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A Two-Echelon Location-inventory Model for a Multi-product Donation-demand Driven Industry

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A TWO-ECHELON LOCATION-INVENTORY MODEL FOR A MULTI-PRODUCT DONATION-DEMAND DRIVEN INDUSTRY

by

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ABSTRACT

ATWO-ECHELON LOCATION-INVENTORY MODEL FOR A MULTI-PRODUCT DONATION-DEMAND DRIVEN INDUSTRY

by

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Under the Supervision of Professor Wilkistar Otieno

This study involves a joint bi-echelon location inventory model for a donation-demand driven industry in which Distribution Centers (DC) and retailers (R) exist. In this model, we confine the variables of interest to include; coverage radius, service level, and multiple products. Each retailer has two classes of product flowing to and from its assigned DC i.e. surpluses and deliveries. The proposed model determines the number of DCs, DC locations, and assignments of retailers to those DCs so that the total annual cost including: facility location costs, transportation costs, and inventory costs are minimized. Due to the complexity of problem, the proposed model structure allows for the relaxation of complicating terms in the objective function and the use of robust branch-and-bound heuristics to solve the non-linear, integer problem. We solve several numerical example problems and evaluate solution performance.

To

My wife

And

Our families

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CHAPTER 1: INTRODUCTION

According to the Council of Supply Chain Management Professionals (CSCMP), "Supply chain management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. It also includes coordination and collaboration between suppliers, intermediaries, third party service providers, and customers. Generally, supply chain management (SCM) integrates supply and demand management within and across companies. SCM is therefore an integrating function with the primary responsibility of connecting major business functions and business processes within and across companies into a comprehensive and effective business model. It includes all of the logistical activities as noted above, as well as manufacturing operations, marketing, sales, product design, finance, and information technology. The primary focus of logistical activities is the planning, implementation, and control of the efficient, effective forward and reverse flow and storage of goods, services and related information between the point of origin and the point of consumption in order to meet customers' requirements. It also encompasses sourcing and procurement, production planning and scheduling, assembly, and customer service". [http://cscmp.org/]

Today's manager increasingly understands that holistic optimization of the logistic system leads to increased cost savings and customer satisfaction. Estimates show that the aggregate cost of any supply chain network typically includes: (i) inventory cost, (ii) cost associated with the establishment of distribution centers, and (iii) freight costs, all of which are interdependent. For example, transportation economics shows there are tradeoffs between the number of fixed service location and the resulting transportation costs since opening many distribution centers may result in lower unit transportation costs, and high customer service, at the expense of higher fixed location costs. Similarly, there are tradeoffs between fixed location costs and inventory costs. Opening fewer distribution centers will result low inventory costs due to ‘risk-pooling’ effects (Eppen, 1979).

Overall, the cost of an integrated supply chain system is said to represent 10-15 percent of the total sales in many companies (Marra, Ho, and Edwards, 2012).Therefore, the ability to optimally integrate these supply chain cost elements is a major challenge. Yet this ability also represents tremendous advantage to a company in the current increasingly competitive market. Strategic decisions such as facility location are long-term and tactical decisions such as inventory management are short-term. Hence, the relationship between the strategic and tactical elements of a supply chain is considered in most supply chain optimization models.

1.1. Components of Supply Chain Management

Supply chain management consists of three components; planning, implementation, and control (Ozsen, 2004). The planning occurs at three levels: strategic, tactical, and operational planning. Figure 1 details the components of planning in the supply chain.

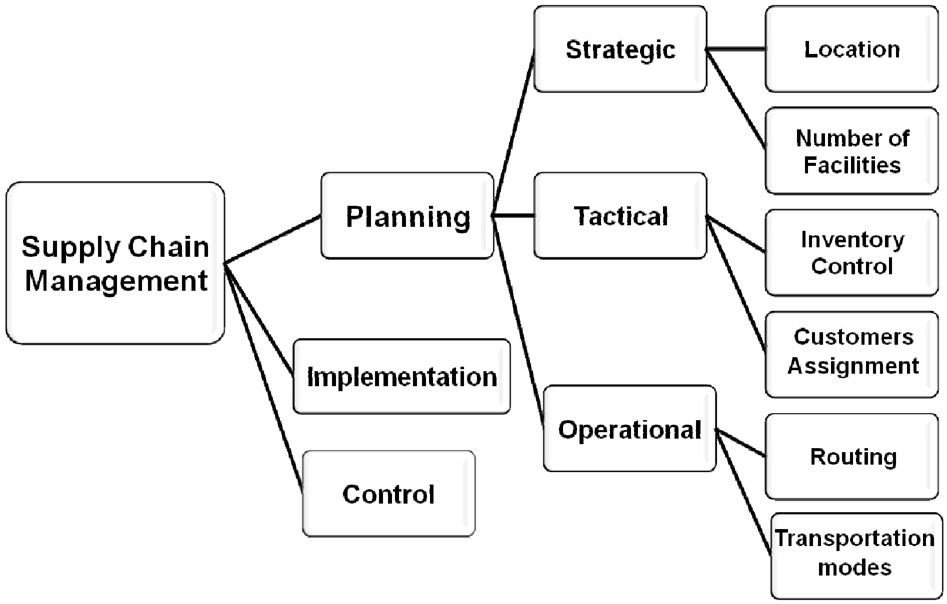


Figure 1-Supply Chain Management Components

1.2 Inventory Management Model with Risk Pooling

This section provides a brief review of some inventory management models that are related to the problem addressed in this work. Detailed discussion about inventory management models appear in a paper by Graves, Rinnoy Kan, and Zipkin (1993).

Figure 2 illustrates the inventory profile in a distribution center (or any stocking facility) for a given product. It can be seen that with time, the inventory level decreases because of the customer demand and increases when inventory is replenished. The reorder point (r) is a specific inventory level and it means that each time when the inventory level decreases to the r, a replenishment order is placed. The time which is needed from placing an order until the inventory replenishment arrives at the DC is defined as the order fulfillment lead time. Generally, the total inventory includes of two portions;

working inventory and the safety stock. The working inventory represents product that has been ordered from the supplier or plant due to demand requirements, but not yet shipped from the distribution center to satisfy customer demand. Safety stock is the inventory level allocated for buffering the system against stock-out given uncertainty in demand during the ordering lead time.

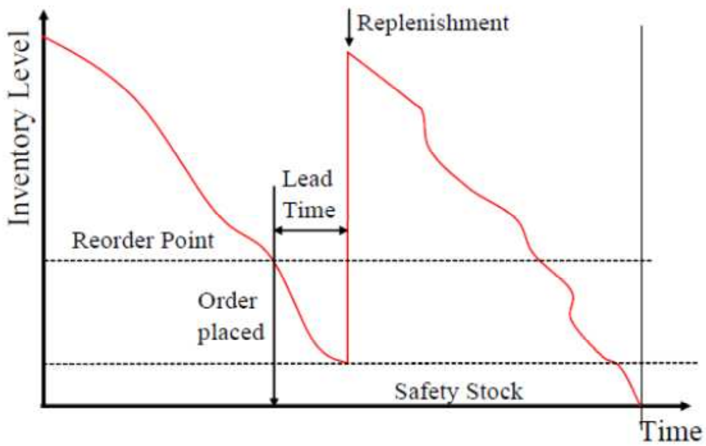


Figure 2- Inventory profile changing with time

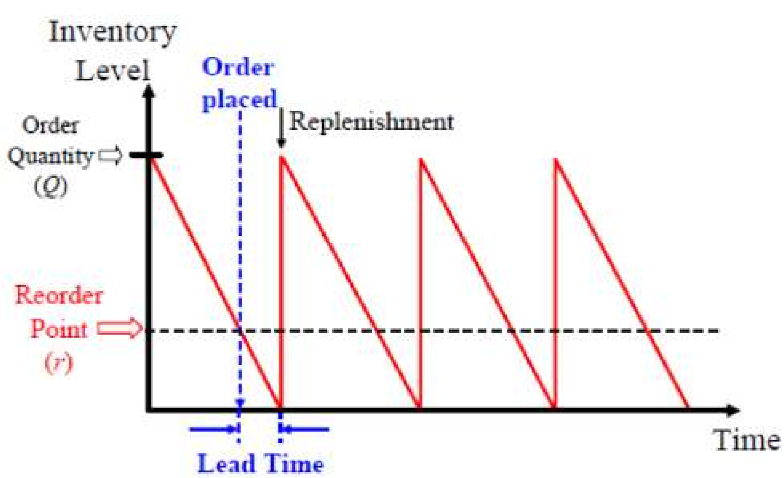


Figure 3- Inventory profile for deterministic demand with (Q, r) policy

A common inventory control policy broadly used is the order quantity/reorder point (Q, r) inventory policy. When using this policy, each time the inventory level decreases to reorder point r, a fixed order quantity Q will be placed for replenishment. When the demand is deterministic with a consistent demand rate, the inventory profile is a series of identical triangles shown in Figure 3. Each of these triangles has the same height (the order quantity *Q*), and the same width denoted as the replenishment time interval. In this case, the optimal order quantity and replenishment time interval can be determined by using an economic order quantity (EOQ) model, which takes into account the trade-off between fixed ordering costs, transportation costs and working inventory holding costs. Although the EOQ model uses the deterministic demands, it has proved to provide very good approximations for working inventory costs of systems using (*Q*, *r*) policy under demand uncertainty (Axsater, 1996).

A typical approach for the (Q, r) inventory policy is addressed by Axsater (1996). First, the stochastic demand is replaced with its mean value and then the optimal order quantity, Q is determined using the deterministic EOQ model. Finally, the optimal reorder point under uncertain demand is calculated based on the order quantity Q.

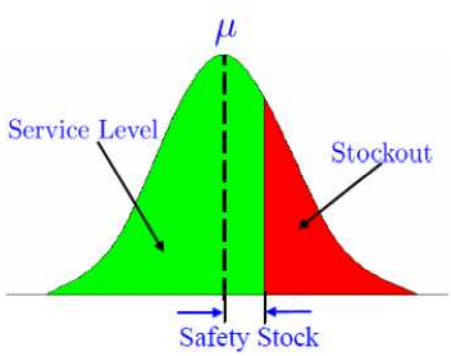


Figure 4- Safety stock and service level under normally distributed demand

A distribution center facing demand uncertainty may not always have enough stock to cushion the volatile demand. If the reorder point (r) in terms of inventory level is less than the demand during the order lead time, stock-out may occur. Type I service level is defined as the probability that the total inventory on-hand exceeds demand (as shown in Figure 4). It requires that if demand is normally distributed with mean μ and standard deviation σ and the ordering lead time is L , the optimal safety stock level to guarantee a service level α is $z_\alpha . \sqrt{L\sigma^2}$ (1) ,

where z_α is a standard normal score such that: $P_r(z \le z_\alpha) = \alpha$ (2)

Eppen (1979) proposes the “risk pooling effect” based on the total safety stock in an inventory system. This effect shows that the safety stock cost can be significantly reduced by aggregating retailers to be fed by a single centralized (or fewer) warehouse(s). Particularly, Eppen considers a single period problem with *N* retailers and one supplier. Each retailer *i* has normally distributed demand with mean μ_i and standard deviation σ_i and the correlation coefficient of demand for retailers *i* and *j* is ρ_{ij} . The order lead time from the supplier to all these retailers is the same and is given as L. Eppen compares two operational orientations of a retailer supply chain; centralized and decentralized mode. In the decentralized mode, each retailer orders independently to minimize its own expected cost. In this mode the optimal safety stock for retailer *i* is $z_\alpha \sigma \sqrt{L}_i$ (3) ,

the total safety stock in the system is calculated by $z_\alpha \sum_{i=1}^{N} \sigma_i \sqrt{L}$ (4)

In the centralized mode, all the retailers are aggregated and a single quantity is ordered for replenishment, so as to minimize the total expected cost of the entire system. In this case the demand at each retailer follows a normal distribution $N(\mu_i, \sigma_i^2)$, the total uncertain demand of the entire system during the order lead time will also follow a normal distribution with mean $L\sum_{i=1}^{N}\mu_i$ (5) ,

and standard deviation $\sqrt{L}\sqrt{\sum_{i=1}^{N}\sigma^2{}_i + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sigma_i\sigma_j\rho_{ij}}$ (6),

therefore, the total safety stock of the distribution centers in the centralized mode is,

$$z_\alpha\sqrt{L}\sqrt{\sum_{i=1}^{N}\sigma^2{}_i + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sigma_i\sigma_j\rho_{ij}} \qquad (7),$$

thus, if the demands of all the N retailers are independent, the optimal safety stock can be expressed by $z_\alpha\sqrt{L}\sqrt{\sum_{i=1}^{N}\sigma^2{}_i}$ (8) (Eppen, 1979)

which is less than $z_\alpha\sqrt{L}\sum_{i=1}^{N}\sigma_i$ (9)

This model illustrates the significant saving in safety stock costs due to risk pooling. As a result, for an inventory system that has multiple distribution centers operating with (Q, r) policy and Type I service level under demand uncertainty, the total inventory cost consists of working inventory costs and safety stock costs. In addition, the optimal working inventory costs can be estimated with a deterministic EOQ model, and the safety stock costs can be reduced by risk pooling. Given the developments above, we now turn attention to the notion of risk pooling in the location modeling literature.

Shen (2000), Shen, Coullard, and Daskin (2003), and Daskin, Coullard, and Shen (2002), developed a location model with risk pooling (LMRP) that considers the impact of working inventory and safety stock costs on facility location decisions. The system in the LMRP context consists of a single facility and multiple retailers some of which are chosen to act as distribution centers (DCs). The DCs maintain safety stock to serve their assigned retailers. The work of these authors is seminal in the sense that order frequencies at the distribution centers are modeled explicitly as decision variables. Integrated location-inventory models prior to the LMRP did not model inventory policies explicitly. Instead the earlier work approximated the inventory-related costs and included these costs in the objective function.

The LMRP succeeds in determining the optimal location of the DCs and the order frequency from the DCs to the customers simultaneously. However, the LMRP assumes infinite capacity at the DCs, which is usually not the case in practice. Having constrained capacity may affect not only the number and location of the DCs, but also the inventory that can be stored at the DCs and consequently the order frequency as well as the assignment of customers to the DCs. Ozsen et.al (2008) developed a LMRP model with capacity constraints in DCs that would be more realistic. They called this model the capacitated facility location model with risk pooling (CLMRP) and are the focus of this thesis.

In the thesis, a joint location-inventory problem for a donation-demand driven service industry setting is proposed. The strategic decisions include facility location decisions, while the tactical issues include assignment of retailers to facilities, amount of inventory to be held in DCs (Warehouses) for repositioning to other retail locations, (deliveries and surplus), and transportation decisions. The objective function of the model involves 3 main components: total facility location costs which is the annual cost for leasing or acquiring DCs in selected nodes (location problem), total transportation costs which includes the annually total product-types movements due to deliveries and surpluses between DCs and their assigned retailers, and total inventory costs, including the average inventory costs and safety stock costs. The model answers these questions such that the total system cost is minimized: How many DCs are needed in the system? Where are the locations of the DCs? And what are the assignments of retailers to these DCs?

In the numerical example section we develop a large set of representative problems based on actual operational data. Three sets of problem sizes are presented: 30, 45, and 60 node problems. Product arrives to the system as donations from consumers who deliver their reusable goods to a donation center. These are the total number of nodes in the company system of donation centers. The donation centers can be an existing retailer center (Sales\Donation centers), Attended Donation centers or ADCs (donation-only centers), and existing Distribution centers or DCs. The model wants to locate a number DCs among all these nodes in a way that minimizes the total system cost. The total system cost includes fixed location costs, transportation costs, and inventory costs. Each node (retailer center, ADC, or existing DC) can be a potential point to locate a new DC. Also each retailer center has two flows to and from its assigned DC for product repositioning (surpluses and deliveries). Both kinds of flows are uncertain.

Product level surpluses materialize when customer donations received at a retail center are higher than retail demand at a specific store location. This often occurs because of the wide variance in retail store size (which limits inventory space), or the need to reposition excess volume of the product by shipping back to the warehouse (DC) for repositioning to other retail locations. As a result, annual surpluses of all product types are measured by the number of Gaylord for the product type that is shipped back to the warehouse in a year. Deliveries are made based upon the demands. When there is a retailer shortage for any product type, the required replenishment volume is picked up on demand from the warehouse and delivered to the retail center; hence annual deliveries of any product type are defined by the number of Gaylord loads for the product that is shipped from the warehouse to the retailer in a year. Also, in spite of different kinds of products in the system, just two of them have the most demands and donations. In this thesis, these product types are referred to as **Hard lines and Soft lines**.

There is no production plant in the proposed supply chain network, so this problem is defined as a two-echelon supply chain design with uncertainties in deliveries and surplus. As far as we know, this study could be the first in the literature that considers both demand and donation (product reuse) in retailer centers for a multi-product system. Another issue of importance is to consider coverage radius, especially from the perspective of a network spanning large geographic regions. Coverage radius is the maximum distance between any retailer and its assigned warehouse. Perishable products such as blood or consumer packaged products face this important attribute of supply chain network design. Additionally, soaring fuel costs and environmental awareness pressure from various governmental and non-governmental entities necessitate the need to include coverage radius in network models, with the aim of decreasing in transportation costs. The broader impact will be a decrease in corporate carbon footprints.

The focal issue which is considered in the proposed model is the minimum number of retailers that can be assigned to a DC. In many actual supply chain contexts, it is not economical to purchase or lease a DC for only two or three retailers, Thus in the spirit of the work by Eppen (1979), the "Risk Pooling" effect factors prominently in stochastic location-inventory problems. Figure 5 illustrates risk pooling effect in details (μ and σ^2 stand for average and variance of demand respectively).

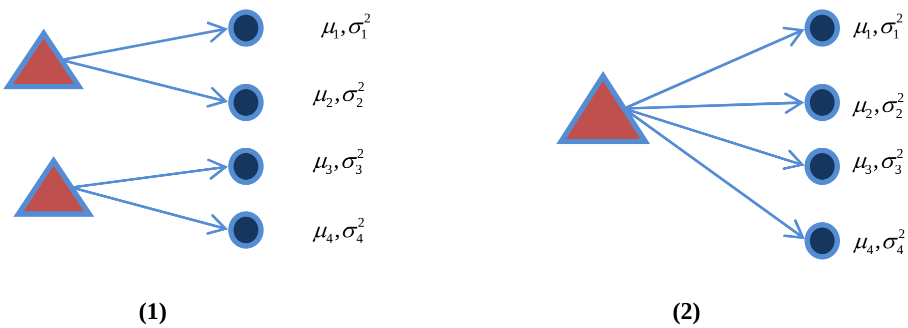


Figure 5- Risk pooling effect

The amounts of Safety stock in 1 and 2 are proportionate with

$$\sqrt{\sigma_1^2 + \sigma_2^2} + \sqrt{\sigma_3^2 + \sigma_4^2} \qquad (10)$$

and $\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2}$ (11)

respectively.

Fundamentally, $\sqrt{\sigma_1^2 + \sigma_2^2} + \sqrt{\sigma_3^2 + \sigma_4^2} \geq \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2}$ (12)

so it follows that the safety stock in (2) is less than safety stock in (1), because of risk pooling effect and centralization of a single warehouse instead of two. This leads to a decrease in total system inventory costs.

CHAPTER 2: LITERATURE REVIEW

Severe contest in today's universal market forces companies to be better in designing and managing their supply chain networks. There are three levels of decision making namely, strategic, tactical, and operational decisions in designing a supply chain network. These decisions are made objectively to decrease operation costs and an increase service level to customers, especially when all the three levels are integrated. Strategic decisions are long-term while tactical and operational decisions are considered mid-term and short-term respectively. In reality these decision are dependent to each other. For example, strategic location decisions have a major effect on shipment and inventory costs, which subsequently affect the operational decisions. Each of these decisions has been considered separately in literature.

