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Production Systems with Deteriorating Product Quality : System-Theoretic Approach

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PRODUCTION SYSTEMS WITH DETERIORATING PRODUCT QUALITY : SYSTEM-THEORETIC APPROACH

by

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ABSTRACT

PRODUCTION SYSTEMS WITH DETERIORATING PRODUCT QUALITY : SYSTEM-THEORETIC APPROACH

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Manufacturing systems with perishable products are widely seen in practice (e.g., food, metal processing, etc.). In such systems, the quality of a part is highly dependent on its residence time within the system. However, the behavior and properties of these systems have not been studied systematically, and, therefore, is carried out in this dissertation. Specifically, it was assumed that the probability that each unfinished part is of good quality is a decreasing function of its residence time in the preceding buffer. Then, in the framework of serial production lines with machines having Bernoulli and geometric reliability models, closed-form formulas for performance evaluation in the two-machine line case were derived, and develop an aggregation-based procedure to approximate the performance measures in $M > 2$-machine lines. In addition, the monotonicity properties of these production lines using numerical experiments were studied. A case study in an automotive stamping plant is described to illustrate the theoretical results obtained. Also, Bernoulli serial lines with controlled parts released was analyzed for both deterministic and stochastic releases. Finally, bottleneck analysis in Bernoulli serial lines with deteriorating product quality were studied.

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Acknowledgments

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support, encouragement, quiet patience and unwavering love were undeniably the fuel that kept me going. Also, I thank my two sisters, and two elder brothers. They were always supporting me and encouraging me with their best wishes.

Chapter 1

INTRODUCTION

1.1 Motivation

Production systems are sets of processing machines and material handling equipments arranged so as to produce desired products. This can be accomplished by maintaining smooth flow of parts throughout the system to prevent production losses. In the last three decades, the increasing competitiveness of the global market has resulted in ever increasing pressure on both the quality of products and the productivity of the systems producing the products. The progress in technology has provided several possibilities for production managers to exercise better control over a production plant's performance, both from the point of view of quality and production logistics. In the past several decades, production systems have been studied extensively and numerous results have been reported. Among these studies, performance analysis and optimization of production systems are mostly investigated. In contrast, system-theoretic properties of production systems have rarely been discussed in the literature. These properties, however, are of importance because they reveal the fundamental principles that characterize the behavior of such systems. This dissertation is intended to provide a contribution in this direction.

Productivity and quality are often considered as the most important metrics of a production system. During the past 60 years, extensive research efforts have been spent in analyzing and improving the productivity of manufacturing systems (see, for instance, monographs [1–6]). On the other hand, there also exist a great amount of results in the literature regarding quality monitoring and control in manufacturing processes (see, for instance, survey [7]). Integrated analysis of productivity and quality in manufacturing systems, however, received limited attention until recent years (see review papers [8] and [9]).

Also, it was assumed in many inventory models that the items can be stored infinitely without any risk of deterioration. However, certain types of items undergo changes while in storage so that, with time, they become partially or entirely unfit for use. Deterioration refers to damage, spoilage, vaporization, or obsolescence of the products. There are several types of items that will deteriorate if stored for extended periods of time. Examples of deteriorating items include metal parts, which are prone to corrosion and rusting, and food items, which are subject to spoilage and decay. Electronic components and fashion clothing also fall into this category, because they can become obsolete over time and their demand will typically decrease drastically. In this work, we consider production systems with reliable machines and finite buffers with deteriorating product quality issues.

The motivation for this study stems from the fact that a more reliable use of buffers in production systems with deteriorating production quality is a prerequisite to get a competitive advantage in factories processing perishable products. Variability in manufacturing environment is one of the obstacles in achieving high production with least waste. In general, the variability is known to be detrimental, but at the same time it is impossible to be eliminated completely. Hence, it is important to identify the sources of variability, measure it accurately, and understand its relationship with the system design factors. Accordingly, the dissertation tries to establish a foundation to investigate how production system design and operation influence productivity and deteriorating product quality by developing conceptual and computational models of two-machines and more systems and performing numerical experiments to evaluate the performance measures and system theoretic properties.

1.2 Related Literature

There is a substantial body of literature on the analysis of asynchronous serial lines with reliable machines; for the last four decades, several researchers have attempted to determine line efficiency and the effect of interstation buffer capacity on various performance measures. The majority of the studies consist of attempts to determine line efficiency measured as throughput either analytically or by utilizing approximate procedures such as predictive equations or simulation models. Exact expressions and numerical methods are developed to determine throughput for lines with a limited length and/or certain processing time distribution functions [10–14]. For the throughput of longer lines with various distribution functions, several approximate expressions and simulation models are proposed [10, 15–20]. Another group of studies search the optimal allocation of buffer capacities to maximize throughput [14, 21–23, 23–27]. Finally, a few researchers examine higher moments of throughput. In this section, only these relevant studies will be reviewed.

Miltenburg [28] presents a Markov analysis to determine the mean and the variance of the number of units produced during a fixed period of time. The stations are considered to be unreliable; thus, three sources of variability, namely, station up and down times and the processing times exist. Due to the large matrices involved for problems of realistic sizes, variance computations are reported for only lines with up to three stations and a total buffer capacity of 14. However, the author recommends his analysis for two-station lines with any buffer capacity and three-station lines with a total buffer capacity of less than 10 units. Even though this approach has limited applicability in industrial settings, it is the first study reported in the literature for variability of interdeparture time.

Chow [29] presents an approximate procedure to determine the throughput and the coefficient of variation (CV) of interdeparture time with coxian type processing time distributions. For a two-station line, regression equations are developed on data obtained from a simulation model to determine the throughput and the CV of the interdeparture time expressions. These expressions are first applied to the first two stations of the line to combine them into a single station. The same process is applied to the combined station and the third station until all the stations in the line are considered. The author also presents an approximate dynamic programming procedure to determine the optimal buffer allocation to achieve a target throughput level. In an example solved, with nonzero buffer capacities at each location, the procedure results in designs that confirm the bowl phenomenon. It is interesting that the results are reported only for the throughput; in a simulation experiment with 10-station lines, most of the relative deviations of the proposed approximate model are within 5%. Unfortunately, the performance of this method is not reported for the CV of interdeparture times.

To the best of our knowledge, the work of Martin and Lau [30] is the first study that examines the properties of interdeparture time distribution for lines with up to 10 stations and buffer capacity of up to 2 per location. According to their approach, lines are partitioned into sub-queues and the moments of interdeparture time for each sub-queue are determined by using regression meta-models. In the simulation experiment to estimate the coefficients of regression equations, the authors consider two levels of CV and several levels for the other system design factors. During simulation experiments, they also note certain relationship between CV and other design factors; CV of interdeparture time increases as the line length, CV, third and fourth moments of the processing times increase. An opposite effect is observed as the buffer capacity at each location increases. In this study, the authors also point out a need for more extensive simulation studies are required to consider other levels of the factors.

Hendricks [31] examines the effects of line length, buffer capacity and buffer allocation on production lines with exponentially distributed processing times using Markov analysis. The performance measures considered are the mean, variance and asymptotic variance of the interdeparture time, and the correlation structure of the output process. The asymptotic variance is defined as the limiting variance, per departure, of the time of the nth departure. Computational findings indicated that for all the line lengths considered (up to 6 stations), the correlations are all less than or equal to zero, as expected. The variance of the interdeparture time increases as the line length increases; however, the asymptotic variance is observed to decrease. Experiments conducted on the effects of buffer capacity and buffer allocation show that as the buffer capacities increase, the variance and the asymptotic variance both decrease and approach to each other. The experiment on the effect of buffer allocation indicates that the optimal buffer allocation to maximize throughput does not always coincide with the one that minimizes the variance. The author also concludes that the difference is not large and could probably be ignored. Another observation reported in the paper is that the reversibility property does hold for the asymptotic variance whereas it does not hold for the variance of the interdeparture time.

In the later work, Hendricks and McClain [31] consider Erlang and uniformly distributed processing times. Skewness of processing time is considered in their simulation model in addition to the factors stated above. Results indicate that the variability of interdeparture time increases as the skewness increases especially for large line lengths. It is also observed that the variability of interdeparture time is completely explained by the processing time variability for large buffer sizes. The other observations are similar to the ones reported in the previous study.

In summary, there are a few studies that examine interdeparture time variability in serial production lines. Even though these studies yield several useful results, there are still a number of issues remained to be addressed. One of the objectives of this paper is to investigate these issues by examining the relationship between several design factors and the interdeparture time variability. Moreover, the problem will be studied for average and variability of *work-in-process* (*WIP*) inventory.

Productivity is an important measure of manufacturing system performance, traditionally estimated including both the reliability of the machining system and its processing speed. However, the influence of configuration on productivity has typically been overlooked. Configuration is the arrangement of operations and their part flow to take a product from raw materials to finished goods. Productivity is defined as the stochastic measure of the production rate of the different operational states of a manufacturing system. As analysts experienced in simulation methods know, treating productivity as stochastic gives information about a configuration's expected long-term production rate, as well as the probability distribution of production rates. This knowledge can be leveraged to take advantage of system configuration to enhance manufacturing line throughput while providing a means to assess a system configuration's value when examining system cost.

On the other hand, topics in quality research have captured the attention of practitioners and researchers since the early 1980's. Statistical Process Control [32], Total Quality Management [33] and Six Sigma [34] theories have been developed for a better control of manufacturing processes, for meeting higher product quality and for continuous improvement of processes. These two fields, productivity and quality, have been extensively studied and reported separately both in the manufacturing systems research literature and the practitioner literature, but there is little research in their intersection. All manufacturers must satisfy these two requirements (high productivity and high quality) at the same time to maintain their competitiveness. The link between these two areas have been very rarely considered at a production system level, even if industrial experience has evidenced the need for jointly considering quality and productivity performance measures while designing the manufacturing system [8]. There are many aspects that prove that quality and production logistics are mutually related. For instance, the production system architecture affects the performance of the quality control system. It has been shown by Gershwin [35] that for a production line with 15 machines, the number of bad parts to be scrapped by the system if inspection stations are poorly allocated, can be 15% higher than the number of bad parts produced with a good allocation of the same number of inspection stations.

Moreover, the results coming from researches carried out in Lean Production area [36], [37] have shown that the reduction of inventory has a positive impact on product quality. However, from the manufacturing system engineering research area, it is known that the production rate of the system is positively affected by the presence of buffers, since they decrease the behavior of the machines, preventing from the propagation of machine disruptions upstream and downstream the line [38]. Some lean manufacturing professionals advocate reducing inventory on the factory floor since the reduction of *WIP* reveals the problems in the production lines [39]. Thus, it can help improve product quality. It is true in some sense: less inventory reduces the time between making a defect and identifying the defect. But it is also true that productivity would diminish significantly without stock [40]. Since there is a tradeoff, there must be optimal stock levels that are specific to each manufacturing environment. In machining and assembly operations, it has been shown that the operating speed is inversely related to the product quality [41]. Thus, improving the machine processing rate has a positive impact on the system production rate but may negatively affect the system yield.

Bottlenecks identification and elimination have been a central topic in control and improvement of production systems and several notions of bottleneck have been proposed in the literature, for instance, [42], [43]. Rigorous study of bottleneck identification in production lines was initiated in [44], which developed an effective arrow-based method to identify the bottleneck in Bernoulli serial lines using the probabilities of machine blockages and starvations. This method is then extended to serial lines with exponential machines in [45–47] .

To consider product quality issues in production systems, various models have been proposed. The simplest model is the Bernoulli quality model, which determines the quality of each part, defective or non-defective, by a series of independent and identical (i.i.d.) Bernoulli random variables. This model is usually applicable where the defects are due to independent reasons, such as dust and scratches in automotive paint shops. Results regarding production systems with Bernoulli quality model can be found in [48–51]. In these studies, the problems of performance analysis; bottleneck identification, placement of inspection stations, operations sequencing, etc., are discussed. Following this direction, a case study at an automotive paint shop was carried out in [52]. While the Bernoulli model can be applied in systems, where the quality of different jobs is independent, it is not applicable when the quality of consecutive parts are closely related, for example, due to tool wear. To model this phenomenon, additional machine states are usually introduced to represent the scenario when the operation is "out-of-control", i.e., when defective parts are being produced. Unlike the operational states (up or down), the quality-related out-of-control states are often assumed to be not immediately observable. Rather, one can only determine if a machine is in an out-of-control state through a local or remote inspection station downstream, where the defective parts and the type of defects are identified using quality control tools such as Statistical Process Control. Representative results in this direction are reported in [53–57]. For production systems with repair/rework, studies have been carried out in, for instance, [58–60]. Specifically, analytical approaches for performance evaluation and bottleneck identification in such systems were developed. A case study at an automotive paint shop is reported in [61]. Another direction of research on product quality in manufacturing systems considers part scrapping in production systems during machine breakdowns (see, for instance, papers [62–64]). However, in these papers, it is assumed that when a machine fails, the part being processed on that machine is immediately scrapped or scrapped with certian probability, regardless of how long the downtime is. Finally, quality issues in multi-product flexible production systems have been discussed in [65], where the effects of product sequencing on product quality is modeled as a Markov chain.

Despite these important results, there are still various situations that the current quality models cannot precisely depict. For example, in the quality models developed above, the product quality is either assumed to be independent of other system parameters or just modeled as part of the machine characteristic, while the interactions with other system factors are not considered. Among these factors, the storage time of parts between consecutive operations is one of the most important issues, especially in systems that produce perishable products (e.g., food, metal processing, etc.). In fact, from a broad perspective, most commodity can be viewed as having deteriorating "quality" or decreasing appeal/value to the customers. These include, but are not limited to, electronics, appliances, fashion goods, computer software, etc. In the current literature, there exist several directions to study the effects of deteriorating part quality in production systems. The first is to introduce the quality deterioration factor into the classical economic order quantity (EOQ) model and economic production quantity (EPQ) model and their variations (see review papers [66–69], and recent publications [70–75]). However, in these studies, the production system is considered as a single entity in the models, and, therefore, the quality issues within the process of production are not addressed. Another direction of studying quality deterioration is in the area of queueing systems with impatient customers (see [76] and [77] for representative results). Unfortunately, all studies in this area have only focused on single-stage queueing systems with parallel servers, while systems with tandem queues have not been investigated. The paper closest to the topic considered in this paper is [78], which studies performance evaluation in a bufferless synchronized production line with machines having geometrically distributed up- and downtimes. The paper assumes that the parts in a machine must be scrapped if the machine is stopped (due to breakdowns or downstream blockages) for a certain amount of time. Nevertheless, production systems with general buffering and non-synchronized operations have not been addressed. This paper is intended to contribute to this end.

1.3 Outline

The remainder of the paper is organized as follows: Chapter 2 introduces the model and defines the performance measures of interest. In Chapter 3, formulas are derived to evaluate the performance measures in the two-machine Bernoulli case and investigate their monotonicity properties. A case study at an automotive stamping plant is discussed. Based on these results, an aggregation-based recursive procedure is developed for performance evaluation in $M > 2$-machine cases. In Chapter 4, releasing parts to the system were controlled and compared with previous study. Chapter 5, formulas are derived to evaluate the performance measures in the two-machine Geometric case and investigate their monotonicity properties. Based on these results, an aggregation-based recursive procedure is developed for performance evaluation in $M > 2$-machine cases. In chapter 6, bottleneck identification in Bernoulli serial lines with perfect quality and non-perfect quality buffers were studied. Finally, the conclusions and topics for future work are given in Chapter 7.

Chapter 2

SYSTEM MODELING AND PROBLEM FORMULATION

2.1 Introduction

As mentioned in Chapter 1, the purpose of this research is to study system-theoretic properties of production lines with deteriorating product quality, general buffering, and non-synchronized operations. Since there are various notions and conventions on production system used in the literature (see review paper [9]), to avoid confusion and to formalize the presentation, this chapter is devoted to define terminologies that are used throughout this work.

2.2 Types of Production Systems

2.2.1 Serial production lines

Serial production line - a group of producing units, arranged in consecutive order, and material handling devices that transport parts (or jobs) from one producing unit to the next.

Figure 2.1 shows the block diagram of a serial production line where circles represents producing units and rectangles are material handling devices.

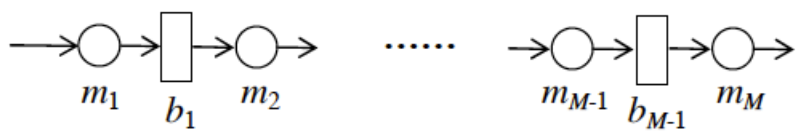


Figure 2.1: Serial production line

The producing units may be either individual machines or departments/shops that have different processes. The material handling devices may be boxes, or conveyers, or vehicles. Whatever the physical appearance may be, we refer to them as buffers, since the most important feature of material handling devices, in this paper, is their storing capacity. The example provided in the figure above is a simple serial production line and the buffers are called *in-process buffers.* There are other types of buffers such as *finished goods buffers* and *empty carrier buffers.* The latter buffer can be seen in lines called *closed with respect to carriers.*

There are other serial lines such as serial lines with product quality inspection where products are checked before getting processed, if they pass they get processed and if they fail they get scrapped, this particular model will be discussed in this paper. Also, there are serial lines with rework, where there is/are quality machines that checks product quality if they fail thus, storing them in buffers to perform rework.

2.2.2 Assembly systems

Assembly system - two or more serial lines, referred to as component lines, one or more merge operations, where the components are assembled, and, perhaps, several subsequent processing operations performed on an assembled part. Figure 2.2 shows typical assembly lines in automative industries.

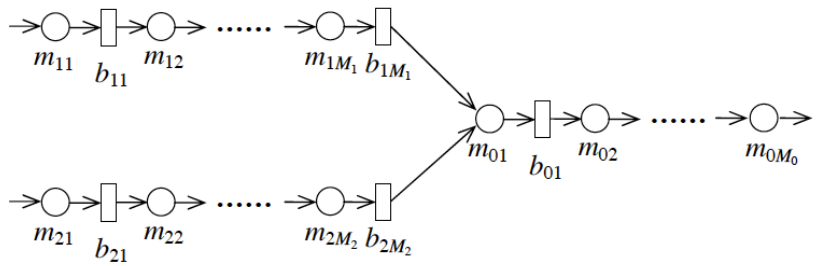


Figure 2.2: Assembly system with single merge operation

Clearly, assembly systems may be viewed as several serial production lines connected through their finished goods buffers. This is clearly one of the simplest serial systems that can be found in the industry while complex assembly systems may carry more complex and advanced lines with quality inspection machines and rework and so on.

2.3 Machine Reliability Models

Machine reliability model - the probability mass function (pmf's) or the probability density function (pdf's) of the up- and downtime of the machine in the slotted or unslotted time, respectively. In this work, some of the following machine reliability models are used:

2.3.1 Reliability models for the slotted time case

Production lines with Bernoulli and Geometric reliability models are usually considered as discrete event systems. The two models addressed:

Bernoulli reliability model (B**):** at the beginning of each time slot, the status of the machine - up or down - is determined by a chance experiment, according to which it is up with probability p and down with probability 1 - p, independently of the status of this machine in all previous time slots. In addition, parameter p is the efficiency of a Bernoulli machine.

This reliability model is simple but practical. Indeed, it is applicable to operations where the unscheduled downtime is, on the average, comparable to the machine cycle time. This often happens in automotive painting and assembly operations, where the downtime is primarily due to quality problems rather than machine breakdowns.

Geometric reliability model (*Geo*): during each time slot, the status of a machine depends on its status in the previous time slot with probabilities of breakdown and repair P and R, respectively as shown in the transition diagram of Figure 2.3

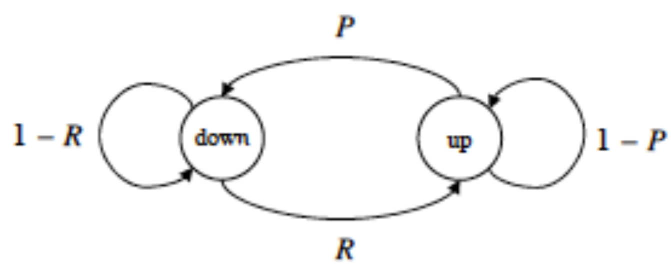


Figure 2.3: Geometric reliability model

It can be shown that the up- and downtime of this machine, denoted as t_{up} and t_{down} are characterized by the following distributions:

$$P[t_{up} = t] = P(1 - P)^{t-1}, \quad t = 1, 2, \ldots$$

$$P[t_{down} = t] = R(1 - R)^{t-1}, \quad t = 1, 2, \ldots$$

Clearly, t_{up} and t_{down} are geometric random variables and we refer to such a machine as a geometric machine, i.e., obeying the geometric reliability model. In addition, it is easy to show that for a geometric machine.

$$T_{up} = \frac{1}{P}, \quad T_{down} = \frac{1}{R},$$

$$e = \frac{T_{up}}{T_{up} + T_{down}} = \frac{R}{P+R}.$$

Methods of analysis of production systems with this reliability model are more complex than in memoryless case. In comparison with the Bernoulli model, this is a more realistic description of a machine.

2.3.2 Reliability models for the continuous time case

The continuous time case is, perhaps, more realistic than the slotted time and, therefore, a larger set of reliability models is addressed. They are as follows:

Exponential reliability model (*exp*): consider a machine in Figure 2.4, which is a continuous time analogue of the geometric machine. Namely, if it is up (respectively, down) at time t, it goes down (respectively, up) during an infinitesimal time δt with probability $\lambda\delta t$ (respectively, $\mu\delta t$). The parameters λ and μ are called the breakdown and repair rates, respectively.

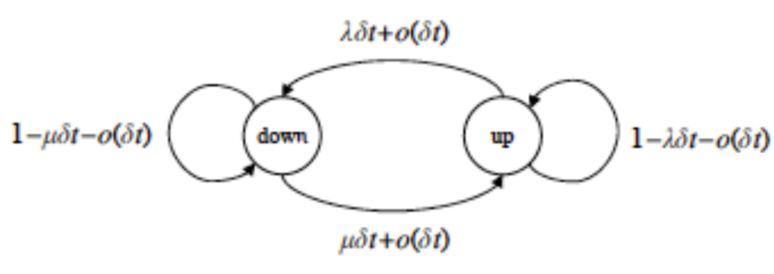


Figure 2.4: Exponential reliability model

It can be shown that the pdf's of the up- and downtime of this machine, denoted as t_{up} and t_{down}, are as follows:

$$f_{t_{up}}(t) = \lambda e^{-\lambda t}, \quad t \geq 0,$$

$$f_{t_{down}}(t) = \mu e^{-\mu t}, \quad t \geq 0.$$

Log-normal reliability model (*LN*): the up- and downtime pdf's of the machine are given by:

$$f_{t_{up}}(t) = \frac{1}{\sqrt{2\pi}\Lambda t} e^{\frac{-(\ln t - \lambda)^2}{2\Lambda^2}}, \quad t \geq 0,$$

$$f_{t_{down}}(t) = \frac{1}{\sqrt{2\pi}M t} e^{\frac{-(\ln t - \mu)^2}{2M^2}}, \quad t \geq 0,$$

where Λ and M are positive numbers. In addition, it can be calculated that for a log-normal machine

$$T_{up} = e^{\lambda + \frac{\Lambda^2}{2}}, \quad T_{down} = e^{\mu + \frac{M^2}{2}},$$

$$CV_{up} = \sqrt{e^{\Lambda^2} - 1}, \quad CV_{down} = \sqrt{e^{M^2} - 1}.$$

2.4 Quality Models

2.4.1 Buffer quality model

The quality deterioration function g is selected from the following three types:

- Type 1: S-shaped function, defined by

$$g(t_r) = \frac{1}{1 + e^{(a \cdot t_r - b)}}, \tag{2.1}$$

where t_r is the residence time of the part in the buffer and a and b are positive constants. Examples of this type of function are shown in Figure 2.5(a). Under this type of quality function, a part maintains a high probability of good quality for short residence time, while the rate of deteriorating is growing as the residence time increases. When the good quality probability becomes already low, the deterioration slows down as well.

- Type 2: L-shaped function defined by

$$g(t_r) = \frac{c}{1 + (c - 1)e^{d \cdot t_r}}, \tag{2.2}$$

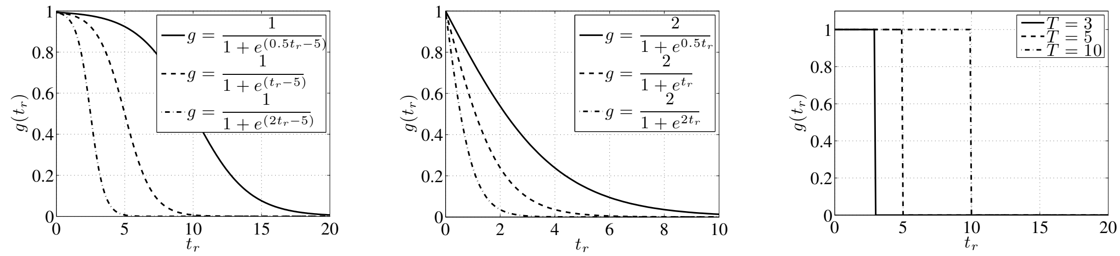
  where c and d are positive constants. Examples of this type of function are shown in Figure 2.5(b). Unlike the functions of Type 1, here, the probability of good quality decreases almost linearly as a function of part residence time before the deterioration slows down after the part resides in the buffer for a relatively long period of time.

- Type 3: Step function defined by

$$g(t_r) = \begin{cases} 1, & \text{if } t_r \le T, \\ 0, & \text{if } t_r > T, \end{cases} \tag{2.3}$$

  where T is a positive constant. Examples of this type of function are shown in Figure 2.5(c). Clearly, parameter T is actually the maximum residence time allowed for a part in the buffer. This type of function is used to imitate the effect of "expiration date" in reality.

While the implication of Type-3 deterioration function is straightforward, the other two may not by immediately intuitive. In fact, both Type-1 and Type-2 deterioration functions are variations of widely used models for quality deterioration. Indeed, the deterioration time, i.e., the time for a product to become defective, is



(a) Type 1 (b) Type 2 (c) Type 3

Figure 2.5: Quality deteriorating functions considered

usually modeled as a random variable subject to gamma or Weibull distributions in the literature for items such as food, fashion goods, technology products, etc. (see, for instance, [67,69,71,73,74]). Thus, the probability of good quality as a function of residence time can be expressed as:

$$g(t_r) = 1 - F(t_r), \tag{2.4}$$

where $F(\cdot)$ is the cumulative distribution function of gamma or Weibull distribution. It can be shown that, depending on the distribution parameters, $g(t_r)$ is either an S-shaped curve or an L-shaped one, similar to the ones shown in Figure 2.5(a) and (b). In this paper, for calculation convenience, we use expressions (2.1) and (2.2) as the quality deteriorating functions to mimic this property.

2.4.2 Machine quality model

In some manufacturing operations, machines can produce defective parts, along with non defective parts. To formalize this situation, we can introduce machine quality models - the pmf or pdf of time intervals during which the machine produces good or defective parts. Listed are some examples of quality models:

Bernoulli quality model: each part produced during a cycle time is good with probability g and defective with probability 1 - g, independent of the quality of parts produced during previous cycles.

Exponential quality model: when up, the intervals of time during which a machine produces good parts or defective parts are distributed exponentially with parameters γ and β, respectively.

2.5 System Considered

In this dissertation, serial production lines, as shown in Figure 2.6 are defined by the following assumptions:

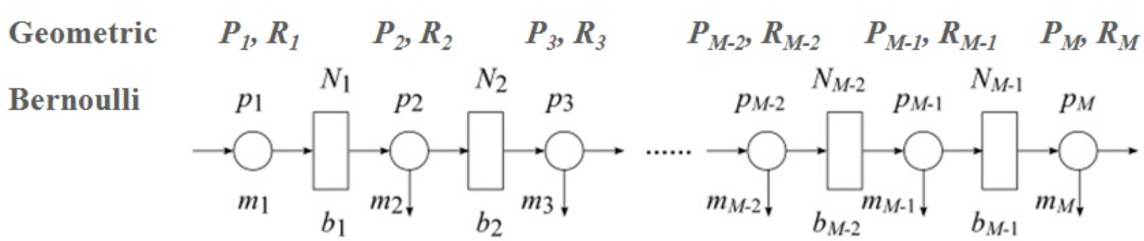


Figure 2.6: Serial production line with deteriorating product quality

(i) The production line consists of M machines (represented by circles) and $M-1$ in-process buffers (represented by rectangles).

(ii) All machines have constant and identical cycle time τ. The time axis is slotted with the slot duration τ. The status of the machines is determined at the beginning of each time slot according to their reliability models.

(iii) Each in-procees buffer, b_i, $i = 1, \ldots, M-1$, is characterized by its capacity, N_i, where $1 < N_i < \infty$. The state of the buffer (i.e., the number of parts in it) is determined at the end of each time slot.

(iv) Machine m_i, $i = 2, \ldots, M$ is starved during a time slot if it is up and buffer b_{i-1} is empty at the beginning of the time slot. It is assumed that machine m_1 is never starved for raw material.

(v) Machine m_i, $i = 1, \ldots, M-1$, is blocked during a time slot if it is up, buffer b_i has N_i parts at the beginning of the time slot and machine m_{i+1} fails to take a part during that time slot. It is assumed that m_M is never blocked.

(vi) The state of the machines is defined by:

(a) If machine m_i, $i = 1, \ldots, M$, when it is neither blocked nor starved, produces a part during a time slot with probability p_i and fails to do so with probability $1 - p_i$. Parameter p_i is referred to as the *efficiency* of m_i. In other words, the machines obey the Bernoulli reliability model.

(b) If machine, m_i, $i = 1, \ldots, M$, when it is neither blocked nor starved and up, it will be down during the next cycle with probability P_i and up with probability $1 - P_i$; if it is down, it will be up during the next cycle with probability R_i and down with probability $1 - R_i$. In other words, the up- and downtime are distributed geometrically with the parameters P_i and R_i respectively.

(vii) The quality of a part deteriorates while residing in the buffers in the sense that the probability that the part is non-defective when exiting buffer b_i, $i = 1, \ldots, M-1$, is a monotonically decreasing function of its residence time in the buffer.

(viii) The quality of parts is identified at each machine after drawn from the previous buffer and the defectives are discarded from the system immediately (represented as the arrows underneath the machines).

Remark 2.1: Note that in large volume production systems, machine cycle time is practically constant or close to being constant. This is the case in most production systems in automotive, electronics, appliance, and other industries. Note also, that the Bernoulli reliability model is applicable to operations where the downtime is, on the average, close to the machine cycle time (see [6], [61] and [79] for practical examples using the Bernoulli model). Systems with machines having other reliability models (e.g., exponential, Weibull, gamma, log-normal, and general, etc.) will be studied in future work.

Remark 2.2: Assumptions (iv), (v) and (vi-b) are formulated in terms of the so-called time-dependant failures, i.e., machines can go down even when blocked or starved [2]. Another possible model is that of operation-dependent failures, were no breakdowns of starved or blocked machines is possible [2], [4]. Both models are practical, depending on the production system at hand: For automated palletized material handling, operation-dependent failures applicable. In case of manual material handling, operation-dependent failures often take place. Both failure modes, however, result in similar behavior. Studies show that thoughputs of a line with time-dependent or operation-dependent failures differ at most by 3 - 4% [2], which is well within the accuracy of the date describing production lines.

Remark 2.3: To reduce the level of complexity, we assume that the parts quality is inspected perfectly, i.e., no good parts are inspected as defective and no defectives are missed.

Remark 2.4: Denote $t_{r,i}$ and g_i as the residence time of a part in buffer b_i and the good quality probability of the part when exiting the buffer. Then, according to assumption (vii), function $g_i(t_{r,i})$ is monotonically decreasing in $t_{r,i}$. As noted above, production systems with residence-time dependent deteriorating quality are widely seen in industries such as food production, metal processing, etc. For instance, in an automotive paint shop, the longer a car body is exposed to plant air, the more probable its surface will be contaminated with dirt and other particles.

Remark 2.5: It should be noted that, in some manufacturing operations, the product quality depends on not only its residence time in the immediate upstream buffer, but also the time in several operations/buffers upstream. The case considered in this paper, however, is also widely observed on the factory floor, where potential quality problems from previous steps are fixed at the operation before the buffer with quality deterioration. For example, in automotive paint shops, wet sanding is the last operation before the car bodies are sent to the paint booths and this operation is designed to thoroughly clean the job surface and have it prepared for painting. However, after being sanded and before being painted, contamination may take place and cause quality problems. Production systems with more complex quality deterioration scenarios will be studied in future work.

