

International Journal of Geospatial and Environmental Research

Volume 1
Number 1 *Geospatial Analysis of Urban Environment*

Article 3

June 2014

Assessing Survivability of the Beijing Subway System


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Recommended Citation

Li, Yan and Kim, Hyun (2014) "Assessing Survivability of the Beijing Subway System," *International Journal of Geospatial and Environmental Research*: Vol. 1 : No. 1 , Article 3.

Available at: <https://dc.uwm.edu/ijger/vol1/iss1/3>

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Assessing Survivability of the Beijing Subway System

Abstract

The Beijing subway system, the third largest in the world, serves more than ten million passengers a day. As Beijing is the capital city of China and thus a booming urban center, its subway system has experienced rapid evolution from a local single line system to a complicated network. Due to its constantly increasing complexity, the system is both a critical asset for a local transit artery and a bridge between intercity transportation modes, increasing the issue of network survivability in the face of potential outages of network components. In this study, we provide a connectivity-based survivability measure with which to explore how potential outages of network components might affect the overall functionality of the Beijing subway system. System survivability is measured from two perspectives: [1] topological connectivity under various simulated failures of transfer stations and [2] variations in passenger flow in response to disruptive factors. Plausible scenarios are constructed using local demographic data and daily shipment reports from subway management companies. To assess the possible range of influences, we develop a *weighted rank-based simulation algorithm* to approximate exact solutions to extreme combinatorial outage instances. The range of potential effects highlights the best and worst-case scenarios to identify critical components and help to prepare corresponding contingency plans. This research will enable planners in urban environments, where infrastructure functionality, particularly that of public transit systems, is critical for maintaining socioeconomic security in times of crisis.

Keywords

weighted rank-based simulation algorithm, network, survivability, subway system, Beijing

1. INTRODUCTION

Transportation systems facilitate the movement of people and goods between origins and destinations across a network. However, disruptions can severely hinder the functionality of paths for that movement (Matisziw et al. 2009). The devastating effect of the 2011 floods in Southeast Queensland (Lee et al. 2013), the 2004 Madrid train bombing, and the 2005 London underground bombing reveal the possibility of natural or human-caused disruptions and the vulnerability of transportation systems confronted with such disruptions (CNN Library 2013a, 2013b). Because transportation systems are deeply embedded into society, disruptions to system components can impair the functionality of the entire system, thereby causing large socio-economic costs, especially when the disruptions result from intentional terrorist attacks (Angeloudis and Fisk 2006, Kim 2009).

With the world population soaring in recent decades, public transportation, as a crucial part of the solution to the world's economy, energy, and environment, is gradually attracting more attention. In 2012, 10.5 billion trips on public transportation were made in the United States, and people boarded public transportation 35 million times each weekday (Publictransportation.org 2013), which indicates that the role of public transportation in fulfilling transportation needs is increasing. In particular, the importance of the public transportation system for China is greatly stressed because the burden of public transportation systems is even heavier outside the United States. In 2010, the average number of motor vehicles owned by one thousand people in the United States was 797, while that in China was 58 (World Bank Group 2013). Under these circumstances, public transportation use in China must be expanded to meet the increasingly urgent demand caused by the fast-growing population, especially in some urban areas, including Beijing, Shanghai, and Shenzhen. Because of the growing importance of public transportation and its tremendous cost recovery, management agencies tend to fortify the system beforehand to prevent potential disasters from disrupting the system. For example, the service of the New York subway system is seasonally disrupted by flooding from rainstorms, and the cost for the system maintenance is large, with \$357 million being used to improve 269 pump rooms since 1992 (Donohue 2007). This begs two questions: (1) How can we prevent disruptions from occurring, and what are the critical components in the system? (2) How can we minimize the aftermath of disruptions? The preliminary approach to tackle these questions is to identify the critical network components and draw feasible scenarios to various situations on the system. Survivability is the capability of a system to fulfill its mission in a timely manner in the presence of threats such as attacks or large-scale natural disasters (Ellison et al. 1997, Mohammad et al. 2006). Assessing survivability and exploring the potential scope of disruptive events are critical in network planning and risk management because the resources to respond to an emergency are limited. Considering their socio-economic importance, transportation systems, such as highways and road networks, have been studied using hypothetical or empirical vulnerability analyses (Jenelius et al. 2006, Matisziw et al. 2009, O'Kelly and Kim 2007, Salmeron et al. 2004).

In this research, we propose an *accessibility-based survivability measure* (ASM) as a measure of survivability considering topological and functional aspects of the effects of

disruptions and develop a *weighted rank-based simulation algorithm* (WRSA) to enable the ASM to be applied to a large network. Compared with the previous survivability measures, the ASM is conceptually simple in measuring survivability while providing better dimensionality to evaluate the multifaceted system's survivability rather than a single perspective, and WRSA efficiently reduces the computational burden when exploring possible outcomes using ASM for various disruption scenarios. The remainder of the paper is organized as follows. After reviewing former survivability research and highlighting the remaining challenges in the second section, we detail the case study area and the data used for analysis. In the fourth section, we provide the research framework coupled with the ASM and WRSA. The fifth section presents the research results followed by our conclusions.

2. BACKGROUND

One of the major concerns in survivability research is exploring appropriate measures to evaluate system survivability. However, the concept of survivability itself does not yet have a commonly accepted definition. The meaning of the term depends on the context (Jenelius et al. 2006). The fundamental assumption of network survivability is that a network is able to maintain part of its characteristics when affected by disruptions to network components (Murray et al. 2008). The scope of the affected network components is either a single node/link or a group of nodes/links.

As well introduced by Murray (2013), any network-based system can fail in various ways, and methods to examine its survivability have been developed based upon the type of systems and approaches. There are two types of measures describing the changes of a disrupted network: binary and fuzzy measures. Binary measures, which represent survivability within a range of values or through certain indices to evaluate the given network system, follow all-or-nothing logic with system operation. In contrast, fuzzy measures assume that network components function within a certain level of operation probability. The probability of a network disruption's occurring, the chance of network components' being disrupted by the event, the degree to which the disrupted components are able to maintain parts of their functionality, etc. can be considered to improve the estimation of survivability. Accordingly, "reliability" and "vulnerability" are two commonly used concepts to define survivability, although their definitions vary and are used differently in the context of analysis. For example, Holmgren (2004) defines vulnerability as a collection of properties of an infrastructure system that may weaken or limit its ability to maintain its intended function when exposed to threats and hazards. Salmeron et al. (2004) compare vulnerability to the system's "cushion" against failed, destroyed, or otherwise unavailable system components, while Berdica (2002) focuses on possible catastrophes by stating that vulnerability is the susceptibility to rare, though big risks. Even though the definition varies with a context of research, vulnerability in the transportation network is commonly seen as the complement of reliability (Berdica 2002). According to Husdal (2004), vulnerability studies primarily focus on the impact or consequence of disruptions, and vulnerability is the non-operability of the network under certain circumstances. On the contrary, he states that reliability is an expression of the

probability that a network will function. Thus, reliability may be regarded as the degree of stability of the quality of service that a system offers. In other words, vulnerability represents the extent to which the system loses its original functionality, while reliability measures the remaining functionality (Bagga et al. 1993, Berdica 2002). The relationship between vulnerability and reliability is complicated (Jenelius et al. 2006). Reliability researchers generally prefer fuzzy measures. In theory, applying a fuzzy measure is more realistic than applying a binary measure because the former considers the different possible malfunctions of network components. For example, Kim (2009) defines reliability as a network's capability to deliver flows or the availability of paths between nodal pairs in a network, which depends on the probability that links or nodes will operate. However, the precondition of applying fuzzy measures is that empirical or hypothetical failure probability of a system or network component is known. Otherwise, the setting of the probability form could be arbitrary.

