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Abstract

This paper presents utilizes Least-Squares Algorithm to obtain more accurate linear models of nonlinear systems using parameter estimation. This approach generates an optimal linear model which is valid over a wide range of trajectories and converges to the desired steady-state value with no errors unlike the existing techniques. The proposed technique is very efficient and does not require storing the data. Therefore, it can easily be used and implemented with limited resources for undergraduate curriculum especially in underdeveloped countries. Most available techniques for linearization of nonlinear system are only valid about the operating point; furthermore, the knowledge of the operating point is required. The advantage of proposed technique is that the linearized model is not sensitive to the operating point; the estimation only requires the order of the system not the operating point. A physical example will be giving to illustrate the linear model of jet engines nonlinear system.

Introduction

It is well known that the standard control design techniques are generally developed for linear systems. However, most practical systems are nonlinear in nature. Furthermore, any derived technique for a particular nonlinear system may not be applicable to other nonlinear systems due to their complicated dynamics. Therefore, in recent years much work has been done in developing proper schemes to obtain a good linear representation of these physical systems to utilize the well-developed linear theories[1], especially, in designing controllers for multivariable systems and their associated applications. Emphasis on this type of work has resulted from a desire to obtain improved system performance over a wider operating envelope.

A particular method used to linearize a nonlinear system about an operating point is the "offset derivative". This technique makes use of the method of finite difference to approximate partial derivatives of a Taylor series expansion of the nonlinear state equations about a nominal operating point or trajectories[1]. A linearized model obtained via the offset derivative technique is valid only for a limited range of operation and often only at the operating point for systems with strong linear characteristics. More importantly, another source of error due to the linearization process is the inability of the linear model to converge to the expected steady-state values. Thus, some modification of the input Jacobian matrix[1] is necessary to eliminate this steady-state error[2]. This leads to the formulation of a technique to generate an optimal linear model which is valid over a wider range of the trajectories and converges to the expected steady-state value, without modification of the input Jacobian matrix. This can be accomplished by estimating the elements of Jacobian matrices using a recursive identification technique[3]. In what follows, the Jacobian estimation is discussed.

Estimation of Jacobians

Let us represent a nonlinear system by the following vector-matrix state equations:

$\dot{x} = f[x(t), u(t), t]$, and $y(t) = g[x(t), u(t), t]$ (1)

Where x(t) represents the $n \times 1$ state vector, u(t) is the $p \times 1$ input vector, f[x(t), u(t), t] denotes an $n \times 1$ function and the output, y(t), is $1 \times k$ vector. In general f is a function of state vector and the input vector. It should be clear that no single linear model can accurately represent the system because of its wide operating range and nonlinear characteristics. At an operating point the system is assumed to be time invariant; thus, the linear model of equation (1) may be written in vector-matrix form.

$\dot{x} = \underline{J}\, x + \underline{G}\, u, \quad y = \underline{C}\, x + \underline{D}\, u$ (2)

Where $\underline{J}$, $\underline{G}$ are the state Jacobians and $\underline{C}$, $\underline{D}$ are input matrices.

In estimation of Jacobians approach the simulation package uses a recursive identification algorithm to directly estimate the Jacobians J and G. The simulation package only has access to derivative evaluations at each timestep and provides as an output the updated system states. The only information that the simulation package requires is the order of the system, the desired timestep and an initial guess of the Jacobians J and G for the estimation process. The algorithm then tracks any changes in the Jacobians J and G to estimate the best fit parameters of the model.

In some situations off-line identification may be impractical or impossible; e.g., where the properties of the system are time-varying, or where it is impossible to perform separate identification (adaptive applications), it is necessary to identify the system in a fairly short time. This type of identification with no special input is referred to as "on-line" identification and does not require storing of all the data. This is due to a recursive algorithm being used for adjusting the estimates of the parameters after each sampling time. Clearly, the amount of computation required for updating the parameter that is the model adjustment is only a fraction of sampling time.

The most common form of on-line identification algorithm is where the current parameter estimate, $\tilde{\theta}_k$, is computed in terms of the previous estimate, $\tilde{\theta}_{k-1}$, as follows:

$\tilde{\theta}_k = \tilde{\theta}_{k-1} + G_{k-1}\, \Phi_{k-t}\, e_k$ (3)

Where $\tilde{\theta}_k$ indicates the parameter estimate at time k, G_{k-1} indicates an algorithm gain (normally a matrix), Φ_{k-t} designates a regression vector containing selected elements from y_{k-t}, u_{k-t} ; t is an integer and e_k is the modeling error created by using the estimated $\tilde{\theta}_{k-1}$. Depending on the actual meanings of G_{k-1}, and e_k, the algorithm can have a variety of different forms[4].

