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An Investigation into Vaccination Behavior: Parametrization of a Samoan Vaccine Scare

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AN INVESTIGATION INTO VACCINATION BEHAVIOR: PARAMETRIZATION OF A SAMOAN VACCINE SCARE

by

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ABSTRACT

AN INVESTIGATION INTO VACCINATION BEHAVIOR: PARAMETRIZATION OF A SAMOAN VACCINE SCARE

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Under the Supervision of Professor Gabriella Pinter

Vaccination behavior can be influenced by many factors. Some examples are vaccine scares, evolutionary game theory, social learning such as media coverage, feedback in the form of infectious cases, and herd immunity. We investigated a previously published model that attempts to explain vaccination behavior based on a game theoretic point of view. The model was applied to a large vaccine scare in the country of Samoa, and a parameter estimation problem was solved for different risk perception scenarios. It was found that the model fit best in the case of no social learning and no feedback. However, adding in these factors did not compromise the models' accuracy. These results confirm that while social learning and feedback may not completely describe vaccinating behavior they are important factors in individuals' decisions to vaccinate or not.

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1 Introduction

Disease has plagued the world since the dawn of time. Age old remedies and medicine have been combating disease ever since. Vaccines, although a newer defense, have helped manage the spread of disease. Still, not all people can or do take advantage of vaccinations. Let us take an investigatory look into vaccinating behavior to help understand this phenomenon. We shall specifically focus on the vaccinating behavior with respect to the measles vaccine.

The measles is an RNA virus from the genus Morbillivirus of the family Paramyxoviridae. The virus is spread from skin to skin contact and through the air. The virus can also remain in the air for hours. Measles has a contact rate of nine out of ten, meaning for every ten people an infected person comes into contact with, nine will contract the disease, making the disease highly contagious. Symptoms of measles include fever, cough, and rash. It may take anywhere from seven to twenty-one days to develop the traditional rash. Therefore, an individual can have the measles and not show signs, which also leads to high infectious rates. Severe complications include permanent brain damage, seizures, or death [7]. The seriousness of the disease led John Enders to invent a vaccine and license it in 1963. Shortly after, in 1968, Maurice Hilleman created an improved vaccine that is still used today [8].

Hilleman's measles vaccine is usually given in combination with the mumps and rubella vaccines, or more recently with the varicella vaccine. The abbreviation for the measles, mumps, and rubella vaccine is MMR and MMRV when the varicella vaccine is included. The vaccine works best in two doses. The first dose is 93% effective and having two doses is approximately 97% effective. It is recommended to receive the first dose at one year of age and the second dose at ages four to six. This leaves children under one year of age susceptible. Also, those who have certain allergies, are pregnant, or have diseases such as HIV/AIDS or cancer that weaken the immune

system may not be able to get the MMR vaccine [9]. Those who are not able to get the vaccine depend on herd immunity to stay protected from the disease [13].

Herd immunity is the concept that if enough of the population is immune, a disease cannot spread, hence an epidemic cannot occur. The immunities may be from vaccination or contraction of the disease and recovery [13]. To better understand herd immunity, the traditional SIR model (susceptible-infected-recovered) may be used. The model is given by

$$\frac{ds}{dt} = -\beta si, \tag{1.1}$$

$$\frac{di}{dt} = \beta si - \gamma i, \tag{1.2}$$

$$\frac{dr}{dt} = \gamma i, \tag{1.3}$$

where $\beta = \frac{6205}{13}$ [1] is the infection rate, and $\gamma = \frac{365}{13}$ [1] is the recovery rate. Furthermore, to prevent an epidemic, the epidemiological threshold, R_0, must be less that one [13], i.e.,

$$R_0 = \frac{\beta s(0)}{\gamma + \delta} < 1, \tag{1.4}$$

where $\delta = 0.02$ [1] is the mortality rate per year and $s(0)$ is the fraction of the population that is initially susceptible. Solving for $s(0)$ yields

$$s(0) < \frac{\gamma + \delta}{\beta}. \tag{1.5}$$

Since

$$s(0) + i(0) + r(0) = 1, \tag{1.6}$$

where $i(0) = 0.0001$ [1] is the fraction of the population that is initially infected and $r(0)$ is the fraction of the population that is vaccinated or immune, we can obtain how

much of the population needs to be vaccinated to avoid the spread of the disease given the vaccine's effectiveness. With one dose of the MMR vaccine, the whole population would need to be vaccinated and the disease would still spread. However with two doses, approximately 97% of the population would need to be vaccinated to obtain herd immunity and prevent an epidemic.

Reaching the herd immunity threshold of 97% is a difficult task. There are some risks when getting the MMR vaccine, which may deter potential vaccinators. Risks include: fever, rash, swelling, seizure, joint pain, low platelet count, allergic reactions, deafness, or brain damage. The more severe side effects are quite rare. For example, only one out of 30,000 doses experience low platelet count [9]. Some individuals may view the risks of vaccination to be greater than the risks of contracting the disease. These views determine vaccinating behavior.

2 Previous Research

Previous research shows how fragile vaccinating behavior can be. Parents want the best for their children and choosing to vaccinate them or not is an important decision that has become a controversial matter. Media coverage, the severity of the disease, vaccine efficiencies, side effects, known infectious cases, herd immunity, and health influence one's vaccinating behavior [15], [1], [2].

2.1 Autism Vaccine Scare

In 1998, Andrew Wakefield et al. published a paper called *"Ileal-lymphoid-nodular hyperplasia, non-specific colitis, and pervasive developmental disorder in children.*" In this paper, Wakefield et al. claimed to have found a link between autism in children and the MMR vaccine [15]. Upon further investigation, it was found that Wakefield et al. had falsified data to get this result [3]. Brian Deer uncovered the truth and

said

> "*Wakefield 'chiseled' the data before him, falsifying medical histories of children and essentially concocting a picture, which was the picture he was contracted to find by lawyers hoping to sue vaccine manufacturers and to create a vaccine scare.*" [3].

As a result of this fraudulent paper, Wakefield lost his medical license in 2010 [3] and the paper has been retracted [3],[4].

Unfortunately, the damage has been done. Wakefield's false study has influenced parents to boycott vaccinations, specifically MMR. To them, the risk of autism exceeds the risk of getting the measles. This resulting vaccine scare has been going on for more than a decade from Wakefield's fraudulent research. I believe media coverage is partially to blame. Most people have heard of the supposed link between MMR and autism, however, they do not know that this study has been retracted, discredited, and Wakefield has lost his medical license. The effects of this scare can be traced in Table 2.1.