In both the traditional field of transportation and the newly emerging field of network science, many approaches exist for assessing the vulnerability of network-based systems (Matisziw et al. 2009). Four widely used network degradation measures are network connectivity, operational cost, capacity, and system flow, which can be classified into three types (Kim 2012, Murray 2013). Network connectivity concerns an available or functional path exists between origin-destination (O-D) pairs belonging to the first type, topological measures. The purpose of a network is to establish and maintain connectivity between a set of interacting elements to facilitate the movement of valuable goods and services across a system (Grubestic et al. 2008). Therefore, the precondition of assessing the vulnerability of a transportation network is connectivity. Regardless of various forms of transportation systems and disruptions, a network can be viewed as a graph consisting of nodes and links. Thus, many indices from graph theory are applicable. For example, Derrible and Kennedy compare "assortativity," which Newman (2002, 2003) proposed and indicates the similarity of adjacent nodes, with a modified cyclomatic number μ to select a better network "robustness" indicator (Derrible and Kennedy 2010). Another type of measure considers how components in a transportation system are linked. Among these components, operational costs, such as time, distance, and tariffs, involve inhibition between origins and destinations (Jenelius et al. 2006, Nicholson 2003). These components may be increased due to network disruptions, as alternative and more expensive routes may become necessary. System capacity, which refers to the maximum of flow that can move between O-D pairs at any given time, is often used as well (Ratliff et al. 1975, Wood 1993). Finally, system flow refers to the existing levels of interaction between O-D pairs and measures the actual function of the network; therefore, the effect of a potential disruption can also be gauged by the magnitude of the flow affected. However, to estimate the amount of traffic flow as accurately as possible, many assumptive conditions have been made in previous studies, indicating that a gap between theoretical assumptions and reality would exist. For example, the public is not aware of the disruptions (Murray-Tuite and Mahmassani 2004); the public is aware of the disruptions and makes a detour, but the total travel demand is constant (Jenelius et al. 2006); or the public is aware of the disruptions and makes a detour, and, at the same time, the travel demand decreases with time (Nicholson 2003). It is obvious that the public awareness of disruptions is a continuous process and needs to be reflected in vulnerability

assessments. For example, the process of informing the public and the fluctuation of travel demand can be considered in the measure to reduce the gap between models and reality.

Central to the assessment of network disruption and associated survivability to such disruptions is the identification of potentially important disruption scenarios, including the best and worst, which delineate the range of disruptive effect. A scenario in this context refers to a set of nodes and/or links affected by disruption, and the effect on the system functionality can be drawn using any survivability measures. To identify scenarios, two exploratory methods deserve attention: mathematical programming and simulation. Mathematical programming approaches are well known for their ability to provide insight into solution bounds (minima/maxima) for a wide range of spatial planning problems such that administrators and managers are more capable of reducing a network's vulnerability to these events (Matisziw et al. 2009; Salmeron et al. 2004). However, constructing the full picture of all potential scenarios to all potential disruption events with exact solutions is limited to a tractable instance. Exploring possible scenarios across a range of disruption levels through simulation approaches can reveal more detail. Unlike mathematical programming, simulation approaches are more flexible in adapting other measures and considering more aspects of network characteristics simultaneously. Note that both binary and fuzzy measures can be used in the simulation approach. For example, Jenelius et al. (2006) remove individual nodes from a highway network and measure the change of O-D flow cost in each scenario. In a recent study, Kim (2009) computes the range of remaining functionality of subway systems when faced with disruption to hub nodes in combinatorial disruption scenarios. Of concern is that although the simulation of the relatively simplistic case where a single network component is impacted is logically simple when the enumeration of all the scenarios is tractable, special algorithms must generally be developed to address large amounts of computation for the simulation of complex scenarios involving multiple network components (Kim 2009). In simulation approaches, therefore, the goal is to evaluate a suitable number of scenarios to obtain an effective characterization of the range of possible effects.

More complex measures to model system loss can be applied to draw the effects of scenarios by removing the time-consuming enumeration of all potential combinatorial scenarios. For example, Murray-Tuite and Mahmassani (2004) apply the travel time cost dynamically determined by the volume through certain paths to address the effect of traffic congestion. When large-scale networks are considered, a mathematical method assumes that the travel time cost is monotonically increasing and differentiable everywhere; accordingly, a simple cost structure is applied in estimating the effect of disruptions (Erath et al. 2009, Luathep et al. 2011).

In this research, we design the ASM to be applied to a relatively large network through a simple process of computation to explore all potential disruption scenarios and use the WRSA to complete the scenarios efficiently. The critical stations are identified through the worst and best scenarios, and the impact of disruptions is defined as the change of the network functionality between the status quo and the condition where that network component stops working (Erath et al. 2009, Luathep et al. 2011). Given the case of the Beijing subway system, we make two assumptions. First, for our scenarios, we assume that common subway system disruptions, such as potential terrorist attacks and

point source air pollution, randomly occur. Thus, the combination of hubs, and thus the set of disrupted stations, is random as well. Second, in our disruptive scenarios, travel demand reaches a new balance rapidly because modern operation and broadcast systems in the Beijing subway system inform passengers of the status of the whole system's functionality rapidly to allow passengers to instantly modify travel decisions and prevent the network from chain and cumulative impacts.

3. DATA

3.1 BEIJING SUBWAY SYSTEM

The Beijing subway system is the rapid transit rail network that serves not only the urban but also the suburban districts of Beijing, one of the largest cities in the world, with more than 20 million residents as of 2012 (BMBS and NSOB 2013a). Beijing is also a major hub for the national highway, expressway, railway, and high-speed rail networks (BMBS and NSOB 2013b). Note that due to its inexpensive, timely, and reliable service and operation, this subway system has become the first choice among transportation modes in Beijing. Its annual ridership ranks third in the world, with 2.46 billion trips in 2012 (BMCT 2013, Liu 2013). As shown in Figure 1, the current network has grown to 17 lines, 227 stations, and 465 km of track in operation since 1969, making it the third longest subway system in the world (Zhu 2013). However, before Beijing won the bid to host the 2008 Summer Olympics, the Beijing subway system only had its first two lines built in the 1970s. The last decade has been a transition period for the Beijing subway system, as its operation mode has changed from single line operation to network operation and experienced the largest effective growth in the world cities (Niedzielski and Malecki 2012, Wang et al. 2012). However, this growth has revealed that the system may suffer from malfunctions and disruptions occurring in other cities' systems, such as the malfunction of the subway system in Shanghai on March 27, 2013 and the accidental crash on September 28, 2011.

Subway stations are often classified into two types based on their roles in the network: non-transfer and transfer stations. Note that unlike a non-transfer station, a transfer station connects different lines to collect traffic flows and to reallocate them, resulting in more frequent use and a heavy concentration of passenger flow. In theory, selected multiple disruptions of transfer stations may easily disconnect the system to an extent, while a single and random disruption rarely causes a critical operational situation (Kim 2009). Shown in Figure 1 with red points are the *hubs* in the Beijing subway system, which are the stations whose degrees are greater than two. The hubs are a special subset of the transfer stations in the system because some of the transfer stations only connect the terminus of two subway lines, and their degrees are equal to two. These transfer terminuses do not serve to collect traffic flow and redirect it, which makes their function similar to the non-transfer stations. Therefore, these special cases are excluded from the candidate set in the analysis. Under this definition, in the Beijing subway system, 34 hubs are identified and are used as a candidate set for disruptions in our analysis.