Remark 2.5: As one may notice, assumptions (i)-(vi-a) define the conventional Bernoulli serial lines, which have been analyzed in [6].

2.6 Performance Measures

In the framework of the model defined above, the productivity performance measures of interest are:

- Production rate, PR: the expected number of finished parts produced by m_M during one time slot in the steady state where $0 < PR < 1$;

- Consumption rate, CR: the expected number of raw parts consumed by m_1 during one time slot in the steady state where $0 < CR < 1$;

- Scrap rate, SR_i: the expected number of defective parts scrapped by m_i during one time slot in the steady state where $0 < SR_i < 1$;

- Work-in-process, WIP_i: the expected number of parts in buffer b_i, $i = 1, \dots, M-1$, in the steady state where $0 < WIP_i < N_i$;

- Machine starvation ST_i: the probability that machine m_i, $i = 2, \dots, M$, is starved in the steady state where $0 < ST_i < 1$;

- Machine blockage BL_i: the probability that machine m_i, $i = 1, \ldots, M-1$, is blocked in the steady state where $0 < BL_i < 1$.

Among these performance measures, while PR, CR, SR, and WIP have been widely used and measured on the factory floor, ST and BL have received significantly less attention. However, as illustrated in [6], ST and BL have important manufacturing implications and are closely related to various issues, such as bottleneck identification and lean design, in manufacturing practice. For systems defined by assumptions (i)-(viii), the above performance measures can be evaluated as follows:

$$PR = P[\{m_M \text{ is up}\} \cap \{b_{M-1} \text{ is non-empty}\}], \quad (2.5)$$

$$CR = P[\{m_1 \text{ is up}\} \cap \{m_1 \text{ is not blocked}\}], \quad (2.6)$$

$$WIP_i = \sum_{j=1}^{N_i} j \cdot P[\{\text{buffer } b_i \text{ contains } j \text{ parts}\}], \quad (2.7)$$

$$ST_i = P[\{m_i \text{ is up}\} \cap \{\text{buffer } b_{i-1} \text{ is empty}\}], \quad (2.8)$$

$$BL_i = P[\{m_i \text{ is up}\} \cap \{\text{buffer } b_i \text{ is full}\} \cap \{m_{i+1} \text{ is neither down nor blocked}\}]. \quad (2.9)$$

2.7 Summary

In this paper, we will develop analytical methods to evaluate these performances measures of the production systems defined above and discuss the effects of quality deterioration on these performances.

It should be noted that a production system is characterized by both steady state and transient performance. Although the quality of each product in the system is dynamic in time, the goal of this work is to study its properties during steady state. Therefore, in this paper, we focus the discussion on the stationary performance of the system. Transient behavior of the system will be investigated in future work.

Chapter 3

BERNOULLI SERIAL LINES WITH DETERIORATING PRODUCT QUALITY

3.1 Two-machine Lines

3.1.1 Performance analysis

In this section, production lines defined by assumptions (i)-(vi-a)-(viii) with $M = 2$ are analyzed. As a matter of fact, conventional two-machine Bernoulli lines, i.e., lines defined by assumptions (i)-(vi-a), have been studied in [6], while two-machine Bernoulli lines with non-perfect quality machines were studied in [50]. However, for the systems considered in this paper, since the quality of parts is dependent on their residence time in the buffer, it is necessary to obtain the distribution of the residence time first.

Lemma 1 *For two-machine Bernoulli lines defined by assumptions (i)-(vi-a)-(viii), the probability distribution of part residence time,* t_r*, is given by*

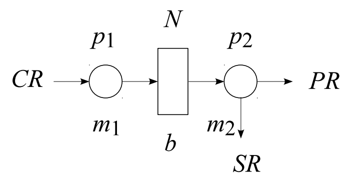


Figure 3.1: Two-machine Bernoulli serial lines

$$P[t_r = t] = p_2 \sum_{i=0}^{\min(t,N-1)} C_t^i \widetilde{P}_i p_2^i (1-p_2)^{t-i}, \quad t = 0, 1, \ldots, \tag{3.1}$$

where

$$C_n^k = \frac{n!}{k!(n-k)!}, \quad 0 \le k \le n,$$

$$\widetilde{P}_0 = \frac{Q(p_1, p_2, N)}{(1-p_1)[1-Q(p_2, p_1, N)]}, \tag{3.2}$$

$$\widetilde{P}_i = \alpha^i(p_1, p_2)\widetilde{P}_0, \quad i = 1, \ldots, N-1, \tag{3.3}$$

$$Q(p_1, p_2, N) = \begin{cases} \frac{(1-p_1)(1-\alpha(p_1,p_2))}{1-\frac{p_1}{p_2}\alpha^N(p_1,p_2)}, & \textit{if } p_1 \neq p_2, \\ \\ \frac{1-p}{N+1-p}, & \textit{if } p_1 = p_2 = p, \end{cases} \tag{3.4}$$

$$\alpha(p_1, p_2) = \frac{p_1(1-p_2)}{p_2(1-p_1)}. \tag{3.5}$$

Proof of Lemma 1: Let P_i, $i = 0, \ldots, N$, denote the steady state probability that the buffer contains i parts at the end of a time slot. Expressions for calculating P_i's are derived in [6]:

$$P_0 = Q(p_1, p_2, N), \quad P_i = \frac{\alpha^i(p_1, p_2)}{1-p_2} P_0,$$

where $Q(p_1, p_2, N)$ and $\alpha(p_1, p_2)$ are given in (3.4) and (3.5), respectively. Introduce the following probability:

$$\widetilde{P}_i = P[\text{buffer has } i \text{ parts when } m_1 \text{ produces a part into the buffer}]. \quad i = 0, \ldots, N-1. \tag{3.6}$$

Thus, $\widetilde{P}_i$, $i = 0, \ldots, N-1$, can be evaluated as follows:

$$\begin{aligned}
\widetilde{P}_0 &= \frac{P[\text{buffer is empty and } m_1 \text{ produces a part}]}{P[\ m_1 \text{ produces a part}]} \\
&= \frac{P_0 p_1 + P_1 p_1 p_2}{p_1[1 - Q(p_2, p_1, N)]} \\
&= \frac{Q(p_1, p_2, N)}{(1-p_1)[1 - Q(p_2, p_1, N)]}, \\
\widetilde{P}_i &= \frac{P[\text{buffer has } i \text{ parts and } m_1 \text{ produces a part}]}{P[\ m_1 \text{ produces a part}]} \\
&= \frac{P_i p_1 (1-p_2) + P_{i+1} p_1 p_2}{p_1[1 - Q(p_2, p_1, N)]} \\
&= \alpha^i(p_1, p_2)\widetilde{P}_0, \quad i = 1, \ldots, N-1.
\end{aligned}$$

According to the total probability formula, the steady state probability distribution of part residence time in systems defined by (i)-(vi-a)-(viii) is given by:

$$\begin{aligned}
P[t_r = t] &= \sum_{i=0}^{\min(t,N-1)} P[\ m_2 \text{ up for } i \text{ cycles in the next } t \text{ time slots}] \cdot \\
&\qquad P[\ m_2 \text{ is up during the } (t+1)\text{th time slots }] \cdot \\
&\qquad P[\text{ the buffer has } i \text{ parts when the new part comes in }] \\
&= p_2 \sum_{i=0}^{\min(t,N-1)} C_t^i \widetilde{P}_i p_2^i (1-p_2)^{t-i}, \quad t = 0, 1, \ldots,
\end{aligned}$$

which completes the proof. ■

Clearly, the quality buy rate of the system, i.e., the probability that a part is non-defective at the output of m_2 can be evaluated as:

$$q = q(p_1, p_2, N, g) = \sum_{t=0}^{\infty} P[t_r = t] g(t_r = t). \tag{3.7}$$

Thus, the performance measures of the two-machine production line can be evaluated using the following:

Theorem 1 *In two-machine Bernoulli lines defined by assumptions (i)-(vi-a)-(viii),*

$$
\begin{aligned}
CR &= p_2[1 - Q(p_1, p_2, N) = p_1[1 - Q(p_2, p_1, N)], && (3.8)\\
PR &= CR \cdot q, && (3.9)\\
SR &= CR \cdot (1 - q), && (3.10)\\
WIP &= \begin{cases} \frac{p_1}{p_2 - p_1\alpha^N(p_1,p_2)}\left[\frac{1-\alpha^N(p_1,p_2)}{1-\alpha(p_1,p_2)} - N\alpha^N(p_1,p_2)\right], & \text{if } p_1 \neq p_2,\\ \frac{N(N+1)}{2(N+1-p)}, & \text{if } p_1 = p_2 = p, \end{cases} && (3.11)\\
BL_1 &= p_1 Q(p_2, p_1, N), && (3.12)\\
ST_2 &= p_2 Q(p_1, p_2, N). && (3.13)
\end{aligned}
$$

where q is defined in (3.7).

Proof of Theorem 1: Follows immediately from Lemma 1 and [50].

■

3.1.2 Monotonicity property

The monotonicity properties of the performance measures are characterized by the following:

Property 1 *In two-machine Bernoulli lines defined by assumptions (i)-(viii),*

- *PR is monotonically increasing in p_2, non-monotonic or monotonically increasing in p_1, and non-monotonic or monotonically decreasing or monotonically increasing in N;*
- *SR is monotonically increasing in p_1 and N, and non-monotonic in p_2;*
- *CR is monotonically increasing in p_i, $i = 1, 2$, and N;*
- *q is monotonically decreasing in p_1 and N, and monotonically increasing in p_2.*

Justifcation of Property 1: To justify these properties, a total of 100,000 production lines were generated with parameters randomly and equiprobably selected from the following sets:

$$p_i \in [0.6, 0.95], \quad N_i \in \{1, 2, 3, 4, 5\}. \tag{3.14}$$

For each line, thus generated, the quality deterioration function g is selected from the three types discussed previously. Specifically, the parameters of these functions are randomly and equiprobably form the following sets:

$$\begin{aligned} &\text{Type 1:} \qquad a \in (0.1, 1.6), \quad b \in (3, 7); \\ &\text{Type 2:} \qquad c \in (1, 2), \quad d \in (0.1, 5); \\ &\text{Type 3:} \qquad T \in \{3, 4, 5, 6, 7, 8\}. \end{aligned} \tag{3.15}$$

Next, we calculate the performance measures of all lines as functions the parameters p_1, p_2, and N using (3.7)-(3.10) and examined whether the corresponding statement of Property 1 holds. As a result, among the 100,000 lines studied, no counterexamples of Property 1 were found. Thus, we conclude that Property 1 indeed takes place.

An illustration of the above properties is given in Figure 3.2, where the quality deterioration function is characterized by an "expiration time", T, as follows:

$$g(t_r) = \begin{cases} 1, & \text{if } t_r \le T, \\ 0, & \text{if } t_r > T. \end{cases} \tag{3.16}$$

As one can see in Figure 3.2, increasing the efficiency of m_1 leads to more defectives, which may result in lower production rate of good parts. Such phenomenon is usually referred to as *quality-quantity coupling* (see [41, 61, 80]). However, in the systems considered here, the decrease of quality is not because of less careful or less precise processing but due to longer residence time in the buffer. On the other hand, improving m_2 always leads to higher production rate and higher quality buy rate. In addition, if p_2 is not significantly smaller than p_1, then the scrap rate can be reduced by increasing p_2. Finally, under quality deterioration function (3.16), larger buffer capacity does not necessarily lead to higher production rate, which is not observed in conventional serial lines defined by assumptions (i)-(vi).

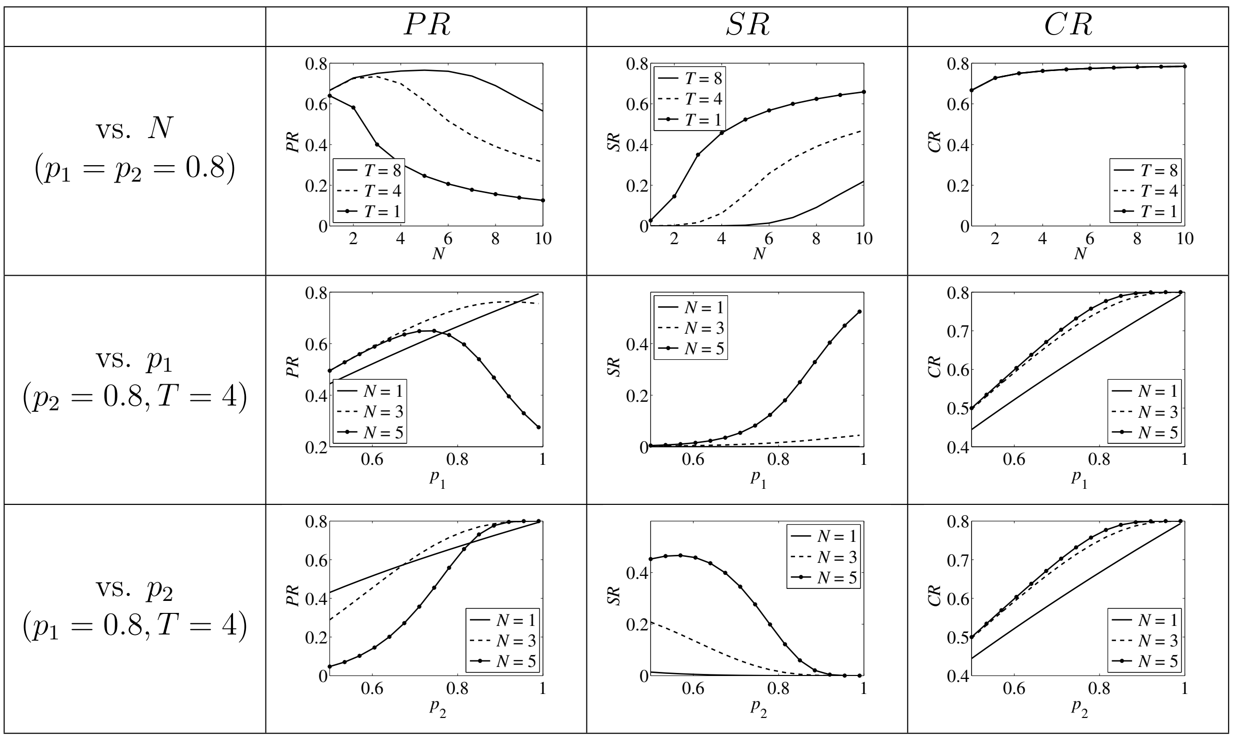


Figure 3.2: PR, SR, and CR as functions of p_1, p_2, N, and T

Due to the lack of monotonicity in PR with respect to buffer capacity N, release of parts into the system needs to be controlled to avoid potentially long residence time. Since the Bernoulli machines are memoryless, the state of the system is just the occupancy of the buffer. Assume that the control point policy is used, i.e.,

$$u(h(n) - h^*) = \begin{cases} 1 & \text{(i.e., release is authorized), if } h(n) \leq h^*, \\ 0 & \text{(i.e., release is denied), otherwise.} \end{cases} \quad (3.17)$$

Therefore, under control (3.17), parameter h^* can be viewed as the virtual capacity of the buffer since its occupancy cannot exceed h^* parts. As a result, the performance measures in the controlled, i.e., closed-loop, systems can be evaluated using (3.8)-(3.10) with N replaced by $\min(N, h^*)$.

To determine the optimal value h^* of control (3.17), the following procedure can be used:

Procedure 3.1: Given the desired production rate PR_d:

(a) For $n = 1$, select $h^*(n) = 1$ and $PR(0) = 0$.

(b) Evaluate $PR(n)$ using (3.9) with N replaced by $h^*(n)$.

(c) If $PR(n-1) < PR(n) < PR_d$ and $h^*(n) < N$, then $h^*(n+1) = h^*(n) + 1$, $n = 1, 2, \ldots$, and return to (b).

(d) if $PR(n) > PR_d$, select $h^* = h^*(n)$ and terminate the algorithm.

(e) If $PR(n) < PR(n-1)$, or $PR(n) < PR_d$ and $h^*(n) = N$, then h^* dose not exist for the given PR_d.

To illustrate the efficacy of the parts release control, consider a Bernoulli line defined by assumptions (i)-(viii) with $p_1 = p_2 = 0.8$, $N = 5$. Assume that the part quality deterioration in the buffer is of Type 3 with $T = 4$. The performance measures of the system are calculated using (3.8)-(3.10) as follows:

$$PR = 0.6154, \quad SR = 0.1538, \quad CR = 0.7692, \quad q = 0.8000.$$

Now, assume that the desired production rate is $PR_d = 0.73$. Then, using Procedure 3.1, the optimal control parameter $h^* = 3$ is obtained and the resulting closed-loop system performances are

$$PR = 0.7338, \quad SR = 0.0162, \quad CR = 0.7500, \quad q = 0.9783.$$

Clearly, with feedback release control, both quantity and quality performances of the system are significantly improved. Specifically, the production rate of good parts is increased by 19%, while the quality buy rate is increased by 17%. Along with these improvements, less raw material is consumed and practically no scraps is observed. Therefore, using optimal feedback control of parts release can improve the productivity and the quality of production systems.

It should be noted that, in practice, there are other techniques that can be used to maintain the quality of work-in-process by, for instance, dispatching and relocating the products. However, controlling the buffer size is often considered as a direct approach, which does not involve additional subsystems (e.g., hoist scheduling). Thus, in this paper, we only consider the effect of controlling buffer capacities on system performance.

3.1.3 Case study

At an automotive stamping plant as shown in Figure 3.3, the raw steel is received via truck in a roll. The blanking press will cut the steel into the required quantity and size for the body panel, and a layer of lubrication (oil) is placed on the blanks of steel during the blanking process. Then the pallet of blanks is shipped to the washer, which cleans debris off the parts and places oil on the blanks at a specified thickness. The washed pallet is placed in a queue waiting for stamping press. The top and bottom blanks on a pallet will be discarded if it has stayed in the queue for more than 4 hours, and the rest is loaded into the press to create the desired body panel. The reason for this discarding is due to that the oil has evaporated after four hours. Such evaporation will result in a bad finished part or die damage. Finally visual inspection is carried out to check obvious defects in the stamping parts. The most critical operations in this process are washing and pressing.

Although the defective panels only accounts for a very small portion of the

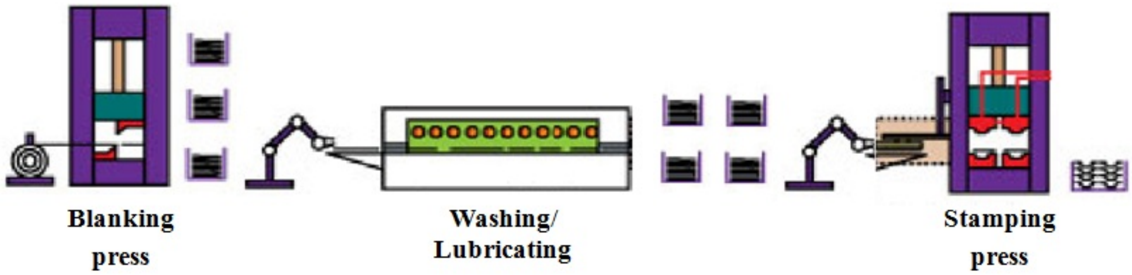


Figure 3.3: Automative stamping plan

entire batch (typically 1-3%), due the large volume of this production system, the scrap can lead to significant unnecessary cost (e.g., additional workforce for quality inspection, material transportation, etc.) even after it is offset through material recycling. Therefore, reducing scrap waste is considered a critical task by the factory floor operators and management personnel in this system. Improvement efforts have been made to increase the thickness of the oil so that no discarding is necessary before 6 hours. To investigate the impact and savings of such effort, a production system model is developed, and using this model, we study the residence time feedback control policy. As described above, the most critical processes, washing and pressing, are included in the model. Thus, a two-machine Bernoulli model is introduced, where the parameters of the machines are identified using the data collected on the factory floor and we obtain:

$$p_1 = 0.768, \quad p_2 = 0.8, \quad \tau = 76.8 \text{ min}.$$

To determine T, note that the maximal residence times for a batch in the buffer are 4 hours and 6 hours, before and after the increase of oil thickness, respectively. Therefore, we assume in the model that $T = 3$ (i.e., residence time $= 3\tau = 3$ hr 50 min) and $T = 5$ (i.e., residence time $= 5\tau = 6$ hr 24 min) for the above two cases, respectively.

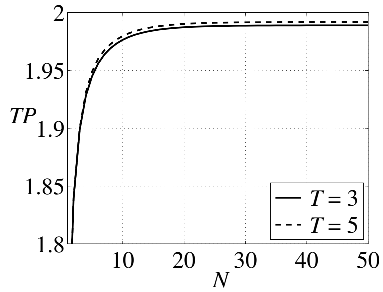
Assume a batch contains 200 blanks on the average. Since only the top and bottom blanks are scrapped for batches residing in the buffer longer than T, the throughput of good blanks of the system is given by:

$$TP = \frac{200(PR + 0.99SR)}{\tau} \quad \text{(blanks/min)}.$$

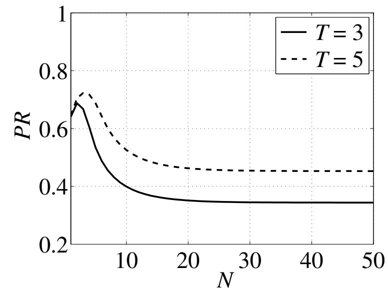
The behavior of the performance measures as functions of buffer capacity, N, is illustrated in Figure 3.4. It can also be observed from the figure that without feedback release control, more than 50% of the washed batches contain defective blanks due to long waiting before being pressed.

Assume now that feedback release controller (3.17) is applied to this system. Since the controller parameter h^* can be viewed as the (virtual) capacity of the buffer, Figure 3.4 can be viewed as the behavior of the performance measures as functions of h^* as well. As one can see from the figure, if the maximal allowed residence time in the buffer is increased to $T = 5$ cycles, then PR, SR, and q can be improved significantly under the same control, with the TP remaining almost the same.

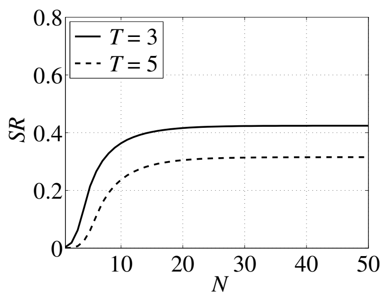
Next, we investigate the optimal feedback release control under desired throughput TP_d for the system at hand. The optimal control parameter h^* can be obtained using Procedure 3.1 and the resulting performance measures as functions of TP_d are illustrated in Figure 3.5. As one can see, both $T = 3$ and $T = 5$ require similar control parameters for the same TP_d. However, significantly less scraps can be produced if the maximal residence time is increased from $T = 3$ to $T = 5$. In addition, as the desired throughput increases, the optimal control parameter also increases to allow more batches into the system, which leads increased scraps. It should be noted that PR is non-monotonic with respect to TP_d due to its non-monotonic behavior with respect to h^* (see Figures 3.4(b) and 3.5(b)).



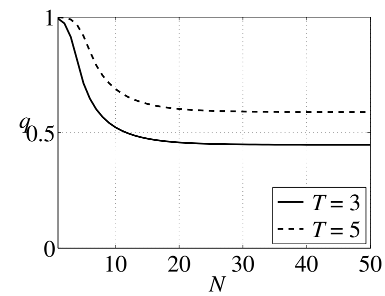
(a) TP vs. N



(b) PR vs. N

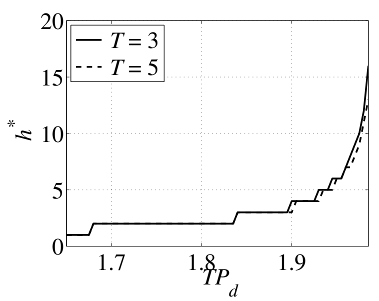


(c) SR vs. N

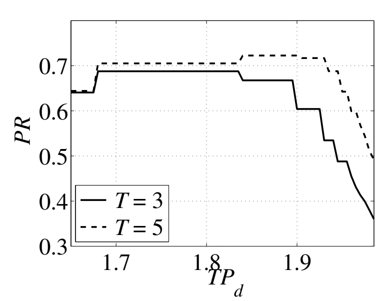


(d) q vs. N

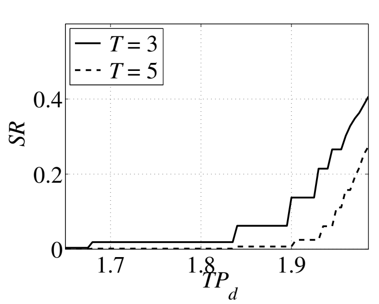
Figure 3.4: Performance measures as functions of N



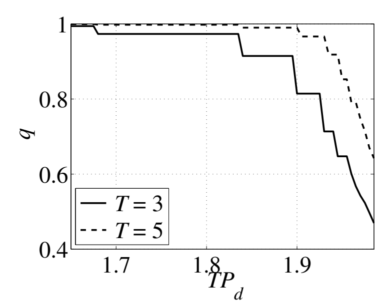
(a) h^* vs. TP_d



(b) PR vs. TP_d



(c) SR vs. TP_d



(d) q vs. TP_d

Figure 3.5: Optimal feedback release control for desired throughput, TP_d

3.2 M > 2-machine Lines

3.2.1 Performance analysis

For Bernoulli serial lines with non-perfect quality machines and inspection machines, an aggregation-based recursive procedure is developed in [50]. Note that the system considered in [50] assumes that the defects are generated at individual machines but independent of the parts residence time in the buffers. As a result, the quality buy rate at each inspection machine can be explicitly calculated by multiplying the quality parameters of all upstream machines until the nearest inspection machine. However, due to the coupling of machines and buffers in the system considered in this paper, the quality buy rates cannot be obtained by explicit calculations. Therefore, the following recursive procedure is proposed to accommodate this feature:

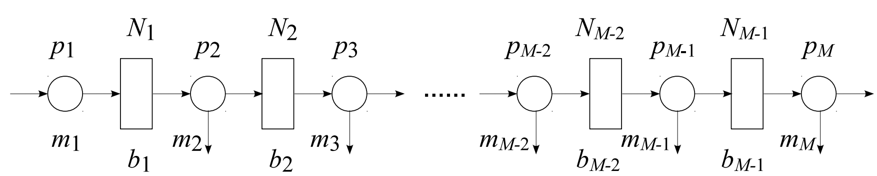


Figure 3.6: M > 2-machine Bernoulli serial lines

Recursive Procedure 3.2:

$$\begin{aligned} p_i^b(s+1) &= p_i\left[1-Q(p_{i+1}^b(s+1),p_i^f(s),N_i)\right], \quad i=1,\dots,M-1, \\ & s=0,1,2,\dots, \qquad (3.18) \\ p_i^f(s+1) &= p_iq(p_{i-1}^f(s+1),p_i^b(s+1),N_{i-1},g_{i-1})\left[1-Q(p_{i-1}^f(s+1),p_i^b(s+1),N_{i-1})\right], \\ & i=2,\dots,M, \quad s=0,1,2,\dots, \qquad (3.19) \end{aligned}$$

with initial conditions

$$p_i^f(0)=p_i, \quad i=1,\dots,M, \qquad (3.20)$$

and boundary conditions

$$p_1^f(s) = p_1, \quad p_M^b(s) = p_M, \quad s = 0, 1, \dots, \tag{3.21}$$

where functions $Q(\cdot)$, $\alpha(\cdot)$ and $q(\cdot)$ are defined in (3.4), (3.5) and (3.7), respectively.

To investigate the convergence of this recursive procedure, we define

$$V(s) = \sum_{i=2}^{M}[p_i^f(s) - p_i^f(s-1)]^2 + \sum_{i=1}^{M-1}[p_i^b(s) - p_i^b(s-1)]^2, \quad s = 1, 2, \dots. \tag{3.22}$$

Numerical Fact 1 *Sequence $V(s)$ is convergent with respect to s with probability $1-\epsilon$, where $\epsilon \ll 1$. In other words, there exist limit V_∞ such that*

$$P\left[\lim_{s\to\infty} V(s) = V_\infty\right] = 1 - \epsilon. \tag{3.23}$$

Justification of Numerical Fact 1: To justify this numerical fact, we studied production lines with $M = 3, 4 \dots, 15$ machines. Specifically, for each $M \in \{3, 4, \dots, 15\}$, a total of 50,000 lines were generated. Therefore, a total of 650,000 production lines were investigated. The efficiencies of the machines and the capacities of the buffers were selected randomly and equiprobably from (3.14). In addition, for each buffer, quality deterioration exists with probability 0.5. In such cases, the quality deterioration function g_i is selected from the three types described above with parameters randomly and equiprobably selected from (3.15). During the justification, we considered sequence $V(s)$ convergent, if there exists $0 < s_0 < 10,000$, such that $|V(s_0) - V(s_0 - 1)| < 10^{-7}$, and terminate the procedure as soon as this inequality is observed.

As a result, sequence $V(s)$ converged in 649,841 lines, i.e., 99.976% of all cases studied. The number of non-convergent cases for each M considered is shown in Figure 3.7. Among the cases, where convergence is observed, two cases are possible:

$V_\infty = 0$ and $V_\infty > 0$. The former implies that sequences $p_i^f(s)$, $i = 2, \ldots, M$, and $p_i^b(s)$, $i = 1, \ldots, M-1$, are also convergent with respect to s with unique limits $\widetilde{p}_i^f$ and $\widetilde{p}_i^b$:

$$\lim_{s \to \infty} p_i^f(s) = \widetilde{p}_i^f, \quad \lim_{s \to \infty} p_i^b(s) = \widetilde{p}_i^b.$$

In the latter, it implies that sequences $p_i^f(s)$ and $p_i^b(s)$ converge with respect to s to limit cycles, i.e., for $s \to \infty$, each sequence oscillates among a set of values, while having $V(s)$ constant everywhere on the cycle. Detailed information regarding this convergence issue for each M considered among the 650,000 lines studied above is summarized in Table 3.1. Clearly, limit cycle convergence appears in a very small portion of all systems studied (usually less than 2%).

Based on these results, we claim that Numerical Fact 1 indeed takes place.

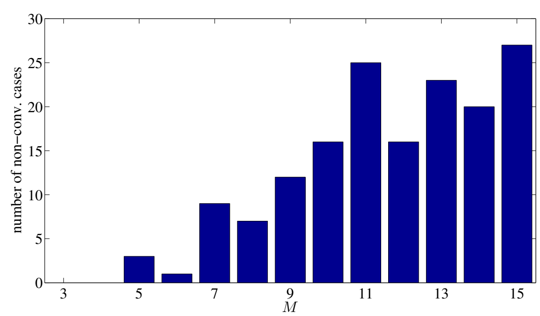


Figure 3.7: Justification of Numerical Fact 1

■

A total of 650,000 randomly generated production lines were used in the justification. As a results, convergence of $V(s)$ is observed in 99.976% of all cases studied. The non-convergent cases often contain system parameters that are dramatically different from one another, which is rarely the case in practical situations. Moreover, for those lines where $V(s)$ converges, two cases are possible: $V_\infty = 0$ and $V_\infty > 0$. The former implies that sequences $p_i^f(s)$, $i = 2, \ldots, M$, and $p_i^b(s)$, $i = 1, \ldots, M-1$,

Table 3.1: Convergence of Recursive Procedure 4.1

	Non-converging	Limit cycles	Unique limits
$M = 3$	0	33	49967
$M = 4$	0	152	49848
$M = 5$	3	351	49646
$M = 6$	1	492	49507
$M = 7$	9	596	49395
$M = 8$	7	726	49267
$M = 9$	12	830	49158
$M = 10$	16	903	49081
$M = 11$	25	927	49048
$M = 12$	16	986	48998
$M = 13$	23	971	49006
$M = 14$	20	940	49040
$M = 15$	27	998	48975

are also convergent with respect to s with unique limits $\widetilde{p}_i^f$ and $\widetilde{p}_i^b$:

$$\lim_{s \to \infty} p_i^f(s) = \widetilde{p}_i^f, \quad \lim_{s \to \infty} p_i^b(s) = \widetilde{p}_i^b. \tag{3.24}$$

In the latter, however, it implies that sequences $p_i^f(s)$ and $p_i^b(s)$ converge with respect to s to limit cycles, i.e., for $s \to \infty$, each sequence oscillates among a set of values, while having $V(s)$ constant everywhere on the cycle. In addition, for those systems with limit cycle convergence, we observed that each limit cycle contains only two values. In this case, we introduce

$$\lim_{s \to \infty} \frac{p_i^f(s) + p_i^f(s-1)}{2} = \bar{p}_i^f, \quad \lim_{s \to \infty} \frac{p_i^b(s) + p_i^b(s-1)}{2} = \bar{p}_i^b. \tag{3.25}$$

Clearly, when sequences $p_i^f(s)$ and $p_i^b(s)$ converge to unique limits (3.24), we have

$$\widetilde{p}_i^f = \lim_{s \to \infty} p_i^f(s) = \lim_{s \to \infty} \frac{p_i^f(s) + p_i^f(s-1)}{2} = \bar{p}_i^f,$$
$$\widetilde{p}_i^b = \lim_{s \to \infty} p_i^b(s) = \lim_{s \to \infty} \frac{p_i^b(s) + p_i^b(s-1)}{2} = \bar{p}_i^b.$$

Therefore, to avoid confusion, in the subsequent discussions, we define

$$p_i^f = \bar{p}_i^f = \lim_{s\to\infty} \frac{p_i^f(s) + p_i^f(s-1)}{2}, \quad p_i^b = \bar{p}_i^b = \lim_{s\to\infty} \frac{p_i^b(s) + p_i^b(s-1)}{2}. \tag{3.26}$$

It should be noted that the limit cycle convergence is not observed in production lines studied in [6, 50].

