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Decision making and trade without probabilities

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Abstract This paper studies trade in a first-price sealed-bid auction where agents know only a range of possible payoffs. The setting is one in which a lemons problem arises, so that if agents have common risk preferences and common priors, then

We dedicate this paper to the memory of our coauthor, John Dickhaut, who passed away on April 10, 2010. Thanks to Geir Asheim, Monica Capra, John Hey, Todd Kaplan, Charles Noussair, Charles Plott, Robin Pope, Ricardo Reis, Tom Rietz, Tomomi Tanaka, Joseph Wang, the conference participants at the 2006 Economic Science Association meetings, the FUR XII and FUR XIV meetings, and the First Nordic Workshop on Behavioral and Experimental Economics, and the workshop participants at Catholic University of Portugal, Carnegie Mellon University, Chapman University, and University of North Carolina.

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expected utility theory leads to a prediction of no trade. In contrast, we develop a model of rational non-probabilistic decision making, under which trade can occur because not bidding is a weakly dominated strategy. We use a laboratory experiment to test the predictions of both models, and also of models of expected utility with heterogeneous priors and risk preferences. We find strong support for the rational non-probabilistic model.

Keywords Knightian uncertainty · Ambiguity · Winners' curse · Market for lemons · Price formation · Auctions

JEL Classification D44 · D82 · D83 · G12

1 Introduction

We present a laboratory experiment on trade in a first-price sealed-bid auction in which traders know ranges of possible payoffs on an asset, but are not told priors on a payoff's likelihood. The setting we have in mind is one where agents trade items that are fundamentally novel, such as shares in research and development intensive firms.

Agents in our market are differentially informed. The initial owner of an asset faces Knightian uncertainty, but has access to better information than potential buyers. We model the initial owners as seeing a private signal, based on which they update their beliefs in an appropriately defined way. The buyers see a coarser signal and are strictly less informed.

Our interest is in the effect of ambiguity on market participation, trade, and price formation. This differs from most experimental work on ambiguity, which chiefly studies individual decision problems. Under risk, our setting leads to a prediction that the market shuts down, essentially due to a lemons problem argument (Akerlof 1970). Before discussing our theory and experiments further, it may be useful to place our work in context.

There is a growing theoretical literature on asset pricing under ambiguity, in which agents' willingness to participate in markets is often of interest. Dow and da Costa Werlang (1992) show that portfolio inertia arises with ambiguity averse investors. Cao et al. (2005), working in a maximin expected utility framework, find that ambiguity leads to limited market participation. Routledge and Zin (2009) have similar findings in a dynamic setting, while Ozsoylev and Werner (2011) find limited participation in a market where expected utility maximizers and traders facing ambiguity coexist. On the other hand, Epstein and Schneider (2008) find that the desire to leave the market is asymmetric, with ambiguity averse traders being more sensitive to bad news than to good news. Because traders do not always react to the information they face, prices in extended notions of rational expectations equilibria may not fully reflect the information in the market. Condie and Ganguli (2011a,b) address this point and discuss settings where rational expectations equilibria might be fully revealing of ambiguous information.

Others have argued that ambiguity can *increase* trade or at least need not lead to market collapse. Bewley (2001) is an example of the former, while Rigotti and

Shannon (2005) is an example of the latter. What seems to be critical is whether initial endowments are ambiguous. de Castro and Chateauneuf (2011) show that the market does not collapse with ambiguous endowments. This is similar to the experiment we present below, as we have high initial levels of uncertainty.

A crucial feature of our experiment is that we combine ambiguity with asymmetric information. The reason is that we wish to distinguish the theoretical predictions of market behavior under ambiguity from the predictions made under risk. We choose a setting where the market shuts down under risk, but can operate under ambiguity. In this respect, our setting is similar to that of de Castro and Yannelis (2010), who study an allocation problem among differentially informed agents. They find that ambiguity can improve allocational efficiency and lead to increased trade.

Our work is related to two streams in the experimental economics literature, one dealing with price setting under ambiguity and another focusing on auctions with a winner's curse (or more precisely a buyer's curse). Examples of the former are Ahn et al. (2010) and Bossaerts et al. (2010), which use heterogeneous ambiguity attitudes to explain trade.

Thaler (1988) provides a detailed review of experiments on the winner's curse. Closest to us are Kagel and Levin (1986) and King (1996), who study auctions with adverse selection. In the Kagel and Levin experiments, disclosing a lower bound on the asset's return avoids a winner's curse by removing the information asymmetry. We extend this idea to a setting with ambiguity: We reveal a lower bound to our bidders, but the current owner of the asset (the seller in our auction) knows a refinement to an upper bound. This preserves some residual uncertainty.

