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Quentin G. Bailey
Embry-Riddle Aeronautical University, baileyq@erau.edu

Charles D. Lane
Berry College

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Relating Noncommutative SO(2,3)$_\star$ Gravity to the Lorentz-Violating Standard-Model Extension

Quentin G. Bailey [1] **and Charles D. Lane** [2,3,*]

1 Physics Department, Embry-Riddle Aeronautical University, Prescott, AZ 86301, USA; baileyq@erau.edu
2 Department of Physics, Berry College, Mount Berry, GA 30149, USA
3 IU Center for Spacetime Symmetries, Indiana University, Bloomington, IN 47405, USA
* Correspondence: clane@berry.edu; Tel.: +1-706-290-2673

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Abstract: We consider a model of noncommutative gravity that is based on a spacetime with broken local SO(2,3)$_\star$ symmetry. We show that the torsion-free version of this model is contained within the framework of the Lorentz-violating Standard-Model Extension (SME). We analyze in detail the relation between the torsion-free, quadratic limits of the broken SO(2,3)$_\star$ model and the Standard-Model Extension. As part of the analysis, we construct the relevant geometric quantities to quadratic order in the metric perturbation around a flat background.

Keywords: Lorentz violation; noncommutative geometry; gravity

1. Introduction

While noncommutative geometry has been studied for more than 70 years [1], it has been especially popular as a possible framework for physics beyond the Standard Model in recent decades [2,3]. In particular, several extensions to general relativity that incorporate noncommutative geometry have been proposed [4–10]. In this paper, we consider one particular model that is based on a flat spacetime with broken SO(2,3)$_\star$ symmetry [11,12].

Any physical model that includes noncommutative effects and that reduces to conventional physics in the proper limit is expected to break Lorentz symmetry [13]. A general framework for the study of Lorentz violation has been developed over the last 30 years [14–18]. Indeed, numerous experimental and observational limits exist already on many different a priori independent types of Lorentz violation [19]. Additionally, this effective-field-theory framework should contain any realistic noncommutative model. This has already been shown for non-gravitational models [13]. In this work, we argue that the noncommutative SO(2,3)$_\star$ gravity model also fits into the gravitational sector of the Standard-Model Extension (SME). This serves as an example of the general notion that the SME contains all specific action-based Lorentz-violating models.

2. Noncommutative SO(2,3)$_\star$ Gravity

Consider a model consisting of a flat four-dimensional spacetime with an SO(2,3) gauge field [12]. Suppose that this symmetry is spontaneously broken along a timelike direction, with the field in that direction achieving a vacuum expectation value ℓ. The corresponding action takes the form of a model of gravity, with pieces corresponding to Einstein–Hilbert terms, cosmological-constant terms, and Gauss-Bonnet terms; this action is symmetric under an SO(1,3) subgroup of the broken SO(2,3) symmetry. If conventional field products are then translated into Moyal–Weyl $\star$-products and a Seiberg–Witten map is used to re-express quantities in terms of commutative products, we get a broken-SO(2,3)$_\star$ gravitational theory.

This process has been carried out in Ref. [12] and we present the relevant results here. This theory may be expressed as a model with noncommutative local SO(1,3)$_\star$ symmetry. The result is expanded in terms of the noncommutative background $\theta^{\alpha\beta}$, with leading terms at second order in this quantity. To display the action, we note that the geometric quantities that will appear use conventional notation: ${e_\mu}^a$ is the vierbein (with determinant e), ${\omega_\gamma}^{ab}$ is the associated spin connection, ${\Gamma^\rho}_{\gamma\alpha}$ are the Christoffel symbols associated with spacetime metric $g_{\alpha\beta}$, $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor, $R_{\alpha\beta}$ is the Ricci tensor, and R is the curvature scalar.

Once spacetime torsion $T_{\lambda\mu\nu}$ is set to zero, the action for the model ([12], Equation (4.2)) may be expressed in the form

$$\begin{aligned} S_{NCR} \quad = \quad & -\tfrac{1}{2\kappa}\int d^4x\, e\left[R - \tfrac{6}{\ell^2}(1+c_2+2c_3)\right] \\ & +\tfrac{1}{16\kappa\ell^4}\int d^4x \sum_{u=1}^{6} e\theta^{\alpha\beta}\theta^{\gamma\delta}C_{(u)}L^{(u)}_{\alpha\beta\gamma\delta} \quad , \end{aligned} \tag{1}$$

where $\kappa = 8\pi G_N$ and ℓ is a length parameter. The antisymmetric coefficients $\theta^{\alpha\beta}$ are to be thought of as a fixed background field describing the degree of noncommutativity of spacetime. Note that natural units are adopted ($\hbar = c = 1$), which implies that ℓ has units of length or inverse mass and θ has units of length squared.

The top row of Equation (1) is the action for conventional general relativity with a cosmological constant $\Lambda = -3\left(\frac{1+c_2+2c_3}{\ell^2}\right)$. (Note that this is the correct value of the cosmological constant only in the commutative limit $\theta = 0$. For $\theta \neq 0$, other terms in the action will also effectively contribute to it.) The parameters c_2 and c_3 describe the relative weights of various contributions to the unbroken SO(2,3)$_\star$ action. Thus, the action S_{CNR} may be thought of as a family of actions parameterized by c_2, c_3, and ℓ. The tensors $L^{(u)}_{\alpha\beta\gamma\delta}$ are geometric quantities; the weights $C_{(u)}$ measure the relative contributions of these quantities to the action. The tensors and their weights are listed in Table 1.