A wide class of linear and nonlinear systems may be expressed by a model of the following form called Deterministic Auto Regression Moving Average (DARMA model)[4]

$y_k = \Phi_{k-1}^{T}\, \theta_0$ (4)

where y_k is the system scalar output at time k, Φ_{k-1} is a linear or nonlinear function of the output sequence $\{y_{k-1}, y_{k-2}, \ldots\}$ and the input sequence $\{u_{k-1}, u_{k-2}, \ldots\}$ in vector forms, and θ_0 denotes a parameter vector (unknown).

A simple example is a first order DARMA model

$Y_k = -ay_{k-1} + bu_{k-1}$

Where, $\Phi_{k-1}^T = [-y_{k-1}, u_{k-1}]$, and $\theta_0^T = [a, b]$

With these notations, the Least-Squares identification algorithm is now introduced.

Least-Squares Algorithm

The standard Least-Square algorithm[4] is given as;

$$\tilde{\theta}_k = \tilde{\theta}_{k-1} + \frac{P_{k-2}\,\Phi_{k-1}}{1+\Phi_{k-1}^T P_{k-2}\,\Phi_{k-1}}\,[y_k - \Phi_{k-1}^T \tilde{\theta}_{k-1}], \quad k \geq 1 \qquad (5)$$

$$P_{k-1} = P_{k-2} - \frac{P_{k-2}\,\Phi_{k-1}\,\Phi_{k-1}^T P_{k-2}}{1+\Phi_{k-1}^T P_{k-2}\,\Phi_{k-1}}, \quad k \geq 1 \qquad (6)$$

With initial estimate, θ_0^T , given and P_{-1} is any positive definite matrix P_0 .

A potential problem with the normal Least-Squares algorithm is the gain reduces drastically when the covariance matrix gets small after a few iterations because of its rapid initial convergence rate. This could be overcome by building some simple logic in the identification algorithm to reset the P matrix at various times when the parameters are experiencing an excessive change. This has the effect of reconstructing the algorithm while maintaining the overall convergence rate. This resetting may be described as follows:

$$\tilde{\theta}_k = \tilde{\theta}_{k-1} + \frac{P_{k-2}\,\Phi_{k-1}}{1+\Phi_{k-1}^T P_{k-2}\,\Phi_{k-1}}\,[y_k - \Phi_{k-1}^T \tilde{\theta}_{k-1}],\ P_{-1} = M_0 I;\ M_0 > 0 \text{ and I is the identity matrix} \qquad (7)$$

Let the sequence $Z_s = \{k_1, k_2, k_3, \ldots.\}$ be the times at which resetting is done; then for k not belonging to the sequence Z_s a normal recursive Least-Squares is used to update the parameters:

That is,

$$P_{k-1} = P_{k-2} - \frac{P_{k-2}\,\Phi_{k-1}\,\Phi_{k-1}^T P_{k-2}}{1+\Phi_{k-1}^T P_{k-2}\,\Phi_{k-1}} \qquad (8)$$

Otherwise, for $k = k_i \,\boldsymbol{\varepsilon}\, \{Z_s\}$. $P_{k(i-1)}$ is reset as follows:

$P_{k(i-1)} = M_i\, I$, where $0 < M_{min} \leq M_i \leq M_{max} < \infty$

Even though, the algorithm has been developed for tracking time-varying systems, it has been shown that the algorithm can retain its convergence properties when used in time-invariant and nonlinear systems as well.

The formulation of Jacobians estimation can be performed as follows:

Assume a nonlinear system of the from;

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}) \qquad (9)$$

The following definition can then be made:

$$\dot{x}_k = J_p\, x_k + G_p\, u_k \qquad (10)$$

The unknown parameter matrix;

$$\theta_{k-1} = [\, J_p^T \quad G_p^T \,]^T \tag{11}$$

the regression vector;

$$\Phi_{k-1}^T = [\, x_k^T \quad u_k^T \,] \tag{12}$$

And the estimated model is then

$$\widetilde{\dot{x}_k^T} = \Phi_{k-1}^T \, \tilde{\theta}_{k-1} \tag{13}$$

Where; x is an n-dimensional state vector, $\boldsymbol{J_p}$ and $\boldsymbol{G_p}$ are $n \times n$ and $n \times m$ (the "best-fit" state space model) matrices, θ is $(n+m) \times n$, and Φ is an (n+m)-dimensional vector. The least squares algorithm with covariance resetting can now be used to estimate the Jacobians, $\boldsymbol{J_p}$ and $\boldsymbol{G_p}$ (the unknown parameters) in the following manner

$$\tilde{\theta}_k = \tilde{\theta}_{k-1} + \frac{P_{k-2}\, \Phi_{k-1}}{1 + \Phi_{k-1}^T P_{k-2}\, \Phi_{k-1}} [\dot{x}_k^T - \widetilde{\dot{x}_k^T}\,] \tag{14}$$