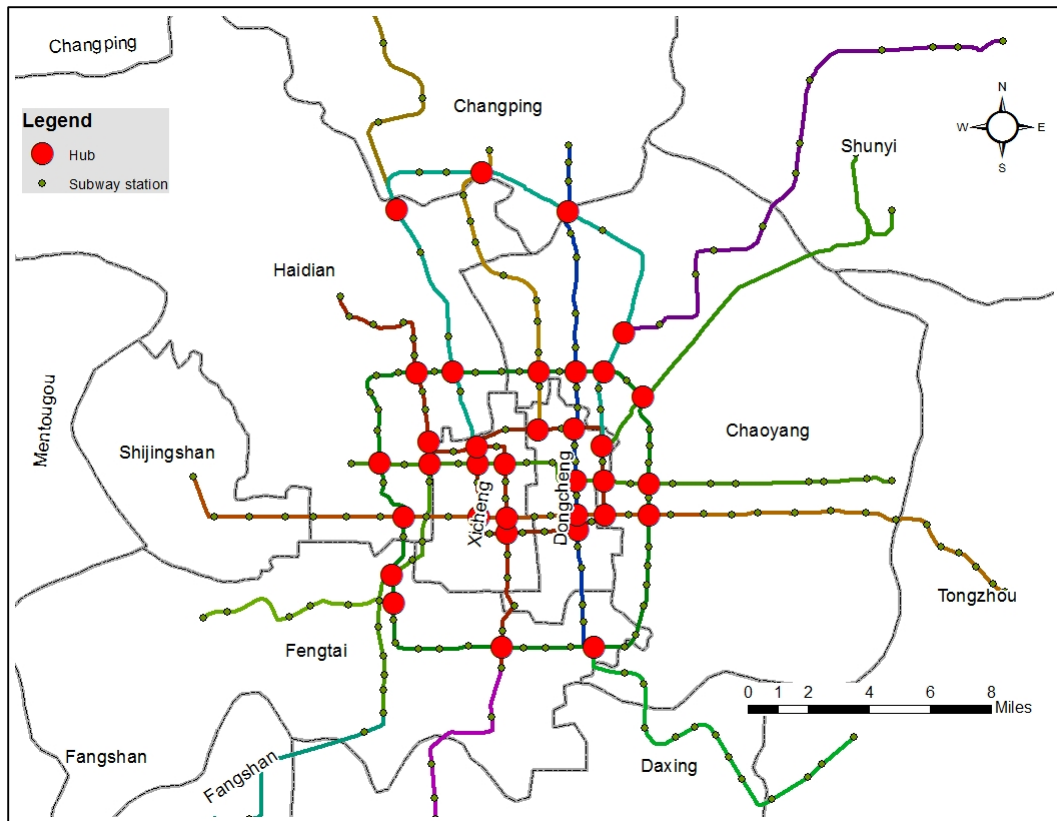


Figure 1. Beijing subway system.

3.2 ESTIMATING PASSENGER FLOW

In our analysis, to measure survivability, we estimate station passenger flow using population distribution. A challenge for the estimation is how to delineate the served area of the subway system, which is defined based on walking distance (Browning et al. 2006, Mohler et al. 2007). Different values have been applied in previous research. Shaw (1991) considers 2,000ft as a reasonable distance to estimate the maximal walking distance for the Miami Tri-Rail system. Kuby et al. (2004) select a round distance of one-half mile for walking distance. Considering the land use in Beijing, we assume that places within a one-hour walk (5 km) are the subway system's served area. As illustrated in Figure 2, we generate a five-kilometer buffer zone. Thiessen polygons are applied to divide the served area into separate zones by station. Then we estimate the potential passengers of each station within its serving area using the Sixth National Population Census taken on November 2011.

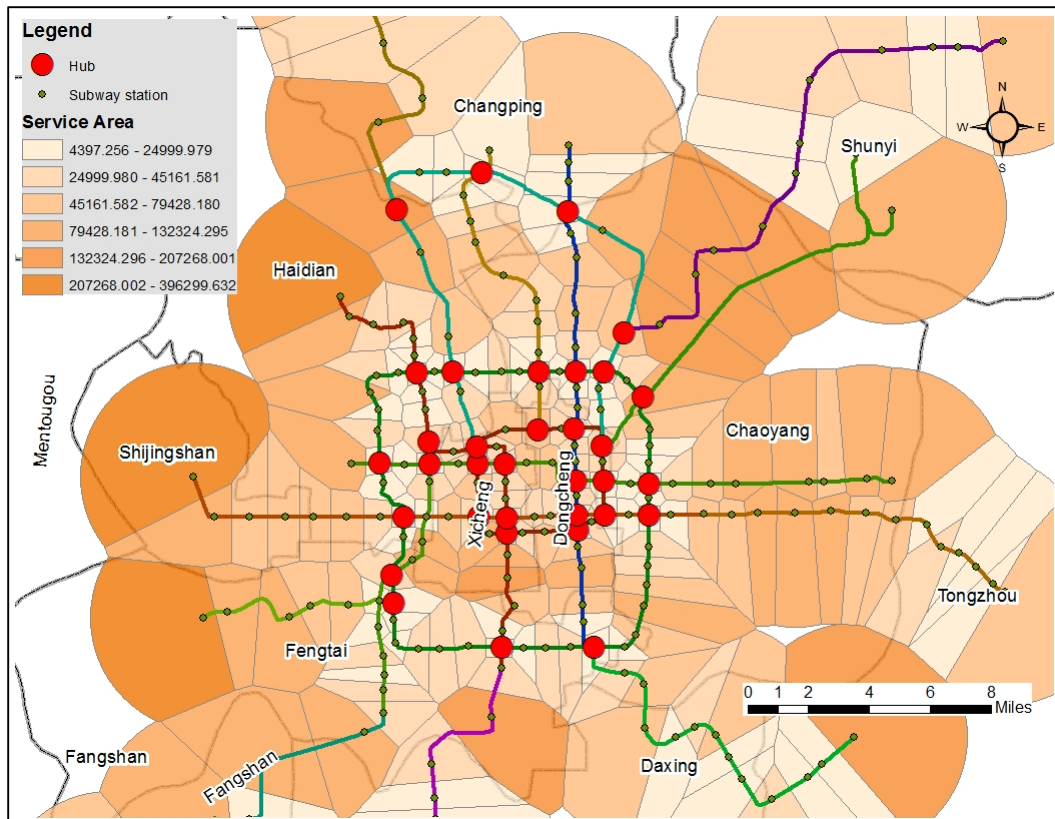


Figure 2. Population served by the Beijing subway system.
(Note: 5 km buffer applied and delineated based on Thiessen polygons).

The estimated passenger flow should be adjusted based on the days of the week, considering that daily ridership and demand fluctuate. For example, the number of passengers using the subway system on Friday is 3 million more than that on Sunday (Beijing Subway 2013). To reflect this fact, we adjust the estimated flows based on the empirical daily ridership of each subway line for two weeks (5/20/2013-6/2/2013). The estimated passenger flow of a station is adjusted by multiplying the ratio between the actual daily ridership and the estimated passenger flow along the line at status quo. Because the estimated passenger flow along a line at status quo is constant, we are able to observe the fluctuation of the actual ridership. In this paper, we select the actual ridership of Friday and Sunday as typical for weekdays and weekend days, respectively. As illustrated in Figure 3, the estimated volume of passenger flows on lines such as Lines 10, 14, 2 and 5 differs greatly on Sunday and Friday, while the volume on other lines remains similar throughout the week. Thus, we explore the survivability results for the Beijing subway system for weekdays and weekends.