Based on Recursive Procedure 4.1 and Numerical Fact 1, the estimates of the performance measures for $M > 2$-machine Bernoulli lines defined by assumptions (i)-(viii) are formulated below:

$$\widehat{PR} = p_M^f, \tag{3.27}$$

$$\widehat{CR} = p_1^b, \tag{3.28}$$

$$\widehat{SR}_i = p_i^b[1 - Q(p_{i-1}^f, p_i^b, N_{i-1})](1 - q_i), \tag{3.29}$$

$$\widehat{WIP}_i = \begin{cases} \frac{p_i^f}{p_{i+1}^b - p_i^f \alpha_i^N(p_i^f, p_{i+1}^b)} \left[\frac{1-\alpha_i^N(p_i^f, p_{i+1}^b)}{1-\alpha(p_i^f, p_{i+1}^b)} - N_i \alpha_i^N(p_i^f, p_{i+1}^b) \right], & \text{if } p_i^f \neq p_{i+1}^b, \\ \frac{N_i(N_i+1)}{2(N_i+1-p_i^f)}, & \text{if } p_i^f = p_{i+1}^b, \end{cases} \tag{3.30}$$

$$\widehat{ST}_i = p_i - \frac{p_i^f}{q_i}, \tag{3.31}$$

$$\widehat{BL}_i = p_i - p_i^b, \tag{3.32}$$

where p_i^f and p_i^b are defined in (3.26).

To evaluate the accuracy of these estimates, we developed a C++ program to simulate the systems considered in this paper and estimated the performance measures of the 649,841 convergent lines generated in the justification of Numerical Fact 1. Specifically, we carried out 20 replications of the simulation code for each line. In each replication, we used the first 20,000 time slots as a warm-up period and the subsequent 400,000 time slots to statistically calculate the average performance. The resulting performance estimates are denoted as PR_{sim}, CR_{sim}, SR_{sim}, $WIP_{i,\text{sim}}$,

$ST_{i,\text{sim}}$ and $BL_{i,\text{sim}}$. Then, we calculated the performance estimates using (3.27)-(3.32) and compared them with those obtained by simulations according to the following metrics:

$$\delta_{PR} = \frac{|PR_{\text{sim}} - \widehat{PR}|}{PR_{\text{sim}}} \cdot 100\%, \tag{3.33}$$

$$\delta_{CR} = \frac{|CR_{\text{sim}} - \widehat{CR}|}{CR_{\text{sim}}} \cdot 100\%, \tag{3.34}$$

$$\delta_{SR} = \frac{|SR_{\text{sim}} - \widehat{SR}|}{CR_{\text{sim}}} \cdot 100\%, \tag{3.35}$$

$$\delta_{WIP} = \frac{1}{M-1} \sum_{i=1}^{M-1} \frac{|WIP_{i,\text{sim}} - \widehat{WIP}_i|}{N_i} \cdot 100\%, \tag{3.36}$$

$$\delta_{ST} = \frac{1}{M-1} \sum_{i=2}^{M} |ST_{i,\text{sim}} - \widehat{ST}_i|, \tag{3.37}$$

$$\delta_{BL} = \frac{1}{M-1} \sum_{i=1}^{M-1} |BL_{i,\text{sim}} - \widehat{BL}_i|. \tag{3.38}$$

The results are summarized in Table 3.2, which also includes the average computation time for the aggregation procedure (also coded as a C++ program), t_{agg}, and the average simulation time t_{sim} for the production lines considered. All computations and simulations were performed on the University of Wisconsin-Milwaukee High Performance Computing Cluster, which consists of 142 Nehalem 5,550 nodes (1,136 cores), with 24 gigabytes of memory per node. As one can see from the table, the errors of the performance estimates (3.27)-(3.32) are increasing as the number of machines in the system M becomes larger. Also, it has been observed during the experiments that the errors of these performance estimates tend to be larger as the number of buffers with quality deterioration increases. However, for all cases, the average errors remain relatively small. Moreover, the time needed by the calculation-based method is significantly shorter than that required by simulations. In addition, despite the lack of guaranteed convergence, the procedure is still convergent with close to 100% probability under practical parameter ranges. Finally, taking into account that the parameters of the machines and buffers are rarely known on the factory floor with accuracy better than 5%-10%, we claim that Recursive Procedure 4.1 and equations (3.27)-(3.32) can be used to approximate the performance of the production systems considered in this paper effectively and efficiently.

Table 3.2: Average accuracy of performance estimates (3.27)-(3.32)

	δ_{PR}	δ_{CR}	δ_{SR}	δ_{WIP}	δ_{ST}	δ_{BL}	t_{agg} (sec)	t_{sim} (sec)
$M = 3$	0.66%	0.56%	0.72%	0.93%	0.0355	0.0014	< 0.01	6.56
$M = 4$	1.26%	0.76%	1.12%	1.46%	0.0342	0.0027	< 0.01	9.11
$M = 5$	1.75%	0.91%	1.44%	1.81%	0.0334	0.0037	0.01	12.17
$M = 6$	2.15%	1.00%	1.70%	2.04%	0.0328	0.0045	0.01	15.11
$M = 7$	2.39%	1.07%	1.87%	2.22%	0.0318	0.0050	0.06	17.68
$M = 8$	2.65%	1.13%	2.03%	2.35%	0.0308	0.0054	0.07	20.27
$M = 9$	2.78%	1.17%	2.13%	2.41%	0.0301	0.0056	0.09	23.48
$M = 10$	2.96%	1.22%	2.25%	2.48%	0.0297	0.0058	0.15	26.26
$M = 11$	3.01%	1.26%	2.30%	2.49%	0.0288	0.0059	0.21	28.70
$M = 12$	3.11%	1.28%	2.34%	2.49%	0.0283	0.0059	0.23	31.55
$M = 13$	3.19%	1.32%	2.39%	2.51%	0.0277	0.0059	0.26	34.42
$M = 14$	3.21%	1.35%	2.41%	2.48%	0.0272	0.0059	0.29	36.78
$M = 15$	3.25%	1.39%	2.43%	2.45%	0.0266	0.0058	0.29	39.75

3.2.2 Monotonicity property

Similar to the two-machine case, we study the monotonicity properties of production lines defined by assumptions (i)-(vi-a)-(viii) with $M > 2$:

Property 1 *In $M > 2$-machine Bernoulli lines defined by assumptions (i)-(viii),*

- *PR is either monotonically increasing in p_i, or non-monotonic in p_i, $i = 1, \dots, M$;*
- *PR is either monotonically increasing in N_i, or monotonically decreasing in N_i, or non-monotonic in N_i, $i = 1, \dots, M-1$;*
- *SR is either monotonically increasing in p_i, or non-monotonic in p_i, $i = 1, \dots, M$;*

- *SR is monotonically increasing in N_i, $i = 1, \dots, M-1$;*
- *CR is monotonically increasing in p_i, $i = 1, \dots, M$, and N_i, $i = 1, \dots, M-1$.*

Again, similar to the two-machine case, due to quality deterioration, the monotonic properties of the performance measures with respect to machine and buffer parameters strongly depend on the location of the machines and buffers in the system.

As an illustration, consider a 5-machine line given in Figure 3.8, where buffer b_4 is the only one with quality deterioration. Assume that the efficiency of the machines are given by $p = [0.93\ 0.78\ 0.90\ 0.75\ 0.81]$, the buffers are of equal capacity $N_i = N$, and the quality deterioration in b_4 is defined by expression (3.16) with $T = 4$. Since the monotonicity properties of CR are simple Figure 3.11, here we only discuss the behavior of PR and SR as functions of p_i's and N_i's for this system. Representative results are shown in Figures 3.9 and 3.10. As one can see, higher efficiency of machine m_4 and/or higher capacity of buffer b_4 may lead to lower production of good parts due to long residence time of parts in buffer b_4, while increasing p_5 can help alleviate the accumulation of work-in-process, and thus, always lead to increasing PR.

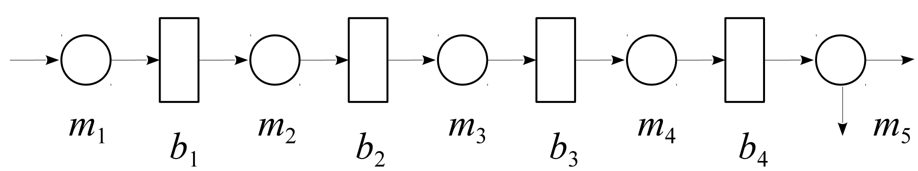


Figure 3.8: 5-machine line example

3.3 Summary

Apparently, the lack of monotonicity in PR makes it more difficult when designing a continuous improvement project for production lines with quality deterioration issues. Intuitively, one would attempt to reduce the residence time of parts in the buffers while

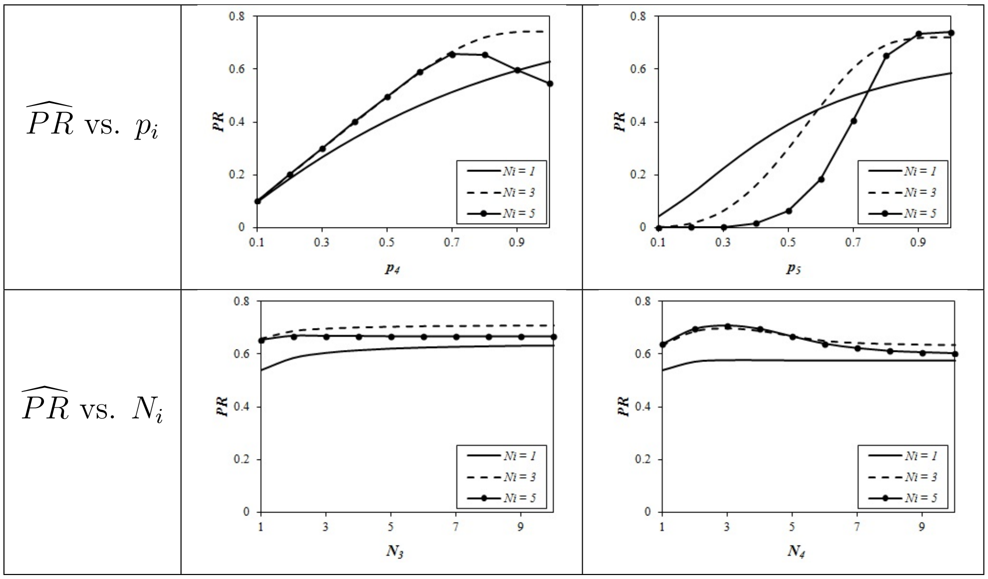


Figure 3.9: $\widehat{PR}$ as functions of p_i and N_i

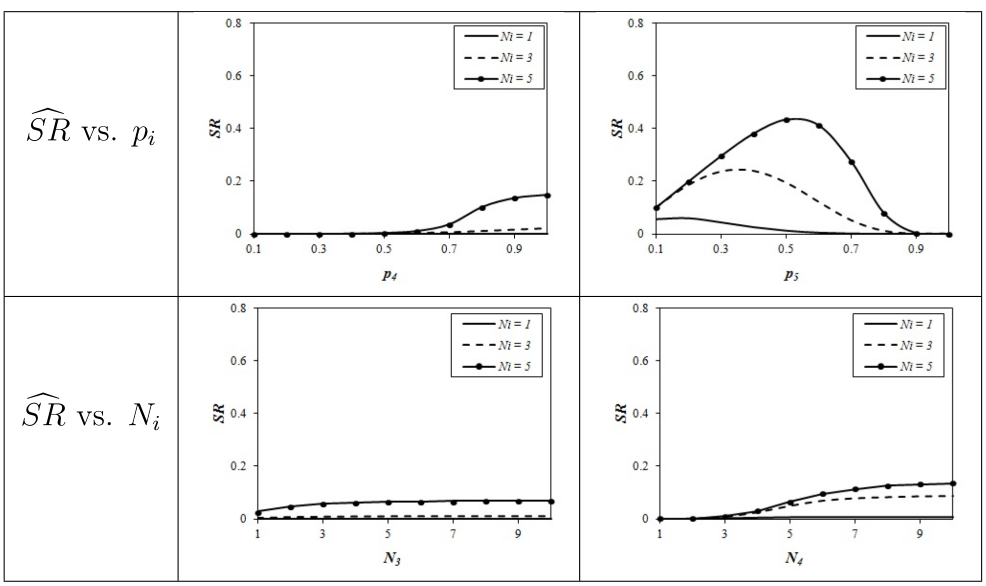


Figure 3.10: $\widehat{SR}$ as functions of p_i and N_i

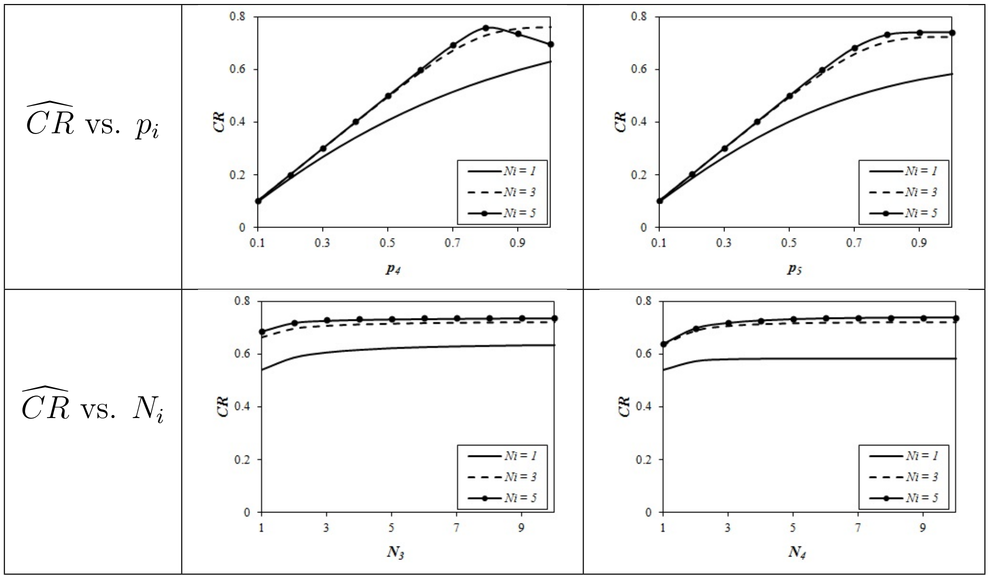


Figure 3.11: $\widehat{CR}$ as functions of p_i and N_i

maintaining sufficient parts flow through the system. However, for practical systems, due to the complicated coupling among machines and buffers, it is all but impossible to "predict" the effect of changing system parameters by using just common-sense. Fortunately, the aggregation-based performance evaluation technique developed in this paper can be used by practitioners as an effective and computationally efficient tool to accomplish this task.

Chapter 4

BERNOULLI SERIAL LINES WITH CONTROLLED PARTS RELEASE

4.1 Introduction

Effective production control systems are those that produces the right parts, at the right time, at a competitive price. Some manufacturers have reported considerable success meeting these objectives by using "pull based" production planning and control systems. The effective production control in any manufacturing system, that is, the management of the total flow of goods through the system, from the acquisition of raw parts to the delivery of final products to customers, is key to the competitiveness of the system. Production control is an optimization problem that typically addresses the question of when and how much to produce in order to achieve a satisfactory production, while keeping low in-process inventories. Difficulties in production control arise because of queueing delays due to variability in production capacity (e.g., due to the failure or maintenance of a machine) and demand for final or intermediate products. In a real production environment, it can often be observed that there are items being scrapped. These scrapped items must be reduced. In all cases, substantial costs are incurred. Therefore, it is more appropriate to take the quality-related cost into account in determining the optimal release policy. Since the recursive procedure used earlier provides estimates, rather than exact values, the results obtained here are also approximate; they provide estimates of the performance measures. The accuracy of these estimates is quantified by simulations and shown to be sufficiently high (well within 3%).

This chapter will consider similar production lines as introduced in Chapter 3 with a minor difference, Figure 4.1 shows the block diagram of a Bernoulli serial production line consists of two machines and the first machine m_0 (grey circle) represents part release control machine (*PRC*) and first buffer b_0 (grey rectangle) is an infinite buffer. In other words, parts released to the system is controlled by m_0 and can be on deterministic or stochastic bases. This study in this chapter was carried out using a simulation model coded as a C++ program. Then we analyze each of them and compare them to one another and to the original system studied. As we shall see, the continuous part release policy is no longer optimal for some cases we covered earlier after using *PRC*.

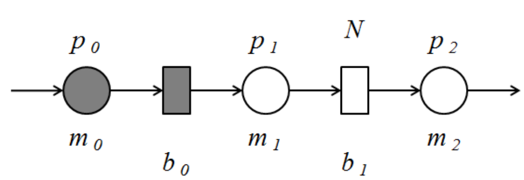


Figure 4.1: Two-machines Bernoulli line with *PRC* machine

4.2 Deterministic Release

The deterministic part is the average, or expected pattern in the absence of any kind of randomness or measurement error (i.e., stochasticity). The system we considered here can be ideally used to model production systems in which PRC_d machine is only interrupted after (approximately) a fixed amount of time since it starts. We will look into the system from different angles starting with; machines efficiencies, buffer capacity, and maximum residence time allowed for parts to stay in the buffer before they become obsolete.

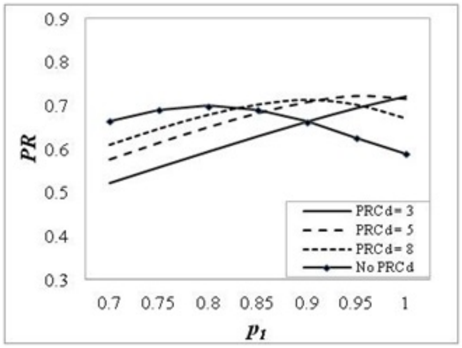
Definition 4.1: Machine m_0 releases x parts to the system to infinite buffer b_0 in a deterministic manner defined by:

$$r_d(x) = \begin{cases} 0, & \text{if } x\%(PRC_d + 1) = 0, \\ 1, & \text{else,} \end{cases} \tag{4.1}$$

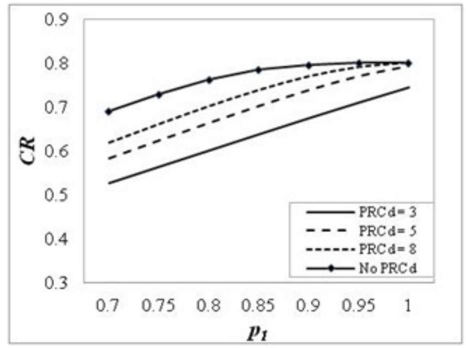
where PRC_d is a positive constant represents how many parts that we want to release in the system before it holds the system from releasing the next part. To analyze the effect of deterministic part release control, series of tests were conducted on two-machine Bernoulli serial line with the following parameters:

$$p_i = 0.80, \quad N = 4, \quad T = 4.$$

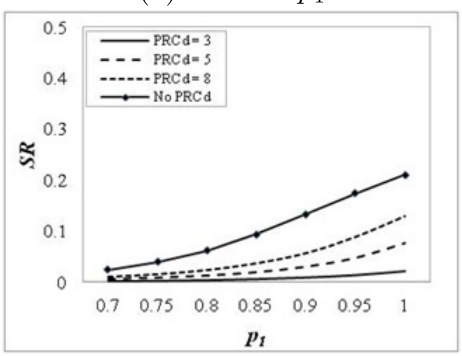
As one can see in Figure 4.2, increasing m_1 efficiency results in monotonically increasing PR for a less frequent release, but as parts released into the system more frequently, production rate becomes non-monotonic with respect to p_1. The non-monotonic behavior is due to the monotonically increasing WIP (average number of parts to be processed in buffer N) with respect to p_1. Also, for a more frequent release, m_0 is allowing more parts into the system, CR improves as well as PR while maintaining low SR. On the other hand, increasing efficiency of m_2, see Figure 4.3, results in monotonically increasing PR and improves with a more frequent release.



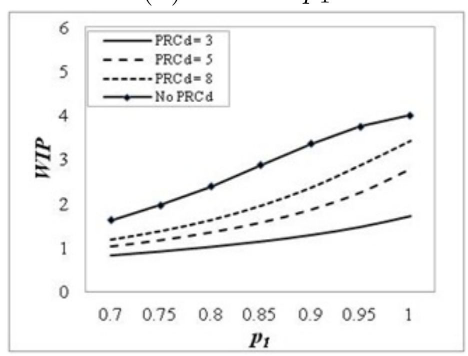
(a) PR vs. p_1



(b) CR vs. p_1



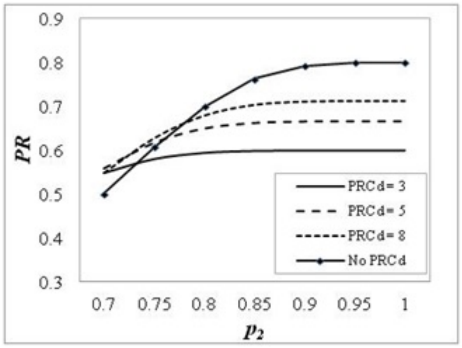
(c) SR vs. p_1



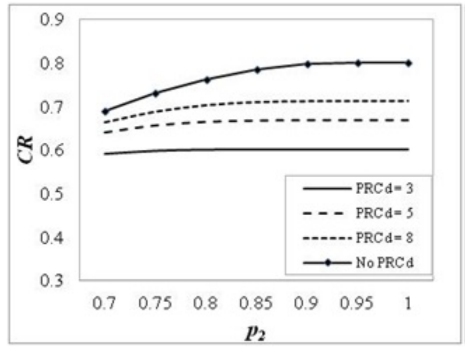
(d) WIP vs. p_1

Figure 4.2: Performance measures as functions of p_1 with deterministic release

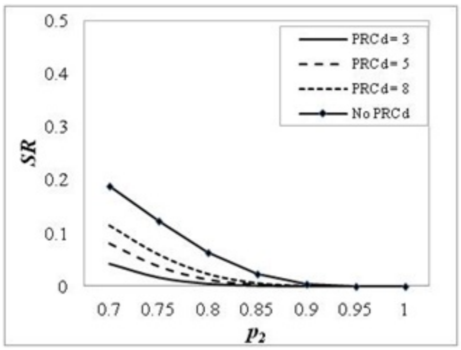
Controlling part release is only sufficient when m_2 efficiency is lower than m_1 efficiency. As mentioned earlier in Chapter 3, the lack of monotonicity in PR with respect to buffer capacity N suggested to control release of parts into the system to avoid potentially long residence time. We can see in Figure 4.4 that, with more frequent part release PR increases until it reaches certain buffer capacity then drops. Finally, Figure 4.5 shows that PR is monotonically increasing in maximum residence time constraint T and how little of a positive effect PRC_d provides for systems with adjustable T.



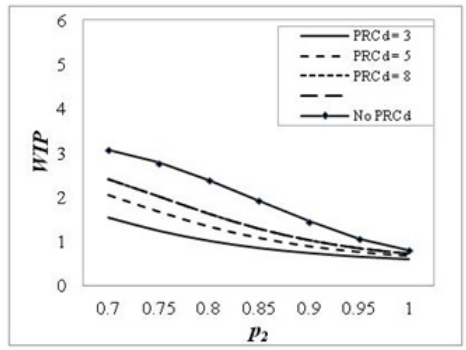
(a) PR vs. p_2



(b) CR vs. p_2

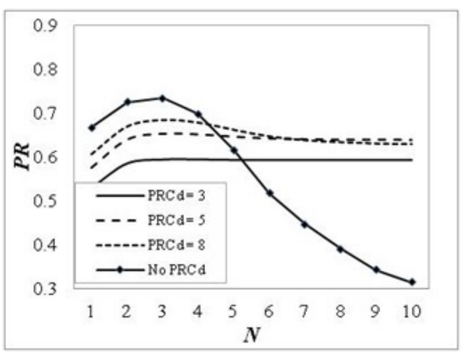


(c) SR vs. p_2

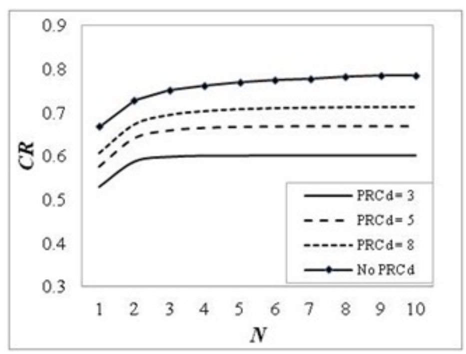


(d) WIP vs. p_2

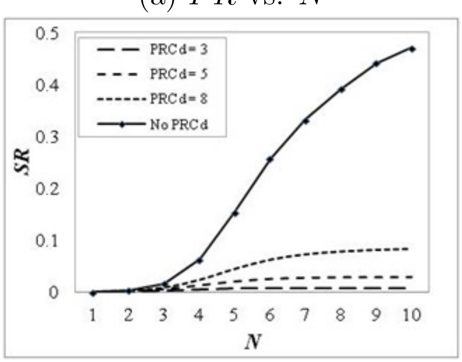
Figure 4.3: Performance measures as functions of p_2 with deterministic release



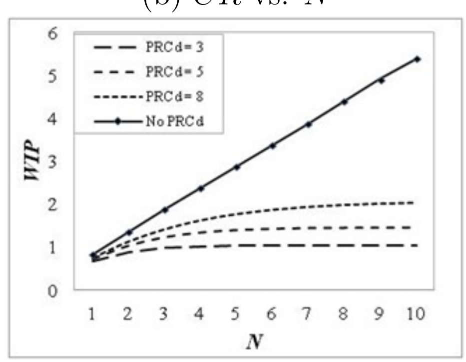
(a) PR vs. N



(b) CR vs. N

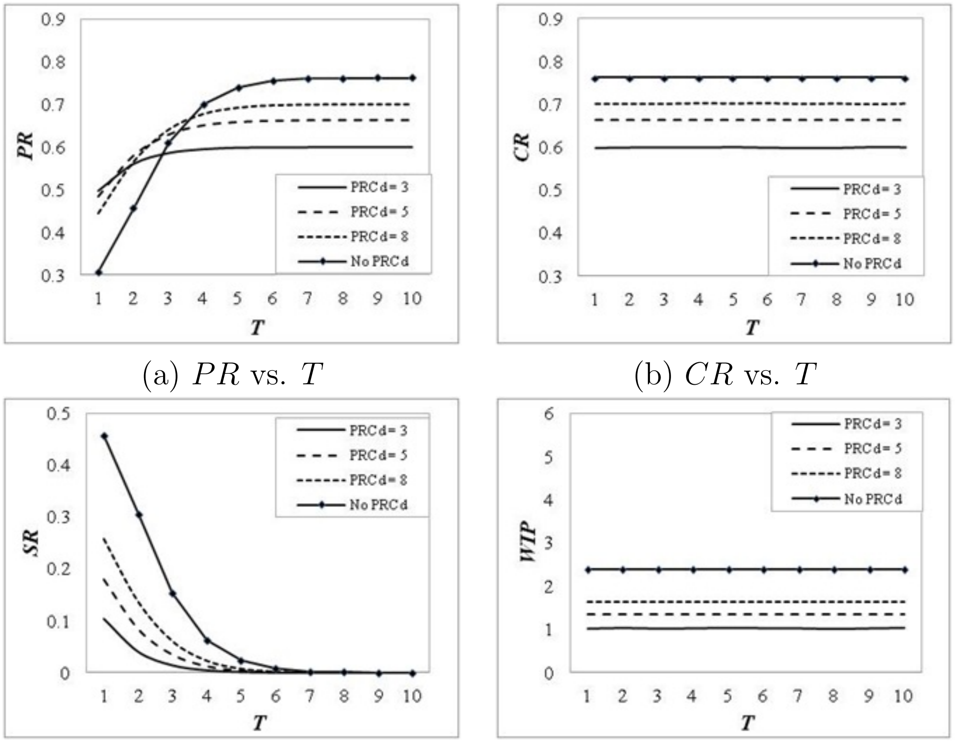


(c) SR vs. N



(d) WIP vs. N

Figure 4.4: Performance measures as functions of N with deterministic release



(a) PR vs. T (b) CR vs. T

(c) SR vs. T (d) WIP vs. T

Figure 4.5: Performance measures as functions of T with deterministic release

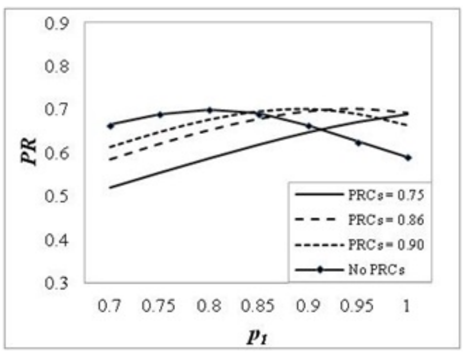
4.3 Stochastic Release

This is the probabilistic counterpart to a deterministic release. The system considered here can be ideally used to model production systems in which PRC_s is only interrupted randomly since it starts based on the probability of release selected. We will look into the system from different angles starting with; machines efficiencies, buffer capacity, and maximum residence time allowed for parts to stay in the buffer before they become obsolete. Results obtained by stochastic release presents similar results studied in the deterministic release and that can be seen in Figures 4.6, 4.7, 4.8, and 4.9.

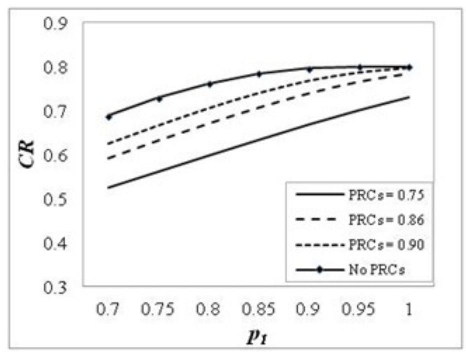
Definition 4.2: Machine m_0 releases x parts to the system to infinite buffer b_0 in a stochastic manner defined by:

$$r_s(x) = \begin{cases} 1, & \text{if } random < PRC_s, \\ 0, & \text{else,} \end{cases} \tag{4.2}$$

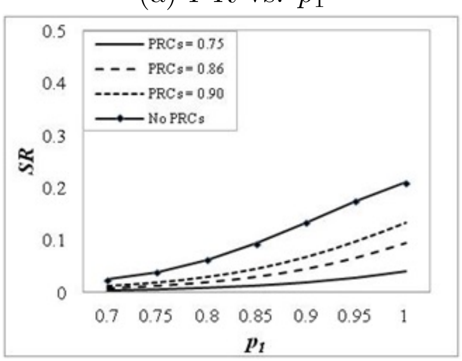
where PRC_s is a positive fraction. This fraction determines whether or not parts are releasing to the system using a random number generation. To analyze the effect of stochastic part release control, series of tests were conducted on two-machine Bernoulli serial line with parameters similar to deterministic part release case.