Table 1. Geometric quantities and their weights that appear in the action S_{CNR}.

u	**Weight** $\mathbf{C_{(u)}}$	**Geometric Quantity** $\mathbf{L^{(u)}_{\alpha\beta\gamma\delta}}$
1	$3c_2 + 16c_3$	$R_{\alpha\beta\gamma\delta}$
2	$-6 - 22c_2 - 36c_3$	$g_{\beta\delta}R_{\alpha\gamma}$
3	$\frac{1}{\ell^2}(6 + 28c_2 + 56c_3)$	$g_{\alpha\gamma}g_{\beta\delta}$
4	$-4 - 16c_2 - 32c_3$	$e^\mu_a e_{\beta b}(\tilde{\nabla}_\gamma e^a_\alpha)(\tilde{\nabla}_\delta e^b_\mu)$
5	$4 + 12c_2 + 32c_3$	$e_{\delta a}e^\mu_b(\tilde{\nabla}_\alpha e^a_\gamma)(\tilde{\nabla}_\beta e^b_\mu)$
6	$2 + 4c_2 + 8c_3$	$g_{\beta\delta}e^\mu_a e^\nu_b[(\tilde{\nabla}_\alpha e^a_\nu)(\tilde{\nabla}_\gamma e^b_\mu) - (\tilde{\nabla}_\gamma e^a_\mu)(\tilde{\nabla}_\alpha e^b_\nu)]$

The adjusted covariant derivatives $\tilde{\nabla}_\mu$ of the vierbein that appear in terms 4 through 6 include contributions from the SO(1,3) connection but not from the Christoffel symbols:

$$\tilde{\nabla}_\gamma {e_\alpha}^a = \partial_\alpha {e_\alpha}^a + {\omega_\gamma}^{ab}e_{ab} = \nabla_\gamma {e_\alpha}^a + {\Gamma^\rho}_{\gamma\alpha}{e_\rho}^a \quad . \tag{2}$$

If the vierbein satisfies the usual compatibility condition $\nabla_\gamma {e_\alpha}^a = 0$, then the adjusted covariant derivative may be expressed as

$$\tilde{\nabla}_\gamma {e_\alpha}^a = {\Gamma^\rho}_{\gamma\alpha}{e_\rho}^a \quad . \tag{3}$$

This implies the explicit appearance of the Christoffel symbols in the Lagrangian, the consequences of which are discussed in the next section.

The model acts like a relativistic theory of gravity in several ways, but there are some issues with interpreting it as such. For example, it is derived with the assumption that $\partial_\alpha\theta^{\mu\nu} = 0$. This assumption is reasonable in the original flat-spacetime context of the model. However, if the model is to be interpreted in curved spacetime, this assumption is clearly coordinate dependent. We may attempt to fix this issue by instead assuming that $\nabla_\alpha\theta^{\mu\nu} = 0$, but even this condition cannot apply in many situations. Nonzero tensor fields with vanishing covariant derivative cannot exist on many manifolds, including, say, spacetime with a Schwarzschild metric [15,20]. Therefore, if we wish to seriously consider action 1 to represent a theory of gravity, then we must consider it to be an approximation to a more realistic model with $\nabla_\alpha \theta^{\mu\nu} \neq 0$. In what follows, we will assume that terms involving derivatives of $\theta^{\mu\nu}$ that may appear in a more-realistic model are negligible in comparison to all other terms.

3. Gravitational Sector of the Lorentz-Violating Standard-Model Extension

The full action [15] describing the gravitational sector of the SME can be expressed as a sum of terms, each of which contracts a coefficient with spacetime indices with geometric quantities such as the Riemann tensor $R_{\alpha\beta\gamma\delta}$, the torsion $T_{\lambda\mu\nu}$, and their covariant derivatives:

$$S_{\text{gravity}} = \frac{1}{2\kappa} \int d^4x\, e \left[(k_T)^{\lambda\mu\nu} T_{\lambda\mu\nu} + (k_R)^{\kappa\lambda\mu\nu} R_{\kappa\lambda\mu\nu} + (k_{DT})^{\kappa\lambda\mu\nu} D_\kappa T_{\lambda\mu\nu} + \cdots \right] \quad . \tag{4}$$

The tensors k_T, k_R, etc. are coefficients for Lorentz and diffeomorphism violation and the ellipses represent terms with higher powers of curvature and torsion and derivative terms [21,22]. Note that a violation of local Lorentz symmetry generically implies a violation of diffeomorphism symmetry, as explained in the literature [15,23]. As with $\theta^{\mu\nu}$, it is not possible for the coefficients to be covariant derivative constants on most spacetime manifolds, and so they must be functions of spacetime position, though we may assume that their partial derivatives are negligible in experimentally relevant frames.

In this work, we consider two limits of this full action: the minimal set of terms necessary for Lorentz violation and the weakly-curved-spacetime limit (or quadratic limit) of the full action.

