# A Biomimetic, Energy-Harvesting, Obstacle-Avoiding, PathPlanning Algorithm for UAVs 

Snorri Gudmundsson

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# A BIOMIMETIC, ENERGY-HARVESTING, OBSTACLE-AVOIDING, PATH-PLANNING ALGORITHM FOR UAVs 

A DISSERTATION SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING
AND THE COMMITTEE ON GRADUATE STUDIES OF EMBRY-RIDDLE AERONAUTICAL UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Snorri Gudmundsson
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# A BIOMIMETIC, ENERGY-HARVESTING, OBSTACLE-AVOIDING, PATH-PLANNING ALGORITHM FOR UAVs 

By<br>Snorri Gudmundsson

This Dissertation was prepared under the direction of the candidate's Dissertation Committee Chair, Dr. Charles Reinholtz, and has been approved by the members of the dissertation committee. It was submitted to the College of Engineering and was accepted in partial fulfilment of the requirements for the Degree of Doctor of Philosophy in Mechanical Engineering.


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#### Abstract

This dissertation presents two new approaches to energy harvesting for Unmanned Aerial Vehicles (UAV). One method is based on the Potential Flow Method (PFM); the other method seeds a wind-field map based on updraft peak analysis and then applies a variant of the Bellman-Ford algorithm to find the minimum-cost path. Both methods are enhanced by taking into account the performance characteristics of the aircraft using advanced performance theory. The combinedapproach yields five possible trajectories from which the one with the minimum energy cost is selected. The dissertation concludes by using the developed theory and modeling tools to simulate the flight paths of two small Unmanned Aerial Vehicles (sUAV) in the 500 kg and 250 kg class. The results show that, in mountainous regions, substantial energy can be recovered, depending on topography and wind characteristics. For the examples presented, as much as $50 \%$ of the energy was recovered for a complex, multi-heading, multi-altitude, 170 km mission in an average wind speed of $9 \mathrm{~m} / \mathrm{s}$. The algorithms constitute a Generic Intelligent Control Algorithm (GICA) for autonomous unmanned aerial vehicles that enables an extraction of atmospheric energy while completing a mission trajectory. At the same time, the algorithm automatically adjusts the flight path in order to avoid obstacles, in a fashion not unlike what one would expect from living organisms, such as birds and insects. This multi-disciplinary approach renders the approach biomimetic, i.e. it constitutes a synthetic system that "mimics the formation and function of biological mechanisms and processes."


## ACKNOWLEDGMENTS

It would have been impossible to complete this work without the help of my dissertation committee. First, I would like to express my appreciation to Dr. Charles Reinholtz, the Chair of the ERAU Mechanical Engineering department. Serving as the PhD committee chair, Dr. Reinholtz' friendly and helpful demeanor was not just exemplary, but made the long winding road toward my PhD considerably less bumpy.

I am also greatly indebted to both Dr. Sergey Drakunov and Dr. Vladimir Golubev. Dr. Drakunov was the committee's first advisor, but also mentored me in the pursuit of my goal. His expertise in mathematics and control theory was crucial in helping me evaluate the merits of the research presented in this dissertation. I am very grateful for his continuous encouragement of the research presented in this work. Dr. Golubev joined the committee later, but immediately became instrumental in pushing me to publish a portion of the work in conference and journal papers. He provided me with key guidance in those areas, something for which I am very thankful. In the capacity of his research background, Dr. Golubev also offered many insightful ideas that were incorporated in the dissertation.

Additionally, I express my great appreciation and thanks to my committee members, Dr. Patrick Currier and Dr. Eric Coyle, both from the ERAU Mechanical Engineering department. I was heartened by the realization that each expressed a deep interest in the problem presented in my research. Their strong understanding of trajectory planning and obstacle avoidance theory helped cement these key areas in the dissertation.

Words cannot express my gratitude for your support in making this dissertation a reality.

Finally, I want to express my gratitude to the former and current chairs of ERAU's Aerospace Engineering department, Dr. Habib Eslami and Dr. Tasos Lyrintzis. I also want to express my most sincere thanks to the current Dean of the College of Engineering, Dr. Maj Mirmirani, for extending the opportunity to complete my PhD in Mechanical Engineering.

## DEDICATION

I decided long ago that, should I ever complete a degree of PhD, I would dedicate my dissertation to three individuals who profoundly impacted my life and approach to aviation. All three were pilots and, sadly, have passed on.

First, there is the late Guð̌mundur Daði Ágústsson (1956-2016), a fellow Icelander, who first introduced me to the operation of real aircraft by inviting me, an eager teenage aircraft enthusiast, on countless flying trips in the mid 1970s. The exposure to real aircraft was enhanced by the myriad of questions regarding aviation that we bounced between us, many of which we were unable to answer. More than anything, this experience reinforced my determination to seek a scientific understanding of the physics of flying.

Second, there is the late Colonel USMC, Robert Overmyer (1936-1996). I worked with Col. Overmyer as a flight test engineer early in my career, during the development of the Cirrus SR2O aircraft. It is impossible to describe the honor of having an opportunity to work with such an experienced aviator. Col. Overmyer was hands down the best pilot with whom I ever had the opportunity to fly. Not only had he flown all the century-series fighters and contemporary fighters in the US arsenal, he also had two space shuttle missions under his belt (STS-5 and STS-51B), as a first officer and pilot-in-command. In spite of this experience, he was both modest and amicable. Tragically, Col. Overmyer died on March $22^{\text {nd }}$ 1996 in the crash of an experimental aircraft he was flight testing.

Third, there is the late Major Scott Anderson (1965-1999), described in Wikipedia as "... a late 20thcentury American polymath: Air National Guard F-16 pilot, instructor pilot, general aviation test pilot, Flight Operations Officer, engineer, inventor, musician, football player, outdoor adventurist, and award winning author." I worked with Maj. Anderson, also as a flight test engineer, during the development of the Cirrus SR20, from 1996-1999. He had taken over Col. Overmyer's position. He was an extremely capable individual, with a perpetual smile that would light up a room. Sadly, Maj. Anderson died while performing a routine flight test on the first production example of the SR2O on March $23^{\text {rd }} 1999$.

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ABBREVIATIONS
ABL Atmospheric Boundary Layer
AC Alternating Current
AFMS Automated Flight Management System
AGL Above Ground Level
AOA Angle-of-Attack
AOY Angle-of-Yaw
AR Aspect Ratio
AUVSI Association of Unmanned Vehicle Systems International
BEC Battery Eliminator Circuit
BPS Best Path Search algorithm
CAS Calibrated AirSpeed
CCA Cold-Cranking Amps
CG Center of Gravity
CIA Central Intelligence Agency
CM Center of Mass
CMF Constant Mass Flow
CNC Computer Numerical Control
CPU Central Processing Unit
CS Control Surface
CSP Concentrated Solar Power
CTOL Conventional Take-Off and Landing
CV Control Volume
DC Direct Current
DEM Digital Elevation Map
DOD Depth-of-Discharge
DOF Degrees-of-Freedom
EAS Equivalent AirSpeed
EMF Electro-Magnetic Force
EOM Equations of Motion
ESC Electronic Speed Controller
FAPF Fuzzy Artificial Potential Fields
FIO Flight-into-Obstacles
FIT Flight-into-Terrain
FPV First-Person View
GA General Aviation
GB Giga Bytes
GHz Giga Hertz
GICA Generic Intelligent Control Algorithm
GPR Gaussian Process Recession (method)
GPS Global Positioning System
GS Glide Slope
HT Horizontal Tail
IAS Indicated AirSpeed
IFR Instrument Flight Rules

```
            ILS Instrument Landing System
            IMU Inertial Measurement System
            IP Integer Programming
            ISR Intelligence, Search, and Reconnaissance
    KCAS Knots, Calibrated AirSpeed
            KE Kinetic Energy
    KEAS Knots, Equivalent AirSpeed
            KIAS Knots, Indicated AirSpeed
            KTAS Knots, True AirSpeed
            LES Large Eddy Simulations
            LIDAR Light Detection and Ranging
            Li-Ion Lithium Ion (battery)
            LiPo Lithium Polymer (battery)
            LiSSA Lift Seek-Sink Avoid algorithm
            LOC Localizer
            LOMS Locally-Optimal Myopic Search (method)
            LQR Linear-Quadratic-Regulator controller
            LSA Light Sport Aircraft
            LTS Lookahead Tree Search
            MAV Micro Aerial Vehicle
            MEM Maximum Endurance Mode
            MEMS Micro-Electro-Mechanical Systems
            MFS Multi-Function Display
            MON Motor Octane Number
            MGC Mean Geometric Chord
                    MNWP Mesoscale Numerical Weather Prediction
                            MRM Maximum Range Mode
                            NASA National Aeronautics and Space Administration
                            NCAR National Center of Atmospheric Research
                            NiMH Nickel-metal-Hydride (battery)
            NLF Natural Laminar Flow
                    NOAA National Oceanic and Atmospheric Administration
    NREL National Renewable Energy Laboratory
NRLMSISE Naval Research Laboratory Mass Spectrometer and Incoherent Scatter
    NWP Numerical Weather Prediction
            OAT Outside Air Temperature
            ODE Ordinary Differential Equation
            PBL Planetary Boundary Layer
            PDE Partial Differential Equation
            PE Potential Energy
            PEM Proton Exchange Membrane
            PFM Potential Flow Method
            PID Proportional-Integral-Derivative controller
            PIV Particle Image Velocimetry
            PV Photo Voltaic
            RAM Rapid Approach Mode
            RC Radio-Control
            RFD Rear-Flank Downdraft
```

RON Research Octane Number<br>RPM Revolutions Per Minute<br>SAS Stability Augmentation System<br>SFC Specific Fuel Consumption<br>S-L Sea Level<br>SPP Shortest Path Problem<br>sUAV Small Unmanned Aerial Vehicle<br>TAS True AirSpeed<br>TALEUS Tactical Long Endurance Unmanned Aerial System<br>TED Trailing Edge Down<br>TEU Trailing Edge Up<br>T-O Take-Off<br>TV Television<br>UAS Unmanned Aerial System<br>UAV Unmanned Aerial Vehicle<br>UCAV Unmanned Combat Aerial Vehicle<br>UUV Unmanned Underwater Vehicles<br>VFR Visual Flight Rules<br>VT Vertical Tail<br>VTOL Vertical Take-Off and Landing<br>WRF Weather Research and Forecasting (model)

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## 1. Introduction

The work presented here details the development of a Generic Intelligent Control Algorithm (from here on abbreviated GICA), intended for use in a variety of vehicles, primarily Small Unmanned Aerial Vehicles (sUAV). The purpose of the algorithm is to enable the vehicle in which it functions to extend its operational range or endurance well beyond its nominal capabilities while simultaneously facilitating navigation around obstacles and terrain. The extension of operational range is achieved by supplementing the energy stored onboard by harvesting convective atmospheric energy and solar energy. The algorithm is intended for multi-role surveillance and reconnaissance aircraft, but will likely find use in many other applications.

A machine controlled by a GICA, in many ways, exhibits mechanical responses resembling that of a living organism, contrasting what one would expect from an "ordinary" computer controlled machine. Such responses are largely caused by the "sequence of decisions" made by the algorithm itself. This sequence is driven by the stochastic nature of its sensory inputs, which originate in the environment in which the machine operates and contrasts "ordinary" pre-programmed machine responses. In this context, industrial robots or CNC machines are classified as "ordinary". The motion of an industrial robot is repetitious; each flight of a sUAV is different. The sUAV is subjected to random environmental inputs requiring the control algorithm to "adapt" and react to multiple sensory signals. This is not a design requirement for ordinary machines; the set of mechanical movements planned for such machines only needs to be optimized once (e.g. consider the installation of components in an assembly by an industrial robot). On the other hand, each flight of a sUAV presents a sequence of multiple optimization problems, each that requires a swift solution where each solution is different.

The GICA developed in this dissertation, requires the corresponding sUAV to be equipped with an Automated Flight Management System (AFMS). The AFMS commands the associated autopilot to control the vehicle in a fashion that conserves and, when possible, harvests energy available in the environment. Such machines, in particular sUAVs, are expected to contend with increasingly challenging atmospheric and mission constraints. Such constraints include the variability of wind strength and direction, avoidance of terrain, navigation around static and dynamic obstacles, and conservation of the propulsive energy stored onboard. It may also include avoidance of geographical regions that are deemed unfavorable to the success of the mission, such as avoidance of weather systems, restricted air spaces, and threat territories.

While the term "intelligent" may be interpreted in a number of ways, here it refers to an approach in which the control algorithm quickly computes numerous options to travel from a vehicle's present location to some desired destination. It does so in a complex, noisy, and non-linear environment. Furthermore, it "learns" about its environment and aligns its response to this learning. In this context, the term refers to the "apparent" intelligence of the algorithm as it "ponders" the best way to complete the mission. For instance, it may select a route that expends the least amount of onboard energy, while considering obstacles to avoid. The subsequent path of the vehicle may be perceived as "surprising" or
even "unintelligible" by a human observer not privy to the inputs that lead to the selection of its path. This definition complies with that presented by White and Sofge [1] and states:

Intelligent control is the use of general-purpose control systems, which learn over time how to achieve goals (or optimize) in complex, noisy, nonlinear environments whose dynamics must ultimately be learned in real time.

From this perspective, it can be considered bio-inspired as such is often the motion of living organism. In fact, the term biomimetic is an adjective that means "the study and development of synthetic systems that mimic the formation, function, or structure of biologically produced substances and materials and biological mechanisms and processes."1 The adjective is commonly used in the literature, for instance see Boslough [2]. The term implies multi-disciplinary activity as is plainly articulated in Reference [3]:

The biomimetic approach to engineering design is inherently a multi-disciplinary activity that results in a highly integrated, multi-functional system (just like real biological systems). Structures, materials, fluid mechanics, controls, power, sensors, etc. all play multiple and interrelated roles, and any meaningful application of biomimetics to engineering design must combine a number of traditionally separate disciplines. ... Clearly, independent, discipline specific research will still be required, but the ultimate applications will demand a multi-disciplinary process if we are to successfully mimic natural systems.

### 1.1 The Focus of the Dissertation

In short, a GICA intended for light aircraft (manned or unmanned) equipped with an AFMS that controls an autopilot is developed in this dissertation. This development calls for the review of methodologies that are used in the algorithm and a demonstration of its effectiveness. The demonstration is accomplished using a flight simulator that accounts for the effect of atmospheric convection in the form of thermals and terrain induced up- and downdrafts. Demonstration through flight simulation is important because it allows us to directly compare the energy cost of a mission that relies on conventional path-planning to one guided by the GICA.

The development of a GICA is truly multi-disciplinary in nature as can be seen in the following outline.

Chapter 1 Presents an introduction to the what, why, and how of the GICA.
Chapter 2 Presents a literature survey of the work that has already been conducted in four pertinent fields; path-planning and obstacle avoidance, modeling of atmospheric phenomena, glide optimization, and energy harvesting.
Chapter 3 Contains a brief overview of the current state of technology of sUAVs, military and civilian. The aircraft used for simulation and experimental work in the dissertation are presented.

[^0]Chapter 4 Presents the mathematical foundation of the several disciplines that must be brought together to form a capable, integrated multi-functional AFMS for autonomous vehicle operation is presented.
Chapter 5 Presents the mathematical tools used to estimate atmospheric properties and convection for prediction and simulation purposes.
Chapter 6 Presents the theory of performance for both powered and unpowered flight, which serves a major role in the utilization of atmospheric energy.
Chapter 7 Methods to model piston and electric engine power are presented. These include methods to estimate energy consumption of both classes of engines.
Chapter 8 Introduces the theory of flight mechanics, as it pertains to the development of the flight simulator software written for this effort.
Chapter 9 Discusses means to harvest external energy and conserve the consumption of onboard energy making it possible to extend range and endurance using the GICA. Methods used to estimate energy cost of traversing a path that accounts for atmospheric convection are developed. Discussion of solar power and piezoelectric is presented as well.
Chapter 10 Describes how the above disciplines are brought together to work seamlessly in a highly capable "intelligent" flight controller (GICA). Introduces the Lift-Seeking-SinkAvoidance (LiSSA) and the Static Obstacle Avoidance algorithms, which form an imperative part of the GICA. Two methods used for trajectory planning through windfields are presented. Also, two methods used for obstacle avoidance are presented. These are tied together in a capable generic path planner, presented in Chapter 11.
Chapter 11 Presents simulation examples that include missions in a variety of topography and weather conditions that are compared to the same mission using contemporary approaches. Power on and off are compared, as well as departure and arrival segments.
Chapter 12 Presentation of concluding remarks.

It is recognized that the physics of many of the aspects of gliding flight, path-planning, and obstacle avoidance have been understood for a long time. However, what sets this work apart is the following:
(1) The combination of fields such as atmospheric energy harvesting, solar energy harvesting, performance optimization, and obstacle avoidance into a single multifunctional algorithm for an autonomous operation is unique.
(2) A trajectory-planner that uses two algorithms (potential flow method and updraft peak identification) to plan an energy efficient mission through a complex wind-field and take advantage of existing convection energy. To the best knowledge of the author, such a hybrid approach is not found in the literature.
(3) Rather than focusing on a single approach the approach presented is hybrid in the sense it generates five possible trajectories. Upon evaluation of the minimum cost of each, the best trajectory is selected. This guarantees the energy recovery of the selected trajectory is always equal or better than the original one.
(4) The algorithm uses advanced performance theory, implemented in a finite difference scheme, to evaluate which of the five possible trajectories yields the greatest energy recovery. The integration of this finite difference scheme includes the predicted atmospheric convection and
leads to an accurate prediction of the fuel consumption, even for highly complex missions that feature multiple headings and altitudes over detailed topography. The incorporation and application of performance theory to supplement the path-planner has not been implemented elsewhere in this capacity to the knowledge of the author.

Generally, the literature presents methods to avoid obstacles or harvest energy through thermals or dynamic soaring or by capturing solar energy.

## 1. 2 Nature's Extraordinary Machines

As stated earlier, the term biomimetic implies the approach to path-planning is sought from Nature. Nature supplies us with valuable information about long distance travel from migrating animals. Migration is a complex aspect of animal ecology and is split into a number of different classes that also includes wandering animals such as the Albatross [4]. Interestingly, herds of migrating animals do not necessarily seek the path of "least" resistance (i.e. with respect to predator avoidance, abundance of water and food, etc.), but rather select the "shortest" and "straightest" route. Biologists have identified five characteristics that appear to apply to most forms of migration; (1) they consist of a protracted movement that carries the animals outside of their original habitat; (2) the movement is largely linear, not jagged; (3) special preparatory behavior (such as overfeeding) and behavior associated with their arrival; (4) they demand special allocation of energy. (5) the animals maintain behavior that is best described as "fervid attentiveness to the greater mission," which keeps them focused and undeterred by the variety of challenges they encounter [5].

In contrast, the motion of wandering animals is less direct and driven by an efficient search for food. It is where we find the source of many energy harvesting techniques. Nature has been working on the problem of migration for millions of years through the evolutionary process. Migration or movement around obstacles is not limited to mammals and birds; small insects, such as ants, demonstrate incredible resolve to overcome obstacles that far exceed the capability of even our most sophisticated machines. Yet, few would attribute the capability of an ant to an innate intellect, at least not the way we commonly understand that term. Something else appears at work; more like a reaction to external excitation in their environment. Another important element is the ability of the ant to adapt its motion to the environment, climbing some obstacles when necessary, while circumventing others. They are capable "machines." Nature presents us with many other examples of organisms that display extraordinary capabilities, attributed to adaptation by natural selection. A few are presented below.

## The Monarch

The first is the Monarch butterfly (Danaus plexippus), arguably one of nature's most intriguing creatures (see Figure 1-1). In the spring, swarms of the butterfly migrate to the northern states of the United States, and even to Canada, after hibernating over winter in Mexico and Southern US. This behavior is driven by its inability to withstand the cold winter temperatures. As soon as warmer days lie ahead, swarms of Monarchs head north to regions where their ideal larval plants (e.g. milkweed) grow. Each butterfly makes it to approximately middle of the continent, where it lays an egg that in a few weeks
hatches and becomes a larva. The larva consumes large amount of Milkweed and grows from the tiny larva of 1 mm to a 25 mm in just a few weeks. The author has personally observed this magnificent transformation in action, documented it through videography. Eventually, the larva attaches itself to the lower side of the Milkweed and forms the well known J-form, before transforming into a cocoon (or chrysalis). A few weeks later a fully formed Monarch hatches from the chrysalis and within a few hours takes off to continue the journey north. In the fall, the Monarchs head back south for hibernation. It has been long established that Monarchs in the eastern part of North America hibernate in the Sierra Madre Mountains of Mexico, while Monarchs in the western part descend on Southern California. Monarchs are the only known species of insects that migrates every year like birds, covering a total distance in excess of 4100 km [6]. There are even examples of Monarchs traveling as far as 5000 km [7].


Figure 1-1: A Monarch butterfly (photo by author)


Figure 1-2: A Black-browed Albatross (Thalassarche melanophrys), also known as the Black-browed Mollymawk, in flight. (Photo by JJ Harrison, Wikimedia Commons)

But how does the Monarch achieve this feat? How does an insect, whose maximum airspeed is well below even a modest breeze, manage to cover such immense distances? It is thought that when the butterflies detect the changing climate it triggers an instinct to travel north or south. The biological processes involved are beyond the scope of this dissertation, other than the actual migration takes advantage of atmospheric convection. This way, air currents may carry southbound Monarchs north, west, or east for some periods, while at other times, they manage to find and get helped by a breeze with a favorable southbound component. Eventually, after a flight that in many ways resembles a forced Brownian motion ${ }^{2}$, swarms of Monarchs show up at the destination over a range of several days or weeks. Even though their average forward "cruising" speed is an order of magnitude less than a typical gust of wind, the mathematical expectation (or average) of the distance covered eventually leads it in the vicinity of an original departure or destination position.

[^1]
## The Albatross

Another example of nature's many magnificent animals is the Albatross (Diomedea exulans), shown in Figure 1-2. The bird has about 60 year life span, mates for life, and has one chick about every other year [8]. Its wingspan extends some 3 to $3.4 \mathrm{~m}(10-11 \mathrm{ft})$ with a wing aspect ratio around 16 [9], the largest of any bird. The albatross, whose habitat is in the southern hemisphere, circumnavigates Antarctica several times a year, often flying 1000 km each day. Sometimes the large bird glides without a single wing flap for hours on end and it even appears capable of sleeping on the wing [8].

The Albatross manages the long range flapless glide largely through a so-called dynamic soaring [10] (see Section 1.3, Fundamentals of Soaring Flight and Figure 1-8). Dynamics soaring is a specific gliding technique in which a bird (or a sailplane) takes advantage of surface wind energy through a complex interaction with its own kinetic and potential energy. It requires the use of a prevailing wind shear profile that often forms on the surface of ocean waves or the leeward side of hills. This soaring method was first recognized and analyzed by Lord Rayleigh as early as 1883 [11]. In 1925 it was confirmed by Idrac [12] that this was indeed the method used by these great birds. An in-depth treatise of the mathematics of the dynamic soaring of the Albatross is given in Cone's pioneering work of Reference [13]. Dynamic soaring is a technique well known to sailplane pilots and pilots of Radio Controlled (RC) sailplanes, allowing some RC sailplanes to reach high subsonic airspeeds. At the time of this writing, the highest speed recorded is $826 \mathrm{~km} / \mathrm{h}$ ( 513 mph ), set by Mr. Spencer Lisenby on 12/22/2015, according to a website that keeps track of such feats ${ }^{3}$. While that airspeed is far above the capability of the Albatross, its flight demonstrates the amount of energy that can be extracted from wind energy. For instance, Wilson [14] developed a simplified model of dynamic soaring. In it he assumes the avian to fly in a wave generated updraft and in which it can accelerate to a specific airspeed, $V_{\max }$, and for which the associated rate-of-descent is precisely balanced by the magnitude of the updraft. If it is further assumed that the minimum airspeed of the Albatross is the stalling speed, $V_{S}$, then the excess kinetic energy between the two states ( $V_{\max }$ and $V_{S}$ ) can be estimated from

$$
\begin{equation*}
\Delta E_{K E}=\frac{1}{2} m\left(V_{\max }^{2}-V_{S}^{2}\right) \tag{1-1}
\end{equation*}
$$

Where $m$ is the mass of the bird. Thus, the Albatross can convert this energy into potential energy per

$$
\begin{equation*}
\Delta E_{P E}=m g h \tag{1-2}
\end{equation*}
$$

Where, $g$ is the gravitational acceleration and $h$ the height between the energy states. Therefore, the bird can use the excess kinetic energy to rise to an altitude of

[^2]\[

$$
\begin{equation*}
\Delta E_{K E}=\Delta E_{P E} \quad \Leftrightarrow \quad h=\frac{\left(V_{\max }^{2}-V_{S}^{2}\right)}{2 g} \tag{1-3}
\end{equation*}
$$

\]

The bird then utilizes this altitude to repeat the cycle.

## Other Birds

The final examples brought up here are that of the Black Vulture (Coragyps atratus - see Figure 1-3), Turkey Vulture (Cathartes aura) and the White Pelican (see Figure 1-4), all which make the southern United States their home, although the White Pelican only resides in winter. The two species of vultures are a common sight in Florida, where they can be seen to glide effortlessly over the flatlands without flapping a wing, exclusively extracting lift from thermals. They are frequently observed circling in thermals, rapidly gaining altitude, or just below Cumulus clouds, sometimes as high as 4000-5000 ft above the ground. Pelicans, on the other hand are common near Florida's beaches, where they glide miles on end, rarely flapping their wings, while taking advantage of lift forming on sand-dunes or on the windward sides of tall buildings sprawled along the beach.


Figure 1-3: A Black Vulture (Coragyps Atratus) in flight. Notice a smaller aspect ratio than that of the Albatross, a consequence of adaptation by natural selection. (Photo by Martien Brand - Wikimedia commons)


Figure 1-4: An American White Pelican (Pelecanus Erythrorhynchos) preparing to land. The ruffled feathers indicate flow separation regions. (Photo by Peter Wallack - Wikimedia commons)

## Lessons for the Design of Energy Harvesting Aircraft

From the standpoint of aircraft design, it is of great interest to ponder the differences between some of these bird and the environmental contributors that guided their evolution. Figure 1-5 compares the planform view of three large birds, the Albatross, the Great White Pelican, and the Andean Condor (Vultur gryphus), which belongs to the same family as the Black and Turkey Vultures; Cathartidae. The upper part of the figure has the three birds superimposed so they all have the same apparent wing span, highlighting the Aspect Ratio $(A R)$ of their wings. $A R$ is defined as the ratio of the wingspan (usually denoted by the letter $b$ ) squared to the area of the wing planform, or wing area (denoted by the letter $S$ ); in other words $A R=b^{2} / S$. The larger this value, the more slender is the wing. It can be seen that the

Albatross has the highest $A R$, about 15-16, followed by the Pelican, about 9, and finally the Andean Condor, about 5.3. Aspect ratio is a fundamental figure of merit in aircraft design. The greater the $A R$, the less is the contribution of lift induced drag to the total drag of the bird (or aircraft). A direct consequence is that the Albatross has the capability of gliding the farthest from a given altitude of the three. But why is there such disparity between the three birds? It is not hard to fathom that great glide range is vital to the survival of the Albatross, but doesn't the same holds for the other birds? If so, why are their $A R$ s some much smaller, in particular that of the Condor?


Figure 1-5: Planform comparison of three great gliding birds.
Other characteristics of these birds must be considered in order to find a clue to this mystery. Two clues come from their habitat and wing loading (i.e. weight divided by wing area). Let's reflect on the latter one first. Knowing the mass of the birds, it is easy to show that the wing loading for the Albatross is approximately $12.7 \mathrm{~kg} / \mathrm{m}^{2}\left(2.61 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}\right)$, the Pelican is $10.4 \mathrm{~kg} / \mathrm{m}^{2}\left(2.13 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}\right)$, and the Condor is 7.76 $\mathrm{kg} / \mathrm{m}^{2}\left(1.59 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}\right)$.

Next, consider the physical constraints large birds have to contend with, in particular in light of their habitat. As an example, cruising speed (which here refers to the speed range at which the birds "tend to travel", since it is unlikely they maintain single airspeed for long as is common in the operation of aircraft), increases with the $\sqrt{ }(W / S)$ the wing loading (e.g. see Equation (6-78)), where $W$ is weight.

Figure 1-6 shows how the wing loading affects the so-called Carson's optimum cruise airspeed (see Section 6.4.2, Cruise) [15] and highlights that these big birds operate at high speeds (relatively speaking). This is a detriment to the Albatross, because of the kinetic energy that must be absorbed during landing on nesting beaches in low wind condition. The impact can be great enough to break bones [13]. The Albatross has truly adapted to high wind strengths. All three birds are subjected to the detriments of the required propulsive thrust-power (i.e. thrust•velocity), $P_{\text {propulsive, }}$ which increases rapidly with the airspeed cubed.

$$
\begin{equation*}
P_{\text {propulsive }}=D V=\frac{1}{2} \rho V^{3} S C_{D} \tag{1-4}
\end{equation*}
$$

Where $D$ is drag, $V$ is airspeed, $\rho$ is air density, and $S$ is reference wing area. This is shown as the specific power (thrust-power per unit square area of wing) in Figure 1-6. This means the power required to propel the bird through wing flapping increases rapidly with airspeed and weight. However, muscle power increases linearly with mass (double the muscle mass, double the power), placing large and heavy birds at serious disadvantage compared to small birds, when comes to prolonged wing flapping. It helps explaining why the large birds are so dependent on efficient use of atmospheric convection for survival.


Figure 1-6: Effect of wing loading and minimum drag coefficient on Carson's airspeed (solid curves) and specific power (dashed curves).

Although the Albatross is adept at conventional soaring [11] it spends large amount of its time gliding the oceans where thermals and updrafts are rare. As shown by Cone [13], effective dynamic soaring
requires high $A R$ and $W / S$. During the accelerated turns, the Albatross may experience as much as 3 g load factor. The maneuver is accompanied by a high Angle-of-Attack (AOA or $\alpha$ ) and, thus, high liftinduced drag. The high $A R$ helps to reduce this drag and its detrimental impact on the achievable altitude gain (potential energy). The Condor, in contrast, inhabits mountainous regions, where updrafts (or slope lift) are common. Its survival depends on taking-off and landing in areas of vegetation and rocky mountain slopes, where low $A R$ and $W / S$ is far more useful. The low $A R$ improves the bird's ability to maneuver in confined spaces and the low $W / S$ reduces its rate-of-descent while gliding, making it possible to take advantage of weaker updrafts than otherwise ${ }^{4}$. The Pelican is sort of an "in-between." The author has made multiple observations of both Black and Turkey vultures over the flatlands of Florida, where thermals are common, as they gain or maintain altitude while gliding. They will glide in this fashion for long periods, without wing flapping. They can also be observed taking advantage of gusty midday winds, through the process of gust soaring. This is a feat of great innate skill, as any pilot of RC gliders can attest to. Possibly, this is permitted by a combination of velocity and acceleration sensing, enabling the birds to detect the motion of the mass of air in which they are flying. It can be argued that what makes these birds so effective at conserving and harvesting energy is a responsive biological control system, well worth our increased understanding, let alone imitation.

One last comment: As any casual observer will notice, it does not appear these animals are greatly influenced by the notion of time; i.e. being on time is not a primary goal; reaching an intended destination (which may be a food source) is. This removes an important constraint (i.e. time) from consideration and makes the mode of travel far more conducive to effective energy harvesting than time-constrained missions. In this context, a mission subjected to time-flexible, adaptive predictorcorrector approach is the focus of this dissertation. It allows autonomous airborne systems to truly take advantage of natural atmospheric convection. This can be accomplished using a flexible set of rules and logic as a part of the navigation computations. Considering the substantial amount of energy often present in atmospheric convection, this approach should allow greater distances to be covered, or alternatively, flights of greater endurance. Of course, the availability of this energy depends on the time of the day, and this affects potential night-time energy harvesting. Regardless, the primary goals of the algorithm is the conservation (and accumulation) of potential energy, as it is so easily converted back to kinetic energy.

## 1. 3 Fundamentals of Soaring Flight

Flying sailplanes and gliders is a very popular past-time for many people. Gliding is the oldest form of heavier than air flight, dating back to the days of Otto Lilienthal $(1848-1896,48)$ who pioneered the craft. For many, the absence of power offers incomparable simplicity and purity that allows the aviator to experience what it is to fly like the birds. However, the apparent simplicity is a veil that covers design sophistication of sailplanes that is far superior to most GA aircraft.

[^3]Soaring is a vital part of the energy harvesting methodology presented in this dissertation. At first, soaring flight may seem trivial to many. The absence of engine power implies reduction in complexity to laypeople and even many engineers. However, reduced technical complexity does not mean reduced operational complexity. The opposite can be argued; the absence of power forces the pilot to consider aspects of flying that engine power renders moot; the extension of flight through atmospheric convection. Many newcomers to the field of soaring flight are surprised to discover how nuanced soaring is when compared to powered flight. Without an engine, the pilot needs a deep understanding of the capability of his airplane and the elements in which it operates. The general lack of understanding of this basic fact renders it necessary to prepare the reader for the complexity of soaring flight by giving a basic introduction. Note that the mathematics of sailplane performance is provided in Chapter 6, Aircraft Performance Theory.

### 1.3.1. Sailplane Fundamentals

Configuration A in Figure 1-7 presents an example of what the typical modern sailplane looks like. Sailplanes generate the least drag of all aircraft; their efficiency is a marvel of engineering. Configuration B is an example of a long endurance UAV, which "borrows" many sailplane features, such as a high $A R$ wing and reduction of wetted area. The typical human operated sailplane carries one to two people, it has a gross weight ranging from 800 to $1800 \mathrm{lb}_{\mathrm{f}}$, wingspan from 35 to 101 ft , wing $A R$ from 10 to 51 , and wing loading from $5-12 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$. At this time, the largest sailplane in the world is the German built Eta, with a wingspan of 101 ft , AR of 51 , and wing loading of $10.44 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$. It is thought to have a best glide ratio around 70 - this means a glide path angle of $0.8^{\circ}$ - or a still air glide range of $213 \mathrm{~km}(115 \mathrm{~nm})$ from an altitude of 3000 m (about 10000 ft ).


Figure 1-7: A sailplane (A) and a powered UAV (B).
Today, serious sailplanes are only fabricated using composite materials that yield the smoothest aerodynamic surfaces possible. The modern sailplane features a tadpole fuselage, whose forward section is shaped to sustain laminar boundary layer naturally (Natural Laminar Flow or NLF) and contracted tail-boom to minimize the wetted area [16]. The skin friction associated with NLF is substantially lower than that of turbulent boundary layer. However, it is difficult to sustain over long distances. Therefore, once it transitions into turbulent boundary layer, the fuselage is reshaped to reduce the surface area. This leads to the distinct shape shown in Figure 1-7. Furthermore, the crosssection of the fuselage is shaped to minimize the frontal area of the vehicle, requiring the pilot to sit in
an inclined position. More efficient sailplanes use a single piece canopy, which can help extend the NLF farther back on the fuselage than a two piece canopy. Additionally, the modern sailplane utilizes a special wing planform style, referred to as a Schuemann planform (see Ref. [17]). This distributes lift more uniformly along the wing to reduce lift-induced drag. It can be considered a "less-costly to manufacture" elliptical wing. Sometimes sailplane wings feature polyhedral dihedral to further reduce lift-induced drag. The most significant contributor to the low drag properties of sailplanes is its wing, and horizontal and vertical tails, all which feature NLF airfoils. Sailplanes usually utilize T-tails to place the Horizontal Tail (HT) outside of the turbulent wake of the fuselage. This helps promote stable NLF over its surface.

### 1.3.2. Operation of Sailplanes

Sailplanes demand a lot from their operators and proper flying techniques require extensive pilot training. Pilots must know how best to position the sailplane "on tow" ${ }^{5}$, how to develop "feel" when searching for lift, how to get the most out of thermals, and how best to manage approach and landing, considering the lack of power reduces the room for error [18]. This requires the pilot to spend long hours sharpening these skills, sitting inclined in a tiny cockpit. Sailplanes are designed to offer the largest glide ratio possible and, ideally, this should be attainable at high airspeed (something that requires an extensive drag bucket, cruise flaps, or jettisonable water ballast). Low rate of descent allows them to stay aloft for long periods, provided atmospheric convection is present. This is possible because even the most anemic atmospheric convection rises faster than the rate of descent for such vehicles.

Sailplane pilots take advantage of four kinds of convection (rising air); thermals, ridge lift, standing mountain waves, and convergence lift. Sailplane pilots refer to these as lift. A thermal refers to air rising due to the ground being heated by the sun. The warmer air is less dense than the surrounding air, which causes it to rise. Thermals can reach altitudes as high as 18000 ft , although 5000-6000 ft is more common. Thermals can often be identified by the cumulus clouds that reside on top of them. Ridge lift results from wind being forced over ground features, such as cliffs, mountains, and ridges. Wave lift is the consequence of oscillatory motion of air, referred to as gravity waves by atmospheric scientists. Sailplanes have reached altitudes in the upper 40000 ft while utilizing such lift. Convergence lift occurs when two masses of air collide, such as sea-breeze and inland air mass.

In addition to these, a gliding technique called dynamic soaring can be employed provided certain atmospheric conditions prevail; (a) formation of a shear layer or (b) the presence of a wind profile in the atmospheric boundary layer (discussed in Chapter 5, Atmospheric Modeling). The former is characterized by two air masses moving at different rates while being separated by a thin shear layer; a fictitious layer characterized by rapid change in wind speed. Such conditions typically exist near the ground between valleys of ocean waves or on the leeward side of ridges. The wind speed in the upper air mass is much greater when compared to the lower one and the two are separated by a steep speed gradient. The actual dynamic soaring consists of a set of maneuvers intended to systematically exchange

[^4]kinetic and potential energy and make up for the energy lost to drag, by taking advantage of the gain in ground speed as the vehicle flies downwind (see Figure 1-8). This way, at 1, the sailplane turns into the wind direction, still below the shear layer where the wind speed is small. At $\mathbf{2}$ it has begun a climb that will take it through the shear layer, where the headwind will now increase rapidly. The true airspeed and, thus, the dynamic pressure acting on the vehicle rise sharply as its inertia drives it through the oncoming wind flow. The rise of lift is instantly transformed into climb to a higher altitude. At the same time, its airspeed is reduced gradually. To prevent too much loss in airspeed, the vehicle banks sharply at $\mathbf{3}$ and begins a dive toward the ground with the wind becoming a tailwind. At 4, the vehicle penetrates the shear layer again, now having greater acceleration with respect to the ground, thanks to the tailwind. At 5, the vehicle begins a new bank to change the heading into the wind and repeat the cycle.


Figure 1-8: The basics of dynamic soaring. (from Reference [17])
Dynamic soaring is utilized by many species of seabirds, some of which use it to travel great distances. However, no bird species rivals the Albatross, who regularly travel thousands of miles in a single trip, with minimal flapping of the wings, effectively gliding across oceans. The method is used both among pilots of sailplanes and radio-controlled aircraft.

Long distance flying in sailplanes depends on taking advantage of atmospheric energy. The pilot will ride the lift as high as possible before proceeding to the next source of rising air. The typical cross-country sailplane flight consists of a climb, followed by a descent, followed by another climb, and so forth. It is possible to reach very high altitudes in the process. Altitudes exceeding 30000 ft is a common occurrence and requires supplemental oxygen for the pilot. The current altitude record in a sailplane is $50722 \mathrm{ft}(15460 \mathrm{~m})$, set on $29^{\text {th }}$ of August, 2006 by the Americans Steve Fossett (1944-2007) and Einar Enevoldson, in a modified Glaser-Dirks DG-505 Open Class sailplane [19]. It was set in a mountain wave lift. The current long range record stands at about $1214 \mathrm{~nm}(2248 \mathrm{~km})$, set by Klaus Ohlman on $2^{\text {nd }}$ of December, 2003, on a Schempp-Hirth Nimbus Open Class sailplane [20].

### 1.3.3. Sailplane Airfoils

The modern sailplane uses sophisticated NLF airfoils, capable of sustaining laminar boundary layer up to $75 \%$ of the chord on the upper surface and $95 \%$ on the lower one. Such airfoils yield lift and drag characteristics that are quite different from "conventional" airfoils and this is evident in the formation of a pronounced "drag bucket". Often this calls for a modification in drag modeling when conducting performance analysis. Many sailplanes feature "cruise" flaps to allow its normal flaps to be raised a few degrees Trailing Edge Up (TEU) above neutral deflection. This shifts the drag polar toward a lower $C_{L}$ allowing the maximum Lift-to-Drag $\left(L D_{\max }\right)$ to be achieved at a higher airspeed, which is very beneficial in long distance competition.

A typical drag polar and L/D graph for a modern sailplane airfoil is shown in Figure 19. The unconventional shape of the L/D curve is evident. Sailplanes generally operate at Reynolds Numbers that places their airfoils close to where the boundary layer is sensitive to transition from laminar to turbulent; the transition region (see Figure 110). It is bounded on the lower end by an expedited transition in high-turbulence environment (e.g. as encountered in wind tunnels) and the upper by what is possible in smooth atmosphere. To take advantage of the low drag associated with laminar boundary layer and to delay the transition of the boundary layer requires very smooth surfaces.


Figure 1-9: Lift and drag characteristics of a typical modern sailplane airfoil. The graph is based on experimental data from Reference [17].

## The Importance of the Drag Bucket

Figure $1-11$ is an idealized representation intended to show the importance of achieving NLF on a hypothetical sailplane. The shape of both the drag polar and $L / D$ curve is classical for all NLF airfoils that feature a distinct two-wall drag bucket, including the double peak shape of the $L / D$ curve. For instance, this characteristic is present in most NACA 65- and 66-series airfoils. The figure shows the impact this has on the glide performance of a hypothetical sailplane. Reducing the $C_{D \min }$ by 30 drag counts (e.g. from 0.013 to 0.010 ) increases the maximum L/D ratio by 4.2 units and shifts its location to a much lower $C_{L}$. Lower $C_{L}$ means the $L D_{\max }$ will be realized at higher airspeed; something very beneficial to a sailplane. Admittedly 30 drag counts are on the high end of drag reduction and often the drag characteristics of the airplane as a whole masks the drag bucket, so a distinct double-peak $L D$ curves is not always achieved for typical applications. Instead, the laminar bucket shifts or widens the range of $C_{L}$, where "near" $L D_{\max }$ performance is found.


Figure 1-10: Sailplanes operate in the transition region and must feature smooth surfaces to delay transition from laminar to turbulent boundary layer. (from Reference [17])

As an example of the flexibility this yields, consider two sailplanes, A and B, which are identical except A does not develop a laminar drag bucket, while B does. Assume a wing loading of $10 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$ and the glide characteristics presented in Figure 1-11. Now consider a scenario in which the sailplanes are towed to an altitude of 1500 ft AGL on a calm, sunny day and set out to reach a thermal some 4 nm away. Naturally, if the thermal is not found the pilots must return to base, but is there enough altitude remaining for the return flight? Using the performance methods of Section 6.4.5, The Descent Maneuver and minimizing altitude loss by maintaining $V_{\mathrm{bg}}$, it is easy to show that Sailplane A achieves its $L D_{\text {max }}$ at 58 KTAS, while B achieves it at 77 KTAS. Covering the distance of 4 nm will take sailplanes $A$ and $B$ some 4 m 09 s and 3 m 07 s , respectively. Sailplane A loses 719 ft of altitude in the process and, upon return will be about 62 ft above the ground, requiring perfect piloting for the entire duration of the flight (see Figure 1-12). Sailplane B loses 640 ft of altitude and will be some 220 ft above ground upon return.


Figure 1-11: Example of the benefit of achieving NLF on a hypothetical sailplane. (from Reference [17])

Naturally, things are more complicated than this, although, this reinforces the point that high $L D_{\max }$ is crucial to the capability of either sailplane. However, it is necessary to keep this in mind as we develop the GICA; its function requires awareness of such "detail."


Figure 1-12: Glide range for two identical sailplanes. Sailplane A does not develop NLF, but B does. (from Reference [17])

### 1.4 Software Development

The method presented in this dissertation has been implemented in flight simulator software offered with the aircraft analysis software SURFACES, which is commercially available. It is also offered as standalone software under the name SURFACES Flight Simulator. The following provides a summary of the capability of this software. Also see user interface in Figure 1-13, Figure 1-14, and Figure 1-15.

- Intended as a research tool.
- Allows any typical airplane to be simulated (twin engines max). Current version uses lookup tables for $C_{L}, C_{D}$, and $C_{m}$, and constant stability derivatives.
- Allows the use of a joystick, although simulation faster than real-time is frequently used.
- Realistic altitude effects up to 276000 ft .
- Typical "World" is about $30 \times 30 \mathrm{~km}(20 \times 20 \mathrm{~nm})$ with topographical definition as small as $80 \times 80$ m , or $250 \times 250 \mathrm{ft}$.
- Offers a tool to create NURBS topography. Terrain collision is detected.
- Allows 6-DOF random winds and thermals to be generated. Thermals distributed using a Voronoi scheme. Surface wind is calculated and altitude corrections applied using a mass flow conservation method.
- Features a PID autopilot with Airspeed-, Altitude-, Heading, Roll-, Yaw- and Position-hold.
- Allows user to create a mission consisting of any number of 3-dim waypoints. Autopilot attempts to climb to or descend to waypoints if power is used.
- Among tools is a strip chart that can display and collect up to 71 parameters, lookup table creator, performance analyzer, and prescribed control input controller.


Figure 1-13: The SURFACES Flight Simulator interface.


Figure 1-14: The SURFACES Flight Simulator primary display window.


Figure 1-15: The SURFACES Flight Simulator Multi-Function Display (MFD) window.

## 1. 5 Features of the GICA

The purpose of this dissertation is to develop elements of an "intelligent decision-making" autopilot for sUAVs. The algorithm could find use in surface vehicles as well, in particular the obstacle avoidance capability. Path planning for typical UAS operations involves the creation of a list of specific geographic points, called waypoints. In this dissertation, the term mission refers to a list of waypoints that begins with a departure waypoint and terminates with an arrival waypoint (see Figure 1-16). These points may be reused for a round trip. A mission consists of no fewer than two waypoints and may contain a large number of waypoints. A line passing through a set of two waypoints is called path segment or simply segment. Thus, a mission consisting of 10 waypoints contains 9 segments. The vehicle's operator (which can be the autopilot) will navigate from one waypoint to the next while attempting to stay as close to the path segment as possible. A waypoint to which the aircraft is headed is called the active waypoint. A point of no return is a virtual position that once passed the onboard energy supplies will not suffice to return to the departure point. The act of creating a mission is referred to as path planning and is detailed in Chapter 10, The Generic Intelligent Control Algorithm.

### 1.5.1. Basic Operation of the GICA

The fundamentals of how the GICA operates will now be presented. It is assumes current state of sensor technology or even future state without specifying what sort of technology that might be. In this context, the dissertation will not explore sensor technology in much depth. Rather, it only assumes the
required inputs are provided by some technology. The operation of the GICA also assumes access to a digitized topographical map of the region in which it operates. A true and practical "fly-and-forget" autonomous sUAV must feature a highly versatile autopilot, capable of making a variety of decisions in real time. Figure 1-17 illustrates how the GICA works using a flow chart and shows how the GICA provides the following capabilities:


Figure 1-16: A mission consisting of a list of ordered waypoints. Here the point of no return is beyond the $N$-th waypoint, indicating the mission can not only be completed, enough fuel remains for the airplane to fly back to its departure waypoint.


Figure 1-17: A flow chart illustrating the basic operation of the GICA.
(1) It allows the user to create an initial flight plan by specifying as many waypoints as desired.
(2) The GICA processes the flight plan in a module called LiSSA (for Lift-Seeking, Sink-Avoidance). The LiSSA analyzes the properties of the atmospheric convection in the terrain in which the vehicle operate and determines topographical regions to avoid (downdraft) and regions to seek for energy harvesting (updraft). The wind field is either obtained a priori from a forecast (which can be uploaded before or en-route), or using an estimation of winds aloft (assuming appropriate corrections) and even the rapid wind field estimation presented in Chapter 5, Atmospheric Modeling. The LiSSA uses two algorithms to determine the best route through the wind field; a Potential Flow Method (PFM) and Best Path Search (BPS) algorithm. Both methods are discussed in detail in Chapter 10, The Generic Intelligent Control Algorithm. By comparing the two methods to the original plan, the LiSSA determines if it is warranted to modify the flight plan to take advantage of available atmospheric convection and, if it concludes so, it automatically modifies the original flight plan accordingly.
(3) Next, the modified path plan is submitted to the Static Obstacle Planner; a module that evaluates en route obstacles that are stored in a database. This includes natural obstacles, such as forests and terrain, and man-made ones, such as tall radio towers and other structures, restricted airspaces, threat territories, and other obstacles whose geographic position and size is known a priori. This static and dynamic obstacle avoidance capability is vital because many missions delegated to sUAVs are destined for operation at low altitudes, where the risk of Flight-into-Terrain (FIT) and Flight-into-Obstacles (FIO) is high. Obstacle avoidance methods are discussed in detail in Chapter 10, The Generic Intelligent Control Algorithm.
(4) Once the preparation of this second-tier flight plan is completed, the GICA activates the plan and flies to the first waypoint in the mission. As shown in Figure 1-17, the algorithm conducts a constant en-route monitoring of static obstacles that have yet to find their way into the database and dynamic obstacles using appropriate sensors (not presented here). Static obstacles not existing in the database are added to it automatically, constituting an important element of an intelligent control system; learning. The operation of the algorithm in this fashion allows the flight plan to be revised instantly, for instance, if triggered by the detection of a previously unknown static or dynamic obstacle.
(5) The loop also keeps track of time since last update and periodically reruns the LiSSA and the Static Obstacle Planner, in case wind aloft have changed or if a new wind field model has been uploaded.
(6) The GICA also has the "smarts" to figure out that if conditions change en-route and, causing a shift in the point-of-no-return (see Figure 1-18) such that completing mission becomes impossible. It will decide whether to return to base. This may be contingent upon winds changing enroute, availability of thermals in addition to updrafts etc. The algorithm will decide whether to activate its automatic return home feature if needed.

Once the vehicle operates in an updraft (slope lift or thermals), the GICA automatically puts the vehicle into a loitering mode to gain as much altitude as mission parameters demand and, thus, gain potential energy that can be converted into kinetic energy. The GICA automatically keeps track of the onboard energy and, whenever possible, substitutes atmospheric energy for it by shutting off engine power (or by throttling gas engines to idle).


Figure 1-18: A mission with a point-of-no-return between too close to departure, implying the mission cannot be completed as shown.

### 1.5.2. Application of GICA in Large Aircraft

At this point, some readers may already be asking if the use of the GICA is only limited to sUAV. It is helpful to provide an answer: Aspects of the algorithm presented in this dissertation can certainly be used in automated control systems of aircraft larger than typical sUAVs. However, if the corresponding aircraft is heavy, atmospheric convection no longer provides enough energy for sustained low power flying. Therefore, a practical application may be limited to aircraft whose wing loading is in the range of 0.05 to $73 \mathrm{~kg} / \mathrm{m}^{2}$ ( 0.01 to $15 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$ ). As shown in Chapter 11 , Simulation Samples, the GICA is applied to two sUAVs with wing loading close to 5 and $11 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$. The higher value represents an upper margin common to manned sailplanes or basic trainer aircraft. This way, it could be used as a primary command module for an Unmanned Aerial Vehicle (UAV) and manned aircraft alike. Manned aircraft include both auto-piloted sailplanes and as an envelope protection for powered airplanes - in particular in case of an engine failure.

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## 2. A Survey of Literature

This chapter presents a survey of scientific literature pertaining to the material detailed in remaining chapters of the dissertation. While the work presented in the dissertation brings new knowledge to the field of path-planning and energy harvesting, the same cannot be said regarding control theory or flight mechanics. Therefore, only literature relevant to key topics of the dissertation is presented in the chapter. This primarily involves obstacle avoidance algorithms, glide optimization, atmospheric modeling, and energy harvesting using UAVs.

### 2.1 Path Planning and Obstacle Avoidance Algorithms

Path planning refers to the preparation of a mission (see Section 1.5, Features of the GICA). There are generally two approaches to obstacle avoidance: (1) Planning path around (known) obstacles (that may call for continuous updating of the plan) and (2) reactively avoiding obstacles (as they are discovered). This effort requires multiple decisions for UAVs: The general route has to be planned, departure, arrival, and intermediary waypoints have to be defined, and corresponding distances and fuel requirements have to be estimated. Furthermore, in-flight maneuvers, such as glide or cruise flight profiles, must be selected and, last but not least, obstacle avoidance must be considered. Obstacles come in many shapes and there are both natural and man-made obstacles to be considered. Even the class of aircraft plays a role in the avoidance and two major classes of aircraft are considered in this context; Conventional T-O and Landing (CTOL) and Vertical T-O and Landing (VTOL). Obstacle avoidance for the former class is far more challenging than the latter, because CTOL aircraft must remain at airspeed greater than some minimum value (the stalling speed). This contrasts VTOL, which, technically, can hover at zero forward airspeed and, thus, are capable of navigating between closely spaced obstacles. The fact that CTOL aircraft are subject to a minimum speed constrains their motion in obstacle fields due to turning radius and bank acceleration (which affects the time it takes to rotate from level to a given bank angle).

Obstacle avoidance remains an active field of research as it has for several decades. The success of this work is, among others, already visible in household items, such as the well known iRobot Roomba vacuum cleaning robot. Fundamentally, sensors onboard a moving vehicle are used to detect the world in which it operates and find a path through a clutter of obstacles. Obstacles are generally classified based on whether they are static or dynamic. Terrain (in particular mountainous) constitutes a static obstacle. So does a radio tower or a high-rise building. The geographic position and size of static obstacles is either known a priori or unknown until discovered en-route, usually by sensors onboard the vehicle. While the position, speed, and size of dynamic obstacles are usually unknown, it may be known in some situations. These two classes of obstacles affect the nature of algorithms used for detection and avoidance. This is important to keep in mind when studying various development efforts for such algorithms.

One of the early research in this area is presented by Moravec [1] who developed an algorithm for a small mobile robot as early as 1980 . The robot was operated by a computer program that used onboard TV cameras to analyze images and gain knowledge about its environment. It would then plan a path
through cluttered space, updating its plan as it discovered new obstacles. This work was intended for surface vehicles designed for extra-terrestrial planetary exploration. It involved a slow moving ground vehicle and was surely hindered by the limited computational power of the early computers. The path planning algorithm used circles to represent obstacles and then found a bypassing path using geometry.

Later work focused on using potential flow theory for obstacle avoidance. The method was suggested as early as 1983-86 by the work of researchers such as Andrews and Hogan [2] (1983) and Khatib [3] (1986). The approach became recognized as the Potential Flow Method (PFM) for obstacle avoidance and was touted for its mathematical elegance and simplicity. The methodology was applied to path planning by researchers such as Akishita et al [4] (1990) and Connolly et al [5] (1990), both who independently developed a global method that used solutions to Laplace's equations to plan a smooth, collision-free path through a collection of obstacles.

Generally, the PFM offers great advantages, such as (1) being superbly established mathematically, as they use theories of complex variables and analytical functions, and (2) permitting a (relatively) simple creation of complex flows by the superposition of multiple elemental flows (see Section 4.1.4, The Use of the Governing Equation for Description of Flow). However, several problems inherent to the method soon began to surface. For instance, as shown by Koren and Borenstein [6] (1991), the space in which a robot operates, and which contains a variety of obstacles, may include regions where the solution of the differential equations that describe the motion of fast and heavy robots becomes oscillatory. Furthermore, they discussed issues with (1) trap situations, caused by local minima of the potential function and that lead to cyclic behavior of the robot. This occurs, for instance, when the robot drives into a U-shaped obstacle. Several solutions have been developed to avoid this predicament. (2) The solution leads to no passage between two closely spaced obstacles (e.g. a door frame) and results from both obstacles (i.e. door frames) to exert combined repelling virtual force that makes the robot turn away when could easily have passed between them. (3) A significant limitation of the method is the presence of oscillations that are caused by the proximity of a collection of obstacles. (4) A related oscillatory issue is caused when the robot operates in a narrow passage.

The work of Kim and Khosla [7] (1992) presented one solution to the problem of "trapping," so detrimental to real-time obstacle avoidance. It turned out the formation of local minima in cluttered environment could be avoided by the use harmonic potential functions, rather than the previously used non-harmonics. To do this, they rewrite the Laplace equation, which is the governing equation for potential flow (see Section 4.1.3, Governing Equation for Irrotational Incompressible Flow) as follows:

$$
\begin{equation*}
\nabla^{2} \phi=\phi_{r r}+\frac{n-1}{r} \phi_{r}+\Delta \theta \tag{2-1}
\end{equation*}
$$

Where $r$ is the distance from the origin, the subscripts $r r$ and $r$ represent the second and first partial derivative of $\phi$ with respect to $r$, and $\Delta \theta$ represents angular terms. The last term becomes zero because the function $\phi=\phi(r)$ only. Then they show that the solution of Equation (2-1) is given by

$$
\phi= \begin{cases}C_{1} \ln r+C_{2} & \text { if } n=2  \tag{2-2}\\ \frac{C_{3}}{r^{n-2}}+C_{4} & \text { if } n>2\end{cases}
$$

Where $C_{i}$ are constants and $n$ is the number of obstacles in the environment. The solution has singularity when $r=0$, but since the Laplace equation is invariant to translation, it is possible to select an origin that is outside of the space of interest. A consequence of this interesting approach is illustrated in Figure 2-1, which is recreated based on Reference [7]. The figure shows the potential field associated with the combination of four obstacles, placed at $(1,0),(0,1),(-1,0)$, and $(0,-1)$. These obstacles are "out-of-sight" in the space, which spans $[-0.5,0.5] \times[-0.5,0.5]$ for both images, but their "repelling" effects are present as the upward slope. It is clear that the harmonic potential on the right does not feature a local minimum that "traps" a robot, like the non-harmonic potential one on the left. Then, the paper goes on to discuss the combination of the approach with the PFM in a fashion similar to what is done in this dissertation (see Section 10.3.2, LiSSA Method 1 - Potential Flow Method).


Figure 2-1: A comparison of using non-harmonic (left) versus harmonic potentials (right). The right potential function does not have a local minimum while the left one does. (Based on Reference [7])

Rew and Kim [8] (2008) presented a new method intended to address the oscillation problem for the path-planning of wheeled mobile robots by using a complex potential flow function and showing it improved path stability by reducing oscillatory amplitudes. Yin et al [9] (2009) present a variation of the PFM that allows a robot to follow a dynamic goal while avoiding dynamic obstacles. Honglun et al [10] present a method to plan the trajectory of a UAV in complex 3-dimensional topography with interspersed threat areas. This method, while sharing some parallels to the method presented here, differs in the absence of energy harvesting.

Considerable activity is also present in the development of path-planning methodologies. A number of interesting algorithms have been developed for this purpose, of which Dijkstra's algorithm (see Section
10.2.2, Dijkstra's Algorithm) is probably best known. Dijkstra is said to have conceived it in 1956, but, interestingly, did not publish it until 1959 (see Dijkstra [11]), after improvements to it had already been published! The algorithm uses weighted edges extended between collections of nodes to determine the shortest (or least costly) path between them (see Figure 2-2).

Originally the algorithm determined the shortest distance between two nodes (e.g. A and H in Figure 2-2), but later version produce shortest path trees from a node to all the other nodes. This allows all solutions of interest to be generated at once. Among shortcomings of Dijkstra's algorithm is its inability to handle edges with negative costs. Several remedies have been developed to improve it, of which Bellman-Ford (see Section 10.2.3, Bellman-Ford (Shimbel's) Algorithm), A*-search, and Prim's algorithms are examples.


Figure 2-2: Dijkstra's algorithm methodically determines the shortest distances between nodes, based on their weighted edges (here with distances).

Other path-planners attempt to solve various constraints in the space in which a robot (or UAV) operates and avoid using Dijkstra's algorithm. For instance, Jun and Raffaello [12] (2003) presented algorithm intended for UAVs operating in adversarial environment, where threat regions can be identified by mapping the regions a priori. They point out that paths associated with threat regions that can be fully defined (or have "perfect information" as they put it) always allows a safe passage to be constructed around those regions. However, when the threat regions are associated with probability, a different approach must be taken. For this they propose a new path-planning algorithm based on the probability map of threats, which they assume can be compiled a priori using surveillance information. In a 2004 paper, Talbot [13] suggest that while Dijkstra's algorithm works for many situations, there are certain complex situations for which it is insufficient. He suggests a dynamical formulation, using Integer Programming (IP) as a more powerful method for such cases. Another example is presented in a 2010 paper by Fernandez-Perdomo, et al [14], who use $A^{*}$-search, which is a heuristic ${ }^{1}$ optimization technique first described by Hart et al [15] in 1968, to help with the path-planning of an Unmanned Underwater Vehicle (UUV). This approach is implemented in a path-planning algorithm called Constant-Time Surfacing A* (CTS-A*). The method is considered an extension of Dijkstra's method, but its performance is much better due to the use of heuristics. The $A^{*}$ algorithm is discussed in detail in Chapter 10, The Generic Intelligent Control Algorithm. Another methodology that also achieves good performance is Bidirectional Dijkstra, which performs simultaneous search from the initial and goal nodes and, once both arrive at the same node, the search and shortest path definition is completed (see Delling et al [16]). This but one of multiple improvements over the original Dijkstra's algorithm, some of which are claimed to be as much as $3 \times 10^{6}$ times faster [16].

[^5]Outside of this, an investigation of the literature reveals the impact harmonic control has had on research in robotics. Some researchers are seeking methods to operate legged robots in areas of unknown environment, while others are investigating controllers that possess some artificial intelligence and, thus, can learn about their environment. For instance, as early as 1996, Huber et al [17] use the harmonic potential function approach to guide a quad-ped (four-legged) robot. The approach is was also presented in a 1997 paper by Feder et al [18], who investigated a use of harmonic potentials for realtime path planning in a dynamic environment in which obstacles present various challenges to the movement of the robot, such as translation, expansion, contraction, and rotation. A year later, in 1998, Zelek [19] suggested parallel computing as a solution to reducing computational time for the solution of the harmonic PFM. He demonstrated that using quad-trees and modified A*-search lead to considerable reduction in computational time. In 2000, Wang and Chirikjian [20] proposed a new artificial PFM for path-planning for self-contained robots, which regards the problem as a heat-flow with minimal thermal resistance. They did this by using the heat equation, which is the governing equation for the distribution of heat in a space as a function of time. By eliminating the time dependency the influence of heat sources and sinks can be represented as

$$
\begin{equation*}
-\nabla \cdot(k \nabla T)=q \tag{2-3}
\end{equation*}
$$

Where $T$ is the temperature, $q$ denots heat sources and sinks, and $k$ is the thermal conductivity. They showed this method leads to path optimization in 3-dimensional space, over complex obstacle geometries. In 2002, Masoud and Masoud [21] proposed a PFM controller they refer to as an Evolutionary, Hybrid, PDE-ODE Controller (EHPC). It is an example of establishing artificial intelligence capability into the controller. A 2003 paper by Alvarez et al [22] presents a non-heuristic approach to the path-planning problem, that produces a real-time incremental development of a path to a goal using sensor information. The method considers the robot dynamics, although the authors specify the high computational load as its primary drawback. Sweeney et al [23] (2003) used harmonic potential to control multiple, independent robots. Kazemi et al [24] (2005) developed a hybrid path-planning technique that uses harmonic functions per PFM coupled with a Probabilistic Roadmap (PRM). The method uses the potential flow field to identify narrow passages in the space. In other words, the search for the shortest-path in an environment of scattered obstacles can sometimes lead the robot to a narrow passage through which it cannot pass. These passages often result in high magnitudes of flow velocity that are then identified and then probabilistic decision is made to avoid them using Dijkstra's method. In 2007, Girau and Boumaza [25] proposed the development of reconfigurable digital circuits to replace hardware implementation of specialized analog controllers. The approach uses harmonic PFMs that can handle dynamic constraints.

Other approaches for robot controllers include fuzzy-logic. Some of this work was published as early as 2000-2003 by researchers such as Walker and Esterline [26] (2000), Tsourveloudis et al [27] (2001), which combines the use of PFMs and fuzzy logic, Tu et al [28] (2003), which uses fuzzy potential energies, and Iraji and Manzuri-Shalmani [29] (2007), whose method uses Fuzzy Artificial Potential Fields (FAPF) to help generate flexible path-planning in real-time.

Most of the aforementioned papers focus on the obstacle avoidance of surface vehicles. Others focus on terrain avoidance for aircraft. Such algorithms typically involve terrain following for military aircraft. For instance, Smith [30] (1969) developed a universal terrain-following algorithm for use in tactical military aircraft, titled as "universal" to indicate its use was not only restricted to such aircraft. The method relied on the use of radar to detect the terrain in front of the aircraft and then executed as pull-up pushover maneuver in sequence within the g-load limitations of the aircraft in which it was used. And in 2012, Oh et al [31] presented an integrated terrain following algorithm for Unmanned Combat Aerial Vehicle (UCAV), using Voronoi diagram to define a horizontal plane path through threat territory, while a cubic spline is used to define the required vertical translation of the UCAV to prevent ground collision.

### 2.2 Gliding Flight

Although various aspects of gliding flight have been thoroughly documented, starting with Otto Lilienthal's 1889 book "Der Vogelflug als Grundlage der Fliegekunst" ${ }^{\text {[ }}$ [32], the field remains active. The optimization of the glide trajectory during powerless flight is a vibrant field as well, with much interesting work being developed. This has resulted in the development of multiple methods to optimize the glide trajectories of sailplanes. Generally, these studies revolve around the optimization and automation of dynamic soaring and gliding flight through thermals.

A very important contribution in this respect was made in 1963 in a pioneering treatise by Cone [33] in which dynamic soaring of the Albatross took center stage. This work explains how aerodynamics is almost certainly the primary driver behind the existing morphological and ecological characteristics of many species of birds. Detailing the importance flight plays in the evolutionary specialization of most birds, the work proceeds to derive a theoretical foundation for the optimal conditions of gliding flight. Just like in case of sailplanes, it is shown that primary parameters governing efficient glide of birds is large lift-to-drag ratio $(L / D)$, low lift coefficient $\left(C_{L}\right)$, and large wing loading $(W / S)$ as this minimizes the cost function of glide which Cone expresses as follows

$$
\begin{equation*}
\frac{1}{L / D} \frac{1}{w}+\sqrt{\frac{\rho}{2}} \sqrt{\frac{C_{L}}{W / S}} \tag{2-4}
\end{equation*}
$$

Where $L$ represents lift, $D$ is drag, $w$ is rate-of-climb (or updraft), $\rho$ density of air, $C_{L}$ is lift coefficient, $W$ is weight, and $S$ reference wing area. Dynamic soaring requires steady horizontal oceanic wind with a boundary layer in which exchange of kinetic and potential energy takes place, with the wind shear making up for the energy loss due to drag. Similar modeling and discussions of dynamic soaring is also provided by Barnes [34] (2005), Denny [35] (2008) and Richardson [36] (2010). However, computer simulation of the flight of the Albatross dates much earlier; at least to 1973 by work by Wood [37]. At any rate, Barnes' paper [34], in particular, is widely cited by many researchers. In a 2010 paper by Sukumar and Selig [38] a commercially available RC flight simulator is used to demonstrate sustained piloted dynamic soaring in high winds over open fields using an Albatross sized RC sailplane with a 3 m

[^6]wingspan, $A R$ of 20 , and 15 kg weight. The formulation used is based on Barnes' approach [34], while the wind profile is the logarithmic model used by researchers like Cone [33] (1964), Wood [37] (1973), Pennycuick [39] (1982), Swolinsky [40] (1986), and Stull [41] (1997) and is given by
\[

$$
\begin{equation*}
U(z)=U_{r e f} \frac{\ln \left(z / z_{0}\right)}{\ln \left(z_{r e f} / z_{0}\right)} \tag{2-5}
\end{equation*}
$$

\]

Where $U(z)$ is the wind speed at altitude $z, U_{\text {ref }}$ is a reference windspeed at altitude $z_{\text {ref }}$, and $z_{0}$ is called the roughness length or roughness factor and depends on the texture of the ground over which the wind flows. The resulting flight trajectories reach altitudes above ground level as high as 185 m and required the vehicle to achieve high airspeed ( $65 \mathrm{~m} / \mathrm{s}$ ) in a wind speeds ranging from $15-20 \mathrm{~m} / \mathrm{s}$. Furthermore, while requiring relatively low pilot workload the maneuver resulted in a high loading of 6-9g. This would be unacceptable for a human occupant, but is less of an issue with unmanned systems, although it may be a concern for fatigue. Shaw-Cortez and Frew [42] 2014 investigated the use of a path planning algorithm to develop dynamic soaring trajectories for a UAV in order to minimize the use of on-board energy in a dynamic soaring environment. In particular, they studied guidance control in which a UAS is either (1) directed to follow a specific trajectory for dynamic soaring, or (2) loiter, in which the vehicle maintains geographic position using dynamic soaring. The wind gradient used is a constant triangular profile, which is unusual for the typical atmospheric boundary layer modeling, which tends to use something closer to the $1 / 7^{\text {th }}$ power law (see Section 5.2.1, Modeling the Planetary (Atmospheric) Boundary Layer). Additional discussion of dynamic soaring, as well as musings of its conversion into practical applications is presented by Pfeifhofer and Tributsch [43] (2014).

Glide optimization pertains to more maneuvers than just dynamic soaring. In 2005, Qi and Zhao [44] presented a study analyzing the flight of a generic UAV flying through a vertically moving thermal cell. Using a 2-dimensional point-mass model of a jet-powered UAV, the authors identify four fundamental parameters of key importance. These are $L D_{\text {max }}$ (the maximum glide ratio), $C_{D \text { min }}$ (minimum drag coefficient), maximum vertical wind speed in a thermal, and the parameter $k$, defined as shown below

$$
\begin{equation*}
\kappa=\frac{\rho g R}{2(W / S)} \tag{2-6}
\end{equation*}
$$

Where $R$ is the radius of the thermal cell and $g$ is the acceleration due to gravity. The parameter typically varies between 0.5 and 25 , where the smaller number applies to small thermals or heavy UAVs, while larger numbers indicate larger thermals or light UAVs. They formulated flights through a thermal cell as nonlinear optimal control problems to minimize the average thrust per unit time and studied the effects of the fundamental parameters on optimal UAV flights. Their results suggest that significant improvements in UAV fuel consumption are possible by taking advantage of thermal energies.

Wolek et al [45] (2015) investigated a problem related to guiding a soaring vehicle (aerial and underwater) while minimizing change in potential energy. This is attempted by ensuring the vehicle operates inside the constraints of its maneuvering envelope. Similar sort of maneuvering considerations
are discussed in References [46] and [47] and is presented in this dissertation in Sections 6.4.6, Analysis of a General Level Constant Velocity Turn and 6.5.12, Circling Flight. These sections consider atmospheric convection as well. Almgren and Tourin [48] (2015) use stochastic optimization to address the problem of uncertain future atmospheric conditions by constructing a nonlinear Hamilton-JacobiBellman equation for the optimal speed to fly. While their study omits the effect of wind (thermals only), it presents an interesting solution of the so-called MacCready problem (see Section 6.5.12, Circling Flight).

Besides the aforementioned, there is other work that involves gliding flight and is worthy of mentioning. One such is a 2012 PhD dissertation by Zhao [49], which presents landing trajectory optimization algorithm that could be adopted to command an autopilot. The algorithm is intended to optimize the landing trajectory for aircraft that have suffered an unanticipated emergency, currently calling for swift decisions to be taken by the pilots. The work uses two actual aircraft accidents to evaluate the robustness of the algorithm. The first is the fatal Swissair's Flight 111 MD11 commercial jetliner, which crashed into the Atlantic Southwest of Halifax International Airport on 09/02/1998. The other is US Airways Flight 1549, which involved the successful ditching of an Airbus A320 into the Hudson River, New York, on 01/15/2009. The author demonstrates the probability of different outcomes for both incidents, had said aircraft been commanded by such an algorithm. Another interesting work related to flight mechanics of large aircraft in wind fields is that by Zhenxing et al [50] (2009), which presents a study on microburst represented by a vortex ring and Rankine vortex principle using a flight simulator.

### 2.3 Atmospheric Modeling

Wind is of central importance in the operation of manned and unmanned aircraft. Its significance can, in part, be seen in the role it plays in aircraft safety. It is estimated that weather contributes to $30 \%$ of all aviation accidents [51], largely due to wind and turbulence. However, wind is important in areas besides safety; for one, it makes energy harvesting possible. Additionally, horizontal winds must be accounted for in any realistic path planning and this is often missing from some of the work presented below. Researchers are learning more and more about the atmosphere every day and atmospheric science remains a vibrant field with substantial number of papers published every year. Many such contributions are useful in the development of the GICA, and atmospheric energy harvesting in general. It is not possible to give this massive field a satisfactory presentation in this survey, so only a handful of papers will be discussed. These papers give idea about research in the planetary boundary layer and help the reader understand the complexity of atmospheric flow. Some of this information is used in the flight simulation section of this dissertation, which was developed to demonstrate the capability of the GICA.

## Thermals

The study of thermals and the atmospheric boundary layer (ABL or planetary boundary layer; PBL) are highly active fields among atmospheric scientists. Understanding of the role convection (mechanism for thermals) plays in the atmosphere is still expanding and much remains to be discovered. Thermals primarily occur in the ABL. They are an essential source of energy for small autonomous vehicles and, thus, thermal science ties directly into the work of researchers that deal with energy harvesting. Some
of the early research in the formation and structure of thermals dates to the 1940s, with work by Woodcock [52]. Contributions were made in the 1950s by researchers such as Haque [53] (1952), Lilly [54] (1960), and Kuo [55] (1961). Their work used perturbation techniques to evaluate the role of convection in cloud formation. A more theoretically driven approach to investigating convection was introduced in 1958 by Ludlam [56] and 1962 by Squires [57] and was soon followed by multitude of other researchers. This work was furthered by the advent of numerical approaches to convection theory as early as 1962 by Lilly [58] and 1963 by Ogura [59], who used dynamic potential flow theory mixed with thermodynamics to predict the growth of a cloud. Much work was done in the 1970s and 80s by researchers such as Lenschow [60] (1970), Manton [61] (1977), and Jensen [62] (1978). Other work of interest is that of Hunt et al [63] from 1988, which investigated the contribution of eddy formation on the development of thermals. The field has been energetic with activity since this research was published and it is not practical to present this development fully in this dissertation, so only a few pertinent papers will be mentioned.

Important experimental work was done by Konovalov [64] in 1970, who used measurements of vertical velocities inside thermals from 377 encounters to construct the two common shapes shown in Figure 2-3. While other shapes were discovered, they were generally without a pronounced maximum and had vertical velocities too low to be of interest and, thus, were omitted from the study. The thermals in Figure 2-3 are referred to as Type-a and -b by Konalov. Type-a thermals are wide and have relatively uniform distribution of strong updraft. Type-b thermals have less strength, but a strong pronounced peak in the center. This work was later used in various capacity by a number of researchers. The author of this work suggests the two types may be caused by different penetration directions. A case in point is the bottom row of images in Figure 2-4 (presented later), which depicts vertical convection as filaments. Flying along a filament would result in a Type-a thermal and flying across a filament would result in Type-b.

Today's computational methods (and power) have ensured enduring activity in the field. Among recent research is work by Allen [65] (2006) who presents an updraft model specifically intended for the simulation of autonomous UAVs. Using simulated thermals in the flight simulation of aircraft is not just a recent development, as shown by the work by Metzger and Hedrick [66] (1974), Reichmann [67] (1993), and Wharington $[68,69]$ (1998). The model is supported by experimental data obtained using various meteorological tools, including weather balloons. A simplified approach is generally proposed by
authors such as Welch et al [70], in which the vertical speed component of a thermal is a function of the airplane's distance from a fictitious core. Such models are discussed in Section 5.3, Thermal Modeling. A case in point is the so-called modified parabolic model, shown below for convenience.


Figure 2-4: Excerpt from Reference [75], showing the complex structure of atmospheric convection. The top row shows $u$-velocities, the center row $v$-velocities, and the bottom one the vertical or w-velocities. (Figure courtesy of Dr. Peter Sullivan)

$$
\begin{equation*}
\frac{V_{T}}{V_{T 0}}=\left[1-(r / R)^{2}\right] \cdot e^{-(r / R)^{2}} \tag{5-39}
\end{equation*}
$$

Where $V_{T}$ is the vertical speed in the thermal at distance $r$ from the center of its core, which has a maximum vertical speed, $V_{T 0}$. The ratio $r / R$ denotes the fractional distance from the center of a thermal whose diameter is $2 R$. As can be seen, the formula is independent of the altitude and assumes that $V_{T 0}$
is known a priori. Allen's method adds a dimension to such models by accounting for a vertical variation using a formula originally developed by Lenschow and Stephens [71] in 1980.

$$
\begin{equation*}
\frac{V_{T}}{V_{T 0}}=\left(\frac{1}{1+\left|k_{1}(r / R)+k_{3}\right|^{k_{2}}}+k_{4}(r / R)+w_{D}\right) \tag{2-7}
\end{equation*}
$$

Where $k_{i}, i=1,2,3,4$ are called shape constants and $w_{D}$ is a downdraft velocity term. The altitude variation is included in $V_{T 0}$ and causes the maximum updraft to be realized at approximately quarter of the distance between the ground and the height of the convective layer (ABL). This is discussed in Chapter 5, Atmospheric Modeling.

Many other studies investigate various aspects of the atmosphere, ranging from downbursts to turbulence in the convection layer of the atmosphere. For instance, in 1984, Chuang et al [72] developed a Mesoscale Atmospheric Simulation System, intended to simulate the temporal development of downburst phenomena, associated with the growth of thunderclouds. This work was intended to provide tools for flight simulators to allow pilots to practice evasive maneuvers once inside such situations. A 2009 study be Yang et al [73] presents an experimental study of flow characteristics around skyscrapers at very high wind speeds. Using Particle Image Velocimetry (PIV) around a model subjected to tornado-like wind speed, they observed significantly different flow characteristics when compared to conventional straight line winds. The study reinforces our knowledge of the complexity of wind flow and, although intended for civil engineering purposes, provides insight into challenges associated with the operation of sUAVs between buildings in high wind conditions. A 2011 paper, Bencatel et al [74] investigates the effect of atmospheric wind shear layers and presents models to represent. Such layers form between wind regions of dissimilar wind direction and strength and can be exploited by UAVs.

The last five years or so have seen a trend toward providing methods to allow more detailed simulation of ground turbulence for wind turbines. A 2010 paper by Saini et al [75] discusses the use of Large Eddy Simulations (LES) for this purpose in lieu of spectral based simulations. The paper presents a method to compress the representation of LES using a so-called orthogonal decomposition. Figure 2-4 shows an excerpt from the reference. The first observation is the detail provided by LES. The two columns represent the original (left) and reproduced (right) predictions of $u, v, w$ components of atmospheric convection over a $5 \times 5 \mathrm{~km}$ area. The first row shows the $u$-component of the wind velocity vector, the second row the $v$-component, and the third the $w$-component, respectively. While current state of computer technology does not permit such patterns to be generated in real time, Figure 2-4 gives a great insight into the complexity of true wind fields. It shows how updraft filaments indicative of classical Rayleigh-Bénard convection dynamics and absence of simplistic round thermal representations.

## Wind Fields by Ground Observation

The determination of wind fields is of great importance in many fields of science and engineering. Atmospheric flow over large terrain features, such as mountains and hills, has been studied extensively by many researchers for applications including optimization of wind turbine placement to determining
the spread of localized atmospheric pollutants, volcanic ash, or ambers from forest fires. There are typically four approaches used for determining the wind field; conceptual, experimental, numerical, and statistical. All have their pros and cons when comes to cost, complexity, and accuracy. Numerical approaches are of importance to the work presented in here. There are generally four methods used to model wind fields numerically; Mass-Consistent models, Jackson-Hunt method (for small hills), Computational Fluid Dynamics (CFD), and Mesoscale Numerical Weather Prediction (Mesoscale NWP).

The theoretical basis of mass-consistent models was developed by Sasaki [76, 77] (1958, 1970) using variational analysis. Later contributions were made by Sherman [78] (1978) and Dickerson [79] (1978), whose work focused on complex terrain represented using local Cartesian coordinate system, while the surface was modeled using stepped altitudes. The basic method defines an integral function whose solution minimizes the variance of the difference between observed and estimated variable values. This calls for a special functional to be defined. Sherman and Dickerson, both, used the functional

$$
\begin{equation*}
E(u, v, w, \lambda)=\int_{V}\left[\alpha_{1}^{2}\left(u-u^{0}\right)^{2}+\alpha_{1}^{2}\left(v-v^{0}\right)^{2}+\alpha_{2}^{2}\left(w-w^{0}\right)^{2}+\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right] d x d y d z \tag{2-8}
\end{equation*}
$$

Where $V$ refers to the computational volume $(x, y)$, the terms $x, y, z$ and $u, v, w$ are conventional spatial and velocity components, $u^{0}, v^{0}, w^{0}$ are corresponding observed velocities, $\lambda(x, y, z)$ is the Lagrange multiplier and $\alpha_{i}$ are called Gauss precision moduli, defined as $\alpha_{i}^{2} \equiv 1 / 2 \sigma_{i}^{-2}$. The Lagrange multiplier is the target of solution in this scheme and is found by solving the PDE

$$
\begin{equation*}
\frac{\partial^{2} \lambda}{\partial x^{2}}+\frac{\partial^{2} \lambda}{\partial y^{2}}+\left(\frac{\alpha_{1}^{2}}{\alpha_{2}^{2}}\right) \frac{\partial^{2} \lambda}{\partial z^{2}}=-2 \alpha_{1}^{2}\left(\frac{\partial u^{0}}{\partial x}+\frac{\partial v^{0}}{\partial y}+\frac{\partial w^{0}}{\partial z}\right) \tag{2-9}
\end{equation*}
$$

using a finite difference scheme solved over the entire computational domain. This approach requires the solution to be adjusted until it matches observation at selected positions in the computational flow, after which the flow elsewhere in the computational domain is thought to be modeled with acceptable precision. Application of this modeling was presented by Davis et al [80] (1984), Davis and Bunker [81] (1984), Connel [82], and Ross et al [83] (1988) and others. Ross et al compared the approach using potential flow theory for simple 3-dimensional topographies and found the two to agree. Computational time is always a concern and an effort to reduce it was presented by Moussiopoulos and Flassak [84] (1986). These efforts would typically involve topographies covering as large as $100 \times 100 \mathrm{~km}$ with $2-4 \mathrm{~km}$ horizontal resolution. Work in this area is still ongoing. For instance, see Ratto et al [85] (1994), Finardi et al [86] (1997), Kim and Patel [87] (2000), Wang et al [88 and 89] (2003 and 2005), and Juarez [90] (2012)). Other approaches of interest in the development of wind fields include that presented by Teixeira and Miranda and Miranda [91,92] (2010) and Zhang and Liu [93] (2014) for wind speed profiles over hills and ridges and the associated reshaping of the boundary layer. This transforms the wind profile from what was described using the methods of Section 5.2.1, Modeling the Planetary (Atmospheric) Boundary Layer, to what, in effect, is best approximated using constant velocity profile. Zhang and Liu support their CFD work with experimental data by Kim et al [94] (1997). Also of interest
are the use of Navier-Stokes solvers in the determination of wind flow and dispersion of contaminants in urban areas (e.g. see Hanna et al [95] (2006) and Camelli and Löhner [96] (2006)).

## Wind Fields by Observation in Flight

The evaluation of wind fields (which is wind velocity, rate of change of wind velocity, and wind gradient) by instruments onboard aircraft (that will take advantage of said wind field if possible) is a focus of interest for great many researchers. The application of this work extends its tenets to a diverse set of fields. In aviation, the flight management system of commercial, business, and military aircraft has offered the capability of determining elements of the wind field in which said aircraft operate for several decades. However, presently, this technological capability is just finding its way into General Aviation (GA) aircraft (e.g. see Myschik and Sachs [97] from 2007), let alone sUAVs.

An understanding of the wind field is of crucial importance for any aircraft engaging in energy harvesting. One of the earlier attempts for estimating the wind field in this fashion dates to a 2005 paper by Kumon et al [98], which presents a simple method for estimating the wind direction using a small UAV featuring a Rogallo wing. The wing configuration renders the vehicle, which hangs down below the articulating wing, easily disturbed by the wind. These disturbances were used to interpret wind direction. The method was then validated in actual flight experiments. A 2008 paper by Palanthandalam-Madapusi et al [99] compares GPS signals to air pressure data and uses a Kalman filter to smooth the digital signal. Their work illustrates the importance of combining heading angle in the post processing of the velocity data to avoid kinematic ambiguity in the wind velocity. Increased sophistication is proposed in a 2011 paper by Petrich and Subbarao [100] who developed a simple method to use GPS data with air data sensors, accelerometers, and magnetometer (to measure the earth's magnetism for aiding with determination of direction) to determine horizontal ground and vehicle velocity components. They also use a Kalman filter to smooth the signal, permitting the determination of the wind speed and direction in real time. A slightly different approach is promoted by Langelaan et al [101] (2011), who developed a method to estimate the wind velocity, rate of change of wind velocity, and wind gradient using the same onboard suite of instruments. As an example, to estimate the components of the wind velocity using the GPS velocity components, they write

$$
\left[\begin{array}{c}
w_{i_{x}}  \tag{2-10}\\
w_{i_{y}} \\
w_{i_{z}}
\end{array}\right]=\left[\begin{array}{c}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{array}\right]_{G P S}-\mathbf{R}^{-1}\left[\begin{array}{c}
u \\
v \\
w
\end{array}\right]
$$

Where $w_{i j}, j=x, y, z$, is the wind speed components in the Earth-fixed inertial frame, $\dot{x}, \dot{y}, \dot{z}$ are the components of the aircraft velocity in the Earth-fixed frame, and $u, v, w$, are the velocity components in the body frame (see Chapter 8, Flight Simulation), and $\mathbf{R}$ is the rotation matrix in terms of the Euler angles, $\phi, \theta$, and $\psi$. They acknowledge the presence of error due to noise in such measurements and then proceed to present a covariance methodology to mitigate it. This work is helpful to simulate the effectiveness of such a detection algorithm and, thus, may be useful to the practical operation of the GICA. Another 2011 paper, by Delahaye and Puechmorel [102] presents two methods for the wind field
estimation using radar. One uses airspeed information with a Kalman filter, while the other assumes only position information, but uses post-turn position to estimate the wind field. Cho et al [103] present a similar method in another 2011 paper. Using a single antenna GPS and Pitot data, smoothed using an extended Kalman filter, they evaluate the wind field by operating an UAV by making it fly in circles. They provide experimental support to the method by an actual implementation in an actual UAV.

Other researchers presenting interesting papers include those of Langelaan et al [104] (2012) and Kampoon et al [105] (2015). The former is a very active researcher in the field of autonomous energy harvesting vehicles. The reference presents method to estimate wind fields that can be used for dynamic soaring. The latter presents a method that utilizes winds aloft information from multiple aircraft to construct the wind field and for which each aircraft is considered a contributing sensor.

### 2.4 Energy Harvesting

Energy harvesting for UAVs is presently a vibrant field of research. Hampered by limited onboard energy storage, the range and endurance of current technology of UAVs can be greatly improved if energy can be supplemented using sources such as atmospheric convection, solar energy, and even structural oscillation. This section reviews contributions to this field, made by several researchers.

## Dynamic Soaring

In a 2002 paper, Boslough [106] presents theoretical calculations and flight simulation of a prototype flight control algorithm intended to control a UAV for sustained flying using dynamic soaring. He makes an important observation in the paper that warrants mentioning:

It is important to recognize that ... behavioral engineering is not a true optimization problem. The types of problems that are likely to be of most use are analytically intractable and involve behavior that operates in a noisy, changing, real-world environment. Any optimization method that is applied to designing for such an environment is going to yield a solution that depends on the simplifying assumptions, ..., chosen by the behavioral engineer. The global optimum may strongly depend on seemingly subtle differences in problem representation. It is therefore important to focus on behaviors that are "good enough" and avoid thinking in terms of "the best," because the best solution determined by some quantitative test will always be an artifact of the problem definition.

This serves to remind us that natural adaption never presents us with organisms optimized for a single situation, but only ones that present the best solution for a multitude of disparate situations.

A 2005 paper by Lissaman [107] discusses further aspects of energy harvesting through dynamic soaring. Inspired by work by Sachs [108] (1994) and Sachs and Mayrhofer [109] (2002), Lissaman normalizes the equations of motion in terms of $L / D$ and cruising speed only, before proceeding to evaluate the minimum wind shear gradient required to sustain dynamic soaring for a specific $L / D$ ratio (of a bird or an aircraft). He also demonstrates that control algorithms developed for dynamic soaring do not need to
be severely restricted. This helps explain why so many different wind-profiles used in the literature are adequate to sustain dynamic soaring. It shows that no particular profile is a necessary condition for this purpose; only as long as there is one. Sachs [110] (2005) also presented a treatise on wind strength required by the Albatross to sustain its dynamic soaring, which is useful in design of energy harvesting aerial systems. He does this using an optimization method that takes into account realistic flight mechanics of the large bird. He concludes that as long as the wind speed at 10 m height above sea level exceeds $8.6 \mathrm{~m} / \mathrm{s}$, the average Albatross can sustain dynamic soaring. A 2012 paper by Mears [111] examines the use of dynamics soaring for increasing the range and endurance of military UAVs and correctly identifies several challenges that must be considered, such as availability of wind conditions and sensor technology required to give the AFMS the ability for precise altitude detection.

## Thermals and Wind Fields

As discussed elsewhere in this chapter, the most obvious source of energy is in thermals. For flatlands, it is effectively the only source of vertical convection. However, this does not mean finding thermals is easy or reliable, at least not until technology will be available that will allow operators to "see" thermals, similar to what is shown in Figure 5-8. Wind fields represent a far more reliable source in hilly and mountainous terrain. In such places, as long as there is horizontal convection, there will be vertical updraft (and unfortunately, downdraft too). Additionally, there is an opportunity to harvest gusts through gust soaring. A multitude of recent papers present methods to extract this source of energy. In a 2008 paper, Bramesfeld et al [112] explore the energy harvesting of turbulent atmosphere, primarily from gusts. The investigation focus on possible drag reduction of small and medium sized UAVs using aeroelastic oscillation of the wing structure using the so-called Knoller-Betz effect (also known as Katzmayr effect) (e.g. see Knoller [113] (1909), Katzmayr [114] (1922), Fick [115] (1925), Halfman [116] (1952), or Jonas and Platzer [117] (1998)). This effect explains the lift and thrust of flapping wings. The authors of the paper claim drag reductions of nearly $30 \%$ is possible for wings that can twist and whose shear center is ahead of the aerodynamic center.

The potential of harvesting energy from thermals applied to autonomous vehicles is exemplified in a 2010 paper by Edwards and Silverberg [118]. The paper details the participation of a 5 kg autonomous sailplane in the so-called Montague Cross-Country Challenge, which took place two years earlier, and in which RC gliders in the 5 kg class compete in terms of speed and distance. All competitor sailplanes are controlled by human operators, except of course the autonomous one. The autonomous glider utilized an autopilot algorithm, called Autonomous Locator of Thermals (ALOFT) and placed third in overall points, while outperforming its manually operated rivals when came to utilizing thermals and in operation in wind conditions. The researchers, like so many others before and after, assume circular thermal structure and, although an inaccurate description of real thermals, yields reasonable energy extraction. Their thermal updraft model is given by

$$
\begin{equation*}
\frac{V_{T}}{V_{T 0}}=e^{-(r / R)^{2}} \tag{2-11}
\end{equation*}
$$

Whose variables have already been defined. Once inside a thermal, the algorithm evaluates when to exit based on altitude gained or thermal updraft strength.

Another example of energy extraction from a wind field is presented by Cutler et al [119] (2010), who studied the feasibility of improving the effectiveness of UAVs by harvesting energy in ridge lift. The work involved a theoretical aerial surveillance mission near a geographic ridge subjected to cross winds and the regions of lift on the windward side. The flow field around the ridge is estimate using potential flow theory and a region of "good lift" is identified. The vehicle is then guided to fly inside this region for energy extraction. The results indicated that mission effectiveness can be improved by exploiting ridge lift, even for UAVs with best glide ratio as low as 11:1.

Up until 2010, work by researchers had focused on soaring flight in wind fields whose characteristics where known a priori. In 2011, Lawrance and Sukkarieh [120, 121] developed a method to estimate and, subsequently, use wind field for soaring flight. The method uses a Gaussian Process Regression (GPR) method to assemble the wind map. In 2013, Nguyen et al [122] investigated a gliding UAV searching for a ground target while simultaneously collecting energy from known thermals, in an attempt to maximize mission effectiveness. Then this information is used by a Lookahead Tree Search (LTS) algorithm, which the authors claim provides a more structured approach to mission path-planning. This method was benchmarked against a Locally-Optimal Myopic Search (LOMS) method, which conducts an exhaustive search for all possible paths, and was found to be more efficient, although the authors acknowledge difficulties with real-time implementation.

## Solar Power and Piezo-Electrics

The use of solar cells and piezo-electrics is not new in airborne vehicles, in particular the former. It can be stated that the many ideas have been spurred by prolific creativity of researchers. For instance, a 1979 paper by Hertzberg and Sun [123] suggests the use of a satellite to relay laser energy obtained using sunlight to power aircraft, equipped with "laser receptors." It is based on the ideas of Myrabo, who in 1985 published this and other controversial (although quite insightful and interesting in this author's view) ideas in his book "The Future of Flight" [124]. That aside, the use of solar energy using technology is intriguing and offers great potential, since aircraft could take-off under their own electric power and cover great distances in glide with the motor partially (or fully) shut off, while charging the onboard battery-pack using solar energy. It must be realized that, at the present, this technology should not be expected to power airplanes that fly much faster than, perhaps, 100-150 knots. Regardless, the technology would provide great potential and utility. It has even spurred interest in terms of solar powered aircraft for exploration of the surface of Mars and Venus (e.g. see Colozza [125] 2004).

Several papers were found from more reputable sources that present the use of solar cells for energy harnessing. One of the earliest one is from 1983 and was co-authored by the late Dr. Paul MacCready [126] and details two solar-powered aircraft, the Sunrise II and Solar Challenger. The former was the first solar-powered aircraft to fly and was a remotely controlled sailplane that managed to climb to an altitude of over 5 km . A year later, Boucher [127, 128], who designed the Sunrise II, presented a fairly complete history of solar powered flight up to that time (albeit sparsely referenced). Further work in

1984 was developed by investigating the structural sizing of solar powered aircraft by Hall and Hall [129]. In 1991, Bruss [130] presents best practices and development of radio controlled solar powered aircraft. In 1995, Brandt and Gilliam [131] introduced design methodology for larger solar powered aircraft, in part based on the approach by Mattingly et al [132] (1987). And in 2001, Voit-Nitschmann [133] presents design considerations for smaller manned solar powered aircraft. A schematic of a component setup required for solar-power, based on his work, is shown in Figure 2-5. Representative component efficiencies are shown, with a total efficiency amounting to $1.5-12.1 \%$, based on aircraft airspeed. Note that this author considers Voit-Nitchmann's presumed propeller efficiency of $87 \%$ highly optimistic, especially for propellers operating at lower Reynolds numbers and lowered it to $80 \%$ (still optimistic). In that context, work by Brandt and Selig [134] (2011) shows small propellers for electrically powered RC aircraft ranges from 0.25 (poor) to 0.65 (efficient propeller). Solar panels with energy conversion efficiency of the order of $30-40 \%$ are being developed.


Figure 2-5: The basic setup of a solar-powered aircraft, showing typical efficiency of each component. Note that the propeller efficiency depends on the airspeed of the aircraft. The total conversion efficiency amounts to 1.512.1\%. Based on Reference [133] with minor modifications.