Two other experimental economics papers should be mentioned here. Chen et al. (2007) study private value auctions with two bidders. Each bidder knows his own value, but faces ambiguity over the other bidder's value. They find bidding behavior that would require ambiguity propensity rather than ambiguity aversion. Their setting differs from ours in that their ambiguity is entirely over the other bidder's private value, whereas our setting studies an unknown common value. Chen and Plott (1998) study a first-price sealed-bid private value auction, which is similar to our setting. Their subjects appear rational, with the sole exception being their formation of beliefs. They attribute subjects' behavior to a form of "limited rationality" and suggest that the answer may lie in a belief-free model. As we show below, our results are broadly consistent with those of Chen and Plott. Moreover, in what follows we view refining a range of possible outcomes, without making a probabilistic assessment, as a rational (if not neoclassical) form of updating. In this sense, the results of Chen and Plott and our results below are consistent with rational non-probabilistic decision making.

The paper proceeds as follows: The next section describes the structure of the experiment and states the predictions of competing theories. Our emphasis is on the differences between expected utility maximization with homogeneous agents and on the theories described above on trade under ambiguity. We also state how the predictions of expected utility theory change when agents are heterogeneous in either their risk attitudes or their priors. As an additional benchmark, we state the predicted behavior of agents who act non-strategically. Section 3 details our experimental procedures. Section 4 reports our results. The final section gives our conclusions.

Table 1 Sequence of events in the experiment

<i>Stage 1</i>	<i>Stage 2</i>	<i>Stage 3</i>	<i>Stage 4</i>	<i>Stage 5</i>
Seller endowed with 1,000 fr	Seller receives information on ρ	Seller sets private reservation price	Buyers learn ρ' and S_0	Buyers bid in first-price sealed bid auction
Possible return $(-0.1, 0.5)$	Possible return (ρ', ρ'') $\subsetneq (-0.1, 0.5)$		Possible return $(\rho', 0.5)$	Buyers can bid up to 1,500 fr
Seller invests S_0				

2 Investment setting

In this experiment, subjects have the opportunity to invest in and trade a financial asset whose return comes from an unknown distribution. Information on the return comes in the form of a range of possible values. A group of subjects consists of four buyers and one seller. The initial range, denoted by $(\underline{\rho}, \bar{\rho})$, is from return of -10 to $+50\%$. This is common knowledge among the buyers and the seller.

The seller is initially endowed with 1,000 francs, while the buyers are initially endowed with 1,500 francs. At step 1, the seller decides how much of his initial endowment to invest. We denote this investment by S_0 . See the first column of Table 1.

At step 2 (column 2 of Table 1), the seller receives updated information in the form of new upper and lower bounds on the possible return. This updated range, which we denote by (ρ', ρ'') , is strictly between the original bounds of -10 percent and $+50$ percent. So $(\rho', \rho'') \subsetneq (\underline{\rho}, \bar{\rho})$.

At step 3 (column 3), the seller privately decides his reservation price, based on the updated information. At step 4 (column 4), the buyers learn the updated lower bound ρ' and the seller's investment S_0 . They do not, however, learn the updated upper bound ρ'' . So at this point, the seller knows that the return $\rho \in (\rho', \rho'')$, while the buyers know that $\rho \in (\rho', \bar{\rho})$.

At step 5 (column 5), the buyers bid for the asset in a first-price, sealed-bid auction. They can bid up to their endowment of 1500 francs. Thus, trade occurs if the highest bid is above the seller's private reservation price, in which case the highest bidder gets the asset and pays the seller his bid. Otherwise, there is no trade, and the seller keeps the asset. The value of ρ is then announced, and all the subjects are paid accordingly.

Our first benchmark is expected utility theory, with homogeneous agents (i.e., with common priors and common risk attitudes). Before the investment is undertaken, the support of the distribution of returns must be some subset of $(\underline{\rho}, \bar{\rho})$. Once the seller receives the revised bounds, the support of the distribution of returns that the seller uses is a subset of (ρ', ρ'') . The buyers, after learning ρ' , update their beliefs, generating a posterior distribution whose support is a subset of $(\rho', \bar{\rho})$.

In the special case of risk-neutral agents, our predictions are quite stark: Sellers invest all or nothing, and, if agents have a common prior, no agent submits a potentially winning bid.

Proposition 1 *A risk-neutral seller never invests if the expected return is negative and invests his entire endowment if the expected return is positive.*

Proof A direct consequence of the definition of risk-neutrality.