Hopp and Spearman (1996), Nahmias (1997), and Perez and Zipkin (1997), focus on inventory control and discuss inventory policies for filling retailer orders. These policies are evaluated based on the service levels, inventory costs, shipping costs and shortage costs. Alternatively, location models tend to focus on determining the number and location of facilities, as well as retailer assignments to each facility. For a review on location modeling, we propose papers by Daskin and Owen (1998, 1999) who are leaders in this area of research. In addition, in their paper, they provide a review for dynamic and stochastic facility location models. Drezner (1995) has extensively worked on location modeling problems as well.

One of the first works in incorporating location models and inventory costs is an article by Baumol and Wolf (1958).They state that inventory costs should add a square root term to the objective function of the uncapacitated fixed charge location problem (UFLP).This condition leads to an NP-Hard problem.

Nozick and Turnquist (1998, 2001a, 2001b) incorporate inventory costs assuming the demands arrive in a Poisson manner and a base stock inventory policy (one-for-one ordering system). In 1998, they use an approximation of inventory costs (a linear function of the number of DCs) into the objective function of the fixed charge location problem (FLP). In 2001, they minimize inventory costs and unfulfilled demands, incorporating them repetitively into the fixed installation costs. Nozick (2001) considers a fixed charge location problem with coverage restriction. Another paper which solves a location model with a fixed inventory cost through Dantzig-Wolfe decomposition is presented by Barahona and Jenson (1998). Erlebacher and Meller (2000) formulate an analytical model for a location-inventory model in which the demand points are continuously placed.

Shen (2000), Shen et al. (2003), and Daskin et al. (2002) present a joint location-inventory model in which location, shipment and nonlinear safety stock inventory costs are included in the same model. In these works, the ordering decisions are based on the EOQ model. Daskin et al. and Shen et al. utilize Lagrangian relaxation and Column Generation respectively to solve this problem. In fact, they present the location model with risk pooling (LMRP). Teo and Shu (2004) introduce a joint location-inventory model that considers a multilevel inventory cost function and solve this problem with column generation.

Miranda and Garrido (2004a, 2004b) present two articles; in the first one, each retailer represents a cluster of final demands. In addition, they present an exciting comparison between traditional approach in which location and inventory decisions are made independently and simultaneous (inventory location decisions). In the second one they consider capacity constraints in the FLP models, limiting the average demand to be allocated to each distribution center.

Eskigun et al. (2005) introduce a location-inventory model that considers pipeline inventory costs based on the expected lead time from plants to the DCs. The lead time is formulated as the function of the amount of demand assigned to that distribution center. For locating cross docking, this model is too efficient. Eppen (1979) investigates the effects of risk pooling and shows that when facing independent demands, the total expected safety stock costs are remarkably less in the centralized state than in the decentralized mode. The inventory costs add a concave function to the objective function of LMRP. In his paper, the inventory policy is based on an estimation of EOQ.

Shen and Qi (2007) develop a model in supply chain system with uncertainty in demands. They determine the number and location of the DCs and also the assignment of retailers' demands to the DCs. They apply routing costs instead of direct shipments which is much more realistic and use Lagrangian relaxation in the solution algorithm. Sourirajan et al. (2007, 2009) develop an integrated network design model that simultaneously considers the operational aspects of lead time (based on queuing analysis) and safety stock. In the first paper, they use Lagrangian relaxation and in the second one, they utilize Genetic algorithm. They then present a comparative analysis of these two algorithms.

Ozsen et al. (2008) develop a capacitated location model with risk pooling in which they consider capacity constraints based on maximum inventory accumulation. They use

Lagrangian relaxation as a solution algorithm. Ozsen et al. (2009) also present a multi-sourcing capacitated location model with risk pooling. Shen (2005) and Balcik (2003) study a multiproduct extension of LMRP.

Most distribution network design models have concentrated on minimizing fixed facility location costs and transportation costs. In literature, some issues related to customer satisfaction, such as lead time, have rarely been studied. Eskigun et al. (2005) propose a supply chain network design considering facility location, lead time, and transportation mode. They use Lagrangian relaxation method to solve the problem and to find efficient solutions in a reasonable amount of time

Uster et al. (2008) present a three level supply chain network in which the decisions variables are the location of a warehouse and inventory replenishment. The objective function is to minimize transportation and inventory costs. In this problem they only consider the location of one warehouse and the inventory replenishment policy is based on power-of-two policy. They utilize the proposed heuristic methods to solve the problem and they show the efficiency of the algorithms. They find solutions within a 6% gap of the lower bound for different experiments.

Ozsen, Daskin, and Coullard (2009) consider a centralized logistics system in which a single company owns the production facility and the set of retailers and establishes warehouses that will replenish the retailers' inventories. They analyze the potential savings that the company will achieve by allowing its retailers to be sourced by more than one warehouse probabilistically, through the use of information technology. They investigate the effect of multi-sourcing in a capacitated location-inventory model that minimizes the sum of the warehouse location costs, the transportation costs, and the inventory costs. The model is formulated as a nonlinear integer-programming problem (INLP) with an objective function that is neither concave nor convex. They solve the model with a Lagrangian relaxation algorithm and test different experiments with various numbers of nodes and finally get the reasonable results in terms of the time and quality of solutions. Ultimately, they conclude that multi-sourcing becomes a more valuable option as transportation costs increase, i.e., constitute a larger portion of the total logistics cost. Additionally, they show that in practice only a small portion of the retailers need to be multi-sourced to achieve significant cost savings.

Ghezavati et al. (2009) present a new model for distribution networks considering service level constraint and coverage radius. To solve this nonlinear integer programming (INLP) model they use a new and robust solution based on genetic algorithm. Another paper was introduced by Sukun Park et al. (2010). They consider a single-sourcing network design problem for a three-tier supply chain consisting of suppliers, distribution centers and retailers, where risk-pooling strategy and lead times are considered. The objective is to determine the number and locations of suppliers and DCs, the assignment of each DC to a supplier and each retailer to a DC, which minimizes the location, transportation, and inventory costs. The problem is formulated as a nonlinear integer programming model, and a two-phase heuristic algorithm embedded in a Lagrangian relaxation method is proposed as a solution procedure. After sensitive analysis, it is shown that the proposed solution algorithm is efficient.

Chen et al. (2011) study a reliable joint inventory-location problem that optimizes facility locations, customer assignments, and inventory management decisions when facilities are under disruption risks (e.g., natural disasters). To avoid high penalty costs due to losing customer service, the customers who were assigned to a failed facility, could be reassigned to an operational facility. The model is formulated as an integer programming model. Objective function, including the facility construction costs, expected inventory holding costs and expected customer costs under normal and failure scenarios, should be minimized. A polynomial-time exact algorithm for the relaxed nonlinear sub-problems embedded in a Lagrangian relaxation procedure is proposed to solve the problem. Numerical examples show the efficiency of the proposed algorithm in computational time and finding near-optimal solutions.

O Berman, D Krass, and MM Tajbakhsh (2012) present a location-inventory model with a periodic-review (*R*, *S*) inventory policy that is taken by selecting the intervals from an authorized choices menu. Two types of coordination are introduced: partial and full coordination where each DC may select its own review interval or the DCs have same review intervals respectively. The problem is to determine the location of the DCs to be opened, the assignment of retailers to DCs, and the inventory policy parameters at the DCs such that the total system cost is minimized. The model is a kind of INLP (integer nonlinear programming) problem and Lagrangian relaxation procedure is performed to solve the problem. Computational results show that location and inventory costs increase due to full coordination. On the other hand, the proposed algorithm seems to be efficient and reliable. As a result, they show that full coordination, while enhancing the practicality of the model, is economically justifiable.

Atamtürk et al. (2012) study several stochastic joint location-inventory problems. In particular, they investigate different issues such as uncapacitated and capacitated facilities, correlated retailer demand, stochastic lead times, and multiple products. This problem is formulated as a conic quadratic mixed-integer problem and they add valid inequalities including extended polymatroid and cover cuts to boost the formulations and also develop computational results. Finally they show that this kind of formulation and solution methods would lead to more general modeling framework and faster solution times.

Hyun-Woong Jin (2012) studies some important issues on the distribution network design such as incorporating inventory management cost into the facility location model. This paper deals with a network model in which decisions on the facility location such as the number of DCs, their locations, and inventory decisions are made. Inventory decisions in their case include order quantity and the level of safety stock at each DC. The difference between this work and previous works is the classification of costs into operational costs and investment costs. A Lagrangian relaxation method is proposed to solve this problem.

Amir Ahmadi Javid and Nader Azad (2012) propose a novel model to simultaneously optimize location, assignment, capacity, inventory, and routing decisions in a stochastic supply chain system. Each customer’s demand is stochastic and follows a normal distribution, and each distribution center keeps a certain amount of safety stock in terms of its assigned customers. They use a two-stage solution algorithm. In the first stage, they reformulate the model as a mixed-integer convex problem and solve it with an exact solution method. Then in the second stage, they apply this solution as an initial point for a heuristic method including “Tabu Search” and “Simulated Annealing” to find the optimum or near optimum solution for the original problem. Different numerical examples show that the proposed solution algorithm works highly effectively and efficiently.

Jae-Hun Kang and Yeong-Dae Kim (2012) present a supply chain network consisting of a single supplier, with a central distribution center (CDC), multiple regional warehouses, and multiple retailers. The decision variables are the location and number of warehouses among a set of candidates, assignments of retailers to the selected warehouses, and inventory replenishment plans for both warehouses and retailers to minimize the objective function. The objective function that comprises of warehouse operation costs, inventory holding costs at the warehouses and the retailers, and transportation costs from the CDC to warehouses as well as from warehouses to retailers. They formulate the problem as a non-linear mixed integer programming (MINLP) model and propose an integrated solution method using Lagrangian relaxation and sub-gradient optimization methods. In the results section, they state that the solution algorithm is relatively efficient because the randomly numerical examples give good solutions in reasonable time.

Hossein Badri, Mahdi Bashiri ,Taha Hossein Hejazi (2012) define a new mathematical model for multiple echelon, multiple commodity Supply Chain Network Design (SCND) and consider different time resolutions for tactical and strategic decisions. Expansions of the supply chain in the proposed model are planned according to cumulative net profits and fund supplied by external sources. Furthermore, some features, such as the minimum and maximum utilization rates of facilities, public warehouses and potential sites for the establishment of private warehouses, are considered. To solve the model, an approach based on a Lagrangian relaxation (LR) method has been developed, and some numerical analyses have been conducted to evaluate the performance of the designed approach.

In another paper, Sri Krishna Kumara, and M.K. Tiwari (2013) consider the location, production–distribution and inventory system design model for a supply chain in order to determine facility locations and their capacity to minimize total network cost. Because the demands are stochastic, the model considers risk pooling effect for both safety stock and RI (Running Inventory). Two cases, due to benefits of risk pooling, are studied in the model; first, when retailers act independently and second, when DCs and retailers are dependent to each other and work jointly. The model is formulated as a mixed integer nonlinear problem and divided into two stages. In the first stage the optimal locations for plants and flow relation between plants-DCs and DCs-retailers are determined. At this stage the problem has been linearized using a piece-wise linear function. In the second stage the required capacity of opened plants and DCs is calculated. The first stage problem is further divided in two sub-problems and in each of them, the model determines the flow between plants-DCs and DCs-retailers respectively using Lagrangian relaxation. Computational results show that main the problem's solution is within the 8.25% of the lower bound and significant amount of cost saving can be achieved for safety stock and RI costs when DCs and retailers work jointly.

Jiaming Qiu and Thomas C. Sharkey (2013) consider a class of dynamic single-article facility location problems in which the facility must determine order and inventory levels to meet the dynamic demands of the customers over a finite horizon. The motivating application of this class of problems is in military logistics and the decision makers in this area are not only concerned with the logistical costs of the facility but also with centering the facility among the customers in each time period, in order to provide other services as well. Both the location plan and inventory plan of the facility in the problem must be determined while considering these different metrics associated with efficiency of these plans. Effective dynamic programming algorithms for this class of problem are provided for both of these metrics. These dynamic programming algorithms are utilized in order to construct the efficient frontier associated with these two metrics in polynomial time. Computational testing indicates that these algorithms can be used in planning activities for military logistics.

In the current competitive business world, leading-edge companies respond to a dynamic environment promptly with various and flexible strategies. These strategies are used to make optimum decision regarding allocation of company income to the major sources including activities or services.

Gharegozloo et al. (2013) present a location-inventory problem in a three level supply chain network under risk uncertainty. The (r,Q) inventory control policy is used for this problem. Additionally, stochastic parameters such as procurement, transportation costs, demand, supply, capacity are presented in this model. Risk uncertainty in this case is due to disasters as well as man-made events. Their robust model determines the locations of distribution centers to be opened, inventory control parameters (r,Q), and allocation of supply chain components simultaneously. This model is formulated as a multi-objective mixed-integer nonlinear programming in order to minimize the expected total cost of such a supply chain network comprising location, procurement, transportation, holding, ordering, and shortage costs. They apply an efficient solution algorithm on the basis of multi-objective particle swarm optimization for solving the proposed model and the final numerical examples and sensitive analysis show the efficiency and performance of the algorithm.

2.1 Research Contribution

As was presented in literature review section, most of the location- inventory models do not consider “coverage radius” constraint as an important parameter in determining service level to end customers. Coverage radius is the maximum distance between any retailer and its assigned warehouse. Increasing fuel cost, supply of perishable products and environmental impact due to transportation, are the most important factors that drive the consideration of coverage radius. In The first contribution in our study is the addition of coverage radius as a constraint. This not only makes the problem and solutions more realistic but also it is specific to the company in the case study.

Secondly, our model is related to a demand-donation driven supply network and we consider the case of an industry in the Southeastern Wisconsin region. In this model, each retailer has two flows, to and from its assigned DC i.e. surpluses (S) and deliveries (D) both with uncertainty. In most previous work, demand is the only flow in all retailer points. Having two flows in the model leads to different inventory levels in warehouses due to the average and standard deviation of difference between surpluses and deliveries for any assigned retailer. The real data from the company in the case study shows that all demands are larger than donations in any retailer point for any product type. We specifically make the proposed model robust enough to accept scenarios in which donations could be larger than demands in any retailer for any product type.

In most literature, multiple products have not been taken into account in a joint location-inventory model. The third contribution is that the proposed model considers multiple commodities in a donation-demand driven network, hence realistic. In addition, our model considers a set of constraints related to the minimum number of retailers that can be assigned to an opened DC for any product type. Because of high annual leasing or purchasing costs for a typical warehouse, this assumption is important. As a result, the research contributions in this study are summarized as follows:

We propose a "***Generalized location-inventory model"*** for a donation-demand driven industrial supply chain network. In this model, we integrate the minimum number of retailers that are assigned to an opened DC and the coverage radius as constraints in a multi-commodity supply chain system. Specific to the company modeled in this study, each retailer point referred to as a donation/demand center is a potential location for opening a DC (distribution center).

CHAPTER 3: PROBLEM DEFINITION, ASSUMPTIONS, AND MODEL FORMULATION

3.1 Problem Definition

As was discussed in the introductory section, this study involves a joint location inventory model using data from a donation-demand driven industry in the Southeastern Wisconsin region. This bi-echelon model involves warehouses (herein also referred to as Distribution Centers (DC)) and retailers (R) (herein also referred to as Donation/Demand Centers). In this model, we restrict our variables to include; coverage radius, service level, and multiple products. Each retailer has two flows to and from its assigned DC i.e. surpluses (S) and deliveries (D). Surpluses result when product-type donations are higher than the demand therefore the excess volume of the product is shipped back to the warehouse (DC) due to limited inventory space in retailer point (herein referred to as a node).Conversely, deliveries result when the product demand is higher than the donations, hence more products should be shipped from the warehouse to the retailer.

Among the retailer nodes, there are specified nodes that are strictly donation only points, as such they do not have any product demand and no products are delivered into them from any warehouse. Such a node is referred to as Attended Donation Centers (ADC). Figure6 is a schematic representation of the company's supply chain network. Here, only three DCs and seven retailers are used for explanation purposes.

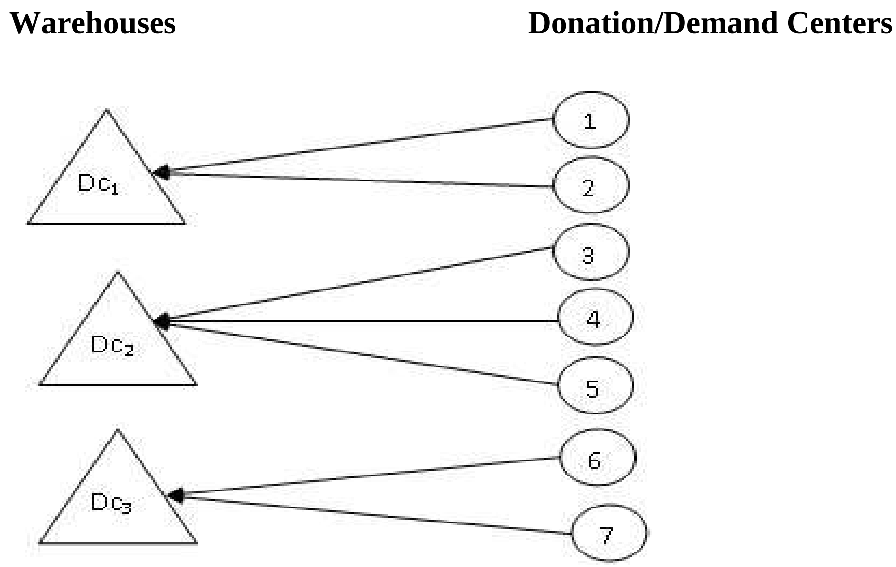


Figure 6- schematic representation of the company's supply chain network

The two flows between each retailer and its assigned DC are completely dependent. This means that in this model, deliveries and surpluses cannot occur simultaneously. Annual deliveries are stochastic, independent and normally distributed (i.n.d). So we can suppose that the deliveries (D) to each retailer (i) from its assigned DC (j) for a given product type (k) is a random variable with average of $\mu_{D_{ik}}$ and variance of $\sigma^2_{D_{ik}}$. Similarly, annual surpluses are also i.n.d. and the surpluses from a retailer (i) to its assigned DC (j) for a given product type (k) are also stochastic with an average and variance of $\mu_{s_{ik}}$ and $\sigma^2_{s_{ik}}$ respectively. Generally, an actual supply chain network for this problem can be represented in Figure 7.

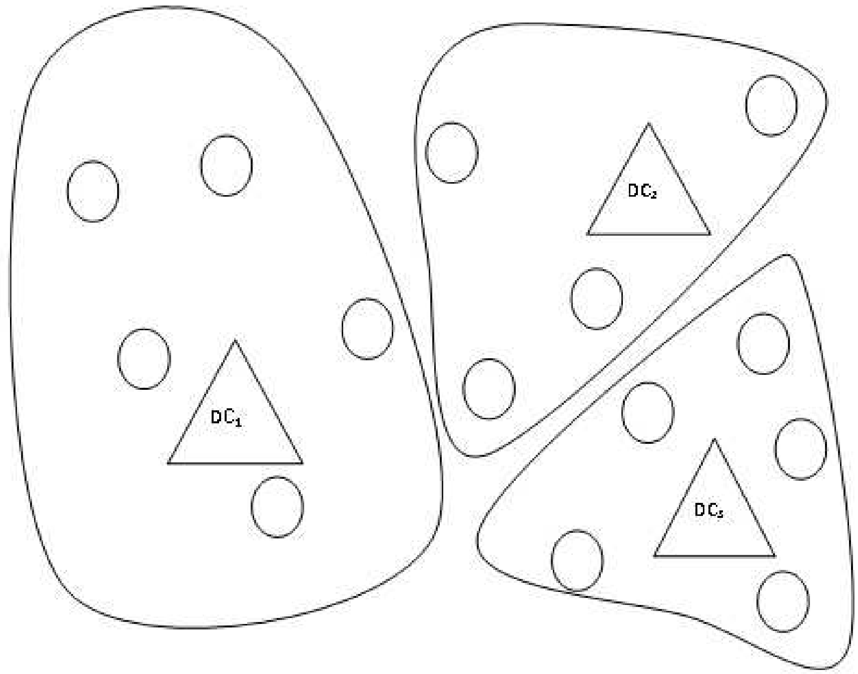


Figure 7- An actual supply chain network for the company

3.1.1 Parameters Description

f_j: Annual fixed location cost for a DC in location j

d_{ji}: Transportation cost for each unit of product type (in Gaylord) per unit distance (miles) between nodes i and j based on current fuel and labor cost

l_{ji}: Distance traveled between node i and j in direct shipment (in miles)

h: Annual holding cost per unit of each product type in DC j

Z_{α}: Normal standardized score with a risk factor of alpha

$\sigma^2_{D_{ik}}$: Annual variance of deliveries for product type k to retailer i

$\sigma^2_{s_{ik}}$: Annual variance of surpluses of product k from retailer i to the assigned DC

σ^2_{ik}: Annual total variance of deliveries and surpluses of a product type k of retailer i

$\mu_{D_{ik}}$: Annual average deliveries of product type k to retailer i

$\mu_{S_{ik}}$: Annual average surpluses of a product type k from retailer i

N : Maximum number of possible DCs in system

M : Minimum number of retailers (R) to be assigned to any DC

$$z_{ji} : \begin{cases} 1 & \text{If candidate DC j can cover retailer i determined by the coverage radius} \\ 0 & \textit{Else} \end{cases}$$

β : Weighted factor assigned to the transportation cost

θ: Weighted factor assigned to the inventory cost

3.2 Assumptions

1. Although the real problem includes various products, for modeling purposes, we only consider two product types with the highest demand and donations i.e. Hard Lines (HL) and Soft Lines (SL).
2. d_{ji} (The transportation cost) includes fuel cost and labor cost. By assuming that each truck has a capacity of 25Gaylord, and transportation cost per unit distance

for each truck is \$2.12 (this includes both fuel and labor costs) [company data], so d_{ji} is \$2.12/25 = \$ 0.0854.