A 2004 paper by Chen and Bernal [135] investigates the design and development of airfoils intended for solar powered aircraft. A thesis by Camacho [136], published in 2007 presents a hybrid convection-solar powered vehicle called the Tactical Long Endurance Unmanned Aerial System (TALEUAS). The control system applies a thermal searching algorithm, while simultaneously capturing solar energy. A paper by Langelaan [137] (2007), presents a system for the autonomous soaring of sUAV. A method to predict the wind field and uses a path planner to extract lift from thermals, orographic lift and dynamic soaring. This work has parallels with the work here; although there are differences (e.g. this work involves random multi-source orographic lift and biomimetic path planning). Noth [138] (2008) presented a thesis containing design guidance for solar powered aircraft. Some of that discussion is presented in Chapter 9, Energy Harvesting. Among other work of interest is that of Wickenheiser and Garcia [139] 2008, who studied the extension of endurance of UAVs using a microwave-/solar-powered flight vehicle, through a continuous harvesting of electromagnetic radiation using onboard antennas and solar panels. Chakrabarty and Langelaan [140] (2009) present a continuation of Langelaan's earlier paper from 2007 and focus on long duration flight using a wind field energy map based on minimum total energy. Langelaan has been a particularly prolific researcher in this field and in a 2010 paper by Chakrabarty and

Langelaan [141, 142] ( $\underline{2010}$ and 2011) discusses a method that uses A* search with an energy related cost function to find the optimum flight trajectory in realistic wind fields. Work by Kagabo and Kolodziej [143] from 2011 considers fuzzy logic algorithm to plan trajectory to a thermal of known position and strength (not likely in real applications). Hajianmaleki [144] (2011) presented a method to perform the conceptual design of solar powered aircraft and introduces method to work in the difference in technology solar power brings. Chakrabarty and Langelaan [145] 2013 introduce further developments in their work by permitting temporal effects, i.e. the change in the wind field as a function of the time of day. This work is largely driven by the use of mesoscale weather prediction software called Weather Research and Forecasting Model (WRF), which, as stated on the associated website "... is a nextgeneration mesoscale numerical weather prediction system designed for both atmospheric research and operational forecasting needs. ${ }^{3 "}$ Further work is presented by Langelaan et al [146] (2013) regarding challenges accompanying fuel efficient flight with respect to the CAFÉ Foundation's Green Flight Challenge of 2011 and by Anton and Inman [147] (2013) who investigated the possibility of harvesting vibration and solar energy in a mini UAV using a radio controlled glider with a 1.8 m wing span. Spangelo and Gilbert [148] (2013) also present flight trajectory optimization for solar-powered aircraft that are constrained to follow a closed ground path, indicative of loitering near a fixed ground point. More work was done by Feiguel et al [149] 2014, who studied the viability of energy harvesting using piezoelectric devices for supplemental power generation using an actual lightweight UAV. They found the energy harvesting capability was insufficient due to the small oscillation frequency and low strain magnitudes of the wing during flight. Nevertheless, they expressed optimism for microscale UASs where power requirements are lower and the oscillation frequency of the wings is higher.

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## 3. A Survey of Automation Technology for sUAVs

The use of drones can be traced directly to military applications, where they have been used in various capacities since 1917. They were first introduced by the US military for operation in World War I [1]. Arguably, one of the first unmanned aircraft actually flew some 21 years earlier, in 1896, when Samuel Pierpont Langley's (1834-1906, 71) steam-powered Aerodrome flew $3 / 4^{\text {th }}$ of a mile; not once, but twice [2]. Of course it hardly qualifies as a contender, as its control was provided by the vehicle's inherent natural stability and could not be changed in flight. Regardless, drones of the early part of the $20^{\text {th }}$ century suffered from poor reliability, something that rendered them more as tools of demonstration than deterrence. Later, in World War II and since, drones saw use as flying gun- and missile targets. The use of such vehicles for surveillance and reconnaissance missions by the world's militaries dates to the Vietnam War of the late 1960s and early 1970s [3]. Drones saw an impressive growth in popularity following their use in the Balkan's in the early 1990s. Their rise has been supported by huge advances in electronic technology, especially over the span of the last 10 years. The jump in miniature electronic technology has pushed drones into the public arena, where they have proliferated among hobbyists and professionals alike. This chapter presents a brief survey of current technology, with emphasis on technology in the public sector.

This chapter reviews selected aspects of the state of technology of UAVs at the time of its writing. The author is aware of the fate of the material presented - obsolescence. Technology progresses so rapidly that yesterday's gadgets are the brunt of today's ridicule. It is recognized this information is bound for obsolescence. Nevertheless, it serves to provide a historical reference to future readers as to how technology has progressed. It will also show the constraints the author had to contend with at the time of writing as well.

## Utility Potential of Unmanned Aerial Systems

Unmanned Aerial Systems (UAS) are not only becoming increasingly sophisticated, their utility potential is expanding as well. As shown in the below summary, current and future applications are many, some of which are discussed by Ball [4] and expanded by the author. Note that the term drone refers to both fixed wing aircraft and rotorcraft, such as quad-copters.

- Agricultural use: The Association of Unmanned Vehicle Systems International (AUVSI) expects $80 \%$ of civil drones will be used in agriculture (e.g. see West [5]) on tasks such as crop health monitoring, harvesting status, and land resource usage.
- Mining industry: Drones are very useful for prospectors looking for minerals, oil and gas, and ferrous metals. They can carry magnetic sensors and other instruments to help detecting ore deposits and anomalies in the gravitational field associated with the presence of minerals. Drones are also used in mining operations, for evaluation of mining site conditions, pit wall inspection, and terrain mapping.
- Construction industry: Drones are used to help monitor progress of projects, providing a new perspective - from above. They allow an effective and rapid quality control, permitting
increasingly precise inspections to be accomplished in hard-to-reach places. For instance, drones can be used to inspect the exterior workmanship of roofs and tall towers far more economically than traditional methods allow.
- Infrastructure monitoring: Drones allow surveillance of electric power lines, oil-pipelines, bridges, plant inspections, and others, for improved and more frequent inspection of places that are otherwise hard to access.
- Scientific research: Drones offer a huge potential in various sciences, including biology, archaeology, geology, aviation, and atmospherics. They provide valuable service monitoring and tracking wildlife. This includes animal population and health assessment, as well as monitoring of hunting and protection from poaching. They can also offer valuable service in sciences such as meteorology. For instance, it is possible to fly specialized drones into dangerous regions of hurricanes and tornadoes to explore its structure. Drones can also be used to track storms and provide access to data that helps with weather forecasting.
- Law enforcement: Drones are used for surveillance of criminal activity, border patrol, and clandestine operations. This is often referred to as Intelligence, Search, and Reconnaissance operations or ISR for short. They can also help with traffic monitoring and support accident investigation.
- Emergency response: The portability of drones allows emergency responders extraordinary means to inspect disaster areas after natural calamities and help with search and rescue. This capability extends to inspection of manmade disasters, such as sites subject to toxic chemical or nuclear contamination. If equipped with infrared detectors, drones can be used to search for missing people. Furthermore, since many disaster areas are strewn with hard-to-cross debris fields, drones can quickly help first responders prioritize rescue efforts. Additional use is associated with detection, monitoring, and evaluation of forest fires.
- Military operations: Situational surveillance in threat territories, monitoring movements of armed forces, and support of combat operations of various kinds. Sometimes there are even unexpected consequences of their use. A case in point are reports that in October 2013 armed supporters of a rebel group called "M23," which took over the city of Goma in the Democratic Republic of Congo, refrained from joining the group in an effort to fight military intervention launched by government forces, because they were certain their movement would be discovered by government drones. The reduced rebel forces allowed the government military to take the city back within two weeks. [6]
- Other uses: News gathering, traffic monitoring, aerial photo- and videography. For instance, drones have become a tool of choice to shoot impressive exterior videos for realtors. The compact size makes them ideal for transportation and operation from unprepared fields; many can simply be hand-launched and hand-retrieved.

This technology is coming of age and has become quite capable, permitted by recent advances of highly sophisticated miniature electronics that can facilitate true 6-degree-of-freedom control capability. Most of these missions call for operational automation, such that a human operator decides where to go, while the vehicle takes care of getting there. At the present time, the state of technology already
permits large scale automation. Nevertheless, development of means to conserve the energy expenditure of autonomous vehicles is an important research topic, as this will permit more to be done with less. More distance can be covered or more time can be spent in the air with less energy. This chapter explores examples of contemporary technologies and looks to the future. The chapter will not detail every single UAV ever developed; for that is not the scope of this work, not to mention there are simply too many examples available. Rather, examples of contemporary technology will be given to make specific points.

### 3.1 UAV Technology

It should first be stated that the scope of current technology of UAVs is vast and is growing larger. It far exceeds what is practical to present here. Readers interested in a more detailed overview of these systems are directed towards Gundlach's text Designing Unmanned Aircraft Systems [7]. The term small Unmanned Aerial Vehicle (sUAV) typically refers to light aircraft such as small rotorcraft (e.g. quadcopters) and fixed wing vehicles of "conventional" and "unconventional" configurations, even morphing aircraft (e.g. see Abdulrahim [8] (2007), Ajaj et al [9] (2014), and Prabhakar et al [10] (2016)). These aircraft are usually powered by small electric motors or gas engines and, being unmanned and small, usually permit design philosophies not possible in manned aircraft. As already stated at length, the work presented in here focuses on fixed wing configurations. Fixed wing aircraft are better suited for energy conservation/harnessing than their rotorcraft counterparts. Fixed wing sUAV feature wing spans that range from approximately 5 m to vehicles rivaling the size a hummingbird or a large moth.

The primary detriment of small UAVs is sensitivity to adverse weather and many suffer from short range, endurance, and low useful load. At the same time, the small size offers many advantages that renders them ideal for many specialized tasks that would be hard, if not impossible, to accomplish using other means. Low acquisition and operational costs, simplicity in use and, recent advent of sophisticated miniature electronics (e.g. camera and surveillance systems, navigation systems, and autopilots) has greatly expanded their utility. Much has been written about this utility potential. For instance, Quigley, et al [11] (2005) present techniques for field deployments of small fixed-wing UAVs (primarily of the flying wing kind), of which automatic take-off and landing play a vital role. Approach to path-planning in search and rescue operations is also presented. Elston et al [12] (2010) discusses the potential use of UAS, consisting of an aircraft, ground station, and the required data relay equipment, that would penetrate a tornado and a meteorological phenomenon called Rear-Flank Downdraft (RFD) thought to be associated with their formation. This system would transmit sorely needed data to further study these dangerous weather occurrences. Others are working on various aspects of sensor technology that can be used for the GICA presented in this writing. An example is a 2011 paper by Johnson and Ivanov [13] that presents a Light Detection and Ranging (LIDAR) technology intended for a future NASA mission to the moon. Methods to post-process LIDAR detection data is presented and give insight into challenges associated with the technology that are being investigated by various researchers.

### 3.2 Selected Contemporary UAVs

The term Unmanned Aerial Vehicle (often abbreviated UAV) applies to a range of unmanned aircraft. In the mind of laypeople, such aircraft are typically used for military applications, although such use does not do justice to the full potential of the application of such vehicles. Nevertheless, the world's militaries operate a large number of different Unmanned Aerial Systems (UAS), as they are often called, when they are not referred to using the term "drones".

Military UAVs include a large range of weight and size, with the largest being the high-altitude, turbofanpowered, reconnaissance aircraft Global Hawk.

## General Atomics MQ-1 Predator

The General Atomics MQ-1 Predator (and the original RQ-1 variant) is a single engine, pusher-propeller, UAV with high $A R$ wing and inverted V-tail (Figure 3-1). Its primary operator is the United States Air Force (USAF) and the Central Intelligence Agency (CIA), where it has served both as a tool for intelligence gathering and, when equipped with Hellfire missiles, in an offensive role. It features a sophisticated infrared camera that allows human bodies to be detected from an altitude of 3000 m . This capability was offered in a search-and-rescue capacity in the aftermath of the Hurricane Katarina, in August 2005 [14].

The MQ-1B has a wingspan of $16.8 \mathrm{~m}(55.1$ ft ), length of $8.22 \mathrm{~m}(27.0 \mathrm{ft})$, an empty weight of $512 \mathrm{~kg}\left(1130 \mathrm{lb}_{\mathrm{f}}\right)$, gross weight of $1020 \mathrm{~kg}\left(2250 \mathrm{lb}_{\mathrm{f}}\right)$, and is powered by a 115 BHP Rotax 914F piston engine [15]. It has a maximum (horizontal) airspeed of 217 km/h (117 knots), cruising speed of $130-165 \mathrm{~km} / \mathrm{h}$ (70-90 knots), stalling speed of $100 \mathrm{~km} / \mathrm{h}$ (54 knots), service ceiling of 7620 m ( 25000 ft ), and endurance of 24 hours. Its design dates to at least the early 1990s, with the maiden flight taking place in 1994.


Figure 3-1: A General Atomics MQ-1 Predator in flight. (Photo by U.S. Air Force photo/Lt Col Leslie Pratt, through Wikipedia Commons)


Figure 3-2: A ScanEagle on its launch catapult. (Photo by U.S. Marine Corps photo/Shannon Arledge, through Wikipedia Commons)

## Boeing Insitu ScanEagle

The Boeing Insitu ScanEagle is a single engine, pusher-propeller, UAV of a flying wing configuration, with a swept aft high $A R$ wing that feature vertical fins/winglet on the tips (Figure 3-2). The UAV, effectively, is a portable system that consists of four aircraft, a ground control system, and launch and recovery system (called Skyhook). This system allows the aircraft to be operated without the use of a runway. The ScanEagle has a wingspan of $3.11 \mathrm{~m}(10.2 \mathrm{ft})$, length of about $1.2 \mathrm{~m}(3.9 \mathrm{ft})$, an empty weight of some 14 $\mathrm{kg}\left(31 \mathrm{lb}_{\mathrm{f}}\right)$, gross weight of $18 \mathrm{~kg}\left(40 \mathrm{lb} \mathrm{b}_{\mathrm{f}}\right)$, and is powered by a 1.5 BHP 2 -stroke piston engine [16]. It has a maximum (horizontal) airspeed of $148 \mathrm{~km} / \mathrm{h}$ ( 80 knots), cruising speed of $111 \mathrm{~km} / \mathrm{h}$ ( 60 knots), service ceiling of $5950 \mathrm{~m}(19500 \mathrm{ft})$, and endurance of more than 24 hours.

## AeroVironment RQ-11 Raven

The RQ-11 Raven is a single engine, pusherpropeller, a hand-launched surveillance UAV of a conventional configuration (see Figure 3-3). It is designed and fabricated by AeroVironment, a company that was founded in 1971 by the late Dr. Paul MacCready (1925-2007, 81), whose rise to fame includes the design of the human powered Gossamer Condor and Gossamer Albatross, the first of such vehicles to cross the English Channel. AeroVironment specializes in the development of unmanned surveillance aircraft for military use. It produces several other similar sUAVs. These aircraft are designed to allow rapid assembly of packaged components in a combat zone. The intent is to allow soldiers to confirm threat and assessment of success (or failure) of engaging that threat.

The Raven has a wingspan of $1.37 \mathrm{~m}(4.5 \mathrm{ft})$, length of about $0.9 \mathrm{~m}(3 \mathrm{ft})$, an operational weight of some $1.9 \mathrm{~kg}\left(4.2 \mathrm{lb}_{\mathrm{f}}\right)$, and is powered by a small electric motor [17]. Its operational airspeed is around $56 \mathrm{~km} / \mathrm{h}$ ( 30 knots), range is 10 km ( 5.4 nm ), and endurance of more than $60-90 \mathrm{~min}^{1}$.


Figure 3-3: A Raven being prepared for a surveillance flight. (Photo by DOD photo/Russell E. Cooley IV, through Wikipedia Commons)

[^9]
## Flyzone Calypso

Now let's consider civilian fixed-wing sUAV. These are primarily intended for hobbyists, although many such aircraft find more serious use in the activities listed at the beginning of this chapter. The sheer volume of different types of such aircraft limits which can be presented here; thus, only two such aircraft will be discussed, both which the author has personally owned and flown. These are the Flyzone Calypso (see Figure 3-4) and Quanum Observer (see Figure 3-5).


Figure 3-4: The Flyzone Calypso is a typical civilian sUAV.
The Calypso is an electrically powered sailplane with a 1.85 m ( 73 inch ) wingspan, wing area of $0.339 \mathrm{~m}^{2}$ ( $3.65 \mathrm{ft}^{2}$ ), and typical ready-to-fly weight is about $0.82 \mathrm{~kg}\left(1.81 \mathrm{lb}_{\mathrm{f}}\right)$. Therefore, the wing loading is about $2.42 \mathrm{~kg} / \mathrm{m}^{2}\left(0.495 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}\right)$ and $A R$ of 10.1 . Observation puts the maximum glide ratio of the order of 1215 at about 17 KCAS. Using Equation (15-123) of Reference [18], this yields a $C_{D \min }$ of about 0.02307 . Flight times around 30-45 minutes are easily achievable using a 1300 mAh 3 S LiPo battery and by the application of short burst of climb to altitude and subsequent glide in typical moderate thermal conditions.

## Quanum Observer

The Quanum Observer is a single-engine, pusherpropeller taildragger aircraft of conventional configuration. It is designed to be a dedicated FPV aircraft. It features a 2.00 m ( 78.8 inch ) wingspan, wing area of $0.480 \mathrm{~m}^{2}\left(5.167 \mathrm{ft}^{2}\right)$, and $A R$ of 8.33 . Typical ready-to-fly weight is about $2.82 \mathrm{~kg}(6.22$ $\mathrm{lb}_{\mathrm{f}}$ ), yielding a wing loading is about $5.87 \mathrm{~kg} / \mathrm{m}^{2}$ ( $1.20 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$ ). Observation puts the maximum glide ratio of the order of $8-10$ at about 21 KCAS. At that airspeed, a maximum flight time of about 4560 minutes is achievable using a 10000 mAh 3 S LiPo battery and by maintaining about 6-7 Amp current draw. The airplane achieves a 27 KCAS cruising speed at 10 A , with about 40 min


Figure 3-5: A Quanum Observer; a dedicated FPV aircraft. (photo by author) endurance. Using Equation (15-123) of Reference [18], this yields a $C_{D \min }$ of about 0.04962 .

## Rotary wing Aircraft

The discussion of UAVs would be incomplete without mentioning rotary wing aircraft; tri-copters, quadcopters, and other multi-rotor copters, which have become enormously popular lately. Their presence here is warranted by the obstacle avoidance capability of the GICA. While the term "drone" includes both fixed and rotary wing aircraft, the term has become synonymous with the latter group among many laypeople. Rotary wings aircraft offer enormous potential due to their VTOL and hovering capabilities, regardless of poor efficiency. Like fixed wings, they benefit from flying in updrafts (due to propeller effects) although harvesting potential is more limited. Similarly, fixed wings cannot compete with rotary wings in the low speed region. These two classes of aircraft should be considered different kind of tools, albeit there is partial overlapping in their capabilities.

Rotary wing sUAV can have a forward speed of as much as $60-70 \mathrm{mph}$ and are superbly agile. They accomplish a variety of specialized maneuvers, including snap rolls, loops, and rapid decelerations. They are also capable of precise maneuvers that would be impossible to achieve in fixed wing aircraft. However, they require large capacity Lithium-Polymer batteries (see Chapter 7, Engine Modeling) that, currently, provide endurance of the order of $30-45$ minutes. The same battery in a fixed wing of similar weight may easily achieve 2-3 times that time.

As stated above, the obstacle avoidance capability of the GICA will find practical use for rotary wing sUAV, although the energy harvesting capability is compromised by their inherent inefficiency. The GICA can find a path through a complex maze of obstacles and terrain (e.g. see Figure 3-7). Rotary wing sUAVs are subject to greater risk of static obstacles due to their use at lower altitudes. The obstacle field can be thought of as forested regions with utility towers, as well as the braces and columns of the truss structure of an oil refinery or other chemical plants. For a technical inspection operator, the GICA would allow a complex path through a plant to be planned, while disregarding complexity of the structure - something that would be taken care of by the GICA.


Figure 3-6: A DJI Inspire 2; a quad-copter for professional videography work. (photo from www.dji.com)


Figure 3-7: The GICA can find a path through a complex obstacle field.

### 3.3 Automatic Flight Management System

An Automatic Flight Management System (AFMS) is a mechanical system designed to assist the human operator of an aircraft. AFMS range widely in complexity and sophistication. The AFMS of the yesteryear were analog in nature, while their modern counterparts are either partially or fully digital in nature. The function of an AFMS may range from that of a simply wing-leveler to the full completion of a preplanned mission, without any control input from the operator (outside of mission planning). The operation of such a vehicle is considered fully autonomous if the AFMS is capable of taking-off, cruising, and landing without receiving any input from the operator en-route (although revising the mission enroute would be considered a necessary feature).

Generally, AFMS have many advantages over human operators. Not only do they prevent pilot fatigue, they can also multi-task, they have short response time, and can make a statically and dynamically unstable systems behave as if they were stable. Among drawbacks is that, generally, current systems are unable to "react" and "improvise" responses to "unexpected" situations. For instance, if an aileron gets jammed in-flight, the AFMS will not "know" how to handle it, unless it has been programmed for such a scenario a priori. A standard response to such a predicament would be to use the rudder (assuming the aircraft feature robust "dihedral effect"), although this could also lead to an uncontrollable predicament, for instance if the aileron got jammed in a fully deflected configuration.

The history of autopilots dates back to 1891 with the proposal of the American/British inventor Sir Hiram Maxim (1840-1916), who is better known for inventing the Maxim gun, the first fully automatic machine gun [19]. Sir Hiram's concept was "intended to secure longitudinal stability by the automatic actuation of elevators in response to disturbances detected by a gyroscope. [20]"

An important advance was made in the early 1910s, with the advent of the Sperry Gyroscopic Stabilizer. The invention of this device is generally attributed to the famous founder of Sperry Gyroscope Company, Elmer Ambrose Sperry (1860-1930) ${ }^{2}$ [21]. The gadget was first demonstrated publically on June $18^{\text {th }}, 1914$, in a Curtiss C-2 biplane at the Concours de la Securité en Aéroplane (Airplane Safety Competition). The aircraft was piloted by Elmer's son Lawrence B. Sperry (1892-1923) [22]. The stabilizer consisted of a quadruple gyroscope that was powered by the engine and rotated at some 12000 RPM. The device actuated the ailerons and elevator and during the demonstration, Lawrence's mechanic, who participated in the flight, walked along the wing to show the capability of the system in maintaining level flight.

The autopilot has undergone enormous technological improvement since those early days. Its history is wrought with gradual sophistication and increase in capability. Today, autopilots guide supersonic fighters at tree-top level and every commercial jetliner from just after take-off to just before landing, and sometimes even those maneuvers are controlled by autopilots. Readers interested in various
${ }^{2}$ It should be noted that a comment from a reader of the reference (which is an online article) claims the honor actually belongs to Martin Lynn Patterson, a chief engineer for Sperry Gyroscope Company.
aspects and historicity of autopilots and flight management systems, are directed towards texts such as those of Collinson [23], McLean [24], Lee and Leffler [25], and Combs et al [26].

### 3.3.1. Basic Classes of Autopilot Systems

This section presents the basics of autopilot systems. As stated earlier, autopilots rely on gyroscopes for the detection of rotational motion. Autopilots for larger aircraft are predominantly mechanical or highly precise ring laser gyros. The most basic is the mechanical gyro, whose axis of rotation is maintained using ball bearings. Laser gyros do not feature moving parts, but rather detect changes in the frequency of light with rotation. For unmanned aircraft (such as military and consumer sUAVs and sometimes spacecraft), electronic gyros, called Inertial Measurement Systems (IMU), are used. Contemporary autopilot systems for sUAVs use miniature IMUs, whose dimensions are a few millimeters and utilize Micro-Electro-Mechanical Systems (MEMS). The most precise gyros are generally those that feature fluid bearings, in particular gas bearings. Such bearings consist of forcing gas between the bearing surfaces, rendering them with extremely low-friction, low-noise and high precision. Such gas-bearing gyros are used in the Hubble space-telescope [27]. AFMS can be classified based on a number of features.

## Primary or Secondary Control Autopilots

These are autopilots whose role is limited to either the primary flight control systems (aileron, elevator, rudder) or secondary flight control systems (throttle, flaps, yaw dampers, etc.). The latter is typically a part of a more sophisticated AFMS that also provides primary controls or a specialized Stability Augmentation System (SAS).

## Longitudinal and Lateral-Directional Autopilots

Autopilots are typically classified as longitudinal or lateral-directional autopilots. The former class will only control the longitudinal motion (pitch) of the aircraft. Such functionality allows the pilot not just to maintain altitude, but also rate of ascent or descent. The latter class controls functions such as wingleveling or yaw-damping.

## Classification based on Number of Axes

In the aviation community, autopilots for aircraft are most often classified by the number of axes they control. A brief summary is provided below. First, the reader must be made familiar with five important aviation terms; VFR, IFR, ILS, GS, and LOC.

VFR stands for Visual Flight Rules. This is how most simple aircraft are flown; by a human operator who relies on own vision to maintain altitude and heading using the horizon as a reference. IFR stands for Instrument Flight Rules and refers to an operation in which the pilot relies on instruments rather than vision when flying. This means flying at night, inside clouds, and above 10000 ft . An aircraft certified to operate under IFR must satisfy a plethora of regulations intended to make the system of instruments in the aircraft reliable. ILS stands for Instrument Landing System. Such systems are designed to allow the pilot (or autopilot) to find a runway and position the airplane on final to that runway while in IFR conditions. ILS requires the system to maneuver the aircraft not only along the correct heading but also, the correct rate of descent. Thus, GS stands for Glide Slope, which is the angle the flight path makes to
the horizontal (typically a $3^{\circ}$ angle) and LOC stands for Localizer, which is the track the airplane must make to the ground (the heading will vary based on the wind the aircraft must contend with).

As stated earlier, sophisticated autopilots, such as those found in commercial jetliners, offer features well beyond what is described here. For instance, such autopilots also control the throttle (referred to as auto-throttle functionality), offer envelope-protection (to prevent the pilot from flying too slowly or too fast), mission planning, and other features beyond the scope of this dissertation. However, contemporary autopilots do not offer means to determine where optimal flight conditions reside. Such work is determined a priori and then numerical algorithms are used to determine deviations based on weight, flight conditions, and other variables.

| Autopilot Type | Comment |
| :--- | :--- |
| Single-axis | The simplest type of autopilot available on the market. Only controls a single axis. These most <br> often come in two types. Either they provide roll or longitudinal (pitch) control. A roll control <br> autopilot allows the pilot to select a roll-stabilization mode (also called wing-leveler), primarily <br> to maintain a zero roll angle (i.e. wings level). This functionality is very helpful when flying IFR. <br> Such autopilots usually allow the pilot to make heading changes and bank the airplane by <br> twisting a small self-centering knob. Then, once the pilot releases the knob, the aircraft rolls <br> back to level flight. The functionality is achieved by adding a mechanical servo into the physical <br> control loop of the aircraft. The autopilot central unit receives signals from other instruments <br> already present in the instrument panel and processes them to "decide" what signal to <br> transmit to the servo. An example of such a control loop is shown in Figure 3-8. This vastly <br> expands the capability of the device, allowing it to maintain a specified heading or helping <br> with the tracking of LOC during ILS approaches. A pitch-control autopilot aids the pilot in <br> maintaining altitude or to control rate-of-ascent (from here on called Rate-of-Climb or, simply, <br> ROC) and Rate-of-Descent (from here on called ROD). Fundamentally, it is similar in setup as <br> the roll control autopilot, with a mechanical servo inserted into the pitch-control loop. Added <br> functionality allows the pilot to maintain GS during ILS approaches. |
| Two-axis | A two-axis autopilot is typically the combination of the two single-axes autopilots described <br> above into a single unit. A typical setup diagram is shown in Figure 3-9. |
| Three-axis | A three-axis autopilot expands the functionality of the autopilot further by adding control of <br> the third axis, the yaw-axis (or the directional axis). At the time of this writing, all civilian <br> aircraft (commercial and general aviation aircraft) and the vast majority of all military aircraft <br> (many would contend all) are designed with an inherent static directional stability. <br> Furthermore, their standard operation calls for directional motion to be relatively quiet when <br> compared to the other two axes. Therefore, the yaw-controller provides added dynamic <br> damping functionality; something referred to as yaw-damping. This suppresses a well known <br> dynamic response called Dutch-roll. Many aircraft, in particular aircraft that feature swept <br> wing and that routinely operate at high altitudes are susceptible to a high-amplitude version of <br> duch-roll that is of great nuisance. Yaw-dampers allow such aircraft to be designed without <br> "awkward" geometry that otherwise would be required to suppress this motion. |



Figure 3-8: A schematic of the equipment required for a single-axis autopilot. HSI stands for Horizontal Situation Indicator, VOR for VHS Omni-directional Radio, GPS for Global Positioning System, and A/P stands for AutoPilot. Other abbreviations are provided in the text.


Figure 3-9: A schematic of the equipment required for a two-axis autopilot.

### 3.3.2. Commercial Lightweight AFMS for Civilian Use

A number of companies specialize in the development of lightweight AFMS for civilian sUAVs. This activity is known as FPV (First Person View) among R/C pilots and consists of the installation of a lightweight video camera and transmitter on the aircraft and a corresponding ground station, which consists of a video receiver and a monitor on which the live feed from the camera can be viewed. In particular, a popular consumer electronic version of such a ground station provides two miniature TV monitors inside special goggles, some which have the receiver built into them and allow users to truly immerse themselves into this experience. An example of such goggles is shown in Figure 3-10. The most popular video frequencies for this purpose are $1.2 \mathrm{GHz}, 2.4 \mathrm{GHz}$, and 5.8 GHz . All have their pros and cons; 1.2 GHz is a long range frequency with good penetration capability, but requires large antennas.

Larger antennas usually mean greater aerodynamic drag (as they have to be exposed to the elements) and may pose installation challenges. 2.4 GHz is a good in-between frequency, with smaller antennas, but suffers from interference from cell phones, which also operate at that frequency. 5.8 GHz uses the smallest antennas, allowing them to be mounted practically anywhere on the aircraft (pending interference due to the aircraft's other electronic systems), but suffers from short range and poor obstacle penetration.


Figure 3-10: A popular consumer version of FPV goggles, manufactured by Fat Shark.
It was to be expected that, once operators could view in-flight footage in real time, people would want to expand the possibility by presenting flight information with the video feed; and the On-Screen Display (OSD) was born. This technology allowed operators to view important information such as battery capacity, voltage, current, GPS position, altitude, and velocities, to signal strength, Line-of-Sight (LOS) distance, flight direction to the operator, and many others. Among well known current manufacturers of this technology are companies like ImmersionRC, Eagletree Systems, Paparazzi, and Ardu. Figure 3-11 shows a screen capture from the author's Eagletree Systems Vector AFMS and OSD unit, installed in a Quanum Observer FPV aircraft. The reader is referred to the product manual for details on legend information [28], which includes signal strength, number of visible GPS satellites, calibrated airspeed, ground speed, throttle setting, LOS distance, rate of climb, barometric and GPS altitudes, geographic location, battery status, just to name a few. The product features a number of autopilot capabilities, such as heading hold, altitude hold, position hold, and return to base auto-flight mode.


Figure 3-11: The author's Eagletree Systems Vector in action.

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## 4. The Use of Potential Flow Theory in the GICA

This chapter presents a key mathematical concept required for the development and operation of the GICA; Potential Flow Theory (PFT). Two versions are presented; the 2-dimensional PFT, which is the foundation of the Potential Flow Method (PFM), deserves a review and 3-dimensional PFT, which is used as comparison to the Constant Mass Flow method for wind simulation, as presented in Chapter 5, Atmospheric Modeling. For this reason, it is appropriate to present its theoretical basis in this chapter.

### 4.1 Two-Dimensional Potential Flow Theory

The Potential Flow Theory (PFT) is a powerful tool for the modeling of fluid flow around objects in a fashion that ensures the resulting flow complies with the natural law of mass conservation. While the theory remains useful for its intended purpose, it can also be utilized for other tasks, for instance, it is the foundation of the Potential Flow Method (PFM) discussed in Chapter 2, A Survey of Literature. However, in this work it is used to construct a virtual force field that can be used for the guidance of a low wing loading aircraft through a wind-field characterized by labyrinth of up- and downdrafts. Thus, it is of vital importance in this work. This is used in the GICA algorithm presented in Section 10.3, The LiftSeeking Sink-Avoidance Algorithm. This feature will be described in more detail later, but this section presents the theoretical foundation on which it rests. The reader is reminded that the ultimate goal of the following discussion is to explain how flow physics can be described using mathematical formulation. Once this has been established, the resulting theory is used to set up "virtual" flow that allows the path planner described in Chapter 10, The Generic Intelligent Control Algorithm to lay out the path of a vehicle such it avoids regions of sink and favors regions of lift.

### 4.1.1. Continuity for Incompressible Flow

Only an approximation for incompressible flow is needed. We must start with a fundamental theorem of fluid flow; the conservation of mass. It requires the familiar relationship between the time rate of change of density, $\rho$, inside a control volume ( $C V$ ) and mass flow in and out of the control surface ( $C S$ ) to hold.

$$
\begin{equation*}
\int_{C V} \frac{\partial \rho}{\partial t} d \Vdash+\int_{C S} \rho \mathbf{V} \cdot \mathbf{n} d A=0 \tag{4-1}
\end{equation*}
$$

We prefer this expression in the differential form, rather than the integral form shown. The easiest way to convert Equation (4-1) into a differential form is through the Divergence Theorem, which states:

$$
\begin{equation*}
\int_{V} \nabla \cdot \mathbf{G} d V=\oint_{A} \mathbf{G} \cdot \mathbf{n} d A \tag{4-2}
\end{equation*}
$$

The circle on the area integral indicates it must be evaluated over the entire surface $A$ that encloses the volume $\forall$. Next, let's denote the two integrals of Equation (4-1) as the density change integral ( $I$ ) and mass flow integral (II) as shown below.

It can be seen that integral $I I$ is analogous to that of the divergence theorem of Equation (4-2). Thus, let $\mathbf{G}=\rho \mathbf{V}$ and substitute for $\rho \mathbf{V} \cdot \mathbf{n}$ :

$$
0=\int_{C V} \frac{\partial \rho}{\partial t} d V+\overbrace{\int_{C V}(\rho \mathbf{V}) \cdot \mathbf{n} d A}^{\text {analogous to } \oint_{A}^{\ell} \mathbf{G} \cdot \mathbf{n} d A}=\int_{C V} \frac{\partial \rho}{\partial t} d \forall+\int_{C V} \nabla \cdot \mathbf{G} d V=\int_{C V} \frac{\partial \rho}{\partial t} d \forall+\int_{C V} \nabla \cdot(\rho \mathbf{V}) d \nvdash
$$

This allows the two integrals to be combined:

$$
\begin{equation*}
\int_{C V}\left[\frac{\partial \rho}{\partial t}+\nabla \cdot(\rho \mathbf{V})\right] d V=0 \tag{4-3}
\end{equation*}
$$

The quantity inside the brackets is the general differential form for the conservation of mass and is called the continuity equation:

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\nabla \cdot(\rho \mathbf{V})=0 \tag{4-4}
\end{equation*}
$$

It follows that if incompressible flow conditions are assumed (for which $\rho$ is treated as a constant), then $\partial \rho / \partial t=0$ and we obtain

$$
\begin{equation*}
\nabla \cdot(\rho \mathbf{V})=\rho \nabla \cdot \mathbf{V}=0 \quad \Leftrightarrow \quad \nabla \cdot \mathbf{V}=0 \tag{4-5}
\end{equation*}
$$

In a 3-dimensional Cartesian coordinate system the incompressible continuity equation becomes:

$$
\begin{equation*}
\nabla \cdot \mathbf{V}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial z}{\partial z}=0 \tag{4-6}
\end{equation*}
$$

Where $u, v, w=$ flow velocity components and $\mathbf{V}=u \mathbf{i}+v \mathbf{j}+w \mathbf{k}$

### 4.1.2. Stream Function, Velocity Potential, and Vorticity

Now, a couple of "simplifications" will be introduced that will come in handy later. Consider the planar form of Equation (4-6) (in the $x-y$ plane), which can be written as follows

$$
\begin{equation*}
\nabla \cdot \mathbf{V}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4-7}
\end{equation*}
$$

It is now possible to define a special function, which we will refer to from now on as the stream function, denoted by $\psi$. It allows the continuity equation to be defined in a compact form using a single function. The velocity components are written using the stream function as shown below:

$$
\begin{equation*}
u=\frac{\partial \psi}{\partial y} \quad \text { and } \quad v=-\frac{\partial \psi}{\partial x} \tag{4-8}
\end{equation*}
$$

Substitution of $u$ and $v$ into Equation (4-7) will indeed add up to zero as shown below

$$
\nabla \cdot \mathbf{V}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right)+\frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right)=\frac{\partial^{2} \psi}{\partial x \partial y}-\frac{\partial^{2} \psi}{\partial y \partial x}=0
$$

We can also define a different function, which from now on will be called the velocity potential, denoted by $\phi$, which allows the velocity, $\mathbf{V}$, to be expressed as

$$
\begin{equation*}
\mathbf{V}=\nabla \phi \tag{4-9}
\end{equation*}
$$

The velocity components are written using the velocity potential as shown below:

$$
\begin{equation*}
u=\frac{\partial \phi}{\partial x} \quad \text { and } \quad v=\frac{\partial \phi}{\partial y} \tag{4-10}
\end{equation*}
$$

Note that the vector presentation of the flow physics invites an investigation of an additional property of the flow field, called vorticity. It is indicative of how fluid parcels may (or may not) distort as they move along streamlines of the flow. The vorticity is defined as the curl of the velocity:

$$
\zeta=\nabla \times \mathbf{V}= \begin{cases}=0 & \text { Irrotational flow }  \tag{4-11}\\ \neq 0 & \text { Rotational flow }\end{cases}
$$

Irrotational flow means the motion of the fluid parcels along a streamline is by translation only, whereas rotational flow means the motion also features angular velocity. The former is associated with flow outside of the boundary layer of an object, whereas the latter is associated with flow inside boundary layers and wakes. Note that for planar flow (where it is assumed that $w=0$ ), Equation (4-11) becomes

$$
\begin{equation*}
\zeta=\nabla \times \mathbf{V}=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \tag{4-12}
\end{equation*}
$$

### 4.1.3. Governing Equation for Irrotational Incompressible Flow

The next step is to derive a "global" or governing equation for the flow. For the purposes of the work in here, we prefer incompressible and irrotational flow. Using the concept of a velocity potential, we can write the incompressible continuity equation in a slightly modified form

$$
\begin{equation*}
\nabla \cdot \mathbf{V}=\nabla \cdot(\nabla \phi)=\nabla^{2} \phi=0 \tag{4-13}
\end{equation*}
$$

We recognize this as Laplace's equation. Thus, in 3-dimensional space, the velocity potential satisfies flow continuity if

$$
\begin{equation*}
\nabla^{2} \phi=\frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}}+\frac{\partial^{2} \phi}{\partial z^{2}}=0 \tag{4-14}
\end{equation*}
$$

The condition of irrotationality is satisfied and this can be shown by substituting the definition of the stream function into Equation (4-12)

$$
\zeta=\nabla \times \mathbf{V}=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)=-\frac{\partial^{2} \psi}{\partial x^{2}}-\frac{\partial^{2} \psi}{\partial y^{2}}
$$

Thus, for irrotational flow, we get

$$
\begin{equation*}
\zeta=\nabla \times \mathbf{V}=0 \quad \Leftrightarrow \quad \frac{\partial^{2} \psi}{\partial x^{2}}+\frac{\partial^{2} \psi}{\partial y^{2}}=0 \tag{4-15}
\end{equation*}
$$

This shows that both the velocity potential, $\phi$, and stream function, $\psi$, satisfy Laplace's equation. This allows us to state that the Laplace equation is the governing equation of the flow.

### 4.1.4. The Use of the Governing Equation for Description of Flow

Equations (4-13) and (4-15) lead to important conclusions:

- All flows that are simultaneously incompressible and irrotational can be described using a velocity potential or a stream function, although the use of stream functions is restricted to 2dimensional flow as stipulated by Equation (4-15).
- Any solution of the Laplace equation (which would return the functions $\phi$ or $\psi$ ) represents a description of some particular flow (depending on the boundary conditions that were used to obtain $\phi$ or $\psi$ ).
- Since the Laplace equation is a linear second order partial differential equation, it follows that the sum of any particular solutions of a linear differential equation is also a solution of the equation through the principle of superposition. This means that if $\phi_{1}, \phi_{2}, \phi_{3}, \ldots$, and $\phi_{n}$ are particular solutions of the Laplace equation, then the following representation is also a solution:

$$
\begin{equation*}
\phi=\sum_{i=1}^{n} \phi_{i} \tag{4-16}
\end{equation*}
$$

### 4.1.5. Boundary Conditions

The implication of Equation (4-25) is profound, because it implies that we can come up with a potential for a complicated flow by considering the individual contributions of a large number of simpler flows. Such flows are called Elementary flows. This fact is the core of potential flow theory. The most common elementary flows are uniform flow, source flow, sink flow, doublet flow, and vortex flow. In interest of space, we will limit the presentation to only the first three, as these suffice to accomplish the task at hand.

The solution of PDEs requires boundary conditions to be specified. The term far-field refers to flow conditions that prevail far from the region of interest. There are typically two boundary conditions that must be considered in the far-field in the context of the work presented in here: (1) There is initial flow velocity in the far-field, such as when we consider flow due to wind. (2) There is no flow in the far-field; all the flow internal to the region of interest is induced by the presence of elementary flows.
(1) Boundary conditions for flow velocity of magnitude $V_{\infty}$ in the far field:

$$
\begin{align*}
& u(\infty)=\frac{\partial \phi}{\partial x}=\frac{\partial \psi}{\partial y}=V_{\infty} \cos \alpha \\
& v(\infty)=\frac{\partial \phi}{\partial y}=-\frac{\partial \psi}{\partial x}=V_{\infty} \sin \alpha \tag{4-17}
\end{align*}
$$

(2) Boundary conditions for no-flow velocity in the far field:

$$
\begin{align*}
& u(\infty)=\frac{\partial \phi}{\partial x}=\frac{\partial \psi}{\partial y}=0 \\
& v(\infty)=\frac{\partial \phi}{\partial y}=-\frac{\partial \psi}{\partial x}=0 \tag{4-18}
\end{align*}
$$

There is an additional boundary condition that must be considered at the surface of any object the flow encloses; the flow must be tangent to it. Another way of presenting this boundary condition is to require the component of the flow normal to the surface to be zero. This means that

$$
\begin{equation*}
\mathbf{V} \cdot \mathbf{n}=\nabla \phi \cdot \mathbf{n}=0 \tag{4-19}
\end{equation*}
$$

### 4.1.6. Elementary Flows for Planar Flows

It is now possible to present a representation of the three elementary flows discussed above (see Figure 4-1). In interest of space, their proof is omitted. Also, no demonstration of their irrotationality will be presented. Interested readers are directed to references [1], [2], [3] and others for more details.

## Elementary Flow 1: Uniform Flow

Consider a uniform flow of velocity of magnitude $V_{\infty}$ and at an angle of inclination (angle-of-attack) of magnitude $\alpha$, measured from the horizontal $x$-axis. Then, the elementary flow using velocity potential is given by

$$
\left.\begin{array}{l}
u=\frac{\partial \phi}{\partial x}=V_{\infty} \cos \alpha  \tag{4-20}\\
v=\frac{\partial \phi}{\partial y}=V_{\infty} \sin \alpha
\end{array}\right\} \Leftrightarrow \phi=V_{\infty}(x \cos \alpha+y \sin \alpha)
$$

Similarly, using the stream function

Uniform

$\phi=V_{\infty} x \quad \psi=V_{\infty} y$

Source

$\phi=\frac{\lambda}{2 \pi} \ln r \quad \psi=\frac{\lambda}{2 \pi} \theta$

$$
V_{r}=\frac{\lambda}{2 \pi r} \quad(\lambda>0)
$$

Sink

$\begin{array}{cc}\phi=\frac{\lambda}{2 \pi} \ln r \quad \psi=\frac{\lambda}{2 \pi} \theta \\ V_{r}=\frac{\lambda}{2 \pi r} & (\lambda<0)\end{array}$

Figure 4-1: The three elementary flows presented in this discussion.

$$
\left.\begin{array}{l}
u=\frac{\partial \psi}{\partial y}=V_{\infty} \cos \alpha  \tag{4-21}\\
v=-\frac{\partial \psi}{\partial x}=V_{\infty} \sin \alpha
\end{array}\right\} \Leftrightarrow \psi=V_{\infty}(y \cos \alpha-x \sin \alpha)
$$

It is possible to represent both functions in polar coordinates as shown below:

$$
\begin{align*}
& \phi=V_{\infty} r(\cos \theta \cos \alpha+\sin \theta \sin \alpha) \\
& \psi=V_{\infty} r(\sin \theta \cos \alpha-\cos \theta \sin \alpha) \tag{4-22}
\end{align*}
$$

## Elementary Flow 2: Source Flow

The concept source is used to represent fluid streaming out of some "opening," for instance, consider water flowing out of a nozzle immersed in a water container. Then, we define the volumetric flow rate, given by $\lambda$, as the mass flow rate (mass of fluid per second), denoted by $\dot{m}$, divided by the mass per unit volume (density), or

$$
\begin{equation*}
\lambda \equiv \frac{\dot{m}}{\rho} \tag{4-23}
\end{equation*}
$$

Typical units of $\lambda$ are in terms per unit depth and are given (in the SI-system) by

$$
\left(\frac{\mathrm{kg} / \mathrm{s}}{\mathrm{~kg} / \mathrm{m}^{3}} \frac{1}{m}=\frac{m^{2}}{s}\right)
$$

Of course, while the aforementioned "nozzle" would have a finite physical diameter, the mathematical representation treats the source as a point entity. If we assume the volumetric flow rate to be uniform
along a circle of radius $r$ around and centered at the source, then the volumetric flow rate per unit depth is given by (further showing the units must be $\mathrm{m}^{2} / \mathrm{s}$ )

$$
\lambda=\frac{\dot{m}}{\rho}=2 \pi r V_{r}
$$

Where $V_{r}$ is the radial speed of the fluid at that radius $r$. It follows this speed is the radial flow velocity contribution of the source:

$$
\begin{equation*}
V_{r}=\frac{\lambda}{2 \pi r} \tag{4-24}
\end{equation*}
$$

Therefore, it is possible to show that the flow in the container outside of the nozzle is given by the velocity potential or stream function as shown below

Velocity potential:

$$
\begin{align*}
& \phi=\frac{\lambda}{2 \pi} \ln r  \tag{4-25}\\
& \psi=\frac{\lambda}{2 \pi} \theta \tag{4-26}
\end{align*}
$$

Note that if the source is placed away from the origin of the coordinate axes, the parameters $r$ and $\theta$ must be adjusted. For instance, if a source is placed at the point ( $a, b$ ), then Equations (4-26) and (4-27) would rewritten as follows:

$$
\begin{align*}
r=\sqrt{(x-a)^{2}+(y-b)^{2}} & \Rightarrow \phi(x, y)=\frac{\lambda}{2 \pi} \ln \sqrt{(x-a)^{2}+(y-b)^{2}}  \tag{4-27}\\
\theta=\tan ^{-1}\left(\frac{y-b}{x-a}\right) & \Rightarrow \psi(x, y)=\frac{\lambda}{2 \pi} \tan ^{-1}\left(\frac{y-b}{x-a}\right) \tag{4-28}
\end{align*}
$$

## Elementary Flow 3: Sink Flow

The concept sink is used to represent fluid streaming into an "opening," for instance, consider water streaming under gravity into an opening at the bottom of a water container. A sink is the "inverse" of a source and the formulation is identical, except the volumetric flow rate is a negative value. Therefore, Equations (4-25) and (4-26) also apply to the sink, except that $\lambda<0$.

### 4.1.7. On the Combination of Elementary Flows

As stated in Section 4.1.4, The Use of the Governing Equation for Description of Flow, the Laplace equation permits all particular solutions to be combined through linear addition. This, in turn, means that any combination of the above elementary flows is a solution too. For instance, consider a flow in which we combine uniform flow with $\alpha=0$ and a source and a sink of equal strength. For instance, using the stream function approach, we can write the entire stream function for such flow with the help of Equation (4-28)

$$
\begin{equation*}
\psi=\sum \psi_{i}=V_{\infty} y+\frac{\lambda_{1}}{2 \pi} \theta_{1}+\frac{\lambda_{2}}{2 \pi} \theta_{2}=V_{\infty} y+\frac{\lambda_{1}}{2 \pi} \tan ^{-1}\left(\frac{y-y_{1}}{x-x_{1}}\right)+\frac{\lambda_{2}}{2 \pi} \tan ^{-1}\left(\frac{y-y_{2}}{x-x_{2}}\right) \tag{4-29}
\end{equation*}
$$

Where the subscripts denote the source (1) and sink (2), respectively. Similarly, using the velocity potential approach, we can write the entire potential for the flow with the help of Equation (4-27)

$$
\begin{equation*}
\phi=\sum \phi_{i}=V_{\infty} x+\frac{\lambda_{1}}{2 \pi} \ln \sqrt{\left(x-x_{1}\right)^{2}+\left(y-y_{1}\right)^{2}}+\frac{\lambda_{2}}{2 \pi} \ln \sqrt{\left(x-x_{2}\right)^{2}+\left(y-y_{2}\right)^{2}} \tag{4-30}
\end{equation*}
$$

An example of such flow is presented in Figure 4-2, in which the source and sink are placed in opposite quadrants equidistance from the origin of the coordinate axes.

Note that the flow velocity at any point can be obtained by differentiating the complete stream function of Equation (4-29); with respect to $y$ to get $u$ (horizontal component) and $x$ to get $v$ (vertical component) per Equation (4-8). Similar approach is used if we use the velocity potential. Thus, if we are interested in the flow velocity at the point $(x, y)$, the contribution of the source located at point $\left(x_{1}, y_{1}\right)$ is given by

$$
V_{r 1}=\frac{\lambda_{1}}{2 \pi r_{1}} \quad \text { where } \quad r_{1}=\sqrt{\left(x-x_{1}\right)^{2}+\left(y-y_{1}\right)^{2}}
$$

The contribution of the source at point $\left(x_{1}, y_{1}\right)$ to the velocity at the point $(x, y)$ are found from:

$$
\begin{equation*}
u_{1}=V_{r 1} \cos \left(\tan ^{-1}\left(\frac{y-y_{1}}{x-x_{1}}\right)\right) \quad \text { and } \quad v_{1}=V_{r 1} \sin \left(\tan ^{-1}\left(\frac{y-y_{1}}{x-x_{1}}\right)\right) \tag{4-31}
\end{equation*}
$$

Identical approach is used to obtain the contribution of the sink at point $\left(x_{2}, y_{2}\right)$ to the velocity at $(x, y)$. Therefore, the global contribution of the sources and sinks to the complete velocity at the point $(x, y)$, which includes the uniform flow, can be calculated as follows:

$$
\left.\begin{array}{l}
u=V_{\infty}+u_{1}+u_{2}=V_{\infty}+\overbrace{V_{r 1} \cos \left(\tan ^{-1}\left(\frac{y-y_{1}}{x-x_{1}}\right)\right)}^{u_{1}}+\overbrace{V_{r 2} \cos \left(\tan ^{-1}\left(\frac{y-y_{2}}{x-x_{2}}\right)\right)}^{u_{2}}) \\
v=0+v_{1}+v_{2}=\underbrace{V_{r 1} \sin \left(\tan ^{-1}\left(\frac{y-y_{1}}{x-x_{1}}\right)\right)}_{v_{1}}+\underbrace{V_{r 2} \sin \left(\tan ^{-1}\left(\frac{y-y_{2}}{x-x_{2}}\right)\right)}_{v_{2}} \tag{4-33}
\end{array}\right\}
$$

Yielding the velocity
A flow of this nature is shown in Figure 4-3 (although $V_{r}$ is zero). In general, the global contribution of $N$ sources (or sinks) to the flow speed induced at a point $(x, y)$ can be calculated as follows:


Figure 4-2: A computational domain showing a velocity field resulting from combining a 2 units/s uniform flow with a source of magnitude 20 units $^{3} / \mathrm{s}$ (in quadrant I) and sink of magnitude -20 units $^{3} / \mathrm{s}$ (in quadrant III) (left figure). The right image shows the corresponding streamlines.


Figure 4-3: The combination of a source and sink induces velocity at some specific point. Note that there is no uniform flow element specified for this flow, or the resultant velocity would look more like what is shown in Figure 4-2.

$$
\begin{align*}
\mathbf{V}=u \mathbf{i}+v \mathbf{j}=\left[V_{\infty} \cos \alpha+\sum_{i=1}^{N}\right. & \left.\frac{\lambda_{i}}{2 \pi r_{i}} \cos \left(\tan ^{-1}\left(\frac{y-y_{i}}{x-x_{i}}\right)\right)\right] \mathbf{i} \\
& +\left[V_{\infty} \sin \alpha+\sum_{i=1}^{N} \frac{\lambda_{i}}{2 \pi r_{i}} \sin \left(\tan ^{-1}\left(\frac{y-y_{i}}{x-x_{i}}\right)\right)\right] \mathbf{j} \tag{4-34}
\end{align*}
$$

### 4.1.8. Non-Lifting Flow about Arbitrary Bodies

Figure 4-2 shows an example of flow resulting from placing an arbitrary position and magnitude of a source and sink in a uniform flow. The flow that resulted was not necessarily realized until after the velocity field and streamlines had been plotted. This raises an interesting question: Is the opposite possible? Is it possible to determine the source strength required to form flow that has certain features? Such a method would allow the flow around bodies of known geometries to be generated and, thus, would be of considerable value. Clearly, the answer is yes, otherwise it would not be brought up. However, the idea requires the concept of elementary flows to be developed further such they are no longer point concepts, but consist of infinitely many and small flows elements arranged side-by-side along a curve. Thus, a curve element of sources would be called a source sheet, and sinks a sink sheet, and so on. Consequently, we would refer to the strength as source (or sink) strength per unit length, denoted by $\lambda=\lambda(s)$, where $s$ refers to a distance along the length of the line element. It has units of volume flow rate per unit depth per unit length ( $\mathrm{m} / \mathrm{s}$ ), in contrast to volume flow rate for the point flows, which are in $\left(\mathrm{m}^{2} / \mathrm{s}\right)$. The strength of a small segment of length $d s$ of the source sheet is $\lambda d s$. Note that even though the text will from now on only refer to sources, the same holds for sinks. After all, a sink is simply the negative of a source. At any rate, this segment induces an infinitesimally small potential, $d \phi$, at some arbitrary point $P=P(x, y)$ given by

$$
\begin{equation*}
d \phi=\frac{\lambda d s}{2 \pi} \ln r \tag{4-35}
\end{equation*}
$$

Where $r$ is the distance from the source segment to the point. If the source sheet curve begins at the point $a$ and terminates at point $b$, the potential induced at $P$ is found from

$$
\begin{equation*}
\phi(x, y)=\int_{a}^{b} \frac{\lambda d s}{2 \pi} \ln r \tag{4-36}
\end{equation*}
$$

If the geometry of the body is complicated, it will be difficult to express it mathematically using functions that also lend themselves to conventional integration. The workaround is to represent the shape using a numerical scheme based on $N$ linear segments, each described as a source sheet, but of varying strengths. These segments are called panels. Thus, the $j$-th panel will induce a potential at $P(x, y)$ as shown below

$$
\begin{equation*}
\Delta \phi_{j}=\frac{\lambda_{j}}{2 \pi} \int_{j} \ln r_{P j} d s_{j} \tag{4-37}
\end{equation*}
$$

Where the distance between a representative point on the $j$-th panel and the $P(x, y)$ is given by

$$
\begin{equation*}
r_{P j}=\sqrt{\left(x-x_{j}\right)^{2}+\left(y-y_{j}\right)^{2}} \tag{4-38}
\end{equation*}
$$

Note that since we will be integrating along each panel, from their starting- to termination-positions, Equation (4-38) represents distance that will vary as the integration progresses between those positions. Furthermore, note that the panel strength, $\lambda_{j}$, is assumed constant over each panel allowing it to be taken outside of the integration. This representation allows the complete potential at $P(x, y)$ due to all $N$ panels to be expressed as follows

$$
\begin{equation*}
\phi(x, y)=\sum_{j=1}^{n} \Delta \phi_{j}=\sum_{j=1}^{n} \frac{\lambda_{j}}{2 \pi} \int_{j} \ln r_{P j} d s_{j} \tag{4-39}
\end{equation*}
$$

At this point it is necessary to bring up the boundary conditions associated with the resulting flow, per earlier discussion. To implement the boundary conditions specified by Equation (4-19), we have to select a point on each panel on which the tangent flow requirement is to be enforced. This point is called the control point or collocation point and is denoted by $(\bar{x}, \bar{y})$. For typical applications, this point is located at the midpoint of the panel, and sometimes $3 / 4$ of the distance between the points $a$ and $b$. In effect, this is like placing the point $P(x, y)$ at $(\bar{x}, \bar{y})$ and adjusting the strength of the source until the velocity component normal to the panel is zero, thus, ensuring only tangential velocity prevails along the panel when the implementation of this scheme takes place. Therefore, we can rewrite Equation (4-39) as shown below:

$$
\begin{equation*}
\phi\left(x_{i}, y_{i}\right)=\sum_{j=1}^{n} \frac{\lambda_{j}}{2 \pi} \int_{j} \ln r_{i j} d s_{j}=\sum_{j=1}^{n} \frac{\lambda_{j}}{2 \pi} \int_{j} \ln \sqrt{\left(\bar{x}_{i}-x_{j}\right)^{2}+\left(\bar{y}_{i}-y_{j}\right)^{2}} d s_{j} \tag{4-40}
\end{equation*}
$$

Where $r_{i j}$ is the distance between a point on the $j$-th panel and the collocation point on the $i$-th panel, denoted by $\left(\bar{x}_{i}, \bar{y}_{i}\right)$, and $s_{j}$ is the elemental distance along the panel. To establish this correctly we will need the component of the far-field velocity, $V_{\infty}=(u, v)$, normal to the panel

$$
\begin{equation*}
\mathbf{V}_{\infty} \cdot \mathbf{n}_{i}=(u \mathbf{i}+v \mathbf{j}) \cdot\left(n_{x} \mathbf{i}+n_{y} \mathbf{j}\right)_{i}=V_{\infty} \cos \beta_{i} \tag{4-41}
\end{equation*}
$$

Where $\mathbf{n}_{i}$ is the normal to the panel and $\beta_{i}$ is the angle between the far-field velocity and the normal (see Figure 4-4). Recall Equation (4-10), which shows how the velocity components are obtained by differentiating with respect to the $x$ - and $y$-axes. Here, in effect, we are dealing with a redefined coordinate system that has been realigned such the $x$-axis is now oriented along the surface (or tangent) of the panel (call it the $s$-axis) and the $y$-axis is aligned with the normal to the panel (call it the $n$-axis). Therefore, the normal component of the velocity vector at the $i$-th panel, induced at ( $x_{i} y_{i}$ ) by the other source panels, is obtained by taking the derivative of Equation (4-40) with respect to the $n$ axis:

$$
\begin{equation*}
V_{n_{i}}=\frac{\partial}{\partial n_{i}}\left[\phi\left(x_{i}, y_{i}\right)\right]=\frac{\partial}{\partial n_{i}}\left[\sum_{j=1}^{n} \frac{\lambda_{j}}{2 \pi} \int_{j} \ln r_{i j} d s_{j}\right] \tag{4-42}
\end{equation*}
$$

When the derivative is evaluated, we will have a case of

$$
\frac{d}{d x} \ln x=\frac{1}{x} \Rightarrow \frac{d}{d x} \ln r_{i j}=\frac{1}{r_{i j}} \frac{d r_{i j}}{d x}
$$

Therefore, the presence of the inverse of $r_{i j}$ will cause a singularity when $i=j$. However, it is possible to show that, when this happens, the contribution of panel $i$ to the velocity on itself is $\lambda_{i} / 2$. Consequently, Equation (4-42) can be rewritten as follows

$$
\begin{equation*}
V_{n_{i}}=\frac{\lambda_{i}}{2}+\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\lambda_{j}}{2 \pi} \int_{j} \frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right) d s_{j} \tag{4-43}
\end{equation*}
$$

When this result is combined with the far-field velocity, the sum must be zero, i.e.

$$
\begin{equation*}
V_{n_{i}}+\mathbf{V}_{\infty} \cdot \mathbf{n}_{i}=V_{n_{i}}+V_{\infty} \cos \beta_{i}=0 \tag{4-44}
\end{equation*}
$$

Figure 4-4: Representation of an arbitrary body using panels. Note that the orientation of panels is clockwise. Light colored diagonal lines represent the underlying far-field velocity isopleths, shown here for reference only.

Substituting Equation (4-43) leads to

$$
\begin{equation*}
\frac{\lambda_{i}}{2}+\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\lambda_{j}}{2 \pi} \overbrace{\int_{j} \frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right) d s_{j}}^{\equiv I_{i j}}+\overbrace{V_{\infty} \cos \beta_{i}}^{\mathbf{v}_{\infty} \cdot \mathbf{n}_{i}}=0 \tag{4-45}
\end{equation*}
$$

Note that the equation has one unknown, namely $\lambda_{i}$. Therefore, for $N$ panels we will have $N$ equations and $N$ unknowns. More details of panels $i$ and $j$ in Figure 4-4 are shown in Figure 4-5. Writing Equation (4-45) for each panel, $i$ $=1,2, \ldots, N$, in this fashion yields a system of equations that are best solved by resorting to matrix inversion techniques, such as Gaussian elimination with partial pivoting (provided in Appendix A.3). Using this numerical scheme, the derivative in the integral can be evaluated using implicit differentiation as follows:

$$
\begin{aligned}
\frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right) & =\frac{1}{r_{i j}} \frac{\partial r_{i j}}{\partial n_{i}} \\
& =\frac{1}{r_{i j}} \frac{\partial}{\partial n_{i}} \sqrt{\left(\bar{x}_{i}-x_{j}\right)^{2}+\left(\bar{y}_{i}-y_{j}\right)^{2}} \\
& =\frac{1}{r_{i j}} \frac{\partial}{\partial n_{i}}\left[\left(\bar{x}_{i}-x_{j}\right)^{2}+\left(\bar{y}_{i}-y_{j}\right)^{2}\right]^{1 / 2}
\end{aligned}
$$



Figure 4-5: Panels $i$ and $j$ in more detail.

Where the collocation point of the $i$-th panel is given by

$$
\begin{align*}
\bar{x}_{i} & =\frac{1}{2}\left(x_{i}+x_{i+1}\right) \\
\bar{y}_{i} & =\frac{1}{2}\left(y_{i}+y_{i+1}\right) \tag{4-46}
\end{align*}
$$

Carrying out the differentiation leads to

$$
\frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right)=\frac{1}{r_{i j}} \frac{1}{2}\left[\left(\bar{x}_{i}-x_{j}\right)^{2}+\left(\bar{y}_{i}-y_{j}\right)^{2}\right]^{-1 / 2} \times\left[2\left(\bar{x}_{i}-x_{j}\right) \frac{d x_{i}}{d n_{i}}+2\left(\bar{y}_{i}-y_{j}\right) \frac{d y_{i}}{d n_{i}}\right]
$$

Where we find that $\frac{d x_{i}}{d n_{i}}=\cos \beta_{i}$ and $\frac{d y_{i}}{d n_{i}}=\sin \beta_{i}$. Note that these are the parallel and tangential components of the unit normal vector to panel $i$ (see Figure 4-5). Simplifying leads to

$$
\frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right)=\frac{1}{2 r_{i j}} \frac{\left\lfloor 2\left(\bar{x}_{i}-x_{j}\right) \cos \beta_{i}+2\left(\bar{y}_{i}-y_{j}\right) \sin \beta_{i}\right\rfloor}{\sqrt{\left(\bar{x}_{i}-x_{j}\right)^{2}+\left(\bar{y}_{i}-y_{j}\right)^{2}}}
$$

Finally, "cleaning this up" yields

$$
\begin{equation*}
\frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right)=\frac{\left(\bar{x}_{i}-x_{j}\right) \cos \beta_{i}+\left(\bar{y}_{i}-y_{j}\right) \sin \beta_{i}}{\left(\bar{x}_{i}-x_{j}\right)^{2}+\left(\bar{y}_{i}-y_{j}\right)^{2}} \tag{4-47}
\end{equation*}
$$

The final step is to use this to evaluate the integral $\int_{j} \frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right) d s_{j}$ of Equation (4-45), denoted by $I_{i j}$. It can be seen from Figure 4-5 that the following relations hold for the $i$-th panel:

$$
\beta_{i}=\Phi_{i}+\frac{\pi}{2} \Rightarrow\left\{\begin{array}{l}
\cos \beta_{i}=\cos \left(\Phi_{i}+\frac{\pi}{2}\right)=-\sin \Phi_{i}  \tag{4-48}\\
\sin \beta_{i}=\sin \left(\Phi_{i}+\frac{\pi}{2}\right)=\cos \Phi_{i}
\end{array}\right.
$$

These allow us to write the panel position along the $j$-th panel in terms of its length, $s_{j}$, as follows

$$
\begin{align*}
& x_{j}(s)=x_{j}+s_{j} \cos \Phi_{j} \\
& y_{j}(s)=y_{j}+s_{j} \sin \Phi_{j} \tag{4-49}
\end{align*}
$$

Substituting Equations (4-47), (4-48), and (4-49) into the integral leads to

$$
\begin{equation*}
\int_{j} \frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right) d s_{j}=\int_{j} \frac{\left(\bar{x}_{i}-x_{j}(s)\right)\left(-\sin \Phi_{i}\right)+\left(\bar{y}_{i}-y_{j}(s)\right)\left(\cos \Phi_{i}\right)}{\left(\bar{x}_{i}-x_{j}(s)\right)^{2}+\left(\bar{y}_{i}-y_{j}(s)\right)^{2}} d s_{j} \tag{4-50}
\end{equation*}
$$

Let's first focus on simplifying the numerator

$$
\begin{aligned}
\left(\bar{x}_{i}-\right. & {\left.\left[x_{j}+s_{j} \cos \Phi_{j}\right]\right)\left(-\sin \Phi_{i}\right)+\left(\bar{y}_{i}-\left[y_{j}+s_{j} \sin \Phi_{j}\right]\right)\left(\cos \Phi_{i}\right) } \\
& =-\left(\bar{x}_{i}-x_{j}\right) \sin \Phi_{i}-\left(y_{j}-\bar{y}_{i}\right) \cos \Phi_{i}+s_{j}\left(\sin \Phi_{i} \cos \Phi_{j}-\cos \Phi_{i} \sin \Phi_{j}\right) \\
& =\underbrace{-\left(\bar{x}_{i}-x_{j}\right) \sin \Phi_{i}+\left(\bar{y}_{i}-y_{j}\right) \cos \Phi_{i}}_{\equiv A}+\underbrace{\left(\sin \Phi_{i}-\sin \Phi_{j}\right)}_{\equiv B} s_{j}=A+B s_{j}
\end{aligned}
$$

Then, let's treat the denominator

$$
\begin{aligned}
\left(\left(\bar{x}_{i}-\right.\right. & \left.\left.x_{j}\right)-s_{j} \cos \Phi_{j}\right)^{2}+\left(\left(\bar{y}_{i}-y_{j}\right)-s_{j} \sin \Phi_{j}\right)^{2} \\
& =\left(\bar{x}_{i}-x_{j}\right)^{2}-2\left(\bar{x}_{i}-x_{j}\right) \cos \Phi_{j} s_{j}+\cos ^{2} \Phi_{j} s_{j}^{2}+\left(\bar{y}_{i}-y_{j}\right)^{2}-2\left(\bar{y}_{i}-y_{j}\right) \sin \Phi_{j} s_{j}+\sin ^{2} \Phi_{j} s_{j}^{2} \\
& =\underbrace{\left(\bar{x}_{i}-x_{j}\right)^{2}+\left(\bar{y}_{i}-y_{j}\right)^{2}}_{\equiv C}+\underbrace{2\left[-\left(\bar{x}_{i}-x_{j}\right) \cos \Phi_{j}-\left(\bar{y}_{i}-y_{j}\right) \sin \Phi_{j}\right] s_{j}+s_{j}^{2}=C+D s_{j}+s_{j}^{2}}_{\equiv D}
\end{aligned}
$$

Thus, the integral of Equation (4-50) can be written as follows

$$
\begin{equation*}
\int_{j} \frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right) d s_{j}=\int_{j} \frac{A+B s_{j}}{C+D s_{j}+s_{j}^{2}} d s_{j}=\underbrace{\int_{j} \frac{A}{C+D s_{j}+s_{j}^{2}} d s_{j}}_{I_{1}}+\underbrace{\int_{j} \frac{B s_{j}}{C+D s_{j}+s_{j}^{2}} d s_{j}}_{I_{2}} \tag{4-51}
\end{equation*}
$$

Where integrals $I_{1}$ and $I_{2}$ have the following solutions

$$
\begin{gather*}
\int_{j} \frac{A}{C+D s_{j}+s_{j}^{2}} d s_{j}=\frac{2 A}{\sqrt{4 C-D^{2}}} \tan ^{-1}\left(\frac{D+2 s_{j}}{\sqrt{4 C-D^{2}}}\right)  \tag{4-52}\\
\int_{j} \frac{B s_{j}}{C+D s_{j}+s_{j}^{2}} d s_{j}=B\left[\frac{\ln \left(C+D s_{j}+s_{j}^{2}\right)}{2}-\frac{D}{\sqrt{4 C-D^{2}}} \tan ^{-1}\left(\frac{D+2 s_{j}}{\sqrt{4 C-D^{2}}}\right)\right] \tag{4-53}
\end{gather*}
$$

Adding the two equations leads to

$$
\begin{equation*}
I_{i j}=\int_{j} \frac{\partial}{\partial n_{i}}\left(\ln r_{i j}\right) d s_{j}=\frac{2 A-B D}{\sqrt{4 C-D^{2}}} \tan ^{-1}\left(\frac{D+2 s_{j}}{\sqrt{4 C-D^{2}}}\right)+\frac{B}{2} \ln \left(C+D s_{j}+s_{j}^{2}\right) \tag{4-54}
\end{equation*}
$$

This equation allows Equation (4-45) to be written as $N$ equations

$$
\begin{equation*}
\frac{\lambda_{i}}{2}+\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\lambda_{j}}{2 \pi} I_{i j}=-\mathbf{V}_{\infty} \cdot \mathbf{n}_{i}=-V_{\infty} \cos \beta_{i} \tag{4-55}
\end{equation*}
$$

and solved for the $N$ unknown source sheet strengths $\lambda_{i}$ for any arbitrary 2-dimensional shape. It is important to remember that the shape will not be a lifting body. For such shapes, we would have to include a vortex sheet. Lifting 2-dimensional shapes are not utilized in this work and, thus, are omitted from further discussion. Similar approach is used to obtain the tangential components of the flow along the panels. The tangential component of the far-field velocity is

$$
\begin{equation*}
\mathbf{V}_{\infty} \cdot \mathbf{s}_{i}=V_{\infty} \sin \beta_{i} \tag{4-56}
\end{equation*}
$$

Where $\mathbf{s}_{i}$ is the tangent to the panel and $\beta_{i}$, again, is the angle between the far-field velocity and the normal (see Figure 4-4). The tangential velocity component is thus

$$
\begin{equation*}
V_{s_{i}}=V_{\infty} \sin \beta_{i}+\sum_{j=1}^{n} \frac{\lambda_{j}}{2 \pi} \int_{j} \frac{\partial}{\partial s}\left(\ln r_{i j}\right) d s_{j} \tag{4-57}
\end{equation*}
$$

This component can be used to estimate the pressure coefficient at the panel surface using the expression below

$$
\begin{equation*}
C_{p, i}=1-\left(\frac{V_{s_{i}}}{V_{\infty}}\right)^{2} \tag{4-58}
\end{equation*}
$$

Also note that the sum of the product of the panel source strengths, $\lambda_{i}$, and the corresponding panel length, $s_{i}$, must add up to zero. This tool can be used for debugging purposes in code development:

$$
\begin{equation*}
\sum_{i=1}^{n} \lambda_{i} s_{i}=0 \tag{4-59}
\end{equation*}
$$

### 4.2 Three-Dimensional Potential Flow Theory

The previous section presented the derivation of the potential flow theory and an application of it to 2dimensional source sheets in space. In this section, we will try to expand the theory to include objects in 3-dimensional space. The purpose is to use the theory to model wind flowing over arbitrary terrain.

### 4.2.1. Adapting the PFT to 3-Dimensional Flow

Consider the volumetric form of Equation (4-6) (in the $x-y$ plane), which can be written as follows

$$
\begin{equation*}
\nabla \cdot \mathbf{V}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{4-60}
\end{equation*}
$$

The 3-dimensional velocity potential, denoted by $\phi$, is given by Equation (4-9), but allows the three components of the velocity to be extracted

$$
\begin{equation*}
u=\frac{\partial \phi}{\partial x}, \quad v=\frac{\partial \phi}{\partial y}, \quad w=\frac{\partial \phi}{\partial z} \tag{4-61}
\end{equation*}
$$

Vorticity in 3-dimensional space is far more complicated than in 2-dimensional. Let the velocity in Cartesian coordinate system is given by $\mathbf{V}=u \mathbf{i}+v \mathbf{j}+w \mathbf{k}$. Then, the curl of the vector field $\mathbf{V}$ is given by:

$$
\zeta=\nabla \times \mathbf{V}=\left|\begin{array}{ccc}
\mathbf{i} & \mathbf{j} & \mathbf{k}  \tag{4-62}\\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
u & v & w
\end{array}\right|
$$

Irrotational flow requires all three vector components to be zero. Expressing the curl explicitly gives

$$
\begin{equation*}
\zeta=\nabla \times \mathbf{V}=\underbrace{\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)}_{=0} \mathbf{i}-\underbrace{\left(\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}\right)}_{=0} \mathbf{j}+\underbrace{\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)}_{=0} \mathbf{k}=\mathbf{0} \tag{4-63}
\end{equation*}
$$

## Laplace's Equation for Irrotational Incompressible Flow

As stated in Section 4.1.3, Governing Equation for Irrotational Incompressible Flow, the Laplace equation is presented as follows in 3-dimensional space

$$
\begin{equation*}
\nabla^{2} \phi=\frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}}+\frac{\partial^{2} \phi}{\partial z^{2}}=0 \tag{4-64}
\end{equation*}
$$

## Boundary Conditions for Wind Flowing over Terrain

Use a source panel for the flow. Boundary conditions for flow velocity of magnitude $\mathbf{V}_{\infty}$ in the far field:

$$
\begin{align*}
& u(\infty)=\frac{\partial \phi}{\partial x}=V_{\infty} \cos \theta \\
& v(\infty)=\frac{\partial \phi}{\partial y}=V_{\infty} \sin \theta  \tag{4-65}\\
& w(\infty)=\frac{\partial \phi}{\partial z}=0
\end{align*}
$$

Where $\theta$ is the wind direction. Again flow tangency with panels (terrain) is accomplished through

$$
\begin{equation*}
\mathbf{V} \cdot \mathbf{n}=\nabla \phi \cdot \mathbf{n}=(u \mathbf{i}+v \mathbf{j}+w \mathbf{k}) \cdot\left(n_{x} \mathbf{i}+n_{y} \mathbf{j}+n_{z} \mathbf{k}\right)=0 \tag{4-66}
\end{equation*}
$$

Where $n_{x}, n_{y}, n_{z}$, are the components of the unit normal to the source panel. This is a sufficient requirement because 0 is returned if the $\mathbf{V}$ is perpendicular to $\mathbf{n}$; which means $\mathbf{V}$ is parallel to the surface at that point. As for the 2-dimensional flow, the combined flow due to all elementary flows must be dotted in this fashion to the panel normal.

### 4.2.2. Elementary Flows for 3-Dimensional Flows

It is now possible to present a 3-dimensional version of the three elementary flows discussed Section 4.1.6, Elementary Flows for Planar Flows, in a 3-dimensional space. Note the definitions of flow components in Figure 4-6.

## Elementary Flow 1: Uniform Flow

Using the nomenclature of Figure 4-6, we get

$$
\left.\begin{array}{l}
u=\frac{\partial \phi}{\partial x}=V_{\infty} \cos \theta  \tag{4-67}\\
v=\frac{\partial \phi}{\partial y}=V_{\infty} \sin \theta \\
w=\frac{\partial \phi}{\partial z}=V_{\infty} \sin \psi
\end{array}\right\} \Leftrightarrow \phi=V_{\infty}(x \cos \theta+y \sin \theta+z \sin \psi)
$$

## Elementary Flow 2: Source Flow

The source is identical to its 2-dimensional counterpart, except the flow is into 3-dimensional space. Given the volumetric flow rate, $\lambda$, as the mass flow rate (mass of fluid per second), denoted by $\dot{m}$, divided by the mass per unit volume (density), or

$$
\begin{equation*}
\lambda \equiv \frac{\dot{m}}{\rho} \quad\left[\frac{\mathrm{~kg} / \mathrm{s}}{\mathrm{~kg} / \mathrm{m}^{3}}=\frac{\mathrm{m}^{3}}{\mathrm{~s}}\right] \tag{4-23}
\end{equation*}
$$

Typical units of $\lambda$ are in terms of per unit volume and are given (in the SI -system) by $\mathrm{m}^{3} / \mathrm{s}$. If we assume the source ejects flow uniformly, the volumetric flow rate has to be uniform over the surface of a
fictitious sphere of radius $r$. Thus, the volumetric flow rate per unit volume is given by (recall that units are $\mathrm{m}^{3} / \mathrm{s}$ )

$$
\lambda=\frac{\dot{m}}{\rho}=4 \pi r^{2} V_{r} \quad\left[m^{2} \frac{m}{s}=\frac{m^{3}}{s}\right]
$$

Where $V_{r}$ is the radial speed of the fluid at radius $r$. It follows that the radial flow velocity, $V_{r}$, at distance $r$ from the source is:

$$
\begin{equation*}
V_{r}=\frac{\lambda}{4 \pi r^{2}} \tag{4-68}
\end{equation*}
$$



Figure 4-6: Definition of velocity components and angles for 3-dimensional flow.
To see what this means, consider volumetric flow rate of $\lambda=100 \mathrm{~m}^{3} / \mathrm{s}$ (recall the flow is ejected in a sphere around the source). Then the flow speed at $r=1 \mathrm{~m}$ amounts to $7.96 \mathrm{~m} / \mathrm{s}$ and at $r=10 \mathrm{~m}$ the speed has dropped to $0.0796 \mathrm{~m} / \mathrm{s}$. Thus, the velocity potential for the 3 -dimensional source can be found as shown below

Velocity potential:

$$
\begin{equation*}
\phi=\int \frac{\lambda}{4 \pi r^{2}} d r=\frac{\lambda}{4 \pi} \int r^{-2} d r=-\frac{\lambda}{4 \pi r} \tag{4-69}
\end{equation*}
$$

Note that the negative sign cancels the minus sign that results from differentiating the potential (required to extract the speed). Also recall that, as before, in order to evaluate the influence of a 3dimensional source located at $\left(x_{s}, y_{s}, z_{s}\right)$ at some distant point $(x, y, z)$, requires Equation (4-25) to be rewritten as follows:

$$
\begin{equation*}
r=\sqrt{\left(x-x_{s}\right)^{2}+\left(y-y_{s}\right)^{2}+\left(z-z_{s}\right)^{2}} \Rightarrow \phi(x, y, z)=-\frac{\lambda}{4 \pi \sqrt{\left(x-x_{s}\right)^{2}+\left(y-y_{s}\right)^{2}+\left(z-z_{s}\right)^{2}}} \tag{4-70}
\end{equation*}
$$

## Elementary Flow 3: Sink Flow

A sink is the "inverse" of a source and the formulation is identical, except the volumetric flow rate is a negative value. Therefore, Equations (4-69) and (4-70) also apply to the sink, except that $\lambda<0$.

### 4.2.3. Non-Lifting 3-Dimensional Flow about Arbitrary Bodies using Point Sources/Sinks

In this section we will create a numerical scheme to estimate 3-dimensional flow over an arbitrary nonlifting body. PFT offers several options for this purpose. For instance, it is possible to represent a body using a collection of point-sources and -sinks and assign boundary conditions at an equal amount of locations, i.e. the collocation points. It is also possible to represent the body using a collection of source and sink panels with an equal amount of collocation points. The difference is that the flow at a point, closer to the surface than about $3-5 \mathrm{X}$ the panel dimensions is better represented using the second approach (e.g. see Katz and Plotkin [4]). The first approach is mathematically less involved, but less accurate near the surface. A numerical scheme for it follows.

Consider the quadrilateral panel in Figure 4-7, for which each vertex has the coordinates shown and no restriction on shape other than the smaller the panel the better, assuming this better approximates the desired geometry without generating too great a truncation error.

## Panel Geometry

Let's first take care of defining the geometric entities associated with the panel. These are the position of the centroid, collocation point, and panel area. The centroid is simply the average of the vertices. Since this is where the source will be located, use the bars to denote the centroid.

$$
\begin{aligned}
& \bar{x}=\frac{1}{4}\left(x_{1}+x_{2}+x_{3}+x_{4}\right) \\
& \bar{y}=\frac{1}{4}\left(y_{1}+y_{2}+y_{3}+y_{4}\right) \\
& \bar{z}=\frac{1}{4}\left(z_{1}+z_{2}+z_{3}+z_{4}\right)
\end{aligned}
$$



Figure 4-7: Definition of a 3-dimensional source panel.

The collocation point is located by centering it with respect to the $x$-axis and if we choose to place it at 0.75 of the length of the panel along the $y$-and $z$-axes, this could be represented as shown below

$$
\begin{align*}
& \hat{x}=x_{s} \\
& \hat{y}=y_{1}+\frac{3}{4}\left(y_{3}-y_{1}\right)=\frac{1}{4} y_{1}+\frac{3}{4} y_{3}  \tag{4-72}\\
& \hat{z}=z_{1}+\frac{3}{4}\left(z_{3}-z_{1}\right)=\frac{1}{4} z_{1}+\frac{3}{4} z_{3}
\end{align*}
$$

The area of the panel can be calculated by breaking it into two triangles and resort to vector algebra to calculate their areas using the cross-product relation. Thus, if $\mathbf{A}$ and $\mathbf{B}$ are two vectors the area of the parallelogram they form is given by

$$
\begin{equation*}
A=|\mathbf{A} \times \mathbf{B}|=|\mathbf{A}||\mathbf{B}| \sin \theta \tag{4-73}
\end{equation*}
$$

Where $\theta$ is the angle between the vectors. When applying this method to the two triangles that constitute the panel, it is necessary to implement it similar to the following (note the orientation of the vertices in Figure 4-7):

$$
A=\frac{1}{2}\left|\begin{array}{ccc}
\mathbf{i} & \mathbf{j} & \mathbf{k}  \tag{4-74}\\
x_{2}-x_{1} & y_{2}-y_{1} & z_{2}-z_{1} \\
x_{3}-x_{1} & y_{3}-y_{1} & z_{3}-z_{1}
\end{array}\right|+\frac{1}{2}\left|\begin{array}{ccc}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
x_{3}-x_{4} & y_{3}-y_{4} & z_{3}-z_{4} \\
x_{2}-x_{4} & y_{2}-y_{4} & z_{2}-z_{4}
\end{array}\right|
$$

## Aerodynamic Coefficients for Panels

Since we are dealing with 3-dimensional point sources and sinks, we do not have to distribute the source strength over the panel area. Rather we create panels that have a source (or sink) at the centroid point per Equation (4-71) and a collocation point per Equation (4-72), as illustrated in Figure 4-8. Thus, the flow speed induced at the collocation point on the $i$-th panel by the source at the centroid of the $j-$ th panel is given by

$$
\begin{equation*}
V_{r_{i j}}=\frac{\lambda_{j}}{4 \pi r_{i j}^{2}}=\frac{\lambda_{j}}{4 \pi\left[\left(\hat{x}_{i}-\bar{x}_{j}\right)^{2}+\left(\hat{y}_{i}-\bar{y}_{j}\right)^{2}+\left(\hat{z}_{i}-\bar{z}_{j}\right)^{2}\right]} \tag{4-75}
\end{equation*}
$$

The velocity induced at the collocation point by the $j$-th panel consists of this magnitude times the components of the unit vector aligned to the vector $\mathbf{r}_{i j}$, i.e.

$$
\begin{equation*}
\mathbf{V}_{r_{i j}}=V_{r_{i j}} \frac{\mathbf{r}_{i j}}{\left|\mathbf{r}_{i j}\right|}=V_{r_{i j}} \frac{\left(\hat{x}_{i}-\bar{x}_{j}\right) \mathbf{i}+\left(\hat{y}_{i}-\bar{y}_{j}\right) \mathbf{j}+\left(\hat{z}_{i}-\bar{z}_{j}\right) \mathbf{k}}{\sqrt{\left(\hat{x}_{i}-\bar{x}_{j}\right)^{2}+\left(\hat{y}_{i}-\bar{y}_{j}\right)^{2}+\left(\hat{z}_{i}-\bar{z}_{j}\right)^{2}}} \tag{4-76}
\end{equation*}
$$

Substitute Equation (4-75) to get

$$
\begin{equation*}
\mathbf{V}_{r_{i j}}=\frac{\lambda_{j}}{4 \pi r_{i j}^{3}}\left[\left(\hat{x}_{i}-\bar{x}_{j}\right) \mathbf{i}+\left(\hat{y}_{i}-\bar{y}_{j}\right) \mathbf{j}+\left(\hat{z}_{i}-\bar{z}_{j}\right) \mathbf{k}\right] \tag{4-77}
\end{equation*}
$$



Figure 4-8: The $\boldsymbol{j}$-th panel induces a velocity at the collocation point of the $\boldsymbol{i}$-th panel.
Again, $r_{i j}=\sqrt{\left(\hat{x}_{i}-\bar{x}_{j}\right)^{2}+\left(\hat{y}_{i}-\bar{y}_{j}\right)^{2}+\left(\hat{z}_{i}-\bar{z}_{j}\right)^{2}}$. The complete velocity at the collocation point of the $i-$ th panel can now be determined by adding all the contributions of the elementary flows to the uniform velocity vector (i.e. the far-field velocity)

$$
\begin{equation*}
\mathbf{V}_{i}=\mathbf{V}_{\infty}+\sum_{j=1}^{N} \mathbf{V}_{r_{i j}} \tag{4-78}
\end{equation*}
$$

In order for the flow to follow the dividing stream-sheet that envelopes the surface of the body, the velocity at the collocation point of all panels representing it must satisfy the tangential requirement, implemented in accordance with Equation (4-19)

$$
\begin{equation*}
\mathbf{V}_{i} \cdot \mathbf{n}_{i}=\left(\mathbf{V}_{\infty}+\sum_{j=1}^{N} \mathbf{V}_{r_{i j}}\right) \cdot \mathbf{n}_{i}=\mathbf{V}_{\infty} \cdot \mathbf{n}_{i}+\sum_{j=1}^{N} \mathbf{V}_{r_{i j}} \cdot \mathbf{n}_{i}=0 \tag{4-79}
\end{equation*}
$$

Thus, we end up with the following relationship for each collocation point

$$
\begin{equation*}
\sum_{j=1}^{N} \mathbf{V}_{r_{i j}} \cdot \mathbf{n}_{i}=-\mathbf{V}_{\infty} \cdot \mathbf{n}_{i} \tag{4-80}
\end{equation*}
$$

This equation has $N$ unknowns, namely the source strengths, $\lambda_{i} i=1,2, \ldots, N$. By writing the expression for the $N$ collocation points we have our $N$ equations with $N$ unknowns, which should allow the entire 3dimensional flow field to be determined using matrix methods. By substituting Equations (4-75) and (476 ) into Equation (4-80), we get $N$ linear equations, each which conforms to

$$
\begin{aligned}
& \sum_{j=1}^{N} \mathbf{V}_{r_{i j}} \cdot \mathbf{n}_{i}=\frac{\lambda_{1}}{4 \pi r_{11}^{3}}\left[n_{x_{1}}\left(\hat{x}_{1}-\bar{x}_{1}\right)+n_{y_{1}}\left(\hat{y}_{1}-\bar{y}_{1}\right)+n_{z_{1}}\left(\hat{z}_{1}-\bar{z}_{1}\right)\right] \\
& +\frac{\lambda_{2}}{4 \pi r_{12}^{3}}\left[n_{x_{1}}\left(\hat{x}_{1}-\bar{x}_{2}\right)+n_{y_{1}}\left(\hat{y}_{1}-\bar{y}_{2}\right)+n_{z_{1}}\left(\hat{z}_{1}-\bar{z}_{2}\right)\right]+\ldots \\
& \ldots+\frac{\lambda_{N}}{4 \pi r_{1 N}^{3}}\left[n_{x_{1}}\left(\hat{x}_{1}-\bar{x}_{N}\right)+n_{y_{1}}\left(\hat{y}_{1}-\bar{y}_{N}\right)+n_{z_{1}}\left(\hat{z}_{1}-\bar{z}_{N}\right)\right]
\end{aligned}
$$

Thus, the set of $N$ equations will be of the form

$$
\begin{align*}
& A_{11} \lambda_{1}+A_{12} \lambda_{2}+\ldots+A_{1 N} \lambda_{N}=-\mathbf{V}_{\infty} \cdot \mathbf{n}_{1} \\
& A_{21} \lambda_{1}+A_{22} \lambda_{2}+\ldots+A_{2 N} \lambda_{N}=-\mathbf{V}_{\infty} \cdot \mathbf{n}_{2}  \tag{4-81}\\
& \vdots \\
& A_{N 1} \lambda_{1}+A_{N 2} \lambda_{2}+\ldots+A_{N N} \lambda_{N}=-\mathbf{V}_{\infty} \cdot \mathbf{n}_{N}
\end{align*}
$$

Where each coefficient is calculated from

$$
\begin{equation*}
A_{i j}=\frac{1}{4 \pi r_{i j}^{3}}\left[n_{x_{i}}\left(\hat{x}_{i}-\bar{x}_{j}\right)+n_{y_{i}}\left(\hat{y}_{i}-\bar{y}_{j}\right)+n_{z_{i}}\left(\hat{z}_{i}-\bar{z}_{j}\right)\right] \tag{4-82}
\end{equation*}
$$

Once the source strengths, $\lambda_{i} i=1,2, \ldots, N$, have been determined, it is possible to calculate the velocity elsewhere in the computational volume as follows

$$
\begin{equation*}
\mathbf{V}(x, y, z)=\mathbf{V}_{\infty}+\sum_{j=1}^{N} V_{r_{j}}=\mathbf{V}_{\infty}+\sum_{j=1}^{N} \frac{\lambda_{j}}{4 \pi r_{j}^{3}}\left[\left(x-\bar{x}_{j}\right) \mathbf{i}+\left(y-\bar{y}_{j}\right) \mathbf{j}+\left(z-\bar{z}_{j}\right) \mathbf{k}\right] \tag{4-83}
\end{equation*}
$$

Where $r_{j}=\sqrt{\left(x-\bar{x}_{j}\right)^{2}+\left(y-\bar{y}_{j}\right)^{2}+\left(z-\bar{z}_{j}\right)^{2}}$. For instance, the velocity at each collocation point is given by

$$
\begin{equation*}
\mathbf{V}\left(\hat{x}_{i}, \hat{y}_{i}, \hat{z}_{i}\right)=\mathbf{V}_{\infty}+\sum_{j=1}^{N} \frac{\lambda_{j}}{4 \pi r_{i j}^{3}}\left[\left(\hat{x}_{i}-\bar{x}_{j}\right) \mathbf{i}+\left(\hat{y}_{i}-\bar{y}_{j}\right) \mathbf{j}+\left(\hat{z}_{i}-\bar{z}_{j}\right) \mathbf{k}\right] \tag{4-84}
\end{equation*}
$$

## Illustrative Example 1

Let's evaluate this approach using the simplified geometry shown in Figure 4-9 and for which each cube is 1-by-1-by-1 units and the far-field velocity is given as $\mathbf{V}_{\infty}=\mathbf{j}$. A source of strength $\lambda$ is placed at the origin. Determine the strength required to force the velocity $\mathbf{V}_{1}$ to be tangent on the panel whose normal is denoted by $\mathbf{n}_{1}$. It should be evident from Figure 4-9 that $\mathbf{V}_{1}$ should have a unit vector whose j component will be positive and $\mathbf{k}$-component will be negative. We start by specifying the centroid,
collocation point, and normal. These are given as ( $0.5,2.5,0.5$ ), ( $0.5,2.75,0.25$ ), and ( $0,0.7071,0.7071$ ), respectively. Then, we tabulate the aerodynamic coefficients per Equation (4-82).

$$
\begin{aligned}
& r=\sqrt{\left(\hat{x}_{1}-\bar{x}\right)^{2}+\left(\hat{y}_{1}-\bar{y}\right)^{2}+\left(\hat{z}_{1}-\bar{z}\right)^{2}}=\sqrt{(0.5-0)^{2}+(2.75-0)^{2}+(0.25-0)^{2}}=\sqrt{\frac{63}{8}} \\
& A_{11}=\frac{1}{4 \pi r^{3}}\left[n_{x_{1}}\left(\hat{x}_{1}-\bar{x}\right)+n_{y_{1}}\left(\hat{y}_{1}-\bar{y}\right)+n_{z_{1}}\left(\hat{z}_{1}-\bar{z}\right)\right]=\frac{1}{4 \pi\left(\sqrt{\frac{63}{8}}\right)^{3}}\left[0+\frac{2.75}{\sqrt{2}}+\frac{0.25}{\sqrt{2}}\right]=\frac{12}{\pi(\sqrt{63})^{3}}
\end{aligned}
$$

The boundary condition on Panel 1 is expressed by Equation (4-19) yields

$$
-\mathbf{V}_{\infty} \cdot \mathbf{n}_{1}=-V_{\infty}(0 \mathbf{i}+1 \mathbf{j}+0 \mathbf{k}) \cdot\left(0 \mathbf{i}+\frac{1}{\sqrt{2}} \mathbf{j}+\frac{1}{\sqrt{2}} \mathbf{k}\right)=-\frac{1}{\sqrt{2}} V_{\infty}
$$

Now, we can write

$$
\begin{aligned}
& \left\lfloor A_{i j} \mid\left\{\lambda_{i}\right\}=-\mathbf{V}_{\infty} \cdot \mathbf{n}_{1}\right. \\
& \quad \Leftrightarrow \overbrace{\frac{12}{\pi(\sqrt{63})^{3}} \cdot \lambda=-\frac{1}{\sqrt{2}} V_{\infty}}^{=A_{11}} \\
& \quad \Leftrightarrow \lambda=-\frac{(\sqrt{63})^{3}}{12 \sqrt{2}} \pi V_{\infty} \approx-92.57 V_{\infty}
\end{aligned}
$$



Figure 4-9: Sample geometry used to demonstrate the 3-dimensional source body.

Let's calculate the velocity at the collocation points to evaluate if flow tangency there is indeed successful. To do this, we use Equation (4-84). Thus, we find

$$
\begin{aligned}
& \mathbf{V}\left(\hat{x}_{1}, \hat{y}_{1}, \hat{z}_{1}\right)=\mathbf{V}_{\infty}+\frac{\lambda}{4 \pi r^{3}}\left[\left(\hat{x}_{1}-\bar{x}\right) \mathbf{i}+\left(\hat{y}_{1}-\bar{y}\right) \mathbf{j}+\left(\hat{z}_{1}-\bar{z}\right) \mathbf{k}\right] \\
& \Rightarrow \mathbf{V}(0.5,2.75,0.25)=\mathbf{j}+\frac{\left(-(\sqrt{63})^{3} /(12 \sqrt{2}) \pi V_{\infty}\right)}{4 \pi(\sqrt{63 / 8})^{3}}\left[\frac{1}{2} \mathbf{i}+\frac{11}{4} \mathbf{j}+\frac{1}{4} \mathbf{k}\right]=\left[-\frac{2}{12} \mathbf{i}+\frac{1}{12} \mathbf{j}-\frac{1}{12} \mathbf{k}\right] V_{\infty}
\end{aligned}
$$

Let's check to see if this velocity is parallel to the panel's plane. If so, the dot product of it and the panel normal must equal the null vector, $\mathbf{0}$.

$$
\mathbf{V}\left(\hat{x}_{1}, \hat{y}_{1}, \hat{z}_{1}\right) \cdot \mathbf{n}_{1}=\left[-\frac{2}{12} \mathbf{i}+\frac{1}{12} \mathbf{j}-\frac{1}{12} \mathbf{k}\right] V_{\infty} \cdot\left(0 \mathbf{i}+\frac{1}{\sqrt{2}} \mathbf{j}+\frac{1}{\sqrt{2}} \mathbf{k}\right)=\mathbf{0}
$$

It is important to recognize that the method only ensures the tangential velocity is, well, tangent to the panel. It will not result in a velocity that is necessarily in the same direction as the wind velocity. This will become more apparent in the next illustrative example.