2.2 Vaccination Dilemma

In 2013, Cardillo et al. studied vaccination behavior in complex networks. They used an SEIR (susceptible-exposed-infected-recovered) model with evolutionary game theory components to model disease spreading. They wanted to examine the vaccine behavior from flu season to flu season. The choice to get vaccinated depended on the previous flu season. They found that when a flu vaccine was 100% effective, individuals were more likely to vaccinate in the next flu season. However, if the vaccine was not perfect, individuals were less likely to vaccinate and the number of infected individuals therefore increased [2].

Year	MMR Coverage
1996	91.5 %
1997	90.8 %
1998	88.3 %
1999	87.6%
2000	87.4 %
2001	84.1 %
2002	81.8 %
2003	79.9%
2004	80.9%
2005	84.1 %
2006	85.2%
2007	84.6 %
2008	84.9 %
2009	88.2 %
2010	89.1%
2011	91.2 %
2012	92.3%

Table 2.1: Vaccination coverage in England [12] after the Wakefield vaccine scare.

2.3 England and Wales Vaccine Scare

In 2012, Chris Bauch and Samit Bhattacharyya wrote a paper called *Evolutionary Game Theory and Social Learning Can Determine How Vaccine Scares Unfold*. They analyzed the MMR coverage data produced from the aftermath of Wakefield's vaccine scare (see Table 2.1) and created a model using social learning and feedback. They found that the model which does not consider either social learning or feedback is the best fit if a particular risk evolution curve is assumed. Running the model including social learning and feedback fits the data under all vaccine risk evolution curves. Therefore, adding social learning and feedback improves the model. Predictive capabilities of the model are also increased in the social learning and feedback case [1].

3 The Model

Bauch and Bhattacharyya's model [1] is the main focus of this thesis. In the next sections, the model is developed in detail, and we describe our efforts in recovering their results. In Chapter 4 the model is applied to vaccinating behavior in Samoa during a vaccine scare.

3.1 Evolutionary Game Theory

The model determined the evolutionary game theory component by considering individuals in the population and their strategies of being for vaccination or opposed to vaccination. The model assumes that each individual samples others in the population at some constant rate, s, and individuals will change strategies if they sample an individual who is playing a strategy that is different than theirs and they are having a higher payoff. The proportionality constant managing the changing of strategies according to expected payoffs is θ. The payoff for vaccination is given by

$$E_v(t) = B(t) - c_v(t), \tag{3.1}$$

where c_v represents the penalty to vaccinate and B represents having ideal health. The payoff for opposing vaccination is given by

$$E_n(t) = B(t) - c_i m L(t), \tag{3.2}$$

where c_i is the penalty of getting infected, m is a proportionality constant characterizing the chances of becoming infected, and $L(t)$ is the number of known cases in the population at some given time t. The difference of the payoffs in 3.1 and 3.2 is given

by

$$\Delta E_{vn}(t) = E_v(t) - E_n(t) = B(t) - c_v(t) - (B(t) - c_i m L(t)) = -c_v(t) + c_i m L(t). \quad (3.3)$$

The proportion of individuals for vaccination, $x(t)$, will switch their strategy to become part of the proportion of non-vaccinators, $1 - x(t)$, if the payoff of opposing vaccination is greater than the payoff of being for vaccination, $E_n(t) > E_v(t)$. Another way to say this is if the penalty of vaccination is greater than the penalty of infection times the number of cases times the chances of becoming infected, then vaccinators will switch their strategy to being opposed to vaccination. Mathematically we can represent the rate individuals for vaccination become opposed to vaccination by

$$\begin{cases} s\theta x(1-x)(c_v - c_i m L) & \text{when } \Delta E_{nv} = c_v - c_i m L > 0 \\ 0 & \text{when } \Delta E_{nv} = c_v - c_i m L \leq 0. \end{cases} \quad (3.4)$$

(Note the t dependence is suppressed in the notation of equations 3.4 and 3.5.)

Similarly, the proportion of individuals opposed to vaccination will switch the strategy they are playing to become part of the proportion of those individuals who are for vaccinating when the payoff for vaccinating is greater than the payoff for opposing vaccination, $E_v(t) > E_n(t)$. In other words, if the penalty of infection times the number of cases times the chances of becoming infected is greater than the penalty of vaccination, non-vaccinators will switch their strategy of play to being for vaccination. The rate that individuals opposed to vaccination become for vaccination can be represented by

$$\begin{cases} s\theta x(1-x)(-c_v + c_i m L) & \text{when } \Delta E_{vn} = -c_v + c_i m L > 0 \\ 0 & \text{when } \Delta E_{vn} = -c_v + c_i m L \leq 0. \end{cases} \quad (3.5)$$

Using these evolutionary game theory components the equations used to determine

the proportion of vaccinators in the population can be generated [1].

3.2 Behavioral Modeling Equations

Four different equations were considered for the behavioral model in [1]. The full behavioral model was obtained by subtracting 3.4 and 3.5 to yield the proportion of those for vaccination, and is given by the equation

$$\frac{dx}{dt} = s\theta x(1-x)(-c_v(t) + c_i m L(t)), \tag{3.6}$$

for $t \geq 0$. The authors simplified this equation by using the substitution $\kappa = s\theta c_i m$ and $\omega = \frac{c_v}{mc_i}$ yielding the equation

$$\frac{dx}{dt} = \kappa x(1-x)(-\omega(t) + L(t)). \tag{3.7}$$

Thus, the full behavioral model factors in social learning, i.e., the proportion of vaccinators depend on what portion plays this strategy in the group, and feedback, i.e., the proportion of vaccinators depends on disease prevalence.

A similar model considered social learning in the absence of feedback. This model was given by the equation

$$\frac{dx}{dt} = x(1-x)(-\omega(t)). \tag{3.8}$$

Another model looked at feedback and no social learning, and is given by the equation

$$x(t) = \rho L(t) - \omega(t). \tag{3.9}$$

The last model determined the proportion of vaccinators without the presence of

social learning or feedback. This equation is given by

$$x(t) = 1 - \omega(t). \tag{3.10}$$

The models were run using five different risk evolution curves, $\omega(t)$. These risk evolution curves were introduced because the ω term is comprised of $c_v(t)$, the perceived penalty to vaccinate, which can change over time during a vaccine scare [1].