3. The holding cost (h) is fixed for both product types.
4. The average demand for a given product type is larger than the average donation of the same product type for any retailer. This assumption stems from two sources: real data from the company and anecdotal, that for any retailer to exist despite seasonal effects, the annual average demand has to exceed the donation. Otherwise the node will become an ADC. However, the proposed model is generalized whereby donation could be larger than demand for a product type or vice versa.
5. For calculating the safety stock cost in the objective function, we need σ_{ik}^2 to be calculated as follows:

$$
\begin{aligned}
&Let:\\
&a = Total\ surplus\ (Pulls)\ of\ a\ product\ to\ a\ DC\ from\ a\ retailer\ in\ year\\
&b = Total\ deliveries\ of\ mentioned\ product\ from\ the\ DC\ to\ that\ retailer\ in\ year\\
&\sigma_{ik}^2 = \text{var}(a-b) = \text{var}(a) + \text{var}(b) - 2\,\text{cov}(a,b) \qquad ;\ (\text{var}(a) = \sigma_{D_{ik}}^2\ ,\ \text{var}(b) = \sigma_{S_{ik}}^2\ )\\
&Also \quad \rho_{a,b} = \frac{\text{cov}(a,b)}{\sigma_a \sigma_b}, \quad \rho_{a,b} = -1 \Rightarrow \text{cov}(a,b) = -\sigma_a . \sigma_b\\
&\Rightarrow \sigma_{ik}^2 = \text{var}(a-b) = \text{var}(a) + \text{var}(b) + 2\sqrt{\text{var}(a).\,\text{var}(b)} \qquad (13)
\end{aligned}
$$

6. We only consider direct shipments i.e. multi-location routing is not allowed.
7. It is assumed that DCs will be located in any of the existing nodes. This assumption follows from discussions with the company experts.
8. The “big circle distance” calculator is used to determine the distance between node i and j. This formula uses the latitudes and longitudes to calculate the

distance between any two locations. For a more realistic estimation of the distances, 14% of the estimated distance is added. l_{ji} is calculated based on the estimated distance multiplying two. The reason for that is because of direct shipment which in a truck leaves node i, reached to j, and then returns to i again.

9. M is the minimum number of retailers that can be assigned to any DC. In this model, we assume that M is five. This value was given by experts within the company. In brief, factors such leasing or purchasing costs of DC facilities were used to determine the realistic value of M.
10. Another factor that is considered in this model is the coverage radius. Normally, coverage radius is prominent in modeling perishable and essential goods. Due to recently soaring fuel prices in recent years, it is inevitable to include coverage radius as one of the main factors in regional facility location models. Besides increasing transportation costs, environmental conditions have an important role in determine the coverage radius, especially given that the model depicts s supply network in U.S.A.'s mid-western region that experiences harsh winters. In addition, environmental pollution policies and penalties also force distributors to ensure minimal transportation in their networks. In this model, 50, 75 and 100 miles are used as case scenarios.

3.3 Model Formulation

Based on the problem definition, parameter description and assumptions, this problem is formulated as a joint location-inventory problem for a bi-level supply chain to determine number of DCs, DC locations, and assignments of retailer to those DCs. The proposed model is minimization problem that seeks to optimize the total annual cost including: fixed facility location costs, transportation costs, and inventory costs. As was discussed before, it is re-emphasized that there are two flows between each retailer and each DC i.e. deliveries from any DC to any retailer and surpluses from any retailer to any DC. On the other hand, there is only surplus flow between any ADC and its assigned DC. Based on the objective function, decision variables in this model are defined as:

$$X_j : \begin{cases} 1 & \text{If a candidate DC is located in j} \\ 0 & \text{Else} \end{cases} \tag{14}$$

$$Y_{jik} : \begin{cases} 1 & \text{If the DC in location j serves retailer i for product type k} \\ 0 & \text{else} \end{cases} \tag{15}$$

So the formulation of model is expressed as follows:

$$Min\ W = \sum_j f_j X_j + \beta \sum_j \sum_i \sum_k l_{ji} . d_{ji} (\mu_{D_{ik}} + \mu_{S_{ik}}) Y_{jik} + \theta h_{com} \sum_j \sum_i \sum_k (\mu_{D_{ik}} - \mu_{S_{ik}}) Y_{jik} + \theta z_\alpha h_{com} \sum_j \sqrt{\sum_i \sum_k \sigma^2{}_{ik} Y_{jik}} \tag{16}$$

$ST:$

$$Y_{jik} \le z_{ji} X_j \qquad \forall i \in I\ ,\ k \in K\ ,\ j \in J \tag{17}$$

$$\sum_j Y_{jik} = 1 \qquad \forall i \in I\ ,\ k \in K \tag{18}$$

$$\sum_i Y_{jik} \ge P \qquad \forall j \in J\ ,\ k \in K \tag{19}$$

$$X_j = 0,1 \qquad \forall j \in J \tag{20}$$

$$Y_{jik} = 0,1 \qquad \forall i \in I\ ,\ k \in K\ ,\ j \in J \tag{21}$$

The objective function consists of four terms. The first term is the total system location costs where f_j is the fixed location cost for any candidate DC. The second term is the total system transportation costs between DCs and retailers for all products types. The third term is the system average inventory costs (for all DCs). The fourth term is the total system safety stock cost i.e. for all and all products types. If the number of DCs increases, total system location and safety stock costs increase while the system transportation cost decreases. However, if the number if DCs decreases, total system location and safety stock costs decrease while the system transportation cost increases. In addition, the average system inventory cost does not change with a change in the number of open DCs. As such, the model is a trade-off between these cost terms in objective function with respect to the model constraints.

The model constraints include: Constraint 17 demonstrates that a retailer can be assigned to any open DC within the coverage radius. Constraint 18 ensures single-sourcing, meaning that only one DC should serve a retailer for any specified of product type. Constraint 19 ensures that the minimum number of retailers that can be assigned to a DC for a given product is met. Lastly, constraints 20 and 21 restrict the decision variables to a binary range.

The model is an INLP (Integer Nonlinear Program) within the family MINLP (Mixed Integer Nonlinear Programs).It is a combinatorial optimization model because it has a finite solution set. However, finding the best solution among all feasible solutions is difficult; hence this problem is an NP-hard because its complexity and the time needed to solve the problem increases exponentially as the number of nodes increases. The solution algorithm is discussed in the next chapter.

3.3.1 Research Contribution: Generalized location-inventory model

The proposed inventory-location model in section 3.3 is specific to the company in our case study. This model assumes that demand is always larger than donation for any retailer and product type. As a result, total deliveries are assumed to always be larger than total surpluses between any DC and its retailers. This assumption could be reasonable, however due to seasonality or other special circumstances, this can be violated. So next, we present a robust generalized model that can accommodate both instances simultaneously.

$$\begin{aligned}
Min\ \ W = & \sum_j f_j X_j + \beta \sum_j \sum_i \sum_k l_{ji}.d_{ji}(\mu_{D_{ik}} + \mu_{S_{ik}})Y_{jik} + \theta\, h_{com} \sum_j t_j (\sum_i \sum_k (\mu_{D_{ik}} - \mu_{S_{ik}})Y_{jik}) \\
& + \theta\, z_\alpha h_{com} \sum_j t_j \sqrt{\sum_i \sum_k \sigma^2{}_{ik} Y_{jik}} + \theta\, h_{com} \sum_{j \in j'} (1-t_j)(\sum_i \sum_k (\mu_{S_{ik}} - \mu_{D_{ik}})Y_{jik}) \\
& + \theta\, z_\alpha h_{com} \sum_{j \in j''} (1-t_j)(\sqrt{\sum_i \sum_k \sigma^2{}_{ik} Y_{jik}} - \sum_i \sum_k (\mu_{S_{ik}} - \mu_{D_{ik}})Y_{jik}) \\
& + \theta\, h_{com} \sum_{j \in j''} (1-t_j)(\sum_i \sum_k (\mu_{S_{ik}} - \mu_{D_{ik}})Y_{jik}) \qquad (22)
\end{aligned}$$

$ST:$

$$Y_{jik} \le z_{ji} X_j \qquad \forall i \in I\ ,\ k \in K\ , j \in J \qquad (17)$$

$$\sum_j Y_{jik} = 1 \qquad \forall i \in I\ ,\ k \in K \qquad (18)$$

$$\sum_i Y_{jik} \ge P \qquad \forall j \in J\ ,\ k \in K \qquad (19)$$

$$B.t_j \ge \sum_i \sum_k (\mu_{D_{ik}} - \mu_{S_{ik}})Y_{jik} \qquad \forall j \in J \qquad (23)$$

$$-(1-t_j).B \le \sum_i \sum_k (\mu_{D_{ik}} - \mu_{S_{ik}})Y_{jik} \qquad \forall j \in J \qquad (24)$$

$$X_j = 0{,}1 \qquad \forall j \in J \qquad (20)$$

$$Y_{jik} = 0{,}1 \qquad \forall i \in I\ ,\ k \in K\ , j \in J \qquad (21)$$

$$t_j = 0{,}1 \qquad \forall j \in J \qquad (25)$$

$$j' : j \in \{\sum_i \sum_k (\mu_{S_{ik}} - \mu_{D_{ik}})Y_{jik}) \ge \sqrt{\sum_i \sum_k \sigma^2{}_{ik} Y_{jik}}\} \qquad (26)$$

$$j'' : j \in \{\sum_i \sum_k (\mu_{S_{ik}} - \mu_{D_{ik}})Y_{jik}) < \sqrt{\sum_i \sum_k \sigma^2{}_{ik} Y_{jik}}\} \qquad (27)$$

In this formulation, B is a large number. For example >10000, which must be larger than the highest difference ($\mu_{D_{ik}} - \mu_{S_{ik}}$) and t_j is a binary decision variable that is 1 for a DC_j if the function $(\sum_i \sum_k (\mu_{D_{ik}} - \mu_{S_{ik}})Y_{jik}) \geq 0$ and is 0 if the function $(\sum_i \sum_k (\mu_{D_{ik}} - \mu_{S_{ik}})Y_{jik}) \leq 0$. These two conditions have been added as constraints (23) and (24). Also, we restate the cost terms in the objective function to include the added model parameters.

CHAPTER 4: SOLUTION ALGORITHM AND PARAMETERS SETTING

4.1 Solution Algorithm

The proposed joint location-inventory model is a nonlinear integer programming where all the decision variables are binary. Besides its combinatorial nature, the nonlinear term is non-convex which makes the optimization model very difficult to solve. First, the original INLP model (**P0**) is reformulated as a mixed-integer nonlinear programming (MINLP) problem with fewer zero-one variables (P1). P1 has concavity in the objective function and linear constraints hence also difficult to solve. P1 is then relaxed of the concavity in the objective function and it is reformulated as a new model with nonlinear constraints and a linear objective function (P2), retaining the properties of problem P1, but simpler to solve. P2can be solved using the “SCIP” solve in GAMS to get optimal or near optimal solutions. The original model (P0) is rewritten as below:

$(P0)$

$$Min\ \ W = \sum_j f_j X_j + \beta \sum_j \sum_i \sum_k l_{ji} . d_{ji} (\mu_{D_{ik}} + \mu_{S_{ik}}) Y_{jik} + \theta\, h_{com} \sum_j \sum_i \sum_k (\mu_{D_{ik}} - \mu_{S_{ik}}) Y_{jik}$$
$$+ \theta\, z_\alpha h_{com} \sum_j \sqrt{\sum_i \sum_k \sigma^2{}_{ik} Y_{jik}} \qquad (16)$$

$ST:$

$$Y_{jik} \le z_{ji} X_j \qquad \forall i \in I\ ,\ k \in K\ , j \in J \qquad (17)$$
$$\sum_j Y_{jik} = 1 \qquad \forall i \in I\ ,\ k \in K \qquad (18)$$
$$\sum_i Y_{jik} \ge P \qquad \forall j \in J\ ,\ k \in K \qquad (19)$$
$$X_j = 0{,}1 \qquad \forall j \in J \qquad (20)$$
$$Y_{jik} = 0{,}1 \qquad \forall i \in I\ ,\ k \in K\ , j \in J \qquad (21)$$

The original INLP model (**P0**) is very difficult to solve especially for large networks due to the potentially large number of binary variables. As shown in proposition 1 below, the assignment variables (Y_{jik}) in the model can be relaxed as continuous variables without changing the optimal integer. This allows us to reformulate (**P0**) as a MINLP problem with fewer binary variables, most of them appearing in linear form.

Proposition1. *The continuous variables* Y_{jik} *take 0-1 binary values when (**P1**) is globally optimized or locally optimized for fixed 0-1 values for* X_j. (You and Grossmann, 2008)

Proposition 1 means that the following problem (**P1**), yields integer values on the assignment variables Y_{jik} when it is globally optimized or locally optimized for fixed binary integer values of X_j, so P0 is reformulated as P1 as below:

$$(P1)$$

$$Min\ W = \sum_j f_j X_j + \beta \sum_j \sum_i \sum_k l_{ji} . d_{ji} (\mu_{D_{ik}} + \mu_{S_{ik}}) Y_{jik} + \theta h_{com} \sum_j \sum_i \sum_k (\mu_{D_{ik}} - \mu_{S_{ik}}) Y_{jik}$$
$$+ \theta z_\alpha h_{com} \sum_j \sqrt{\sum_i \sum_k \sigma^2{}_{ik} Y_{jik}} \qquad (16)$$

$$ST:$$

$$Y_{jik} \le z_{ji} X_j \qquad \forall i \in I\ ,\ k \in K\ , j \in J \qquad (17)$$

$$\sum_j Y_{jik} = 1 \qquad \forall i \in I\ ,\ k \in K \qquad (18)$$

$$\sum_i Y_{jik} \ge P \qquad \forall j \in J\ ,\ k \in K \qquad (19)$$

$$X_j = 0{,}1 \qquad \forall j \in J \qquad (20)$$

$$Y_{jik} \ge 0{,}1 \qquad \forall i \in I\ ,\ k \in K\ , j \in J \qquad (28)$$

Another problem that exists in model P1 is that the objective function has concavity which is complicated to solve. P1 is therefore relaxed into another model (P2) that does not have concavity in objective function; hence another non-negative continuous variable "U_j "is defined to replace the square root term in objective function. This variable is described as follow:

$$U_j^2 = \sum_i \sum_k \sigma^2{}_{ik} Y_{jik} \qquad j \in J \qquad (29)$$

$$U_j \geq 0 \qquad (30)$$

Because the non-negative variable U_j has a positive coefficient in the objective function, and this problem is a minimization problem, (29) can be further relaxed using the following inequality:

$$-U_j^2 + \sum_i \sum_k \sigma^2{}_{ik} Y_{jik} \leq 0 \qquad j \in J \qquad (31)$$

The reformulated model is expressed as P2 below:

$$(P2)$$

$$\begin{aligned}
Min\ W = &\sum_j f_j X_j + \beta \sum_j \sum_i \sum_k l_{ji} . d_{ji} (\mu_{D_{ik}} + \mu_{S_{ik}}) Y_{jik} + \theta h_{com} \sum_j \sum_i \sum_k (\mu_{D_{ik}} - \mu_{S_{ik}}) Y_{jik} \\
&+ \theta z_\alpha h_{com} \sum_j U_j \qquad (32)
\end{aligned}$$

$$ST:$$

$$\begin{aligned}
&Y_{jik} \leq z_{ji} X_j \quad \forall i \in I, k \in K, j \in J && (17) \\
&\sum_j Y_{jik} = 1 \quad \forall i \in I, k \in K && (18) \\
&\sum_i Y_{jik} \geq P \quad \forall j \in J, k \in K && (19) \\
&-U_j^2 + \sum_i \sum_k \sigma^2{}_{ik} Y_{jik} \leq 0 \quad \forall j \in J && (31) \\
&X_j = 0,1 \qquad \forall j \in J && (20) \\
&Y_{jik} \geq 0,1 \qquad \forall i \in I\ ,\ k \in K\ ,\ j \in J && (28) \\
&U_j \geq 0 \qquad \forall j \in J && (30)
\end{aligned}$$

P2 and P1 can be trivially shown to be equal but with linear objective function and quadratic terms in the constraints. As shown by You and Grossmann (2008), the following proposition can be established for problem P2.

Proposition2. *In the global optimal solution of problem P2 or a local optimal solution with fixed binary values for* X_j *, all the continuous variables* Y_{jik} *take on integer values (0 or 1).*

Now we just need to solve P2 to get the global optimal or near optimal solutions for P1 and P0. This is accomplished using "SCIP" solver in GAMS. In the next section, the SCIP solver, used to solve P2 is briefly presented.

4.1.1 SCIP Solver in GAMS

SCIP (Solving Constraint Integer Programs) was developed at the Konrad-Zuse-Zentrum fuerr Informationstechnik in Berlin (ZIB). SCIP is only available for users with a GAMS academic license. SCIP is a framework for solving Constrained Integer Programming, especially to address the needs of Mathematical Programming experts who want to have total control of the solution process and access all internal information of the solver. SCIP can also be used as a pure MIP solver or as a framework for branch-cut-and-price. Within GAMS, the MIP and MIQCP solving facilities of SCIP are available. SCIP has different features and plugins to handle constrained integer programming. In the following discussion, we briefly present these plugins and their roles in solving constraints integer programming through SCIP solver (Achterberg, 2007).

Constraint handlers

Each constraint handler provides algorithms to handle constraints with the same class.

The initial task is to check a given solution for feasibility with respect to all constraints of its type existing in the problem instance. So the resulting procedure would be a complete enumeration of all potential solutions because no additional information is available. Also to improve the efficiency in finding a solution, the constraint handlers may use pre-solving methods, propagation methods, linear relaxation, and branching decisions.

Presolvers

In addition to constraint based pre-solving algorithms, SCIP perform dual pre-solving reductions with respect to the objective function.

Cut Separators

In SCIP, there are two different types of cutting planes. The first type involve constraint-based cutting planes, that are valid inequalities or even facets of the polyhedron described by a single constraint or a subset of the constraints of a single constraint class. The second type of cutting planes is general purpose cuts, which use the current LP relaxation and the integrality conditions to generate valid inequalities. Generating those cuts is the task of cut separators.