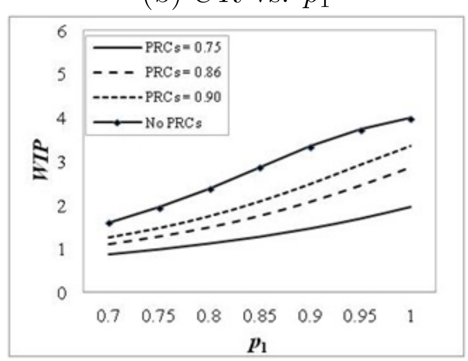
(a) PR vs. p_1



(b) CR vs. p_1

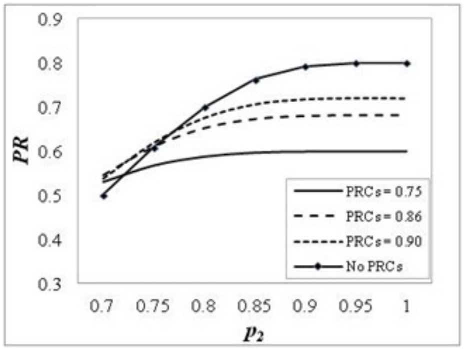


(c) SR vs. p_1

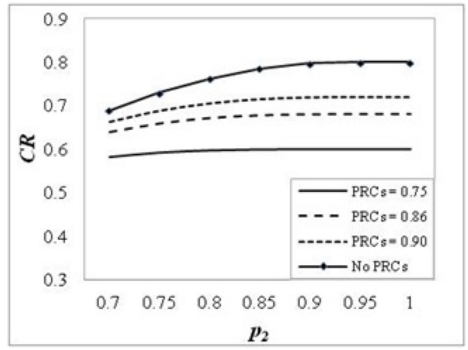


(d) WIP vs. p_1

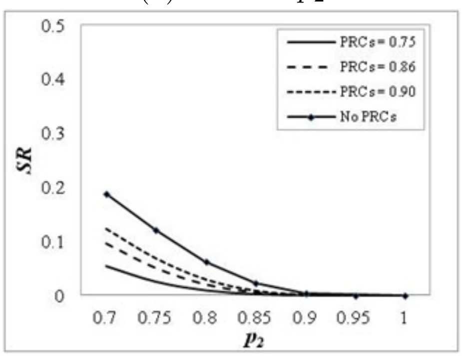
Figure 4.6: Performance measures as functions of p_1 with stochastic release



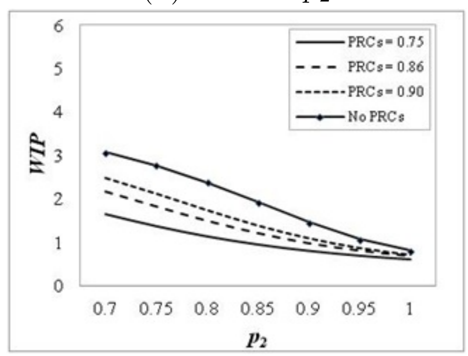
(a) PR vs. p_2



(b) CR vs. p_2

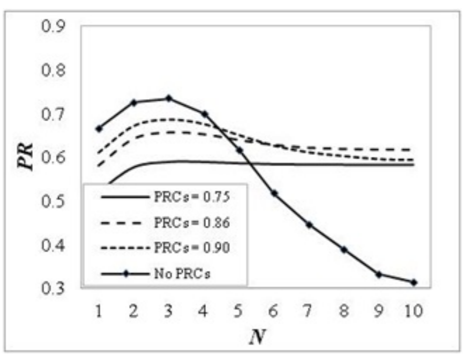


(c) SR vs. p_2

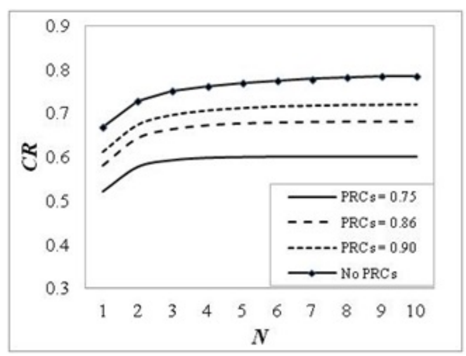


(d) WIP vs. p_2

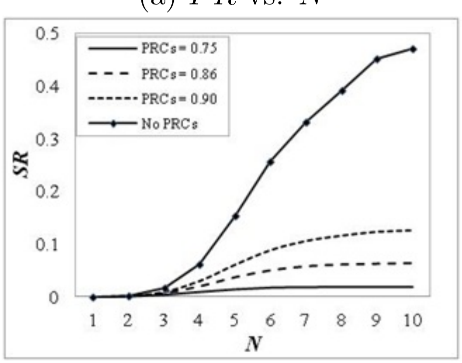
Figure 4.7: Performance measures as functions of p_2 with stochastic release



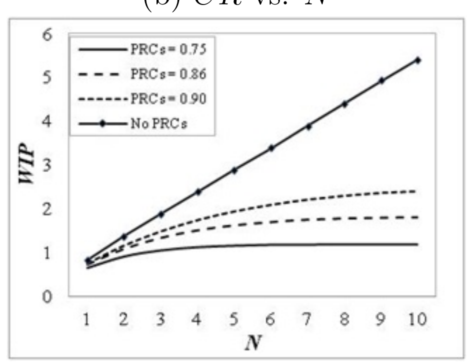
(a) PR vs. N



(b) CR vs. N

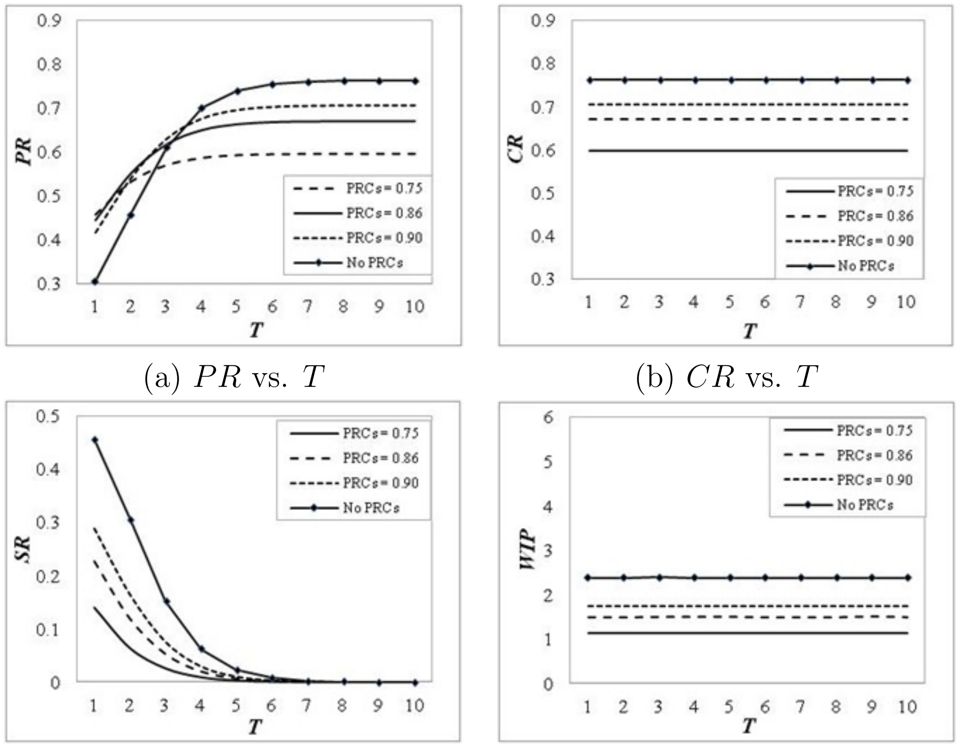


(c) SR vs. N



(d) WIP vs. N

Figure 4.8: Performance measures as functions of N with stochastic release



(a) PR vs. T (b) CR vs. T

(c) SR vs. T (d) WIP vs. T

Figure 4.9: Performance measures as functions of T with stochastic release

4.4 Summary

Increasing the efficiency of m_1 results in lower production rate of good parts. The decrease of production rate is not because of less careful or precise processing but due to longer residence time in the buffer. By applying deterministic part release control, significantly lower scrap rate is discovered. Even though the production rate is not as high as continuous release due to lower consumption rate, the decease in average number of parts to be processed in the buffer and the lower scrap rate may justify the improvement. On the contrary, increasing the efficiency of m_2 always leads to higher production rate and higher quality buy rate. In addition, if p_2 is not significantly smaller than p_1, then the scrap rate can be reduced by increasing p_2. Therefore, part release control doesn't help as much in this scenario. Due to the lack of monotonicity in PR with respect to buffer capacity N, the part release control provides a perfect solution to keep the PR significantly high while increasing buffer capacity.

We also noticed that the system behavior in both deterministic and stochastic release is similar to one another. The only difference lies in the application of each approach. Deterministic release can be used in production lines where very tight SR is desired while knowing the shipping schedules. On the other hand, stochastic release can be implemented in production lines where shipping schedules are unknown with considerably high demand.

Chapter 5

GEOMETRIC SERIAL LINES

5.1 Perfect Buffers Quality

In such serial production lines, it is assumed that the quality of items residing in the buffer while waiting to be processed is not affected by time. This will allow us to examine the theoretic properties of the geometric serial lines and have a better understanding of each parameter effect on the system at hand.

5.1.1 Two-machine lines

Performance analysis

In this section, the production system considered here is shown in Figure 5.1 and defined by assumptions (i)-(vi-b) with $M = 2$ are analyzed.

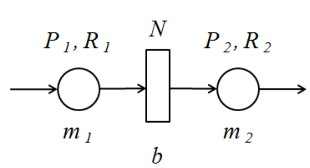


Figure 5.1: Geometric two-machine case

Theorem 2 *[81] The production rate in a serial production line defined by assumptions (i)-(vi-b) with $M = 2$ is given by*

$$PR = e_2[1 - Q(P_1, R_1, P_2, R_2, N)], \tag{5.1}$$

where

$$e_i = \frac{R_i}{P_i + R_i}, \quad i = 1, 2,$$

$$Q(P_1, R_1, P_2, R_2, N) = \begin{cases} \frac{P_1\beta_2}{(R_1+R_2-R_1R2)(R_1+P_1)}, & \text{if } N = 1, \\ \\ \frac{P_1\alpha_1\alpha_2{\beta_2}^2(R_2+P_2)}{A+B+C+D}, & \text{if } N > 1, \end{cases} \tag{5.2}$$

and

$$\begin{aligned}
\alpha_1 &= P_1 + P_2 - P_1P_2 - R_1P_2, \\
\alpha_2 &= P_1 + P_2 - P_1P_2 - R_2P_1, \\
\beta_1 &= R_1 + R_2 - R_1R_2 - P_1R_2, \\
\beta_2 &= R_1 + R_2 - R_1R_2 - P_2R_1, \\
\sigma &= \frac{\alpha_2\beta_1}{\alpha_1\beta_2}, \\
A &= P_1R_2\alpha_1\alpha_2\beta(P_2 + \beta_2), \\
B &= P_1R_1R_2\alpha_2\Big[{\beta_2}^2 + (\alpha_1 + \beta_1)(\alpha_2 + 2\beta_2)\Big], \\
C &= \sum_{k=2}^{N-1} P_1P_2R_1R_2(\alpha_2 + \beta_2)^3\sigma^{k-1}, \\
D &= P_2R_1\alpha_1\beta_2\Big[R_2(\alpha_1 + \beta_1) + \alpha_2(P_1 + R_1)\Big]\sigma^{N-1}.
\end{aligned}$$

Moreover, the average probability of the buffer occupancy, WIP, and the probabilities of manufacturing starvation of m_2, ST, and blockage of m_1, BL, are given *by*

$$WIP = \begin{cases} \frac{P_1[(R_1+R_2-R_1R_2)(P_2+R_2)+P_1P_2]}{(R_1+R_2-R_1R_2)(R_1+P_1)(R_2+P_2)}, & N=1, \\ \\ \frac{B+\sum_{k=2}^{N-2} P_1P_2R_1R_2(\alpha_2+\beta_2)^3\sigma^{k-1}+ND}{A+B+C+D}, & N>1, \end{cases} \tag{5.3}$$

$$ST = \begin{cases} \frac{P_1R_1\beta_2}{(R_1+R_2-R_1R_2)(R_1+P_1)(R_2+P_2)}, & N=1, \\ \\ \frac{P_1R_2\alpha_1\alpha 2\beta_2^2}{A+B+C+D}, & N>1, \end{cases} \tag{5.4}$$

$$BL = \begin{cases} \frac{P_2R_1\beta_1}{(R_1+R_2-R_1R_2)(R_1+P_1)(R_2+P_2)}, & N=1, \\ \\ \frac{B+\sum_{k=2}^{N-2} P_1P_2R_1R_2(\alpha_2+\beta_2)^3\sigma^{k-1}+ND}{A+B+C+D}, & N>1, \end{cases} \tag{5.5}$$

Proof of Theorem 2: [81] The proof of this theorem consists of the following three steps:

Step 1: Derivation of the steady state balance equations.

First, introduce the following steady state probabilities

$$\begin{aligned} Y_{k,s_1s_2} &= Prob\{k \text{ parts in the buffer, } m_1 \text{ and } m_2 \text{ are in states } s_1 \text{ and } s_2 \\ &\quad \text{respectively at the beginning of the slot}\}, \quad k=0,1,\ldots,N, \end{aligned}$$

where

$$s_i = \begin{cases} 1, & m_i \text{ is up,,} \\ 0, & m_i \text{ is down,} \quad \text{i = 1, 2..} \end{cases}$$

Next, write the balance equations for empty buffer, buffer occupancy equaled to 1 ($N = 1$, $N > 1$, respectively), buffer occupancy equaled to k, $1 < k < N$, and the full buffer, respectively.

Step 2: Analysis of case $N = 1$.

- Write $Y_{0,00}$, $Y_{0,10}$, $Y_{0,01}$, $Y_{1,11}$, $Y_{1,10}$, $Y_{1,01}$, $Y_{1,00}$ in terms of $Y_{0,11}$.

- From the fact that the total probabilities is equal to 1, calculate $Y_{0,11}$ and $Q(P_1,$ $R_1,$ $P_2,$ $R_2,$ $N)$.

$$Y_{0,11} = \frac{R_1 R_2 P_1 \beta_2}{(R_1 + R_2 - R_1 R_2)(R_1 + P_1)(R_2 + P_2)}.$$

It follows then that

$$Q(P_1, R_1, P_2, R_2, N) = \frac{Y_{0,11}}{e_2 R_1} = \frac{P_1 \beta_2}{(R_1 + R_2 - R_1 R_2)(R_1 + P_1)}.$$

Step 3: Analysis of the case $N > 1$.

- Write $Y_{k,11}$, $Y_{k,10}$, $Y_{k,01}$, $Y_{k,00}$, $k = 1, \ldots, N$, interms of $Y_{0,11}$.

- From the fact that the total probability os equal to 1, calculate $Y_{0,11}$ and $Q(P_1,$ $R_1,$ $P_2,$ $R_2,$ $N)$.

$$\begin{aligned} Y_{0,11} &= P_1 R_1 R_2 \alpha_1 \alpha_2 \beta_2^2 \left[P_1 R_2 \alpha_1 \alpha_2 \beta_2 (P_2 + \beta_2) \right. \\ &\quad + P_1 R_1 R_2 \alpha_2 \left[\beta_2^2 + P_2(\alpha_1 + \beta_1)(\alpha_2 + 2\beta_2) \right] \\ &\quad + \sum_{k=2}^{N-1} P_1 P_2 R_1 R_2 (\alpha_2 + \beta_2)^3 \sigma^{k-1} \\ &\quad \left. + P_2 R_1 \alpha_1 \beta_2 \left(R_2(\alpha_1 + \beta_1) + \alpha_2(P_1 + R_1) \right) \sigma^{N-1} \right]^{-1} \\ &= \frac{P_1 R_1 R_2 \alpha_1 \alpha_2 \beta_2^2}{A + B + C + D}, \end{aligned}$$

where,

$$\begin{aligned} A &= P_1 R_2 \alpha_1 \alpha_2 \beta (P_2 + \beta_2), \\ B &= P_1 R_1 R_2 \alpha_2 \Big[{\beta_2}^2 + (\alpha_1 + \beta_1)(\alpha_2 + 2\beta_2) \Big], \\ C &= \sum_{k=2}^{N-1} P_1 P_2 R_1 R_2 (\alpha_2 + \beta_2)^3 \sigma^{k-1}, \end{aligned}$$

$$D = P_2 R_1 \alpha_1 \beta_2 \Big[R_2(\alpha_1 + \beta_1) + \alpha_2(P_1 + R_1) \Big] \sigma^{N-1}.$$

It follows then that,

$$Q(P_1, R_1, P_2, R_2, N) = \frac{P_1 \alpha_1 \alpha_2 {\beta_2}^2 (R_2 + P_2)}{A + B + C + D}.$$

- Calculate WIP, ST_2, BL_1, where

$$\begin{aligned} WIP &= \sum_{k=1}^{N} k(Y_{k,11} + Y_{k,10} + Y_{k,01} + Y_{k,00}), \\ ST_2 &= Y_{0,11} + Y_{0,01}, \\ BL_1 &= Y_{N,10}. \end{aligned}$$

Theorem 2 is proved. ■

Monotonicity property

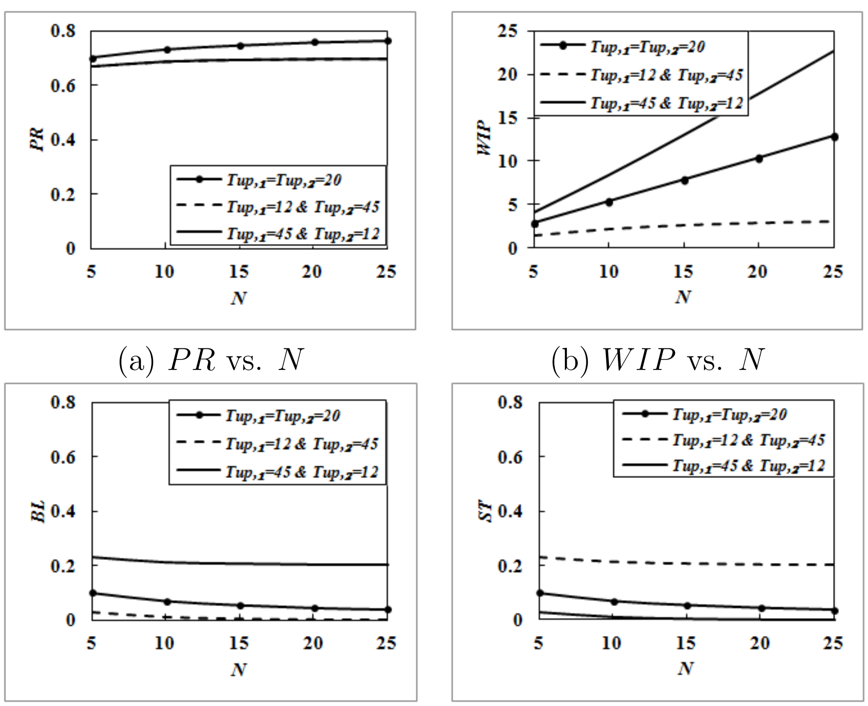
The monotonicity properties of the performance measures are characterized by the following:

Property 2 *In two-machine geometric lines defined by assumptions (i)-(vi-b),*

- PR *is monotonically increasing in* N, $T_{up,1}$, *and* $T_{up,2}$, *and monotonically decreasing in* $T_{down,i}$;
- WIP_i *is monotonically increasing in* N, $T_{up,1}$, *and* $T_{down,2}$, *and monotonically decreasing in* $T_{up,2}$ *and* $T_{down,1}$;
- BL_i *is monotonically decreasing in* N, $T_{up,2}$, *and* $T_{down,1}$, *and monotonically increasing in* $T_{up,1}$ *and* $T_{down,2}$;
- ST_i *is monotonically decreasing in* N, $T_{up,1}$, *and* $T_{down,2}$*), and monotonically increasing in* $T_{up,2}$ *and* $T_{down,1}$;

A more interesting prospective of this system theoretic property was revealed by studying Figures 5.3, 5.4, 5.5 and 5.6. It was found that shorter up- and downtime lead to a higher production rate than longer ones, even if the machines' efficiency remains the same. This phenomenon takes place because finite buffers protect against shorter downtime better than against longer ones. Mathematically, this phenomenon is due to the fact that the probabilities of buffer being empty and full are larger for machines with longer up- and downtime.

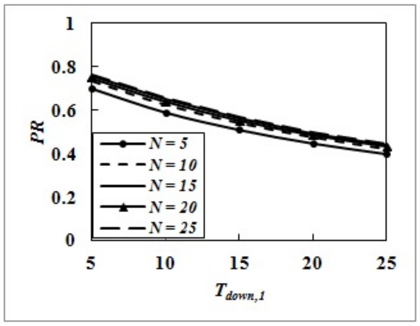
Clearly, production rate can be improved by either increasing the uptime of a machine or decreasing its downtime. Is it more beneficial to increase the uptime, say by a factor, or decrease its downtime by the same factor? It was found that decreasing downtime by given factor leads to a larger production rate than increasing uptime by the same factor [82].



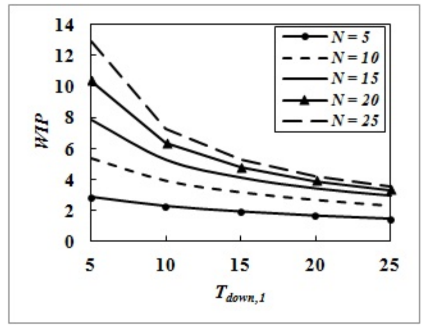
(a) PR vs. N (b) WIP vs. N

(c) BL_1 vs. N (d) ST_2 vs. N

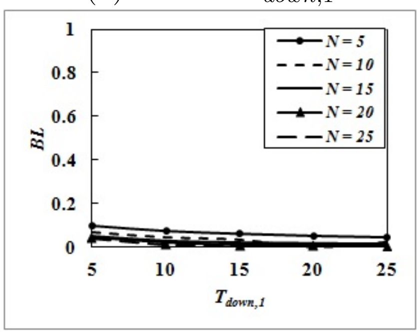
Figure 5.2: Performance measures as a function of N with $T_{down,i} = 5$



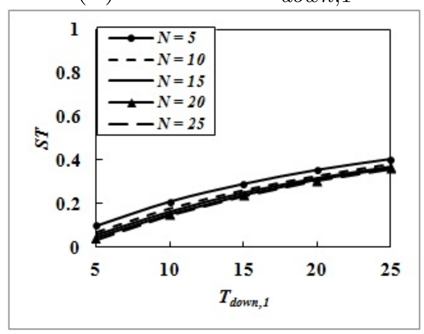
(a) PR vs. $T_{down,1}$



(b) WIP vs. $T_{down,1}$

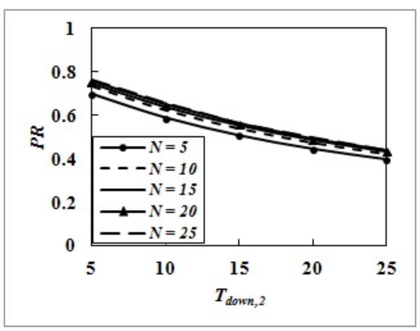


(c) BL_1 vs. $T_{down,1}$

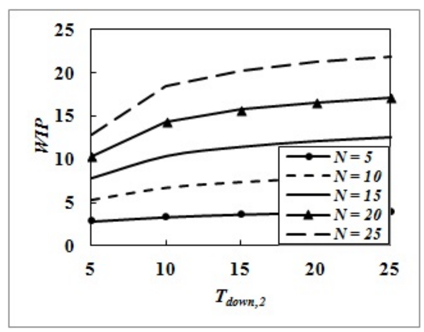


(d) ST_2 vs. $T_{down,1}$

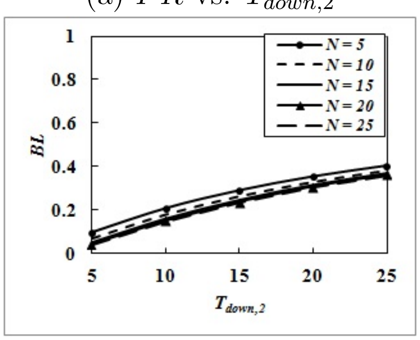
Figure 5.3: Performance measures as a function of $T_{down,1}$ with $T_{up,i}$=20 and $T_{down,2}$=5



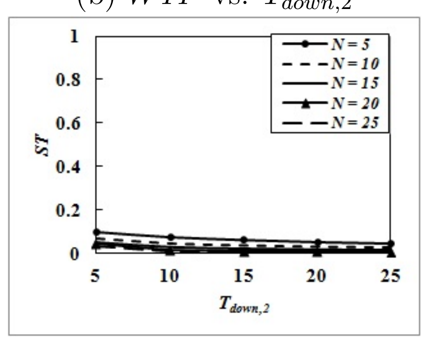
(a) PR vs. $T_{down,2}$



(b) WIP vs. $T_{down,2}$

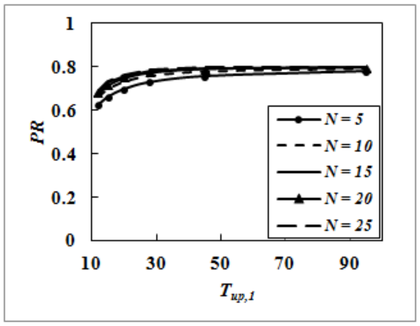


(c) BL_1 vs. $T_{down,2}$

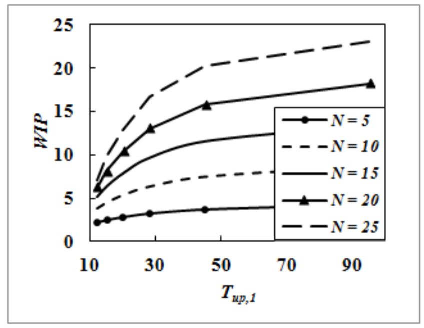


(d) ST_2 vs. $T_{down,2}$

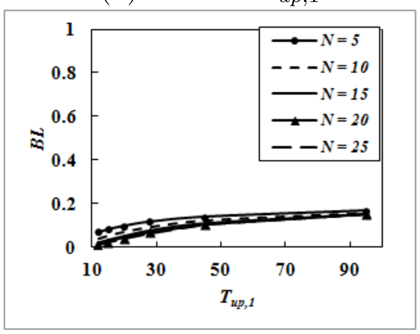
Figure 5.4: Performance measures as a function of $T_{down,2}$ with $T_{up,i}$=20 and $T_{down,1}$=5



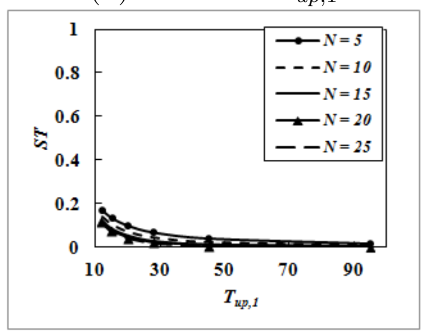
(a) PR vs. $T_{up,1}$



(b) WIP vs. $T_{up,1}$

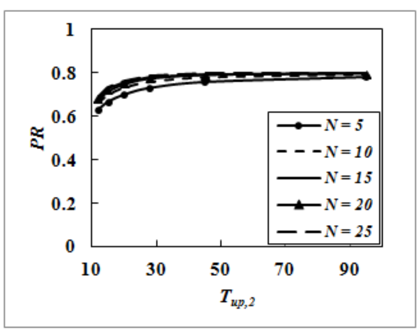


(c) BL_1 vs. $T_{up,1}$

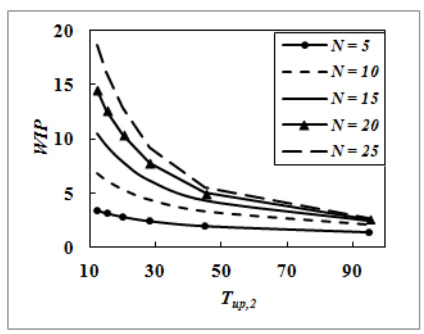


(d) ST_2 vs. $T_{up,1}$

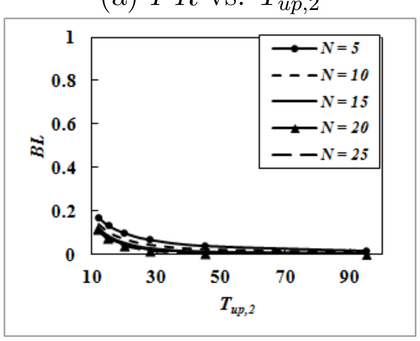
Figure 5.5: Performance measures as a function of $T_{up,1}$ with $T_{down,i}$=5 and $T_{up,2}$=20



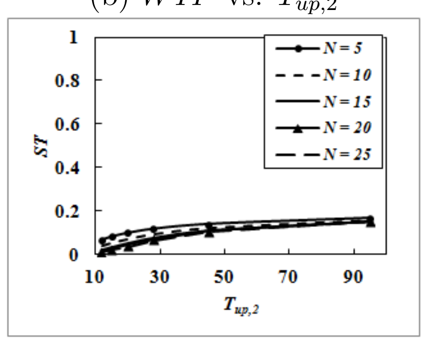
(a) PR vs. $T_{up,2}$



(b) WIP vs. $T_{up,2}$

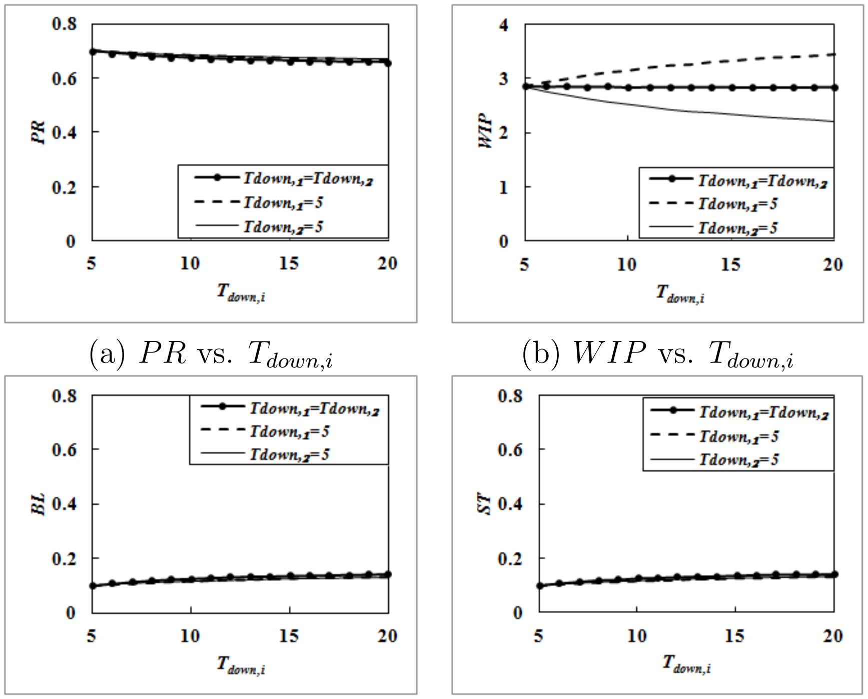


(c) BL_1 vs. $T_{up,2}$



(d) ST_2 vs. $T_{up,2}$

Figure 5.6: Performance measures as a function of $T_{up,2}$ with $T_{down,i}$=5 and $T_{up,1}$=20



(a) PR vs. $T_{down,i}$ (b) WIP vs. $T_{down,i}$

(c) BL_1 vs. $T_{down,i}$ (d) ST_2 vs. $T_{down,i}$

Figure 5.7: Performance measures as a function of $T_{down,i}$ with $T_{up,i} = 20$

Justifcation of Property 2: To justify these properties, a total of 100,000 production lines were generated with parameters randomly and equiprobably selected from the following sets:

$$e_i \in [0.6, 0.95], \quad N \in \{5, 10, 15, 20, 25\},$$
$$T_{down,i} \in [5, 20], \quad T_{up,i} \in [20, 35]. \tag{5.6}$$

Next, we calculate the performance measures of all lines as functions the parameters N, e_i, $T_{up,i}$, and $T_{down,i}$; using equations (5.1)-(5.5) and examined whether the corresponding statement of Property 2 holds. As a result, among the 100,000 lines studied, no counterexamples of Property 2 were found. Thus, we conclude that Property 2 indeed takes place.