## Illustrative Example 2

Let's evaluate the approach using the simplified geometry shown in Figure 4-10 and for which each square is 1-by-1 units and the far-field velocity is given as $\mathbf{V}_{\infty}=\mathbf{j}$. We start with tabulating the centroids, collocation points, and normals (see Table 4-1). Then, we tabulate the aerodynamic coefficients per Equation (4-82). For instance, if $i=1$ and $j=2$, we get

Table 4-1: Geometric parameters

|  | Centroid |  |  | Collocation |  |  | Normal |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel | $\boldsymbol{x}_{\boldsymbol{i}}$ | $\boldsymbol{y}_{\boldsymbol{i}}$ | $\boldsymbol{z}_{\boldsymbol{i}}$ | $\boldsymbol{x}_{\boldsymbol{i}}$ | $\boldsymbol{y}_{\boldsymbol{i}}$ | $\boldsymbol{z}_{\boldsymbol{i}}$ | $\boldsymbol{n}_{\boldsymbol{x}}$ | $\boldsymbol{n}_{\boldsymbol{y}}$ | $\boldsymbol{n}_{\boldsymbol{z}}$ |
| 1 | 0.5 | 0.5 | 0.5 | 0.5 | 0.75 | 0.75 | 0 | -0.7071 | 0.70711 |
| 2 | 0.5 | 1.5 | 1 | 0.5 | 1.75 | 1 | 0 | 0 | 1 |
| 3 | 0.5 | 2.5 | 0.5 | 0.5 | 2.75 | 0.25 | 0 | 0.70711 | 0.70711 |

$$
\begin{aligned}
r_{12} & =\sqrt{\left(\hat{x}_{1}-\bar{x}_{2}\right)^{2}+\left(\hat{y}_{1}-\bar{y}_{2}\right)^{2}+\left(\hat{z}_{1}-\bar{z}_{2}\right)^{2}}=\sqrt{(0.5-0.5)^{2}+(0.75-1.5)^{2}+(0.75-1)^{2}}=\frac{\sqrt{10}}{4} \\
A_{12} & =\frac{1}{4 \pi r_{12}^{3}}\left[n_{x_{1}}\left(\hat{x}_{1}-\bar{x}_{2}\right)+n_{y_{1}}\left(\hat{y}_{1}-\bar{y}_{2}\right)+n_{z_{1}}\left(\hat{z}_{1}-\bar{z}_{2}\right)\right] \\
& =\frac{1}{4 \pi(\sqrt{10} / 4)^{3}}\left[0-\frac{1}{\sqrt{2}}(0.75-1.5)+\frac{1}{\sqrt{2}}(0.75-1)\right]=\frac{2}{\pi(\sqrt{5})^{3}}
\end{aligned}
$$

Similar treatment of the boundary condition on Panel 1 and expressed by Equation (4-19) yields

$$
-\mathbf{V}_{\infty} \cdot \mathbf{n}_{1}=-V_{\infty}(0 \mathbf{i}+1 \mathbf{j}+0 \mathbf{k}) \cdot\left(0 \mathbf{i}-\frac{1}{\sqrt{2}} \mathbf{j}+\frac{1}{\sqrt{2}} \mathbf{k}\right)=\frac{V_{\infty}}{\sqrt{2}}
$$

Expressing all coefficients in a matrix form leads to

$$
\left[A_{i j}\right]\left\{\lambda_{i}\right\}=-\mathbf{V}_{\infty} \cdot \mathbf{n}_{i} \Leftrightarrow\left[\begin{array}{ccc}
0 & 0.05694 & 0.02037 \\
0.01631 & 0 & 0.05433 \\
0.009700 & 0.009083 & 0
\end{array}\right]\left\{\begin{array}{l}
\lambda_{1} \\
\lambda_{2} \\
\lambda_{3}
\end{array}\right\}=\left\{\begin{array}{c}
0.7071 \\
0 \\
-0.7071
\end{array}\right\}
$$



Figure 4-10: Sample geometry used to demonstrate the 3-dimensional source body.
From which we find (using standard matrix solution methods) that the panel strengths are

$$
\left\{\begin{array}{l}
\lambda_{1} \\
\lambda_{2} \\
\lambda_{3}
\end{array}\right\}=\left\{\begin{array}{c}
-76.80 \\
4.171 \\
23.05
\end{array}\right\}
$$

Let's calculate the velocity at the collocation points to evaluate if flow tangency there is successful. To do this, we use Equation (4-84). Thus, we find

$$
\begin{aligned}
\mathbf{V}(0.5,0.75,0.75) & =\mathbf{j}+\frac{1}{4 \pi}\left[\frac{-76.80}{(0.3536)^{3}}[0.25 \mathbf{j}+0.25 \mathbf{k}]+\frac{4.171}{(0.7906)^{3}}[-0.75 \mathbf{j}-0.25 \mathbf{k}]+\frac{23.05}{(1.768)^{3}}[-1.75 \mathbf{j}+0.25 \mathbf{k}]\right] \\
& =-34.66 \mathbf{j}+34.66 \mathbf{k}
\end{aligned}
$$

Results of such modelling are presented in Chapter 5, Atmospheric Modelling.

## REFERENCES

${ }^{1}$ Anderson, John D., Jr., Fundamentals of Aerodynamics, $4{ }^{\text {th }}$ Ed., McGraw-Hill, 2007.
${ }^{2}$ Granger, Robert A., Fluid Mechanics, $1{ }^{\text {st }}$ Ed., Dover Publications, Inc., 1985.
${ }^{3}$ Houghton, E.L., Carpenter, P.W., Collicott, Steven H., and Valentine, Daniel T., Aerodynamics for Engineering Students, $6{ }^{\text {th }}$ Ed., Butterworth-Heinemann an imprint of Elsevier, 2013.
${ }^{4}$ Katz, Joseph, and Plotkin, Allen, Low-Speed Aerodynamics, $2{ }^{\text {nd }}$ Ed., Cambridge University Press, 2001.

## 5. Atmospheric Modeling

It is vital to evaluate the GICA in a realistic flight simulation environment. This, in turn, requires a realistic estimation of the fundamental properties of the atmosphere (pressure, density, and temperature) and wind (horizontal and vertical, steady and unsteady). It is also essential in the operation of the algorithm itself, where it is used to evaluate energy recovery of a particular flight trajectory using performance theory. Realistic estimation also allows the effect of altitude and deviations from standard pressure and temperature to be considered. The determination of atmospheric properties is limited to macroscopic averages. Microscopic deviations are ignored on the basis that some will benefit performance while others will not. As an example, consider the macroscopic average temperature in the "world" in which a simulation takes place to be, say, $15^{\circ} \mathrm{C}$ at sea-level. However, there may be microscopic (or localized) variations from this on a mountain slope, near sealevel, facing the sun, where temperature might be $18^{\circ} \mathrm{C}$. Such microscopic deviations are ignored in the simulation work and this is justified because these deviations are not large enough to have large influence on the path selection.

This chapter presents several of the many mathematical tools required to evaluate the characteristics of the atmosphere. Most of these tools are used directly by the GICA for prediction purposes, while others are used in the flight simulation section. These are the topic discussed in this chapter

- Theory of atmospheric properties
- Modeling of the atmospheric (or planetary) boundary layer
- Modeling of horizontal convection, steady and unsteady
- Modeling of vertical convection
- Modeling of thermal regions


### 5.1 Theory of Atmospheric Properties

Methods to estimate the average properties of the atmosphere as functions of altitude have been developed by a number of organizations [1]. One of the best known of these is the U.S. Standard Atmosphere 1976 [2], developed by the National Oceanic and Atmospheric Administration (NOAA). Although not the most sophisticated among such methods (in terms of number of inputs), it lends itself well to the rapid coding required by the GICA. The discussion in here is limited to the U.S. Standard Atmosphere 1976 and only mathematical expressions pertaining to it are presented. These allow the atmospheric properties to be calculated to 85 km ( 278000 ft ).

This discussion would not be complete without mentioning briefly one of the "sophisticated" atmospheric models, the NRLMSISE-00 (Naval Research Laboratory Mass Spectrometer and Incoherent Scatter, where E means from surface of the Earth to the Exosphere). This model requires input data in the form of year, day, time of day, altitude, geodetic latitude and longitude, and many others. It returns information such as temperature, mass density, and molecular densities of Oxygen $\left(\mathrm{O}_{2}\right)$, Nitrogen $\left(\mathrm{N}_{2}\right)$,
mono-atomic Oxygen (O) and Nitrogen (N), Argon (Ar), and Hydrogen (H). These are used to estimate other properties, such as Specific Gas Constant, $R$, pressure, and the ratio of specific heats (typically denoted by $\gamma$ ). Among numerous applications, this model is used to predict the orbital decay of satellites due to atmospheric drag and to study the effect of atmospheric gravity waves [1]. While this capability is not needed in the execution of the GICA, it was considered but deemed not required. Thus, in this dissertation, all atmospheric data is based on the US Standard Atmosphere 1976, unless otherwise specified. An interesting comparison of the temperature rendered by the U.S. Standard Atmosphere 1976 and the NRLMSISE-00 models is shown in Figure 5-1.

### 5.1.1. Classification of Atmospheric Layers

The atmosphere is divided into several layers based on some specific characteristics (see Table 5-1). The Troposphere extends from the ground to some 11-16 km (6.8-10 mi). It is where most clouds form and winds and precipitation is most active. The next layer is a thin region called the Tropopause, where the Troposphere transitions to the third layer, the Stratosphere. Air temperature increases in this layer. The bulk of the atmosphere is found within these three lowest layers. Above the Stratosphere is the Mesosphere, where temperature begins to decrease again with altitude. Layers above the Stratosphere are rarely used for aerodynamic flight. The lonosphere follows the Mesosphere. It is also recognized as the Thermosphere, because of the relatively higher temperatures that prevail. The final layer is the Exosphere, which extends to some 9600 km (about 6000 mi ) and is the outer limit of the atmosphere. It should be emphasized that the GICA algorithm is primarily intended for flight in the Troposphere, at altitudes below $11 \mathrm{~km}(36089 \mathrm{ft})$. Note that the derivation of most of the following expressions is provided in Reference [1].

Table 5-1: Layer Classification of the Atmosphere.

| Name of Layer | Altitude in km | Altitude in statute miles |
| :--- | :---: | :---: |
| Troposphere ${ }^{1}$ | $0-11 \mathrm{~km}$ | $0-6.8 \mathrm{sm}$ |
| Tropopause | $11-11.5 \mathrm{~km}$ | $6.8-7.1 \mathrm{sm}$ |
| Stratosphere | $11.5-46 \mathrm{~km}$ | $11.5-29 \mathrm{sm}$ |
| Stratopause | $46-51 \mathrm{~km}$ | $29-32 \mathrm{sm}$ |
| Mesosphere | $51-85 \mathrm{~km}$ | $32-53 \mathrm{sm}$ |
| lonosphere (Thermosphere) | $85-640 \mathrm{~km}$ | $53-400 \mathrm{sm}$ |
| Exosphere | $640-9600 \mathrm{~km}$ | $400-6000 \mathrm{sm}$ |

[^10]

Figure 5-1: A comparison of temperature changes with altitude up to 85 km , using the US Standard Atmosphere 1976 and NRLMSISE-00 atmospheric models. The former represents standard conditions, whereas the latter is at a geodesic location $\mathbf{N} 45^{\circ}$ W80 on January $1^{\text {st }}$, 2012. (from Ref. [1])

### 5.1.2. Atmospheric Ambient Temperature

The change in air temperature, $T$, with altitude can be approximated using a linear function:

$$
\begin{equation*}
T=T_{0}+a\left(h-h_{0}\right) \tag{5-1}
\end{equation*}
$$

An alternative form of Equation (5-1) is:

$$
\begin{equation*}
T=T_{0}(1+\kappa \cdot h) \tag{5-2}
\end{equation*}
$$

Where:

$$
\begin{aligned}
& a=\text { Lapse rate } \\
& h=\text { Altitude in } \mathrm{ft} \text { or } \mathrm{m} \\
& h_{0}=\text { Reference altitude } h_{0}
\end{aligned}
$$

$T=$ Temperature at altitude $h$
$T_{0}=$ Temperature at reference altitude $h_{0}$ $\kappa=$ lapse rate constant $=a / T_{0}$

A shortcoming of this model is that it does not account for temperature inversion, which is a phenomenon that results from the ground being colder than the air above it. This causes the temperature to increase with altitude up to a specific elevation. Such non-standard effects are ignored in this investigation.

### 5.1.3. Atmospheric Pressure and Density for Altitudes below 36089 ft ( 11000 m )

The atmospheric pressure, $p$, and density, $\rho$, are determined as functions of altitude, $h$, assuming hydrostatic equilibrium and that the ideal gas law holds. Hydrostatic equilibrium requires $d p / d h=-\rho g$ and the ideal gas law is given by $p=\rho g R_{0} T=\rho R T$. Therefore, we get:

$$
\left.\begin{array}{rl}
d p & =-\rho g d h  \tag{5-3}\\
p & =\rho g R_{0} T
\end{array}\right\} \Leftrightarrow \frac{d p}{p}=\frac{-\rho \cdot g \cdot d h}{\rho \cdot g \cdot R_{0} T}=-\frac{d h}{R_{0} T}
$$

Then, differentiate Equation (5-1) with respect to the altitude $h$ and substitute for $d h$ in Equation (5-3)

$$
\begin{equation*}
\frac{d p}{p}=-\frac{d h}{R_{0} T}=-\frac{d T / a}{R_{0} T}=-\frac{1}{a R_{0}} \frac{d T}{T} \tag{5-4}
\end{equation*}
$$

Then, integrate with respect to $T$ to get

$$
\int \frac{d p}{p}=-\frac{1}{a R_{0}} \int \frac{d T}{T} \Leftrightarrow[\ln p]_{p_{o}}^{p}=-\frac{1}{a R_{0}}[\ln T]_{T_{o}}^{T}
$$

Finally, after some algebraic manipulations, we substitute Equation (5-1) for $T$, yielding

$$
\begin{equation*}
\ln \left(\frac{p}{p_{0}}\right)=\ln \left(\frac{T}{T_{0}}\right)^{-\frac{1}{a R_{o}}} \Leftrightarrow \frac{p}{p_{0}}=\left(\frac{T}{T_{0}}\right)^{-\frac{1}{a R_{o}}} \tag{5-5}
\end{equation*}
$$

For formulation in the Troposphere, substitute standard day coefficients, i.e.:

$$
\begin{aligned}
& a=\text { Lapse rate }=-0.0065 \mathrm{~K} / \mathrm{m}=-0.00356616^{\circ} \mathrm{F} / \mathrm{ft} \\
& h=\text { Altitude in } \mathrm{m} \text { or } \mathrm{ft} \\
& h_{0}=0 \mathrm{~m} \mathrm{or} \mathrm{ft} \\
& T_{0}=15+273.15=288.15 \mathrm{~K}=59+459.67=518.67^{\circ} \mathrm{R} \\
& R_{0}=29.26 \mathrm{~m} / \mathrm{K}=53.35 \mathrm{ft} /{ }^{\circ} \mathrm{R}
\end{aligned}
$$

Therefore:

$$
-\frac{1}{a R_{0}}=-\frac{1}{(-0.0065)(29.26)}=5.2561
$$

Thus, we can rewrite Equation (5-5) as follows:

$$
\begin{equation*}
p=p_{0}(1+\kappa \cdot h)^{5.2561} \tag{5-6}
\end{equation*}
$$

An expression for density as a function of altitude is obtained by rewriting the ideal gas law in terms of density:

$$
p=\rho g R_{0} T \Rightarrow \rho=\frac{p}{g R_{0} T}=\frac{p}{R T}
$$

Then, substitute the Equations (5-1) and (5-5), expand and simplify to get:

$$
\begin{equation*}
\rho=\rho_{0}(1+\kappa \cdot h)^{4.2561} \tag{5-7}
\end{equation*}
$$

### 5.1.4. Density of Air Deviations from a Standard Atmosphere

Atmospheric conditions often deviate from models shown above. Such deviations are treated as shown below. A derivation is presented in Reference [1].

UK-system:

$$
\begin{equation*}
\rho=\frac{352.6(1+\kappa \cdot h)^{4.2561}}{\left(T+\Delta T_{I S A}\right)} \tag{5-8}
\end{equation*}
$$

Where $h$ is the reference altitude in m or $\mathrm{ft}, T$ is the standard day temperature at the given altitude per the International Standard Atmosphere (in degrees K ). $\Delta T_{I S A}$ is the deviation from International Standard Atmosphere in ${ }^{\circ} \mathrm{C}$ or K , or ${ }^{\circ} \mathrm{F}$ or ${ }^{\circ} \mathrm{R}$. For non-standard atmosphere, use a negative sign for colder and a positive sign for warmer than ISA when determining $\Delta T_{I S A}$.

### 5.1.5. Atmospheric Property Ratios

The pressure, density, and temperature often appear in formulation as fractions of their baseline values. Consequently, they are identified using special characters and are called: pressure ratio, density ratio, and temperature ratio.

Temperature ratio:

$$
\begin{equation*}
\theta=\frac{T}{T_{0}} \tag{5-10}
\end{equation*}
$$

Pressure ratio:

$$
\begin{equation*}
\delta=\frac{p}{P_{0}}=\theta^{5.2561} \tag{5-11}
\end{equation*}
$$

Density ratio:

$$
\begin{equation*}
\sigma=\frac{\rho}{\rho_{0}}=\theta^{4.2561}=\frac{\delta}{\theta} \tag{5-12}
\end{equation*}
$$

### 5.1.6. Pressure and Density Altitudes below 36089 ft ( 11000 m )

Sometimes the pressure or density ratios are known for one reason or another. It is then possible to determine the altitudes to which they correspond. For instance, if the pressure ratio is known, we can calculate the altitude to which it corresponds. The altitude is then called Pressure Altitude. Similarly, from the density ratio we can determine the Density Altitude.

Pressure altitude in ft :

$$
\begin{align*}
& h_{P}=145442\left[1-\left(\frac{p}{p_{0}}\right)^{0.19026}\right]  \tag{5-13}\\
& h_{\rho}=145442\left[1-\left(\frac{\rho}{\rho_{0}}\right)^{0.234957}\right] \tag{5-14}
\end{align*}
$$

### 5.1.7. Speed of Sound and Mach Number

The speed of sound is retrieved from the expression below:

Speed of Sound:

$$
\begin{gather*}
a_{0}=\sqrt{\gamma R T}  \tag{5-15}\\
M=\frac{V}{a_{0}} \tag{5-16}
\end{gather*}
$$

Mach Number:

Where $R$ is the universal gas constant ( $1716 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{slug} \cdot{ }^{\circ} \mathrm{R}$ ), $\gamma$ is the ratio of specific heats $=1.4$ for air.

### 5.1.8. Atmospheric Properties from S-L to Upper Mesosphere

As stated earlier, determination of atmospheric properties from S-L to the Upper Mesosphere are detailed in the document US Standard Atmosphere 1976, published by NOAA, the US Air Force, and the National Aeronautics and Space Administration (NASA). A sample of the formulation, from S-L to altitude of 36089 ft , is presented in Table 5-2. A Visual Basic code to implement this in the SURFACE Flight Simulator is presented in Appendix A.1. It is based on the work of Reference [3] and allows atmospheric properties to be evaluated using intrinsic functions, from S-L to an altitude of 85 km ( 278000 ft ).

Table 5-2: Formulation for the US Standard Atmosphere 1976 (from Ref. [1]).

| $0 \leq h \leq 36089 \mathrm{ft}$ | $h \leq 6.8 \mathrm{mi} \quad$ Troposphere |
| :--- | :--- |
| Temperature ratio: | $\theta=(1-0.0000068756 h)=(1-h / 145442)$ |
| Pressure ratio: | $\delta=(1-0.0000068756 h)^{5.2561}=(1-h / 145442)^{5.2561}$ |
| Density ratio: | $\sigma=(1-0.0000068756 h)^{4.2561}=(1-h / 145442)^{4.2561}$ |

### 5.2 Modeling Atmospheric Convection

The complexity of atmospheric flow forces meteorologists to classify observation based on the size (or scale) of the phenomenon being studied. One of the most common such sub-division is attributed to Orlanski [4] (1975). He defined macro-scale (also called synoptic-scale or cyclonic-scale) as phenomena whose horizontal dimensions exceed 2000 km . Meso-scale refers to atmospheric flow over terrain ranging from 2 km to 2000 km in scale, while micro-scale to flow of lesser dimensions than 2 km . Orlanski further divided the meso-scale into meso- $\alpha$ (200-2000 km), meso- $\beta$ (20-200 km), and meso- $\gamma$ (2$20 \mathrm{~km})$. Using these definitions, this work primarily focuses on orographic ${ }^{2}$ phenomenon on the meso- $\gamma$ scale.

As stated in Section 2.3, Atmospheric Modeling, atmospheric convection should be modeled as realistically as possible in flight simulation software. However, high-fidelity reproduction of atmospheric flow is not easy to accomplish, due to the required computational effort. This fact drives the programmer to simplified approaches. The effect of time-averaged values of winds, steady or not, with or without up- and downdrafts, significantly affects the flight path of aircraft and directly dictates the inputs necessary to control altitude, speed, and heading (and attitude of the aircraft). A fundamental realism in simulation requires the wind vector includes three components, each which should display random, unsteady characteristics. Of course, there are limits to the detail that can be modeled; a practical balance must be struck between realism and computational intensity. This section presents methods to achieve acceptable level of realism in flight simulation, without severely taxing computer resources.

A basic illustration of the Earth's lower atmosphere, extending from the equator (left) to the pole (right), is shown in Figure 5-2. It depicts the change in the ceiling of the troposphere with latitude. The Planetary Boundary Layer (PBL), also called the Atmospheric Boundary Layer (ABL), is a sub-layer within the Troposphere and this is where most sUAVs operate. The thickness of the PBL depends on the time of day (solar heating) and intensity of the convection that is present. There are significant challenges associated with simulating flow in the troposphere, as evident from Figure 5-3. The satellite image shows how large-scale topographical features complicate flow dynamics, exemplified in the generation of Von Kárman streets, which extend 100s of km downwind of where initiated. The details of this flow are very difficult to capture (when possible) using current multi-processor computer technology, even with our most sophisticated flow solvers. To account for the complexity of atmospheric wind in flight simulation software requires the flow dynamics to be averaged; an approach that puts computational efficiency above global accuracy. However, this can be justified based on the small size of the vehicle when compared to the mass and dimensions of the flow features: The flight simulation software used to demonstrate the capability of the GICA does not have to capable of forecasting weather, but only simulate the effect of meso- $\gamma$ convection to a reasonable extent.

[^11]

Figure 5-2: The basics of winds and weather. Most sUAVs and MAVs operate inside the planetary boundary layer, although a few have busted into the Stratosphere.

### 5.2.1. Modeling the Planetary (Atmospheric) Boundary Layer

The Planetary Boundary Layer (PBL), mentioned earlier refers to the region of the troposphere closest to the ground. Stull [5] (1997) defines it as "...that part of the troposphere that is directly influenced by the presence of the earth's surface, and responds to surface forcings [sic]with a timescale of about an hour or less." It extends from the ground to perhaps 500 to 2000 m (1500-6000 ft) above the ground during the day, but overnight, the cooling of the air causes it to contract to approximately 100-300 m (300-900 ft ) [6]. The thickness largely depends on the characteristics of the terrain below; it is thick in desert regions and thin over cooler regions, such as bodies of water and moist soil. The PBL is highly tubulent in nature and dependent on the surface features over which it flows. Thus, the boundary layer associated with flow over the ocean differs from that flowing through a city or urban areas, as illustrated in Figure 5-4. As will be shown later, the boundary layer is also modified by topographical features, such as hills and escarpments.

The shape of the wind profile is important for many reasons, including the simulation of dynamic soaring. The development of such profiles are sought in boundary layer theory and the representative formulation has theoretical basis in the Navier-Stokes equations. The mathematical derivation of such formulas is beyond the scope of this discussion, but interested readers can refer to texts by Schlichting [7], Young [8], and many others, for instance, Cengel and Cimbala [9], who give a broad and clear introduction to the topic. A frequently used empirical approximation for the time averaged velocity profile of a turbulent flow over a flat plate is the $1 / 7^{\text {th }}$-power law given by the approximation [9]


Figure 5-3: A highly complex interaction of topography and fluid dynamics yields the formation of several "von Kárman streets" behind islands in the Atlantic ocean, with Madeira (top center) and the Canary island archipelago (center) off the west coast of Africa. (NASA image by Jeff Schmaltz, LANCE/EOSDIS)


Figure 5-4: Idealized wind speed profiles in the planetary boundary layer depends on surface features.

$$
\begin{equation*}
U(z)=U_{r e f}\left(\frac{z}{z_{r e f}}\right)^{1 / 7} \tag{5-1}
\end{equation*}
$$

Where $U(z)$ is the wind speed at altitude $z, U_{r e f}$ is a reference wind speed at altitude $z_{r e f}$. Other versions of the power law raise the ratio in the parenthesis to a different power to better represent various ground "textures". For instance, the following versions are commonly used to represent city, urban, and ocean profiles.

Urban:

$$
\begin{equation*}
U(z)=U_{\text {ref }}\left(\frac{z}{z_{\text {ref }}}\right)^{2 / 7} \tag{5-18}
\end{equation*}
$$

City:

$$
\begin{equation*}
U(z)=U_{r e f}\left(\frac{z}{z_{r e f}}\right)^{1 / 2} \tag{5-19}
\end{equation*}
$$

Ocean:

$$
\begin{equation*}
U(z)=U_{\text {ref }}\left(\frac{z}{z_{\text {ref }}}\right)^{1 / 7} \tag{5-17}
\end{equation*}
$$

Barnes [10] (2004) disavowed the use of such formulae because (1) the gradient $d u / d z$ is infinite at $z=0$, while the true gradient is thought to be finite, and (2) because the gradient is finite at $z=z_{\text {ref }}$, where the actual gradient vanishes. Therefore, the wind velocity calculated using the $1 / 7^{\text {th }}$-power law continues to increase with altitude. Instead, Barnes proposed the following wind profile, which he considered more representative of the ocean flow profile the Albatross has to contend with:

$$
\begin{equation*}
U(z)=U_{r e f}\left(1-e^{-\frac{a}{z_{r e f}} z}\right) \tag{5-20}
\end{equation*}
$$

Where $a$ is a constant whose value depends on surface type. Barnes uses $a=7$ to represent oceanic boundary layer. In their approach to demonstrate the practicality of dynamic soaring over open land, Sukumar and Selig [11] (2010) presented a modified formula for the wind profile that assumes the reference wind speed is taken at an altitude that corresponds to a typical height of a male RC flyer - at eye level ( 1.83 m or 6 ft ). This approach is askew with the traditional approach of Equations (5-17) and (5-20), which considers the height above the surface at which the wind speed becomes $99 \%$ of the wind speed at higher altitudes (in accordance with boundary layer theory). Their wind profile is based on the work by Stull [5] and is given by

$$
\begin{equation*}
U(z)=U_{\text {ref }} \frac{\ln \left(z / z_{0}\right)}{\ln \left(z_{\text {eye }} / z_{0}\right)} \tag{5-21}
\end{equation*}
$$

Where $U_{\text {ref }}$ is a reference wind speed at eye level $z_{\text {eye }}$ and $z_{0}$ is called the roughness length or roughness factor and depends on the texture of the ground over which the wind flows. Sukumar and Selig [11] state a value $z_{0}=0.05 \mathrm{~m}$ matches wind profiles over open farm fields. However, an inspection of the formulation reveals the value of $z_{0}$ must be about 0.08 m to equal the value of $10 \mathrm{~m} / \mathrm{s}$ at 200 m where it closely resembles the $1 / 7^{\text {th }}$ power law. These profiles are shown in Figure 5-5. Note that all profiles represent a condition for which the wind speed at 200 m altitude above ground level reaches $10 \mathrm{~m} / \mathrm{s}$. Above 200 m , Barnes' profile is $10 \mathrm{~m} / \mathrm{s}$, while the other two keep increasing.


Figure 5-5: Three wind speed profiles for wind speed of $10 \mathrm{~m} / \mathrm{s}$ at 200 m altitude.

### 5.2.2. On the Modeling of Horizontal Wind with Stochastic Variation

Atmospheric wind is rarely, if ever, steady. Rather, its speed and direction varies randomly with time. A mathematical description of this variation is important in many engineering disciplines, for instance in wind power engineering, where it is used to predict nominal power generated by wind turbines (e.g. see Wang et al [12] (2011)). An idealized profile of wind strength (wind speed) as a function of time is shown in Figure 5-6. While the figure appears to show discrete changes is wind speed, real change is continuous and differentiable, although it can be sharp. It is typical of unsteady wind speed to vary around some average value and the variation can be described using a probability density function. Typically, Gaussian, Rayleigh, or Weibull probability density functions are used, of which the last one compares best with observation (e.g. see Justus et al [13] (1978), Stevens and Smulders [14] (1979), Gupta [15] (1986), Rohatgi et al [16] (1989), Rehman et al [17], and Garcia et al [18]). This section presents two of these approaches; Gaussian and Weibull. The former is used by the current version of the SURFACES Flight Simulator, although Weibull is being incorporated. Note that it can be argued based on the conclusions in Chapter 12, that both lead to similar results.


Figure 5-6: A representation of the nature of wind strength as a stationary Gaussian random process. Similar holds for the wind direction.

## Gaussian Distribution

Mathematically, wind velocity is a 2-dimensional time-dependent random variable, i.e. direction and strength vary with time in a process called a stationary Gaussian random process. Such processes ${ }^{3}$ represent a statistical distribution of samples $X_{t} \mid t \in\left\{t_{1}, t_{2}, \ldots\right\}$ and for which any linear combination has a joint Gaussian distribution. Thus, the joint distribution of wind speed, denoted by $V_{w}$, and wind direction, denoted by $\theta_{w}$, can be represented as a joint bivariate Gaussian probability density function, as shown below

$$
\begin{equation*}
p(x, y)=\frac{1}{2 \pi \sigma_{x} \sigma_{y} \sqrt{1-r^{2}}} e^{-\frac{1}{1-r^{2}}\left[\frac{1}{2}\left(\frac{x}{\sigma_{x}}\right)^{2}-r\left(\frac{x}{\sigma_{x}}\right)\left(\frac{y}{\sigma_{y}}\right)+\frac{1}{2}\left(\frac{y}{\sigma_{y}}\right)^{2}\right]} \tag{5-22}
\end{equation*}
$$

[^12]Where $x$ and $y$ are two processes, and $r$ is the correlation between them and ranges from 0 (no correlation) to 1 (perfect correlation). The variables $\sigma_{x}$ and $\sigma_{y}$ are the standard deviations associated with the two events. The term "stationary" refers to how it is idealized as infinite in duration, with properties (e.g. expectation, standard deviation, etc.) that are invariant of where or when sampled. The term bivariate means that $x$ and $y$ are related in some fashion, as indicated by $r$. The random process described by Equation (5-22) can be adapted to the wind speed and direction as follows

$$
\begin{equation*}
p\left(V_{w}, \theta_{w}\right)=\frac{1}{2 \pi \sigma_{V_{w}} \sigma_{\theta_{w}} \sqrt{1-r^{2}}} e^{-\frac{1}{1-r^{2}}\left[\frac{1}{2}\left(\frac{V_{w}-\bar{\zeta}_{w}}{\sigma_{V_{w}}}\right)^{2}-r\left(\frac{V_{w}-\bar{V}_{w}}{\sigma_{V_{w}}}\right)\left(\frac{\theta_{w}-\bar{\theta}_{w}}{\sigma_{\theta_{w}}}\right)+\frac{1}{2}\left(\frac{\theta_{w}-\bar{\theta}_{w}}{\sigma_{\theta_{w}}}\right)^{2}\right]} \tag{5-23}
\end{equation*}
$$

Where $\bar{V}_{w}$ and $\bar{\theta}_{w}$ are the average values of the wind speed and direction, respectively. Since the maximum and minimum wind speed and directions can occur anywhere across the spectrum, it is reasonable to assume there is no correlation between the two. This renders $r=0$ and reduces Equation (5-23) to

$$
\begin{equation*}
p\left(V_{w}, \theta_{w}\right)=\frac{1}{2 \pi \sigma_{V_{w}} \sigma_{\theta_{w}}} e^{-\frac{1}{2}\left[\left(\frac{V_{w}-\bar{V}_{w}}{\sigma_{V_{w}}}\right)^{2}+\left(\frac{\theta_{w}-\bar{\theta}_{w}}{\sigma_{\theta_{w}}}\right)^{2}\right]} \tag{5-24}
\end{equation*}
$$

This representation yields the classical Gaussian dome, positioned around the averages, as shown in Figure 5-7 for a token average wind speed of $10 \mathrm{~m} / \mathrm{s}$ with $\sigma_{V_{w}}=3 \mathrm{~m} / \mathrm{s}$ and $10^{\circ}$ wind direction for which $\sigma_{\theta_{w}}=3^{\circ}$. The probability of a specific wind speed at some specific wind direction is obtained from the probability distribution as follows:

$$
\begin{equation*}
F\left(V_{w}, \theta_{w}\right)=\int_{-\infty}^{V_{w}} \int_{-\infty}^{\theta_{w}} p\left(V_{w}, \theta_{w}\right) d V_{w} d \theta_{w}=\frac{1}{2 \pi \sigma_{V_{w}} \sigma_{\theta_{w}}} \int_{-\infty}^{V_{w}} \int_{-\infty}^{\theta_{w}} e^{-\frac{1}{2}\left[\left(\frac{V_{w}-\bar{V}_{w}}{\sigma_{V_{w}}}\right)^{2}+\left(\frac{\theta_{w}-\bar{\theta}_{w}}{\sigma_{\theta_{w}}}\right)^{2}\right]} d V_{w} d \theta_{w} \tag{5-25}
\end{equation*}
$$

## Weibull Distribution

The Weibull density function finds wide use in various disciplines. Its rise to fame is in material science, where it is used to predict time-to-failure in alloys. The Weibull probability density function is given by the three parameter expression

$$
p(x)=\left\{\begin{array}{lc}
\frac{k}{\lambda}\left(\frac{x-\theta}{\lambda}\right)^{k-1} e^{-(x / \lambda)^{k}} & x \geq 0  \tag{5-26}\\
0 & x<0
\end{array}\right.
$$



Figure 5-7: A probability density representing modest wind gusts with a $10 \mathrm{~m} / \mathrm{s}$ average wind speed and $10^{\circ}$ average wind direction, with a standard deviations of $3 \mathrm{~m} / \mathrm{s}$ and $3^{\circ}$, obtained using Equation (5-23).

Where $k>0$ is the shape parameter, $\lambda>0$ is the scale parameter, and $\theta$ is the location parameter. Thus, if the value of $k<1$ then the function describes as situation in which frequency of failures decreases with time, if $k=1$ the frequency are constant with time (exponential distribution), and if $k>1$ then failure frequency increases with time. If $k=2$ then $p(x)$ is Rayleigh distribution, commonly used to describe wind velocity. Just like the Gaussian distribution presented earlier, it is possible to represent the spread of wind speed and directions using a Weibull distribution function as shown below

$$
\begin{equation*}
p\left(V_{w}, \theta_{w}\right)=\overbrace{\left[\frac{k}{\lambda}\left(\frac{V_{w}-\bar{V}_{w}}{\lambda}\right)^{k-1} e^{-\left(\left(V_{w}-\bar{V}_{w}\right) / \lambda\right)^{k}}\right]}^{\text {Wind }} \overbrace{\left[\frac{l}{\eta}\left(\frac{\theta_{w}-\bar{\theta}_{w}}{\eta}\right)^{l-1} e^{\left.-\left(\left(\theta_{w}-\bar{\theta}_{w}\right) / \eta\right)^{\gamma}\right)}\right]}^{\text {speed }} \overbrace{[\text { dind direction }} \tag{5-27}
\end{equation*}
$$

Where $k>0$ is the shape parameter for the wind speed, $l>0$ is the shape parameter for the wind direction, and $\lambda, \eta>0$ are the scale parameters for the wind speed and direction, respectively. The use of Weibull distribution to represent the distribution of wind speed is favored in wind farm design and other wind related fields of engineering (e.g. see Carta et al [19] (2009) or Monahan et al [20] (2011) and many others). Simplifying leads to

$$
\begin{equation*}
p\left(V_{w}, \theta_{w}\right)=\frac{k}{\lambda} \frac{l}{\eta}\left(\frac{V_{w}-\bar{V}_{w}}{\lambda}\right)^{k-1}\left(\frac{\theta_{w}-\bar{\theta}_{w}}{\eta}\right)^{l-1} e^{-\left(\left(V_{w}-\bar{V}_{w}\right) / \lambda\right)^{k}+\left(\left(\theta_{w}-\bar{\theta}_{w}\right) / \eta\right)^{l}} \tag{5-28}
\end{equation*}
$$

## Hybrid Gaussian Weibull

It is also possible to describe a random process using a combination of dissimilar distribution functions. For instance, as stated earlier, it is desirable that the wind speed complies with a Weibull distribution. However, it is reasonable to assume the direction complies with Gaussian distribution. Therefore, the joint distribution of the two phenomena can be presented using a hybrid Weibull-Gaussian distribution function

$$
\begin{equation*}
p\left(V_{w}, \theta_{w}\right)=\overbrace{\frac{1}{\sigma_{\theta_{w}} \sqrt{2 \pi}} e^{-\frac{1}{2}\left(\frac{\theta_{w}-\bar{\theta}_{w}}{\sigma_{\theta_{w}}}\right)^{2}}}^{\text {Wind direction }} \overbrace{\left.\frac{k}{\lambda}\left(\frac{V_{w}-\bar{V}_{w}}{\lambda}\right)^{k-1} e^{-\left(\left(V_{w}-\bar{V}_{w}\right) / \lambda\right)^{k}}\right]}^{\text {Wind speed }} \tag{5-29}
\end{equation*}
$$

Simplifying leads to

$$
\begin{equation*}
p\left(V_{w}, \theta_{w}\right)=\frac{k}{\lambda \sigma_{\theta_{w}} \sqrt{2 \pi}}\left(\frac{V_{w}-\bar{V}_{w}}{\lambda}\right)^{k-1} e^{-\frac{1}{2}\left[\left(\frac{\theta_{w}-\bar{\theta}_{w}}{\sigma_{\theta_{w}}}\right)^{2}+2\left(\frac{V_{w}-\bar{V}_{w}}{\lambda}\right)^{k}\right]} \tag{5-30}
\end{equation*}
$$

### 5.2.3. Determination of Wind using Discrete Gaussian Variation

The aforementioned discussion describes the observed nature of the variation of wind speed and direction. However, this does not describe how this is implemented in a flight simulator. This section addresses how this is accomplished. As shown in Chapter 8, Flight Simulation, the simulator kernel operates in an infinite do-loop ${ }^{4}$. The execution of the simulation code can be implemented in two ways; (1) by specifying a constant time increment, $\Delta t$, or (2) by keeping track of system time and calculate a "variable" $\Delta t$. The former, typically, causes the simulator to run at faster pace than real-time. The latter causes the simulator to run in real-time (assuming it can keep up with other tasks, such as the display of data and outside view of the world). At any rate, to simulate the randomness of natural unsteady wind, a random seed is generated each time the loop is iterated and used as basis for the wind change, as explained below.

Computer operating systems typically return random numbers that have a uniform distribution. If a Gaussian distribution is required, these random numbers, called seeds, must be transformed into values that have Gaussian distribution. The most common way of doing this is using the Box-Muller-Wiener (BMW) algorithm (see Toral and Chakrabarti [21] (1992)). In this method, two random seeds, $\xi_{1}$ and $\xi_{2}$, obtained from the operating system and which are uniformly distributed in the interval [0,1], are transformed into the random numbers $x_{1}$ and $x_{2}$, which have a Gaussian distribution of mean 0 and variance 1 , using the following expressions

[^13]\[

$$
\begin{align*}
& x_{1}=\sqrt{-2 \ln \left(\xi_{1}\right)} \sin \left(2 \pi \xi_{2}\right) \\
& x_{2}=\sqrt{-2 \ln \left(\xi_{1}\right)} \cos \left(2 \pi \xi_{2}\right) \tag{5-31}
\end{align*}
$$
\]

Note that any random numbers $z$ of mean $\mu$ and variance $\sigma^{2}$ can be generated by the linear relation $z=$ $\mu+\sigma x$. The advantage of the BMW algorithm is that it produces an unbiased Gaussian distribution. However, the number of operations required to obtain the number is a major disadvantage and is computationally slow. While faster methods exist, for instance the one presented in Ref. [21], it was not deemed necessary to abandon the BMW method. The flight simulator generates one such random seed during each iteration and uses it with the one from the previous iteration.

The application of these random numbers is used to calculate the change in wind speed and direction during each iteration. In this approach, it is recognized that during a given time period, the wind speed has some average value, $\bar{V}_{w}$, and from which the instantaneous wind speed varies inside some limits of $\pm \Delta V_{g u s t}$. This leads to a maximum and minimum wind speed value, $V_{g \text { max }}$ and $V_{g \text { min }}$, respectively (see Figure 5-6). Therefore, at least at first glance, it would appear that the instantaneous wind could be represented as shown below

$$
\begin{equation*}
V_{w}(t)=\bar{V}_{w}+G R N D(t) \Delta V_{g u s t} \quad \text { and } \quad V_{g \min } \leq V_{w}(t) \leq V_{g \max } \tag{5-32}
\end{equation*}
$$

Where $t$ is time, $G R N D(t)$ is a Gaussian random number generator function, which returns a value between -1 and 1 . For instance, if $G R N D(t)=-1$, then we get $V_{w}(t)=\bar{V}_{w}-\Delta V_{\text {gust }}$, if $G R N D(t)=0$, then $V_{w}(t)=\bar{V}_{w}$, and if $G R N D(t)=1$, then $V_{w}(t)=\bar{V}_{w}+\Delta V_{g u s t}$. However, upon closer inspection, there is a serious problem with Equation (5-32); abrupt changes. When the expression is implemented in a discrete numerical environment, it is possible the wind speed changes from $\bar{V}_{w}$ to $V_{g \max }$ or $V_{g \min }$, instantly. This does not reflect natural behavior. Thus, the wind speed is better represented using a scheme that reflects a historical trend, like that obtained using a random-walk (aka Brownian random) motion. Such a model is presented below:

$$
\begin{equation*}
V_{w}(t, i)=V_{w}(t, i-1)+\operatorname{sgn}(G R N D(t)) \frac{\Delta V_{g u s t}}{M} \tag{5-33}
\end{equation*}
$$

Where $V_{w}(0,0)=\bar{V}_{w}$ and $V_{w}(t) \in\left[V_{g \min }, V_{g \max }\right]$. In this model, $i$ is an index and implies that the current instantaneous wind strength depends on the wind strength from the previous iteration and the addition or subtraction of a small value (or discrete speed change). Whether the discrete speed is added or subtracted depends on whether a or - sign is returned using the $\operatorname{sgn}()$ function, which returns a value of -1 or +1 depending on the value returned from the random function $G R N D(t)$. The size of the discrete speed change is controlled by the user using the variable $M$, and uses the random function $G R N D(t)$ to govern the sign (+ or) of the change. If the value of $M$ is small (e.g. 10) the wind speed fluctuates rapidly between the limits $V_{g \text { max }}$ to $V_{g \min }$, whereas if $M$ is large (e.g. 100) the fluctuations will
be smaller, and if $M$ is very large (e.g. $\rightarrow \infty$ ) the fluctuations will be non-existent; we will have steady wind. Furthermore, to convenience the user in a software implementation, we can define the average wind as follows

$$
\begin{equation*}
\bar{V}_{w}=\frac{1}{2}\left(V_{g \min }+V_{g \max }\right) \tag{5-34}
\end{equation*}
$$

A similar approach is used for wind direction, which too will vary between a maximum and minimum angular value, $\theta_{g \text { max }}$ and $\theta_{g \text { min, }}$, respectively, with an average direction of $\bar{\theta}_{w}$. This can be applied using similar expression, i.e.

$$
\begin{equation*}
\theta_{w}(t)=\bar{\theta}_{w}+\operatorname{sgn}(G R N D(t)) \frac{\Delta \theta_{\text {gust }}}{M} \tag{5-35}
\end{equation*}
$$

Where $\theta_{w}(0,0)=\bar{\theta}_{w}$ and $\theta_{w}(t) \in\left[\theta_{g \text { min }}, \theta_{g \text { max }}\right]$ and

$$
\begin{equation*}
\bar{\theta}_{w}=\frac{1}{2}\left(\theta_{g \text { min }}+\theta_{g \text { max }}\right) \tag{5-36}
\end{equation*}
$$

As stated earlier, a model of this nature represents a stationary Gaussian process that is both homogeneous (independent of flight path) and isotropic (independent of vehicle attitude).

### 5.3 Thermal Modeling

As stated in Section 1.3, Fundamentals of Soaring Flight, there are typically four forms of convection that lead to rising air; thermals, orographic (or ridge) lift, standing mountain waves, and convergence lift (caused by the collision of two air masses, such as sea-breeze and inland air mass). In addition there is horizontal convection with the associated boundary layer wind profile (which permits dynamic soaring). Strictly speaking, there also is an additional opportunity for soaring using a method called gust soaring, which takes advantage of sharp gusts and turbulence (for instance see Boslough [22] (2002)). The work presented in here only treats lift due to thermals, orographic lift, and horizontal convergence. It treats convergence lift in the same manner as thermals; its detection results in identical autopilot response (i.e. a decision to activate position hold mode while gaining altitude). This section provides mathematical formulation used in the analysis of thermals, in particular. Such formulation must account for (1) thermal velocity profile (vertical speed of air inside of and in the vicinity outside the thermal) and (2) topographical distribution.

### 5.3.1. Thermal Velocity Profiles

In its most simplistic terms, the word thermal refers to a mass of air that is warmer than its surrounding air and, thus, less dense, which causes it to rise vertically. Since the flow of this air must satisfy the conservation of mass, its transportation from low to high altitude must be replaced with an equal mass of air. This replacement air comes from various directions, including the flow outside of the thermal
moving downward (downdraft). Thermals are commonly described by laypeople as columns of updraft that extend from the ground to the upper edge of the planetary boundary layer or that they are like bubbles that float upward. Neither description is accurate. A far more realistic display of thermal shapes is illustrated in Figure 5-8, depicting a highly complex structure. The image shows dry convection thermals on a calm day (zero average horizontal wind). However, a simplified picture better helps the understanding of the velocity field inside of and in the vicinity of a thermal. The 3-dimensional velocity field generated by the presence of a thermal has the following characteristics:
(1) air flows upward inside the thermal,
(2) air flows downward outside of the thermal (albeit at lesser rate than the upward flow),
(3) air flows horizontally toward the thermal (see Figure 5-9). This leads to an increase in wind speed on the "windward" side and reduction on the "leeward" side, something used by experienced flyers of RC sailplanes to identify the presence of a thermal, and
(4) thermals drift with the wind.
(5) thermals form tall irregularly shaped pillars of random sizes (see Figure 5-8).


Figure 5-8: Examples of thermals generated solving the Navier-Stokes equations. Blue represents updraft and red downdraft. Planar dimensions are $5 \times 5 \mathrm{~km}$. (Image courtesy of Dr. Peter Sullivan of NCAR)

Classical thermal modeling often assumes the thermal resembles that of a column with a specified diameter inside which the air flow upward, and outside of which it flows downward. The 3-dimensional shape of thermals is of vital importance to the design and operation of sailplanes. Since thermals are of finite dimension, the sailplane pilot must assertively bank inside it and ideally circle around its core while gaining as much altitude as possible. Of course, an idealized core does not really exist, but rather there
are regions of stronger versus weaker updrafts and this is considered a "core." Small turning radius allows the maximum lift to be extracted out of the thermal. However, a smaller turning radius is associated with a steeper bank angle and this, unavoidably, comes at the cost of reduced climb rate. Too shallow a bank will fly the sailplane out of the thermal. Too steep a bank increases the rate of descent and reduces potential altitude gain. Too steep a bank may even cause altitude loss in strong thermals. Being able to mathematically describe the vertical velocity inside the thermal is thus fundamental to determine the optimum bank angle, given the distance of the sailplane from the core. The mathematics of this predicament is presented in Section 6.5.12, Circling Flight.

Welch and Irving [23] present several thermal updraft profiles, of which three are shown in Figure 5-10. These are the Power-Law, Spherical Bubble, and Modified Parabolic models. Of these, the author recommends the last one, as it leads to a modest downdraft outside of the thermal, something reported by sailplane pilots (e.g. see Welch and Irving [23], Konovalov [24], and Reichmann [25]). Generally, the vertical speed in a thermal, denoted by $V_{T}$, will be greatest at its core. This maximum speed is denoted by $V_{T 0}$. Even though the following mathematical models assume symmetrical thermal shape, this is not necessarily so in real thermals. The three thermal profiles are defined mathematically below. The ratio $r / R$ denotes the fractional distance from the center of a thermal whose diameter is $2 R$. Of the three presented, the Power-Law, using $n=2$ is sometimes used for competition handicapping purposes [23], assuming a thermal radius of $R=$ 1000 ft and with a core strength $V_{T 0}=4.2$ knots.


Figure 5-9: Horizontal effect of the presence of a thermal, looking from above down to a horizontal plane. The thermal is at the center and horizontal wind is blowing from the left. There is increased wind speed on the "windward" side and reduced on the "leeward" side of the thermal.

Power-Law Velocity Profile: $\quad \frac{V_{T}}{V_{T 0}}=1-(r / R)^{n}$

Spherical Bubble Model:
$\frac{V_{T}}{V_{T 0}}=\frac{1-(r / R)^{2}}{\left[1+2(r / R)^{2}\right]^{2.5}}$
$\frac{V_{T}}{V_{T 0}}=\left[1-(r / R)^{2}\right] \cdot e^{-(r / R)^{2}}$

The second element of thermal velocity profiles is their strength. Carmichael [26] (1954) defines thermal strength in as described below:
(1) Strong thermal has a maximum vertical speed of $6.1 \mathrm{~m} / \mathrm{s}$ ( $20 \mathrm{ft} / \mathrm{s}$ or $\approx 12$ knots) that falls to 3.05 $\mathrm{m} / \mathrm{s}(10 \mathrm{ft} / \mathrm{s})$ when $r=61 \mathrm{~m}(200 \mathrm{ft})$.
(2) Weak thermal has a maximum vertical speed of $3.05 \mathrm{~m} / \mathrm{s}(10 \mathrm{ft} / \mathrm{s}$ or $\approx 6$ knots) that falls to 1.5 $\mathrm{m} / \mathrm{s}(5 \mathrm{ft} / \mathrm{s})$ when $r=61 \mathrm{~m}(200 \mathrm{ft})$.
(3) Wide thermal has a maximum vertical speed of $4.6 \mathrm{~m} / \mathrm{s}(15 \mathrm{ft} / \mathrm{s}$ or $\approx 9$ knots) that falls to $2.3 \mathrm{~m} / \mathrm{s}$ $(7.5 \mathrm{ft} / \mathrm{s})$ when $r=122 \mathrm{~m}(400 \mathrm{ft})$.

Updraft and size corrections per Allen [27] (2006) can be incorporated. These modify the updraft strength and thermal radius as a function of altitude, $z$. The method uses formulation originally developed by Lenschow and Stephens [28] in 1980, and, thus, is referred to as Lenschow's method in this dissertation. This correction defines two scaling parameters, convective velocity, $w^{*}$, and convective mixinglayer thickness, $z_{i}$, whose values vary with the time of year, as shown in Table 5-3, determined by experiment. These values are used to calculate the average updraft velocity, $\bar{w}$, using the following expression


Figure 5-10: Common models used to approximate the vertical speed profile inside (and outside) a thermal.

$$
\begin{equation*}
\bar{w}=w^{*}\left(\frac{z}{z_{i}}\right)^{1 / 3}\left(1-1.1 \frac{z}{z_{i}}\right) \tag{5-40}
\end{equation*}
$$

The thermal radius is computed as shown below

$$
\begin{equation*}
R=\max \left(10,0.102 \cdot z_{i} \cdot\left(\frac{z}{z_{i}}\right)^{1 / 3}\left(1-0.25 \frac{z}{z_{i}}\right)\right) \tag{5-41}
\end{equation*}
$$

Table 5-3: Monthly Convective Scales for Hours between Sunrise and Sunset near Desert Rock Airport, Nevada. Reproduced from [27].

|  | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dec |  |  |  |  |  |  |  |  |  |  |  |
| Mean $w^{*}, \mathrm{~m} / \mathrm{s}$ | 1.14 | 1.48 | 1.64 | 1.97 | 2.53 | 2.38 | 2.69 | 2.44 | 2.25 | 1.79 | 1.31 |
| $z_{i}$ for mean $w^{*}, \mathrm{~m}$ | 504 | 666 | 851 | 1213 | 1887 | 1728 | 1975 | 1755 | 1382 | 893 | 627 |
| Max $w^{*}, \mathrm{~m} / \mathrm{s}$ | 3.59 | 3.97 | 4.89 | 5.53 | 5.49 | 5.51 | 6.3 | 5.64 | 5.97 | 4.57 | 4.55 |
| $z_{i}$ for $\max w^{*}, \mathrm{~m}$ | 1800 | 1970 | 3900 | 2380 | 3833 | 4027 | 3962 | 4940 | 2460 | 3285 | 1783 |

As an example, consider a thermal in the month of July. According to Table 5-3, considering average values first, the mean thermal updraft velocity at 1000 m above ground level is obtained from Equation (5-40) as show below

$$
\bar{w}=w^{*}\left(\frac{z}{z_{i}}\right)^{1 / 3}\left(1-1.1 \frac{z}{z_{i}}\right)=2.69\left(\frac{1000}{1975}\right)^{1 / 3}\left(1-1.1 \frac{1000}{1975}\right)=0.950 \mathrm{~m} / \mathrm{s}
$$

The radius of the thermal at that altitude is found from Equation (5-41)

$$
R=\max \left(10,0.102 \cdot 1975 \cdot\left(\frac{1000}{1975}\right)^{1 / 3}\left(1-0.25 \frac{1000}{1975}\right)\right)=140 \mathrm{~m}
$$

### 5.3.2. Topographical Distribution of Thermals

The distribution of thermals over terrain is of great importance in the operation of sailplanes, as discussed in Sections 6.5.8 through 6.5.12. The filament distribution pattern visible in Figure $5-8$ and the bottom row of Figure 2-4 are of great importance in this respect. The color contours in both figures represents the vertical speed of the air at some distance above the ground plane (note that the height of the plane is $z>0$ ). The shape of the updraft contours changes with altitude and time (maximum strength is typically around noon). The blue shades in Figure 5-8 indicate updraft, while yellow to red indicate downdraft. The filaments are indicative of classical Rayleigh-Bénard convection dynamics (e.g. see Eckert et al [29] or Moeng and Rotunno [30]), which describes flow resulting from natural convection (vertical transportation) of fluids near a horizontal plane. The ground plane constrains the flow and turns vertical flow in a horizontal direction. This, in turn, causes the formation of distinct cells of updraft, separated by regions of downdrafts, as the horizontal flow components collide with one another, forming another region of updraft, ultimately resulting in very complicated regions of up and downdrafts.

It is important to remember that, at this time, thermal structure and evolution is an active field of research and it is not likely that simple formulae that describe this complex phenomenon will surface. The thermal structure in Figure 5-8 was generated by solving the Navier-Stokes equations using Large Eddy Simulation (LES). The complexity hits home when one considers the solution shown neglects the effect of moisture, horizontal winds, and the presence of random topographical effects such as the heating of one side of a mountain or hillside. It is evident this complexity can neither be recreated in a practical fashion in a flight simulator or the computational module of the GICA. The SURFACES Flight Simulator wind simulator approximates the thermal terrain distribution using a Voronoi diagram. To the knowledge of the author, this methodology has not been applied before in this fashion. The application of this approach is justified on the basis it results in terrain distribution that has many parallels to the pattern in a paper by Saini et al [31] and which explains the visible distribution of cumulus clouds, associated with thermal formation. For instance, it results in similar randomly sized downdraft cells, surrounded by straight-walled updrafts.

## Voronoi Diagrams

A Voronoi diagram is attributed to the Ukrainian mathematician Georgy Fedosevich Voronoy (18681908,40 ). It can be defined mathematically in the following fashion. Let $\mathbf{X}$ be a metric space with a distance function $r$ and $P_{i}$ be an ordered collection of points (a tuple) in $\mathbf{X}$. Then, a Voronoi cell (or tile), $C_{i}$, associated with $P_{i}$ is defined as follows:

$$
\begin{equation*}
C_{i}=\left\{x \in \mathbf{X} \mid r\left(x, P_{i}\right) \leq r\left(x, P_{j}\right) \quad \forall j \neq i\right\} \tag{5-42}
\end{equation*}
$$

What this means is that any point $x$ in $\mathbf{X}$ whose distance from $P_{i}$ is less or equal than its distance from $P_{j}$ is a member of the set $C_{i}$. A typical Voronoi diagram is shown in Figure 5-11. The figure helps relaying the true meaning of the diagram using the following analogy. Imagine the square represents a geographic region, perhaps a city, and that each point (called seed) is a Starbucks retailer inside the city limits. It can be argued that most people in the area, looking to buy a cup of joe at Starbucks, will most likely visit the one closest to their residence. The distance of any point inside the cell will be closer to its seed, than any of other seeds in the figure. In other words, the closest Starbucks for the inhabitants inside a cell is the one inside that cell. It turns out that this idea (i.e. Voronoi diagrams) has many practical applications in sciences as disparate as ecology, biology, astrophysics, and many others, including replication of the distribution of thermals. Voronoi diagrams is related to Delaunay triangulation in that it represents the triangles formed by adjacent closest seeds.

Voronoi diagrams for a continuous space can be generated by several means, of which Fortune's algorithm is one of the better known. The generation of Voronoi diagrams for discrete spaces is relatively straight forward and can be accomplished for 2-dimensional space as follows:

STEP 1: Define space $\mathbf{X}$ in terms of $M, N$ discrete and uniformly shaped rectangles, whose indexes are denoted by $i, j$, respectively.
STEP 2: Using a uniform probability density function, randomly distribute $K$ seeds, $P_{k}$, in $\mathbf{X}$, where $k=1, \ldots, K$.
STEP 3: From $i=1, \ldots, M$ and $j=1, \ldots, N$ calculate the distance from each $x_{i j}$ in $\mathbf{X}$ to each $P_{k}$. STEP 4: Select the shortest distance for each $P_{k}$ and having assigned a color code (or other means of identification) to each seed, color the rectangle.


Figure 5-11: A Voronoi diagram can be used to simulate Rayleigh-Bénard formation of thermals.

This algorithm was used to generate the Voronoi diagram like in Figure 5-11. Additional step is required to complete the distribution of thermals; the determination of the tile boundaries. This is along which the thermals will be placed. A simple method to do this is to scan the tiles and identify where one
changes to another. The application of this methodology is illustrated in Figure 5-12 through Figure 5-14. For added realism, the thermals are of random strength and size, as discussed earlier.


Figure 5-12: A Voronoi diagram superimposed on the topography, here using 1000 seeds to ensure small enough tile .


Figure 5-13: Boundary tracing completed. It is along these boundaries that the thermals are strewn.


Figure 5-14: Distribution of thermals completed. More than 4200 individual thermals, of randomly varying position, strength, and size, are strewn across the topography. This sample "world" is where the flight simulation takes place. It is approximately $30 \times 30 \mathrm{~km}$ in dimensions. The tallest mountain is about $\mathbf{2 4 0 0} \mathbf{~ m}$ above sea-level.

### 5.4 Modeling Flow over Large Terrain Features

Atmospheric flow over large topographical features, such as mountains and escarpments, has been studied extensively by many researchers. The search for this knowledge has been driven by practical applications, such as design of wind farms, large infrastructure engineering, and even emergency purposes, such as to determining the spread of localized atmospheric pollutants, volcanic ash, or embers from forest fires. It is important to recognize that terrain wind flow is very complex and a thorough study of the detail is beyond the scope of this work. However, many trends in such flow are relatively easy to understand. For instance, there is always updraft on the windward side of a mountain, although wind can also flow around the sides of an isolated hill, reducing the updraft (blockage effect). On the leeward side there is either very strong downdraft or fully separated wake, both of which should be avoided by any flying vehicle, small and large. The size, velocity field, and turbulence level of this wake, including its dimensions, is hard to predict and it is further compounded when certain wind condition cause a temporal fluctuation in the wind velocity, causing standing mountain waves. The generation of precise mathematical description of such flow requires approaches such as the solution of the NavierStokes equations using LES. When faster computations are required, researchers have resorted to mass conservation methods such as the Consistent Mass Method, or CMM for short (e.g. see Ratto et al [32] (1994), Finardi et al [33] (1997), Kim and Patel [34] (2000), Wang et al [35 and 36] (2003 and 2005), and Juarez [37] (2012)). Regardless, such methods are far too computationally intensive and time consuming to be practical in a small onboard sUAV, demanding faster methods to be resorted to, of course, at the cost of accuracy in detail.

The wind flow over topography can be approximated with acceptable accuracy without resorting to solving the Navier-Stokes equations. Such methods are practical for use as a wind generator for flight simulation. The algorithm detects and avoids flying inside leeward mountain winds; it sticks to the windward face of a mountain. This reduces the need to analyze the shape of the leeward wind field in detail. Two methods for this purpose are presented here; 3-dimensional Potential Flow Theory (PFT, see Chapter 4, The Use of Potential Flow Theory in the GICA) and Constant Mass Flow method (CMF).

### 5.4.1. Capability of Wind Simulation

Before discussing either method in detail, it is helpful to consider some capabilities that a wind simulator must offer. First, it should ensure wind can flow around, as well as over a mountain (see Figure 5-15). This is necessary to account for change in the horizontal wind direction that occurs in real flow.

Second, the wind simulator should provide a variation in wind velocity with altitude. The effect is illustrated in Figure 5-16, where the left side shows region airplanes should avoid at all cost. This region is characterized by strong downdraft and turbulence. The downdraft can easily exceed the aircraft's maximum rate of climb. It also features dangerous eddies or rotors that can surprise even the most alert pilot and call for swift upset recovery control. The geometry of such downdraft regions is subject to many variables, including prevailing wind strength, terrain characteristics, and influences of the surrounding topography.


Figure 5-15: Surface streamlines show flow "over and around" an isolated mountain.

It is also of interest to point out a common occurrence in such flow; a quasi-static boundary whose shape, in part, depends on the glide characteristics of the aircraft. Consider an aircraft gliding at constant airspeed and rate of descent. Since the flow of air accelerates over the peak of the mountain, the wind speed updraft may equal the airspeed and ROD over a narrow band (distance-wise) above the mountain. In such a situation, and assuming such wind conditions prevail for a period, the aircraft would remain more or less stationary over the terrain, at least it would make limited progress. Considering Figure 5-16, if the airplane drifted to the right, it would begin to rise and make headway again. However, if it drifted to the left, it would quickly find itself moving toward the downdraft of the leeward wind. It would call for a dive or engine power to bring it out of this predicament. Thus, this represents a boundary beyond which the pilot (or autopilot) should avoid traversing.

Third, as shown by Stull [5], flow over even an isolated hill displays different stability based on the Froude number, $F_{r}=V /(N L)$, where $V$ is the wind speed, $N$ the Brunt-Väisälä frequency, and $L$ the length scale of the hill. Associated complexity is caused by blockage effect mentioned earlier, for instance see Reinecke and Durran [38] and Petersen et al [39]. All these are concerns that present great
challenges to computational methods. While the windward side is easier to work with (and it is where we want to harvest atmospheric energy), the updraft may be less than expected because of blocked flow. In accordance with conservation laws, air has no choice but to flow upward, giving rise to a harvestable updraft, although this is subject to blockage. Another study, by Zhang and Liu [40] (2014), investigated wind speed profiles over hills and ridges and the associated reshaping of the boundary layer. This transforms the wind profile from what was described using the methods of Section 5.2.1, Modeling the Planetary (Atmospheric) Boundary Layer, to what, in effect, is best approximated using constant velocity profile. Zhang and Liu support their CFD work with experimental data by Kim et al [41] and an example figure from this work is shown in Figure 5-17. Such approaches have even found their way into building codes for structures that have to withstand steady high-speed winds in mountainous regions (exemplified in Figure 5-18 based on Reference [42]).


Figure 5-16: A simple illustration of the windward and leeward sides of a mountain.

### 5.4.2. The Application of the 2-Dimensional PFT in Mountainous Terrain

One way of understanding the nature of atmospheric flow over topographical features is to analyze it using 2-dimensional PFT. This gives insight into such flow in the absence of viscous effects and associated computational challenges. Figure 5-19 exemplifies such flow over an idealized mountain range, some 7 km ( 4.2 statute miles) long with the highest peak $1000 \mathrm{~m}(3000 \mathrm{ft}$ ) above sea level. The PFT analysis implements the method of Section 4.1.8, Non-Lifting Flow about Arbitrary Bodies. As stated in the caption, the isopleths show regions of updraft ranging from $0.1 \times V_{w}$ (farthest from mountain) to $1 \times V_{w}$ (closest to mountain), separated by a step distance of $0.1 \times V_{w}$, where $V_{w}$ is the prevailing average wind (ignoring the boundary layer). It is evident that the disturbance of the mountain has more or less dissipated at altitude of 5000 m (approximately 16000 ft ) and slope lift is in limited supply above 2500 m
(8000 ft). It can also be seen that the best slope lift is (as expected) on the windward side of the two peaks, with strong downdraft on the leeward sides.


Figure 5-17: An excerpt from Ref [40], showing wind profiles over 2-dimensional hills, based on experiment and CFD predictions.


Figure 5-18: An example of the change in velocity profile of atmospheric winds flowing over a ridge commonly found in building codes.

While Figure 5-19 is helpful in illustrating the power of the PFT, the computational complexity of such analysis should not be underestimated. Namely, in solving for the panel strengths for the mountain range, the tangential flow conditions over the flat segments (i.e. the horizontal surfaces which could represent water) are best achieved by including its mirror image. This is illustrated in Figure 5-20. This doubles the number of panels that must be included in the analysis. More precisely, if $N$ panels are use to represent the mountain, the total number panels representing the body of Figure $5-20$ is $2 N$. This quadruples the number of aerodynamic influence coefficients, which, in turn, increases the computational time more than $8 N^{3}$-fold ${ }^{5}$ ! Thus, flow over a 4 panel half-circle requires 8 panels, which requires 64 influence coefficients (an $8 \times 8$ matrix), which requires at least $8^{3}$ or 512 math-operations. We

[^14]have thus identified a potential issue with implementing the PFT in a flight simulator in 3-dimensional space; memory requirements, as will be shown shortly.


Figure 5-19: Atmospheric flow over a mountain range, predicted using potential flow theory. The contours show regions of updraft ranging from $0.1 \times V_{w}$ (farthest from mountain) to $1 \times V_{w}$ (closest to mountain), separated by a step distance of $0.1 \times V_{w}$.


Figure 5-20: Complication in the generation of wind fields using PFT - The tangential flow over the horizontal segments to the left and right requires the model of the mountain range in Figure 5-19 to include its mirror image, substantially taxing computational resources.

### 5.4.3. The Application of the 3-Dimensional PFT in Mountainous Terrain

It is of considerable value to implement the 3-dimensional PFT described in Section 4.2, ThreeDimensional Potential Flow Theory. This allows its accuracy, reliability, and cost of computational resources to be assessed. The isolated island mountain in Figure 5-21 was used to perform this evaluation. The geometry consists of 2500 square panels ( $50 \times 50$ panels), each which is $2000 \times 2000 \mathrm{ft}$ in size. In order to speed up the solution, the ocean panels were omitted. However, the mountain had to
be mirrored, which resulted in 1080 panels, each featuring a constant-strength source. Thus, the geometry of the mathematical model resembles a flying saucer, with the upper and lower halves consisting of some 540 panels each. The resulting aerodynamic influence matrix consists of $1080 \times 1080$ elements and requires approximately 24 seconds of computational time using a typical contemporary laptop computer with 2.20 GHz Intel Core i3-2330M CPU, 4 GB installed memory, and Windows 10 operating system. Another rendering of this topography consisted of even greater $1000 \times 1000 \mathrm{ft}$ panel resolution, requiring a $4144 \times 4144$ influence matrix. The computational time jumped to about 30 minutes. This renders the method impractical for the GICA, except for very simple topographies.


Figure 5-21: This mathematically generated island mountain is used to evaluate the 3 -dimensional PFT. The peak reaches approximately 3000 m above sea-level. Streamlines extend from approximately $600-8000 \mathrm{~m}$ ( 2000 to 26000 ft ). The wind speed is $20 \mathrm{~m} / \mathrm{s}$ and the representative Froude number is about 0.6 . Note the erroneous trajectory of the lowest streamline - a consequence of the PFT modeling so close to the ground.

More results from the PFT analysis of the $50 \times 50$ panel topography are illustrated in Figure 5-22 using streamlines with wind speed annotations. While the prediction of air parcel movement at altitudes above 2000 ft appears reasonable, the trends in close proximity to the ground are poor. As discussed by Katz and Plotkin [43], this is expected and is caused by the point source idealization of the 3-dimensional PFT. A parcel of air near the terrain become more influenced by the singularities that are, relatively speaking, closer to it than singularities farther away. This can cause streamlines generated by parcel tracing to follow an incorrect path when closer to the ground. In other words, the PFT will cause unrealistic wind directions and speeds when close to the ground and this is not acceptable, unless the simulation takes place well above the ground only. Improvements are to be had using the distributed source idealization that was implemented for the 2-dimensional PFT of Section 4.1.8, Non-Lifting Flow about Arbitrary Bodies and whose results are illustrated in Figure 5-20. However, it is important to realize that an unpredictable nature of streamlines near the ground is only one problem; the computational time required for the solution is another one and renders the approach prohibitive for use in the GICA. This is compounded by the fact that during simulation, the wind conditions at the position of the aircraft are calculated many times each second. The PFT solution requires extensive multiplication operations involving the aerodynamic coefficients, which severely taxes the computational resources for large, complex topographies.

Another shortcoming of the 3-dimensional PFT is shown in Figure 5-23. While the boundary conditions $(\mathbf{V} \cdot \mathbf{n}=0)$ are satisfied at each collocation point, i.e. surface velocities are tangential to the panel surface, their directions are not. Since the wind direction is not truly accounted for in the boundary conditions, the velocities point in random directions over the mountain. Recall that a vector in the plane, perpendicular to the plane's normal can possess any angle between 0 and $2 \pi$. This result renders it impossible to estimate flow near the ground reliably. This shortcoming is exacerbated with terrain complexity. The topography uses the same panel resolution as the isolated island mountain before, but required a $3523 \times 3523$ influence matrix, which took close to 18 minutes to solve.


Figure 5-22: Streamlines associated with a $20 \mathrm{~m} / \mathrm{s}$ wind speed at $90^{\circ}$ (out of East), showing how the wind speed changes with position and altitude.


Figure 5-23: While the surface winds resulting from the 3-dimensional PFT satisfy the tangential flow conditions, they still flow in random directions.

### 5.4.4. Constant Mass Flow (CMF) Model for Wind Simulation

We have seen the implementation of a 3-dimensional PFT in the previous section. In this section, an alternative model is offered for wind simulation in the SURFACES Flight Simulator, although PFT is also
offered. The primary goal of the method is to improve the computational speed over the PFT. It is essential to understand the method is a hybrid terrain following/fluid dynamics method. And while providing acceptable accuracy in many situations, they are suspect in other cases, for instance in complicated canyon type topography. The same can be argued of the PFT. CMF also avoids computational issues associated with the use of PFT, which arises in the presence of complex topography, with multiple mountain peaks and valleys, and which can lead to erroneous results for the entire computational domain.

The basic premise of the CMF is that it follows the terrain, something implemented by projecting the general wind direction onto the surface tangent. This terrain hugging is gradually terminated at some selected altitude, called the fictitious ceiling, $H_{f}$. Thus, the shape of the flow will change from that tangent to the surface to the original, user specified, wind speed and direction at $H_{f}$. Then wind speed can be accounted for by assuming constant mass flow rate through an elemental area extending from the surface to $H_{f}$. A mathematical description will now be presented.

## Mathematics of Terrain Following Flow

Let $\zeta=\zeta(x, y)$ be a topographical surface subject to a far-field wind velocity $\mathbf{V}_{\mathbf{w}}$, where $\zeta=0$, which is given by

$$
\begin{equation*}
\mathbf{V}_{w}=w_{x} \mathbf{i}+w_{y} \mathbf{j}+w_{z} \mathbf{k} \tag{5-43}
\end{equation*}
$$

Then the unit normal to the surface, denoted by $\mathbf{n}$, is given by

$$
\begin{equation*}
\mathbf{n}=\frac{\nabla \zeta}{|\nabla \zeta|}=\frac{1}{|\nabla \zeta|}\left[\frac{\partial \zeta}{\partial x} \mathbf{i}+\frac{\partial \zeta}{\partial y} \mathbf{j}+\frac{\partial \zeta}{\partial z} \mathbf{k}\right] \tag{5-44}
\end{equation*}
$$

Under most circumstances we assume that the $z$ component of in the far-field is zero, thus


Figure 5-24: The projection of the wind velocity onto the normal of the surface. Note that all the vectors shown are coplanar.

$$
\begin{equation*}
\mathbf{V}_{w}=w_{x} \mathbf{i}+w_{y} \mathbf{j}=V_{w} \cos \theta \mathbf{i}+V_{w} \sin \theta \mathbf{j} \tag{5-45}
\end{equation*}
$$

Where $\theta$ is the direction of the horizontal far-field wind vector and $V_{w}$ is the wind speed (note that both are specified by the user). Therefore, the projection of the wind vector onto the tangent to the point $(x, y, z)$ on the topography is found from vector analysis and is obtained as follows. First, determine the parallel projection of $\mathbf{V}_{\mathbf{w}}$ on $\mathbf{n}$, shown below

$$
\begin{equation*}
\mathbf{V}_{\mathbf{A}_{\|}}=\frac{\mathbf{n} \cdot \mathbf{V}_{w}}{\mathbf{n} \cdot \mathbf{n}} \mathbf{n} \tag{5-46}
\end{equation*}
$$

This can be simplified, since $\mathbf{n}$ is a unit normal. Thus, we can write

$$
\begin{equation*}
\mathbf{V}_{\mathbf{A}_{\|}}=\left(\mathbf{n} \cdot \mathbf{V}_{w}\right) \mathbf{n} \tag{5-47}
\end{equation*}
$$

Substituting Equations (5-44) and (5-45) into Equation (5-47) leads to

$$
\begin{equation*}
\mathbf{V}_{\mathbf{A}_{\|}}=\frac{V_{w}}{|\nabla \zeta|^{2}}\left(\cos \theta \frac{\partial \zeta}{\partial x}+\sin \theta \frac{\partial \zeta}{\partial y}\right)\left(\frac{\partial \zeta}{\partial x} \mathbf{i}+\frac{\partial \zeta}{\partial y} \mathbf{j}+\frac{\partial \zeta}{\partial z} \mathbf{k}\right) \tag{5-48}
\end{equation*}
$$

It follows that the normal to the surface normal, is the difference between this vector and the wind velocity, i.e.

$$
\begin{equation*}
\mathbf{V}_{\mathbf{A}_{\perp}}=\mathbf{V}_{\mathbf{w}}-\mathbf{V}_{\mathbf{A}_{\|}}=\frac{\left(\mathbf{n} \times \mathbf{V}_{\mathbf{w}}\right) \times \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}}=\left(\mathbf{n} \times \mathbf{V}_{\mathbf{w}}\right) \times \mathbf{n} \tag{5-49}
\end{equation*}
$$

This vector is tangent to the surface and in the direction of the wind velocity. It represents the direction the flow makes as it flow over the surface. The next step turns this vector into a unit wind velocity and is given by

$$
\begin{equation*}
\mathbf{u}_{\mathbf{A}_{\perp}}=\frac{\mathbf{V}_{\mathbf{A}_{\perp}}}{\left|\mathbf{V}_{\mathbf{A}_{\perp}}\right|} \tag{5-50}
\end{equation*}
$$

Where the lower case u denotes the unit vector. The final step is to determine the surface wind speed (assuming absence of boundary layer, i.e. inviscid flow) and present methods to estimate the wind velocity at altitude.

## Constant Mass Flow Model

Consider ideal gas flowing over a large flat plate at speed $V_{\infty}$ with constant velocity profile (which means we ignore the boundary layer). Also, assume that, some distance $H_{f}$ (from here on called fictitious ceiling) above the plate, the shape of a streamline is unaffected by the shape of the obstructions on the plate (from here on called terrain) and, thus, can be approximated by a straight line as illustrated in Figure 5-25. Then, according to Bernoulli's law, the speed of the air flowing along that streamline is constant. We call this the dividing streamline as it separates the upper and lower regions in the flow. In the lower region, between the dividing streamline and terrain, the presence of obstacles on the plate will cause the flow to accelerate. Considering the flow is quasi-two-dimensional (i.e. assuming it can be approximated as a 2-dimensional flow), the mass flow rate in the far-field (left side of Figure 5-25) can be approximated as

$$
\begin{equation*}
\dot{m}_{\infty}=\bar{\rho} V_{\infty} A_{\infty}=\bar{\rho} V_{\infty} H_{f} d w \tag{5-51}
\end{equation*}
$$



Figure 5-25: The wind simulator is based on the conservation of mass flow between the dividing streamline at height $H_{f}$ and the terrain.

Where $\bar{\rho}$ is the "average" density between the surface and $H_{f}$ and $A_{\infty}$ is the cross-sectional area, given by $A_{\infty}=H_{f} d w$, where $d w$ is an infinitesimal width. Since the flow must to comply with the mass flow rate at any point and assuming the absence of flow separation over the terrain (since the fluid is assumed ideal), it follows that in the near-field, for instance over the top of the terrain in Figure 5-25, the mass flow rate is given by

$$
\begin{equation*}
\dot{m}(z)=\bar{\rho} \bar{V}_{0}\left(z_{m}\right)\left(H_{f}-z_{m}\right) d w \tag{5-52}
\end{equation*}
$$

Where $z_{m}$ is the elevation of the terrain. Conservation of mass requires this to be equal to that in the farfield, per Equation (5-51). Thus, the flow speed over the terrain complies with

$$
\begin{equation*}
\dot{m}(z)=\dot{m}_{\infty} \Leftrightarrow \bar{\rho} \bar{V}_{0}\left(z_{m}\right)\left(H_{f}-z_{m}\right) d w=\bar{\rho} V_{\infty} H_{f} d w \quad \Leftrightarrow \quad \bar{V}_{0}\left(z_{m}\right)=V_{\infty} \frac{H_{f}}{H_{f}-z_{m}} \tag{5-53}
\end{equation*}
$$

This simple representation does not satisfy the aforementioned assumption that the speed along the dividing streamline is constant. It also violates the results from the PFT and in which the wind speed near the top of the island mountain is higher near the surface than at altitude, as shown in Figure 5-22. Thus, the wind speed $\bar{V}_{0}$ is really the average speed over the constriction. An approximation of this wind profile is accomplished in the wind generator as follows. As stated earlier, it is assumed the flow velocity, $V_{\infty}$, is uniform along the dividing streamline. At the surface of the terrain, it has a value $V_{0}$, as shown in Figure 5-26. Thus, the average of any given speed profile $V(z)$ is obtained using the average value theorem of calculus as shown below

$$
\begin{equation*}
\bar{V}_{0}\left(z_{m}\right)=\frac{1}{H_{f}-z_{m}} \int_{z_{m}}^{H_{f}} V(\zeta) d \zeta \tag{5-54}
\end{equation*}
$$



Figure 5-26: Wind speed profile analysis for the wind simulator used by the flight simulator.
Where $\zeta$ is the integration variable. This theorem can be used to obtain the value of the wind speed at the terrain level. The simplest profile assumes a linear change between the extremes and offers reasonable similarities to the results obtained using the PFT. It is simple enough to allow the value of the surface wind speed to be derived using algebra. The average speed $\bar{V}_{0}$ is the average of the surface speed, $V_{0}$ and the speed at the dividing streamline, $V_{\infty}$, i.e.

$$
\begin{equation*}
\bar{V}_{0}=\frac{1}{2}\left(V_{\infty}+V_{0}\right)=V_{\infty} \frac{H_{f}}{H_{f}-z_{m}} \Leftrightarrow V_{0}=V_{\infty}\left(\frac{2 H_{f}}{H_{f}-z_{m}}-1\right) \tag{5-55}
\end{equation*}
$$

The speed is the maximum speed in the profile. This allows us to develop the following parametric expression for the wind velocity as a function of altitude $z$ and terrain altitude. First, we define the parameter $\xi$ as shown below

$$
\begin{equation*}
\xi=\frac{z-z_{m}}{H_{f}-z_{m}} \tag{5-56}
\end{equation*}
$$

Yielding the parametric equation

$$
\begin{equation*}
V(\xi)=V_{\infty}\left(\frac{2 H_{f}}{H_{f}-z_{m}}-1\right)(1-\xi)+V_{\infty} \xi=V_{\infty}\left[\left(\frac{2 H_{f}}{H_{f}-z_{m}}-1\right)(1-\xi)+\xi\right] \tag{5-57}
\end{equation*}
$$

If we substitute the parameter $\xi$ and simplify, this becomes

$$
\begin{equation*}
V(z)=\frac{2 V_{\infty}}{H_{f}-z_{m}}\left[H_{f}\left(\frac{H_{f}-z}{H_{f}-z_{m}}-1\right)-\frac{z_{m}}{2}+z\right] \tag{5-58}
\end{equation*}
$$

Conversely, if the value of $V_{0}$ has been calculated a priori, this becomes

$$
\begin{equation*}
V(\xi)=V_{0}(1-\xi)+V_{\infty} \xi \tag{5-59}
\end{equation*}
$$

As an example, if $H_{f}$ is $35000 \mathrm{ft}, z_{m}$ is 10000 ft , and $V_{\infty}$ is $20 \mathrm{ft} / \mathrm{s}$, then the average and surface wind speeds should be

$$
\begin{aligned}
& \bar{V}_{0}(10000)=V_{\infty} \frac{H_{f}}{H_{f}-z_{m}}=20 \frac{35000}{25000}=28 \mathrm{ft} / \mathrm{s} \\
& V_{0}=V_{\infty}\left(\frac{2 H_{f}}{H_{f}-z_{m}}-1\right)=20\left(\frac{70000}{25000}-1\right)=36 \mathrm{ft} / \mathrm{s}
\end{aligned}
$$

And the wind speed at $z$ of 15000 ft is given by

$$
\begin{aligned}
& \xi=\frac{z-z_{m}}{H_{f}-z_{m}}=\frac{15000-10000}{35000-10000}=\frac{1}{5} \\
& \Rightarrow V\left(\frac{1}{5}\right)=V_{\infty}\left[\left(\frac{2 H_{f}}{H_{f}-z_{m}}-1\right)(1-\xi)+\xi\right]=20\left[\left(\frac{70000}{15000}-1\right)\left(1-\frac{1}{5}\right)+\frac{1}{5}\right] \approx 32.8 \mathrm{ft} / \mathrm{s}
\end{aligned}
$$

Note that other wind profiles may be developed besides the linear one and used for the wind simulation. Also note that in typical terrain (with sporadic peaks among flatlands, e.g. like that illustrated in Figure 5-12 through Figure 5-14) the value of $H_{f}$ may range from 45000 to 50000 ft , representing the height of the troposphere.

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## 6. Aircraft Performance Theory

The flight of conventional aircraft can be broken into a number of distinct phases: Take-Off, Climb, Cruise, Descent, and Landing. Aircraft Performance Theory is the mathematical analysis of these phases. It allows parameters such as best rate of climb or the airspeed of longest glide to be determined. This chapter presents a compressed version of aircraft performance theory. The formulae provided will be essential in route selection, as well as directing the autopilot to use the most efficient maneuver ${ }^{1}$, based on atmospheric convection. The scope of aircraft performance theory is extensive, so only methods that are practical for the CIGA are considered here. For this reason, take-off and landing analysis are omitted because these do not constitute maneuvers used and they would therefore expand the scope of this work beyond what is practical. Only methods applicable to climb, cruise, range, endurance, descent, and selected maneuvers that involve acceleration will be presented.

This dissertation does not consider flight phases associated with the operation of helicopters, or UAVs like tri-, quad-, or hexa-copters and similar, of which hover is an example. The generation of lift of such vehicles is very inefficient and, thus, they do not lend themselves well to the capability of a CIGA algorithm. Rather, it is assumed that aircraft utilizing the CIGA feature fixed wing of high aspect ratio, low drag, and low wing loading. The operation of such aircraft largely consists of short time periods of acceleration and deceleration, followed by long periods of relatively constant airspeed. Regardless, such aircraft can perform a large number of maneuvers that involve acceleration of one kind or another; for instance, turning flight, rolls, loops, and many others. Turning flight is important to the CIGA; rolls, loops, and other aerobatic maneuvers are not.

The chapter also presents a more specialized branch of performance theory; the performance of gliding flight. This topic is vital for the flight management system, as the CIGA will turn off engine power if conditions for gliding are favorable. In short, the topics of this chapter include:

- Airspeed Measurement Theory
- Fundamentals of Lift and Drag Coefficients
- Classical Aircraft Performance Theory
- Performance Theory of Gliding Flight

As an example of the utility obtained from this chapter consider this: All fixed-wing aircraft have specific optimal maneuvering speeds. For instance there is an airspeed at which the aircraft will have its fastest rate of climb, or minimum power required, or farthest glide distance, and so on. The only issue is that, many of these airspeeds change if the airplane is operating in head- or tailwind, or in up- or downdraft. The theory detailed in this chapter allows these changes to be accounted for, something that is used by
${ }^{1}$ Note that not all maneuvers are loops or Immelmans. A maneuver is a specific control operation required to accomplish some task. Thus, the T-O is a maneuver, so is climb, or cruise, or banking. In this dissertation the term maneuver may refer to a complicated aerobatic maneuver or it may refer to a simple climb.
the SURFACES Flight Simulator, when using the so-called "SmartPilot" function. The simulator quickly completes performance analysis and pinpoints these optimal speeds in real-time. An example analysis performed by this software feature is shown in Figure 6-1.



Figure 6-1: An example performance analysis performed by the SURFACES Flight Simulator. Here, showing the rate-of-descent (in $\mathrm{ft} / \mathrm{min}$ or fpm ) as a function of true airspeed (in knots) for a sample aircraft. The blue curve represents a baseline, standard day, no-wind, no-convection condition. The red curve represents the same aircraft at $\mathbf{6 0 0 0} \mathrm{ft}$, in a $\mathbf{1 0}$ knot headwind, in 150 fpm updraft, while banking $45^{\circ}$.

### 6.1 Airspeed Measurement Theory

The operation of aircraft hinges on maintaining specific airspeeds for longer or shorter periods. For instance, a specific airspeed must be maintained to achieve a maximum Rate-of-Climb (ROC). Another specific airspeed results in the minimum amount of energy consumed; a third one allows a maximum range in pure glide to be accomplished, and so on. Some of these airspeeds vary depending on the heador tailwind or on whether the airplane is in an updraft (lift) or downdraft (sink).

### 6.1.1. Types of Airspeeds

Generally, these specialized airspeeds are given definite names and are denoted by the letter $V$ followed by a subscript. They are commonly referred to as $V$-speeds. Table 6-1 lists the airspeeds that are of importance to the GICA method (and assumes low speed, low drag aircraft).

Then there are types of airspeed that depend on how they are measured. There is an indicated, calibrated, equivalent, and true airspeed. There is also ground speed and Mach number. These airspeeds are of crucial importance in the operation of the aircraft and, thus, must be included in this discussion. They are presented below.

Table 6-1: V-Speeds of Importance

| $V$-speed | Description |
| :---: | :--- |
| $V_{B A}$ | Minimum Rate-of-Descent airspeed, which yields the least altitude lost in a unit time. |
| $V_{B G}$ | Best glide speed. Minimum or Best Angle-of-Descent airspeed. This speed will result in <br> the shallowest glide angle and will yield the longest range, should the airplane lose <br> engine power. |
| $V_{H}$ or $V_{\max }$ | Maximum level airspeed. |
| $V_{N E}$ | Never-exceed speed or maximum structural airspeed. |
| $V_{N O}$ | Normal operating speed; also called maximum structural cruising speed. This speed <br> should not be exceeded except in smooth air, and then only with caution. |
| $V_{R}$ | Rotation speed. The speed at which the airplane's nose wheel loses contact with the <br> ground. It is high enough to ensure the aircraft can reach T-O safety speed (also called <br> $V_{2}$ ) at 50 ft (GA) or 35 ft (commercial) in case of an engine failure on a multiengine <br> aircraft. |
| $V_{R E F}$ | Landing reference speed or threshold crossing speed, typically 1.2•V $\mathrm{V}_{\text {so }}$ to 1.3•V ${ }_{\text {so }}$. The <br> factor 1.2 is typically used for military aircraft, but 1.3 for civilian aircraft per $\S 23.73$, <br> Reference Landing Approach Speed. |
| $V_{S}$ | Stalling speed or minimum steady flight speed for which the aircraft is still controllable. |
| $V_{X}$ | The best angle of climb airspeed (max altitude gain per unit distance). |
| $V_{Y}$ | The best rate of climb airspeed (max altitude gain per unit time). |

### 6.1.2. Instrument Airspeed, $V_{I A S}$

The instrument airspeed the pilot reads off the airspeed indicator. The reading can be affected by three kinds of error:
(1) Indication error (due to flaws in the instrument itself),
(2) Position error (due to incorrect location of static or Pitot sensors), and
(3) Pressure lag error (due to rapid change in pressure, such as when a fighter climbs so rapidly the indication system doesn't keep up with the change in pressure and "lags").

Instrument airspeed is denoted by the variable $V_{I A S}$. In the SI -system, the units are typically $\mathrm{m} / \mathrm{s}$ or kmh . In the UK-system, the units are $\mathrm{ft} / \mathrm{s}, \mathrm{mph}$, or knots. It is useful to identify this type of a measurement using the unit KIAS, which stands for Knots, Indicated AirSpeed.

### 6.1.3. Speed of Sound

The speed of sound is the rate at which a pressure wave propagates through fluid. For air, it can be estimated in terms of $\mathrm{ft} / \mathrm{s}$ using the following expression:

$$
\begin{equation*}
a=\sqrt{\gamma R T} \tag{6-1}
\end{equation*}
$$

Where $R$ is the universal gas constant, $\gamma$ is the ratio of specific heats, and $T$ is the temperature of the air in which the aircraft is operating. For the troposphere, the ratio of specific heats is 1.4 and the universal gas constant is $286.9 \mathrm{~m}^{2} /\left(\mathrm{K} \cdot \mathrm{s}^{2}\right)$ or $1716 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} /$ slug. ${ }^{\circ} \mathrm{R}$.

### 6.1.4. Calibrated Airspeed, $V_{C A S}$

If the error, $\Delta$ error, in the airspeed indicator is known, the calibrated airspeed is defined as:

$$
\begin{equation*}
V_{C A S}=V_{I A S}+\Delta \text { error } \tag{6-2}
\end{equation*}
$$

### 6.1.5. Equivalent Airspeed, $V_{E A S}$

The equivalent airspeed is what the airplane would have to maintain at Sea-Level in order to generate the same dynamic pressure as that experienced at the specific flight condition (altitude and true airspeed). It relates to the true airspeed as follows:

$$
\begin{equation*}
V_{E A S}=V_{T A S} \sqrt{\rho / \rho_{S L}}=V_{T A S} \sqrt{\sigma} \tag{6-3}
\end{equation*}
$$

Where $\rho$ is the density of air and $\delta$ is the density ratio, as is discussed in Chapter 5, Atmospheric Modeling. The equivalent airspeed can also be calculated if the calibrated airspeed in know, using the following expression:

$$
\begin{equation*}
V_{E A S}=V_{C A S} \sqrt{\frac{p}{p_{0}}} \sqrt{\frac{\left(1+q_{c} / p\right)^{0.286}-1}{\left(1+q_{c} / p_{0}\right)^{0.286}-1}} \tag{6-4}
\end{equation*}
$$

Note that the power 0.286 is shorthand for the ratio $1 / 3.5$. Conversely, if the equivalent airspeed is known, the calibrated airspeed can be calculated as follows:

$$
\begin{equation*}
V_{C A S}=V_{E A S} \sqrt{\frac{p_{0}}{p}} \sqrt{\frac{\left(1+q_{c} / p_{0}\right)^{0.286}-1}{\left(1+q_{c} / p\right)^{0.286}-1}} \tag{6-5}
\end{equation*}
$$

Where $p_{0}$ and $p$ are the static pressures at S-L and altitude, respectively, and $q_{c}$ is the compressible dynamic pressure is given by:

$$
\begin{equation*}
q_{c}=P\left[\left(1+0.2 M^{2}\right)^{3.5}-1\right] \tag{6-6}
\end{equation*}
$$

Using the Mach number:

$$
\begin{equation*}
M=V / a \tag{6-7}
\end{equation*}
$$

If the calibrated airspeed is known, the Mach number for compressible flow conditions can be determined from:

$$
\begin{equation*}
M=\sqrt{\frac{2}{\gamma-1}\left(\left[\frac{1}{\delta}\left[\left\{1+\frac{\gamma-1}{2}\left(\frac{V_{C A S}}{661.2}\right)^{2}\right\}^{\frac{\gamma}{\gamma-1}}-1\right]+1\right]^{\frac{\gamma-1}{\gamma}}-1\right)} \tag{6-8}
\end{equation*}
$$

Where the constant 661.2 is the standard day speed of sound at S -L in knots, $V_{\text {CAS }}$ is in KCAS, $\delta$ is the pressure ratio, and $\gamma$ is the specific heat ratio (1.4).

### 6.1.6. True Airspeed, $V_{T A S}$

The true airspeed is the rate at which the air molecules in the far-field pass the aircraft (since the local molecules accelerate or decelerate as they pass the airplane). The following expression is used to convert equivalent airspeed to true airspeed:

$$
\begin{equation*}
V_{T A S}=\frac{V_{E A S}}{\sqrt{\sigma}} \approx \frac{V_{C A S}}{\sqrt{\sigma}} \tag{6-9}
\end{equation*}
$$

The right approximation is valid only if the Mach number is low ( 0.3 or less) at low altitudes, as the equivalent and calibrated airspeeds are close to one another.

### 6.1.7. Ground Speed, $V_{G S}$

The ground speed is the rate at which the aircraft moves along the ground. This speed equals the true airspeed if there is no wind aloft (perfectly calm). However, if windy, the component of the wind parallel to the direction of the aircraft will either add (tailwind) or subtract (headwind) from the true airspeed. If this parallel wind component, denoted by $V_{w}$, is known, then the following expression is used to convert the true airspeed to ground speed:

$$
\begin{equation*}
V_{G S}=V_{T A S}+V_{w} \tag{6-10}
\end{equation*}
$$

### 6.2 Fundamentals of Lift and Drag Coefficients

Any object that moves through a fluid induces a pressure field in its vicinity (see Figure 6-2). The pressure field changes the pressure on its surface and induces a resultant pressure force, $R$, which acts on the object. Then, we define Lift, $L$, as the component of this force that is normal to the trajectory (or flight path). Similarly, Drag, $D$, is defined as the component of this force tangent to the trajectory. In addition to the pressure force, viscous friction adds to the total drag force.

Note that it is a convention in the literature to denote forces and moments for 2-dimensional geometry by a lower case letter but capitalization when referring to 3-dimensional geometry. This way, lift, drag, and moment for an airfoil would be denoted by $l, d$, and $m$, respectively, but using $L, D$, and $M$ for a 3dimensional wing. The difference between the two, fundamentally, is that a wing has a finite Aspect Ratio ( $A R$ ), whereas an airfoil can be considered like a wing of infinite span and, thus, infinite AR. This convention will be adhered to in this dissertation. Consequently, the lift, drag, and moment coefficients for an airfoil are written using a lower case identifiers; $C_{l}, C_{d}, C_{m}$. Capitalized identifiers are used for 3D wings or an aircraft as a whole: $C_{L}, C_{D}, C_{M}$, although the last one is also written as $C_{m}$. Performance analysis depends on the use of lift and drag coefficients, denoted by $C_{L}$ and $C_{D}$, respectively. It is of crucial importance to give a brief overview of these coefficients in support of the discussion that follows.


Figure 6-2: An object moving in fluid induces pressure field. (from Ref. [3])

### 6.2.1. Representation of Forces and Moments

The total force (or resultant force) generated by a wing depends on several parameters; the wing's geometry, density of air, airspeed, and the angle the chord line of the wing's airfoils make to the flow of air, the Angle-of-Attack (from here on also referred to as AOA). While the wing is 3-dimensional, it is usually treated as a set of two 2-dimensional geometric features; the airfoil ( $x-z$ plane as shown in Figure $6-3$ ) and planform ( $x-y$ plane). It can be shown by dimensional analysis (i.e. Buckingham's $\Pi$-Theorem) that the equation describing this resultant force, $r$, is given by:

$$
\begin{equation*}
r=\frac{1}{2} \rho V^{2} \cdot S \cdot C_{r} \tag{6-11}
\end{equation*}
$$

Where $\rho$ is density of air, $V$ is airspeed, $S$ is reference area, typically wing area, and $C_{r}$ is a nondimensional coefficient that relates AOA to the force.


