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Numerical simulation of nonlinear elastic wave propagation in piecewise homogeneous media

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Abstract

Systematic experimental work [S. Zhuang, G.Ravichandran and D. Grady, J. Mech. Phys. Solids 51 (2003) 245-265] on laminated composites subjected to high velocity impact loading exhibits the dispersed wave field and the oscillatory behavior of waves with respect to a mean value. Such a behavior is absent in homogeneous solids. An approximate solution to the plate impact in layered heterogeneous solids has been developed in [X. Chen, N. Chandra and A.M. Rajendran, Int. J. Solids Struct. 41 (2004) 4635-4659]. The influence of the particle velocity on many process characteristics was demonstrated. Based on earlier results [A. Berezovski, J. Engelbrecht and G. A. Maugin, Arch. Appl. Mech. 70 (2000) 694-706], numerical simulations of one-dimensional wave propagation in layered nonlinear heterogeneous materials have been performed. The formulated problem follows a conventional experimental configuration of a plate impact. An extension of the high-resolution finite volume wave-propagation algorithm [R.J. LeVeque, Finite Volume Methods for Hyperbolic Problems, Cambridge University Press (2002)] is used. The speed of sound depends nonlinearly on a current stress value in each layer but also on the mismatch properties of layers. Results of numerical simulations capture the experimental data rather well.

Key words: Nonlinear elastic waves, numerical simulation, finite-volume method, heterogeneous solids

1 Introduction

Wave propagation in solids can also be characterized as the thermomechanical response of the media. In this context scattering, dispersion and attenuation

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play an essential role. These phenomena are mostly attributed to material heterogeneity but also to a number of nonlinearities. The nonlinear effects in their turn certainly depend on material properties. However, also impedance and geometric mismatch at various length scales have an effect on nonlinearities together with the initial energy of excitation. Although there is a progress in the analysis of wave propagation in heterogeneous materials, the phenomenon of material and geometric dispersion in such media is not fully understood.

The impulsive shock loading in homogeneous media may be divided into three regimes: strong shocks or high pressure, weak shocks or intermediate pressure and elastic or low pressure; the corresponding behavior of solids are respectively hydrodynamic, finite-strain plastic and linear elastic [1]. Though the stress response has been very well understood for homogeneous materials, the same cannot be said for heterogeneous systems. In heterogeneous media, scattering due to interfaces between dissimilar materials plays an important role for shock wave dissipation and dispersion [2].

Diagnostic experiments for the dynamic behavior of heterogeneous materials under impact loading are usually carried out using a plate impact test configuration under a one-dimensional strain state. These experiments are recently reviewed in [3, 4]. For almost all the experiments, stress response has shown a sloped rising part followed by an oscillatory behavior with respect to a mean value [3, 4]. Such a behavior in the periodically layered systems is consistently exhibited in the systematic experimental work by Zhuang and coworkers [5]. The specimens used in the shock compression experiments [5] were periodically layered two-component composites prepared by repeating a composite unit as many times as necessary to form a specimen with the desired thickness. A buffer layer of the same material as the soft component of the specimen was used at the other side of the specimen. A window in contact with the buffer layer was used to prevent the free surface from serious damage due to unloading from shock wave reflection at the free surface. Shock compression experiments were conducted by employing a powder gun loading system, which could accelerate a flat plate flyer to a velocity in range of 400 m/s to about 2000 m/s. In order to measure the particle velocity history at the specimen window surface, a velocity interferometry system was constructed, and to measure the shock stress history at selected internal interfaces, the manganin stress technique was adopted. Four different materials, polycarbonate, 6061-T6 aluminum alloy, 304 stainless steel, and glass, were chosen as components. The selection of these materials provided a wide range of combinations of shock wave speeds, acoustic impedance and strength levels. The influence of multiple reflections of internal interfaces on shock wave propagation in the layered composites was clearly illustrated by the shock stress profiles measured by manganin gages. The origin of the observed structure of the stress waves was attributed to material heterogeneity at the interfaces. For high velocity impact loading conditions, it was fully realized that material nonlinear effects

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may play a key role in altering the basic structure of the shock wave.

Among the modeling efforts, the mechanical behavior of composites has been extensively investigated using the homogenization approach [6]. Since this approach does not directly consider the interfaces, it is limited in examining the impact behavior, where the wave interactions can be very important.

An approximate solution for layered heterogeneous materials subjected to high velocity plate impact has been developed in [3, 4]. For laminated systems under shock loading, shock velocity, density and volume were related to the particle velocity by means of equation of state. The elastic analysis was extended to shock response by incorporating the nonlinear effects through computing shock velocities of the wave trains and superimposing them.

As pointed out in [5], stress wave propagation through layered media made of isotropic materials provides an ideal model to investigate the effect of heterogeneous materials under shock loading, because the length scales, e.g., thickness of individual layers, and other measures of heterogeneity, e.g., impedance mismatch, are well defined.

Since the impact velocity in shock experiments is sufficiently high, various nonlinear effects may affect the observed behavior. That is why we apply numerical simulations of finite-amplitude nonlinear wave propagation to study of scattering, dispersion and attenuation of shock waves in layered heterogeneous materials. The main goal of the paper is to investigate the applicability of the nonlinear description to the shock response of heterogeneous materials.

2 Formulation of the problem

The geometry of the problem follows the experimental configuration [5] (Fig.1). We consider the initial-boundary value problem of impact loading of a heterogeneous medium composed of alternating layers of two different materials. The impact is provided by a planar flyer of the length f which has an initial velocity v_0. A buffer of the same material as the soft component of the specimen is used to eliminate the effect of wave reflection at the stress-free surface. The densities of the two materials are different, and the materials response to compression is characterized by the distinct stress-strain relations $\sigma(\varepsilon)$. Compressional waves propagating in the direction of layering are modeled by the one-dimensional hyperbolic system of conservation laws

$$\rho\frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial x}, \qquad \frac{\partial \varepsilon}{\partial t} = \frac{\partial v}{\partial x}, \tag{1}$$

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where $\varepsilon(x,t)$ is the strain and $v(x,t)$ the particle velocity.

Initially, stress and strain are zero inside the flyer, the specimen, and the buffer, but the initial velocity of the flyer is nonzero:

$$v(x,0) = v_0, \quad 0 < x < f, \tag{2}$$

where f is the size of the flyer. Both left and right boundaries are stress-free.

