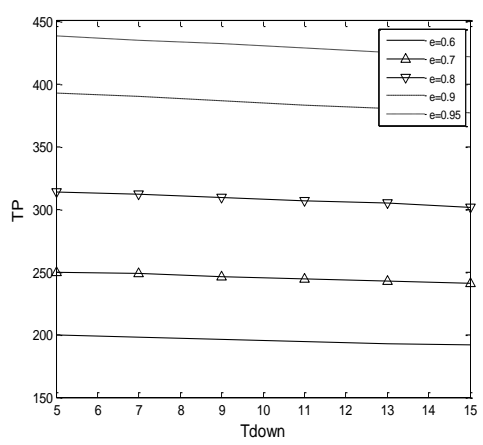
K

M

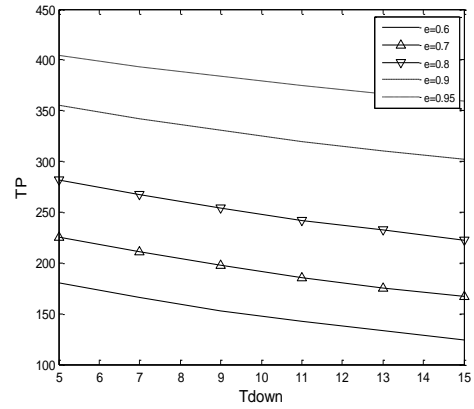
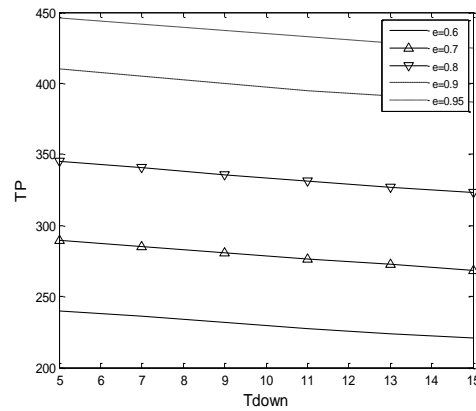
3

10

1



3



5

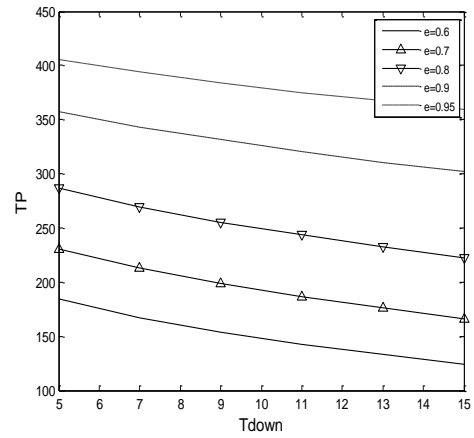
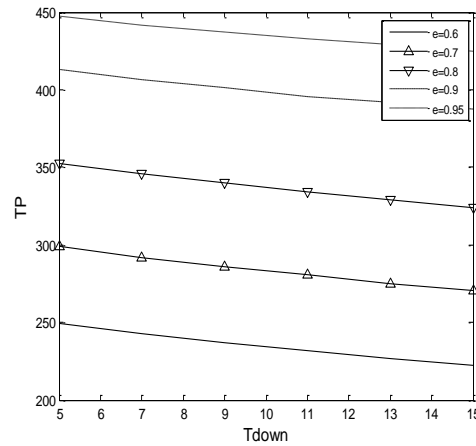


Figure 20 Effects of T_{down} on Total Production of Exponential Lines

Analysis of TP: Figure 20 shows the graphs of TP vs. T_{down} for various K and M.

- 1) T_{down} has negative impact on TP, larger T_{down} leads to smaller TP. The relationship is linear.
- 2) The maximum variation of TP due to T_{down} changes is less than 10% at a fixed e.

3.5 Production Loss

Production loss (PL) is a measure of change in total production comparing with that in steady state. This ratio contains information of relative production loss. However, it does

not translate directly into the value of total production. Therefore, low PL does not necessarily imply high TP.

3.5.1 Effects of e

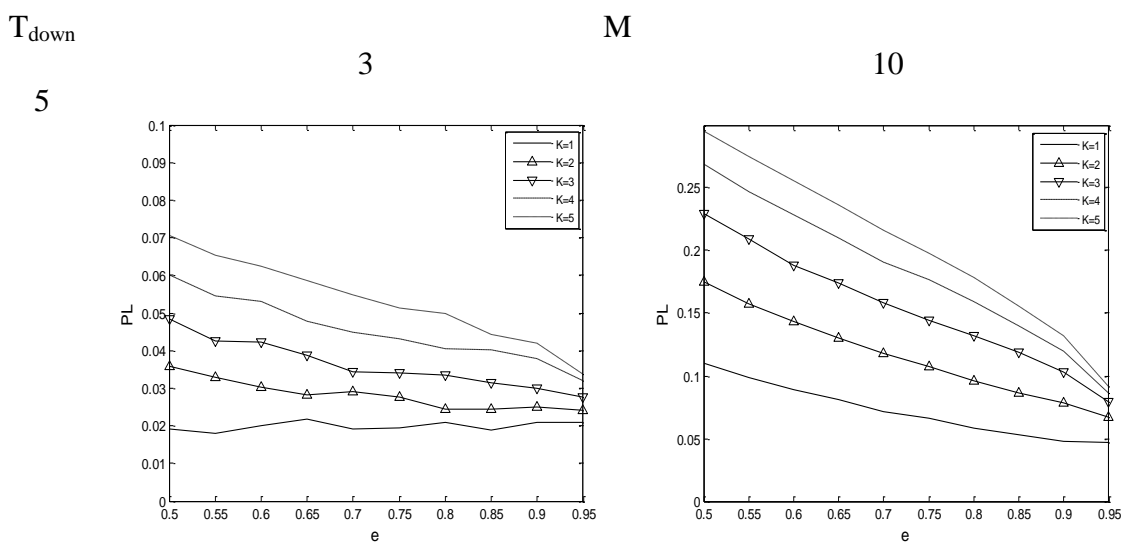
In order to analyze the effects of efficiency (e) on PL, simulations are implemented with the following system parameters:

Table 12 Exponential System Parameters (Effects of e on PL)

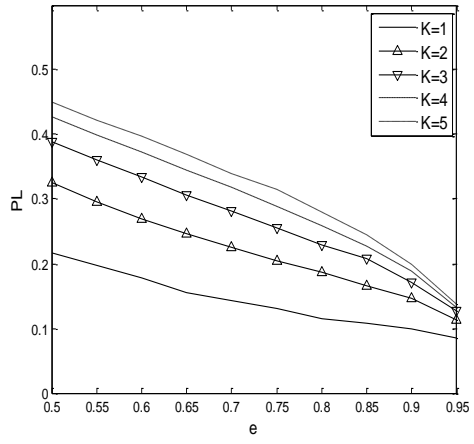
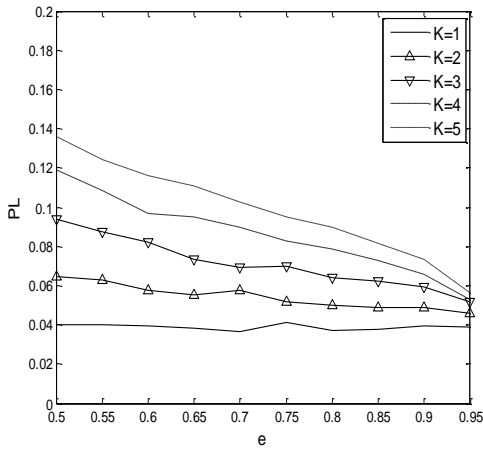
Parameters	e	M	K	T _{down}
Value	[0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95]	[3, 10]	3	[5, 10, 15]

Analysis of PL: Figure 21 shows the graphs of PL vs. e for various T_{down} and M.

- 1) e has negative impact on PL, larger e leads to a smaller PL, the relationship is close to linear.
- 2) When e is equal, the more machine, the larger production loss. Machine efficiency is 0.95 when there are 10 machines in the system, for instance, the production loss could be as much as 40% if T_{down} is 15.
- 3) PL increases if there are more machines in the system.



10



15

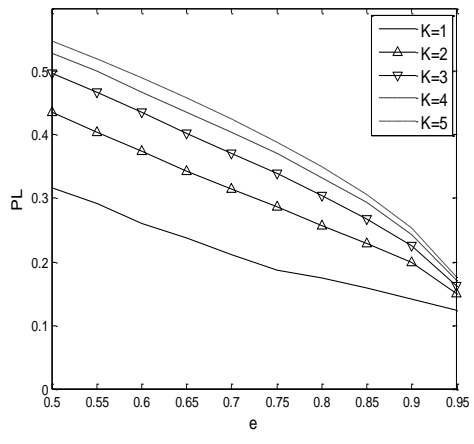
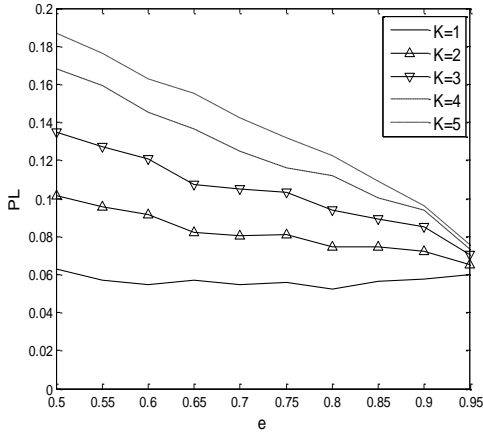


Figure 21 Effects of e on Production Loss of Exponential Lines

3.5.2 Effects of K

In order to analyze the effects of K on PL, simulations are implemented with the following system parameters:

Table 13 Exponential System Parameters (Effects of K on PL)

Parameters	e	M	K	T _{down}
Value	[0.6, 0.7, 0.8, 0.9, 0.95]	[3, 10]	[1, 2, 3, 4, 5]	[5, 10, 15]

Analysis of PL: Figure 22 shows the graphs of PL vs. K for various T_{down} and M.

1) K has positive impact on PL, larger K leads to a larger PL. However, the slope decreases as K increases, PL saturates at a certain point when K increases (The larger e, the smaller K).

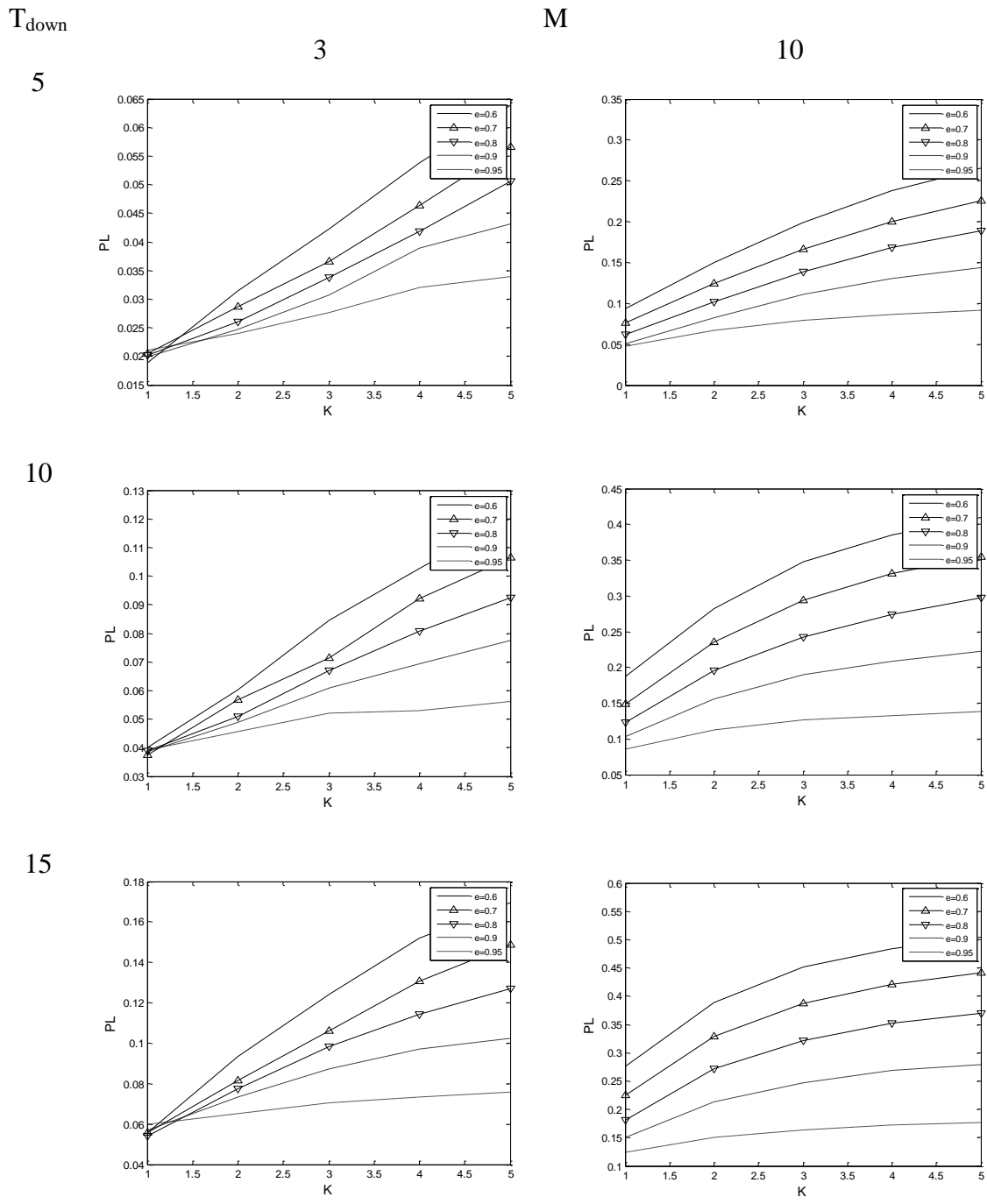


Figure 22 Effects of K on Production Loss of Exponential Lines

3.5.3 Effects of T_{down}

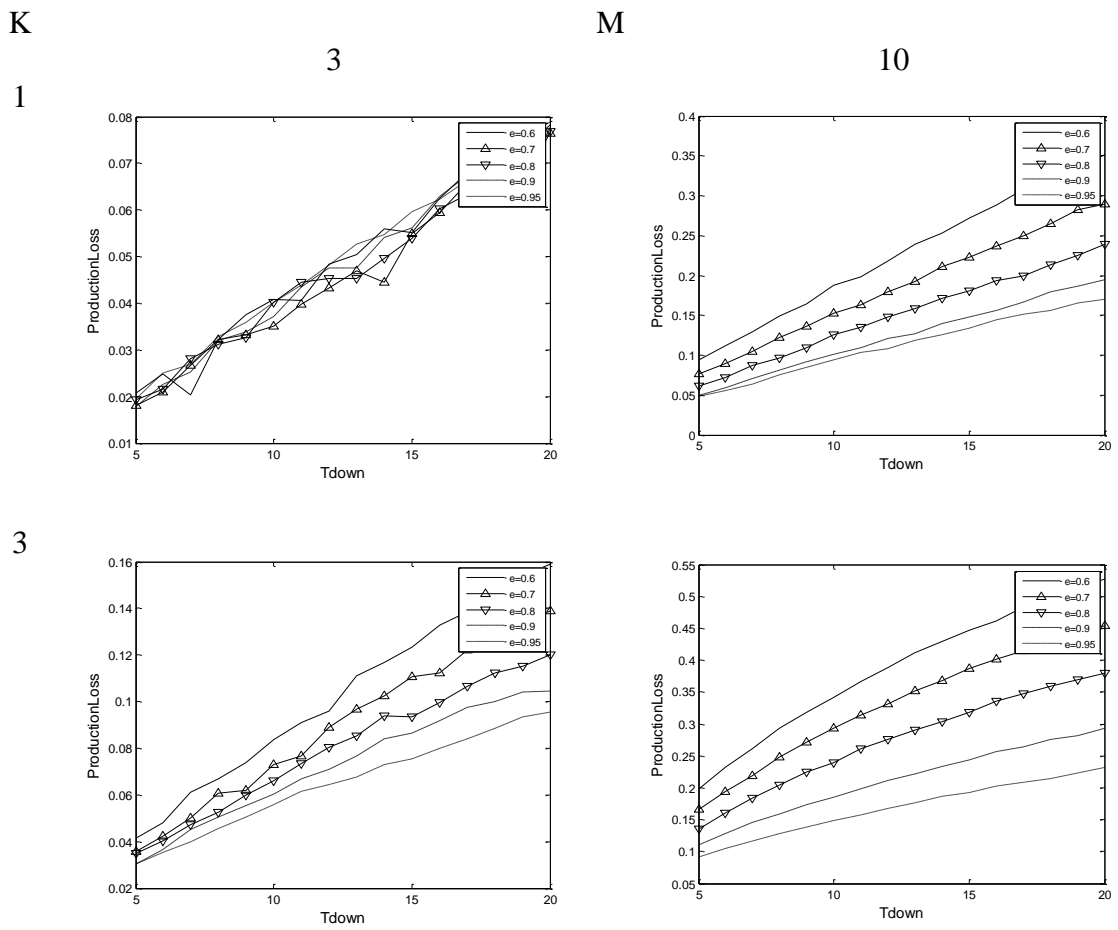
In order to analyze the effects of T_{down} on the PL, simulations are implemented with the following system parameters:

Table 14 Exponential System Parameters (Effects of T_{down} on PL)

Parameters	e	M	K	T_{down}
Value	[0.6, 0.7, 0.8, 0.9, 0.95]	[3, 10]	[1, 3, 5]	[5,6,7,8,9,10,11,12,13,14,15,16,17,18, 19,20]

Analysis of PL: Figure 23 shows the graphs of PL vs. T_{down} for various K and M.

- T_{down} has positive impact on PL, larger T_{down} leads to larger PL. It is close to linear. When K and M are larger, the slope decreases as T_{down} increases.



5

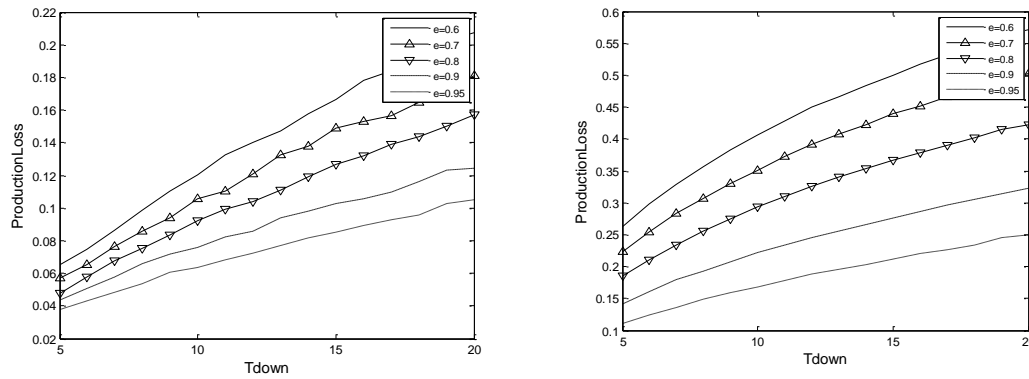


Figure 23 Effects of T_{down} on Production Loss of Exponential Lines

3.6 Summary

This chapter investigates the transient of an individual exponential machine and bufferless exponential serial line. Results show that the transient of an individual exponential machine is characterized by two factors. One is the distance between the initial condition and the steady state, the other is the system mode $e^{-(l+m)t}$. For bufferless exponential serial line, in general condition, the system throughput should be slower than the machine with the slowest transients.

We investigate if the transients of serial lines with exponential machines could be transformed into the system with geometric machines. Results show that the geometric line-based formula has very good accuracy in approximating the transient production rate of an exponential line.

The transient serial production line with machine reliability model satisfying exponential distribution are also analyzed. Simulations are implemented to analyze the effects of system parameters, including e , K , and T_{down} on the transient performance settling time

(t_{sPR} and t_{sWIP}), total production (TP) and production loss (PL). Based on the simulation results, the overall effects of system parameters are summarized in Table 15.