Figure 6-3: Forces and moments acting on an airfoil (left) and the definition of Normal and Chordwise Force on an airfoil at a high AOA (right). (from Ref. [3])

Figure 6-3 shows that the lift (force normal to the airspeed), drag (force parallel to the airspeed), and pitching moment (which all are assumed to act at the quarter chord) can be defined as follows:

$$
\begin{align*}
& l=\frac{1}{2} \rho V^{2} \cdot S \cdot C_{l}=\frac{1}{2} \rho V^{2} \cdot S \cdot C_{r} \cos \alpha \\
& d=\frac{1}{2} \rho V^{2} \cdot S \cdot C_{d}=\frac{1}{2} \rho V^{2} \cdot S \cdot C_{r} \sin \alpha  \tag{6-12}\\
& m=\frac{1}{2} \rho V^{2} \cdot S \cdot c \cdot C_{m}
\end{align*}
$$

Sometimes it is of importance to consider the normal and chordwise forces, $f_{n}$ and $f_{c}$, respectively (see Figure 6-3). The normal force is perpendicular to the wing plane (formed by the span- and chordwise vectors), while the chordwise force is parallel to the chord plane. At low angles-of-attack the magnitude of $f_{c}$ is close to the drag force and points toward the trailing edge of the airfoil. However, at high angles-of-attack $f_{c}$ actually points forward, toward the leading edge. Figure 6-3 shows that the normal and chordwise forces can be defined as follows:

$$
\begin{align*}
& f_{n}=\frac{1}{2} \rho V^{2} \cdot S \cdot\left(C_{l} \cos \alpha+C_{d} \sin \alpha\right) \\
& f_{c}=\frac{1}{2} \rho V^{2} \cdot S \cdot\left(C_{d} \cos \alpha-C_{l} \sin \alpha\right) \tag{6-13}
\end{align*}
$$

For 3-dimensional objects like aircraft, the representation of forces and moments that correspond to Equation (6-12) is given by:

$$
\begin{align*}
& L=\frac{1}{2} \rho V^{2} \cdot S \cdot C_{L} \\
& D=\frac{1}{2} \rho V^{2} \cdot S \cdot C_{D}  \tag{6-14}\\
& M=\frac{1}{2} \rho V^{2} \cdot S \cdot C_{M G C} \cdot C_{M}
\end{align*}
$$

Where $L, D, M$ refer to the 3-dimensional lift, drag, and pitching moment. This way, $C_{L}$ is the 3dimensional lift coefficient, $C_{D}$ the drag coefficient, and $C_{M}$ the pitching moment coefficient of the complete aircraft. These will be treated in more detail in Chapters 9,11 , and $15 . C_{M G C}$ is the wing's Mean Geometric Chord and $S$ is the reference wing area. Refer to Reference [3] for more details.

### 6.2.2. The Lift Coefficient

The lift coefficient relates the AOA to the lift force. If the lift force is known at a specific airspeed the lift coefficient can be calculated from:

$$
\begin{equation*}
C_{L} \equiv \frac{2 L}{\rho V^{2} S} \tag{6-15}
\end{equation*}
$$

## Properties of the 3-Dimensional Lift Curve

Figure 6-4 shows a typical 3-dimensional lift curve. It represents the behavior of the entire aircraft (wings, fuselage, HT, VT, etc.). The contribution of the individual components of the aircraft to the overall non-linear shape of the curve may make it very different from that of the selected airfoils.

## Lift Curve Slope, $C_{L \alpha}$

The lift curve slope is a measure of how rapidly the wing generates lift with unit change in AOA. The theoretical maximum of airfoils is $2 \pi$. The lift curve slope of a 3 -dimensional wing is always less than that of the airfoils it features. Once a certain AOA has been achieved the wing will display a pronounced reduction in the lift curve slope (see Figure 6-4). This point is called stall and, although only shown in one place in the figure, occurs both at a positive and negative angle-of-attack. The lift at stall dictates how much wing area the aircraft must feature for a desired stalling speed. See Section 8.4.3, Lift Related Stability Derivatives for methods to estimate the derivative.

## Maximum and Minimum Lift Coefficients, $C_{L \max }$ and $C_{L \text { min }}$

The largest and smallest magnitudes of the lift coefficient are denoted by $C_{L \max }$ and $C_{L \min }$, respectively. It indicates at what angle-of-attack the airplane will achieve its minimum airspeed (stalling speed), or what wing area is required for a desired stalling speed. As for the airfoil, the stall is defined as the flow conditions that follow the first lift curve peak, which is where the $C_{L \max }$ (or $C_{L \min }$ ) occurs [1]. Both values are required when generating aerodynamic loads for the structures group.

## $C_{L}$ at Zero AOA, $C_{L o}$

Is the value of the lift coefficient of the wing at zero AOA. It is of great importance in the scheme of things, because it affects the Angle-of-Incidence at which the wing must be mounted. Generally this
value ranges from 0.0 (for symmetric airfoils) to 0.6 (for highly cambered airfoils). It is negative for under-cambered airfoils (e.g. airfoils used near the root of high subsonic jet aircraft).


Figure 6-4: Important properties of the lift curve. (from Ref. [3])

## Angle-of-Attack at Zero Lift, $\alpha_{z L}$

This is the angle at which the wing generates no lift. For positively cambered airfoils this angle is always negative, unless some specific components (e.g. cambered fuselage) affect it greatly. For symmetrical airfoils it is always $0^{\circ}$.

## Linear Range

The linear range is analogous to that of the airfoil, except it applies to the entire aircraft. In this range, the following equation of a line can be used to describe how lift varies with AOA.

$$
\begin{equation*}
C_{L}=C_{L_{0}}+C_{L_{\alpha}} \alpha \tag{6-16}
\end{equation*}
$$

Equation (6-16) allows the AOA corresponding to a specific lift coefficient to be determined provided the lift curve slope known:

$$
\begin{equation*}
\alpha=\frac{C_{L}-C_{L_{0}}}{C_{L_{\alpha}}} \tag{6-17}
\end{equation*}
$$

## Angle-of-Attack for Maximum Lift Coefficient, $\alpha_{\text {stall }}$

Once a certain AOA is reached, a pronounced reduction in lift curve takes place with further increase in $A O A$; this is the stall.

## Angle-of-Attack where Lift Curve becomes Non-Linear, $\alpha_{N L}$

Once a certain AOA is reached the wing begins to display a pronounced reduction in the lift curve slope. This always happens before the stall AOA is reached.

### 6.2.3. The Drag Coefficient

The drag coefficient relates the drag to the lift force. If the drag force is known at a specific airspeed the drag coefficient can be calculated from:

$$
\begin{equation*}
C_{D} \equiv \frac{2 D}{\rho V^{2} S} \tag{6-18}
\end{equation*}
$$

The drag coefficient is called a drag model.

## Basic Drag Modeling

Basic drag modeling is the mathematical combination of all sources of drag for a vehicle, such that the effect of changing its orientation with respect to its path of motion and fluid velocity is realistically replicated. This modeling culminates in the determination of the total drag coefficient, $C_{D}$. The total drag coefficient consists of basic pressure drag, skin friction, lift-induced drag, wave drag, and other sources, commonly referred to as miscellaneous drag. Typically, basic pressure drag, skin friction drag, and miscellaneous drag are lumped together into a single number, called the minimum drag. Only aircraft that operate at high subsonic or supersonic airspeeds have to contend with wave drag. Finally, miscellaneous drag refers to drag caused the numerous protrusions and discontinuities in the generally smooth surfaces of the aircraft.

## Minimum Drag, $C_{D \text { min }}$

The drag coefficient associated with the minimum drag will from now be referred to as $C_{D \min }$. It is also referred to as a profile drag or parasitic drag or zero-lift drag, although this dissertation will only use the term minimum drag. The minimum drag coefficient represents the lowest drag the vehicle will generate.

## Quadratic Drag Modeling

A standard way to present the drag coefficient is to relate it to the lift coefficient using a quadratic polynomial. The method can provide an accurate prediction over a range of low lift coefficients, although the accuracy drops rapidly at the extremes of the drag polar. A standard "simplified" quadratic presentation for the drag coefficient is:

$$
\begin{equation*}
C_{D}=C_{D_{\min }}+\frac{C_{L}^{2}}{\pi \cdot A R \cdot e}=C_{D_{\min }}+k \cdot C_{L}^{2} \tag{6-19}
\end{equation*}
$$

Where $A R$ is the reference Aspect Ratio (see Chapter 1), $e$ is called the Oswald efficiency, and $k$ is the lift-induced drag constant. A far more "realistic" presentation for the drag coefficient is the adjusted drag model.

$$
\begin{equation*}
C_{D}=C_{D_{\min }}+k \cdot\left(C_{L}-C_{L_{\min D}}\right)^{2} \tag{6-20}
\end{equation*}
$$

Where $C_{L \min D}$ is the lift coefficient where drag becomes a minimum.

## Lift-Induced Drag Constant, k

Is the constant whose product with the lift coefficient squared yields the lift-induced drag. It is given by:

$$
\begin{equation*}
k=\frac{1}{\pi \cdot A R \cdot e}=\frac{1}{\pi \cdot A R_{e}} \tag{6-21}
\end{equation*}
$$

The product $A R \cdot e$, or $A R_{e}$, is often referred to as the Effective Aspect Ratio. It can be considered a factor that renders the $A R$ less effective than the geometric value would indicate. Methods to estimate $e$ are provided in Reference [3].

## $C_{D}$ Dependency on $\alpha$ and $\beta$

Sometimes, $C_{D \min }$ is treated as if constant with respect to $\alpha$ and $\beta$. This is not true in real airflow. Changes in $\alpha$ and $\beta$ will move the laminar to turbulent flow transition line and reshape flow separation regions. This changes the pressure drag, modifying the minimum drag coefficient.

## Compressibility Effects

The effect of compressibility is accounted for by modifying $C_{D \min }$ at high subsonic airspeeds using a special correction factor. However, for the aircraft considered in this document such corrections may be omitted.

## Drag Counts

A "drag count" is the drag coefficient multiplied by a factor of 10000. For instance, 250 drag counts is equivalent to a $C_{D}=0.0250 ; 363$ drag counts is equivalent to a $C_{D}=0.0363$, and so on.

## Equivalent Flat Plate Area (EFPA)

The Equivalent Flat Plate Area (denoted by $f$ ) is used when comparing the relative drag of different aircraft. It is simply the product of the minimum drag coefficient and the reference area, as shown below. Alternatively, this is nothing but the minimum drag force at the given airspeed, $D_{\min }$, divided by the dynamic pressure, $q$ :

$$
\begin{equation*}
f=S_{r e f} \times C_{D \min }=\frac{D_{\min }}{q} \tag{6-22}
\end{equation*}
$$

The concept assumes the drag of the airplane is equivalent to that of a fictitious plate that has a drag coefficient $C_{D}=1.0$. This way, if the flat plate area of an airplane is $10 \mathrm{ft}^{2}$, it means its drag amounts to that of a flat plate of the same area moving normal to the flight path. The concept is bogus in many respects. For instance, it disregards the effect of Reynolds and Mach numbers, the $C_{D}$ of a flat plate at

Re around $10^{5}$ is actually closer to 1.17 and no notion is given as to the true geometry of this "plate" (i.e. is it rectangular or circular or any other shape?).

## Limitations of the Quadratic Drag Model

The drag of some aircraft cannot be accurately represented with the quadratic drag model. A sailplane may be "clean" enough to provide superior aerodynamics and Natural Laminar Flow (NLF) with a drag bucket (although this will only manifest itself at low AOAs and higher airspeed). Consequently, the quadratic approximation will give erroneous values of max lift-to-drag ratio and where it occurs. The quadratic model works well for airplanes that do not have a noticeable drag bucket, except at very high or very low lift coefficients (see Figure 6-5). It is vital to be aware of this limitation as it leads to erroneous prediction of best endurance and range airspeeds, in particular of airplanes with very low wing loading (LSA aircraft or RC aircraft).


Figure 6-5: Curve-fitting the true drag polar. Note that if the airfoil (or vehicle) features a camber, the simplified drag polar is no longer a valid representation of the drag polar. (from Ref. [3])

## Approximating the Drag Coefficient at High Lift Coefficients

Figure 6-5 reveals a critical problem in all drag modeling; at higher $C_{L} \mathrm{~S}$, the drag model deviates drastically from the actual drag and under predicts it severely. The deviation is caused by a rapid growth of flow separation with AOA and the second order approximation cannot keep up with the resulting increase in drag. This is a serious problem when estimating aircraft performance at low airspeeds. Therefore, important airspeeds, such as best angle of climb, minimum rate of descent, and others, are shifted to lower airspeeds than observed in practice. This shortcoming can be remedied as follows.

Consider the hypothetical wind tunnel test data in Figure 6-6. A satisfactory approximation to the data cannot be provided by the simplified or adjusted drag models at high (or low) lift coefficients. The graph
shows the test data begins to deviate sharply from the curve starting at $C_{L}=1.15$ (and $C_{L}=-0.45$ ). Of course we are primarily interested in the positive lift coefficient, as this is needed for low speed performance predictions. In order to work around this predicament, we add a spline to the segment of the polar where the deviation becomes pronounced. In this method, once the $C_{L}$ exceeds a certain value, which here will be called $C_{L m}$, a quadratic (or cubic) spline is created to simply replace the values of the adjusted drag model.


Figure 6-6: The same hypothetical drag polar, showing the improvement in prediction accuracy at higher lift coefficients by the introduction of the quadratic spline, $C_{D \text { mod }}$. (from Ref. [3])

The method presented here effectively defines a new quadratic polynomial and splices it to the adjusted drag model at $C_{L m}$. Other splines are certainly possible; however, the advantage of the quadratic spline is the simplicity of its determination and the acceptable accuracy it provides. The method allows the spline to blend smoothly with the underlying adjusted drag model. The first step is to define a modified drag coefficient to be used for $C_{L}>C_{L m}$. It can be represented with:

$$
\begin{equation*}
C_{D \text { mod }}=A C_{L}^{2}+B C_{L}+C \tag{6-23}
\end{equation*}
$$

Ultimately, the task is to define the constants $A, B$, and $C$ so $C_{D \bmod }$ can be evaluated. To do this, two conditions have to be satisfied at $C_{L \mathrm{~m}}$ :

$$
\begin{array}{ll}
\text { (1) Equal drag at } C_{L m}: & C_{D \bmod }\left(C_{L m}\right)=C_{D}\left(C_{L m}\right) \text { and } \\
\text { (2) Equal slope at } C_{L m}: & \left.\frac{\partial C_{D \bmod }}{\partial C_{L}}\right|_{a t}=\left.\frac{\partial C_{D}}{\partial C_{L}}\right|_{a t}
\end{array}
$$

A third condition is needed to finalize the determination of the coefficients; it is that the value of $C_{D \bmod }$ at $C_{L \text { max }}$ must match that of the wind tunnel data. That aside, the function for $C_{D \bmod }$ has three constants
( $A, B, C$ ), so three equations are required to determine them. One of the equations requires the derivative of both $C_{D}$ and $C_{D \text { mod }}$ to be determined. These are presented below:

Slope of the adjusted drag model: $\quad \frac{\partial C_{D}}{\partial C_{L}}=\frac{\partial}{\partial C_{L}}\left[C_{D \min }+k\left(C_{L}-C_{L \min D}\right)^{2}\right]=2 k\left(C_{L}-C_{L \min D}\right)$
Slope of the modified drag model: $\quad \frac{\partial C_{D \text { mod }}}{\partial C_{L}}=\frac{\partial}{\partial C_{L}}\left[A C_{L}^{2}+B C_{L}+C\right]=2 A C_{L}+B$

Now the three equations that allow the constants $A, B$, and $C$ can be written as follows:

Equation (1):

$$
A C_{L m}^{2}+B C_{L m}+C=C_{D \min }+k\left(C_{L m}-C_{L \min D}\right)^{2}
$$

Equation (2):
$2 A C_{L m}+B=2 k\left(C_{L m}-C_{L \text { min } D}\right)$
Equation (3):
$A C_{L \text { max }}^{2}+B C_{L \text { max }}+C=C_{D \text { stall }}$

Rearranging this in a matrix form yields the following expression that allows $A, B$, and $C$ to be determined using any matrix method; Cramer's rule or matrix inversion methods:

$$
\left[\begin{array}{ccc}
C_{L m}^{2} & C_{L m} & 1  \tag{6-24}\\
2 C_{L m} & 1 & 0 \\
C_{L \max }^{2} & C_{L \max } & 1
\end{array}\right]\left\{\begin{array}{l}
A \\
B \\
C
\end{array}\right\}=\left\{\begin{array}{c}
C_{D \min }+k\left(C_{L m}-C_{L \min D}\right)^{2} \\
2 k\left(C_{L m}-C_{L \min D}\right) \\
C_{D s t a l l}
\end{array}\right\}
$$

Then, once $A, B$, and $C$ have been determined the drag model is further refined as follows:

$$
C_{D}=\left\{\begin{array}{cc}
C_{D \min }+k\left(C_{L}-C_{L \min D}\right)^{2} & \text { if } C_{L} \leq C_{L m} \\
A C_{L}^{2}+B C_{L}+C & \text { if } C_{L}>C_{L m}
\end{array}\right.
$$

This model has been implemented in Figure 6-6. The improvements in the capability of the drag model is demonstrated by how the quadratic spline, represented by $C_{D \bmod }$, smoothly follows the wind tunnel data starting at $C_{L m}$. Note that although wind tunnel data is assumed here, the same approach can also be used for analytical estimation of the drag coefficient. For such work, it is reasonable to select $C_{L m}$ as the value $1 / 2\left(C_{L \min D}+C_{L \max }\right)$ as a first guess.

### 6.3 Fundamentals of Propeller Thrust

This section presents the basics of thrust generated using a propeller, powered either by a piston engine or an electric motor. This discussion is warranted because most sUAVs are powered by propellers and it explains how the SURFACES Flight Simulator calculates propeller thrust during simulation.

### 6.3.1. Basic Propeller Geometry and Nomenclature

A three-bladed propeller is shown in Figure 6-7, rotating about the axis of rotation. The propeller blades generate the thrust by developing lift analogous to a cantilevered wing that moves in a circular path, rather than along a straight one. Just like an airplane's wing, the planform of the propeller blade has a profound impact on the magnitude of the thrust created, as well as the amount of power required to rotate it, not to mention side effects such as noise. The spinner is an aerodynamically shaped cover that reduces the drag of the propeller hub and protects it from the elements. The pressure differential between the front and aft faces of the propeller blades sheds a tip vortex that is carried back by the airflow. This forms the classical helical shape shown in the image. Only one vortex is shown for clarity; two additional ones are also formed by the other blades. A frontal projection of the three-bladed propeller is shown in Figure 6-8, where $R$ is the blade radius, $r$ is the radius to an arbitrary blade station, and $\Omega$ is the rotation rate, typically in radians per second or minutes. Note that the propeller theory is presented in detail in Reference [3].


Figure 6-7: Propeller helix. (from Ref. [3])


Figure 6-8: Propeller geometry. (from Ref. [3])

### 6.3.2. Fixed versus Constant Speed Propellers

Propeller efficiency is an indicator as to how much engine power is being converted into propulsive power (Thrust x Airspeed). Thus, if a particular propeller is 0.80 efficient at a specific condition, then $80 \%$ of the engine power is converted into propulsive power. The propeller efficiency is a function of airspeed and the RPM at which it rotates. Generally, propellers for aircraft are designed for a particular
airspeed or range of airspeeds specified by the airframe manufacturer. In this case the following rules of thumb apply:

> For low speed operations use a low pitch.

For high speed operations use a large (coarse) pitch².

The blade pitch angle on a fixed pitch propeller is permanently set. Such propellers are simple, light, and inexpensive. However, their optimum efficiency is achieved at a particular airspeed only, so the designer (or operator) must decide whether to emphasize climb or cruise performance and select a prop that favors one or the other. This gives rise to the terms "climb" and "cruise" propellers. Fixed pitch climb propellers achieve maximum propeller efficiency at a relatively low airspeed, making them ideal for use in airplanes where climb performance is important. Conversely, fixed pitch cruise propellers achieve optimum efficiency at higher airspeeds, making them suitable for higher cruise performance. An airplane with a climb-propeller has a lower maximum speed than the same aircraft with a corresponding cruise propeller.

A constant speed propeller combines the performance of a climb and cruise propeller by allowing pitch of the blades to be changes in flight. Figure 6-9 shows how the propeller efficiency typically varies with airspeed for these three propeller types. The green and blue curves represent the efficiency of propellers whose pitch is fixed. Constant speed propellers are only used in expensive aircraft. The aircraft on which this work focuses are likely to feature the lighter, less expensive, fixed-pitch propellers.


Figure 6-9: Two kinds of "fixed" pitch propellers versus a "constant speed" propeller. (from Ref. [3])

### 6.3.3. Power Required

Propulsive power is the power required to move a vehicle at a specific speed using specific force or thrust. Power is defined by:

$$
\text { Power }=\frac{\text { Work }}{\text { Time }}
$$

Since work is defined as the application of a force over a specific distance (Force $x$ Distance) we get:

[^15]$$
\text { Power }=\frac{\text { Work }}{\text { Time }}=\frac{\text { Force } \times \text { Distance }}{\text { Time }}=\text { Force } \times \frac{\text { Distance }}{\text { Time }}=\text { Force } \times \text { Speed }
$$

We therefore define required power as follows:

$$
\begin{equation*}
P_{R}=D \cdot V=\frac{1}{2} \rho V^{3} S C_{D} \tag{6-25}
\end{equation*}
$$

All variables have already been defined. The subscript $R$ stands for Required as this term represents the power required in the performance analysis of later sections.

### 6.3.4. Converting Power to Thrust

As stated above, aircraft performance analysis relies on the evaluation of required and available power. The former was determined above. Here we will show how to obtain the latter. First, we must review how to convert engine power into propeller thrust. In the UK system, the thrust is calculated as follows, using airspeed (in $\mathrm{ft} / \mathrm{s}$ ), engine power (in BHP), $\mathrm{P}_{\text {Bнр }}$, and propeller efficiency (dimensionless)

$$
\begin{equation*}
\text { Piston engine power (UK-system): } \quad T=\frac{\eta_{p} P}{V}=\frac{\eta_{p} \times 550 \times P_{B H P}}{V} \quad\left[\text { units } \mathrm{lb}_{\mathrm{f}}\right. \text { ] } \tag{6-26}
\end{equation*}
$$

Where $\eta_{p}$ is the propeller efficiency. The factor 550 converts the BHP into $\mathrm{ft} \cdot \mathrm{lb} / \mathrm{s} / \mathrm{s}$. In the SI system the conversion is not necessary as power is given in kW . In the SI-system (airspeed in $\mathrm{m} / \mathrm{s}$ and engine power in Watts), the following expression applies

Piston engine power (SI-system): $\quad T=\frac{\eta_{p} P}{V}=\frac{\eta_{p} P_{W}}{V} \quad$ [units N]

The expression indicates that as the airspeed $V$ approaches zero the thrust will trend toward infinity. This is physically impossible. What happens in reality can be understood by noting that the magnitude of the velocity through the propeller disc requires a constant flow of air toward it and efficiency trends toward zero. This, in turn, means $V$ is never zero. Thrust at zero forward airspeed can be estimated using the Momentum Theory or Blade Element Theory, both which are presented in References [2] or [3], to name a few. Typical propeller efficiencies for a range of propellers are shown in Table 6-2.

### 6.3.5. Approximation of Thrust as a Function of Airspeed

Mathematically swift and accurate methods are preferred when determining the characteristics of natural phenomenon, although this is, understandably, not always possible. However, such methods are particularly desirable in simulation, where many equations must be solved during a time-step. A method, called the Quadratic Interpolation Method is developed in Reference [3] and allows rapid estimation of propeller thrust with acceptable accuracy. It combines the momentum theory with knowledge of propeller efficiency at the optimum airspeed at fixed RPM, effectively allowing thrust to be presented as a function of airspeed only. The method states that the variation of thrust between the
static thrust condition and the optimum point for which the propeller was designed (i.e. maximum value of $\eta_{p}$ and the airspeed at which it occurs) can be described using a quadratic (or cubic) spline, as shown below

Table 6-2: Typical Values of the Propeller Efficiency

| Propeller Type | Phase | Value | Comments |
| :---: | :---: | :---: | :---: |
| High quality constant speed propeller | Idle | 0-0.1 | Assumes a propeller for standard manned aircraft. |
|  | T-O | 0.6 |  |
|  | Climb | 0.75 |  |
|  | Cruise | 0.88 |  |
| Constant speed propeller to use during conceptual design phase | Idle | 0-0.1 | Assumes a propeller for standard manned aircraft. |
|  | T-O | 0.55 |  |
|  | Climb | 0.70 |  |
|  | Cruise | 0.85 |  |
| High quality fixed pitch propeller | Idle | 0-0.05 | Assumes a propeller for standard manned aircraft. |
|  | T-O | 0.5 |  |
|  | Climb | 0.65 |  |
|  | Cruise | 0.8 |  |
| Typical fixed pitch propeller | Idle | 0-0.05 | Assumes a propeller for standard manned aircraft. |
|  | T-O | 0.45 |  |
|  | Climb | 0.6 |  |
|  | Cruise | 0.75 |  |
| Typical fixed pitch propeller for RC aircraft | Idle | 0-0.025 | Assumes a propeller for standard RC aircraft. |
|  | T-O | 0.3 |  |
|  | Climb | 0.4 |  |
|  | Cruise | 0.6 |  |

$$
\begin{equation*}
T(V)=\left(\frac{T_{\text {STATIC }}-2 T_{\max }}{V_{\max }^{2}}\right) V^{2}+\left(\frac{3 T_{\max }-2 T_{\text {STATIC }}}{V_{\max }}\right) V+T_{\text {STATIC }} \tag{6-28}
\end{equation*}
$$

Where $T_{\max }$ is the thrust at the airspeed, $V_{\max }$, where $\eta_{p}$ becomes maximum. It can be estimated using Equation (6-26) or (6-27). $T_{\text {STATIC }}$ is the static thrust for the propeller at the same power and RPM, when $V=0$. It is calculated using the expression

$$
\begin{equation*}
T_{\text {STATIC }}=P^{2 / 3}\left(2 \rho A_{2}\right)^{1 / 3} \tag{6-29}
\end{equation*}
$$

Where $P$ is the power (e.g. in $\mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s}$ ) and $A_{2}$ is the propeller disc area. As a rule of thumb, Equation (629) overestimates static thrust for large aircraft by some $15-20 \%$ for various reasons, such as blockage effects, the presence of the hub, which reduces the disk area, reduction of lift distribution near the tip and hub, and so on. Therefore, for design purposes, for typical aircraft, estimate $T_{\text {STATIC }}$ by the following empirical correction:

$$
\begin{equation*}
T_{\text {STATCC }}=0.85 P^{2 / 3}\left(2 \rho A_{2}\right)^{1 / 3}\left(1-\frac{A_{\text {spimer }}}{A_{2}}\right) \tag{6-30}
\end{equation*}
$$

Where $A_{\text {spiner }}$ is the frontal area of the spinner. For small sUAV use the following correction based on the work of authors such as Merchant and Miller [4] (2006), Ol and Zeune [5] (2008), Brandt and Selig [6] (2011), Oliveira et al [7] (2012), Deters et al [8] (2014), and others, the maximum propeller efficiency of typical propeller is close to $0.5-0.55$. For this reason, it is necessary to reduce the factor of 0.85 even further, to 0.47

$$
\begin{equation*}
T_{\text {STATIC }}=0.47 P^{2 / 3}\left(2 \rho A_{2}\right)^{1 / 3}\left(1-\frac{A_{\text {spinner }}}{A_{2}}\right) \tag{6-31}
\end{equation*}
$$

This value compares favorably to unpublished experiments for the Quanum Observer (see Chapter 3, $A$ Survey of Automation Technology for sUAVs). The application of $E=12.4 \mathrm{~V}$ and $I=22.5 \mathrm{Amps}$, using a 10 inch two-bladed propeller with a 1 inch spinner, resulted in a measured static thrust of 14.6 N ( 3.27 $\left.1 \mathrm{~b}_{\mathrm{f}}\right)$. This compares to $3.29 \mathrm{lb}_{\mathrm{f}}$ static thrust using Equation (6-31).

### 6.4 Classical Aircraft Performance Theory

This section presents important mathematical methods used to estimate the performance of the aircraft. These are called classical because they include standard methods taught to all aerospace or aeronautical engineers. However, it is important to realize two things: (1) their purpose is to introduce vital concepts of aircraft performance and (2) the accuracy of the methods is limited to the quality of the drag model employed. This is discussed in more detail in Section 6.4.1, The Climb Maneuver below. Note that for completeness, a few formulas pertain to jet aircraft. Jet engines are commercially available for sUAV, although their use is not treated in the energy harvesting discussion the follows in Chapter 9, Energy Harvesting. The following topics will be presented in this section:

- Climb Performance,
- Cruise Performance,
- Range and Endurance,
- Descent Performance, and
- Accelerated Flight Performance.


### 6.4.1. The Climb Maneuver

Classical aircraft performance theory studies the motion of the airplane using a free-body diagram like the one in Figure 6-10. The climb constitutes a set of pilot operations (high power setting, trim for and maintain a particular airspeed). Since it requires a dedicated control input to perform, it is truly a maneuver. The free-body diagram shows an airplane moving along a flight path. The $x$ - and $z$-axes pass through the Center of Gravity (CG) of the airplane such that the $x$ - and $z$-axis is a tangent and normal to
the flight path, respectively. The velocity is tangent to the flight path. It is balanced in terms of inertia, mechanical, and aerodynamic forces. However, a similar balance exists for the moments, although this is not shown in the diagram. The lift is the component of the resultant aerodynamic force generated by the aircraft that is perpendicular to the flight path (along its $z$-axis). The $d r a g$ is the component of the aerodynamic force that is parallel (along its $x$-axis). These are balanced by the weight, $W$, and the corresponding components of the thrust, $T$. For this reason, the resulting motion is one of steady state. A Datum is shown, which passes through the CG and is parallel to the chord line of the wing's Mean Geometric Chord (MGC). The angle between the datum and the tangent to the flight path ( $x$-axis) is the Angle-of-Attack, denoted by $\alpha$. The thrust may be at an angle $\varepsilon$ with respect to the $x$-axis. This angle is 0 for some aircraft, while for others it is aligned to approximately pass through the CG. This can be the consequence of the application of thrust (or thrustline) being above or below the vertical position of the CG. The purpose is to minimize the effect of thrust changes on the pitch attitude of the aircraft.

The coordinate system moves with the aircraft and its orientation with respect to the Horizon changes depending on the maneuver being performed (see Figure 6-10). The Climb Angle, denoted by $\theta$, is the angle between the horizon and the $x$-axis. If $\theta>0$, then the aircraft is said to be climbing. If $\theta=0$, then the aircraft is said to be flying straight and level (cruising). If $\theta<0$, then the aircraft is said to be descending. With this diagram in mind, it is possible to derive the Planar Equations of Motion (PEOM) for the airplane. These assume no or steady rotation about $y$-axis. They are obtained by summing the forces depicted in Figure 6-1 about the $x$ - and $z$-axes as follows:

$$
\begin{align*}
L-W \cos \theta+T \sin \varepsilon & =\frac{W}{g} \frac{d V_{Z}}{d t}  \tag{6-32}\\
-D-W \sin \theta+T \cos \varepsilon & =\frac{W}{g} \frac{d V_{X}}{d t} \tag{6-33}
\end{align*}
$$



Figure 6-10: A 2-dimensional free-body of the airplane in climbing flight (from Ref [3]).

The derivation of these equations can be found in a variety of references, for example [9], [10], [3], [11], and [12], and are used to model the various maneuvers of the airplane as shown below. The climb is analyzed assuming a steady symmetrical motion by making the following assumptions:
(1) Steady motion implies $d V / d t=0$.
(2) The climb angle, $\theta$, is a non-zero quantity.
(3) The Angle-of-Attack, $\alpha$, is small.
(4) The thrust angle, $\varepsilon$, is $0^{\circ}$

This allows the PEOM of Equations (6-32) and (6-33) to be modified to represent a steady climb:

$$
\begin{gather*}
L-W \cos \theta=0 \quad \Rightarrow \quad L=W \cos \theta  \tag{6-34}\\
-D-W \sin \theta+T=0 \quad \Rightarrow \quad T-D=W \sin \theta \tag{6-35}
\end{gather*}
$$

Equation (6-34) shows that lift in climb is actually less than the weight (the difference is balanced by the vertical component of the thrust):

$$
\begin{equation*}
L=W \cos \theta \tag{6-36}
\end{equation*}
$$

The lift coefficient at this condition is thus:

$$
\begin{equation*}
C_{L}=\frac{2 W \cos \theta}{\rho V^{2} S}=\frac{W \cos \theta}{q S} \tag{6-37}
\end{equation*}
$$

The drag force, using the simplified drag model, is given by:

$$
\begin{equation*}
D=q S\left(C_{D \min }+k \cdot C_{L}^{2}\right) \tag{6-38}
\end{equation*}
$$

Inserting Equation (6-37) into Equation (6-38) yields:

$$
\begin{align*}
& D=q S\left(C_{D \min }+k \cdot\left(\frac{W \cos \theta}{q S}\right)^{2}\right)  \tag{6-39}\\
& D=\left(q S C_{D \min }+k \cdot \frac{W^{2} \cos ^{2} \theta}{q S}\right) \tag{6-40}
\end{align*}
$$

Expanding:

Note that the drag, $D$, calculated by Equation (6-40) should be used with Equations (6-46) and (6-48). However, this would call for an iterative scheme to solve for $R O C$ and $\theta$. Mair and Birdsall [13]
demonstrated that if $\cos \theta \approx 1$ then eliminating the cosine from the equation yields an acceptable accuracy for modest climb angles. Discussion of expected error is also given in Ref [3].

## Horizontal and Vertical Airspeed

We want to use the methods in this section to evaluate characteristics like best Rate-of-Climb (ROC), best (largest) Angle-of-Climb (AOC), and the associated airspeeds. The first important concept is the Horizontal Airspeed. It is used to estimate the horizontal distance covered during climb:

$$
\begin{equation*}
V_{H}=V \cos \theta \tag{6-41}
\end{equation*}
$$



Figure 6-11: Airspeed components during climb. (from Ref [3]).

Note that this is not the same as the maximum horizontal airspeed of Table 6-1. Then, we define the Vertical Airspeed, also called the Rate-of-Climb (ROC):

$$
\begin{equation*}
V_{V}=V \sin \theta \tag{6-42}
\end{equation*}
$$

Both can be derived by observation from Figure 6-11. Note that in terms of calibrated airspeed, $V$ is the airspeed indicated on the airspeed indicator, $V_{V}$ is observed on the vertical speed indicator (VSI), and $V_{H}$ is the ground speed.

## Power Available, Power Required and Excess Power

These three concepts define the climb capability of the aircraft. Note that for aircraft propelled with jet engines, the power available is estimated by multiplying its thrust by the airspeed. For aircraft powered with propellers, the power is obtained by multiplying the engine power by the propeller efficiency ( $\eta_{p}$ ). Since the engine power is usually presented in terms of BHP or SHP, this number must be converted from horsepower to $\mathrm{ft} \cdot \mathrm{l} \mathrm{b}_{\mathrm{f}} / \mathrm{s}$ by multiplying by a factor of 550, if using the UK-system. If using the SIsystem, the horsepower number must be multiplied by a factor of 745.7 to convert it to Watts.

$$
\begin{array}{ll}
\text { Power Available: } & P_{A V} \equiv \text { Force } \times \text { Speed }=T V=\eta_{p} \cdot P_{E N G} \\
\text { Power Required: } & P_{R E Q} \equiv \text { Force } \times \text { Speed }=D V \\
\text { Excess Power: } & P_{E X} \equiv P_{A V}-P_{R E Q} \tag{6-45}
\end{array}
$$

## Vertical Airspeed in Terms of Thrust or Power

The vertical airspeed can be estimated if thrust or power and drag characteristics of the aircraft are known. To do this, we first multiply Equation (6-35) by $V / W$ to get:

$$
\frac{V}{W}(-D-W \sin \theta+T)=0
$$

Manipulating algebraically yields:

$$
-\frac{V}{W} D-\frac{V}{W} W \sin \theta+\frac{V}{W} T=0 \Rightarrow \frac{T V}{W}-\frac{D V}{W}=V \sin \theta
$$

This is expressed as follows for jet aircraft and propeller aircraft:

For jets:

$$
\begin{equation*}
V_{V} \equiv \frac{T V-D V}{W}=V \sin \theta \tag{6-46}
\end{equation*}
$$

For propellers:

$$
\begin{equation*}
V_{V} \equiv \frac{P_{A V}-P_{R E Q}}{W}=\frac{\eta_{p} \cdot P_{E N G}-P_{R E Q}}{W}=V \sin \theta \tag{6-47}
\end{equation*}
$$

Note that the above expressions are some of the most important equations in the entire climb analysis methodology. Ultimately, we want to determine some specific values of $V_{V}$, for instance the maximum value, or the one that results in the steepest climb possible, and so on. Additionally, the formulation shows that in order for an airplane to increase its altitude, its thrust power ( $T V$ ) or available power ( $P_{A V}$ ) must be larger than the drag power ( DV ) or required power $\left(P_{R E Q}\right)$ for level flight.

## Rate of Climb ( $R O C$ )

If the thrust and drag are known at a given flight condition, the instantaneous $R O C$ is calculated as follows:

$$
\begin{equation*}
R O C \equiv 60\left(\frac{T V-D V}{W}\right)=60\left(\frac{P_{A V}-P_{R E Q}}{W}\right) \tag{6-48}
\end{equation*}
$$

Note that the units for $R O C$ in Equation (6-48) are commonly presented as $\mathrm{ft} / \mathrm{min}$ or fpm . This is why it is multiplied by 60 to convert the $R O C$ in $\mathrm{ft} / \mathrm{s}$ into fpm. In the SI system, ROC is usually in terms of $\mathrm{m} / \mathrm{s}$, rendering the factor 60 unnecessary. The reader must be aware of the difference in the representation of time between the UK and SI systems. Unless otherwise specified the $R O C$ is in fpm. It is also possible that the ROC might be given in $\mathrm{ft} / \mathrm{s}$.


Horizontal distance, ft
Figure 6-12: Distance components during climb.
(from Ref [3]).

## Climb Gradient

Climb is sometimes expressed in terms of $\%$ climb gradient and used in lieu of fpm or $\mathrm{m} / \mathrm{s}$. The concept assumes no wind conditions and is defined as follows (see Figure 6-12):

$$
\text { Climb Gradient }=\frac{\text { Vertical Distance }}{\text { Horizontal Distance }}=\frac{\text { Vertical Distance } / \Delta \mathrm{t}}{\text { Horizontal Distance } / \Delta \mathrm{t}}
$$

Consider an airplane whose climb gradient is 0.1 at $100 \mathrm{KTAS}(\mathrm{nm} / \mathrm{hr})$ in no wind conditions. Its rate of climb in fpm would be:

$$
R O C=0.1 \times \frac{100 \mathrm{~nm} / \mathrm{hr}}{60 \mathrm{~min} / \mathrm{hr}} \times 6076 \mathrm{ft} / \mathrm{nm}=1013 \mathrm{ft} / \mathrm{min}
$$

## General Rate of Climb

Substitute Equation (6-40) into (6-46) and manipulate algebraically leads to

$$
V \sin \theta=\frac{T V-D V}{W}=\frac{T V}{W}-\frac{\left(q S V C_{D \min }+k \cdot V \frac{W^{2} \cos ^{2} \theta}{q S}\right)}{W}
$$

Then, simplify and rearrange to get the following general expression to estimate the $R O C$ based on Thrust-to-Weight ratio and wing loading:

$$
\begin{equation*}
V_{V}=V \sin \theta=V\left(\frac{T}{W}-q\left(\frac{S}{W}\right) C_{D \min }-k \cdot\left(\frac{W}{S}\right) \frac{\cos ^{2} \theta}{q}\right) \tag{6-49}
\end{equation*}
$$

It assumes the simplified drag model and returns the vertical airspeed in terms of $\mathrm{ft} / \mathrm{s}$ or $\mathrm{m} / \mathrm{s}$.

## General Climb Angle

A general expression for the climb angle, which assumes the simplified drag model, is obtained by dividing by $V$ on either side of Equation (6-46) and solving for $\sin \theta$ :

General angle of climb: $\quad \sin \theta=\frac{T}{W}-\frac{1}{L / D}$

## Determination of the Airspeed for Best Rate of Climb for a Propeller Aircraft

The determination of the best $R O C$ airspeed, denoted by $V_{Y}$, for propeller aircraft is not as simple as one might expect at first glance. While Equation (6-59) is a noble attempt, it requires an accurate value of the propeller efficiency, which depends on airspeed and propeller RPM, among others. A more sophisticated method is presented below. If thrust can be estimated using a method such as the one
presented in Equation (6-28), then it is possible to do this more effectively as follows. First, assume we can write the thrust of a propeller equation as a function of airspeed, $V$, as follows

$$
T(V)=A_{0}+A_{1} V+A_{2} V^{2}
$$

Where the coefficients $A_{i}$ correspond to those of Equation (6-28). Also assume that drag can be expressed using the adjusted drag model of Equation (6-20), repeated below for convenience

$$
C_{D}=C_{D \text { min }}+k\left(C_{L}-C_{L \text { min } D}\right)^{2}
$$

Where $C_{L}=\frac{2 W}{\rho V^{2} S}$. Therefore, we can rewrite the drag coefficient as

$$
C_{D}=C_{D \min }+k\left(C_{L}-C_{L \min D}\right)^{2}=C_{D \min }+k\left(\frac{2 W}{\rho V^{2} S}-C_{L \min D}\right)^{2}
$$

We can now write the available and required power as follows:

Available power: $\quad P_{A} \equiv T \cdot V=A_{0} V+A_{1} V^{2}+A_{2} V^{3}$
Required power: $\quad P_{R} \equiv D \cdot V=\frac{1}{2} \rho V^{2} S C_{D} \cdot V=\frac{1}{2} \rho V^{3} S\left[C_{D \min }+k\left(\frac{2 W}{\rho V^{2} S}-C_{L \min D}\right)^{2}\right]$

Where

$$
\left(\frac{2 W}{\rho V^{2} S}-C_{L \min D}\right)^{2}=\frac{4 W^{2}}{\rho^{2} V^{4} S^{2}}-2 \frac{2 W C_{L \min D}}{\rho V^{2} S}+C_{L \min D}^{2}
$$

Therefore, the rate of climb can be expressed as

$$
\dot{h}=\frac{P_{A}-P_{R}}{W}=\frac{\left(A_{0} V+A_{1} V^{2}+A_{2} V^{3}\right)-\left(\frac{1}{2} \rho V^{3} S\left[C_{D \min }+k\left(\frac{4 W^{2}}{\rho^{2} V^{4} S^{2}}-2 \frac{2 W C_{L \min D}}{\rho V^{2} S}+C_{L \min D}^{2}\right)\right]\right)}{W}
$$

After some algebraic manipulations, this can be written in terms of the airspeed, V

$$
\dot{h}=\frac{1}{W}\left[-\frac{2 k W^{2}}{\rho S} \frac{1}{V}+\left[A_{0}+2 k W C_{L \min D}\right] V+A_{1} V^{2}+\left[A_{2}-\frac{1}{2}\left(C_{D \min }+k C_{L \min D}^{2}\right) S \rho\right] V^{3}\right]
$$

Then, we can determine the maximum rate of climb by taking the derivative with respect to $V$ and set to 0 and then solve for what airspeed leads to the best rate of climb:

$$
\frac{d \dot{h}}{d V}=\frac{1}{W}\left[\frac{2 k W^{2}}{\rho S V^{2}}+\left[A_{0}+2 k W C_{L \min D}\right]+2 A_{1} V+\left[3 A_{2}-\frac{3}{2}\left(C_{D \min }+k C_{L \min D}^{2}\right) S \rho\right] V^{2}\right]
$$

The maximum is achieved when the term inside the bracket is zero, i.e. $d \dot{h} / d V=0$. This leads to the following $4^{\text {th }}$ order polynomial

$$
\begin{equation*}
\frac{2 k W^{2}}{\rho S}+\left[A_{0}+2 k W C_{L \min D}\right] V^{2}+2 A_{1} V^{3}+\left[3 A_{2}-\frac{3}{2}\left(C_{D \min }+k C_{L \min D}^{2}\right) S \rho\right] V^{4}=0 \tag{6-51}
\end{equation*}
$$

This equation can be solved for $V$ in a number of ways. For instance, one can use the analytical method presented by Tuma [14] for the solution of quartic equations. It is also possible to solve using numerical methods, such as Regula Falsi, Newton-Raphson, or the Bisection method. If the simplified drag model is assumed, then $C_{L \min D}=0$ and this becomes:

$$
\begin{equation*}
\frac{2 k W^{2}}{\rho S}+A_{0} V^{2}+2 A_{1} V^{3}+\left[3 A_{2}-\frac{3}{2} S C_{D \min } \rho\right] V^{4}=0 \tag{6-52}
\end{equation*}
$$

## Summary of Useful Equations for the Climb Maneuver

The following list contains several useful equations for the analysis of climbing flight. Derivations are omitted to conserve space, but interested readers are directed to Reference [3] for the details. All the expressions assume the simplified drag model. Note that formulation for jets is included, since some sUAVs are powered by miniature jet engines.

Best (max) climb angle, $\theta_{\max }$, for a jet: $\quad \theta_{\max } \approx \sin ^{-1}\left(\frac{T_{\max }}{W}-\sqrt{4 \cdot C_{D \min } \cdot k}\right)$

Airspeed for $\theta_{\text {max }}$ for a jet:

$$
\begin{equation*}
V_{X}=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{C_{D \min }}} \cos \theta_{\max }} \tag{6-54}
\end{equation*}
$$

$R O C$ for $\theta_{\max }$ for a jet:

$$
\begin{equation*}
R O C_{X}=60 \cdot V_{X} \cdot \sin \theta_{\max } \tag{6-55}
\end{equation*}
$$

Airspeed for best $R O C$ for a jet:

$$
\begin{equation*}
V_{Y}=\sqrt{\frac{(T / W)(W / S)}{3 \rho C_{D \min }}\left[1+\sqrt{1+\frac{3}{L D_{\max }^{2}(T / W)^{2}}}\right]} \tag{6-56}
\end{equation*}
$$

Best $R O C$ for a jet: $\quad R O C_{\max }=\sqrt{\frac{(W / S) \zeta}{3 \rho C_{D \min }}}\left(\frac{T}{W}\right)^{3 / 2}\left(1-\frac{\zeta}{6}-\frac{3 \cos ^{2} \theta}{2(T / W)^{2} L D_{\max }^{2} \zeta}\right)$

Where

$$
\zeta=1+\sqrt{1+\frac{3}{L D_{\max }^{2}(T / W)^{2}}}
$$

Airspeed for $\theta_{\text {max }}$ for a propeller powered airplane, denoted by $V_{X}$ :

$$
\begin{equation*}
V_{X}^{4}+\frac{\eta_{p} \cdot 550 \cdot B H P}{\rho S C_{D \min }} V_{X}-\left(\frac{W}{S}\right)^{2} \frac{4 k}{\rho^{2} C_{D \min }}=0 \tag{6-58}
\end{equation*}
$$

Airspeed for Best $R O C$ for a propeller powered airplane:

$$
\begin{equation*}
V_{Y}=V_{E_{\max }}=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{3 \cdot C_{D \min }}}} \tag{6-59}
\end{equation*}
$$

Best $R O C$ for a propeller powered airplane:

$$
\begin{equation*}
V_{Y} \sin \theta=\frac{\eta_{p} P}{W}-\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{3 \cdot C_{D \min }}}} \frac{1.1547}{L D_{\max }} \tag{6-60}
\end{equation*}
$$

If the best $R O C$ airspeed, $V_{Y}$, is known and the value of $R O C$ is desired in terms of $f p m$, it can be calculated from:

$$
\begin{equation*}
R O C_{\max }=60\left(\frac{\eta_{p} P}{W}-V_{Y} \frac{1.1547}{L D_{\max }}\right) \tag{6-61}
\end{equation*}
$$

Note that the power, $P$, must be in terms of $\mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s}$. For this reason, if power is given in BHP, it must be be converted to the proper units by multiplying by the factor 550.

Time to Altitude
The time required to increase altitude, $h$, can be determined by noting that $R O C$ is the rate of change of altitude $d h / d t$, that is:

$$
\begin{equation*}
\frac{d h}{d t}=R O C \Leftrightarrow d t=\frac{d h}{R O C} \Rightarrow t=\int \frac{d h}{R O C} \tag{6-62}
\end{equation*}
$$

If the $R O C$ is in terms of fpm , it will return time in minutes. If $R O C$ is in terms of $\mathrm{ft} / \mathrm{s} \mathrm{or} \mathrm{m} / \mathrm{s}$ then the time will be in seconds. If the airplane is operating at some initial altitude, $h_{0}$, and $h_{1}$ is the target altitude the expression is presented using these as limits. The minimum time to altitude is achieved if the pilot maintains the best $R O C$ airspeed $\left(V_{Y}\right)$ through the entire climb maneuver.

## Rapid Approximation of Time to Altitude

For mathematical simplicity it may be convenient to assume a constant value of $R O C$ and take it out of the integral sign. The value of the $R O C$ should be a representative value between the initial and final altitude, denoted by the symbol $R O C_{a}$ and called the representative $R O C$. In the absence of a better value, the average of the $R O C$ between the initial and final altitudes can be used, although the true value should be biased toward the higher altitude as the aircraft will spend more time completing the last half of the climb than the first one. Since this approach treats the representative $R O C$ as a constant, it can be taken out of the integral of Equation (6-62) yielding the following expression:

$$
\begin{equation*}
t=\int_{h_{0}}^{h_{1}} \frac{d h}{R O C}=\frac{1}{R O C_{a}} \int_{h_{0}}^{h_{1}} d h=\frac{\left(h_{1}-h_{0}\right)}{R O C_{a}} \tag{6-63}
\end{equation*}
$$

If the $R O C$ is known as a function of altitude and given by $R O C(h)=A \cdot h+B$, then, using Equation (665 ) below, the value of $R O C_{a}$ can be found from the expression:

$$
\begin{equation*}
R O C_{a}=\frac{A\left(h_{1}-h_{0}\right)}{\ln \left(A h_{1}+B\right)-\ln \left(A h_{0}+B\right)} \tag{6-64}
\end{equation*}
$$

If the initial and final altitudes are close, the $R O C_{a}$ can be approximated as the average of the $R O C$ at the initial and final altitudes.

## Linear Approximation of Time to Altitude

If a particular airspeed, such as $V_{Y}$, is maintained through the climb, the $R O C$ will decrease in a reasonably linear fashion. In this case, the $R O C$ can be approximated with an equation of a line: $R O C(h)$ $=A \cdot h+B$. In this case the time to altitude is given by:

$$
\begin{equation*}
t=\int_{h_{0}}^{h_{1}} \frac{d h}{A h+B}=\frac{\ln \left(A h_{1}+B\right)-\ln \left(A h_{0}+B\right)}{A} \tag{6-65}
\end{equation*}
$$

## Absolute/Service Ceiling Altitude

Absolute and service ceilings are two common performance parameters. The absolute ceiling is the maximum altitude the airplane can maintain level flight. The service ceiling is the altitude at which the aircraft is capable of some 100 fpm rate-of-climb. Each ceiling is actually dependent on the aircraft weight at condition and atmospheric conditions and can deviate thousands of feet (or meters) from the calculated values. Also, the service ceiling may be established using aviation regulations. The theoretical absolute and service ceilings can be computed by the following method:

1. Compute $R O C_{\max }$ at a number of altitudes.
2. Create a trendline in the form of a line or a polynomial.
3. Solve the trendline for $R O C=100 \mathrm{fpm}$. This is the Service Ceiling.
4. Solve the trendline for $R O C=0 \mathrm{fpm}$. This is the Absolute Ceiling.

### 6.4.2. Cruise

The cruise consists of maintaining level flight at some given airspeed. It is a maneuver that renders the airplane a true transport vehicle. The airspeed depends on a given power setting (e.g. 75\% power) and the maintenance of level flight at some desired altitude. This section summarized methods to estimate various cruise characteristics of the airplane.

## Planar Equations of Motion (Assumes No Rotation about Y-axis)

A general free-body diagram for steady level flight is presented in Figure 6-13. It is assumed that all forces act at the CG and all moments are balanced, resulting in a steady motion for which the climb angle, $\theta$, is $0^{\circ}$, the $\alpha$ is small, and the thrust angle, $\varepsilon$, is $0^{\circ}$. These assumptions allow the PEOM of Equations (6-32) and (6-33) to be modified to yield the familiar form:

$$
\begin{gather*}
L=W  \tag{6-66}\\
D=T \tag{6-67}
\end{gather*}
$$



Figure 6-13: A 2-dimensional free-body of the airplane in level flight. (from Ref. [3])

## Summary of Useful Equations for Steady, Level Cruise

The following list contains several useful equations for the analysis of cruising flight. Derivations are omitted to conserve space, but interested readers are directed to Reference [3] for the details. All the expressions assume the simplified drag model:

Airspeed in terms of thrust:

$$
\begin{align*}
& V=\sqrt{\frac{T \pm \sqrt{T^{2}-4 C_{D \min } k W^{2}}}{\rho S C_{D \min }}}  \tag{6-68}\\
& V_{\min }=\sqrt{\frac{T_{\max }-\sqrt{T_{\max }^{2}-4 C_{D \min } k W^{2}}}{\rho S C_{D \min }}} \tag{6-69}
\end{align*}
$$

Minimum Airspeed, $V_{\min }$ :

Note that if the minimum speed is smaller than the stalling speed of the aircraft, then the stalling speed becomes the $V_{\min }$.

Level, 1 g stalling Speed, $V_{S}: \quad \quad V_{S}=\sqrt{\frac{2 W}{\rho S C_{L_{\max }}}}$

The stalling speed, $V_{S}$, is defined as the theoretical minimum speed at which an airplane can maintain altitude. If the load factor, $n$, acting on the airplane differs from 1 g , the equation is written as follows:

Level stalling speed at load factor $\mathrm{n}: \quad V_{S}=\sqrt{\frac{2 n W}{\rho S C_{L_{\text {max }}}}}$
If an airplane banks at a bank angle $\phi$ while maintaining altitude (level constant speed turn), the load factor acting on it increases. Thus, the stalling speed at any given angle of bank $\phi$ can be estimated as follows:

Stalling speed at angle of bank $\phi: \quad V_{S}=\sqrt{\frac{2 W}{\rho S C_{L_{\max }} \cos \phi}}=\frac{V_{S_{\text {level }}}}{\sqrt{\cos \phi}}$
Where $V_{\text {Slevel }}$ is the stalling speed with wings level. Airspeed for a given $C_{L}$ at bank angle $\phi$

$$
\begin{equation*}
V=\sqrt{\frac{2 W}{\rho S C_{L}}\left(\frac{1}{\cos \phi}\right)} \tag{6-73}
\end{equation*}
$$

Airspeed of Minimum Power Required, $V_{P R \min }$, or Maximum Endurance speed, $V_{E m a x}$, is obtained when

$$
\begin{equation*}
\left(\frac{C_{L}^{1.5}}{C_{D}}\right)_{\max }=\frac{1}{4}\left(\frac{3}{k \cdot C_{D \min }^{1 / 3}}\right)^{3 / 4} \tag{6-74}
\end{equation*}
$$

The lift coefficient at this condition can be obtained from:

$$
\begin{equation*}
C_{L}=\sqrt{\frac{3 C_{D \min }}{k}} \tag{6-75}
\end{equation*}
$$

Maximum Endurance Airspeed for a Propeller Powered Aircraft, $V_{\text {Emax }}$ :

$$
\begin{equation*}
V_{P R_{\min }}=V_{E_{\max }}=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{3 \cdot C_{D \min }}}} \tag{6-76}
\end{equation*}
$$

Note that Equation (6-76) is also presented as Equation (6-60), where it is used to determine the best $R O C$ for a propeller aircraft. In other words; $V_{P R_{\min }}=V_{E_{\max }}=V_{Y}$ for a propeller aircraft.

Power Required: $\quad P_{R E Q}=\sqrt{\frac{2 W^{3} C_{D}^{2}}{\rho S C_{L}^{3}}}$

Airspeed of Minimum Thrust Required, $V_{T R \min }$, or Best Glide Speed, $V_{B G}, V_{L D \max }$ :

Best glide speed:

$$
\begin{equation*}
V_{\mathrm{TR} \min }=V_{B G}=V_{L D_{\max }}=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{C_{D \min }}}} \tag{6-78}
\end{equation*}
$$

Maximum L/D Ratio:

$$
\begin{equation*}
L D_{\max }=\left(\frac{C_{L}}{C_{D}}\right)_{\max }=\frac{1}{\sqrt{4 \cdot C_{D \min } \cdot k}} \tag{6-79}
\end{equation*}
$$

Best Range Airspeed for a Jet, $\mathrm{V}_{\text {Rmax }}$, also called the Carson's speed, $V_{C A R}$, is obtained (for both a jet and a propeller aircraft) when

$$
\begin{equation*}
\left(\frac{C_{L}^{0.5}}{C_{D}}\right)_{\max }=\frac{3}{4}\left(\frac{1}{3 k \cdot C_{D \min }^{3}}\right)^{1 / 4} \tag{6-80}
\end{equation*}
$$

Best Range Airspeed for a Jet:

$$
\begin{equation*}
V_{R_{\max }}=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{3 k}{C_{D \min }}}} \tag{6-81}
\end{equation*}
$$

In a paper titled Fuel Efficiency of Small Aircraft [15], Carson discusses the mismatch between the amount of power required for climb and cruise in small propeller-driven aircraft. He then proceeds to demonstrate that this excess power can be used more efficiently by bringing it closer to the so-called Gabrielli-Von Kárman limit for vehicle performance [16]. Carson stated that (1) fuel economy of aircraft is directly proportional to the L/D ratio, whose optimum is typically achieved at unacceptably low airspeeds. (2) Power required for climb results in aircraft airspeeds well beyond this optimum. (3) This results in greater fuel penalties than otherwise. In short, Carson suggests that flying at a speed faster than the airspeed for $L D_{\text {max }}$ is more advantageous, as the flying public generally values time enroute more than fuel efficiency. From a certain point of view, this airspeed can be considered the "fastest efficient airspeed" to fly.

The most frequently cited measure of efficiency is the so-called transport efficiency, defined as $W V / P$, where $W$ is the vehicle weight, $V$ its speed of travel, and $P$ is the installed power. Expanding on this idea, Carson derives a relation for a specific airspeed that is about 32\% larger than the best glide airspeed (see Equation (6-78)). This speed is now recognized as Carson's airspeed. See Smith [17] for additional discussion on the Carson airspeed.

Carson's airspeed:

$$
\begin{equation*}
V_{C A R}=3^{0.25} V_{L D \max } \approx 1.32 V_{L D \max } \tag{6-82}
\end{equation*}
$$

Maximum Level Airspeed, $V_{\max }: \quad V_{\max }=\sqrt{\frac{T_{\max }+\sqrt{T_{\max }^{2}-4 C_{D \min } k W^{2}}}{\rho S C_{D \min }}}$

This airspeed is also denoted by the variable $V_{H}$ in Table 6-1.

## Special Case: Propeller Aircraft

It is unlikely that the thrust for a propeller powered aircraft will be known at $V_{\max }$, as it is a function of the airspeed itself. For this reason, the airspeed must be determined by iteratively solving the equation below.

$$
\begin{equation*}
\rho S C_{D \min } V_{\max }^{3}=550 \eta_{p} P_{B H P}+\sqrt{\left(550 \eta_{p} P_{B H P}\right)^{2}-4 W^{2} V_{\max }^{2} C_{D \min } k} \tag{6-84}
\end{equation*}
$$

Equation (6-84) can be solved using a multitude of methods, for instance the Bisection Method, Regula Falsi, or others. These functions require a single function to be solved, in which case the equation can be rewritten as the function $f\left(V_{\text {max }}\right)$ :

$$
\begin{equation*}
f\left(V_{\max }\right)=\rho S C_{D \min } V_{\max }^{3}-550 \eta_{p} P_{B H P}-\sqrt{\left(550 \eta_{p} P_{B H P}\right)^{2}-4 W^{2} V_{\max }^{2} C_{D \min } k} \tag{6-85}
\end{equation*}
$$

A possible initial condition would then be written for $V_{\max }=0$ as $f(0)=-1100 \eta_{p} P_{B H P}$. Note that for other values of Equation (6-84), terms under the radical require the following to hold:

$$
\left(550 \eta_{p} P_{B H P}\right)^{2}>4 W^{2} V_{\max }^{2} C_{D \min } k
$$

### 6.4.3. Range Analysis

Range analysis is an investigation of how far an aircraft can fly, how quickly, and at what cost. The analysis does not only consider what airspeed an airplane must maintain in order to obtain optimum range, but also evaluates consequences of selecting other airspeeds. It also allows various sensitivities to be evaluated, such as the effect of fuel weight and altitude.

## Basic Cruise Segment for Range Analysis

The basic cruise segment is shown in Figure 6-14. The aircraft begins the cruise at some initial weight, $W_{\text {ini }}$, and after covering some distance, $R$, its range will have changed to some final weight, $W_{\text {fin }}$. This weight is less than the initial weight only if the aircraft uses fossil fuels. It is assumed unchanged if the source of energy is electric power. Both energy sources are considered here. Note that the three curves in the figure indicate the aircraft may initially burn more (blue curve) or less fuel (lavender curve) than later in the segment. However, for simplicity, a linear fuel burn is usually assumed, represented with the straight line. This approximation is accurate enough for most analyses. A cruise segment consisting of multiple power settings is simply broken into smaller segments, for which the linear assumption holds.


Figure 6-14: The basic cruise segment. (from Ref. [3])

## Basic Definitions

For mathematical convenience it is useful to transpose the axes in Figure 6-14, to what is shown in Figure 6-15, displaying the range from 0 at $W_{i n i}$ to the final range, $R_{f i n}$, at $W_{f i n}$. We use this representation to define the change in range:

$$
\begin{equation*}
\frac{d R}{d W}=\frac{\text { Rate of change of distance }}{\text { Rate of change of weight }}=\frac{V}{-c_{t} T} \tag{6-86}
\end{equation*}
$$

Where $c_{t}=$ thrust specific fuel consumption (in $1 / \mathrm{sec}$ ), $V=$ airspeed in $\mathrm{ft} / \mathrm{s}$ or $\mathrm{m} / \mathrm{s}$, and $T=$ thrust in $\mathrm{lb}_{\mathrm{f}}$ or N . During cruise it is reasonable to assume that $T=D$ and $D=W /(L / D)$, where $L$ is the lift and $D$ the drag. Therefore

$$
\begin{equation*}
\frac{d R}{d W}=\frac{V}{-c_{t} T}=\frac{V}{-c_{t} D}=\frac{V(L / D)}{-c_{t} W} \tag{6-87}
\end{equation*}
$$

Recall that expressions (6-86) and (6-87) are only valid for aircraft that burn fossil fuels. Electrically powered aircraft do not change their weight with distance covered.


Figure 6-15: The basic cruise segment with transposed axes. (from Ref. [3])

## The "Breguet" Range Equation

Equation (6-87) is solved for the range by integration, in which the limits are the initial and final weight during that segment. It was originally developed by one of the early pioneers of aviation, the French aircraft designer Louis Charles Breguet (1880-1955, 75), and, thus, it is referred to as the "Breguet" Range Equation:

$$
\begin{equation*}
R=\int_{W_{i n i}}^{W_{i n i}-W_{f}} \frac{V(L / D)}{-c_{t} W} d W=\int_{W_{\text {fin }}}^{W_{\text {ini }}} \frac{V}{c_{t}} \frac{C_{L}}{C_{D}} \frac{1}{W} d W \tag{6-88}
\end{equation*}
$$

The solution of Equation (6-88) requires the dependency of $V, L / D$, and $c_{t}$ on $W$ to be specified, but these depend on the "sort" of cruise flown. While the Breguet equation lends itself well to numerical integration, closed form solutions exist for specific "types" of cruise. A selection of such solutions are presented below.

## Thrust Specific Fuel Consumption for a Jet

In the UK-system, the specific fuel consumption $\left(c_{\text {jel }}\right)$ for a jet is given in terms of $\mathrm{lb}_{\mathrm{f}} / \mathrm{hr} / \mathrm{l}_{\mathrm{f}}$. But since $V$ is in terms of $\mathrm{ft} / \mathrm{s}$ the units must be made consistent.

$$
c_{t}=c_{j e t}\left\langle\frac{l b_{f} / h r}{l b_{f}}\right\rangle\left\langle\frac{1}{3600 \mathrm{sec} / h r}\right\rangle=\left(\frac{c_{j e t}}{3600}\right)\left\langle\frac{l b_{f} / \mathrm{sec}}{l b_{f}}\right\rangle
$$

Therefore, the $T S F C$ for a jet is given by (unit is $1 / \mathrm{s}$ or $\mathrm{l} \mathrm{b}_{\mathrm{f}} / \mathrm{sec} / \mathrm{lb}_{\mathrm{f}}$ ):

$$
\begin{equation*}
c_{t}=\left(\frac{c_{j e t}}{3600}\right) \tag{6-89}
\end{equation*}
$$

## Thrust Specific Fuel Consumption for a Piston Engine

In the UK-system, the specific fuel consumption ( $c_{\text {bhp }}$ ) for a piston engine is given in terms of $\mathrm{lb}_{\mathrm{f}} / \mathrm{hr} / \mathrm{BHP}$ (in terms of power). So, it must be converted to reflect thrust specific fuel consumption. The resulting expression for the TSFC for a piston engine aircraft is given by (units is $1 / \mathrm{s}$ ):

$$
\begin{equation*}
c_{t}=\frac{c_{\text {bhp }} V}{1980000 \eta_{p}} \tag{6-90}
\end{equation*}
$$

A derivation of this expression is provided in Reference [3].

## Mission Profiles

Generally, airplanes follow three different types of operation during cruise, based on selected combinations of the following physical and mathematical interpretations shown in Table 6-3. Closed form solutions are provided for the combinations of parameters in Table 6-4. It is helpful to keep Equation (6-14) in mind when considering these combinations, repeated here for convenience:

$$
\begin{equation*}
L=\frac{1}{2} \rho V^{2} \cdot S \cdot C_{L} \tag{6-14}
\end{equation*}
$$

Table 6-3: Physical and Mathematical Interpretation of Parameters for Mission Analysis.

| Physical Interpretation | Mathematical Interpretation |
| :--- | :--- |
| Constant Airspeed implies... | $\ldots$ V = Constant |
| Constant Altitude implies... | $\ldots \rho=$ Constant |
| Constant Attitude (i.e. AOA) implies... | $\ldots C_{L} / C_{D}=$ Constant |

Table 6-4: Sample Table for Determining the Best Range for an Electrically Powered Aircraft.

| Type of Cruise | $\boldsymbol{V}$ | $\boldsymbol{\rho}$ | $\boldsymbol{C}_{\boldsymbol{L}} / \boldsymbol{C}_{\boldsymbol{D}}$ |
| :--- | :---: | :---: | :---: |
| Constant Airspeed/Altitude | Constant | Constant |  |
| Constant Attitude/Altitude |  | Constant | Constant |
| Constant Airspeed/Attitude | Constant |  | Constant |
| Constant Weight (electric aircraft) | Constant | Constant | Constant |

## Range Profile 1: Constant Airspeed/Constant Altitude Cruise

This type of cruise requires the airspeed and altitude to be maintained between Points 2 and 3 in Figure 6-16. The reduction in weight en-route requires the AOA (or the aircraft's attitude) to be reduced, which reduces the lift coefficient. Note that Equation (6-91) below only estimates the distance covered between Points 2 and 3 . Here, the thrust specific fuel consumption, $c_{t}$, and the airspeed, $V$, are constant. However, since $C_{L} / C_{D}$ is dependent on the change in weight this must remain inside the integral. Therefore, we write Equation (6-88) as follows

$$
R=\int_{W_{\text {fin }}}^{W_{i n i}} \frac{V}{c_{t}} \frac{C_{L}}{C_{D}} \frac{1}{W} d W=\frac{V}{c_{t}} \int_{W_{\text {fin }}}^{W_{\text {ini }}} \frac{C_{L}}{C_{D}} \frac{1}{W} d W
$$

Insert the expression for drag (using the simplified drag model of Equation (6-19) and expand to get

$$
R=\frac{1}{c_{t}} \frac{\rho V^{3} S}{2 k} \int_{W_{\text {fin }}}^{W_{\text {ini }}} \frac{1}{\left(C_{D \min } \frac{\rho^{2} V^{4} S^{2}}{4 k}+W^{2}\right)} d W
$$

Then, define $a$ such that:

$$
a=\frac{\sqrt{C_{D \min }}}{\sqrt{k}} \frac{\rho V^{2} S}{2} \Leftrightarrow a^{2}=C_{D \min } \frac{\rho^{2} V^{4} S^{2}}{4 k}
$$

And note the integral is of the form:

$$
\int \frac{d u}{a^{2}+u^{2}}=\frac{1}{a} \tan ^{-1} \frac{u}{a}
$$

Therefore we get;

$$
\begin{equation*}
R=\frac{V}{c_{t} \sqrt{k C_{D \text { min }}}}\left[\tan ^{-1}\left(\frac{2 \sqrt{k}}{\rho V^{2} S \sqrt{C_{D \min }}} W_{i n i}\right)-\tan ^{-1}\left(\frac{2 \sqrt{k}}{\rho V^{2} S \sqrt{C_{D \text { min }}}} W_{f i n}\right)\right] \tag{6-91}
\end{equation*}
$$



Figure 6-16: Constant airspeed/constant altitude cruise mission. (from Ref. [3])

## Range Profile 2: Constant Altitude/Constant Attitude Cruise

This type of cruise requires the altitude and attitude (AOA) to be maintained between Points 2 and 3 (the cruise segment) in Figure 6-17. This requires the airspeed to be reduced en-route, as can be seen from Equation (6-14). Here the airspeed, $V$, is a variable and $C_{L} / C_{D}$ is a constant and, thus, can come outside of the integral.

$$
R=\int_{W_{\text {fin }}}^{W_{i n i}} \frac{V}{c_{t}} \frac{C_{L}}{C_{D}} \frac{1}{W} d W=\frac{1}{c_{t}} \frac{C_{L}}{C_{D}} \int_{W_{\text {fin }}}^{W_{i n i}} \frac{V}{W} d W
$$

Which becomes

$$
R=\frac{1}{c_{t}} \frac{C_{L}}{C_{D}} \frac{\sqrt{2}}{\sqrt{\rho S C_{L}}} \int_{W_{f n n}}^{W_{\text {ini }}}(W)^{-1 / 2} d W=\frac{1}{c_{t}} \frac{C_{L}}{C_{D}} \frac{\sqrt{2}}{\sqrt{\rho S C_{L}}}\left[\frac{\sqrt{W}}{1 / 2}\right]_{W_{f n}}^{W_{\text {ini }}}
$$

Therefore: $\quad R=\frac{1}{c_{t}} \frac{\sqrt{C_{L}}}{C_{D}} \frac{2 \sqrt{2}}{\sqrt{\rho S}}\left(\sqrt{W_{\text {ini }}}-\sqrt{W_{f i n}}\right)=\frac{1}{c_{t} C_{D}} \sqrt{\frac{8 C_{L}}{\rho S}}\left(\sqrt{W_{i n i}}-\sqrt{W_{f i n}}\right)$


Figure 6-17: Constant altitude/constant attitude cruise mission. (from Ref. [3])

## Range Profile 3: Constant Airspeed/Constant Attitude Cruise

The third cruise profile is the Constant Airspeed/Constant Attitude cruise. In this cruise mode, the airplane must climb as the fuel is consumed to ensure reduction in lift (per Equation (6-14) between Points 2 and 3 (the cruise segment) in Figure 6-18. For this case, the airspeed, $V$, and $C_{L} / C_{D}$ are constant and, thus, can come outside of the integral. Therefore

Which leads to

$$
\begin{gather*}
R=\int_{W_{\text {fin }}}^{W_{\text {ini }}} \frac{V}{c_{t}} \frac{C_{L}}{C_{D}} \frac{1}{W} d W=\frac{V}{c_{t}} \frac{C_{L}}{C_{D}} \int_{W_{\text {fin }}}^{W_{\text {ini }}} \frac{1}{W} d W=\frac{V}{c_{t}} \frac{C_{L}}{C_{D}}[\ln (W)]_{W_{\text {fin }}}^{W_{\text {ini }}} \\
R=\frac{V}{c_{t}} \frac{C_{L}}{C_{D}} \ln \left(\frac{W_{\text {ini }}}{W_{\text {fin }}}\right) \tag{6-93}
\end{gather*}
$$



Figure 6-18: Constant airspeed/attitude cruise mission. (from Ref. [3])

## Range Profile 4: Cruise Range in the Absence of Weight Change

Electric airplanes differ from ordinary aircraft in that their weight is independent of range, rendering the Breguet formulation inapplicable. A rudimentary estimate of the range of an electric airplane can be expressed as the product of its airspeed and flight time, per the profile of Figure 6-16. True range and endurance is harder to get as it involves the battery discharge curves discussed in Section 7.4.3, Discharge Curves and calls for a detailed description of power discharge (in terms of current). Thus, a rudimentary estimate of the range would be obtained as follows. Let $m_{b}$ be the mass (or weight) of the battery system and $E_{B A T T}$ the energy density of the battery. Then, the total energy stored in the battery system is $E=m_{b} \times E_{B A T T}$. Suppose the power of the motor is $P_{\text {ELECTRIC }}$, then the time to run the motor is:

$$
\begin{equation*}
t_{\text {TOT }}=\frac{E}{P_{\text {ELECTRIC }}}=\frac{m_{b} \times E_{\text {BATT }}}{P_{\text {ELECTRIC }}} \tag{6-94}
\end{equation*}
$$

In order to determine maximum range, the designer should tabulate power required for a range of airspeeds similar as set up in Table 6-5 and then extract the best range. Note that $P_{\text {REQ }}$ is the power the electric motor must generate and is thus a direct indication of the power setting required. Also, although not directly shown in the table, it is assumed the proper conversion factors are employed to ensure the units displayed.

Table 6-5: Sample Table for Determining the Best Range for an Electrically Powered Aircraft.

| $\begin{gathered} V \\ \text { KTAS } \end{gathered}$ | $C_{L}$ | $C_{D}$ | $D$ | $\begin{gathered} P_{\text {REQ }}=D \cdot V \\ \text { Watts } \end{gathered}$ | $\begin{aligned} & \boldsymbol{t}_{\text {TOT }} \\ & \text { hrs } \end{aligned}$ | $\begin{gathered} R \\ \mathrm{~nm} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{1}$ | $C_{L 1}$ | $C_{D 1}$ | $D_{1}$ | $D_{1} \cdot V_{1}$ | $E /\left(D_{1} \cdot V_{1}\right)$ | $t_{\text {TOT1 }} \cdot V_{1}$ |
| $V_{2}$ | $C_{L 2}$ | $C_{D 2}$ | $D_{2}$ | $D_{2} \cdot V_{2}$ | $E /\left(D_{2} \cdot V_{2}\right)$ | $t_{\text {TOT2 }} \cdot V_{2}$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $V_{N}$ | $C_{L N}$ | $C_{D N}$ | $D_{N}$ | $D_{N} V_{N}$ | $E /\left(D_{N} \cdot V_{N}\right)$ | $t_{\text {TOTN }}{ }^{\prime} V_{N}$ |

## Specific Range

Specific Range $(S R)$ is the distance an airplane can fly on a given amount of fuel. This quantity is important when comparing the efficiency of different aircraft types or different airspeed for an individual aircraft, for instance, when determining at which airspeed a particular airplane is the most efficient.

$$
\begin{equation*}
S R \equiv \frac{\text { Distance }}{\text { Quantity of Fuel }} \tag{6-95}
\end{equation*}
$$

$S R$ is analogous to the term gas mileage as used for cars, the primary difference is that when used for airplanes one usually specifies fuel quantity in terms of $\mathrm{lb}_{\mathrm{f}}$ rather than gallons. Knowing the range flown and weight of the fuel consumed during that segment, we can calculate the Average $S R$ from the following expression:

$$
\begin{equation*}
S R=R / W_{f} \tag{6-99}
\end{equation*}
$$

We can also compute the Instantaneous $S R$ as follows:

$$
\begin{equation*}
S R=\frac{\Delta R / \Delta t}{W_{f} / \Delta t}=\frac{V_{\text {TAS }}}{\dot{w}_{\text {fuel }}}=\frac{\text { True Airspeed }}{\text { Fuel Weight Flow }} \tag{6-97}
\end{equation*}
$$

### 6.4.4. Endurance Analysis

Endurance is the amount of time an airplane can stay aloft while consuming a specific amount of fuel. Like range, this amount of time is of great importance in aircraft design, particularly for some military aircraft, such as fighters, tankers, and UAVs.

## Basic Cruise Segment for Endurance Analysis

As in the case of the range, endurance is considered in terms of a cruise segment. Such a segment is shown in Figure 6-19 and is identical to Figure 6-15, except for the vertical axis features time. Begin by defining the Thrust Specific Fuel Consumption as follows:

$$
\begin{equation*}
c_{t} \equiv \frac{\dot{w}_{\text {fuel }}}{T}=\frac{d W / d t}{T} \Rightarrow d W / d t=-c_{t} T \tag{6-98}
\end{equation*}
$$

The inverse of the term $d W / d t$ is simply the rate of change in time with respect to weight. This allows us to write the change in time aloft as follows:

$$
\begin{equation*}
\frac{d W}{d t}=-c_{t} T \quad \Leftrightarrow \quad d t=\frac{d W}{-c_{t} T} \tag{6-99}
\end{equation*}
$$

Expand this expression by introducing the same assumptions as for the range, i.e. noting that $T=D=$ $W /(L / D)$ as done for Equation (6-87), which yields:

$$
\begin{equation*}
d t=\frac{1}{-c_{t} T} d W=\frac{1}{-c_{t} D} d W=\frac{(L / D)}{-c_{t} W} d W \tag{6-100}
\end{equation*}
$$

In order to solve Equation (6-100) we must incorporate the dependency of $V, L / D$, and $c_{t}$ on $W$.

## The "Breguet" Endurance Equation

As for the range, Equation (6-100) is solved for the endurance by integration, in which the limits are the initial and final weight during that segment. It is referred to as the "Breguet" Endurance Equation:

$$
\begin{equation*}
E=\int_{W_{i n i}}^{W_{\text {ini }}-W_{f}} \frac{(L / D)}{-c_{t} W} d W=\int_{W_{\text {fin }}}^{W_{\text {ini }}} \frac{1}{c_{t}} \frac{C_{L}}{C_{D}} \frac{1}{W} d W \tag{6-101}
\end{equation*}
$$



Figure 6-19: The basic cruise segment in terms of time of flight.(from Ref. [3])
The solution of Equation (6-101) requires the dependency of $V, L / D$, and $c_{t}$ on $W$ to be established in the same manner as for the range. As before, closed form solutions exist for the same cases as for the range and are based on similar assumptions. However, in interest of space, only the results will be presented here. Derivations are provided in Reference [3]. An inspection of the resulting equations demonstrates a certain commonality between all. In other words, in order to achieve a long endurance capability the airplane must be designed with the following features in mind:
(1) It must have a high operational L/D ratio. In other words, it must feature as low drag as possible.
(2) It must have a low Specific Fuel Consumption (or low Thrust Specific Fuel Consumption).
(3) If driven by a propeller it must feature the highest possible propeller efficiency and operate at a low altitude (high $\rho$ ).
(4) It must carry as much fuel as possible.

Formulation for the estimation of endurance will now be developed assuming the same profiles used for the development of range. The results are valid only for the simplified drag model.

Endurance Profile 1: Constant Airspeed/Constant Altitude Cruise

$$
\begin{equation*}
E=\frac{1}{c_{t} \sqrt{k C_{D \min }}}\left[\tan ^{-1}\left(\frac{2 \sqrt{k}}{\rho V^{2} S \sqrt{C_{D \min }}} W_{i n i}\right)-\tan ^{-1}\left(\frac{2 \sqrt{k}}{\rho V^{2} S \sqrt{C_{D \min }}} W_{f i n}\right)\right] \tag{6-102}
\end{equation*}
$$

## Endurance Profile 2: Constant Attitude/Altitude Cruise

For a jet:

$$
\begin{equation*}
E=\frac{1}{c_{t}} \frac{C_{L}}{C_{D}} \ln \left(\frac{W_{i n i}}{W_{f i n}}\right) \tag{6-103}
\end{equation*}
$$

For a propeller:

$$
\begin{equation*}
E=\left(\frac{1980000 \eta_{P}}{c_{b h p}}\right) \sqrt{2 \rho S}\left(\frac{C_{L}^{1.5}}{C_{D}}\right)\left[\frac{1}{\sqrt{W_{f i n}}}-\frac{1}{\sqrt{W_{i n i}}}\right] \tag{6-104}
\end{equation*}
$$

## Endurance Profile 3: Constant Airspeed/Attitude Cruise

$$
\begin{equation*}
E=\frac{1}{c_{t}} \frac{C_{L}}{C_{D}} \ln \left(\frac{W_{i n i}}{W_{f i n}}\right) \tag{6-105}
\end{equation*}
$$

### 6.4.5. The Descent Maneuver

The descent is a maneuver that brings the aircraft closer to the ground. Typical control inputs constitute a low power setting and trim for and maintenance of a particular airspeed. It differs from a better known maneuver called the dive in which power may or may not be reduced and usually lasts for much shorter time. When comes to optimized glide performance, the descent is ideally established to achieve the longest possible glide distance (best glide) or longest possible glide time (minimum rate-of-descent). Each requires a particular airspeed to be selected and maintained. This section presents methods to determine the optimized airspeeds and other glide parameters of interest, such as the actual rate-ofdescent and glide distance.

Similar to Figure 6-10, the free-body diagram of Figure 6-20 is also balanced in terms of inertia, mechanical, and aerodynamic forces. Figure 6-20 is used to derive the Planar Equations of Motion for the descent maneuver. The equations of motion for gliding flight are given by:

$$
\begin{align*}
L-W \cos \theta+T \sin \varepsilon & =\frac{W}{g} \frac{d V_{Z}}{d t}  \tag{6-106}\\
-D+W \sin \theta+T \cos \varepsilon & =\frac{W}{g} \frac{d V_{X}}{d t} \tag{6-107}
\end{align*}
$$

Note the subtle difference between Equation (6-33) and (6-107). The equations of motion can be adapted to descending flight by making the following assumptions:
(1) Steady motion implies $d V / d t=0$.
(2) The descent angle, $\theta$, is a non-zero quantity.
(3) The Angle-of-Attack, $\alpha$, is small.
(4) The thrust angle, $\varepsilon$, is $0^{\circ}$


Figure 6-20: A 2-dimensional free-body of the airplane in a powered gliding flight (from Ref [3]).
Equations of motion for a steady unpowered $(T=0)$ descent:

$$
\begin{gather*}
L-W \cos \theta=0 \quad \Rightarrow \quad L=W \cos \theta  \tag{6-108}\\
-D-W \sin \theta=0 \quad \Rightarrow \quad D=W \sin \theta \tag{6-109}
\end{gather*}
$$

Equations of motion for a steady powered $(T>0)$ descent:

$$
\begin{align*}
L-W \cos \theta=0 & \Rightarrow \quad L=W \cos \theta  \tag{6-110}\\
-D+W \sin \theta+T=0 & \Rightarrow \quad D=T+W \sin \theta \tag{6-111}
\end{align*}
$$

Vertical Airspeed:

$$
\begin{equation*}
V_{V}=V \sin \theta \tag{6-112}
\end{equation*}
$$

AOD is also known as Angle-of-Glide (AOG) or glide angle.

## General Angle of Descent (AOD)

The AOD in unpowered descent is the flight path angle with respect to the horizontal and is obtained by dividing Equation (6-109) by (6-108):

$$
\begin{equation*}
\frac{D}{L}=\frac{W \sin \theta}{W \cos \theta} \Leftrightarrow \tan \theta=\frac{D}{L}=\frac{1}{L / D} \approx \frac{D}{W} \tag{6-113}
\end{equation*}
$$

If the airplane uses power during the descent, the AOD is obtained from Equation (6-35):

$$
\begin{equation*}
D=T+W \sin \theta \quad \Leftrightarrow \quad \sin \theta=\frac{D-T}{W}=\frac{D}{W}-\frac{T}{W} \tag{6-114}
\end{equation*}
$$

The right approximations ( $\approx$ ) are valid for low descent angles, $\theta$, and when the CG is not to far forward, as this can put a high load on the stabilizing surface and invalidate the approximation $L \approx W$.


Figure 6-21: Airspeed components during climb (from Ref [3]).

## General Rate of Descent

The rate at which an aircraft reduces altitude is obtained by multiplying Equation (6-109) by $V$, and then rewrite $V \sin \theta$ using Equation (6-112):

$$
D=W \sin \theta \quad \text { and } \quad V_{V}=V \sin \theta \quad \Rightarrow \quad D V=W V \sin \theta=W V_{V} \quad \Leftrightarrow \quad V_{V}=\frac{D V}{W}
$$

Therefore, we get:

$$
\begin{equation*}
V_{V}=\frac{D V}{W}=\frac{V}{\left(C_{L} / C_{D}\right)} \tag{6-115}
\end{equation*}
$$

The above expression has units of $\mathrm{ft} / \mathrm{s}$ or $\mathrm{m} / \mathrm{s}$. Generally, the units preferred by pilots is in terms of feet per minute or fpm for General Aviation, Commercial Aviation, and Military, but $\mathrm{m} / \mathrm{s}$ for sailplanes and some nations, that use the metric system. To convert Equation (6-115) into units of fpm multiply by 60.

## Summary of Important Equations for the Descent Maneuver

The following list of useful equations is presented below for convenience. Derivations are omitted to conserve space, but interested readers are directed to Reference [3] for the details. All the expressions assume the simplified drag model:

Equilibrium Glide Speed: $\quad V=\sqrt{\frac{2 \cos \theta}{\rho C_{L}} \frac{W}{S}}$

Straight and level sink rate:

$$
\begin{equation*}
V_{V}=\sqrt{\frac{2}{\rho\left(C_{L}^{3} / C_{D}^{2}\right)} \frac{W}{S}}=\frac{C_{D}}{C_{L}^{3 / 2}} \sqrt{\frac{2}{\rho} \frac{W}{S}} \tag{6-117}
\end{equation*}
$$

Sink rate while banking at $\phi$ :

$$
\begin{equation*}
V_{V}=\sqrt{\frac{2}{\rho\left(\left(C_{L} \cos \phi\right)^{3} / C_{D}^{2}\right)} \frac{W}{S}}=\frac{C_{D}}{C_{L}^{3 / 2} \cos ^{3 / 2} \phi} \sqrt{\frac{2}{\rho} \frac{W}{S}} \tag{6-118}
\end{equation*}
$$

Airspeed of Minimum Sink Rate, $V_{B A}$ :

$$
\begin{equation*}
V_{B A}=V_{E \max }=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{3 \cdot C_{D \min }}}} \tag{6-119}
\end{equation*}
$$

Note that the lift coefficient associated with the airspeed of minimum sink rate can be found as follows:

$$
C_{L}=\left(\frac{W}{S}\right) \frac{2}{\rho V^{2}} \Leftrightarrow C_{L_{B A}}=\left(\frac{W}{S}\right) \frac{2}{\rho\left[\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{3 \cdot C_{D \min }}}\right]}=\sqrt{\frac{3 \cdot C_{D \min }}{k}}
$$

Minimum Angle of Descent:

$$
\tan \theta_{\min }=\frac{1}{L D_{\max }}=\sqrt{4 \cdot k \cdot C_{D \min }}
$$

Best Glide Speed, $V_{B G}$ :

$$
\begin{equation*}
V_{B G}=V_{L D \max }=\sqrt{\frac{2}{\rho} \sqrt{\frac{k}{C_{D \min }}} \frac{W}{S}} \tag{6-121}
\end{equation*}
$$

Glide Distance (see Figure 6-22): $\quad R_{\text {glide }}=h \cdot\left(\frac{L}{D}\right)=h \cdot\left(\frac{C_{L}}{C_{D}}\right)$


Figure 6-22: Distance covered during glide can be estimated using the L/D ratio.(from Ref. [3])

| Where: | $C_{D}=$ Drag coefficient |
| :--- | :--- |
|  | $C_{D \min }=$ Mininmum drag coefficient |
|  | $C_{L}=$ Lift coefficient |
|  | $D=$ Drag |
|  | $h=$ Altitude |
|  | $k=$ Lift-induced drag constant |
|  | $L=$ Lift |
|  | $L D_{\max }=$ Max lift-to-drag ratio |
|  | $R_{g l i d e}=$ Glide range |
|  | $S=$ Reference wing area |

```
\(V=\) Airspeed
\(V_{B A}=\) Airspeed of minimum rate of descent
\(V_{B G}=\) Best glide airspeed (where \(L D_{\max }\) occurs)
\(V_{V}=\) Vertical airspeed
\(W=\) Weight
\(\phi=\) Bank angle
\(\theta=\) Glide angle
\(\theta_{\text {min }}=\) Minimum glide angle
\(\rho=\) Air density
```

It is of interest to co-plot the minimum sink rate (or minimum $R O D$ ) as a function of wing loading ( $W / S$ ) for the three large birds discussed in Chapter 1 and the two RC aircraft introduced in Chapter 3, A Survey of Automation Technology for sUAVs. This is done in Figure 6-23 and Figure 6-24 for wing loading ranging from 0 to $10 \mathrm{lb}_{f} / \mathrm{ft}^{2}$ and $C_{D \min }$ ranging from 0.0100 to 0.0250 . There are clear trends between the curves, outside of the obvious influence of wing loading on the glide characteristics. For instance, Figure 6-23 shows an inverse trend between the $C_{D \min }$ and the airspeed at which the minimum $R O D$ occurs: The less "draggy" the airplane, the higher the airspeed for minimum $R O D$. Figure $6-24$ shows alternative trend: The less "draggy" the airplane, the lower is the actual value of the minimum ROD. For effective energy harvesting, this clearly favors sleek aircraft, for not only can such aircraft gain or maintain altitude in lighter updrafts, they also do so at higher airspeed, making them more tolerant of headwind.

Figure 6-23 and Figure 6-24 are also interesting with respect to the evolutionary features of the three large birds shown. They are much larger and heavier than either one of the RC aircraft. Their capability at harvesting atmospheric convection serves to remind us of the latent energy available for harvesting. It is also of interest to compare the Andean Condor and Albatross in Figure 6-23 and Figure 6-24. The Condor descends at $60-90 \mathrm{fpm}$ lower rate than the Albatross and at airspeed that is 6-7 knots lower. This would favor the Condor's low wing loading in the rocky mountainous regions it inhabits. The higher $L D_{\max }$ of the Albatross favors the dynamic soaring it exercises in its habitat.

### 6.4.6. Analysis of a General Level Constant Velocity Turn

The previous sections present analysis methods for aircraft un-accelerated level flight. In this section a number of common analysis methods intended for accelerated flight will be introduced. Among maneuvers considered are turning flight, pull-up (loop), and accelerated rate-of-climb. All utilize the simplified drag model.

Consider the aircraft in Figure 6-25, which is banking at an angle $\phi$. In order to maintain altitude (no slipping or skidding) its lift must balance the weight while generating a centripetal force component that balances the centrifugal force component. The resulting motion causes the airplane to undergo a steady heading change. This also requires the magnitude of the lift to be larger than the weight of the aircraft (or the airplane will lose altitude). Thus, the airframe is subjected to higher loads than realized in level flight, as represented in the load factor, defined as $n=L / W$. The set of equations describing the motion of the airplane in this condition is written as follows:


Figure 6-23: Airspeed of minimum ROD as a function of wing loading.


Figure 6-24: Minimum ROD as a function of wing loading.


Figure 6-25: Forces on an aircraft in a level constant velocity turn. (from Ref [3]).

Fore-aft forces:

$$
\begin{equation*}
T-D=0 \tag{6-123}
\end{equation*}
$$

Vertical forces:
$L \cos \phi-W=0$

Lateral forces:
$L \sin \phi-\frac{W}{g} \frac{V^{2}}{R_{\text {turn }}}=0$

Using these we can now analyze the level constant velocity turn as shown here. First we note that since the load factor is defined as $n=L / W$ we readily see from Figure $6-25$ that it relates to the bank angle $\phi$ as follows:

$$
\begin{equation*}
W=n W \cos \phi \quad \Leftrightarrow \quad \cos \phi=\frac{1}{n} \quad \Leftrightarrow \quad \phi=\cos ^{-1}\left(\frac{1}{n}\right) \tag{6-126}
\end{equation*}
$$

Next, we divide Equation (6-124) by (6-123) to get:

$$
\frac{L \cos \phi-W=0}{T-D=0} \Rightarrow \frac{L \cos \phi}{D}=\frac{W}{T} \Rightarrow \frac{L}{D} \frac{1}{n}=\frac{W}{T}
$$

Therefore the load factor is related to the thrust, $T$, weight, $W$, lift, $L$, and drag, $D$, as show below

Load factor:

$$
\begin{equation*}
n=\frac{1}{\cos \phi}=\left(\frac{T}{W}\right)\left(\frac{L}{D}\right) \tag{6-127}
\end{equation*}
$$

Centrifugal force corresponding to the force diagram in Figure 6-25 can be found from the standard curvilinear relation $m V^{2} / R$ :

$$
m \frac{V^{2}}{R_{t u r n}}=\left(\frac{W}{g}\right) \frac{V^{2}}{R_{t u r n}}=n W \sin \phi
$$

Manipulating algebraically leads to the following

$$
\left(\frac{W}{g}\right) \frac{V^{2}}{R_{t u r n}}=n W \sin \phi \Rightarrow \frac{V^{2}}{R_{\text {turn }}}=n g \sin \phi
$$

This yields the following expression for the turn radius

Turn radius:

$$
\begin{equation*}
R_{t u r n}=\frac{V^{2}}{n \cdot g \cdot \sin \phi}=\frac{V^{2}}{g \sqrt{n^{2}-1}} \tag{6-128}
\end{equation*}
$$

The distance the airplane covers in the turn at an airspeed $V$ in time $t_{\psi}$ is equal to the standard arc length of a circle of radius $R_{\text {turn }}$ through the angle $\psi$. In other words

$$
R_{t u r n} \times \psi=V \times t_{\psi}
$$

By solving for $t_{\psi}$ in the above expression and noting the angle to be used must be in radians (note the conversion factor $\pi / 180$ ) we get the expression for the time to turn.

Time to turn $\psi$ degrees: $\quad t_{\psi}=\frac{R_{\text {turn }}}{V}\left(\psi \frac{\pi}{180}\right)$

Turn rate is the change in heading with respect to time and can be written as follows:

$$
\dot{\psi}=\frac{d \psi}{d t} \cong \frac{\psi}{t_{\text {turn }}} \Rightarrow \frac{\psi}{\frac{R_{\text {turn }} \times \psi}{V}}=\frac{V}{R_{\text {turn }}}=\frac{V}{\frac{V^{2}}{g \sqrt{n^{2}-1}}}=\frac{g \sqrt{n^{2}-1}}{V}
$$

This gives the following expression for the turn rate in radians/sec:

$$
\begin{equation*}
\dot{\psi}=\frac{g \sqrt{n^{2}-1}}{V}=\frac{V}{R_{t u r n}} \tag{6-130}
\end{equation*}
$$

The level constant velocity turn requires thrust to equal the drag of the airplane in the turn, in other words (assuming the simplified drag model):

$$
T_{R}=D=q S C_{D} \Rightarrow T_{R}=q S\left[C_{D \min }+k C_{L}^{2}\right]=q S\left[C_{D \min }+k\left(\frac{n W}{q S}\right)^{2}\right]
$$

Since elevator is required to trim the airplane in the turn, the increase in trim (trim drag) should be considered if it is significant. This term, $D_{\text {trim, }}$, is shown in Equation (6-131).