It was interesting to observe how the system reacts to all parameters specifically $T_{up,i}$ and $T_{down,i}$ and their interrelation with one another with respect to e_i, where e_i

$= \frac{T_{up,i}}{T_{up,i}+T_{down,i}}$. With fixed values of $T_{up,i}$ and $T_{down,i}$, system behave as expected, PR is monotonically increasing in N and $T_{up,i}$ and monotonically decreasing in $T_{down,i}$. On the other hand, when $T_{down,i}$ is unknown and calculated using its relationship with $T_{up,i}$ and e_i, they system behaves differently, PR is monotonically decreasing in $T_{up,i}$. The reversed effect of increasing $T_{up,1}$ and $T_{up,2}$ on WIP, BL, and ST in each case, was expected as well. Also, detailed study of different $T_{down,i}$ values were investigated, which lead to similar general results.

5.1.2 M > 2-machine lines

No closed form expression for PR in $M-$machine line is available. Therefore, an aggregation procedure, based on the results of the previous subsections. Specifically, the first two machines into a single machines, m_2^f , with R_2^f defined as

$$R_2^f = R_2[1 - Q(P_1, R_1, P_2, R_2, N_1)],$$

and P_2^f selected so that

$$\frac{R_2^f}{P_2^f + R_2^f} = \frac{R_2}{P_2 + R_2}[1 - Q(P_1, R_1, P_2, R_2, N_1)],$$

i.e.,

$$P_2^f = P_2 + R_2 Q(P_1, R_1, P_2, R_2, N_1),$$

where Q(.) is defined in 5.2. Next m_2^f is aggregated with m_3 to result in m_3^f, with the parameters defined as shown above, and so on until all machines are aggregated in a single ones, m_M^f. This continues to forward aggregation (subscript f is used to denote this fact). Then, in the backward aggregation, the last machine, m_M, is aggregated with m_{M-1}^f to result in m_{M-1}^b and so until all machines are again aggregated in a single machine, m_1^b [81]. Then the procedure is repeated again. Formally, this process can be represented as follows:

Recursive Procedure 5.1:

$$
\begin{aligned}
R_i^b(s+1) &= R_i - R_i Q\left(P_{i+1}^b(s+1), R_{i+1}^f(s), P_i^f(s), R_i^f(s) N_i\right), \quad i=1,\ldots,M-1, \\
&\quad s=0,1,2,\ldots, \\
P_i^b(s+1) &= P_i + R_i Q\left(P_{i+1}^b(s+1), R_{i+1}^f(s), P_i^f(s), R_i^f(s) N_i\right), \quad i=1,\ldots,M-1, \\
&\quad s=0,1,2,\ldots, \\
R_i^f(s+1) &= R_i - R_i Q\left(P_{i-1}^b(s+1), R_{i-1}^f(s), P_i^f(s+1), R_i^f(s+1) N_{i-1}\right), \\
&\quad i=1,\ldots,M-1, \quad s=0,1,2,\ldots, \\
P_i^f(s+1) &= P_i + R_i Q\left(P_{i-1}^b(s+1), R_{i-1}^f(s), P_i^f(s+1), R_i^f(s+1) N_i i-1)\right), \\
&\quad i=1,\ldots,M-1, \quad s=0,1,2,\ldots,
\end{aligned}
$$

with initial conditions

$$P_i^f(0) = P_i, \quad R_i^f(0) = R_i, \quad i=1,\ldots,M,$$

and boundary conditions

$$
\begin{aligned}
P_1^f(s) = P_1, &\quad R_1^f(s) = R_1, \\
P_M^b(s) = P_M, &\quad R_M^b(s) = R_M, \\
&\quad s=0,1,\ldots,
\end{aligned}
$$

where Q(.) is defined in 5.2.

The equation of convergence of the resulting sequences $P_i^b(s)$, $R_i^b(s)$, $P_i^f(s)$, $i=1,\ldots,$ M, $s=1,\ldots,$ is answered in the following:

Theorem 3 *[81] Under function $Q(P_1, R_1, P_2, R_2, N)$, $N > 1$, is monotonically increasing with respect to P_1 and R_2, and decreasing with respect to P_2 and R_1, the recursive procedure 5.1 is convergent and, therefore, the following limits exist:*

$$\lim_{s\to\infty} P_i^f(s) = \widetilde{P}_i^f, \quad \lim_{s\to\infty} P_i^b(s) = \widetilde{P}_i^b,$$
$$\lim_{s\to\infty} R_i^f(s) = \widetilde{R}_i^f, \quad \lim_{s\to\infty} R_i^b(s) = \widetilde{R}_i^b, \tag{5.7}$$
$$i = 1, \ldots, M$$

Moreover, the following relationship holds:

$$\frac{R_M^f}{P_M^f} = \frac{R_1^b}{P_1^b}. \tag{5.8}$$

Proof of Theorem 3: [81] Under the assumptions of the Theorem, since the sequences $P_j^f(s)$ and $P_i^b(s)$ are monotonically increasing and sequences $R_j^f(s)$ and $R_i^b(s)$ are monotonically increasing and bounded from above and below, they are convergent. This proves (5.7). To prove (5.8), consider the steady state equations of the recursive procedure(1) and define

$$e_i^f = \frac{R_i^f}{R_i^f + P_i^f}, \quad i = 1, \ldots, M,$$
$$e_i^b = \frac{R_i^b}{R_i^b + P_i^b}, \quad i = 1, \ldots, M.$$

The following property holds (see Li and Meerkov 2000c):

$$\frac{e_i^f e_i^b}{e_i} = \frac{e_j^f e_j^b}{e_j}, \quad i, j = 1, \ldots, M, \in i \neq j.$$

Therefore, $\frac{R_M^f}{P_M^f} = \frac{R_1^b}{P_1^b}$. Theorem 3 is proved.

■

The limits in 5.7 can be used to define estimates of performance measures for production line with assumptions (i)-(vi-b). Production rate can be estimated as [81]

$$\widehat{PR}(P_1, R_1, \ldots, P_M, R_M, N_1, \ldots, N_{M-1}) = \frac{R_M^f}{P_M^f + R_M^f} = \frac{R_1^b}{P_1^b + R_1^b}. \tag{5.9}$$

$$\delta_{PR} \quad = \quad \frac{|PR_{\text{sim}} - \widehat{PR}|}{PR_{\text{sim}}} \cdot 100\%. \tag{5.10}$$

To evaluate the accuracy of the estimate 5.9, we developed a C++ program to simulate the system defied by assumptions (i)-(vi-b) with various machine and buffer parameters assumed. Twenty of them with 3 - 8 machines, are shown in Table 5.1. This simulation was used to evaluate the performance measures specifically PR. Confidence intervals have been evaluated with 20 runs. The 95% confidence intervals were consistently around ±0.0015. In Table 5.1, PR denotes the actual production rate obtained by simulation, whereas $\widehat{PR}$ denotes the estimate of production rate calculated according to 5.9 [81]. As it can be seen from Table 5.1, the estimate results in relatively high precision, comparable with [4], [83], [84], and [85].

Monotonicity property

To investigate the monotonicity properties of the performance measures for $M >$ 2-machine geometric serial lines, the following sets of serial lines with five machines, i = 1,. . . , 5, four buffers, i = 1,. . . , 4, were introduced:

Set 1: Machines' up-times vs. buffer capacities: This set of lines was created to evaluate the effect of machines' up-times with respect to buffer capacities.

L_1: T_{up_i} = [45,45,45,45,45], $T_{down,i} = 5$, $N_i = \{5,\ldots,25\}$,

L_2: T_{up_i} = [45,0.28,20,15,12], $T_{down,i} = 5$, $N_i = \{5,\ldots,25\}$,

L_3: T_{up_i} = [12,15,20,28,45], $T_{down,i} = 5$, $N_i = \{5,\ldots,25\}$,

L_4: T_{up_i} = [45,28,12,28,45], $T_{down,i} = 5$, $N_i = \{5,\ldots,25\}$,

Table 5.1: [81]Numerical justification of production rate estimation for $M > 2$-machine using 5.10)

	P_i	R_i	N_i	PR	$\widehat{PR}$	δ_{PR}
$M = 3$	0.06 0.07 0.08	0.27 0.28 0.29	1 2	0.561	0.562	0.21%
$M = 3$	0.12 0.15 0.10	0.43 0.46 0.50	2 2	0.606	0.605	0.18%
$M = 3$	0.05 0.10 0.10	0.50 0.45 0.40	2 2	0.5627	0.629	0.32%
$M = 3$	0.10 0.10 0.10	0.42 0.42 0.42	3 3	0.668	0.668	0.04%
$M = 3$	0.10 0.05 0.20	0.90 0.85 0.75	3 2	0.776	0.786	1.24%
$M = 3$	0.10 0.02 0.06	0.60 0.04 0.09	1 1	0.360	0.349	3.11%
$M = 4$	0.11 0.08 0.08 0.11	0.40 0.41 0.41 0.40	2 3 2	0.608	0.606	0.39%
$M = 4$	0.11 0.12 0.13 0.10	0.39 0.38 0.36 0.43	4 5 3	0.606	0.605	0.20%
$M = 4$	0.08 0.09 0.07 0.06	0.37 0.43 0.41 0.39	3 2 2	0.644	0.645	0.22%
$M = 4$	0.15 0.04 0.30 0.02	0.50 0.80 0.40 0.70	3 2 3	0.550	0.557	1.26%
$M = 4$	0,06 0.08 0.05 0.10	0.36 0.39 0.42 0.37	3 3 4	0.682	0.689	0.94%
$M = 4$	0.04 0.07 0.10 0.13	0.40 0.43 0.37 0.46	2 3 3	0.646	0.645	0.13%
$M = 4$	0.10 0.07 0.09 0.12 0.11	0.40 0.35 0.33 0.42 0.39	4 4 4 3	0.607	0.607	0.15%
$M = 5$	0.10 0.12 0.13 0.11 0.12	0.45 0.42 0.43 0.46 0.44	4 3 4 3	0.613	0.612	0.03%
$M = 5$	0.12 0.09 0.12 0.09 0.12	0.41 0.36 0.41 0.36 0.41	3 4 3 4	0.627	0.631	0.54%
$M = 5$	0.05 0.09 0.13 0.17 0.21	0.42 0.45 0.48 0.51 0.54	2 2 2 2	0.542	0.540	0.31%
$M = 6$	0.80 0.80 0.80 0.80 0.80 0.80	0.42 0.42 0.42 0.42 0.42 0.42	3 3 3 3 3	0.638	0.644	0.97%
$M = 6$	0.06 0.08 0.07 0.01 0.12 0.09	0.43 0.46 0.45 0.48 0.47 0.44	2 2 3 2 3	0.616	0.617	0.13%
$M = 7$	0.06 0.08 0.07 0.10 0.12 0.10 0.07	0.35 0.37 0.32 0.38 0.39 0.41 0.36	3 2 3 4 3 2	0.544	0.547	0.58%
$M = 8$	0.06 0.07 0.09 0.10 0.12 0.08 0.11 0.09	0.43 0.42 0.41 0.41 0.43 0.45 0.44 0.40	3 3 2 3 4 3 2	0.575	0.582	1.34%

L_5: T_{up_i} = [12,28,45,28,12], $T_{down,i} = 5$, $N_i = \{5,\dots,25\}$,

L_6: T_{up_i} = [12,45,12,45,12], $T_{down,i} = 5$, $N_i = \{5,\dots,25\}$,

L_7: T_{up_i} = [45,12,45,12,45], $T_{down,i} = 5$, $N_i = \{5,\dots,25\}$,

L_8: T_{up_i} = [15,15,95,15,15], $T_{down,i} = 5$, $N_i = \{5,\dots,25\}$.

Set 2: Machines' downtimes vs. buffer capacities: This set of lines was created to evaluate the effect of machines' downtimes with respect to buffer capacities.

L_1: $T_{up_i} = 20$, $T_{down,i}$ = [5,5,5,5,5], $N_i = \{5,\dots,25\}$,

L_2: $T_{up_i} = 20$, $T_{down,i}$ = [5,10,15,20,25], $N_i = \{5,\dots,25\}$,

L_3: $T_{up_i} = 20$, $T_{down,i}$ = [25,20,15,10,5], $N_i = \{5,\dots,25\}$,

L_4: $T_{up_i} = 20$, $T_{down,i}$ = [5,10,20,10,5], $N_i = \{5,\dots,25\}$,

L_5: $T_{up_i} = 20$, $T_{down,i}$ = [20,10,5,10,20], $N_i = \{5,\dots,25\}$,

L_6: $T_{up_i} = 20$, $T_{down,i}$ = [20,5,20,5,20], $N_i = \{5,\dots,25\}$,

L_7: $T_{up_i} = 20$, $T_{down,i}$ = [5,20,5,20,5], $N_i = \{5,\dots,25\}$,

L_8: $T_{up_i} = 20$, $T_{down,i}$ = [20,20,5,20,20], $N_i = \{5,\dots,25\}$.

Set 3: Machines' up-times vs. machines' downtimes: This set of lines was created to evaluate the effect of machines' up-times with respect to downtimes.

L_1: T_{up_i} = [45,45,45,45,45], $T_{down,i} = \{5,\dots,25\}$, $N_i = 5$,

L_2: T_{up_i} = [45,0.28,20,15,12], $T_{down,i} = \{5,\dots,25\}$, $N_i = 5$,

L_3: T_{up_i} = [12,15,20,28,45], $T_{down,i} = \{5,\dots,25\}$, $N_i = 5$,

L_4: T_{up_i} = [45,28,12,28,45], $T_{down,i} = \{5,\dots,25\}$, $N_i = 5$,

L_5: T_{up_i} = [12,28,45,28,12], $T_{down,i} = \{5,\dots,25\}$, $N_i = 5$,

L_6: T_{up_i} = [12,45,12,45,12], $T_{down,i} = \{5,\dots,25\}$, $N_i = 5$,

L_7: T_{up_i} = [45,12,45,12,45], $T_{down,i} = \{5,\dots,25\}$, $N_i = 5$,

L_8: T_{up_i} = [15,15,95,15,15], $T_{down,i} = \{5,\dots,25\}$, $N_i = 5$,

Set 4: Machines' downtimes vs. machines' up-times: This set of lines was created to evaluate the effect of machines' downtimes with respect to up-times.

L_1: $T_{up_i} = \{12,\dots,95\}$, $T_{down,i}$ = [5,5,5,5,5], $N_i = 5$,

L_2: $T_{up_i} = \{12,\ldots,95\}$, $T_{down,i} = [5,10,15,20,25]$, $N_i = 5$,

L_3: $T_{up_i} = \{12,\ldots,95\}$, $T_{down,i} = [25,20,15,10,5]$, $N_i = 5$,

L_4: $T_{up_i} = \{12,\ldots,95\}$ $T_{down,i} = [5,10,20,10,5]$, $N_i = 5$,

L_5: $T_{up_i} = \{12,\ldots,95\}$, $T_{down,i} = [20,10,5,10,20]$, $N_i = 5$,

L_6: $T_{up_i} = \{12,\ldots,95\}$, $T_{down,i} = [20,5,20,5,20]$, $N_i = 5$,

L_7: $T_{up_i} = \{12,\ldots,95\}$, $T_{down,i} = [5,20,5,20,5]$, $N_i = 5$,

L_8: $T_{up_i} = \{12,\ldots,95\}$, $T_{down,i} = [20,20,5,20,20]$, $N_i = 5$.

The reasons for selecting these particular lines, shown in Figure 5.8, are as follows: Line 1 illustrates the behavior of systems with identical machines. Lines 2 and 3 represent systems with increasing and decreasing machines, respectively; clearly L_3 is the reverse of L_2. Lines 4 and 5 illustrate systems with machine allocated according to a bowl and an inverted bowl patterns, respectively. Lines 6 and 7 exemplify systems with "oscillating" machine allocation. Finally, Line 8 is selected to illustrate the case of a good machine surrounded with low ones. To illustrate more, these lines were used in each set differently. In sets 1 and 3, lines introduced in the form of machines' efficiencies. While, in sets 2 and 4, lines introduced in the form of machines' downtimes.

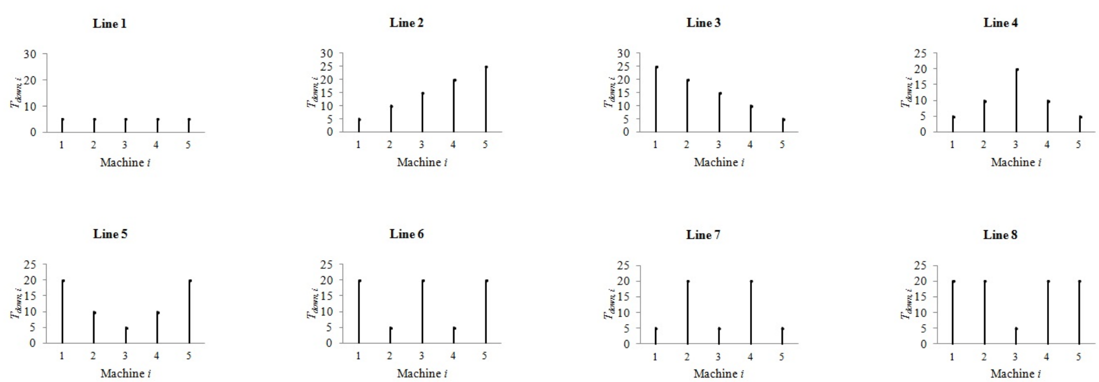


Figure 5.8: Lines proposed for studying the system behavior

Property 3 *In $M > 2$-machine geometric lines defined by assumptions (i)-(vi-b),*

- *PR is monotonically increasing in N_i and $T_{up,i}$, and monotonically decreasing in $T_{down,i}$;*
- *WIP_i is monotonically increasing in N_i, and monotonically increasing or monotonically decreasing in $T_{up,i}$ and $T_{down,i}$;*
- *BL_i is monotonically decreasing in N_i and $T_{up,i}$, and monotonically increasing in $T_{down,i}$;*
- *ST_i is monotonically decreasing in N_i and $T_{up,i}$, and monotonically increasing in $T_{down,i}$.*

Illustration of the property was analyzed in Figures 5.9, 5.10, 5.11, and 5.12. Similar results were found in $M > 2$-machine serial lines as in two-machine serial lines. Also, it was found that increasing $T_{up,i}$ with fixed $T_{down,i}$ increases PR while increasing $T_{up,i}$ with variable $T_{down,i}$ leads to decreasing PR.

Justifcation of Property 3: To justify these properties, a total of 100,000 production lines were generated with parameters randomly and equiprobably selected from the following sets:

$$\begin{aligned} e_i \in [0.6, 0.95], \quad N \in \{5, 10, 15, 20, 25\}, \\ T_{down,i} \in [5, 20], \quad T_{up,i} \in [20, 35]. \end{aligned} \tag{5.11}$$

Next, we calculate the performance measures of all lines as functions the parameters N, e_i, $T_{up,i}$, and $T_{down,i}$; using simulation model and examined whether the corresponding statement of Property 3 holds. As a result, among the 100,000 lines studied, no counterexamples of Property 3 were found. Thus, we conclude that Property 3 indeed takes place.

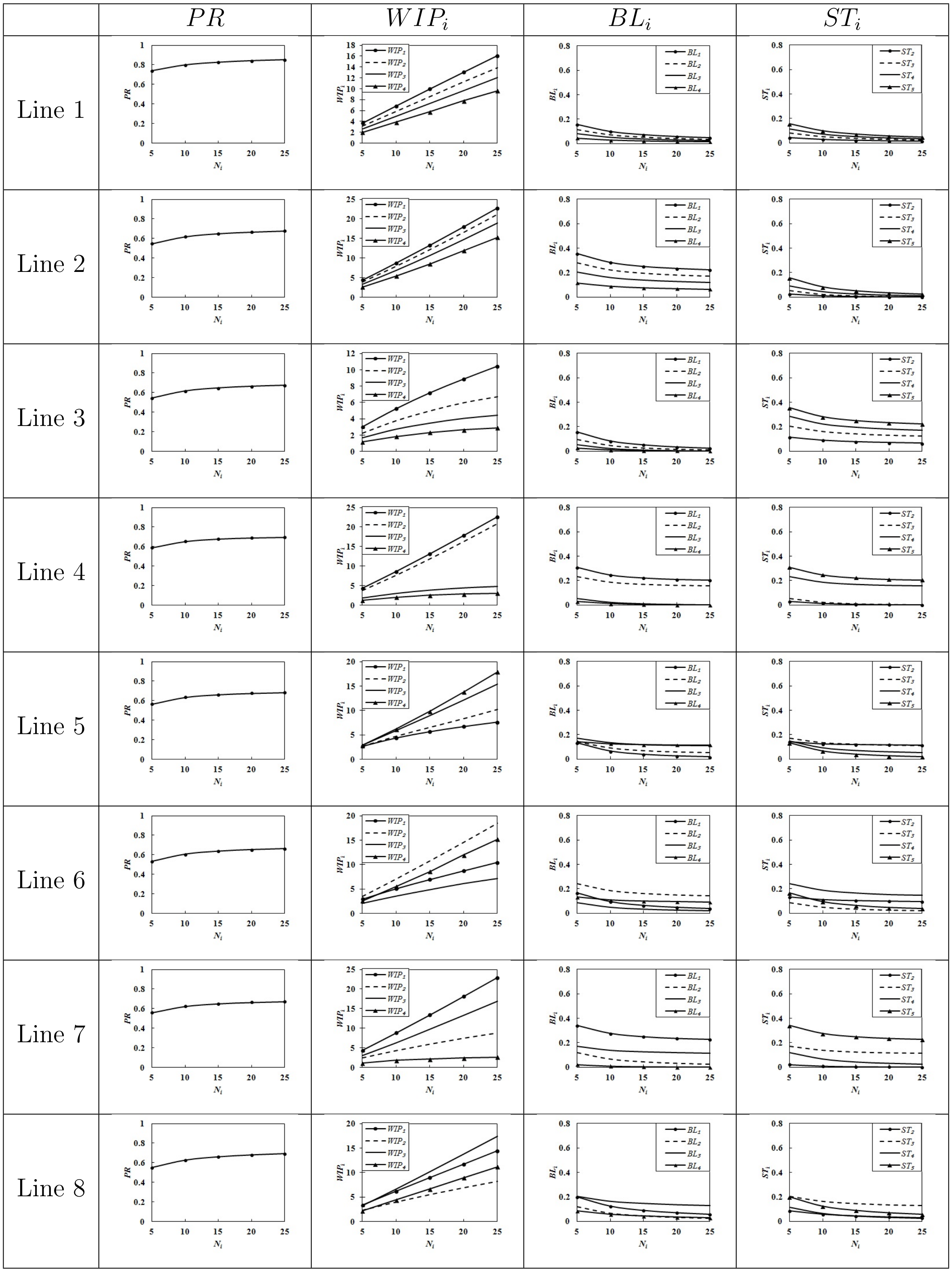


Figure 5.9: Performance measures of Set 1: machines' up-times vs. buffer capacities

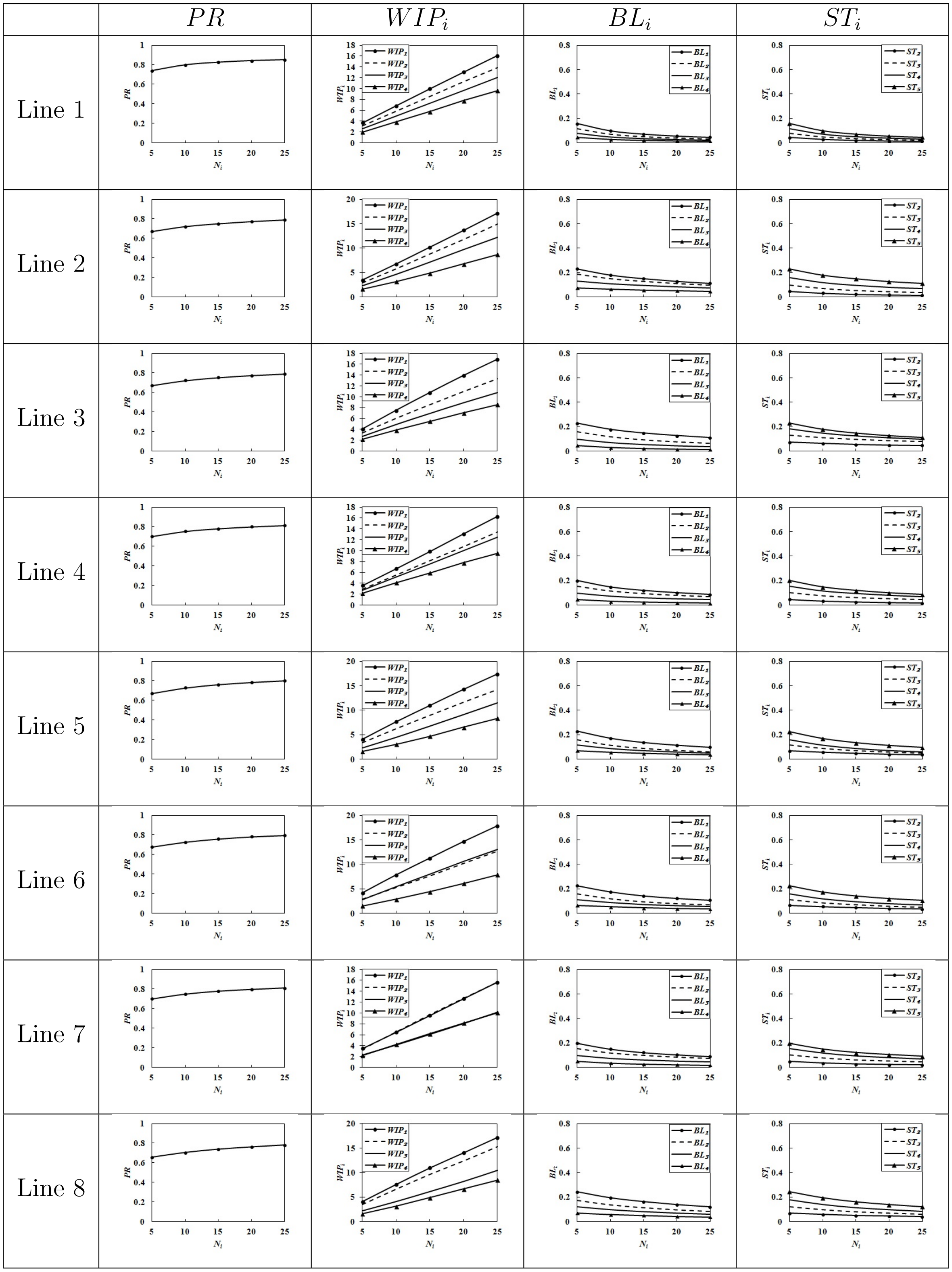


Figure 5.10: Performance measures of Set 2: machines' downtimes vs. buffer capacities

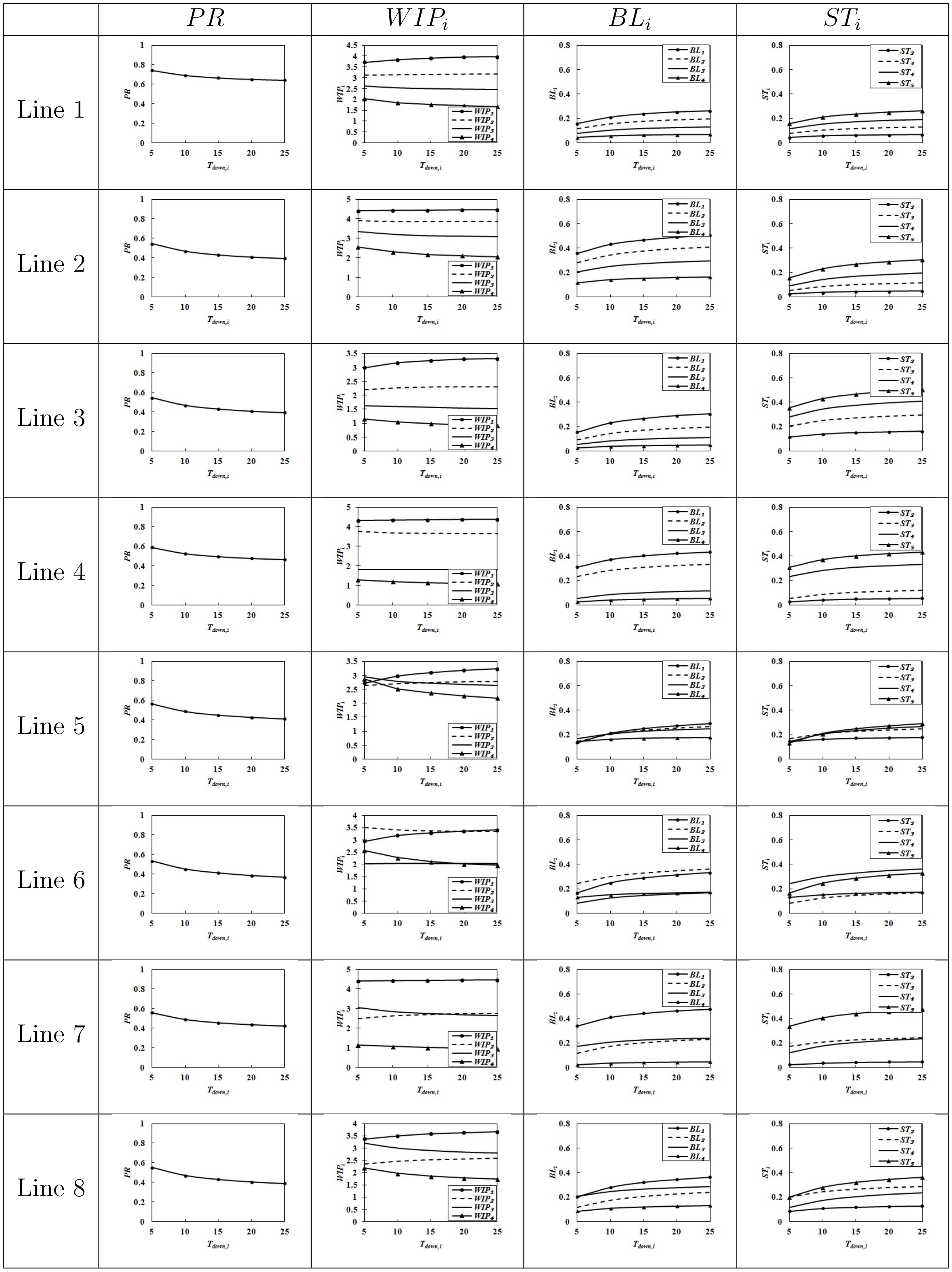


Figure 5.11: Performance measures of Set 3: machines' up-times vs. machines' down-times

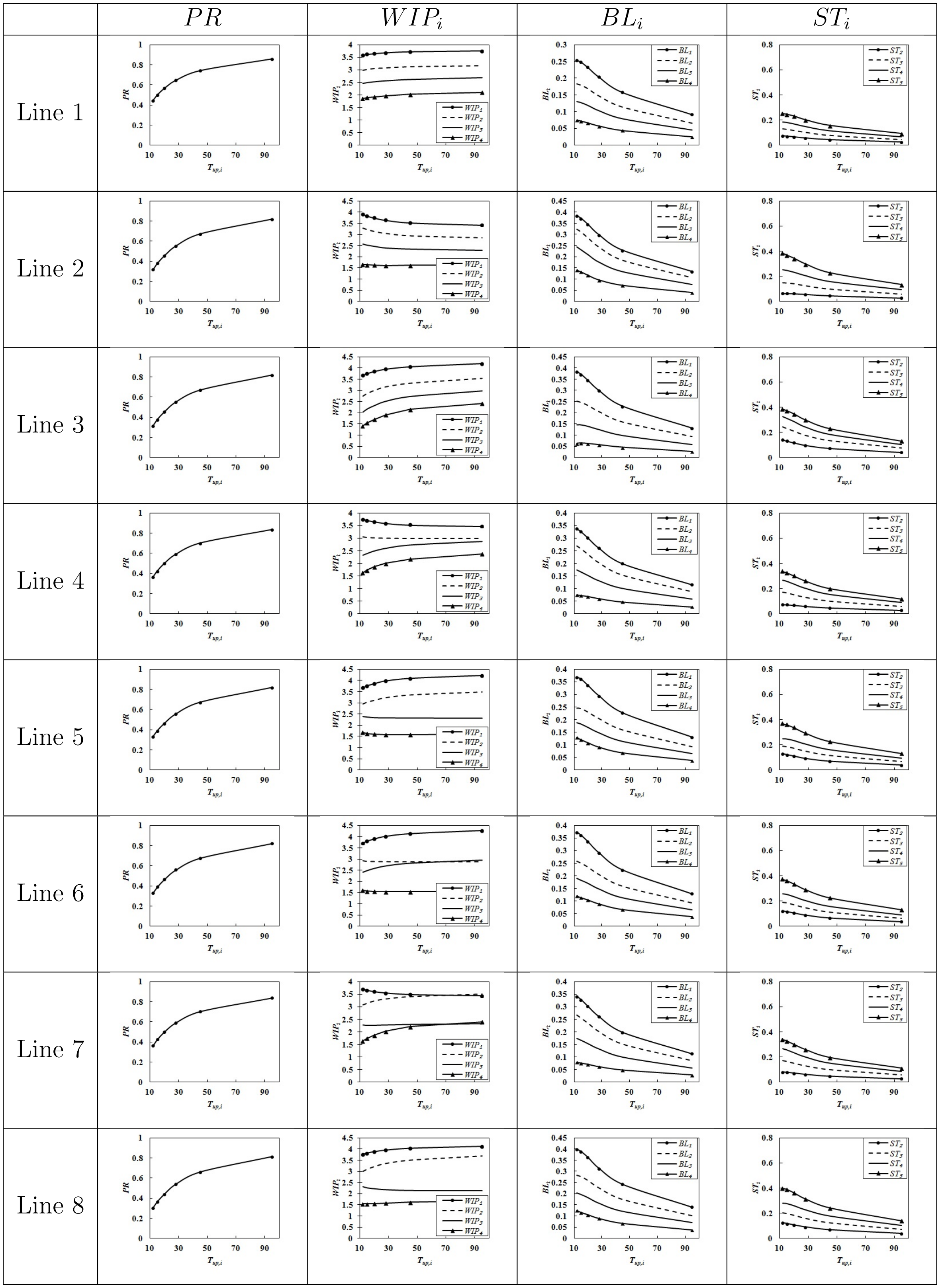


Figure 5.12: Performance measures of Set 4: machines' downtimes vs. machines' up-times

5.2 Deteriorating Quality Buffers (DQB)

In such serial production lines, it is assumed that the quality of items residing in the buffer while waiting to be processed is affected by time, i.e., the longer the item stays in the buffer, the higher risk of its quality to deteriorate over time.