3.1. Covariant Match

In the gravity sector of the fully observer-covariant SME, the minimal set of terms that arises are given by the action [15],

$$S_{LV,\text{cov}} = \frac{1}{2\kappa} \int d^4x\, e \left[R + (k_R)_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \right] , \tag{5}$$

where $(k_R)_{\alpha\beta\gamma\delta}$ are the 20 (background) coefficients for local Lorentz and diffeomorphism violation. It is clear that there is overlap with the noncommutative model Equation (1). However, there are no terms in the SME containing explicit dependence on the non-tensorial connection coefficients $\Gamma^\alpha{}_{\beta\gamma}$.

It is important at this stage to distinguish two types of symmetry transformations. The first is called an *observer* diffeomorphism, or general coordinate transformation, which is a diffeomorphism that affects both the background, $(k_R)_{\alpha\beta\gamma\delta}$ and the dynamical fields $e_\mu{}^a$. The second is called a *particle* diffeomorphism, which is a diffeomorphism that leaves the background $(k_R)_{\alpha\beta\gamma\delta}$ unchanged while the dynamical fields $e_\mu{}^a$ transform in the usual way. It is this second type of symmetry breaking, *particle* diffeomorphism symmetry breaking, that is described by the SME approach and is broken by the second term in Equation (5). Because the action terms in the SME are scalars under general coordinate transformations, they trivially satisfy observer symmetry. These points are discussed in more detail in the literature [15,24,25].

The explicit appearance of $\Gamma^\rho{}_{\gamma\alpha}$ in terms 4–6 of Equation (1) implies that each of these terms is not symmetric under observer diffeomorphisms. Whether the model can be massaged into an observer covariant form, for example by a special choice of the parameters c_2 and c_3, remains to be shown. Note that the model does appear covariant under *observer* local Lorentz transformations while breaking *particle* local Lorentz symmetry. Despite the difficulty, we can proceed at the quadratic-action level, where a model that breaks observer diffeomorphism invariance cannot be distinguished from a model that breaks particle diffeomorphism invariance.

3.2. Linearized Lorentz-Violating Standard-Model Extension

If we restrict the full SME to a version with equations of motion that are linear in $h_{\mu\nu}$ [26–28], then the action takes the form, after a rescaling by $1/2\kappa$,

$$S = \int d^4x \left[\mathcal{L}_0 + \tfrac{1}{8\kappa} h_{\mu\nu} \sum_d \widehat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma} \right] \quad . \tag{6}$$

In this expression, $\mathcal{L}_0 = e(R - 2\Lambda)/2\kappa$ is the usual quadratic Einstein–Hilbert Lagrange density and $h_{\mu\nu} := g_{\mu\nu} - \eta_{\mu\nu}$ is the metric perturbation, assumed to be small. The $\widehat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$ are general derivative operators formed from background coefficients and derivatives. The summation is over the mass dimension d of the operators. In general, apart from surface terms, this sum includes 14 classes of irreducible representations involving tensors and derivative operators, all detailed in [28].

A primary goal of this paper is to argue that the non-commutative (broken-)SO(2,3)$_\star$ action S_{NCR} in the linearized limit is a special case of this general linearized Lorentz-violating action. We explicitly calculate the map that shows this correspondence. We will show that the subset of the operator terms in the action Equation (6) that occur in the non-commutative model Equation (1) can be written as

$$\begin{aligned} S_{LV,NC} \quad = \quad & \tfrac{1}{8\kappa} \int d^4x\, h_{\mu\nu} \big\{ \big[s^{(4)\mu\rho\alpha\nu\sigma\beta} + s^{(4,1)\mu\rho\nu\sigma\alpha\beta} + s^{(4,2)\mu\rho\alpha\nu\sigma\beta} + k^{(4,3)\mu\alpha\nu\beta\rho\sigma} \big] \partial_\alpha \partial_\beta \\ & + s^{(2,1)\mu\rho\nu\sigma} + k^{(2,1)\mu\nu\rho\sigma} \big\} h_{\rho\sigma}. \end{aligned} \tag{7}$$

Each of these terms has distinct tensor symmetries described by a particular Young tableau [29]. The coefficients with the $(4,\#)$ label are coefficients for mass dimension 4 operators, while those without derivatives labeled $(2,\#)$ are coefficients for mass dimension 2 operators. The latter represent an arbitrary mass matrix for the gravitational fluctuations $h_{\mu\nu}$. Incidently, none of the terms in Equation (1) contain odd mass dimension operators, and therefore the CPT symmetry is maintained.

4. Connecting NC SO(3,2)$_\star$ Gravity to the SME

The action for any linearized theory of gravity is quadratic in the perturbation $h_{\mu\nu}$. Therefore, we need to calculate each of the quantities that appears in S_{NCR} to second order in $h_{\mu\nu}$. Calculations of these quantities to first order are widespread in the literature, but calculations to second order are not, so we summarize the key results in the Appendix. With these formulæ, we may expand the noncommutative action S_{NCR} in powers of $h_{\mu\nu}$. The results may then be manipulated into the form of the linearized action Equation (6).