Thrust required at a load factor $n$ :

$$
\begin{equation*}
T_{R}=q S\left[C_{D \min }+k\left(\frac{n W}{q S}\right)^{2}\right]+D_{t r i m} \tag{6-131}
\end{equation*}
$$

First note that, typically, $D_{\text {trim }}$ is around 1-2\% of the total drag of the airplane and, thus, ignoring it will yield acceptable accuracy. Use Equation (6-131), assuming $D_{t r i m}=0$ and solve for the load factor, $n$.

$$
T_{R}=q S\left[C_{D \min }+k\left(\frac{n W}{q S}\right)^{2}\right] \Rightarrow \frac{T_{R}}{q S}-C_{D \min }=k\left(\frac{n W}{q S}\right)^{2} \Rightarrow n=\frac{q S}{W} \sqrt{\frac{1}{k}\left(\frac{T_{R}}{q S}-C_{D \min }\right)}
$$

The load factor that can be sustained at a given thrust, $T$, and airspeed, $V$, can be obtained as shown below. Note that inserting the maximum thrust will yield the maximum load factor at a given airspeed:

$$
\begin{equation*}
n=\frac{q S}{W} \sqrt{\frac{1}{k}\left(\frac{T}{q S}-C_{D \min }\right)} \tag{6-132}
\end{equation*}
$$

This expression is used to plot a part of the banking constraint diagram of Figure 6-26.
Airspeed for a given $C_{L}: \quad V=\sqrt{\frac{2 W}{\rho S C_{L}}\left(\frac{1}{\cos \phi}\right)}$

A common way to present the turn performance of an aircraft can be seen in the Banking Constraint Diagram of Figure 6-26 and the Turn Performance Map of Figure 6-27. Both present a convenient way to show the banking capability of an aircraft. First, consider the banking constraint diagram of Figure 6-26, here based on the SR22 general aviation aircraft. The straight dashed horizontal line shows the limit load factor of 3.8 gs . The two vertical dashed lines show the clean stalling speed at normal 1 g loading ( $V_{S}$, to the left) and the normal operating speed ( $V_{N O}$, to the right). These two lines effectively enclose the normal speed range of the aircraft. The solid curve, labeled "Max stall load factor," shows the stalling speed of the aircraft at various load factors. This way, it can be seen that at a load factor of 3 g , the airplane will stall at about 120 KCAS. This curve is a part of the standard V-n diagram. The dotted curve, labeled "Max banking load factor," shows the maximum gs the aircraft can bank at while maintaining altitude. It can be seen that between $V_{S}$ and 120 KCAS, the airplane will simply stall before achieving its maximum "theoretical" banking load factor. This way, at 100 KCAS, if the airplane didn't stall first (at $n \approx$ 2.1 g ) it could achieve $n=2.7 \mathrm{~g}$ before it would begin to lose altitude. At airspeeds beyond 120 KCAS , the airplane cannot achieve 3.8 gs (its limit load factor) while banking and maintaining altitude. This means that when flight testing the aircraft for structural flight tests (e.g. per 14 CFR 23.307, Proof of structure), means other than constant altitude banking may have to be considered.


Figure 6-26: A Banking constraint diagram for the SR22. The maximum stall load factor is calculated by solving Equation (6-71) for the load factor. The maximum banking load factor is calculated using Equation (6-132). (from Ref. [3])

The turn performance map of Figure 6-27 is a cross-plot of Equation (6-130). It is generated by plotting curves for constant turn radius (the straight lines) and then for constant load factors (the curves). Since the equation does not involve any variables dependent on particular aircraft geometry, it is valid for all aircraft, although Figure 6-27 has been drawn up for aircraft that comply with 14 CFR Part 23 (and the stall boundary varies from airplane to airplane). The map shows how rapidly an aircraft can maneuver at specific airspeeds. The maneuvering speed is where the stall boundary intersects the curve for the limit load factor. It is also called the corner speed and is the lowest airspeed where the airplane achieves its maximum bank angle, most rapid heading change, and minimum turning radius.

As an example of how these expressions can be used, estimate the bank angle, turn radius, and time to complete full circle for an airplane flying at 200 KTAS at 10000 ft , while enduring a 2 g load factor. This can be solved as show below:

Angle of bank:

$$
\phi=\cos ^{-1}\left(\frac{1}{n}\right)=\cos ^{-1}\left(\frac{1}{2}\right)=60^{\circ}
$$

Turn radius:

$$
R_{t u r n}=\frac{V^{2}}{g \sqrt{n^{2}-1}}=\frac{(200 \times 1.688)^{2}}{(32.174) \sqrt{(2)^{2}-1}}=2045 \mathrm{ft}
$$

Time to turn $360^{\circ}$ :

$$
t_{\psi}=\frac{R_{\text {turn }}}{V}\left(\psi \frac{\pi}{180}\right)=t_{\psi}=\frac{2045}{(200 \times 1.688)}(2 \pi)=38 \mathrm{sec}
$$



Figure 6-27: A turn performance map is constructed using Equation (6-130).(from Ref. [3])

### 6.4.7. Extremes of Constant Velocity Turns

## Maximum Sustainable Load Factor, $\boldsymbol{n}_{\text {max }}$

The maximum load factor that the aircraft can sustain without stalling is obtained from Equation (6-127) when the thrust-to-weight and lift-to-drag ratios are at their maximum values:

Max Sustainable Load Factor: $\quad n_{\text {max }}=\left(\frac{T_{\text {max }}}{W}\right) L D_{\text {max }}$

## Maximum Sustainable Turn Rate, $\dot{\Psi}_{\text {max }}$

The maximum turn rate is a very important indicator of an airplane's maneuverability. A large $T / W$ and $A R \cdot e$ combined with a low $W / S$ and altitude yield the smallest turning radius. This is the fastest heading change the airplane can perform and is given by the following relation:

Turn rate in radians/sec:

$$
\begin{equation*}
\dot{\psi}_{\max }=\frac{g \sqrt{n_{\max }^{2}-1}}{V_{\max \dot{\psi}}} \tag{6-135}
\end{equation*}
$$

Where $V_{\max }=$ Fastest turn velocity, given by Equation (6-78), repeated below for convenience.

$$
\begin{equation*}
V_{\max \dot{\psi}}=V_{T R \min }=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{C_{D \min }}}} \tag{6-78}
\end{equation*}
$$

A derivation of this result is given by Asselin [18].

## Minimum Sustainable Turning Radius

The minimum sustainable turning radius is another important indicator of an airplane's maneuverability. A large $T / W$ and $A R \cdot e$ combined with a low $W / S$ and altitude yield the smallest turning radius. It can be calculated from the following relation:

Turn radius:

$$
\begin{equation*}
R_{\min }=\frac{V_{R_{\min }}^{2}}{g \sqrt{n_{R_{\min }}^{2}-1}} \tag{6-136}
\end{equation*}
$$

Where: $\quad n_{R_{\min }}=\sqrt{2-1 / n_{\max }^{2}}=$ Load factor for minimum turning radius

$$
V_{R_{\min }}=2 \sqrt{\frac{(W / S)}{(T / W)} \frac{k}{\rho}}=\text { Airspeed for minimum turning radius }
$$

A derivation of this result is given by Asselin [18].

## Maximum Bank Angle

This is the maximum angle the aircraft can bank while maintaining altitude (provided it has enough power or thrust) and sustain the limit load factor it has been designed to. It can simply be determined using Equation (6-126) with $n_{\text {lim }}$ being the limit load factor:

Maximum Level Bank angle: $\quad \phi_{\max }=\cos ^{-1}\left(\frac{1}{n_{\lim }}\right)$

## Airspeed for Maximum Bank Angle

Consider Equation (6-132) for turning load factor: $\quad n=\frac{q S}{W} \sqrt{\frac{1}{k}\left(\frac{T}{q S}-C_{D \min }\right)}$

Begin by solving for the dynamic pressure when banking at the limit load, $n_{\text {lim }}$ :

$$
\begin{aligned}
& n_{\lim }^{2}=\left(\frac{q_{\lim } S}{W}\right)^{2}\left(\frac{1}{k}\left(\frac{T}{q_{\lim } S}-C_{D \min }\right)\right) \Leftrightarrow \frac{k n_{\lim }^{2} W^{2}}{S^{2}}=\left(q_{\lim }^{2}\right)\left(\frac{T}{q_{\lim } S}-C_{D \min }\right) \\
\Rightarrow & k\left(\frac{n_{\lim } W}{S}\right)^{2}=\frac{T}{S} q_{\lim }-C_{D \min } q_{\lim }^{2} \Rightarrow C_{D \min } q_{\lim }^{2}-\frac{T}{S} q_{\lim }+k\left(\frac{n_{\lim } W}{S}\right)^{2}=0
\end{aligned}
$$

This can be solved as a quadratic equation as shown below:

$$
q_{\mathrm{lim}}=\frac{1}{2} \rho V_{\lim }^{2}=\frac{\frac{T}{S} \pm \sqrt{\left(\frac{T}{S}\right)^{2}-4\left(C_{D \min }\right) k\left(\frac{n_{\mathrm{lim}} W}{S}\right)^{2}}}{2 C_{D \min }}
$$

Further manipulations lead to:

$$
V_{\lim }=\sqrt{\frac{\frac{T}{S} \pm \sqrt{\left(\frac{T}{S}\right)^{2}-4\left(C_{D \min }\right) k\left(\frac{n_{\lim } W}{S}\right)^{2}}}{\rho C_{D \min }}}=\sqrt{\frac{\left(T \pm \sqrt{T^{2}-4 k C_{D \min }\left(n_{\lim } W\right)^{2}}\right)}{\rho S C_{D \min }}}
$$

Using the simplified drag model, the airspeed required to reach the limit load factor for a given thrust setting is given by the following expression:

$$
\begin{equation*}
V_{\lim }=\sqrt{\frac{\left(T \pm \sqrt{T^{2}-4 k C_{D \min }\left(n_{\lim } W\right)^{2}}\right)}{\rho S C_{D \min }}} \tag{6-138}
\end{equation*}
$$

The expression will return two airspeeds, one for each sign. These represent a low and high speed conditions.

### 6.4.8. Energy State

## Energy Height

The total energy of an airplane whose mass and weight is given by $m$ and $W$, respectively, flying at altitude $h$ and airspeed $V$ is a linear combination of its potential and kinetic energy and can be computed from:

$$
\begin{equation*}
E_{\text {total }}=m g h+\frac{1}{2} m V^{2}=W h+\frac{1}{2} \frac{W}{g} V^{2} \tag{6-139}
\end{equation*}
$$

## Specific Energy and Energy Height

The Specific Energy is defined as the total energy per unit weight and can be computed as follows:

$$
\begin{equation*}
H_{E} \equiv \frac{E_{\text {total }}}{W}=h+\frac{V^{2}}{2 g} \tag{6-140}
\end{equation*}
$$

Since the units of specific energy is that of height (ft or m) it is also called Energy Height. This concept highlights that the maneuvering of an airplane can be considered an exchange of potential and kinetic energy. To explain what this means, consider an airplane cruising at an altitude of 10000 ft at airspeed of 236 KTAS ( $400 \mathrm{ft} / \mathrm{s}$ ) as shown in Figure 6-28. Its specific energy is then $10000+400^{2} /(2 \cdot 32.174)=12768$ ft . This means that if the entire kinetic energy was converted into potential energy, by raising the nose
of the aircraft and allowing it to climb until the airspeed drains to zero (this is a maneuver called zooming), the airplane would reach an altitude of 12768 ft .

The graph of Figure 6-28 is called a Constant Energy Height Map. It consists of isopleths of constant energy height that extend from the vertical to the horizontal axis. The airspeed at any altitude can be calculated for a given energy height by solving for the airspeed in Equation (6-140) as follows:

$$
\begin{equation*}
V=\sqrt{2 g\left(H_{E}-h\right)} \tag{6-141}
\end{equation*}
$$

Equation (6-141) was used to create the constant energy height map of Figure 6-28. The figure shows isopleth for energy heights $\left(\mathrm{H}_{\mathrm{E}}\right)$ of $5000,10000,15000$, and 20000 ft , with the one of 12768 ft shown as a dashed line. Furthermore, the exchange from the initial altitude of 10000 ft and 250 KTAS to 12768 and 0 KTAS is shown as well. The plot applies to all aircraft, regardless of weight. A more type dependent representation is obtained by determining and plotting the Specific Excess Power contour plots (see below).


Figure 6-28: Constant energy height isobars.(from Ref. [3])

## Specific Excess Power

Just like the specific energy was defined as the total energy per unit weight, we also define Specific Excess Power as the excess power (per Equation (6-45)):

$$
\begin{equation*}
P_{S} \equiv \frac{P_{E X}}{W}=\frac{T V-D V}{W} \tag{6-142}
\end{equation*}
$$

Begin with the dynamic version of the equations of motion, i.e. Equation (6-33), repeated here for convenience (assuming the thrust angle $\varepsilon=0$ ):

$$
\begin{equation*}
-D-W \sin \theta+T=\frac{W}{g} \frac{d V}{d t} \tag{6-33}
\end{equation*}
$$

This can be rewritten as follows: $\quad T-D=W \sin \theta+\frac{W}{g} \frac{d V}{d t}=W\left(\sin \theta+\frac{1}{g} \frac{d V}{d t}\right)$

Multiply by $V / W$ :

$$
\begin{equation*}
\frac{T V-D V}{W}=\frac{P_{E X}}{W}=P_{S}=V \sin \theta+\frac{V}{g} \frac{d V}{d t} \tag{ii}
\end{equation*}
$$

Noticing that $V \sin \theta=$ Rate-of-Climb $=d h / d t$ we can rewrite Equation (ii) as follows:

$$
\begin{equation*}
P_{S}=\frac{d h}{d t}+\frac{V}{g} \frac{d V}{d t} \tag{6-143}
\end{equation*}
$$

This expression is the differential form of the specific excess power. It shows that the specific excess power of an airplane is the combination of its Rate-of-Climb $(d h / d t)$ and forward acceleration ( $V / g \cdot d V / d t$ ). This way, if $d V / d t=0$ (steady state), the specific excess power is simply the ROC of the airplane. Likewise, if $d h / d t=0$, the specific excess power is simply its acceleration. An inspection of Equation (6140) reveals its time derivative equals the specific excess power, that is:

$$
\begin{equation*}
\frac{d}{d t}\left(h+\frac{V^{2}}{2 g}\right)=\frac{d H_{e}}{d t}=\frac{d h}{d t}+\frac{V}{g} \frac{d V}{d t} \tag{6-144}
\end{equation*}
$$

In short, the specific excess power is the time rate of change of the Energy Height.

$$
\begin{equation*}
P_{S}=\frac{d H_{e}}{d t} \tag{6-145}
\end{equation*}
$$

### 6.5 Sailplane Performance Theory

This section presents elements of sailplane performance theory. The operation of sailplanes is far more influenced by the presence of rising and sinking air, as well as head- and tailwinds, than that of ordinary powered aircraft. Sound understanding of glide performance is vital for sailplane pilots, regardless of whether attempting to maximize range and endurance. It must be emphasized that the powered and unpowered flight performance is crucial for any energy harvesting and proper management of airspeeds will make or break any algorithm designed for this purpose. Note that it is vital for the discussion in this section to recall the reference to sailplanes A and B originate in Section 1.3.3, Sailplane Airfoils.

The discussion and theory presented in here is largely based on references such as Reichmann [19], Welch and Irving [20], Thomas [21], Stewart [22], and Scull [23]. The images and discussion is largely based on Gudmundsson [3]. In order to help explain the fundamentals of glide performance, the properties of some imaginary sailplane will be utilized. A number of concepts require a simplified quadratic drag model to be used. So, let's assume we are given the drag polar $C_{D}=0.010+0.01498 \cdot C_{L}{ }^{2}$. Also, discussions involving time is represented using a format in which 6.25 minutes would be written as 6 m 15 s .

### 6.5.1. Glide Range and Glide Endurance

The glide range is the horizontal distance a gliding aircraft covers between two given altitudes. The maximum range in still air is obtained by maintaining the airspeed for minimum glide angle (or best angle of descent), denoted by $V_{B G}$ (e.g. see Equation (6-121)). The glide endurance is the time it takes a gliding aircraft to descent between two given altitudes. The maximum endurance in still air is obtained by maintaining the airspeed for minimum rate of descent, denoted by $V_{B A}$ (e.g. see Equation (6-119)). This is shown schematically in Figure 6-29. It can be seen that the range and time aloft achieved from a given altitude is highly dependent on the airspeed the pilot maintains. If launched from an altitude of 1000 ft and assuming still air, maintaining $V_{B G}$ will always yields the longest range and $V_{B A}$ the longest time aloft. Values are obtained using a flight polar like the one in Figure 6-30.


Figure 6-29: A simple schematic showing the impact of any particular airspeed on glide range and time aloft of the example sailplane. (from Ref [3])

### 6.5.2. The Basic Speed Polar - Optimum Glide in Still Air

Sailplane glide performance is determined using the speed polar; a diagram that shows the Rate-of-Descent ( $R O D$ ) as a function of airspeed. It is obtained by plotting the product $-60 \cdot \mathrm{D} \cdot \mathrm{V} / \mathrm{W}$ versus airspeed, $V$.

Consider the basic speed polar of Figure 6-30, which shows the glide characteristics in still air and in the absence of lift or sink. Note that if the sailplane loses altitude, the value of the $R O D$ is negative. A positive $R O D$ means the sailplane is gaining altitude (climbing), perhaps due to the lift in a thermal. Two important parameters are indicated in Figure 6-30; the airspeeds of minimum $R O D, V_{B A}$, and minimum glide angle, $V_{B G}$.


Figure 6-30: Basic speed polar for Sailplane A at S-L. (from Ref [3])

It can be seen that the sailplane achieves a minimum $R O D$ of 131 fpm at airspeed of $46 \mathrm{KCAS}\left(V_{B A}\right)$ and $L D_{\text {max }}$ of 40.9 is achieved at $60 \mathrm{KCAS}\left(V_{B G}\right)$. The $R O D$ at $V_{B G}$ is 149 fpm . Maintaining $V_{B A}$ yields the longest time aloft (endurance), and $V_{B G}$ the greatest range. These are optimum values in no-wind, nothermal conditions only. It is important to keep this plot in mind when reflecting on speed polars for which the sailplane is subject to lift and sink and head- or tailwind.

### 6.5.3. The Speed Polar with Variable Wing Loading

The speed polar is often prepared with one or two specific and frequently used weights in mind. Operating at the higher weight shifts the speed polar to a higher airspeed (see Figure 6-31). The general rule-of-thumb is that there is no change in the $L D_{\max }$, only in the airspeed at which it occurs. On the other hand, and as is to be expected, the magnitude of $R O D_{\text {min }}$ increases, as does the airspeed at which it occurs. The figure shows this is akin to sliding the polar along the sloped line, although it also "expands" as shown. In the case of Sailplane A, there is no change in the magnitude of the $L D_{\max }$, but its airspeed increases from 60 to 69 KCAS with a $30 \%$ increase in weight. The magnitude of $R O D_{\min }$ increases from 131 to 149 fpm and its airspeed from 46 to 52 KCAS.

### 6.5.4. Optimum Glide in Sinking Air

If the sailplane enters a column of air sinking at some average rate, say 200 fpm ( $\approx 2$ knots), this is akin to shifting the speed polar downward as shown in Figure 6-32, left. This does not affect $V_{B A}$. However, $V_{B G}$ is shifted to a higher airspeed, here, from 60 to 77 KCAS. As stated before, in the absence of lift or sink, the normal ROD at these two airspeeds is 131 (at $V_{B A}$ ) and 148 fpm (at $V_{B G}$ ), respectively. The effective $L D_{\text {max }}$ (defined as horizontal speed/vertical speed) is reduced from 40.9 to 18.8 . Note that it is not important whether the polar is shifted downward or the origin is shifted upward, as shown in the right
graph of Figure 6-32. The right graph, thus, presents a clever method to determine the best airspeed-tofly using a polar made for standard conditions only. This is discussed in more detail later.

To help understand why it is beneficial to increase the airspeed, assume the sailplane glides right through the center of a column of uniformly sinking air, whose diameter is 1 nm , and average rate is 200 fpm. At 60 KCAS, it will take the sailplane one minute flat to fly through the column, during which it is descends at $131+200=$ 331 fpm . In other words, it loses 331 ft of altitude in the process. At 77 KCAS, the sailplane descends at $148+200=348 \mathrm{fpm}$ and it will take 47 seconds to cruise through the column, during which it loses (47/60) x $348=273 \mathrm{ft}$. Therefore, less altitude is lost by flying at the higher airspeed. While the difference ( 58 ft ) may seem trivial, it is of crucial importance to the precision piloting required to fly long distances competitively. Figure 6-33 shows how the downdraft affects the glide from a different perspective; presenting it as


Figure 6-31: Basic speed polar for Sailplane A at S-L, while subject to $\mathbf{3 0 \%}$ increase in weight. (from Ref [3]) $L / D$ versus airspeed. The peak $L / D$ is reduced and shifted to a higher airspeed.


Figure 6-32: The speed polar for Sailplane A, assuming it enters a column of air sinking at $\mathbf{2 0 0} \mathbf{f p m}(\approx 2$ knots or 1 $\mathrm{m} / \mathrm{s})$. Both graphs display the same information. By shifting the origin of the vertical axis in the right graph up by 200 fpm , exactly the same answer is obtained as in the left graph.(from Ref. [3])

### 6.5.5. Optimum Glide in Rising Air

If the sailplane enters a column of air rising at some average rate, say 200 fpm , this corresponds to shifting the speed polar upward by that amount (see Figure 6-34). Again, this has no effect on $V_{B A}$. Similarly, $V_{B G}$ becomes the best-angle-of-climb airspeed (denoted by $V_{X}$ ) and it should be maintained in straight and level flight inside a thermal to achieve the steepest climb angle (although thermaling usually requires circling flight, so this is not as simple as that). Shifting the polar downward or the origin upward yields the same answer (see Figure 6-32).

### 6.5.6. Optimum Glide in Headwind or Tailwind

Consider Sailplane A gliding in a 10 knot headwind (or tailwind) while the pilot maintains a constant calibrated airspeed of 60 KCAS. In terms of ground speed, the aforementioned glide speeds will occur 10 knots slower (or faster). This way, the normal best angle of glide speed of 60 KCAS ( $V_{B G}$ ) will actually correspond to 50 (or 70 ) KGS (Knots Ground Speed). If the pilot maintains 60 KCAS, the glide range of the sailplane (descending at 148 fpm ) will vary greatly. The same holds for the glide angle, $\theta$, with respect to the ground, which can be determined as follows:

In a 10 knot tailwind:
In no-wind conditions:
In a 10 knot headwind:

$$
\begin{array}{ll}
\theta=\tan ^{-1}[(148 / 60) /(70 \times 1.688)]=1.20^{\circ} & \Delta \theta=-0.20^{\circ} \\
\theta=\tan ^{-1}[(148 / 60) /(60 \times 1.688)]=1.40^{\circ} \\
\theta=\tan ^{-1}[(148 / 60) /(50 \times 1.688)]=1.67^{\circ} & \Delta \theta=+0.27^{\circ}
\end{array}
$$

Where $\Delta \theta$ represents the difference between the still air and windy glide-angle condition. This is shown schematically in Figure 6-35. If the sailplane begins its glide 1000 ft above the ground, it will touch down in 1000/148 $=6 \mathrm{~m} 45 \mathrm{~s}$ in all three cases, however, the range in headwind will be approximately ( 50 $\mathrm{nm} / 60 \mathrm{~min}) \times(6.75 \mathrm{~min})=5.63 \mathrm{~nm}, 6.76 \mathrm{~nm}$ in still air, and 7.88 nm in tailwind.

This begs the question: In headwind, is there an airspeed other than 60 KCAS that yields range greater than 5.63 nm ? To answer this question, assume that this time Sailplane $A$ is being flown in a 10 knot headwind at 63 KCAS. The ROD at this airspeed is 156 fpm . The ground speed will be 53 KGS and the glide will last for $1000 / 156=6 \mathrm{~m} 24 \mathrm{~s}$. In that time, it will cover ( $53 \mathrm{~nm} / 60 \mathrm{~min}$ ) x $(6.40$ $\mathrm{min})=5.65 \mathrm{~nm}$, greater than 5.63 nm at 60 KCAS. This shows that increasing the airspeed (up to a certain point) in headwind, yields greater range. The inverse is true for tailwind.


Figure 6-33: Standard and "effective" L/D curves for Sailplane A at S-L. The sink rate is 200 fpm.


Figure 6-34: The speed polar for Sailplane A, assuming it enters a lift of 200 fpm ( $\approx \mathbf{2}$ knots or $\mathbf{1 ~ m} / \mathrm{s}$ ). Both graphs display the same information. By shifting the origin of the vertical axis in the right graph down by 200 fpm , exactly the same answer is obtained as that in the left graph.(from Ref. [3])

It should be clear that a sailplane gliding in headwind equal to its forward speed in magnitude will not make any headway with respect to the ground. Rather it would descend vertically - its glide angle would be $90^{\circ}$. Horizontal distance can only be achieved by a forward glide speed faster than the headwind. The airspeed that yields the greatest range can be determined by shifting the origin of the speed polar horizontally by a distance that equals the wind speed, and then draw a tangent to the speed polar. Like the previous discussion demonstrates, in headwind, the origin of the coordinate system is shifted sideways to the right, while for tailwind it is shifted to the left, as shown in Figure 6-36.

### 6.5.7. Speed-to-Fly

The preceding discussion shows that a speed polar for no wind, no thermal conditions can be used with ease to determine the proper airspeed to fly in any non-standard conditions in order to maximize the range of the sailplane. The particular airspeed obtained this way is referred to as Speed-to-Fly by sailplane pilots. It is determined by shifting the origin around as shown in Figure 6-38. For headwind, the origin is shifted to the right. For headwind and sink, it is shifted to the right and up, and so on.

### 6.5.8. Average Cross-Country Speed

Physics dictates that while cruising toward a thermal a sailplane will exchange altitude for distance. Of course, the idea is that once inside the thermal the altitude will eventually be recovered. The total time consumed to travel to the thermal and "get back" to the original altitude is a figure of merit not just for sailplanes, but also piloting skills in long distance competitions. Consider the sailplane depicted in Figure $6-37$, where the segment $A-B$ is the glide segment and $B-C$ the climb segment. The average cross-country speed, denoted by $V_{\text {avg }}$ (also shown in Figure 6-38) can be defined as the distance travelled to the
thermal divided by the total time it takes to reach it and recover the lost altitude. Mathematically, this can be represented as follows:


Figure 6-35: A simple schematic showing the impact of head- or tailwind on the glide range assuming the pilot maintains the same indicated (or calibrated) airspeed in all three cases.(from Ref. [3])


Figure 6-36: The speed polar for Sailplane A, for a glide in a 10 knot tail- and headwind. By shifting the origin of the horizontal axis to the left or right by 10 knots, an ideal airspeed for glide is obtained.(from Ref. [3])

$$
\begin{equation*}
V_{a v g}=\frac{R_{\text {glide }}}{t}=\frac{R_{\text {glide }}}{t_{\mathrm{glide}}+t_{\mathrm{climb}}} \tag{6-146}
\end{equation*}
$$

Where $t_{\text {glide }}$ and $t_{\text {climb }}$ is the time spent in the glide and climb phases, respectively, and $R_{\text {glide }}$ is the total distance covered. The three variables ( $R_{\text {glide }}, t_{\text {glide }}$, and $t_{\text {climb }}$ ) are further defined as follows:

$$
\begin{equation*}
R_{g l i d e}=\left(\frac{V_{G S}}{V_{S}}\right) H \quad t_{\text {glide }}=\frac{H}{V_{S}} \quad t_{\text {climb }}=\frac{H}{V_{C}} \tag{6-147}
\end{equation*}
$$

$\begin{array}{ll}\text { Where: } & V_{S}=\text { Vertical speed in glide } \\ & V_{G S}=\text { Arbitrary horizontal (forward) glide speed (see Figure 6-37) } \\ & V_{C}=\text { Vertical speed in climb }\end{array}$


Figure 6-37: Definition of a cross-country model. (Adapted from Ref [21])
Substitute these equations into Equation (6-146) and manipulate algebraically to yield:

$$
\begin{equation*}
V_{a v g}=\frac{V_{G S} V_{C}}{V_{C}+V_{S}} \Rightarrow \frac{V_{a v g}}{V_{G S}}=\frac{V_{C}}{V_{C}+V_{S}} \tag{6-148}
\end{equation*}
$$

The speed of climb, $V_{C}$, is the difference between the thermal strength (the rate at which air is rising), denoted by $V_{T}$, and the rate of sink of the sailplane as it circles inside the thermal, $V_{S C}$ :

$$
\begin{equation*}
V_{C}=V_{T}-V_{S C} \Rightarrow \frac{V_{a v g}}{V_{G S}}=\frac{V_{T}-V_{S C}}{V_{T}-V_{S C}+V_{S}} \tag{6-149}
\end{equation*}
$$

### 6.5.9. Optimum Speed-to-Fly between Thermals in Still Air

The preceding discussion pertains to the optimization of distance flown in still or moving air. It does not answer what optimum airspeed to maintain when flying between thermals and this must be answered for still or moving air as well.


Figure 6-38: Putting it all together - here for Sailplane A. The appropriate directions in which to move the origin of the polar based on wind and thermal properties are shown. Then, a tangent from the offset origin to the polar is drawn to reveal the Speed-to-Fly and average cross-country speed. (from Ref. [3])

Consider Sailplane A in Figure 6-39 at point A, some 2000 ft above the ground, as it begins its 4 nm journey toward a thermal. Further, assume the thermal strength is known to be 400 fpm . Say the pilot considers 3 airspeeds to fly; $V_{1}=V_{B G}=60 \mathrm{KCAS}, V_{2}=80 \mathrm{KCAS}$, and $V_{3}=100 \mathrm{KCAS}$. Each will indeed lead to different results. Clearly, maintaining $V_{B G}$ while cruising toward the thermal will lead to the longest travel time, however, it also leads to the least amount of altitude to be made up. Conversely, maintaining $V_{3}$ leads to the earliest arrival time, but the greatest altitude to be made up. Details of this speed selection is shown in Table 6-6, which assumes uniform S-L atmospheric properties and that $V_{B A}$ is maintained in the thermal in all three cases (in straight and level flight). It can be seen that the second airspeed, $V_{2}=80 \mathrm{KCAS}$, is superior to the other two, as it leads to the least amount of total time required to reach the original altitude of 2000 ft . Consequently, its $V_{\text {avg }}$ is the fastest.


Figure 6-39: A sailplane headed to a thermal whose strength is known to be 400 fpm.(from Ref. [3])
Table 6-6: Summary of Trip Parameters (from Ref. [3])

| Route | V | L/D | $\mathrm{V}_{\mathrm{s}}, \mathrm{fpm}$ |  | $\Delta H_{\text {cruise }}$ | $\Delta t_{\text {cruise }}$ | $\Delta t_{\text {climb }}$ | $\Delta \mathrm{t}_{\text {total }}$ | $\mathrm{V}_{\text {avg }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | KCAS |  | in cruise | in thermal | ft | min | min | min | KCAS |
| $\mathrm{A}-\mathrm{B}_{\mathrm{BG}}-\mathrm{C}_{\mathrm{BG}}$ | 60 | 40.9 | -148 | 269 | 595 | 4.00 | 2.21 | 6.21 | 38.7 |
| $\mathrm{A}-\mathrm{B}_{2}-\mathrm{C}_{2}$ | 80 | 35.1 | -230 | 269 | 693 | 3.00 | 2.57 | 5.57 | 43.1 |
| $\mathrm{A}-\mathrm{B}_{3}-\mathrm{C}_{3}$ | 100 | 26.2 | -386 | 269 | 928 | 2.40 | 3.45 | 5.85 | 41.1 |

In addition to the airspeed, $V$, Table 6-6 shows the lift-to-drag ratio, vertical speed, $V_{S}$, in fpm, altitude lost enroute, $\Delta H_{\text {cruise }}$, in ft , time in cruise, $\Delta t_{\text {cruise }}$, time to climb back to $2000 \mathrm{ft}, \Delta t_{\text {climb, }}$, and total time, $\Delta t_{\text {total }}$, all in minutes. The last column is an indication of progress made during the glide and subsequent climb. It is the average cross-country speed, here 4 nm divided by the total time, $\Delta t_{\text {total }}$.

Figure 6-40 shows how $V_{\text {avg }}$ varies with $V_{\text {speed-to-fly }}$ for Sailplane A on an idealized no-wind day with a thermal of strength 400 fpm . It is assumed that the pilot maintains $V_{B A}$ once entering the thermal and that the thermal is large enough to allow shallow bank angle to be maintained. The right graph shows how the speed polar can be used to extract $V_{\text {speed-to-fly }}$, while allowing $V_{\text {avg }}$ to be extracted at the same
time. Shift the origin to 269 fpm , which is the thermal strength ( 400 fpm ) added to the ROD at $V_{B A}(-131$ fpm ) to read 82 and 43 KCAS to be read, respectively. The mathematical derivation of why this leads to the correct result is beyond the scope of this dissertation, but an interested reader is directed to Reference 19.


Figure 6-40: Left graph shows how $\mathrm{V}_{\text {avg }}$ varies with the Speed-to-Fly for Sailplane A under specific conditions. The right graph shows how the speed polar can be used to extract $\mathrm{V}_{\text {speed-to-fly }}$ and $\mathrm{V}_{\text {avg. }}$. (from Ref. [3])

### 6.5.10. Optimum Speed-to-Fly between Thermals in Moving Air

If the sailplane is subject to lift or sink, as well as head- or tailwind, the $V_{\text {avg }}$ and $V_{\text {speed-to-fly }}$ can be determined by moving the origin of the flight polar to a new position dictated by the wind and the expected climb rate in the thermal as explained earlier and as shown in the right graph of Figure 6-40. This airspeed can also be determined analytically using Equation (6-148), which leads to the following solution that requires an iterative scheme to solve for the optimum lift coefficient, $C_{\text {Lopt }}$, given some expected rate-of-climb, $V_{C}$ :

Optimum lift coefficient: $\quad C_{D \text { min }}-k C_{\text {Lopt }}^{2}-V_{C} \sqrt{\frac{\rho S}{8 W}} C_{\text {Lopt }}^{3 / 2}=0$
With $C_{\text {Lopt }}$ known, the average cross-country speed can be calculated from the below expression, which is actually applicable to any lift coefficient, $C_{L}$ :

Average cross-country speed: $\quad V_{\text {avg }}=\frac{V_{C} C_{L}^{-1 / 2}}{V_{C} \sqrt{\frac{\rho S}{2 W}}+\frac{C_{D \min }+k C_{L}^{2}}{C_{L}^{3 / 2}}}$

The derivation of these two equations requires Equation (6-151) to be treated before Equation (6-150). First note that the glide speed for a glide angle close to zero is given by Equation (6-116). Using small angle relations this is:

$$
V_{G S}=\sqrt{\frac{2 \cos \theta}{\rho C_{L}} \frac{W}{S}} \approx \sqrt{\frac{2}{\rho C_{L}} \frac{W}{S}}
$$

The rate of descent is given by Equation (6-117): $\quad V_{V}=V_{S}=\frac{C_{D}}{C_{L}^{3 / 2}} \sqrt{\frac{2}{\rho} \frac{W}{S}}$

Replacing the corresponding terms in Equation (6-148) leads to:

$$
V_{\text {avg }}=\frac{V_{G S} V_{C}}{V_{C}+V_{S}}=\frac{V_{C} \sqrt{\frac{2}{\rho C_{L}} \frac{W}{S}}}{V_{C}+\frac{C_{D}}{C_{L}^{3 / 2}} \sqrt{\frac{2}{\rho} \frac{W}{S}}}=\frac{V_{C} C_{L}^{3 / 2} C_{L}^{-1 / 2} \sqrt{\frac{2}{\rho} \frac{W}{S}}}{C_{L}^{3 / 2} V_{C}+C_{D} \sqrt{\frac{2}{\rho} \frac{W}{S}}}=\frac{V_{C} C_{L}^{-1 / 2}}{\frac{V_{C}}{\sqrt{\frac{2}{\rho} \frac{W}{S}}+\frac{C_{D}}{C_{L}^{3 / 2}}}}=\frac{V_{C} C_{L}^{-1 / 2}}{V_{C} \sqrt{\frac{\rho S}{2 W}}+\frac{C_{D}}{C_{L}^{3 / 2}}}
$$

If the drag is represented using the simplified drag polar, $C_{D}=C_{D \min }+k \cdot C_{L}{ }^{2}$, this becomes:

$$
\begin{equation*}
V_{a v g}=\frac{V_{C} C_{L}^{-1 / 2}}{V_{C} \sqrt{\frac{\rho S}{2 W}}+\frac{C_{D \min }+k C_{L}^{2}}{C_{L}^{3 / 2}}}=\frac{V_{C} C_{L}^{-1 / 2}}{V_{C} \sqrt{\frac{\rho S}{2 W}}+\frac{C_{D \min }}{C_{L}^{3 / 2}}+k C_{L}^{1 / 2}} \tag{i}
\end{equation*}
$$

This is Equation (6-151). The optimum $C_{L}$ is obtained by differentiating Equation (i) with respect to $C_{L}$ and setting the derivative to zero. Using the quotient rule of calculus we get

$$
\begin{aligned}
& f=V_{C} C_{L}^{-1 / 2} \Rightarrow d f=-\frac{V_{C} C_{L}^{-3 / 2}}{2} \\
& g=V_{C} \sqrt{\frac{\rho S}{2 W}}+\frac{C_{D \min }}{C_{L}^{3 / 2}}+k C_{L}^{1 / 2} \Rightarrow d g=-\frac{3}{2} C_{D \min } C_{L}^{-5 / 2}+\frac{1}{2} k C_{L}^{-1 / 2}
\end{aligned}
$$

Using this with the derivative of the function $f / g$ (as stipulated by the quotient rule) we get:

$$
\frac{d V_{a v g}}{d C_{L}}=\frac{\left(-\frac{V_{C} C_{L}^{-3 / 2}}{2}\right)\left(V_{C} \sqrt{\frac{\rho S}{2 W}}+\frac{C_{D \min }}{C_{L}^{3 / 2}}+k C_{L}^{1 / 2}\right)-\left(V_{C} C_{L}^{-1 / 2}\right)\left(-\frac{3}{2} C_{D \min } C_{L}^{-5 / 2}+\frac{1}{2} k C_{L}^{-1 / 2}\right)}{\left(V_{C} \sqrt{\frac{\rho S}{2 W}}+\frac{C_{D \min }}{C_{L}^{3 / 2}}+k C_{L}^{1 / 2}\right)^{2}}=0
$$

Or more conveniently:

$$
\left(-\frac{V_{C} C_{L}^{-3 / 2}}{2}\right)\left(V_{C} \sqrt{\frac{\rho S}{2 W}}+\frac{C_{D \min }}{C_{L}^{3 / 2}}+k C_{L}^{1 / 2}\right)-\left(V_{C} C_{L}^{-1 / 2}\right)\left(-\frac{3}{2} C_{D \min } C_{L}^{-5 / 2}+\frac{1}{2} k C_{L}^{-1 / 2}\right)=0
$$

Some algebraic aerobatics of this equation leads to Equation (6-150).

### 6.5.11. The MacCready Speed Ring

Being able to accurately maintain the proper airspeed in a sailplane is vital for anyone striving to maximize the range. The optimized airspeed requires constant pilot awareness of the atmospheric convection. For this reason, long distance flying calls for continuous adjustment of the airspeed. To help, a special device called the MacCready ring is mounted to the variometer (ROC indicator) in the sailplane. The device is a dial or a ring, on which airspeeds for various sink or lift conditions are marked. It is rotated such that its index arrow indicates the lift expected in the next thermal. This rotates the airspeed markings such the needle of the variometer points at $V_{\text {speed-to-fly, }}$ allowing the pilot to quickly read the without having to resort to the speed polar. The name of the device is attributed to the late Dr. Paul MacCready (1925-2007, 82). More details on the operation of this device can be gleaned from references such as Reichman [19]).

### 6.5.12. Circling Flight

Once in a thermal, the pilot flies the sailplane in circles to take advantage of the rising air. Unfortunately, this increases the sink rate ( $V_{S C}$ ) over that in straight and level flight ( $V_{S}$ ). The steeper the bank, the greater the sink rate and less potential energy is gained per unit time. It also results in smaller turning radius, allowing the pilot to stay closer to the thermal core. This implies that an optimum bank angle exists that maximizes the rate of climb, given a specific turning radius.

First, we must develop formulation that allows the sink rate to be assessed based on bank angle and turning radius. For this, consider Figure 6-41, which shows the forces acting on the sailplane banking at an angle $\phi$, while flying at airspeed $V$. $L$ is the lift, $W$ the weight, $m$ the mass, and $R$ is the turn radius. If these are known, the sinking speed in circling flight can be determined from:

$$
\begin{equation*}
V_{S C}=\frac{C_{D}}{C_{L}^{3 / 2}} \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)} \frac{1}{\left[1-\left(\frac{2}{\rho}\left(\frac{W}{S}\right) \frac{1}{R \cdot g \cdot C_{L}}\right)^{2}\right]^{3 / 4}} \tag{6-152}
\end{equation*}
$$

The freebody diagram of Figure 6-41 shows that: $\quad \tan \phi=\frac{m V^{2} / R}{m g}=\frac{V^{2}}{R g}$

Therefore:

$$
\begin{align*}
V & =\sqrt{R g \tan \phi}  \tag{ii}\\
R & =\frac{V^{2}}{g \tan \phi} \tag{iii}
\end{align*}
$$

or


Figure 6-41: Forces acting on the sailplane as it is banked in a circling flight. (from Ref. [3])
Using these equations, any of the variables $V, \phi$, and $R$, can be estimated if the two others are known. Then, Equation (6-72), repeated here for convenience, can be used to estimate the speed of the airplane as a function of the lift coefficient, $C_{L}$, and bank angle, $\phi$ :

$$
\begin{equation*}
V=\sqrt{\frac{2 W}{\rho S C_{L}}\left(\frac{1}{\cos \phi}\right)} \tag{6-72}
\end{equation*}
$$

More conveniently, the equation can be used to extract the lift coefficient, $C_{L}$, required during bank at a given airspeed, from which the drag coefficient, $C_{D}$, can be determined. Using Equation (ii) a relationship between the airspeed, bank angle, and turning radius can be established:

$$
V^{2}=\frac{2 W}{\rho S C_{L}}\left(\frac{1}{\cos \phi}\right)=R g \tan \phi=R g \frac{\sin \phi}{\cos \phi}
$$

Which leads to:

$$
\sin \phi=\frac{2 W}{\rho S C_{L} R g}
$$

Using the trigonometric identity $\cos ^{2} x+\sin ^{2} x=1$, it is now possible to write:

$$
\cos \phi=\sqrt{1-\left(\frac{2 W}{\rho S C_{L} R g}\right)^{2}}
$$

This relates the turning radius to the bank angle. Substituting this into Equation (6-118) yields Equation (6-152). Using Equation (6-152) the map shown in Figure 6-42 can be utilized to evaluate the turning performance of the sailplane, which is imperative for circling flight inside thermals. The diagram shows that if a given bank angle is maintained, the turn radius reduces only if the sailplane slows down. Similarly, for a fixed airspeed, the turn radius can only be reduced by banking steeper - which increases
the sink rate further. It also shows that, for instance for a $60^{\circ}$ bank angle, the least sink rate is to be had around 67 KCAS, resulting in a turning radius of about 225 ft . Both styles of curves are plotted using Equation (6-152). The solid curves are generated by first calculating $C_{L}$ for a range of airspeeds using Equation (6-73) and fixed $\phi$. This is used to calculate $C_{D}$ using the drag polar. The turning radius is also computed using Equation (iii) in the following derivation. Finally, these are inserted into Equation (6152). The dashed curves are calculated for a range of turning radii and fixed airspeeds. First, the bank angle is calculated using Equation (i) in the derivation below. Then this is used to calculate $C_{L}$ and $C_{D}$ as before. Again, these are substituted into Equation (6-152).


Figure 6-42: Turn performance map for Sailplane A, shows its rate-of-descent while banking at the specific angles and airspeeds. (from Ref. [3])

Note that Equation (6-152) can be used during the design stage to help shape parameters, such as wing area, $A R$, and drag characteristics, in an attempt to contour the turn performance curves towards a desirable turn radius and bank angle inside a thermal of specific characteristics. Of course, this must take into account the net rate of climb ( $V_{T}+V_{S C}$, assuming $V_{S C}$ has a negative value). This requires thermals to be modeled mathematically, as presented below.

This information can be combined with the turn performance map to create a representation displaying the optimum bank angle given specific airspeed. This is shown for Sailplane A in Figure 6-43. The optimum climb for the selected airspeeds is easily identifiable. The map also shows that only airspeeds below 80 KCAS will result in climb in this condition and that exceeding $30^{\circ}$ of bank is detrimental to the climb performance.


Figure 6-43: Turn performance map for Sailplane A assuming a maximum thermal radius of 1000 ft and core strength of 4.2 knots used to evaluate best ROC and the corresponding speed and bank angle. (from Ref. [3])

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## 7. Engine Performance Modeling

This chapter presents information and mathematical models to predict the performance of gas engines and electric motors. The methods presented are of crucial importance in predicting consumption of onboard energy and determine the energy savings to be had through energy harvesting during flight. The chapter focuses on the following topics:

- Review of energy concepts
- Review of gas powered engines
- Review of battery power


### 7.1 Introduction

Typical engines for light aviation use include the normally aspirated piston engine and electric motors. Piston engines typically run on gasoline (or Aviation Gasoline or Avgas) or Diesel. It is of interest to ponder the pros and cons of three power-plant options: gas-piston, diesel-piston, and electric motors. Figure 7-1 shows typical installations of a single-cylinder piston engine and a corresponding installation of an outrunner electric motor. It should be evident that the installation of the electric motor is substantially less complex and lighter. Electric motors have more reliable start-stop-start characteristics than piston engines. Also, they produce less noise and, provided the propeller is propeller balanced, run vibration-free, much like turbomachinery. Their primary disadvantage is the low energy density of the current battery technology. This renders long flights a considerable challenge. This is compounded by the long recharge time and limited number of recharging cycles permitted by current technology. The battery can be recharged several hundred times before it must be replaced, at great cost. It should be mentioned that (1) the technology is rapidly developing when this is written and more capable batteries are on the horizon, and (2) the battery technology presents more hindrance to manned aircraft than smaller unmanned ones.


Figure 7-1: Installation of a typical RC piston engine and electric motor (photo by www.greatplanes.com).

## Dry Weight and Power-to-Weight Ratio of Aircraft Engines

Of the three options, the dry weight (engine only) of the gas-piston and diesel-piston per power is greater than that of a comparable electric motor. For instance, a popular 74.6 kW ( 100 BHP ) piston engine (gas or diesel) for aircraft weighs around $55-60 \mathrm{~kg}\left(120-135 \mathrm{lb}_{f}\right)^{1}$. This constitutes a power-toweight ratio of about $1.22-1.36 \mathrm{~kW} / \mathrm{kg}$. In comparison, per a 2015 press release, the German industrial giant Siemens revealed a new technology electric aircraft motor that delivers a maximum continuous power of 260 kW (equivalent to 349 BHP ) and weighs $50 \mathrm{~kg}\left(110 \mathrm{lb}_{f}\right)$ [1]. This corresponds to a power-toweight ratio of $5.20 \mathrm{~kW} / \mathrm{kg}$. An example of a (slightly) more mature engine for aircraft use is a 40 kW (54 BHP equivalent) electric motor produced by the Chinese firm Yuneec, which weighs about $20 \mathrm{~kg}\left(44 \mathrm{lb}_{\mathrm{f}}\right)$. This corresponds to a power-to-weight ratio of $2.00 \mathrm{~kW} / \mathrm{kg}$.

## Installation Complexity

The installation complexity of gas and Diesel engines is substantially greater than that of electric motors. Such engines require additional components such as fuel tanks, fuel pumps, collector tanks, fuel sealant, fuel filters, air vents, drain valves, baffles, and so on. The installation of the electric motor requires batteries, wiring, and Electronic Speed Control (ESC) to convert the DC current from the battery to AC used by the motor. However, the electric installation is clean and free of toxic fumes, contrasting the piston engine installation. While the installation in Figure 7-1 is that for RC aircraft, and installation for larger aircraft is more complex, the electric motor installation is still simpler. That aside, the mechanical complexity of a piston engine is much greater than the electric motor, increasing the probability of mechanical failure and in-flight engine shutdown. Additionally, piston engines are more expensive to acquire and maintain than electric motors.

## Energy and Energy Storage

The energy density ( $\mathrm{J} / \mathrm{kg}$ ) of Avgas is substantially greater than that of even the best modern battery. The energy density of Diesel oil is about $10 \%$ greater energy than Avgas. The specific energy of several types of fuel is shown in Table 7-1. The specific energy of Lithium-Polymer (LiPo) battery is included in the table for comparison. Diesel fuel is safer than gasoline because the vapors are less volatile; they do not ignite or explode as easily as gasoline. Gasoline engines fall into a class of engines called rich-burning engines: The combustion takes place in environment that provides fuel-to-oxygen ratio close to the ideal stochiometric ratio. Too much or too little gas in the mixture will prevent the engine from running. In contrast, Diesel engines are lean-burning engines. The combustion inside such engines takes place in an oxygen-rich environment, but this requires higher compression ratio. In fact, Diesel fuel self-ignites because of the higher pressure inside the cylinder. Thus, a spark plug is not needed for continuous operation like for gas engines, although a glow plug is required to initially start the engine. A typical Diesel engine has a compression ratio of $14: 1$ to $18: 1$, versus $8: 1$ to $10: 1$ for gas engines. The higher energy content of the fuel, higher compression ratio, and leaner fuel mixture makes Diesel engines more efficient than gasoline engines. Thus, a Diesel engine will have between 20-30\% lower fuel consumption than a comparable gasoline engine. As an example, an automobile that gets 30 miles per gallon using

[^16]ordinary car might get 40 miles per gallon with a comparable Diesel engine. Among drawbacks of Diesel engines is the "dirty" fuel whose combustion produces known carcinogens, not to mention it is more expensive than AvGas at the present. Additionally, it is prone to "runaway" failure. This is a direct consequence of the self-ignition property of the Diesel fuel; the engine will keep running as long as fuel is supplied.

Table 7-1: Comparison of Specific Energy of Several Fuel Types.

|  | Density |  | Specific Energy |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\mathbf{k g} / \mathrm{L}\left(\mathrm{or} \mathrm{g} / \mathrm{cm}^{\mathbf{3}}\right)$ | $\mathbf{W}$-hr/kg | $\mathbf{k J} / \mathbf{k g}$ |  |
| Gasoline/E10 | 0.721 | 12069 | 43445 |  |
| Low Sulfur Diesel | 0.832 | 11986 | 43147 |  |
| Biodiesel | 0.880 | 10544 | 37956 |  |
| Propane (LPG) | 0.504 | 12974 | 46704 |  |
| Liquefied Natural Gas (LNG) | 0.450 | 3663 | 13187 |  |
| Ethanol/E100 | 0.789 | 7509 | 27029 |  |
| Methanol | 0.792 | 5610 | 20196 |  |
| LiPo Battery | - | $100-265$ | $360-950$ |  |

Primary source: http://www.afdc.energy.gov/fuels/fuel_comparison_chart.pdf

### 7.2 Review of Energy Concepts

This section presents topics that are vital to any discussion of energy; the energy contained in typical fuel for piston engines and that contained in batteries. The emphasis is more on electric energy as many sUAVs are powered with electric motors. Note that discussion of jets is omitted, as its use is less likely for the type of vehicles presented here. Far more fuel efficiency is to be had with pistons and electric engines (in particular) than with jets, in one part because of the lower speed at which the GICA process is practical. For instance, using a jet engine on a slow moving, low flying aircraft with a low wing loading is a recipe for fuel inefficiency. Using it on a high speed vehicle calls for higher wing loading, which does not lend itself to taking advantage of available thermal or slope lift.

### 7.2.1. The Basics of Energy, Work, and Power

The basics of energy, power, and torque are tabulated in Table 7-2 for convenience, as a familiarity with various energy and power concepts is essential for the discussion that follows.

### 7.2.2. Basic Formulas of Electricity

The following basic equations of electric current, voltage, resistance, and power are used when solving various problems that involve electrical power.

Table 7-2: The Basics of Energy, Work, and Power

| Concept | Formulation | Units |  |
| :---: | :---: | :---: | :---: |
|  |  | SI-system | UK-system |
| Energy <br> The conservation of mass-energy is one of the fundamental conservation laws of physics. It says that energy can neither be created nor destroyed, but it changes form. The form of energy refers to potential, kinetic, electrical, nuclear, chemical, and other forms of energy. | Kinetic Energy: $K E=\frac{1}{2} m V^{2}$ <br> Potential Energy: $P E=m g h$ | $\begin{gathered} \text { Joules (J) } \\ \text { kWh } \\ 1 \mathrm{kWh}=3.6 \times 10^{6} \mathrm{~J} \end{gathered}$ | BTU |
| Work <br> Work is defined as the product of force applied to move an object a given distance. Work is also the same as Torque. | Work $\equiv$ Force $\times$ Distance | Joules <br> N•m | $\mathrm{ft} \cdot 1 \mathrm{~b}_{\mathrm{f}}$ |
| Power <br> Power is defined as the amount of work done in a given time. It is also possible to define it as shown. | $\begin{aligned} \text { Power } & \equiv \frac{\text { Work }}{\text { Time }} \\ & \equiv \frac{\text { Force } \times \text { Distance }}{\text { Time }} \\ & \equiv \text { Force } \times \text { Speed } \\ & \equiv \frac{\text { Torque }}{\text { Time }} \end{aligned}$ | W <br> $\mathrm{J} / \mathrm{sec}$ <br> $\mathrm{N} \cdot \mathrm{m} / \mathrm{s}$ | $\begin{gathered} \mathrm{hp} \\ \mathrm{ft} \cdot \mathrm{lb} / \mathrm{f} / \mathrm{sec} \end{gathered}$ |
| One "Horsepower" |  | $\begin{gathered} 746 \mathrm{~W} \\ 0.746 \mathrm{~kW} \end{gathered}$ | $33000 \mathrm{ft} \cdot 1 \mathrm{lb}_{\mathrm{f}} / \mathrm{min}$ $550 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{sec}$ |

$$
\begin{array}{ll}
\text { Voltage: } & V=\left\{\begin{array}{c}
I \times R \\
P / I \\
\sqrt{P \times R}
\end{array}\right.
\end{array} \text { Volts }
$$

$$
\begin{array}{ll}
\text { Current: } & I=\left\{\begin{array}{l}
\sqrt{P / R} \\
P / V \\
V / R
\end{array}\right. \\
\text { Power: } & P=\left\{\begin{array}{l}
V^{2} / R \\
R \times I^{2} \\
V \times I
\end{array} \quad\right. \text { Watts }
\end{array}
$$

Where:

$$
\begin{array}{l|l}
I=\text { current (Amps) } & R=\text { resistance (Ohms } \\
P=\text { power (Watts or VoltAmps) } & V=\text { voltage (Volts) }
\end{array}
$$

## Energy Density

Energy density is of paramount importance when comes to batteries. It is the amount of energy stored in a unit weight of a battery. It is denoted by $e_{B A T T}$ and is typically given in terms of Watt-hours $/ \mathrm{kg}$ or

Wh/kg for short. The energy density of even the best contemporary batteries is substantially lower than that of fossil fuels, about 60 times less [2]. The energy density for typical aircraft battery packs is given in terms of kWh (kilowatt-hours). A 5.6 kWh battery can deliver 5600 W over a period of an hour. This corresponds to the energy required to keep a 100 Watt light-bulb lit for 56 hours ( 2.3 days) or a 1500 W microwave running for 3.7 hours.

## Specific Energy and Specific Power

Specific Energy:

$$
\begin{align*}
e_{B A T T} & =\frac{V \cdot I \cdot t}{m}  \tag{7-1}\\
p_{B A T T} & =\frac{V \cdot I \cdot t}{m} \tag{7-2}
\end{align*}
$$

Where $t$ is time and $m$ is the mass of the battery.

### 7.3 The Fundamentals of Piston Engines for sUAV Aircraft

This section and the next present a brief review of topics that relate to the use of piston and electric engines for aircraft. The subject is vital to help the reader understand the differences and commonalities between the two.

### 7.3.1. Fossil Fuel Basics

This section discusses the basic concepts of aircraft gasoline, commonly known as Avgas. It should be noted that many sUAVs do not use avgas, but rather variants that include other chemical. The true properties of such variations are not always available in the literature. Nevertheless, in this document and where applicable, avgas is assumed. Basic properties of avgas are presented in Table 7-3.

Table 7-3: The Basics of Avgas (from Ref. [3])

| Parameter | SI-system | UK-system |
| :--- | :---: | :---: |
| Density | $0.71 \mathrm{~kg} / \mathrm{liter}$ | $5.9-6.0 \mathrm{lb}_{\mathrm{f}} / \mathrm{gallon}^{2}$ |
| Energy Content | $1 \mathrm{~kW} \sim 1019 \times$ mass flow (in kg/s) | $1 \mathrm{hp} \sim 620 \times \mathrm{mass}$ flow (in lb $\mathrm{m} / \mathrm{s}$ ) |
| Ideal (stochiometric) ratio $^{3}$ | Air:Fuel = 14.7:1 |  |

## Fuel Octane Rating and Fuel Grades for Piston Engines

Fuel octane rating is a measure of the capability of a fuel to resist compression before it spontaneously self-ignites. This way, fuel with higher octane number can withstand greater pressure inside the cylinder before igniting in this fashion. For this reason high octane number fuel must be used in highcompression (high-performance) engines or they will suffer from engine knocking. That aside, the

[^17]concept is one that is often misunderstood, as there are a number of different octane ratings (e.g. Research Octane Number - RON, Motor Octane Number - MON, etc.). These definitions are outside the scope of this book.

## Specific Fuel Consumption (SFC)

Specific Fuel Consumption (SFC) is one of the most important metrics employed in aviation. The SFC indicates how efficiently the power plant converts chemical energy into mechanical energy. While there usually is not a great variation in SFC between engines within a specific class of power plants, there is a huge variation between the classes. This way, piston engines are generally more efficient than turbo machinery, which is far more efficient than, say, rockets. Naturally, other properties besides efficiency affect engine selection, e.g. target speed and altitude.

It is crucial for the engineer compare the efficiency of different power plants for design purposes and this is accomplished by the definition of Fuel Consumption as the quantity of fuel burned in a unit time (lbs/hr, kg/min, etc.). This is sometimes referred to as Fuel Flow (FF). We then define Specific Fuel Consumption (SFC) as the quantity of fuel burned in unit time required to produce a given engine output. SFC is a technical figure of merit that indicates how efficiently the engine converts fuel into power.

The fuel consumption of piston engines is always measured in terms of mass or weight of fuel flow per unit time per unit of power. For instance, a small piston engine may burn some $100 \mathrm{lb}_{\mathrm{f}}(45 \mathrm{~kg})$ of fuel in a matter of an hour at a T-O power setting. If the power of the engine is known in BHP or kW, the SFC can be computed as shown below:

UK-system: $\quad S F C=c_{b h p} \equiv \frac{\text { weight of fuel in } \mathrm{lbs} / \text { hour }}{\text { power in brake horsepower }}=\frac{\mathrm{lb}_{\mathrm{f}} / \mathrm{hr}}{\mathrm{BHP}}$

SI-system:

$$
\begin{equation*}
S F C=c_{w s} \equiv \frac{\text { mass of fuel in grams } / \mathrm{sec}}{\text { power in Watts }}=\frac{\mathrm{g}}{W \cdot \mathrm{sec}}=\frac{\mathrm{g}}{\mathrm{~J}} \tag{7-4}
\end{equation*}
$$

As an example, consider a piston engine that consumes 10 gallons of fuel per hour while generating 150 BHP. If the fuel weight is $6 \mathrm{lb}_{f}$ per gallon the values of $c_{b h p}$ and $c_{w s}$ (in the UK and SI-systems, respectively) can be found as follows:

$$
\begin{gathered}
c_{b h p}=\frac{10 \mathrm{gallons} / \mathrm{hr}}{150 \mathrm{BHP}}=\frac{60 \mathrm{lb}_{\mathrm{f}} / \mathrm{hr}}{150 \mathrm{BHP}}=0.400 \frac{l b_{f} / \mathrm{hr}}{\mathrm{BHP}} \\
c_{w s}=\frac{(27.18 \mathrm{~kg} / \mathrm{hr})}{150 \mathrm{BHP}}=\frac{(0.00755 \mathrm{~kg} / \mathrm{sec})}{111.9 \mathrm{~kW}}=\frac{(7.55 \mathrm{~g} / \mathrm{sec})}{111.9 \mathrm{~kW}}=\frac{(7550 \mathrm{mg} / \mathrm{sec})}{111900 \mathrm{~W}}=0.06747 \frac{\mathrm{mg}}{\mathrm{~W} \cdot \mathrm{sec}}
\end{gathered}
$$

### 7.3.2. Piston Engine Basics

## Brake Horsepower (BHP)

BHP usually refers to the amount of power delivered at the engine output shaft of a piston engine. It is measured using an instrument called a dynamometer, which is either a mechanical or electric braking device. In the UK-system the horsepower corresponds to the work required to raise a weight of 33000 $\mathrm{lb}_{\mathrm{f}}$ one ft in one minute. This also corresponds to the work required to raise a weight of $550 \mathrm{lb}_{\mathrm{f}}$ one ft in one second. This is written as follows: $1 \mathrm{hp}=33000 \mathrm{ft} \cdot \mathrm{lb}_{f} / \mathrm{min}=550 \mathrm{ft} \cdot \mathrm{lb}_{f} / \mathrm{s}$. Horsepower can be converted to Watts ( $\mathrm{J} / \mathrm{s}$ ) in the SI system by multiplying by a factor of 746 . In other words:

$$
1 \mathrm{hp}=746 \mathrm{~W}=0.746 \mathrm{~kW}
$$

## Thrust Horsepower (THP)

THP refers to the amount of power used to propel an aircraft through the air in terms of horsepower. The thrust of turbojets, turbofans, pulsejets, and rockets is generated by accelerating the fluid directly. For that reason, such engines are always rated in terms of the thrust they generate. This contrasts piston engines and turboprops, whose mechanical work is used to rotate a propeller, which then creates the thrust. This way, it is not the engine per se that generates the thrust, but the propeller attached to it and this is why it is more appropriate to rate such engines in terms of their power. For instance, one can mount two different types of propellers on the same engine and generate two different magnitudes of thrust at the same power output. This is not the case for a turbofan or a turbojet. One of the nuisances about this difference is that sometimes it is helpful to convert the thrust into horsepower, for instance, to compare the effective power of a piston or a turboprop to a turbofan. This must be done by multiplying the thrust T of the turbofan (or turbojet or pulsejet, etc.) with the airspeed V at which it is flying. If working in the UK-system the thrust is given in $\mathrm{Ib}_{\mathrm{f}}$ and the airspeed in $\mathrm{ft} / \mathrm{s}$. The unit for the power is thus in terms of $\mathrm{ft} \cdot \mathrm{l} \mathrm{b}_{\mathrm{f}} / \mathrm{s}$, which can be converted to horsepower (i.e. THP) by dividing the product by 550 , using the expression below:

$$
\begin{array}{ll}
\text { UK-system }(\mathrm{T} \text { in } \mathrm{lb} \mathrm{f}, \mathrm{~V} \text { in } \mathrm{ft} / \mathrm{s}): & T H P=T V / 550 \\
\text { Sl-system }(\mathrm{T} \text { in } \mathrm{N}, \mathrm{~V} \text { in } \mathrm{m} / \mathrm{s}): & T H P=T V / 746 \tag{7-6}
\end{array}
$$

## Two-Stroke versus Four-Stroke Engines

Two-stroke engines are valve-less so they are simpler, lighter, and less expensive to manufacture than their four-stroke counterparts. They are less durable than four-stroke engines because they lack a dedicated lubrication system. Instead, they require oil to be mixed in with the gas (about 4 oz . per gallon of gas). For this reason they burn considerable amount of oil when compared to the four-stroke engine.
A two-stroke engine will exhaust combustion gases and draw in a fresh fuel/air mixture on the downstroke. It will then compress and ignite the mixture on the up-stroke. A four-stroke engine will ignite with a subsequent down-stroke. On the following up-stroke the combustion gases are forced out of the cylinder. As the piston's next down-stroke begins, the fuel/air mixture will be drawn into the cylinder and be compressed and ignited on the subsequent up-stroke. This way, ignition occurs once every revolution in a two-stroke engine, but once every other revolution in a four-stroke. This gives the two-
stroke engine a significant power boost and allows it, potentially, to double the power for the same displacement engine.

The operation of a two-stroke engine is less efficient than that of a four-stroke engine. This results from the use of cleaner gasoline in a four-stroke engine, which is not mixed with oil like fuel for two-stroke engines. Therefore, the combustion burns the fuel more completely and at a higher temperature than possible in a two-stroke engine and both lead to higher efficiency. Additionally, the two-stroke approach leaves remnants of combusted gases inside the cylinder during compression and ignition and forces unburned gas in the exhaust, resulting in greater emission of environmentally harmful chemicals.

## Air-to-Fuel Ratio

As stated above, the ideal stochiometric (air-to-fuel) ratio is 14.7:1 and it yields the least amount of carbon monoxide emissions. However, the engine's maximum power is generally achieved at about 12:1 to 13:1 (rich mixture). On the other hand, minimum fuel consumption is achieved at approximately 16:1 (lean mixture).

## Compression and Pressure Ratios

The Compression Ratio is defined as the ratio between the volume of the cylinder with the piston in the bottom position, $V_{\text {bottom }}$ (largest volume), and in the top position, $V_{\text {top }}$ (smallest volume). The higher this ratio, the greater will be the power output from a given engine. It is generally in the 6-10 range.

Similarly, the Pressure Ratio is defined as the ratio of the pressure inside the cylinder with the piston in the top or bottom position, denoted by $p_{\text {top }}$ and $p_{\text {bottom }}$, respectively. Assuming adiabatic compression inside the cylinder (no heat energy is added when compressing the gas), the relation between the pressure and volume can be shown to comply with the following expression:

$$
\begin{equation*}
p_{\text {bottom }} V_{\text {bottom }}^{\gamma}=p_{\text {top }} V_{\text {top }}^{\gamma} \Leftrightarrow \frac{p_{\text {top }}}{p_{\text {bottom }}}=\left(\frac{V_{\text {bottom }}}{V_{\text {top }}}\right)^{\gamma} \tag{7-7}
\end{equation*}
$$

## Displacement

Displacement is the total volume of combustion chamber in the piston engine. The diameter of each cylinder is called a bore. The total distance a piston moves is called a stroke. The displacement of an engine with $N$ cylinders is defined as follows:

$$
\begin{equation*}
V_{\text {disp }}=\frac{\pi}{4} N\left(\text { bore }^{2} \times \text { stroke }\right) \tag{7-8}
\end{equation*}
$$

The displacement represents the maximum volume of the combustion chambers of all the cylinders assuming the piston is simultaneously at the bottom of the stroke for all (an impossible scenario).

## Typical Specific Fuel Consumption for Piston Engines

The fuel consumption of piston engines varies by type as shown in Table 7-4:

Table 7-4: Specific Fuel Consumption of Typical Piston Engines for Aircraft.

|  | Normally Aspirated Piston Engines |  |
| :--- | :---: | :---: |
|  | $\mathbf{l b}_{\mathbf{f}} / \mathbf{h r} / \mathbf{B H P}$ | $\mathbf{g r} / \mathbf{k W} / \mathbf{h r}$ |
| Two-stroke | $0.83-1.80$ | $280-600$ |
| Four-stroke | $0.42-0.60$ | $140-205$ |

A typical breakdown of how energy is wasted in piston engines can be seen in Table 7-5.
Table 7-5: Energy Wasted in a Piston Engine (Based on Reference [4]).

| Cause | Percentage |
| :--- | :---: |
| Available in fuel | 100 |
| Heat lost to oil | -2 |
| Heat lost to cooling air | -11 |
| Heat lost to radiation | -5 |
| Heat lost to exhaust | -52 |
| Mechanical losses | -5 |
| SUM | $\approx \mathbf{2 5 \%}$ |

## Effect of Airspeed on Engine Power

Generally, the power generated by a piston engine is assumed constant with airspeed, making it an airspeed independent power plant. Effectively this means that if a piston engine produces 10 BHP at a specific power setting, say, at stalling speed, it will also generate 10 BHP at the same power setting at its maximum airspeed. In real applications, however, the power output from the engine depends on the pressure recovery at the manifold. If the pressure recovery is airspeed dependent, for instance due to changes in the attitude of the aircraft, then the engine power will become airspeed dependent. However, for all intents and purposes for analysis work, piston engine power-output can be considered independent of airspeed.

## Effect of Altitude on Engine Power

The power output of normally aspirated engines depends on how efficiently the mixture of air and fuel burns inside the cylinder during combustion; a process that sharply increases the pressure inside the cylinder and pushes the piston in the opposite direction. This, in turn, depends on the total quantity of Oxygen molecules $\left(\mathrm{O}_{2}\right)$ initially inside the cylinder as the piston begins the compression stroke. The quantity of molecules inside the enclosed volume of the cylinder, of course, is the density and it is directly related to the initial pressure in the cylinder, as realized through the equation of state. For this reason, pressure and density are fundamental variables in the operation of piston engines. This, of course, implies that such engines are highly dependent on the density of air, which is a function of altitude.

Initial pressure in the cylinders can be increased by two means: (1) By recovering as much ram air pressure as possible in the engine manifold (pertains to normally aspirated engines) and (2) by artificially increasing the pressure in the manifold. The former is achieved by ensuring the intake is not blocked and
is located in an area where air is allowed to stagnate with minimum losses. The latter can be done through the process of turbo-charging or turbo-normalizing.

To estimate the impact of altitude on the power output of an engine some specialized models are applied. The simplest one, presented below, assumes that the engine power is directly dependent on the density ratio:

Simple altitude dependency model: $\quad P=P_{S L}\left(\frac{\rho}{\rho_{S L}}\right)=P_{S L} \sigma$

Where $P, \rho$ and $\sigma$ are power, density, and density ratio at altitude, respectively, and $P_{S L}$ and $\rho_{S L}$ correspond to S-L values. Another more accurate altitude model for piston engines is the so-called Gagg and Ferrar model [5], here presented in its three most frequently encountered forms:

$$
\begin{equation*}
P=P_{S L}\left(\sigma-\frac{(1-\sigma)}{7.55}\right)=P_{S L}(1.132 \sigma-0.132)=P_{S L} \frac{(\sigma-0.117)}{0.883} \tag{7-10}
\end{equation*}
$$

Where $P_{S L}$ is power in terms of BHP at S-L and $\sigma=$ Density ratio.

The two above expressions are used with normally aspirated engines only. Figure 7-2 shows a comparison between the simple altitude-dependency model and the Gagg and Ferrar models. The horizontal axis shows the percentage power and the vertical altitude in ft. One way of reading the graph is to ask, how much power does a piston engine deliver at a given altitude? For instance, how much power does an engine rated as 200 BHP at S-L deliver at full throttle at 15000 ft ? By tracing the horizontal line extending from 15000 ft to the point where it intersects the thick curve of the Gagg and Ferrar model, it can be seen it delivers approximately $57.5 \%$ power, or $0.575 \times 200=115$ BHP. By the same token, consider the same engine, at S-L, at some arbitrary throttle setting that generates 100 BHP. At the same altitude it will deliver mere 57.5 BHP at the same throttle setting. The Gagg and Ferrar model matches manufacturer's data far better than the former and is recommended for design work.

Straight lines representing 55\%, 65\%, and 75\% power have been plotted in Figure 7-2, but these represent typical power settings reported by manufacturers or widely known publications such as Jane's All the World's Aircraft. It can be seen that, regardless of engine, once at 8283 ft , even at full throttle the maximum engine power will not exceed $75 \%$ of its rated S-L value as long as it is normally aspirated. Corresponding altitudes for $65 \%$ and $55 \%$ are shown as 12106 ft and 16324 ft , respectively. In order prevent this sort of power loss with altitude; the air flowing into the manifold must be introduced at a higher pressure. It must be pre-pressurized. Such pre-pressurization is called turbocharging or turbonormalizing or supercharging and is beyond the scope of this work. Refer to Ref. [3] for more information about this aspect of engine operation.

## Effect of Temperature on Engine Power

Since power is affected by density and pressure, it follows suit that it is also influenced by temperature. The following expression is used to correct power at a nonstandard temperature conditions:

$$
\begin{equation*}
\frac{P}{P_{s t d}}=\sqrt{\frac{T_{s t d}}{T_{O A T}}}=\sqrt{\frac{518.67(1-\kappa h)}{T_{O A T^{\circ} R}}}=\sqrt{\frac{273.15(1-\kappa h)}{T_{O A T ~ K}}} \tag{7-11}
\end{equation*}
$$

Where:

$$
\begin{aligned}
& P_{s t d}=\text { Stndrd power at altitude and ISA } \\
& \kappa=\text { Lapse rate constant } \\
& T_{s t d}=\text { Standard day temperature }
\end{aligned}
$$

$T_{O A T}=$ Outside air temperature at condition $h=$ Pressure altitude at condition

Note that Equation (7-11) is recommended by several piston engine manufacturers without specifically presenting derivations for it. As demonstrated in the following example, it over-estimates the engine power when compared to the Gagg and Ferrar model with density ratio based on the ideal gas law. This author recommends the latter method as it yields more conservative performance estimations.


Figure 7-2: A comparison showing the difference between several models used to describe how piston engine power is affected by change in altitude. (from Ref. [3])

To illustrate the use of this formulation, consider the problem of estimating the power of a piston engine rated at 100 BHP operated at full power at 10000 ft on a day on which the OAT is $30^{\circ} \mathrm{F}$ (or $30^{\circ} \mathrm{R}$ ) higher than ISA. Compare the use of Equation (7-11) to the use of the ideal gas law and Equation (7-10).

First, calculate the Lapse rate factor: $\quad(1-\kappa h)=(1-0.0000068756 \times 1000)=0.9312$

Then, estimate standard day temperature and density ratio at 10000 ft :

Temperature:

$$
T_{s t d}=518.67 \times 0.9312=483.0^{\circ} \mathrm{R}
$$

Density ratio:

$$
\sigma=0.9312^{4.2561}=0.7385
$$

Maximum power at 10000 ft per Gagg and Ferrar:

$$
P=P_{S L}(1.132 \sigma-0.132)=100(1.132 \times 0.7385-0.132)=70.4 \mathrm{BHP}
$$

This is further reduced by the warmer than normal day per Equation (7-11) as follows:

$$
P=P_{\text {std }} \sqrt{\frac{T_{s t d}}{T_{O A T}}}=70.4 \sqrt{\frac{483.0}{483.0+30}}=68.3 \mathrm{BHP}
$$

Next, calculate the power using the ideal gas law and the Gagg and Ferrar model:

Pressure at 10000 ft :

$$
p=2116(1-k h)^{5.2561}=2116 \times 0.9312^{5.2561}=1455 \mathrm{psf}
$$

Density at 10000 ft :

$$
\rho=\frac{p}{R T}=\frac{1455}{1716 \times(483+30)}=0.001653 \mathrm{slugs} / \mathrm{ft}^{3}
$$

Density ratio at $10000 \mathrm{ft}: \quad \sigma=\frac{\rho}{\rho_{0}}=\frac{0.001653}{0.002378}=0.6951$

$$
\text { Gagg and Ferrar: } \quad P=100(1.132 \times 0.6951-0.132)=65.5 \mathrm{BHP}
$$

About 3 BHP difference remains between the two methods.

## Effect of Manifold Pressure and RPM on Engine Power

The relationship between the manifold pressure and RPM is complex and is usually presented by the piston engine manufacturer in the form of a special graph called an engine performance chart. These effects are detailed in References [3] and [6].

### 7.3.3. Model for Energy Consumption of Gas Powered Engines

As stated earlier, the specific fuel consumption indicates the weight of fuel consumed per unit time to produce a specific amount of horsepower.

$$
\begin{equation*}
\dot{w}=P \cdot S F C=[B H P]\left[\frac{\mathrm{lb}_{\mathrm{f}} / \mathrm{hr}}{\mathrm{BHP}}\right]=\mathrm{lb}_{\mathrm{f}} / \mathrm{hr} \tag{7-12}
\end{equation*}
$$

Therefore, the instantaneous amount of fuel being consumed is found from

$$
\begin{equation*}
d \dot{w}=P \cdot S F C \cdot d t \tag{7-13}
\end{equation*}
$$

It follows that if we know the time history of the application of power and, furthermore, do not assume a constant SFC, the total quantity of fuel consumed at time $t$ is given by

$$
\begin{equation*}
\dot{w}(t)=\int d \dot{w}=\int_{0}^{t} P(\tau) \cdot S F C(\tau) \cdot d \tau \tag{7-14}
\end{equation*}
$$

To treat complicated missions, Equation (7-14) can be split into any number of smaller integrals that each deals with a specific portion of the flight for which the SFC can be considered constant.

### 7.4 Fundamentals of Electric Powerplant for sUAV Aircraft

This section presents practical information regarding electric power and electric energy in the context of electrically powered vehicles. In particular, it presents facts about the battery (specifically LithiumPolymer batteries) and electric motors. Mathematics used to estimate various discharge properties of batteries is presented in Section 7.4.3, Discharge Curves. Note that the more general information regarding batteries is obtained from the online resource "The Electropedia," maintained by the Woodbank Communications, Ltd [7] and Buchmann [8], the founder of the online resource "Battery University" (Ref. [9]). Hipperle [10] also presents a good discussion of the potential of electric flight. When comes to general knowledge about batteries (albeit with limited mathematical presentation), the author considers theses sources in all, convenient, practical, and reliable.

### 7.4.1. The Battery

A battery is a container of two or more chemicals, which when combined undergo a chemical reaction of which a current of electrons is a primary byproduct. In that sense, a battery is a container of electric energy (it is a "container" because it allows this energy to be carried around). The complete history of the battery is both a long and an interesting read, although it is beyond the scope of this dissertation. Some claim the history of the battery dates back to the so called Parthian batteries, at the beginning of the Common Era, some 2000 years ago. While contested, we know with certainty that the modern battery dates back to the late 1700s and is recognized as an invention by Alessandro Volta (1745-1827, 82). In fact, it dates at least to earlier experiments performed by Luigi Galvani (1737-1798, 61), although the phenomenon was first explained by Volta, who considered it the consequence of joining two dissimilar metals [11]. In interest of space, a compact history of the development of the battery is presented as a timeline in Figure 7-3.

## The Basics

All batteries have two terminals, of which one has a positive charge (lack of electrons) and is called the cathode, while the other has a negative charge (abundance of electrons) and is called the anode. Generally, batteries fall into two classes; primary (disposable) and secondary (rechargeable). In primary
cells, the chemical reaction that generates the electric current is not reversible. The constituent chemicals change permanently during the discharge and electrical energy is available until the chemical reaction has fully completed. Therefore, the battery can only be used once. In contrast, the chemical reaction in secondary cells is reversible, making it possible to reconstitute the original constituent materials by the generation of an electrical potential between the anode and cathode. This permits the battery to be discharged and recharged multiple times.


Figure 7-3: A timeline of battery development.

## Terminology for Batteries

Familiarity with concepts related to battery use is vital when comes to discussion of propulsive energies. The following list presents a useful list of definitions.

| Term | Definition |
| :--- | :--- |
| Anode | The terminal of negative charge (abundance of electrons) |
| Battery capacity | Quantity that indicates the amount of current-time stored in the battery. Batteries are rated <br> in Amp-hours (or Ah or mAh). Thus, a 1 Ah (or 1000 mAh ) battery means that 1 Amp current <br> can be drawn steadily for 1 hour, 2 Amps for 30 minutes, and so on. |
| Cathode | The terminal of positive charge (lack of electrons) |
| Cold-Cranking Amps (CCA) | Term used for batteries used to start piston engines. Such batteries are typically marked with <br> a CCA value, which indicates the current (in Amps) the battery can deliver at $-18^{\circ} \mathrm{C}\left(0^{\circ} \mathrm{F}\right)$. |
| C-rating | The " $\mathrm{C}^{\prime \prime}$ in C-rating stands for Coulomb and represents the SI unit for electric charge, which <br> equals to the quantity of electricity transported each second by a 1 Amp (A) current. Thus, 1C <br> equals 1 A•s. At 1C, a battery rated at 1000 mAh would be fully charged in 1 hour using a 1000 <br> mA current. At 1C it would discharge 1000 mA in approximately an hour. Similarly, a 35 C <br> battery allows a maximum of 35 A to be discharged each second. |


| Term | Definition |
| :---: | :---: |
| Cycle life | Refers to the number of times a secondary battery can be charged and discharged. Depends on parameters such as chemical stability, environmental factors, operating temperatures, and typical DOD. |
| Depth-of-Discharge (DOD) | The ratio of the quantity discharged from a battery to its rated charge capacity. For instance, if 500 mAh are consumed from a 1500 mAh battery, the DOD is $500 / 1500=0.333$. A low value of DOD in rechargeable (secondary) batteries results in exponentially greater cycle life of the battery (see [7]). |
| Electrolyte | Ionic conductor that resides inside the battery and serves as a medium used to transfer electric charge as ions between the anode and cathode. |
| Energy density | The amount of energy (e.g. Joules) stored in a system (e.g. battery) per unit volume of the system. It is a goal in battery design to get as high an energy density as possible. Often taken to mean Specific energy. |
| Energy efficiency | Energy efficiency is the ratio of the energy contained in a source, such as gasoline, coal, or a battery, to what can be harnessed. Generally, the charge and discharge efficiencies of batteries are very high when compared to other sources of power. Charge efficiency is close to $100 \%$ and discharge efficiency is close to $95 \%$. The discharge efficiency of fuel cells is $20-$ $60 \%$. The energy efficiency of typical gas engines is about $25 \%$. |
| Impedance | Impedance is to AC current what resistance is to DC current. Caused by the combination of ohmic resistance and reactance. |
| Internal impedance | Internal "resistance" in the battery The internal impedance is not only due to the resistivity of the active materials in the cell, it also depends of the quality of the contacts between the individual electrode particles. For this reason particles should be small with a regular shape. Conductivity is increased by coating the cells with a very thin layer of conducting compound and by using conductive binders. As also noted above, introducing additives can reduce the cell capacity. |
| Load | The amount of current (in Amps) being drawn from a battery. |
| Memory effect | Refers to a behavior in some batteries that progressively reduces their charge capacity. It is reversible and occurs in NiCad and to a lesser extent in NiMH batteries. It is caused by a growth of crystalline formation from a fine (desirable) to a large structure, caused when the cell is recharged before it is fully discharged. |
| Nominal voltage | Refers to a "typical" voltage of a battery during use. Since most discharge curves are neither linear nor flat (see Figure 7-6), a typical value is generally taken which is close to the voltage during actual use. |
| Power | Battery power is specified in Watts (W) or Volt-Amps (VA). |
| Self-discharge | The inevitable and undesirable chemical reaction that takes place inside the battery due to current leakage through the electrolyte and reduces its charge during periods of storage. Can be reduced using smaller micro-pores in the separator, although a byproduct is increase in the cell's internal impedance. Self discharge is temperature dependent. |
| Separator | Component that isolates the anode and cathode. |


| Term | Definition |
| :--- | :--- |
| Specific energy | Amount of energy (e.g. Joules or Watt-hours; Wh) stored in a system (e.g. battery) per unit <br> mass of the system. Thus, the specific energy of a battery might be $100 \mathrm{~Wh} / \mathrm{kg}$. |
| Specific power | Amount of power (e.g. Watts; W) stored in a system (e.g. battery) per unit mass of the system. <br> Thus, the specific energy of a battery might $10 \mathrm{~W} / \mathrm{kg}$. As an example, batteries for electric RC <br> aircraft offer high specific power, moderate specific energy, and very low internal resistance. <br> Alkaline batteries have high specific energy (Wh/kg) but poor specific power (W/kg), while the <br> opposite holds for a supercapacitor. |
| Storage capacity | See Battery capacity |
| Thermal runaway | A condition in which a battery will overheat and destroy itself through internal heat <br> generation. Typically caused by overcharging or excessive current discharge and similar abuse. |

## Current State of Technology

Battery technology has changed drastically over the last five years or so. Even though the NiCad battery was invented in 1899, it didn't see great use until the 1970s or 80s, when small high use electronics became widely available. The popularity of such devices called for rechargeables to keep battery costs to a minimum. The advent of the laptop computer and small cellular communication devices called for rechargeables with even greater capability, a need that was initially met by the use of Nickel-metalHydride (NiMH) batteries and later by Lithium-Ion (Li-Ion) batteries. The need to power such tools for hours helped drive the development of batteries of greater endurance. The current high capacity rechargeable battery is the derivative of the Li-Ion battery called the Lithium-Polymer or LiPo for short. It has caused a technical jump in aviation. These batteries have substantially greater energy capacity than the batteries of the yesteryear, permitting constant high-power usage for 5-10 minutes on highspeed RC aircraft, and 45+ minutes in low-speed RC gliders. This battery is what is used in electrically powered RC aircraft and the kind of aircraft this dissertation focuses on.

The ideal battery for use in airplanes should be light, rechargeable, have a long durability, and with the highest energy density possible. The current state of technology has been largely driven by demand for laptop computers and cell phones, where bright screens and long endurance is of the essence. Batteries of the kind people are mostly familiar with, such as those used for flashlights ( $D, C, A A$, and AAA style) or conventional car batteries suffer from low energy density and high weight. The modern Lithium-ion battery marks a huge improvement over the batteries of the past, although it is in fact marginal for manned aircraft. Currently, two types of batteries are suitable for use in aircraft: $\mathrm{LiFePO}_{4}$ and $\mathrm{LiCoO}_{2}$. Both have their pros and cons. The current battery technology consists of the types of batteries listed in Table 7-6:

## Shortcomings of the Modern Battery

Several issues concerning batteries are of great importance and must be kept in mind:

Table 7-6: Common Battery Types

| Battery Type | Comment |
| :--- | :--- |
| Lead-Acid | Best known as "car-battery." Not suited for use in aircraft. |
| NiCad | Used to be popular for Radio-Controlled aircraft. Largely obsolete. |
| NiMH | Popular as rechargeable batteries for robots. |
| Li-lon | Lithium-ion battery, best known as battery packs for laptop computers. |
| LiPo | Current power packages for electric aircraft. |

(1) Low energy density when compared to fossil fuels. A lot of battery power requires a lot of battery - i.e. a lot of weight.
(2) Energy durability - shelf life. There is more to battery capability than just energy density. For instance $\mathrm{LiCO}_{2}$ batteries have a higher initial energy density than, say, $\mathrm{LiFePO}_{4}$ batteries. However, after a year of frequent recharge-discharge cycles, the $\mathrm{LiFePO}_{4}$ battery has similar residual energy density than the $\mathrm{LiCO}_{2}$ battery. In two years it does better. This describes the essence of battery durability.
(3) Discharge voltage depends on the remaining charge and battery temperature. The initial discharge voltage is usually high, but diminishes with the energy used. This means that initially after a battery recharge is completed, the battery appears to "contain a lot of power." However, this drops rapidly. For aircraft this means that a fully charged battery yields a reported T-O distance, but the first touch-n-go requires a much longer runway. This could be remedied using a control circuit that limits maximum current based on battery voltage. However, the drawback is that there would be reduces power available for use by the operator. This is also common for turbo-machinery and selected piston engines, where it is referred to as flat-rating.
(4) Some contemporary battery technology poses fire hazards.

The Ragone plot in Figure 7-4 is used to compare the performance of a range of electrochemical devices. It shows that Fuel Cells can store large amounts of energy but have a relatively low power output, while the opposite holds true for ultracapacitors (supercapacitors). These can deliver very high power in a short amount of time, but have a very limited storage capacity.

### 7.4.2. The Lithium Polymer Battery (LiPo)

The current state-of-the-art battery for high energy use, such as that involved in the operation of sUAVs, is a variant of the Lithium-Ion battery, called Lithium polymer or LiPo battery for short. Its use warrants a focused presentation. The LiPo battery is similar to the Li-lon battery, but uses a polymer for electrolyte, which is leak-resistant and more damage-resistant. The use of the polymer for electrolyte eliminates the need for a heavy protective casing and allows the battery to be formed in sheets and even irregularly shaped geometry for optimum spatial use. Contrary to common belief, LiPos can sustain severe physical abuse, as long as it does not start chemical reaction between the Lithium and oxygen. A listing of pros and cons of LiPos is provided below.


Figure 7-4: Ragone diagram of battery types (based on Ref. [7])

## Pros:

- High voltage ( $3.7 \mathrm{~V} /$ cell) compared to other batteries. For example, a LiPo cell has 3 X the voltage of a NiCad or NiMH battery (1.2V/cell). One LiPo cell will do what requires three NiCad or NiMH cells.
- No electrolyte eliminates the risk of leaking.
- High energy density compared to other batteries.
- High power density compared to other batteries.
- Light weight compared to other batteries.
- High discharge rate (40C or more). Low self-discharge rate (can retain charge for over 10 years).
- Maintains constant voltage over $80 \%$ of its discharge capacity (see Figure 7-6).
- No memory effect.
- Long cycle-life that does not need reconditioning (like Nickel-based batteries).


## Cons:

- Chemical stability of Lithium requires care in their operation. Susceptible to thermal runaway.
- More expensive (i.e. \$/weight) than many other types of batteries.
- Higher impedance than for batteries such as NiCad.
- Degrades if discharged below 2V/cell or if exposed to high temperature environment.
- Requires protective circuits that, that for instance, calls for recharging "algorithm" (constantcurrent, constant voltage charging system) to be followed with associated cell balancing for multi-cell batteries.


Figure 7-5: An example of a typical 3-cell (or 3S) LiPo battery as used for electric RC aircraft. The packaging reveals the battery has a nominal voltage of 11.1 V , allowable discharge rate of $\mathbf{2 0 C}$, and a capacity of 5000 milliAmp-hours (mAh), or 5 Ah.

LiPos should be charged regularly. The fully charge voltage of a LiPo cell is typically 4.2 V , but charging to $4.1 \mathrm{~V} /$ cell will extend cycle life (albeit reduce capacity by some $10 \%$ ).

### 7.4.3. Discharge Curves

The Voltage-Discharge plot in Figure 7-6 shows how the voltage reduces with discharge state. Such curves are of vital importance in the operation battery powered vehicles. Most of the batteries suffer from a rapid initial voltage drop, which then tends to reduce at a relatively slow rate. In fact, both Li-Ion (LiPo) and NiCad batteries maintain relatively constant voltage over $80 \%$ of their discharge capacity. Then, toward the end of their capacity there is a rapid reduction in voltage. This is important to keep in minde when operating vehicles that require large capacity discharges, such as reconnaissance platforms. It is a common occurrence among operators of RC aircraft to permit the voltage of the LiPo cells to drop dangerously close to this "cliff" with the aircraft (perhaps) far away from the landing strip. The plot of Figure 7-6 shows that it is inadvisable to discharge LiPo batteries to lower than 3.7 V per cell. This would indicate that some $90 \%$ of the nominal charge capacity has been consumed.

## Temperature Effects - the Arrhenius Equation

The best operating temperature for most batteries is between $10^{\circ} \mathrm{C}$ and $35^{\circ} \mathrm{C}\left(50^{\circ} \mathrm{F}\right.$ to $\left.95^{\circ} \mathrm{F}\right)$. Intuitively, higher temperature will expedite the chemical reaction inside the battery, improving battery performance. An unfortunate side-effect is a corresponding reduction in battery-life. The relationship between temperature and the rate of the chemical reaction that takes place inside a battery is typically described using the Arrhenius equation, attributed to the Swedish scientist Svante Arrhenius (18591927, 68). Consider a chemical reaction (synthesis reaction) involving two chemicals $M_{1}$ and $M_{2}$ that yields the combined form $M_{12}$


Figure 7-6: Discharge curves for several types of batteries (based on Ref. [7]).

$$
\begin{equation*}
a M_{1}+b M_{1} \rightarrow c M_{12} \tag{7-15}
\end{equation*}
$$

Where $a, b$, and $c$ are the associated stochiometric coefficients. The rate at which the reaction takes place (in moles/unit time) can then be expresses as shown below [12]

$$
\begin{equation*}
r=-\frac{1}{a} \frac{d\left[M_{1}\right]}{d t}=-\frac{1}{b} \frac{d\left[M_{2}\right]}{d t} \tag{7-16}
\end{equation*}
$$

This allows the rate to be expressed more concisely as follows:

$$
\begin{equation*}
r=k(T)\left[M_{1}\right]^{m}\left[M_{2}\right]^{n} \tag{7-17}
\end{equation*}
$$

Where $\left[M_{1}\right]$ and $\left[M_{2}\right]$ are the molar concentrations of the two materials and $m$ and $n$ are empirical constants that depend on the sequence of how the reaction takes place. The term $k$ is called the rate constant and is a function of the absolute temperature, $T$. It is calculated using the Arrhenius equation

$$
\begin{equation*}
k(T)=A e^{-E_{a} /(R T)} \tag{7-18}
\end{equation*}
$$

Where $A$ is called the prefactor, $E_{a}$ is the activation energy, $R$ the universal gas constant, and $T$ the absolute temperature in degrees Kelvin (K). Arrhenius introduced the term activation energy, $E_{a}$, to describe the minimum amount of energy that must be present to help "start" a chemical reaction. It is often described as an energy barrier that prevents a chemical reaction from taking place. In short,
increasing temperature will increase the fraction of molecules that have energies in excess of the activation energy [13]. Their unit is typically in terms of $\mathrm{kJ} / \mathrm{mol}$ or $\mathrm{kcal} / \mathrm{mol}$. The universal gas constant, $R$, has a value of $8.314 \mathrm{~J} /(\mathrm{K} \mathrm{mol})$. A common rule-of-thumb, supported by Equation (7-18), is that the rate of many chemical reactions at room temperature doubles with an increase of $10^{\circ} \mathrm{C}$ temperature.

## Discharge Effects - Peukert's Law

Additional effect must be considered when discharging batteries; a greater discharge rate leads to reduction in the available capacity. The discovery of this effect is attributed to the German scientist Wilhelm Peukert (1855-1932, 77). It is an important law in the operation of electric vehicles, many which demand high current discharge. The effect can be phrased in the following fashion. Consider a 5 Ah battery being discharged at a constant 10 Amp draw. Then, one would expect the battery to be fully drained in $5 \mathrm{Ah} / 10 \mathrm{~A}=0.5$ hours. However, in practice the battery will "run out" in less time, perhaps only 0.4 hours. The effect, which is called Peukert's Law, is expressed mathematically as shown below.

$$
\begin{equation*}
C=I^{k} t \tag{7-19}
\end{equation*}
$$

Where $C$ is the discharge capacity (in Ah), $I$ the current (in Amps), $k$ is the Peukert's constant, and $t$ the time (in hours) it takes to discharge the battery. The value of $k$ depends on the type of battery and varies between 1 (ideal battery) to 2 (terrible battery). Typical values for lead-acid batteries are between 1.1 and 1.3 and for LiPo batteries it varies from 1.00 to 1.28 (see Omar et al [14]).

Several researchers have suggested that Peukert's law should be applied with care and its reliability requires constant current draw and limited internal temperature rise due to discharge (e.g. see [14] and Doerfell et al. [15]). The effect of internal temperature rise is an increase in charge capacity (see "Temperature Effects - the Arrhenius Equation" above) that counteract Peukert's law. These researchers emphasize that operators regard the battery as "a complex system, where the capacity is a function of current rate, depth of discharge and temperature." [14] The following linearized expression is proposed to better describe the actual relationship

$$
\begin{equation*}
C=\left[\frac{\partial C}{\partial I} C_{I}+\frac{\partial C}{\partial T} C_{T}+\frac{\partial C}{\partial D O D} C_{D O D}\right]+C_{0} \tag{7-20}
\end{equation*}
$$

Where $C_{I}$ is the capacity at current draw $I, C_{T}$ is the capacity at temperature $T, C_{D O D}$ is the capacity at depth-of-discharge percentage $D O D$, and $C_{0}$ is constant. Naturally, the problem is to determine the derivatives for the battery being used.