Proposition 2 *Suppose there is a consensus, atomless initial probability distribution with support $(\underline{\rho}, \bar{\rho})$. If agents are risk neutral, then there will be no trade in the auction, and the buyers will never submit bids above $(1 + \rho')S_0$.*

Proof After learning the revised upper and lower bounds on the return, the seller forms a reservation price based on ρ' and ρ'' , whereas the buyers only know ρ' . If a buyer submits a bid above $(1 + \rho')S_0$ and the seller is risk neutral, then trade occurs only if the bid exceeds the seller's private expectation; the latter is the expectation conditional upon all the information in the economy. That is, a bid above $S_0(1 + \rho')$ wins only if it has negative expected value. Consequently, a risk-neutral buyer will only bid $S_0(1 + \rho')$ or less, and this must be strictly below the risk-neutral seller's reservation price. There will therefore be no trade.

The no-trade prediction of Proposition 2 extends from setting of risk neutrality to any setting with homogeneous risk aversion and a common prior. When agents are heterogeneous in their risk aversion, trade can occur, but only in highly restricted ways; in particular, the ranking of bids should be consistent across auctions. Heterogeneous priors are more nuanced: If there is sufficient dispersion among subjects' valuations, then those with higher valuations may bid. But as the experiment progresses, the subjects will have more shared observations, and consequently, their posterior beliefs will become more similar (for sufficiently well-behaved priors). If beliefs become sufficiently close, the potential advantages to bidding vanish. Summing up, heterogeneous priors should lead to preserved rankings of subjects' bids, provided the market does not collapse.

We test these predictions of expected utility theory against those of trade under ambiguity. We focus on the weakest possible form of updating with ambiguity: New information restricts the set of possible outcomes, but we do not impose any further probabilistic structure on an agent's updated beliefs. We refer to this as rational non-probabilistic belief revision, or as the ambiguity setting.

Returning to our experiment, we note that the predictions of rational non-probabilistic belief revision differ sharply from those of expected utility theory. A rational seller in the ambiguity setting can invest anything between 0 and 1000 francs and can set his reservation price anywhere in $[(1 + \rho')S_0, (1 + \rho'')S_0]$. For example, a seller with maximin preferences would choose a reservation price at exactly $(1 + \rho')S_0$.

The indeterminacy in the seller's reservation price in the ambiguity model completely alters the optimal strategy for the buyers from that of the expected utility model. Under ambiguity, the buyers rationally bid anything in the interval $[(1 + \rho')S_0, (1 + \bar{\rho})S_0]$. Recall that under the expected utility model, the seller would privately set a reservation price at the expected value of the asset, so that any buyer who submits a potentially winning bid would have a negative expected return. In the non-probabilistic setting, however, there is no unique expected value available to the seller. Moreover,

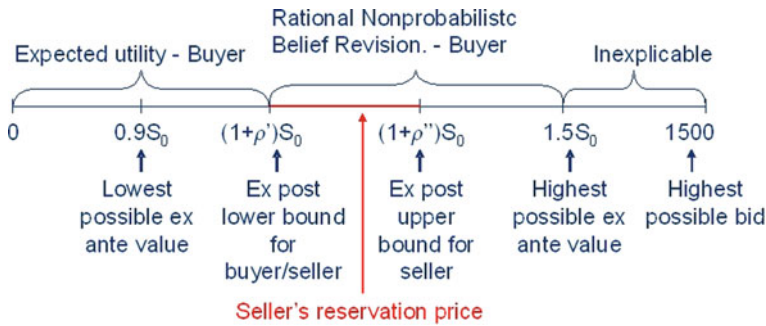


Fig. 1 Range of possible bids

since the seller can rationally set his reservation price to $(1 + \rho')S_0$ (again in contrast with the expected utility setting, where the reservation price must be strictly above this lower bound), any bid below this ex post lower bound is weakly dominated by a bid at $(1 + \rho')S_0$. A buyer bidding less than this can never expect to win the asset from a rational seller, while a buyer bidding exactly this amount has some chance of purchasing the asset and if so will purchase an asset guaranteed to make a positive return (though one that can become arbitrarily small).

The argument stated above shows the following:

Proposition 3 *Suppose that the agents do not form subjective beliefs and have incomplete preferences, which are increasing over non-overlapping possible payoffs. Then, the seller's reservation price will belong to the interval $[(1 + \rho')S_0, (1 + \rho'')S_0]$, and the buyers' bids will belong to the interval $[(1 + \rho')S_0, (1 + \bar{\rho})S_0]$.*

Remark 1 Proposition 3 indicates that trade can occur, but trade still need not occur. The indeterminacy of the seller's reservation price makes trade possible, but cannot guarantee trade.

A direct consequence of Proposition 3 is the following:

Corollary 1 *Under the assumptions of Proposition 3, if trade occurs, the market price will belong to the same interval as the buyers' possible bids, namely $[(1 + \rho')S_0, (1 + \bar{\rho})S_0]$.*

Proof A rational seller's reservation price must be at least $(1 + \rho')S_0$, so no trade can occur below this price. A rational buyer's bid must be below $(1 + \bar{\rho})S_0$, since the asset is guaranteed to return less than this amount. Between the seller's lowest possible bid and the buyers' highest possible price, behavior is indeterminate, so trade is possible.