Instead of the equation of state like used in [3, 4], we apply a more simple nonlinear stress-strain relation $\sigma(\varepsilon, x)$ for each material (cf. [7])

$$\sigma = \rho c^2 \varepsilon (1 + A\varepsilon), \tag{3}$$

where ρ is the density, c is the conventional longitudinal wave speed, A is a parameter of nonlinearity, values of which are supposed to be different for hard and soft materials.

3 Numerical simulations

It is easy to see that the cross-differentiation of equations (1) leads to the conventional wave equation, solution of which is well-known if corresponding fields are smooth. Assumptions about the smoothness of solutions are not valid near discontinuities in the material parameters. Therefore, standard methods often fail completely if the parameters vary drastically on the grid size. By contrast, the recently developed high-resolution wave-propagation algorithm [8] has been found well suited for the modeling of wave propagation in rapidly-varying heterogeneous media [9]. Within the wave propagation algorithm, every discontinuity in parameters is taken into account by solving the Riemann problem at each interface between discrete elements. The reflection and transmission of waves at each interface are handled automatically for the considered inhomogeneous media.

High-resolution finite-volume methods were originally developed for capturing shock waves in solutions to nonlinear systems of conservation laws, such as the Euler equations of gas dynamics [10]. However, they are also well suited to solving nonlinear wave propagation problems in heterogeneous media containing many sharp interfaces where coefficients in the equation have discontinuities. Recently, the wave-propagation algorithm was successfully applied to the one-dimensional nonlinear elastic waves in a heterogenous periodic medium consisting of alternating thin layers of two different materials [11, 12].

An improved composite wave propagation scheme where a Godunov step is performed after several second-order Lax-Wendroff steps was successfully applied for the two-dimensional thermoelastic wave propagation in media with rapidly-varying properties [13, 14, 15]. This scheme is also applied here for the solution of the problem (1)-(3). The approximate Riemann solver for the nonlinear elastic media (3) is similar to that used in [11, 12]. This means that a modified sound velocity, $\hat{c}$, following the nonlinear stress-strain relation (3) is applied at each time step

$$\hat{c} = c\sqrt{1 + 2A\varepsilon} \qquad (4)$$

instead of the constant value corresponding to the linear case. Calculations are performed with the Courant number equal to one. Results of the numerical simulations compared with experimental data [5] are presented in the next section.

4   Comparison with experimental data

Figure 2 shows the measured and calculated stress time history in the composite, which consists of 8 units of polycarbonate, each 0.74 mm thick, and of 8 units of stainless steel, each 0.37 mm thick. The material properties of components are extracted from [5]: the density $\rho = 1190$ kg/cm^3, the sound velocity $c = 1957$ m/s for the polycarbonate and $\rho = 7890$ kg/cm^3, $c = 5744$ m/s for the stainless steel. The stress time histories correspond to the distance 0.76 mm from the impact face. Calculations are performed for the flyer velocity 561 m/s and the flyer thickness 2.87 mm.

Results of numerical calculations depend crucially on the choice of the parameter of nonlinearity A. We choose this parameter from the conditions to match the numerical simulations to experimental results (see Section 5 for the discussion).

Time histories of particle velocity for the same experiment are shown in Figure 3. It should be noted that the particle velocity time histories correspond to the boundary between the specimen and the buffer. As one can see both stress and particle velocity time histories are well reproduced by the nonlinear model with the same values of the nonlinear parameter A.

As it is pointed out in [5], the influence of multiple reflections of internal interfaces on shock wave propagation in the layered composites is clearly illustrated by the shock stress time histories measured by manganin gages. Therefore, we focus our attention on the comparison of the stress time histories.

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Figure 4 shows the stress time histories in the composite, which consists of 16 units of polycarbonate, each 0.37 mm thick, and of 16 units of stainless steel, each 0.19 mm thick. The stress time histories correspond to the distance 3.44 mm from the impact face. Calculations are performed for the flyer velocity 1043 m/s and the flyer thickness 2.87 mm.

The nonlinear parameter A is chosen here to be equal 2.80 for polycarbonate and zero for stainless steel. Additionally, the stress time history corresponding to linear elastic solution (i.e., nonlinear parameter A is zero for both components) is shown. It can be seen, that the stress time history computed by means of the considered nonlinear model is very close to the experimental one. It reproduces three main peaks and decreases with distortion, as it is observed in the experiment [5].

In Figure 5 the same comparison is presented for the same composite as in Figure 4, only the flyer thickness is different (5.63 mm). This means that the shock energy is approximately twice higher than that in the previous case. The nonlinear parameter A is also increased to 4.03 for polycarbonate and remains zero for stainless steel. As a result all 6 experimentally observed peaks are reproduced well.

In Figure 6 the comparison of stress time histories is presented for the composite, which consists of 16 0.37 mm thick units of polycarbonate and 16 0.20 mm thick units of D-263 glass. The material properties of D-263 glass are [5]: the density $\rho = 2510$ kg/cm^3, the sound velocity $c = 5703$ m/s. The distance between the measurement point and the impact face is 3.41 mm. Corresponding flyer velocity is 1079 m/s and the flyer thickness is 2.87 mm. The nonlinear parameter A is chosen to be equal 5.025 for polycarbonate and zero for D-263 glass. Again, the stress time history corresponding to linear elastic solution (i.e., nonlinear parameter is zero for both components) is shown. As one can see, the stress time history corresponding to the nonlinear model reproduces all 5 peaks with the same amplitude as observed experimentally.

Figure 7 shows the comparison of stress time histories for composite, which consists of 7 units of polycarbonate, each 0.74 mm thick, and 7 units of float glass, each 0.55 mm thick. The material properties of float glass are slightly different from those for D-263 glass [5]: the density $\rho = 2500$ kg/cm^3, the sound velocity $c = 5742$ m/s. The stress profiles correspond to the distance 3.37 mm from the impact face, to the flyer velocity 563 m/s, and to the flyer thickness 2.87 mm. The nonlinear parameter A is equal 3.04 for polycarbonate and zero for float glass. The result of numerical simulation coincides with experiment both in amplitude and in shape.

In Figure 8 the same comparison is presented for the same composite, only the flyer velocity is almost twice higher, namely, 1056 m/s. The value of the

nonlinear parameter A is 5.53 for polycarbonate and zero for float glass. It can be seen, that the result of numerical simulation is very close to experimental data. The complicated shape of the experimental stress time history is reproduced as well.

As it can be seen, the agreement between results of calculations and experiments is achieved by the adjustment of the nonlinear parameter A.