In this dissertation, derivative versions of Peukert's expression of Equation (7-19) will be used with conservative value of the Peukert's constant, whose value varies widely between brands of batteries. Such derivative formulations are helpful to predict the discharge capacity of a battery based on the expected current draw ( $I$ ) and commercially rated values (assuming these can be trusted). The following expression permits this to be done

$$
\begin{equation*}
C=C_{\text {rated }}\left(\frac{I_{\text {rated }}}{I}\right)^{k-1} \tag{7-21}
\end{equation*}
$$

Where $C$ is the discharge capacity realized during battery use, $C_{\text {rated }}$ and $C_{\text {rated }}$ are the rated (maximum) discharge capacity (e.g. 100 Ah ) and discharge current (e.g. 35 A ), $I$ the actual discharge current (in Amps). A modified discharge model, based on Peukert's law, is presented by Fuller [16]. It applies to systems that operate on constant power $(P=V I)$, rather than constant current. Since the voltage drops as a function of capacity remaining, constant power implies that current must be increased. The approach rewrites Equation (7-21) to permit $C$ to be determined based on current draw. Thus, since

$$
\frac{C}{C_{\text {rated }}}=\left(\frac{I_{\text {rated }}}{I}\right)^{k-1}=\frac{I \cdot t}{I_{\text {rated }} \cdot t_{\text {rated }}}=\frac{I \cdot t}{I_{\text {effective }} \cdot t} \Rightarrow \quad I_{\text {effective }}=I\left(\frac{I}{I_{\text {rated }}}\right)^{k-1}
$$

Therefore, for operation that assumes variable current (e.g. to ensure constant power) the effective capacity is the sum of time segments during which an effective current is applied, i.e.

$$
\begin{equation*}
C_{\text {effective }}=\sum_{i=1}^{N} I_{\text {effective }}^{i} \text { } \cdot \Delta t_{i}=\sum_{i=1}^{N} I_{i}\left(\frac{I_{i}}{I_{\text {rated }}}\right)^{k-1} \cdot \Delta t_{i} \tag{7-22}
\end{equation*}
$$

## Tremblay's Method

Finally, Tremblay et al [17] developed a simple model to estimate the open-circuit voltage of secondary (rechargeable) battery based on the state of charge. The approach makes several assumptions, including the basic plot of Figure 7-7:

- No voltage recovery.
- Constant internal resistance.
- Discharge characteristics are the reverse of the charging characteristics.
- No Peukert effect
- No temperature effects.
- No self-discharge, and
- No memory effects

Then, Tremblay's method is the following expression:

$$
\begin{equation*}
V_{O C}(C)=V_{0}-\left(\frac{\kappa \cdot C_{c u t}}{C_{c u t}-C}\right)+A e^{-B \cdot C} \tag{7-23}
\end{equation*}
$$

Where $A \equiv V_{\text {full }}-V_{\text {exp }}$ and $B \equiv \frac{3}{C_{\text {exp }}}$ and


Figure 7-7: Discharge curve to use with Tremblay's method (based on Ref. [17]).

$$
\begin{equation*}
\kappa=\frac{\left(V_{\text {full }}-V_{\text {nom }}+A\left(e^{-B \cdot C_{\text {nom }}}-1\right)\right)\left(C_{\text {cut }}-C_{\text {nom }}\right)}{C_{\text {nom }}}=\left(V_{\text {full }}-V_{\text {nom }}+A\left(e^{-B \cdot Q_{\text {nom }}}-1\right)\right)\left(\frac{C_{\text {cut }}}{C_{\text {nom }}}-1\right) \tag{7-24}
\end{equation*}
$$

and

$$
\begin{equation*}
V_{0}=V_{\text {full }}+\kappa+\left(R_{C} \cdot I_{0}\right)-A \tag{7-25}
\end{equation*}
$$

Other variables are:
$C=$ Capacity discharged (e.g. 345 mAh )
$C_{\text {exp }}=$ Capacity discharged at the end of the exponential range (e.g. 220 mAh )
$C_{n o m}=$ Capacity discharged at the end of the nominal range (e.g. 1700 mAh )
$C_{c u t}=$ Capacity discharged at cut-off (e.g. 2200 mAh )
$V_{\text {full }}=$ Fully charged potential (e.g. 12.54 V )
$V_{\text {exp }}=$ Potential at the end of the exponential range (e.g. 11.9 V )
$V_{\text {nom }}=$ Potential at the end of the nominal range (e.g. 11.3 V )
$V_{\text {cut }}=$ Cut-off potential (e.g. 9.8 V )
$I_{0}=$ Specified discharge current (e.g. 10 Amps )
$R_{C}=$ Internal resistance (e.g. $2 \times 10^{-3} \mathrm{Ohms}$ )

An example of Tremblay's method for a typical 2200 mAh 3 LiPo battery for an RC aircraft is presented in Figure 7-8. The graph uses the data shown in the parenthesis next to the above variables.


Figure 7-8: Discharge curve prediction for a typical 3-cell LiPo for an RC aircraft.

### 7.4.4. Fuel Cells

A fuel cell is a device that produces electricity by combining hydrogen and oxygen, forming water and heat as byproducts (see Figure 7-9). The fuel cell is superior to batteries in many ways. It has a potential of being a zero emission device (if the electrical energy is generated using renewable energy) and it overcomes a serious drawback of chemical batteries in which voltage reduces as a function of the battery charge. For batteries, this effectively means that as its charge drains it is no longer possible to get maximum power obtained when it was "freshly" charged. For airplanes this means reduced and "variable" top performance; one which depends on charge remaining. In order for electric aircraft to be truly compatible with a conventional gas powered aircraft, it is necessary that maximum power can be drawn regardless of the state of charge. This is not an issue with fuel cells and renders them particularly attractive for use in electric aircraft.


Figure 7-9: The workings of a fuel cell (see text).

While there are several types of fuel cells, this dissertation only considers ones that consist of a thin membrane, called a Proton Exchange Membrane (PEM). One side of it is exposed to pure Hydrogen gas $\left(\mathrm{H}_{2}\right)$ and the other to Oxygen $\left(\mathrm{O}_{2}\right)$. The PEM catalytically strips the electrons off the Hydrogen,
converting it into Hydrogen ions $\left(\mathrm{H}^{+}\right)$. Furthermore, it ensures the ions can only pass through it in one direction; to the side that is bathed in oxygen. The electrons that were stripped off take a different path and flow through the Anode to the Cathode, generating electric current in the process. At the same time, the Hydrogen ions that pass through the PEM encounter Oxygen and the electrons flowing through the Cathode and react to form $\mathrm{H}_{2} \mathrm{O}$, completing the process.

```
At the Anode: \(\mathrm{H}_{2} \rightarrow 2 \mathrm{H}^{+}+2 \mathrm{e}^{-}\)
At the Cathode: \(1 / 2 \mathrm{O}_{2}+2 \mathrm{H}^{+}+2 \mathrm{e}^{-} \rightarrow \mathrm{H}_{2} \mathrm{O}\)
```


### 7.4.5. The Electric Motor

Electric motors are no longer only for parasailing and ultra-lights but are quickly becoming an important trend in the light aircraft industry, particularly in Light Sport Aircraft (LSA). Electric motors, capable of delivering power in excess of $80 \mathrm{HP}(60 \mathrm{~kW})$, are now being used to power aircraft that can carry as many as 4 people.

Two recent electric aircraft types are of great interest because they reveal some of the challenges of this emerging technology. The first is the two seat Yuneec e430, acclaimed as the "World's first commercially produced 'Electric Aircraft' [18]." It is a two-seat powered sailplane that has the appearance of a high wing trainer. It has a glide ratio of 24, about two times that of typical avgas powered aircraft. According to product information, its empty weight without batteries is $377 \mathrm{lb}_{\mathrm{f}}(172 \mathrm{~kg})$, empty weight with batteries is $561 \mathrm{lb}_{\mathrm{f}}(255 \mathrm{~kg})$, and maximum T-O weight is $946 \mathrm{lb}_{\mathrm{f}}(430 \mathrm{~kg})$. The battery pack weighs $184 \mathrm{lb}_{\mathrm{f}}$ ( 83.5 kg ) and is a Lithium-ion Polymer (LiPo) that takes $3-4$ hours to recharge at 220 Volts (double or triple that for 110 Volts) and provides about 2 hrs of flying time. The e 430 stalls at 35 KCAS, cruises at 52 KTAS, and has a top speed of 80 KTAS.

The second aircraft is the Electraflyer-C (see Figure 7-10). It is a modified single-seat Monnett Moni motorglider that has been equipped with a $13.5 \mathrm{~kW}(18 \mathrm{hp})$ electric motor. Its empty weight with a battery pack is $380 \mathrm{lb}_{\mathrm{f}}\left(172 \mathrm{~kg}\right.$ ) and maximum T-O weight is $625 \mathrm{lb}_{\mathrm{f}}(283 \mathrm{~kg})$. It cruises at 61 KTAS and has a maximum level airspeed of 78 KTAS. It can stay aloft for 1.5 hours using a 5.6 kWh LiPo battery pack. Such a pack weighs about $78 \mathrm{lb}_{\mathrm{f}}$ ( 36 kg ).

## Hybrid Electric Aircraft

The low energy density of electric motor installations has led to the introduction of engine configurations that bridge the gap between gas engines and electric motors through hybrid functionality. A hybrid electric aircraft is one that features a combination of electric motor and some other type of power plant. Strictly speaking, such a power plant can be any of the other types discussed in this section. However, in this dissertation the use of the term will be limited to aircraft driven by a propeller powered by a combination of an electric motor or a piston or a gas turbine. In this context we further classify hybrid electric aircraft into the following types:
(1) A parallel hybrid has an electric motor and a gas engine connected to the same drive-train. This way, the same propeller can be swung using either electrical or gas power or use both simultaneously.
(2) In a series hybrid the propeller is swung using the electric motor only. However, it has a gas engine that runs a generator that charges the battery when it runs low.


Figure 7-10: An experimental Electraflyer C (a modified Moni motor glider), powered by a 13.5 kW ( $\mathbf{1 8} \mathbf{~ h p}$ ) directdrive electric motor. The photo to the right shows that the installation of electric motors is particularly clean and simple when compared to other engines. (Courtesy of www.electraflyer.com).

## The Pure Electric Aircraft

Electric propulsion has a number of advantages over propulsion that depends on fossil fuel. Among those is a very quiet operation of such engines. This is compounded using large diameter, low RPM propellers that also generate relatively low noise. This contrasts noisy piston or gas turbine engines and propellers associated with conventional installations. The operation of electric motors is practically without vibration as long as the propeller is well balanced. The other usual nuisances of gasoline operating engines are absent as well. There are no chemical residues, odors, or stains associated with the operation of such motors - they are extremely clean. Additional advantages include a very simple and reliable start and operation of the motor. The motor itself is a very reliable device that requires minimal maintenance when compared to a piston engine. Tune-ups and expensive overhauls are not required. Another important consideration is pilot and passenger safety; since no fossil fuels are consumed there is no chance of Carbon Monoxide poisoning. Another advantage is that batteries can be recharged by simply plugging them into a household outlet and, at this time, recharging batteries is inexpensive. Electric airplanes are also environmentally friendly and don't emit greenhouse gases, although this is offset by the fact that in many places the production of electricity releases harmful greenhouse chemicals into the environment. This holds for electricity produced by oil or coal electric plants. Renewable energy is of course the answer, giving the electric airplane a unique potential as an environmentally friendly transportation vehicle.

Unfortunately, the production of electricity is not the only drawback. One of the most important one is the storage of the energy on board an aircraft. There are primarily two ways electrical energy is provided to drive the motor: via Batteries or Fuel Cells. At this time, both options are very heavy and a
high toll must be paid in terms of reduction in useful load. In fact, as will be discussed later, an airplane really must be specifically designed for an effective use of electric propulsion; it is impractical to convert existing gas powered aircraft into electric ones. One of the issues with batteries is their relatively low energy content. For longer flights this calls for a large amount of matter (i.e. mass) to be carried around, which reduces the useful load of the airplane. Fuel cells, on the other hand, require large quantities of Hydrogen to be carried in highly pressurized bottles, sometimes as high as 5000-10000 psi. In comparison, the pressure inside the combustion chamber of Space Shuttle Main Engine is in the 3000 psi range. This means that if the Hydrogen bottles burst, they pose a serious threat to the airplane and its occupants.

## Terminology for Electric Motors

Familiarity with concepts related to electric motors use is vital when comes to discussion of propulsive energies. The following list presents a useful list of definitions. Refer to Figure 7-11 for illustration of a typical electric motor system for RC aircraft.

| Term | Definition |
| :--- | :--- |
| Brushless electric <br> motor | An electric motor that lacks a commutator or slip ring. Such motors require <br> alternating current to turn, either from an AC supply, or an electronic circuit. <br> Such electronic circuits in RC aircraft are called ESC for Electronic Speed <br> Controller. |
| Motor kV-rating | 1000 kV means that the rpm of the motor increases by 1000 rpm for each <br> volt increase. So the rating is the slope for the motor's rpm-versus-Volt <br> graph. |
| Inrunner motor | Refers to a brushless electric motor for which the stators (to which the <br> copper wire is wound) are outside an inner core that rotates. See "Outrunner <br> motor" for comparison and Figure 7-12. |
| Outrunner motor | Refers to a brushless electric motor for which the stators (to which the <br> copper wire is wound) is inside of an outer shell that rotates (see Figure 7- <br> 12). Typically used in electric RC model aircraft. Such engines rotate at lower <br> rpm than inrunner motors, but generate more torque. This makes them ideal <br> for driving propellers, as the configuration reduces weight, complexity, and <br> inefficiency of its inrunner counterpart, in addition to eliminating the need <br> for a reduction drive gear (since the rpm is lower). |
| Electronic Speed |  |
| Controller (ESC) | A vital device for modern brushless, outrunner, electric motors. In effect, it is <br> an inverter that receives DC current from the battery and transforms it into <br> AC, which is what the motor requires. It is connected to the receiver module <br> of the aircraft's remote control system and allows the pilot to control the <br> speed of the motor. |

\(\left.\begin{array}{|l|l|}\hline Term \& Definition <br>
\hline Is an extra circuit provided in most modern ESCs to allow the receiver to be <br>
powered using the same battery as the motor. This eliminates having to use <br>
one battery for the motor and a separate one for the receiver and servos. <br>
The current for the receiver is carried in the same 3-wire (signal-current- <br>
Ground) cable that goes to the throttle slot in standard receivers. The <br>
drawback of the BEC is that if the electric connection is disrupted, power to <br>
both the motor and servos is lost. However, the convenience of a single <br>
battery usually weighs more. When the BEC detects voltage drop associated <br>
with low charge, it will cut off power to the motor, allowing for the <br>

possibility of a dead stick landing.\end{array}\right\}\)| A predecessor of the modern BEC. An external circuitry, in contrast to the |
| :--- |
| BEC, which is built into the ESC. Although the BEC provides "cleaner" voltage |
| Signal and is less expensive to manufacture due to reduced component |
| Count, the UBEC is more efficient, can withstand higher voltage, and |
| generates less heat. |

### 7.4.6. Modeling Energy Consumption of Electric Engines

Consider a battery being used to power an electric motor, such the current draw amounts to $I$ amps over a time segment $\Delta t$. This means the battery capacity is being reduced by amount

$$
\begin{equation*}
\Delta C=I \Delta t \tag{7-26}
\end{equation*}
$$

If the time segment is shortened to an infinitesimal length, the capacity reduction can be written as

$$
\begin{equation*}
d C=I d t \tag{7-27}
\end{equation*}
$$

This allows us to estimate the total battery capacity consumed as a function of the time history of the current, through integration of Equation (7-27)

$$
\begin{equation*}
C_{u s e d}=\int_{0}^{t} I(\tau) d \tau \tag{7-28}
\end{equation*}
$$

The typical true operation of an electric motor in an RC aircraft is shown in Figure 7-13. It is evident that the voltage remains relatively constant over the duration of the two T-Os and landings performed in the airplane during that experiment. What changes is the current: The pilot controls engine power by adjusting current draw and not voltage. And per Section 7.2.2, Basic Formulas of Electricity, the power delivered to the engine is a function of time and is given by:

$$
\begin{equation*}
P(t)=V(t) I(t) \tag{7-29}
\end{equation*}
$$



Figure 7-11: Typical electric motor setup for an RC aircraft. (photo by author)


Figure 7-12: A schematic of the mechanical difference between an outrunner and inrunner electric motors.
It is important to recognize that the current draw usually recorded by telemetry (e.g. such as that shown in Figure 7-13) is the combination of that used to operate systems, such as the servos, cameras, video transmitter, receiver, and the telemetry system, and that used by the motor. For instance, the average current draw for systems for the Quanum Observer, shown in Figure 7-13, amounts to about 0.45 Amps . This extra current draw is largely constant, although it could also depend on time. Therefore we must define total current draw, $I_{t o t}$, and system current draw, $I_{\text {sys }}$, and define the motor current draw, $I(t)$, as follows:

$$
\begin{equation*}
I_{t o t}(t)=I_{s y s}(t)+I(t) \tag{7-30}
\end{equation*}
$$

The total current draw dictates the total battery capacity consumed, $C_{\text {used }}$. And it (i.e. $C_{\text {used }}$ ) dictates the open circuit voltage of the battery. Thus, we must rewrite Equation (7-28) in the following fashion

$$
\begin{equation*}
C_{u s e d}=\int_{0}^{t}\left[I_{s y s}(\tau)+I(\tau)\right] d \tau \tag{7-31}
\end{equation*}
$$

Most of the time, the current directed to system operation is constant. It can, therefore, be integrated separately, yielding

$$
\begin{equation*}
C_{u s e d}=I_{s y s} t+\int_{0}^{t} I(\tau) d \tau \tag{7-32}
\end{equation*}
$$

Using Tremblay's method of Equation (7-23), the engine power can now be written as follows:

$$
\begin{equation*}
P(t)=V(t) I(t)=\left[V_{0}-\left(\frac{\kappa \cdot C_{c u t}}{C_{c u t}-C_{u s e d}(t)}\right)+A e^{-B \cdot C_{u s e d}(t)}\right] I(t) \tag{7-33}
\end{equation*}
$$

This allows us to establish the engine power as a function of time, provided a time-history of the current is available or can be established

$$
\begin{equation*}
P(t)=\left[V_{0}-\left(\frac{\kappa \cdot C_{c u t}}{C_{c u t}-I_{s y s} t-\int_{0}^{t} I(\tau) d \tau}\right)+A e^{-B \cdot\left(I_{s y s} t+\int_{0}^{t} I(\tau) d \tau\right)}\right] I(t) \tag{7-34}
\end{equation*}
$$

In a simulation environment, Equation (7-34) would typically be implemented using a numerical summing function, where $\tau$ is discrete time. Therefore, it would be rewritten as follows

$$
\begin{equation*}
P\left(t_{i+1}\right)=\left[V_{0}-\left(\frac{\kappa \cdot C_{c u t}}{C_{c u t}-I_{s y s} \sum_{i, N} t_{i}-I_{i} \cdot \Delta t}\right)+A e^{-B \cdot\left(I_{s y s} \sum_{i, N} t_{i}+I_{i} \cdot \Delta t\right)}\right] I(t) \tag{7-35}
\end{equation*}
$$

Where $i$ is a time step index, $N$ is the current number of iterations, and $\Delta t$ is the time step (here assumed constant).


Figure 7-13: Typical consumption of electric power for an FPV aircraft (file Quanum Observer 12-11-2015.xlsx).

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## 8. Flight Simulation

This chapter presents a review of flight mechanics and how it is implemented to permit flight simulation to take place. In particular, details of the flight simulation software developed to demonstrate the GICA will be presented from a theoretical perspective. So far, it has been referred to as the SURFACES Flight Simulator, as its development is linked to the aircraft design software SURFACES. The simulation is 6-Degrees-of-Freedom (DOF) and allows the dynamics of large and small aircraft to be simulated (although this work focuses on smaller aircraft). It solves the differential equations that describe relations between forces, moments, translation, and rotation of the aircraft being modeled. The time-history of the solution is accomplished using numerical methods, such as Euler integration or higher order AdamsBashforth integration (see Appendix B). The simulator allows the aircraft to be controlled manually (using a joystick) or using an autopilot. The latter permits far more refined control inputs and, thus, better suited for the analysis work presented later. Selected introductory details of the simulator are presented in Section 1.4, Software Development. The chapter approaches the topic in the following fashion:

- Definition of coordinate systems and velocity vector
- Presentation of the rigid body equations of 6-DOF motion
- Formulation of forces and moments
- Formulation of necessary force and moment coefficients
- Outline of flight simulator
- Presentation of the autopilot functions that make up the AFMS


### 8.1 Coordinate Systems

A 6-DOF motion includes translation along and rotation about the $x$-, $y$-, and $z$-axes. In order to describe the motion of an airplane, it is necessary to establish coordinate systems to which the required formulation refers. In this sense, the term world refers to the geographic space in which the motion of the airplane takes place. Some of the coordinate systems refer to this "world," while others refer to the aircraft itself. In this dissertation the following coordinate systems are used.

### 8.1.1. Vehicle Coordinate System (VCS)

Generally, problems that involve aerodynamics and solid mechanics (structures) use a special coordinate system, called the Vehicle Coordinate System (VCS), in which the $x$-axis points from the nose to the tail, the $y$-axis points in a right spanwise direction, and the $z$-axis points upward (see Figure $8-1$ ). Its origin may be placed at the CG, as shown, or any other fixed reference point, for instance, the firewall and even in front of the airplane. It is a right-handed coordinate system, for which lift and drag have a positive sign, while thrust and weight have negative signs. The convenience of this system is that the $x$ location of components like the vertical tail (keeping in mind the aircraft in Figure 8-1) is a larger numeric value than the location of, say, the engine. This corresponds to the common notion of a
forward (nose) and aft (tail) portion of an aircraft. For this reason, it is often used to specify the position of components inside the aircraft.

### 8.1.2. Body Axes Coordinate System (BCS)

A Body Axes Coordinate System (BCS) is a reference frame whose origin is the airplane's CG and is oriented such the $x_{B}$-axis points forward and lies in the plane of symmetry, $y_{B}$-axis points outward toward the right wingtip, and the $z_{B}$-axis points downwards and is also in the plane of symmetry (see Figure $8-2$ ). The subscript " $B$ " is used to denote the $B C S$. The convenience of the BCS is primarily in terms of allowing aerodynamic forces to be specified using the aircraft as a reference frame.


Figure 8-1: A Vehicle Coordinate System.


Figure 8-2: Earth Fixed and Body Coordinate System.

### 8.1.3. Earth Fixed Coordinate System (NED)

An Earth Fixed System is a reference frame whose origin is attached to some point on the ground and traditionally oriented such the $x_{E}$-axis points North, $y_{E}$-axis points East, and the $z_{E}$-axis points down toward the center of the Earth (see Figure 8-2). For this reason, this reference system is often labeled NED for North-East-Down. The subscript " $E$ " is used to denote the NED coordinate system. This has been transposed in the SURFACES Flight Simulator, such that the $x_{E}$-axis points East and the $y_{E}$-axis
points North. This coordinate system is used to determine the position of the aircraft in the "world" in which it is flying.

### 8.1.4. Stability Coordinate System (SCS)

A Stability Coordinate System is a reference frame whose origin is the airplane's CG and is oriented such the $x_{S}$-axis points forward and lies in the plane of symmetry. However, the axis is aligned with the component of the far-field velocity that lies in the plane-of-symmetry. This way it is tilted by an angle that corresponds to the Angle-of-Attack (AOA). The $y_{S}$-axis points outward toward the right wingtip and the $z_{S}$-axis points downwards and remains in the plane of symmetry, but is perpendicular to the far-field velocity (see Figure 8-3). The subscript " $S$ " is used to denote the SCS. This reference is used whether or not the airplane is actually yawing. This system is traditionally used for stability analysis of the aircraft.


Figure 8-3: Stability Coordinate System.

### 8.1.5. Wind Axes Coordinate System (WAS)

A Wind Axes System is a reference frame whose origin is the airplane's CG and is oriented such the $x_{W^{-}}$ axis points forward, although it is no longer necessarily in the plane of symmetry, but rather aligned with the component of the far-field velocity that is projected on to the $x y$-plane of the BCS. This component is tilted by an angle that corresponds to the Angle-of-Yaw (AOY). The $y_{W}$-axis points outward toward the right wingtip and the $z_{W}$-axis lies in the plane of symmetry and is equal to the $z_{S}$-axis (see Figure $8-4$ ). The wind axis is important because it defines the orientation of the airplane with respect to the wind.

### 8.1.6. Definition of Airspeed Components

Just like the position of the airplane is specified using three Cartesian coordinates, the airspeed is specified in the same fashion. Figure 8-4 and Figure 8-5 reveals that only two angles-of-orientation are needed to determine the magnitude of the aerodynamic forces that act on the airplane; $\alpha$ and $\beta$. By inspection these angles can be related to the components of the velocity as follows:

Angle-of-Attack:

$$
\begin{equation*}
\tan \alpha=\frac{w}{u} \tag{8-1}
\end{equation*}
$$

Angle-of-Yaw:

$$
\begin{align*}
& \sin \beta=\frac{v}{V_{\infty}}  \tag{8-2}\\
& V_{\infty}=\sqrt{u^{2}+v^{2}+w^{2}} \tag{8-3}
\end{align*}
$$

Strictly speaking, the aerodynamic forces also depend on the rotation rates of the aircraft about its CM , and these dependencies will be described later.

### 8.1.7. Transformation of Coordinate Systems

As shown above, the far-field velocity defines the wind axis. However, for convenience, we are interested in establishing the equations of motion along the body axis. For instance, $V_{\infty}$ is always specified along the wind axis, but the equations use velocity components along the body axes. Therefore, it is of importance to establish transformations that allow us to go from one coordinate system to the next. First, we want to define the components of the far-field airspeed, $V_{\infty}$, to the BCS as $u$, $v$, and $w$, respectively. The component " $u$ " refers to the component of the airspeed along the $x_{B}$-axis, " $v$ " along the $y_{B}$-axis, and " $w$ " along the $z_{B}$-axis (see Figure 8-5).


Figure 8-4: Wind Axes Coordinate System.

## Converting Stability Axes to Wind Axes

By inspection, comparing Figure 8-5 and Figure 8-4 reveals the following relationships hold:

$$
\left[\begin{array}{l}
x_{W}  \tag{8-4}\\
y_{W} \\
z_{W}
\end{array}\right]=\left[\begin{array}{ccc}
\cos \beta & \sin \beta & 0 \\
-\sin \beta & \cos \beta & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
x_{S} \\
y_{S} \\
z_{S}
\end{array}\right]
$$

This expression is referred to as Euler angle rotation.


Figure 8-5: The definition of airspeed components.

## Converting Body Axes to Stability Axes

By inspection, comparing Figure 8-4 and Figure 8-5 reveals the following relationships hold:

$$
\left[\begin{array}{l}
x_{S}  \tag{8-5}\\
y_{S} \\
z_{S}
\end{array}\right]=\left[\begin{array}{ccc}
\cos \alpha & 0 & \sin \alpha \\
0 & 1 & 0 \\
-\sin \alpha & 0 & \cos \alpha
\end{array}\right]\left[\begin{array}{l}
x_{B} \\
y_{B} \\
z_{B}
\end{array}\right]
$$

## Converting Body Axes to Wind Axes

Converting body axes to wind axes requires the two previous transformations to be multiplied:

$$
\left[\begin{array}{c}
x_{W}  \tag{8-6}\\
y_{W} \\
z_{W}
\end{array}\right]=\left[\begin{array}{ccc}
\cos \alpha \cos \beta & \sin \beta & \sin \alpha \cos \beta \\
-\cos \alpha \sin \beta & \cos \beta & -\sin \alpha \sin \beta \\
-\sin \alpha & 0 & \cos \alpha
\end{array}\right]\left[\begin{array}{l}
x_{B} \\
y_{B} \\
z_{B}
\end{array}\right]
$$

## Converting Velocity Vector from Wind Axes to Body Axes

It is a necessary step to transform the velocity vector to the body axes system to calculate the aerodynamic forces. This is accomplished using the 3-2-1 sequence or Euler rotations, using the compiled rotation matrix below

$$
\left\{\begin{array}{l}
x  \tag{8-7}\\
y \\
z
\end{array}\right\}=\left[\begin{array}{ccc}
C \psi C \theta & S \psi C \theta & -S \theta \\
-S \psi C \phi+C \psi S \theta S \phi & C \psi C \phi+S \psi S \theta S \phi & C \theta S \phi \\
S \psi S \phi+C \psi S \theta C \phi & -C \psi S \phi+S \psi S \theta C \phi & C \theta C \phi
\end{array}\right]\left\{\begin{array}{l}
x_{\text {old }} \\
y_{\text {old }} \\
z_{\text {old }}
\end{array}\right\}
$$

Where $\mathrm{C}(\cdot)=\cos (\cdot)$ and $\mathrm{S}(\cdot)=\sin (\cdot)$. In this case the velocity vector is given as $\left\{V_{\infty}, 0,0\right\}^{T}$ and, thus, is transformed to velocity components aligned with the body coordinate system by noting that the Euler angles $\{\phi, \theta, \psi\}$ correspond to the angles of attack and yaw as $\{0, \alpha,-\beta\}$, where the minus sign " - " is used to denote that a positive side-slip angle is negative yaw angle. Substituting these terms into Equation (8-7) and simplifying leads to

$$
\left\{\begin{array}{l}
u  \tag{8-8}\\
v \\
w
\end{array}\right\}=\left[\begin{array}{ccc}
\cos \alpha \cos \beta & -\cos \alpha \sin \beta & -\sin \alpha \\
\sin \beta & \cos \beta & 0 \\
\sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha
\end{array}\right]\left\{\begin{array}{c}
V_{\infty} \\
0 \\
0
\end{array}\right\}=\left[\begin{array}{c}
V_{\infty} \cos \alpha \cos \beta \\
V_{\infty} \sin \beta \\
V_{\infty} \sin \alpha \cos \beta
\end{array}\right]
$$

Recall that $u$, $v$, and $w$ are the components of $V_{\infty}$ in the body axis system (see Figure 8-5).

### 8.2 The Rigid-Body Equations of Motion

A description of the motion of any 6-DOF vehicle requires a set of equations to express the translation along and rotation about the three axes of the Cartesian coordinate system. These equations are called the Equations of Motion (EOM). In its most simple portrayal they are derived using Newton's second law of motion. While it is possible to include flexible structures in the equations, the form presented here assumes rigid-bodies. The associated simplification will not affect the results presented, because the maneuvers commanded by the GICA induce relatively low g-loads on the airframe and, thus, limited airframe deformation.

The EOM allow the position and orientation of the vehicle to be determined through time integration. They consist of four groups, each containing three equations. One group contains force equations that return linear accelerations, a second group contains moment equations that return angular accelerations, a third one contains rotation rate equations, and the fourth Euler rotation rates. These angular equations are required to keep track of the orientation of the aircraft and on which the forces and moments depend. These constitute a set of 12 equations, some of which are coupled. The derivations of these equations will take too much space in this dissertation and, thus, must be omitted. However, the derivation is provided in a number of references, for instance, by Nelson [1], Etkin [2], Napolitano [3], Allerton [4], Stevens and Lewis [5], Perkins and Hage [6], and Roskam [7]. Additional insights are provided by Abzug and Larrabee [8]. This section presents the EOM in a form suitable for use in flight simulators.

$$
\begin{array}{ll} 
& X-m g \sin \theta=m(\dot{u}+q w-r v) \\
\text { Force Equations: } & Y+m g \cos \theta \sin \phi=m(\dot{v}+r u-p w) \\
& Z+m g \cos \theta \cos \phi=m(\dot{w}+p v-q u) \\
\text { Moment Equations: } & L=I_{x x} \dot{p}-I_{x z} \dot{r}+\left(I_{z z}-I_{y y}\right) q r-I_{x z} p q+\left(q h_{S R z}-r h_{S R y}\right) \\
& M=I_{y y} \dot{q}+\left(I_{x x}-I_{z z}\right) p r+I_{x z}\left(p^{2}-r^{2}\right)+\left(r h_{S R x}-p h_{S R z}\right) \\
& N=I_{z z} \dot{r}-I_{x z} \dot{p}+\left(I_{y y}-I_{x x}\right) p q+I_{x z} q r+\left(p h_{S R y}-q h_{S R x}\right) \\
& p=\dot{\phi}-\dot{\psi} \sin \theta \\
\text { Rotation Rate Equations: } \quad q=\dot{\theta} \cos \phi+\dot{\psi} \cos \theta \sin \phi  \tag{8-11}\\
& r=\dot{\psi} \cos \theta \cos \phi-\dot{\theta} \sin \phi
\end{array}
$$

Euler Rate Equations:

$$
\begin{align*}
& \dot{\theta}=q \cos \phi-r \sin \phi \\
& \dot{\phi}=p+q \sin \phi \tan \theta+r \cos \phi \tan \theta  \tag{8-12}\\
& \dot{\psi}=q \sin \phi \sec \theta+r \cos \phi \sec \theta
\end{align*}
$$

Where: $h_{x}, h_{y}, h_{z}=$ Angular momentums about the $x, y$, and $z$ axes, respectively.
$h_{S R x}, h_{S R y}, h_{S R z}=$ Angular momentums of spinning rotors about the $x, y$, and $z$ axes, respectively.
$m=$ Mass of the vehicle.
$p, q, r=$ Rotation rates about $x, y$, and $z$ axes, respectively.
$u, v, w=$ Speeds along $x, y$, and $z$ axes, respectively.
$X, Y, Z=$ Aerodynamic (including propulsive) forces along $x, y$, and $z$ axes, respectively.
$\phi, \theta, \psi=$ Roll, pitch, and yaw components of the Euler angles, respectively.

Where ( $X, Y, Z$ ) are the aerodynamic forces in the $x, y, z$ directions (respectively) of the WAS coordinate system attached to the vehicle's center of gravity. ( $L, M, N$ ) are the aerodynamic moments about said axes, $I_{i j}$ are the moments and products of inertia, $(p, q, r)$ are the rotation rates and $(\phi, \theta, \psi)$ are the roll, pitch, and yaw components of the Euler angles, respectively. Note that while Euler angles are required to transform forces and moments from wind- to body-axes (and vice versa), they are kept track of in the simulator using quaternions (see Appendix D). This is done to avoid the condition of gimbal-locking. Figure 8-6 shows a flow chart giving a generalized illustration of how flight simulators are organized. It can be seen that the EOM are the core of such software and is fed information from an aerodynamic and engine databases. The output of the core is then used to display information such as the outside environment and the performance of the aircraft.


Figure 8-6: The equations of motion are the center of any flight simulator. (Based on Ref. [4])

### 8.3 Formulation of Aerodynamic Forces and Moments

This section discusses how aerodynamic forces and moments are obtained for a model whose dynamics are being simulated. The following expressions are used throughout this section in the derivation of various relationships. Note that in the UK- and SI-systems of units forces are in $\mathrm{lb}_{f}$ or N , and moments are in $\mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}}$ or $\mathrm{N} \cdot \mathrm{m}$, respectively.

### 8.3.1. Total Forces Acting on the Vehicle

The difficulty in analyzing forces on aircraft is that they depend on the orientation of the vehicle with respect to the airflow. Strictly speaking, two classes of forces act on a body moving through fluid; gravitational and aerodynamic. The former is relatively easy to treat, the latter not so. If we prefer to consider the gravitational force as a single vector, then the aerodynamic force should also be considered a single resultant vector. Regardless, it is convenient to break it into two mutually orthogonal vector components depending on, frankly, what is of interest to us. For instance, for structural reasons we may be interested in the force component normal and parallel to the wing plane, because it can be used to estimate the spanwise and chordwise bending moments the wing must react. In this case, the direction of the force components would be independent of the orientation of the aircraft with respect to the airflow; only their magnitudes would change. However, the more common breakdown involves defining the two force components parallel and normal to the tangent of the trajectory of the motion. We call the former drag and the latter lift. This definition is of great convenience, for instance, when extracting empirical data about the aircraft using wind tunnel or flight testing experiments. The cost of this convenience is that we must be ready to transform the force components for other situations. This section discusses such transformations.

As stated above, traditionally, we obtain the lift and drag of the airplane assuming the stability coordinate system of Figure 8-3. We will then represent these two force components in terms of dynamic pressure, denoted by $q$, reference area, $S$, the lift and drag coefficients, $C_{L}$ and $C_{D}$, respectively, and the side force, denoted by $C_{y}$. Furthermore, we may represent a propulsion force, $T$, in a coefficient format as follows

Thrust force

$$
\begin{equation*}
T=q S \cdot C_{T} \tag{8-13}
\end{equation*}
$$

Where $C_{T}$ is the thrust coefficient. Here the thrust is assumed to act parallel to the $x$-axis. Sometimes it is helpful to represent the forces in terms of the body coordinate system of Figure 8-2, using these force coefficients. If these non-dimensional force coefficients along the $x$-, $y$-, and $z$-axes are denoted by $C_{x}, C_{y}$, and $C_{z}$, the aerodynamic forces in the body coordinate system are defined as follows:

$$
\begin{array}{ll}
\text { Force along x-axis } & X=q S \cdot C_{x} \\
\text { Force along y-axis } & Y=q S \cdot C_{y} \\
\text { Force along z-axis } & Z=q S \cdot C_{z} \tag{8-16}
\end{array}
$$

Where the coefficients $C_{x}, C_{y}$, and $C_{z}$ are obtained from the expression below, where they are resolved on the body axes using the rotational transformation (note that only the $x$ - and $z$-axes are rotated)

$$
\begin{align*}
& C_{x}=C_{T}+C_{L} \sin \alpha-C_{D} \cos \alpha \\
& C_{y}=C_{y_{\beta}} \beta  \tag{8-17}\\
& C_{z}=-C_{L} \cos \alpha-C_{D} \sin \alpha
\end{align*}
$$

Of these, $C_{L}$ and $C_{D}$ were introduced in Section 6.2, Fundamentals of Lift and Drag Coefficients, but how they are calculated in the flight simulation code is presented in Section 8.4.1, Common Expressions for Selected Aerodynamic Coefficients. Note that sometimes it is convenient to express Equation (8-17) using small angle relations. These are only applicable for small radian angles, such that $\sin \alpha \approx \alpha$ and cos $\alpha \approx 1$ and must be used with care. While such simplifications are not used in the 6-DOF flight simulator, they are handy for a variety of other situations. The simplification converts the exact equations to

$$
\begin{align*}
& C_{x}=C_{T}+C_{L} \alpha-C_{D} \\
& C_{y}=C_{y \beta} \beta  \tag{8-18}\\
& C_{z}=-C_{L}-C_{D} \alpha
\end{align*}
$$

### 8.3.2. Total Moments Acting on the Vehicle

The following moments represent the summation of moments about the three body axes. A stable and trimmed aircraft requires the summation of the moments about a given axis to equal zero. If the nondimensional moment coefficients along the $x-, y$-, and $z$-axes are denoted by $C_{x}, C_{y}$, and $C_{z}$, respectively, the aerodynamic forces are defined as follows:

| Rolling moment (about $x$-axis): | $L=M_{x}=q S b \cdot C_{l}$ |
| :--- | :--- |
| Pitching moment (about $y$-axis): | $M=M_{y}=q S C_{M G C} \cdot C_{m}$ |
| Yawing moment (about $z$-axis): | $N=M_{z}=q S b \cdot C_{n}$ |

It is important not to confuse the rolling moment coefficient with that of the lift coefficient of a 2dimensional geometry.

Where: $\quad q=1 / 2 \rho V^{2}=$ is the dynamic pressure
$\rho=$ Density of air
$V=$ True airspeed
$b=$ Reference span
$S=$ Reference area
$C_{M G C}=$ Reference mean geometric chord
$C_{l}=$ Rolling moment coefficient
$C_{m}=$ Pitching moment coefficient
$C_{n}=$ Yawing moment coefficient

### 8.3.3. Lookup Tables

The simulation performed here primarily involves airspeeds ranging from the airplane's stalling speed to its maximum horizontal airspeed. Assuming $\beta=0$, higher airspeeds are associated with low $\alpha$ for which the airplane orientation presents limited flow separation, permitting many force and moment coefficients to be approximated using linear relations. This excludes the drag coefficient; its change is always non-linear. Conversely, lower airspeed requires higher $\alpha$, resulting in extensive flow separation and non-linear changes in the force and lift coefficients. Often, the non-linearity can be approximated using simple polynomials, but at other times, these are not sufficiently precise. Some more complex
functions are required, but these unavoidably increase the time required to calculate a given coefficient and this, in turn, may start to slow down the simulation. For instance, while a Fourier series could be used to estimate such coefficients, the iteration required to achieve acceptable approximation is prohibitive. The work-around for this detriment is to extract the values of the coefficient using lookup tables. The SURFACES Flight Simulator uses this technique for the lift coefficient, $C_{L}$, drag coefficient, $C_{D}$, and the pitching moment coefficient, $C_{m}$. At this time, all other coefficients assume linear relations.

A lookup table typically lists the relation of interest using a 1-dimensional vector. Thus, the independent (e.g. $x$ ) and the associated dependent variable (e.g. $y$ ) are stored as two column vector (or matrix). Then, a specific value of the dependent variable (e.g. $y(x)$ ) is obtained by looking for the value in the lookup table. When the value of the independent variable that falls between two stored values, the dependent variable is estimated using linear interpolation. Thus, the use of lookup tables calls for a rapid interpolation method to extract the coefficient. To better explain the use of lookup tables in the SURFACES Flight Simulator, consider the sample table shown in Figure 8-7. It lists the values of $\alpha$ in center column and corresponding $C_{L}$ in the right column. The left column is an index and is used for programmatic purposes. Typically, such tables are empirical. For instance, consider the task of estimating the value of $C_{L}$ at $\alpha=4.5$ degrees. The resulting value will require an interpolation between the known values at $\alpha=4^{\circ}$ and $\alpha=5^{\circ}$, displayed by the shaded region. The following method allows for a quick interpolation of such values, using parametric line interpolation. A generic version of this table is shown in Figure 8-8.

First, consider a general table with $N$-rows of independent $(x)$ and dependent $(y)$ variables and determine the two coordinates enclosing the solution, $x_{i}, y_{i}$, and $x_{i+1}, y_{i+1}$. Given the target value of $x$, calculate the parameters $s$ from:

$$
\begin{equation*}
s=\frac{x-x_{i}}{x_{i+1}-x_{i}} \tag{8-22}
\end{equation*}
$$

Then, calculate the interpolated value $y$, using the following parametric presentation

$$
\begin{equation*}
y=(1-s) y_{i}+s \cdot y_{i+1} \tag{8-23}
\end{equation*}
$$

A Visual Basic code to perform this interpolation is presented in Appendix A.2.

### 8.4 A Summary of Stability Derivatives

The determination of the aerodynamic forces and moment requires the use of static and dynamic stability derivatives. These can be obtained by a number of means, for instance, through wind tunnel testing or by analytical methods. The following summary of stability derivatives are of key importance in any flight simulation.

| Index | AOA, $\alpha$ <br> in degrees | Lift Coefficient, $C_{L}$ |
| :---: | :---: | :---: |
| 1 | -2 | 0.04 |
| 2 | -1 | 0.12 |
| 3 | 0 | 0.2 |
| 4 | 1 | 0.28 |
| 5 | 2 | 0.36 |
| 6 | 3 | 0.44 |
| 7 | 4 | 0.52 |
| 8 | 5 | 0.6 |
| 9 | 6 | 0.68 |
| 10 | 7 | 0.76 |
| 11 | 8 | 0.84 |

Figure 8-7: A sample 1-dimensional lookup table.

| Index | AOA, $\alpha$ <br> in degrees | Lift Coefficient, $C_{L}$ |
| :---: | :---: | :---: |
| 1 | $\alpha_{1}$ | $C_{L 1}$ |
| 2 | $\alpha_{2}$ | $C_{L 2}$ |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  | $C_{L i}$ |
| $i$ | $\alpha_{i}$ | $C_{L(i+1)}$ |
| $i+1$ | $\alpha_{i+1}$ |  |
|  |  | $C_{L \mathrm{~N}}$ |
|  | $\alpha_{\mathrm{N}}$ |  |

Figure 8-8: A generic 1-dimensional lookup table.

### 8.4.1. Common Expressions for Selected Aerodynamic Coefficients

In the formulation that follows, the lift, drag, and moment for the entire aircraft are expressed in terms of coefficients that are linear combinations of various contributions. Note that the moment is always taken about the CG:

$$
\begin{equation*}
C_{L}=C_{L_{0}}+C_{L_{\alpha}} \alpha+C_{L_{\dot{\alpha}}} \frac{\dot{\alpha} C_{M G C}}{2 V}+C_{L_{\beta}} \beta+\eta_{H T} \frac{S_{H T}}{S} C_{L_{\delta_{e}}} \delta_{e}+C_{L_{\delta_{f}}} \delta_{f}+C_{L_{\delta_{\text {spoiler }}}} \delta_{\text {spoiler }}+\cdots \tag{8-24}
\end{equation*}
$$

The total lift coefficient can also be obtained using the familiar expression

$$
\begin{equation*}
C_{L}=\frac{2 L}{\rho V^{2} S} \tag{6-15}
\end{equation*}
$$

Of course, for steady flight, the two will be equal. Similarly, the lift coefficient of the tail is given by:

$$
\begin{equation*}
C_{L_{H T}}=\eta_{H T}\left[C_{L_{0_{H T}}}+C_{L_{\alpha_{H T}}} \cdot \alpha_{H T}+C_{L_{\delta_{e}}} \cdot \delta_{e}+\cdots\right] \tag{8-25}
\end{equation*}
$$

The drag coefficient can be represented as follows:

$$
\begin{equation*}
C_{D}=C_{D \min }+C_{D \alpha} \alpha+C_{D \beta} \beta+\eta_{H T} \frac{S_{H T}}{S} C_{D \delta_{e}} \delta_{e}+C_{D \delta_{f}} \delta_{f}+C_{D \delta_{\text {spoiler }}} \delta_{\text {spoiler }}+\cdots \tag{8-26}
\end{equation*}
$$

The drag coefficient can also be represented in terms of the $C_{L}$ in the following fashion, using the adjusted drag model discussed in Section 6.2.3, The Drag Coefficient, and as long as the $C_{L}$ is below the flow separation lift coefficient $C_{L M}$

$$
\begin{equation*}
C_{D}=C_{D \min }+k\left(C_{L}-C_{L \min D}\right)^{2} \tag{6-20}
\end{equation*}
$$

This assumes the coefficient $k$ provides a reasonable response of drag with respect to the $C_{L}$, but this is primarily used for low values of $\alpha$. See Reference [9] for more details on drag analysis. The SURFACES Flight Simulator uses look-up tables for both $C_{L}$ and $C_{D}$. The thrust coefficient is given by Equation (813), here shown solved for $C_{T}$

$$
\begin{equation*}
C_{T}=\frac{2 T}{\rho V^{2} S} \tag{8-27}
\end{equation*}
$$

### 8.4.2. Expressions for the Aerodynamic Force Coefficients

The forward force coefficient, $C_{x}$, represented in Equation (8-17) is modified by substituting Equations (8-24) and (8-26) to account for control inputs. The expression becomes quite a bit longer as shown below:

$$
\begin{align*}
C_{x}=C_{T}\left(\delta_{T}\right)+ & {\left[C_{L_{0}}+C_{L_{\alpha}} \alpha+C_{L_{\beta}} \beta+\eta_{H T} \frac{S_{H T}}{S} C_{L_{\delta_{e}}} \delta_{e}+C_{L_{\delta_{f}}} \delta_{f}+C_{L_{\delta_{\text {spoiler }}}} \delta_{\text {spoiler }}+\cdots\right] \sin \alpha } \\
& -\left[C_{D \min }+C_{D \alpha} \alpha+C_{D \beta} \beta+\eta_{H T} \frac{S_{H T}}{S} C_{D \delta_{e}} \delta_{e}+C_{D \delta_{f}} \delta_{f}+C_{D \delta_{\text {spoiler }}} \delta_{\text {spoiler }}+\cdots\right] \cos \alpha \tag{8-28}
\end{align*}
$$

Where $\delta_{T}$ is a throttle parameter to allow power settings to be included. The side force coefficient, $C_{y}$, must also account for control inputs, yielding

$$
\begin{equation*}
C_{y}=\overbrace{C_{y_{\alpha}}}^{=0 \text { typ. }} \alpha+C_{y \beta} \beta+C_{y_{\delta_{r}}} \delta_{r}+\ldots \tag{8-29}
\end{equation*}
$$

The vertical force coefficient can be represented as shown below:

$$
\begin{align*}
C_{z}=-\left[C_{L_{0}}\right. & \left.+C_{L_{\alpha}} \alpha+C_{L_{\beta}} \beta+\eta_{H T} \frac{S_{H T}}{S} C_{L_{\delta_{e}}} \delta_{e}+C_{L_{\delta_{f}}} \delta_{f}+C_{L_{\delta_{\text {spoiler }}}} \delta_{\text {spoiler }}+\cdots\right] \cos \alpha \\
& -\left[C_{D_{\min }}+C_{D_{\alpha}} \alpha+C_{D \beta} \beta+\eta_{H T} \frac{S_{H T}}{S} C_{D \delta_{e}} \delta_{e}+C_{D \delta_{f}} \delta_{f}+C_{D \delta_{\text {spoiler }}} \delta_{\text {spoiler }}+\cdots\right] \sin \alpha \tag{8-30}
\end{align*}
$$

The rolling moment coefficient can be represented as shown below:

$$
\begin{equation*}
C_{l}=\overbrace{C_{l_{o}}}^{=0}+\overbrace{C_{l_{\alpha}}}^{\text {typ. }} \cdot \alpha+C_{l_{\beta}} \cdot \beta+C_{l_{p}} \frac{p b}{2 V}+C_{l_{r}} \frac{r b}{2 V}+C_{l_{\delta_{\alpha}}} \delta_{a}+C_{l_{\delta_{r}}} \delta_{r}+\cdots \tag{8-31}
\end{equation*}
$$

The pitching moment coefficient can be represented as shown below:

$$
\begin{equation*}
C_{m}=C_{m_{0}}+C_{m_{\alpha}} \cdot \alpha+{\stackrel{C}{C_{\beta}}}_{=0 \text { twp. }}^{\text {tw }^{2}} \cdot \beta+C_{m_{q}} \frac{q C_{M G C}}{2 V}+C_{m_{\dot{\alpha}}} \frac{\dot{\alpha} C_{M G C}}{2 V}+C_{m_{\delta_{e}}} \delta_{e}+C_{m_{\delta_{f}}} \delta_{f}+\cdots \tag{8-32}
\end{equation*}
$$

The yawing moment coefficient can be represented as shown below:

$$
\begin{equation*}
C_{n}=\overbrace{C_{n_{0}}}^{=0 \text { typ. }}+\overbrace{C_{n_{\alpha}}}^{=0 \text { typ. }} \cdot \alpha+C_{n_{\beta}} \cdot \beta+C_{n_{p}} \frac{p b}{2 V}+C_{n_{r}} \frac{r b}{2 V}+C_{n \delta_{r}} \delta_{r}+C_{n \delta_{a}} \delta_{a}+\cdots \tag{8-33}
\end{equation*}
$$

Where:

|  | $C_{L 0}=$ Zero $\alpha$ lift coefficient |
| :---: | :---: |
|  | $C_{L \alpha}=$ Lift curve slope, dependency on $\alpha$ |
|  | $C_{L_{\dot{\alpha}}}=$ Lift damping due to time rate of change of $\alpha$ |
|  | $C_{L \beta}=$ Lift curve slope, dependency on $\beta$ |
|  | $C_{L \delta e}=$ Change in lift coefficient of the tail with elevator deflection |
|  | $C_{L \delta \delta}=$ Change in lift coefficient with flap deflection |
|  | $C_{L \delta s p o i l e r}=$ Change in lift coefficient with deflection of spoiler |
| HT Derivatives | $C_{L H T}=$ Lift coefficient of the HT |
|  | $C_{L O H T}=$ Zero $\alpha$ lift coefficient of the HT |
|  | $C_{L a H T}=$ Lift curve slope of the HT |
|  | $C_{\text {Dmin }}=$ Minimum drag coefficient |
|  | $C_{D \alpha}=$ Drag curve slope, dependency on $\alpha$ |
|  | $C_{D \beta}=$ Drag curve slope, dependency on $\beta$ |
|  | $C_{D \delta \text { e }}=$ Change in drag coefficient of the tail with elevator deflection |
|  | $C_{D \delta f}=$ Change in drag coefficient with flap deflection |
|  | $C_{\text {DSspoiler }}=$ Change in drag coefficient with deflection of spoiler |
|  | $C_{L \text { min }}=$ Lift coefficient corresponding to the minimum drag |
| Side Force Derivatives | $C_{y \alpha}=$ Side force derivative due to $\alpha$ |
|  | $C_{y \beta}=$ Side force derivative due to $\beta$ |
|  | $C_{y \delta r}=$ Side force change due to rudder deflection $\delta_{r}$ |
|  | $C_{l o}=$ Rolling moment coefficient at zero $\phi$ (typically 0) |
|  | $C_{l \alpha}=$ Rolling moment coefficient dependency on $\alpha$ (typically 0) |
|  | $C_{l \beta}=$ Rolling moment coefficient dependency on $\beta$ (also called dihedral effect) |
|  | $C_{l p}=$ Change in rolling moment coefficient due to roll rate $p$ (roll damping) |
|  | $C_{l r}=$ Change in rolling moment coefficient due yaw rate $r$ |
|  | $C_{l \delta a}=$ Change in rolling moment coefficient due to aileron deflection $\delta_{a}$ |
|  | $C_{l \delta r}=$ Change in rolling moment coefficient due to rudder deflection $\delta_{r}$ |
|  | $C_{m o}=$ Pitching moment coefficient at zero $\alpha$ |
|  | $C_{m \alpha}=$ Pitching moment coefficient dependency on $\alpha$ |
|  | $C_{m \beta}=$ Pitching moment coefficient dependency on $\beta$ (typically 0) |
|  | $C_{m q}=$ Pitch damping due to roll rate $q$ |
|  | $C_{m_{\dot{\alpha}}}=$ Pitch damping due to time rate of change of $\alpha$ |
|  | $C_{m \text { ¢е }}=$ Pitching moment coefficient dependency on elevator deflection $\delta_{e}$ |
|  | $C_{m \delta f}=$ Pitching moment coefficient dependency on flap deflection $\delta_{f}$ |


|  | $C_{n o}=$ Yawing moment coefficient at zero $\beta$ (typically 0 ) |
| :---: | :---: |
|  | $C_{n \alpha}=$ Yawing moment coefficient dependency on $\alpha$ (typically 0) |
|  | $C_{n \beta}=$ Yawing moment coefficient dependency on $\beta$ (directional stability) |
|  | $C_{n \mathrm{p}}=$ Change in yawing moment coefficient due roll rate $p$ |
|  | $C_{n \mathrm{r}}=$ Change in yawing moment coefficient due yaw rate $r$ (yaw damping) |
|  | $C_{n \delta \bar{a}}=$ Yawing moment coefficient dependency on aileron deflection $\delta_{a}$ |
|  | $C_{n \delta r}=$ Yawing moment coefficient dependency on rudder deflection $\delta_{r}$ |
|  | $\delta_{a}=$ Aileron deflection |
|  | $\delta_{e}=$ Elevator deflection |
|  | $\delta_{f}=$ Flap deflection |
|  | $\delta_{r}=$ Rudder deflection |
|  | $\delta_{T}=$ Throttle control input |
|  | $\delta_{\text {spoiler }}=$ Spoiler deflection |
| $\begin{aligned} & \text { n } \\ & \stackrel{0}{n} \\ & \stackrel{0}{7} \end{aligned}$ | $b=$ Reference span (typically the wing span) |
|  | $C_{M G C}=$ Reference Mean Geometric Chord |
|  | $C_{T}=$ Thrust coefficient |
|  | $k=$ Lift-induced drag constant |
|  | $p=$ Roll rate |
|  | $q$ = Pitch rate |
|  | $r=$ Yaw rate |
|  | $S$ = Reference area (typically the wing area) |
|  | $S_{H T}=$ Reference area for the horizontal stabilizer |
|  | $T$ = Thrust |
|  | $V=$ Airspeed |
|  | $\eta_{H T}=$ Horizontal tail efficiency, a consequence of propwash over the HT, typ. 1.05-1.15 |

### 8.4.3. Lift Related Stability Derivatives

## Basic Constants

$C_{L_{0}}$ is the intercept of the linear relation to the $Y$-axis (which represents $C_{L}$ ). See Section 6.2.2, The Lift Coefficient for more detail.

## Change in Lift due to $\alpha$ and $\beta$

$C_{L_{u}}$ is the slope of the lift curve and is normally called the lift curve slope. Strictly, it is defined as follows

$$
\begin{equation*}
C_{L_{\alpha}} \equiv \frac{\partial C_{L}}{\partial \alpha} \tag{8-34}
\end{equation*}
$$

Reference [9] presents several methods to determine the lift curve slope in the linear region of the lift curve, using the geometry of the wing. It derives the slope for an elliptical wing planform shape using Prandtl's lifting line theory. The following expression applies to such wings only:

Lift curve slope for an elliptical wing: $\quad C_{L \alpha}=\frac{C_{l \alpha}}{1+\frac{C_{l \alpha}}{\pi \cdot A R}}$

A common (but not always correct) assumption is that the lift curve slope of an airfoil, denoted by $C_{l \alpha}$, is $2 \pi$. Substituting this into Equation (8-35) leads to

Elliptical wing with $C_{l \alpha}=2 \pi: \quad C_{L \alpha}=2 \pi \frac{A R}{A R+2}$
References [10] and [11] present methods to make Equation (8-36) suited for $A R<4$, but these are generally unwieldy. The following expression allows Equation (8-35) to be used for arbitrary wing shapes by the application of a correction factor, $\tau$ :

Lift curve slope for an arbitrary wing: $\quad C_{L \alpha}=\frac{C_{l \alpha}}{1+\frac{C_{l \alpha}}{\pi \cdot A R}(1+\tau)}$

The factor $\tau$ is a function of the Fourier coefficients determined using the lifting line theory and represents the following correction to the induced AOA, as shown by Dommasch [12]. The actual value of $\tau$ is calculated and provided by Glauert [13]

$$
\alpha_{i}=\frac{C_{L}(1+\tau)}{\pi \cdot A R}
$$

By making some approximations and determining the downwash at the $3 / 4$ chord station of the airfoil, rather than the $1 / 4$ station, Helmbold [14] derived the expression below for an arbitrary wing shape:

Hembold equation:

$$
\begin{equation*}
C_{L \alpha}=\frac{2 \pi \cdot A R}{2+\sqrt{A R^{2}+4}} \tag{8-38}
\end{equation*}
$$

Polhamus modified the Hembold equation as shown in Reference [15] for an arbitrary non-curved swept wing planform with a mid-chord sweep angle $\Lambda_{c / 2}$, subjected to compressible flow, yielding the expression below. It is also presented in Reference [16]. The expression accounts for compressibility (M $\leq 0.8)$ and deviation from the $2 \pi$ airfoil lift curve slope:

Polhamus equation:

$$
\begin{equation*}
C_{L \alpha}=\frac{2 \pi \cdot A R}{2+\sqrt{\left(\frac{A R \cdot \beta}{\kappa}\right)^{2}\left(1+\frac{\tan ^{2} \Lambda_{C / 2}}{\beta^{2}}\right)+4}} \tag{8-39}
\end{equation*}
$$

Where $A R$ is the wing Aspect Ratio, $\beta$ is called the Mach number parameter (Prandtl-Glauert) $=(1-$ $\left.M^{2}\right)^{0.5}, \kappa$ is the ratio of 2-dimensional lift curve slope to $2 \pi$, and $\Lambda_{c / 2}$ is the sweepback angle of the airfoil's mid-chord. Of the above methods, Equations (8-38) and (8-39) compare well with experiment.
The derivative $C_{L_{\dot{\alpha}}}$ is the change in lift due to the rate of change of $\alpha$ and is often referred to as the downwash lag. This transient effect derivative accounts for the fact that it takes a short time for airflow to "adapt" to a new orientation of the aircraft with respect to flow direction. This transient effect must
be treated for many applications using Theodorsen's complex function $C(k)=F(k)+i \quad G(k)$ per Theordorsen [17] and also Tobak [18]. Here, $F(k)$ and $G(k)$ are Bessel functions of the first and second kind, and $k$ is the reduced frequency, given as $k=C \omega / 2 V$, where $C$ is the wing chord, $\omega$ the frequency of oscillation, and $V$ the airspeed. However, for conventional configurations, the HT plays a large role in the magnitude of the effect and a more practical approximation for the effect is available. It is shown in [2] that downwash lag of lift due to the tail is approximated with acceptable accuracy by

$$
\begin{equation*}
C_{L_{\dot{\alpha}}} \approx\left(\frac{\partial C_{z}}{\partial \dot{\alpha}}\right)_{H T}=\frac{\partial C_{z}}{\partial\left(\frac{\dot{\alpha} C}{2 V}\right)}=-2 C_{L_{\alpha}} V_{H T} \frac{\partial \varepsilon}{\partial \alpha} \tag{8-40}
\end{equation*}
$$

The derivative $C_{L_{\beta}}$ is the rate of change of lift coefficient with yaw angle, $\beta$. It is usually a small number.

## Change in Lift due to Control Surface Deflections

The derivative $C_{L_{\delta_{e}}}$ is the rate of change in the lift coefficient due to deflection of the elevator $\delta_{e}$.

$$
C_{L_{\delta_{e}}} \equiv \frac{\partial C_{L}}{\partial \delta_{e}}=\left\{\begin{array}{cl}
\frac{\partial C_{L_{w b}}}{\partial \delta_{e}} & \text { For tailless aircraft }  \tag{8-41}\\
\frac{S_{H T}}{S} \frac{\partial C_{L_{H T}}}{\partial \delta_{e}} & \text { For conventional aircraft }
\end{array}\right.
$$

Where $S$ is the planform area of the wing and $S_{H T}$ is the planform area of the Horizontal Tail (HT), $C_{L_{w b}}$ is the lift coefficient of the wing-body combination and only applies to tailless aircraft (e.g. flying wings), and $C_{L_{H T}}$ is the lift coefficient of a conventional tail configuration and is typically expressed as shown below

$$
\begin{equation*}
C_{L_{H T}}=C_{L_{\alpha_{H T}}} \alpha_{H T}+\frac{\partial C_{L_{H T}}}{\partial \delta_{e}} \delta_{e} \tag{8-42}
\end{equation*}
$$

Where $\alpha_{H T}$ is the angle-of-attack of the horizontal tail, $C_{L_{\alpha_{H T}}}$ is the lift curve slope of the HT.

The derivative $C_{L_{\delta_{f}}}$ is the rate of change in the lift coefficient due to deflection of flaps or other high lift devices, $\delta_{f}$, if the aircraft features such devices.

$$
\begin{equation*}
C_{L_{\delta_{f}}} \equiv \frac{\partial C_{L}}{\partial \delta_{f}} \tag{8-43}
\end{equation*}
$$

The derivative $C_{L_{\delta_{\text {spoiler }}}}$ is the rate of change in the lift coefficient due to deflection of spoilers or other drag increasing controls, $\delta_{\text {spoiler }}$, if the aircraft features such devices. A spoiler will decrease lift over the
portion of the wing from which it deploys. This must be made up by increasing AOA or airspeed or a combination of both. The derivative is defined as follows

$$
\begin{equation*}
C_{L_{\delta_{\text {spoiler }}}} \equiv \frac{\partial C_{L}}{\partial \delta_{\text {spoiler }}} \tag{8-44}
\end{equation*}
$$

### 8.4.4. Drag Related Stability Derivatives

## Basic Constants

$C_{D \text { min }}$ and $C_{L \min D}$ are the minimum drag coefficient and the lift coefficient of minimum drag, respectively. They are not derivatives, but rather constants that are a part of the same function as the derivatives and, thus, help define the total drag coefficient. See Section 6.2.3, The Drag Coefficient for more detail. Reference [9] presents details of typical values of these coefficients

## Change in Drag due to $\alpha$ and $\beta$

The derivative $C_{D_{\alpha}}$ is the rate of change in the drag coefficient due to change in $\alpha$. If the drag coefficient is represented using the adjusted drag model, then we may write:

$$
\begin{aligned}
C_{D} & =C_{D_{\min }}+k\left(C_{L}-C_{L_{\min D}}\right)^{2}=C_{D_{\min }}+k\left(C_{L_{\alpha}} \alpha-C_{L_{\min D}}\right)^{2} \\
& =C_{D_{\min }}+k\left(C_{L_{\alpha}}^{2} \alpha^{2}-2 C_{L_{\alpha}} \alpha C_{L_{\min D}}+C_{L_{\min D}}^{2}\right)
\end{aligned}
$$

Consequently, the $\alpha$-derivative of the drag coefficient is given by:

$$
\frac{\partial C_{D}}{\partial \alpha}=\frac{\partial}{\partial \alpha}\left[k C_{L_{\alpha}}^{2} \alpha^{2}-2 k C_{L_{\alpha}} \alpha C_{L_{\min D}}+k C_{L_{\min D}}^{2}\right]=2 k C_{L_{\alpha}}^{2} \alpha-2 k C_{L_{\alpha}} C_{L_{\min D}}
$$

Therefore

$$
\begin{equation*}
C_{D_{\alpha}}=2 k C_{L_{\alpha}}\left(C_{L_{\alpha}} \alpha-C_{L_{\min D}}\right) \tag{8-45}
\end{equation*}
$$

The derivative $C_{D_{\beta}}$ is the rate of change in the drag coefficient due to change in $\beta$. This describes the rise in the drag coefficient caused by yaw of the airplane. The product $C_{D_{\beta}} \beta$ is always a positive number (that increases drag).

$$
\begin{equation*}
C_{D_{\beta}} \equiv \frac{\partial C_{D}}{\partial \beta} \tag{8-46}
\end{equation*}
$$

While the pilot is usually completely oblivious of its existence, he or she will sometimes use it to aid in the approach for landing of aircraft, by deliberately yawing it on approach for landing using rudder deflection.

## Change in Drag due to Control Surface Deflections

The derivative $C_{D_{\delta_{e}}}$ is the rate of change in the drag coefficient due to deflection of the elevator $\delta_{e}$ and is defined as

$$
\begin{equation*}
C_{D_{\delta_{e}}} \equiv \frac{\partial C_{D}}{\partial \delta_{e}} \tag{8-47}
\end{equation*}
$$

The product $C_{D_{\delta_{e}}} \delta_{e}$ is always a positive number (that increases drag), however, there may be an offset in the minimum, for instance, due to the longitudinal stabilizing surface being installed with an incidence angle.

The derivative $C_{D_{\delta_{f}}}$ is the rate of change in the drag coefficient due to deflection of flaps or other high lift devices to the deflection angle, $\delta_{f}$, assuming the aircraft features such devices. It always increases the total drag of the airplane.

$$
\begin{equation*}
C_{D_{\delta_{f}}} \equiv \frac{\partial C_{D}}{\partial \delta_{f}} \tag{8-48}
\end{equation*}
$$

The derivative $C_{D_{\delta_{\text {sppoiler }}}}$ is the rate of change in the drag coefficient due to deflection of spoilers or other drag increasing controls, $\delta_{\text {spoiler, }}$, if the aircraft features such devices.

$$
\begin{equation*}
C_{D_{\delta_{\text {spoiler }}}} \equiv \frac{\partial C_{D}}{\partial \delta_{\text {spoiler }}} \tag{8-49}
\end{equation*}
$$

### 8.4.5. Side Force Related Stability Derivatives

The derivative $C_{y_{\beta}}$ is the side force derivative is analogous to the lift curve slope, $C_{L_{\alpha}}$. In fact, if the airplane does a knife-edge maneuver (flying on its side), this derivative dictates the side-slip angle $\beta$ the airplane must make to maintain altitude.

$$
\begin{equation*}
C_{y_{\beta}} \equiv \frac{\partial C_{y}}{\partial \beta} \tag{8-50}
\end{equation*}
$$

The derivative $C_{y_{\delta_{r}}}$ is the change in the side force with rudder deflection, $\delta_{r}$. For this reason, the derivative is analogous to the derivative $C_{L_{\delta_{e}}}$.

### 8.4.6. Roll Related Stability Derivatives

Recall that the rolling moment coefficient is denoted by $C_{l}$ as depicted by Equation (8-31). The first term, $C_{l_{0}}$, is a constant and is always zero, unless the airplane is asymmetrically shaped or if, for some reason, it is loaded such the CG is no longer in the plane of symmetry, or it must react asymmetric thrust (e.g. multi-engine aircraft with one engine inoperative). The second derivative, $C_{l_{\alpha}}$, means that a rolling moment is induced with changes in AOA. While it is usually zero for normal aircraft, it might be used to represent anomaly, perhaps due to damage while in flight.

The derivative $C_{l_{\beta}}$ is called dihedral effect and is a very important derivative that represents rolling moment that is generated if the airplane yaws. It is a figure of merit for the presence of roll stability. Its magnitude depends on geometric shape of the aircraft, such as vertical location of the wing (low, mid, high) on the fuselage and wing sweep angle. It must be negative, for restoring (stabilizing) moments to be generated. It has an important contribution to the airplane's Dutch-roll characteristics.

$$
\begin{equation*}
C_{l_{\beta}} \equiv \frac{\partial C_{l}}{\partial \beta} \tag{8-51}
\end{equation*}
$$

While the function of this derivative is not too hard to relate to, it is not necessarily easy to predict. Methods to predict it are provided in References [1, 2, 3, 6, and 7].

The derivative $C_{l_{p}}$ is known as roll damping. It generates a moment that counters the roll capability of the aircraft and is the reason for why all aircraft have a steady-state roll rate. With respect to magnitude, consider the difference between the roll performance of an aerobatic aircraft like the Pitts Special and a typical sailplane. Since the span of the sailplane is substantially larger, it resists roll rate far more effectively than the Pitts. Consequently, the sailplane has a much more sluggish roll response, while the Pitts is very responsive.

The roll damping can be estimated analytically for wing planform shapes that can be described using continuous and discontinuous functions. In general, if $S$ and $b$ are the reference wing area and span, $c(y)$ is the wing chord as a function of the spanwise station $y$, and $q$ is the dynamic pressure, we can calculate the rolling moment can be calculated using the following expression:

$$
\begin{equation*}
L_{p}=\frac{2 q p\left(c_{l_{\alpha}}+c_{d o}\right)}{V} \int_{0}^{b / 2} y^{2} \cdot c(y) d y \tag{8-52}
\end{equation*}
$$

Where $C_{l \alpha}$ is the airfoil's lift-curve slope and $c_{d o}$ is the minimum drag coefficient of the airfoil, $p$ is the roll rate, and $V$ is the airspeed. The expression can be used to determine the rolling moment coefficient due to roll damping in terms of $(p b / 2 V)$ is then calculated as shown below:

$$
\begin{equation*}
C_{l}=\frac{L_{p}}{q S b}=-\left(\frac{p b}{2 V}\right) \frac{4\left(c_{l_{\alpha}}+c_{d o}\right)}{S b^{2}} \int_{0}^{b / 2} y^{2} \cdot c(y) d y \tag{8-53}
\end{equation*}
$$

The change in rolling moment coefficient due to roll rate $p$ is called the roll damping coefficient and can be found from:

$$
\begin{equation*}
C_{l_{p}}=-\frac{4\left(c_{l_{\alpha}}+c_{d o}\right)}{S b^{2}} \int_{0}^{b / 2} y^{2} \cdot c(y) d y \tag{8-54}
\end{equation*}
$$

The closed form solution for two common wing planform shapes is given below. Derivation is presented in Reference [9], which also provides analytic solutions for two following wing planform shapes:

CASE 1: Straight tapered wing with Taper Ratio $\lambda$ :

$$
\begin{equation*}
C_{l_{p}}=-\frac{\left(c_{l_{\alpha}}+c_{d o}\right) \cdot C_{R} b}{24 S}[1+3 \lambda] \tag{8-55}
\end{equation*}
$$

CASE 2: Rectangular wing ( $\lambda=1$ ):

$$
\begin{equation*}
C_{l_{p}}=-\frac{c_{l_{\alpha}}+c_{d o}}{6} \tag{8-56}
\end{equation*}
$$

The derivative $C_{l_{r}}$ is the yaw coupling derivative. It indicates the rolling moment that is induced when the airplane is yawed. Once the yaw rate disappears, the tendency does so as well. It is defined as

$$
\begin{equation*}
C_{l_{r}} \equiv \frac{\partial C_{l}}{\partial r} \tag{8-57}
\end{equation*}
$$

The contribution due to the geometry of the VT can be estimated as follows

$$
\begin{equation*}
\left(C_{l_{r}}\right)_{V T}=C_{L_{\alpha_{V T}}} \frac{S_{V T}}{S} \frac{z_{F}}{b}\left(2 \frac{l_{V T}}{b}+\frac{\partial \sigma}{\partial(r b / 2 V)}\right) \tag{8-58}
\end{equation*}
$$

The derivative $C_{l_{\delta_{a}}}$ is called the roll authority derivative. It indicates the change in rolling moment that result from deflecting the ailerons. In essence, it is the derivative that generates the rolling moment that the roll damping opposes. It can be estimated using the so-called the Strip Integration Method. However, it requires change in the airfoil's lift coefficient with aileron deflection (denoted by $c_{l_{\delta_{\alpha}}}$ ) to be known. Note that capitalization is used to separate $C_{l_{\delta_{a}}}$ (aileron authority) from $c_{l_{\delta_{a}}}$ (change in the lift coefficient with aileron deflection). The strip integration method is presented below

$$
\begin{equation*}
C_{l_{\delta_{a}}}=\frac{d C_{l}}{d \delta_{a}}=\frac{2 c_{\delta_{\delta_{a}}}}{S b} \int_{b_{1}}^{b_{2}} c \cdot y \cdot d y \tag{8-59}
\end{equation*}
$$

Where $b$ is the wing span, in ft or $\mathrm{m}, S$ is the wing area, $c$ the wing chord, and $y$ the wing station.

The dimensions $b_{1}$ and $b_{2}$ can be seen in Figure 8-9. Note that the expression overestimates the roll authority by not accounting for end effects on either side of each aileron. An analytic solution of Equation (8-59) for two common wing planform shapes can be provided.


Figure 8-9: Definition of the aileron geometry. (from Ref. [9])

CASE 1: Straight Tapered Wing with Taper Ratio $\lambda$ :

$$
\begin{equation*}
C_{l_{\delta_{a}}}=\frac{c_{l_{\delta_{a}}} C_{R}}{S b}\left[\left(b_{2}^{2}-b_{1}^{2}\right)+\frac{4(\lambda-1)}{3 b}\left(b_{2}^{3}-b_{1}^{3}\right)\right] \tag{8-60}
\end{equation*}
$$

CASE 2: Rectangular Wing $(\lambda=1): \quad C_{l_{\delta_{a}}}=\frac{c_{l_{\delta_{a}}}\left(b_{2}^{2}-b_{1}^{2}\right)}{b^{2}}$

Note that once the roll damping and authority derivatives have been determined, it is possible to estimate the steady state roll rate as follows. The steady state roll helix angle is determined from:

$$
\begin{equation*}
\frac{p b}{2 V}=-\frac{C_{l_{\delta_{a}}}}{C_{l_{p}}} \delta_{a} \tag{8-62}
\end{equation*}
$$

Which allows the steady-state roll rate (in rad/sec) to be determined by solving for $p$

$$
\begin{equation*}
p=-\frac{C_{l_{\delta_{a}}}}{C_{l_{p}}} \delta_{a}\left(\frac{2 V}{b}\right) \tag{8-63}
\end{equation*}
$$

Finally, the derivative $C_{l_{\delta_{r}}}$ is the roll due to rudder deflection derivative. It indicates the change in rolling moment resulting from deflecting the rudder. It can be used with the dihedral effect to generate bank capability using rudder only and, as such, provides a backup control function for the pilot. Methods to estimate it are too involved to present here and the effort would be reduced by modeling the aircraft using panel methods or similar techniques.

### 8.4.7. Pitch Stability Derivatives

## Basic Constants

$C_{m_{0}}$ is the intercept of the pitching moment curve to the $C_{m}$ axis ( $y$-axis) and, while treated as a constant, it is actually a function of control surface deflections, for instance, the elevator deflection $\delta_{e}$, which shifts it up and down.

The longitudinal stability is denoted by $C_{m_{\alpha}}$ and represents the slope of the pitching moment curve. In the linear region of the curve it can be written in terms of the lift curve slope as shown below

$$
\begin{equation*}
C_{m_{\alpha}}=C_{L_{\alpha}}\left(h-h_{n}\right)=C_{L_{\alpha}} \frac{\left(x_{C G}-x_{\text {neu }}\right)}{C_{M G C}} \tag{8-64}
\end{equation*}
$$

It is possible that yawing the airplane will affect its longitudinal stability. This is known to happen for airplanes with V-tails, where it is referred to as pitch-yaw coupling. If this tendency is present, it is denoted using the variable $C_{m_{\beta}}$. This effect is much less noticeable for conventional aircraft and can be neglected.

The derivative $C_{m_{q}}$ is called pitch-damping. It is caused by the pitch rate of the airplane and can be thought of as the "paddle-effect". The derivative can be obtained through analytical methods, such as those shown by Etkin [2] and Napolitano [3], potential flow methods such as vortex-lattice or doubletlattice, or from specialized rotary balance wind tunnels. References [2] and [3] give the following expression for the pitch damping, which ignores contribution from the fuselage and wing

$$
\begin{equation*}
C_{m_{q}} \approx-2 C_{L_{\alpha_{H T}}} \eta_{H T} \frac{S_{H T}}{S}\left(\frac{x_{A C_{H T}}-x_{C G}}{C_{M G C}}\right)^{2}=-2 C_{L_{\alpha} H T} \eta_{H T} \frac{S_{H T}}{S}\left(\frac{l_{H T}}{C_{M G C}}\right)^{2} \tag{8-65}
\end{equation*}
$$

Where $x_{A C H T}$ is the physical location of the aerodynamic center of the HT (approximately $0.25 C_{M G C H T}$ ) and $x_{C G}$ is the physical location of the CG of the airplane.

The derivative $C_{m_{\dot{\alpha}}}$ is the change in pitching moment due to the rate of change of $\alpha$. The source of the effect was discussed in detail with the derivative $C_{L_{\dot{\alpha}}}$, earlier. It can be shown in [2], that for conventional configurations, where the HT plays a major role in the magnitude of the effect the contribution of the horizontal tail is approximated as follows

$$
\begin{equation*}
C_{m_{\dot{\alpha}}} \approx \frac{l_{H T}}{C}\left(\frac{\partial C_{z}}{\partial \dot{\alpha}}\right)_{H T}=\frac{\partial C_{z}}{\partial\left(\frac{\dot{\alpha} C}{2 V}\right)}=-2 C_{L_{\alpha_{H T}}} V_{H T} \frac{l_{H T}}{C} \frac{\partial \varepsilon}{\partial \alpha} \tag{8-66}
\end{equation*}
$$

## Change in Moment due to Control Surface Deflections

The derivative $C_{m_{\delta_{e}}}$ is the elevator authority; a crucial derivative that allows the pilot to control the trim airspeed of the aircraft.

$$
C_{m_{\delta_{e}}} \equiv \frac{\partial C_{m}}{\partial \delta_{e}}=\left\{\begin{array}{cl}
C_{L_{\delta_{e}}} \frac{\left(x_{C G}-x_{n e u}\right)}{C_{M G C}}+\frac{\partial C_{m_{a c}}}{\partial \delta_{e}} & \text { For tailless aircraft }  \tag{8-67}\\
C_{L_{\delta_{e}}} \frac{\left(x_{C G}-x_{n e u}\right)}{C_{M G C}}-V_{H T} \frac{\partial C_{L_{H T}}}{\partial \delta_{e}} & \text { For conventional aircraft }
\end{array}\right.
$$

The derivative $C_{m_{f}}$ represents the contribution of flap deflection to the longitudinal stability of the aircraft. Typical trailing edge high-lift system will increase the nose-down pitching moment of the aircraft, requiring increased TEU deflection of the elevator. The magnitude of the derivative depends on the deflection angle of said high-lift system; it is 0 when retracted and $<0$ for deflected values.

### 8.4.8. Yaw Stability Derivatives

The yawing moment coefficient is denoted by $C_{n}$ as depicted by Equation (8-33). The first term, $C_{n_{0}}$, is a constant and is always zero, unless the airplane is asymmetrically shaped or if, for some reason, it is loaded such the CG is no longer in the plane of symmetry, or it must react asymmetric thrust (e.g. multiengine aircraft with one engine inoperative). The second derivative, $C_{n_{\alpha}}$, means that a yawing moment is induced with changes in AOA. While it is usually zero for normal aircraft, it might be used to represent anomaly, perhaps due to damage while in flight.

The derivative $C_{n_{\beta}}$ is called directional stability and is a very important derivative that represents yawing moment that is generated if the airplane yaws. It is a figure of merit for the presence of yaw stability. Its magnitude depends on geometric shape of the aircraft, of which the vertical tail is the primary contributor. Its numerical value must be positivetive for restoring (stabilizing) moment to be generated. Like the dihedral effect, it has an important contribution to the airplane's Dutch-roll characteristics.

$$
\begin{equation*}
C_{n_{\beta}} \equiv \frac{\partial C_{n}}{\partial \beta} \tag{8-68}
\end{equation*}
$$

The derivative $C_{n_{p}}$ is the yaw due to roll cross derivative. It generates a yawing moment because of roll; it is one manifestation of how roll and yaw are coupled ( $C_{l_{r}}$ is another). It is largely caused by the increase in the drag of one wing due to roll, but high-sitting HT (e.g. T-tail) may contribute to the tendency as well. The contribution of the tail can be expressed as follows

$$
\begin{equation*}
\left(C_{n_{p}}\right)_{V T}=C_{L_{\alpha T}} V_{V T}\left(2 \frac{z_{F}}{b}-\frac{\partial \sigma}{\partial(p b / 2 V)}\right) \tag{8-69}
\end{equation*}
$$

Where $z_{F}$ is the span of the $\mathrm{VT}, \sigma$ is the sidewash due to roll. The derivative $C_{n_{r}}$ is the yaw damping derivative. It is always negative, unless there is excessive wing sweepback $\left(>60^{\circ}\right)$. The contribution of the VT can be expressed as follows

$$
\begin{equation*}
\left(C_{n_{r}}\right)_{V T}=-C_{L_{\alpha_{V T}}} V_{V T}\left(2 \frac{l_{V T}}{b}-\frac{\partial \sigma}{\partial(r b / 2 V)}\right) \tag{8-70}
\end{equation*}
$$

Finally, the derivative $C_{n_{\delta_{r}}}$ is the yaw-due-to-rudder deflection derivative. It indicates the change in yawing moment due to rudder deflection. Methods to estimate it are too involved to present here and the effort would be reduced by modeling the aircraft using panel methods or similar techniques.

### 8.5 State Vector and State-Space Representation

The EOM of Equations (8-9) through (8-12) are computed in the flight simulator. During execution, it calculates a number of important information and stores it in an array called the state vector. The required size of this vector is 12 , although it is expanded in the SURFACES Flight Simulator to allow the collection of information of interest during each time step. This information allows various characteristic of the aircraft model to be plotted and analyzed in real time. In fact, the simulator stores two types of the state vector; its normal and time-derivative forms. The state vectors is defined as follows

Normal form:

$$
\begin{align*}
& \mathbf{X} \equiv x(i) \quad i=1 \ldots 12  \tag{8-71}\\
& \dot{\mathbf{X}} \equiv \dot{x}(i) \tag{8-72}
\end{align*}
$$

Where $i$ is index to a specific member in the vector. The indexes refer to the characteristics shown in Tables 8-1 and 8-2.

Table 8-1: Break-down of the state vector in the normal form

Velocity

$$
\begin{align*}
& u=x_{1}=\int \dot{x}_{1} d t=\int \dot{u} d t \\
& v=x_{2}=\int \dot{x}_{2} d t=\int \dot{v} d t  \tag{8-73}\\
& w=x_{3}=\int \dot{x}_{3} d t=\int \dot{w} d t \\
& \phi=x_{4}=\int \dot{x}_{4} d t=\int \dot{\phi} d t \\
& \theta=x_{5}=\int \dot{x}_{5} d t=\int \dot{\theta} d t  \tag{8-74}\\
& \psi=x_{6}=\int \dot{x}_{6} d t=\int \dot{\psi} d t
\end{align*}
$$

Rotation rates | $p=x_{7}$ | $=\int \dot{x}_{7} d t=\int \dot{p} d t$ |
| ---: | :--- |
| $q$ | $=x_{8}=\int \dot{x}_{8} d t=\int \dot{q} d t$ |
| $r$ | $=x_{9}=\int \dot{x}_{9} d t=\int \dot{r} d t$ |
| $N$ | $=x_{10}=\int \dot{x}_{10} d t=\int \dot{N} d t$ |
| $E$ | $=x_{11}=\int \dot{x}_{11} d t=\int \dot{E} d t$ |
| $h$ | $=x_{12}=\int \dot{x}_{12} d t=\int \dot{h} d t$ |

Table 8-2: Break-down of the state vector in the time-derivative form

| Acceleration | Rotational <br> acceleration of Euler <br> angles | Rotational <br> acceleration of <br> rotation rates | Ground speed <br> components |
| :---: | :---: | :---: | :---: |
| $\dot{x}_{1}=\dot{u}$ | $\dot{x}_{4}=\dot{\phi}$ | $\dot{x}_{7}=\dot{p}$ | $\dot{x}_{10}=\dot{N}$ |
| $\dot{x}_{2}=\dot{v}$ | $\dot{x}_{5}=\dot{\theta}$ | $\dot{x}_{8}=\dot{q}$ | $\dot{x}_{11}=\dot{E}$ |

### 8.6 A Roadmap to a Flight Simulator

This section presents the building blocks of the flight simulator developed for this work. In interest of space, many program details are omitted. However, enough is presented for a basic flight simulator to be programmed. Even though such computer programming may be of no interest to the reader, the presentation will forge an understanding of the code that must be executed in order to accomplish a realistic simulation. As will be shown, many of the tools developed in the previous chapters will be utilized. Also note that this simulator assumes the UK-system of units ( $\mathrm{ft}-\mathrm{sec}-\mathrm{lb}_{\mathrm{f}}$ ). This is not an endorsement of that system - the SI-system is preferred. At this time, the UK-system prevails in aviation in the US and, thus, is applied for courtesy reasons in this work. It will simply be easier for many US readers to relate to the results presented later.

Consider Figure 8-10, which shows a block diagram that represents the flow of the flight simulator code. The set of equations that must be solved in each numbered block will now be detailed. The solution involves numerical time-integration, using the methodologies presented in Appendix B. Note that in the form shown, ground forces (e.g. due to take-off or landing) are not accounted for. This is not a required detail for the work presented in here: the simulation presumes the aircraft is already airborne.


Figure 8-10: A block diagram showing how the flight simulator operates.

### 8.6.1. Initialization

The flight simulator requires initial conditions and these are presented in the normal and time-derivative state vectors. For instance, it is reasonable to begin the simulation in a steady state form. Note that this is not necessary and this simulator allows unusual orientation and accelerations as initial conditions. However, if steady state is chosen, then most state vector members will be zero. For instance, if we assume the airplane begins its flight at the steady state forward airspeed $(u)$ of $100 \mathrm{ft} / \mathrm{s}$ and at altitude of 10000 ft with a North-bound heading, the state vector of Equations ( $8-71$ ) and ( $8-72$ ) will be initialized as follows:

$$
\begin{aligned}
& \mathbf{X}: x(1)=u=100 \quad x(6)=\psi=\frac{\pi}{2} \quad x(12)=H=10000 \quad x(i)=0 \quad i=2,3,4,5,7,8,9,10,11 \\
& \dot{\mathbf{X}}: \dot{x}(i)=0 \quad i=1 \ldots 12
\end{aligned}
$$

Then, the quaternions must be initialized to ensure the angular motion of the aircraft can be tracked. Note that as shown in Appendix D, this is accomplished using the expression below:

$$
\begin{align*}
& e_{0}= \pm \cos \left(\frac{\phi}{2}\right) \cos \left(\frac{\theta}{2}\right) \cos \left(\frac{\psi}{2}\right)+\sin \left(\frac{\phi}{2}\right) \sin \left(\frac{\theta}{2}\right) \sin \left(\frac{\psi}{2}\right) \\
& e_{1}= \pm \sin \left(\frac{\phi}{2}\right) \cos \left(\frac{\theta}{2}\right) \cos \left(\frac{\psi}{2}\right)-\cos \left(\frac{\phi}{2}\right) \sin \left(\frac{\theta}{2}\right) \sin \left(\frac{\psi}{2}\right) \\
& e_{2}= \pm \cos \left(\frac{\phi}{2}\right) \sin \left(\frac{\theta}{2}\right) \cos \left(\frac{\psi}{2}\right)+\sin \left(\frac{\phi}{2}\right) \cos \left(\frac{\theta}{2}\right) \sin \left(\frac{\psi}{2}\right)  \tag{8-78}\\
& e_{3}= \pm \cos \left(\frac{\phi}{2}\right) \cos \left(\frac{\theta}{2}\right) \sin \left(\frac{\psi}{2}\right)-\sin \left(\frac{\phi}{2}\right) \sin \left(\frac{\theta}{2}\right) \cos \left(\frac{\psi}{2}\right)
\end{align*}
$$

It is now possible to enter the infinite loop of the program kernel and begin the simulation.

### 8.6.2. Block 1 - Atmospheric Properties

Atmospheric properties must be updated every iteration as the airplane changes altitude. In this block the property is the density, denoted by $\rho$ or rho. The properties are calculated using versions Equations (5-1), (5-6), and (5-7), although the simulator actually calls a predefined function that allows density to be calculated to altitude of $85 \mathrm{~km}(278000 \mathrm{ft})$. This is performed using the code snippet in Appendix A. 1 Routine to Determine Atmospheric Properties. This is calculated in the block using a statement like

```
'Density at altitude (which is stored in X(12))
rho = AtmosProperty(X(12), 12) 'Mode 12 returns density in slugs/ft3
```


### 8.6.3. Block 2 - Mass Properties

Mass properties must also be reevaluated every iteration, in particular if the aircraft operates using fossil fuels, whose consumption reduces the weight of the aircraft. Then, the current weight, $W_{\text {curr }}$, is used to calculate the current mass, to be used later with the EOM. In the current version of the program, the associated change in moments and products of inertia are not included. As already discussed in Chapter 7, Engine Performance Modeling, electric airplanes are not subject to such changes. This is implemented by subtracting the weight of the fuel consumed, $W_{f c}(t)$, from the initial weight of the aircraft, $W_{\text {ini }}$, as shown below

$$
\begin{gather*}
W_{\text {curr }}=W_{i n i}-W_{f c}(t)  \tag{8-79}\\
m_{\text {curr }}=\frac{W_{\text {curr }}}{g} \tag{8-80}
\end{gather*}
$$

Where $g$ is the acceleration due to gravity. The "Miscellaneous Flight Management" block keeps track of the fuel quantity and if used up, it automatically shuts the engine off.

### 8.6.4. Blocks 3-5: Control Input Blocks

The next three blocks are not detailed here because they involve treatment of control inputs, either by user using a joystick, or by the available autopilot functions through the AFMS (see Section 8.7, Autopilot Functions). This works through extracting the joystick position through the computer operating system and transform it into corresponding control surface deflections.

### 8.6.5. Block 6 - Airspeeds

The next step is to evaluate the current total airspeeds. This is done for airspeed planar to the $x z$-plane (the AOA plane), $V_{u w}$, and the total $x y z$-airspeed, $V_{T}$. The former is used in several places and calculating it only once increases the computational speed. The latter is used in multiple places as well and is used for determination of coefficients. They are given by

$$
\begin{gather*}
V_{u w}=\sqrt{u^{2}+w^{2}}  \tag{8-81}\\
V_{T}=\sqrt{u^{2}+v^{2}+w^{2}} \tag{8-82}
\end{gather*}
$$

The velocity components $u, v, w$ are extracted from the state vector of Equation (8-73).

### 8.6.6. Block 7 - Angular Orientation

With the airspeeds determined, it is possible to update the angles of attack and yaw, and their corresponding accelerations. Note that the angular accelerations can play a role in the total moment of the aircraft through the stability derivatives $C_{m \dot{\alpha}}$ and $C_{m \dot{\beta}}$. As shown by Stevens and Lewis [5] and Allerton [4], these can be calculated in following fashion

$$
\begin{array}{ll}
\text { Angle-of-attack, } \alpha: & \alpha=\tan ^{-1}(w / u) \\
\text { Rate of change of } \alpha: & \dot{\alpha}=\frac{u \cdot \dot{w}-w \cdot \dot{u}}{V_{u w}} \\
\text { Angle-of-yaw, } \beta: & \beta=\tan ^{-1}\left(v / V_{u w}\right) \\
\text { Rate of change of } \beta: & \dot{\beta}=\frac{\dot{v} \cdot V_{u w}-v(u \cdot \dot{u}-w \cdot \dot{w})}{V_{u w} \cdot V_{t}^{2}} \tag{8-86}
\end{array}
$$

The accelerations $\dot{u}, \dot{v}, \dot{w}$ are extracted from the state vector of Equation (8-73).

### 8.6.7. Block 8 - Aerodynamic Force Coefficients

The importance of obtaining the angles $\alpha$ and $\beta$ is realized in this block, as they are used to obtain the aerodynamic forces through calls to dedicated subroutines. These routines are implemented using Equations (8-24), (8-26), and (8-29), by performing programmatic calls like

```
'(8) Compute aerodynamic force coefficients: CL, CY, CD
CL = AERO_Get_CL(AOA, AOY, deltaAIL, deltaELV, deltaRDR, deltaFLP, deltaSPL)
```

```
CY = AERO_Get_CY(AOA, AOY, deltaAIL, deltaELV, deltaRDR, deltaFLP, deltaSPL)
CD = AERO_Get_CD(AOA, AOY, deltaAIL, deltaELV, deltaRDR, deltaFLP, deltaSPL)
```

Where the variables deltaAil, deltaELV, deltaRDR, deltafle, deltaSpl passed to the routines refer to the control surfaces (AIL = ailerons, ELV = elevator, RDR = rudder, FLP = flaps, and SPL = spoiler). Note that the flight simulator actually uses a lookup table inside the calls for the lift and drag coefficients, to increase the effect of the nonlinearities associated with those coefficients. This is accomplished using the method of Section 8.3.3, Lookup Tables.

### 8.6.8. Block 9 - Aerodynamic Moment Coefficients

Similar routines are called to obtain the aerodynamic moment coefficients, as implemented in Equations (8-31), (8-32), and (8-33). Note the naming of the variables is intended to prevent the force coefficient being confused with the force coefficients.

```
'(9) Compute aerodynamic moment coefficients: CmomL, CmomM, CmomN
CmomL = AERO_Get_momCL (AOA, AOY, deltaAIL, deltaELV, deltaRDR, deltaFLP, deltaSPL)
CmomM = AERO Get momCM(AOA, AOY, deltaAIL, deltaELV, deltaRDR, deltaFLP, deltaSPL)
CmomN = AERO_Get_momCN(AOA, AOY, deltaAIL, deltaELV, deltaRDR, deltaFLP, deltaSPL)
```


### 8.6.9. Block 10 - Body Frame Forces

With the aerodynamic force coefficients known, the next step involves calculating the aerodynamic body frame forces. This is done using Equations (8-14) through (8-16)

$$
\begin{align*}
& F_{\text {lift }}=q S C_{L} \\
& F_{\text {side }}=q S C_{Y}  \tag{8-87}\\
& F_{\text {drag }}=q S C_{D}
\end{align*}
$$

Where $q=1 / 2 \rho V_{T}{ }^{2}$ and $S$ is the reference wing area. Note that these forces still refer to the aircraft's body axes.

### 8.6.10. Block 11 - Engine Forces and Moments

The forces and moments generated by the engine can be complex in nature and is bound to have components acting along and about each of the three axes of the body frame coordinate system. The estimation of the magnitude of this contribution depends on the type of powerplant used; is it a turbojet, turbofan, turboprop, pistonprop, or electroprop? The scope of possibilities is large and in the interest of space, the reader is directed to Reference [9] for thrust and moment methodologies.

In general, the engine will generate pressure force due to $\alpha$ and $\beta$. The pressure force that results is usually referred to as a normal and side-force, respectively. In the case of a jet engine, these forces are caused by the pressure force acting on the nacelle (or shroud). For propellers, it is caused by the asymmetric thrust loading of the propeller disc. In addition, one must consider thrustline effects. If the engine sits high above the CG, it will generate a nose pitch-down moment that affects the elevator deflection required to trim the aircraft. It is important to include these effects for added realism. The

SURFACES Flight Simulator does this by allowing the user to specify the spatial position of the engine. It is assumed that the total resulting force vector components returned are denoted by $T_{E N G x}, T_{E N G y}$, and $T_{E N G z}$, and that the moments are denoted by $M_{E N G x}, M_{E N G y}$, and $M_{E N G z}$. All are aligned to the body frame system. The change in the weight of the fuel that is consumed is also taken care of in this block.