3.3 Risk Evolution Curves

The model equations use the function $\omega = \omega(t)$ to represent how $c_v(t)$, the perceived penalty to vaccinate, is changing over time. They assumed that $\omega(t)$ is constant until a point in time where a vaccine scare occurs. Then, $\omega(t)$ increases linearly, plateaus at a constant for some period of time, decreases linearly, and finally returns to the original constant. Prior to the vaccine scare, $\omega(t) = \omega_{pre}$. The length of the time interval where $\omega(t)$ increases is denoted by $D_{increase}$. The function $\omega(t)$ plateaus at a level given by $\sigma\omega_{pre}$, where $\sigma > 1$, and D_{max} denotes the length of the time interval where $\omega(t) = \sigma\omega_{pre}$. Finally, the function $\omega(t)$ decreases on the time interval of $D_{decrease}$ [1].

The authors examined five risk evolution curves by manipulating the various parameters $D_{increase}$, D_{max}, and $D_{decrease}$, which are listed below. Let t_s denote the starting time of the vaccine scare, $D_{inc} = D_{increase}$, and $D_{dec} = D_{decrease}$. The first curve, $\omega_1(t)$, sets both D_{inc} and D_{max} equal to zero and is given by

$$\omega_1(t) = \begin{cases} \omega_{pre} & \text{if } t < t_s \text{ or } t \geq t_s + D_{dec} \\ \dfrac{\sigma\omega_{pre} - \omega_{pre}}{-D_{dec}}(t - (t_s + D_{dec})) + \omega_{pre} & \text{if } t_s \leq t < t_s + D_{dec}. \end{cases} \tag{3.11}$$

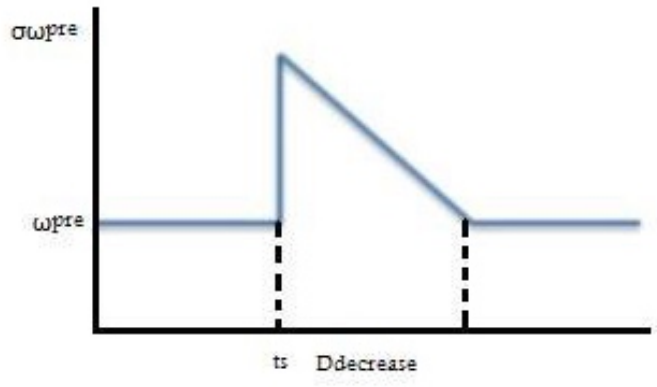

Figure 3.1: Graph of the first risk evolution curve ω_1.

The second curve, $\omega_2(t)$, sets both D_{inc} and D_{dec} equal to zero and has the piecewise function of

$$\omega_2(t) = \begin{cases} \omega_{pre} & \text{if } t < t_s \text{ or } t \geq t_s + D_{max} \\ \sigma\omega_{pre} & \text{if } t_s \leq t < t_s + D_{max}. \end{cases} \tag{3.12}$$

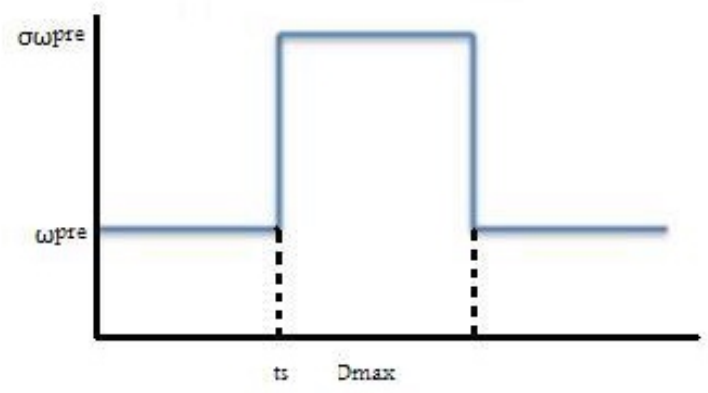

Figure 3.2: Graph of the second risk evolution curve ω_2.

The third curve, $\omega_3(t)$, sets D_{inc} equal to zero and is given by,

$$\omega_3(t) = \begin{cases} \omega_{pre} & \text{if } t < t_s \text{ or } t \geq b \\ \sigma\omega_{pre} & \text{if } t_s \leq t < t_s + D_{max} \\ \dfrac{\sigma\omega_{pre} - \omega_{pre}}{-D_{dec}}(t-b) + \omega_{pre} & \text{if } t_s + D_{max} \leq t < b \end{cases} \tag{3.13}$$

where $b = t_s + D_{max} + D_{dec}$.

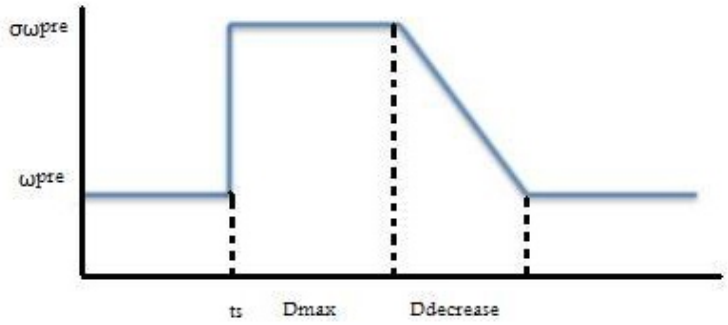

Figure 3.3: Graph of the third risk evolution curve ω_3.

The fourth curve, $\omega_4(t)$, sets D_{dec} equal to zero, and has the piecewise function

$$\omega_4(t) = \begin{cases} \omega_{pre} & \text{if } t < t_s \text{ or } t \geq t_s + D_{inc} + D_{max} \\ \dfrac{\sigma\omega_{pre} - \omega_{pre}}{D_{inc}}(t - t_s) + \omega_{pre} & \text{if } t_s \leq t < t_s + D_{inc} \\ \sigma\omega_{pre} & \text{if } t_s + D_{inc} \leq t < t_s + D_{inc} + D_{max}. \end{cases} \tag{3.14}$$

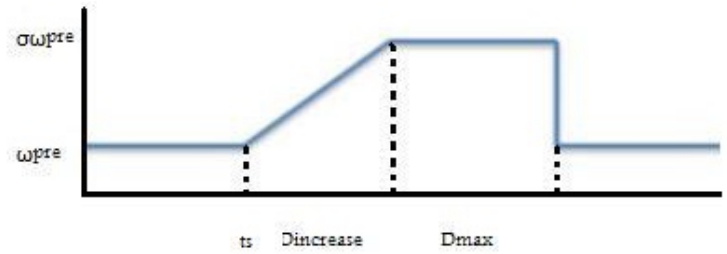

Figure 3.4: Graph of the fourth risk evolution curve ω_4.