Table 15 Effects of system parameters (e , K , and T_{down}) on the t_{sPR} and t_{sWIP} , TP and PL

	t_{sPR}	t_{sWIP}	TP	PL
e	Negative	Convex function	Positive (close to linear)	Negative (linear)
K	No significant impact	Positive (close to linear)	Positive (saturated around $K=2$)	Positive (saturated)
T_{down}	Positive (linear)	Positive (linear)	Negative (linear)	Positive(linear)

For exponential serial production line with identical machines and buffers, the following conclusions are obtained from the simulation results.

- 1) Increasing machines efficiency will reduce the settling time of production rate and production loss, and increase the total production. However, e has a convex effect on t_{sWIP} . Too large or too small e will lead to longer t_{sWIP} . Appropriate e has to be selected to obtain the shortest t_{sWIP} . In the experiment, shortest t_{sWIP} 's are obtained when machines efficiency falls in the range of (0.7, 0.8).
- 2) K has positive effect on t_{sWIP} , TP and PL. It implies that if T_{down} is fixed in a system, increasing buffer capacity (N) will lead to longer settling time of WIP, more total production and more production loss. According to the simulation results, TP is saturated around $K=2$. Therefore, the increase of K when $K>2$ does not lead a significantly increase of TP, but results in a relative large increase of

PL. So $K=2$ is a good tradeoff point when the system optimization goal is to achieve a large TP but not sacrificing too much on PL.

- 3) T_{down} has positive effect on t_{sPR} , t_{sWIP} and PL and negative effect on TP. Reducing T_{down} will reduce the settling time of the system and production loss, and increase TP. T_{down} is the only system parameter that all the four performance measures improve at the same time by changing this parameter.
- 4) According to the simulation results, when there are more machines M in the serial production line, t_{sPR} , t_{sWIP} and PL are larger and TP is smaller at the same simulation conditions. That is because the more machines in the system, the more complexity of the system. Consequently, the system requires longer time to approach steady state and loses more production in the same duration. Furthermore, the more machines in the system, the lower efficiency of the system. Therefore, the system produces less production in the same duration.
- 5) A serial production line may have different optimization targets.

If the optimization goal is to reduce settling time t_{sPR} , then reducing T_{down} or increasing e are both effective ways, because T_{down} has linear positive impact on t_{sPR} , e has negative impact on t_{sPR} and K does not have significant impact on t_{sPR} .

If the optimization goal is to reduce settling time t_{sWIP} , then reducing T_{down} or K are both effective ways, meanwhile an appropriate e has to be selected to obtain the shortest t_{sWIP} due to the convex function.

If the optimization goal is to increase TP, then increasing e or reducing T_{down} are both effective ways. Increasing K can also be effective when $K \leq 2$ but not significant when $K > 2$.

If the optimization goal is to reduce PL, then increasing e or reducing T_{down} are both effective ways. Increasing K can also be effective when K is small but not significant when K is large.

- 6) Another observation from the simulation results is that, to reduce the settling time or increase total production for exponential serial lines, buffers should have more protections to the system. For Bernoulli machine line, to reduce the settling time, all buffers initial condition are suggested half full. For exponential serial production line, it is suggested that all the buffer should be filled in the initial condition to reduce the settling time and production loss, and increase the total production.

Chapter 4 Weibull, Gamma, Log-Normal

Systems

This chapter investigates the transients of serial production line with machines reliability model satisfying Weibull, Gamma, and Log-Normal distribution, respectively. The serial production line is operated under the assumptions (a-f) in section 2.3.2. The parameters of the three distributions are determined by the system parameters table in each section. This chapter utilizes simulation method to analyze the effects of system parameters, including e , K , T_{down} , CV_{up} and CV_{down} on the transient performance measures which are settling time, total production and production loss.

4.1 Transient Performance Analysis

To analyze the system's transient performance, settling times, t_{sPR} and t_{sWIP} , one has to know the behavior of PR and WIP as a function of t . Therefore, in this section, we first analyze the trajectories of $PR(t)$ and $WIP(t)$.

The system parameters in the simulation are shown in Table 16.

Table 16 Non-Exponential System Transient Performance Analysis Values

Parameters	E	M	K	Tdown	CVup	CVdown
Range	[0.7,0.9]	[3,10]	[1,5]	[3,9]	[0.4,0.7]	[0.3,0.6]
Default Value	0.8	3	3	5	0.4	0.3

4.1.1 Effects of e

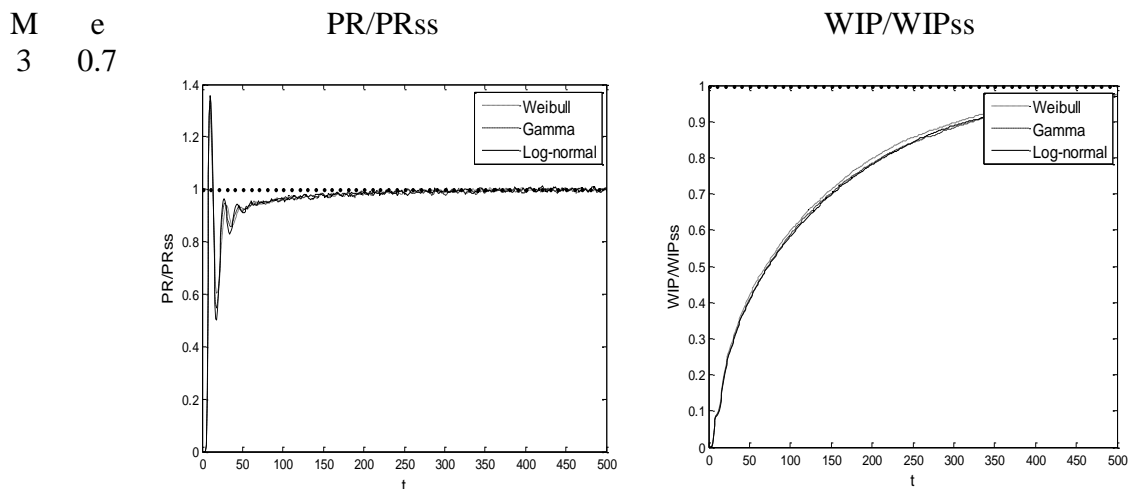
In order to investigate the effects of efficiency (e) on the transient performance, PR and WIP are simulated with the following system parameters.

Table 17 Non-Exponential System Parameters (Effects of e on transient performance)

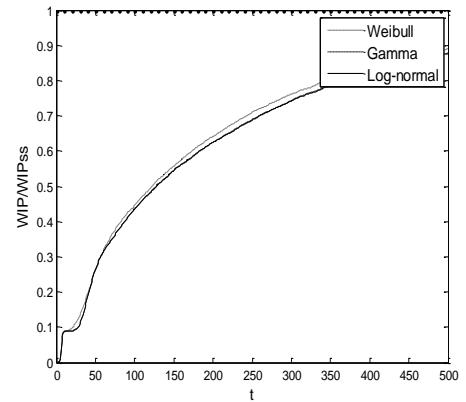
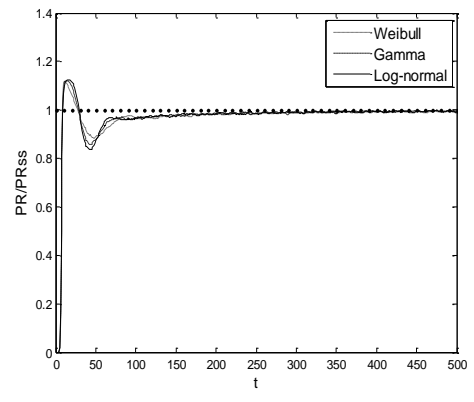
Parameters	e	M	K	T_{down}	CV_{up}	CV_{down}
Value	[0.7,0.9]	[3,10]	3	5	0.4	0.3

To compare the transient performances of PR and WIP, Figure 24 shows the graphs of PR/PR_{ss} and WIP/WIP_{ss} for various e and M . The following conjectures are observed:

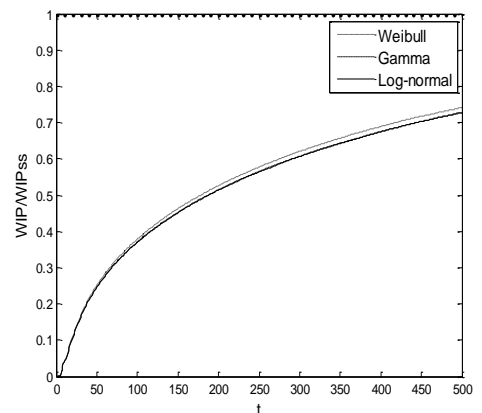
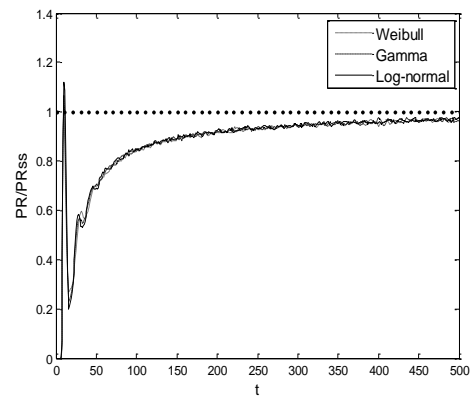
- 1) As long as system parameters are the same for the three continuous reliability models, there is no significant difference in performances regarding PR/PR_{ss} , WIP/WIP_{ss} and settling time.
- 2) PR and WIP both approach steady state slower when there are more machines in the system. For instance, when there are three machines in the systems, machines' efficiency is 0.7, the settling time of PR is close to 80. In contrast, the settling time of PR is more than 200 when there are ten machines in the system.
- 3) PR approaches steady state value faster when e increases. In contrast, WIP needs longer time to reach the steady state as e increases.



0.9



10 0.7



0.9

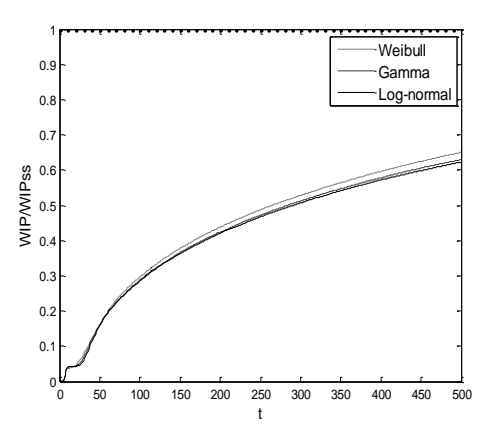
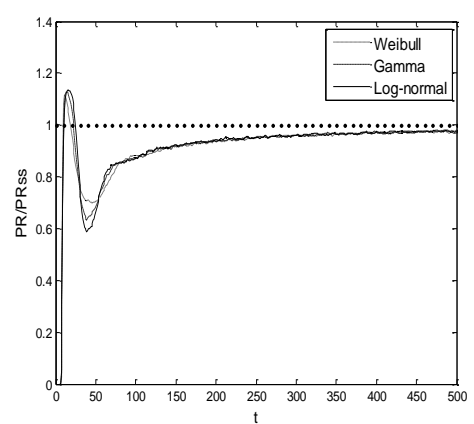


Figure 24 Effects of e in Transient Process of Non- Exponential Lines

- 4) The production rate (PR) approaches steady state value faster than the work-in-process (WIP) in the same system. As e becomes larger, the difference becomes more pronounced.

- 5) PR during transient period may be higher than its steady state level, while no such bump is captured in WIP.

4.1.2 Effects of K

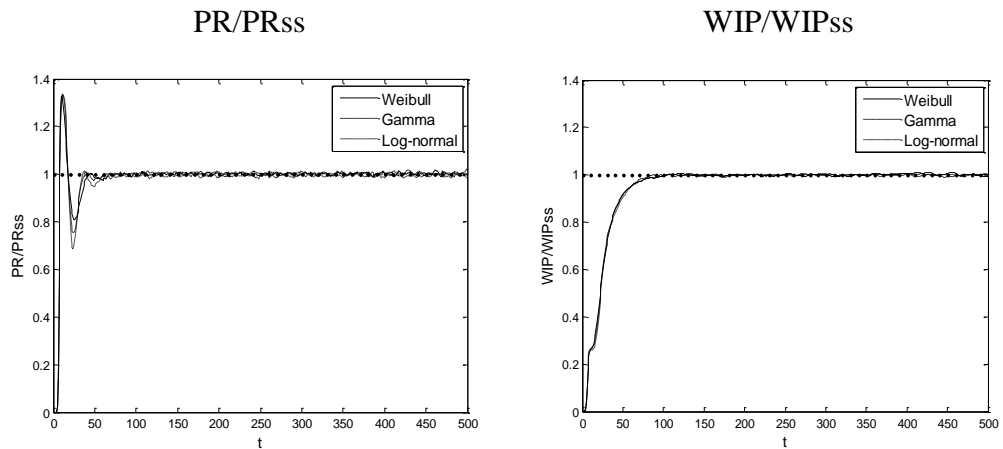
In order to investigate the effects of K on the transient performance, PR and WIP are simulated with the following system parameters:

Table 18 Non-Exponential System Parameters (Effects of K on transient performance)

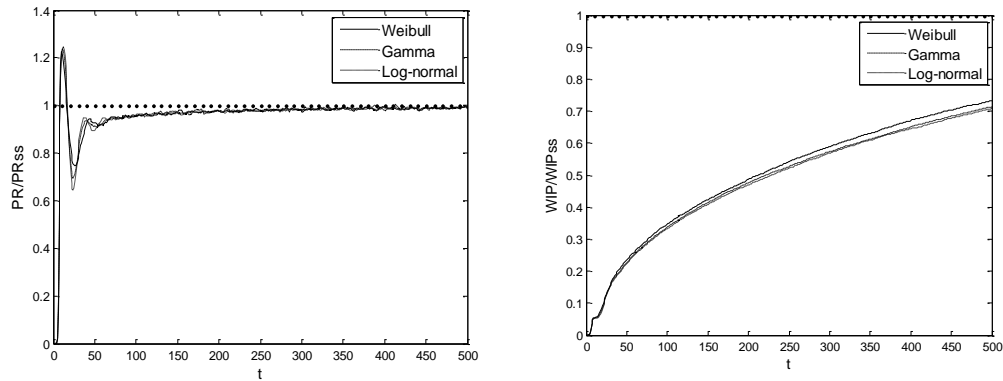
Parameters	E	M	K	T _{down}	CV _{up}	CV _{down}
Value	0.8	[3,10]	[1,5]	5	0.4	0.3

Analysis of PR and WIP: To compare the transients of PR and WIP, Figure 25 shows the

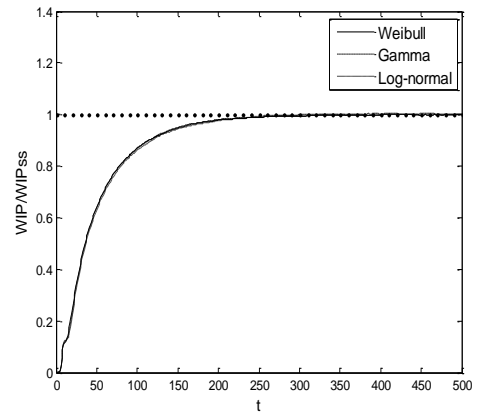
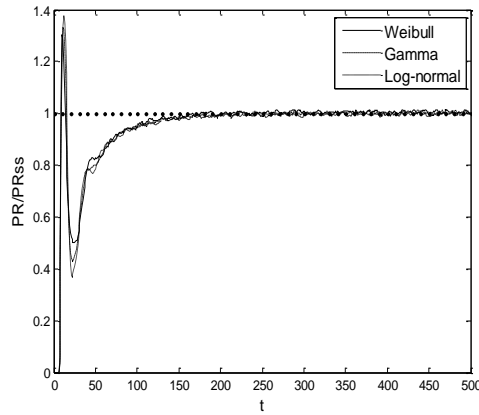
M K
3 1



5



10 1



5

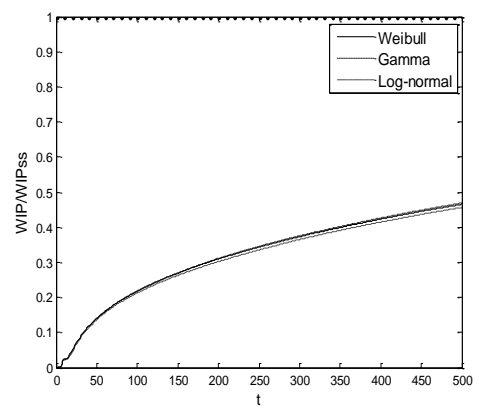
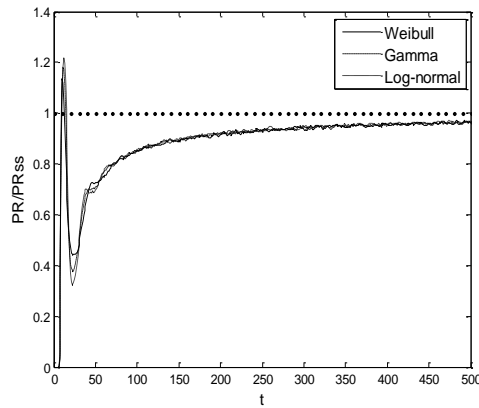


Figure 25 Effects of K in Transient Process of Non- Exponential Lines

graphs of PR/PR_{ss} and WIP/WIP_{ss} for various K and M .the following conjecture is observed:

- 1) PR and WIP both approach steady state value slower when K increases.

4.1.3 Effects of T_{down}

In order to analyze the effects of T_{down} on the transient performance. PR and WIP are simulated with the following system parameters:

Table 19 Non-Exponential System Parameters (Effects of Tdown on transient performance)

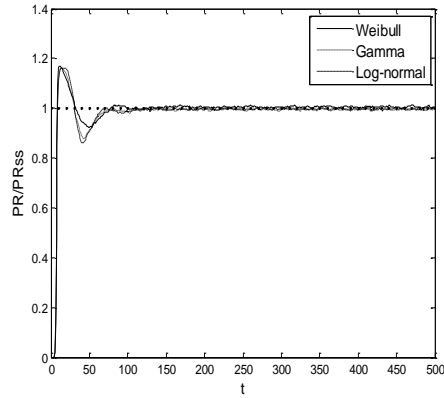
Parameters	E	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	[3,10]	3	[3, 9]	0.4	0.3

To compare the transients of PR and WIP, Figure 26 shows the graphs of PR/PR_{ss} and WIP/WIP_{ss} for various K and M , the following conjecture is observed:

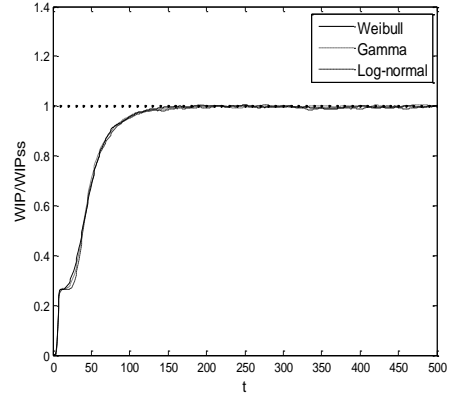
- 1) PR and WIP both approach steady state value slower when T_{down} increases.