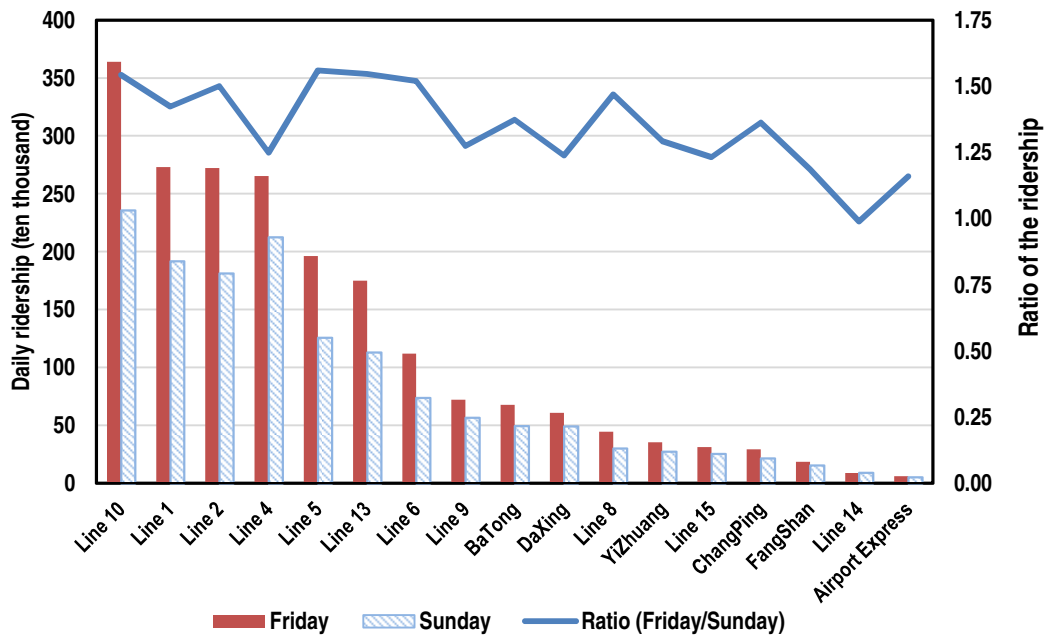


Figure 3. Daily ridership of subway lines.

4. METHODOLOGY

In this paper, we define the term *survivability* as the ability of a network system to maintain its topological and functional state when a certain level of disruptions on stations occurs simultaneously. Note that disruptions affecting the same number of stations are considered as being on the same disruption level. For example, the m^{th} disruption level represents the situations that the number of disrupted stations is m out of the 34 hub stations so that the number of possible scenarios at that level is enumerated by ${}_{34}C_m$. To explore the potential effect of combinatorial disruptions at a set of stations, we consider disruption scenarios at all levels. The ASM examines the network survivability of the Beijing subway system from no disruption (status quo) to a total of 34-hub disruptions from two perspectives: [1] system topological loss (T_m^{loss}) and [2] system flow loss (F_m^{loss}).

4.1 SYSTEM TOPOLOGICAL LOSS

As introduced from graph theory, a connectivity matrix (hereafter *C*-Matrix) is used to represent the topological connectivity of a network system (G). Let us define the 1st level *C*-Matrix (C^1) to represent the adjacent connection matrix of a network. If a node is connected to another node, then we define its connectivity as 1 (otherwise 0) in C^1 . Then,

the k^{th} level C-Matrix (C^k) is the k -step connection matrix of a network, which results from $C^{k-1} \times C^1$ ($k \geq 2$), $C^{k-1} = C^{k-2} \times C^1$ ($k \geq 3$), and so forth. Each entry (c_{ij}^k) in C^k represents the number of possible paths with length k between nodal pairs (nodes i and j). Then, the sum of the entries in all the connectivity matrices C^k indicates the topological connection of a network ($k = [1, d]$, $d =$ the diameter of the network), where all the possible paths with k steps ($k = [1, d]$) are recorded. Note that as k increases, the number of paths that visit nodes repeatedly increase as well. To treat this problem, a constant $\alpha = 0.5$ is employed to mitigate the attenuation effect (Taaffe et al. 1996). The α is applied as α^k , which imposes more weight on shorter k -step paths than longer steps paths, which can have meaningless round trips. We define this measure as a system's total connectivity (T^{sys}), as expressed below:

$$T^{\text{sys}} = \sum_{k=1}^d \sum_{i=1}^n \sum_{j=1}^n \alpha^k c_{ij}^k \quad (1)$$

where

α : the constant to mitigate the attenuation;

d : the diameter of the network G ;

n : the number nodes in the network G ;

c_{ij}^k : the entry at the i^{th} column and the j^{th} row in C^k ($i = 1$ to n , $j = 1$ to n).

Larger values of T^{sys} indicate a more complex and more highly connected network. Disruptions occurring at subway stations affect the topological relationship in the network by removing all links to the disrupted station, resulting in a decrease in the total connectivity of the network, T^{sys} . The system topological loss to the number of m nodal disruptions (T_m^{loss}) is calculated by the disparity between the total connectivity index in this scenario (T_m^{sys}) and the original T^{sys} at status quo (T_0^{sys}):

$$T_m^{\text{loss}} = T_0^{\text{sys}} - T_m^{\text{sys}} \quad (2)$$

A larger T_m^{loss} value represents a less survivable subway system. By comparing all T_m^{loss} values at the m^{th} disruption level, the critical scenarios when m stations are disrupted are revealed.

4.2 SYSTEM FLOW LOSS

The second measure is to depict system flow loss ($F_{m,\text{day}}^{\text{loss}}$). Since T^{sys} may include the attenuation effect, we consider the shortest distance but exclude round-trips among nodes to examine system flow loss in the disruption of nodes. We assume that the original passenger flow is redirected to alternative subway paths rapidly so that the passengers do not experience any significant delay due to the disruption of stations, and the system flow therefore reaches a new balance, although the volume of passengers decreases because of the increased travel distance for rerouting. This practice is well applied in the Beijing subway system because its broadcast system informs passengers in real time if any

operational problems occur. In this paper, we assume that the degree of flow loss is assessed based on the logic of spatial interaction, which means the total number of passengers between two stations will decrease with increased travel distance due to the disrupted stations. The shortest distances among the nodes are calculated simultaneously when applying the bookkeeping process of the C-Matrix series to reduce extra computational burden. To estimate the loss of passenger flows considering the m^{th} disruption scenario and the difference in ridership between weekdays and weekends, we use four steps. First, suppose that there are two stations with potential flows P_i and P_j on the network and that their distance from one another is (d_{ij}); then, the volume of passenger flows V_{ij} is calculated using

$$V_{ij} = \frac{(P_i \times P_j)}{d_{ij}} \quad (3)$$

Then, the total volume of the passenger flow at station i (V^S_i) is calculated by summing V_{ij} from station i to all other stations:

$$V^S_i = \sum_{j \in S} V_{ij} \quad (4)$$

where

S : the set of all stations in the Beijing subway system except station i itself.

Second, the passenger flow to a subway line a (V^L_a) is calculated by summing V^S_i of the stations along this line:

$$V^L_a = \sum_{i \in A} V^S_i \quad (5)$$

where

V^S_i : the estimated volume of passenger flow of station i ;

a : the index of the subway line in the Beijing subway system;

A : the set of subway stations on the subway line a .