5 Discussion

Though the parameter of nonlinearity A looks like a material constant in the equations (3) and (4), numerical simulations show that this parameter depends also on the structure of the specimen. The values of the nonlinear parameter together with the used experimental conditions are given in Table 1. In the table, PC denotes polycarbonate, GS - glass, SS - 304 stainless steel; the number following the abbreviation of the component material represents the layer thickness in hundredths of a millimeter.

It appears that the application of the nonlinear model to only soft material (polycarbonate) is sufficient to reproduce stress profiles at the gage position about 3.4 mm; any hard material can be treated as linear elastic one.

The comparison of the conditions of experiments 110501 and 110502 as well as 120201 and 120202 and the corresponding values of the parameter of non-linearity A demonstrates the dependence of the parameter A on the impact energy. The influence of the impedance mismatch is clearly follows from the results of simulations corresponding to experiments 110501 and 112301. The dependence on the number of layers is not clear: the difference between the values of the nonlinear parameter in the simulations of experiments 112301 and 120202 can be attributed to the slightly different material properties of float glass and D-263 glass. The effect of the thickness ratio of the layers mentioned in [3] cannot be investigated on the basis of the discussed experimental data, since the thickness ratio was unchanged in the experiments [5].

It follows that the nonlinear behavior of the soft material is affected not only by the energy of the impact but also by the scattering induced by internal interfaces. It should be noted that the influence of the nonlinearity is not necessarily small. In the numerical simulations, that match with the experiments, the increase of the actual sound velocity of polycarbonate follows. It may be up to two times in comparison with the linear case. This conclusion is really surprising but supported by the stress time histories. Such an effect but at smaller scale has also been shown by [11, 12].

To be able to compare the results of experiments with different geometry and loading conditions we need to have a similarity in the experimental setting. However, experimental data in [5] correspond to various impact energies, impedance mismatches, and number and thickness of units. Therefore, we need to normalize the experimental conditions. First of all, we choose one of the experiments as a representative one. For example, we can choose experiment marked as 110501 as a representative one. Then we relate all other experimental conditions to the conditions of the representative experiment. This means that the impact energy for each experiment should be normalized with respect to the impact energy corresponding to the experiment 110501 resulting in the normalized impact energy $\check{E}$. Similarly, the impedance ratio of hard and soft materials should be normalized with respect to the corresponding ratio for the experiment 110501 to obtain the normalized impedance ratio $\check{Z}$. The geometrical factor can be introduced as follows:

$$G = \frac{m h_2}{h_1 + h_2},$$ (5)

where m is gage position, h_1 and h_2 are thicknesses of soft and hard layers, respectively. Its normalized value $\check{G}$ is obtained as described above.

Then we can compute a modified parameter of nonlinearity $\check{A}$

$$\check{A} = A \sqrt{\frac{\check{Z}}{\check{E}\check{G}}}.$$ (6)

The results of calculations are given in Table 2. As one can see, the modified values of the parameter of nonlinearity deviate from the mean value (equal to 2.806) less than by 3.5 %.

The possibility to calculate the single value of the parameter of nonlinearity means that there exists a similarity in the process under different impact energies, impedance mismatches and geometry. Therefore, the value of the parameter of nonlinearity can be calculated following simple similarity relation (6) from one set of experimental conditions with respect to another.

It should be also noted that the equation of state suggested for the simulation of the plate impact test in [3, 4] is simply an approximation of the relation (4) in the case of very small deformations. In fact,

$$\hat{c} = c\sqrt{1 + 2A\varepsilon} \sim c(1 + A\varepsilon) \quad \text{for} \quad A\varepsilon \ll 1.$$ (7)

The nonlinear part $A\varepsilon$ can be represented as Av/c at least under condition

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$dx/dt = c$, which leads to the equation of state

$$\hat{c} = c + Av, \tag{8}$$

mentioned above. Such kind of equation of state is also condition-dependent since the particle velocity v depends definitely on the structure of a specimen.

Thus, application of nonlinear stress-strain relation for materials in numerical simulations of the plate impact problem of a layered heterogeneous medium shows that a good agreement between computations and experiments can be obtained by adjusting the values of the parameter of nonlinearity. In the numerical simulations of the finite-amplitude shock wave propagation in heterogeneous composites, the flyer size and velocity, impedance mismatch of hard and soft materials, as well as the number and size of layers in a specimen were the same as in experiments [5]. Moreover, a nonlinear behavior of materials was also taken into consideration. This means that combining of scattering effects induced by internal interfaces and physical nonlinearity in materials behavior into one nonlinear parameter provides the possibility to reproduce the shock response in heterogeneous media observed experimentally. In this context, parameter A is actually influenced by (i) the physical nonlinearity of the soft material and (ii) the mismatch of elasticity properties of soft and hard materials. The mismatch effect is similar to the type of nonlinearity characteristic to materials with different moduli of elasticity for tension and compression. The mismatch effect manifests itself due to wave scattering at the internal interfaces, and therefore, depends on the structure of a specimen. The variation of the parameter of nonlinearity confirms the statement that the nonlinear wave propagation is highly affected by interaction of the wave with the heterogeneous substructure of a solid [5].

The relation between different values of the parameter of nonlinearity is found by means of the normalization of experimental conditions. The obtained similarity means that the same physical mechanism can manifest itself differently depending on the particular heterogeneous substructure.

Acknowledgments

References

[1] R.A. Graham, Solids under High-Pressure Shock Compression: Mechanics, Physics, and Chemistry, Springer, New York, 1993.

[2] D. Grady, J. Mech. Phys. Solids 46 (1998) 2017–2032.

[3] X. Chen, N. Chandra, Comp. Sci. Technol. 64 (2004) 1477–1493.

[4] X. Chen, N. Chandra, A.M. Rajendran, Int. J. Solids Struct. 41 (2004) 4635–4659.

[5] S. Zhuang, G.Ravichandran, D. Grady, J. Mech. Phys. Solids 51 (2003) 245–265.

[6] Z. Hashin, J. Appl. Mech. 50 (1983) 481–505.

[7] T. Meurer, J. Qu, L.J. Jacobs, Int. J. Solids Struct. 39 (2002) 5585–5614.

[8] R.J. LeVeque, J. Comp. Physics 131 (1997) 327–353.

[9] T.R. Fogarty, R.J. LeVeque, J. Acoust. Soc. Amer. 106 (1999) 17–28.

[10] R.J. LeVeque, Finite Volume Methods for Hyperbolic Problems, Cambridge University Press, 2002.

[11] R.J. LeVeque, Int. J. Numer. Meth. Fluids 40 (2002) 93–104.

[12] R.J. LeVeque, D.H. Yong, SIAM J. Appl. Math. 63 (2003) 1539–1560.