Figure 1 presents an overview of the possible bids and possible reservation prices under each model.

The discussion above indicates that trade can occur if buyers and sellers are fully rational but do not specify a unique probability distribution over returns, given what they know. It is also possible that trade can occur if buyers are boundedly rational, in the sense of calculating expected returns naïvely rather than considering the seller's

incentives. We consider two forms of naïve buyers. To get clear predictions, we focus attention on the cases where agents treat the return as uniformly distributed over an appropriate interval.

Since the buyer knows only that the asset's value is in $((1 + \rho')S_0, (1 + \bar{\rho})S_0)$, a simple calculation would be to treat ρ as uniformly distributed between ρ' and $\bar{\rho}$, giving the buyer an expected value at $(1 + (\rho' + \bar{\rho})/2)S_0$. On the other hand, if the buyer considers the fact that the seller knows ρ'' but does not consider the seller's strategic choice, the buyer would recognize that $\rho'' \in (\rho', \bar{\rho})$. The principle of insufficient reason would then suggest that the buyer's expectation of ρ'' is $(\rho' + \bar{\rho})/2$ and that the buyer's expectation of ρ would consequently be $(3\rho' + \bar{\rho})/4$.

Note that the seller's behavior cannot be said to be naïve in the same way that the buyers' behaviors can. This is because, under expected utility theory, the seller optimally chooses a reservation price at the expected value of the asset and does not benefit from choosing a different reservation price based on conjectures about the buyers' strategy.

Summing up, a naïve buyer may be expected to set a reservation price at either the midpoint of the interval of possible values, or at a corrected midpoint that adjusts for an expected value of ρ'' . Bids that are outside these values are inconsistent with boundedly rational risk-neutral expected utility maximization but can be consistent with rational non-probabilistic belief revision. However, any bids above $(1 + \bar{\rho})S_0 = 1.5S_0$ are inexplicable under the theories we compare.

3 Experimental procedure

The experiment was run with 20 subjects, using the z-Tree software (Fischbacher 2007). Each subject was grouped with four other participants and was in the same group throughout the experiment. There were 20 rounds, and the group composition remained unaltered across rounds.

In each round, one subject in each group was randomly selected to be the seller; other subjects in the group were designated as buyers. The seller received 1,000 francs (currency for the experiment) as an initial endowment. He was reminded that the maximum possible return on the investment in every round was 0.5 and that maximum possible loss was -0.1 ; these values had also been presented in the instructions, which were read aloud to all the subjects before the start of the experiment. After having been reminded of the possible returns and of his endowment, the seller was required to decide how much of the 1,000 francs to invest. We denote this investment choice S_0 .

After the seller made his investment decision, he privately saw on his computer screen a revised minimum return and a revised maximum return, along with the corresponding range of values in francs. The revised minimum return ρ' was always at least as high as the initial minimum return, and the revised maximum return ρ'' was never higher than the initial maximum return. After seeing the revised returns, the seller was told to set the minimum selling price for his asset.

The treatment of the buyers was as follows: in each round, each buyer received 1,500 francs as an endowment for that round. The buyers were told the investment made by the seller, S_0 , and the revised minimum return, ρ' , and were reminded of the initial maximum return of 0.5. The buyers were also shown the corresponding levels of possible payoffs in francs. The buyers then entered their bids privately on their computers.

After all the buyers submitted their bids, the computer would select the highest bid and compare it to the seller's minimum selling price. If this highest bid was at least as high as the seller's minimum price, trade occurred; otherwise, trade did not occur. When trade occurred, the highest bidder received the asset, and the seller received the winning bid. In case of a tie, one buyer was randomly selected to be the winning bidder.

The true return was determined, and payoffs were calculated as follows: If trade did not occur, seller's payoff was set to 1000 francs – the seller's investment + the actual payoff from the asset – the minimum possible ending balance (900 francs):

$$\begin{aligned} \text{Seller's Payoff if No Trade} &= (1000 - S_0) \\ &+ (1 + \rho)S_0 - 900 = 100 + \rho S_0. \end{aligned} \quad (1)$$

If trade occurred, seller's payoff was set to 1,000 francs – the seller's investment + the selling price – the minimum possible ending balance (900 francs):

$$\begin{aligned} \text{Seller's Payoff if Trade} &= (1000 - S_0) \\ &+ \text{Highest Bid} - 900 = 100 + \text{Highest Bid} - S_0. \end{aligned} \quad (2)$$