Third, to reflect the different volume of ridership between weekdays (Friday) and weekends (Sunday), V^S_i should be adjusted by the ratio between the actual ridership ($r_{a,day}$) and the estimated passenger flows (V^L_a) at *status quo* for each line a . For the m^{th} disruption scenarios, the adjusted passenger flows at station i (noted $V'_{m,day,i}$) are calculated using

$$V'_{m,day,i} = V^S_{m,i} \times \sum_{a \in U^L} \frac{r_{a,day}}{V^L_{a,0}} \quad (6)$$

where

m : the index of the disruption level;

$r_{a,day}$: actual ridership along subway line a on a certain day (Friday or Sunday);

V^L_a : the estimated volume of the passenger flow along subway line a under the status quo;

$V^S_{m,i}$: the estimated volume of the passenger flow of station i at the m^{th} disruption level;

U^L : the set of subway lines passing station i because more than one line passes transfer stations.

The fourth step is to sum the adjusted volume of passenger flow at all the stations to calculate the total volume of passenger flow in the whole system for the m^{th} disruption scenario ($F_{m,\text{day}}$):

$$F_{m,\text{day}} = \sum_{i \in S} V'_{m,\text{day},i} \quad (7)$$

The final step is to calculate the system flow loss ($F_{m,\text{day}}^{\text{loss}}$) by finding the difference in the estimated passenger flows between F_{day} at status quo and $F_{m,\text{day}}$ for the m^{th} disruption scenario.

4.3 METHOD OF EXPLORING NETWORK RESILIENCE

When the number of disruptive stations is fixed, the extent to which a network system will degenerate varies with the disruptive stations' topological importance and the scale of passengers served. Now that both topological attributes and station service functions differ, we can draw a range delineated by the best and worst scenarios for any given number of disruptive stations. Therefore, one efficient way to identify a group of critical stations and examine the resilience of the network system is to draw such a range. Prior attempts have been made to characterize simulation results using the concept of an *envelope* (Kim 2009).

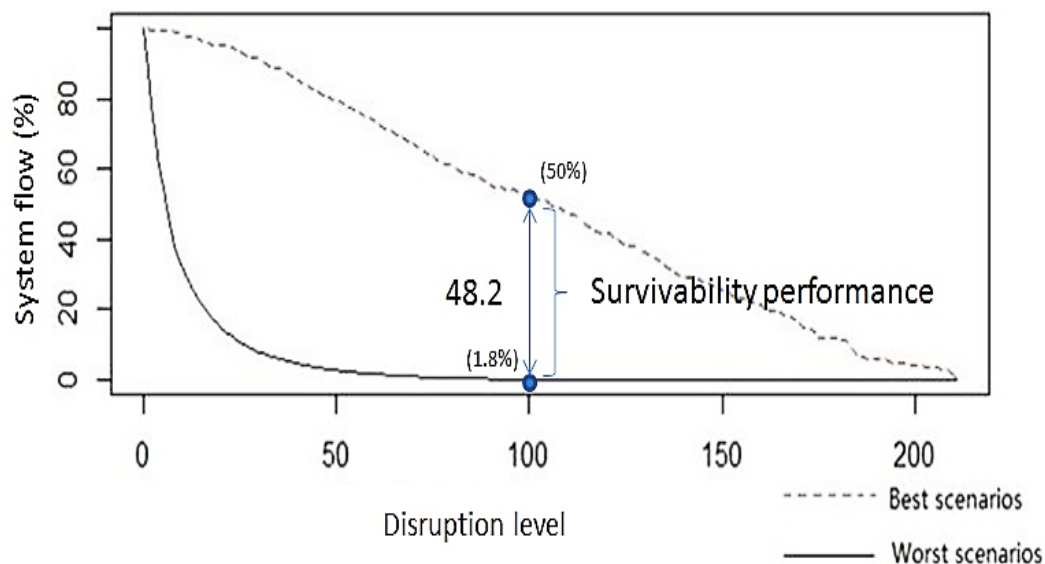


Figure 4. Survivability envelope.

As illustrated in Figure 4, named *survivability envelope*, the y-axis represents the level of network survivability, while the x-axis represents the m^{th} disruption levels. The range

of disruption impact is drawn from the status quo (0^{th} level) to the level where all transfer stations are disrupted (n^{th} level). At each level, the gap between the best and worst scenarios (minimum and maximum negative impacts, respectively) is the *survivability performance*. For example, if the best and worst scenarios are 50% and 1.8% at the 100^{th} level, then the survivability performance is 48.2%. The range will be narrowed with the levels.

A significant challenge in drawing the envelopes is the complexity of the computation needed to complete the simulation, which depends heavily on the network size and disruption levels. Although we only consider 34 hubs when generating scenarios, doing so is computationally burdensome for a wide range of disruption scenarios, and simulating scenarios between the 5^{th} and 29^{th} levels is not well applicable. For instance, the computation of the 1^{st} level disruption for the enumerated 34 scenarios (${}_{34}C_1 = 34$) takes 85 seconds using the Windows 7 32-bit OS platform with Intel® Core™ i5-2.60GHz. However, simulating all the scenarios at the 5^{th} level (${}_{34}C_5 = 278,256$) requires more than 8 days. The computational complexity significantly increases until it reaches the 17^{th} level, where an explicit numeration would take 184 years. After that, the computation complexity decreases with the number of disrupted stations until the 29^{th} level, at which the time needed is the same as that at the 5^{th} level. To tackle this problem, the algorithm, named WRSA, is developed to traverse the potential disruption scenarios. The WRSA consists of two procedures. At lower complexity levels (i.e., 0 to 4 and 30 to 34), all the possible scenarios are enumerated and the best and worst scenarios are identified. However, for other levels, the algorithm searches the scope of the best and worst scenarios by constructing a set of candidate nodes as a subset of all hubs to the disruption levels. The selection of candidate nodes is made using the global rank index ($GRI_{m,i}$) of the hubs at each level. The steps to calculate $GRI_{m,i}$ ($1 \leq m \leq 34$) are as follows. Suppose that the best and worst scenarios are identified at the m^{th} level. The algorithm sorts all scenarios at the level from the best (most survivable) to the worst (least survivable) case in terms of the criteria, T_m^{loss} and $F_{m,\text{day}}^{\text{loss}}$. The next step is to construct the most and least critical hub sets based on the best and worst scenarios from both criteria. WRSA evaluates the criticality of the hubs according to the degree of influence at each scenario. In other words, each hub has two different ranks, named $rt_{m,i}$ and $rf_{m,i}$, corresponding to the criteria T_m^{loss} and F_m^{loss} at the m^{th} level, and the algorithm constructs local rank index ($LRI_{m,i}$) which evaluates the criticality of the hubs based on $rt_{m,i}$ and $rf_{m,i}$ using formula (8). The $LRI_{m,i}$ is used to calculate the $GRI_{m,i}$ for each hub i at the m^{th} level using formulas (8) and (9) together.

$$LRI_{m,i} = w \times rt_{m,i} + (1 - w) \times rf_{m,i} \quad (0 \leq w \leq 1, m \geq 1) \quad (8)$$

$$\begin{cases} GRI_{1,i} = LRI_{1,i} \\ GRI_{m,i} = LRI_{m,i} + 0.5 \times GRI_{m-1,i} \quad (m \geq 2) \end{cases} \quad (9)$$

where

w : the weight to calibrate the importance of the two survivability measures.