First, we show the match to the SME for the *massive* $u = 3$ term. Expanding this term from Equation (1) in the quadratic action limit, we obtain

$$\begin{aligned} S_{\text{NC,Mass}} \quad &= \quad \tfrac{C_{(3)}}{16\kappa\ell^6} \int d^4x e \theta^{\alpha\beta}\theta^{\gamma\delta} g_{\alpha\gamma} g_{\beta\delta} \\ &= \quad \tfrac{1}{8\kappa} \int d^4x \Big\{ \tfrac{C_{(3)}}{2\ell^6} \Big(\theta^2 + \Big[\tfrac{1}{2}\theta^2\eta^{\mu\nu} + 2\theta_\alpha{}^\mu\theta^{\alpha\mu} \Big] h_{\mu\nu} \Big) \\ & \qquad + \tfrac{C_{(3)}}{16\ell^4} h_{\mu\nu} \left[\theta^2\eta^{\mu\nu}\eta^{\rho\sigma} - 2\theta^2\eta^{\mu\rho}\eta^{\nu\sigma} + 8\theta_\alpha{}^\mu\theta^{\alpha\nu}\eta^{\rho\sigma} + 8\theta^{\mu\rho}\theta^{\nu\sigma} \right] h_{\rho\sigma} \Big\}, \end{aligned} \tag{8}$$

where $\theta^2 := \theta_{\mu\nu}\theta^{\mu\nu}$. Note that all indices on the right-hand sides of these expressions are raised and lowered with η, as they are considered to act in the flat spacetime with field $h_{\mu\nu}$. The first term with just θ^2 is a constant and irrelevant for dynamics, while the second term linear in $h_{\mu\nu}$ acts as a constant contribution to the stress-energy tensor (of the form of a cosmological constant). The last line can be matched to the last two terms in Equation (7) using Young tableau projections. The coefficients appearing, $s^{(2,1)\mu\rho\nu\sigma}$ and $k^{(2,1)\mu\nu\rho\sigma}$, correspond to the Young tableaus $\begin{array}{|c|c|}\hline \mu & \nu \\ \hline \rho & \sigma \\ \hline \end{array}$ and $\begin{array}{|c|c|c|c|}\hline \mu & \nu & \rho & \sigma \\ \hline \end{array}$, respectively. The explicit results we find are

$$\begin{aligned} s^{(2,1)\mu\rho\nu\sigma} &= \tfrac{C_{(3)}}{12\ell^4}\big[2\eta^{\mu\nu}\theta^{\rho\alpha}\theta^{\sigma}{}_{\alpha} + 2\eta^{\rho\sigma}\theta^{\mu\alpha}\theta^{\nu}{}_{\alpha} - 2\eta^{\rho\nu}\theta^{\sigma\alpha}\theta^{\mu}{}_{\alpha} - 2\eta^{\mu\sigma}\theta^{\rho\alpha}\theta^{\nu}{}_{\alpha} \\ &\qquad + 2\theta^{\rho\nu}\theta^{\sigma\mu} + 4\theta^{\rho\mu}\theta^{\sigma\nu} + 2\theta^{\mu\nu}\theta^{\rho\sigma} + (\eta^{\rho\sigma}\eta^{\mu\nu} - \eta^{\rho\nu}\eta^{\sigma\mu})\,\theta^2\big] \quad , \\ k^{(2,1)\mu\nu\rho\sigma} &= \tfrac{C_{(3)}}{48\ell^4}\big[4\eta^{\mu\nu}\theta^{\rho\alpha}\theta^{\sigma}{}_{\alpha} + 4\eta^{\rho\sigma}\theta^{\mu\alpha}\theta^{\nu}{}_{\alpha} + 4\eta^{\rho\nu}\theta^{\sigma\alpha}\theta^{\mu}{}_{\alpha} + 4\eta^{\mu\sigma}\theta^{\rho\alpha}\theta^{\nu}{}_{\alpha} \\ &\quad + 4\eta^{\sigma\nu}\theta^{\rho\alpha}\theta^{\mu}{}_{\alpha} + 4\eta^{\rho\mu}\theta^{\sigma\alpha}\theta^{\nu}{}_{\alpha} - (\eta^{\rho\sigma}\eta^{\mu\nu} + \eta^{\rho\nu}\eta^{\sigma\mu} + \eta^{\rho\mu}\eta^{\sigma\nu})\,\theta^2\big] \quad . \end{aligned} \tag{9}$$

We classify the remaining terms in Equation (1) as *kinetic* terms that only involve mass dimension 4 operators. After expanding these terms in the quadratic-action limit and manipulating the result into the form of Equation (7), we obtain

$$S_{\text{NC,Kin}} = \frac{1}{8\kappa}\int d^4x\, h_{\mu\nu}(K_{\text{NC}})^{\mu\nu\rho\sigma\alpha\beta}\partial_\alpha\partial_\beta h_{\rho\sigma}, \tag{10}$$