5.2.1 Two-machine lines

In this section, production lines defined by assumptions (i)-(vi-b)-(viii) with $M = 2$ are analyzed as shown in Figure 5.13. As a matter of fact, conventional two-machine geometric lines, i.e., lines defined by assumptions (i)-(vi-b), have been introduced in the previous section that was studied in [81]. Since the quality of parts is dependent on their residence time in the buffer, simulation model of the system was used to evaluate the system theoretic properties.

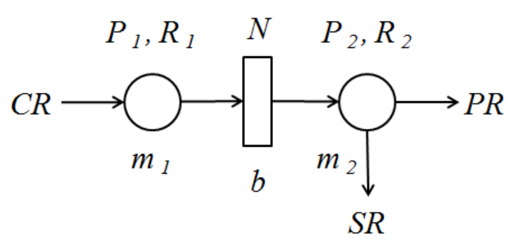


Figure 5.13: Two-machine geometric serial line with deteriorating quality buffer

Monotonicity property

To investigate the monotonicity property of the performance measures for $M =$ 2-machine geometric serial lines with deteriorating product quality, the following sets of serial lines of two-machines were introduced:

Set 1: M **= 2-Identical machines:**

L_1: $T_{up,i}$ = [20,20], $T_{down,i}$ = [5,5], $N = \{5,10,15,20,25\}$, $T = \{1,4,8,20\}$,

L_2: $T_{up,i}$ = [40,40], $T_{down,i}$ = [10,10], $N = \{10,20,30,40,50\}$, $T = \{1,4,8,40\}$,

L_3: $T_{up,i} = \{12, \ldots, 95\}$, $T_{down,i}$ = [5,5], $N = 5$, $T = \{1,4,8\}$,

L_4: $T_{up,i} = \{23, \ldots, 190\}$, $T_{down,i}$ = [10,10], $N = 10$, $T = \{1,4,8\}$,

L_5: $T_{up,i}$ = [20,20], $T_{down,i} = \{5, \ldots, 20\}$, $N = \{5,10,15,20,25\}$, $T = \{1, \ldots, 20\}$,

L_6: $T_{up,i}$ = [20,20], $T_{down,i} = \{5, \ldots, 20\}$, $N = 5$, $T = \{1,4,8\}$,

L_7: $T_{up,i} = \{12, \ldots, 95\}$, $T_{down,i} = 5$, $N = \{5,10,15,20,25\}$, $T = 4$.

Set 2: M **= 2-Different machines:**

L_1: $T_{up,i}$ = [20,45], $T_{down,i}$ = [5,5], $N = \{5,10,15,20,25\}$, $T = \{1,4,8,20\}$,

L_2: $T_{up,i}$ = [45,20], $T_{down,i}$ = [5,5], $N = \{5,10,15,20,25\}$, $T = \{1,4,8,20\}$,

L_3: $T_{up,1} = 12$, $T_{up,2} = \{12, \ldots, 95\}$, $T_{down,i}$ = [5,5], $N = 5$, $T = \{1,4,8\}$,

L_4: $T_{up,1} = \{12, \ldots, 95\}$, $T_{up,2} = 12$, $T_{down,i}$ = [5,5], $N = 5$, $T = \{1,4,8\}$,

L_5: $T_{up,i}$ = [20,20], $T_{down,1} = 5$, $T_{down,2} = \{5, \ldots, 20\}$, $N = 5$, $T = \{1,4,8\}$,

L_6: $T_{up,i}$ = [20,20], $T_{down,1} = \{5, \ldots, 20\}$, $T_{down,2} = 5$, $N = 5$, $T = \{1,4,8\}$.

The reason for selecting these particular lines was to examine different possible combinations of the system parameters to carefully analyze its behavior. Therefore, the monotonicity properties of the performance measures are characterized by the following:

Property 4 *In two-machine geometric lines defined by assumptions (i)-(vi-b)-(vii-viii),*

- *PR is monotonically increasing in T and $T_{up,2}$, monotonically decreasing in $T_{down,1}$, monotonically increasing or monotonically decreasing in $T_{up,1}$ and $T_{down,2}$, and monotonically decreasing or non-monotonic in N;*

- *SR is monotonically increasing in N, $T_{up,1}$, and $T_{down,1}$, and monotonically decreasing in T, monotonically increasing and decreasing in $T_{up,2}$ and $T_{down,2}$;*
- *CR is constant and increasing in T, monotonically increasing in N, $T_{up,1}$, and $T_{up,2}$, and monotonically decreasing in $T_{down,1}$, and $T_{down,2}$;*

Illustration of the property was analyzed in was analyzed in Figures 5.14 and 5.15. As buffer capacity increases production rate decreases monotonically in relation with increasing the average downtime in the buffer. The more Items allowed to reside in the buffer, the higher possibility that some items might reside longer in the buffer, therefore higher scrap rate. The non-monotonic behavior of PR in relation with N appears with higher residence time. This provided an inter-relation between residence time constraint T and buffer capacity. Also, the expiration behavior associated with residence time constraint T suggests that the product is less scrapped when items are allowed to stay in the buffer for longer period of time therefore higher production rate.

5.2.2 M > 2-machine lines

In this section, production lines defined by assumptions (i)-(vi-b)-(viii) with $M >$ 2-machine are analyzed as shown in Figure 5.16. As a matter of fact, conventional $M >$ 2-machine geometric lines, i.e., lines defined by assumptions (i)-(vi-b), have been introduced in the previous section. Since the quality of parts is dependent on their residence time in the buffer, simulation model of the system was used to evaluate the system theoretic properties.

Monotonicity property

To investigate the monotonicity properties of the performance measures for $M >$ 2-machine geometric serial lines with deteriorating product quality, the following sets of serial lines with three and five identical machines were introduced:

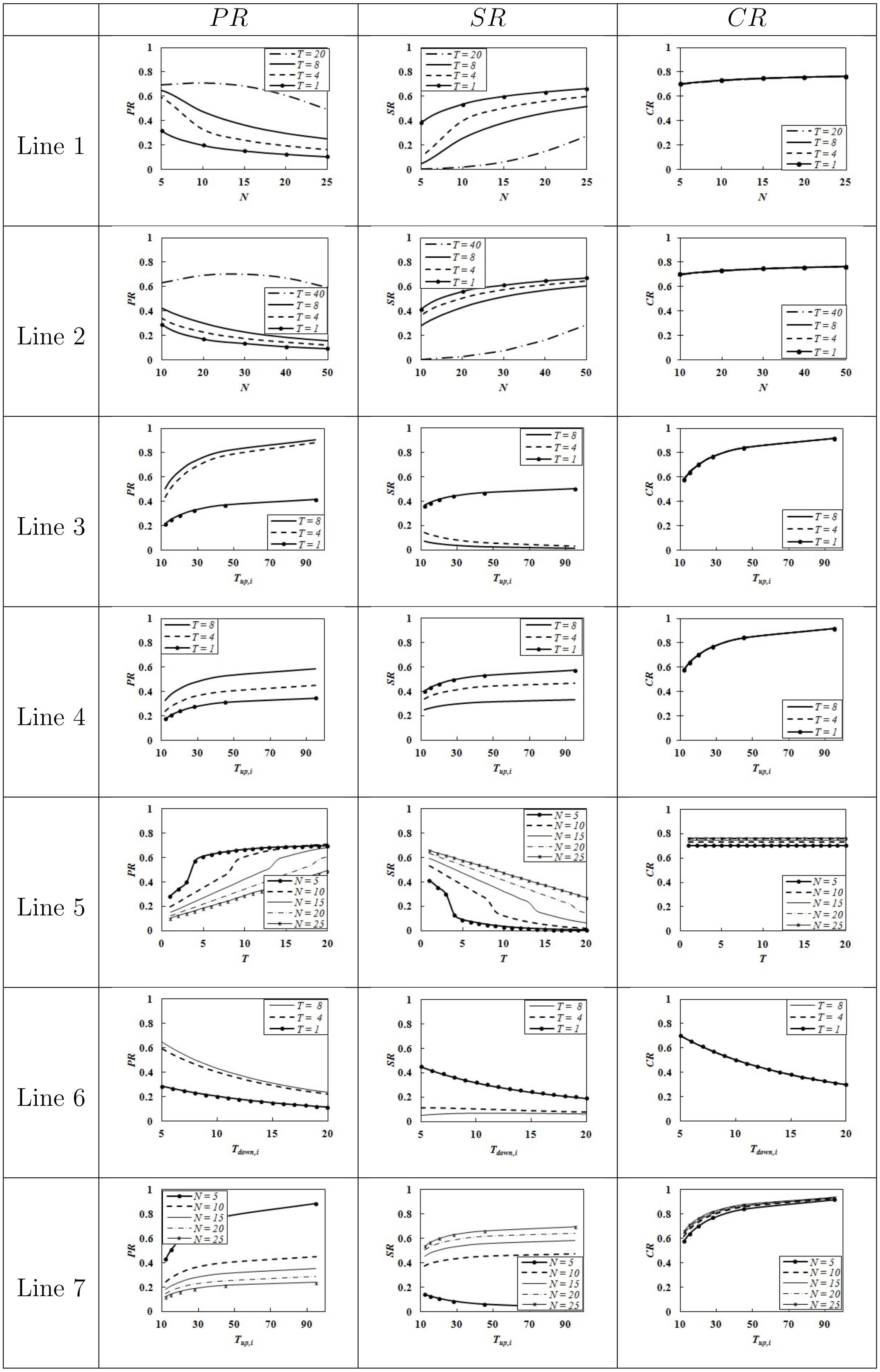


Figure 5.14: Performance measures of Set 1: M = 2-identical machines

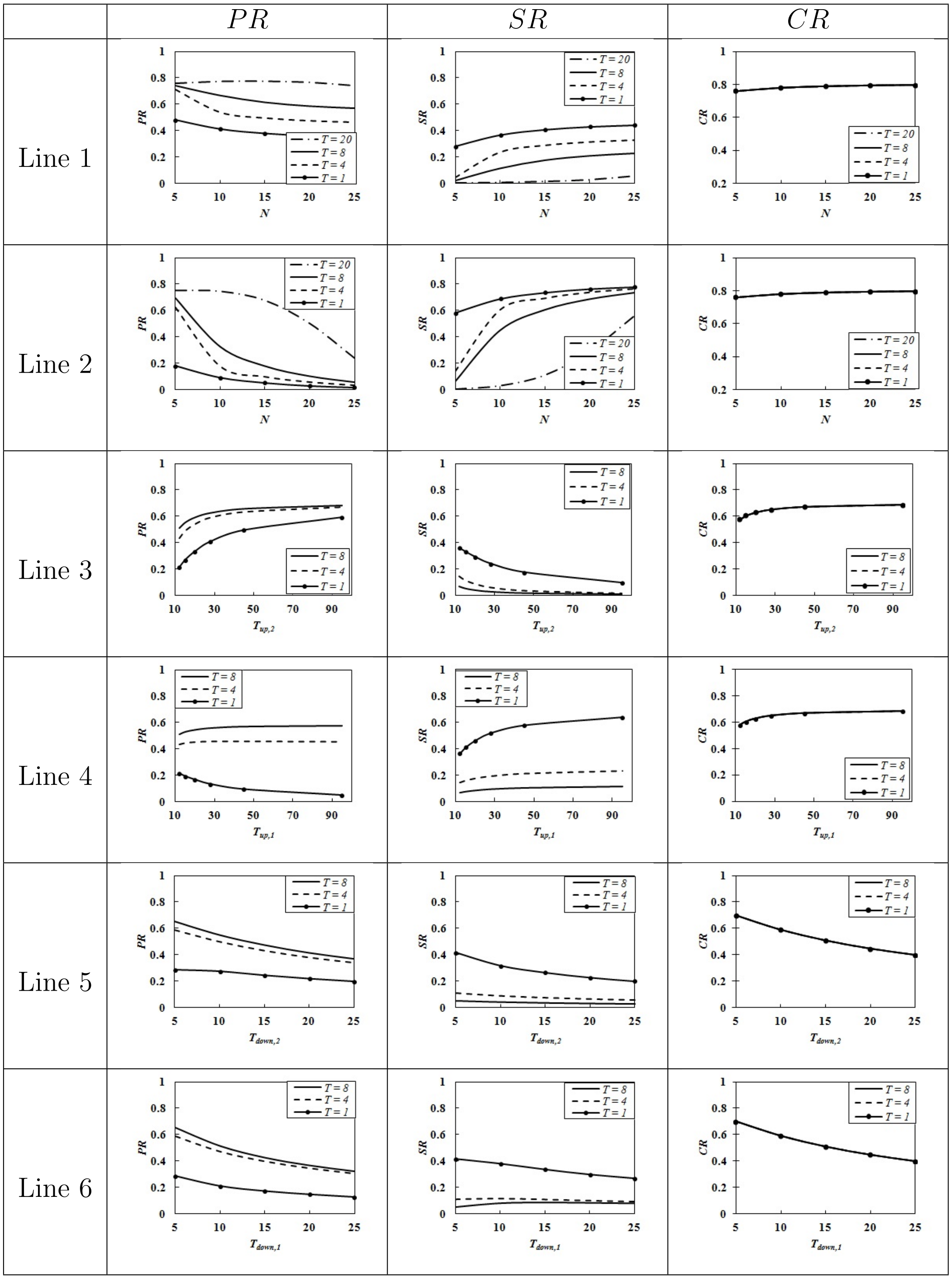


Figure 5.15: Performance measures of Set 2: M = 2-different machines

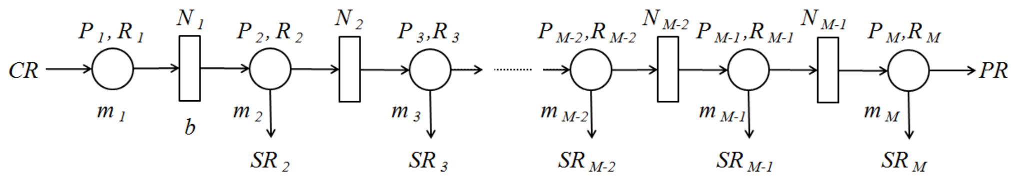


Figure 5.16: Two-machine geometric serial line with deteriorating quality buffer

Set 1: M = 3-machines vs. N_i: this particular set was created to identify the effect of machines' up-times on the system, whether both are deteriorating quality buffers or one of them. The difference between lines 1-3 and lines 4-6 is the downtimes and buffer capacities to accommodate one downtime in the buffer.

$T_{up,i} = [20,20,20]$, $T_{down,i} = [5,5,5]$, $N_i = \{5,10,15,20,25\}$, $T_i = \{1,4,8,20\}$,

L_1: $DQB = N_i$, L_2: $DQB = N_1$, L_3: $DQB = N_2$,

$T_{up,i} = [40,40,40]$, $T_{down,i} = [10,10,10]$, $N_i = \{5,10,15,20,25\}$, $T_i = \{1,4,8,20\}$,

L_4: $DQB = N_i$, L_5: $DQB = N_1$, L_6: $DQB = N_2$.

Set 2: M = 3-machines vs. $T_{up,i}$: the set was created to identify the effect of machines' up-times on the system, whether the system with both deteriorating quality buffers or one of them. Again, the difference between lines 1-3 and lines 4-6 is the downtimes and buffer capacities to accommodate one downtime in the buffer.

$T_{up,i} = \{12, \ldots, 95\}$, $T_{down,i} = [5,5,5]$, $N_i = [5,5]$, $T_i = \{1,4,8\}$,

L_1: $DQB = N_i$, L_2: $DQB = N_1$, L_3: $DQB = N_2$,

$T_{up,i} = \{23, \ldots, 190\}$, $T_{down,i} = [10,10,10]$, $N_i = [10,10]$, $T_i = \{1,4,8\}$,

L_4: $DQB = N_i$, L_5: $DQB = N_1$, L_6: $DQB = N_2$.

Set 3: M = 3-machines vs. $T_{down,i}$: this set was created to identify the effect of machines' downtimes on the system, whether the system with both deteriorating quality buffers or one of them.

$T_{up,i}$ = [20,20,20], $T_{down,i} = \{5,\ldots,20\}$, N_i = [5,5], $T_i = \{1,4,8\}$,

L_1: $DQB = N_i$, L_2: $DQB = N_1$, L_3: $DQB = N_2$.

Set 4: M = 3-machines vs. $T_{up,i}$ and N_i: this particular set was created to determine the effect of machines' up-times and buffer capacities on the system in 3-machines case, whether the system with both deteriorating quality buffers or one of them.

$T_{up,i} = \{12, \ldots,95\}$, $T_{down,i}$ = [5,5,5], $N_i = \{5,10,15,20,25\}$, $T_i = \{1,4,8\}$,

L_1: $DQB = N_i$, L_2: $DQB = N_1$, L_3: $DQB = N_2$.

Set 5: M = 3-machines vs. T_i: the set was created to analyze the effect of residence time constraint T_i on the system, whether the system with both deteriorating quality buffers or one of them.

$T_{up,i} = \{12, \ldots,95\}$, $T_{down,i}$ = [5,5,5], $N_i = \{5,10,15,20,25\}$, $T_i = \{1,4,8\}$,

L_1: $DQB = N_i$, L_2: $DQB = N_1$, L_3: $DQB = N_2$.

Set 6: M = 5-machines vs. N_i: this particular set was created to identify the effect of buffer capacities on the system, whether the system with all deteriorating quality buffers or one of them.

$T_{up,i}$ = [20,20,20,20,20], $T_{down,i}$ = [5,5,5,5,5], $N_i = \{5,10,15,20,25\}$, $T_i = \{1,4,8,20\}$,

L_1: $DQB = N_i$, L_2: $DQB = N_1$, L_3: $DQB = N_2$, L_4: $DQB = N_3$, L_5: $DQB = N_4$,

Set 7: M = 5-machines vs. $T_{up,i}$: the set was created to analyze the effect of machines' up-times on the system, whether the system with all deteriorating quality buffers or one of them.

$T_{up,i} = \{12, \ldots, 95\}$, $T_{down,i} = [5,5,5,5,5]$, $N_i = \{5,10,15,20,25\}$, $T_i = \{1,4,8\}$, L_1: $DQB = N_i$, L_2: $DQB = N_1$, L_3: $DQB = N_2$, L_4: $DQB = N_3$, L_5: $DQB = N_4$.

Set 8: M = 5-machines with one DQB vs. $T_{up,i}$: this particular set was generated to identify the effect of machines' up-times on the system, whether the system with all deteriorating quality buffers or one of them. To be specific, this particular set will be checking the effect of increasing the machine up-time downstream of the DQB.
$T_{up,i} = 20$, $T_{down,i} = [5,5,5,5,5]$, $N_i = [5,5,5,5]$, $T_i = \{1,4,8\}$,
L_1: $T_{up,1} = \{0.7, \ldots, 0.95\}$,$DQB = N_1$, L_2: $T_{up,2} = \{0.7, \ldots, 0.95\}$,$DQB = N_2$, L_3: $T_{up,3} = \{0.7, \ldots, 0.95\}$,$DQB = N_3$, L_4: $T_{up,4} = \{0.7, \ldots, 0.95\}$,$DQB = N_4$.

Set 9: M = 5-machines with one DQB vs. $T_{up,i+1}$: this set was created to identify the effect of machines' up-times on the system, whether the system with all deteriorating quality buffers or one of them. To be specific, this particular set will be checking the effect of increasing the machine efficiency upstream of the DQB.
$T_{up,i} = 20$, $T_{down,i} = [5,5,5,5,5]$, $N_i = [5,5,5,5]$, $T_i = \{1,4,8\}$,
L_1: $T_{up,2} = \{0.7, \ldots, 0.95\}$,$DQB = N_1$, L_2: $T_{up,3} = \{0.7, \ldots, 0.95\}$,$DQB = N_2$, L_3: $T_{up,4} = \{0.7, \ldots, 0.95\}$,$DQB = N_3$, L_4: $T_{up,5} = \{0.7, \ldots, 0.95\}$,$DQB = N_4$.

Set 10: M = 5-machines vs. $T_{up,i}$ and N_i: the set was put together to identify the effect of machines' up-times and buffer capacities on the system in 5-machines case, whether the system with both deteriorating quality buffers or one of them.
$T_{up,i} = \{12, \ldots, 95\}$, $T_{down,i} = [5,5,5,5,5]$, $N_i = [5,5,5,5]$, $T_i = \{1,4,8\}$.

Set 11: M = 5-machines vs. $T_{down,i}$: this particular set was created to identify the effect of machines' downtimes on the system in 5-machines case.

$T_{up,i} = \{12, \ldots, 95\}$, $T_{down,i} = \{5, \ldots, 20\}$, $N_i = [5,5,5,5]$, $T_i = \{1,4,8\}$.

Set 12: M = 5-machines vs. T_i: this particular set was generated to identify the effect of residence time constraint T_i on the system in 5-machines case. Then, compare the effect of $T_i$on each buffer separately.

$T_{up,i} = \{12, \ldots, 95\}$, $T_{down,i} = [5,5,5,5,5]$, $T_i = \{1, \ldots, 20\}$,

L_1: $DQB = N_i$, $N_i = \{5,10,15,20,25\}$, L_2: $DQB = N_1, N_2, N_3, N_4, N_i$, $N_i = [5,5,5,5]$.

The monotonicity properties of the performance measures are characterized by the following:

Property 5 *In $M > 2$-machine geometric lines defined by assumptions (i)-(vi-b)-(vii-viii),*

- *PR is monotonically increasing in T, monotonically decreasing in $T_{down,i}$, monotonically increasing or monotnoically decreasing in $T_{up,i}$, and monotonically decreasing or non-monotonic in N_i;*
- *SR is monotonically increasing in N_i and $T_{down,i}$, monotonically decreasing in T, and monotonically increasing or monotonically decreasing in $T_{up,i}$;*
- *CR is monotonically increasing in N_i, $T_{up,i}$ and $T_{down,i}$, and monotonically decreasing or constant in T;*

As buffer capacity increases, production rate decreases monotonically. This is due to the average downtime in the buffer increases as buffer capacity increases. The results suggests that the best allocation of the deteriorating quality buffer is toward the end of the line to ensure the highest production rate of the line. Also, increasing the up-time of the machine following the deteriorating quality buffer provides less scrap rate then increasing up-time of the machine preceding the deteriorating quality buffer.

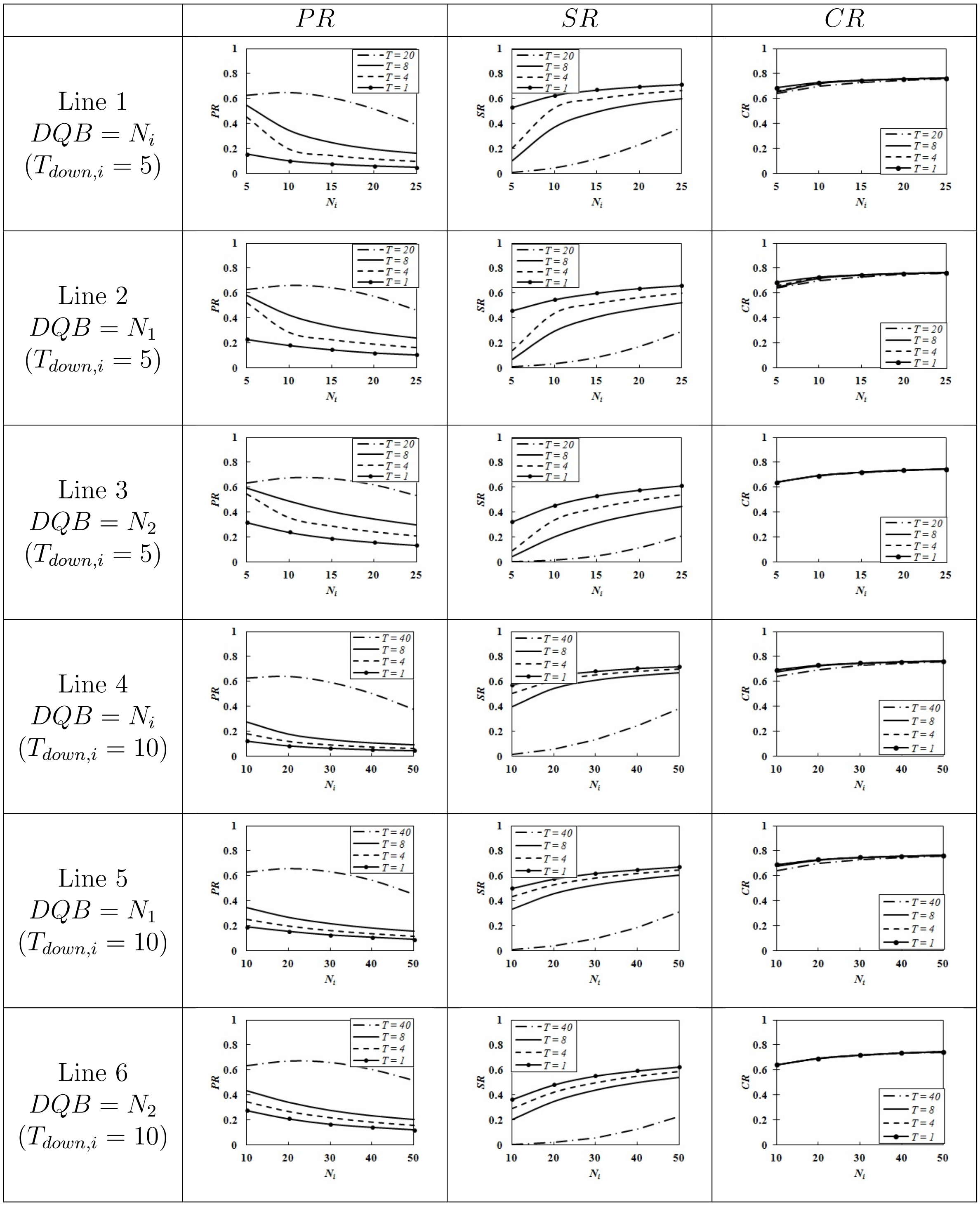


Figure 5.17: Performance measures of Set 1: M = 3-machines vs. N_i

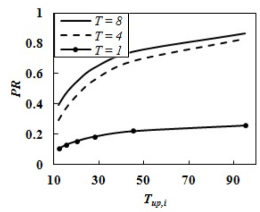
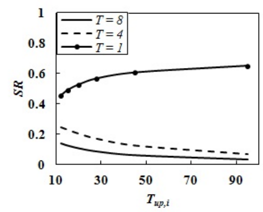
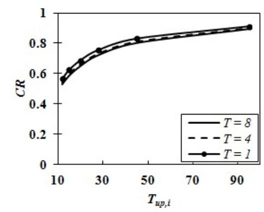
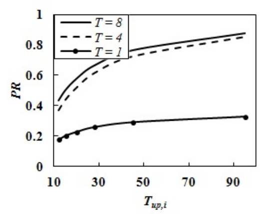
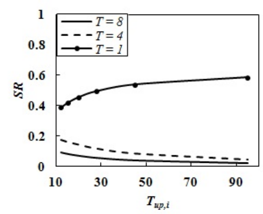
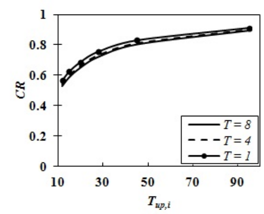
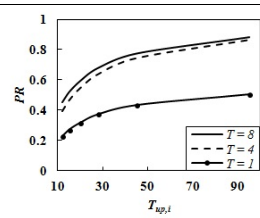
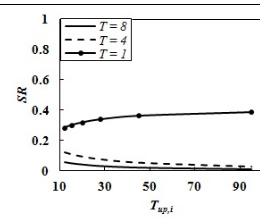
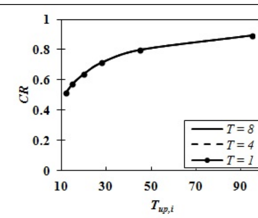
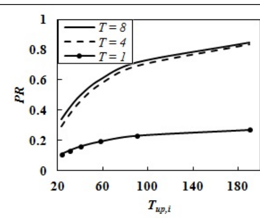
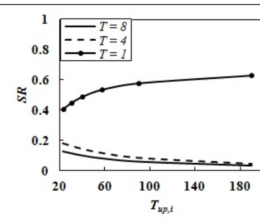
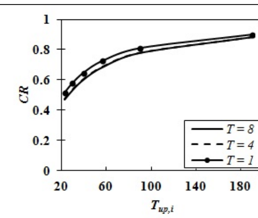
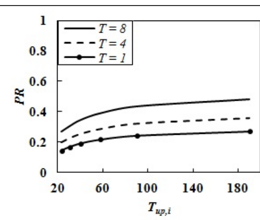
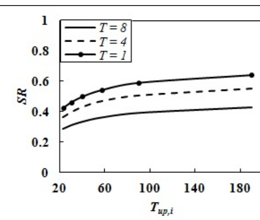
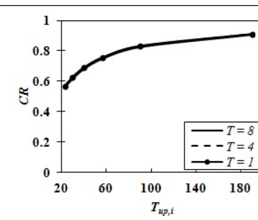
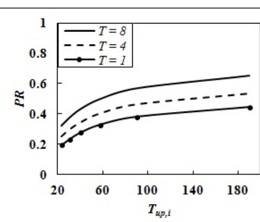
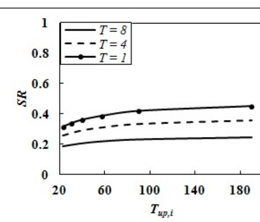
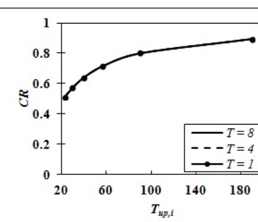
	PR	SR	CR
Line 1 $DQB = N_i$ $(T_{down,i} = 5)$			
Line 2 $DQB = N_1$ $(T_{down,i} = 5)$			
Line 3 $DQB = N_2$ $(T_{down,i} = 5)$			
Line 4 $DQB = N_i$ $(T_{down,i} = 10)$			
Line 5 $DQB = N_1$ $(T_{down,i} = 10)$			
Line 6 $DQB = N_2$ $(T_{down,i} = 10)$			