M T_{down}
 3 3

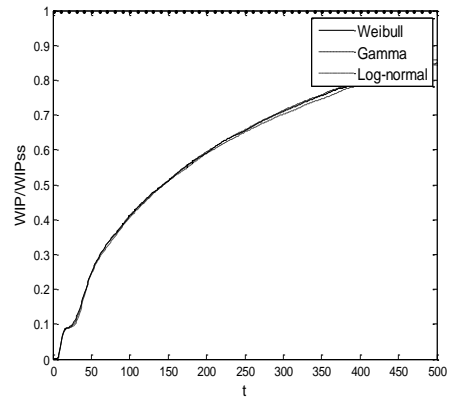
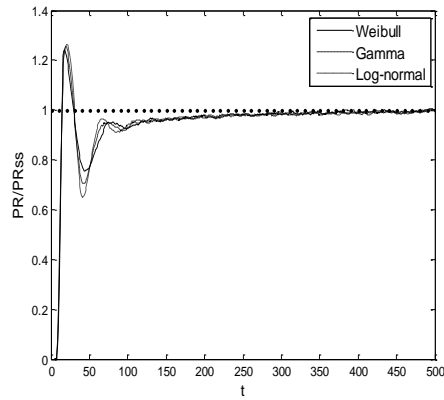
PR/PR_{ss}



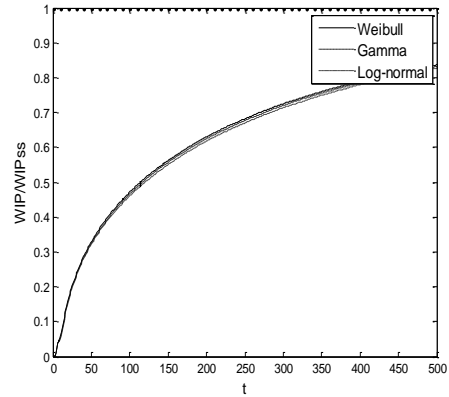
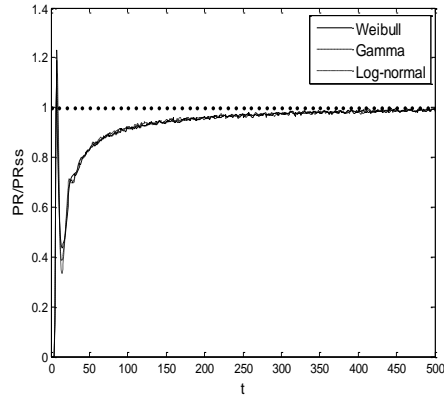
WIP/WIP_{ss}



9



10 3



9

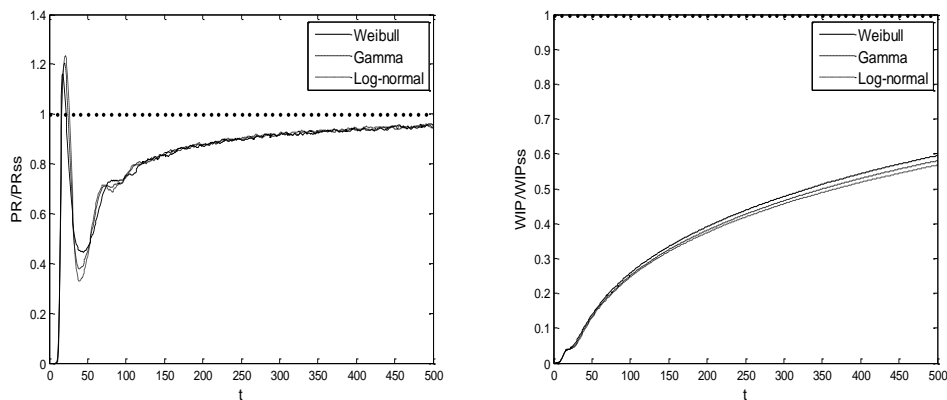


Figure 26 Effects of T_{down} in Transient Process of Non- Exponential Lines

4.1.4 Effects of CV_{up}

In order to analyze the effects of CV_{up} on the transient performance. PR and WIP are simulated with the following system parameters:

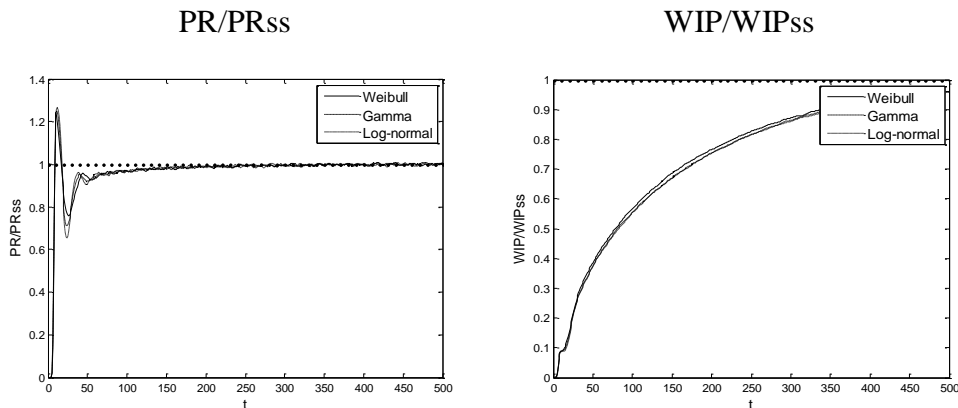
Table 20 Non-Exponential System Parameters (Effects of CV_{up} on transient performance)

Parameters	E	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	[3,10]	3	5	[0.4,0.7]	0.3

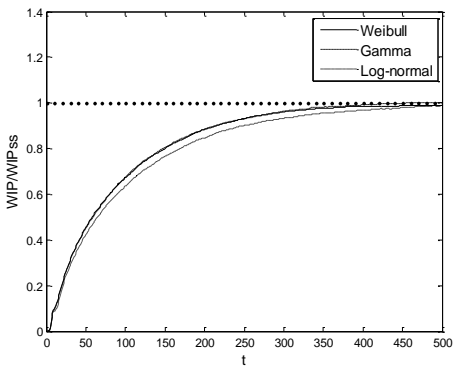
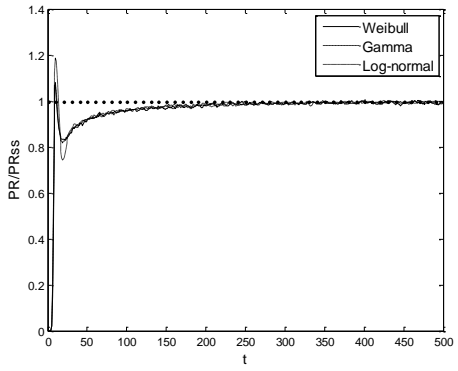
To compare the transients of PR and WIP, Figure 27 shows the graphs of PR/PR_{ss} and WIP/WIP_{ss} for various CV_{up} and M, the following conjecture is observed:

- 1) PR and WIP both approach steady state value slower when CV_{up} increases.

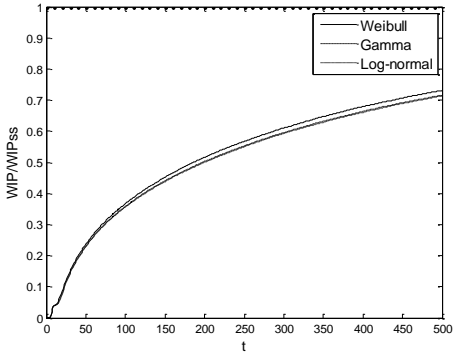
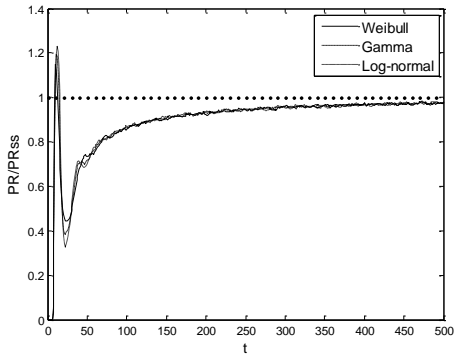
M CV_{up}
3 0.4



0.7



10 0.4



0.7

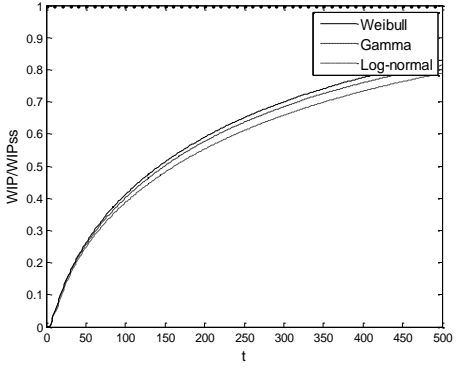
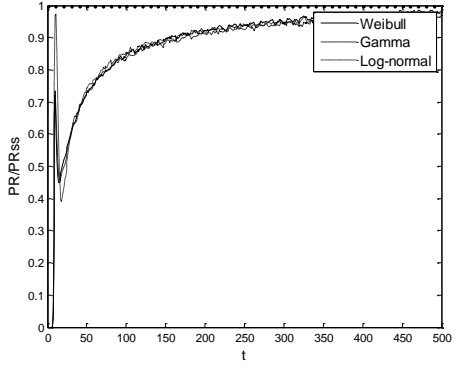


Figure 27 Effects of CV_{up} in Transient Process of Non- Exponential Lines

4.1.5 Effects of CV_{down}

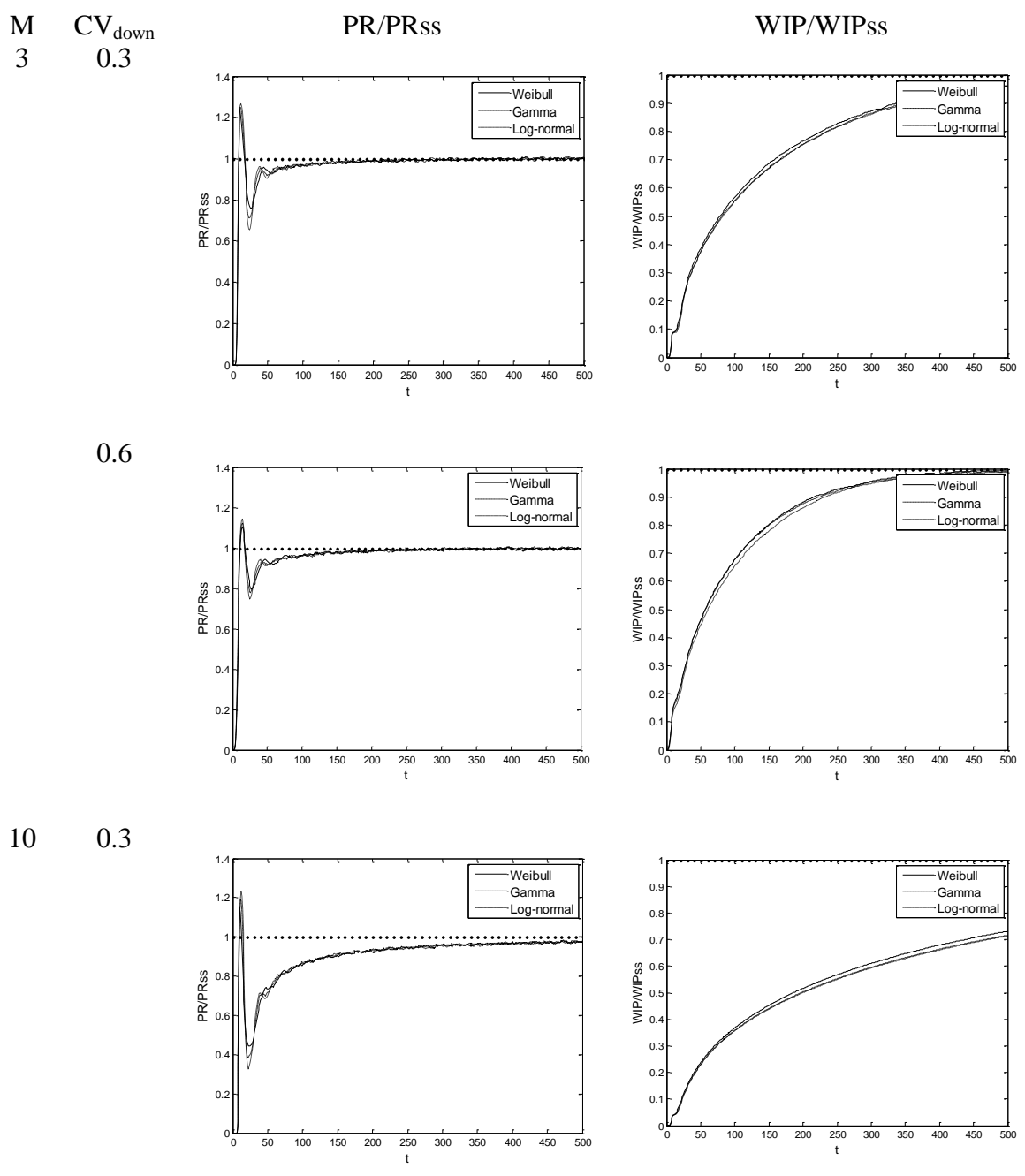
In order to analyze the effects of CV_{down} on the transient performance. PR and WIP are simulated with the following system parameters:

Table 21 Non-Exponential System Parameters (Effects of CV_{down} on transient performance)

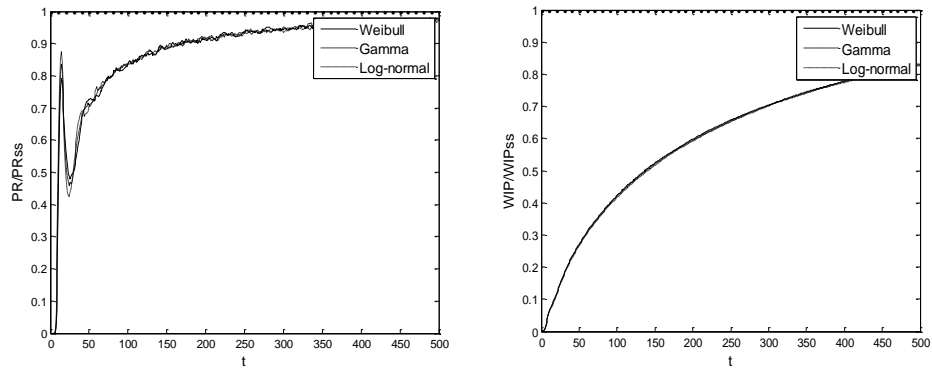
Parameters	E	M	K	T _{down}	CV _{up}	CV _{down}
Value	0.8	[3,10]	3	5	0.4	[0.3,0.6]

To compare the transients of PR and WIP, Figure 28 shows the graphs of PR/PR_{ss} and WIP/WIP_{ss} for various CV_{up} and M, the following conjecture is observed:

- 1) PR and WIP both approach steady state value slower when CV_{down} increases.



0.6

Figure 28 Effects of CV_{down} in Transient Process of Non- Exponential Lines

4.2 Settling Time

To justify the conjectures in section 5.1, simulations are implemented and the system parameters are shown in each subsection.

4.2.1 Effects of e

In order to analyze the effects of efficiency (e) on the settling time t_{sPR} and t_{sWIP} , simulations are implemented with the following system parameters:

Table 22 Non-Exponential System Parameters (Effects of e on t_{sPR} and t_{sWIP})

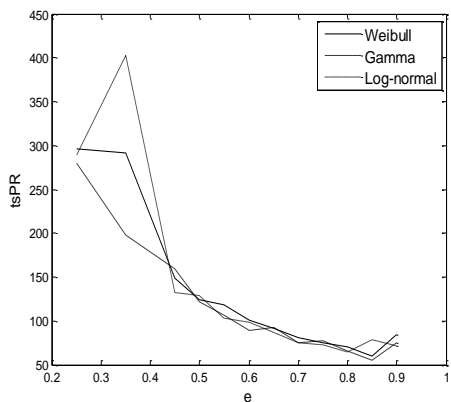
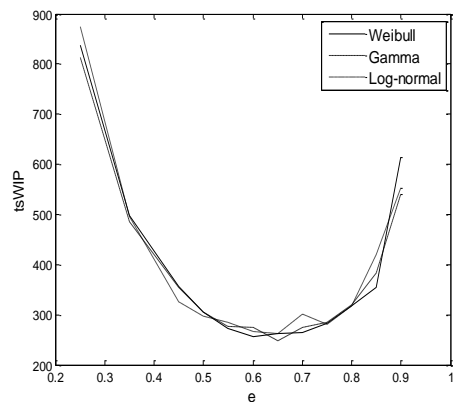
Parameters	e	M	K	T_{down}	CV_{up}	CV_{down}
Value	[0.25,0.35,0.45,0.5,0.55,0.6,0.65,0.7,0.75,0.8,0.85,0.9,0.95]	3	3	5	[0.4,0.7]	[0.3,0.6]

Figure 29 shows the graphs of t_{sPR} and t_{sWIP} vs. e for various CV_{up} and CV_{down} .

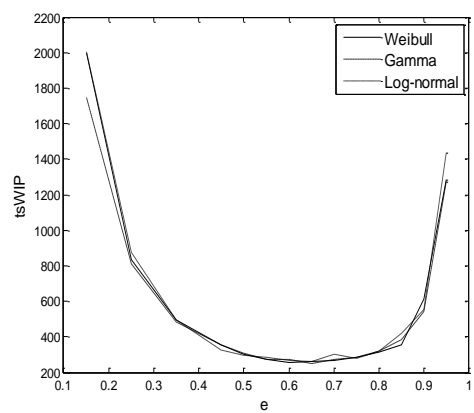
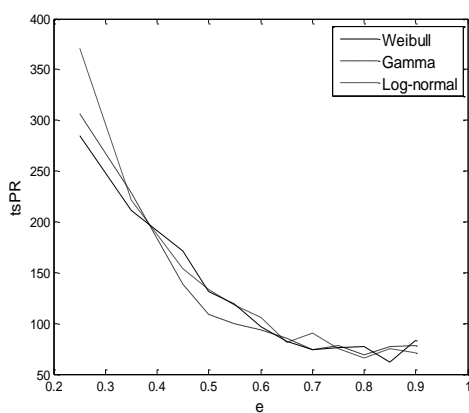
- 1) PR has shorter settling time than WIP in the same system ($t_{sPR} < t_{sWIP}$).
- 2) t_{sPR} becomes shorter as e increases. In contrast, t_{sWIP} is a convex function of e .

$$\frac{CV_{up}}{CV_{down}}$$

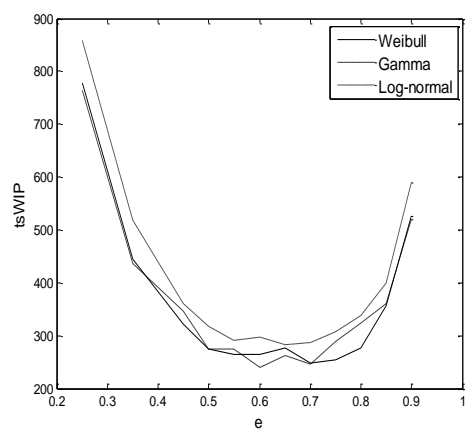
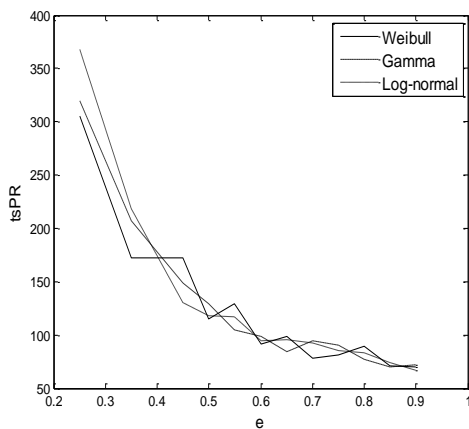
0.4/
0.3

 t_{sPR}

 t_{sWIP}


0.4/
0.6



0.7/
0.3



0.7/
0.6

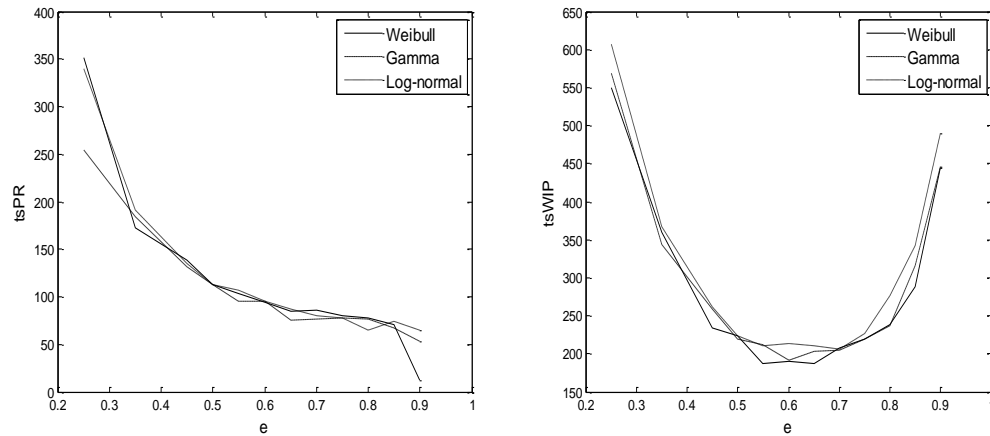


Figure 29 Effects of e on t_{sPR} and t_{sWIP} of Non-Exponential Lines

4.2.2 Effects of K

In order to analyze the effects of K on the settling time t_{sPR} and t_{sWIP} , simulations are implemented with the following system parameters:

Table 23 Non-Exponential System Parameters (Effects of K on t_{sPR} and t_{sWIP})

Parameters	E	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	3	[1,2,3,4,5]	5	[0.4,0.7]	[0.3,0.6]

Analysis of t_{sPR} and t_{sWIP} : Figure 30 shows the graphs of t_{sPR} and t_{sWIP} vs. K for various CV_{up} and CV_{down} . The following conclusions are observed.