In this research, we use $w = 0.5$ is applied to treat both measures being equal.

As shown in Figure 5, the algorithm includes all hubs in the candidate set for the 1st to the 4th levels and 30th to the 34th levels, which are applicable for explicit enumeration. For the other levels, WRSA generates a disruption candidate set for the $(m+1)^{th}$ level according to $GRI_{m,i}$. Specifically, WRSA requires four steps. The first step is explicit enumeration of the potential combinatorial disruption scenarios for candidates at this level. T_m^{loss} and $F_{m,day}^{loss}$ is calculated for each scenario. In the second step, WRSA calculates the criticality for the selected stations using $LRI_{m,i}$ and $GRI_{m,i}$. In the third step, WRSA sorts the stations based on $GRI_{m,i}$ from the most critical to the least critical stations. These candidate sets are used to complete the scenario. The final step selects n top stations and n bottom ones in the set of candidate stations for the next level scenario. When m equals the number of the hubs in the system, the algorithm terminates with all potential disruption levels explored.

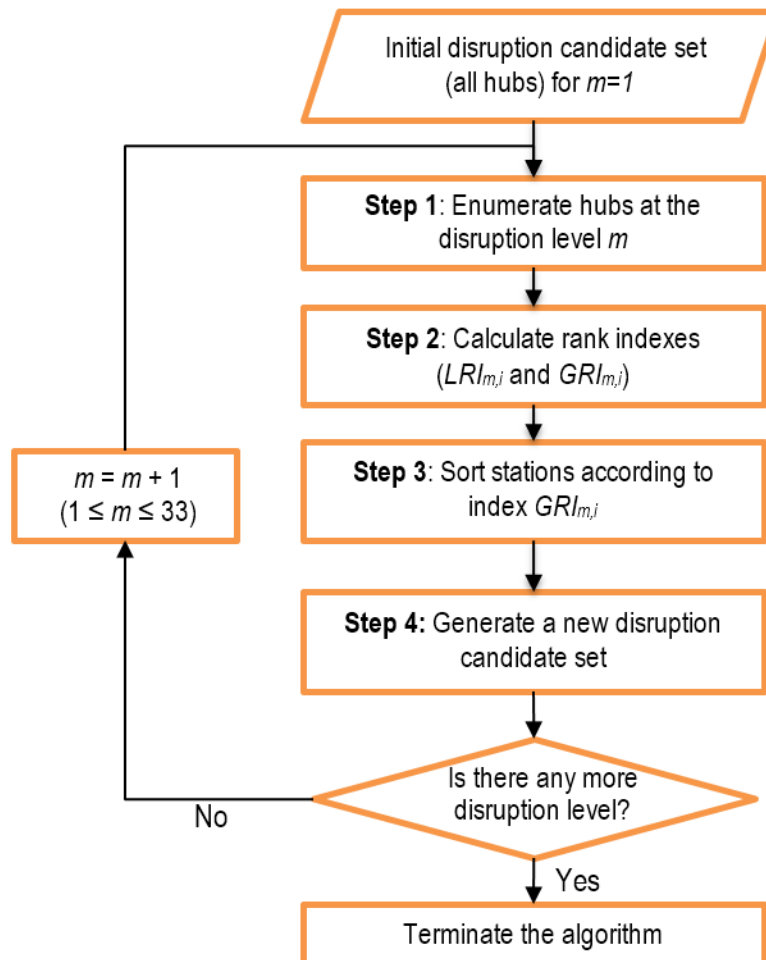


Figure 5. Weighted rank-based simulation algorithm.

5. RESULTS

By applying ASM, the enumeration of potential disruption scenarios becomes applicable. Following ASM and the logic that the potential worst or best scenarios possibly consist of stations in the extreme scenarios for other levels, the number of potential scenarios in each level is limited to 20,000 to accelerate the computation and to make the more in-depth exploration of combinatorial disruption scenarios possible. Two items are worth noting based on the completed scenarios.

5.1 STATION CRITICALITY

Our first analysis focuses on evaluating network survivability at an individual hub in terms of ASM. Based on the computation, the criticality of the hubs is ranked according to their T_1^{loss} , $F_{1, Friday}^{loss}$ and $F_{1, Sunday}^{loss}$. A higher rank indicates the hub is more critical. Table 1 summarizes the ranks of the top ten critical hubs and the values of their survivability measures. In terms of T_1^{loss} , Xizhimen, Dongzhimen, and Chegongzhuang are worthy of note. Both of Xizhimen and Dongzhimen are the hubs connecting Line 2 and Line 13. Line 2 is the only loop line serving the city center, which is the traditional “inner city” while Line 13 is a special line serving only the northern part of the city and intersecting with four branch lines in the north. In particular, the degree of node of Xizhimen is five, which is the largest in the system, followed by Dongzhimen, whose degree is four. Chegongzhuang is the hub next to Xizhimen on Line 2.

Table 1. The survivability results of the first level disruptions

Rank	T_1^{loss}		$F_{1, Friday}^{loss}$		$F_{1, Sunday}^{loss}$	
	Hubs	Value	Hubs	Value (x1000)	Hubs	Value (x1000)
1	Xizhimen	143.76	Gongzhufen	1052.14	Gongzhufen	737.21
2	Dongzhimen	117.57	Guomao	922.18	Guomao	644.54
3	Chegongzhuang	100.98	Wangjingxi	815.70	Wangjingxi	569.54
4	Gongzhufen	93.31	Xierqi	772.61	Xierqi	533.20
5	Chaoyangmen	89.13	Jiaomenxi	709.50	Liuliqiao	526.58
6	Songjiazhuang	87.49	Liuliqiao	708.38	Jiaomenxi	516.10
7	Jianguomen	86.81	Haidian-huangzhuang	667.74	Hujialou	498.14
8	Yonghegong	85.30	Hujialou	655.62	Haidian-huangzhuang	445.91
9	Dongdan	83.20	Songjiazhuang	602.77	Songjiazhuang	425.20
10	Shaoyaoju	81.32	Xizhimen	580.40	Xizhimen	406.48

However, the results based on $F_{1, Friday}^{loss}$ and $F_{1, Sunday}^{loss}$ produced a different ranking of hub criticality. For example, Gongzhufen, Guomao, Wangjingxi, and Xierqi are not highly ranked in T_1^{loss} , but they are in $F_{1, Friday}^{loss}$ and $F_{1, Sunday}^{loss}$. For seven hubs, disruptions will cause more than 6% system flow loss, while the worst connectivity loss is less than 6%. Compared with T_1^{loss} , which weighs all the stations the same, $F_{1, Friday}^{loss}$ and $F_{1, Sunday}^{loss}$ reveal more serious potential influences of one-hub disruption.