Figure 5.18: Performance measures of Set 2: M = 3-machines vs. $T_{up,i}$

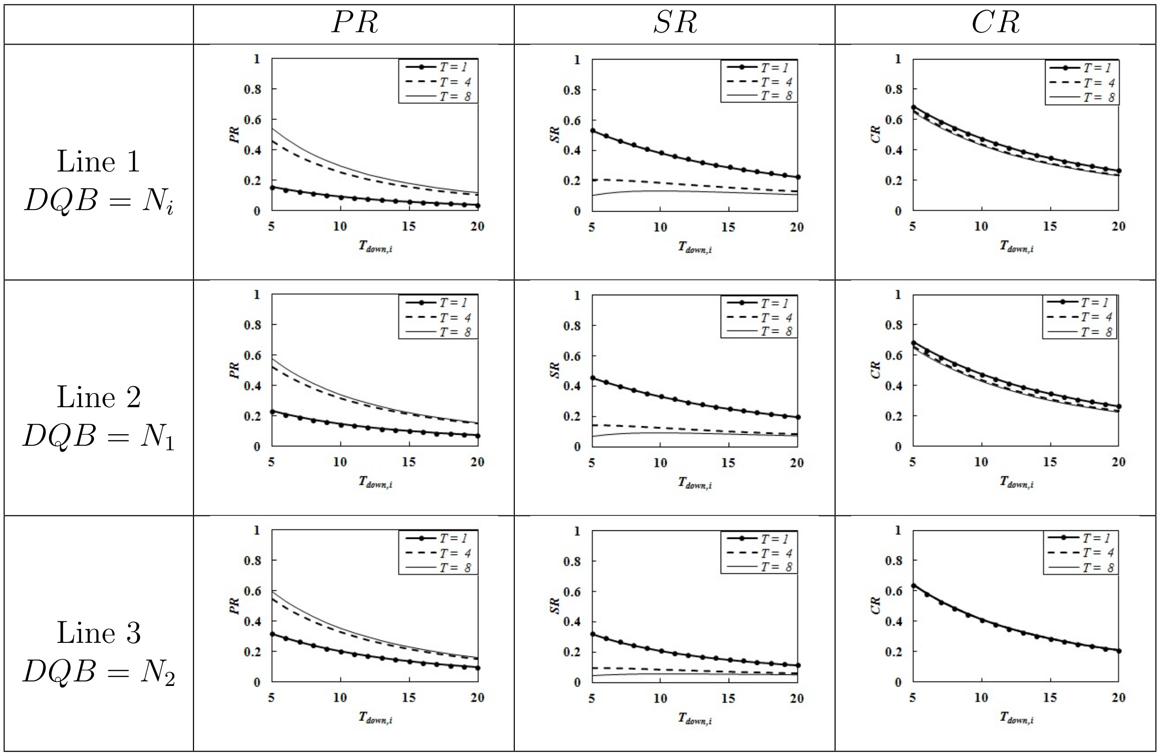


Figure 5.19: Performance measures of Set 3: M = 3-machines vs. $T_{down,i}$

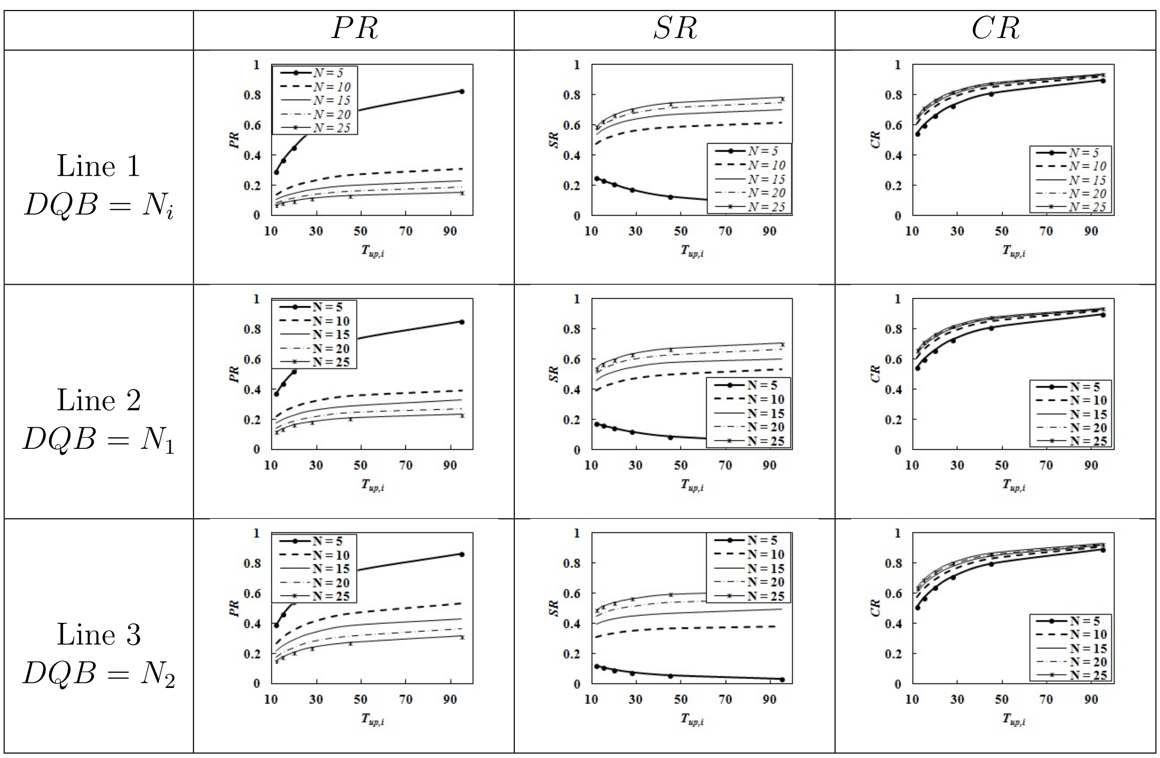


Figure 5.20: Performance measures of Set 4: M = 3-machines vs. $T_{up,i}$ and N_i

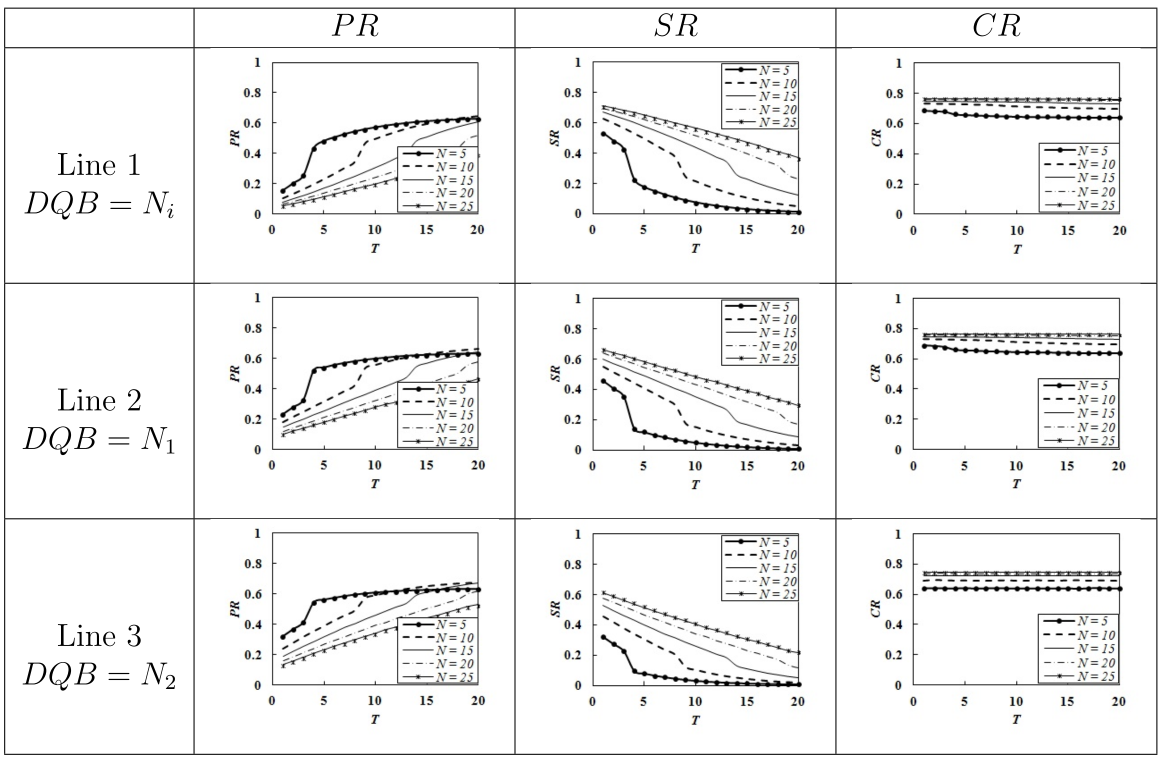


Figure 5.21: Performance measures of Set 5: M = 3-machines vs. T

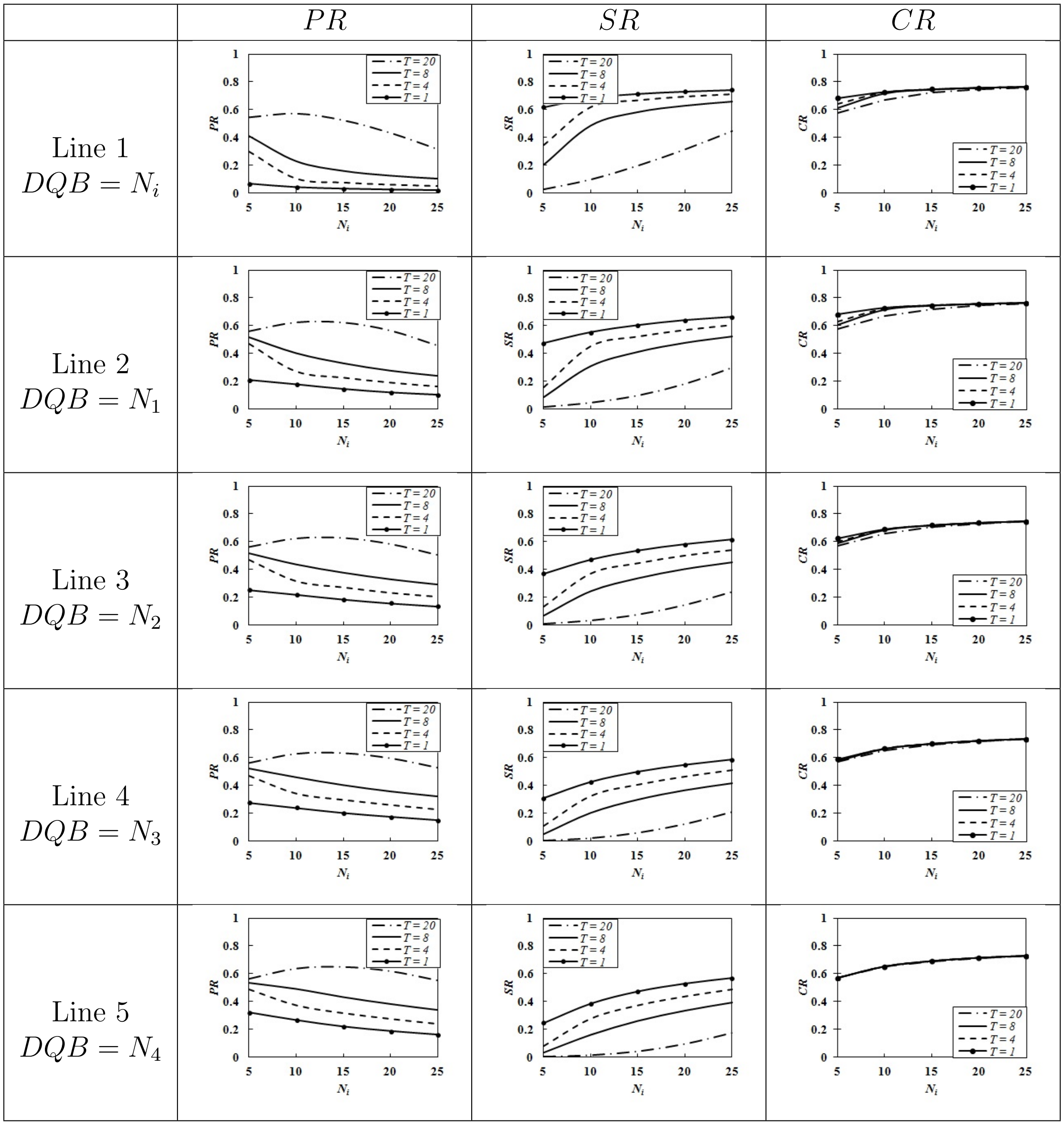


Figure 5.22: Performance measures of Set 6: M = 5-machines vs. N_i

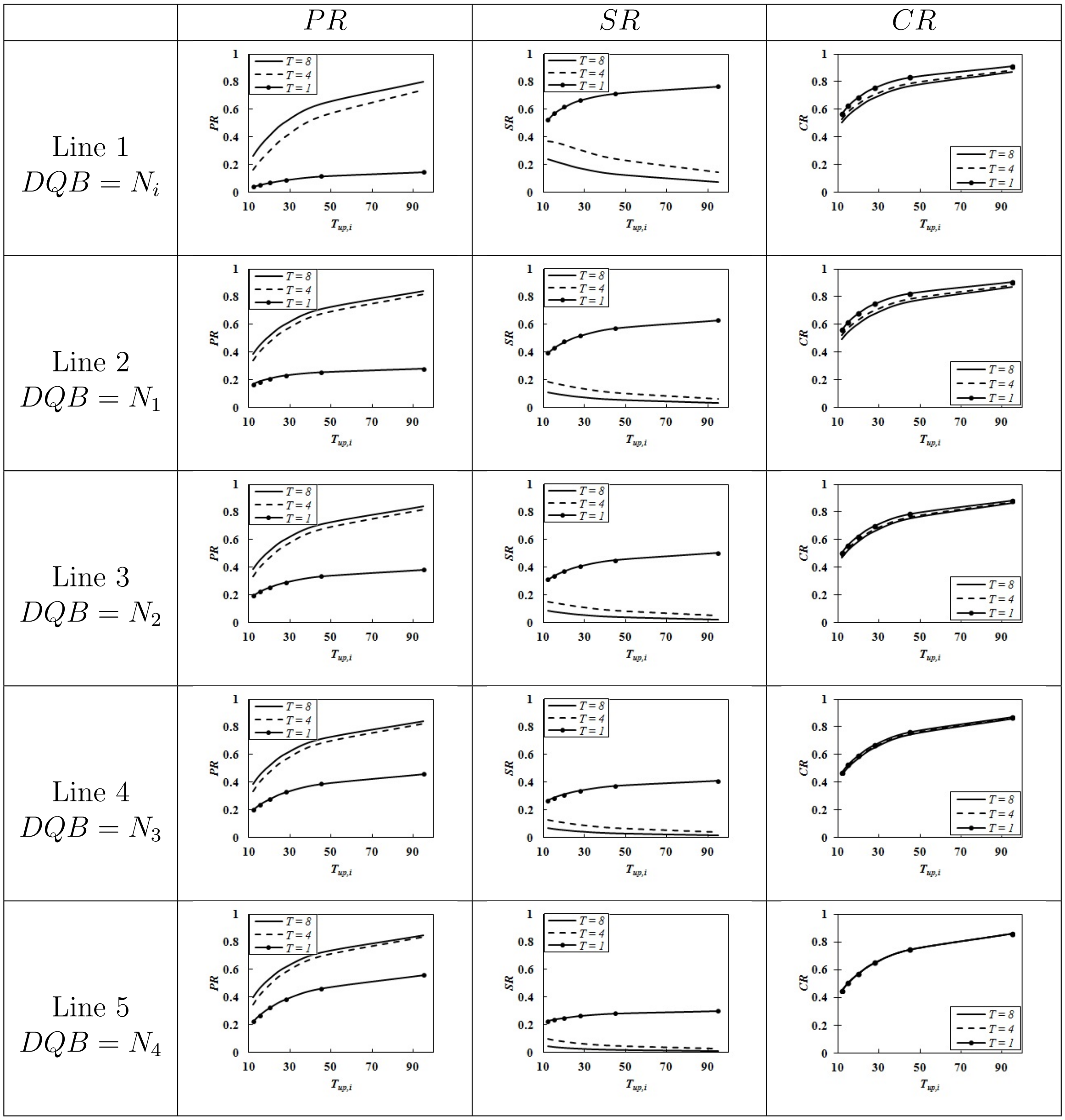


Figure 5.23: Performance measures of Set 7: M = 5-machines vs. $T_{up,i}$

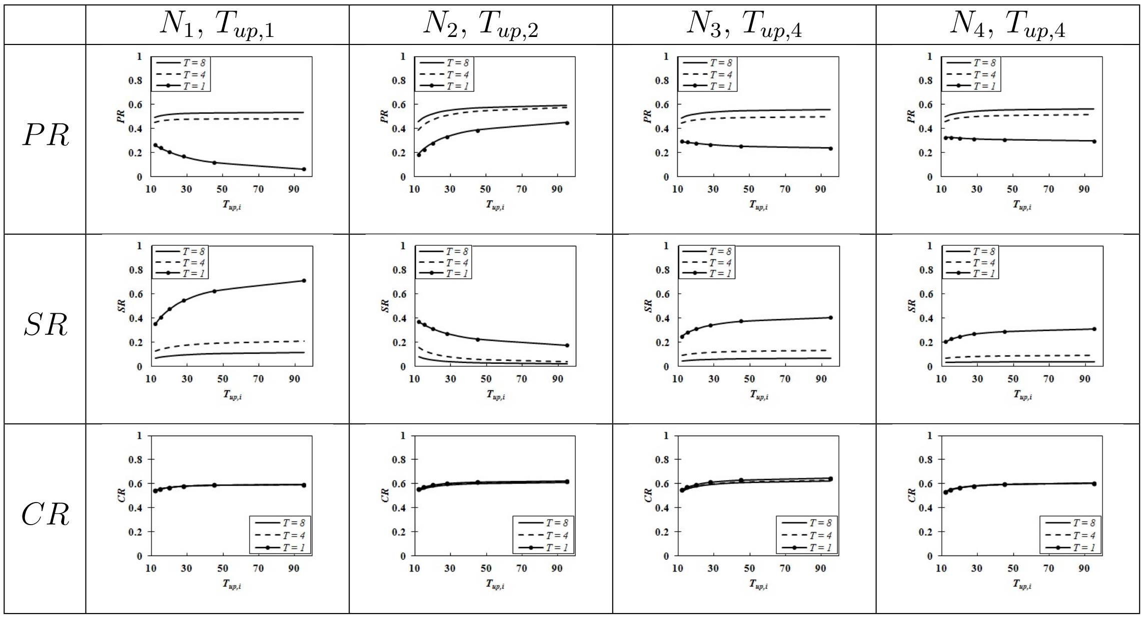


Figure 5.24: Performance measures of Set 8: M = 5-machines with one DQB and previous machine's $T_{up,i}$

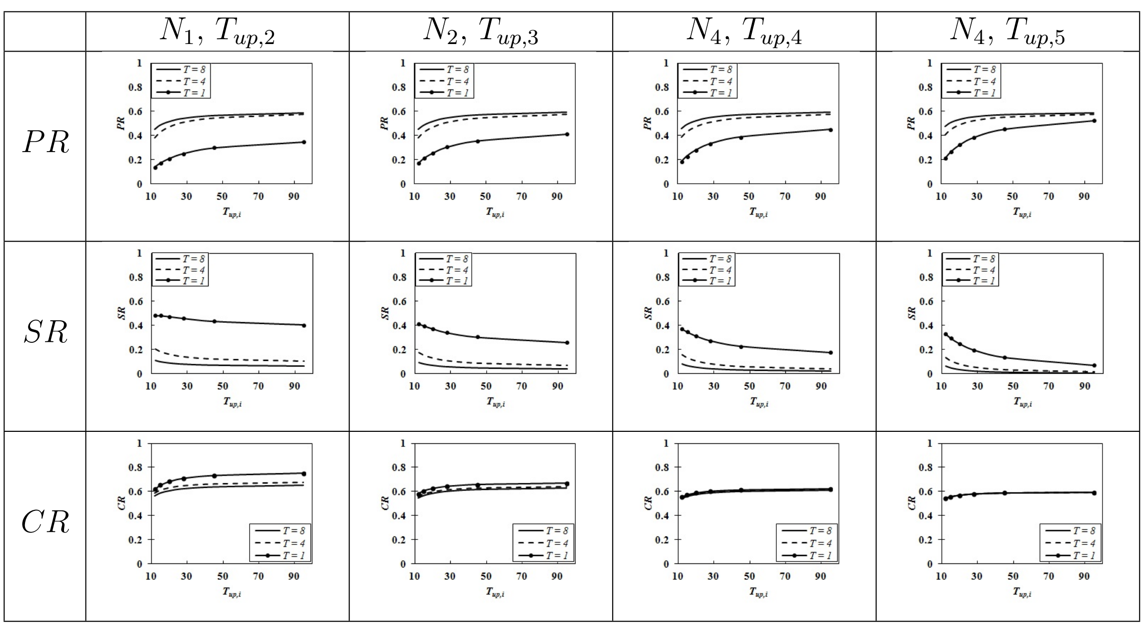


Figure 5.25: Performance measures of Set 9: M = 5-machines with one DQB and following machine's $T_{up,i}$

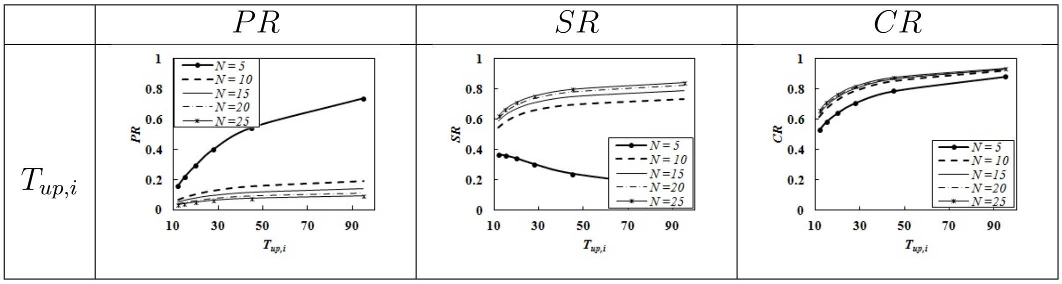


Figure 5.26: Performance measures of Set 10: M = 5-machines vs. $T_{up,i}$ and N_i

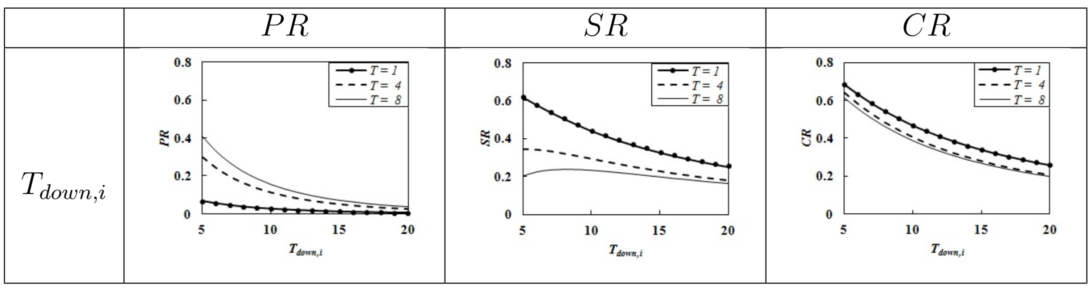


Figure 5.27: Performance measures of Set 11: M = 5-machines vs. $T_{down,i}$

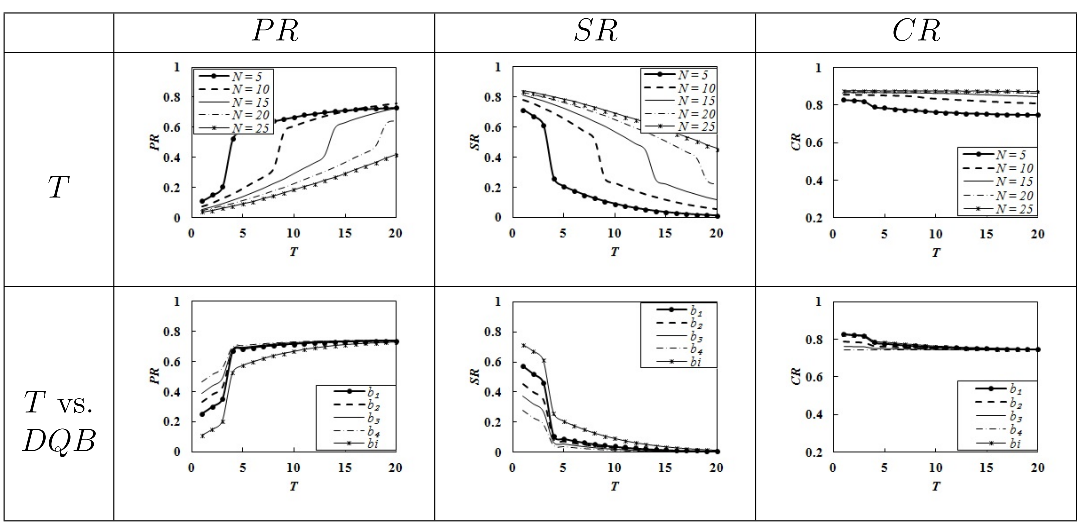


Figure 5.28: Performance measures of Set 12: M = 5-machines vs. T

To elaborate more on the monotonicity property of a geometric serial line with deteriorating product quality for longer lines, consider a 5-machine line given in Figure 5.29, where b_3 is the buffer with quality deterioration. Assume that line parameters are as follows:

$$T_{up,i} = [45, 30, 50, 40, 45], \quad T_{down,i} = [5, 5, 5, 5, 5], \quad N_i = [5, 5, 5, 5], and \quad T = 3.$$

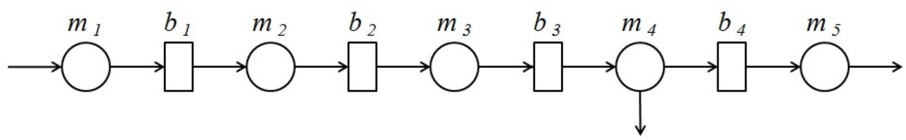


Figure 5.29: 5-machine geometric line example

The results are shown in Figures 5.30, 5.30, and 5.32. As one can see, higher uptime of m_3 and/or higher capacity of buffer b_3 may lead to lower production of good parts due to long residence time of parts in buffer b_3, while increasing uptime of m_4 can help alleviate the accumulation of work-in-process, and thus, always lead to increasing PR.

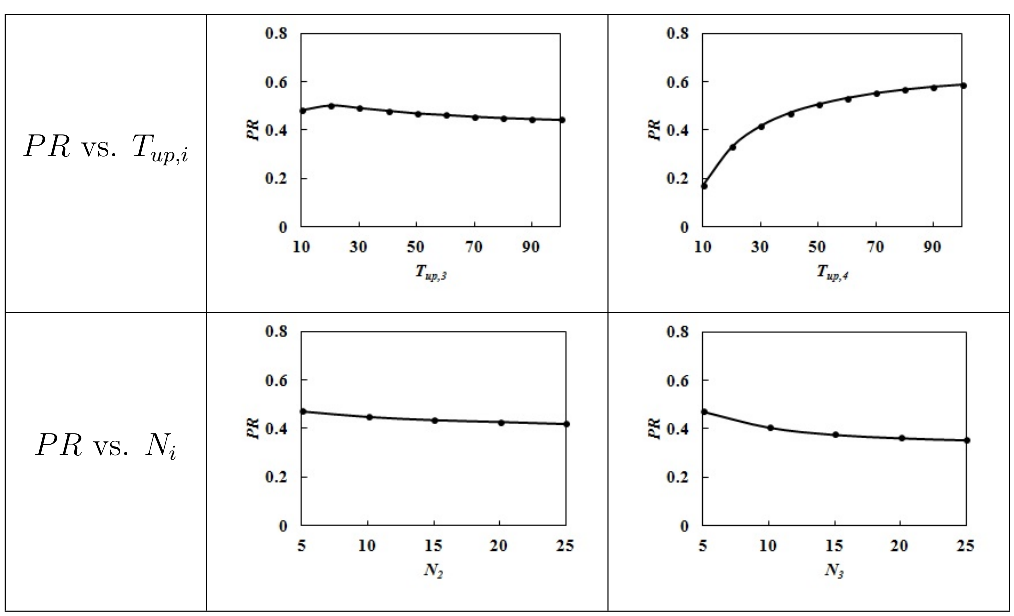


Figure 5.30: PR as functions of $T_{up,i}$ and N_i

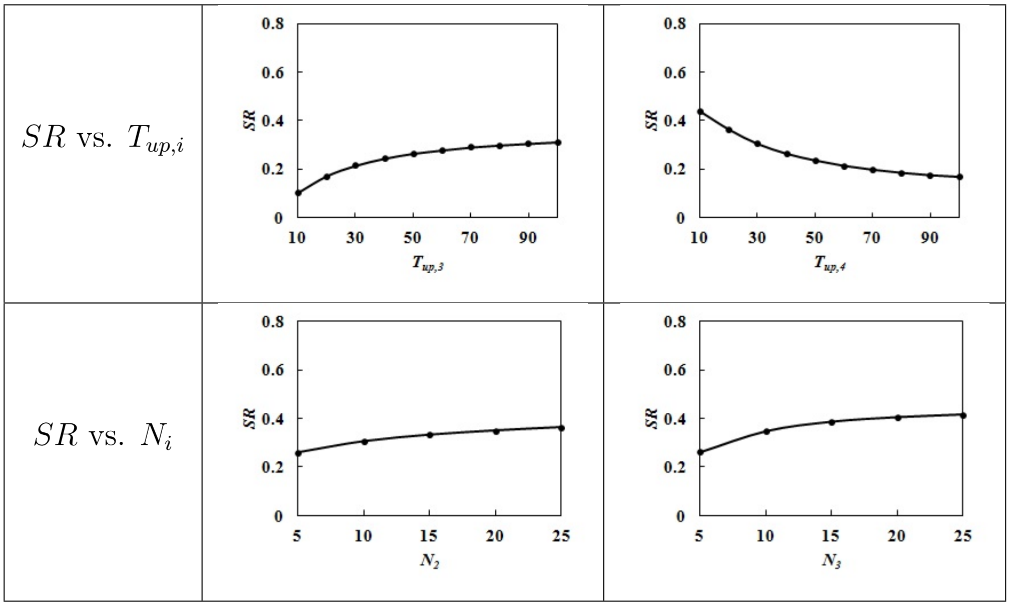


Figure 5.31: SR as functions of $T_{up,i}$ and N_i

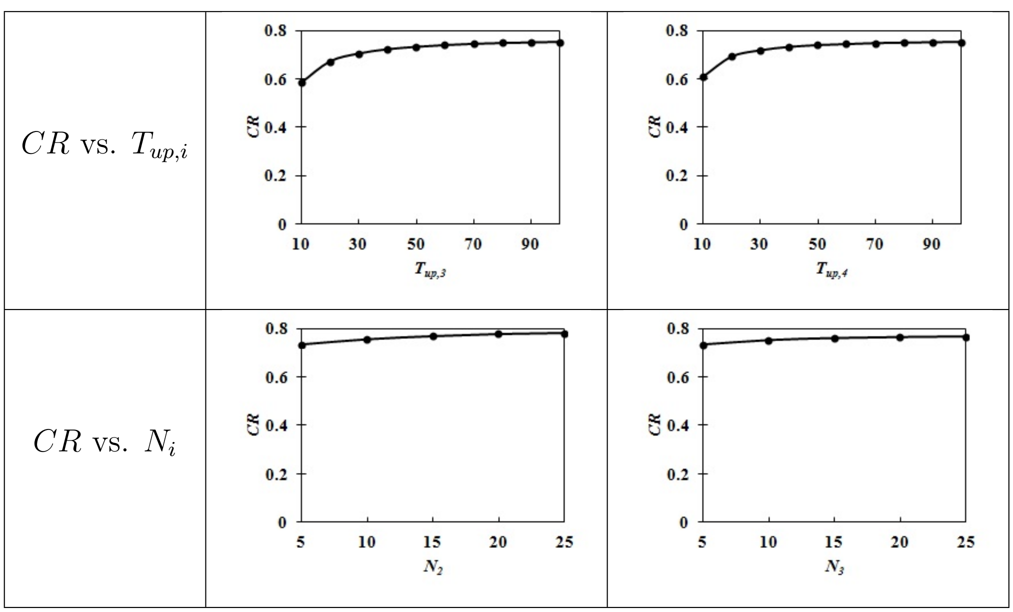


Figure 5.32: CR as functions of $T_{up,i}$ and N_i

5.3 Summary

- Throughput of a geometric serial line is monotonic with respect to machine and buffer parameters.
- Shorter up- and downtime lead to a higher production rate (or throughput) than longer ones, even if machine efficiency remains constant.
- A decrease in downtime leads to higher throughput than a similar increase in uptime.
- The aggregation procedure introduced provides a very acceptable error less than 3.15% compared to the simulation model for the system.
- Deteriorating quality buffer (DQB) must be placed towards the end of the line to ensure the largest throughput.
- More efficient machine after the deteriorating quality buffer decreases scrap rate and therefore improves throughput.
- The non-monotonic behavior of PR in N_i suggests that a part release control may help in SR reduction.