Where

$$\widetilde{\dot{x}_k^T} = \Phi_{k-1}^T \tilde{\theta}_{k-1} \text{ and } P_{k-1} = P_{k-2} - \frac{P_{k-2}\, \Phi_{k-1}\, \Phi_{k-1}^T P_{k-2}}{1 + \Phi_{k-1}^T P_{k-2}\, \Phi_{k-1}} \tag{15}$$

with initial estimate $\tilde{\theta}_0$ and P_0 given. In the parameter identification scheme, the following assumption is made. 1) The parameters of the system are slowly time-varying but the variation speed is lower that identification speed. 2) Measurement noise is negligible. 3) The state variables x_k and $\dot{x}_k$ of equation (10) are measurable.

The formulation of this technique is easily achieved by the block diagram of Figure-1.

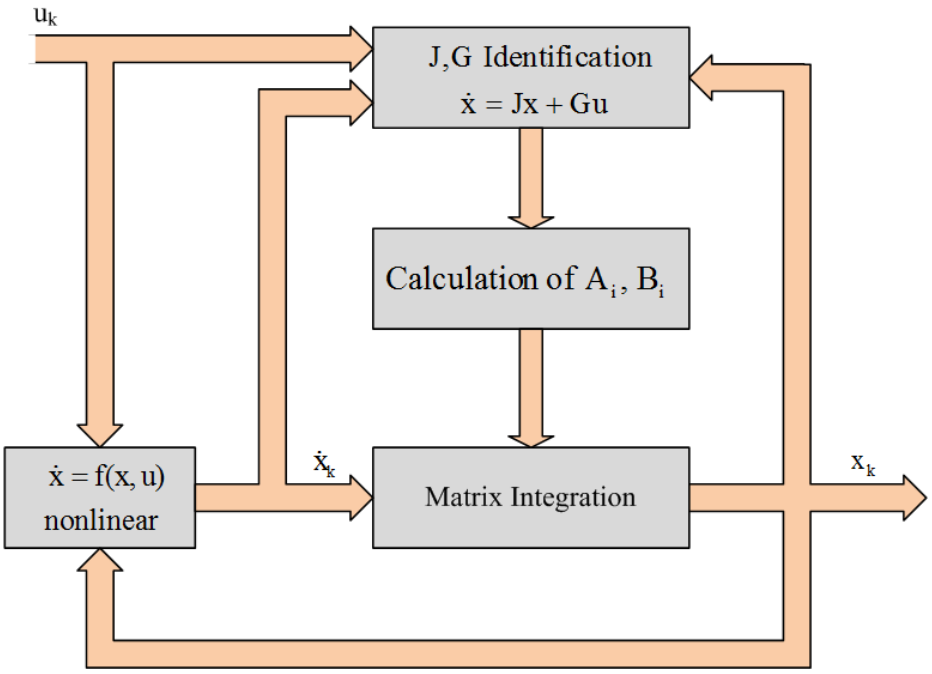

Figure-1 Jacobian Estimation

Simulation using physical example

As an illustrated example of the linear-model generation procedure, a nonlinear model is studied to show how the proposed method may deal with large negative eigenvalues of varying magnitude combined with complex eigenvalues.

Third Order Turbojet

The nonlinear system used in this study is the third order stiff nonlinear model of Brennan and Leake turbojet engine model described by the following nonlinear differential equations[5]. This third order model is obtained by reducing the complexity of a larger system using proper linearization and approximation.

$$T_3 = 0.64212 + 0.35788N^2 \quad (16)$$

$$\dot{W}_3 = 1.3009N - 0.13982[P_4^2 - \sqrt{(P_4 + 0.41688N - 0.0899PN)}\,] \quad (17)$$

$$\frac{dP_4}{dt} = (0.93586\,\frac{P_4}{\rho_B} + 31.486)\dot{W}_f + 21.435\dot{W}_3 T_3 - 53.86\,\frac{P_4^2}{\rho_B} \quad (18)$$

$$\frac{d\rho_B}{dt} = 37.78\,\dot{W}_3 - 38.448P_4 + 0.66849\,\dot{W}_f \quad (19)$$

$$\frac{dN}{dt} = (\frac{1.258}{N})(\frac{P_4^2}{\rho_B} - \dot{W}_3 N^2\,) \quad (20)$$

where P_4 is Combustor Pressure, ρ_B is Combustor Density, N is Rotor Speed, $\dot{W}_3$ is Compressor Discharge Mass Flow, T_3 is Compressor Discharge Temperature, $\dot{W}_f$ is the fuel input mass rate. The system is normalized about $\dot{W}_f$ =1.0 with initial conditions:

$P_4 = 0.53831$, $\rho_B = 1.77504$, N=0.54589 and states; $x_1 = P_4$, $x_2 = \rho_B$, $x_3 = N$.