Payoffs to the buyers were calculated analogously: If trade did not occur, each buyer's payoff was set to 1,500 francs – the minimum possible ending balance (900 francs). If trade did occur, the payoff to the winning buyer was 1,500 francs – the purchase price + the actual payoff from the asset – the minimum possible ending balance (900 francs); the payoffs to the other buyers were 1,500 francs – the minimum possible ending balance (900 francs) = 600 francs:

$$\begin{aligned} \text{Highest Bidder's Payoff if Trade} &= 1500 - \text{Highest Bid} \\ &+ (1 + \rho)S_0 - 900 \\ &= 600 - \text{Highest Bid} + (1 + \rho)S_0. \end{aligned} \quad (3)$$

$$\begin{aligned} \text{Other Buyers' Payoffs if Trade} &= \text{All Buyers' Payoffs if No Trade} \\ &= 1500 - 900 \\ &= 600. \end{aligned} \quad (4)$$

Note that the minimum possible payoff to the seller assumes that the seller does not sell the asset for a return below the ex ante lower bound of –10%. Analogously, the minimum possible payoff to the buyers assumes that the buyers do not bid above the ex ante upper bound of 50%.

The earnings from each round were put into a bank account and were unavailable for use in trade in subsequent rounds. At the end of the experiment, each subject's total earnings were converted to Canadian dollars, at the exchange rate of 4 francs to 1¢.

An experimenter read the instructions aloud to the subjects, while the subjects followed their own copies of the instructions. The experimenter then asked the subjects to answer written questions about the experiment. The CIRANO Research Institute in Montreal recruited the subjects and ran the experiment.

We sought in our experiment to insure that in some sense, the distributions of revised bounds and actual returns were unknown to the subject and to the experimenter. To achieve this result, we used quantum bits (qbits). As noted in Calude (2004, p. 10) and Calude and Svozil (2006, p. 6), real-world quantum random number generators do not typically produce qbits with the idealized probability of 1/2, even when subjected to unbiasing algorithms. Moreover, Calude and Svozil argue that no sequence of qbits is Turing-computable. Nevertheless, the sampling distribution we employed is difficult to distinguish from a uniform distribution (details available upon request). In our analysis, we assess the degree to which the data are consistent with a subject's assuming a uniform distribution.

The grouping warrants some elaboration. As we note in Sect. 2, heterogeneity in risk preference or in beliefs about the distribution of ρ could lead expected utility maximizing subjects to submit potentially winning bids, but only if the ordering of bids is preserved throughout the experiment, or at least until the heterogeneity becomes small and the market collapses. To test whether subjects' bids were driven by heterogeneity, it was necessary to keep the same subjects grouped together. This potentially leads to considerations of repeated play and hence to the bids not being independent across rounds within a group. We address this below.

4 Results

The experiment produced 80 observations of sellers' investment decisions and 312 observations of bidding across the four groups of subjects.¹ The average subject earned C\$26.90.

The sellers' average investment was typically not all or nothing: only one group-round (1.3%) had nothing invested, while in 14 group-rounds (17.5%), the sellers invested everything. The average investment was 606.55 francs. It was over 500 francs in all but the first round (where average investment was 444.75 francs), and under 900 francs in all but the last round (where it was 923.75), and had a general upward trend through the experiment.² Overall, the investment decisions were therefore inconsistent with risk-neutral expected utility maximization.

¹ There was a data entry error in Round 12 for Group 1, making the bids impossible to analyze for that group-round. The seller in Group 3 during Round 13 chose not to invest. The remaining 78 group-rounds generated four bids each, or 312 total. In an earlier version of this paper, we analyzed the data with Rounds 13–20 for Group 1 omitted. The results (available upon request) were similar to those we present here.

² An OLS regression of average contribution on the round number had an intercept of 553.25 francs and a slope of 13.12 francs, indicating that average investment typically rose 13.12 francs per round. The R^2 was 0.34.

Table 2 Frequency of bids by category

Rounds	# Bids	EU only (%)	Maximin (%)	NP only (%)	Inexplicable (%)
1–10	160	31.9	3.1	51.9	13.1
11–20	152	39.5	2.0	55.3	3.3
Total	312	35.6	2.6	53.5	8.3
1–10	139	36.7	3.6	59.7	
11–20	147	40.8	2.0	57.1	
Total	286	38.8	2.8	58.4	

Among the buyers, 35.6% of the bids were strictly below the ex post lower bound $(1 + \rho')S_0$ and were thus in a range consistent with risk-neutral expected utility maximization and inconsistent with non-probabilistic belief revision. Only 2.5% of the bids were exactly at $(1 + \rho')S_0$, the only point consistent with both theories. We observed 56.1% of the bids in the interval $((1 + \rho')S_0, 1.5S_0)$, consistent with rational non-probabilistic belief revision but not with risk-neutral expected utility maximization. The remaining 8.3% of the bids were inexplicable by either of these alternatives, as they were at or above $1.5S_0$.