where the quantity $(K_{\text{NC}})^{\mu\nu\rho\sigma\alpha\beta}$ is given by

$$\begin{aligned} (K_{\text{NC}})^{\mu\nu\rho\sigma\alpha\beta} &= \tfrac{1}{16\ell^4}(2C_{(1)} - 2C_{(2)} + C_{(4)})(\eta^{\alpha\beta}\theta^{\rho\nu}\theta^{\sigma\mu} + \eta^{\alpha\beta}\theta^{\rho\mu}\theta^{\sigma\nu}) \\ &+ \tfrac{1}{64\ell^4}(4C_{(1)} - 2C_{(2)} + C_{(4)})\big(\{(\eta^{\nu\alpha}\theta^{\beta\rho}\theta^{\sigma\mu} - \eta^{\sigma\alpha}\theta^{\beta\mu}\theta^{\rho\nu} + \eta^{\nu\beta}\theta^{\alpha\rho}\theta^{\sigma\mu} - \eta^{\sigma\beta}\theta^{\alpha\mu}\theta^{\rho\nu}) \\ &\qquad + (\rho \rightleftharpoons \sigma)\} + \{\mu \rightleftharpoons \nu\}\big) \\ &+ \tfrac{1}{16\ell^4}(2C_{(1)} + C_{(2)} - C_{(5)})(\eta^{\mu\nu}\theta^{\rho\alpha}\theta^{\beta\sigma} + \eta^{\rho\sigma}\theta^{\mu\alpha}\theta^{\beta\nu} + \eta^{\mu\nu}\theta^{\rho\beta}\theta^{\alpha\sigma} + \eta^{\rho\sigma}\theta^{\mu\beta}\theta^{\alpha\nu}) \\ &+ \tfrac{1}{16\ell^4}(C_{(2)} - C_{(6)})\big(\{(\tfrac{1}{2}\eta^{\sigma\alpha}\eta^{\beta\nu}\theta^{\rho\gamma}\theta^{\mu}{}_{\gamma} + \tfrac{1}{2}\eta^{\sigma\beta}\eta^{\alpha\nu}\theta^{\rho\gamma}\theta^{\mu}{}_{\gamma} - \eta^{\sigma\nu}\eta^{\alpha\beta}\theta^{\rho\gamma}\theta^{\mu}{}_{\gamma}) + (\mu \rightleftharpoons \nu) \\ &\qquad + \eta^{\rho\nu}\eta^{\sigma\mu}\theta^{\alpha\gamma}\theta^{\beta}{}_{\gamma}\} + \{\rho \rightleftharpoons \sigma\} - 2\eta^{\rho\sigma}\eta^{\mu\nu}\theta^{\alpha\gamma}\theta^{\beta}{}_{\gamma}\big) \\ &+ \tfrac{1}{16\ell^4}C_{(1)}\big(\{(\eta^{\sigma\nu}\theta^{\rho\alpha}\theta^{\beta\mu} + \eta^{\sigma\nu}\theta^{\rho\beta}\theta^{\alpha\mu}) + (\mu \rightleftharpoons \nu)\} + \{\rho \rightleftharpoons \sigma\}\big) \quad . \end{aligned} \tag{11}$$

At this stage, one can project Equation (11) into the irreducible tensors that appear in Equation (7).

Consider the first coefficients, $s^{(4)\mu\rho\alpha\nu\sigma\beta}$, for which the operator it is contracted with, $\sim h\partial\partial h$, is a gauge invariant combination (invariant under the transformation $\delta h_{\mu\nu} = -\partial_\mu\xi_\nu - \partial_\nu\xi_\mu$). Calculation with Young Tableau projection P_Y reveals

$$\begin{aligned} s^{(4)\mu\rho\alpha\nu\sigma\beta} &= P_Y^{\begin{array}{|c|c|}\hline \mu & \nu \\ \hline \rho & \sigma \\ \hline \alpha & \beta \\ \hline \end{array}}(K_{\text{NC}})^{\mu\nu\rho\sigma\alpha\beta} \\ &= \tfrac{1}{36\ell^4}(2C_{(1)} - 3C_{(2)} + C_{(4)} + C_{(5)})\big(\tfrac{1}{2}\eta^{\rho\sigma}\theta^{\alpha\nu}\theta^{\beta\mu} - \tfrac{1}{2}\eta^{\nu\rho}\theta^{\alpha\sigma}\theta^{\beta\mu} + \eta^{\rho\sigma}\theta^{\alpha\mu}\theta^{\beta\nu} + \ldots\big), \end{aligned} \tag{12}$$

where the ellipses stand for the remaining symmetrizing terms. The explicit terms are not shown for brevity and because this contribution can be more profitably expressed using an equivalent two-tensor set of coefficients defined by

$$\bar{s}_{\gamma\delta} = -\tfrac{1}{36}\epsilon_{\mu\rho\alpha\gamma}\epsilon_{\nu\sigma\beta\delta}s^{(4)\mu\rho\alpha\nu\sigma\beta}. \tag{13}$$

Employing this, the portion of the Lagrangian containing the $s^{(4)}$ coefficients can be expressed as

$$L_{\text{LV,NC}} \supset \tfrac{1}{4\kappa}\int d^4x h_{\mu\nu}\bar{s}_{\kappa\lambda}\mathcal{G}^{\mu\kappa\nu\lambda}, \tag{14}$$

where, for the non-commutative model under study, we have

$$\bar{s}_{\kappa\lambda} = -\tfrac{1}{24\ell^4}(2C_{(1)} - 3C_{(2)} + C_{(4)} + C_{(5)})\left(\theta_{\kappa\alpha}\theta_{\lambda}{}^{\alpha} - \tfrac{1}{4}\eta_{\kappa\lambda}\theta^2\right), \tag{15}$$

and we have removed the trace of these coefficients since they contribute only as a scaling of GR at this level. This result shows that the non-commutative model overlaps with, in part, the minimal SME gravity sector in the weak-field limit. In this model, the nine coefficients $\bar{s}_{\kappa\lambda}$ are evidently controlled by the six non-commutative parameters $\theta^{\alpha\beta}$. Note that the size of these coefficients depends on the relative size of the non-commutative parameters and the the length parameter ℓ.