Domain Propagators

As same as “Cut Separators”, there are two different Domain Propagations: Constraint based (primal) algorithms, and objective function (dual) based algorithms. An example is the simple objective function propagator that tightens the variables’ domains with respect to the objective bound $c^T x < \hat{c}$ with $\hat{c}$ being the objective value of the current best primal solution.

Variable Pricers

Several optimization problems are modeled with a huge number of variables. In this case, the full set of variables cannot be generated in advance. Instead, the variables are added dynamically to the problem whenever they may improve the current solution. In mixed integer programming, this technique is called column generation. SCIP supports dynamic variable creation by variable pricers. They are called upon during sub-problem processing and have to generate additional variables that reduce the lower bound of the sub-problem. If they operate on the LP relaxation, they would usually calculate the reduced costs of the not yet existing variables with a problem specific algorithm and add some or all of the variables with negative reduced costs. Note that since variable pricers are part of the model, they are always problem class specific. Therefore, SCIP does not contain any "default" variable pricers.

Branching Rules

If the LP solution of the current subproblem is fractional, the integrality constraint handler calls the branching rules to split the problems into subproblems. Usually, a branching rule creates two subproblems by splitting a single variable's domain.

Node Selectors

Node selectors decide which of the leaves in the current branching tree is selected as next sub-problem to be processed. This choice can have a large impact on the solver's performance, because it influences the search speed for the feasible solutions and the development of the global dual bound.

Primal Heuristics

SCIP provides specific infrastructure for diving and probing heuristics. Diving heuristics iteratively resolves the LP after making a few changes to the current sub-problem, usually aiming at driving the fractional values of integer variables to integrality. Probing heuristics are even more sophisticated. Besides solving LP relaxations, they may call the domain propagation algorithms of the constraint handlers after applying changes to the variables' domains, and they can undo these changes by backtracking. Other heuristics such as rounding heuristics, objective diving heuristic, and improvement heuristics are also used in SCIP solver.

Relaxation Handlers

SCIP provides specific support for LP relaxations: constraint handlers implement callback methods for generating the LP, additional cut separators may be included to further tighten the LP relaxation, and there are a lot of interface methods available to access the LP information at the current subproblem.

SCIP also contains other plugins such as "Event Handlers", "Conflict Handlers", "Dialog Handlers", and "Message Handlers". For example "Conflict Handlers" can be applied to learn from infeasible sub-problems. SCIP uses additional relaxations (e.g., semidefinite relaxations or Lagrangian relaxations) working in parallel or interleaved. Another important feature of SCIP is the dynamic memory management which reduces the number of operation system calls with automatic memory leakage detection in debug mode.

4.2 Parameters setting

As mentioned earlier, solving P2 is sufficient to get a global or local optimum for the original problem P0. Before using SCIP in GAMS to solve P2, parameters settings are needed to test different scenarios in our problem. Some of these parameter settings are shown in Table 1.

Parameters	Values
f_j	Uniformly distributed random numbers between [80,120]. (see Table 4)
d_{ji}	0.0854
l_{ji}	2(1.14)Great circle distance between i & j
h	12
Z_α	1.64, 1.96
M	5
z_{ji}	50,75,100

Table 2- Parameters setting values

The annual average and variance of surpluses and deliveries for all candidate nodes (60 nodes) for any product-type are taken from company data. Also some missing data and coefficient of variations of all nodes are randomly generated because of lack of data. There is no average and variance for existing DCs (i.e. no demand/donations in the current DCs) and no demand in existing ADCs. These values are derived from the annual number of trips from Oct 2011 to Sep 2012. Also as advised by the company sources, deliveries and surplus percentage for any product type are different in various months. During Sep-May, deliveries and surpluses are about 80% and 20% respectively, but during June-Aug, these percentages change to 40% and 60% respectively. Also the product-types ratios are different from one month to the other. During Sep-May, HL and SL ratios are 66% and 34% respectively and in June, July, and August, these ratios change to 35% and 65% respectively. These ratios and assumptions are used to calculate the annual average of deliveries and surpluses in terms of the number of Gaylord for any product type (herein HL and SL) in all nodes. For the stores without information about the number of trips, the annual number of trips is a uniformly distributed random number generated with mean 125 and standard deviation of 46. As mentioned in assumptions section, 25 Gaylord of any product type is shipped in each trip, equal to the capacity of a truck.

The coefficients of variation (CV) are used to calculate the annual standard deviation of deliveries and surpluses in terms of the number of Gaylord for any product type in all nodes. CV is generated as a uniformly distributed random number between 0.1-0.4. This range is reasonable based on the literature review. According to equation (1) in the problem definition section, the total variance of difference between surpluses and deliveries for all product types is calculated. For illustrative purposes, Table 2 shows a summary of only 10 nodes in the system including annual average # of trips, annual average # of deliveries and surplus, and mean CV of deliveries and surplus.

Fixed location costs (f_j), of 10 nodes are also presented in Table 3 for illustration

purposes only. Similarly to the fixed costs in Table 3, values for all 60 nodes are randomly generated as uniformly distribution in the [80,120] interval. These interval limits are representative of the range of warehouse fixed costs. As was mentioned in parameter description section, l_{ji} is the total distance travelled, which is double the estimated distance between any two nodes.

STORE CODE	ANNUALIZED SEP'12 - SEP'11	# of TRIPS According to Retail and ADC Ratios		Coefficient of Variation (CV) Assumption: CV ~ U(0.1 - 0.4)	
		MU - Delivery	MU - Surplus	CV - Delivery	CV - Surplus
GW03	119	83	36	0.24	0.25
GW05	79	55	24	0.18	0.26
GW07	136	0	136	0.25	0.24
GW09	99	69	30	0.32	0.27
GW11	99	69	30	0.21	0.24
GW13	99	69	30	0.36	0.30
GW15	263	184	79	0.25	0.39
GW17	117	82	35	0.28	0.19
GW19	108	76	32	0.37	0.22
GW21	102	71	30	0.32	0.35

Table 3- Annual average # of trips, Deliveries, Surplus

Store Code	f_j
GW01	**115**
GW02	**88**
GW03	**86**
GW04	**111**
GW05	**94**
GW06	**106**
GW07	**115**
GW08	**101**
GW09	**90**
GW10	**89**

Table 3- Fixed location costs for 10 nodes

CHAPTER 5: NUMERICAL EXAMPLES, RESULTS, CONCLUSION, AND FUTURE RESEARCH

5.1 Numerical Examples

Three set of nodes are tested for numerical examples; 30, 45, and 60 nodes. The 30-node set includes the odd-numbered nodes (GW01, GW03, GW05, etc) only. The 45-node set includes the first 30 nodes, in addition to 15 other nodes in multiples of four (i.e. GW04, GW08, etc). The 60-node set includes all nodes in the supply chain system. For any problem set, different settings of β, θ , coverage radius, and z_α are used as experimental scenarios to test the problem. These scenarios (numerical examples) were run using the relaxation model P2, written in GAMS. β and θ take the values 0.1, 0.01, and 0.001, so the total number of combinations (β, θ) is nine. Coverage radius is chosen from {50, 75,100} in miles and z_α is chosen from {1.64, 1.96}. So the total number of experiments for any set of nodes is 9*3*2 (i.e. 54).

Model outputs include: solution gap, solution time, annual facility location cost, total annual transportation cost, annual average inventory cost, total annual safety stock cost, total system cost (objective function value), opened DCs, retailer assignments. We note that in SCIP solver the solution gap is the difference between upper bound (feasible solution) and lower bound (the infeasible heuristic solution). Tables 4 a-d, 5 a-d, and 6a-d present all numerical examples in the model for 30, 45, and 60- node sets respectively.

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
1	30	0.001	0.001	50	1.96	1	0	10	244	265	361	139	1,010
2	30	0.001	0.001	50	1.64	1	0	12	244	265	361	117	987
3	30	0.001	0.001	75	1.96	1	0	0.18	166	266	361	116	909
4	30	0.001	0.001	75	1.64	1	0	0.14	166	266	361	97	890
5	30	0.001	0.001	100	1.96	1	0	0.47	166	266	361	116	909
6	30	0.001	0.001	100	1.64	1	0	0.65	166	266	361	97	890
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
7	30	0.001	0.01	50	1.96	0.1	0	131	244	303	3,613	1,311	5,471
8	30	0.001	0.01	50	1.64	0.1	0	85	244	288	3,613	1,111	5,256
9	30	0.001	0.01	75	1.96	0.1	0	49	166	266	3,613	1,160	5,204
10	30	0.001	0.01	75	1.64	0.1	0	34	166	266	3,613	970	5,015
11	30	0.001	0.01	100	1.96	0.1	0	52	166	266	3,613	1,160	5,204
12	30	0.001	0.01	100	1.64	0.1	0	43	166	266	3,613	970	5,015
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
13	30	0.001	0.1	50	1.96	0.01	0	995	244	303	36,126	13,112	49,784
14	30	0.001	0.1	50	1.64	0.01	0	896	244	303	36,126	10,971	47,644
15	30	0.001	0.1	75	1.96	0.01	0	175	195	631	36,126	10,212	47,163
16	30	0.001	0.1	75	1.64	0.01	0	181	195	631	36,126	8,545	45,496
17	30	0.001	0.1	100	1.96	0.01	0	6	111	829	36,126	8,201	45,266
18	30	0.001	0.1	100	1.64	0.01	0	5	111	829	36,126	6,862	43,928
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
19	30	0.01	0.001	50	1.96	10	0	14	480	1,886	361	176	2,903
20	30	0.01	0.001	50	1.64	10	0	12	480	1,886	361	147	2,874
21	30	0.01	0.001	75	1.96	10	0	21	476	1,884	361	176	2,897
22	30	0.01	0.001	75	1.64	10	0	23	476	1,884	361	147	2,869
23	30	0.01	0.001	100	1.96	10	0	31	476	1,884	361	176	2,897
24	30	0.01	0.001	100	1.64	10	0	28	476	1,884	361	147	2,869
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
25	30	0.01	0.01	50	1.96	1	0	133	316	2,401	3,613	1,350	7,680
26	30	0.01	0.01	50	1.64	1	0	133	389	2,130	3,613	1,308	7,439
27	30	0.01	0.01	75	1.96	1	0	616	166	2,661	3,613	1,160	7,600
28	30	0.01	0.01	75	1.64	1	0	547	299	2,339	3,613	1,130	7,380
29	30	0.01	0.01	100	1.96	1	0	776	166	2,661	3,613	1,160	7,600
30	30	0.01	0.01	100	1.64	1	0	748	299	2,339	3,613	1,130	7,380

Table 4a- Gap/Time/Costs in 30 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
31	30	0.01	0.1	50	1.96	0.1	0	415	316	2,481	36,126	13,352	52,276
32	30	0.01	0.1	50	1.64	0.1	0	370	316	2,481	36,126	11,172	50,096
33	30	0.01	0.1	75	1.96	0.1	0	1146	166	2,661	36,126	11,595	50,549
34	30	0.01	0.1	75	1.64	0.1	0	1177	166	2,661	36,126	9,702	48,656
35	30	0.01	0.1	100	1.96	0.1	0	1836	166	2,661	36,126	11,595	50,549
36	30	0.01	0.1	100	1.64	0.1	0	1778	166	2,661	36,126	9,702	48,656
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
37	30	0.1	0.001	50	1.96	100	0	7	480	18,859	361	176	19,877
38	30	0.1	0.001	50	1.64	100	0	8	480	18,859	361	147	19,848
39	30	0.1	0.001	75	1.96	100	0	9	481	18,807	361	176	19,826
40	30	0.1	0.001	75	1.64	100	0	12	481	18,807	361	147	19,797
41	30	0.1	0.001	100	1.96	100	0	20	481	18,807	361	176	19,826
42	30	0.1	0.001	100	1.64	100	0	23	481	18,807	361	147	19,797
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
43	30	0.1	0.01	50	1.96	10	0	17	480	18,859	3,613	1,760	24,712
44	30	0.1	0.01	50	1.64	10	0	13	480	18,859	3,613	1,473	24,425
45	30	0.1	0.01	75	1.96	10	0	28	481	18,807	3,613	1,761	24,662
46	30	0.1	0.01	75	1.64	10	0	19	481	18,807	3,613	1,474	24,375
47	30	0.1	0.01	100	1.96	10	0	42	481	18,807	3,613	1,761	24,662
48	30	0.1	0.01	100	1.64	10	0	43	481	18,807	3,613	1,474	24,375
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
49	30	0.1	0.1	50	1.96	1	0	182	480	18,859	36,126	17,605	73,070
50	30	0.1	0.1	50	1.64	1	0	129	480	18,859	36,126	14,731	70,196
51	30	0.1	0.1	75	1.96	1	0	1227	429	20,485	36,126	15,719	72,759
52	30	0.1	0.1	75	1.64	1	0	1140	481	18,807	36,126	14,737	70,152
53	30	0.1	0.1	100	1.96	1	0	1278	429	20,485	36,126	15,719	72,759
54	30	0.1	0.1	100	1.64	1	0	1284	481	18,807	36,126	14,737	70,152

Table 4b- Gap/Time/Costs in 30 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
1	30	0.001	0.001	50	1.96	1	{19,27,45}	{**19:** 19,31,47,49,55} {**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**45:** 5,7,11,15,29,43,45,53}
2	30	0.001	0.001	50	1.64	1	{19,27,45}	{**19:** 19,31,47,49,55} {**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**45:** 5,7,11,15,29,43,45,53}
3	30	0.001	0.001	75	1.96	1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
4	30	0.001	0.001	75	1.64	1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
5	30	0.001	0.001	100	1.96	1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
6	30	0.001	0.001	100	1.64	1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	DCs	Assignments
7	30	0.001	0.01	50	1.96	0.1	{19,27,45}	{**19:** 7,15,19,29HL,31,45,47HL,49,53SL,55SL} {**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**45:** 5,11,29SL,43,47SL,53HL,55HL}
8	30	0.001	0.01	50	1.64	0.1	{19,27,45}	{**19:** 7,15,19,29HL,31,47,49,53SL,55} {**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**45:** 5,11,29SL,43,45,53HL}
9	30	0.001	0.01	75	1.96	0.1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
10	30	0.001	0.01	75	1.64	0.1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
11	30	0.001	0.01	100	1.96	0.1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
12	30	0.001	0.01	100	1.64	0.1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	DCs	Assignments
13	30	0.001	0.1	50	1.96	0.01	{19,27,45}	{**19:** 7,15,19,29HL,31,45,47HL,49,53SL,55SL} {**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**45:** 5,11,29SL,43,47SL,53HL,55HL}
14	30	0.001	0.1	50	1.64	0.01	{19,27,45}	{**19:** 7,15,19,29HL,31,45,47HL,49,53SL,55SL} {**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**45:** 5,11,29SL,43,47SL,53HL,55HL}
15	30	0.001	0.1	75	1.96	0.01	{19,35}	{**19:** 5,19,47SL,49,53HL,55} {**35:** All rest of retailers and products}
16	30	0.001	0.1	75	1.64	0.01	{19,35}	{**19:** 5,19,47SL,49,53HL,55} {**35:** All rest of retailers and products}
17	30	0.001	0.1	100	1.96	0.01	{23}	{**23:** ALL retailers and products}
18	30	0.001	0.1	100	1.64	0.01	{23}	{**23:** ALL retailers and products}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	DCs	Assignments
19	30	0.01	0.001	50	1.96	10	{9,27,37,47,55}	{**9:** 1,3,9,11,35} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39,51} {**47:** 7,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
20	30	0.01	0.001	50	1.64	10	{9,27,37,47,55}	{**9:** 1,3,9,11,35} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39,51} {**47:** 7,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
21	30	0.01	0.001	75	1.96	10	{3,27,37,47,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**47:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
22	30	0.01	0.001	75	1.64	10	{3,27,37,47,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**47:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
23	30	0.01	0.001	100	1.96	10	{3,27,37,47,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**47:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
24	30	0.01	0.001	100	1.64	10	{3,27,37,47,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**47:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	DCs	Assignments
25	30	0.01	0.01	50	1.96	1	{15,27,55}	{**15:** 7,11,15,29,31,45,47,53} {**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**55:** 5,19,43,49,55}
26	30	0.01	0.01	50	1.64	1	{9,27,47,55}	{**9:** 1HL,3SL,9,11,35,51} {**27: 1SL,3HL,**13,17,21,25,27,33,37,39,41,57} {**47:** 7,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
27	30	0.01	0.01	75	1.96	1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}

Table 4c-DC locations and assignments in 30 nodes

28	30	0.01	0.01	75	1.64	1	{27,47,55}	{**27: 1,3,9,**13,17,21,25,27,33,35,37,39,41,51,57} {**47:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
29	30	0.01	0.01	100	1.96	1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
30	30	0.01	0.01	100	1.64	1	{27,47,55}	{**27: 1,3,9,**13,17,21,25,27,33,35,37,39,41,51,57} {**47:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	**DCs**	**Assignments**
31	30	0.01	0.1	50	1.96	0.1	{15,27,55}	{**15:** 7,11,15,19HL,29,31,45,47,53SL} {**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**55:** 5,19SL,43,49,53HL,55}
32	30	0.01	0.1	50	1.64	0.1	{15,27,55}	{**15:** 7,11,15,19HL,29,31,45,47,53SL} {**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**55:** 5,19SL,43,49,53HL,55}
33	30	0.01	0.1	75	1.96	0.1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
34	30	0.01	0.1	75	1.64	0.1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
35	30	0.01	0.1	100	1.96	0.1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
36	30	0.01	0.1	100	1.64	0.1	{27,53}	{**27:** 1,3,9,13,17,21,25,27,33,35,37,39,41,51,57} {**53:** 5,7,11,15,19,29,31,43,45,47,49,53,55}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	**DCs**	**Assignments**
37	30	0.1	0.001	50	1.96	100	{9,27,37,47,55}	{**9:** 1,3,9,11,35} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39,51} {**47:** 7,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
38	30	0.1	0.001	50	1.64	100	{9,27,37,47,55}	{**9:** 1,3,9,11,35} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39,51} {**47:** 7,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
39	30	0.1	0.001	75	1.96	100	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
40	30	0.1	0.001	75	1.64	100	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
41	30	0.1	0.001	100	1.96	100	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
42	30	0.1	0.001	100	1.64	100	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	**DCs**	**Assignments**
43	30	0.1	0.01	50	1.96	10	{9,27,37,47,55}	{**9:** 1,3,9,11,35} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39,51} {**47:** 7,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
44	30	0.1	0.01	50	1.64	10	{9,27,37,47,55}	{**9:** 1,3,9,11,35} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39,51} {**47:** 7,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
45	30	0.1	0.01	75	1.96	10	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
46	30	0.1	0.01	75	1.64	10	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
47	30	0.1	0.01	100	1.96	10	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
48	30	0.1	0.01	100	1.64	10	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	**DCs**	**Assignments**
49	30	0.1	0.1	50	1.96	1	{9,27,37,47,55}	{**9:** 1,3,9,11,35} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39,51} {**47:** 7,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
50	30	0.1	0.1	50	1.64	1	{9,27,37,47,55}	{**9:** 1,3,9,11,35} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39,51} {**47:** 7,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
51	30	0.1	0.1	75	1.96	1	{17,25,29,55}	{**17:** 1,3,9,13,17,27,35,37,41,51} {**25:** 21,25,33,39,57} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
52	30	0.1	0.1	75	1.64	1	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
53	30	0.1	0.1	100	1.96	1	{17,25,29,55}	{**17:** 1,3,9,13,17,27,35,37,41,51} {**25:** 21,25,33,39,57} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}
54	30	0.1	0.1	100	1.64	1	{3,27,29,37,55}	{**3:**1,3,9,35,51} {**27:** 13,17,27,41,57} {**37:** 21,25,33,37,39} {**29:** 7,11,15,29,31,45,47,53} {**55:** 5,19,43,49,55}