### 8.6.11. Block 12 - Total Body Forces

The total force acting on the aircraft in the body frame can now be determined by a summation of the aerodynamic and mechanical forces. The inclusion of the force of gravity and rotation rates will be accounted for in the next block. Note that the lift and drag forces must be resolved onto the body axes because, as is, they are aligned with the wind axes (similar to Equation (8-17) and as discussed in Section 8.1.5, Wind Axes Coordinate System (WAS)). Therefore, they are transformed in Block 12 as shown below

$$
\begin{align*}
& F_{X}=F_{l i f t} \sin \alpha-F_{\text {drag }} \cos \alpha+T_{E N G x} \\
& F_{Y}=F_{\text {side }}+T_{E N G y}  \tag{8-88}\\
& F_{Z}=-F_{l i j t} \cos \alpha-F_{\text {drag }} \sin \alpha+T_{E N G z}
\end{align*}
$$

### 8.6.12. Block 13 - Body Frame Accelerations

With the total forces in the body frame determined it is possible to compute the corresponding linear accelerations in accordance with Equation (8-9):

$$
\begin{align*}
& \dot{x}=\frac{F_{X}}{m}-q w+r v-g \sin \theta \\
& \dot{y}=\frac{F_{Y}}{m}-r u+p w+g \sin \phi \cos \theta  \tag{8-89}\\
& \dot{z}=\frac{F_{Z}}{m}-p v+q u+g \cos \theta \cos \phi
\end{align*}
$$

Where $p, q, r$ are the rotation rates and $\phi, \theta$ are the roll and pitch components of the Euler angles, respectively.

### 8.6.13. Block 14 - Body Frame Velocities

Recall that up to this point, the velocity components $u, v, w$ were those obtained in the previous time step. It is now possible to update these components and this is done in Block 14 using the numerical integration scheme of choice. In particular, the ones presented in Appendix B are recommended. These combine accuracy with speed, especially the two Adams-Bashforth schemes. Note that the SURFACES Flight Simulator allows the user to select one of three schemes for this purpose; Euler integration, Adams-Bashforth $2^{\text {nd }}$ order, and Adams-Bashforth $4^{\text {th }}$ order. The first scheme should use a time-step no larger than 0.01 sec for stability. In effect, if the numerical scheme of choice was Euler integration, it would be implemented as shown below

$$
\begin{align*}
& u_{i+1}=u_{i}+\dot{x}_{i+1} \Delta t \\
& v_{i+1}=v_{i}+\dot{y}_{i+1} \Delta t  \tag{8-90}\\
& w_{i+1}=w_{i}+\dot{z}_{i+1} \Delta t
\end{align*}
$$

Where the index $i$ refers to the value during the previous iteration and $i+1$ the current iteration, separated by time-step $\Delta t$.

### 8.6.14. Block 15 - Wind and Turbulence

The effects of atmospheric convection are added at this point. Realistic effects should affect the velocity and angular rate components. A method for this purpose is shown in Chapter 5, Atmospheric Modeling. Lets first consider the atmospheric wind vector, $V_{W}$, contribution, which consists of the components $V_{W N}, V_{W E}$, and $V_{W H}$, where the subscripts stand for North, East, and Height (altitude). These components must be added to the true airspeed components of the airplane. Rotational components are added to the rotation component of the state vector as well.

### 8.6.15. Block 16 - Earth Velocities

The position of the aircraft with respect to the ground requires the ground speed to be known. These are obtained using the rotation matrix (see Section 8.6.23, Block 24 - Compute Direction Cosine Matrix below) and by accounting for the contribution of the atmospheric wind vector, $\mathbf{V}_{\mathbf{w}}$, which consists of the components $V_{W N}, V_{W E}$, and $V_{W H}$, where the subscripts stand for North, East, and Height (altitude), respectively. Therefore, the ground speed is computed from

$$
\begin{align*}
& \dot{N}=u a_{11}+v a_{12}+w a_{13}-V_{W N} \\
& \dot{E}=u a_{21}+v a_{22}+w a_{23}-V_{W E}  \tag{8-91}\\
& \dot{H}=u a_{31}+v a_{32}+w a_{33}-V_{W H}
\end{align*}
$$

### 8.6.16. Block 17 - Earth Position

The Earth position can be obtained by continuous integration of the ground speed vector. Again, assuming the numerical scheme of choice is Euler integration, this scheme would be implemented as shown below

$$
\begin{align*}
& N_{i+1}=N_{i}+\dot{N} \Delta t \\
& E_{i+1}=E_{i}+\dot{E} \Delta t  \tag{8-92}\\
& H_{i+1}=H_{i}+\dot{H} \Delta t
\end{align*}
$$

Where the index $i$ refers to the value during the previous iteration and $i+1$ the current iteration, separated by time-step $\Delta t$.

### 8.6.17. Block 18 - Body Rates in the Stability Coordinate System

The next few steps treat the moments to which the airplane is subjected. Since the moment coefficients correspond to the stability coordinate system and the rotation rates $p$ and $r$ refer to the body coordinate system, it is necessary to convert these to the stability system using Equation (8-5).

$$
\begin{align*}
& p_{S}=p \cos \alpha+r \sin \alpha \\
& r_{S}=r \cos \alpha-p \sin \alpha \tag{8-93}
\end{align*}
$$

Where the subscript $S$ refers to the Stability Coordinate System.

### 8.6.18. Block 19 - Body Frame Moments in the Stability Coordinate System

Using the result from Block 18, it is now possible to calculate the aerodynamic moments acting on the aircraft in the stability coordinate system. This is done using Equations (8-19) through (8-21)

$$
\begin{align*}
& L_{S}=\frac{1}{2} \rho V_{T}^{2} S b C_{m o m L}+\frac{1}{4} \rho V_{T} S b^{2}\left(C_{l_{p}} p_{S}+C_{l_{r}} r_{S}\right) \\
& M_{S}=\frac{1}{2} \rho V_{T}^{2} S C_{M G C} C_{m o m M}+\frac{1}{4} \rho V_{T} S C_{M G C}^{2}\left(C_{m_{Q}} q+C_{m_{\dot{\alpha}}} \dot{\alpha}\right)  \tag{8-94}\\
& N_{S}=\frac{1}{2} \rho V_{T}^{2} S b C_{m o m N}+\frac{1}{4} \rho V_{T} S b^{2}\left(C_{n_{p}} p_{S}+C_{n_{r}} r_{S}\right)
\end{align*}
$$

Where $b$ is the wing span and $C_{M G C}$ is the mean geometric chord of the wing. Other variables are detailed in Section 8.4, A Summary of Stability Derivatives.

### 8.6.19. Block 20 - Body Frame Moments in the Body Frame Coordinate System

Next we must convert the above moments to the body frame so the angular accelerations can be determined and, subsequently, the angular rates before the next iteration. At this point, we also apply the moments generated by the power plant and calculated in Block 11. We must also account for the moment that is generated by the lift and drag being offset from the aerodynamic center of the airplane. The offset is the difference of the physical location of the CG, denoted by $X_{C G}$, the aerodynamic center, denoted by $X_{R E F}$. Its value of this offset is often close to ( $X_{C G}-C_{M G} / 4$ ), assuming the distance $X_{C G}$ refers to the leading edge of the $C_{M G C}$. Thus, we can transform the moments in the stability system to the body system as shown below

$$
\begin{align*}
& L_{B}=L_{S} \cos \alpha-N_{S} \sin \alpha+M_{E N G_{x}} \\
& M_{B}=M_{S}+\left(F_{\text {lift }} \cos \alpha+F_{\text {drag }} \sin \alpha\right)\left(X_{C G}-X_{R E F}\right)+M_{E N G_{y}}  \tag{8-95}\\
& N_{B}=N_{S} \cos \alpha+L_{S} \sin \alpha-F_{\text {side }}\left(X_{C G}-X_{R E F}\right)+M_{E N G_{z}}
\end{align*}
$$

Where the subscript $B$ refers to the Stability Coordinate System.

### 8.6.20. Block 21 - Body Frame Angular Accelerations

Now we are able to compute the angular accelerations in the body frame system.

$$
\begin{align*}
& \dot{p}=\frac{L_{B}+\left(I_{y}-I_{z}\right) q r+I_{x z}(\dot{r} p+p q)}{I_{x}} \\
& \dot{q}=\frac{M_{B}+\left(I_{z}-I_{x}\right) r p+I_{x z}\left(r^{2}+p^{2}\right)}{I_{y}}  \tag{8-96}\\
& \dot{r}=\frac{N_{B}+\left(I_{x}-I_{y}\right) p q+I_{x z}(\dot{p}-q r)}{I_{z}}
\end{align*}
$$

Where the variables $I_{x}, I_{y}, I_{z}$, and $I_{x z}$ are the moments and products of inertia about the body axes.

### 8.6.21. Block 22 - Body Frame Angular Rates

With the angular accelerations known, it is possible to determine the orientation of the aircraft at this instant. As before, the accelerations must be integrated over the time-step. As before, if the numerical scheme of choice was Euler integration, it would be implemented as shown below

$$
\begin{align*}
& p_{i+1}=p_{i}+\dot{x}_{i+1} \Delta t \\
& q_{i+1}=q_{i}+\dot{y}_{i+1} \Delta t  \tag{8-97}\\
& r_{i+1}=r_{i}+\dot{z}_{i+1} \Delta t
\end{align*}
$$

### 8.6.22. Block 23 - Compute the Quaternions

The quaternions must be updated to keep track of the orientation of the aircraft. To do this, follow this two step process (as shown in Appendix D). First compute the quaternion rates for this iteration using Equation (8-98). The values of the quaternions at this point are the ones computed in the previous iteration.

$$
\begin{align*}
& \lambda=1-\left(e_{0}^{2}+e_{1}^{2}+e_{2}^{2}+e_{3}^{2}\right) \\
& \dot{e}_{0}=-\frac{1}{2}\left(e_{1} p+e_{2} q+e_{3} r\right)+\lambda e_{0} \\
& \dot{e}_{1}=\frac{1}{2}\left(e_{0} p+e_{2} q-e_{3} r\right)+\lambda e_{1}  \tag{8-98}\\
& \dot{e}_{2}=\frac{1}{2}\left(e_{0} p+e_{3} q-e_{1} r\right)+\lambda e_{2} \\
& \dot{e}_{3}=\frac{1}{2}\left(e_{0} p+e_{1} q-e_{2} r\right)+\lambda e_{3}
\end{align*}
$$

Then, the quaternions are obtained using the chosen numerical integration scheme by integration over the time-step to get the current values of the quaternions. If terms of the Euler scheme, this would be accomplished in the following fashion

$$
\begin{align*}
& \left(e_{0}\right)_{i+1}=\left(e_{0}\right)_{i}+\left(\dot{e}_{0}\right)_{i+1} \Delta t \\
& \left(e_{1}\right)_{i+1}=\left(e_{1}\right)_{i}+\left(\dot{e}_{1}\right)_{i+1} \Delta t \\
& \left(e_{2}\right)_{i+1}=\left(e_{2}\right)_{i}+\left(\dot{e}_{2}\right)_{i+1} \Delta t  \tag{8-99}\\
& \left(e_{3}\right)_{i+1}=\left(e_{3}\right)_{i}+\left(\dot{e}_{3}\right)_{i+1} \Delta t
\end{align*}
$$

Where the index $i$ has already been defined.

### 8.6.23. Block 24 - Compute Direction Cosine Matrix

With the quaternions calculated, we now compute the elements of the rotation matrix (DCM) using the expression below

$$
R=\left[\begin{array}{ccc}
e_{0}^{2}+e_{1}^{2}-e_{2}^{2}-e_{3}^{2} & 2\left(e_{1} e_{2}+e_{0} e_{3}\right) & 2\left(e_{1} e_{3}-e_{0} e_{2}\right)  \tag{8-100}\\
2\left(e_{1} e_{2}-e_{0} e_{3}\right) & e_{0}^{2}-e_{1}^{2}+e_{2}^{2}-e_{3}^{2} & 2\left(e_{2} e_{3}+e_{0} e_{1}\right) \\
2\left(e_{1} e_{3}+e_{0} e_{2}\right) & 2\left(e_{2} e_{3}-e_{0} e_{1}\right) & e_{0}^{2}-e_{1}^{2}-e_{2}^{2}+e_{3}^{2}
\end{array}\right]=\left[\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}\right]
$$

Note that this established the current value of the matrix elemens, $a_{i j}$, used in earlier blocks.

### 8.6.24. Block 25 - Compute the Euler Angles

For the next iteration we will need the current values of the Euler angles. Therefore, perform this transformation using Equation (8-101):

$$
\begin{align*}
& \phi=\tan ^{-1}\left(\frac{a_{32}}{a_{33}}\right) \\
& \theta=\sin ^{-1}\left(-a_{31}\right)  \tag{8-101}\\
& \psi=\tan ^{-1}\left(\frac{a_{21}}{a_{11}}\right)
\end{align*}
$$

At this stage, a complete iteration has been completed. The flight simulator can now check if the user has triggered a flag indicating the termination of the code. If not, the code is redirected to Block 1, where the process is repeated. Before entering that point in the code, it is possible to execute various tasks, such as display refresh, which may include pilot instruments and outside world view.

### 8.7 Autopilot Functions

The SURFACES Flight Simulator comes with an AFMS that consists of a number of individual autopilot functions that work either on their own, independently from each other, or in harmony using the AFMS that operates through a program function called the SMARTPILOT. This section details how the numerous autopilot functions are implemented in the simulator. The program allows the user to apply a sophisticated set of autopilot functionality by selecting the options on the Flight Control Console, shown in Figure 8-11.


Figure 8-11: The Flight Control Console allows the flight simulation to be controlled. The AFMS is operated using the buttons inside the A/P MODE (autopilot mode) frame.

The autopilot functionality is provided using classical Proportional-Integral-Derivative (PID) controllers, implemented using the expression below;

$$
\begin{equation*}
\delta_{e}(t)=K_{P} \Delta V(t)+\int_{0}^{t} \Delta V(\tau) d \tau+K_{D} \frac{d}{d t} \Delta V(t) \tag{8-102}
\end{equation*}
$$

Where $K_{P}$ is the proportional constant, $K_{I}$ is the integral constant, and $K_{D}$ is the derivative constant. The implementation of this controller is shown in Figure 8-12. It is precisely the form implemented in the SURFACES Flight Simulator.


Figure 8-12: A flowchart showing the PID controller applied to elevator deflection.

## Airspeed Hold (KCAS HOLD)

This makes the autopilot maintain a specific (calibrated) airspeed that can be entered directly into the textbox below the KCAS HOLD button. The new airspeed is activated once the user presses the enter key. The airspeed can also be selected directly from a popup menu (through right-click), which contains a list of an assortment of optimum speeds (see Figure 8-13). These airspeeds are calculated an instant before the menu is displayed and will depend on the altitude, outside air temperature, weight, and
other factors that pertain directly to the aircraft model being simulated. The button is disabled when the SMARTPILOT is pressed, as it uses the GICA to figure out which airspeed to select. The airspeed hold function uses the elevator to set the airspeed, using the PID expression below

$$
\begin{equation*}
\delta_{e}(t)=K_{P_{e}} \Delta V(t)+K_{I_{e}} \int_{0}^{t} \Delta V(\tau) d \tau+K_{D_{e}} \frac{d}{d t} \Delta V(t) \tag{8-103}
\end{equation*}
$$

Where $\delta_{e}$ is the elevator deflection angle, $K_{P e}, K_{I e}$, and $K_{D e}$ are the proportional, integral, and derivative constants, $V$ is the airspeed, and $t$ is time. The constants play an important role in helping with speed stabilization and facilitate reduction of time between commanded airspeed changes. For instance, the best glide speed of an airplane is a function of the convection in which the airplane operates; head- or tailwind and lift or sink modify this airspeed. However, the dynamics of the airplane, as it passes through random regions of convection, makes sudden airspeed changes impossible to achieve and, if left to own devices, would render the autopilot preoccupied with airspeed changes. A better approach is to use the stochastic expectation of that airspeed sampled over some time interval. This is done in the flight simulator.


Figure 8-13: The AFMS with the airspeed menu displayed.

## Heading Hold (HDG HOLD)

This function makes the autopilot maintain a heading that can be entered directly into the textbox below the HDG HOLD button. The new heading is activated once the user presses the enter key. The autopilot will bank through the shortest angle to the desired heading. It will limit bank angle to the value entered in the (blue) textbox below the label " $\varnothing$ max $\left({ }^{\circ}\right)$ ". The heading can also be selected directly from a popup menu (through right-click), which contains a list of an assortment of optimum speeds (see Figure 8-14).

$$
\begin{equation*}
\delta_{a}(t)=K_{P_{a}} \Delta \psi(t) \quad \text { and } \quad \Delta \psi=\psi(t)-\psi_{\text {target }} \tag{8-104}
\end{equation*}
$$

Where $\delta_{a}$ is the aileron deflection angle, $\psi$ is the heading angle, $\psi_{\text {target }}$ the target heading angle, and $K_{P a}$ is the proportional constant.


Figure 8-14: The AFMS with the heading menu displayed.

## Altitude Hold (ALT HOLD)

The altitude hold makes the autopilot maintain a specific altitude that can be entered directly into the textbox below the ALT HOLD button. The autopilot uses the throttle to maintain altitude using the PID function

$$
\begin{equation*}
\delta_{\mathrm{THR}}(t)=K_{P_{\mathrm{THR}}} \Delta h(t)+K_{I_{\mathrm{THR}}} \int_{0}^{t} \Delta h(\tau) d \tau+K_{D_{\mathrm{THR}}} \frac{d}{d t} \Delta h(t) \tag{8-105}
\end{equation*}
$$

Where $\delta_{\text {THR }}$ is the throttle lever deflection angle, $K_{P T H R}, K_{I T H R}$, and $K_{D T H R}$ are the proportional, integral, and derivative constants, $h$ is the altitude, and $t$ is time.

## Roll Hold (ROLL HOLD)

This function allows the autopilot to maintain a specific bank angle that can be entered directly into the text box below the ROLL HOLD button.

$$
\begin{equation*}
\delta_{a}(t)=K_{P_{a}} \Delta \phi(t) \quad \text { and } \quad \Delta \phi=\phi(t)-\phi_{\text {target }} \tag{8-106}
\end{equation*}
$$

Where $\delta_{a}$ is the aileron deflection angle, $\phi$ is the roll angle, $\phi_{\text {target }}$ the target roll angle, and $K_{P a}$ is the proportional constant.

## Yaw Hold (YAW HOLD)

This makes the autopilot maintain a specific yaw angle that can be entered directly into the text box below the YAW HOLD button. If the value is left as zero (0), the function works like a yaw damper and suppresses nuisance characteristics such as poorly damped dutch roll. The functionality is achieved using the PID function below

$$
\begin{equation*}
\delta_{r}(t)=K_{P_{r}} \Delta \beta(t) \quad \text { and } \quad \Delta \beta=\beta(t)-\beta_{\text {target }} \tag{8-107}
\end{equation*}
$$

Where $\delta_{r}$ is the rudder deflection angle, $\beta$ is the heading angle, $\phi_{\text {target }}$ the target yaw angle, and $K_{P r}$ is the proportional constant.

### 8.8 The Making of Worlds

When comes to the development of the GICA, it is not enough just to make a flight simulator. The making of the world (i.e. topography) in which the airplane operates, is also required. The SURFACES Flight Simulator provides tools to generate rudimentary topography that makes it possible to fly the model above mountainous or flat terrain. This section presents the elements of the methodology. The result of this work is topography of the kind shown in Figure 8-15.


Figure 8-15: A virtual world, $\mathbf{3 0 \times 3 0} \mathbf{~ k m}$ in size, boasting mountain ranges as high as $\mathbf{2 4 0 0} \mathbf{m}(\mathbf{8 0 0 0} \mathrm{ft})$ above S-L and detail some $75 \mathrm{~m}(250 \mathrm{ft})$ across.

The terrain can be created and edited by the user. The topography consists of quadrilateral polygons, simply referred to as panels, which are stored in memory using a structured variable. Each such panel contains the four vertices of the polygon, its centroid, and the color, which depends on the orientation of the panel to the "Sun". If all panel vertices have a zero vertical value, the panel is painted in blue color to represent water. In the current version of the program, all four vertices of a panel are stored to make it possible to display the landscape using algorithms such as those described by Eberly [19] or Lengyel [20].

The panel creation is implemented as follows:

STEP 1: For panel $i=1, j=1$, calculate vertices $x_{1}, y_{1}, z_{1} \ldots x_{4}, y_{4}, z_{4}$.
STEP 2: For panel $i>1, j=1, \quad$ let $z_{1}(i, j)=z_{2}(i-1, j)$
let $z_{3}(i, j)=z_{4}(i-1, j)$
calculate $z_{2}(i, j)$ and $z_{4}(i, j)$
STEP 3: For panel $i>1, j>1, \quad$ let $z_{1}(i, j)=z_{3}(i, j-1)$
let $z_{2}(i, j)=z_{4}(i, j-1)$
calculate $z_{3}(i, j)$ and $z_{4}(i, j)$

The values of the $z$-vertex can be made using some functions or other means. For instance, coupling this algorithm with a NURBS (Non-Uniform Rational B-Splines) surface yielded the topography shown in

Figure 8-17. The mathematics of NURBS is outside the scope of this work, but a tool to generate such surfaces is provided in the SURFACES Flight Simulator.


Figure 8-16: The detail of the topography, showing how terrain and water is generated.


Figure 8-17: The plotting of the panels explained.

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## 9. Energy Harvesting

This chapter develops methods that the GICA path planner uses to command the autopilot when harvesting energy in the environment in which the airplane operates. These methods make it possible for the GICA to quickly assess whether to revise the current trajectory. The complexity of the wind field in which the sUAV operates renders it necessary to break any potential trajectory into elements and, using a finite difference scheme, evaluate the total cost of traversing it. The resulting cost functions are treated in Chapter 10, The Generic Intelligent Control Algorithm; however the methodology by which these assess the cost of traversing a segment is developed here.

In terms of vehicular motion, the expression energy harvesting refers to the use of supplemental energy that is not a part of the original energy stored onboard the vehicle. There are two kinds of energy harvesting; passive and active. Passive energy harvesting is accomplished through the operation of the aircraft and does not require additional systems (outside of an autopilot). Active energy harvesting required additional systems, such as solar panels or controllers associated with piezo-electrics or similar systems.


Figure 9-1: This chapter presents methods the GICA uses in its LiSSA module.

### 9.1 Introduction

The term hypermiling is used in the world of automobiles. It is an example of energy harvesting that is easy to understand. The term was allegedly coined by Mr. Wayne Gerdes [1], an avid automotive fuel economy enthusiast. It is defined as "the practice of making adjustments to a vehicle or using driving techniques that will maximize the vehicle's fuel economy." ${ }^{1}$ It is the art of taking advantage of a car's

[^18]kinetic energy and the application of a number of common-sense fuel saving techniques. An example of such techniques includes a slow acceleration of the vehicle to the desired travel speed, enhanced by the careful use of the car's own momentum. Once the car's speed drops to some minimum speed, the driver accelerates again, slowly, and repeats the process, conserving the car's momentum for as long as possible. Hypermiling involves an assortment of activities that range from parking without the need to reverse and turning off the engine if idling exceeds 7 seconds, to always taking advantage of gravity when moving downhill and careful timing of acceleration-deceleration between traffic lights, always using the momentum to cover as much distance as possible to minimize fuel consumption. It is a disappointing and disheartening consequence of this activity that other drivers, some who are less inclined toward energy conservation, regard it unfavorably. Regardless, the author has experimented with this sort of driving in limited capacity, for instance, by using the car's cruise control when driving on flatlands. Even such rudimentary effort increased the mileage of the author's car from about 29 miles per gallon (mpg) to 31.5 mpg , with 32.5 mpg being the maximum. This constituted about $12 \%$ increase in mileage. However, this is not particularly impressive when compared to Mr. Gerdes' achievements, who in May, 2008, drove 800 miles from Chicago to New York in a Toyota Prius using only 8.9 gallons of fuel [2]. According to Guinness World Records, Mr. Gerdes holds the official world record in low fuel consumption for a car visiting all 48 contiguous US states. Between June $22^{\text {nd }}$ and July $7^{\text {th }} 2015, \mathrm{Mr}$. Gerdes drove some 13498 km ( 8387 miles) in a 2015 VW Golf TDI, with a fuel consumption of 2.89 liters per $100 \mathrm{~km}(81.17 \mathrm{mpg})$ [3]. Passive energy harvesting truly has a great potential.

An example of passive energy harvesting in aviation is an electrically powered sailplane. Such aircraft can be equipped with solar panels to supplement their battery with energy from the Sun, making it capable of staying aloft for extended periods (an active energy harvesting). This possibility is not a flight of the imagination: At the time of this writing, the Solar Impulse 2 has just completed a series of flights around the world on solar power only. The capture of the Sun's radiation in flight and subsequent transformation into electric energy is a compelling example of energy harvesting. However, it is not the only way and it and several other methods are the topic of this chapter. These possibilities include structural oscillations (piezo-electrics), wireless energy transfer using high-energy lasers (e.g. see Summerer and Purcell [4] (2009)), but primarily atmospheric convection (thermals and updrafts). The first two represent areas of intrigue and research and, thus, will only be discussed briefly. However, the atmospheric convection is much better understood and offers practical passive supplemental energy at the disposal of any suitable aircraft. The application of this possibility is twofold; through passive energy harvesting and energy conservation through a strategic operation of the vehicle. Of course, neither is simple nor easy to accomplish.

The operation of aircraft, in particular ones with low wing loading, presents more options for energy harvesting than cars. In aircraft, this is accomplished through conservation, replenishing, and harvesting. Conservation is the act of limiting the use of energy through a strategic operation of the vehicle, for instance, by resorting to soaring rather than powered flight when possible. Thus, energy conservation is the strategic operation of a vehicle such that it consumes less energy than it would otherwise. Energy conservation does not increase the onboard energy, but rather reduces the rate of its consumption. Replenishing is the capture, conversion, and accumulation of energy present in the environment in
which the vehicle operates. For instance, if powered by an electric motor, solar energy can be captured, converted to electric energy, and stored in the onboard battery. Thus, replenishing is the collection of one form of energy to be converted later into another form of energy. Harvesting is the use of energy that is not readily converted to (in this case) battery energy. This includes the increase of the potential energy of the vehicle through the updrafts. The potential energy can readily be converted to kinetic energy. Thus, energy harvesting is the act of increasing the energy state of the vehicle without converting it into another form of energy. This chapter focuses on the following means to harness energy:

- Energy conservation through performance prediction (Section 9.2)
- Energy harvesting through thermals (Section 9.3)
- Energy harvesting through lift seeking (Section 9.3)
- Energy conservation through sink avoidance (Section 9.3)
- Energy replenishing using solar, piezo-electric, and wireless laser energy (Section 9.4)


### 9.1.1. Classification of Time-Priority and Energy-Consumption Priority Missions

The term mission is defined in Chapter 10, The Generic Intelligent Control Algorithm, as a list of waypoints a vehicle is intended to follow. In this context, it is helpful to classify missions as:

## Class 1: A mission for which departure and arrival time is of primary priority. <br> Class 2: A mission for which departure and arrival time is of secondary importance.

Time is always a primary objective in commercial transportation of passengers. Not just because of the passengers, but also because being on time is a pre-requisite for effective management of the operational costs associated with transportation vehicles. However, time is not always the primary objective for many missions. In particular, consider a reconnaissance mission, such as forest fire surveillance, for which endurance is limited only by the onboard energy. In such operations, the mission might be modified en-route based on surveillance observations. For instance, if a region of forest fire is discovered, and assuming onboard energy stores permit, the operator might change the current flight plan to investigate said region, perhaps obtaining the geographic location and size. This "unforeseen" activity would be challenging to change were time to play a primary role in the completion of said mission. In this fashion, a transportation paradigm independent of a specific arrival time allows for energy available in the atmosphere to be utilized more extensively than one for which arrival time is a priority. This work focuses on Class 2 missions. The goal is an efficient trip; a trip in which onboard energy is carefully consumed and, ideally, supplemented by energy available in the environment. This does not mean that arrival time is not important; after all, we desire to eventually arrive at our destination, preferably as quickly as possible. But rather, arrival with minimum energy cost is of greater importance.

### 9.1.2. Evolutionary Solutions for Energy Harvesting

It is of interest to consider how Nature solves the energy harvesting problem for birds. While it is a stretch to state the flight of birds exemplifies the execution of a predefined mission, it can be argued that if it were, it would be a Class 2 mission. Anyone who has carefully observed birds of prey, such as Albatrosses or Vultures, can attest to they hardly follow a strict time schedule. The Vulture glides effortlessly for long periods until a carcass is discovered. The Albatross utilizes dynamic soaring to cover long distances over the south Pacific, until coming across a squid near the surface. Both bird species take advantage of convection in the atmosphere, conserving own energy. But both species also present two different solutions to the problem of energy harvesting, for why does the Albatross have a high aspect ratio wing and wing loading, while the Vulture has low aspect ratio and wing loading? How could evolution have come to such two different solutions for flight that involves long endurance flying with minimum energy consumption? Our knowledge of aircraft performance theory, presented in Chapter 6, Aircraft Performance Theory, gives us an important insight into the answer.

Various energy harvesting approaches have been studied experimentally in aircraft. The best known and most practical of these are gliding flight. Sailplanes are capable of achieving great range and altitudes when their operators find updrafts in the form of thermals or slope lift. Chapter 1, Introduction cited several examples of how birds and insects have mastered taking advantage of energy available in the atmosphere. However, there are other ways besides thermals of supplementing the energy of powered aircraft of light wing loading. Among those is careful use of airspeeds (which is crucial in conserving energy), energy replenishing (e.g. using solar cells and piezo-electrics), and strategic search for lift and avoidance of sink.

### 9.2 Energy Conservation through Performance Optimization

This section presents how it is possible to conserve substantial amount of energy through performance optimization. Before reading this section, the reader is advised to review the methodologies presented in Chapter 6, Aircraft Performance Theory, in particular that of sailplane performance. It reveals that, in an airplane, there is more than one way to travel from point $A$ to $B$ and it all depends on the airspeed selected. Thus, in order to conserve the energy stored onboard, the problem boils down to the determination of an optimum airspeed for selected maneuvers. Methods for this purpose are presented in Section 6.4.2, Cruise. There is a caveat; optimum airspeeds also depend on wind conditions. They are affected by head- or tailwinds and lift or sink. Powerful techniques to correct for atmospheric convection of this sort are presented in Section 6.5, Sailplane Performance Theory. However, there is a stipulation: (1) Since the airspeed for minimum power required is usually a low value compared to the aircraft's maximum level airspeed, it will inevitably take longer to complete the mission than otherwise, and (2), for the same reason, the airplane will have less tolerance for headwind and this inflicts an additional operational limitation. There is not much to do about this - this is the cost of energy conservation. Luckily, not every day brings excessive winds.

Consider a straight line mission between points $\mathbf{A}$ and $\mathbf{B}$ (see Figure 9-2). Clearly, the most efficient trip is one in which no onboard energy is consumed. While a trip may require some consumption of stored
energy, e.g. to reach an initial cruise altitude, it may be possible to complete the trip using only that initial burst of energy. For instance, in hilly landscape, orographic (slope) lift may be used to maintain or gain altitude, allowing the destination to be reached in gliding flight. In that case, the mission profile might consist of a climb followed by subsequent glide, as will be discussed shortly.


Figure 9-2: Mission from Point A to Point B.
A more likely travel profile in an aircraft is that shown in Figure 9-3. It illustrates important facts about most missions; segmenting, consisting of a dedicated climb, cruise, and descent. The question we must ask is; what is the most energy efficient way of conducting the flight in each of these segments? Again, the short answer is that it depends on the airspeed and the atmospheric convection present. A more practical answer warrants a brief review of how to determine the amount of energy required to complete each.


Figure 9-3: A more practical mission profile between points $A$ and $B$.

### 9.2.1. Notes on Power Required

If the drag polar of an aircraft is known, it is possible to determine the amount of power its engine(s) must produce in order to maintain any part of the flight. The first step is to identify the airspeed and altitude at which the airplane is operated, as well as its weight, $W$. These are used to calculate the lift coefficient at the flight condition. It, in turn, is used to calculate the drag coefficient at the flight condition. This information allows the power required for level flight to be estimated. It is shown in Reference [5] that if the simplified drag model of Section 6.2.3, The Drag Coefficient is assumed, the power required, $P_{\text {REQ }}$, can be estimated as follows:

$$
\begin{equation*}
P_{R E Q}=T_{R E Q} V=D_{R E Q} V=\sqrt{\frac{2 W^{3} C_{D}^{2}}{\rho S C_{L}^{3}}} \tag{6-77}
\end{equation*}
$$

Where $W$ is the weight of the airplane, $T_{R E Q}$ is the thrust required to maintain altitude at the given airspeed $V, C_{D}$ and $C_{L}$ are the drag and lift coefficient, respectively, at $V, \rho$ is density of air, and $S$ is reference area. Note that here, distance is denoted using lower case' $s$ ', whereas capital ' $S$ ' denotes wing area. The same reference also shows that the thrust of a propeller powered aircraft is given by

Thrust if power is in BHP: $\quad T=\frac{\eta_{p} 550 P_{B H P}}{V}$

Where $P_{B H P}$ is power in brake-horsepower and $\eta_{p}$ is the propeller efficiency at the given airspeed (in $\mathrm{ft} / \mathrm{s}$ ). This means that the power supplied by the engine (or motor) will not be completely converted to thrust-power ( $T V$ ), but rather will be reduced by the factor $\eta_{p}$. Recall that $\eta_{p}$ is a function of airspeed (and RPM and propeller pitch angle) and varies from 0 to 0.9 , or so, as explained in Section 6.3, Fundamentals of Propeller Thrust. Brake horsepower is the standard unit for piston engines, although they are also rated in SI-units using Watts. The advantage of estimating thrust using power as Watts and airspeed in $\mathrm{ft} / \mathrm{s}$ is that we can compare piston and electric power plants directly. If the propeller is driven using power specified in watts, $P_{W}$, then Equation (9-1) transforms to

Thrust if power is in Watts:

$$
\begin{equation*}
T=\frac{\eta_{p} 0.7376 P_{W}}{V}=\frac{\eta_{p} 0.7376 \mathrm{IE}}{V} \tag{9-2}
\end{equation*}
$$

Where $I$ is current in amps and $E$ is the potential (voltage) in volts, and $V$ is in $\mathrm{ft} / \mathrm{s}$. Now, consider a scenario in which the airplane is operating in level horizontal flight at some steady airspeed. We want to be able to estimate the electric current required to maintain that airspeed, given some voltage (which we assume stays relatively constant). Thus we equate

$$
\begin{equation*}
P_{R E Q}=T_{R E Q} V \Leftrightarrow\left(\frac{\eta_{p} 0.7376 I E}{V}\right) V=\eta_{p} 0.7376 I E=\sqrt{\frac{2 W^{3} C_{D}^{2}}{\rho S C_{L}^{3}}} \tag{9-3}
\end{equation*}
$$

Manipulating leads to

$$
\begin{equation*}
I=\frac{1}{\eta_{p} 0.7376 E} \sqrt{\frac{2 W^{3} C_{D}^{2}}{\rho S C_{L}^{3}}} \tag{9-4}
\end{equation*}
$$

The current can also be represented in the following form:

$$
\begin{equation*}
I=\frac{1}{\eta_{p} 0.7376 E} \sqrt{\frac{2 W}{\rho S C_{L}} \frac{W^{2} C_{D}^{2}}{C_{L}^{2}}} \Rightarrow I=\frac{V W}{\eta_{p} 0.7376 E}\left(\frac{C_{D}}{C_{L}}\right) \tag{9-5}
\end{equation*}
$$

### 9.2.2. Notes on Engine Energy Consumption

If we could formulate the complete time history of the flight path of a vehicle in the 3-dimensional space in which it operates, for instance using parametric formulation of time, it would be possible to document its consumption of onboard energy. If we can write the path as follows

$$
\begin{equation*}
\mathbf{r}(x, y, z, t)=x(t) \mathbf{i}+y(t) \mathbf{j}+z(t) \mathbf{k} \tag{9-6}
\end{equation*}
$$

It follows the velocity is given by

$$
\begin{equation*}
\mathbf{v}(x, y, z, t)=\dot{x}(t) \mathbf{i}+\dot{y}(t) \mathbf{j}+\dot{z}(t) \mathbf{k} \tag{9-7}
\end{equation*}
$$

The acceleration is

$$
\begin{equation*}
\mathbf{a}(x, y, z, t)=\ddot{x}(t) \mathbf{i}+\ddot{y}(t) \mathbf{j}+\ddot{z}(t) \mathbf{k} \tag{9-8}
\end{equation*}
$$

With the acceleration known, we can determine the net force acting on the aircraft

$$
\begin{equation*}
\mathbf{F}(t)=m(t) \mathbf{a}(x, y, z, t)=m(t)[\ddot{x}(t) \mathbf{i}+\ddot{y}(t) \mathbf{j}+\ddot{z}(t) \mathbf{k}] \tag{9-9}
\end{equation*}
$$

This force is a combination of aerodynamic, mechanical, and gravitational forces, i.e.

$$
\begin{equation*}
\mathbf{F}(t)=\mathbf{F}_{\mathbf{A}}(t)+\mathbf{F}_{\mathbf{M}}(t)+\mathbf{F}_{\mathbf{g}}(t) \Leftrightarrow \mathbf{F}_{\mathbf{M}}(t)=\mathbf{F}(t)-\mathbf{F}_{\mathbf{A}}(t)-\mathbf{F}_{\mathbf{g}}(t) \tag{9-10}
\end{equation*}
$$

We are interested in the mechanical force, because energy consumption is directly dependent on its use, i.e. the application of mechanical power, $\mathbf{P}$ :

$$
\begin{equation*}
\mathbf{P}(t)=\mathbf{F}_{\mathbf{M}}(t) \cdot \mathbf{v}(t) \Rightarrow \dot{E} \propto|\mathbf{P}(t)| \tag{9-11}
\end{equation*}
$$

This, of course, means the time history of the application of mechanical power is a direct measure of the amount of mechanical energy consumed. Traditionally, we use the mass (SI-system) or weight (UKsystem) of the consumed fuel as an indicator of this energy consumption. If considering piston engine power, we apply the following approximation, using the concept of Specific Fuel Consumption (SFC), specified in terms of kg of fuel per kW per hour ( $\mathrm{kg} /(\mathrm{kW} \cdot \mathrm{hr})$ ) in the SI-system or $\mathrm{lb}_{\mathrm{f}}$ of fuel per BHP per hour ( $\left(\mathrm{b}_{\mathrm{f}} /(\mathrm{BHP}-h r)\right.$ ) using the UK-system. Thus, if we know the time history of the use of engine power we can write

SI-system:

$$
\begin{align*}
& m_{\text {fuel }}=\int \dot{m} d t=\int_{0}^{t} P_{k W}(t) \cdot S F C_{k W}(t) d t  \tag{9-12}\\
& w_{\text {fuel }}=\int \dot{w} d t=\int_{0}^{t} P_{B H P}(t) \cdot S F C_{B H P}(t) d t \tag{9-13}
\end{align*}
$$

The sheer complexity of the time history renders it impractical to solve such integral analytically, although flight simulators keep track of energy consumption by conducting numerical integration. Luckily, we do not have to solve the above integrals directly. Instead, we write them in terms of
idealized operation of the aircraft in each of the three segments of flight, mentioned earlier (i.e. climb, cruise, descent) and use these to evaluate the energy cost associated with flying from point $\mathbf{A}$ to $\mathbf{B}$.

### 9.2.3. Energy Consumption during Climb

Consider the climb segment in Figure 9-3, which starts at altitude $h_{0}$, is completed at altitude $h_{1}$, and is conducted at constant calibrated airspeed $V_{\text {CAS }}$ throughout the maneuver. Then, let's use Equations (663 ) and (6-64) to evaluate the time, $t_{c}$, to complete the segment

$$
\begin{equation*}
t_{c}=\frac{1}{\dot{h}_{a}} \int_{h_{0}}^{h_{1}} d h=\frac{2\left(h_{1}-h_{0}\right)}{\dot{h}_{0}+\dot{h}_{1}} \tag{9-14}
\end{equation*}
$$

Where the $R O C$ (or $\dot{h}$ ) at either altitude is determined using Equation (6-48). Thus, the energy consumed during the climb segment is obtained from Equations (9-12) and (9-13) for a piston engine and an electric motor as follows:

Piston engine, in kW

$$
\begin{align*}
& m_{\text {fuel }}=\int_{0}^{t_{c}} P_{k W}(t) \cdot S F C_{k W}(t) d t=\frac{2\left(h_{1}-h_{0}\right) \cdot P_{k W} \cdot S F C_{k W}}{\dot{h}_{0}+\dot{h}_{1}}  \tag{9-15}\\
& w_{\text {fuel }}=\int_{0}^{t_{c}} P_{B H P}(t) \cdot S F C_{B H P}(t) d t=\frac{2\left(h_{1}-h_{0}\right) \cdot P_{B H P} \cdot S F C_{B H P}}{\dot{h}_{0}+\dot{h}_{1}} \tag{9-16}
\end{align*}
$$

Even though the Specific Fuel Consumption (SFC) is a function of time (as well as a function of power) it can often be represented as a constant value. A typical value for piston engines is $0.5-0.6 \mathrm{lb}_{\mathrm{f}} /(\mathrm{BHP} \cdot \mathrm{hr})$. In order to utilize these equations, the first step is to establish power during the segment. Next, determine the $R O C$ at the initial and final altitudes and, then, the amount of energy (or its substitute values in terms of kg or $\mathrm{lb}_{\mathrm{f}}$ ).

For an electric motor, the equation must return the battery capacity consumed in Amp-hours. This is accomplished as follows. First recall that the voltage of a battery varies with the capacity remaining (see Section 7.4.3, Discharge Curves). For this reason, its power output, $P=I E$ (in W) varies (reduces) with time - the only way to maintain constant power is to increase the current draw as a function of time. Of course, this is only possible as long as the battery is being discharged below its maximum discharge rate. As far as the airplane performance is concerned, this means that the ROC will decrease with time. However, let's assume the climb segment lasts a relatively short time, during which the power more or less remains constant. In that case, we can write the battery capacity consumed as show below:

$$
\begin{equation*}
P=I E \quad \Leftrightarrow \quad I=\frac{P}{E} \Rightarrow C_{c}=I t_{c}=\left(\frac{P}{E}\right) \frac{2\left(h_{1}-h_{0}\right)}{\dot{h}_{0}+\dot{h}_{1}} \tag{9-17}
\end{equation*}
$$

Where, again, the subscript ' $c$ ' refers to the climb segment in Figure 9-3. When the duration of the segment involving power usage is long, it is necessary to take into account the reduction in power and this introduces added complexity to the analysis. To do this, it is helpful to resort to Tremblay's method
(see Section 7.4.3, Discharge Curves) to estimate the battery voltage as a function of remaining capacity allows the following polynomial approximation to be made

$$
\begin{equation*}
E(C)=a_{0}+a_{1} C+a_{2} C^{2}+a_{3} C^{3}+a_{4} C^{4}+a_{5} C^{5} \tag{9-18}
\end{equation*}
$$

From the definition of the capacity we get $\quad C=\int I d t \Rightarrow I(t)=\frac{d C}{d t}=\frac{P(t)}{E(t)}$

At any instant $t$, the voltage obtained by the time history must be equal to the voltage obtained from the capacity consumed. In other words, we must have

$$
E(t)=E(C)=a_{0}+a_{1} C+a_{2} C^{2}+a_{3} C^{3}+a_{4} C^{4}+a_{5} C^{5}
$$

Thus, if we know the time history of the power usage, we can relate it and the remaining battery capacity

$$
P(t)=\frac{d C}{d t} E(t) \quad \Rightarrow \quad P(t)=\frac{d C}{d t}\left(a_{0}+a_{1} C+a_{2} C^{2}+a_{3} C^{3}+a_{4} C^{4}+a_{5} C^{5}\right)
$$

By separation of variables we may write

$$
\int P(t) d t=\int\left(a_{0}+a_{1} C+a_{2} C^{2}+a_{3} C^{3}+a_{4} C^{4}+a_{5} C^{5}\right) d C
$$

Whose solution is given by

$$
\begin{equation*}
\int P(t) d t=a_{0} C+\frac{1}{2} a_{1} C^{2}+\frac{1}{3} a_{2} C^{3}+\frac{1}{4} a_{3} C^{4}+\frac{1}{5} a_{4} C^{5}+\frac{1}{6} a_{5} C^{6} \tag{9-19}
\end{equation*}
$$

### 9.2.4. Energy Consumption in Cruise

Consider the cruise segment in Figure 9-3 conducted at altitude $h_{1}$ at constant calibrated airspeed $V_{\text {CAS }}$ throughout the maneuver. If we know the distance covered is $s_{d}$ and the horizontal wind during the maneuver is given by $V_{w}\left(>0\right.$ if tailwind, $<0$ if headwind) then the duration of the cruise $t_{d}$ is found from

$$
\begin{equation*}
t_{d}=\frac{s_{d}}{V_{C A S} / \sqrt{\sigma}+V_{W}} \tag{9-20}
\end{equation*}
$$

Where the subscript ' $d$ ' refers to the cruise segment in Figure $9-3, \sigma$ it the density ratio, so the radical will convert the calibrated airspeed to true airspeed. Note that it requires vector analysis to determine the component of the horizontal wind parallel to the velocity vector (and denoted by $V_{w}$ ). Once the time
for cruise is known, the energy consumption during the segment is again obtained from Equations (9-12) and (9-13) for a piston engine and an electric motor as follows:

Piston engine, in kW

$$
\begin{equation*}
m_{f u e l}=\int_{0}^{t_{d}} P_{k W}(t) \cdot S F C_{k W}(t) d t=\frac{s_{d} \cdot P_{k W} \cdot S F C_{k W}}{V_{C A S} / \sqrt{\sigma}+V_{W}} \tag{9-21}
\end{equation*}
$$

Piston engine, in BHP

$$
\begin{equation*}
w_{\text {fuel }}=\int_{0}^{t_{d}} P_{B H P}(t) \cdot S F C_{B H P}(t) d t=\frac{s_{d} \cdot P_{B H P} \cdot S F C_{B H P}}{V_{C A S} / \sqrt{\sigma}+V_{W}} \tag{9-22}
\end{equation*}
$$

For electroprops, the battery capacity consumed, $C_{d}$ (do not confuse with an airfoil's drag coefficient), is found from

$$
\begin{equation*}
P=I E \quad \Leftrightarrow \quad I=\frac{P}{E} \quad \Rightarrow \quad C_{d}=I t_{d}=\frac{P}{E}\left(\frac{s_{d}}{V_{C A S} / \sqrt{\sigma}+V_{W}}\right) \tag{9-23}
\end{equation*}
$$

### 9.2.5. Energy Consumption during Descent

Power consumption during descent is generally low; however, this does not preclude it from having to be accounted for. Conservation of fuel through extended periods of in-flight piston-engine shutdown can invite engine restarting problems and, for many engines, are not recommended. This requires affected engines to be running through-out the duration of the mission. The associated fuel consumptions during descent is estimated using Equation (9-15) or (9-16). Different story holds for an electrically powered airplane; the motor could reliably be shut off during descent. However, even with the motor shut off, there is still power draw to run onboard systems and this must be accounted for. As an example, the Quanum Observer mentioned in Chapter 3 and in this chapter, draws some 0.45 Amps for its flight controller and video camera and feed operation.

### 9.3 The Basics of the Climb-Glide Cruise Profile

In this section we will consider an unorthodox range profile, especially designed with electric sailplane configurations in mind. Consider the previous path planning problem in which we intend to fly from point A to point Busing a climb-glide profile like the one illustrated in Figure 9-4. For instance, we wish to begin at point $\mathbf{A}$ by climbing to a particular altitude, $h$, and then turn off the engine and glide the remaining distance to point $\mathbf{B}$. Thus, the climb segment will cover some portion of the total distance and the remainder will be covered in glide. The questions of interest are, how high should we climb to cover some specific distance? Once the airplane has climbed to altitude $h$, how much distance will remain to point $\mathbf{B}$ ? What is the time to travel? How much fuel is consumed?

To address this problem, we have to make some simplifying assumptions. The first one involves the time taken to change between flight modes (or airspeeds). It is vital to keep in mind that airplanes do not stabilize instantaneously when their airspeed changes from one to another. For instance, switching from
the best rate-of-climb airspeed to the minimum rate-of-descent airspeed might take, say, 10-30 seconds before the new airspeed is "stable." For this reason it is impractical to immediately change airspeed when going through intermittent regions of lift or sink that are relative small across. If the autopilot was commanded to switch rapidly from one speed to the next, rapid and continuous accelerationdecelerations would ensue due to the vehicles innate Phugoid mode ${ }^{2}$. In such situations, it is better to select a single airspeed, sort of like a best "speed to fly" as common in the operation of sailplanes. At any rate, such transients are ignored in the discussion below.


Figure 9-4: Basic climb-glide cruise profile from Point A to Point B.
This is justified on the basis that the time required to change from one airspeed to another is negligible in comparison to the total time of travel between the ground positions $\mathbf{A}$ and $\mathbf{B}$, so it will be ignored. Further, it is assumed the airspeed maintained during the climb ( $V_{c}$ ) and glide ( $V_{e}$ ) segments are constant and may or may not be equal. Most of the time these are different airspeeds and we assume we can change between the two instantly without exciting the Phugoid mode. Also note that while the subscripts used here are those of the best rate of climb (minimum time to altitude) and best glide (longest distance covered from altitude), any airspeed within the speed range of the airplane can be selected. However, once selected, they have to be maintained over the duration of the segment. It is further assumed the weight of the airplane, $W$, changes negligibly during the climb segment.

### 9.3.1. Climb Segment

The time to climb to altitude $h$ is $t_{c}$ and time to glide to $\mathbf{B}$ is given by $t_{e}$. From Equations (6-108) and (6109), $L=W \cos \theta$ and $D=W \sin \theta$. With this in mind and per Figure 9-4, the distance covered during climb, $s_{c}$, can be written as

[^19]\[

$$
\begin{equation*}
s_{c}=V_{c} \cos \theta_{c} t_{c}=V_{c} \cos \theta_{c}\left(\frac{h}{\dot{h}}\right)=V_{c} \cos \theta_{c}\left(h \frac{W}{P_{A}-P_{R}}\right) \tag{9-24}
\end{equation*}
$$

\]

Where $V_{c}$ is the airspeed of the aircraft, $\theta_{c}$ is the climb angle, $t_{c}$ time it takes to reach altitude $h, R O C$ is the rate of climb, and $P_{A}$ and $P_{R}$ are the available and required power, respectively. Substituting the definitions of $P_{A}$ and $P_{R}$ and manipulating leads to

$$
s_{c}=V_{c} \cos \theta_{c}\left(\frac{W}{P_{A}-P_{R}}\right) h=\frac{W V_{c} \cos \theta_{c}}{T V_{c}-D V_{c}} h
$$

Further manipulations lead to

$$
s_{c}=\frac{W \cos \theta_{c}}{T-D} h=\frac{L}{T-D} h=\frac{q S C_{L_{c}}}{q S C_{T}-q S C_{D_{c}}} h
$$

Where the lift and drag coefficients during climb (at airspeed $V_{c}$ ) are denoted by $C_{L c}$ and $C_{D c}$. Therefore, the climb distance is

$$
\begin{equation*}
s_{c}=\frac{C_{L_{c}}}{C_{T}-C_{D_{c}}} h \tag{9-25}
\end{equation*}
$$

Where $C_{T}$ is the thrust coefficient during the climb segment and is calculated for a piston engine using:

$$
\begin{equation*}
C_{T} \equiv \frac{T}{q S}=\frac{\eta_{p} 550 P_{B H P}}{q S V}=\frac{\eta_{p} 550 P_{B H P}}{\frac{1}{2} \rho V^{3} S}=\frac{1100 \eta_{p} P_{B H P}}{\rho V^{3} S} \tag{9-26}
\end{equation*}
$$

Where $P_{B H P}$ is the power in BHP. In case of an electric engine, where the power is given in kilowatts $(\mathrm{kW})$, the thrust coefficient is given by

$$
\begin{equation*}
C_{T} \equiv \frac{T}{q S}=\frac{\eta_{p} 550 P_{B H P}}{q S V}=\frac{\eta_{p} 550\left(1.341022 \cdot P_{k W}\right)}{\frac{1}{2} \rho V^{3} S}=\frac{1475 \eta_{p} P_{k W}}{\rho V^{3} S} \tag{9-27}
\end{equation*}
$$

If the power is given in watts, rather than kW , as is common for UAVs, Equation (9-27) is rewritten as follows:

$$
\begin{equation*}
C_{T} \equiv \frac{1.475 \eta_{p} P_{W}}{\rho V^{3} S} \tag{9-28}
\end{equation*}
$$

The climb angle can be found from

$$
\begin{equation*}
\theta_{c}=\sin ^{-1}\left(\frac{\dot{h}}{V_{c}}\right)=\sin ^{-1}\left(\frac{P_{A}-P_{R}}{W V_{c}}\right) \tag{9-29}
\end{equation*}
$$

The time to cover the climb distance $s_{c}$ is given by:

$$
\begin{equation*}
t_{c}=\frac{s_{c}}{V_{c} \cos \theta_{c}}=\frac{h}{\dot{h}} \tag{9-30}
\end{equation*}
$$

## Glide Segment

The descent segment is analyzed in a similar manner. The distance covered during glide is given by

$$
\begin{equation*}
s_{e}=V_{e} \cos \theta_{e} t_{e}=\left(\frac{L}{D}\right) h=\left(\frac{C_{L_{e}}}{C_{D_{e}}}\right) h \tag{9-31}
\end{equation*}
$$

Where $V_{e}$ is the glide airspeed of the aircraft (parallel to the descent vector), $\theta_{e}$ is the glide angle, $t_{e}$ is time it takes to reach $\mathbf{B}$ from altitude $h$, and $C_{L e}$ and $C_{D e}$ are the lift and drag coefficients during descent (at airspeed $V_{e}$ ), respectively. Additionally, the time to cover the glide distance $s_{e}$ is given by:

$$
\begin{equation*}
t_{e}=\frac{s_{e}}{V_{e}}=\frac{C_{L_{e}}}{C_{D_{e}}} \frac{h}{V_{e}} \tag{9-32}
\end{equation*}
$$

## Combined Climb and Glide Segments

The complete climb-glide segment distance is thus expressed rather elegantly as follows

$$
\begin{equation*}
s=s_{c}+s_{e}=\frac{C_{L_{c}}}{C_{T}-C_{D_{c}}} h+\left(\frac{C_{L_{e}}}{C_{D_{e}}}\right) h=\left[\frac{C_{L_{c}}}{C_{T}-C_{D_{c}}}+\frac{C_{L_{e}}}{C_{D_{e}}}\right] h \tag{9-33}
\end{equation*}
$$

And the total flight time is

$$
\begin{equation*}
t_{T O T}=t_{c}+t_{e}=\left(\frac{1}{\dot{h}}+\frac{C_{L_{e}}}{C_{D_{e}}} \frac{1}{V_{e}}\right) h \tag{9-34}
\end{equation*}
$$

Therefore, the trip airspeed is $V_{\text {trip }}=\frac{s}{t_{T O T}}=\frac{\frac{C_{L_{c}}}{C_{T}-C_{D_{e}}}+\frac{C_{L_{e}}}{C_{D_{e}}}}{\frac{1}{\dot{h}}+\frac{C_{L_{e}}}{C_{D_{e}}} \frac{1}{V_{e}}}$
The expression does not really simplify algebraically to a more suitable form. It should be considered akin to an average airspeed for the profile. The optimal trip airspeed is when the airplane climbs at its best rate-of-climb airspeed $\left(V_{y}\right)$ and descends at its best glide speed $\left(V_{B G}\right)$. It is also of interest to ask what minimum altitude the vehicle has to climb to in order to cover a given distance. This is easy to answer by solving Equation (9-33) for the altitude:

$$
\begin{equation*}
h=\frac{s}{\left[\frac{C_{L_{c}}}{C_{T}-C_{D_{c}}}+\frac{C_{L_{e}}}{C_{D_{e}}}\right]} \tag{9-36}
\end{equation*}
$$

Of particular interest is the distance associated with the best glide ratio, $L D_{\text {max }}$, for which $V_{e}$ must be the best glide speed. In this case, Equations (9-33), (9-35), and (9-36) can be written as

$$
\begin{align*}
& s=\left[\frac{C_{L_{c}}}{C_{T}-C_{D_{c}}}+L D_{\max }\right] h  \tag{9-37}\\
& V_{\text {trip }}=\frac{\frac{C_{L_{c}}}{C_{T}-C_{D_{c}}}+L D_{\max }}{\frac{1}{\dot{h}_{\max }}+\frac{L D_{\max }}{V_{e}}}  \tag{9-38}\\
& h=\frac{s}{\left[\frac{C_{L_{c}}}{C_{T}-C_{D_{c}}}+L D_{\max }\right]} \tag{9-39}
\end{align*}
$$

As an example of how these expressions are used, consider the following worked example.

## Worked Example 1:

The aircraft used here is the Quanum Observer, a popular radio controlled aircraft used for FPV (First-Person-View) flying. It is chosen for the examples in this chapter to permit comparison with an existing aircraft. It is an electroprop, powered by a 3-cell LiPo battery ( 12.4 V max voltage) and at max power draws 22.5 Amps (12.0V x $22.5 \mathrm{~A}=270$ Watts). It swings a 10 -inch diameter propeller and has the following properties:

| Weight | $W=2.82 \mathrm{~kg}=6.22 \mathrm{lb}_{\mathrm{f}}$ |
| :--- | :--- |
| Wing span | $b=2.00 \mathrm{~m}=6.56 \mathrm{ft}$ |
| Wing Mean Geometric Chord | $C_{M G C}=0.240 \mathrm{~m}=0.787 \mathrm{ft}$ |
| Wing area | $S=0.480 \mathrm{~m}^{2}=5.167 \mathrm{ft}^{2}$ |
| Wing Aspect Ratio | $A R=b^{2} / S=8.333$ |
| Oswald's span efficiency | $e=1.78\left(1-0.045 A R^{0.68}\right)-0.64=0.8013$ |
| Lift induced drag constant | $k=1 /(\pi A R e)-0.64=0.04767$ |
| Minimum drag coefficient | $C_{D \min }=0.05040$ |
| Lift coefficient of $C_{D \min }$ | $C_{L \operatorname{minD}}=0.0$ |
| Propeller efficiency at $V_{y}$ | $\eta_{p}=0.35$ |

Assume density is invariant of the low altitude and is 0.002378 slugs $/ \mathrm{ft}^{3}$. Note that this problem is intended to highlight various aspects of the climb-cruise profile and, thus, makes several simplifications. In practice, effects of non-linearities in power delivery and high AOA attitude of the airplane are ignored. These are handled automatically in the flight simulation tool.
(a) Estimate the climb distance and time to climb from S-L to 1000 ft at $V_{y}$.
(b) Estimate the glides distance and time to glide from 1000 ft to $\mathrm{S}-\mathrm{L}$ at $V_{B G}$.
(c) Estimate the total distance and time covered in (a) and (b).
(d) Estimate the total battery capacity consumed, assuming the current and voltage used above is constant during the climb segment.
(e) Estimate battery capacity consumed per unit distance.
(f) Determine the current $I$ required to maintain level flight at $V_{B G}$, assuming a constant 12.0 V potential and a propeller efficiency of 0.35 at condition.
$(\mathrm{g})$ Estimate what altitude the aircraft must climb to in order to complete a 10 nm mission. How long will it take to climb to that altitude and how much energy will be consumed?


Figure 9-5: The Quanum Observer, a long range FPV aircraft.

## Solution:

(a) First note that since $C_{L \operatorname{minD}}=0$ the simplified drag model can be used. Thus, the drag polar can be written as follows:

$$
C_{D}=C_{D \min }+k C_{L}^{2}=0.05040+0.04767 C_{L}^{2}
$$

Let's use this to estimate the best climb and glide characteristics of the Observer. First, determine the maximum lift-to-drag ratio using Equation (6-79):

$$
L D_{\max }=\frac{1}{\sqrt{4 k C_{D \min }}}=\frac{1}{\sqrt{4(0.04767)(0.05040)}}=10.20
$$

Determine the airspeed for the best $R O C$ for the airplane using Equation (6-78):

$$
V_{y}=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{3 \cdot C_{D \min }}}}=\sqrt{\frac{2}{0.002378}\left(\frac{6.22}{5.167}\right) \sqrt{\frac{0.04767}{3 \times 0.05040}}}=23.8 \mathrm{ft} / \mathrm{s} \quad(14.1 \mathrm{KCAS})
$$

This value is erroneous because the resulting $C_{L}$ is too high (1.781) due to assumptions made about changes in available power with airspeed. A more refined analysis using Equation (6-51) yields $32.0 \mathrm{ft} / \mathrm{s}$ (19.0 KCAS) with $C_{L}=0.9883$ and this value will be used here. Using this value, estimate the $R O C$ at $V_{y}$ using Equation (6-61) assuming the LiPo battery delivers constant 270 Watts of power during the climb phase. Let's convert the power to equivalent horsepower and then to $\mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s}$.

$$
P=\frac{P_{\text {watts }}}{745.7 \mathrm{~W} / \mathrm{hp}} \times 550=\frac{270 \mathrm{~W}}{745.7 \mathrm{~W} / \mathrm{hp}} \times 550=0.362 \times 550=199.1 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s}
$$

Therefore, the resulting maximum $R O C$ is

$$
\dot{h}_{\max }=R O C_{\max }=60\left(\frac{\eta_{p} P}{W}-V_{Y} \frac{1.1547}{L D_{\max }}\right)=60\left(\frac{(0.35)(199.1)}{6.22}-(32.0) \frac{1.1547}{10.20}\right)=455 \mathrm{fpm}
$$

This means it will take a little over two minutes to climb to 1000 ft . The author has observed similar values for the ROC in practice and, thus, argues these calculated results are supported by experiment, at least for the first few minutes. It should be noted that this ROC can only be maintained as long as the voltage and current draw remain constant. For real applications this is not possible. The full-charge voltage of a 3 LiPo ( 12.4 Volts) drops rapidly at first (see Section 7.4.3, Discharge Curves), then, as the voltage falls below 12.0 Volts, it reduces at a slower rate until the voltage reaches about 10.8-11.1V, after which it drops sharply to its cutoff voltage. It is therefore unlikely the airplane could maintain this climb rate for two minutes. Regardless, it serves as an idealized baseline to keep in mind. Next, in order to determine the climb distance covered, we must first calculate the lift, drag, and thrust coefficients.

Lift coefficient:

$$
C_{L_{c}}=\frac{2 W}{\rho V_{y}^{2} S}=\frac{2(6.22)}{(0.002378)(32.0)^{2}(5.167)}=0.9983
$$

Drag coefficient:

$$
C_{D_{c}}=C_{D \min }+k C_{L_{c}}^{2}=0.05040+0.04767 C_{L_{C}}^{2}=0.09696
$$

Thrust coefficient:

$$
C_{T}=\frac{1.475 \eta_{p} P_{W}}{\rho V_{y}^{3} S}=\frac{1.475(0.35)(270)}{(0.002378)(32.0)^{3}(5.167)}=0.3462
$$

Therefore, per Equation (9-25), while climbing to 1000 ft , the airplane covers

$$
s_{c}=\frac{C_{L_{c}}}{C_{T}-C_{D_{c}}} h=\frac{0.9983}{0.3462-0.09696}(1000)=3965 \mathrm{ft} \quad(0.6526 \mathrm{~nm})
$$

The flight time to climb to 1000 ft is obtained using Equation (9-30):

$$
t_{c}=\frac{h}{R O C}=\frac{1000}{455}=2.196 \mathrm{~min}
$$

(b) The best glide airspeed is estimated using Equation (6-78):

$$
V_{B G}=\sqrt{\frac{2}{\rho} \sqrt{\frac{k}{C_{D \min }}} \frac{W}{S}}=\sqrt{\frac{2}{0.002378} \sqrt{\frac{0.04767}{0.05040}}\left(\frac{6.22}{5.167}\right)}=31.4 \mathrm{ft} / \mathrm{s} \quad(18.6 \mathrm{KCAS})
$$

We use Equation (9-31) to estimate the distance the airplane covers while gliding back to S-L from an altitude of 1000 ft :

$$
s_{e}=\left(\frac{C_{L_{e}}}{C_{D_{e}}}\right) h=(10.20)(1000)=10200 \mathrm{ft} \quad(1.679 \mathrm{~nm})
$$

The flight time associated with this is obtained using Equation (9-34):

$$
t_{e}=\frac{C_{L_{e}}}{C_{D_{e}}} \frac{h}{V_{B G}}=(10.20) \frac{1000}{31.4}=5.419 \mathrm{~min}
$$

This corresponds to an average rate of descent of $1000 \mathrm{ft} / 5.419=185 \mathrm{fpm}(3.08 \mathrm{ft} / \mathrm{s})$.
(c) Therefore, the total distance covered amounts to

$$
s=s_{c}+s_{e}=14166 \mathrm{ft} \quad(2.331 \mathrm{~nm})
$$

The total flight time is

$$
\begin{aligned}
& t_{\text {TOT }}=t_{c}+t_{e}=7.616 \mathrm{~min} \\
& V_{\text {trip }}=\frac{s}{t_{\text {TOT }}}=\frac{2.331}{7.616 / 60}=18.4 \mathrm{KTAS}
\end{aligned}
$$

The trip airspeed is:
(d) Since the battery energy is only consumed during the climb segment and assuming the current and voltage used above is constant, the capacity consumed is computed using Equation (7-28)

$$
C_{\text {used }}=\int_{0}^{t} I(\tau) d \tau=[I]_{0}^{t}=[22.5 \tau]_{0}^{2.196 / 60}=0.8236 \mathrm{Ah}=823.6 \mathrm{mAh}
$$

(e) The capacity consumed per unit distance is $\frac{C_{\text {used }}}{s}=\frac{823.6 \mathrm{mAh}}{2.331 \mathrm{~nm}}=353.3 \frac{\mathrm{mAh}}{\mathrm{nm}}$
(f) Lift and drag coefficients at $V_{B G}$ are

Lift coefficient:

$$
C_{L_{e}}=\frac{2 W}{\rho V_{B G}^{2} S}=\frac{2(6.22)}{(0.002378)(31.4)^{2}(5.167)}=1.028
$$

Drag coefficient: $\quad C_{D_{e}}=C_{D \text { min }}+k C_{L_{e}}^{2}=0.05040+0.04767 C_{L_{e}}^{2}=0.1008$

Alternatively

$$
C_{D_{e}}=\frac{C_{L_{e}}}{L D_{\max }}=\frac{1.028}{10.20}=0.1008
$$

Current required for maintaining $V_{B G}$ in level flight:

$$
I=\frac{1}{\eta_{p} 0.7376 E} \sqrt{\frac{2 W^{3} C_{D}^{2}}{\rho S C_{L}^{3}}}=\frac{1}{(0.35) 0.7376(12.0)} \sqrt{\frac{2(6.22)^{3}(0.1008)^{2}}{(0.002378)(5.167)(1.028)^{3}}}=5.40 \mathrm{~A}
$$

(g) To complete a 10 nm mission, the vehicle must climb to an altitude of

$$
h=\frac{s}{\left[\frac{C_{L_{c}}}{C_{T}-C_{D_{c}}}+\frac{C_{L_{e}}}{C_{D_{e}}}\right]}=\frac{10 \times 6076}{\left[\frac{0.9883}{0.3462-0.09696}+\frac{1.028}{0.1008}\right]}=4289 \mathrm{ft}
$$

If a constant 455 fpm climb rate is assumed (erroneous) it will take 4289/455 $=9.420$ minutes to climb to that altitude. The battery capacity consumed is computed using Equation (7-28)

$$
C_{\text {used }}=\int_{0}^{t} I(\tau) d \tau=[I]_{0}^{t}=[22.5 \tau]_{0}^{9.420 / 60}=3.533 \mathrm{Ah}=3533 \mathrm{mAh}
$$

It should be stressed that while this result is unrealistic because of the aforementioned unavoidable reduction in power output of the battery, it gives a helpful insight into the capability of a typical electric sUAV.

### 9.3.2. The Basic Climb-Glide Cruise Profile - Non-linear Approach

The last result of Worked Example 1 reveals a serious flaw if the vehicle needs to climb to great altitudes. Since the voltage drop that results from reduction in battery capacity, which is inevitable in the use of any battery, reduces power, the $R O C$ at a given airspeed drops as well. This means it takes longer to reach the required altitude, during which more battery capacity is consumed. It is therefore desired to incorporate the more realistic non-linearity. For instance, among variables that affect the true performance are:
(1) Density drops with altitude. This affects $R O C$ and $V_{T A S}$, both which affect the distance covered and time to altitude.
(2) Voltage drops with reduced battery capacity. Therefore, with wide open throttle, current draw reduces rapidly with the accompanying reduction in power. A procedural change is to climb at a throttle setting below wide open, allowing the battery to maintain close to constant current draw. This will affect maximum power, but it will allow constant current to be maintained over longer period. Constant power requires the product $I \cdot E$ to be constant.

Consider the expression for rate of climb, here shown for engine power, required power, and weight, all which are functions of the altitude and the airspeed.

$$
\begin{equation*}
\dot{h}=\frac{d h}{d t}=\frac{P_{A}-P_{R}}{W}=\frac{P_{E N G}(h, V)-q(h, V) S C_{D} V}{W(h)} \tag{9-40}
\end{equation*}
$$

Thus, for instance, time to altitude would be obtained by performing integration as shown in Equation (6-62), i.e.

$$
\begin{equation*}
t=\int \frac{d h}{\dot{h}}=\int \frac{W(h)}{\left(P_{E N G}(h, V)-q(h, V) S C_{D} V\right)} d h \tag{9-41}
\end{equation*}
$$

Consider the use of a normally aspirated piston engine, for which the engine power is expressed as shown below

$$
\begin{equation*}
P_{E N G}(h, V)=550 P_{B H P_{\max }}(1.132 \sigma(h)-0.132) \eta_{p}(V, R P M) \tag{9-42}
\end{equation*}
$$

Where $P_{B H P_{\max }}$ is the max rated power in brake-horsepower (BHP), $\sigma$ is the density ratio as a function of altitude, and $\eta_{p}$ is the propeller efficiency as a function of the airspeed and $R P M$. Since this expression assumes the power is given as BHP, the constant 550 converts it to $\mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s}$. To get a better idea of the complexity this introduces, let's substitute the standard day expression of Equation (5-7) for the density ratio and a separated cubic polynomial for the dependency of $\eta_{p}$ on $V$ and $R P M$.

$$
\begin{equation*}
P_{E N G}(h, V)=550 P_{B H P_{\max }}\left(1.132(1+\kappa h)^{4.2561}-0.132\right)\left[\phi(R P M)\left(\xi_{0}+\xi_{1} V+\xi_{2} V^{2}+\xi_{3} V^{3}\right)\right] \tag{9-43}
\end{equation*}
$$

Where $\xi_{i,} i=0 \ldots 3$ are constants and $\phi(R P M)$ is a function that modifies the cubic polynomial to yield the correct propeller efficiency $\eta_{p}$ based on $R P M$. Similarly, the dynamic pressure must reflect dependency on $h$ and $V$ as shown below:

$$
\begin{equation*}
q(h, V)=\frac{1}{2} \rho_{0} \sigma V^{2}=\frac{1}{2} \rho_{0}(1+\kappa h)^{4.2561} V^{2} \tag{9-44}
\end{equation*}
$$

And the weight dependency on altitude would reflect the reduction in weight affects aircraft using fossil fuels due to fuel being consumed per unit time. Consider the aircraft increasing its altitude from some initial altitude $h_{i}$ to $h_{i}+\Delta h$ over time $\Delta t$, while consuming $c_{B H P}$ weight units per unit BHP per unit time.

$$
\begin{equation*}
W\left(h_{i}+\Delta h\right)=W\left(h_{i}\right)-P_{E N G}(\bar{h}, V) c_{B H P} \Delta t=W\left(h_{i}\right)-P_{E N G}(\bar{h}, V) c_{B H P} \frac{\Delta h}{\dot{\bar{h}}} \tag{9-45}
\end{equation*}
$$

The bar over $h$ indicates it is an average value of the power and rate of climb over the altitude band. Substituting these into Equation (9-40), leads to

$$
\begin{align*}
\dot{h}=\left\{550 P_{B H P_{\max }}\right. & \left(1.132(1+\kappa h)^{4.2561}-0.132\right)\left[\phi(R P M)\left(\xi_{0}+\xi_{1} V+\xi_{2} V^{2}+\xi_{3} V^{3}\right)\right] \\
& \left.-\frac{1}{2} \rho_{0}(1+\kappa h)^{4.2561} V^{3} S C_{D}\right\} /\left\{W\left(h_{i}\right)-P_{E N G}(\bar{h}, V) c_{B H P} \frac{\Delta h}{\dot{\bar{h}}}\right\} \tag{9-46}
\end{align*}
$$

It should be clear that to integrate this analytically per Equation (9-41) is challenging, to say the least, and there is no guarantee a closed form solution exists. An attempt at improve the "realism" of the solution (here, the time-to-altitude) is better accomplished using computational tools, such as a flight simulator or a stepwise numerical integration using selected altitudes, $h_{i} \rightarrow h_{i}+\Delta h$.

### 9.3.3. Comparing Climb-Glide and Climb-Constant Airspeed/Altitude Cruise Profiles

Consider a scenario in which the operator of a powered aircraft wants to compare the difference in energy consumption between flying a given distance using the previous climb-glide profile versus the more traditional approach of climbing to altitude $h$ and maintain constant airspeed and constant altitude using engine power (see Figure 9-6). Such profiles are presented in Section 6.4.3, Range Analysis. Note that Figure 9-6 is produced in color, where red indicates cruise or climb segments that depends on the consumption of onboard energy, whereas the green segment is a power-off glide. We now want to answer the following questions:

1. Which of the two profiles leads to longer range, assuming all onboard energy is consumed?
2. Which profile leads to longest endurance assuming all onboard energy is consumed?
3. What is the amount of energy consumed assuming fixed distance is flown (specific range)?
4. What is the time required to reach the destination, assuming fixed distance is flown?
5. What values of $L / D, h_{1}-h_{0}$, and $V_{C R Z}$ will make the climb-glide cruise profile more efficient than the constant airspeed/altitude cruise profile?


Figure 9-6: Traveling from Point A to Point B using a constant airspeed/altitude cruise or a climb-glide cruise profile.

It is important to remember that the climb segments are all covered at maximum power, whereas the constant airspeed/altitude cruise segment is at a lower power setting. Thus, if the goal is to maximize distance (range), the power setting would be that associated with the best range airspeed (e.g. see Equation (6-82)) and the airspeed for glide would be the best glide airspeed of Equation (6-78). Conversely, if the goal is to maximize time aloft (endurance), the power setting would that associated with minimum power (e.g. see Equation (6-76)) and the glide airspeed would be the minimum sink airspeed of Equation (6-119).

To investigate the profiles we first note that the initial climb segment is identical in both cases, thus, it can be omitted from further consideration; the climb-glide profile begins the first time the aircraft reaches altitude $h_{1}$, at which time we assume both aircraft begin the mission. Furthermore, we assume the altitude band for the climb-glide profile is $\Delta h=h_{1}-h_{0}$. Figure 9-6 shows that even though the comparison starts with a glide from $h_{1}$, it is followed by a climb segment from $h_{0}$ back to $h_{1}$. Thus, the methodology of the previous section still applies. Furthermore, the total distance covered from the top of the initial climb will be some real-valued fraction of the climb-glide segment (e.g. " 8.357 climb-glide segments"). Therefore, it suffices to compare the two profiles over one climb-glide segment, as shown in Table 9-1. Note that the comparison assumes the use of the simplified drag model presented in Section 6.2.3, The Drag Coefficient.

Table 9-1: Cruise Profile Comparison for an Electric Airplane

|  | Constant Speed/Constant Altitude | Climb-Glide Profile |
| :--- | :---: | :---: |
| Time to climb | Climb phase has already completed when <br> cruise begins | $t_{c}=\frac{\Delta h}{\dot{h}}$ |
| Time to glide | Cruise phase is terminated at the end-of- <br> cruise | $t_{e}=\frac{C_{L_{e}}}{C_{D_{e}}} \frac{\Delta h}{V_{e}}$ |
| Time in cruise | $t_{C R Z}=\frac{s}{V_{C R Z}}$ | No constant altitude cruise segment |
| Best climb airspeed | Climb phase has already completed when <br> cruise begins | Equation (6-51) or similar |
| Best range airspeed | $V_{R}=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{3 k}{C_{D \text { min }}}}}$ | No constant altitude cruise segment |
| Best endurance airspeed | $V_{E \max }=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right) \sqrt{\frac{k}{3 \cdot C_{D \min }}}}$ | No constant altitude cruise segment |
| Power for best range | $P_{R E Q}=\sqrt{\frac{2 W^{3} C_{D}^{2}}{\rho S C_{L}^{3}}}$ | No constant altitude cruise segment |
| Energy consumed | $C_{u s e d}$ | $=\int_{0}^{t_{C R Z} I_{C R Z}(\tau) d \tau}$ |

## Worked Example 2:

The aircraft of Worked Example 1 is flown at its best range airspeed ( $V_{B G}=31.4 \mathrm{ft} / \mathrm{s}$ ) over a total distance of $s=2.331 \mathrm{~nm}$, obtained in its solution. Determine the amount of battery capacity consumed and capacity consumed per unit distance.

## Solution:

Time to complete distance: $\quad t_{C R Z}=\frac{s}{V_{C R Z}}=\frac{14166}{31.4}=415.6 \mathrm{sec}=7.526 \mathrm{~min}=0.1254 \mathrm{hrs}$

The current required is already calculated as item (f) of Worked Example 1 and was found to equal 5.40 Amps. This value is assumed constant in this problem. Therefore, the battery capacity consumed is

$$
\begin{aligned}
& \qquad C_{\text {used }_{C R Z}}=\int_{0}^{t_{C R Z}} I_{C R Z}(\tau) d \tau=5.40 \times 0.1254=0.6774 \mathrm{Ah}=677.4 \mathrm{mAh} \\
& \text { The capacity consumed per unit distance is } \frac{C_{u s e d_{C R Z}}}{S}=\frac{677.4 \mathrm{mAh}}{2.331 \mathrm{~nm}}=290.6 \frac{\mathrm{mAh}}{\mathrm{~nm}}
\end{aligned}
$$

### 9.3.4. Influence of Sinking or Rising Air on the Climb-Glide Profile

Now that we have developed formulation to predict endurance and range of both the climb and descent segments, it is appropriate to incorporate the effect of atmospheric convection; lift and sink and headand tailwind. We will proceed in a manner similar to that presented in Section 6.5, Sailplane Performance Theory, and present each effect separately, starting with sink and lift, followed by the effect of head- and tailwind.

The basic influence of operating an aircraft in an up- and downdraft are illustrated in Figure 9-7. It can be seen that in updraft, the climb-distance is shortened (aircraft gets to altitude $h$ faster than the baseline), while the opposite holds for climb in downdraft. Similarly, the glide distance in updraft is greater, while it is shorter in the downdraft. What we will develop here is formulation to account for these effects on the profile properties. Also, recall that the vertical convection affects the calibrated airspeed at which optimum airspeeds such as best angle-of-climb and best glide are achieved.

## Climb Segment

Introducing only lift or sink to the climb profile will not affect ground speed (over what it is in no convection), only the vertical speed. Recall, that the attitude (or pitch angle) of the aircraft during climb is the same in all three situations. However, the path (or climb) angle is different. If the airplane is subjected to vertical wind component $w$ (which is $>0$ for lift and $<0$ for sink) during climb, then its rate-of-climb must be adjusted by

$$
\begin{equation*}
R O C=\dot{h}_{c}+w=\frac{P_{A}-P_{R_{c}}}{W}+w=\frac{P_{A}-P_{R_{c}}+W w}{W} \tag{9-47}
\end{equation*}
$$



Figure 9-7: Effect of operating an aircraft in a climb-glide profile in up- or downdraft on range. DN = Downdraft, UP = Updraft.

Where the subscript ' $c$ ' refers to the climb segment in Figure 9-7. This differs from the calm-day condition by the introduction of the vertical wind component, $W w$. Thus, time to altitude $h$ is given by

$$
\begin{equation*}
t_{c}=\frac{h}{\dot{h_{c}}+w}=\frac{W}{P_{A}-P_{R_{c}}+W w} h \tag{9-48}
\end{equation*}
$$

The climb angle can then be estimated by the appropriate modification of Equation (9-29), i.e.

$$
\begin{equation*}
\theta_{c}=\sin ^{-1}\left(\frac{\dot{h}_{c}+w}{V_{c}}\right)=\sin ^{-1}\left(\frac{P_{A}-P_{R_{c}}+W w}{W V_{c}}\right) \tag{9-49}
\end{equation*}
$$

And the climb distance $s_{c}$ covered is obtained by modifying Equation (9-24)

$$
\begin{equation*}
s_{c}=V_{c} \cos \theta_{c} \overbrace{\left(\frac{W}{P_{A}-P_{R_{c}}+W w}\right)}^{=t_{c}} h=\frac{W V_{c} \cos \theta_{c}}{T V_{c}-D V_{c}+W w} h \tag{9-50}
\end{equation*}
$$

Note that Equation (9-47) has additional utility. Consider a situation in which we want to climb to a specific altitude in the presence of vertical wind speed. The equation permits the estimation of the power the engine must be capable of generating, allowing us to check a priori if it is possible to climb to said altitude. First, solve Equation (9-47) for the available power

$$
\begin{equation*}
P_{A}=W \cdot \dot{h}_{c}-W w+P_{R_{c}} \tag{9-51}
\end{equation*}
$$

Then observe that the power off glide performance can be used to extract the power required. Since the rate-of-descent, $R O D$, can be written as

$$
\begin{equation*}
\dot{h}_{e}=\frac{-P_{R_{e}}}{W} \tag{9-52}
\end{equation*}
$$

It follows that

$$
\begin{equation*}
P_{R_{e}}=-W \cdot \dot{h}_{e} \tag{9-53}
\end{equation*}
$$

We can obtain $R O D$ at the given airspeed directly using classical performance analysis. Thus, substituting this into Equation (9-51) allows us to determine the required magnitude of the power available:

$$
\begin{equation*}
P_{A}=W \cdot \dot{h}_{c}-W w-W \cdot \dot{h}_{e}=W\left(\dot{h}_{c}-w-\dot{h}_{e}\right) \tag{9-54}
\end{equation*}
$$

Where $\dot{h}_{c}$ and $\dot{h}_{e}$ are the rate of ascent and descent, respectively. The above expression can be used to evaluate the fraction of maximum power required to either sustain a desired altitude, or climb at a given rate. As an example, suppose a $500 \mathrm{lb}_{\mathrm{f}} \mathrm{sUAV}$, operating at 50 KCAS (at sea-level) on a calm day, is capable of climbing 530 fpm , while descending at 230 fpm in unpowered glide (assuming the propeller does not windmill). This means the available thrust-power ( $T \cdot V$ ) is

$$
P_{A}=W\left(\dot{h}_{c}-\dot{h}_{e}\right)=500\left(\frac{530}{60}-\frac{(-230)}{60}\right)=6333 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s} \quad(=11.5 \mathrm{BHP})
$$

If equipped with a 20 BHP engine, this means the propeller is about 11.5/20 $=0.575$ efficient at that condition. Now, consider a situation in which this sUAV, again at 50 KCAS, is required to climb at 250 fpm to clear terrain, while subjected to a constant 100 fpm downdraft. This means the propeller thrustpower must amount to

$$
P_{A}=W\left(\dot{h}_{c}-w-\dot{h}_{e}\right)=500\left(\frac{250}{60}-\frac{(-230)}{60}-\frac{(-100)}{60}\right)=4833 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s} \quad(=8.8 \mathrm{BHP})
$$

Assuming the propeller is being operated at the same RPM, the power the engine must deliver is $20(4833 / 6333)=15.3$ BHP ( $76 \%$ power). We conclude the vehicle should be capable of achieving the 250 fpm climb (which amounts to 350 fpm on a calm day).

## Glide Segment

The descent segment is analyzed in a similar manner as shown earlier per Equation (9-31). First, an unpowered glide is assumed. The rate-of-descent is given by:

$$
\begin{equation*}
R O D=\dot{h}_{e}+w=\frac{-P_{R_{e}}}{W}+w=\frac{W w-P_{R_{e}}}{W} \tag{9-55}
\end{equation*}
$$

Where the subscript ' $e$ ' refers to the descent segment. Where, again, the vertical wind component is denoted by $w$ ( $>0$ for lift and $<0$ for sink). Thus, the time from altitude $h$ is given by

$$
\begin{equation*}
t_{e}=-\frac{h}{\dot{h}_{e}+w}=-\frac{W}{W w-P_{R_{e}}} h \tag{9-56}
\end{equation*}
$$

Where the negative sign is required to yield positive value of time. The descent angle can then be estimated by the appropriate modification of Equation (9-29), i.e.

$$
\begin{equation*}
\theta_{e}=\sin ^{-1}\left(\frac{\dot{h}_{e}+w}{V_{e}}\right)=\sin ^{-1}\left(\frac{W w-P_{R_{e}}}{W V_{e}}\right) \tag{9-57}
\end{equation*}
$$

The time required to cover the glide segment is thus

$$
\begin{equation*}
s_{e}=V_{e} \cos \theta_{e} \overbrace{\left(\frac{W}{W w-P_{R_{e}}}\right)}^{=t_{e}} h=\frac{W V_{e} \cos \theta_{e}}{W w-D V_{e}} h \tag{9-58}
\end{equation*}
$$

Where $V_{e}$ is the glide airspeed of the aircraft (parallel to the descent vector), $\theta_{e}$ is the glide angle, $t_{e}$ is time it takes to reach B from altitude $h$.

## Worked Example 3:

Using data from previous worked examples for the Quanum Observer, estimate the following for a vertical wind speed of $w=-1,0$, and $+1 \mathrm{ft} / \mathrm{s}$ :
(a) the time to climb from 0 to 500 ft , (b) the associated climb distances, (c) battery capacity consumed (assuming constant current draw of 22.5 Amps ), (d) time to glide from 500 ft to 0 ft at the best glide speed ( $L D_{\text {max }}=10.20$ ), (e) the associated glide distances, (f) total distance achieved, and (g) distance per mAh consumed of climb and total distances, assuming the following properties from Worked Example 1 hold:

Weight:
Lift coefficient in climb:
Lift coefficient in descent:
Drag coefficient in climb:
Drag coefficient in descent:
Density at condition:
Climb speed:
Glide speed:
Rate-of-Climb:
Rate-of-Descent:
Prop efficiency in climb:
Climb current:
Max power:
Power available:
Power required in climb:

$$
\begin{aligned}
& W=6.22 \mathrm{lb}_{\mathrm{f}} \\
& C_{L c}=0.9983 \\
& C_{L e}=1.028 \\
& C_{D c}=0.09696 \\
& C_{D e}=C_{L e} / L D_{\max }=0.1008 \\
& \rho=0.002378 \mathrm{slugs} / \mathrm{ft}^{3} \\
& V_{c}=V_{y}=32.0 \mathrm{ft} / \mathrm{s} \\
& V_{e}=V_{B G}=31.4 \mathrm{ft} / \mathrm{s} \\
& \dot{h}_{c}=455 \mathrm{fpm}=7.58 \mathrm{ft} / \mathrm{s} \\
& \dot{h}_{e}=-185 \mathrm{fpm}=-3.08 \mathrm{ft} / \mathrm{s} \\
& \eta_{p}=0.335 \\
& I_{B}=22.5 \mathrm{~A} \\
& P=199.1 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s} \\
& P_{A}=\eta_{p} P=0.335 \times 199.1=66.7 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s} \\
& P_{R c}=1 / 2 \rho V_{c}^{3} S C_{D c}=19.5 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s}
\end{aligned}
$$

Power required in descent: $\quad P_{R e}=1 / 2 \rho V_{e}^{3} S C_{D e}=19.1 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s}$

## Solution:

(a) Time to climb in $\operatorname{sink}(w=-1 \mathrm{ft} / \mathrm{s})$, no convection ( $w=0 \mathrm{ft} / \mathrm{s}$ ), and lift ( $w=+1 \mathrm{ft} / \mathrm{s}$ ):

$$
t_{c}=\frac{W}{P_{A}-P_{R_{c}}+W w} h=\left\{\begin{array}{ccc}
75.9 \mathrm{sec} & \text { if } & w=-1 \mathrm{ft} / \mathrm{s} \\
65.9 \mathrm{sec} & \text { if } & w=0 \mathrm{ft} / \mathrm{s} \\
58.2 \mathrm{sec} & \text { if } & w=+1 \mathrm{ft} / \mathrm{s}
\end{array}\right.
$$

(b) To determine the distance to climb, we first calculate the climb angle per Equation (9-49)

$$
\theta_{c}=\sin ^{-1}\left(\frac{\dot{h}_{c}+w}{V_{c}}\right)=\left\{\begin{array}{ccc}
11.9^{\circ} & \text { if } & w=-1 \mathrm{ft} / \mathrm{s} \\
13.7^{\circ} & \text { if } & w=0 \mathrm{ft} / \mathrm{s} \\
15.6^{\circ} & \text { if } & w=+1 \mathrm{ft} / \mathrm{s}
\end{array}\right.
$$

Therefore, the climb distance is given by Equation (9-50)

$$
s_{c}=V_{c} \cos \theta_{c} t_{c}=\left\{\begin{array}{ccc}
2378 \mathrm{ft} & \text { if } & w=-1 \mathrm{ft} / \mathrm{s} \\
2049 \mathrm{ft} & \text { if } & w=0 \mathrm{ft} / \mathrm{s} \\
1795 \mathrm{ft} & \text { if } & w=+1 \mathrm{ft} / \mathrm{s}
\end{array}\right.
$$

(c) Battery capacity consumed assuming constant current draw of 22.5 A .

$$
C_{c}=I_{c} t_{c}=\left\{\begin{array}{lcc}
1708 \mathrm{mAh} & \text { if } & w=-1 \mathrm{ft} / \mathrm{s} \\
1483 \mathrm{mAh} & \text { if } & w=0 \mathrm{ft} / \mathrm{s} \\
1310 \mathrm{mAh} & \text { if } & w=+1 \mathrm{ft} / \mathrm{s}
\end{array}\right.
$$

(d) Time to descend to 0 ft in $\operatorname{sink}(w=-1 \mathrm{ft} / \mathrm{s})$, no convection ( $w=0 \mathrm{ft} / \mathrm{s}$ ), and lift ( $w=+1 \mathrm{ft} / \mathrm{s}$ ):

$$
t_{e}=-\frac{W}{W w-P_{R_{e}}} h=\left\{\begin{array}{ccc}
123 \mathrm{sec} & \text { if } & w=-1 \mathrm{ft} / \mathrm{s} \\
163 \mathrm{sec} & \text { if } & w=0 \mathrm{ft} / \mathrm{s} \\
241 \mathrm{sec} & \text { if } & w=+1 \mathrm{ft} / \mathrm{s}
\end{array}\right.
$$

(e) To determine the glide distance, we first calculate the descent angle per Equation (9-57)

$$
\theta_{e}=\sin ^{-1}\left(\frac{\dot{h_{e}}+w}{V_{e}}\right)=\left\{\begin{array}{ccc}
-7.46^{\circ} & \text { if } & w=-1 \mathrm{ft} / \mathrm{s} \\
-5.63^{\circ} & \text { if } & w=0 \mathrm{ft} / \mathrm{s} \\
-3.79^{\circ} & \text { if } & w=+1 \mathrm{ft} / \mathrm{s}
\end{array}\right.
$$

Therefore, the glide distance is given by Equation (9-58)

$$
s_{e}=V_{e} \cos \theta_{e} t_{e}=\left\{\begin{array}{ccc}
3816 \mathrm{ft} & \text { if } & w=-1 \mathrm{ft} / \mathrm{s} \\
5075 \mathrm{ft} & \text { if } & w=0 \mathrm{ft} / \mathrm{s} \\
7540 \mathrm{ft} & \text { if } & w=+1 \mathrm{ft} / \mathrm{s}
\end{array}\right.
$$

(f) The total distance is

$$
s_{\text {TOT }}=s_{c}+s_{e}=\left\{\begin{array}{ccc}
6193 \mathrm{ft} & \text { if } & w=-1 \mathrm{ft} / \mathrm{s} \\
7124 \mathrm{ft} & \text { if } & w=0 \mathrm{ft} / \mathrm{s} \\
9335 \mathrm{ft} & \text { if } & w=+1 \mathrm{ft} / \mathrm{s}
\end{array}\right.
$$

(g) The total distance per mAh consumed is

$$
\frac{s_{c}}{C_{c}}=\left\{\begin{array}{l}
1.39 \mathrm{ft} / \mathrm{mAh} \\
1.38 \mathrm{ft} / \mathrm{mAh} \\
1.37 \mathrm{ft} / \mathrm{mAh}
\end{array} \quad \text { and } \quad \frac{s_{T O T}}{C_{c}}=\left\{\begin{array}{l}
3.63 \mathrm{ft} / \mathrm{mAh} \\
4.80 \mathrm{ft} / \mathrm{mAh} \\
\text { if } \\
7.12 \mathrm{ft} / \mathrm{mAh} \\
\text { if } \\
\text { if } \\
w=-1 \mathrm{ft} / \mathrm{s} \\
w=+1 \mathrm{ft} / \mathrm{s}
\end{array}\right.\right.
$$

### 9.3.5. Influence of Head- or Tailwind on the Climb-Glide Profile

Let's extend the discussion to horizontal winds. The basic influence of operating an aircraft in a head- or tailwind is illustrated in Figure 9-8. It can be seen that in headwind, the climb-distance is shortened, although the aircraft gets to altitude $h$ at precisely the same time as the baseline. The opposite holds for climb in downdraft. Similarly, the glide distance in headwind is shorter, while it is greater in tailwind.

## Climb Profile

As presented in Section 6.5, Sailplane Performance Theory, introducing head- or tailwind to the climb profile only affects the ground speed (over its value in no convection) and not the vertical speed. This changes the path (or climb) angle, but the time to altitude is unchanged. If the airplane is subjected to horizontal wind component $V_{w}$ (which is $>0$ for tailwind and $<0$ for headwind) during climb, then the time to altitude is


Figure 9-8: Effect of operating an aircraft in a climb-glide profile in head- or tailwind on range. HW = Headwind, TW = Tailwind.

$$
\begin{equation*}
t_{c}=\frac{h}{\dot{h}_{c}}=\frac{W}{P_{A}-P_{R_{c}}} h \tag{9-59}
\end{equation*}
$$

Where the subscript ' $c$ ' refers to the climb segment. The climb angle can then be estimated by the appropriate modification of Equation (9-29) as follows

$$
\begin{equation*}
\theta_{c}=\sin ^{-1}\left(\frac{\dot{h}_{c}}{V_{c}+V_{w}}\right)=\sin ^{-1}\left(\frac{P_{A}-P_{R_{c}}+W w}{W\left(V_{c}+V_{w}\right)}\right) \tag{9-60}
\end{equation*}
$$

Note that this form will lead to a larger denominator in tailwind and, thus, shallower climb angle. And the climb distance $s_{c}$ covered is obtained by modifying Equation (9-24)

$$
\begin{equation*}
s_{c}=\left(V_{c}+V_{w}\right) \cos \theta_{c} \overbrace{\left(\frac{W}{P_{A}-P_{R_{c}}}\right)}^{\epsilon_{c}} h=\frac{W\left(V_{c}+V_{w}\right) \cos \theta_{c}}{(T-D)\left(V_{c}+V_{w}\right)} h \tag{9-61}
\end{equation*}
$$

## Glide Profile

If the airplane is subjected to horizontal wind component $V_{w}$ during glide, then the time from altitude is

$$
\begin{equation*}
t_{e}=\frac{h}{\dot{h_{e}}}=\frac{W}{P_{R_{e}}} h \tag{9-62}
\end{equation*}
$$

Where the subscript ' $e$ ' refers to the glide segment. The glide angle can then be estimated by the appropriate modification of Equation (9-29) as shown below

$$
\begin{equation*}
\theta_{e}=\sin ^{-1}\left(\frac{\dot{h}_{e}}{V_{e}+V_{w}}\right)=\sin ^{-1}\left(\frac{-P_{R_{e}}}{W\left(V_{e}+V_{w}\right)}\right) \tag{9-63}
\end{equation*}
$$

The glide distance $s_{e}$ covered is obtained by modifying Equation (9-24)

$$
\begin{equation*}
s_{e}=\left(V_{e}+V_{w}\right) \cos \theta_{e} \overbrace{\left(\frac{W}{-P_{R_{e}}}\right)}^{=t_{e}} h=-\frac{W\left(V_{e}+V_{w}\right) \cos \theta_{e}}{D\left(V_{e}+V_{w}\right)} h \tag{9-64}
\end{equation*}
$$

### 9.3.6. Combined Influence of Lift or Sink and Head- or Tailwind on the Climb-Glide Profile

We can now combine the convection effects into the following set of equations that are used by the GICA when optimizing trajectory through wind fields:

## Climb Profile

Rate-of-Climb: $\quad R O C=\dot{h}_{c}+w=\frac{P_{A}-P_{R_{c}}+W w}{W}$

Time to altitude: $\quad t_{c}=\frac{h}{\dot{h}_{c}+w}=\frac{W}{P_{A}-P_{R_{c}}+W w} h$
Climb angle: $\quad \theta_{c}=\sin ^{-1}\left(\frac{\dot{h}_{c}+w}{V_{c}+V_{w}}\right)=\sin ^{-1}\left(\frac{P_{A}-P_{R_{c}}+W w}{W\left(V_{c}+V_{w}\right)}\right)$
Climb distance: $\quad s_{c}=\left(V_{c}+V_{w}\right) \cos \theta_{c} t_{c}=\frac{W\left(V_{c}+V_{w}\right) \cos \theta_{c}}{(T-D)\left(V_{c}+V_{w}\right)+W w} h$

## Glide Profile

Rate-of-descent: $\quad R O D=\dot{h}_{e}+w=\frac{-P_{R_{e}}}{W}+w=\frac{W w-P_{R_{e}}}{W}$
Time to glide: $\quad t_{e}=-\frac{h}{\dot{h}_{e}+w}=-\frac{W}{W w-P_{R_{e}}} h$
Descent angle: $\quad \theta_{e}=\sin ^{-1}\left(\frac{\dot{h}_{e}+w}{V_{e}+V_{w}}\right)=\sin ^{-1}\left(\frac{W w-P_{R_{e}}}{W\left(V_{e}+V_{w}\right)}\right)$
Glide distance: $\quad s_{e}=\left(V_{e}+V_{w}\right) \cos \theta_{e} t_{e}=\frac{W\left(V_{e}+V_{w}\right) \cos \theta_{e}}{W w-D\left(V_{e}+V_{w}\right)} h$

### 9.4 Additional Topics involving Energy Harvesting

This section will discuss additional means to improve potential for energy harvesting. These topics are design guidelines for aircraft intended for energy harvesting, the use of solar panels, and piezo-electrics.