[13] A. Berezovski, J. Engelbrecht, G.A. Maugin, Arch. Appl. Mech. 70 (2000) 694–706.

[14] A. Berezovski, G.A. Maugin, J. Comp. Physics 168 (2001) 249–264.

[15] A. Berezovski, J. Engelbrecht, G.A. Maugin, Eur. J. Mech. A/Solids 22 (2003) 257–265.

Table 1. Experimental conditions and values of the parameter of nonlinearity

Exp.	Specimen soft/hard	Units	Flyer velocity (m/s)	Flyer thickness (mm)	Gage position (mm)	A (PC)	A other
110501	PC37/SS19	16	1043	2.87 (PC)	3.44	2.80	0
110502	PC37/SS19	16	1045	5.63 (PC)	3.44	4.03	0
112301	PC37/GS20	16	1079	2.87 (PC)	3.41	5.025	0
120201	PC74/GS55	7	563	2.87 (PC)	3.37	3.04	0
120202	PC74/GS55	7	1056	2.87 (PC)	3.35	5.53	0

Table 2. Normalized experimental conditions and nonlinearity parameters

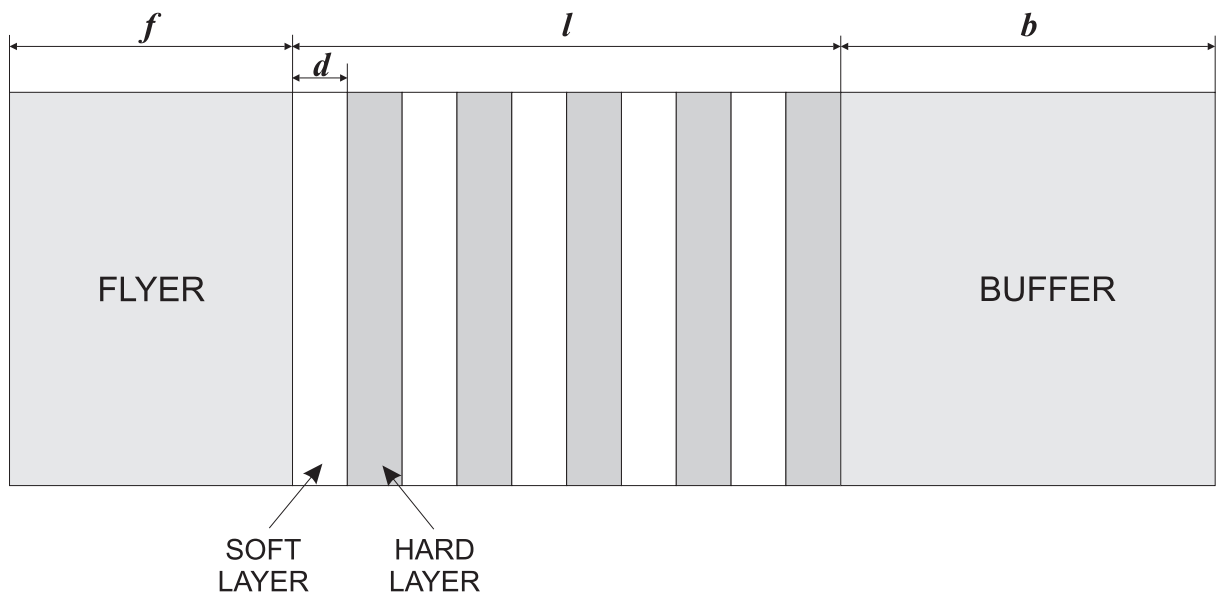| Exp.	Specimen soft/hard	Normalized flyer energy $\check{E}$	Relative impedance mismatch $\check{Z}$	Geometrical factor $\check{G}$	A (PC)	$A\sqrt{\frac{\check{Z}}{\check{E}\check{G}}}$
110501	PC37/SS19	1.00	1.00	1.00	2.80	2.80
110502	PC37/SS19	1.97	1.00	1.00	4.03	2.87
112301	PC37/GS20	1.07	0.316	1.02	5.025	2.71
120201	PC74/GS55	0.29	0.316	1.226	3.04	2.87
120202	PC74/GS55	1.025	0.316	1.226	5.53	2.78

Figure captions

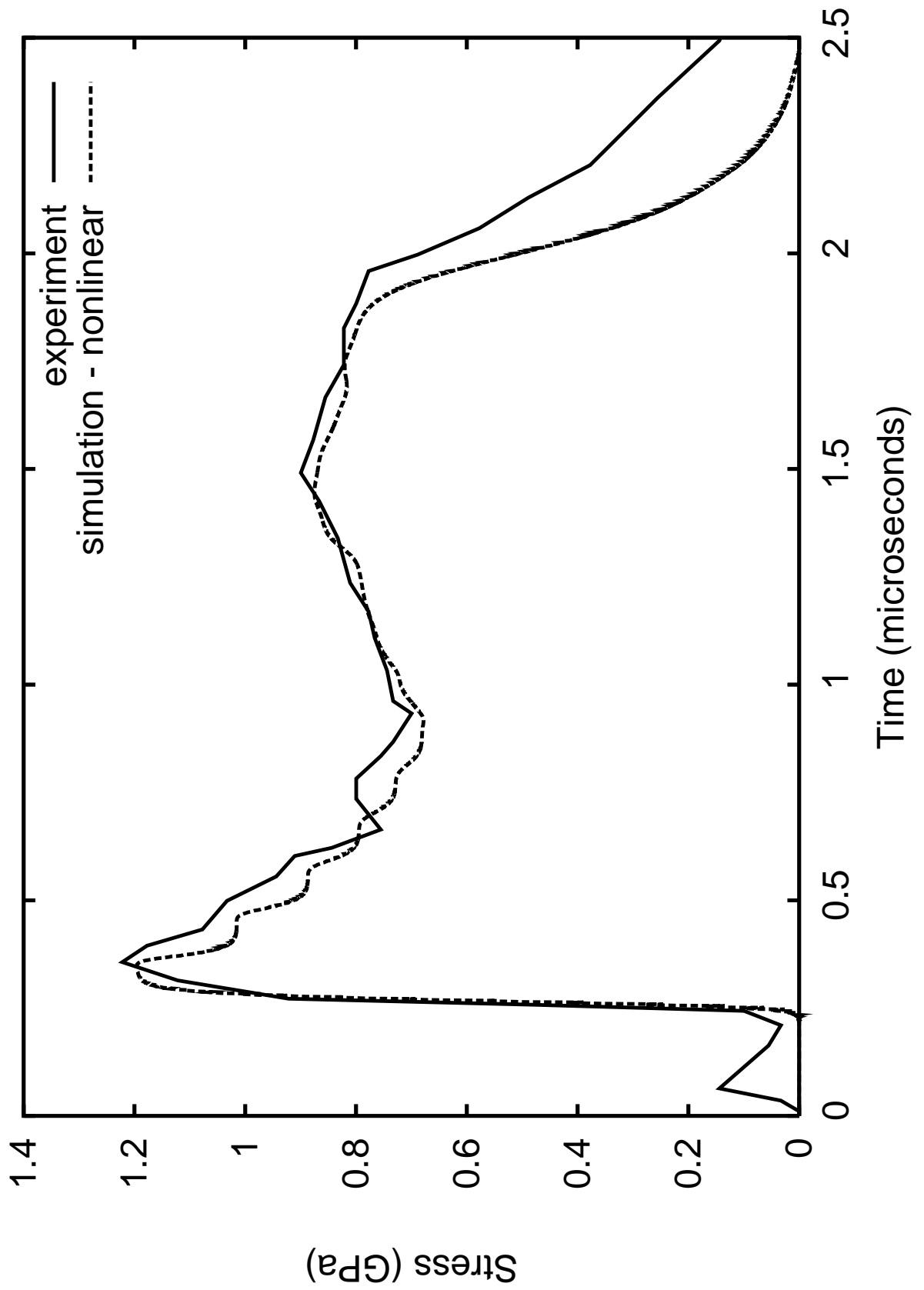Fig. 1. Geometry of the problem.