Table 4d- DC locations and assignments in 30 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
1	45	0.001	0.001	50	1.96	1	0	7	267	1,272	434	157	1,272
2	45	0.001	0.001	50	1.64	1	0	23	267	1,272	434	131	1,246
3	45	0.001	0.001	75	1.96	1	0	1	166	410	434	137	1,147
4	45	0.001	0.001	75	1.64	1	0	1	166	410	434	115	1,125
5	45	0.001	0.001	100	1.96	1	0	2	166	410	434	137	1,147
6	45	0.001	0.001	100	1.64	1	0	2	166	410	434	115	1,125
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
7	45	0.001	0.01	50	1.96	0.1	0	1,606	267	416	4,342	1,562	6,586
8	45	0.001	0.01	50	1.64	0.1	0	546	267	416	4,342	1,562	6,586
9	45	0.001	0.01	75	1.96	0.1	0	757	166	410	4,342	1,370	6,288
10	45	0.001	0.01	75	1.64	0.1	0	479	166	410	4,342	1,146	6,064
11	45	0.001	0.01	100	1.96	0.1	0	852	166	410	4,342	1,370	6,288
12	45	0.001	0.01	100	1.64	0.1	0	595	166	410	4,342	1,146	6,064
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
13	45	0.001	0.1	50	1.96	0.01	0	7,098	285	487	43,415	15,264	59,451
14	45	0.001	0.1	50	1.64	0.01	0	9,312	285	487	43,415	12,772	56,959
15	45	0.001	0.1	75	1.96	0.01	0	1,803	183	1,004	43,415	11,753	56,355
16	45	0.001	0.1	75	1.64	0.01	0	3,076	183	1,004	43,415	9,834	54,437
17	45	0.001	0.1	100	1.96	0.01	0	21	111	1,221	43,415	9,687	54,434
18	45	0.001	0.1	100	1.64	0.01	0	37	111	1,221	43,415	8,105	52,853
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
19	45	0.01	0.001	50	1.96	10	0	474	692	2,444	434	247	3,817
20	45	0.01	0.001	50	1.64	10	0	534	692	2,444	434	207	3,777
21	45	0.01	0.001	75	1.96	10	0	339	666	2,406	434	251	3,757
22	45	0.01	0.001	75	1.64	10	0	180	666	2,406	434	210	3,716
23	45	0.01	0.001	100	1.96	10	0	751	666	2,406	434	251	3,757
24	45	0.01	0.001	100	1.64	10	0	262	666	2,406	434	210	3,716
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
25	45	0.01	0.01	50	1.96	1	12.39	3,005	586	2,671	4,342	2,274	9,873
26	45	0.01	0.01	50	1.64	1	10.28	3,758	687	2,538	4,342	2,046	9,613
27	45	0.01	0.01	75	1.96	1	20.04	3,628	560	2,633	4,342	2,315	9,850
28	45	0.01	0.01	75	1.64	1	17.02	3,731	666	2,406	4,342	2,100	9,514
29	45	0.01	0.01	100	1.96	1	20.34	9,173	666	2,406	4,342	2,510	9,923
30	45	0.01	0.01	100	1.64	1	16.45	3,810	666	2,406	4,342	2,100	9,514

Table 5a- Gap/Time/Costs in 45 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
31	45	0.01	0.1	50	1.96	0.1	12.4	3,713	285	4,216	43,415	15,471	63,388
32	45	0.01	0.1	50	1.64	0.1	10.12	4,232	285	4,216	43,415	12,945	60,862
33	45	0.01	0.1	75	1.96	0.1	18.92	8,963	302	4,647	43,415	15,563	63,927
34	45	0.01	0.1	75	1.64	0.1	17.39	3,670	279	4,307	43,415	12,854	60,855
35	45	0.01	0.1	100	1.96	0.1	13.8	7,008	184	4,313	43,415	13,699	61,612
36	45	0.01	0.1	100	1.64	0.1	9.52	6,664	184	4,313	43,415	11,463	59,375
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
37	45	0.1	0.001	50	1.96	100	0	37	770	23,598	434	267	25,069
38	45	0.1	0.001	50	1.64	100	0	31	770	23,598	434	223	25,026
39	45	0.1	0.001	75	1.96	100	0	71	777	23,265	434	268	24,744
40	45	0.1	0.001	75	1.64	100	0	76	777	23,265	434	224	24,701
41	45	0.1	0.001	100	1.96	100	0	103	777	23,265	434	268	24,744
42	45	0.1	0.001	100	1.64	100	0	85	777	23,265	434	224	24,701
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
43	45	0.1	0.01	50	1.96	10	0	1,592	770	23,598	4,342	2,670	31,380
44	45	0.1	0.01	50	1.64	10	0	777	770	23,598	4,342	2,234	30,944
45	45	0.1	0.01	75	1.96	10	2.7	4,180	754	23,424	4,342	2,671	31,191
46	45	0.1	0.01	75	1.64	10	0	3,494	777	23,265	4,342	2,242	30,625
47	45	0.1	0.01	100	1.96	10	2.95	3,624	777	23,265	4,342	2,679	31,063
48	45	0.1	0.01	100	1.64	10	3.87	5,783	777	23,265	4,342	2,242	30,625
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Thet	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
49	45	0.1	0.1	50	1.96	1	23/28	18963/383	702	25,667	43,415	24,576	94,361
50	45	0.1	0.1	50	1.64	1	17.5/22.5	18323/325	692	24,441	43,415	20,658	89,207
51	45	0.1	0.1	75	1.96	1	30	955	711	24,853	43,415	24,740	93,720
52	45	0.1	0.1	75	1.64	1	24	4,090	711	24,853	43,415	20,701	89,681
53	45	0.1	0.1	100	1.96	1	29	2,842	692	24,441	43,415	24,689	93,238
54	45	0.1	0.1	100	1.64	1	22.82	3,195	675	24,310	43,415	21,062	89,462

Table 5b- Gap/Time/Costs in 45 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
1	45	0.001	0.001	50	1.96	1	{11,16,27}	{11: 9HL,11,20,36SL,43,48} {16: 5,7,15,16,28,29,31,32,36HL,45,47,49,52,53,55} {27: 1,3,4,8,9SL,12,13,17,19,21,24,25,27,33,35,37,39,40,41,44,51,56,57,59}
2	45	0.001	0.001	50	1.64	1	{11,16,27}	{11: 9HL,11,20,36SL,43,48} {16: 5,7,15,16,28,29,31,32,36HL,45,47,49,52,53,55} {27: 1,3,4,8,9SL,12,13,17,19,21,24,25,27,33,35,37,39,40,41,44,51,56,57,59}
3	45	0.001	0.001	75	1.96	1	{27,53}	{53: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers and products}
4	45	0.001	0.001	75	1.64	1	{27,53}	{53: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers and products}
5	45	0.001	0.001	100	1.96	1	{27,53}	{53: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers and products}
6	45	0.001	0.001	100	1.64	1	{27,53}	{53: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers and products}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
7	45	0.001	0.01	50	1.96	0.1	{11,16,27}	{11: 9HL,11,20,24SL,36SL,43HL,48} {16: 5,7,15,16,28,29,31,32,36HL,43SL,45,47,49,52,53,55} {27: 1,3,4,8,9SL,12,13,17,19,21,24HL,25,27,33,35,37,39,40,41,44,51,56,57,59}
8	45	0.001	0.01	50	1.64	0.1	{11,16,27}	{11: 9HL,11,20,24SL,36SL,43HL,48} {16: 5,7,15,16,28,29,31,32,36HL,43SL,45,47,49,52,53,55} {27: 1,3,4,8,9SL,12,13,17,19,21,24HL,25,27,33,35,37,39,40,41,44,51,56,57,59}
9	45	0.001	0.01	75	1.96	0.1	{27,53}	{53: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers and products}
10	45	0.001	0.01	75	1.64	0.1	{27,53}	{53: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers and products}
11	45	0.001	0.01	100	1.96	0.1	{27,53}	{53: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers and products}
12	45	0.001	0.01	100	1.64	0.1	{27,53}	{53: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers and products}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
13	45	0.001	0.1	50	1.96	0.01	{16,27,43}	{16: 5,28SL,29SL,32HL,47SL,49,53HL,55HL} {43: 7,11,15,16,20,28HL,29HL,31,32SL,36,43,45,47HL,52,53SL,55SL} {27: ALL THE REST}
14	45	0.001	0.1	50	1.64	0.01	{16,27,43}	{16: 5,28SL,29SL,32HL,47SL,49,53HL,55HL} {43: 7,11,15,16,20,28HL,29HL,31,32SL,36,43,45,47HL,52,53SL,55SL} {27: ALL THE REST}
15	45	0.001	0.1	75	1.96	0.01	{19,48}	{19: 5,19,28SL,49,53HD,55} {48: All the rest of retailers}
16	45	0.001	0.1	75	1.64	0.01	{19,48}	{19: 5,19,28SL,49,53HD,55} {48: All the rest of retailers}
17	45	0.001	0.1	100	1.96	0.01	{23}	{23: all the retailers}
18	45	0.001	0.1	100	1.64	0.01	{23}	{23: all the retailers}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
19	45	0.01	0.001	50	1.96	10	{16,24,25,27,37,43,47}	{16:7,16,28,49,55} {24:1,9,24,35,48} {25: 21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37: 4,33,37,51,59} {43: 5,11,20,43,45} {47:15,29,31,32,36,47,52,53}
20	45	0.01	0.001	50	1.64	10	{16,24,25,27,37,43,47}	{16:7,16,28,49,55} {24:1,9,24,35,48} {25: 21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37: 4,33,37,51,59} {43: 5,11,20,43,45} {47:15,29,31,32,36,47,52,53}
21	45	0.01	0.001	75	1.96	10	{16,24,25,27,37,45,47}	{16:5,16,28,49,55} {24:1,9,24,35,48} {25:21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37:4,33,37,51,59} {45:7,11,20,36,43,45,52} {47:15,29,31,32,47,53}
22	45	0.01	0.001	75	1.64	10	{16,24,25,27,37,45,47}	{16:5,16,28,49,55} {24:1,9,24,35,48} {25:21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37:4,33,37,51,59} {45:7,11,20,36,43,45,52} {47:15,29,31,32,47,53}
23	45	0.01	0.001	100	1.96	10	{16,24,25,27,37,45,47}	{16:5,16,28,49,55} {24:1,9,24,35,48} {25:21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37:4,33,37,51,59} {45:7,11,20,36,43,45,52} {47:15,29,31,32,47,53}
24	45	0.01	0.001	100	1.64	10	{16,24,25,27,37,45,47}	{16:5,16,28,49,55} {24:1,9,24,35,48} {25:21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37:4,33,37,51,59} {45:7,11,20,36,43,45,52} {47:15,29,31,32,47,53}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
25	45	0.01	0.01	50	1.96	1	{16,24,27,37,43,47}	{16: 7,16,28,49,55} {24: 1,9,24,35,48} {27: 3,8,12,13,17,19,27,40,41,44,56,57} {37: 4,21,25,33,37,39,51,59} {43: 5,11,20,43,45} {47: 15,29,31,32,36,47,52,53}
26	45	0.01	0.01	50	1.64	1	{3,16,24,25,27,43,47}	{3: 3,4,9,51,59} {16: 7,16,28,49,55} {24: 1,24,35,40,48} {25: 21,25,33,39,44} {27: 8,12,13,17,19,27,37,41,56,57} {43: 5,11,20,43,45} {47: 15,29,31,32,36,47,52,53}
27	45	0.01	0.01	75	1.96	1	{16,24,27,37,45,47}	{16: 5,16,28,49,55} {24: 1,9,24,35,48} {27: 3,8,12,13,17,19,27,40,41,44,56,57} {37: 4,21,25,33,37,39,51,59} {45: 7,11,20,36,43,45,52} {47: 15,29,31,32,47,53}
28	45	0.01	0.01	75	1.64	1	{16,24,25,27,37,45,47}	{16:5,16,28,49,55} {24:1,9,24,35,48} {25:21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37:4,33,37,51,59} {45:7,11,20,36,43,45,52} {47:15,29,31,32,47,53}
29	45	0.01	0.01	100	1.96	1	{16,24,25,27,37,45,47}	{16:5,16,28,49,55} {24:1,9,24,35,48} {25:21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37:4,33,37,51,59} {45:7,11,20,36,43,45,52} {47:15,29,31,32,47,53}
30	45	0.01	0.01	100	1.64	1	{16,24,25,27,37,45,47}	{16:5,16,28,49,55} {24:1,9,24,35,48} {25:21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37:4,33,37,51,59} {45:7,11,20,36,43,45,52} {47:15,29,31,32,47,53}

Table 5c- DC locations and assignments in 45 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
31	45	0.01	0.1	50	1.96	0.1	{16,27,43}	{16: 7,15,16,28,29,31,32,36HL,45,47,49,52,53,55SL} {43: 5,11,20,36SL,43,55HL} {27: ALL THE REST}
32	45	0.01	0.1	50	1.64	0.1	{16,27,43}	{16: 7,15,16,28,29,31,32,36HL,45,47,49,52,53,55SL} {43: 5,11,20,36SL,43,55HL} {27: ALL THE REST}
33	45	0.01	0.1	75	1.96	0.1	{1,3,47}	,39SL,40HL,41SL,48HL,56HL,59SL} {3: 1HL,3,4,8,9,12,13,17,21,24HL,25,27SL,33,35,37,39HL,40SL,41HL,44,48SL,51,56SL,57,59HL} {47: 5,7,11SL,15,16,19,20,28,29,31,32,36
34	45	0.01	0.1	75	1.64	0.1	{16,27,47}	6: 5SL,7,15,16,20,28,31,32SL,36HL,43,45,47HL,49,52,55} {27: 1,3,4,8,9,12,13,17,19,21,24,25,27,33,35,37,39,40,41,44,48,51,56,57,59} {47: 5HL,11,29,32HL,36SL,47SL,5
35	45	0.01	0.1	100	1.96	0.1	{27,47}	{47: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers}
36	45	0.01	0.1	100	1.64	0.1	{27,47}	{47: 5,7,11,15,16,20,28,29,31,32,36,43,45,47,49,52,53,55} {27: All rest of retailers}
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
37	45	0.1	0.001	50	1.96	100	{1,11,16,25,27,37,47,52}	1,12,24,35,40} {11: 9,11,20,43,48} {16: 5,16,28,49,55} {25:21,25,39,44,57} {27: 3,8,13,17,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,36,45,52,
38	45	0.1	0.001	50	1.64	100	{1,11,16,25,27,37,47,52}	1,12,24,35,40} {11: 9,11,20,43,48} {16: 5,16,28,49,55} {25:21,25,39,44,57} {27: 3,8,13,17,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,36,45,52,
39	45	0.1	0.001	75	1.96	100	{ 16,17,24,25,27,37,47,52}	:5,16,28,49,55} {17: 3,8,12,17,40} {24: 1,9,24,35,48} {25:21,25,39,44,57} {27: 13,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,11,20.36,43,45,52,
40	45	0.1	0.001	75	1.64	100	{ 16,17,24,25,27,37,47,52}	:5,16,28,49,55} {17: 3,8,12,17,40} {24: 1,9,24,35,48} {25:21,25,39,44,57} {27: 13,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,11,20.36,43,45,52,
41	45	0.1	0.001	100	1.96	100	{ 16,17,24,25,27,37,47,52}	:5,16,28,49,55} {17: 3,8,12,17,40} {24: 1,9,24,35,48} {25:21,25,39,44,57} {27: 13,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,11,20.36,43,45,52,
42	45	0.1	0.001	100	1.64	100	{ 16,17,24,25,27,37,47,52}	:5,16,28,49,55} {17: 3,8,12,17,40} {24: 1,9,24,35,48} {25:21,25,39,44,57} {27: 13,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,11,20.36,43,45,52,
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
43	45	0.1	0.01	50	1.96	10	{1,11,16,25,27,37,47,52}	1,12,24,35,40} {11: 9,11,20,43,48} {16: 5,16,28,49,55} {25:21,25,39,44,57} {27: 3,8,13,17,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,36,45,52,
44	45	0.1	0.01	50	1.64	10	{1,11,16,25,27,37,47,52}	1,12,24,35,40} {11: 9,11,20,43,48} {16: 5,16,28,49,55} {25:21,25,39,44,57} {27: 3,8,13,17,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,36,45,52,
45	45	0.1	0.01	75	1.96	10	{13,16,24,25,27,37,45,47}	12,13,40,41,56} {16: 5,16,28,49,55} {24: 1,9,24,35,48} {25:21,25,39,44,57} {27: 3,8,17,19,27} {37: 4,33,37,51,59} {45: 7,11,20,36,43,45,52} {47: 15,29,31,32,
46	45	0.1	0.01	75	1.64	10	{ 16,17,24,25,27,37,47,52}	5,16,28,49,55} {17: 3,8,12,17,40} {24: 1,9,24,35,48} {25:21,25,39,44,57} {27: 13,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,11,20.36,43,45,52,
47	45	0.1	0.01	100	1.96	10	{ 16,17,24,25,27,37,47,52}	:5,16,28,49,55} {17: 3,8,12,17,40} {24: 1,9,24,35,48} {25:21,25,39,44,57} {27: 13,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,11,20.36,43,45,52,
48	45	0.1	0.01	100	1.64	10	{ 16,17,24,25,27,37,47,52}	:5,16,28,49,55} {17: 3,8,12,17,40} {24: 1,9,24,35,48} {25:21,25,39,44,57} {27: 13,19,27,41,56} {37: 4,33,37,51,59} {47: 15,29,31,32,47} {52: 7,11,20.36,43,45,52,
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
49	45	0.1	0.1	50	1.96	1	{ 16,17,24,37,43,47,56}	6: 7,16,19,28,49,55} {17: 3,8,12,13,17,27} {24: 1,9,24,35,48} {37: 4,21,25,33,37,39,51,59} {43: 5,11,20,43,45} {47: 15,29,31,32,36,47,52,53} {56: 40,41,44,56,5
50	45	0.1	0.1	50	1.64	1	{16,24,25,27,37,43,47}	{16:7,16,28,49,55} {24:1,9,24,35,48} {25: 21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37: 4,33,37,51,59} {43: 5,11,20,43,45} {47:15,29,31,32,36,47,52,53}
51	45	0.1	0.1	75	1.96	1	{ 16,17,24,25,37,43,47}	16: 7,16,19,28,49,55} {17: 3,8,12,13,17,27,40,41,56} {24: 1,9,24,35,48} {25: 21,25,39,44,57} {37: 4,33,37,51,59} {43: 5,11,20,43,45} {47: 15,29,31,32,36,47,52,53}
52	45	0.1	0.1	75	1.64	1	{ 16,17,24,25,37,43,47}	16: 7,16,19,28,49,55} {17: 3,8,12,13,17,27,40,41,56} {24: 1,9,24,35,48} {25: 21,25,39,44,57} {37: 4,33,37,51,59} {43: 5,11,20,43,45} {47: 15,29,31,32,36,47,52,53}
53	45	0.1	0.1	100	1.96	1	{16,24,25,27,37,43,47}	{16:7,16,28,49,55} {24:1,9,24,35,48} {25: 21,25,39,44,57} {27:3,8,12,13,17,19,27,40,41,56} {37: 4,33,37,51,59} {43: 5,11,20,43,45} {47:15,29,31,32,36,47,52,53}
54	45	0.1	0.1	100	1.64	1	{24,25,27,28,29,37,52}	4: 1,9,24,35,48} {25: 21,25,39,44,57} {27: 3,8,12,13,17,19,27,40,41,56} {28: 5, 7SL,16,28,49,53,55} {29: 15,29,31,32,47} {37: 4,33,37,51,59} {52: 7HL,11,20,36,43,45,5