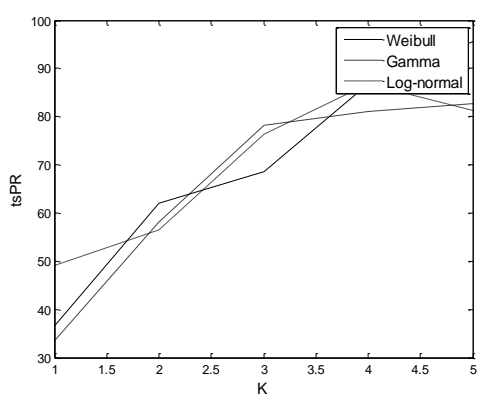
- 1) K has positive impact on t_{sPR} , larger K leads to a longer t_{sPR} . The relationship is close to linear.
- 2) K has positive impact on t_{sWIP} , larger K leads to a longer t_{sWIP} . The slope increases as K becomes larger.

Note the variation of K in this scenario is equivalent to the variation of N (T_{down} is fixed,

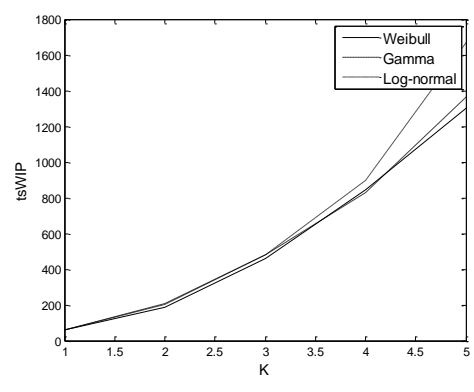
$= \frac{N}{T_{down}}$), so the analysis and conclusions also apply to effects of N when T_{down} is fixed.

$CV_{up}/$
 CV_{down}
0.4/
0.3

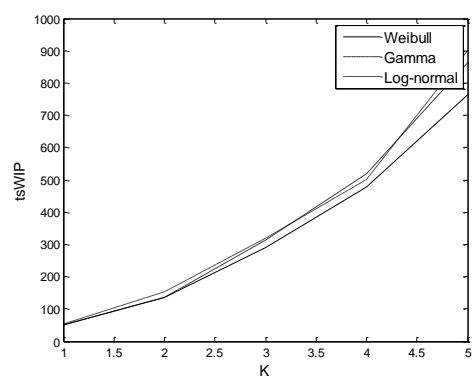
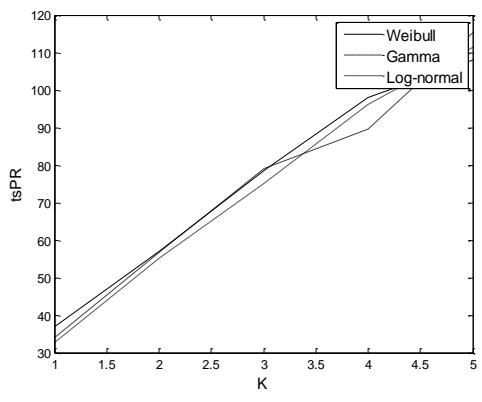
t_{sPR}



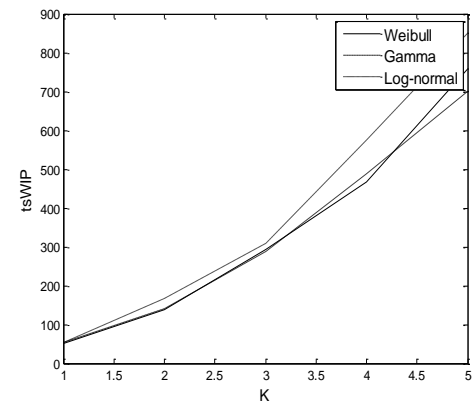
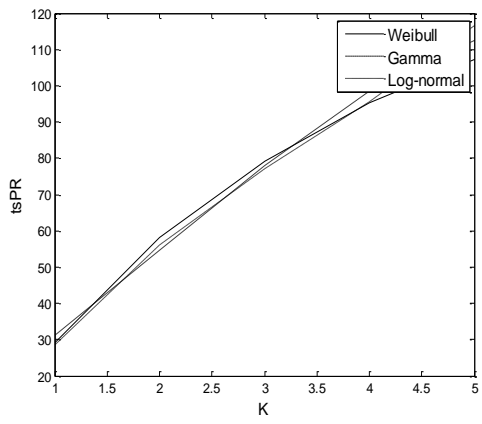
t_{sWIP}



0.4/
0.6



0.7/
0.3



0.7/
0.6

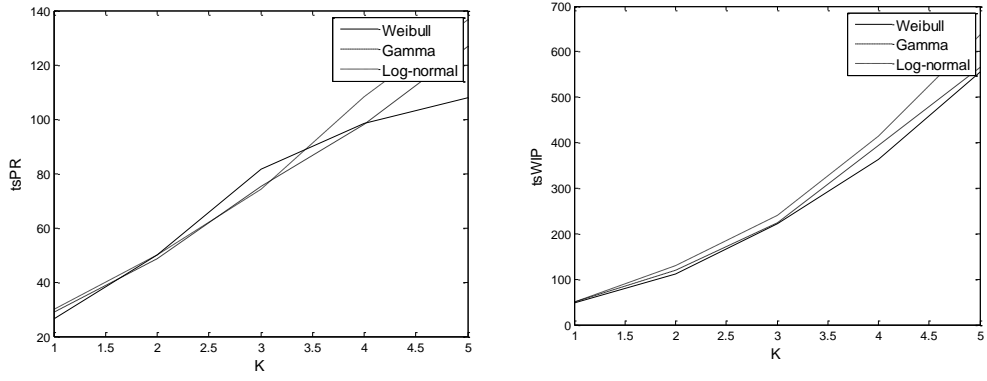


Figure 30 Effects of K on t_{sPR} and t_{sWIP} of Non-Exponential Lines

4.2.3 Effects of T_{down}

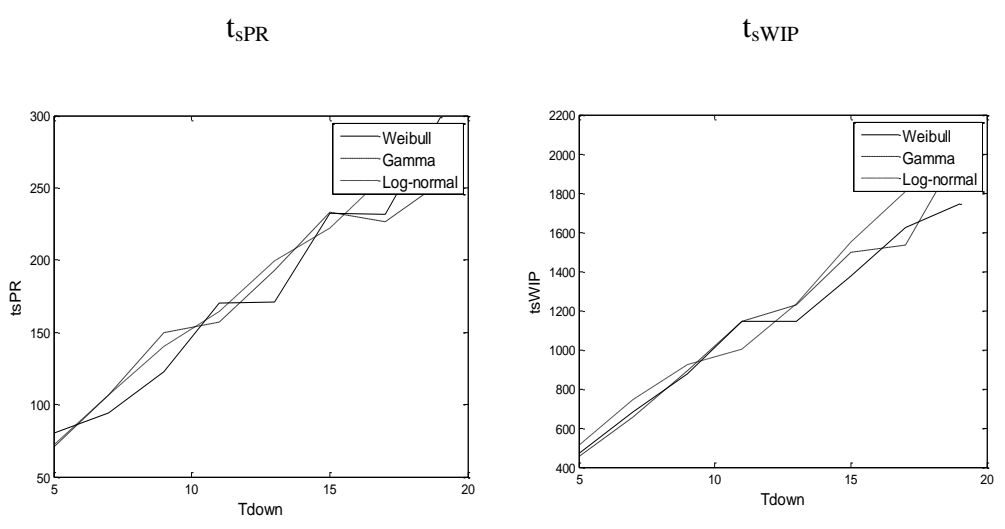
In order to analyze the effects of T_{down} on the settling time t_{sPR} and t_{sWIP} , simulations are implemented with the following system parameters:

Table 24 Non-Exponential System Parameters (Effects of T_{down} on t_{sPR} and t_{sWIP})

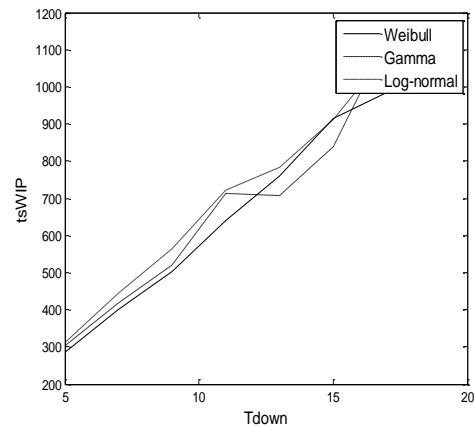
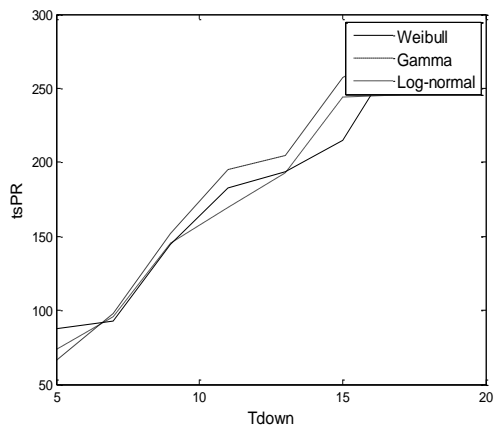
Parameters	E	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	3	3	[5,7,9,11,13,15,17,19]	[0.4,0.7]	[0.3,0.6]

Figure 31 shows the graphs of t_{sPR} and t_{sWIP} vs. T_{down} for various CV_{up} and CV_{down} . The

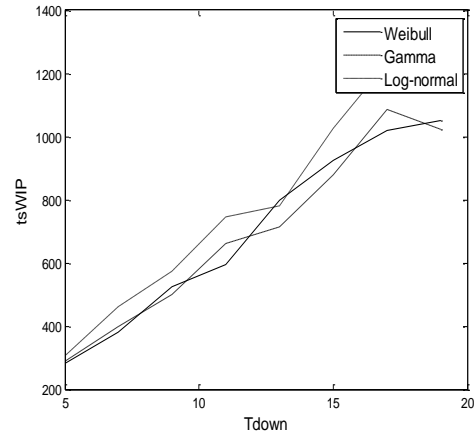
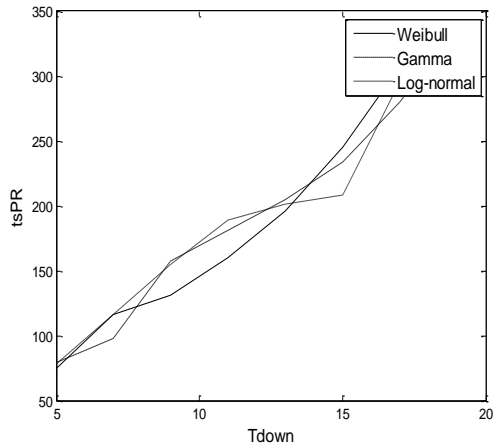
$CV_{up}/$
 CV_{down}
0.4/
0.3



0.4/
0.6



0.7/
0.3



0.7/
0.6

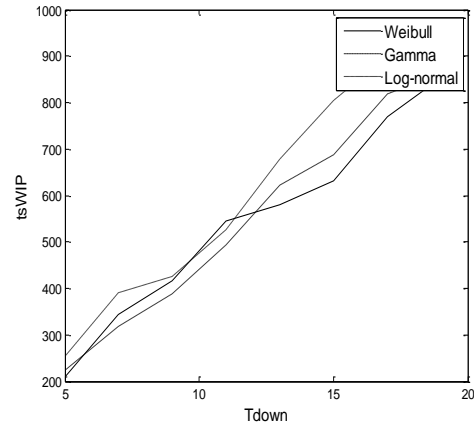
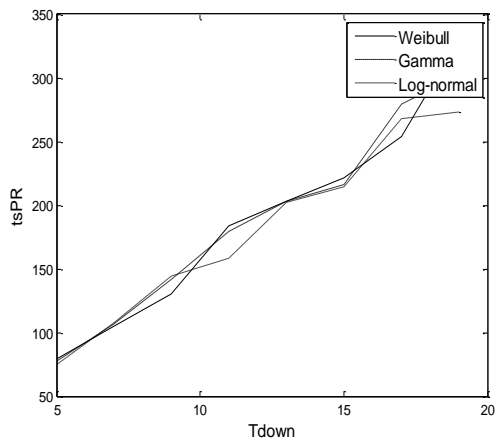


Figure 31 Effects of T_{down} on t_{sPR} and t_{sWIP} of Non-Exponential Lines

following conclusion is observed.

- 1) T_{down} has positive impact on both t_{sPR} and t_{sWIP} , larger T_{down} leads to longer t_{sPR} and t_{sWIP} . The relationship is close to linear.

4.2.4 Effects of CV_{up}

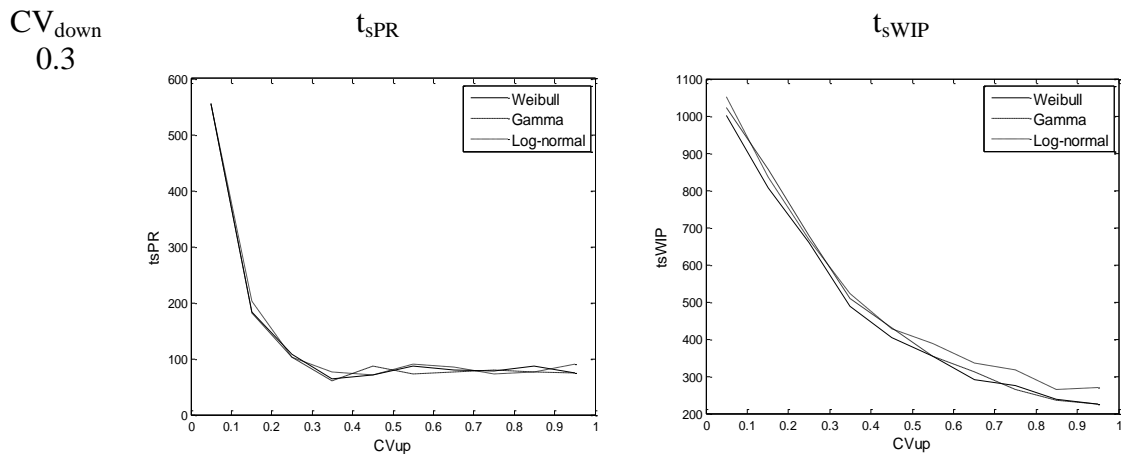
In order to analyze the effects of CV_{up} on the settling time t_{sPR} and t_{sWIP} , simulations are implemented with the following system parameters:

Table 25 Non-Exponential System Parameters (Effects of CV_{up} on t_{sPR} and t_{sWIP})

Parameters	e	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	3	3	5	[0.05,0.15,0.25,0.35,0.45,0.55,0.65,0.75,0.85,0.95]	[0.3,0.6]

Figure 32 shows the graphs of t_{sPR} and t_{sWIP} vs. CV_{up} for various CV_{down} .

- CV_{up} has negative impact on t_{sPR} , larger CV_{up} leads to a smaller t_{sPR} . However, t_{sPR} saturate around $CV_{\text{up}} = 0.25$. The decrease of t_{sPR} from $CV_{\text{up}} = 0.05$ to $CV_{\text{up}} = 0.25$ is significant; the impact on t_{sPR} for $CV_{\text{up}} > 0.25$ is negligible.
- CV_{up} has negative impact on t_{sWIP} , larger CV_{up} leads to a smaller t_{sWIP} , the relationship is close to linear.



0.6

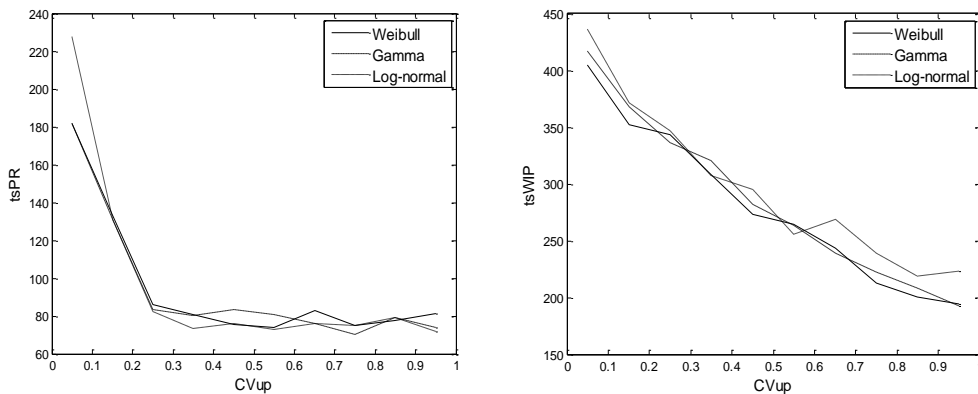


Figure 32 Effects of CV_{up} on t_{sPR} and t_{sWIP} of Non-Exponential Lines

4.2.5 Effects of CV_{down}

In order to analyze the effects of CV_{down} on the settling time t_{sPR} and t_{sWIP} , simulations are implemented with the following system parameters:

Table 26 Non-Exponential System Parameters (Effects of CV_{down} on t_{sPR} and t_{sWIP})

Parameters	e	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	3	3	5	[0.4, 0.7]	[0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95]

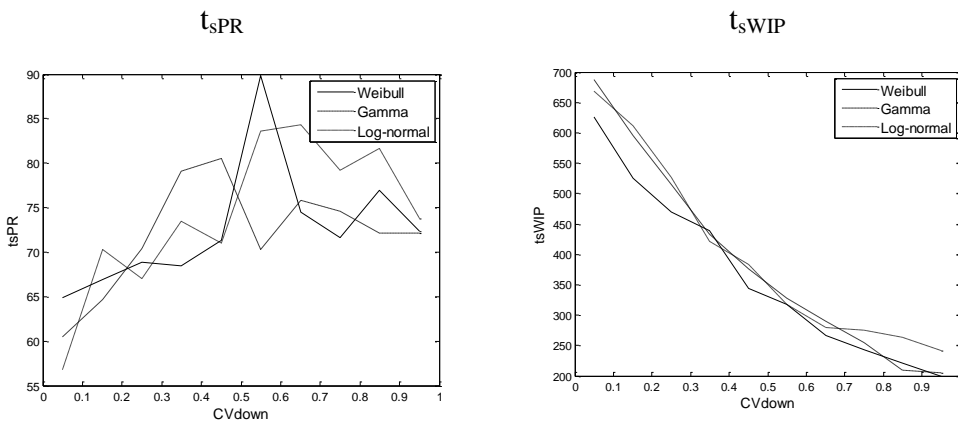
Figure 33 shows the graphs of t_{sPR} and t_{sWIP} vs. CV_{down} for various CV_{up} .

1) CV_{down} has no significant impact on t_{sPR} .