Fig. 2. Comparison of shock stress time histories corresponding to the experiment 112501 [5].
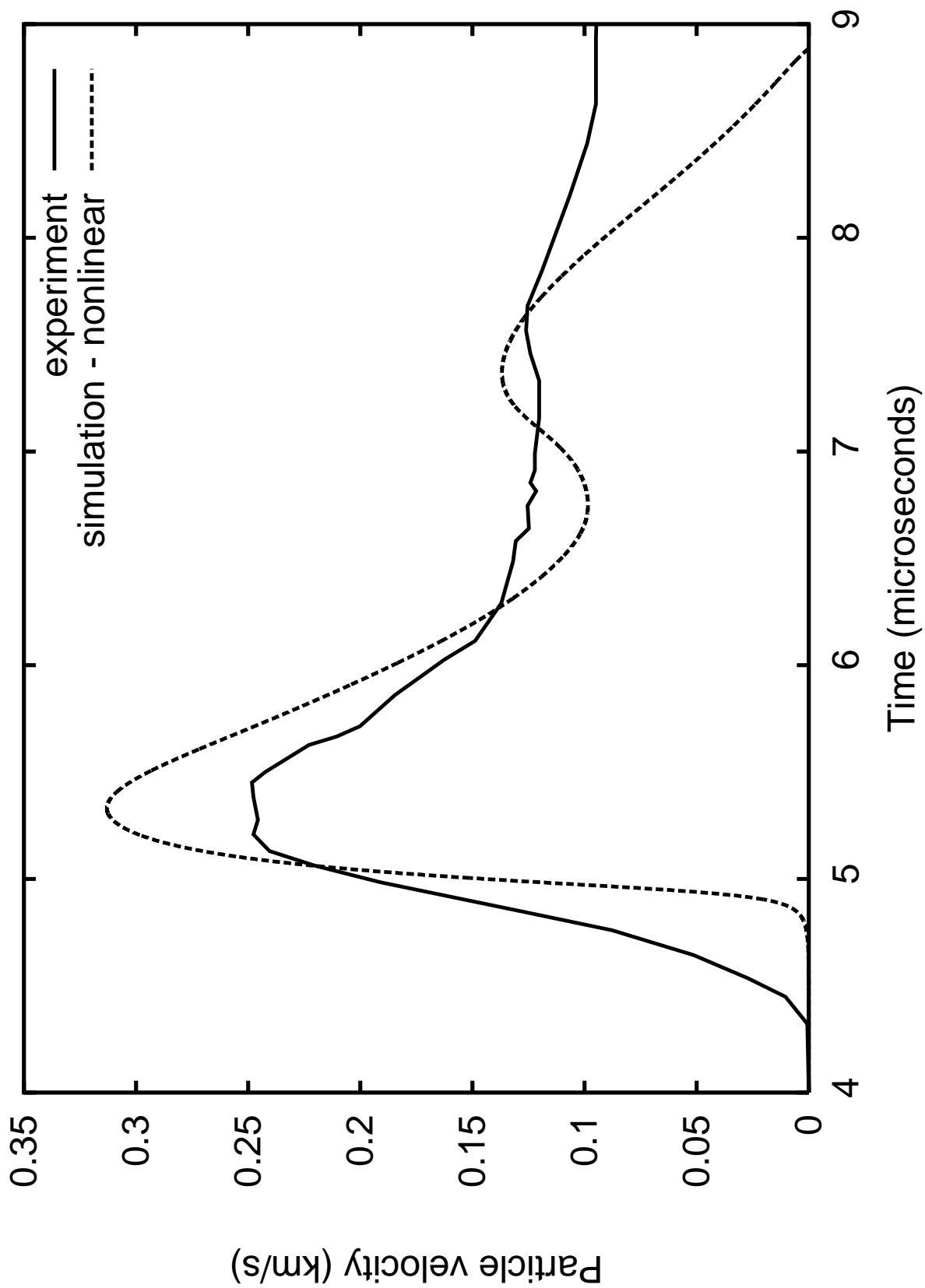
Fig. 3. Comparison of particle velocity time histories corresponding to the experiment 112501 [5].
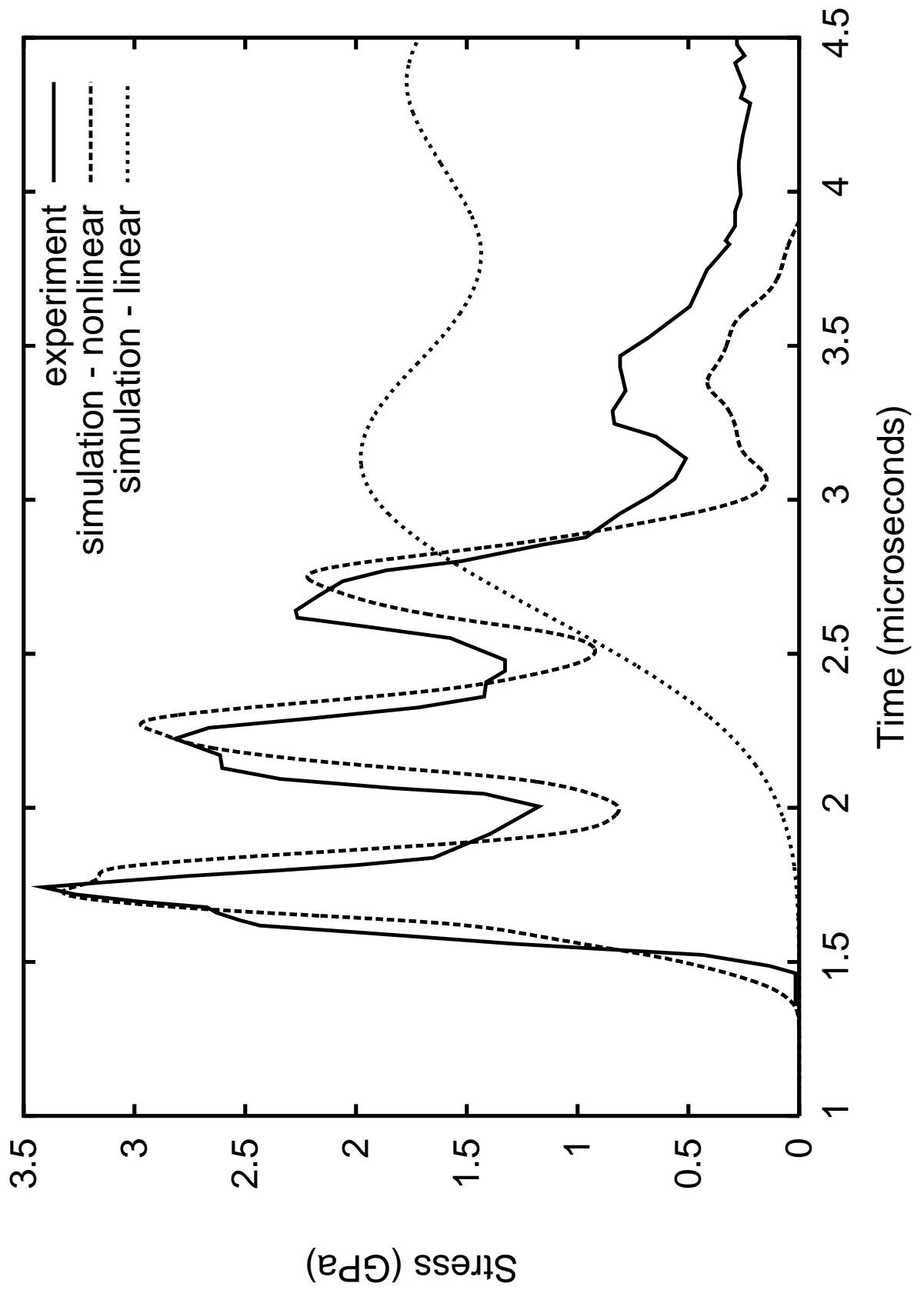
Fig. 4. Comparison of shock stress time histories corresponding to the experiment 110501 [5].