For the other classes of coefficients appearing in Equation (7), we can proceed in a similar fashion with the Young Tableau projection. All terms are summarized in Table 2 below. The explicit expressions for the Young projections are lengthy and omitted here for brevity, but they can be calculated with standard methods [29].

Table 2. Young Projections for the *kinetic* portion of the NC action.

SME Coefficients	Young Projection
$s^{(4)\mu\rho\alpha\nu\sigma\beta}$	$P_Y^{\begin{smallmatrix}\mu & \nu \\ \rho & \sigma \\ \alpha & \beta\end{smallmatrix}}(K_{\rm NC})^{\mu\nu\rho\sigma\alpha\beta}$
$s^{(4,1)\mu\rho\nu\sigma\alpha\beta}$	$P_Y^{\begin{smallmatrix}\mu & \nu & \alpha & \beta \\ \rho & \sigma & & \end{smallmatrix}}(K_{\rm NC})^{\mu\nu\rho\sigma\alpha\beta}$
$s^{(4,2)\mu\rho\alpha\nu\sigma\beta}$	$P_Y^{\begin{smallmatrix}\mu & \nu & \beta \\ \rho & \sigma & \\ \alpha & & \end{smallmatrix}}(K_{\rm NC})^{\mu\nu\rho\sigma\alpha\beta}$
$k^{(4,3)\mu\alpha\nu\beta\rho\sigma}$	$P_Y^{\begin{smallmatrix}\mu & \nu & \rho & \sigma \\ \alpha & \beta & & \end{smallmatrix}}(K_{\rm NC})^{\mu\nu\rho\sigma\alpha\beta}$

5. Conclusions, Prospects for Further Work

We have shown that the model proposed in Ref. [12], in its quadratic limit, is a subset of the Lorentz- and diffeomorphism-violating Standard-Model Extension. The main results are understood as a series of Young Tableau maps described in Section 4.

One consequence of the match obtained relates to experimental and observational constraints on the noncommutative model considered. For the gauge-preserving portion of the Lagrangian, for which the observable effects are controlled by the $\overline{s}_{\mu\nu}$ coefficients, an extensive study of phenomenology has been performed [30–34]. To date, numerous experiments and observations have reported measurements on these coefficients [19,35,36]. The best current astrophysical limits come from a recent comparison of the arrival times of electromagnetic and gravitational waves from a pair of colliding neutron stars [37]. Lunar laser ranging and ground-based gravimetry also place limits on these coefficients [38–41]. The best limits imply constraints on the order of $\overline{s}_{\mu\nu} < 10^{-14}$. Heuristically then, this would imply that the non-commutivity coefficients $\theta^{\alpha\beta}$ and the length parameter ℓ are related by $\theta^2/\ell^4 < 10^{-15}$. However, a more precise statement would require a thorough phenomenological analysis of the diffeomorphism-violating terms in Section 4 above.

It would be of interest to explore the role of additional terms in the non-commutative model, as in Ref. [11] that involve higher derivatives. These terms have been generally classified in the SME approach and a match should exist [28].

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Appendix A. Geometric Quantities to 2nd Order in the Metric Perturbation

Consider a pair of theories. The first operates in curved-spacetime, including a manifold $\mathcal{M}$, a metric $g_{\mu\nu}$, a local flat metric for tangent spaces η_{ab}, and a set of vierbein $e_\mu{}^a$ that relate the metrics through $g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab}$. (Equivalently, the vierbein may be thought of as a position-dependent change-of-basis matrix that relates a manifold coordinate basis $\{\vec{v}_\mu\}$ to a local tangent-space basis $\{\vec{u}_a\}$.) The second theory operates in a flat spacetime with an auxiliary field $h_{\mu\nu}$. For this theory, the manifold is simply $\mathbb{R}^4$, the manifold metric is $\eta_{\mu\nu}$, the tangent-space metric is η_{ab}, and global coordinates may be found so that the vierbein is just the Kronecker delta $\delta_\mu{}^a$.

A perturbation scheme is a map

$$(\mathcal{M}, g_{\mu\nu}, \eta_{ab}, e_\mu{}^a) \to (\mathbb{R}^4, \eta_{\mu\nu}, \eta_{ab}, \delta_\mu{}^a) + h_{\mu\nu} \tag{A1}$$

between these theories so that they approximately describe the same physical effects. In particular, we will consider situations where $g_{\mu\nu} \approx \eta_{\mu\nu}$, so that the map may be nicely approximated by a power series in $g_{\mu\nu} - \eta_{\mu\nu}$. We wish to calculate an action in terms of $h_{\mu\nu}$ that mimics the physical effects of the original theory up to order h^2.