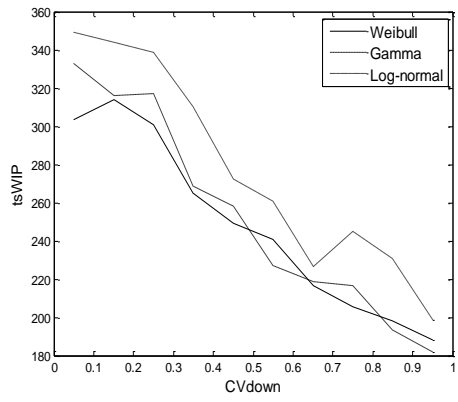
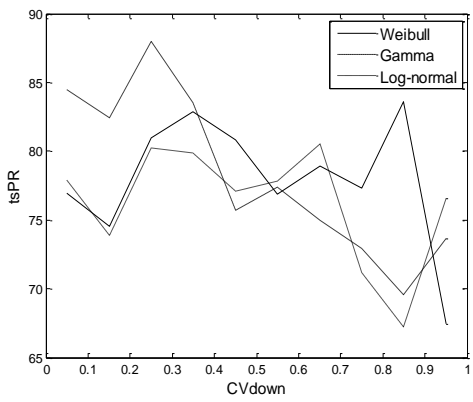
CV_{up}

M=3

0.4

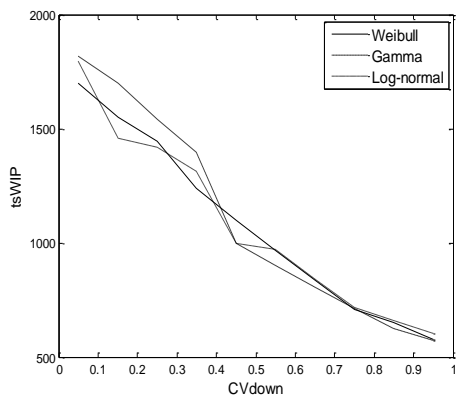
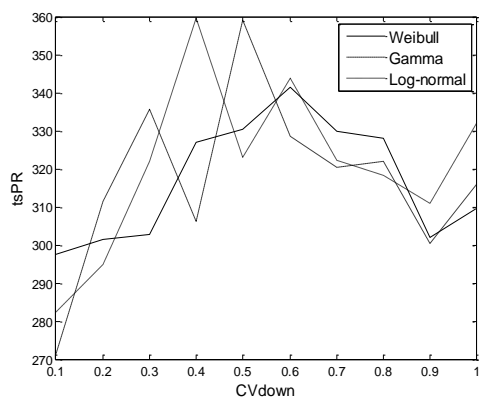


0.7



M=10

0.4



0.7

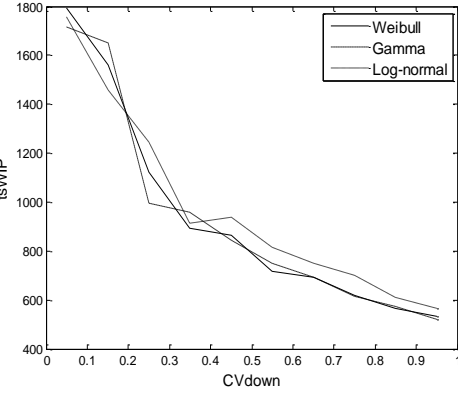
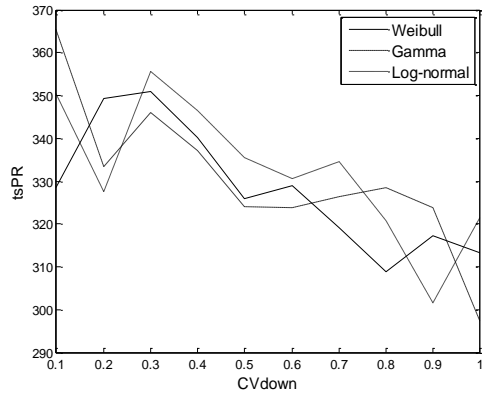


Figure 33 Effects of CV_{down} on t_{sPR} and t_{sWIP} of Non-Exponential Lines

2) CV_{down} has negative impact on t_{sWIP} , larger CV_{down} leads to a smaller t_{sWIP} , the relationship is close to linear.

4.3 Total Production

The analysis in this section will investigate the impact on TP due to system parameters of serial production line with machine reliability model satisfying Weibull, Gamma, and Log-Normal distributions.

4.3.1 Effects of e

In order to analyze the effects of efficiency (e) on TP, simulations are implemented with the following system parameters:

Table 27 Non-Exponential System Parameters (Effects of e on TP)

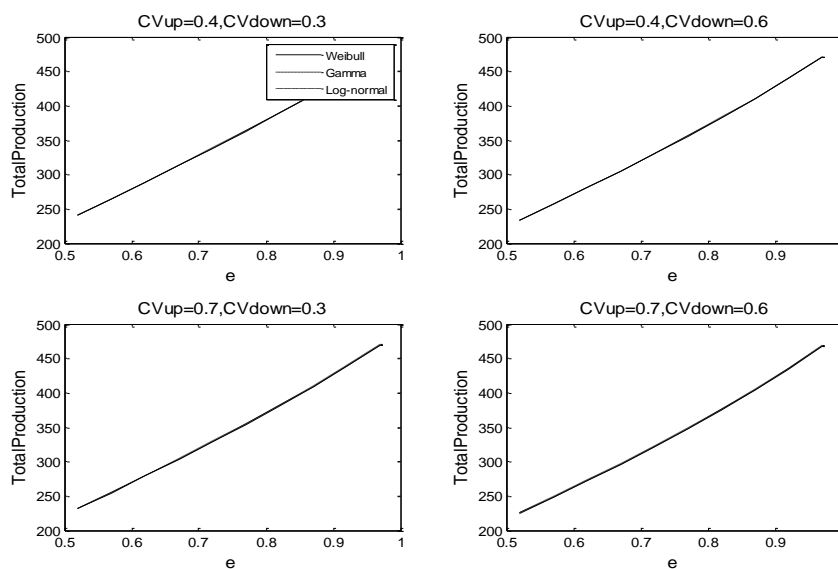
Parameters	e	M	K	T_{down}	CV_{up}	CV_{down}
Value	[0.52,0.57,0.62,0.67,0.72,0.77,0.82,0.87,0.92,0.97]	3	3	[5,10,15]	[0.4,0.7]	[0.3,0.6]

Analysis of TP: Figure 34 shows the graphs of TP vs. e for various CV_{up} , CV_{down} , and

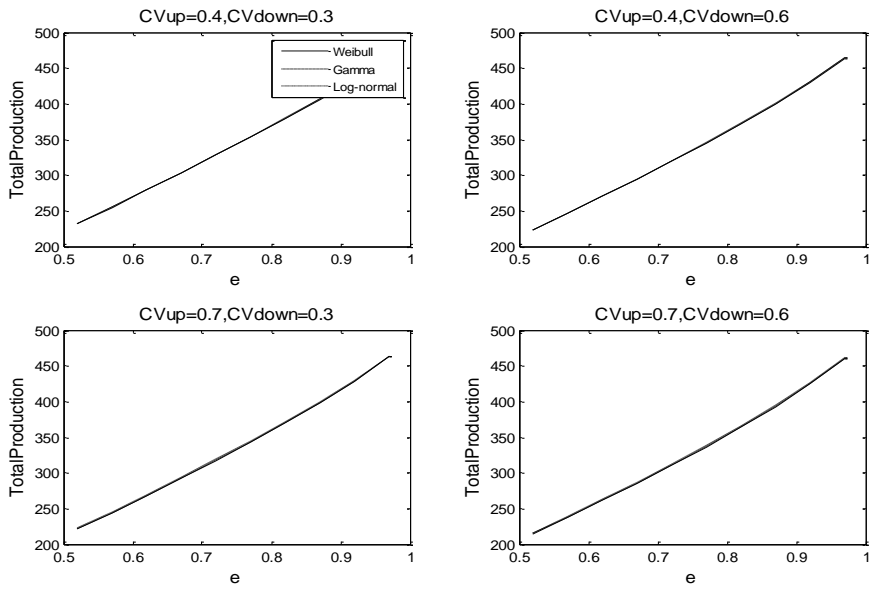
T_{down} .

T_{down}
5

M=3



10



15

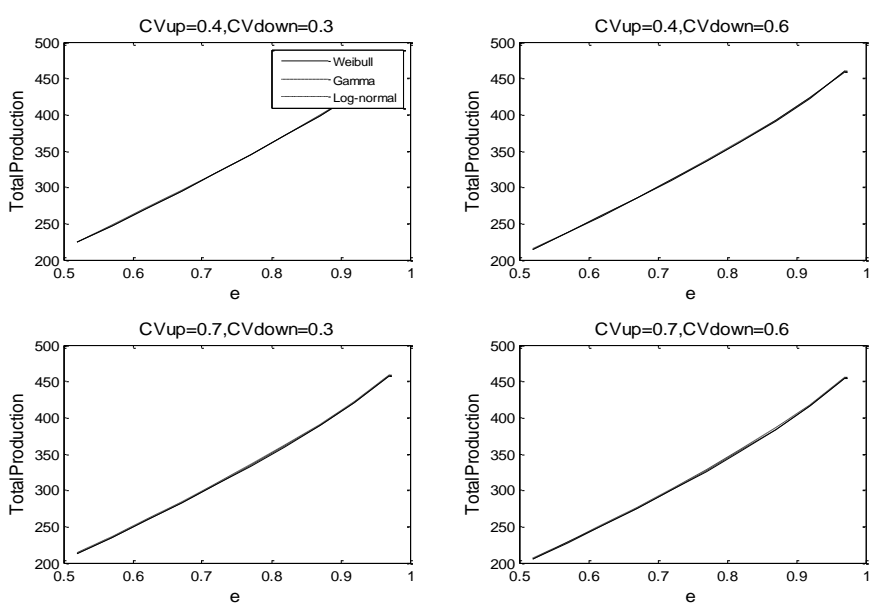


Figure 34 Effects of e on Total Production of Non-Exponential Lines

- 1) e has positive impact on TP. The relationship is close to linear, and is not affected by the variations of CVup, CVdown, and Tdown. The slope is almost the same.

4.3.2 Effects of K

In order to analyze the effects of K on TP, simulations are implemented with the following system parameters:

Table 28 Non-Exponential System Parameters (Effects of K on TP)

Parameters	E	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	3	[1,2,3,4,5]	[5,10,15]	[0.4,0.7]	[0.3,0.6]

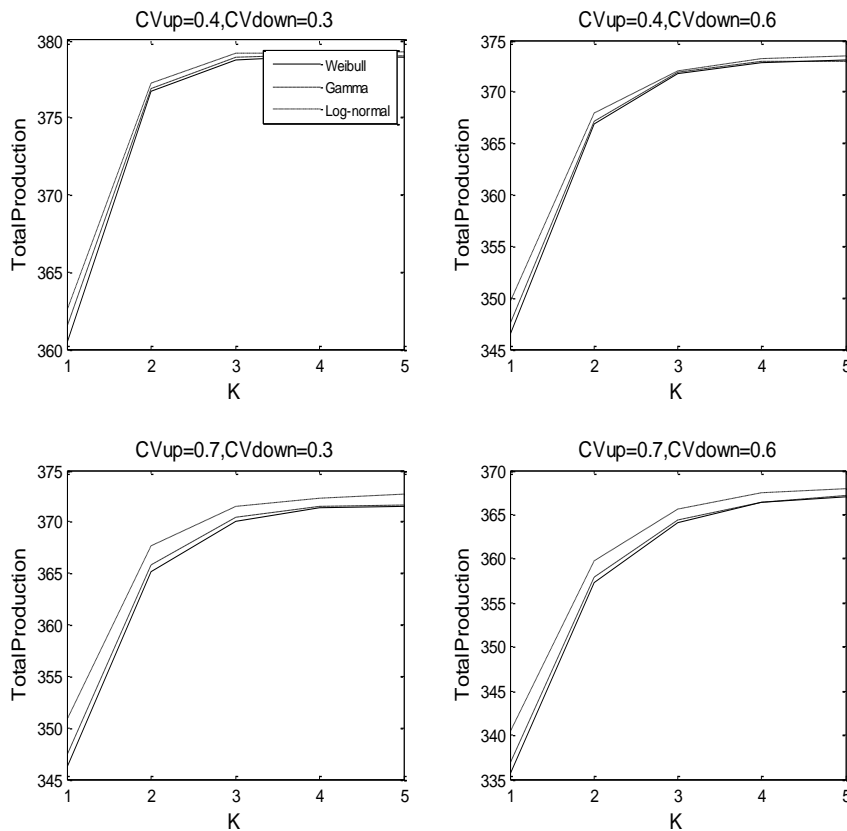
Analysis of TP: Figure 35 shows the graphs of TP vs. K for various CV_{up} , CV_{down} , and T_{down} .

- 1) K has positive impact on TP, larger K leads to a larger TP. However, TP saturates around K=2.

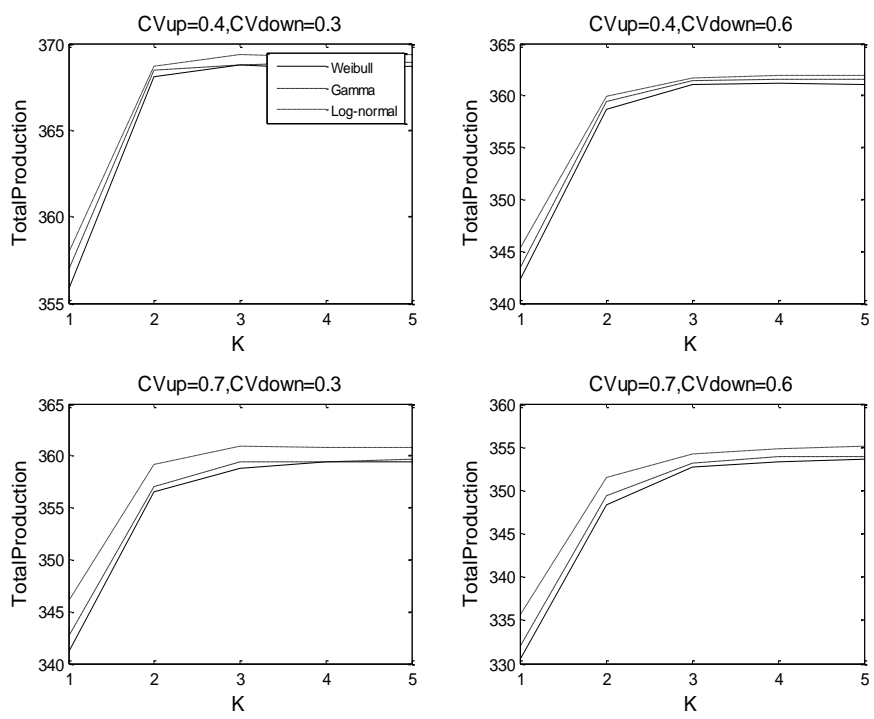
T_{down}

M=3

5



10



15

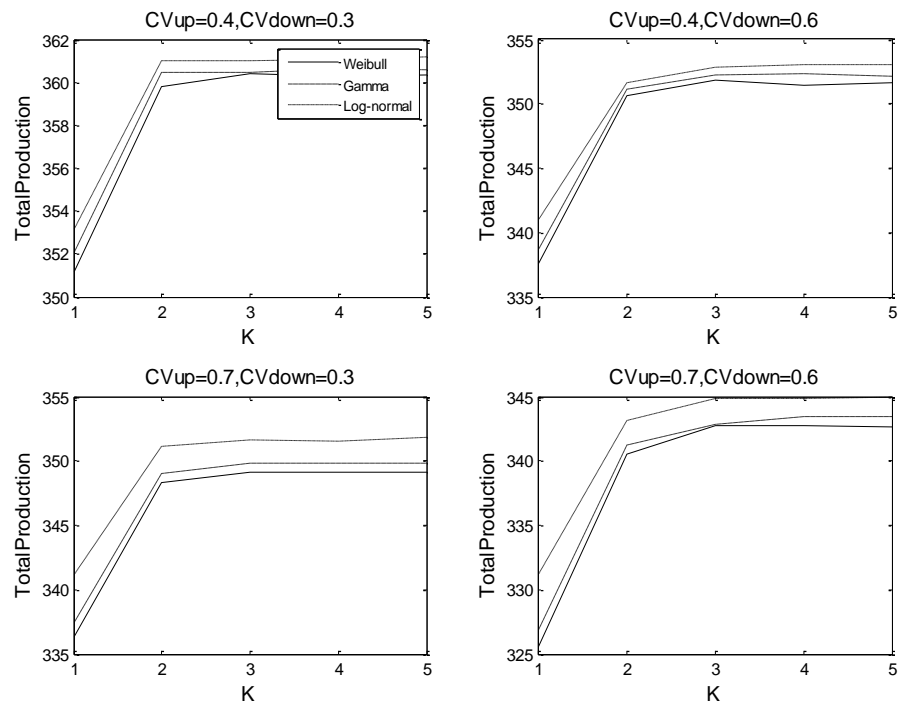


Figure 35 Effects of K on Total Production of Non-Exponential Lines

4.3.3 Effects of T_{down}

In order to analyze the effects of T_{down} on the TP, simulations are implemented with the following system parameters:

Table 29 Non-Exponential System Parameters (Effects of T_{down} on TP)

Parameters	E	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	3	[1,3,5]	[5, 7, 9, 11, 13, 15, 17,19]	[0.4,0.7]	[0.3,0.6]

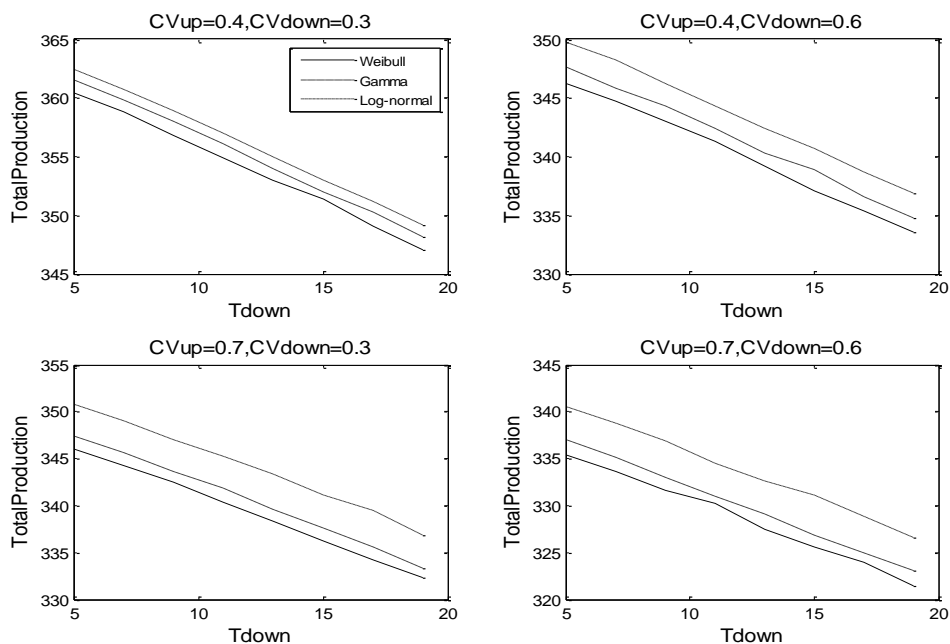
Analysis of TP: Figure 36 shows the graphs of TP vs. T_{down} for various K.

- T_{down} has negative impact on TP, larger T_{down} leads to smaller TP. The relationship is linear.
- Increase of CV_{up} , CV_{down} results in a smaller TP at the same T_{down} , and a shifted-down TP vs. T_{down} curve. In contrast, increase of K results in a larger TP at the same T_{down} , and the slope of TP vs. T_{down} curve increases.