Fig. 5. Comparison of shock stress time histories corresponding to the experiment 110502 [5].

Fig. 6. Comparison of shock stress time histories corresponding to the experiment 112301 [5].

Fig. 7. Comparison of shock stress time histories corresponding to the experiment 120201 [5].
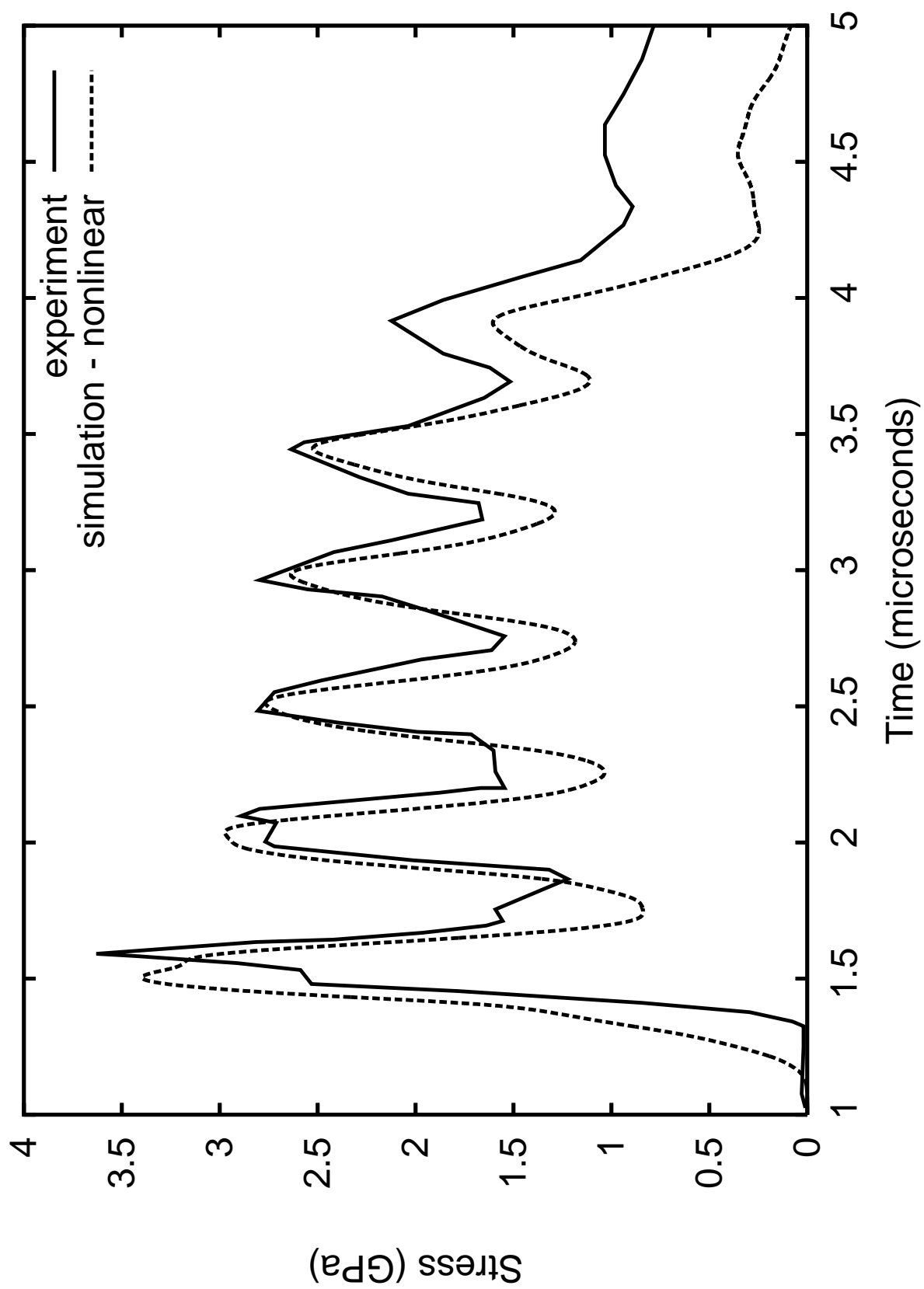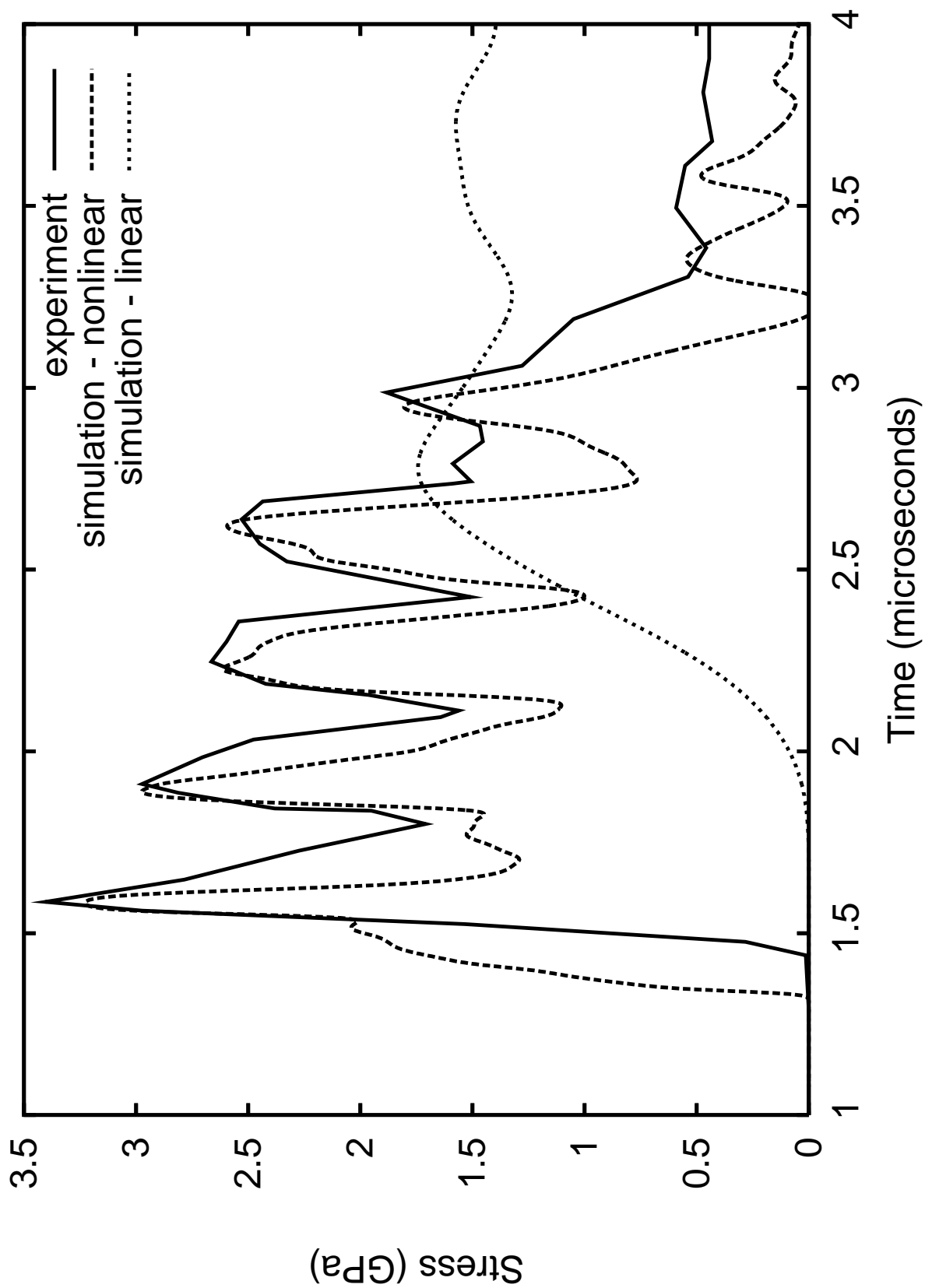
Fig. 8. Comparison of shock stress time histories corresponding to the experiment 120202 [5].

Fig. 1. Geometry of the problem.

Fig. 2. Comparison of shock stress time histories corresponding to the experiment 112501 [5].

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Fig. 3. Comparison of particle velocity time histories corresponding to the experiment 112501 [5].

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Fig. 4. Comparison of shock stress time histories corresponding to the experiment
110501 [5].

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Fig. 5. Comparison of shock stress time histories corresponding to the experiment 110502 [5].