The first piece of the map is defined by the correspondence

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad . \tag{A2}$$

This is the definition of $h_{\mu\nu}$ and hence is correct to all orders in h. Our goal in this section is to find expressions for other geometric quantities $g^{\mu\nu}$, $e_\mu{}^a$, and so on that appear in the action of the full theory. The formulas for these quantities should only involve the flat-spacetime tensors $h_{\mu\nu}$, $\eta_{\mu\nu}$, η_{ab}, and $\delta_\mu{}^a$.

It is important to note that the defining map Equation (A2) is not a tensor equation in the original spacetime. This implies that indices on $h_{\mu\nu}$ cannot be raised and lowered like the indices of true tensors. That is, $h^{\mu\nu}$ is *not* equal to $g^{\mu\alpha}g^{\nu\beta}h_{\alpha\beta}$. The geometry of the original manifold does not by itself define a unique value of such quantities, and we have some freedom in choosing our definition of them. The most convenient choice is defining them so that $h_{\mu\nu}$ acts like a true tensor in the flat spacetime. That is, we pick $h^\mu{}_\nu := \eta^{\mu\alpha}h_{\alpha\nu}$, $h^{\mu\nu} := \eta^{\mu\alpha}\eta^{\nu\beta}h_{\alpha\beta}$, etc. Similarly, we may choose to relate global and tangent-space indices with the flat-space veirbein $\delta^\mu{}_a$: $h_{\mu a} := h_{\mu\nu}\delta^\nu{}_a$, $h_\mu{}^a := h_{\mu\nu}\eta^{\nu\lambda}\delta^a{}_\lambda$, etc.

The raised-index metric $g^{\mu\nu}$ may then be evaluated to second order in $h_{\mu\nu}$ through the following strategy. The fundamental definition of $g^{\mu\nu}$ is that it is the matrix inverse of $g_{\mu\nu}$:

$$\delta_\mu{}^\lambda = g_{\mu\nu}g^{\nu\lambda} \quad . \tag{A3}$$

We proceed by using the ansatz $g^{\nu\lambda} = \eta^{\nu\lambda} + j^{\nu\lambda} + k^{\nu\lambda} + o(h^3)$ where $j^{\nu\lambda}$ is first order in h and $k^{\nu\lambda}$ is second order. If we insist that Equation (A3) hold order-by-order in h, then we need

$$j^{\alpha\lambda} = -\eta^{\alpha\mu}\eta^{\nu\lambda}h_{\mu\nu} \quad \text{and} \quad k^{\alpha\lambda} = \eta^{\alpha\mu}\eta^{\nu\beta}\eta^{\lambda\gamma}h_{\mu\nu}h_{\beta\gamma} \quad . \tag{A4}$$

Using the definitions of upper-index h quantities described in the previous paragraph, we may then write

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + h^{\mu\alpha}h_\alpha{}^\nu + o(h^3) \quad . \tag{A5}$$

Note again that $g^{\mu\nu} \neq \eta^{\mu\nu} + h^{\mu\nu}$ as the breakdown of $g_{\mu\nu}$ into $\eta_{\mu\nu} + h_{\mu\nu}$ is not a true tensor operation.

The quadratic approximation for the vierbein may be calculated by using the ansatz $e_\mu{}^a = \delta_\mu{}^a + f_\mu{}^a + \ell_\mu{}^a + o(h^3)$, where f is first order in h and ℓ is second order, and insisting that the exact relation

$$g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab} \tag{A6}$$

hold order-by-order in h. This results in the expression

$$e_\mu{}^a = \delta_\mu{}^a + \tfrac{1}{2}h_\mu{}^a - \tfrac{1}{8}h_{\mu\lambda}h^{\lambda a} + o(h^3) \quad , \tag{A7}$$

where again h quantities are related to each other with the flat-spacetime metrics $\eta_{\mu\nu}, \eta_{ab}$ and flat-spacetime vierbein $\delta_\mu{}^a$. Explicitly, $h_\mu{}^a := \eta^{\nu\rho}\delta_\rho{}^a h_{\mu\nu}$ and $h^{\lambda a} := \eta^{\lambda\mu}\eta^{\nu\rho}\delta_\rho{}^a h_{\mu\nu}$.

Once we have these, calculations of other geometric quantities are rather straightforward if tedious.