The fifth curve, $\omega_5(t)$, follows the general form of $\omega(t)$, where it begins as a constant, increases linearly, plateaus, decreases linearly, and returns to the starting

constant. No time intervals are equal to zero in $\omega_5(t)$ and it is given by the following piecewise function,

$$\omega_5(t) = \begin{cases} \omega_{pre} & \text{if } t < t_s \text{ or } t \geq c \\ \dfrac{\sigma\omega_{pre} - \omega_{pre}}{D_{inc}}(t - t_s) + \omega_{pre} & \text{if } t_s \leq t < t_s + D_{inc} \\ \sigma\omega_{pre} & \text{if } t_s + D_{inc} \leq t < t_s + D_{inc} + D_{max} \\ \dfrac{\sigma\omega_{pre} - \omega_{pre}}{-D_{dec}}(t - c) + \omega_{pre} & \text{if } t_s + D_{inc} + D_{max} \leq t < c. \end{cases} \tag{3.15}$$

where $c = t_s + D_{inc} + D_{max} + D_{dec}$.

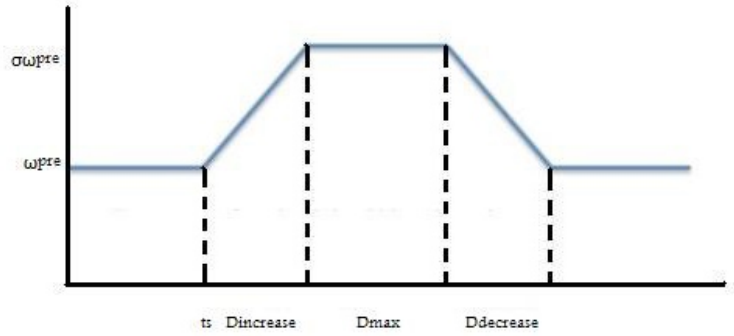

Figure 3.5: Graph of the fifth risk evolution curve ω_5.

3.4 Model Replication

Bauch and Bhattacharyya's model was replicated using their given parameters for each of the risk evolution curves under three cases of social learning and feedback. The case that included feedback but no social learning could not be replicated using the given parameters due to issues that will be discussed in Section 5.2. Therefore this case was omitted. Bauch's parameters for each of the remaining cases are listed in Tables 3.2, 3.3, and 3.4. For the case involving feedback, the disease incidence data, $L(t)$, is listed in Table 3.1 and the vaccine data coverage is listed in Table 2.1. The solutions can be seen in Figures 3.6, 3.7, and 3.8.

Year	Cases
1996	112
1997	177
1998	56
1999	92
2000	100
2001	70
2002	319
2003	437
2004	188
2005	78
2006	740
2007	990
2008	1370
2009	1144
2010	380
2011	1087
2012	2030
2013	1843

Table 3.1: Known infectious cases of the measles in England and Wales [6].

Curve	ω_{pre}	σ	$D_{increase}$	D_{max}	$D_{decrease}$	κ
ω_1	20	125.3202	0	0	8	$9.18*10^{-5}$
ω_2	13.9864	100.3957	0	6.5	0	$1.15*10^{-4}$
ω_3	16	100.608	0	4.9597	2.0389	$1.05*10^{-4}$
ω_4	11	100.0988	3	4	0	$1.94*10^{-4}$
ω_5	19.7	99.9456	3.4374	1.3556	1.6089	$1.11*10^{-4}$

Table 3.2: Bauch parameter values for social learning and feedback under the five risk evolution curves [1].

Curve	ω_{pre}	σ	$D_{increase}$	D_{max}	$D_{decrease}$
ω_1	$1.02*10^{-3}$	200.0474	0	0	6.7691
ω_2	$5.95*10^{-4}$	199.9999	0	6.1842	0
ω_3	$7.52*10^{-4}$	200.0022	0	4.3253	1.0205
ω_4	$5.95*10^{-4}$	200.0811	1	5	0
ω_5	$8.79*10^{-4}$	199.9777	2.3671	1	2.4731

Table 3.3: Bauch parameter values for social learning and no feedback under the five risk evolution curves [1].

Curve	ω_{pre}	σ	$D_{increase}$	D_{max}	$D_{decrease}$
ω_1	0.1131	1.633	0	0	10
ω_2	0.1197	1.4918	0	4.5	0
ω_3	0.1	1.6674	0	6.6058	3.6602
ω_4	0.0999	1.7289	6.6458	4.3542	0
ω_5	0.0871	2.1803	5.6049	1	8.4948

Table 3.4: Bauch parameter values for no social learning and no feedback under the five risk evolution curves [1].

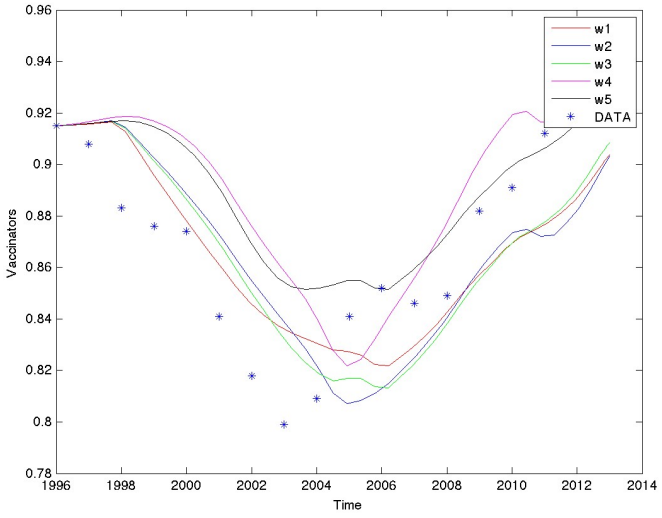

Figure 3.6: Graph of social learning and feedback using Bauch's parameters [1].