The distribution of bids among these categories was generally consistent across the experiment, with the exception of decreasing frequencies of bids in the inexplicable range. Table 2 aggregates the bids in each range across group-rounds for the first and last half of the experiment. As noted above, the last ten rounds had 8 fewer observable bids than the first 10 rounds. The proportion of inexplicable bids is significantly lower in the last 10 rounds on a two-tailed test: A 95% confidence interval, including the continuity correction, on the difference in the proportions is from 3.3 to 16.6%. None of the other proportions differ significantly between the first and last ten rounds.

We did not find strong support for the models of boundedly rational expected utility maximization described in Sect. 2. Recall that a naïve buyer who considers only the interval of possible returns would estimate an expected value of $(1 + (\rho' + \bar{\rho})/2)S_0$, while a naïve buyer who considers the seller's upper bound but not the seller's strategy would estimate an expected value of $(1 + (3\rho' + \bar{\rho})/4)S_0$. Only four bids (1.3%) were within a rounding error of either of these naïve expectations. If we consider all bids weakly between either of these naïve expectations as consistent with boundedly rational expected utility maximization, we have 17.3% of the bids consistent with the theory. Two subjects bid weakly between the naïve expectations more than half the time (52.9% of the bids for Subject 7 and 58.8% of the bids for Subject 11). These two subjects were responsible for 35.2% of the bids that were between the two naïve expectations.

The equilibrium market behavior was as follows: In 14 of the 78 group-rounds with possible trade (17.9%), there was no trade, which on the surface is consistent with expected utility maximization under homogeneous risk preference and homogeneous beliefs. Among these 14, there was one round where the highest bid was in the inexplicable range, but where the seller's reservation price was even higher in the inexplicable range. Another 12 had bids strictly above $(1 + \rho')S_0$, but did not have trade because

Table 3 Market behavior by group-rounds

Rounds	# Group-rounds	EU only (%)	Maximin (%)	NP only	Inexplicable
1–10	40	0	0	60.0% (12.5%)	40.0% (2.5%)
11–20	38	2.6	2.6	81.6% (21.1%)	13.2%
Total	78	1.3	1.3	70.5% (16.7%)	26.9% (1.3%)

Table 4 Seller reservation prices

Rounds	# Reservation prices	Low	Optimal	High
1–10	40	27.5% (30.0%)	45.0% (40.0%)	27.5% (30.0%)
11–20	38	31.6% (34.2%)	57.9% (55.3%)	10.5%
Total	78	29.5% (32.1%)	51.3% (47.4%)	19.2% (20.5%)

the highest bid was below the seller's reservation price. Only one round had no trade and all bids below the $(1 + \rho')S_0$. One additional round had trade because of a low reserve price, but three of four bids were below $(1 + \rho')S_0$, and one was exactly at this maximin point. Thus, *only 2 of 78 group-rounds (2.6%) were consistent with expected utility under homogeneity.*

There were 20 group-rounds with prices in the inexplicable range. Along with the one group-round with an inexplicably high bid and an even higher reservation price, this means that 26.9% of the group-rounds were not explained by any theory we put forth. This number is high in part because bids above $1.5S_0$ were likely to win, unless another bidder or the seller was even more aggressive in the same group-round. Out of these 21 group-rounds, 16 occurred in the first ten rounds. Ten of the inexplicable prices were driven by a single bidder.

There were 55 (70.5%) group-rounds consistent with non-probabilistic beliefs. Among these, 13 had no trade due to a seller's reservation price above the highest bid, while 42 (53.8%) had prices consistent with non-probabilistic beliefs. Table 3 summarizes these results. The percentages represent the frequency with which the highest bid was in each category. The numbers in parentheses indicate the frequency of no trade occurring when the highest bid was in each category.

The market behavior shows no tendency to converge toward the predictions of expected utility and shows no indication of concentration on the maximin point. The results of the market pricing reflect that the inexplicable bids became less prevalent as the experiment progressed.

As noted above, the market collapsed in 14 of the 78 group-rounds (17.9%). Among these, 6 observations were in rounds 1–10 (out of 40 group-rounds, or 15%), and 8 observations were in rounds 11–20 (out of 38 group-rounds, or 21.1%). This may seem surprising, given that the frequency of inexplicable bids fell in the second half of the experiment. However, the sellers also reduced their frequency of setting their reservation prices inexplicably high. Table 4 provides details. A price below $(1 + \rho')S_0$ was below the optimal value either under expected utility or non-probabilistic beliefs. A price above $(1 + \rho'')S_0$ was above the optimal value under either theory. An expected

Table 5 Frequency of violations of Condorcet ranking

Rounds	# of Group-rounds	Violations of Condorcet ranking (%)
1–10	40	62.5
11–20	38	63.2
Total	78	62.8

utility maximizer's optimal price would be strictly between these values, while a seller who treats the information as non-probabilistic could choose either endpoint, depending on his or her level of optimism. Numbers in the table indicate the percentages under the assumptions of non-probabilistic beliefs, with the corrections for the assumptions of expected utility maximization in parentheses.