Metric:

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu} \quad , \\ g^{\mu\nu} &= \eta^{\mu\nu} - h^{\mu\nu} + h^{\mu\alpha}h_\alpha{}^\nu + o(h^3) \quad . \end{aligned} \tag{A8}$$

Vierbein:

$$\begin{aligned} e_\mu{}^a &= \delta_\mu{}^a + \tfrac{1}{2}h_\mu{}^a - \tfrac{1}{8}h_{\mu\lambda}h^{\lambda a} + o(h^3) \quad , \\ e_{\mu a} &= \eta_{\mu a} + \tfrac{1}{2}h_{\mu a} - \tfrac{1}{8}h_{\mu\lambda}h^\lambda{}_a + o(h^3) \quad , \\ e^{\mu a} &= \eta^{\mu a} - \tfrac{1}{2}h^{\mu a} + \tfrac{3}{8}h^\mu{}_\lambda h^{\lambda a} + o(h^3) \quad , \\ e^\mu{}_a &= \delta^\mu{}_a - \tfrac{1}{2}h^\mu{}_a + \tfrac{3}{8}h^{\mu\lambda}h_{\lambda a} + o(h^3) \quad , \\ e := \det(e_\mu{}^a) &= 1 + \tfrac{1}{2}h_\mu{}^\mu + \tfrac{1}{8}(h_\mu{}^\mu h_\nu{}^\nu - 2h_\mu{}^\nu h_\nu{}^\mu) + o(h^3) \quad . \end{aligned} \tag{A9}$$

Note again that the expressions for the vierbein quantities cannot be related to each other simply by raising and lowering indices: $e^{\mu a} \neq \eta^{\mu\lambda}e_\lambda{}^a$, etc. Note also that the index placement in the definition of e is important: $\det(e^\mu{}_a) = \frac{1}{\det(e_\mu{}^a)}$.

Connection coefficients:

$$\begin{aligned} \Gamma_{\alpha\mu\nu} &= \tfrac{1}{2}(\partial_\mu h_{\nu\alpha} + \partial_\nu h_{\mu\alpha} - \partial_\alpha h_{\mu\nu}) \quad , \\ \Gamma^\alpha{}_{\mu\nu} &= \tfrac{1}{2}(\eta^{\alpha\sigma} - h^{\alpha\sigma})(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu}) + o(h^3) \quad , \\ \omega_\mu{}^{ab} &= \Big[-\tfrac{1}{2}\partial^a h_\mu{}^b + -\tfrac{1}{8}h^{a\lambda}\partial_\mu h_\lambda{}^b + \tfrac{1}{4}h^{a\lambda}\partial_\lambda h_\mu{}^b - \tfrac{1}{4}h^{a\lambda}\partial^b h_{\lambda\mu}\Big] - \Big[a \rightleftharpoons b\Big] + o(h^3) \quad . \end{aligned} \tag{A10}$$

Derivative compatibility:

$$\begin{aligned} \nabla_\gamma g_{\mu\nu} &= 0 \quad , \\ \nabla_\gamma e_\mu{}^a &= 0 \quad . \end{aligned} \tag{A11}$$

Riemann tensor:

$$\begin{aligned} R_{\alpha\beta\mu\nu} = \Big[\Big(&-\tfrac{1}{2}\partial_\alpha\partial_\mu h_{\beta\nu} - \tfrac{1}{8}\partial_\alpha h_{\mu\lambda}\partial_\beta h_\nu{}^\lambda - \tfrac{1}{8}\partial_\mu h_{\alpha\lambda}\partial_\nu h_\beta{}^\lambda - \tfrac{1}{8}\partial_\lambda h_{\alpha\mu}\partial^\lambda h_{\beta\nu} \\ &-\tfrac{1}{4}\partial_\alpha h_{\mu\lambda}\partial_\nu h_\beta{}^\lambda + \tfrac{1}{4}\partial_\alpha h_{\mu\lambda}\partial^\lambda h_{\beta\nu} + \tfrac{1}{4}\partial_\mu h_{\alpha\lambda}\partial^\lambda h_{\beta\nu}\Big) - \Big(\alpha \rightleftharpoons \beta\Big)\Big] - \Big[\mu \rightleftharpoons \nu\Big] + o(h^3). \end{aligned} \tag{A12}$$

Ricci tensor:

$$\begin{aligned} R_{\alpha\mu} = g^{\beta\nu}R_{\alpha\beta\mu\nu} = \Big[&\tfrac{1}{2}\partial_\alpha\partial_\lambda h_\mu{}^\lambda - \tfrac{1}{4}\partial_\alpha\partial_\mu h_\lambda{}^\lambda - \tfrac{1}{4}\partial_\lambda\partial^\lambda h_{\alpha\mu} \\ &-\tfrac{1}{2}h^{\lambda\rho}\Big(\partial_\alpha\partial_\lambda h_{\mu\rho} - \tfrac{1}{2}\partial_\alpha\partial_\mu h_{\lambda\rho} - \tfrac{1}{2}\partial_\lambda\partial_\rho h_{\alpha\mu}\Big) \\ &+\Big(\tfrac{1}{4}\partial_\lambda h_\rho{}^\rho - \tfrac{1}{2}\partial^\rho h_{\rho\lambda}\Big)\Big(\partial_\alpha h_\mu{}^\lambda - \tfrac{1}{2}\partial^\lambda h_{\alpha\mu}\Big) \\ &-\tfrac{1}{4}(\partial_\lambda h_\mu{}^\rho)\Big(\partial_\rho h_\alpha{}^\lambda - \tfrac{1}{2}\partial^\lambda h_{\alpha\rho}\Big) + \tfrac{1}{8}(\partial_\alpha h_{\lambda\rho})(\partial_\mu h^{\lambda\rho})\Big] + \Big[\alpha \rightleftharpoons \mu\Big] + o(h^3). \end{aligned} \tag{A13}$$

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