In on-line estimation, the least squares algorithm with covariance resetting (12, 13) is used as the recursive identification algorithm to readjust the Jacobians with $P_0 = 0.3*I$, $M_i=0.3$, and $Z_s = \{5\ 10\ 15\}$. Clearly, the initial estimate of the Jacobians and P_0 (a measure of confidence in startup) play an important role in the identification of Jacobians. This algorithm seems to work reasonably well in nonlinear applications.

The converged Jacobians are:

$$J = \begin{bmatrix} -52.7214 & 23.4486 & 21.0545 \\ -47.2648 & -0.2340 & 46.9170 \\ 2.2266 & -0.9686 & -3.8325 \end{bmatrix} \quad \text{and} \quad G = \begin{bmatrix} 8.2966 \\ 0.5421 \\ 2.5738 \end{bmatrix} \quad (21)$$

Simulation of both estimated linear model compared to the nonlinear model using Adams-Bashforth two-step method[6] with stepsize of 0.00002 seconds is shown in Figure (2).

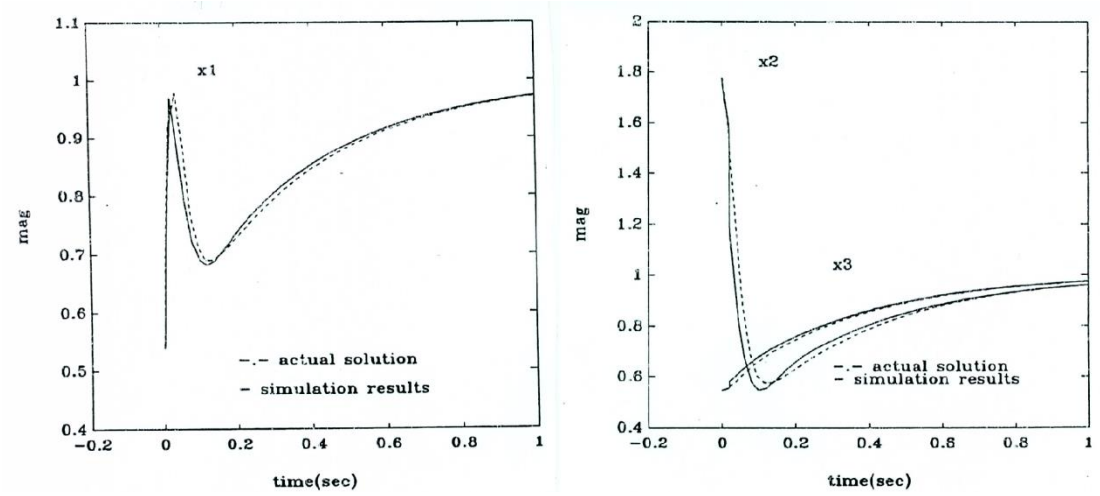

Figure 2 Simulation of linearized third order Brennan and Leak Engine model with h=0.0002 sec.

Conclusion

A linearization technique using Jacobian estimation was developed for nonlinear systems. This technique utilizes an adaptive algorithm to track any changes in Jacobians J_P and G_P and readjust accordingly the parameters to maintain the model as accurate as possible. More importantly, no modification of input Jacobain, G_P, is necessary to compensate for steady-state error. The only information that the algorithm requires is the order of the system, the desired timestep and an initial guess of the Jacobians with no knowledge of the operating point. The algorithm then tracks any changes in the Jacobians J_P and G_P to estimate the best fit model.

A linear model for a third order nonlinear jet engine using the proposed method was derived. The simulation of this linear model indicates that the proposed method significantly copes with the spatially varying parameters of a given system. Such adaptive characteristics allow its use for time-invariant, time-varying, and nonlinear systems. Furthermore, the developed method results in more accurate simulation that the linearized version about the operating point with the same sampling time. In fact, this linearized model can be viewed as the optimal linear model for the system which is valid over a wider range of the trajectory. Additionally, the variation of the eigenvalues can be detected as the system evolves.

Clearly, linear models derived about the operating point have unsatisfactory transient response (large startup transient) due to inaccurate eigenvalues. Furthermore, experience has shown that a large startup transient can completely overshadow the effects of small input.

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