To evaluate whether heterogeneity in risk preference or in beliefs could drive the bidding, we calculate the number of times each subject outbid each other subject in the same group when neither was the seller. This method provides a Condorcet ranking of the bidders, potentially with ranking cycles. In three of the four groups, there was a well-defined Condorcet ranking, that is, there were no cycles, though two of these groups had two subjects who tied. The remaining group (Group 2) had a Condorcet cycle: Subject 4 outbid Subject 9 in the majority of the rounds where both were buyers, Subject 9 outbid Subject 10 in a majority of the rounds where both were buyers, and Subject 19 outbid Subject 4 in a majority of the rounds where both were sellers. All three in this cycle outbid the other two subjects a majority of the time, and those two outbid each other equally often. Thus, Group 2 had a top cycle and a pair with lower Condorcet valuations.

Table 5 shows the number of rounds where the bidding order among the four buyers violated their Condorcet rankings. It is clear from the table that the ordering was routinely violated and that the frequency of violations did not change over the course of the experiment. Because of the Condorcet cycle in Group 2, we counted a violation of the ranking only if one of the two subjects in the bottom group outbid a subject in the top cycle. All groups had at least one round where the Condorcet winner had the lowest bid, and three of the four groups had the Condorcet lower with the highest bid at least once. Since the heterogeneity explanation requires subjects to preserve a rank ordering, the data appear inconsistent with heterogeneity as the driver of the bidding behavior.

5 Conclusion

This experiment provides insight into how people behave when they have enough information about a problem to know what is possible but insufficient data to form meaningful probabilities. We contrast the predictions of expected utility theory with those of models of decision making under ambiguity.

In our experiment, trade would not occur in any round if subjects are expected utility maximizers with homogeneous risk preferences and common priors, while under rational non-probabilistic belief revision, trade is possible though not a given. These predictions are similar in spirit to those of [de Castro et al. \(2010, 2011\)](#).

Our main focus is on testing the predictions of two competing theories: expected utility with homogeneous agents versus rational non-probabilistic belief revision. Our design, however, enables us to test two additional alternatives: Agents may be heterogeneous (either in risk attitudes or beliefs), or agents may be boundedly rational, naïvely calculating an expected value but ignoring strategic considerations. To test for heterogeneity, we looked for consistency in the ranking of subjects' bids across rounds. We kept our subjects in fixed groups throughout the experiment, so comparing the ranking of their bids across rounds was straightforward. We tested for boundedly rational behavior by constructing naïve estimates of an expected value in two different ways. We looked at how often subjects made bids within a rounding error of either of these estimates, and how often subjects made bids in between either of these estimates.

Approximately 35–40% of our buyers made bids consistent only with expected utility theory, while 55–60% were consistent only with non-probabilistic belief revision. Our rank-order tests do not show support for bidding based on heterogeneity.

There was not strong support for a naïve model of boundedly rational expected utility maximization: About 1% of the bids were within a rounding error of either naïve expectation, and around 17% were in between these two values; 35% of these bids were generated by two subjects, and even these two subjects made almost half of their bids outside of this range.

In the early rounds, there were some bids that were inexplicable under either theory. These diminished over the experiment, while the other bidding behavior was highly stable. These results are similar to [Harrison and List \(2008\)](#), who find that most subjects do not fall prey to the winner's curse and that susceptibility to the winner's curse diminishes with familiarity and experience.

Consistent with the predictions of [Bewley \(2001\)](#); [Rigotti and Shannon \(2005\)](#); [de Castro and Yannelis \(2010\)](#) and [de Castro and Chateauneuf \(2011\)](#), we find that ambiguity does not lead to market collapse, even in the presence of asymmetric information.

One might also view this result in contrast to those of [Quiggin \(2007\)](#), who argues that information asymmetries are a chief reason for ambiguity aversion. The results are not in conflict: [Quiggin](#), working in the setting of [Machina \(2004, 2005\)](#), is concerned with asymmetries between the subjects and the experimenter. By withholding information to create ambiguity, an experimenter may make the subjects suspicious of the experimenter's motives. In our setting, the most prominent information asymmetry is between the seller and the buyers, all of whom face the same informational disadvantage with respect to the experimenter. This difference leads to the contrast between our results and those [Quiggin](#) predicts. For discussion of ways to avoid information asymmetries between subjects and experimenters in laboratory studies, see [Stecher et al. \(2011\)](#).