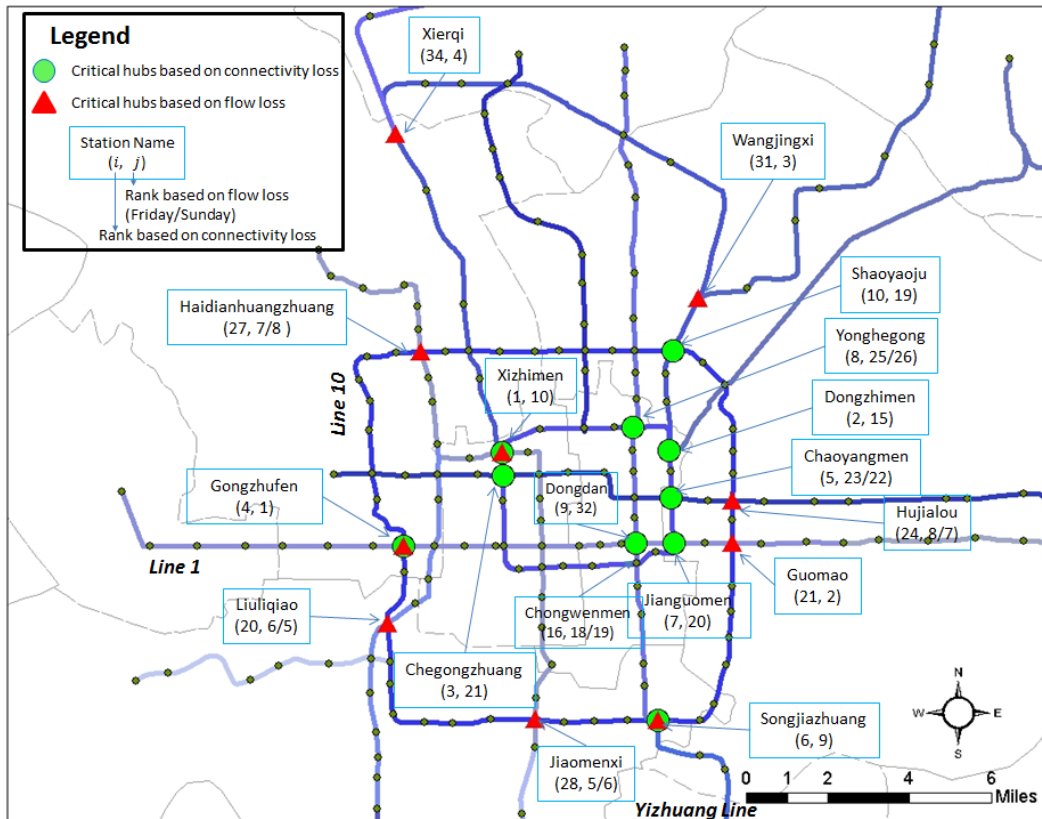


Figure 6. The locations of the top 10 critical hubs based on topological loss and flow loss.

Second, notice that all the critical hubs based on $F_{1, Friday}^{loss}$ and $F_{1, Sunday}^{loss}$, except Xizhimen, are *bridges*. The bridge here is defined as a station with the degree of node larger than two and acting as the only connection for a subway line to the rest of the system. This fact is very significant in terms of network survivability because excluding one of the bridges can result in losing the entire passenger flow from or to an entire branch line. For example, the disruption of Gongzhufen disconnects Line 1 from Line 10, causing the west part of Line 1 to be completely excluded from the system and decreasing the estimated passenger flow by nearly 11% on both Friday and Sunday.

Third, in our analysis, 11 out of 34 (33%) hubs would cause a system flow loss of more than 5% and an average system flow loss of 4.2% on both days for one-hub

disruption level. In other words, given the system’s daily ridership, the expected number of affected passengers confronted with a one-hub disruption is nearly 427,000 on Friday and 300,000 on Sunday, demonstrating that even a minor disruption in a large subway system such as Beijing’s can inconvenience a large number of customers. Finally, the members of the top 10 critical hubs indicated by $F_{1, Friday}^{loss}$ and $F_{1, Sunday}^{loss}$ are identical, although there are some changes in the lower rankings among them. The fluctuation of the passenger flow across a week does not affect the criticality for the top five hubs. However, the changes in the rank of the remainder of the stations also imply that some stations play more important roles on weekends or vice versa.

5.2 NETWORK RESILIENCE

In addition to the station criticality displayed when facing single station disruptions, network resilience for multiple station disruptions is also important. According to our scenarios, multiple station disruptions would decrease the system’s functionality dramatically depending on what particular set of hubs stations are disrupted. Figure 7 shows the survivability envelope for system connectivity and disruption levels. The bar represents the survivability performance of system connectivity. From the 8th to the 26th level, the survivability performance is almost constant and less than 20%. The decreasing rates of system connectivity in the best and the worst scenarios are similar as well. When all 34 hubs are disrupted, 30% system connectivity remains.

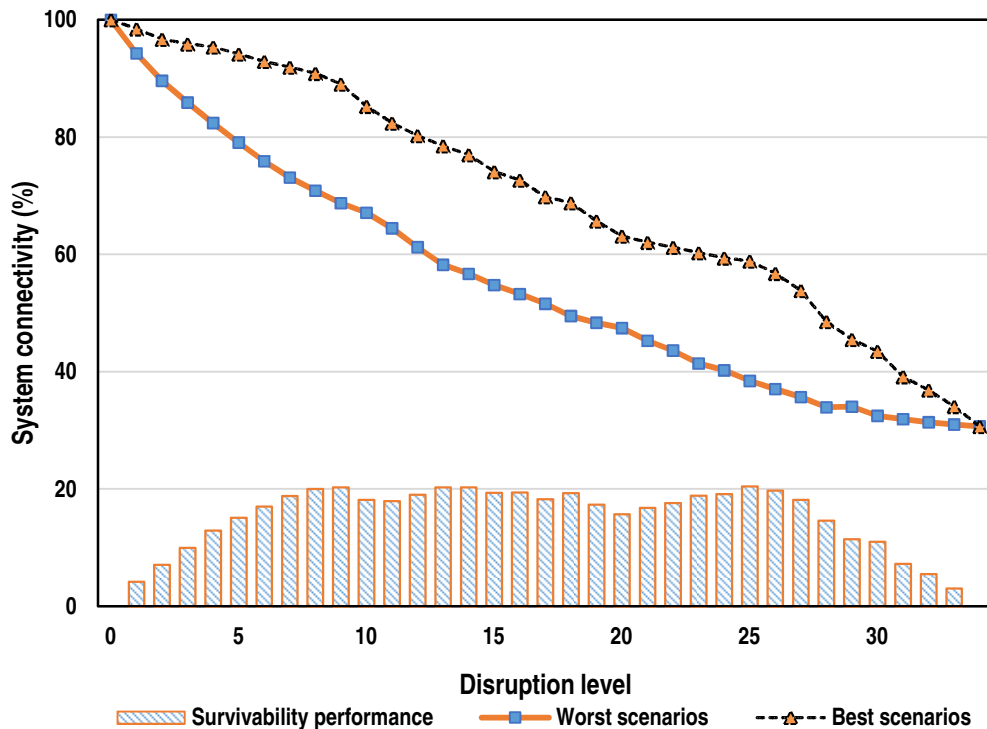


Figure 7. Survivability envelopes of system connectivity.

Recall that system connectivity reflects only a network's topological characteristics, so this measure is less able to reveal potential survivability issues hidden by other geographic factors. Figure 8 shows the survivability envelope of the normalized estimated system flow (%) and the disruption levels for Friday and Sunday. Compared with the survivability envelope of system connectivity in Figure 7, the envelopes in Figure 8 clearly show that the system flow is more sensitive to disruptions than is system connectivity. In terms of the worst scenarios drawn in solid lines, one additional disrupted station would cost a 7-10% system flow loss on average, and the system only maintains half of its original flows until six stations are disrupted. However, in terms of best scenarios, the system can maintain half of its passenger flows until 17 station disruptions occur. Notice that the survivability performance, the gap between the best and worst scenarios, ranges from a few percent to as high as 46% at the 8th disruption level. As the disruption level increases, the gap first increases until it reaches its peak at the 8th level and then decreases slowly. In other words, the largest gap, indicating degree of consequences, between a targeted attack and that of a random disruption is expected at the 8th level. This wide gap also clearly indicates that the subway system is more vulnerable to targeted disruptions than random accidents. Comparing the survivability envelopes of Friday and Sunday, the difference in survivability is minimal by the 15th disruption level, which is consistent with our finding in the stations' criticality that the fluctuation of system flow is not strong enough to affect the criticality of some stations. As disruptions affect more stations, the remaining system flow on Friday is less than that on Sunday in both the best and worst scenarios. Finally, when all the hubs are disrupted, the remainder of the system flow is nearly 10% of the original, which means the 34 hubs determine the main functionality of the system with 227 stations.