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Fig. 6. Comparison of shock stress time histories corresponding to the experiment 112301 [5].
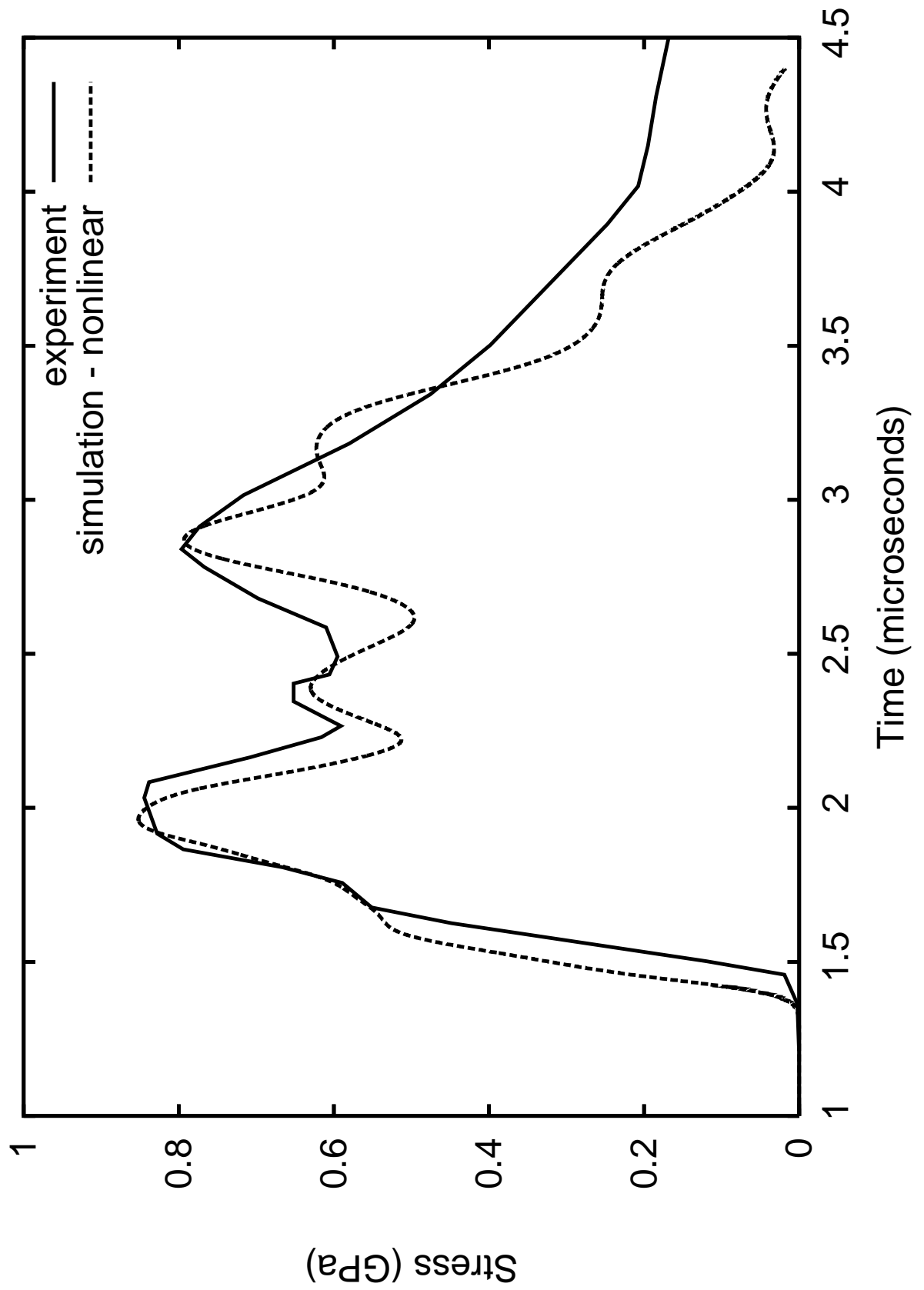
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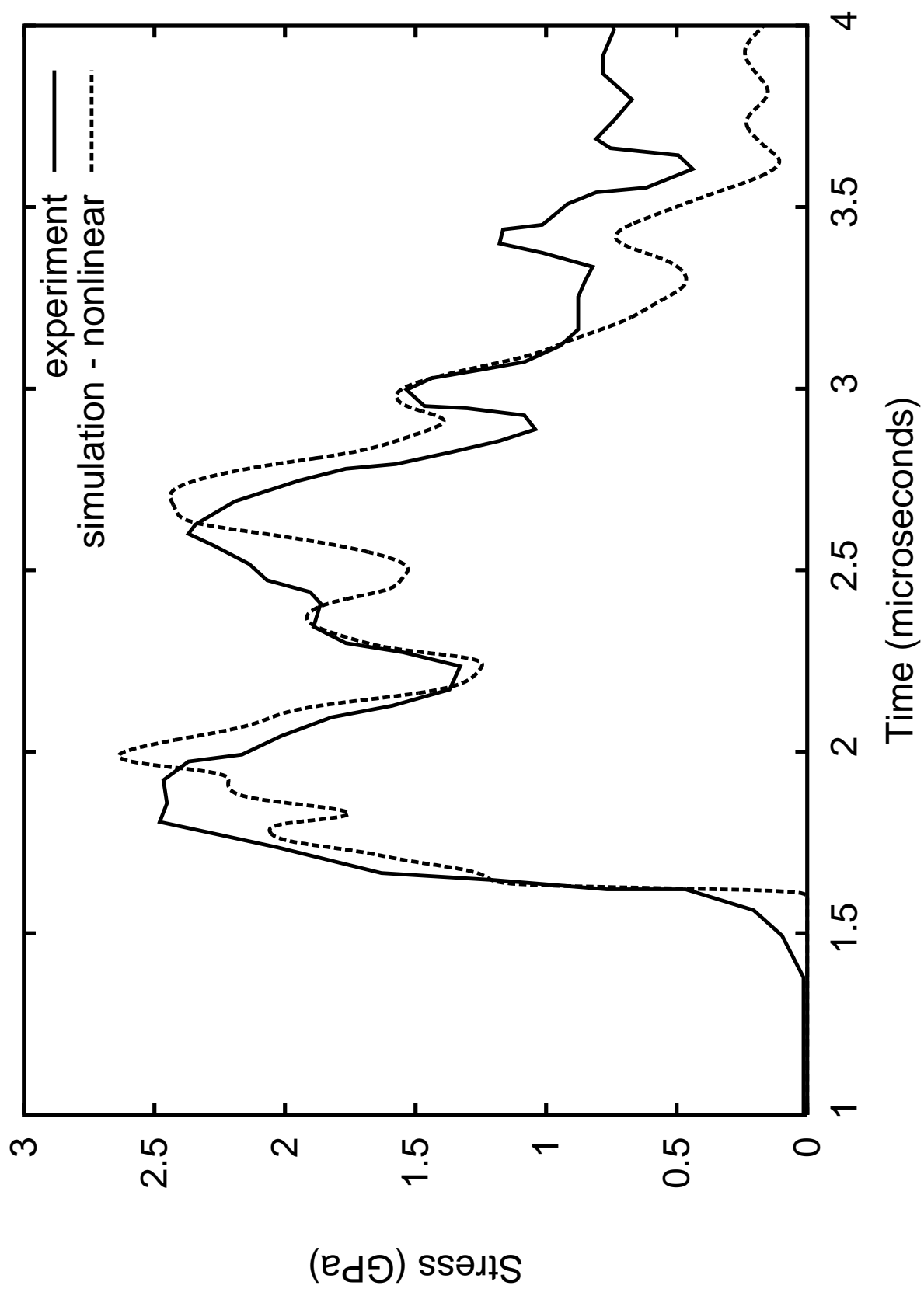Fig. 7. Comparison of shock stress time histories corresponding to the experiment 120201 [5].

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Fig. 8. Comparison of shock stress time histories corresponding to the experiment 120202 [5].

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