References

- Ahn, D., Choi, S., Gale, D., Kariv, S.: Estimating ambiguity aversion in a portfolio choice experiment. Working paper, University of California at Berkeley, University College London, and New York University (2010)
- Akerlof, G.A.: The market for "lemons": Quality uncertainty and the market mechanism. *Q J Econ* **84**(3), 488–500 (1970)

- Bewley, T.F.: Market innovation and entrepreneurship: a Knightian view. In: Debreu, G., Neufeind, W., Trockel, W. (eds.) *Economics Essays: A Festschrift for Werner Hildenbrand*, pp. 41–58. Berlin: Springer (2001)
- Bossaerts, P., Ghirardato, P., Guarnaschelli, S., Zame, W.: Ambiguity in asset markets: theory and experiments. *Rev Financial Stud* **23**(4), 1325–1359 (2010)
- Calude, C.S.: Algorithmic randomness, quantum physics, and incompleteness. In: Margenstern, M. (ed.) *Proceedings of the conference “machines, computations and universality” MCU’2004*, number 3354 in *Lecture notes in computer science*, pp. 1–17. Springer, New York (2004)
- Calude, C.S., Svozil, K.: Quantum randomness and value indefiniteness. CDMTCS 291, University of Auckland Centre for Discrete Mathematics and Theoretical Computer Science (2006)
- Cao, H.H., Wang, T., Zhang, H.H.: Model uncertainty, limited market participation, and asset prices. *Rev Financial Stud* **18**(4), 1219–1251 (2005)
- Chen, K.-Y., Plott, C.R.: Nonlinear behavior in sealed bid first price auctions. *Games Econ Behav* **25**(1), 34–78 (1998)
- Chen, Y., Katusčák, P., Ozdenorenc, E.: Sealed bid auctions with ambiguity: theory and experiments. *J Econ Theory* **136**(1), 513–535 (2007)
- Condie, S., Ganguli, J.: Ambiguity and rational expectations equilibrium. *Rev Econ Stud*, forthcoming (2011a)
- Condie, S., Ganguli, J.: Informational efficiency with ambiguous information. *Econ Theory*, this issue (2011b)
- de Castro, L.I., Chateaufneuf, A.: Ambiguity aversion and trade. *Econ Theory*, this issue (2011)
- de Castro, L.I., Pesce, M., Yannelis, N.C.: Core and equilibria under ambiguity. *Econ Theory*, this issue (2011)
- de Castro, L.I., Pesce, M., Yannelis, N.C.: A new perspective to rational expectations: maximin rational expectations equilibrium. Working paper, Northwestern University Department of Managerial Economics and Decision Sciences (2010)
- de Castro, L.I., Yannelis, N.C.: Ambiguity aversion solves the tradeoff between efficiency and incentive compatibility. Working paper, Northwestern University and University of Illinois at Urbana-Champaign (2010)
- Dow, J., da Costa Werlang, S.R.: Uncertainty aversion, risk aversion, and the optimal choice of portfolio. *Econometrica* **60**(1), 197–204 (1992)
- Epstein, L., Schneider, M.: Ambiguity, information quality, and asset pricing. *J Finance* **63**(1), 197–228 (2008)
- Fischbacher, U.: z-Tree: Zurich toolbox for ready-made economic experiments. *Exp Econ* **10**(2), 171–178 (2007)
- Harrison, G.W., List, J.: Naturally occurring markets and exogenous laboratory experiments: a case study of the winner’s curse. *Econ J* **118**(528), 822–843 (2008)
- Kagel, J.H., Levin, D.: The winner’s curse and public information in common value auctions. *Am Econ Rev* **76**(5), 894–920 (1986)
- King, R.R.: Reputation formation for reliable reporting: an experimental investigation. *Account Rev* **71**(3), 375–396 (1996)
- Machina, M.J.: Almost-objective uncertainty. *Econ Theory* **24**(1), 1–54 (2004)
- Machina, M.J.: ‘Expected utility / subjective probability’ analysis without the sure-thing principle or probabilistic sophistication. *Econ Theory* **26**(1), 1–62 (2005)
- Ozsoylev, H., Werner, J.: Liquidity and asset prices in rational expectations equilibrium with ambiguous information. *Econ Theory*, this issue (2011)
- Quiggin, J.: Ambiguity and the value of information: an almost-objective events analysis. *Econ Theory* **30**(3), 409–414 (2007)
- Rigotti, L., Shannon, C.: Uncertainty and risk in financial markets. *Econometrica* **73**(1), 203–243 (2005)
- Routledge, B., Zin, S.: Model uncertainty and liquidity. *Rev Econ Dyn* **12**(4), 543–566 (2009)
- Stecher, J., Shields, T., Dickhaut, J.: Generating ambiguity in the laboratory. *Manag Sci* **57**(4), 705–712 (2011)
- Thaler, R.H.: Anomalies: the winner’s curse. *J Econ Perspect* **2**(1), 191–202 (1988)