Chapter 6

BOTTLENECK ANALYSIS

6.1 Introduction

Bottlenecks within a production line significantly reduce the productivity. Because in practice bottlenecks are almost certain to exist [86], and because the existence of bottlenecks is a major factor in line performance and management [87], [47], it is important to improve the bottleneck. By improvement, we mean increasing the effective throughput capacity of the current bottleneck, which in turn permits greater production rate for the entire production line. However, before the bottleneck can be improved, it must be located.

If quantitative performance evaluation is carried out at all, then in almost any case simulation is the only tool used. Machine and buffer optimization problems are mainly solved through simple trial-and-error approaches, which suffer from the severe drawbacks of being both very time-consuming and providing solutions that are usually far from optimal. The practitioner must also know when to stop improving. Improvement at non-bottleneck resources does not increase system capacity. Bottleneck analysis is of high interest in manufacturing operations and in recent years a great deal of research has focused on the area of bottleneck detection [88].

It was noted that a small change of data or system characteristics may generate a considerably different behavior of the system under study. For example, slightly changing the processing time at a station may shift the bottleneck of the system with the need to rearrange the buffers completely. As every production line is obviously unique, it jeopardizes the economic efficiency if a flow line planner relies completely upon experience gathered from observations of other production lines. Therefore, tools are required that can provide system-specific performance measures in a fast and reliable manner. Quick and correct identification of the bottleneck locations can lead to an improvement in the operation management of utilizing finite manufacturing resources, increasing the system production rate, and minimizing the total cost of production. This chapter will be focusing on identifying bottleneck in Bernoulli serial line with perfect quality and deteriorating quality buffers.

6.2 Bernoulli Serial Lines with Perfect Quality Buffer

Bottleneck machine: Consider a serial production line with M Bernoulli machines defined by parameters p_i, i= 1 ,. . . , M and M-1 buffers with capacities N_i, i = 1 ,. . . , M-1. Assume that the line operates according to assumptions (i)-(vi-a)
Let, as before, PR, denote the production rate of the system, i.e., $PR = PR(p_1, \ldots,$ p_M, N_1,. . . ,N_{M-1})

Definition 6.1: [44] Machine m_i, $i \in 1, \ldots, M$, is the bottleneck machine (BN-m) of a Bernoulli line defined by assumptions $(i) - (vi - a)$ if

$$\frac{\delta PR}{\delta p_i} > \frac{\delta PR}{\delta p_j}, \qquad \forall j \neq i. \tag{6.1}$$

Due to the monotonicity properties of PR with respect to $p_i's$, both derivation in (6.1) are positive. Thus, definition implies that m_i is the BN-m if its infinitesimal improvement leads to largest increase of the production rate, as compared with a similar improvement of any other machine in the system [44] .

Furthermore, a machine with the smallest p_i is not necessarily the BN-m in the sense of Definition 6.1. Indeed, consider the production lines shown in Figure 6.2, where the numbers in the circles and the rectangles are p_i and N_i, respectively, and the row of numbers under the machines represent the estimates of partial derivation $\frac{\delta PR}{\delta p_i}$ evaluated by numerical simulations. Clearly, the bottleneck machines are m_2 (in Figure 6.1) and also m_2 (in Figure 6.2), none of which corresponds to the worst machine (i.e., the machine with the smallest p_i). In fact, m_2 in Figure 6.1 is the best machine in the system.

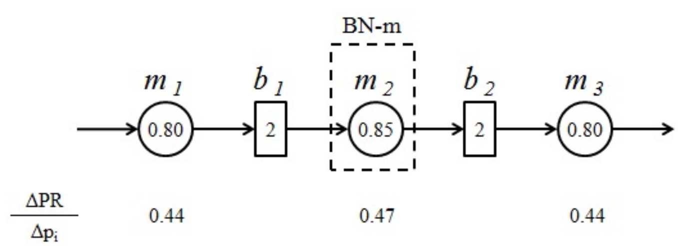


Figure 6.1: The best machine is the bottleneck in Bernoulli lines

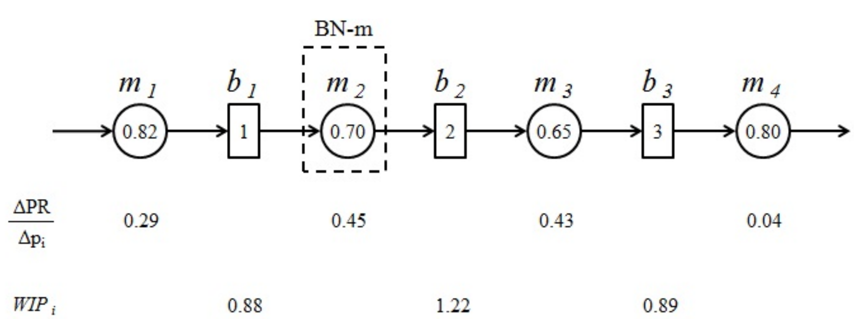


Figure 6.2: The worst machine is not the bottleneck in Bernoulli lines

Similarly, a machine with the largest work-in-process inforont of it is not necessarily the bottleneck. An example is given in Figure 6.2, where m_3 has the largest WIP to be processed, while the BN-m is m_2.

Bottleneck buffer: While the term "bottleneck machine" is widely used in practice, the term "bottleneck buffer" is not. This is due to focusing on machine efficiencies happens to be believed more important than the effect of buffer capacity adjacent to these machines on the overall production rate of the system. On the contrary, the buffers "shock absorbers" are of importance. Therefore, bottleneck buffer must be introduced in order to explore all means of system improvements.

Definition 6.2: [44] Buffer b_i, i $\in$ 1 ,. . . , M-1, is the bottleneck buffer (BN-b) of a Bernoulli line defined by assumptions $(i) - (vi - a)$ if

$$\begin{aligned} &PR(p_1, \ldots, p_M, N_1, \ldots, N_i + 1, \ldots, N_{M-1}) \\ &\quad > PR(p_1, \ldots, p_M, N_1, \ldots, N_j + 1, \ldots, N_{M-1}), \qquad \forall j \neq i. \end{aligned} \tag{6.2}$$

In other words, BN-b is the buffer, which leads to the largest increase in PR if its capacity is increased by 1, as compared with increasing other buffers in the system. An example is shown in Figure 6.3, where the numbers under each buffer corresponds to the PR of the system obtained by simulations when the capacity of this buffer is increased by one. Moreover, a buffer with the smallest capacity is not necessarily the BN-b. To identify the BN-b using Definition 6.2, one would have to experiment with the system by increasing each buffer and measuring the resulting production rate, which is hardly possible in practice.

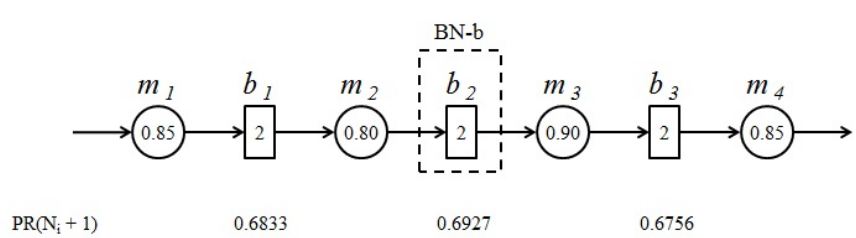


Figure 6.3: Example of bottleneck buffer in Bernoulli lines

To make these definitions practical, [44] reformulated them in terms of quantities, which are either available through measurements on the factory floor or through analytical calculations or both:

Theorem 4 *[44] For two-machine Bernoulli lines, the inequality*

$$\frac{\delta PR}{\delta p_1} > \frac{\delta PR}{\delta p_2} \qquad (respectively, \frac{\delta PR}{\delta p_1} < \frac{\delta PR}{\delta p_2}) \tag{6.3}$$

takes place if and only if

$$BL_1 < ST_2 \qquad (respectively, BL_1 > ST_2).$$

This result relates the "non-measurable" and "non-calculable" partial derivatives of PR with the "measurable" and "calculable" probabilities of blockages and starvations. In addition, it states that the BN-m can be identified without even knowing parameters of the machines and buffer, but just by measuring ST_2 and BL_1.

Inspired by this theorem, an arrow-based method has been developed to identify the BN in longer lines: arrange the probabilities of starvations (ST_i) and blockages (BL_i) under each machine as shown in Figure 6.4 and place arrows directed from one machine to another according to the following method [44]

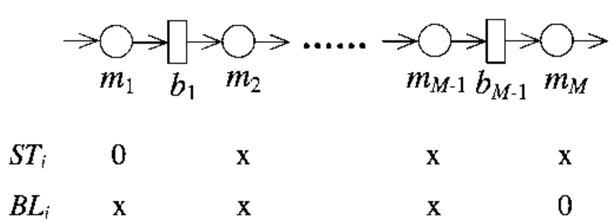


Figure 6.4: BN identification in M-machine lines

Arrow assignment method: [44] This method derives its name from the practice of drawing arrows pointing left or right showing which machines have higher blockage and starvation compared to adjacent machines and uses two related rules to locate the bottleneck. The first rule is the bottleneck indicator rule, composed of two related parts, which says that:

(a) If $BL_i > ST_{i+1}$, assign the arrow pointing from m_i to m_{i+1},

(b) If $BL_i < ST_{i+1}$, assign the arrow pointing from m_{i+1} to m_i.

In a Bernoulli line with $M > 2$-machines,

- if there is a single machine with no emanating arrows, it is the BN-m;
- if by this rule there are multiple machines with no emanating arrows, then the primary bottleneck is determined by using of the second rule, the one with the largest severity if the Primary BN-m (PBN-m), where the severity of each (local) BN-m is defined by

$$\begin{aligned} S_i &= \mid ST_{i+1} - BL_i \mid + \mid ST_i - BL_{i-1} \mid, \quad i = 2, \ldots, M-1, \\ S_1 &= \mid ST_2 - BL_1 \mid, \\ S_M &= \mid ST_M - BL_{M-1} \mid; \end{aligned} \tag{6.4}$$

- the BN-b is the buffer immediately upstream of the BN-m (or PBN-m) if it is more often starved than blocked, or immediately downstream the BN-m (or PBN-m) if it is more often blocked than starved.

According to this method, m_2 and b_2 are the bottlenecks in Figure 6.5, which indicates that there is a single bottleneck machine in the system. On the other hand, m_2 and b_2 are the PBN-m and BN-b in Figure 6.6, where multiple bottleneck machines are available in the system.

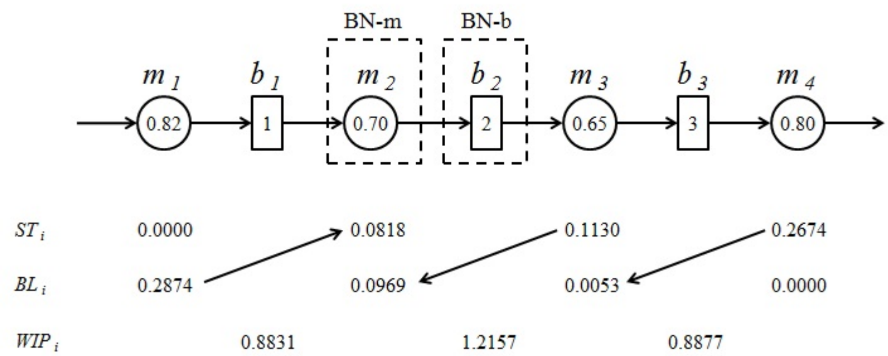


Figure 6.5: Illustration of a Bernoulli line with a single bottleneck machine

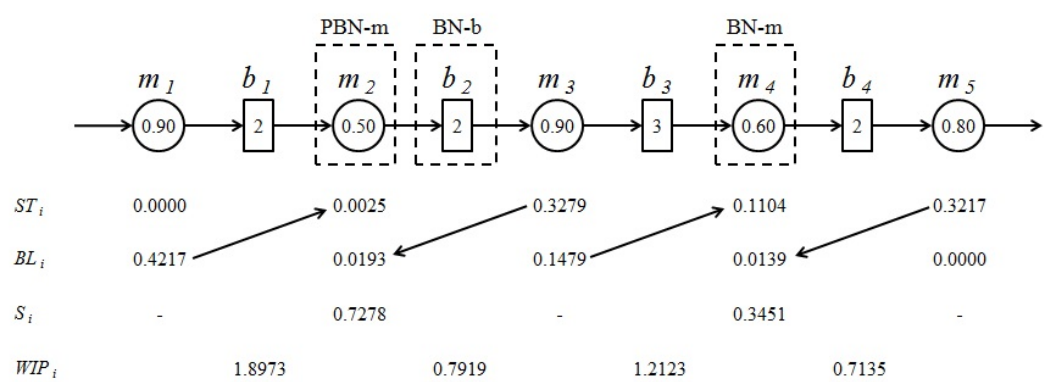


Figure 6.6: Illustration of a Bernoulli line with multiple bottleneck machines

6.3 Bernoulli Serial Lines with Deteriorating Quality Buffer (DQB)

Consider a serial production line with M Bernoulli machines defined by parameters p_i, i= 1 ,..., M and M-1 buffers with capacities N_i, i = 1 ,..., M-1. Assume that the line operates according to assumptions (i)-(vi-a)-(viii) Let, as before, PR, denote the production rate of the system, i.e., $PR = PR(p_1, \ldots, p_M, N_1, \ldots, N_{M-1})$

Definition 6.3: Machine m_i, i ∈ 1 ,..., M, is the bottleneck machine (BN-m) of a Bernoulli line defined by assumptions $(i) - (vi - a) - (viii)$ if

$$\left|\frac{\delta PR}{\delta p_i}\right| > \left|\frac{\delta PR}{\delta p_j}\right|, \qquad \forall j \neq i. \tag{6.5}$$

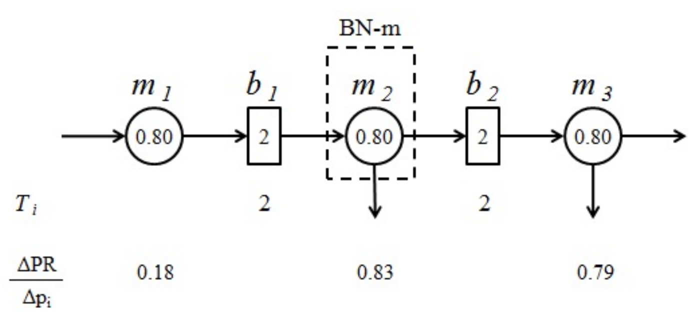


Figure 6.7: Illustration of 3-machine Bernoulli line with DQB

This definition is similar to Definition 6.1 for production lines with perfect quality buffers. The only difference is that absolute values of partial derivatives are used in (6.5), because it is not *priori* clear that PR in systems defined by assumptions (i)-(vi-a)-(viii) are both monotonic and non-monotonic in some cases with respect to $p_i's$.

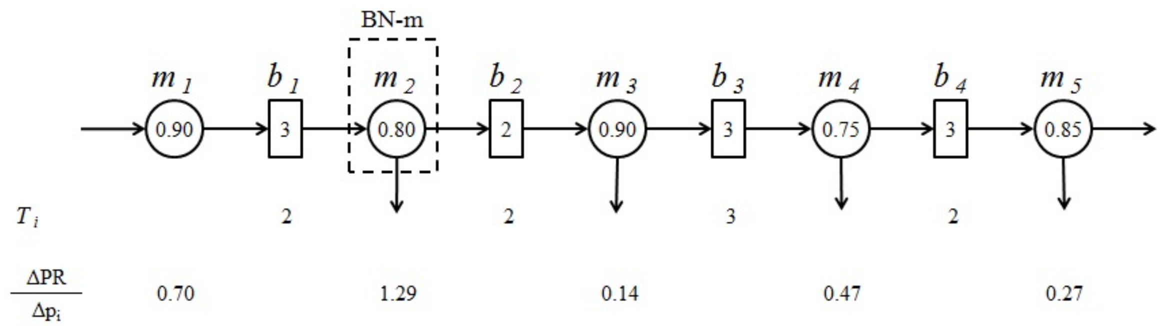


Figure 6.8: Illustration of 5-machine Bernoulli line with DQB

Figures 6.7 and 6.8, where the numbers under each machine corresponds to the $\left|\frac{\delta PR}{\delta p_i}\right|$ of the system obtained by simulation when the machine efficiency of this machine is increased by a factor (all machines' efficiencies are increased by the same factor).

Definition 6.4: Buffer b_i, i $\in$ 1 ,..., M-1, is the bottleneck buffer (BN-b) of a Bernoulli line defined by assumptions $(i) - (vi - a) - (viii)$ where

$$PR(p_1, \ldots, p_M, N_1, \ldots, N_i \pm 1, \ldots, N_{M-1}, T_1, \ldots, T_{M-1})$$

if

$$\left|\frac{\delta PR}{\delta N_i}\right| > \left|\frac{\delta PR}{\delta N_j}\right|, \qquad \forall j \neq i. \tag{6.6}$$

In other words, BN-b is the buffer, which leads to the largest increase in PR if its capacity is increased or decreased by 1, as compared with increasing or decreasing other buffers in the system. The definition is set to accommodate the monotonicity property of Bernoulli serial lines with deteriorating quality buffer; PR is monotonically decreasing in buffer capacity N_i. An example is shown in Figures 6.9 and 6.10, where the numbers under each buffer corresponds to the T_i, followed by PR of the system obtained by simulations when the buffer capacities N_i is increased by one, and followed by PR of the system obtained by simulations when the residence time constraint T_i of this buffer is increased by one.

Definition 6.5: Buffer b_i, i $\in$ 1 ,..., M-1, is the quality bottleneck buffer (QBN-b) of a Bernoulli line defined by assumptions $(i)-(vi-a)-(viii)$ if

$$\begin{aligned} &PR(p_1,\ldots,p_M,N_1,\ldots,N_{M-1},T_1\ldots,T_i+1,\ldots,T_{M-1}) \\ &\quad > PR(p_1,\ldots,p_m,N_1,\ldots,N_{M-1},T_1,\ldots,T_j+1,\ldots,T_{M-1}), \qquad \forall j \neq i. \end{aligned} \tag{6.7}$$

Similarly, QBN-b is the buffer, which leads to the largest increase in PR if its residence time constraint is increased by 1, as compared with increasing other buffers in the system. An example is shown in Figure 6.9, where the numbers under each buffer corresponds to the T_i, followed by PR of the system obtained by simulations when the buffer capacities N_i is increased by one, and followed by PR of the system obtained by simulations when the residence time constraint T_i of this buffer is increased by one. Note that, the system may have both QBN-b and BN-b represented in one buffer or separate buffers. Finally, another indicator that can be utilized to predict the QBN-b, the machine with the highest SR_i is downstream of the QBN-b.

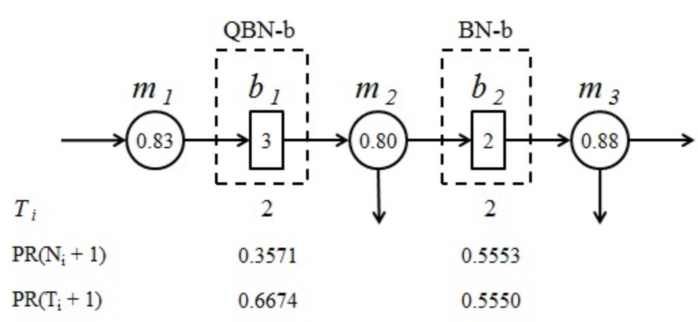


Figure 6.9: Illustration of 3-machine Bernoulli line with DQB

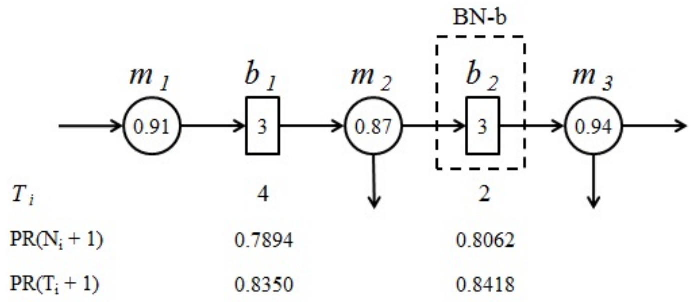


Figure 6.10: Illustration of 3-machine Bernoulli line with DQB

6.4 Summary

- The machine with the smallest p_i is not necessarily the BN-m.
- The buffer with the smallest capacity N_i is not necessarily the BN-b.
- The bottleneck in a Bernoulli serial line with prefect quality buffer can be identified by an arrow assignment method using machine blockages and starvations.
- The bottleneck in a Bernoulli serial line with DQB can be analyzed using simulation.
- The QBN-b can be identified easily by identifying the highest SR_i in the system of all machines m_i following the buffers with deteriorating quality issues.

Chapter 7

CONCLUSION AND FUTURE WORK

7.1 Conclusion

Manufacturing systems with perishable products are widely seen in practice. In this dissertation, system-theoretic properties of production lines are described. Specifically, performance evaluation, monotonicity property, and bottleneck identification.

Introducing the mathematical models of machines and buffers are necessary, in particular, for calculating performance measures of production systems at hand. The performance evaluation in Bernoulli serial lines with deteriorating product quality were introduced using Markovian analysis, closed-form expressions are provided to calculate the performance measures for two-machine lines, and a recursive procedure based on aggregation is developed for longer lines. Based on these techniques, the monotonicity properties of good part production rate, scrap rate, and raw material consumption rate are discussed for Bernoulli serial lines. A case study in an automotive stamping plant is described to illustrate the efficacy of the method developed. For all systems studies in this part, the production rate is always monotonic to all machines and buffers except for lines with deteriorating quality buffers.

It was noticed that the system reacts similarly in both deterministic and stochastic part release. The notable difference between both PRC approaches relies in the application. Deterministic release can be relevant in production lines where a very strict SR is required while knowing shipping schedules. On the contrary, stochastic release best implemented in production lines where shipping schedules are unknown with considerably high demand. Due to the lack of monotonicity in PR with respect to buffer capacity N, the part release control provides a perfect solution to keep the PR significantly high while increasing buffer capacity.

Similarly, the performance evaluation in geometric serial lines with perfect quality buffers were introduced for two-machine lines, and a recursive procedure based on aggregation is developed for longer lines. To verify the accuracy of the aggregation procedures introduced in both lines, a simulation models were introduced to to each system and was extremely accurate with error less than 3.15%. These results provided a more logical way to introduce the geometric serial line with deteriorating quality buffer and study its behavior. Based on these techniques, the monotonicity properties of good part production rate, scrap rate, and raw material consumption rate are discussed for geometric serial lines with deteriorating quality buffers. For all systems studies in this part, the production rate is always monotonic to all machines and buffers.

The bottleneck studied in this dissertation is defined as the machine (or buffer), which has the largest effect on the system performance. For Bernoulli serial lines with perfect quality buffers, an arrow assignment method is described to identify the bottleneck machine and bottleneck buffer. Then, for Bernoulli serial lines with deteriorating quality buffers, bottleneck machine, bottleneck buffer, and quality bottleneck buffer were defined.

7.2 Future Work

Future work in this direction includes:

- Investigation of the structural properties of system performance with respect to machine and buffer parameters to ensure fast and robust search of high quality feedback release controllers in $M > 2$-machine lines;
- Investigation of continuous improvement and lean design in Bernoulli serial lines with quality deterioration;
- Investigation of continuous improvement, bottleneck identification, and lean design in geometric serial lines with quality deterioration;
- Investigation of transient behavior of the production system with quality deterioration in both Bernoulli and geometric serial lines;
- Investigation of the impact of production control rules (Kanban, Basestock and Conwip) on production lines with quality deterioration;
- Investigation of production lines with more than one down state of the machines such loss of usefulness, which includes but not limited to obsolescence, surface degradation and accidents;
- Extension of the results of quality deterioration to production systems with different topologies, e.g., parallel lines, hybrid lines, assembly systems, closed lines, re-entrant lines, lines with rework, etc;
- Extension of the results of quality deterioration to systems with machines having other reliability models such as exponential, Weibull, gamma, log-normal, etc;
- Extension of the results to systems with other quality models and complex deteriorating characteristics;
- Extension of the results to systems with non-perfect quality machines;

- Customization of system to fit real manufacturing facilities, there might be one or several constrains like equipment restrictions, facility layout restrictions, buffer allocation and stations length which essentially differ from plant to plant;
- Applications of the results to real manufacturing systems.

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CURRICULUM VITAE

RAED AHMAD H. NAEBULHARAM

301 W Coventry Court, Apt 310, Glendale, WI 53217
cell: (323)423-6672 e-mail: range78@gmail.com

OBJECTIVE

A full-time position in industrial engineering at one of the top universities with an emphasis on production systems engineering and quality control.

EDUCATION

Doctor of Philosophy, Industrial Engineering
University of Wisconsin, Milwaukee, WI — May 2014
GPA 3.667/4.0

Master of Science, Engineering Management
California State University, Northridge, CA — September 2008
GPA 3.82/4.0

Bachelor of Science, Industrial Engineering
King Abdulaziz University, Jeddah, Saudi Arabia — September 2001
GPA 4.28/5.0

RELATED COURSES

Production Systems Engineering, Product Realization, Statistical Quality Control, Innovation & Commercialization, Simulation, Design of Experiments, and Operations Research.

DISSERTATION TITLE

Production Systems with Deteriorating Product Quality: System-Theoretic Approach

ACADEMIC EXPERIENCE

University of Wisconsin, Milwaukee, WI January 2013 – May 2013
Teaching Assistant - Introductory Statistics for Physical Sciences & Engineering

- Secondary course instructor, covered a class and helped students with understanding course related problems.
- Graded course homework and kept student grade records.

University of Wisconsin, Milwaukee, WI September 2010 – December 2012
Teaching Assistant - Innovation and Commercialization

- Prepared weekly class materials and organized materials online for students easy access and reference.
- Assisted students with their project management and problem solving skills.
- Mentored and provided advice to students during their class project.
- Communicate with guest speakers and facilitate access to them.
- Responsible of overall grade records and reporting them to students.
- Evaluated class activities and report attendance.

University of Wisconsin, Milwaukee, WI September 2009 – May 2010
Teaching Assistant - Product Realization

- Helped students with their project management skills.
- Organized online class materials for students easy access and reference.
- Mentored and provided advice to students during their class project.

King Abdulaziz University, Jeddah, Saudi Arabia September 1998– December 2001
Student Admission Coordinator / Computer Lab Technician - Industrial Engineering Department

- Supervised junior/senior undergrad students on class registration and graduation plans.
- Helped students with job training applications.
- Responsible of computer lab maintenance and software updates.
- Help in the process of developing ORACLE reports and database for the department.
- Helped the department in process of filing for ABET.

PROFESSIONAL EXPERIENCE

Savola Group, Jeddah, Saudi Arabia September 2005– June 2006
Panda & HyperPanda Promotions' General Manager

- Led the marketing launch activities during the opening of new stores.
- Led the marketing team in planning the calendar of activities, and executing the weekly leaflet for Panda and HyperPanda.
- Planned and led the execution of Mega Promotions, responsible for design, mechanics, costing, sponsors, prizes, and raffle.
- Led the development of the identity manuals of the own label and private label packaging designs starting from briefing the advertising agency till launching the product on shelves.
- Led the in-store communication team to ensure proper installation of end gondola promotional items and special vendors in-store activities.
- Reviewed promotions performance and monitored competition activities and their pricing strategy.
- Negotiated annual contract with advertising agencies and saved the company 15% in annual costs of photography by establishing an in-house studio that is responsible of shooting new products and managing image banks.

Savola Group, Jeddah, Saudi Arabia June 2004– August 2005
Panda & HyperPanda Promotions' Manager

- Assisted in the development of brand identity standards and contribute to brand CVP developments, communication strategy developments, and calendar of activities developments.
- Responsible for planning the calendar of activities, and executing the weekly leaflet for Panda and responsible of improving the leaflet creative, develop and ensure smooth promotion process.
- Assisted in Planning 2 Mega Promotion events and responsible for design, mechanics, costing, prizes, and raffle.
- Responsible for in-store screen network and materials advertised in it.
- Led the marketing team in several projects, increasing the produce sales during the summer by 25%, raised the produce image by emphasizing the produce selection process, and public relations campaign. I was awarded a special bonus for achieving the targets of these projects.

Savola Group, Jeddah, Saudi Arabia January 2003– May 2004
Assistant Promotion Manager

- Responsible of executing the weekly leaflet for Panda andHyperPanda.
- Developed the promotional leaflet process.
- Assisted in two Mega Promotion events in the design, prizes, and raffle.

Savola Group, Jeddah, Saudi Arabia January 2002– December 2003
Assistant Project Manager (opening the first HyperPanda in Saudi Arabia)

- Managed meetings with consultants from UK, France, and UAE.
- Assisted in Managing the opening project and reviewing all suppliers' contracts against their yearly performance in the business. I was awarded a special bonus for achieving the targets of these projects.
- Developed standard induction manual for all new comers.

Savola Group, Jeddah, Saudi Arabia October 2001– December 2001
Business Analyst

- Extensive analysis on seasonal promotion products (Ramadan, Eid, Hajj, and Back to School).
- Proposed the key seasonal products with prices and forecast selling quantities.
- Reported the impact of promotion on seasonal products' sales and overall revenue.

COMPUTER SKILLS

Experienced in Microsoft Office, Microsoft Project 2000, SPSS, Matlab, Minitab, ProModel, C/C++, and LATEX.

PUBLICATIONS

- Naebulharam, R. and Zhang, L. (2012). Bernoulli serial lines with deteriorating product quality : Performance evaluation and system-theoretic properties. International Journal of Production Research - under revision (TPRS-2012-IJPR-0727).
- Naebulharam, R. and Zhang, L. (2012) Performance Analysis of Serial Production Lines with Deteriorating Product Quality submitted to the 2013 IFAC Conference on Manufacturing, Modeling, Management, and Control, (MIM 2013).

CONFRENCES

- INFORMATICS RISING, Phoenix, AZ, October 14 - 17, 2012 (chair and speaker of a special session on "Process Engineering" at INFORMS Annual Meeting).

AWARDS

- University of Wisconsin - Milwaukee Chancellor's Graduate Student Awards 2009-2012.
- California State University - Northridge Honor Award 2008.
- Savola Group - Outstanding Performance Team Building Award 2005.
- King Abdulaziz University - Jeddah Honor Award 2001.
- King Abdulaziz University - Jeddah Excellence Award 1998-2001.

TRAINING COURSES

- Six Sigma Green Belt Certified 2007
- Basics of Marketing in Competitive Environment 2005
- Unleashing the Power Within 2005
- Performance Management 2005
- Rethinking Marketing 2005
- Creative Thinking and Problem Solving 2003
- Building a High Performance Team 2003
- The 7 Habits of Highly Effective People 2001
- Setting SMART Objectives 2001
- Big Rocks System 2001

MEMBERSHIPS

- Institute for Operations Research and the Management Sciences (INFORMS).
- Institute of Industrial Engineers (IIE).

LANGUAGES

- Arabic: Native speaker.
- English: Fluent.
- Turkish: Very good command.

INTERESTS

Enjoy reading, research, cooking, and traveling.