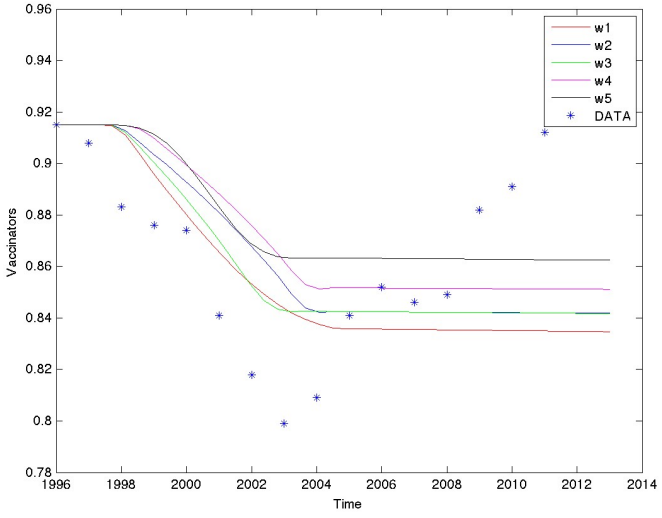

Figure 3.7: Graph of social learning and no feedback using Bauch's parameters [1].

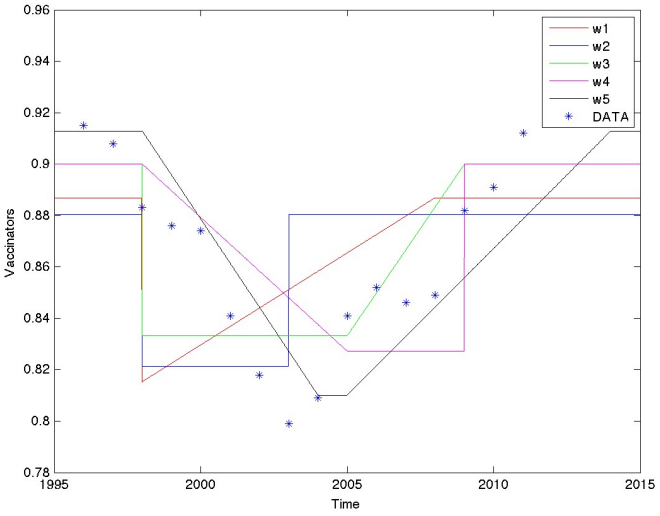

Figure 3.8: Graph of no social learning and no feedback using Bauch's parameters [1].

4 Parametrization

To determine whether Bauch's model for social learning and feedback could fit vaccination behavior in other cases, we investigated another vaccine scare under each of the risk evolution curves for the different models.

4.1 Data Collection

To parametrize the model, various vaccine coverage data from around the world was examined. The vaccinator equations model the number of vaccinators in the population after a vaccine scare [1], hence a country whose vaccine coverage data exemplified a large vaccine scare was sought after. The country of Samoa was chosen as the coverage data showed a large vaccine scare in the population. See Table 4.1, and Figure 4.1. Note that effects of the vaccine scare began in 2003, so it was assumed to have started in 2002 [16]. It is unknown what caused the vaccine scare. Although, it could be speculated that the lack of infectious cases, $L(t)$, see Table 4.2, caused

the decrease of vaccinations as there were no cases of measles from 2000 to 2009 [5]. This data was interpolated from a graph from GIDEON Informatics Inc. [5].

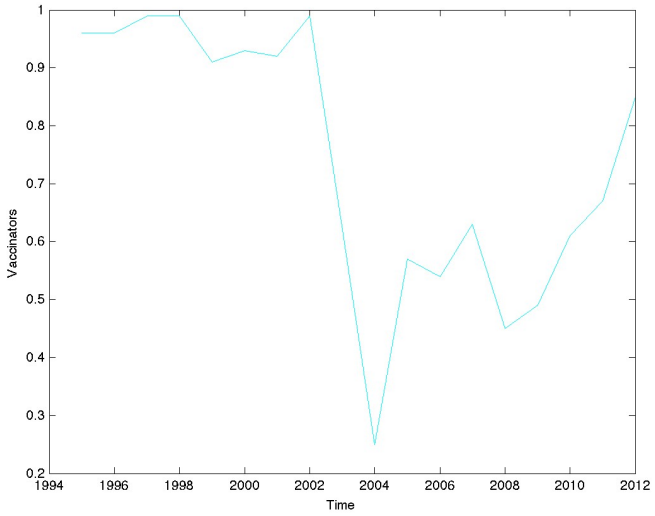

Figure 4.1: Graph of the Samoan MMR vaccine coverage data.

4.2 Error

An ordinary differential equation solver (ode solver) from MATLAB R2014A was used to solve each of the differential equations for the cases of social learning and feedback and social learning with no feedback. The error of the model was calculated by taking the sums of the squares of the difference between the fit of the model and the vaccine coverage data divided by the data squared, with the requirement that all parameters should be positive.

For the case of no social learning or feedback, the error was calculated by taking the sums of the squares of the difference of the fit of the model and the data, with the restriction that all parameters needed to be positive.

The error is used to determine the best fit model. The error for each curve can be seen in Tables 5.1, 5.2, and 5.3 as the RSS_{vac} values.

Year	MMR Vaccine Coverage
1995	96%
1996	96%
1997	99%
1998	99%
1999	91%
2000	93%
2001	92%
2002	99%
2003	62%
2004	25%
2005	57%
2006	54%
2007	63%
2008	45%
2009	49%
2010	61%
2011	67%
2012	85%

Table 4.1: Samoan measles vaccine coverage data [16].

Year	Measles Cases
1995	0
1996	100
1997	0
1998	10
1999	5
2000	0
2001	0
2002	0
2003	0
2004	0
2005	0
2006	0
2007	0
2008	0
2009	0
2010	8
2011	0
2012	1

Table 4.2: Number of infectious cases of the measles in Samoa [5].

4.3 Parameter Search and Solutions

An optimization routine, fminsearch, was run in MATLAB R2014a to determine the values of ω_{pre}, σ, $D_{increase}$, D_{max}, $D_{decrease}$, in all cases, and κ in the case with both social learning and feedback, that minimizes the error. The differential equations were then solved using an ode solver in MATLAB with the newly found parameters and their solutions were graphed. The results can be seen below.