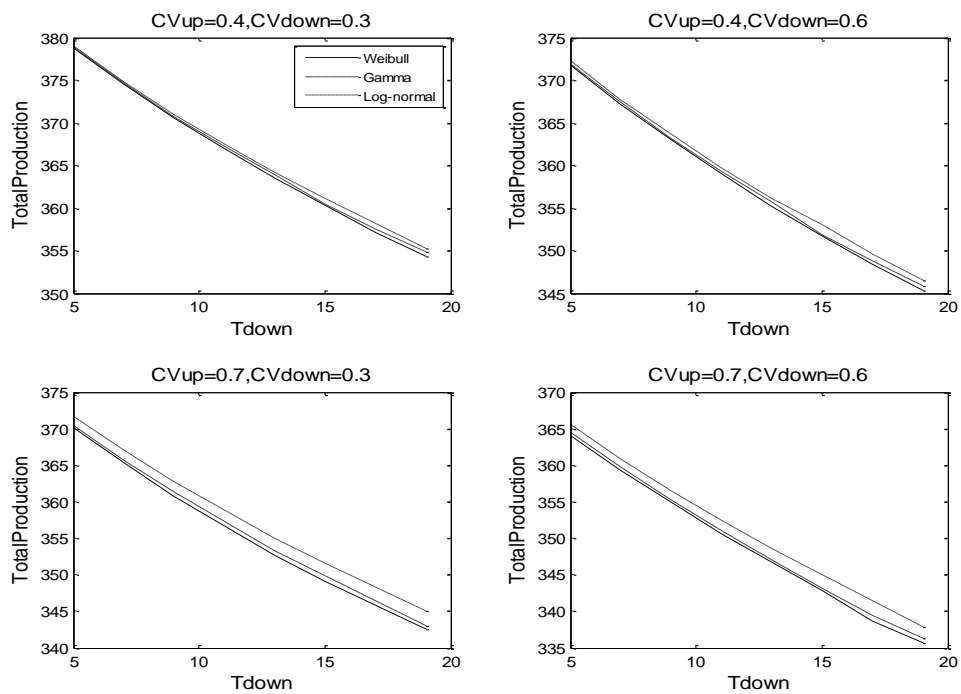
K

M=3

1



3



5

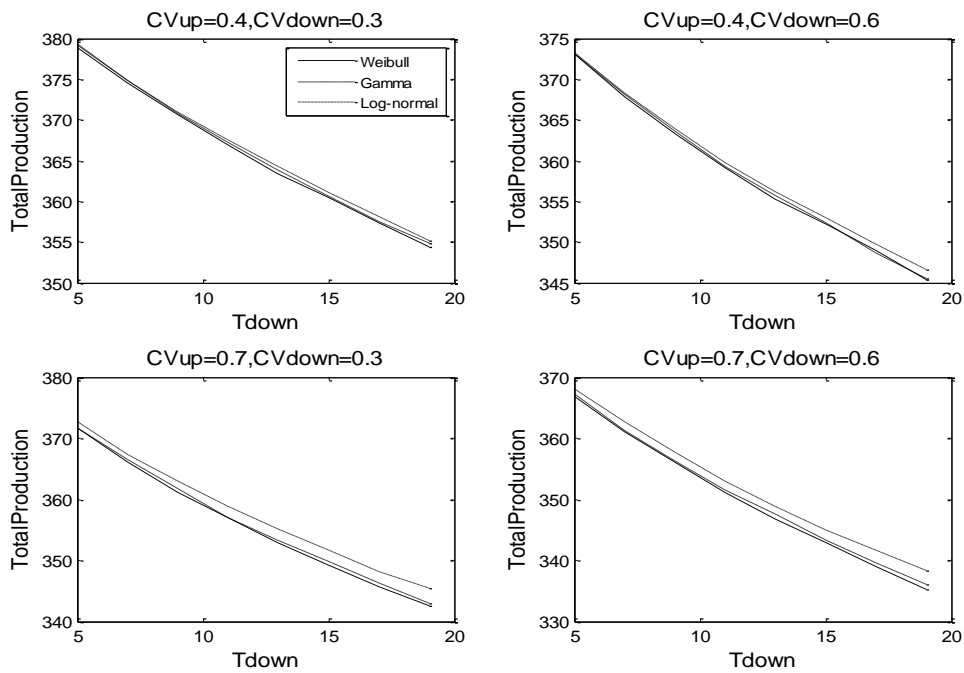


Figure 36 Effects of T_{down} on Total Production of Non-Exponential Lines

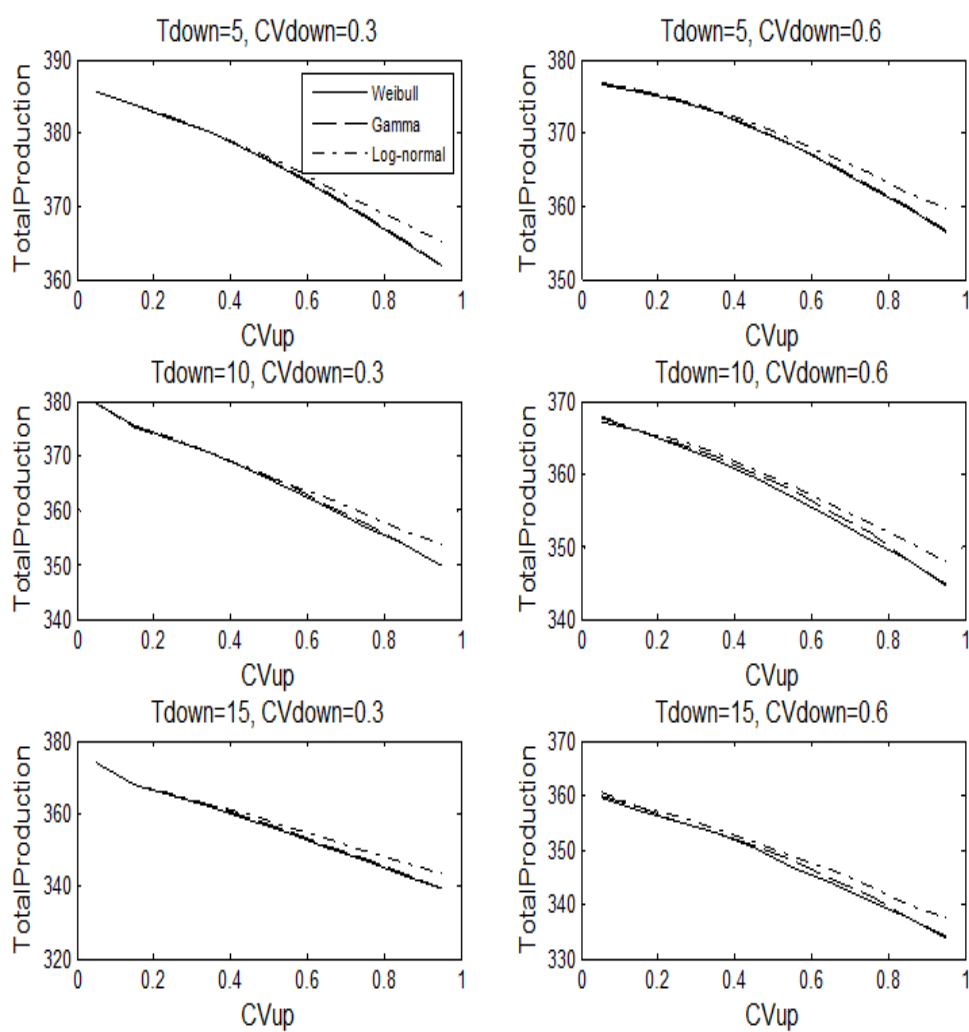
4.3.4 Effects of CV_{up}

In order to analyze the effects of CV_{up} on TP, simulations are implemented with the following system parameters:

Table 30 Non-Exponential System Parameters (Effects of CV_{up} on TP)

Parameters	e	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	[3,10]	3	[5,10,15]	[0.05,0.15,0.25,0.35,0.45,0.55,0.65,0.75,0.85,0.95]	[0.3,0.6]

M=3



M=10

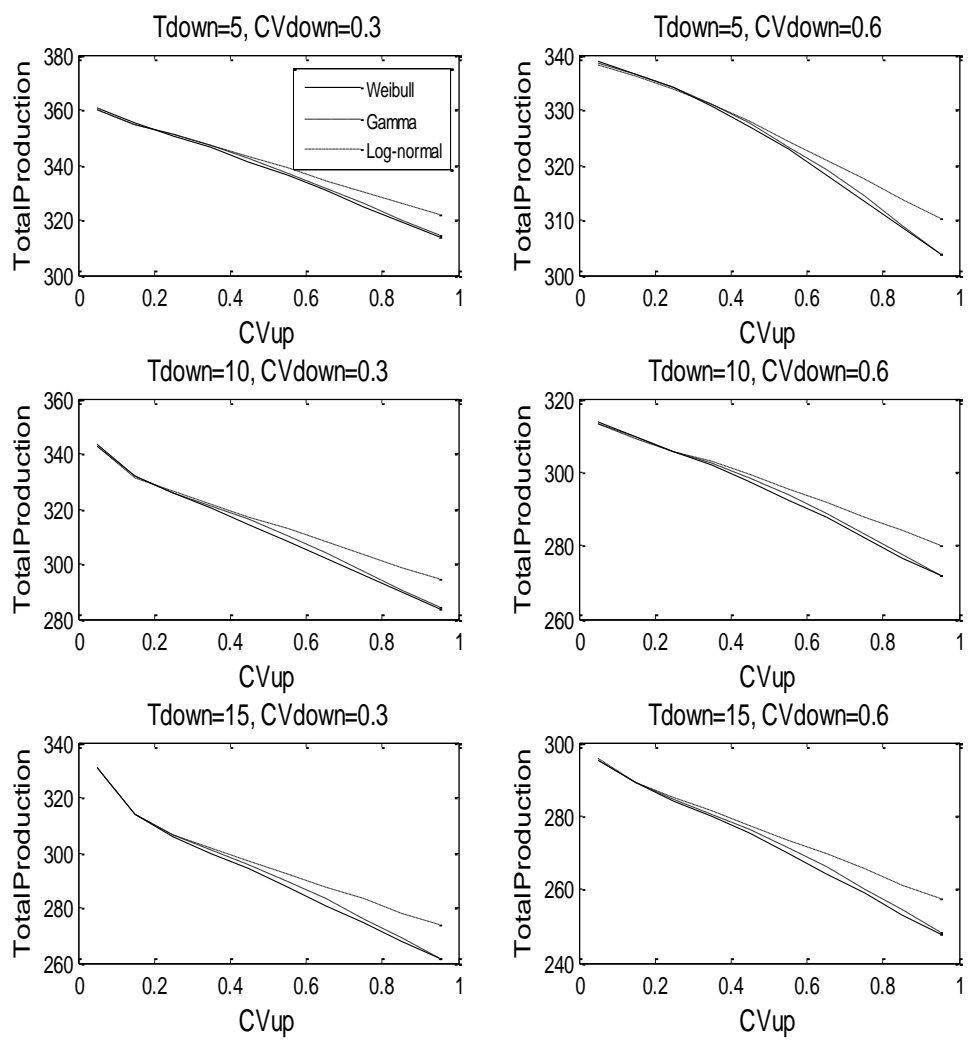


Figure 37 Effects of CV_{up} on Total Production of Non-Exponential Lines

Analysis of TP: Figure 37 shows the graphs of TP vs. CV_{up} for various CV_{down} and T_{down} .

- 1) CV_{up} has negative impact on TP, larger CV_{up} leads to a smaller TP. The relationship is close to linear.
- 2) Increase of CV_{down} or T_{down} results in a smaller TP at the same CV_{up} , and a shifted-down TP vs. CV_{up} curve.

4.3.5 Effects of CV_{down}

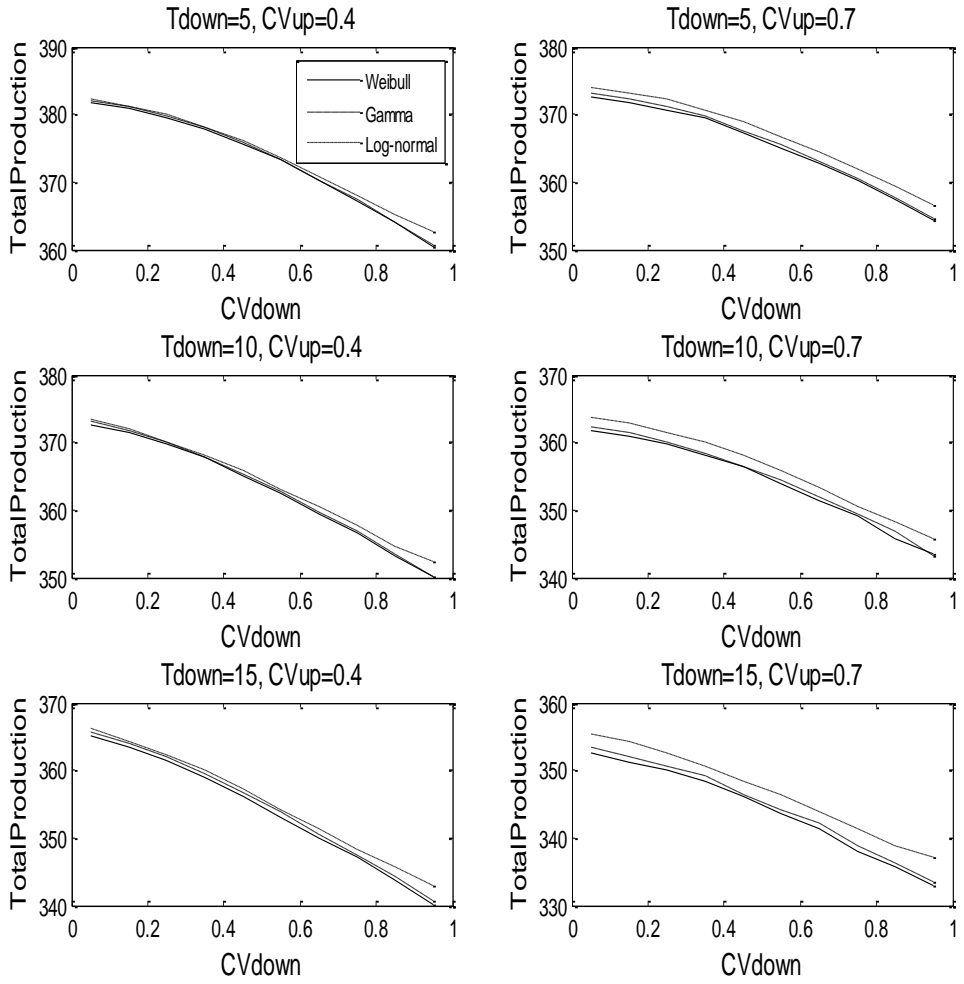
In order to analyze the effects of CV_{down} on TP, simulations are implemented with the following system parameters:

Table 31 Non-Exponential System Parameters (Effects of CV_{down} on TP)

Parameters	e	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	3	[3,10]	[5,10,15]	[0.4,0.7]	[0.05,0.15,0.25,0.35,0.45,0.55,0.65,0.75,0.85,0.95]

Figure 38 shows the graphs of TP vs. CV_{down} for various CV_{up} and T_{down} .

M=3



M=10

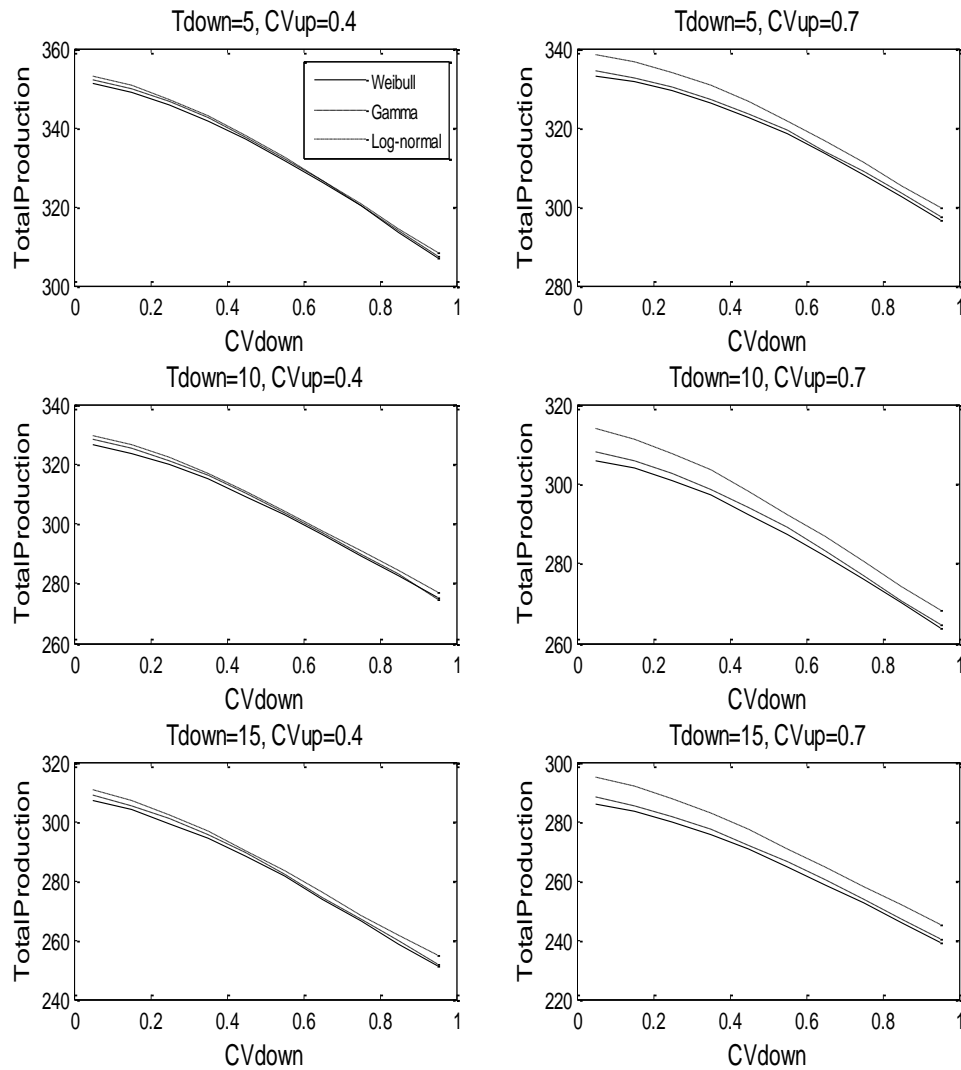


Figure 38 Effects of CV_{down} on Total Production of Non-Exponential Lines

- 1) CV_{down} has negative impact on TP, larger CV_{down} leads to a smaller TP. The relationship is close to linear.
- 2) Increase of CV_{up} or T_{down} results in a smaller TP at the same CV_{down} , and a shifted-down TP vs. CV_{down} curve.

4.4 Production Loss

The analysis in this section will investigate the impact on PL due to system parameters of serial production line with machine reliability model satisfying Weibull, Gamma, and Log-Normal distribution, respectively.

4.4.1 Effects of e

In order to analyze the effects of efficiency (e) on PL, simulations are implemented with the following system parameters:

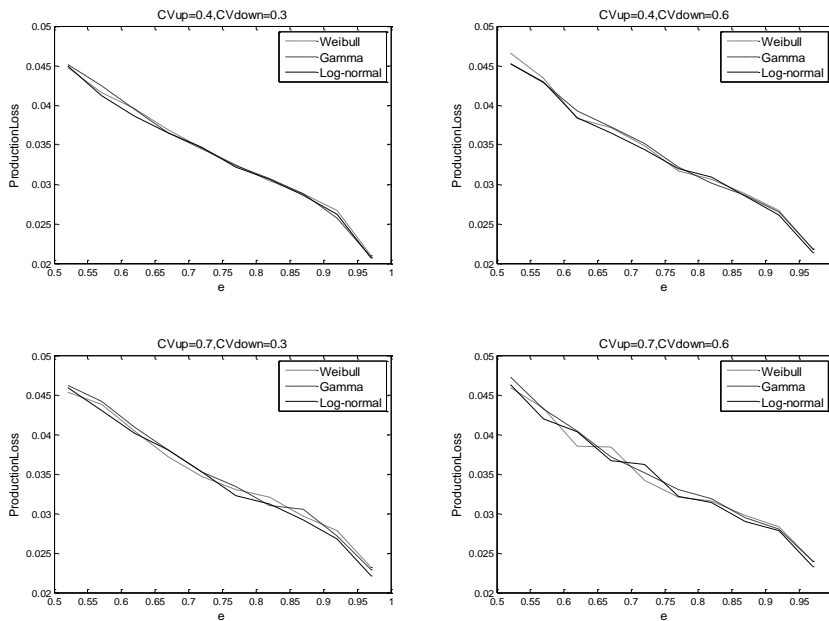
Table 32 Non-Exponential System Parameters (Effects of e on PL)

Parameters	e	M	K	T _{down}	CV _{up}	CV _{down}
Value	[0.52,0.57,0.62,0.67,0.72,0.77,0.82,0.87,0.92,0.97]	3	3	[5,10,15]	[0.4,0.7]	[0.3,0.6]

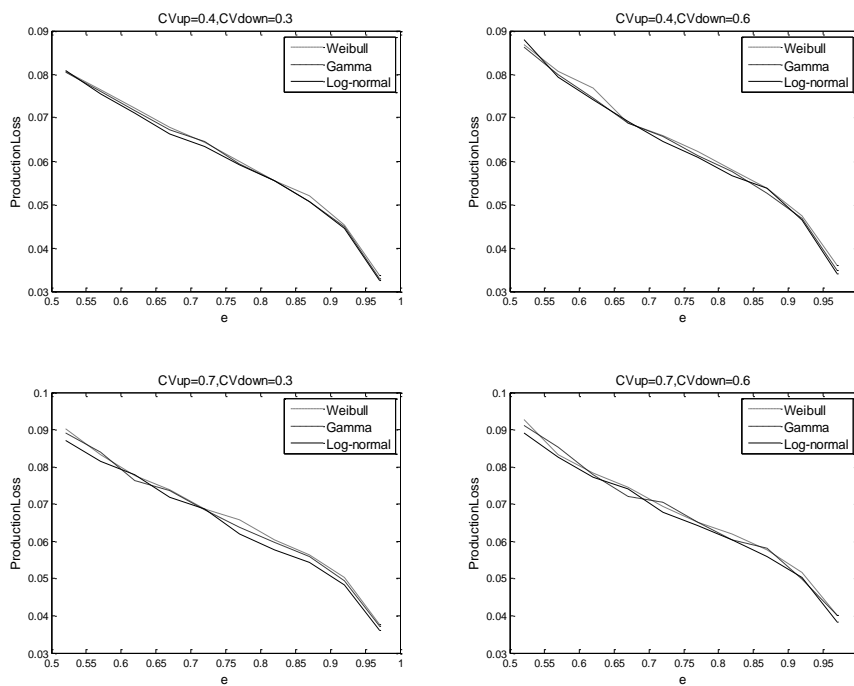
Analysis of PL: Figure 39 shows the graphs of PL vs. e for various CV_{up}, CV_{down} and T_{down}.