Table 5d- DC locations and assignments in 45 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
1	60	0.001	0.001	50	1.96	1	0	419	267	635	549	187	1,639
2	60	0.001	0.001	50	1.64	1	0	315	356	528	549	173	1,607
3	60	0.001	0.001	75	1.96	1	0	5	166	616	549	160	1,492
4	60	0.001	0.001	75	1.64	1	0	2	166	616	549	134	1,465
5	60	0.001	0.001	100	1.96	1	0	35	166	616	549	160	1,492
6	60	0.001	0.001	100	1.64	1	0	2	166	616	549	134	1,465
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
7	60	0.001	0.01	50	1.96	0.1	5.99	9,357	267	642	5,494	1,837	8,241
8	60	0.001	0.01	50	1.64	0.1	3.5	10,270	267	642	5,494	1,537	7,941
9	60	0.001	0.01	75	1.96	0.1	0	3,887	166	616	5,494	1,605	7,881
10	60	0.001	0.01	75	1.64	0.1	3.09	4,697	166	616	5,494	1,343	7,619
11	60	0.001	0.01	100	1.96	0.1	1.59	3,853	166	616	5,494	1,605	7,881
12	60	0.001	0.01	100	1.64	0.1	0	2,051	166	616	5,494	1,343	7,619
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
13	60	0.001	0.1	50	1.96	0.01	7.11	11,989	290	806	54,944	17,622	73,662
14	60	0.001	0.1	50	1.64	0.01	4.78	68,468	267	646	54,944	15,360	71,217
15	60	0.001	0.1	75	1.96	0.01	3.03	8,829	188	1,394	54,944	13,470	69,997
16	60	0.001	0.1	75	1.64	0.01	2.18	7,616	188	1,394	54,944	11,271	67,797
17	60	0.001	0.1	100	1.96	0.01	0	1,533	80	1,637	54,944	11,353	68,014
18	60	0.001	0.1	100	1.64	0.01	0	564	80	1,637	54,944	9,500	66,161
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
19	60	0.01	0.001	50	1.96	10	3.92	4,567	944	3,117	549	346	4,957
20	60	0.01	0.001	50	1.64	10	2.8	4,807	944	3,117	549	290	4,900
21	60	0.01	0.001	75	1.96	10	4.76	1,422	915	3,100	549	351	4,915
22	60	0.01	0.001	75	1.64	10	3.22	8,185	915	3,100	549	294	4,858
23	60	0.01	0.001	100	1.96	10	4.87	1,207	915	3,100	549	351	4,915
24	60	0.01	0.001	100	1.64	10	4.27	2,004	915	3,100	549	294	4,858
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
25	60	0.01	0.01	50	1.96	1	29	7,576	944	3,117	5,494	3,464	13,020
26	60	0.01	0.01	50	1.64	1	24.22	7,234	838	3,357	5,494	2,741	12,431
27	60	0.01	0.01	75	1.96	1	29.98	6,613	819	3,307	5,494	3,349	12,970
28	60	0.01	0.01	75	1.64	1	25.63	3,497	835	3,343	5,494	2,753	12,426
29	60	0.01	0.01	100	1.96	1	30	3,608	829	3,436	5,494	3,186	12,945
30	60	0.01	0.01	100	1.64	1	25	5,053	814	3,301	5,494	2,791	12,401

Table 6a- Gap/Time/Costs in 60 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
31	60	0.01	0.1	50	1.96	0.1	39	6,065	833	5,098	54,944	27,440	88,316
32	60	0.01	0.1	50	1.64	0.1	32.88	4,939	746	4,456	54,944	23,301	83,448
33	60	0.01	0.1	75	1.96	0.1	24.22	17,100	421	6,066	54,944	18,968	80,399
34	60	0.01	0.1	75	1.64	0.1	20.42	6,929	197	8,845	54,944	13,427	77,414
35	60	0.01	0.1	100	1.96	0.1	28.82	9,304	105	16,569	54,944	11,353	82,972
36	60	0.01	0.1	100	1.64	0.1	28.56	10,541	105	16,569	54,944	9,500	81,119
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
37	60	0.1	0.001	50	1.96	100	0	617	1,158	29,872	549	369	31,948
38	60	0.1	0.001	50	1.64	100	0	525	1,158	29,872	549	308	31,887
39	60	0.1	0.001	75	1.96	100	0	546	1,127	29,277	549	368	31,321
40	60	0.1	0.001	75	1.64	100	0	462	1,127	29,277	549	308	31,261
41	60	0.1	0.001	100	1.96	100	0	725	1,127	29,277	549	368	31,321
42	60	0.1	0.001	100	1.64	100	0	657	1,127	29,277	549	308	31,261
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
43	60	0.1	0.01	50	1.96	10	7.89	4,042	1,158	29,872	5,494	3,686	40,210
44	60	0.1	0.01	50	1.64	10	6.64	3,775	1,158	29,872	5,494	3,084	39,608
45	60	0.1	0.01	75	1.96	10	8.25	3,630	1,099	29,438	5,494	3,685	39,716
46	60	0.1	0.01	75	1.64	10	6.62	2957/39000	1,102	29,367	5,494	3,074	39,037
47	60	0.1	0.01	100	1.96	10	6.37/8.18	36899/3470	1,127	29,276	5,494	3,680	39,577
48	60	0.1	0.01	100	1.64	10	5.06/7.1	37273/3745	1,083	29,508	5,494	3,074	39,160
EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	Gap %	Time (sec)	DC_Cost	Trans_Cost	Mean inv. Cost	Service_Cost	Obj. Value
49	60	0.1	0.1	50	1.96	1	35.47	2693/37000	1,026	31,276	54,944	34,170	121,416
50	60	0.1	0.1	50	1.64	1	30	4,112	1,026	31,276	54,944	28,592	115,838
51	60	0.1	0.1	75	1.96	1	37	6,127	953	31,196	54,944	35,046	122,139
52	60	0.1	0.1	75	1.64	1	31	5,683	1,073	29,589	54,944	30,709	116,315
53	60	0.1	0.1	100	1.96	1	37	8,804	1,102	29,388	54,944	36,700	122,134
54	60	0.1	0.1	100	1.64	1	31.45	3,210	1,102	29,388	54,944	30,709	116,142

Table 6b- Gap/Time/Costs in 60 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
1	60	0.001	0.001	50	1.96	1	{11,16,27}	{11: 11,20,22,34,48} {16: 5,7,14,15,16,19,28,29,30,31,32,36,38,43,45,47,49,52,53,54,55} {27: 1,2,3,4,6,8,9,10,12,13,17,18,21,24,25,26,27,33,35,37,39,40,41,42,44,46,50,51,56,57,58,59}
2	60	0.001	0.001	50	1.64	1	{11,16,27,54}	{11: 9HL,11,20,22SL,35HL,43,48} {16: 5,14,16,19,28,30SL,49,55} {27: 1,2,3,4,6,8,9SL,10,12,13,17,18,21,24,25,26,27,33,35,37,39,40,41,42,44,46,50,51,56,57,58,59} {54: 7,15,22HL,29,30HL,31,32,34,36,38,45,47,52,53,54}
3	60	0.001	0.001	75	1.96	1	{27,53}	{53: 5,7,11,14,15,16,19,20,22,28,29,30,31,32,34,36,38,43,45,47,49,52,53,54,55} {27: All rest of retailers and products}
4	60	0.001	0.001	75	1.64	1	{27,53}	{53: 5,7,11,14,15,16,19,20,22,28,29,30,31,32,34,36,38,43,45,47,49,52,53,54,55} {27: All rest of retailers and products}
5	60	0.001	0.001	100	1.96	1	{27,53}	{53: 5,7,11,14,15,16,19,20,22,28,29,30,31,32,34,36,38,43,45,47,49,52,53,54,55} {27: All rest of retailers and products}
6	60	0.001	0.001	100	1.64	1	{27,53}	{53: 5,7,11,14,15,16,19,20,22,28,29,30,31,32,34,36,38,43,45,47,49,52,53,54,55} {27: All rest of retailers and products}
EXP #	**Nodes**	**Beta**	**Theta**	**CR**	**Zalpha**	**Beta/Theta**	**DCs**	**Assignments**
7	60	0.001	0.01	50	1.96	0.1	{11,16,27}	{11: 11,20,22,24SL,43HL,48} {16: 5,7,14,15,16,19,28,29,30,31,32,34,36,38,43SL,45,47,49,52,53,54,55} {27: 1,2,3,4,6,8,9,10,12,13,17,18,21,24HL,25,26,27,33,35,37,39,40,41,42,44,46,50,51,56,57,58,59}
8	60	0.001	0.01	50	1.64	0.1	{11,16,27}	{11: 11,20,22,24SL,43HL,48} {16: 5,7,14,15,16,19,28,29,30,31,32,34,36,38,43SL,45,47,49,52,53,54,55} {27: 1,2,3,4,6,8,9,10,12,13,17,18,21,24HL,25,26,27,33,35,37,39,40,41,42,44,46,50,51,56,57,58,59}
9	60	0.001	0.01	75	1.96	0.1	{27,53}	{53: 5,7,11,14,15,16,19,20,22,28,29,30,31,32,34,36,38,43,45,47,49,52,53,54,55} {27: All rest of retailers and products}
10	60	0.001	0.01	75	1.64	0.1	{27,53}	{53: 5,7,11,14,15,16,19,20,22,28,29,30,31,32,34,36,38,43,45,47,49,52,53,54,55} {27: All rest of retailers and products}
11	60	0.001	0.01	100	1.96	0.1	{27,53}	{53: 5,7,11,14,15,16,19,20,22,28,29,30,31,32,34,36,38,43,45,47,49,52,53,54,55} {27: All rest of retailers and products}
12	60	0.001	0.01	100	1.64	0.1	{27,53}	{53: 5,7,11,14,15,16,19,20,22,28,29,30,31,32,34,36,38,43,45,47,49,52,53,54,55} {27: All rest of retailers and products}
EXP #	**Nodes**	**Beta**	**Theta**	**CR**	**Zalpha**	**Beta/Theta**	**DCs**	**Assignments**
13	60	0.001	0.1	50	1.96	0.01	{2,16,43}	{2: 1,2,3,4,6,8,9,10,12,13,17,18,21,22,24,25,26,27,33,35,37,39,40,41,42,44,46,48,50,51,56,57,58,59} {16: 5HL,14,28SL,29SL,47SL,49,53HL,55HL} {43: 5SL,7,11,15,16,19,20,28HL,29HL,30,31,32,34,36,38,43,45,47HL,52,53SL,54,55SL}
14	60	0.001	0.1	50	1.64	0.01	{11,16,27}	{11: 11,20,22,24SL,35HL,43,48HL} {16: 5,7,14,15,16,19,28,29,30,31,32,34,36,38,45,47,49,52,53,54,55} {27: 1,2,3,4,6,8,9,10,12,13,17,18,21,24HL,25,26,27,33,35,37,39,40,41,42,44,46,48SL,50,51,56,57,58,59}
15	60	0.001	0.1	75	1.96	0.01	{14,48}	{14: 5,14,19,49,55} {48: the rest of retailers and products}
16	60	0.001	0.1	75	1.64	0.01	{14,48}	{14: 5,14,19,49,55} {48: the rest of retailers and products}
17	60	0.001	0.1	100	1.96	0.01	{34}	{34: all retailers}
18	60	0.001	0.1	100	1.64	0.01	{34}	{34: all retailers}
EXP #	**Nodes**	**Beta**	**Theta**	**CR**	**Zalpha**	**Beta/Theta**	**DCs**	**Assignments**
19	60	0.01	0.001	50	1.96	10	{3,12,16,25,31,35,37,41,43,54}	{3: 3,8,9,17,50,51} {12: 1,2,12,18,40,42} { 16: **14,16,19,28,49,55**} {25: 21,25,39,44,46,57,58} {31: 29,30,31,32,47} {35: 22,24,26,35,48} {37: 4,6,33,37,59} {41: 10,13,27,41,56} {43: 5,11,20,43,45} {54: 7,15,34,36,38,52,53,54}
20	60	0.01	0.001	50	1.64	10	{3,12,16,25,31,35,37,41,43,54}	{3: 3,8,9,17,50,51} {12: 1,2,12,18,40,42} { 16: **14,16,19,28,49,55**} {25: 21,25,39,44,46,57,58} {31: 29,30,31,32,47} {35: 22,24,26,35,48} {37: 4,6,33,37,59} {41: 10,13,27,41,56} {43: 5,11,20,43,45} {54: 7,15,34,36,38,52,53,54}
21	60	0.01	0.001	75	1.96	10	{3,22,25,28,31,37,41,42,54,55}	{3: 3,8,9,17,50,51} {22: 11,20,22,34,48} {25: 21,25,39,44,46,57,58} { 28: 7,16,28,43,45} {31: 29,30,31,32,47} {37: 4,6,33,37,59} {41: 10,13,27,41,56} {42: 1,2,12,18,24,26,35,40,42} {54: 15,36,38,52,53,54} {55: 5,14,19,49,55}
22	60	0.01	0.001	75	1.64	10	{3,22,25,28,31,37,41,42,54,55}	{3: 3,8,9,17,50,51} {22: 11,20,22,34,48} {25: 21,25,39,44,46,57,58} { 28: 7,16,28,43,45} {31: 29,30,31,32,47} {37: 4,6,33,37,59} {41: 10,13,27,41,56} {42: 1,2,12,18,24,26,35,40,42} {54: 15,36,38,52,53,54} {55: 5,14,19,49,55}
23	60	0.01	0.001	100	1.96	10	{3,22,25,28,31,37,41,42,54,55}	{3: 3,8,9,17,50,51} {22: 11,20,22,34,48} {25: 21,25,39,44,46,57,58} { 28: 7,16,28,43,45} {31: 29,30,31,32,47} {37: 4,6,33,37,59} {41: 10,13,27,41,56} {42: 1,2,12,18,24,26,35,40,42} {54: 15,36,38,52,53,54} {55: 5,14,19,49,55}
24	60	0.01	0.001	100	1.64	10	{3,22,25,28,31,37,41,42,54,55}	{3: 3,8,9,17,50,51} {22: 11,20,22,34,48} {25: 21,25,39,44,46,57,58} { 28: 7,16,28,43,45} {31: 29,30,31,32,47} {37: 4,6,33,37,59} {41: 10,13,27,41,56} {42: 1,2,12,18,24,26,35,40,42} {54: 15,36,38,52,53,54} {55: 5,14,19,49,55}
EXP #	**Nodes**	**Beta**	**Theta**	**CR**	**Zalpha**	**Beta/Theta**	**DCs**	**Assignments**
25	60	0.01	0.01	50	1.96	1	{3,12,16,25,31,35,37,41,43,54}	{3: 3,8,9,17,50,51} {12: 1,2,12,18,40,42} { 16: **14,16,19,28,49,55**} {25: 21,25,39,44,46,57,58} {31: 29,30,31,32,47} {35: 22,24,26,35,48} {37: 4,6,33,37,59} {41: 10,13,27,41,56} {43: 5,11,20,43,45} {54: 7,15,34,36,38,52,53,54}
26	60	0.01	0.01	50	1.64	1	{3,12,16,31,35,37,41,43,54}	{3: 3,8,9,17,50,51} {12: 1,2,12,18,40,42} { 16: 14,16,19,28,49,55} {25: 21,25,39,44,46,57,58} {31: 29,30,31,32,47} {35: 22,24,26,35,48} {37: 4,6,21,25,33,37,39,46,58,59} {41: 10,13,27,41,44,56,57} {43: 5,11,20,43,45} {54: 7,15,34,36,38,52,53,54}
27	60	0.01	0.01	75	1.96	1	{3,22,25,28,31,33,41,42,54}	{3: 3,8,9,17,50,51} {22: 11,20,22,34,48} {25: 6,21,25,39,44,57,58} { 28: **5,7,14,16,19,28,43,49,55**} {31: 29,30,31,32,47} {33: 4,33,37,46,59} {41: 10,13,27,41,56} {42: 1,2,12,18,24,26,35,40,42} {54: 15,36,38,45,52,53,54}
28	60	0.01	0.01	75	1.64	1	{21,22,31,37,38,41,42,51,55}	{21: 21,25,39,46.59} {22: 11,20,22,34,48} {31: 29,30,31,32,47} {37: 6,17,33,37,58} {38: 7,15,16HL,28,36,38,43,45,52,53,54} {41: 10,13,27,41,44,56,57} {42: 1,12,18,24,26,35,40,42} {51: 3,4,8,9,50,51} {55: 5,14,16SL,19,49,55}
29	60	0.01	0.01	100	1.96	1	{22,27,37,41,42,46,51,54,55}	{22: 11,20,22,34,48} {27: 2,13,17,27,57} {37: 6,8,33,37,58} {41: 10,40,41,44,56} {42: 1,12,18,24,26,35,40,42} {46: 21,25,39,46,59} {51: 3,4,9,50,51} {54: 7,15,16HL,28,29,30,31,32,36,38,43,45,47,52,53,54} {55: 5,14,16SL,19,49,55}
30	60	0.01	0.01	100	1.64	1	{16,21,22,27,31,41,42,51,55}	{16: 5,14,16,19,28,43,49,55} {21: 6,21,25,33,39,46,59} {22: 11,20,22,34,48} {27: 8,13,17,27,58} {31: 29,30,31,32,47} {41: 10,41,44,56,57} {42: 1,2,12,18,24,26,35,40,42} {51: 3,4,9,37,50,51} {54: 7,15,36,38,45,52,53,54}