Curve	ω_{pre}	σ	$D_{increase}$	D_{max}	$D_{decrease}$	κ
ω_1	37.155	258.771	0	0	1.779	$4.23*10^{-4}$
ω_2	33.361	818.736	0	1.301	0	$7.915*10^{-5}$
ω_3	19.277	421.051	0	0.764	1.028	$3.054*10^{-4}$
ω_4	142.481	153.876	0.017	1.091	0	$1.114*10^{-4}$
ω_5	143.703	302.783	0.354	0.002	1.233	$8.011*10^{-5}$

Table 4.3: Samoan parameter values for social learning and feedback under the five risk evolution curves.

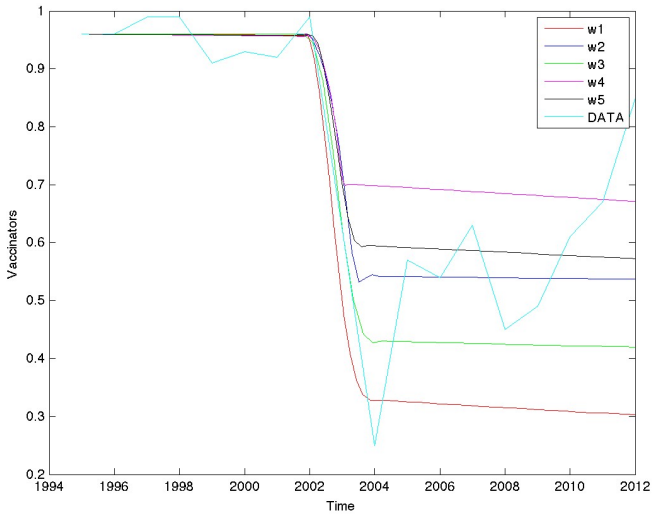

Figure 4.2: Graph of the Samoan social learning and feedback models.

Curve	ω_{pre}	σ	$D_{increase}$	D_{max}	$D_{decrease}$
ω_1	0.002	1303.837	0	0	1.798
ω_2	0.004	541.807	0	1.314	0
ω_3	0.004	560.490	0	1.307	0.008
ω_4	$5.361*10^{-4}$	946.684	$2.504*10^{-6}$	4.892	0
ω_5	0.028	85.005	0.046	0.431	1.208

Table 4.4: Samoan parameter values for social learning and no feedback under the five risk evolution curves.

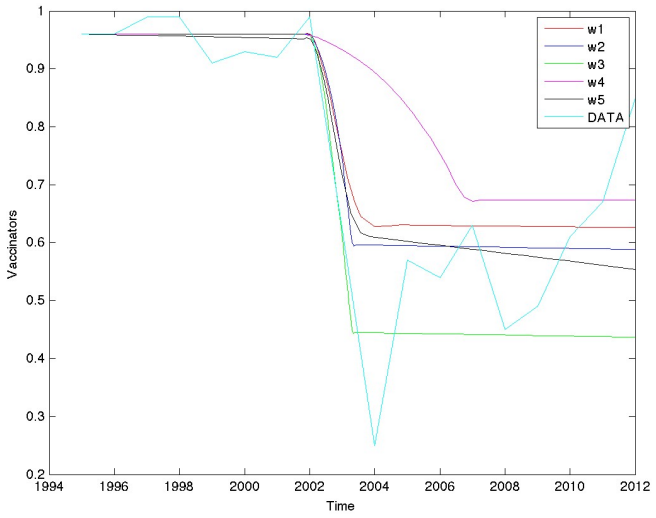

Figure 4.3: Graph of the Samoan social learning and no feedback models.

Curve	ω_{pre}	σ	$D_{increase}$	D_{max}	$D_{decrease}$
ω_1	0.049	8.413	0	0	120.022
ω_2	0.061	6.825	0	9.438	0
ω_3	0.049	8.807	0	8.457	2.107
ω_4	0.144	3.399	0.810	5.308	0
ω_5	0.041	12.400	1.387	5.739	3.949

Table 4.5: Samoan parameter values for no social learning and no feedback under the five risk evolution curves.

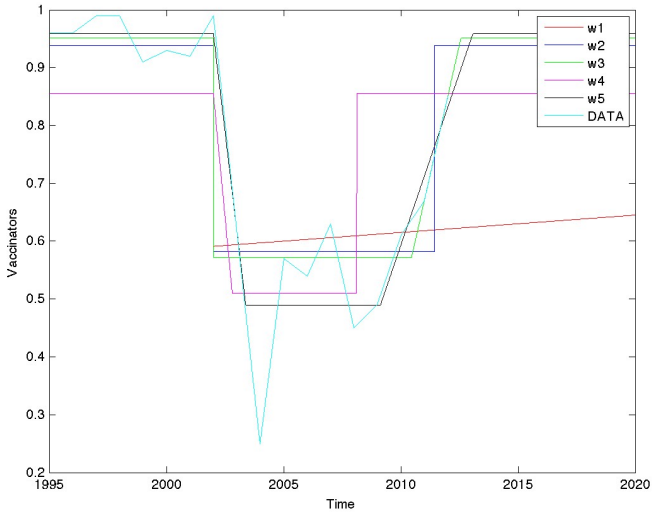

Figure 4.4: Graph of the Samoan no social learning and no feedback models.

5 Results

The resulting best fit model for each case of the Samoan vaccine scare was not consistent with previous research in all facets. Under no social learning or feedback the findings were consistent, however it appears that social learning in the absence of feedback is a better model than social learning and feedback for most ω risk evolution curves. Disease incidence data shows no increase after the substantial drop of vaccinations in 2003 so it is no surprise that including feedback in the model does not improve the fit. The reason vaccinations picked up again after 2003 must have an alternate explanation.

To account for no disease incidence data, another parametrization was done to see if we could better capture the data by incorporating another parameter, R. This parameter is a regulation term or a compliance term, as individuals feel better when they follow the rules or comply. Incorporating this term will imitate feedback since for a period of nine years, no infectious cases were present in the population. The resulting parameters and graph of solutions can be seen in Table 5.1 and Figure 5.1.