T_{down}
5

M=3



10



15

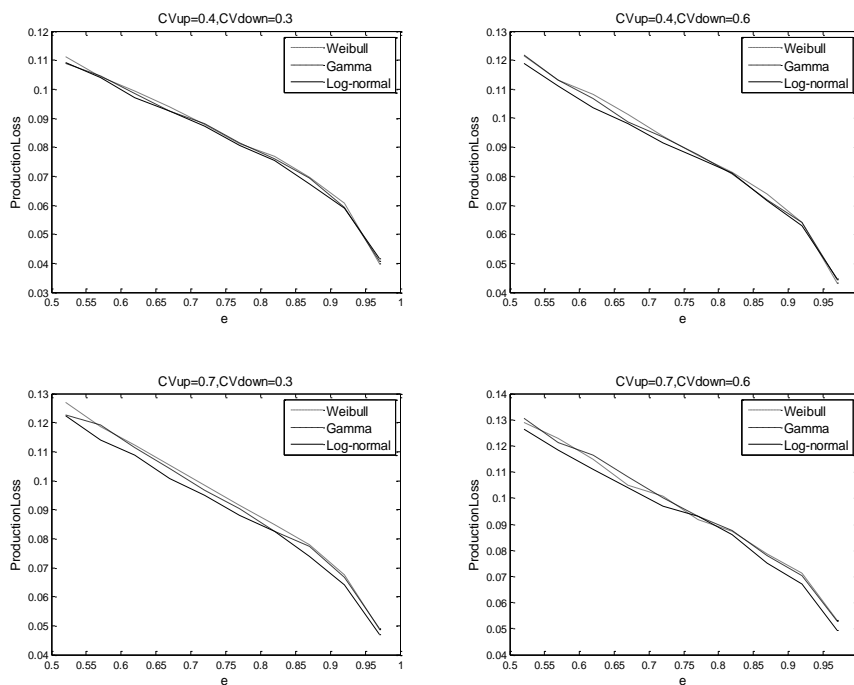
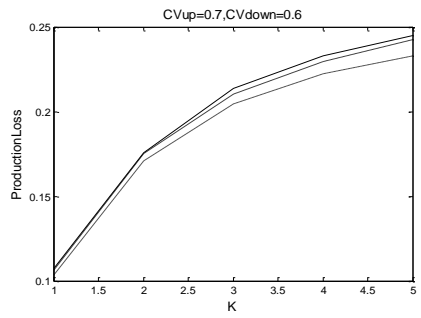
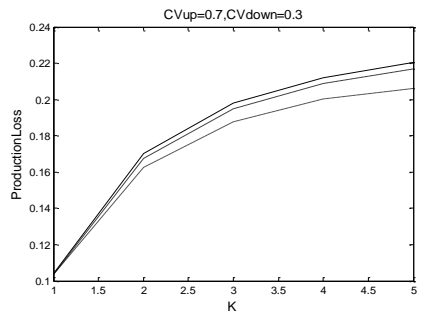
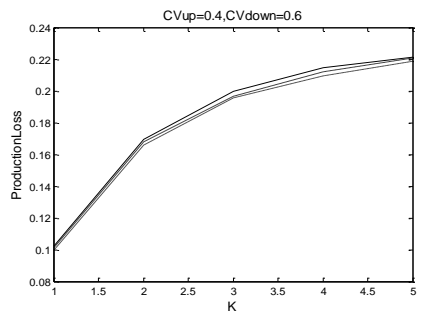
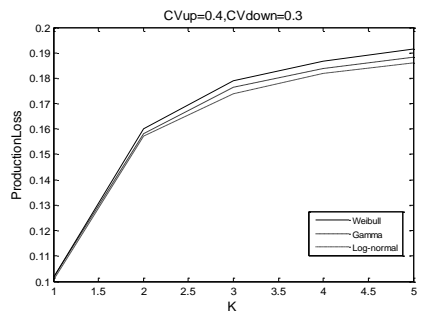


Figure 39 Effects of e on Production Loss of Non-Exponential Lines

10



15

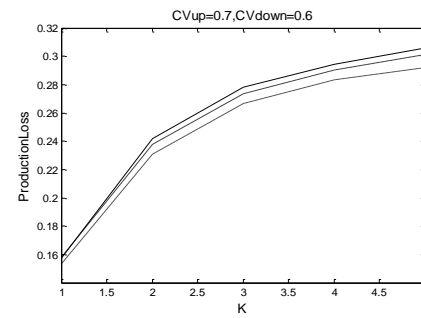
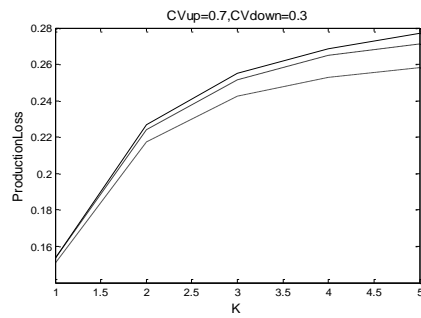
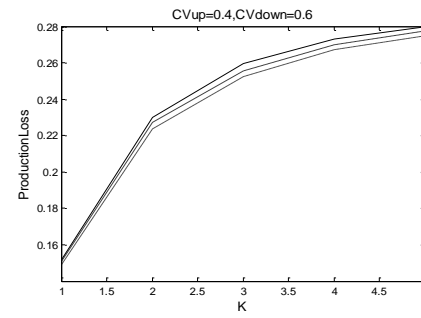
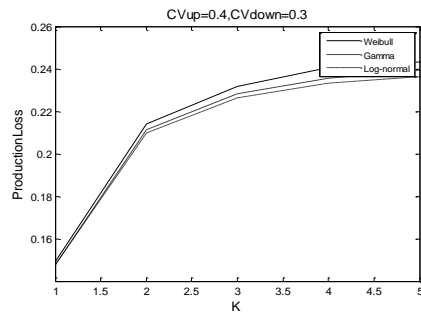


Figure 40 Effects of K on Production Loss of Non-Exponential Lines

- 1) K has positive impact on PL, larger K leads to a larger PL. The curve becomes saturated at larger K ; in other words, the curve slope decreases as K becomes larger.

4.4.3 Effects of T_{down}

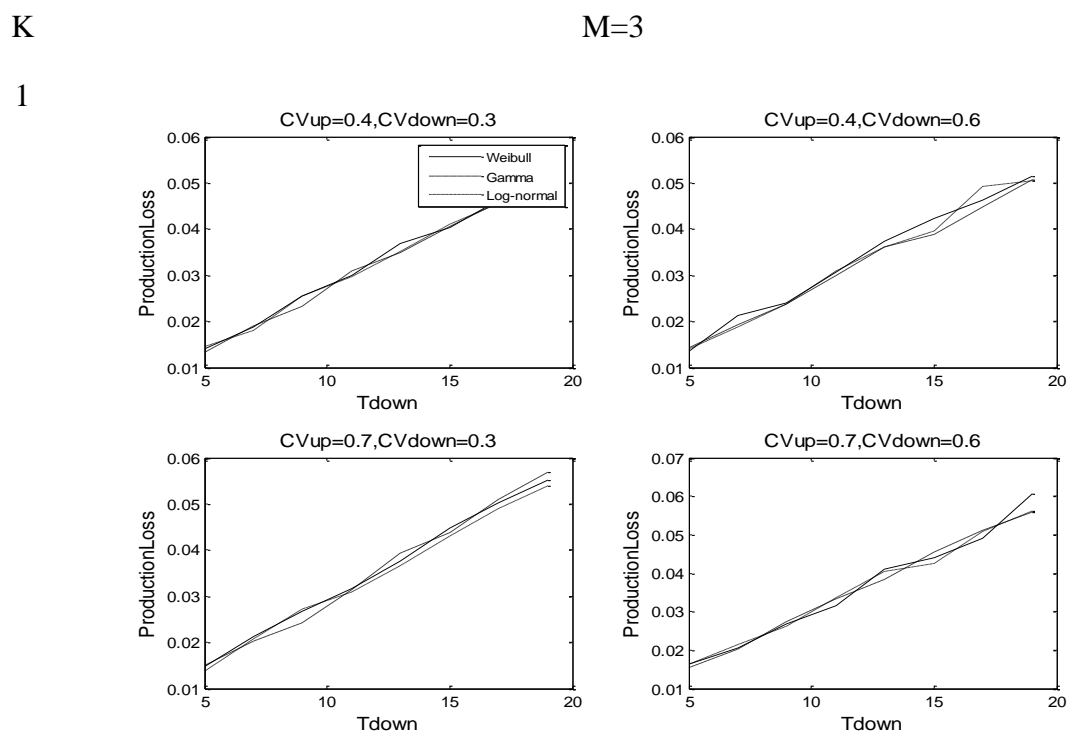
In order to analyze the effects of T_{down} on the PL, simulations are implemented with the following system parameters:

Table 34 Non-Exponential System Parameters (Effects of T_{down} on PL)

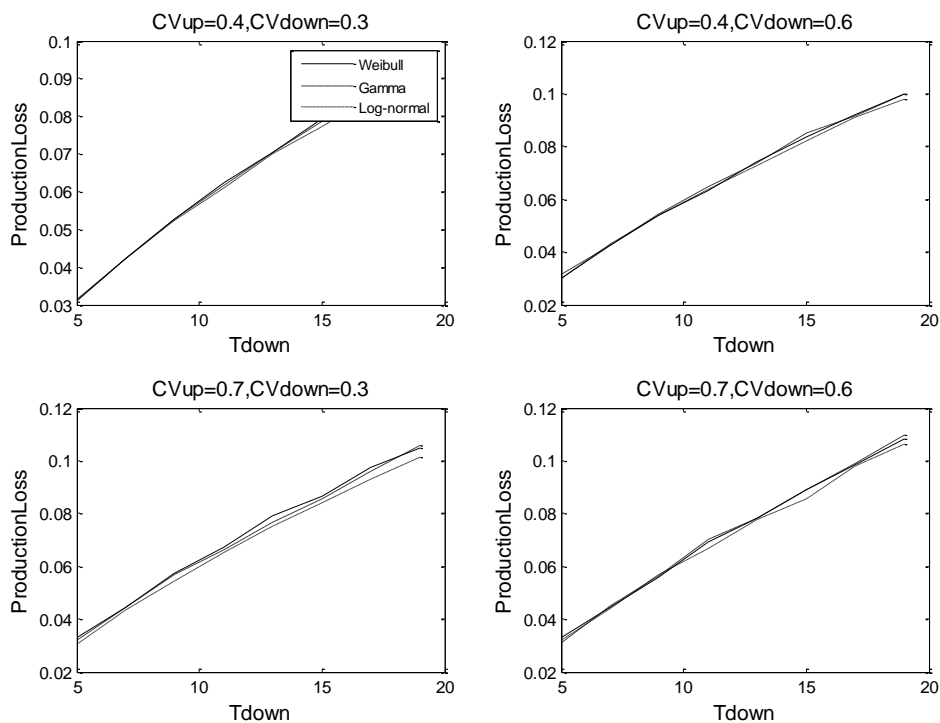
Parameters	e	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	3	[1,3,5]	[5, 7, 9, 11, 13, 15, 17,19]	[0.4,0.7]	[0.3,0.6]

Figure 41 shows the graphs of PL vs. T_{down} for various CV_{up} , CV_{down} , and K .

- 1) T_{down} has positive impact on PL, larger T_{down} leads to larger PL. The relationship is linear. The slope of the line increases when K increases.



3



5

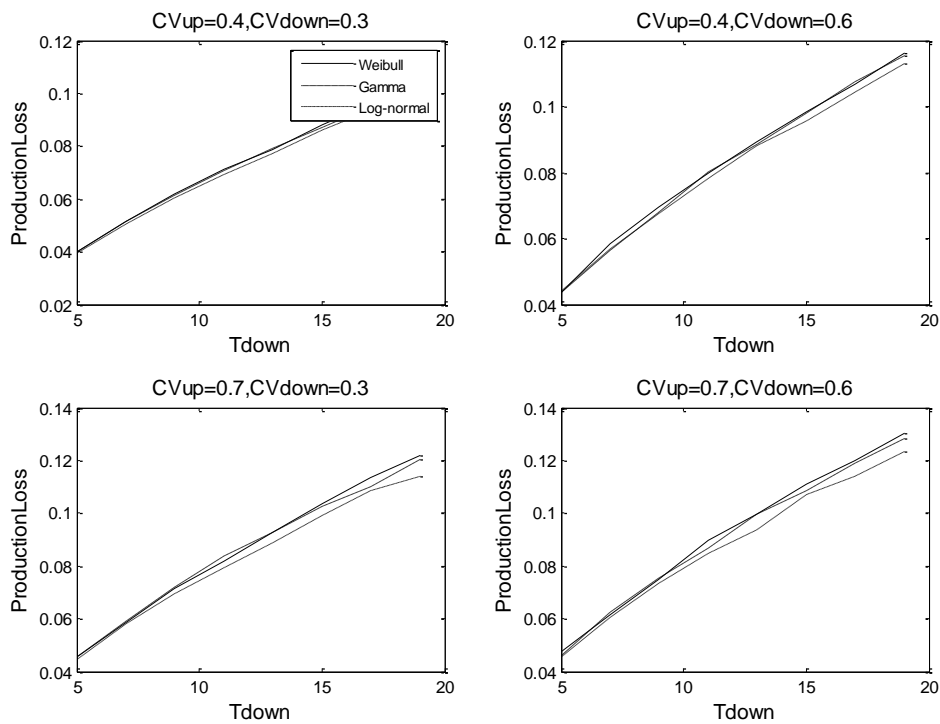


Figure 41 Effects of T_{down} on Production Loss of Non-Exponential Lines

4.4.4 Effects of CV_{up}

In order to analyze the effects of CV_{up} on PL, simulations are implemented with the following system parameters:

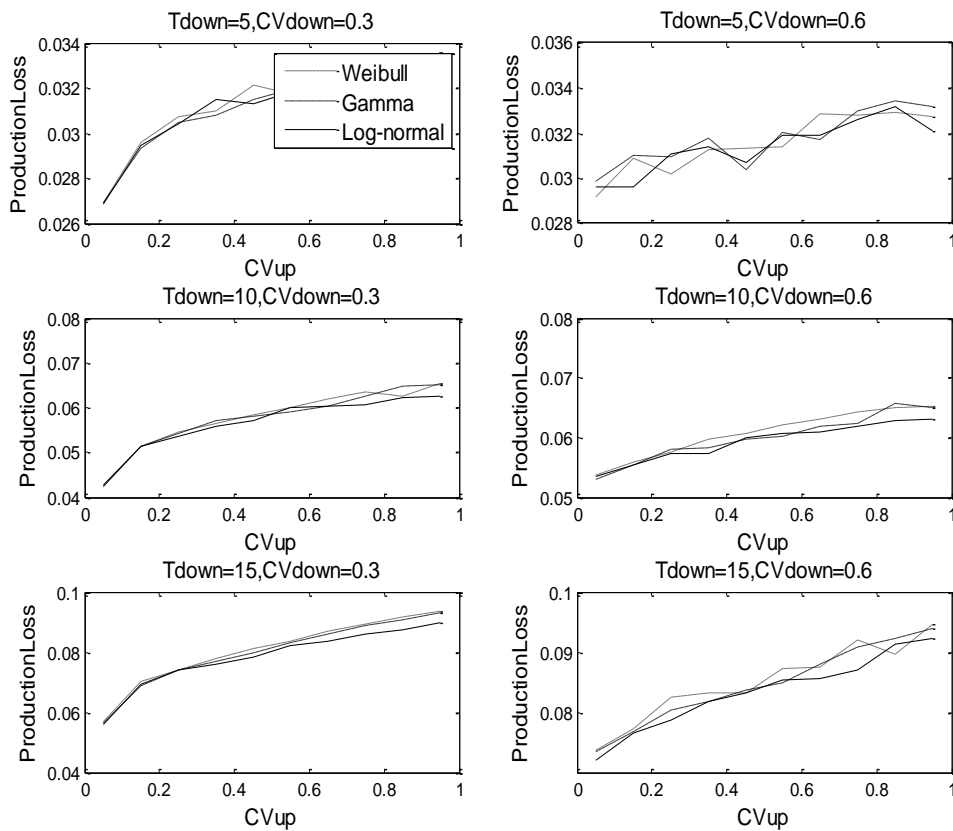
Table 35 Non-Exponential System Parameters (Effects of CV_{up} on PL)

Parameters	E	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	3	3	[5,10,15]	[0.05,0.15,0.25,0.35,0.45,0.55,0.65,0.75,0.85,0.95]	[0.3,0.6]

Figure 42 shows the graphs of PL vs. CV_{up} for various CV_{down} and T_{down} .

- CV_{up} has positive impact on PL, larger CV_{up} leads to a larger PL. The relationship is close to linear. When the number of machines increase, the linearity becomes more obvious.