Table 6c- DC locations and assignments in 60 nodes

EXP #	Nodes	Beta	Theta	CR	Zalpha	Beta/Theta	DCs	Assignments
31	60	0.01	0.1	50	1.96	0.1	{8,15,16,33,35,43,52,56}	{8: Rest of retailers} {15: 15,16HL,19,22HL,28HL,31,32SL,34,38,47HL,52SL,54,55SL} {16: 14,16SL,28SL,32HL,47SL,49,53HL,55HL} {33: 3SL,6HL,21SL,33HL,39,50SL,58HL,59} {35: 1SL,2HL,18HL,22SL,24SL,26HL,42,48} {43: 5,11,20,43,45} {52: 7,29,30,36,52HL,53SL} {56: 2SL,12HL,27HL,40,41,56HL,57SL,58SL}
32	60	0.01	0.1	50	1.64	0.1	{12,19,31,35,37,43,54,57}	{12: 1SL,2,12HL,18HL,40,42,50SL} {19: 14,16,19,28SL,49,55HL} {31: 29,30,32HL,36SL,47,53} {35: 9HL,22SL,24,26,35,48} {37: Rest of retailers} {43: 5,11,20,43,45} {54: 7,15,22HL,28HL,31,32SL,34,36HL,38,52,54,55SL} {57: 3SL,27HL,39SL,41,56HL,57SL,58,59HL}
33	60	0.01	0.1	75	1.96	0.1	{7,27,51,55}	{7: 7,11SL,15,16,19,20,22,28HL,29,30,31,32,34,36,38,43,45,47HL,52,53,54,55SL} {27: 1,2,3HL,4,6,8,9,10,12,13,17,18,21,24HL,25,26,27,33,35,37,39,40SL,41,44,46,48SL,50HL,51,56SL,57,58} {51: 3SL,24SL,40HL,42,48HL,50SL,56HL,59} {55: 5,11HL,14,28SL,47SL,49,55HL}
34	60	0.01	0.1	75	1.64	0.1	{12,55}	{55: 5,7,14,15,16,19,20,28,29,30,31,32,36,38,43,45,47,49,52,53,54,55} {12: rest of retailers}
35	60	0.01	0.1	100	1.96	0.1	{RDCD}	{RDCD: all retailers}
36	60	0.01	0.1	100	1.64	0.1	{RDCD}	{RDCD: all retailers}
EXP #	**Nodes**	**Beta**	**Theta**	**CR**	**Zalpha**	**Beta/Theta**	**DCs**	**Assignments**
37	60	0.1	0.001	50	1.96	100	{15,25,27,32,33,35,40,43,51,54,55}	{**15:** 15,22,29,34,36} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {32: 28,30,31,32,47} {33: 6,33,37,46,59} {35: 24,26,35,42,48} {40: 1,2,10,12,18,40} {43: 5,11,20,43,45} {51: 3,4,9,50,51} {54: 7,38,52,53,54} {55: 14,16,19,49,55}
38	60	0.1	0.001	50	1.64	100	{15,25,27,32,33,35,40,43,51,54,55}	{**15:** 15,22,29,34,36} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {32: 28,30,31,32,47} {33: 6,33,37,46,59} {35: 24,26,35,42,48} {40: 1,2,10,12,18,40} {43: 5,11,20,43,45} {51: 3,4,9,50,51} {54: 7,38,52,53,54} {55: 14,16,19,49,55}
39	60	0.1	0.001	75	1.96	100	{7,22,25,27,31,33,35,40,51,54,55}	{7: 7,16,28,43,45,53} {22: 11,20,22,34,48} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {35: 1,24,26,35,42} {40: 2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,36,38,52,54} {55: 5,14,19,49,55}
40	60	0.1	0.001	75	1.64	100	{7,22,25,27,31,33,35,40,51,54,55}	{7: 7,16,28,43,45,53} {22: 11,20,22,34,48} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {35: 1,24,26,35,42} {40: 2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,36,38,52,54} {55: 5,14,19,49,55}
41	60	0.1	0.001	100	1.96	100	{7,22,25,27,31,33,35,40,51,54,55}	{7: 7,16,28,43,45,53} {22: 11,20,22,34,48} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {35: 1,24,26,35,42} {40: 2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,36,38,52,54} {55: 5,14,19,49,55}
42	60	0.1	0.001	100	1.64	100	{7,22,25,27,31,33,35,40,51,54,55}	{7: 7,16,28,43,45,53} {22: 11,20,22,34,48} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {35: 1,24,26,35,42} {40: 2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,36,38,52,54} {55: 5,14,19,49,55}
EXP #	**Nodes**	**Beta**	**Theta**	**CR**	**Zalpha**	**Beta/Theta**	**DCs**	**Assignments**
43	60	0.1	0.01	50	1.96	10	{15,25,27,32,33,35,40,43,51,54,55}	{**15:** 15,22,29,34,36} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {32: 28,30,31,32,47} {33: 6,33,37,46,59} {35: 24,26,35,42,48} {40: 1,2,10,12,18,40} {43: 5,11,20,43,45} {51: 3,4,9,50,51} {54: 7,38,52,53,54} {55: 14,16,19,49,55}
44	60	0.1	0.01	50	1.64	10	{15,25,27,32,33,35,40,43,51,54,55}	{**15:** 15,22,29,34,36} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {32: 28,30,31,32,47} {33: 6,33,37,46,59} {35: 24,26,35,42,48} {40: 1,2,10,12,18,40} {43: 5,11,20,43,45} {51: 3,4,9,50,51} {54: 7,38,52,53,54} {55: 14,16,19,49,55}
45	60	0.1	0.01	75	1.96	10	{25,27,28,31,33,34,35,40,51,54,55}	{25: 1,25,39,44,58} {27: 8,13,17,27,41,56,57} {28: 7,16,28,43,45} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {34: 11,20,22,34,36} {35: 24,26,35,42,48} {40: 1,2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,38,52,53,54} {55: 5,14,19,49,55}
46	60	0.1	0.01	75	1.64	10	{22,25,27,28,31,33,35,40,51,54,55}	{22: 11,20,22,34,48} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {28: 7,16,28,43,45SL,53HL} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {35: 1,24,26,35,42} {40: 2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,36,38,45HL,52,53SL,54} {55: 5,14,19,49,55}
47	60	0.1	0.01	100	1.96	10	{7,22,25,27,31,33,35,40,51,54,55}	{7: 7,16,28,43,45,53} {22: 11,20,22,34,48} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {35: 1,24,26,35,42} {40: 2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,36,38,52,54} {55: 5,14,19,49,55}
48	60	0.1	0.01	100	1.64	10	{22,25,27,28,31,35,37,40,51,54,55}	{22: 11,20,22,34,48} {25: 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {28: 7,16,28,43,45} {31: 29,30,31,32,47} {35: 1,24,26,35,42} {37: 6,33,37,46,59} {40: 2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,36,38,52,53,54} {55: 5,14,19,49,55}
EXP #	**Nodes**	**Beta**	**Theta**	**CR**	**Zalpha**	**Beta/Theta**	**DCs**	**Assignments**
49	60	0.1	0.1	50	1.96	1	{6,17,31,35,40,41,43,51,54,55}	{6: 6,21,25,33,37,39,46,58,59} {17: 2,8,13,17,27} {31: 29,30,31,32,47} {35: 22,24,26,35,48} {40: 1,12,18,40,42} {41: 10,41,44,56,57} {43: 5,11,20,43,45} {51: 3,4,9,50,51} {54: 7,15,28,34,36,38,52,53,54} {55: 14,16,19,49,55}
50	60	0.1	0.1	50	1.64	1	{6,17,31,35,40,41,43,51,54,55}	{6: 6,21,25,33,37,39,46,58,59} {17: 2,8,13,17,27} {31: 29,30,31,32,47} {35: 22,24,26,35,48} {40: 1,12,18,40,42} {41: 10,41,44,56,57} {43: 5,11,20,43,45} {51: 3,4,9,50,51} {54: 7,15,28,34,36,38,52,53,54} {55: 14,16,19,49,55}
51	60	0.1	0.1	75	1.96	1	{1,22,27,28,31,41,46,51,54,55}	{1: 1,2,12,18,24,26,35,40,42} {22: 11,20,22,34,48} {27: 8,13,17,27,58} {28: 7,16,28,43,45SL,53HL} {31: 29,30,31,32,47} {41: 10,41,44,56,57} {46: 6,21,25,33,39,46,59} {51: 3,4,9,37,50,51} {54: 15,36,38,45HL,52,53SL,54} {55: 5,14,19,49,55}
52	60	0.1	0.1	75	1.64	1	{12,22,25,27,28,31,33,35,51,54,55}	{12: 2,10,12,18,40} {22: 11,20,22,34,48} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {28: 7,16,28,43,45SL,53HL} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {35: 1,24,26,35,42} {51: 3,4,9,50,51} {54: 15,36,38,45HL,52,53SL,54} {55: 5,14,19,49,55}
53	60	0.1	0.1	100	1.96	1	{22,25,27,28,31,33,35,40,51,54,55}	{22: 11,20,22,34,48} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {28: 7,16,28,43,45SL,53HL} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {35: 1,24,26,35,42} {40: 2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,36,38,45HL,52,53SL,54} {55: 5,14,19,49,55}
54	60	0.1	0.1	100	1.64	1	{22,25,27,28,31,33,35,40,51,54,55}	{22: 11,20,22,34,48} {**25:** 21,25,39,44,58} {27: 8,13,17,27,41,56,57} {28: 7,16,28,43,45SL,53HL} {31: 29,30,31,32,47} {33: 6,33,37,46,59} {35: 1,24,26,35,42} {40: 2,10,12,18,40} {51: 3,4,9,50,51} {54: 15,36,38,45HL,52,53SL,54} {55: 5,14,19,49,55}

Table 6d- DC locations and assignments in 60 nodes

5.2 Results

5.2.1 Analysis of 30-Node Set Results

Based on the outputs of model, the model parameters can be studied in more details. Considering the 30 nodes, in all experiments, the gap is zero which means that the optimal solution is found for all experiment performed using model P2. The maximum time to solve the problem is about 1836 seconds, which is quite efficient using the SCIP solver. The objective function consists of four terms namely; total annual facility location cost (DC_cost), total system annual transportation cost (Trans_cost), average annual inventory cost (Mean inv.cost), and total system annual safety stock cost (Service_cost).

The first six experiments closely represent the problem in reality because the four cost terms have a similar scaling, hence can be used for actual company costs assessments. For example, in experiment 1, the total objective function is $1010, proportioned as 244, 265, 361, and 139 for DC_cost, Trans_cost, Mean inv.cost, and Service_cost respectively. In this case, β and θ are 0.001 and 0.001 respectively and their ratio is 1. Considering model parameter scaling, the total system annual cost for the company with coverage radius 50 and service level 1.96, is about $1010. In actual sense, this value is $1,010,000 and the recommended number of DCs to be opened in three.

A decrease in the $\frac{\beta}{\theta}$ ratio indicates that inventory costs are assigned more weight than transportation costs. As shown from Table 4 a-d, this decrease results in the centralization of DCs, due to risk pooling effect. On the other hand, having coverage radius as a constraint forces the model to increase the number of DCs. Such a paradox presents a natural trade-off between the inventory, location and transportation costs.

Also, as presented in Table 4 a-d, mean inv. cost changes with changing θ. However this change is not affected by centralizing or decentralizing DCs. Despite having a similar ratio of $\frac{\beta}{\theta}$, system configurations change depending on the scale of β and θ. For instance when the β and θ are 0.001, centralization takes place more than when the β and θ are 0.1 while the ratio for both of them is one. This occurrence stems from the interconnection among facility location, transportation, and inventory costs. For example, in experiments 1-6, this ratio is the same with experiments 25-30, but the number of DCs and assignment are not completely the same. In experiments 25-30, due to the scaling differences among the costs, compared to the first six experiments, number of DCs increases. We note that in experiments 25-30, the scale of facility location costs is about on tenth each of other cost terms.

Overall, service level does not affect system configuration much. However, looking at experiments 25-30 and 51-54, service level does change the system configuration. In both cases, ratio $\frac{\beta}{\theta}$ is 1 and only the transportation and safety stock costs seem to affect the system because. In this case, slight changes in any of these terms would lead to different configuration, for instance service level changes from 1.96 to 1.64. Although this change seems insignificant, its effect on system configuration is highly felt due to the comparatively low value of facility location cost.

5.2.2 Analysis of 45-Node Set Results

In the 45-node set, as shown in table 5 a-d, there appears to be local optimums or near optimum solutions in some experiments. This is unlike the 30-node set which exhibited only global optima. This happens because of an increase in the number of nodes hence making the problem large and complicated. Once again, coverage radius and $\frac{\beta}{\theta}$ ratio are the most important factors to determine system configuration and the objective function solution.

The most experiments that consume more time are difficult to converge (wider gap), are the ones with a ratio of 1 especially when β and θ are both either 0.1 or 0.01. Experiments 31-36 also exhibit difficulty in finding a solution because the ratio is 0.1 with β and θbeing 0.01 and 0.1 respectively. In both cases, total system facility location cost is much less than transportation and inventory costs, so the only trade off is between the two later terms. The model takes a longer time to solve because of not incorporating facility location cost which has the opposite algorithmic direction of the transportation cost. This is also exhibited in experiments 49 and 50 which indicate that with time, the model solution shows no significant improvement (considering the solution gap and run time).

5.2.3 Analysis of 60-Node Set Results

In the 60-nodeset experiments as presented in 6 a-d, most of the examples cannot reach a global optimum with the SCIP solver. The most important reason for this is the increase in the number of nodes which exponentially increases the model run time. Overall the equivalent parameter changes seem to result in reasonably similar solution trends to those from the 30 and 45-node sets. Similar to 30 and 45 nodes, experiments 1-6 results in a 0% solution gap due to equivalence in the cost scaling. Once more, these experiments could be useful as actual industrial cost estimation and system configuration.

We note here that in experiments 35 and 36, the only open DC in the system is the RDC. This is a true exhibition of the current system configuration of the industry, where only one DC exists—the RDC. Also in most experiments, both HL and SL deliveries and surpluses for any retailer are assigned to the same DC. In some cases, it happens that for any retailer, HL and SL are assigned to different DCs. Although holding cost and transportation cost are equal for both product-types, the average and variance of deliveries and surpluses are different.

5.2.4 Overall System results Analyses (30, 45 & 60-Node Sets)

Figure 8 shows the relationship between the experiments and objective function values in all three set of nodes simultaneously. The highest objective function is about $120,000 from the 60-node set problem when β, θ, coverage radius and service level are 0.1, 0.1,75 miles and 1.96 respectively. Experiments 49-54, 31-36, and 13-18 have the higher objective function solution. In all these experiments, θ is 0.1.Figure 8 also shows that coverage radius and service level do not change objective function value significantly for any of the node set as seen in the experiments.

Figure 9 presents the interrelationship between the experiments and solution time in all three set of nodes. Experiment 14 in the 60-node set has the longest solution time. However, the solution times especially for the 60-node set presented in this table are not a complete representation of actual solution time, due to lack of algorithmic convergence. As a result, in some experiments, in spite of a significant increase in solution time, the solution gap does not decrease significantly.

Figure 10 and 11 show the number of DCs and solution gap for all experiments in all the three set of nodes. The maximum number of open DCs for 30, 45, and 60-node sets are 5, 8 and11respectively. According to figure 11, all experiments in 30 nodes set are global optima, so the solution gap is zero. The maximum gap is about 40% in one of the 60-node set experiments, when β , θ, coverage radius, and service level are 0.01, 0.1, 50, and 1.96 respectively. Once again, this is due to the low coverage radius and the insignificant effect of the facility location cost as already addressed earlier in the 60-node set result analysis.