Curve	ω_{pre}	σ	$D_{increase}$	D_{max}	$D_{decrease}$	κ	R
ω_1	38.135	278.981	0	0	1.665	$4.228*10^{-4}$	223.163
ω_2	30.349	225.295	0	1.091	0	$4.181*10^{-4}$	105.125
ω_3	10.973	402.969	0	0.675	1.046	$6.283*10^{-4}$	60.357
ω_4	17.692	644.306	$6.403*10^{-9}$	1.571	0	$1.640*10^{-4}$	50.699
ω_5	55.290	323.846	0.326	0.452	0.515	$1.872*10^{-4}$	38.441

Table 5.1: Samoan parameter values for social learning and feedback with regulation under the five risk evolution curves.

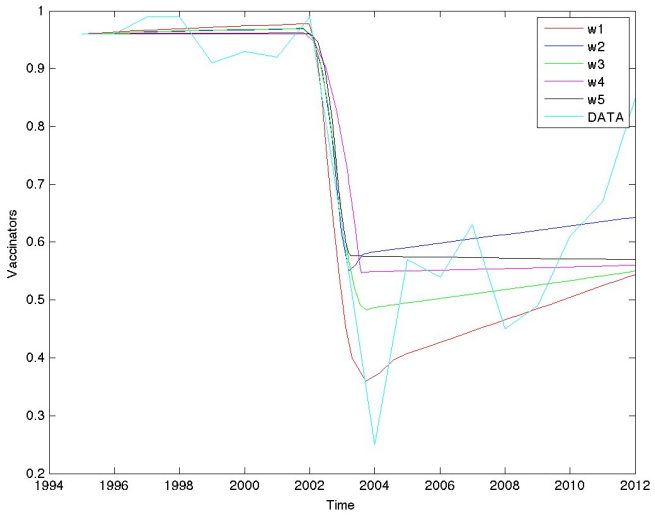

Figure 5.1: Graph of the Samoan social learning and feedback with regulation models.

5.1 Akaike Information Criterion

To ensure that the Samoan models were a good fit and parsimonious, I evaluated the Akaike Information Criterion (AIC). The AIC numbers take into account the model's fit and the number of parameters used. While it may be possible to achieve a better fit with more parameters the AIC 'penalizes' the use of too many parameters and aims at finding the most parsimonious model that is still a good fit [1].

The AIC value factors in the number of data points N and the number of parameters l. It is given by

$$AIC = -2\ln(M) + 2l + (2l(l+1))/(N-l-1), \tag{5.1}$$

where $\ln(M)$ is the natural logarithm and M is the likelihood estimator,

$$M = e^{-N/2}/(2\pi RSS_{vac}/N)^{N/2}. \tag{5.2}$$

The RSS_{vac} term is the error. In all of these calculations, the number of data points, N, was 18.

The best model with the social learning and feedback was under the ω_1 curve having an AIC value of -22.526858704651655. The ω_2 curve AIC value was quite close having an AIC score of -22.439183905732566. See Table 5.2.

Curve	l	RSS_{vac}	AIC
ω_1	4	0.162938561236933	-22.526858704651655
ω_2	4	0.163734141985326	-22.439183905732566
ω_3	5	0.163676074463980	-18.522491727802098
ω_4	5	0.161893555800294	-18.719596454847469
ω_5	6	0.162464325641427	-14.019883855300751

Table 5.2: The RSS_{vac}, l, and resulting AIC values under the five risk evolution curves for both social learning and feedback.

Under the social learning no feedback equation, the best model was the ω_2 risk evolution curve which had an AIC score of -25.848763435691843. The ω_1 risk evolution curve was a close second with an AIC value of -25.822419181596707. See Table 5.3.

Curve	l	RSS_{vac}	AIC
ω_1	3	0.163546883501212	-25.822419181596707
ω_2	3	0.163307696318826	-25.848763435691843
ω_3	4	0.163291668154259	-22.487892806207217
ω_4	4	0.163674371657626	-22.445755915107529
ω_5	5	0.159596319589236	-18.976842226865251

Table 5.3: The RSS_{vac}, l, and resulting AIC values under the five risk evolution curves for social learning and no feedback.

The best model for no social learning or feedback was ω_5 with an AIC score of -27.810271405563945. This model was also the best of all the cases. As seen in Table 5.4, the rest of the models using the other ω curves were not very good. ω_1 was the worst model under no social learning or feedback having a positive AIC score of 10.299974236643381.

Curve	l	RSS_{vac}	AIC
ω_1	3	0.3874	10.299974236643381
ω_2	3	0.3320	-13.077784313658706
ω_3	4	0.3144	-10.695587301527208
ω_4	4	0.4193	-5.513016616337229
ω_5	5	0.0977	-27.810271405563945

Table 5.4: The RSS_{vac}, l, and resulting AIC values under the five risk evolution curves for no social learning and no feedback.

The best model for social learning and feedback with regulation was under the ω_2 risk evolution curve having an AIC value of -17.9699. Since this AIC value is larger than the best AIC values from the other cases, it shows that adding in this regulation term does not necessarily improve the model as we get penalized for having excess parameters.

Curve	l	RSS_{vac}	AIC
ω_1	5	0.1742	-17.4035
ω_2	5	0.1688	-17.9699
ω_3	6	0.1694	-13.2726
ω_4	6	0.1648	-13.7626
ω_5	7	0.1640	-8.2889

Table 5.5: The RSS_{vac}, l, and resulting AIC values under the five risk evolution curves for social learning and feedback with regulation.

5.2 Limitations and Complications

The model has some limitations. This model only seems to apply to vaccine coverage data that has a true vaccine scare. The model was not a good fit when applied to the United States MMR vaccine coverage data so this route was quickly abandoned. The U.S. data only fluctuates between 90% - 93% from 1996 to 2013 [11], hence there is no vaccine scare present in the data.

When simulating the Bauch model complications arose. I was not able to reproduce Bauch and Bhattacharyya's results from their parameters in the social learning and feedback case. The model still appears to be a good fit, but it does not yield the same results they got using the given values. In effort to correct this, I ran a parametrization on that case. The resulting parameters were close to what they had found. These results can be seen below in Table 5.6 and Figure 5.2. The social learning and no feedback case and no social learning and no feedback case were duplicated without any difficulties. However, the feedback without social learning case was not replicable from the given parameters. The parameters given in the supplemental information were inconsistent with the claims of the paper. For example, $\sigma > 1$ is a condition, however, in the table of parameters, all of the σ values were 0.001, which is clearly less than 1. Finally, when recreating the model, tables in the original paper were mislabeled which made it challenging to decipher parameter values.