M=3



M=10

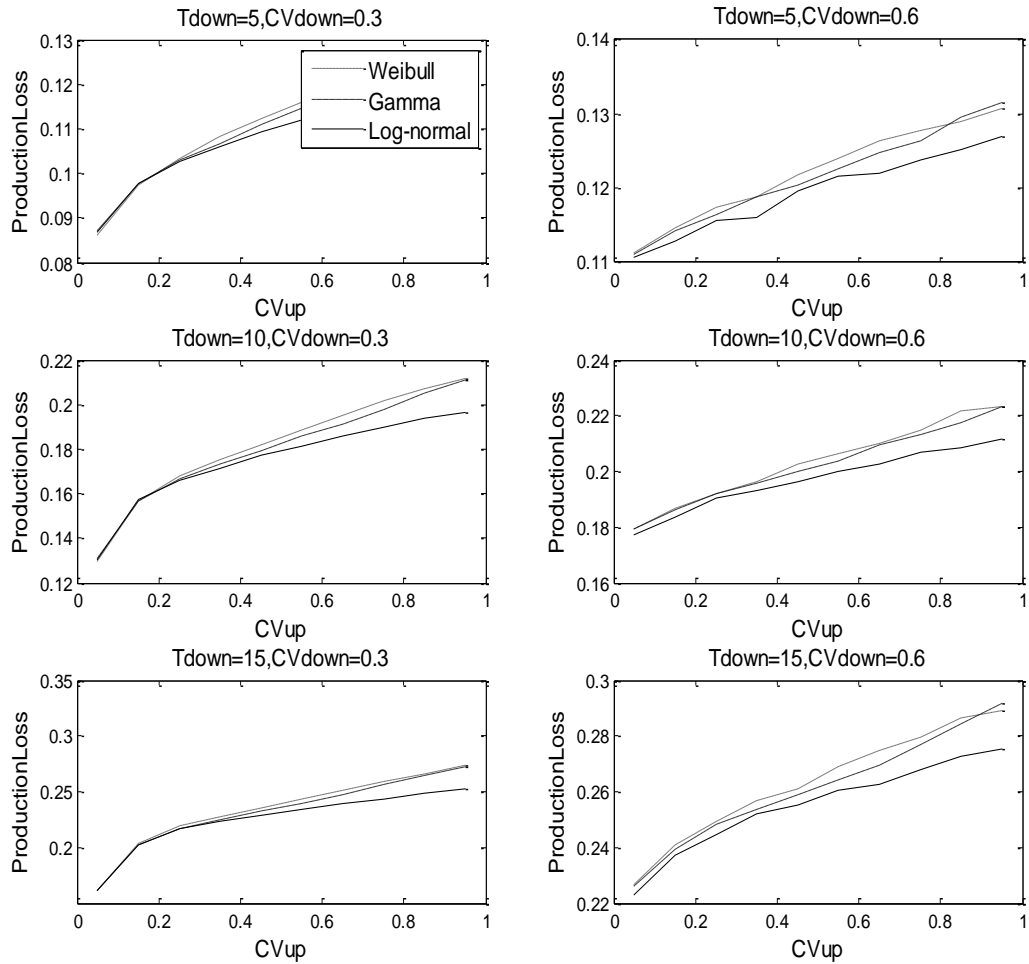


Figure 42 Effects of CV_{up} on Production Loss of Non-Exponential Lines

4.4.5 Effects of CV_{down}

In order to analyze the effects of CV_{down} on PL, simulations are implemented with the following system parameters:

Table 36 Non-Exponential System Parameters (Effects of CV_{down} on PL)

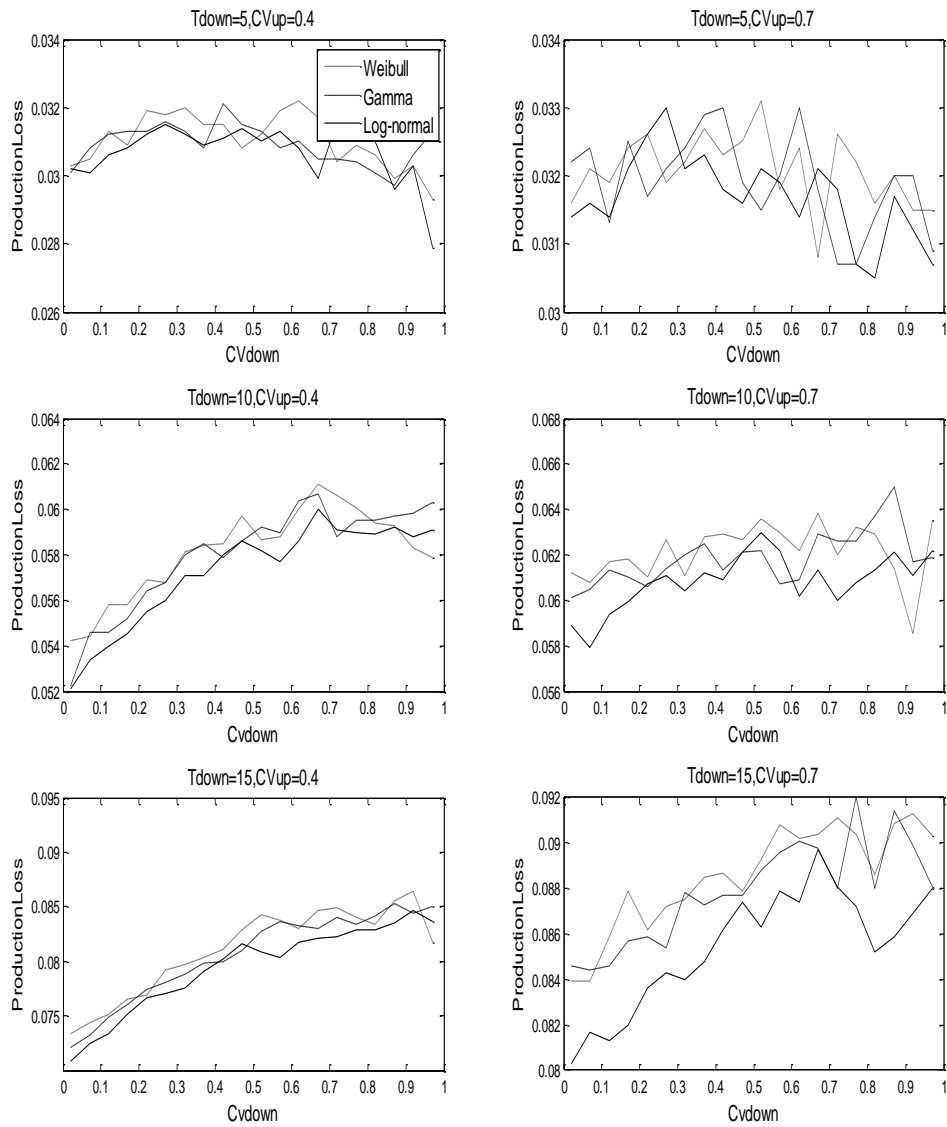
Parameters	E	M	K	T_{down}	CV_{up}	CV_{down}
Value	0.8	[3,10]	3	[5,10,15]	[0.4,0.7]	[0.05,0.15,0.25,0.35,0.45,0.55,0.65,0.75,0.85,0.95]

Figure 43 shows the graphs of PL vs. CV_{down} for various CV_{up} , T_{down} , and M .

The trends becomes more obvious when there are ten machines in the system.

1. CV_{down} has positive impact on PL, larger CV_{down} leads to a larger PL. The relationship is close to linear.

$M=3$



M=10

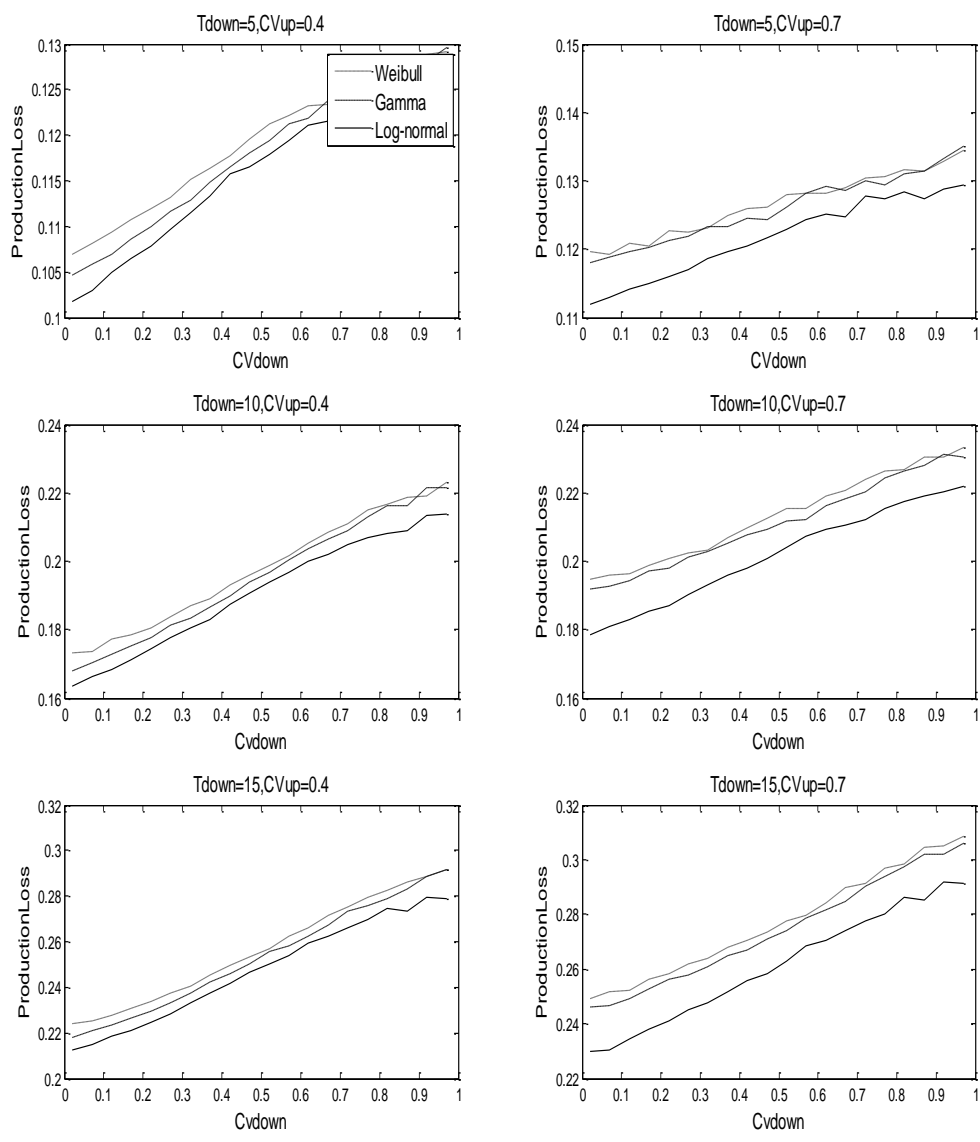


Figure 43 Effects of CV_{down} on Production Loss of Non-Exponential Lines

4.5 Summary

This chapter investigates the transient serial production line with machines reliability model satisfying Weibull, Gamma, and Log-Normal distribution, respectively.

Simulations are implemented to analyze the effects of system parameters, including e , K , T_{down} , CV_{up} and CV_{down} on the transient performance measures settling time (t_{sPR} and t_{sWIP}), total production (TP) and production loss (PL).

Simulation results show that as long as system parameters are the same for the three continuous reliability models, there is no significant difference in performances regarding total production, production loss and settling time.

Based on the simulation results, the overall effects of system parameters are summarized in Table 37.

Table 37 Effects of system parameters (e , K , and T_{down}) on the t_{sPR} and t_{sWIP} , TP and PL

	t_{sPR}	t_{sWIP}	TP	PL
e	Negative	Convex function	Positive (linear)	Negative (linear)
K	Positive (linear)	Positive (slop increases)	Positive (saturated around $K=2$)	Positive (saturated)
T_{down}	Positive (linear)	Positive (linear)	Negative (linear)	Positive (linear)
CV_{up}	Negative (saturated around $CV_{\text{up}}=0.25$)	Negative (linear)	Negative (linear)	Positive (linear)
CV_{down}	No significant impact	Negative (linear)	Negative (linear)	Positive (linear)

Comparing the results in Table 37 with Table 15, for continuous serial production line with machine reliability models of Weibull, Gamma and Log-normal, the effects of system parameters (e , K and T_{down}) on performance t_{sPR} , t_{sWIP} , TP and PL are almost the same as the effects in Exponential model. Please refer to Section 3.6 for the detailed conclusions for Exponential model. The only difference is that, for serial production line with Weibull, Gamma and Log-normal machine models, K has a linear positive impact

on t_{sPR} , so reducing K does accelerate the setting time t_{sPR} . Whereas in Exponential model, K does not have significant impact on t_{sPR} .

Therefore, the conclusions in Section 3.6 for exponential serial production line also apply to Weibull, Gamma and Log-normal serial production lines. These general conclusions based on the simulation results in both Chapter 3 and Chapter 4 provides guidelines for a real production system to improve the system performance even though the real machine reliability model is unknown.

Moreover, for serial production line with identical machines reliability model that satisfies Weibull, Gamma and Log-normal. The following conclusions are obtained from the results.

- 1) CV_{up} and CV_{down} have linear effect of t_{sWIP} , TP and PL. Smaller CV_{up} or CV_{down} leads to larger TP, smaller PL but longer t_{sWIP} .
- 2) CV_{up} has negative impact on t_{sPR} , t_{sPR} saturate around $CV_{up} = 0.25$. CV_{down} does not have significant impact on t_{sPR} .

Chapter 5 Conclusions and Future Works

This dissertation investigates the effects of system parameters on performance measures for transient serial production line with multiple identical machines having continuous reliability and identical buffers with finite capacity. The reliability models investigated are continuous machine reliability models, including exponential and no-exponential (Weibull, Gamma, Log-normal). The system parameters include machine efficiency e , ratio of N and $T_{\text{down}}(K)$, machines' average downtime T_{down} , and coefficient of variation CV on different performance measures. The performance measures include settling time of production rate (t_{sPR}), settling time of work-in-process (t_{sWIP}), total production (TP), production loss (PL).

General Effect of e , K , T_{down} and CV (apply to all continuous models considered in this work)

Table below shows the relationship between the performance measures and system parameters.

1. Simulation results show that as long as system parameters are the same for the three continuous reliability models, there is no significant difference in performances regarding total production, production loss and settling time.
2. For continuous serial production line with machine reliability models of Weibull, Gamma and Log-normal, the effects of system parameters (e , K and T_{down}) on performance t_{sPR} , t_{sWIP} , TP and PL are almost the same as the effects in Exponential model.

	t_{sPR}	t_{sWIP}	TP	PL
e	Negative	Convex function	Positive (linear)	Negative (linear)
K	Positive (linear) for Non-Exponential line	Positive (slop increases)	Positive (saturated around K=2)	Positive (saturated)
	No significant impact for Exponential line			
T_{down}	Positive (linear)	Positive (linear)	Negative (linear)	Positive (linear)
CV_{up}	Negative (saturated around $CV_{up} = 0.25$)	Negative (linear)	Negative (linear)	Positive (linear)
CV_{down}	No significant impact	Negative (linear)	Negative (linear)	Positive (linear)

3. Depending on availability and cost of resources, decision makers will be able to give fast response on the directions to improve system performance in terms of TP, PL and settling time of PR and WIP.

1) If the optimization goal is to reduce settling time t_{sPR} , there are four ways:

- Increasing e;
- Reducing K for Non-Exponential machine line;
- Reducing T_{down} ;
- Increasing CV_{up} .

2) If the optimization goal is to reduce settling time t_{sWIP} , there are four ways:

- Selecting an appropriate e;
- Reducing T_{down} ;
- Reducing K;

- Increasing CV.
- 3) If the optimization goal is to increase TP, there are four ways:
- Increasing e ;
 - Increasing K can be effective when $K \leq 2$ but not significant when $K > 2$.
 - Reducing T_{down} ;
 - Reducing CV.
- 4) If the optimization goal is to reduce PL, there are four ways:
- Increasing e ;
 - Reducing K ;
 - Reducing T_{down} ;
 - Reducing CV.

The future work of this research includes:

- 1) Quantitatively analyze the effects of system parameters to system performance in continuous production lines;
- 2) Extend the analysis to serial production line with other machine numbers, such as five machines, in order to justify the conclusion got from this research working generally.
- 3) Change initial condition of machines and buffers, to get a more general working rule.
- 4) Quantitatively analyze the effects of system parameters to system performance in continuous production lines;

- 5) Extend the analysis to systems with other machine reliabilities, such as Rayleigh and Erlang.
- 6) Extend the analysis to systems with non-identical machines and buffers.

Bibliography

- [1] J. Li, and S. M. Meerkov (2009), *Production Systems Engineering*, Springer.
- [2] J. A. Buzacott and J. G. Shanthikumar, *Stochastic Models of Manufacturing Systems*. Prentice Hall, Englewood Cliff, NJ, 1993.
- [3] H. T. Papadopoulos, C. Heavy, and J. Browne, *Queueing Theory in Manufacturing Systems Analysis and Design*. Chapman & Hill, London, UK, 1993.
- [4] S. B. Gershwin, *Manufacturing Systems Engineering*. Prentice Hall, Englewood Cliff, NJ, 1994.
- [5] D. D. Yao, *Stochastic Modeling and Analysis of Manufacturing Systems*. Springer-Verlag, New York, NY, 1994.
- [6] T. Altiok, *Performance Analysis of Manufacturing Systems*. Springer-Verlag, New York, NY, 1997.
- [7] Y. Dallery and S. B. Gershwin, “Manufacturing flow line systems: a review of models and analytical results,” *Queueing Systems: Theory and Applications*, vol. 12, no. 1–2, pp. 3–94, 1992.
- [8] H. T. Papadopoulos and C. Heavy, “Queueing theory in manufacturing systems analysis and design: A classification of models for production and transfer lines,” *European Journal of Operational Research*, vol. 92, no. 1, pp. 1–27, 1996.
- [9] S. M. Meerkov and L. Zhang, ‘Transient behavior of serial production lines with Bernoulli machines’, *IIE Transactions* 40(3), 297–312, 2008
- [10] G. Chen, L. Zhang, J. Arinez and S. Biller, “Feedback control of machine startUp for energy- efficient manufacturing in Bernoulli serial lines”,

Proceedings of 7th IEEE Conference on Automation Science and Engineering, Trieste, Italy, pp. 666–671, 2011

- [11] S.-Y. Chiang, C.-T. Kuo and S. M. Meerkov, “Bottlenecks in Markovian production lines: a systems approach”, *IEEE Transactions on Robotics and Automation* 14(2), 352–359, 1998
- [12] S.V. Volsema, W. Dullaert and H.V. Landeghem (2007), “An evolutionary algorithm and discrete event simulation for optimizing inspection strategies for multi-stage processes”, *European Journal of Operational Research* 179(3), 621–633.
- [13] R. Alvarez-Vargas, Y. Dallery and R. David, “A study of the continuous-flow model of production lines with unreliable machines and finite buffers”, *Journal Of Manufacturing Systems*, 13(3), 221–234, 1994.
- [14] S. Ching, S. M. Meerkov and L. Zhang, “Assembly systems with non-exponential machines: throughput and bottlenecks”, *Nonlinear Analysis* 69(3), 911–917, 2008.
- [15] T. Baris, “Effects of variability on the due-time performance of a continuous materials flow production system in series”, *International Journal of Production Economics* 54, 87–100, 1998.
- [16] D. Jacobs and S. M. Meerkov (1995), “Mathematical theory of improvability for production systems”, *Mathematical Problems in Engineering* 1(2), 95–137.

- [17] B. Tan and S. Yeralan (1997), “A decomposition model for continuous materials flow production systems”, *International Journal of Production Research* 35(10), 2759–2772.
- [18] J. Li and S. M. Meerkov (1995), “Evaluation of throughput in serial production lines with non-exponential machines”, in E. K. Boukas and R. Malhame, eds, “Analysis Control and Optimization of Complex Dynamic Systems”, Kluwer Academic Publisher, chapter 4, pp. 55–82.
- [19] S. M. Meerkov, N. Shimkin and L. Zhang (2010), “Transient behavior of two-machine geometric production lines”, *IEEE Transactions on Automatic Control* 55(2), 453–458.
- [20] T. Baris and G. Stanley, “Analysis of a general markovian two-stage continuous-flow production system with a finite buffer”, *International Journal of Production Economics* 120, 327–339, 2009.
- [21] S. Bruno, “A finite buffer fluid queue driven by a markovian queue”, *Queueing Systems* 38, 213–220, 2001
- [22] D. S. Kim, D. M. Kulkarny and F. Lin (2002), “An upper bound for carriers in a three-workstation closed serial production system operating under production blocking”, *IEEE Transactions on Automatic Control* 47(7), 1134–1138.
- [23] L. Zhang, “System-theoretic properties of production lines”, Dissertation submitted in the University of Michigan, 2009.

- [24] M. Colledani, and T. Tolio, “Impact of statistical process control (SPC) on the performance of production systems”, Proceedings of 5th International Conference on the Analysis of Manufacturing Systems, pp. 77–92, 2005.
- [25] S. B. Gershwin, and M. H. Burman (2000), “A decomposition method for analyzing inhomogeneous assembly/disassembly systems”, Annals of Operations Research 93(1-4), 91–115.
- [26] J. Li, (2005), “Overlapping decomposition: A system-theoretic method for modeling and analysis of complex manufacturing systems”, IEEE Transactions on Automation Science and Engineering, 2(1), 40–53.
- [27] N. Viswanadham and Y. Narahari (1992), Performance Modeling of Automated Manufacturing Systems, Prentice Hall, Englewood Cliff, NJ.
- [28] L. Zhang, C. Wang, J. Arinez and S. Biller (2013), “Transient analysis of Bernoulli serial lines: Performance evaluation and system-theoretic properties”, IIE Transactions 45(5), 528–543.