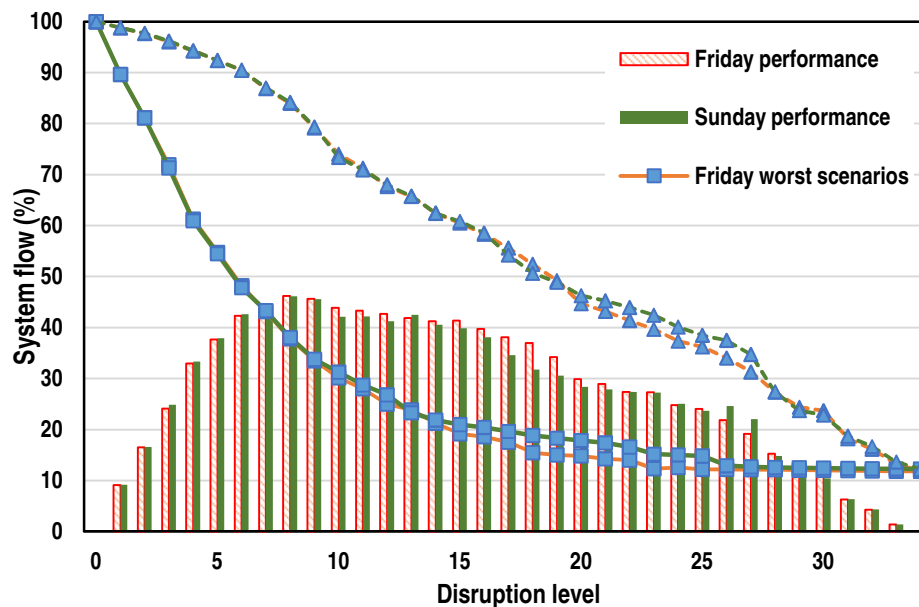


Figure 8. Survivability envelopes of system flow for Weekdays (Friday) and Weekends (Sunday).

Now that the effect of system flow fluctuation is clear after a disruption involving more than 15 stations, we take the worst scenarios at the 20th level as an example to explore how they affect the system from a geographic perspective. Figure 9 shows the worst scenarios and marks the locations of the stations on Friday (triangles) and Sunday (squares). There are five stations that are different between the two days, and these stations are labeled with frames. If a city axis is drawn from north to south, the six different stations on Friday are all located in the east part of the system, while there are two stations on Sunday, Cishousi and Jiaomenxi, located on the southwest corner of the system with no match on Friday. Consistent with the finding in the previous section, five out of the ten different stations are bridges, namely Lishuiqiao, Hujialou, Jiaomenxi, Cishousi, and Xierqi, and these stations have a large effect on the system's survivability. To highlight the system flow fluctuation, the width of the branch lines in Figure 9 represents the ratio between the flow on Friday and that on Sunday. The ratios of the Line 5, Line 6, Line 1, BaTong, and Changping Lines are obviously much larger than those of other lines. Thus, on Friday, the percentage of passengers using the branch lines in the east part of the system is larger than that on Sunday. In other words, the difference between the ridership of the branch lines in the east part and that of the southwest corner is more apparent on Friday than on Sunday. The change in relative weight affects the location of the disrupted bridges and thus the hubs in the center region.

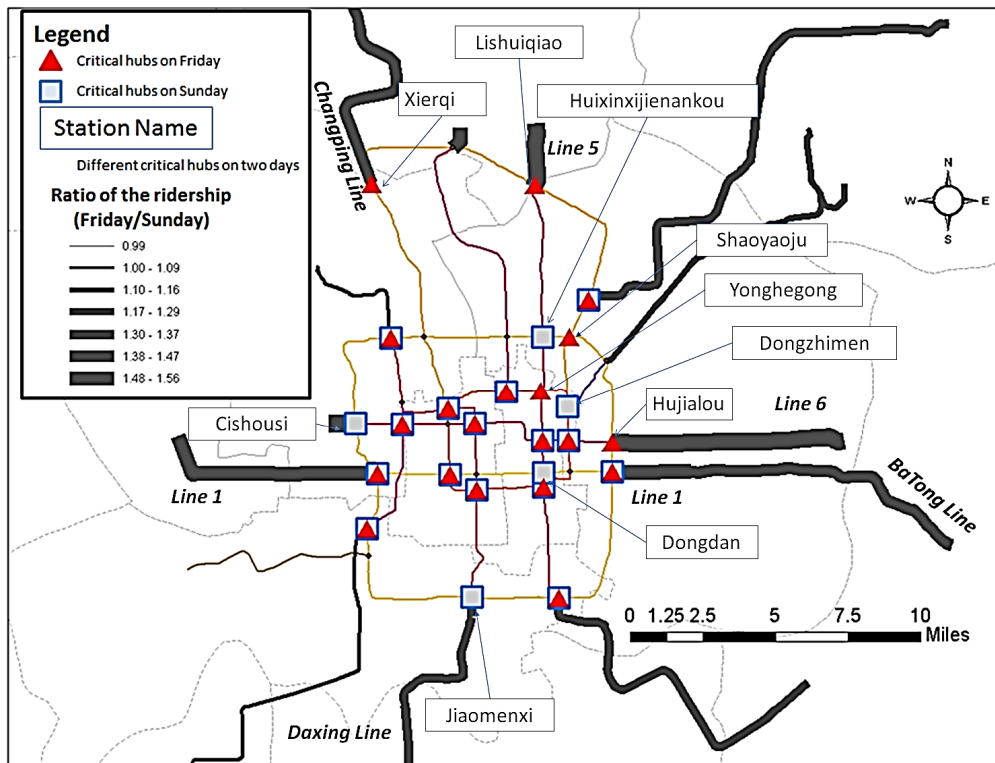


Figure 9. The worst scenario where 20 stations are disrupted.

6. CONCLUSIONS

The Beijing subway system serves as a crucial means of public transportation for a mass of people. Its growing importance, as well as its geographic and functional features, requires specifically designed research on its survivability to allow for better network protection.

Differently to previous network survivability research, we develop and test an accessibility-based measure. This measure considers both topological and functional aspects of the consequence of disruptions, which capture the differences in consequence from two perspectives. This study also highlights several important findings. First, when facing disruptions at a single station, the Beijing subway system shows strong survivability. Only 33% of one-hub disruptions will cost more than an estimated 5% passenger loss for the entire system. Second, system performance in the face of multiple disrupted stations varies with the disruption level and is highly dependent on the combination of hub stations. This finding is consistent with the results of previous research. The protection of hubs acting as gateways for branch lines to the rest of the network is extremely important, indicating that providing more belt lines or alternative lines would improve the system's ability to maintain survivability. Third, even though the fluctuation of ridership throughout the week to some extent affects network survivability, a well-prepared emergency plan for the entire week is currently acceptable. The effect of the ridership change remains limited to certain disruption levels, at least in the Beijing subway system.

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