Curve	ω_{pre}	σ	$D_{increase}$	D_{max}	$D_{decrease}$	κ
ω_1	19.1469	123.0163	0	0	7.7963	$9.64*10^{-5}$
ω_2	13.7971	95.8915	0	5.6103	0	$1.30*10^{-4}$
ω_3	18.5281	107.0292	0	2.6977	2.2378	$1.25*10^{-4}$
ω_4	17.4493	128.9044	0.5688	2.8721	0	$1.41*10^{-4}$
ω_5	19.4281	121.3795	0.5636	1.6462	1.8429	$1.46*10^{-4}$

Table 5.6: Parameter estimation values for social learning and feedback under the five risk evolution curves [1].

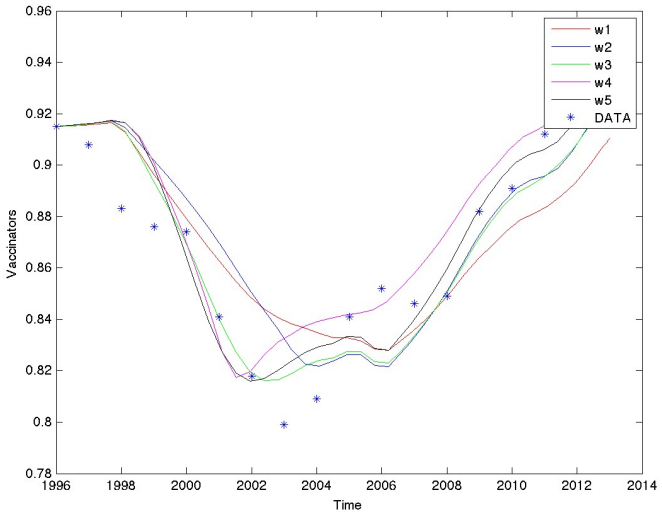

Figure 5.2: Graph of social learning and feedback under new parameters.

5.3 Further Research

Further research should be done to examine vaccinating behavior. It would be of interest to develop a model that could determine vaccinating behavior without needing a true vaccine scare. Investigating vaccinating behavior of other preventable diseases, such as the varicella vaccine or the human papillomavirus (HPV) vaccine, may be of interest. The varicella vaccine is relatively new, 1995 [14], and in 2012 approximately 74.9% of those who have not had the chickenpox had two doses of the vaccine [10]. Also in 2012, only 53.8% of females and 20.8% of males had just one dose of the HPV vaccine. Similarly, only 33.4% of females and 6.8% of males have had three or more doses of the HPV vaccine [10]. Investigating why these percentages are so low would be worthy of some research.

6 Concluding Remarks

The vaccinating behavior of individuals is a pertinent, controversial issue in our society. I believe that vaccinating for preventable diseases should be an avid practice for all of those who are able to vaccinate. It is our duty to maintain the threshold of herd immunity to ensure that those who are not able to vaccinate are protected from these preventable diseases.

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APPENDIX

SAMPLE CODES

Parameter estimation of social learning and feedback:

ω_5

```
clear all
global X wcp
X=[1995 0 1996 100 1997 0 1998 10 1999 5 2000 0 2001 0 2002 0 2003 0
2004 0 2005 0 2006 0 2007 0 2008 0 2009 0 2010 8 2011 0 2012 1];
X=reshape(X,2,length(X)/2);
tspand=1998:2013;
wcp=[.96 .96 .99 .99 .91 .93 .92 .99 .62 .25 .57 .54 .63 .45 .49 .61
.67 .85];
t0=1995;
tf=2012;
x0=0.96;
%p0=[0.0001 80 200 1 2 1];
p0=[0.0001 75 200 1 2 1];
p=fminsearch(@errorSAMOA_SLF5,p0)
[t,v]=ode45(@vac_SAMOA_SLF5,[t0 tf],x0,[],p);
figure(1)
subplot(2,1,1)
plot(1995:2012,wcp,'r')
axis([1995 2012 0 1])
hold on
plot(t,v,'b')
legend('vaccine coverage', 'model5')
MSLF5=sum((v(1:1+17)-wcp').^2)/sum(wcp.^2)
p =
1.0e+02 *
Columns 1 through 3
0.000000801146833   1.437027625499285   3.027831138888646
Columns 4 through 6
0.003535918948115   0.000019157816402   0.012328834179898
```

Error code of social learning and feedback:

ω_5

```
function err=errorSAMOA_SLF5(p);
global X wcp
t0=1995;
tf=2012;
x0=0.96;
tspan=[t0:1:tf];
[t,v]=ode23s(@vac_SAMOA_SLF5,tspan,x0,[],p);
err=sum((v(1:1+17)-wcp').^2)/sum(wcp.^2)+1000*(p(1)<0)+1000*(p(2)<0)+
1000*(p(3)<0)+1000*(p(4)<0)+1000*(p(5)<0)+1000*(p(6)<0);
```

Vaccinator code of social learning and feedback:

ω_5

```
function vprime=vac_SAMOA_SLF5(t,v,p)
global X wcp
dx=p(1)*v*(1-v)*(-SAMOA_GENERIC1piecewiseSLF5(t,p(2:end))+interp1(X
(1,:),X(2,:),t));
vprime=dx;
```

Piecewise function:

ω_5

```
function omega5=SAMOA_GENERIC1piecewiseSLF5(t,p);
global X wcp
wpre=p(1);
sigma=p(2);
Dinc=p(3);
Dmax=p(4);
Ddec=p(5);
if t<2002 | t>=2002+Dinc+Dmax+Ddec
    omega5=wpre;
elseif 2002<=t & t<2002+Dinc
    omega5=((((sigma*wpre)-wpre)/(Dinc))*(t-2002))+wpre;
elseif 2002+Dinc<=t & t<2002+Dinc+Dmax
    omega5=sigma*wpre;
elseif 2002+Dinc+Dmax<=t & t<2002+Dinc+Dmax+Ddec
    omega5=((((sigma*wpre)-wpre)/(-Ddec))*(t-(2002+Dinc+Dmax+Ddec)))+wpre;
end
```