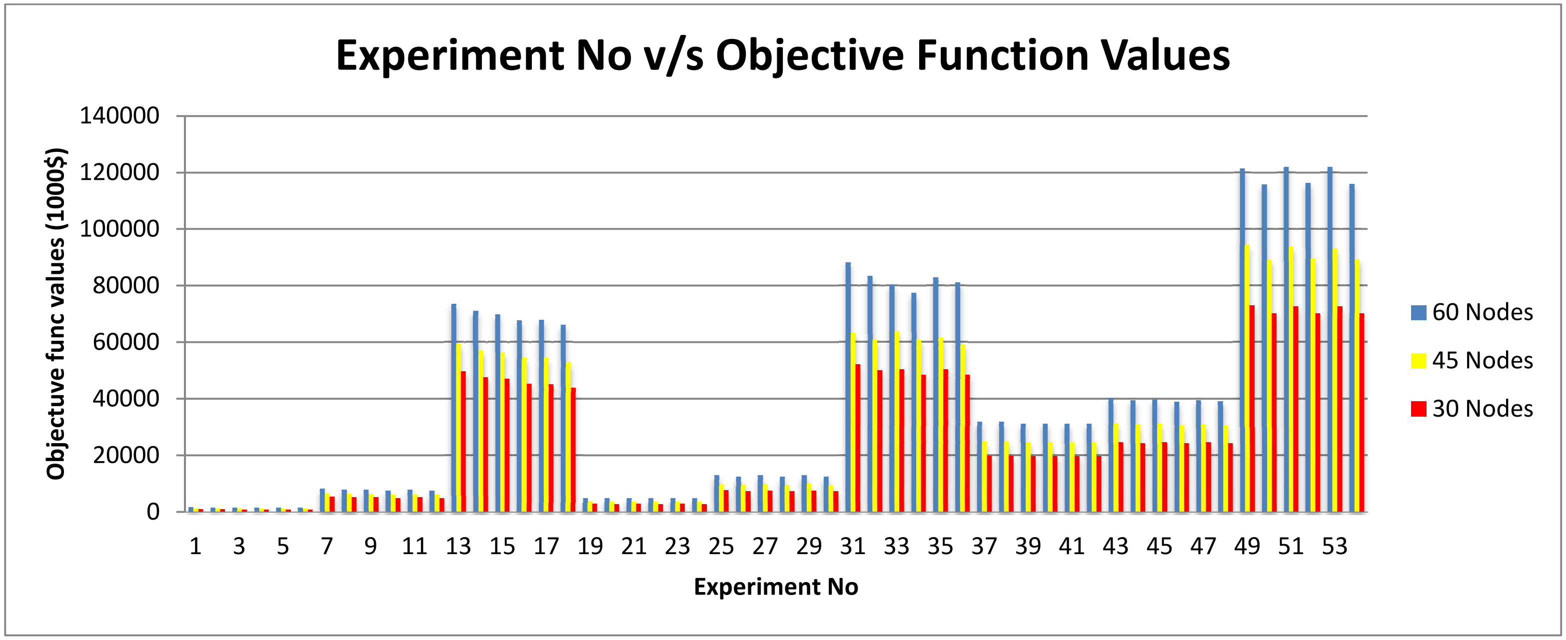


Figure 8- Experiment No v/s Objective Function Values for 60, 45, and 30 nodes

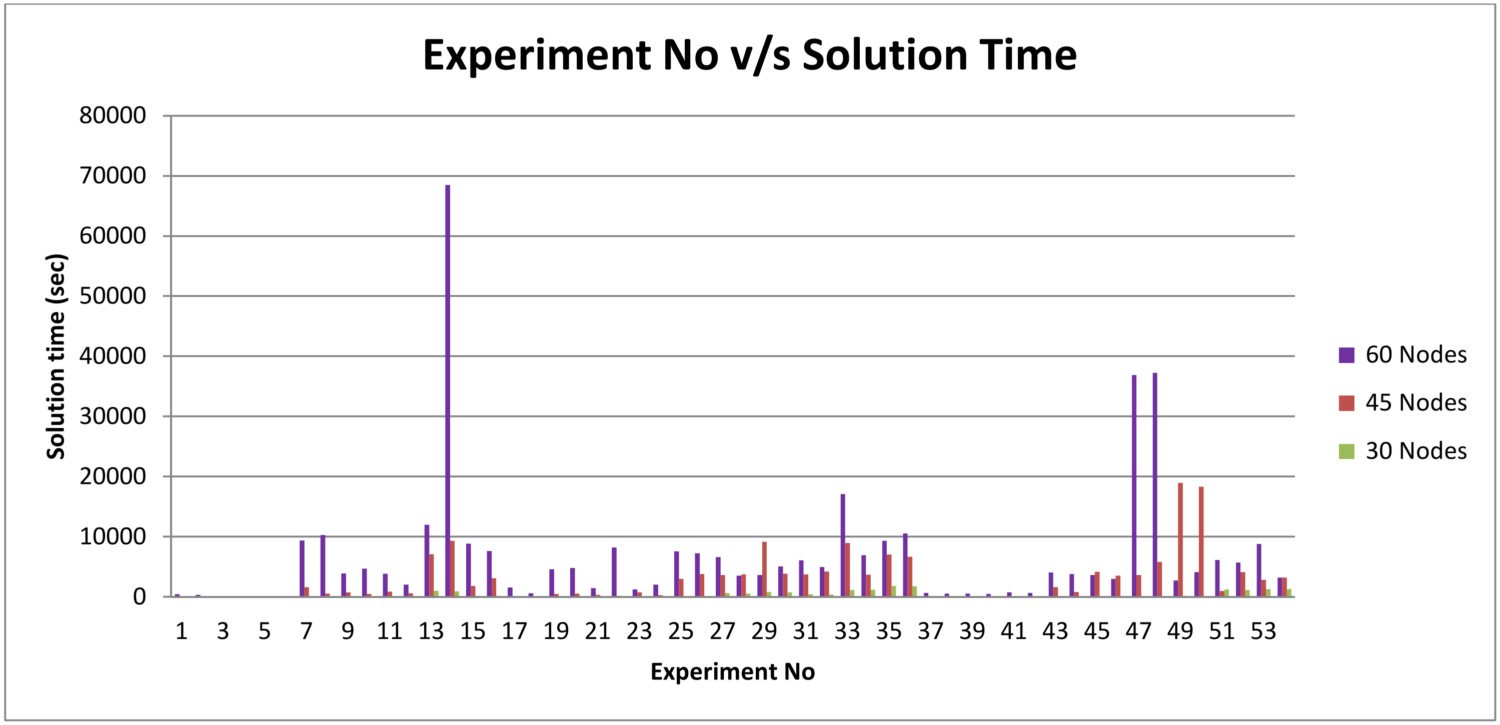


Figure 9- Experiment No v/s Solution Time for 60, 45, and 30 nodes

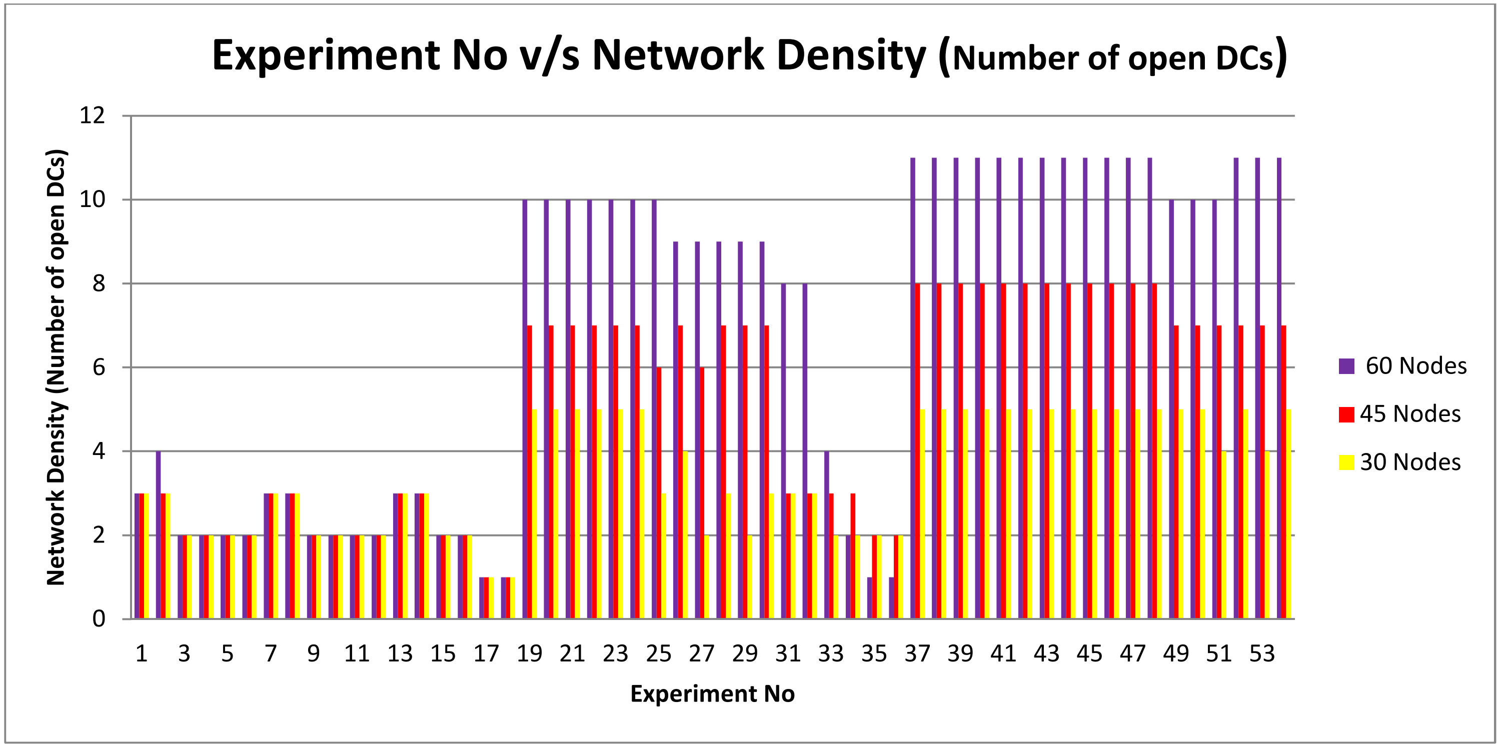


Figure 10- Experiment No v/s Network Density for 60, 45, and 30 nodes

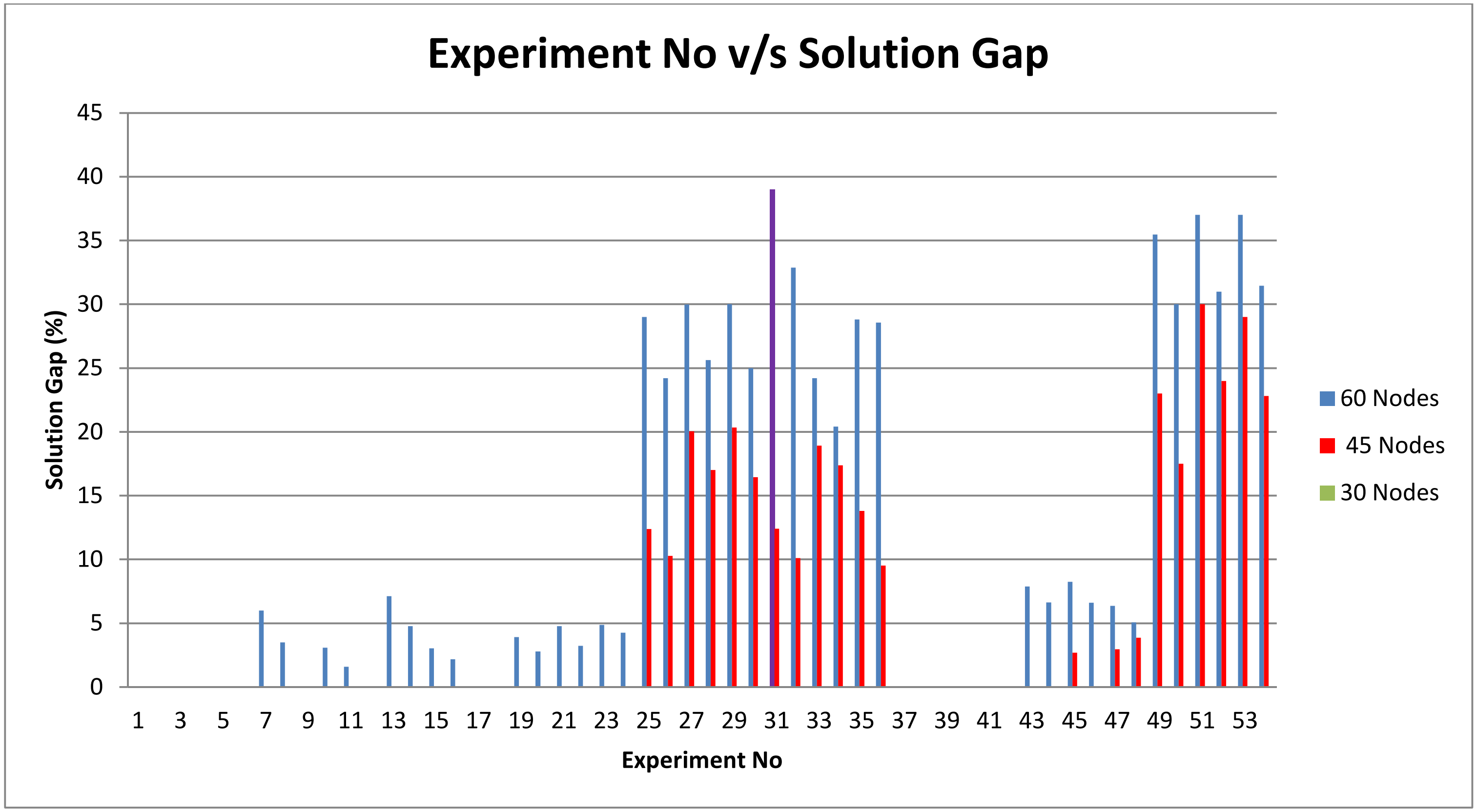
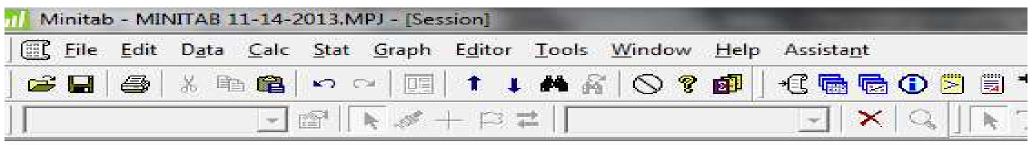


Figure 11- Experiment No v/s Solution Gap for 60, 45, and 30 nodes

5.2.5 ANOVA Test

Analysis of Variance (ANOVA) for the objective function as a response value is performed in MINITAB to analyze the effects of the different parameters and their interactions. In the proposed model, ratio ($\frac{\beta}{\theta}$), coverage radius, and service level are the parameter considered in the ANOVA analysis. The results are presented as follows:




```
ANOVA: Obj. Value versus Beta/Theta, Coverage radius, Z_alpha

Factor           Type    Levels  Values
Beta/Theta       random       5    0.01,    0.10,    1.00,   10.00, 100.00
Coverage radius  random       3   50,  75, 100
Z_alpha          random       2  1.64, 1.96

Analysis of Variance for Obj. Value

Source                      DF           SS          MS        F      P
Beta/Theta                   4  19499583132  4874895783  1235.21  0.000 x
Coverage radius              2      9022993     4511497     1.69  0.244 x
Z_alpha                      1      2005850     2005850     1.55  0.280 x
Beta/Theta*Coverage radius   8     21329150     2666144   310.74  0.000
Beta/Theta*Z_alpha           4      5156204     1289051   150.24  0.000
Coverage radius*Z_alpha      2        20167       10083     1.18  0.357
Error                        8        68640        8580
Total                       29  19537186136

x Not an exact F-test.

S = 92.6282   R-Sq = 100.00%   R-Sq(adj) = 100.00%
```

Figure 12-ANOVA test for objective function in 60 nodes

Based on the results from ANOVA test and P-values, it can be seen that coverage radius and service level do not significantly affect the objective function, but ratio significantly affects the objective function. Also based on results in Figure 12, the interaction between ratio and both coverage radius and service level significantly affect the objective function. As expected the interaction between coverage radius and service level does not affect the objective function significantly. As a result, the ratio is the most important factor among all factors as depicted in the interaction plot (Figure 13). The main effects plot and residual plots for objective function are also presented in Figures14 and 15 respectively. There is a high likelihood that the ratio has a quadratic relationship with the objective function values in this study. This is evidenced by the concavity of the ratio effects plot and the unusual residual plots.

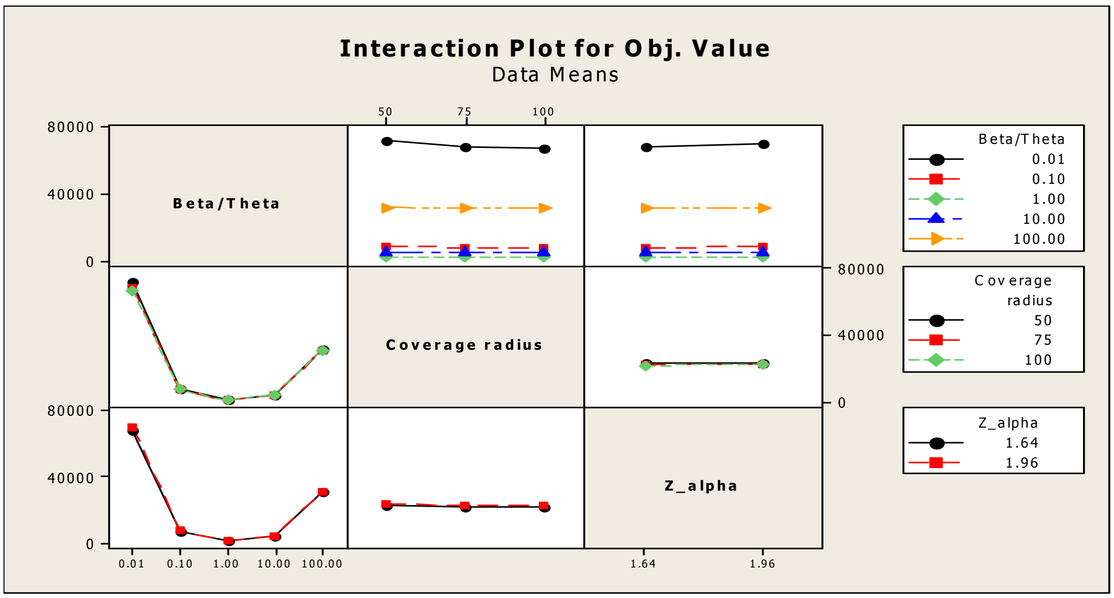


Figure 13-Interaction Plot for objective function in 60 nodes

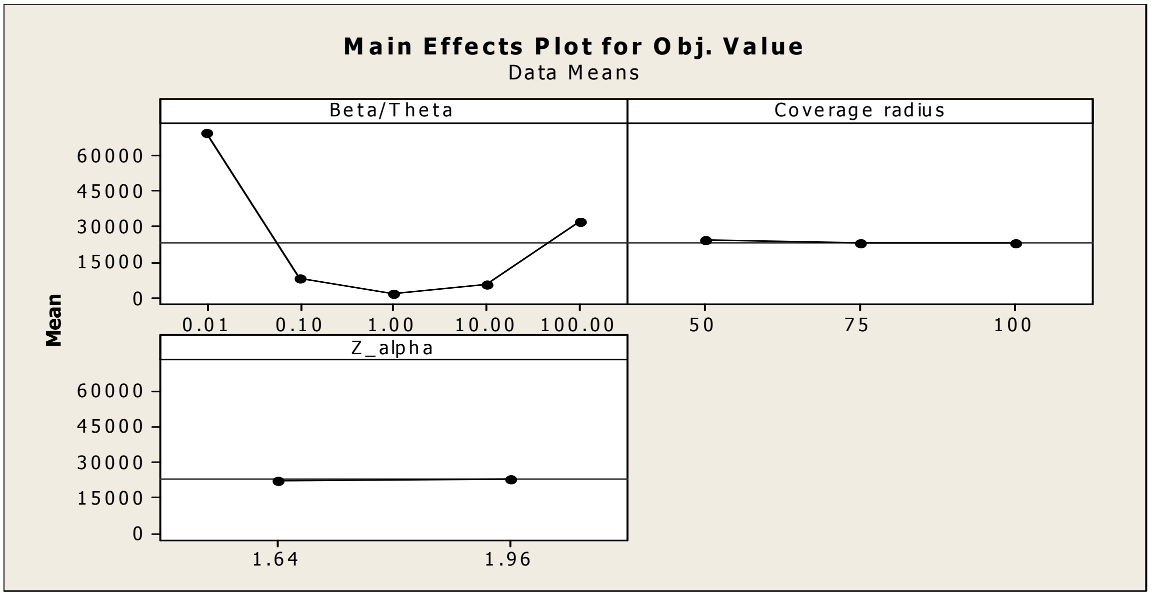


Figure 14- Main effects plot for objective function in 60 nodes

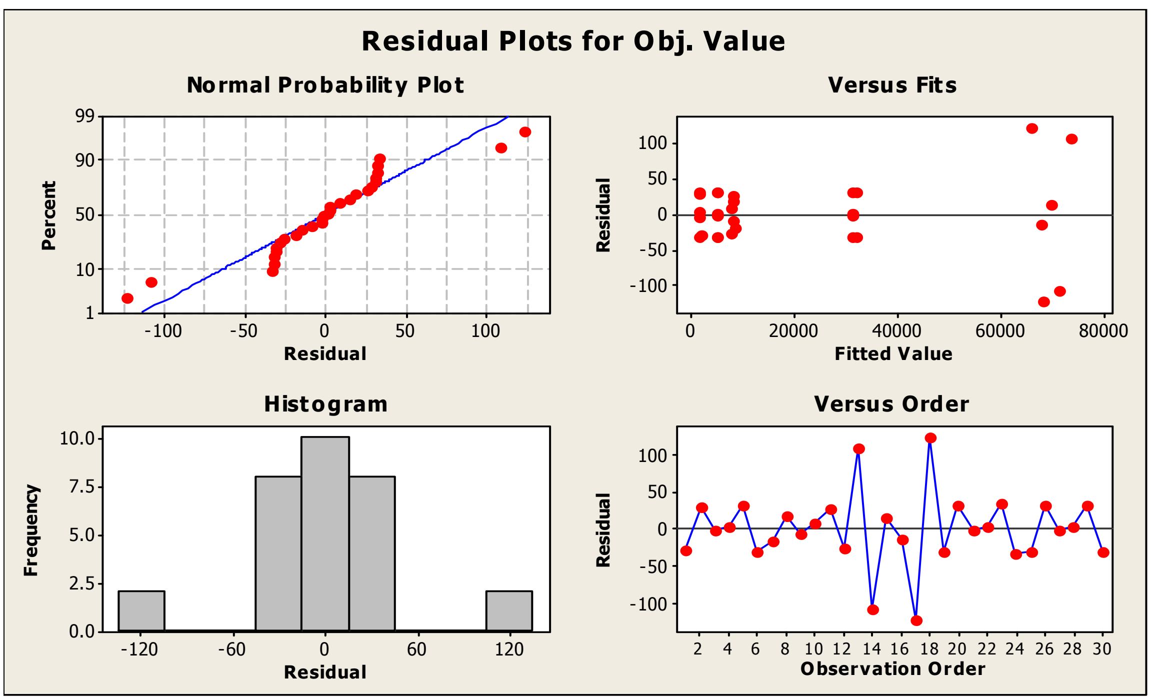


Figure 15-Residual plots for objective function in 60 nodes

5.3 Conclusion

In this study, a joint location-inventory model for a donation-demand driven industry is proposed. This bi-echelon model involves warehouses (DC) and retailers (R) also referred to as Donation/Demand Centers. The model also considers coverage radius, service level, and multiple products. Each retailer has two flows, to and from its related DC i.e. surpluses (S) and deliveries (D). Surpluses result when product-type donations are higher than the demand therefore the excess volume of the product is shipped back to the warehouse (DC) due to limited inventory space in retailer point. Conversely, deliveries result when the product demand is higher than the donations, hence more products are shipped from the warehouse to the retailer. Among all retailers, there are some nodes that just serve as a donation centers; they are called ADCs.

The proposed cost minimization model output include: the recommended number of open DCs, DC locations, assignments of retailer to open DCs and the objective function solution (total annual system cost). The total system cost has three components, namely; fixed facility location cost, transportation cost, and inventory cost. As was discussed in the research contribution section, we suggest a "***Generalized location-inventory model"*** for a donation-demand driven industrial supply chain network. We integrate the minimum number of retailers that are assigned to an opened DC and the coverage radius as constraints in a multi-commodity supply chain system. Specific to the company modeled in this study, each retailer point referred to as a donation/demand center is a potential location for opening a DC (distribution center).

Because of complexity of original model here referred to as P0, we use an efficient algorithm proposed by You et al. (2008) to relax the original problem into two revised models referred to as P1 and P2. As a result, the relaxations lead to model P2which has: (1) fewer binary (0, 1) assignment variables; (2) linear objective functional; and (3) quadratic constraints. So model P2 is much simpler to solve compared to the original P0 model.

GAMS-SCIP solver, which uses branch, cut, and price algorithms, is used to solve the proposed model. We present three case-study scenarios, 30, 45, and 60-node sets problems with different parameter settings. The model parameters used in our problem include: (1) transportation and inventory costs weighting factors β and θ respectively, the coverage radius, and service level. The results show the efficiency of proposed solver to our model especially for 30 and 45-node sets. In these two cased, the solver spews good solutions (small solution gaps) in reasonable times (time within which there is significant convergence).

5.4 Future Research

First, as discussed earlier in the ANOVA results, the ratio-effects results using MINITAB indicated that the ratio potentially has a quadratic effect on the total system cost. This is evidenced by the concavity of the ratio-effects plot, the variable interaction plot and the unusual residual plots. In future, further research will be done to find a credible rational to include this ratio quadratic term into the objective function.

Second, a natural extension to our model would be to consider “truck routing” instead of direct shipments. However, in practice, the shipments from a DC to the assigned retailers often involve a “traveling-salesman-like” tour. Thus, a better approximation of the shipment costs (e.g., the approximations developed by Daganzo (1991)) could be incorporated in our model.

Third, another extension to the proposed model would be to formulate the model as a dynamic programming problem. This extension is important because it will render the model robust enough to consider seasonality in the network. For instance, the average donations and demands for each product-type may easily vary from one season to another. In addition, considering tactical and operational decision variables may be allowed to change with time. These variables include: retailer assignments, average inventory level in DCs, safety stock level in DCs, transportation modes and fuel cost, vehicle routing. This list is by no means exhaustive.

Fourth, we note that in the proposed research shipment is only between DCs and retailers. In future we propose that transshipments among DCs should also be added. This will lead to less safety stock due to pooling the assigned retailers of both DCs simultaneously. This extension is very useful especially when the weighted inventory cost is much larger than weighted transportation cost.

Fifth, multi-sourcing which allows retailers to source and ship multiple product-types to any of their assigned DCs should be included in the model.

Once the proposed changed are effected, a detailed comparative analysis should be carried out to compare performance of the proposed relaxations to others such as Lagrangian relaxation. In addition, further comparative analyses should be done to compare the performance the proposed algorithm to other meta-heuristics algorithms such as Tabu Search and Simulated Annealing.

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