### 9.4.1. Design Guidelines for Efficient Energy Harvesting Aircraft

By comparing the result from the above example to the one from Worked Example 1(e), we see that the range of the Quanum Observer is not improved by flying the climb-glide profile ( 353.3 versus 290.6 $\mathrm{mAh} / \mathrm{nm})$. The climb-glide profile can be made to consume less energy than the conventional constant altitude cruise at minimum power requires its $L / D$ at the condition to be higher, begging the question: What $L / D$ ratio is required to achieve to make the climb-glide profile more efficient than the constant airspeed-constant altitude profile?

To present this problem more formally, consider Figure 9-9, which shows two airplanes, A and B, in the process of covering the same total distance, $s$, using the two cruise profiles. When the comparison begins, at Point 1, Airplane A is already at altitude $h$, where it covers the total distance $s$ using constant engine power, $P_{B H P A}$, associated with some fixed airspeed, denoted by $V_{A}$. Airplane B begins its mission below Airplane A at Point 1 and climbs to altitude $h$ using maximum power, $P_{B H P}{ }_{B}$, at some fixed airspeed, denoted by $V_{c}$. As soon as Airplane B reaches $h$, at Point 2, its engine is shut off and it begins a glide at some fixed airspeed, denoted by $V_{e}$. Ignoring time and distance lost to the stabilization of the Phugoid mode, it holds that the total distance $s$ must consist of a complete climb and subsequent descent to the initial altitude (which does not have to be at ground level, as shown in Figure 9-9).

Additionally, the altitude $h$ must be small enough to allow altitude variations of performance to be ignored. Assuming the specific fuel consumption is equal for the power settings involved, we can estimate the fuel consumed by both planes as follows:


Figure 9-9: Two ways to complete a mission from Point 1 to Point 3. Starting at altitude $\boldsymbol{h}$, aircraft A travels at constant airspeed and altitude in powered flight. Aircraft B only uses power to climb to $h$ and then glides back to its initial altitude.

Fuel consumed by Airplane A:

$$
\begin{align*}
& W_{f_{A}}=c_{B H P} P_{B H P_{A}}\left(h, V_{A}\right) \Delta t=c_{B H P} P_{B H P_{A}}\left(h, V_{A}\right) \frac{s}{V_{A}}  \tag{9-73}\\
& W_{f_{B}}=c_{B H P} P_{B H P_{B}}\left(\bar{h}, V_{B}\right) \Delta t=c_{B H P} P_{B H P_{B}}\left(\bar{h}, V_{B}\right) \frac{s_{c}}{V_{B}} \tag{9-74}
\end{align*}
$$

Where $\bar{h}$ is an altitude of "average engine properties," e.g. around $h / 2$ if $h$ is not too large. It should be clear that airplane B consumes more fuel over the distance $s_{c}$ than airplane A. Thus, aircraft B must make up for this deficiency by maximizing its glide distance $s_{e}$, where it descends without engine power. Thus, the following must hold over the total distance $s$ :

Simplifying yields:

$$
\begin{align*}
W_{f_{B}} \leq W_{f_{A}} \Leftrightarrow & c_{B H P} P_{B H P_{B}}\left(\bar{h}, V_{c}\right) \frac{S_{c}}{V_{c}} \leq c_{B H P} P_{B H P_{A}}\left(h, V_{A}\right) \frac{s}{V_{A}} \\
& \frac{P_{B H P_{B}}\left(\bar{h}, V_{c}\right)}{P_{B H P_{A}}\left(h, V_{A}\right)} \frac{V_{A}}{V_{c}} \leq \frac{s}{s_{c}} \tag{9-75}
\end{align*}
$$

Where $s=s_{c}+s_{e}$ and $s_{e}=\frac{C_{L_{e}}}{C_{D_{e}}} h=L D_{e} h$. Substituting this into Equation (9-75) yields

$$
\begin{equation*}
\frac{s}{s_{c}}=\frac{s_{c}+s_{e}}{s_{c}}=1+\frac{s_{e}}{s_{c}}=1+\left(\frac{h}{s_{c}}\right) L D_{e} \Rightarrow \frac{P_{B H P_{B}}\left(\bar{h}, V_{c}\right)}{P_{B H P A}\left(h, V_{A}\right)} \frac{V_{A}}{V_{c}} \leq 1+\left(\frac{h}{s_{c}}\right) L D_{e} \tag{9-76}
\end{equation*}
$$

This allows us to determine the minimum lift-to-drag ratio Airplane B must achieve to beat Airplane A in fuel efficiency.

$$
\begin{equation*}
L D_{e} \geq \frac{S_{c}}{h}\left[\frac{P_{B H P_{B}}\left(\bar{h}, V_{c}\right)}{P_{B H P_{A}}\left(h, V_{A}\right)} \frac{V_{A}}{V_{c}}-1\right] \tag{9-77}
\end{equation*}
$$

If we are interested in the same optimization for an electric airplane, we have to determine the energy consumed in terms of, say, mAh consumed.
mAh consumed by Airplane A:

$$
\begin{align*}
& C_{A}=\int_{0}^{t_{A}} I_{A}(\tau) d \tau \approx I_{A} \frac{s}{V_{A}}  \tag{9-78}\\
& C_{B}=\int_{0}^{t_{B}} I_{B}(\tau) d \tau \approx I_{B} \frac{s_{c}}{V_{c}} \tag{9-79}
\end{align*}
$$

Where $t_{A}$ is the total time between points 1-2-3 for airplane A and $t_{B}$ is the total time between points 1-2 for airplane B. Proceeding in a similar fashion as before, we write

$$
C_{B} \leq C_{A} \quad \Leftrightarrow \quad I_{B} \frac{s_{c}}{V_{c}} \leq I_{A} \frac{s}{V_{A}} \Leftrightarrow \frac{I_{B}}{I_{A}} \frac{V_{A}}{V_{c}} \leq 1+\frac{s_{e}}{s_{c}}=1+\left(\frac{h}{s_{c}}\right) L D_{e}
$$

This calls for a lift-to-drag ratio for an electric aircraft of at least

$$
\begin{equation*}
L D_{e} \geq \frac{s_{c}}{h}\left[\frac{I_{B}}{I_{A}} \frac{V_{A}}{V_{c}}-1\right] \tag{9-80}
\end{equation*}
$$

Equations (9-77) and (9-80) can be used for initial design of aircraft intended for energy harvesting that uses climb-glide profiles.

## Worked Example 4:

It was shown in Worked Examples 1 and 2, that the $L D_{\text {max }}$ of the Quanum Observer is not high enough to make the climb-glide profile more efficient than its best range profile. Determine the minimum value of the glide ratio required to make it so, using Equation ( $9-80$ ). What design guidance can be gleaned from the result?

## Solution:

Using the values given and determined in Worked Examples 1 and 2, we get:

| Climb distance: | $s_{c}=3965 \mathrm{ft}$ |
| :--- | :--- |
| Altitude: | $h=1000 \mathrm{ft}$ |
| Cruise current: | $I_{A}=5.40 \mathrm{~A}$ |
| Climb current: | $I_{B}=22.5 \mathrm{~A}$ |
| Cruise speed: | $V_{A}=V_{B G}=31.4 \mathrm{ft} / \mathrm{s}$ |
| Climb speed: | $V_{c}=V_{y}=32.0 \mathrm{ft} / \mathrm{s}$ |

Therefore, the Quanum Observer must at least be capable of achieving the following lift-to-drag ratio:

$$
L D_{e} \geq \frac{S_{c}}{h}\left[\frac{I_{B}}{I_{A}} \frac{V_{A}}{V_{c}}-1\right]=\frac{3965}{1000}\left[\frac{22.5}{5.40} \frac{31.4}{32.0}-1\right]=12.2
$$

The Observer needs a drag cleanup if the climb-glide profile is to be used. Other than that, the minimum value of 12.2 is not too hard to achieve.

### 9.4.2. Active Means for Energy Replenishing - Solar Energy

Solar powered flight dates back to experiments conducted by Colonel H. J. Taplin of the UK in June 1957, who launched the first electrically powered (RC) aircraft, called the "Radio Queen." (Noth [6] (2008)) This flight is remarkable because it demonstrated aircraft could indeed be electrically powered -a precursor to electric flight using solar power. The first truly solar powered aircraft, called the Sunrise I, took off from the dry lake bed at Camp Irwin, California, on November $4^{\text {th }}, 1974$ (Boucher [7, 8]). Since then, there has been substantial activity in the field, as already discussed in the introduction of this chapter.

The conversion of sunlight to electric energy has great potential in aircraft. Solar power is usually harnessed in two ways; either using lenses and mirrors through Concentrated Solar Power (CSP) or using Photo-Voltaics (PV), which is a technology that converts photons from the Sun directly to electricity. The conversion mechanism is voltage produced between two dissimilar materials when their common junction is illuminated with photons (e.g. see Lerner and Trigg [9]). The PV is what is used to generate electricity using solar cells, which are rated by their output power (in Watts) and the voltage between the cathode and anode ports. Note that the incorporation of this technology results in weight increase of the vehicle, which increases the rate-of-descent, offsetting some of the benefits.

## Efficiency of PV

As intuition would hold, a primary parameters of concern is how much of the photons are converted to electric energy. Figure 9-10 shows a radiation ${ }^{3}$ spectrum for the Sun. A solar cell that would be capable

[^20]of utilizing the entire spectrum would be $100 \%$ efficient. Current solar cells use semiconductors that only convert radiation for selected range of wavelengths making such efficiency impossible presently. While the physics of the conversion is very interesting (e.g. see Krane [10] and Huang [11]), it is outside the scope of this work. For instance, the maximum theoretical radiative energy available on a sunny day is $1368 \mathrm{~W} / \mathrm{m}^{2}$ (e.g. see Huang [11]). This quantity is called the solar constant. In industry setting, the solar constant is assumed $1000 \mathrm{~W} / \mathrm{m}^{2}$. Of course this value varies with latitude, time of the day, and presence of clouds. In this context, the term PV efficiency, $\eta_{\max }$, refers to the amount of power that can be extracted from a unit area of a solar panel. It is defined as shown below


Figure 9-10: Solar radiation spectrum (Source: Wikipedia Commons)

$$
\begin{equation*}
\eta_{\max } \equiv \frac{\text { Max power output }}{\text { Incident radiation flux } \times \text { Collector area }}=\frac{P_{\max }}{E_{i} \times A_{c}} \tag{9-81}
\end{equation*}
$$

Where $P_{\max }$ is the maximum power output of the solar cell, $E_{i}$ is the energy available per unit area (e.g. $1000 \mathrm{~W} / \mathrm{m}^{2}$ ) and $A_{c}$ is the area usable for energy collection. Presently, the typical consumer solar panels are rated as high as $21.5 \%$ efficient (e.g. see X-series Solar Panels [12]), although there is annual deterioration (for instance, efficiency typically reduces by between $5-20 \%$ over 25 years of use). Figure 9-11 gives an insight into how this technology is advancing; with top-rated solar cells currently as high as $46 \%$ efficient. Efficiency should be treated as a primary design variable in the design of solar powered aircraft.

## Types of Solar Cells

There are many types of solar cells, although cells based on Silicon ( Si ) semiconductors constitute the vast majority, with nearly $90 \%$ marketshare [13]. Silicon semiconductors are generally available in three forms; monocrystalline, polycrystalline, and amorphous. Their pros and cons are listed in Table 9-2.


Figure 9-11: Trends in solar panel efficiency. The graph shows how the maximum efficiency of solar panels has increased from about 15\% in 1980 to 46\% in 2015 (research technology) (Source: National Renewable Energy Laboratory - NREL, http://www.nrel.gov/ncpv/images/efficiency_chart.jpg)

## Current and Voltage of a Solar Cell

Figure 9-12 shows a classical shape illustrating how the current output by a solar cell varies with voltage. When the cell pads are not connected the circuit is open (OC) and the voltage measured across them amounts to $V_{O C}$. When short circuited (SC), the voltage drops to zero and the current flow reaches its maximum value, denoted by $I_{S C}$. At these extremes the power, $P=V \cdot I$, is zero. However, between these values, and as shown in Figure 9-12, the power reaches maximum, when the voltage is $V_{P \max }$ and current is $I_{P \max }$. This marks the condition at which the solar cell should be operated. The current curve in Figure 9-12 can be approximated using the expression below

$$
I=\left\{\begin{array}{cl}
I_{S C} & \text { if } \quad V<V_{N L}  \tag{9-82}\\
I_{S C} \cos ^{n}\left(\frac{\pi}{2}\left(\frac{V-V_{N L}}{V_{O C}-V_{N L}}\right)\right) & \text { if } \quad V \geq V_{N L}
\end{array}\right.
$$

Where $I$ is the current from the circuit, $I_{S C}$ is the short circuit current, $V_{N L}$ is the voltage where curve becomes non-linear, $V_{O C}$ is the open circuit voltage, $V$ is the circuit voltage, and $n$ is an exponential, which here varies linearly from 0.1 for $100 \%$ irradiance to 0.25 for $50 \%$ irradiance.

Table 9-2: Summary of Pros and Cons of Silicon-based Solarcells

| Type | Pros | Cons |
| :--- | :--- | :--- |
|  | Environmentally friendly as they are void of | Low efficiency. Low space-efficiency |
| (toxic heavy metals, such as lead. Simple to |  |  |
| (sometimes 4x less than monocrystalline). |  |  |
| This increases the cost of PV-equipment (e.g. |  |  |
| mass-produce. Inexpensive. Efficiency is |  |  |
| support structure and cables). Degrade faster |  |  |
| than mono- and polycrystalline solar panels |  |  |
| (so they typically come with a shorter |  |  |
| warranty). |  |  |




Figure 9-12: Current-versus-voltage for a typical solar cell (left) and effect of irradiance on.

## Factors of Importance

The most important parameter in the use of solar cells is the efficiency that can be achieved. A solar cell capable of, say, $20 \%$ conversion efficiency may operate at much lower value in practice. Factors that ultimately determine the operational efficiency include (but are not limited to) shading, orientation, environmental temperature, and cleanliness. Shading will not just bring down the efficiency of the solar cells; partial shading can cause the system of cells to fail. The efficiency also depends on the orientation of the panel with respect to sunlight. The extraction of maximum achievable power requires the cell to be oriented such it is perfectly normal to the incoming sunlight. Additionally, efficiency drops slightly in excessive heat. Cleanliness is important because dust and grime can reduce the penetration of sunlight.

Finally, the Icaré 2 solar powered sailplane shown in Figure 9-13 is a culmination of the technological advancement of solar panels. Designed and built by the faculty of the department of Aero- and Spacetechnology of the University of Stuttgart, Germany, the carbon-fiber-honeycomb sandwich airplane has racked up various accolades, including six long range world records. The longest of those was $518.3 \mathrm{~km}(280 \mathrm{~nm})$ along a pre-defined course. The airplane illustrates important elements of solar airplane design; high wing aspect ratio, tadpole fuselage, and laminar flow surfaces. The airplane can maintain flight with solar radiation as lows as $500 \mathrm{~W} / \mathrm{m}^{2}$. It can take off under own power, using energy from an onboard battery, and climb at almost $2 \mathrm{~m} / \mathrm{s}$ ( 400 fpm ). Its wing span is $25 \mathrm{~m}(82 \mathrm{ft})$, empty and gross weight is $300 \mathrm{~kg}\left(662 \mathrm{lb}_{\mathrm{f}}\right)$ and $390 \mathrm{~kg}\left(860 \mathrm{lb}_{\mathrm{f}}\right)$ respectively, best glide ratio is 36 , and it generates a maximum of 3600 W of power using solar cells capable of $17 \%$ efficiency. It electric motor is capable of 12 kW (16 BHP).


Figure 9-13: The Icaré $\mathbf{2}$ in flight. It is a single seat fully solar powered aircraft. Note the folding propeller mounted on the top of its vertical tail. (Photo courtesy of Icare)

### 9.4.3. Active Means for Energy Replenishing - Piezo-Electric Energy

Piezo-electrics refer to a property of selected solid materials that become charged when subjected to mechanical stress. In other words; such materials develop electricity when placed in compression or tension or while accelerating (as this deforms the material). The phenomenon was first observed by the

Swedish botanist Carl Linnaeus (1707-1778, 70) and the German physicist Franz Aepinus (1724-1802, 77) in the 1750s, although it first appears in a publication written by the French physicist brothers Jacques (1856-1941, 84) and Pierre Curie (1859-1906, 46) in 1880 [14]. The theory of how piezo-electrics work is beyond the scope of this work. Piezo-electrics have been considered a potential source of electricity for use in sUAV and MAV, even complementing other sources such as solar panels. The advantage of such devices is that they permit energy to be replenished regardless of availability of sunlight (on which solar cells depend) (e.g. see Anton and Inman [15]).

Often, the energy extracted from a piezo-electric is stored in a capacitor. A typical piezo-electric generator circuit is shown in Figure 9-14. Thus, the energy $E$ (in Joules or W•s) stored in a capacitor is given by

$$
\begin{equation*}
E=\frac{1}{2} C V^{2} \tag{9-83}
\end{equation*}
$$

Where $C$ is the capacitance in Farads ( $1 \mathrm{~F}=1 \times 10^{6} \mu \mathrm{~F}$ ) and $V$ is the voltage across the capacitor. Then, a sharp acceleration will generate a stream of electrons that will cause a spike in voltage $\Delta V$, which, in turn, will add energy of amount $\Delta E$ to the capacitor

$$
\begin{equation*}
\Delta E=\frac{1}{2} C \Delta V^{2} \tag{9-84}
\end{equation*}
$$

As an example, consider a typical consumer capacitor of $250 \mu \mathrm{~F}$, subjected to a voltage spike $\Delta V=0.5 \mathrm{~V}$ when the piezo-electric is tapped. Using Equation (9-84) we get

$$
\Delta E=\frac{1}{2} C \Delta V^{2}=\frac{1}{2}\left(\frac{250}{1 \times 10^{6}}\right)(0.5)^{2}=0.00003125 \quad \mathrm{~W} \cdot \mathrm{~s}
$$



Figure 9-14: A simplified electric circuit diagram of a piezo-electric generator
In comparison, a 3-cell, 2000 mAh LiPo battery stores energy that amounts to $2 \mathrm{Ah}=2 \times 3600 \mathrm{~A} \cdot \mathrm{~s}=7200$ A.s. At nominal voltage of 11.1 V the energy in the battery is $7200 \times 11.1=79920 \mathrm{~W} \cdot \mathrm{~s}$. Thus, one would expect the "number" of taps required to charge up the battery to be in the excess of 79920/0.00003125
$=2.56 \times 10^{9}$. A piezo-electric generator of this sort, oscillating at 1000 Hz would charge the battery in about 710 hours ( 30 days)! Of course a well engineered generator would consist of multiple units that would bring this time down, although this number highlights the challenges with the current technology.

As stated in the literature survey, a number of papers discussing the use of piezo-electrics in sUAV and MAV can be found. As stated in the survey, Anton and Inman [15] experimented with a sUAV and found that charging a capacitor using piezo-electrics was possible, although the amount was limited and was "...orders of magnitude less than that of the solar panels,...". Feiguel et al [16] also studied the viability of energy harvesting using piezoelectric devices using an sUAV and reported insufficient energy harvesting capability due to the small oscillation frequency and low strain magnitudes of the wing during flight. Both authors recommend actions that can help improve the recharging potential. Presently, the use of piezo-electrics for this purpose is largely experimental and it is a technology that must be given more time to mature.

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## 10. The Generic Intelligent Control Algorithm

This chapter discusses the remaining topics required to permit the creation of the GICA; the path planning and obstacle avoidance modules. It will also discuss the modules that make up the GICA. Figure 10-1 shows a flow chart that depicts how these modules interact with one another. While the SURFACES Flight Simulator will eventually have all of these modules, it is not necessary to have all of them functional to demonstrate the energy harnessing effectiveness of the algorithm, which is the primary focus of this work. Examples of energy harnessing effectiveness are the center of Chapter 11, Simulation Samples. The static obstacle planner has already been coded and its functionality demonstrated, as evidenced later in this chapter. Like all computer codes, this work remains in development and it remains to be tied in with the functionality of the flight simulator's autopilot.


Figure 10-1: This chapter presents the operation of the GICA.

### 10.1 Basics of Trajectory Planning

Path planning and obstacle avoidance are closely entwined; by default, a proper path-planning is not completed until all "known" obstacles are avoided. The operator of the sUAV in which the GICA functions is not expected to spend much effort planning a mission in such detail - that is what the GICA is for. The work of the operator is expected to be limited to specifying departure and destination waypoints, as well as any geographic locations of interest. The polishing of this flight plan, from here on referred to as the user-mission, takes place in the two modules highlighted in Figure 10-2. The LiSSA module, short for Lift-Seeking-Sink Avoidance, is explained in Section 10.3, The Lift-Seeking Sink-

Avoidance Algorithm. The Static Obstacle Planner is discussed in Section 10.4, The Static Obstacle Avoidance Algorithm. However, first it is necessary to establish some mathematics and important concepts.


Figure 10-2: The LiSSA module and Static Obstacle Planner highlighted.
In terms of aerial navigation, trajectory planning is the preparation and specification of an intended path from some geographical departure position to a destination position. This is ordinarily accomplished by specifying the latitude and longitude of a position on the surface of the Earth. Rather than using position coordinates specified in degrees and minutes, the SURFACES Flight Simulator uses distance coordinates that reference the origin of the topography used (see below). As we recall from Section 1.5, Features of the GICA, the resulting list of waypoints constitutes the mission (see Figure 10-4). The LiSSA module will break the entire mission into small elements and determine the cost of traversing each element using performance theory. An visual depiction of the a typical match between the performance prediction and simulation is shown in Figure 10-3.

Among parameters returned is the total energy consumption (fuel weight or battery capacity) required to complete the mission, total travel time, and final altitude (which is a measure of potential energy), just to name a few. This is carried out for the original mission and for the PFM and BPS optimization methods. Then, the LiSSA selects the most efficient path, while making all of the user's original waypoints. As will be shown in Chapter 11, Simulation Samples, this can result in substantial energy savings. The LiSSA also checks for ground proximity and if the trajectory violates altitude minimums, it adjusts waypoints to prevent ground collision. Upon the completion of this work, the GICA directs the refined mission through the Static Obstacle Planner, which determines if any static obstacles pose problems en-route. If so, it adjusts the waypoints around the obstacle. This mission modification can result in reduction in the resulting energy recovery.


Figure 10-3: This view compares a portion of the "predicted" versus the simulated trajectories. The vertical bars represent the performance prediction and the dashed lines are straight segments between waypoints. The simulation is mostly superimposed on the predicted path, although a distinct deviation can be seen near the rightmost waypoint. This deviation is largely due to the tight turn the simulated aircraft had to make, whereas the performance prediction is forced along the intended trajectory.

## Definitions

A mission always consists of at least two points; departure and destination waypoints, but can contain any number of waypoints. Each waypoint is a 3-dimensional entity; with an East ( $x$ ), North (y), and Elevation ( $z$ ) ordinates. The first waypoint in the list is always the departure waypoint and the last one is always the arrival waypoint. The number of waypoints permitted in SURFACES Flight Simulator is limited only by computer resources. A line passing through two waypoints is the path segment or simply a segment; a mission consisting of 10 waypoints contains 9 segments. The segment is used to establish the direction in which the operator must "point" the vehicle; this direction is the heading angle or simply the heading. The vehicle's autopilot will then navigate from one waypoint to the next, while attempting to stay as close to the path segment as possible. A waypoint to which the aircraft is headed is called the active waypoint. The act of creating a mission is referred to as path planning. In order to improve the accuracy of segment following, the SURFACES Flight Simulator uses virtual offset correction; in which a virtual waypoint is inserted close to the "previous" waypoint to help the vehicle intercept the segment (see Figure 10-4). This is necessitated by the fact that when the aircraft arrives at the active waypoint, it inevitably overshoots the waypoint and, thus, once it set the heading to the next waypoint, it may be at an angle with respect to the segment. Overshooting of this nature is particularly pronounced in high tailwind conditions.

In this document, the mission always begins after take-off, when the vehicle is put on a heading toward the first waypoint in the list. The Automated Flight Management System (AFMS) (see Section 3.3, Automatic Flight Management System) keeps track of the distance between the active waypoint and the current position of the airplane. Eventually, this distance becomes small enough to consider the airplane having arrived at (or visited) the waypoint. The exact minimum distance for arrival depends on various parameters, for instance speed, turn radius, but it may as well be some arbitrary distance. Upon arrival, the next waypoint is activated, which causes the autopilot to turn and head toward it. Once the airplane arrives at that waypoint, the next one is activated, and so forth, until the list is exhausted. When this happens, the mission is


Figure 10-4: An example mission. considered completed.

Consider the operation of a surveillance sUAV, intended to patrol a geographic region. An example of such a hypothetical surveillance flight plan is shown in Figure 10-5. Block 1 allows the operator to store previously created "standard" patrol missions on a computer, "reload" them, and "run" on a need-tobasis using the AFMS. Reckless mission planning in mountainous topography may result in the trajectory penetrating terrain. For this reason, this block will check for this situation, using the mathematics presented below. Figure 10-5 shows the consequence of such a check and if printed in color, yellow and red segments can be seen superimposed on blue dashed segment lines in selected areas. The yellow color indicates regions for which the segment is closer than 500 ft to the ground. The red color indicates regions where the segment goes through the terrain. This gives the operator an instant feedback about the quality of the flight plan. Regardless, even if unnoticed, the GICA will not permit the airplane to collide with the terrain and automatically avoids it, as explained in Block 3 (pg. 337).

As stated in Section 10.1, Basics of Trajectory Planning, a mission segment is a line passing through two waypoints. An example of such a segment is shown in Figure 10-6. This definition allows for a mathematical description of the segment and convection anywhere along it to be estimated. Consider an arbitrary segment passing through the waypoints $i$ and $i+1$. Then, any point along the segment can now be determined using the parametric representation

$$
\begin{align*}
& x=x_{i}(1-t)+x_{i+1} t \\
& y=y_{i}(1-t)+y_{i+1} t  \tag{10-1}\\
& z=z_{i}(1-t)+z_{i+1} t
\end{align*}
$$



Figure 10-5: An example of a poorly planned flight trajectory, destined to direct the vehicle through several mountains.


Figure 10-6: Definition of a segment stretching between two waypoints.
Where $i$ is the segment index and $t$ is a parameter that varies from 0 (segment start waypoint) to 1 (segment end waypoint). The heading angle, denoted by $\theta_{i}$, is the angle of the segment $i$ in the $x-y$ plane and is determined as follows:

$$
\begin{equation*}
\theta_{i}=\tan ^{-1}\left(\frac{y_{i+1}-y_{i}}{x_{i+1}-x_{i}}\right) \tag{10-2}
\end{equation*}
$$

The length of a segment is the norm of the vector representing it and is given by

$$
\begin{equation*}
s_{i}=\sqrt{\left(x_{i+1}-x_{i}\right)^{2}+\left(y_{i+1}-y_{i}\right)^{2}} \tag{10-3}
\end{equation*}
$$

The total range of the mission is

$$
\begin{equation*}
s_{1 \rightarrow N}=\sum_{i=1}^{N-1} s_{i}=\sum_{i=1}^{N-1} \sqrt{\left(x_{i+1}-x_{i}\right)^{2}+\left(y_{i+1}-y_{i}\right)^{2}} \tag{10-4}
\end{equation*}
$$

Where the subscript $1 \rightarrow N$ denotes the segment extending from waypoint 1 to $N$. The angle of ascent along the $z$-axis is called the required climb angle. The angle of descent along the $z$-axis is called the required descent angle. Both are calculated using the expression below.

$$
\begin{equation*}
\gamma_{i}=\tan ^{-1}\left(\frac{z_{i+1}-z_{i}}{l_{i}}\right) \tag{10-5}
\end{equation*}
$$

If the arrival at a waypoint is dictated by when the aircraft is closer than the turn radius, $R$, at a given airspeed, then Equation (6-128) can be used to determine this distance as follows

$$
\begin{equation*}
R=\frac{V^{2}}{g \sqrt{n^{2}-1}}=\frac{V^{2}}{g \sqrt{\cos ^{-2} \phi-1}} \tag{10-6}
\end{equation*}
$$

Where $\phi$ is the bank angle. This means a maximum bank angle (e.g. $10^{\circ}, 20^{\circ}$, etc) could be established and this used to derive a distance $R$ that would be used to trigger a bank to the next waypoint in the mission. This approach is used by the SURFACES Flight Simulator.

With the mission defined, each segment is now broken into a number of elementary segments. Consider Figure 10-6, which shows the segment $s_{i}$ between the waypoints $i$ and $i+1$. We break the segment into $M$ equally long finite elements, such that the segment $s_{i}$ is defined as

Where

$$
\begin{gather*}
s_{i}=\sum_{j=1}^{M} \Delta s_{i j}  \tag{10-7}\\
\Delta s_{i j}=\frac{\sqrt{\left(x_{i+1}-x_{i}\right)^{2}+\left(y_{i+1}-y_{i}\right)^{2}}}{M} \tag{10-8}
\end{gather*}
$$

With the finite elements defined, using Equation (10-1), we can define the endpoint of each element as follows. If the segment consists of $M$ elements, then each element can be associated with the parameter $t$ of Equation (10-1) using the elemental parameter, defined as follows:

$$
\begin{equation*}
\Delta t=\frac{\Delta s_{i j}}{s_{i}}=\frac{\frac{\sqrt{\left(x_{i+1}-x_{i}\right)^{2}+\left(y_{i+1}-y_{i}\right)^{2}}}{M}}{\sqrt{\left(x_{i+1}-x_{i}\right)^{2}+\left(y_{i+1}-y_{i}\right)^{2}}}=\frac{1}{M} \tag{10-9}
\end{equation*}
$$

Using the elemental parameter, the distance from the segment's starting point can be expressed as

$$
\begin{equation*}
s_{i}(j)=\sum_{k=1}^{j} \Delta s_{i k}=\sum_{k=1}^{j} k \cdot \Delta t \tag{10-10}
\end{equation*}
$$

Where $k$ is an index. For instance, a unit long segment consisting of 10 elements has $\Delta t=0.1$. Thus, when $j=1$, the segment is 0.1 units, when $j=7$, the segment is 0.7 units, and so on. And the value of the parameter at index $j$ is given by

$$
\begin{equation*}
t_{j}=\sum_{k=1}^{j} k \cdot \Delta t \tag{10-11}
\end{equation*}
$$

Therefore, the position of a point on the mission segment associated with index $j$ is

$$
\begin{align*}
& x_{j}=x_{i}\left(1-t_{j}\right)+x_{i+1} t_{j} \\
& y_{j}=y_{i}\left(1-t_{j}\right)+y_{i+1} t_{j}  \tag{10-12}\\
& z_{j}=z_{i}\left(1-t_{j}\right)+z_{i+1} t_{j}
\end{align*}
$$

This position can be compared to the elevation of the topography at the point to evaluate if it is above or below the terrain.

Using the tiling methodology in the construction of the NURBS topography, in which the "world" plane is defined as the rectangle $\left(X_{\min }, Y_{\min }\right)-\left(X_{\max }, Y_{\max }\right)$ with tile size $(\Delta X, \Delta Y)$, the geographical position expressed by Equation (10-12) can be related to any tile with indices $1, \varphi$, as follows:

$$
\begin{align*}
& \mathrm{\imath}=\operatorname{int}\left(\frac{x_{j}-X_{\text {min }}}{\Delta X}\right)+1=\operatorname{int}\left(\frac{x_{i}\left(1-t_{j}\right)+x_{i+1} t_{j}-X_{\min }}{\Delta X}\right)+1 \\
& \varphi=\operatorname{int}\left(\frac{y_{j}-Y_{\text {min }}}{\Delta Y}\right)+1=\operatorname{int}\left(\frac{y_{i}\left(1-t_{j}\right)+y_{i+1} t_{j}-Y_{\text {min }}}{\Delta Y}\right)+1 \tag{10-13}
\end{align*}
$$

Since the convection is known at each tile that defines the topography (and as described in Section 5.4.4, Constant Mass Flow (CMF) Model for Wind Simulation), this allows this GICA to optimize the airspeeds along each segment of the mission, something of importance for use in Block 2.

### 10.2 Trajectory Planning

In this work, the term trajectory planning refers to the sequential placement of waypoints on topography to permit an autopilot to complete a mission by conducting waypoint following. Consider a situation in which we want to determine the shortest possible route from City A to City B. One way of doing this is to determine the length of every road-segment we find on a map and then create paths
using every single segment found on the map. If enough road segments exists, such as is the case in large countries, we would be confronted with the assembly and evaluation of millions of possible paths. Most of the segments are likely unworthy of consideration, because they would simply result in very long distances. We want to consider methods that exclude such paths, leaving only ones that yield practical paths.

### 10.2.1. The Shortest Path Problem

The Shortest Path Problem (SPP) refers to a mathematical problem that involves the determination of the shortest path through a collection of path segments that consists of nodes (or vertices) that are connected using lines, or edges, each that has been assigned a specific value, called weight (e.g. see Figure 2-2). These weights can be thought of as distances, or the products of distance and speed, or they can represent any other suitable cost function. The path begins with a departure node and terminates with a destination node. The sum of weighted edges from the departure to the destination that yields the lowest (or highest) sum is the solution to the problem. The problem is solved using a branch of mathematics called graph theory, which is the study of graphs, where the term graph refers to a set of objects that are interconnected or linked. The SPP is solved using various methods or algorithms, of which Dijkstra's algorithm is probably best known. Other path algorithms presented here are the more practical version of Dijkstra's, the Bellman-Ford algorithm, and a modification of it called the Edge Exclusion algorithm. The SPP is of great importance here, because in some cases the GICA must select between a few possible paths. It will then regard those paths as a shortest path problem to decide which to select. The problem here is further compounded by the fact that the selection of proper airspeeds (for climb, cruise, and descent) may result in large differences in the weight of an edge.

In mathematics, the term directed graph refers to a set of vertices connected by edges that have a direction associated with them. It is important to realize that an edge either goes into or out of a vertex (waypoint). This is an important distinction, because it dictates whether it is possible to arrive at or depart from (or both) a particular vertex. In this dissertation, an edge is considered positively vectored if it is directed out of a vertex otherwise it is negatively vectored. Consider a mission defined on a directed graph $G$ as the set of $N$ orderly waypoints $M:\left\{p_{1}, p_{2}, \ldots, p_{N}\right\}$, where $p_{i} \in \mathbb{R}^{3}$ is the triplet $\left(x_{i}, y_{i}, z_{i}\right)$. Define the segment $i$ as the vector that extends from $p_{i}$ to $p_{i+1}$. The number of segments is $N-1$. These definitions are used in the discussion that follows.

### 10.2.2. Dijkstra's Algorithm

As briefly mentioned in Chapter 2, Dijkstra's algorithm is attributed to the Dutch computer scientist Edsger W. Dijkstra (1930-2002, 72), considered by many one of the founders of software engineering (e.g. see Hashagen et al [1] or Henderson [2]). However, it was first published Leyzorek, Gray, Johnson, Ladew, Meaker, Petry, and Seitz in 1957 (Schrijver [3]). Dijkstra published this algorithm in 1959 (see Dijkstra [4]). The algorithm is described as follows: Given the weighted, directed graph $G=(V, E)$, with weight function $w: E \rightarrow \mathbb{R}$, mapping edges to real-valued weights. The weight of path $M:\left\{p_{1}, p_{2}, \ldots\right.$, $\left.p_{N}\right\}$ is the sum of the weights of its constituent edges:

$$
\begin{equation*}
w(p)=\sum_{i} w\left(p_{i+1}-p_{i}\right) \tag{10-14}
\end{equation*}
$$

Then, we define the shortest-path weight from vertex $u$ to vertex $v$ by

$$
\delta(u, v)=\left\{\begin{array}{c}
\min \{w(p): u \stackrel{p}{\rightarrow v}\}  \tag{10-15}\\
\infty
\end{array}\right.
$$

This holds as long as such a path exists. If not, then the shortest path from vertex $u$ to vertex $v$ is any path $M$ with weight $w(p)=(u, v)$. It has a worst case running time of $\mathrm{O}(\mathrm{E}+\mathrm{V} \log \mathrm{V})$

### 10.2.3. Bellman-Ford (Shimbel's) Algorithm

The primary drawback of Dijkstra's algorithm is that it only permits positive edge weights. This limits its practicality for use in the GICA, because energy recovery (potential energy gained or lost) can be positive or negative. The remedy is offered by the Bellman-Ford algorithm, which permits both. The algorithm was first proposed by Shimbel in 1954, elaborated on by Moore in 1957, and independently rediscovered twice, first by Woodbury and Dantzig in 1957 and, second, later by the American mathematician Richard Bellman (1920-1984, 63) in 1958, who used Lester Ford's (1927-) formulation of edge relaxation (Erickson [5]). The algorithm is best described in an example. This will also permit an opportunity for the key differences between it and the modification made to it to be discussed.

## Example of Bellman-Ford Algorithm

Consider the weighted graph of Figure 10-7 and for which nodes $A$ and $H$ are the initial and target nodes, respectively. The weights of each edge are shown too. The complete implementation of the algorithm is shown in Figure 10-8 (as it applies to Figure 10-7) and consists of the following steps.


Figure 10-7: An example weighted graph for shortest path analysis.

STEP 1: Consider line 1, column $A$ (representing vertex $A$ ) of Figure 10-8. Traversing from $A$ to $A$ is a trivial solution with weight of 0 . Note that the subscripts represent the path to the particular point. Thus, $A B$ means from $A$ to $B$, or $A \rightarrow B$, and so forth. Only three vertices can be reached from vertex $A ; B$,
$C$, and $D$. The weight of edge $A B$ is 4 (i.e. $4_{A B}$ ), edge $A C$ is $1\left(1_{A C}\right)$, and edge $A D$ is $3\left(3_{A D}\right)$. The rest of the vertices cannot be accessed at this point and this is indicated by the infinity signs in the remaining columns in line 1 of Figure 10-8. Furthermore, the traverse from A to A is a complete step, and this is indicated by shading the cell. We call this cell closed, indicating the trip from vertex A to that vertex (in this case $A$ ) is complete. The next step requires us to identify the cell with the lowest value that is not yet closed. This is found in column $C$, representing the segment from $A \rightarrow C$, whose weight is 1 . This cell is used to start the next step and we indicate so by placing $C$ (i.e. $1_{A C}$ ) in line 2.

| Line |  | A | B | C | D | E | F | G | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | A | $0_{\text {AA }}$ | $4_{\text {AB }}$ | $1_{\text {AC }}$ | $3_{\text {AD }}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| 2 | C |  | $4_{\text {AB }}$ | $1_{\text {AC }}$ | $2_{\text {ACD }}$ | $4_{\text {ACE }}$ | $\infty$ | $\infty$ | $\infty$ |
| 3 | D |  | $3_{\text {ACDB }}$ |  | $2_{\text {ACD }}$ | $3_{\text {ACDE }}$ | $5_{\text {ACDF }}$ | $7_{\text {ACDG }}$ | $\infty$ |
| 4 | E |  | $3_{\text {ACDB }}$ |  |  | $3_{\text {ACDE }}$ | $5_{\text {ACDF }}$ | $4_{\text {ACDEG }}$ | $\infty$ |
| 5 | B |  | $3_{\text {ACDB }}$ |  |  |  | $5_{\text {ACDF }}$ | $4_{\text {ACDEG }}$ | $\infty$ |
| 6 | G |  |  |  |  |  | $5_{\text {ACDF }}$ | $4_{\text {ACDEG }}$ | $9_{\text {ACDEGH }}$ |
| 7 | F |  |  |  |  |  | $5_{\text {ACDF }}$ |  | $7_{\text {ACDFH }}$ |
| 8 | H |  |  |  |  |  |  |  | $7_{\text {ACDFH }}$ |

Figure 10-8: An example of how Dijkstra's and Bellman-Ford algorithms process. See text for details.
STEP 2: Starting with column C, line 2, enter $1_{A C}$. Only two open vertices can be reached from vertex C; D and $E$. Column $A$ is closed so nothing is written there. Vertex $B$ cannot be reached from $C$, so transfer $4_{A B}$ from line 1 to there. Vertex $C$ completes the path from $A \rightarrow C$, so fill the cell with grey to indicate it has been closed. Vertex $D$ can be reached from $C$ through $A \rightarrow C \rightarrow D$ with weight $1+1=2$. This is less than the previous weight of $3_{A D}$, so replace it by entering $2_{A C D}$ there. Vertex $E$ can also be reached from $C$ through $A \rightarrow C \rightarrow E$, so write $1+3=4_{A C E}$ there. The remaining vertices $(F, G, H)$ are out of reach. The lowest open value in this line is $2_{\text {ACD }}$ in column $D$.

STEP 3: Starting with column $D$, write $2_{A C D}$ in line 3. Four open vertices can be reached from vertex $D ; B$, $E, F$, and $G$. Column $A$ is closed. Vertex $B$ can be reached from vertex $D$ through $A \rightarrow C \rightarrow D \rightarrow B$, with weight $1+1+1=3$. This is less than the previous weight of $4_{A B}$, so enter $3_{A C D B}$ in column $B$, line 3 . Column $C$ is closed. Column $D$ terminates in this line, so fill the cell with grey to close it. Vertex $E$ can be reached from vertex $D$ through $A \rightarrow C \rightarrow D \rightarrow E$, with weight $1+1+1=3$. This is less than the previous weight of $4_{A C E}$, so replace it by entering $3_{A C D E}$ in line 3 . The minimum weight for vertex $F$ through $D$ is $A \rightarrow C \rightarrow D \rightarrow F$, with weight $1+1+3=5_{\text {ACDF }}$. The minimum weight for vertex $G$ through $D$ is $A \rightarrow C \rightarrow D \rightarrow G$, with weight $1+1+5=$ $7_{A C D G}$. Vertex $H$ is out of direct reach from vertex $D$. The lowest open value in line 3 occurs in columns $B$ and $E$. We can select either one; here let's select column $E$.

STEP 4: Starting with column E, write $3_{\text {ACDE }}$ in line 4. Only two open vertices can be reached from vertex $E ; D$ and $G$. Column $A$ is closed. Vertex $B$ cannot be reached from $E$, so line 4, column $B$ retains its previous weight of $3_{\text {ACDB }}$. Columns $C$ and $D$ are closed. Column $E$ terminates in this line, so fill the cell with grey to close it. Vertex $F$ cannot be reached from $E$, so line 4, column F retains its previous weight of $5_{A C D F}$. Vertex $G$ can be reached from vertex $E$ through $A \rightarrow C \rightarrow D \rightarrow E \rightarrow G$, with weight $1+1+1+1=4_{A C D G}$.

This is less than the previous weight of $7_{A C D D}$, so replace it by entering $4_{A C D G}$ there. Vertex H remains out of reach. The lowest open value in line 4 occurs in columns B.

STEP 5: Starting with column $B$, write $3_{\text {ACDB }}$ in line 5 . Only one open vertex can be reached from vertex $B$; F. Column A is closed. Column B terminates in this line, so fill the cell with grey to close it. Columns C, D, and $E$ are closed. Vertex $F$ can be reached from $B$ through $A \rightarrow B \rightarrow F$, with weight $4+6=10$. This is higher than the current weight of $5_{\text {ACDF }}$, so write the previous value in line 5 . Vertex $G$ cannot be reached from vertex $B$, so write the previous value in line 5 . Vertex $H$ is out of reach. The lowest open value in line 5 occurs in column G .

STEP 6: Starting with column $G$, write $4_{\text {ACDEG }}$ in line 6 . Only two open vertices can be reached from vertex G; F and H. Column A through E are closed. Vertex $F$ can be reached from $G$ through $A \rightarrow C \rightarrow D \rightarrow E \rightarrow G \rightarrow F$, with weight $1+1+1+1+4=8$. This is higher than the current weight of $5_{A C D F}$, so write the previous value in line 6 . Column G terminates in this line, so fill the cell with grey to close it. Vertex H can be reached from G through $\mathrm{A} \rightarrow \mathrm{C} \rightarrow \mathrm{D} \rightarrow \mathrm{E} \rightarrow \mathrm{G} \rightarrow \mathrm{H}$, with weight $1+1+1+1+5=9$. Enter $9_{\text {ACDEGH }}$ in line 6 , column H . The lowest open value in line 6 occurs in column $F$.

STEP 7: Starting with column F, write $5_{\text {ACDF }}$ in line 7. Only H can be reached from vertex G . Column A through $E$ are closed. Vertex $H$ can be reached from $F$ through $A \rightarrow C \rightarrow D \rightarrow F \rightarrow H$, with weight $1+1+3+2=$ 7. This is lower than the current weight of $9_{\text {ACDEGH }}$, so enter $7_{\text {ACDFH }}$ in line 7 . Column $F$ terminates in this line, so fill the cell with grey to close it. The lowest open value in line 7 occurs in column H . This is also the last vertex in the table. Enter it in line 8 as shown in Figure 10-8 and fill with grey to close.

We have thus not just determined the shortest path from vertex $A$ to vertex $H$ (which goes through $A \rightarrow C \rightarrow D \rightarrow F \rightarrow H$ ), but also determined the shortest path from vertex $A$ to all the other vertices.

### 10.2.4. Edge Exclusion Algorithm

While the Bellman-Ford algorithm is practical for use by the GICA, it yields more information than needed. As illustrated in Figure 10-8, the shortest paths to all vertices have been determined. This is not a necessary requirement for the LiSSA. Only the best path that "progresses" to the arrival waypoint needs to be determined. Also, if we want to ensure the vehicle is always moving toward the arrival waypoint, a step has to be incorporated to make this possible. As can be seen in Section 10.3.3, LiSSA Method 2 - Best Path Search Method, updraft peaks are located and their geographic position is always random. Thus, in order to guarantee motion toward the destination, the trick is to order them based on distance from the departure waypoint. This step is detailed in Section 10.3.3. We will now present an algorithm that only considers paths that connect that departure and arrival vertices.

Given the weighted, directed graph $G=(V, E)$, with weight function $w: E \rightarrow \mathrm{R}$, which map real-valued weights for any edge $j$ that are positively vectored (recall the edge go out of vertices) for any vertex $p_{i}$ $\forall j<i$ and negatively vectored for any vertex $p_{i} \forall j>i$ (as illustrated for a specific example in Figure 10-9).

The vertices of path $M:\left\{p_{1}, p_{2}, \ldots, p_{N}\right\}$ are sorted by projected distance along the directrix from $p_{1}$ to $p_{N}$, where the weight of path $M$ is the sum of the weights of its constituent edges:

$$
\begin{equation*}
w(p)=\sum_{i} w\left(p_{i+1}-p_{i}\right) \tag{10-16}
\end{equation*}
$$

Then, we define the shortest-path weight from vertex $u$ to vertex $v$ by

$$
\delta(u, v)=\left\{\begin{array}{c}
\min \left\{w(p): u \stackrel{p}{\rightarrow}^{w}\right\}  \tag{10-17}\\
\infty
\end{array}\right.
$$

This holds as long as such a path exists. If not, then the shortest path from vertex $u$ to vertex $v$ is the path $M:\left\{p_{1}, p_{N}\right\}$.


Figure 10-9: An example weighted graph for shortest path analysis.
To demonstrate this algorithm in action, again, consider Figure 10-9. It can be seen that any particular edge weight is denoted by the indexes of its constituent vertices. Thus $w_{14}$ refers to the weight of the edge stretching from vertex 1 and 4, and so on. We can now define multitude of paths, for instance, one such path is shown in Figure 10-10 ( $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8$ ). We can also consider the path that skips the $2^{\text {nd }}$ vertex; $1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8$, or skips the $2^{\text {nd }}$ and $3^{\text {rd }}$ vertices; $1 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8$, and so on. Since we demand we move toward the destination and the vertices are ordered by distance, we can break the path into selected paths so we do not have to reconsider path segments that are not a part of the solution. For instance, starting at vertex 1 , there are $N-1$ paths; $1 \rightarrow 2,1 \rightarrow 3, \ldots, 1 \rightarrow N$. From vertex 2 , there are $N-2$ paths; $2 \rightarrow 3,2 \rightarrow 4, \ldots, 2 \rightarrow N$. From vertex 3 , there are $N-3$ paths; $3 \rightarrow 4,3 \rightarrow 5, \ldots, 3 \rightarrow N$, and so forth. This is tabulated in Table 10-1 for the 8 vertices in Figure 10-9. This means that if we determine that the weight associated with direct travel from, say, vertex 1 to 4 is less than either $1 \rightarrow 2$ or $1 \rightarrow 3$, then the problem is reduced to considering the remaining vertices 5 through 8 only. Note that an unfortunate consequence of this is that the resulting path may not be the most efficient one.


Figure 10-10: An example weighted graph for shortest path analysis.
Table 10-1: Possibilities for 1-directional paths from each vertex.

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | $w_{12}$ | $w_{13}$ | $w_{14}$ | $w_{15}$ | $w_{16}$ | $w_{17}$ | $w_{18}$ |
| $\mathbf{2}$ |  | 0 | $w_{23}$ | $w_{24}$ | $w_{25}$ | $w_{26}$ | $w_{27}$ | $w_{28}$ |
| $\mathbf{3}$ |  |  | 0 | $w_{34}$ | $w_{35}$ | $w_{36}$ | $w_{37}$ | $w_{38}$ |
| $\mathbf{4}$ |  |  |  | 0 | $w_{45}$ | $w_{46}$ | $w_{47}$ | $w_{48}$ |
| $\mathbf{5}$ |  |  |  |  | 0 | $w_{56}$ | $w_{57}$ | $w_{58}$ |
| $\mathbf{6}$ |  |  |  |  |  | 0 | $w_{67}$ | $w_{68}$ |
| $\mathbf{7}$ |  |  |  |  |  |  | 0 | $w_{78}$ |
| $\mathbf{8}$ |  |  |  |  |  |  |  | 0 |

Let's apply this algorithm to the analysis table shown in Figure 10-11. Here, let's assume the table contains positive and negative weights and that we are interested in the largest sum of weights. First we see that (in line 1) the segment 1-4 has the greatest gain; 12 . This means we must move down to line 4 to determine where to traverse from vertex 4 . We see that the segment $4-7$, albeit negative, has the highest value in line 4 . This means we move to line 7 to determine where to traverse from vertex 7 . Of course, only one vertex remains; vertex 8 . Therefore, the total cost of traversing $1 \rightarrow 4 \rightarrow 7 \rightarrow 8$ is 12-3-13 $=-4$.

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{2}$ | $\mathbf{7}$ | $\mathbf{1 2}$ | -1 | -27 | -25 | -37 |
| $\mathbf{2}$ |  | 0 | -5 | -3 | -32 | -7 | -15 | -23 |
| $\mathbf{3}$ |  |  | 0 | -6 | 7 | -9 | -17 | -28 |
| $\mathbf{4}$ |  |  |  | 0 | -27 | -4 | -3 | -23 |
| $\mathbf{5}$ |  |  |  |  | 0 | -15 | -12 | -16 |
| $\mathbf{6}$ |  |  |  |  |  | 0 | -11 | -37 |
| $\mathbf{7}$ |  |  |  |  |  |  | 0 | -13 |
| $\mathbf{8}$ |  |  |  |  |  |  |  | 0 |

Figure 10-11: An example weighted graph for shortest path analysis.

### 10.3 The Lift-Seeking Sink-Avoidance Algorithm

This section presents two methods used by the GICA to improve operator generated trajectories in mountainous regions. The term for this approach is what has been mentioned in numerous places as Lift-Seeking Sink Avoidance or LiSSA for short. The two methods are the Potential Flow Method (PFM) and Best Path Search (BPS). The PFM uses the potential flow theory presented in Chapter 4, The Use of Potential Flow Theory in the GICA to generate trajectory through the wind field with a high energy recovery. It shares analogies with the Potential Flow Method introduced in Chapter 2, A Survey of Literature. The BPS identifies peaks of up- and downdrafts and uses a variation of the Bellman-Ford algorithm to find a path through the wind field with the highest energy recovery. Both methods use the performance theories presented in Chapter 6, Aircraft Performance Theory and Chapter 9, Energy Harvesting to evaluate and compare the cost of traversing the modified to the original trajectory.

The LiSSA algorithm uses predictions of up- and downdraft regions in a given topography based on prevailing wind conditions. This is accomplished in the SURFACES Flight Simulator using the built-in wind simulator, which uses the constant mass flow method presented in Chapter 5, Atmospheric Modeling. When more computationally intensive and accurate prediction methods are required, the wind field can be uploaded to the GICA wirelessly, while in flight. Of course, it is to be expected that future development of computers will permit this to be done in an onboard flight computer. At any rate, advanced Navier-Stokes methodologies are beyond the scope of this work and, thus, will not be considered further. The assumption made here is that the GICA has access to an accurate wind field model through software functionality. It is of greater important here is how the LiSSA uses it than how it was obtained.

For demonstration purposes in this work, the constant mass flow method allows the LiSSA to estimate wind conditions at the altitude at which the aircraft operates. As the reader may recall, the shortcoming of the method is its inability to predict flow separation on the leeward side (mountain wakes) and possible formation of mountain waves are not modelled (e.g. see Stull [6]). This can be justified in the path-planning method that re-routes the airplane, placing it above the windward side and deliberately avoiding the leeward regions. It is acknowledged that this simplification does not always apply and viscous effects, which play a major role in wind flow over terrain, are ignored.

### 10.3.1. Development of Cost Function for Energy Harvesting

Cost functions are used to evaluate the loss or gain of conducting a particular action. In the context of flight trajectory optimization, the term "action" can refer to the cost of climbing, cruising, or gliding flight. Largely, we seek a function that assigns a value to some property of interest. It is also convenient to consider cost functions for pure unpowered gliding flight and powered flight separately.

One of the simplest cost function for direct flight along a segment $i$ of the mission estimates the altitude lost or gained while traversing segment. This will amount to the sum of the rate of descent (or ascent) and vertical speed due to atmospheric convection, as shown below

$$
\begin{equation*}
\operatorname{Cost}_{i}=\dot{h}_{i}+w_{z_{i}} \tag{10-18}
\end{equation*}
$$

Where $\dot{h}_{i}$ is the rate-of-change of altitude (rate-of-climb or rate-of-descent) and $w_{z_{i}}$ is the up- or downdraft in which the airplane operates. The expression represents the instantaneous cost. Clearly, a climb in an updraft will return a large positive value, while descending in a downdraft may return a large negative number.

This cost function can be modified to represent the altitude lost or gained along a segment, by integrating with respect to time

$$
\begin{equation*}
\Delta H=\int_{0}^{t}\left(\dot{h}_{i}+w_{z_{i}}\right) d t \tag{10-19}
\end{equation*}
$$

The above cost function can be modified to take speed of traversing into account. Consider the airplane traversing two segments for which the cost function of Equation (10-19) returns an equal value, but one takes less time than the other. If our goal is to maximize the cost, we could incorporate the less time as follows:

$$
\begin{equation*}
\operatorname{Cost}_{i}=\frac{\Delta H}{t}=\frac{V_{i}}{s_{i}} \int_{0}^{t}\left(\dot{h}_{i}+w_{z_{i}}\right) d t \tag{10-20}
\end{equation*}
$$

Where $V_{i}$ is the speed along the segment, assumed to be constant, and $s_{i}$ is the length of the segment. Although time is not always a primary priority for energy harvesting, the higher speed might be favored. It is important to keep in mind that, while time is not a primary priority, some applications may consume low level of energy (idle energy consumption) in soaring flight. This renders it possible for the vehicle to consume more energy during a long segment of soaring flight at idle power, than a shorter segment in powered flight. The time to traverse a segment depends on the ground speed, rather than just the airspeed. In the presence of head- or tailwinds, denoted by $V_{w}$, it is appropriate to modify Equation (1020) as shown below

$$
\begin{equation*}
\operatorname{Cos}_{i}=\frac{\overbrace{\left(V+V_{w}\right)_{i}}^{s_{i}}}{\text { ground speed }} \int_{0}^{t}\left(\dot{h}_{i}+w_{z_{i}}\right) d t \tag{10-21}
\end{equation*}
$$

What matters is the sign of $V_{w}$. This formulation ensures that the cost of traversing a mission segment in headwind yields less cost (worse) than tailwind. It must be kept in mind that the influence of head- or tailwind can have a profound impact on the situation. For instance, if the airplane finds itself in sink, while operating in headwind, the time to traverse will increase and more altitude will be lost. The same holds for lift; less altitude will be gained. Naturally, the units for Equations (10-18) and (10-21) are different, $\mathrm{m} / \mathrm{s}$ versus $\mathrm{m}^{2} / \mathrm{s}$. However, this is irrelevant, because we are interested in the magnitude and cost functions are bound to have odd units. Consistency comes from applying the same cost function to
all mission segments. At any rate, Equation (10-21) returns higher cost (which is desirable if we are maximizing) for the faster of two airplanes given the same value of $\dot{h}_{i}+w_{z_{i}}$.

What is also of importance in the scheme of things is the consumption of fuel along the segment of the mission. In many cases, onboard energy must be consumed to complete a mission segment. While higher speed is incorporated in the cost function by multiplication, the opposite holds for energy consumption. We want this to amount to as low a value as possible. This requires the following modification to our cost function

$$
\begin{equation*}
\operatorname{Cost}_{i}=\frac{\left(V+V_{w}\right)_{i}}{\left(1+\Delta W_{f_{i}}\right) s_{i}} \int_{0}^{t}\left(\dot{h}_{i}+w_{z_{i}}\right) d t \tag{10-22}
\end{equation*}
$$

Where $\Delta W_{f_{i}}$ is the fuel consumed along the mission segment. The " 1 " in the denominator is necessary to avoid a singularity when no fuel is consumed. In case of fossil fueled aircraft, $\Delta W_{f_{i}}$ can be the weight of fuel consumed in kg or $\mathrm{lb}_{\mathrm{f}}$. In case of electric aircraft, $\Delta W_{f_{i}}$ should be the battery energy consumed in Ah.

In order to determine the total cost of traversing a particular path, the cost of the constituent mission segments is summed, as shown below

$$
\begin{equation*}
\operatorname{COST}=\sum_{i} \operatorname{Cost}_{i}=\sum_{i=1}^{N} \frac{\left(V+V_{w}\right)_{i}}{\left(1+\Delta W_{f_{i}}\right) s_{i}} \int_{0}^{t}\left(\dot{h}_{i}+w_{z_{i}}\right) d t \tag{10-23}
\end{equation*}
$$

Where $N$ is the number of mission segments. The integration is best performed numerically.

Powered flight in this context largely revolves around maintaining prescribed altitude, which can involve climb to a new altitude, maintenance of constant altitude, or a descent to a new altitude. Descent can require power when it is desired to maintain airspeed higher than what the descent itself would. The most suitable cost function to compare flight paths is specific range, better known by laypeople as "fuel mileage." This is what the SURFACES Flight Simulator uses for this purpose. It is expressed as follows:

$$
\begin{equation*}
C O S T=\sum_{i} \operatorname{Cost}_{i}=\sum_{i=1}^{N} \frac{s_{i}}{\Delta W_{f_{i}}} \tag{10-24}
\end{equation*}
$$

## Thoughts on Idle Power

We must consider the implications of "idle power" on the energy harvesting scheme. This is not an issue for electrically powered aircraft, which have superbly reliable restart characteristics. The same does not always hold for gas powered engines. The energy harnessing methods usually result in greater distances and time en route, since the vehicle must navigate from energy source to energy source. And, as evident
by the simulation samples of Chapter 11, Simulation Samples, the energy consumption can be substantially reduced (which of course is our primary goal). While this leads to greater energy savings in electrically powered aircraft (because the motor is fully shut off), idle power for piston engines adds complexity to the scheme. Shutting off piston engines en route may be objectionable, because such engines are thought to be subject to "shock-cooling" (a controversial phenomenon, which may be more perception than reality), but more realistically, extensive time off at high altitude may cool the engine enough between restarts to cause restart difficulties. For such engines, the operator may choose to run the engine at idle-power (or other power setting that does not cause propeller wind milling drag to increase). This means the engine is consuming fuel at all times, reducing the fuel saving potential. This necessitates the LiSSA to compare the actual fuel consumption (e.g. kilograms of fuel consumed) for the optimized trajectory to the original user-defined trajectory. For this reason, the LiSSA may select the user-defined trajectory over the optimized ones, despite the cost in $\mathrm{km} / \mathrm{liter}$ or $\mathrm{nm} /$ gallon is favorable for the latter ones.

### 10.3.2. LiSSA Method 1 - Potential Flow Method

The implementation of the PFM places a strong source at the departure waypoint and its negative (i.e. a sink) at the destination waypoint. This accomplishes two functions: (1) The general flow direction is from the departure to the destination waypoint and (2) if there is no mechanical convection, the vehicle will simply follow the planned straight path between the waypoints. In the presence of convection, sources (of much lesser strength) are placed where downdraft is detected and sinks where updraft is detected. The effect of the sources is to "repel" the vehicle, while the sinks "attract" it. This is illustrated in Figure 10-12. The strengths of these elementary flows are prorated based on actual strength of the up- or downdraft to magnify the attraction to strong updrafts and repulsion from strong downdrafts. This effect becomes particularly significant when a large region contains only sources or only sinks, such as that associated with the windward or leeward side of a mountain. Using potential flow theory, we can express the "virtual flow" from the departure waypoint (A) to the destination waypoint (B) that is biased toward updrafts using the potential flow expressions below

$$
\begin{align*}
& u^{*}=\frac{\Lambda_{A}}{2 \pi R_{A}} \cos \theta_{A}+\frac{\Lambda_{B}}{2 \pi R_{B}} \cos \theta_{B}+\sum_{i=1}^{N} \frac{\lambda_{i}}{2 \pi r_{i}} \cos \theta_{i} \\
& v^{*}=\frac{\Lambda_{A}}{2 \pi R_{A}} \sin \theta_{A}+\frac{\Lambda_{B}}{2 \pi R_{B}} \sin \theta_{B}+\sum_{i=1}^{N} \frac{\lambda_{i}}{2 \pi r_{i}} \sin \theta_{i} \tag{10-25}
\end{align*}
$$

Where $u^{*}$ and $v^{*}$ are the virtual velocity components, $\lambda$ are the source strengths, $\Lambda$ is the source strength at the departure or destination waypoints, and the subscript $A$ and $B$ refer to the source and sink at the departure and destination waypoints, respectively. Other variables are specified in Section 4.1, Two-Dimensional Potential Flow Theory. The "heading" associated with the streamline can be obtained from


Figure 10-12: Left figure shows the potential field during calm wind conditions. The right shows the effect of introducing updraft (quadrant II) and downdraft (quadrant IV) in the form of a sink and source, respectively. See text for detail.

$$
\begin{equation*}
\psi=\tan ^{-1}\left(\frac{v^{*}}{u^{*}}\right) \tag{10-26}
\end{equation*}
$$

Figure 10-13 illustrates how the PFM "avoids" the regions of sink and is "attracted" to regions of lift. The two waypoints in the left image represent typical means by which a mission segment might be planned. Note the user can specify a Field-of-View (FOV) in order to affect which sources will be selected to represent the wind field. This limits the trajectory planning to include only mechanical convection inside the FOV sector and reduces the "contamination" the results from accounting for sources and sinks that are behind or far away from the vehicle. As an example, in this world the theoretical maximum number of sources and sinks is 160000 . However, they are rarely all used. It is typical the analysis includes a few thousand sources, although the solution may easily include 40-60 thousand. The user can also limit the magnitude of the up- and downdraft being considered. This is helpful to exclude lift or sink whose magnitude is too low to be of concern.

The PFM will now generate a streamline through the computational domain, starting at Waypoint 1. The algorithm keeps track of each element constituting the streamline and plants waypoints at regular intervals (e.g. 1 km apart) between the departure and arrival points. It extracts only the flow direction from the solution - it marks the heading the autopilot will need to maintain at that position; the speed of this fictitious flow is ignored. It should be stated that the PFM will sometimes generate results that brings the path through regions of downdraft. The inbound trajectory is different from the outbound one. An approach that prevents this from happening has already been developed and may be presented at a later time.

Once the streamline through the flow field has been established, the LiSSA analyses the cost of this path by running it through a "cost analyzer," which converts each segment into elements to which it applies performance theory. The same routine also analyzes the user- and BPS-missions.


Figure 10-13: A top view of the energy map corresponding to the topography and average wind speed of $9 \mathrm{~m} / \mathrm{s}$ at $135^{\circ}$ (SE). The left image shows the energy map before the PFM finds a trajectory with good energy recovery. Note that the unplanned path brings the airplane through indiscriminate regions of lift and sink. In the right image the PFM has created trajectory that ensures the airplane will predominantly fly through regions of lift. Waypoints have been placed at fixed intervals and these constitute a mission. North is up.

### 10.3.3. LISSA Method 2 - Best Path Search Method

The BPS method is fundamentally different from the PFM. Once the wind field has been established in the "world" in which the aircraft operates, the method creates an energy map, like the one shown in Figure 10-14. The energy map, in effect, is a surface that shows the distribution of updrafts (peaks) and downdrafts (valleys). The map in Figure 10-14 was generated for the world used elsewhere in this dissertation, subjected to $30 \mathrm{ft} / \mathrm{s}$ wind at $135^{\circ}$. Using optimization theory, the BPS identifies multiple local maxima of updraft on the map and uses this to find the best path to travel. The number of such vertices depends on complexity of the topography and, as one would expect, the larger number usually leads to missions with better energy recovery. Only vertices inside the "field-of-view", shown in Figure 10-15, will be transformed into a list of waypoints that constitutes the mission. Once the maxima have been identified, the method sorts them based on the distance from the departure waypoint and connects using weighted edges. The algorithm does this by "traversing" along each edge at some specified airspeed, while integrating the energy recovery cost using Equation (10-23). This step is of crucial importance and takes into account the effect of the wind, performance, and propulsion characteristics of the airplane at the altitude at which it travels. It also detects whether the airplane has
collided with terrain along the edges, if headwind is greater than forward speed (a maximum headwind constraint), and the amount of onboard energy consumption (fuel or battery capacity).


Figure 10-14: The energy potential in the mechanical convection is shown here in form of an energy map. The LiSSA identifies the updraft peaks and plans the optimum flight by sort of an island hopping.


Figure 10-15: A top view of the energy map corresponding to the topography and wind speed of $9 \mathrm{~m} / \mathrm{s}$ at $135^{\circ}$ (SE). The left image shows the original direct flight path and the right image shows the optimized mission (1-2-4-10-10) based on the modified Bellman-Ford algorithm.

As already discussed, Equation (10-23) favours altitude gain, ground speed, and low fuel consumption and is easily adapted to electric power. The resulting cost constitutes the weight of edge $j$ (or the energy recovery parameter), which can be returned in terms of total altitude change, consumption of onboard energy, time en route, and so on. Then, the method applies a variant of the Bellman-Ford algorithm, discussed in Section 10.2.4, Edge Exclusion Algorithm, to the collection of edges and determines the
path of maximum energy recovery, while forcing progression toward the destination waypoint. This results from giving importance to arrival time, even though energy harvesting is of primary importance. Ultimately, the goal is to arrive at the destination waypoint as efficiently and fast as possible. An application of unmodified Bellman-Ford may cause the sUAV to move along trajectories that bring it closer to the departure waypoint, risking substantial increase in travel time. Note that since the original trajectory is also an edge, it is possible it will be the selected, for it may well be the best path too. For this reason, the selected path will never be worse than the original path.

### 10.4 Static Obstacle Avoidance Algorithms

In mathematics, the term obstacle avoidance refers to "the task of satisfying some control objective subject to non-intersection or non-collision position constraints." Expressed more clearly, this refers to the modifications that must be made to the velocity of a vehicle to prevent a collision with terrain or man-made obstructions. Avoidance of such obstacles increases with proximity to the ground, but highaltitude avoidance could consist of avoidance of inclement weather, restricted air space, or threatregions. The term obstacle applies to many things; terrain, buildings, towers, trees, regions of inclement weather, regions of noise restrictions, restricted airspaces, threat territory, and so on. A capable mission planner is one that automatically modifies the path to avoid obstacles. An example of a GICA-modified path plan is shown in Figure 10-16. The left image shows the original user plan, the center image shows the underlying obstacles, and the right one shows a modified trajectory, which visits all user waypoints while avoiding obstacles.

The center image in Figure 10-16 shows obstacles that may have been stored in an obstacle database or based on surveillance data, superimposed on the plan. The user mission directs the vehicle to pass through some of them, most likely unbeknownst to the operator. Clearly, if this risk was to be ignored, the mission would terminate at the first collision. The right image in Figure 10-16 shows the modification the GICA has made to the user mission - it ensures the vehicle bypasses all obstacles by a preset distance (called path-distance, as discussed shortly), set by the operator. In this example, the user mission consisted of 8 waypoints, but required additional 20 waypoints to avoid the en-route obstacles. The hatched circles represent regions in which the area density of obstacles is large enough to prevent safe passage. In other words, consider a fixed wing aircraft of a given wingspan. If the separation of two obstacles is less than the wingspan, the aircraft will not be able to pass between them - the two obstacles may as well be combined and considered a single obstacle. Also, it must be recognized (and as shown in Figure 10-4 and Figure 10-16) that the actual track of the vehicle may not always be precisely on top of the intended track. The airplane cannot turn "on a dime," but rather will overshoot each segment as it turns from one segment to the next. The magnitude of the overshooting depends on the speed and inertia of the vehicle and the commanded turn radius.


Figure 10-16: An example mission consisting of 8 waypoints or 7 segments (left). The plot thickens when we superimpose a collection of random obstacles with which the vehicle is at risk of colliding (center). The GICA automatically creates an alternative plan by finding the "shortest" path around the obstacles, while always arriving at each waypoint.

### 10.4.1. Types of Obstacles

As already discussed, the GICA algorithm plans the flight path in an attempt to recover energy from the atmosphere when possible. The LiSSA may stray far from the "desired" flight path, possibly exposing the airplane to obstacles, fixed or moving, that were not a threat had it followed the user mission. For this reason, it is crucial to provide the algorithm with some rudimentary obstacle avoidance capability. This is the focus of this section. In this context, the three following types of obstacles will be considered

- Type I: Static obstacles of "known" position (structures, vegetation, terrain)
- Type II: Static obstacles of "unknown" position
- Type III: Dynamic obstacles (other aircraft, thunderstorms, etc.)

The algorithm stores static obstacles in a database and considers this when planning the trajectory. Such obstacles are referred to as Type I obstacles in this dissertation. Type II obstacle is a static obstacle, whose position is not found in the obstacle database. An example of such obstacle is a tall radio tower not found on maps. To avoid such obstacles, the GICA algorithm must receive an input from a sensor, for instance, stereo cameras or similar, that submits a position and size inside the field of view of the airplane. This signal places the previously unknown obstacle in the database and triggers a flight-plan replanning event. Type III obstacles are dynamic obstacles that the GICA keeps track of and predicts future position near the flight path. Information about dynamic obstacles triggers a forecasting event and a subsequent re-planning event, allow the flight path to be modified such a direct encounter is avoided.

### 10.4.2. Representation of Obstacles

In this dissertation, obstacles come in two geometric forms; circles and polygon. The former is referred to as circular obstacles, the latter as patch obstacles. These types do not preclude the use of other mathematical forms, only that these are not considered here. This section presents algorithms to treat both types, as well as the necessary mathematics. Both types are treated as 2-dimensional (or planar) constructs.

## Circular Obstacles

A collection of objects of relatively simple geometries and/or environment in which objects are distributed in a sparse manner can be represented using circles. In the SURFACES Flight Simulator, each circle has geometric property of position, $(x, y)$, radius, $r$, and height above ground, $h$. The circles are ideal to represent individual trees in a relatively sparse forest, or the occasional radio tower the vehicle might encounter en route, and many other obstacles. The true cross-sectional (or planar) shape of said obstacle is in fact irrelevant, as long as the circle that represents it encloses it completely. There is no need for a vehicle to bypass an obstacle by a few meters or feet. Too many variables affect the available precision of a flight path. This renders it necessary to define a minimum path width, which takes into account factors that affect path precision, such as wind gusts, airspeed deviations, and dynamics of the aircraft. This path width is denoted by $2 \Delta w$. In fact, a collection of closely spaced trees can be replaced by encircling them with a single circle. This process is called consolidation, and the GICA treats all static circular obstacles of Type I in this fashion. Thus, the following requirement holds for the generation of circular obstacles:

$$
\begin{equation*}
\delta(i, j)=\sqrt{\left(x_{j}-x_{i}\right)^{2}+\left(x_{j}-x_{i}\right)^{2}}<2 \Delta w \tag{10-27}
\end{equation*}
$$

Where $\Delta w$ is $1 / 2$ of the necessary path width and d is the distance between. As shown in Figure $10-17$, in order to avoid colliding with a circular obstacle, the path must be offset by distance

$$
\begin{equation*}
r_{\min }=r_{j}+\Delta w \tag{10-28}
\end{equation*}
$$

Thus, the offset distance from the center of the obstacle must be

$$
\begin{equation*}
r_{o f f}=\frac{r_{\min }}{\cos \theta} \tag{10-29}
\end{equation*}
$$

Where $\theta$ is the included angle. With the offset dimension defined, simple vector algebra can be used to place the auxiliary point such the vehicle never gets closer to the obstacle than the distance $\Delta w$.

## Navigation through a Collection of Circular Obstacles

The circular obstacles can be circumvented using several algorithms. Three means of accomplishing this are presented in Figure 10-18. These are called the Gate-Search, Centroid-Search, and Path-Search. The Gate-Search method treats the collection obstacles as triangles. Starting at the current position (called a
reference point) of the vehicle, it looks ahead and determines which two obstacles are closest. These three entities are considered a triangle. Then, the algorithm calculates the midpoint along the edge closest to the destination point. This point becomes the new reference point. It will be used to look ahead to the next two obstacles, forming the next triangle. This is how it works its way toward the destination waypoint. When implemented along the collection, a path similar to the one shown in the second tile of Figure 10-18 may be generated. The Centroid-Search method operates in a similar manner, except it defines the centroid of the triangle as the new reference point. Both algorithms work well for many collections of sparsely distributed obstacles. However, for large number of confined obstacles, for which the separation reduces, the path has a propensity to converge to form tight turns that results in unavoidable collision with obstacles. This problem can be partially remedies using the Path-Search method, which always considers the radial toward the destination and places an offset (as considered above) at the first obstacle from the reference point. This offset point becomes the new reference point, after which a new radial is defined. This method is reliable for most collections of obstacles, sparse or not, and results in the shortest path of the three. Path-Search is analogous to a continuous-space A* algorithm.


Figure 10-17: Bypassing a circular obstacle through offsetting.

## Patch Obstacles

The second type of obstacle is the patch obstacle. It allows features too large to be considered circular to be represented. Examples of patch obstacles include terrain cross-sections, regions of downdrafts (sink), restricted airspaces, and threat territories. The polygons can also be used to represent a large array of complex buildings, when planning trajectories for slower and more maneuverable vehicles, such as multi-copters. A step-by-step procedure to avoid such obstacles is presented in Figure 10-19. The patch obstacle is a polygon stored in the obstacle database. It is defined using a set of $N$ vertices oriented in a clockwise fashion.


Figure 10-18: Three methods to find a path through a collection of obstacles.

### 10.5 Dynamic Obstacle Avoidance Algorithm

Dynamic obstacle avoidance can be incorporated in the GICA in many ways. A common way to do this involves Line-of-Sight methodologies, in which Blocks 5 and 8 (see Figure 10-1) would be utilized. Dynamic obstacle avoidance requires advanced sensors, whose description is beyond the scope of this work. Such sensors are required to detect and help with estimation of the position and velocity of the risk entity. In the GICA, the Dynamic Obstacle Detector (Block 5) triggers a signal that a potential risk entity has been detected and returns current position and velocity. It then evaluates if this entity is a threat by calculating if it intercepts any of the mission segments. If so, it calculates the time of interception and if our aircraft is anywhere near at that time. If the proximity of the two is within a given distance (effectively inside a "sphere" of radius $R$ ), the sphere is treated as a virtual static obstacle that is added as a temporary static obstruction in the obstacle database. This triggers the operation flow to update the flight plan. The remaining mission will now be replanned, going through the LiSSA and Static Obstacle Planner. Since the entire mission is replanned in less than a second, this operation is repeated as long as the Dynamic Obstacle Monitor (Block 8) concludes the threat remains active.

STEP 1: Generate a constant-distance offset geometry around the obstacle (dashed line). Note that each vertex will become a potential bypass-waypoint, to be inserted later.

STEP 2: Determine the locations where the intended path segment (solid arrow) intersects the offset geometry. Here, this occurs in four locations. It can be seen that there is one entry point, two internal points, and one exit point.

STEP 3: Delete the internal points. This is accomplished by sorting the distance of the points from the start point of the original segment. The closest point becomes the entry point and the farthest one the exit point.

STEP 4: The original segment is shortened and terminated at the entry point. This is where new path segments will be added.

STEP 5: Create two temporary paths; both begin and end at the entry and exit points, respectively. However, one path traverses in a clockwise fashion (bottom figure), while the other on traverses in a counterclockwise manner (center figure). It can be seen that the former will consist of 5 waypoints, the latter of 10 .

STEP 6: Determine the length (or cost) of each path, in effect using the aforementioned cost analysis. Here, the clockwise path is shorter, distance wise. However, when available lift is accounted for in an edge weight cost function, there is no guarantee it will always be selected.

STEP 7: Splice the shorter path to the original path.


Figure 10-19: Bypassing a patch obstacle through offsetting and shortest path selection.

### 10.6 Generic Intelligent Control Algorithm

This section brings to close to the development of the elements that are required to create the Generic Intelligent Control Algorithm, GICA. It discusses how the interaction of these elements result in a capable functionality whose action takes into account more than "just" energy conservation, energy harvesting, or obstacle avoidance, but all, simultaneously; the synthesis of these capabilities makes the algorithm intelligent. Figure $10-2$ shows a complete operational flow chart for the GICA. The bold numbers are used for referencing in this dissertation. The section will also highlight the following elements:

- Energy management through energy conservation
- Energy management through energy harvesting
- Energy management through performance optimization
- Obstacle avoidance (adaptable and fixed static obstacles)
- Return to home feature


### 10.6.1. Basic Operation of the GICA

With reference to Figure 10-2, the basic operation of the GICA involves blocks 1 through 8 . These will now be described in more detail. The GICA can be thought of as an event director. An analogy for the GICA is a conductor for a classical music orchestra. The conductor dictates the tempo of the music as well as when specific instruments should enter the music and how "intense" this entry is. Similarly, the GICA decides what maneuver to perform, when to perform it, and how intense the maneuver must be. Rather than dishing out commands to an orchestra, it hands them out to the autopilot. Each maneuver is called an event and is initiated through triggering, a command that tells the autopilot to initiate the maneuver. These commands are reduced to the set of basic maneuvers listed in Table 10-2. As an example, when the GICA decides that an altitude increase is required, it triggers Maneuver 3. Once the target altitude is achieved, it will switch over to Maneuver 1 , until the next maneuver is commanded. If the aircraft is efficient enough to utilize the climb-glide profile described in Section 9.3, The Basics of the Climb-Glide Cruise Profile, it will repeatedly trigger Maneuver 3 followed by Maneuver 4 followed by Maneuver 3 again, and so on until the segment distance is completed. The events are executed in a linear fashion, which means that once an even is triggered, its input value will not just override, but replace the user-entered input values in the control console (see Figure 8-11) used by the autopilot.

### 10.6.2. Description of Individual Blocks Comprising the GICA

This section details the operation of each of the blocks comprising the GICA in Figure 10-1.

## Block 1: Process Initial Flight Plan

In this block, the operator enters an intended flight trajectory to the AFMS. As detailed in Chapter 10, The Generic Intelligent Control Algorithm, the waypoints used by the SURFACE Flight Simulator are 3dimensional: i.e. denoted using ( $x, y, z$ ) coordinates. In this representation, the $x$-ordinate corresponds to the East position, the $y$-ordinate corresponds to the North position and the $z$-ordinate corresponds to the Altitude position in the NED coordinate system, as described in Section 8.1.3, Earth Fixed Coordinate

System (NED). This 3-dimensional representation is essential for allowing the GICA to apply optimal performance schemes, as will be described in Block 4 of Section 10.6.2, Description of Individual Blocks Comprising the GICA.

Table 10-2: Summary of Basic Maneuvers

| ID | Maneuver Name | Input | Output | Comment |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Maintain altitude | $h, V_{\text {CAS }}$ | $\delta_{\text {THR, }}, \delta_{e}$ | Appropriate engine power ( $\delta_{T H R}$ ) and elevator deflection ( $\delta_{e}$ ) is used to maintain altitude $h$ at airspeed $V_{\text {CAS }}$. Power output controls altitude. Elevator controls airspeed. |
| 2 | Maintain heading | $\psi$ | $\delta_{a}$ | Aileron deflection $\left(\delta_{a}\right)$ is used to maintain (wings level) heading. |
| 3 | Climb to altitude | $h, V_{C A S}$ | $\delta_{\text {THR, }}, \delta_{e}$ | Preset engine power is supplied ( $\delta_{T H R}$ ) and elevator deflection ( $\delta_{e}$ ) is used to maintain airspeed $V_{C A S}$, which is typically $V_{y}$. This results in the fastest possible climb and, thus, minimum consumption of energy to reach altitude $h$. |
| 4 | Descent to altitude | $h, V_{C A S}$ | $\delta_{\text {THR, }} \delta_{e}$ | Preset engine power is supplied ( $\delta_{T H R}$ ) and elevator deflection ( $\delta_{e}$ ) is used to maintain airspeed $V_{\text {CAS, }}$, which may or may not be $V_{B G}$. If gas powered, the engine is brought to idle. If electrically powered, the engine is shut off. |
| 5 | Emergency descent | $h, V_{\text {CAS }}$ | $\delta_{\text {THR, }}, \delta_{e}$ | Assumes no engine power. Elevator deflection $\left(\delta_{e}\right)$ is used to maintain airspeed $V_{C A S}=V_{B G}$. |
| 6 | Maintain roll angle | $\phi$ | $\delta_{a}$ | Aileron deflection $\left(\delta_{a}\right)$ is used to establish and maintain roll angle $\phi$. |
| 7 | Maintain position | marked <br> $x_{\text {hold }}, y_{\text {hold }}$ | $\delta_{a}$ | Aileron deflection ( $\delta_{a}$ ) is used to establish and maintain a user entered maximum or limiting roll angle $\phi_{\text {lim, }}$ resulting in a circling flight about the geographic position $x_{\text {hold }}, y_{\text {hold }}$. |

## Block 2: LiSSA

Upon entering this block, the GICA formally takes over the planning of the mission trajectory. The output variables from Block 1 are the mission itself; delivered in an ordered list of 4-dimensional coordinates, consisting of Cartesian points; the original waypoints, and the calibrated airspeed to fly. The block presented here contains the LiSSA, described in Section 10.3, The Lift-Seeking Sink-Avoidance Algorithm. The LiSSA performs only one primary task: It looks at one mission segment at a time and determines the most energy efficient path through the wind field using the wind field information. In doing so, it creates four additional missions: (1) PFM mission using the user selected airspeeds, (2) PFM mission using GICA selected airspeeds, (3) BPS mission using the user selected airspeeds, and (4) a BPS mission using GICA selected airspeeds. If it determines any of these additional mission to achieve greater energy recovery, it modifies the intended mission segment by inserting new waypoints on a need-to-basis. An example of this is shown in Figure 10-13. It illustrates how the initial two-waypoint mission was transformed into a 31-waypoints trajectory. Thus, LiSSA "pushes" the segment to take advantage of regions of updrafts,
while avoiding locales of downdrafts. This block returns a modified list of ordered waypoints. The original waypoint list is returned only if LiSSA determines the original path has greater energy recovery than the modified one. This is unlikely, but possible. A pseudo-code showing the process is presented below

```
For i=1 to N-1
    Identify waypoint i
    Identify waypoint i+1
    Calculate properties for segment i }->\mathrm{ i+1
    Break segment into M elements by calculating \Deltas
    For j=1 to M
        Calculate for element j:
                Trajectory 1: Energy recovery by User Mission
                Trajectory 2a: Energy recovery by PFM Mission
                Trajectory 2b: Energy recovery by PFM Mission (LiSSA speeds)
                Trajectory 3a: Energy recovery by BPS Mission
                Trajectory 3b: Energy recovery by BPS Mission (LiSSA speeds)
    Next j
    Sum (energy recovery for all trajectories)
Next i
Select max energy recovery trajectory
Create additional waypoints
```


## Block 3: Static Obstacle Planner

This block receives a modified waypoint list from Block 2. Its primary purpose is to modify the LiSSA optimized trajectory, if necessary, in order to prevent the vehicle from colliding with terrain or static obstacles in the database. If operating around restricted airspace, radio towers, threat regions, or other obstacles, the waypoint list will be modified further, possibly producing a substantially more complex trajectory than initially supplied by the operator. Terrain avoidance can usually be accomplished by increasing the altitude of the waypoints. Such avoidance is already built into the LiSSA Trip Optimizer that accompanies the SURFACES Flight Simulator. The advantage of this method is that it minimally increases the range of the trip. Redirecting the trajectory around the topography, such as mountain ranges or escarpments can add substantial travel distances to the plan. Using performance theory, the Trip Optimizer determines the proximity between the aircraft and the terrain and warns the user if this is less than 500 ft and allows the situation to be repaired by removing suspect waypoints. In the actual GICA module, the Static Obstacle Planner will make that call autonomously.

## Block 4: Active Flight Plan Controller

This block is the heart of the GICA and performs multiple specialized tasks. These tasks are associated with specific flight maneuvers, such as the basic maneuvers specified in Table 10-2. These tasks convert the state inputs detailed in Table 10-3 into the corresponding control outputs. The controller can be operated in two modes: Standard and Smartpilot. The standard mode operates the controller using the airspeed commanded by the operator. The Smartpilot mode overrides these airspeeds and, instead, uses optimized airspeeds, such as $V_{y}$ when climbing and $V_{b g}$ when gliding. These values are adjusted by sensor obtained altitude, outside air temperature, and wind field strength.

The Smartpilot will also attempt to harvest additional energy from thermals or mechanical convection by triggering position hold mode. Whether this takes place depends in part on the updraft strength and frequency of occurrences along the flight path. It is not possible to rely on the presence of thermals with certainty and, thus, the algorithm will keep track of stochastic properties of their occurrences and assessing the expectation of encountering further thermals when optimizing the route. Then, once encountering a thermal, the algorithm will try to estimate its size and position and attempt to take advantage of available lift by establishing circular flight pattern. Examples of theoretical treatment of thermals include Allen [7], Welch et al [8] and Scull [9].

Table 10-3: Matrix of State Inputs and Corresponding Control Outputs

|  | State Inputs |  |  |  |  | Control Outputs |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ID | $h$ | $V_{C A S}$ | $\psi$ | $\phi$ | $\begin{aligned} & x_{C A S}, \\ & y_{C A S} \end{aligned}$ | $\delta_{\text {THR }}$ | $\delta_{a}$ | $\delta_{e}$ | $\delta_{r}$ | $\delta_{\text {spoil }}$ |
| 1 | 0 | ( |  |  |  | ( |  | ( |  |  |
| 2 |  |  | (0) |  |  |  | ( |  | NTB |  |
| 3 | (0) | ( |  |  |  | ( 0 |  | (0) |  |  |
| 4 | 0 | (0) |  |  |  | ( 0 |  | (0) |  | NTB |
| 5 |  | (0) |  |  |  |  |  | (0) |  |  |
| 6 |  |  |  | ( |  |  | ( |  | NTB |  |
| 7 |  |  |  |  | ( 0 |  | ( 0 |  | NTB |  |

## Performance Optimization through Stochastic Analysis

As the GICA is operates en route, it continuously collects sensor information, such as position, airspeeds, and wind field data obtained, for instance, using Langelaan's method through GPS data as shown in Equation (2-10) or through a datalink from a ground station. When operating over relatively flat terrain, the stochastic expectation of wind speed and direction is calculated using

$$
\begin{equation*}
\mathbf{V}_{w}=\frac{1}{N} \sum_{i=1}^{N} \mathbf{V}_{w_{i}} \tag{10-30}
\end{equation*}
$$

Where $\mathbf{V}_{w}$ is the wind vector and $N$ is the number of observations included. This vector is used by the SmartPilot in the performance analysis routine.

## Sensor Requirements

It should be clear that achieving some of the inputs required for analysis calls for a number of specialized sensors. Such sensors are not the focus of this work. Some may not even yet exist. Among sensors that already exist are position- (GPS), speed- (GPS, Pitot-static), acceleration- (accelerometers), orientation- (gyros) and atmospheric condition (OAT, altimeter) sensors. Among sensor that do not yet exist might be a future obstacle sensor (e.g. LIDAR or similar) that can detect obstacles far enough ahead
of the airplane's position to permit "gentle" rather than "abrubt" evasive maneuvers for obstacle avoidance. Their operational range would affect the speed at which the vehicle should move; a short range sensor of this nature would require the airplane to fly at lower airspeed than a long range one. The algorithm will analyze the current state vector for the vehicle, take into account conditions the vehicle has experienced in the past, and use stochastic methodologies coupled with control theory to select an appropriate sequence of "built-in" maneuvering commands to help take advantage of atmospheric energy. In this sense, the algorithm is environmentally adaptive.

## Block 5: Dynamic Obstacle Detection

In this block, the GICA receives signals from dynamic obstacle sensors. These sensors, whose details are beyond the scope of this work, must return position and velocity of any number of entities that may or may not pose threat to the motion of our sUAV. Little else can be said at this point, but a discussion of this block is included to emphasize this capability as a future goal of the effort.

## Block 6: Timer: Update Plan

This block serves a simple purpose. Keep track of time elapsed and trigger intermittent trajectory updates. This is necessary if onboard sensors indicate changes in the winds aloft change or if the GICA receives a revised wind field through uploading (e.g. transmitted by a ground station that use WRF data that includes time-dependent forecasting or similar).

## Block 7: Return to Home?

In this block, the distance to the departure waypoint (also called Home) is continuously calculated and the quantity of fuel (or $A h$ ) required for returning to it in direct flight is estimated assuming Carson's airspeed (see Equation (6-82)). This fuel quantity is compared to the remaining fuel. If the remaining fuel is insufficient to complete the planned trajectory the autopilot will override it and set course for the flight home. This function is commonly referred to as a RTH in commercially available (civilian) AFMS and is either triggered by the operator (pilot) or if the radio signal between the pilot radio controller and the onboard receiver is lost. In civilian flight, this is caused by reasons such as the pilot flew too far ("out of range"), the pilot lost the video feed and, thus, lost track of the orientation of the airplane and, thus, control of it, or because the radio signal suffers through interference of other signal sources. The GICA is more sophisticated in that it automatically determines if this is required by remaining fuel or battery capacity, necessitated by the fact it is fully autonomous.

## Block 8: Monitor Dynamic Obstacles

This block, just like Block 5 , is included to emphasize future capability of the GICA. This block is intended to analyze the signal from Block 5 (position and velocities of potential risk) and evaluate which, if any, pose real threat.

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## 11. Simulation Samples

This chapter presents flight simulation examples that put the primary potential of the GICA to the test; its energy harvesting capability. This calls for a realistic flight simulation tool, such as the SURFACES Flight Simulator (detailed in Chapter 8, Flight Simulation). Furthermore, results for two sUAVs are presented; the first one, called the Shadow Observer, resembles an existing operational sUAV in the 500 kg class with wing loading of approximately $11 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$. The other, called the Sparrow Hawk, is used to demonstrate the effectiveness of the GICA for a hypothetical sUAV in the 250 kg class. For added realism, the vehicles are subjected to a complex mission, rather than a simple "from A to B" style mission (see Figure 11-1). The complexity reflects a hypothetical surveillance mission for border control, forest fire patrol, or drug enforcement operation and subjects the vehicles to (1) multiple altitudes and airspeeds and (2) multitude of wind speeds and directions, just as it would real aircraft. This mission, which from here on will be referred to as the baseline mission (is is the user mission), will also be optimized using the PFM and BPS methods, provided in the simulation software. The resulting energy recovery is compared to the baseline mission (which is planned ignoring the presence of the wind field). A simple cost function, such as the number of km per kg (or nm per gallon) of fuel is all that is needed to evaluate energy recovery provided by the GICA. In fact, here it simply suffices to compare the weight of fuel consumed.

The simulation will be conducted at wind speed of $9 \mathrm{~m} / \mathrm{s}(30 \mathrm{ft} / \mathrm{s})$ for eight general wind directions of $0^{\circ}$, $45^{\circ}, 90^{\circ}, 135^{\circ}, 180^{\circ}, 225^{\circ}, 270^{\circ}$, and $315^{\circ}$. Since the mission shown in Figure $11-1$ results in multiple headings, the exposure to the elements is realistic. Then, several sensitivity analyses are presented that will answer a number of questions. For instance, how do the LiSSA missions compare? How do reduced wind speeds affect the results? How does the introduction of thermals affect the results? How does stochastic wind variability affect the results? These and other sensitivity simulations will be presented here, completing the demonstration of the GICA.


Figure 11-1: A complex surveillance mission in a mountainous region subjects the sUAV to various wind conditions, is presented in this chapter. The total range of this mission is $169 \mathbf{~ k m ~ ( 9 1 . 2 ~ n m})$.

### 11.1 Aircraft Model Descriptions

One of the primary functions of flight simulation is to allow the evaluation of both existing and hypothetical aircraft. It is does not matter which, as long as the modeling of the flight mechanics is realistic and representative of the class of aircraft under consideration. When considering "real" aircraft for which an aerodynamic model is not available (it rarely is, because manufacturers refrain from releasing the aerodynamic properties of their aircraft), the flight characteristics must be defined using classical analysis methods. Naturally, these methods apply equally to brand new aircraft. Historically, these methods have proven successful and agree well with both wind tunnel and flight test data. That said, practically all existing sUAV of the size class for which the GICA is intended (ranges approximately from 1 to $73 \mathrm{~kg} / \mathrm{m}^{2}$ or 0.2 to $15 \mathrm{lb} / \mathrm{ft}^{2}$ ) and would be ideal for comparative analysis, are military vehicles. Understandably, aerodynamic and inertia models for such aircraft are not in the public domain. The availability of such data is even less common for civilian sUAVs, because the companies involved often lack the expertise to develop them (the development of aerodynamic and inertia models is highly specialized). For this reason, two hypothetical aircraft were designed to allow the GICA to be evaluated. The first one, the Shadow Observer, is based on the geometry of an existing sUAV, although the aerodynamic and interial models were created using classical design methods. The second one, the Sparrow Hawk, is an original design that weighs about one-half of the Shadow Obserever. Both are presented below.

### 11.1.1. sUAV in the $\mathbf{5 0 0} \mathbf{~ k g}$ Class - The Shadow Observer

The Shadow Observer (see Figure 11-2) is a hypothetical aircraft, although it is based on an existing sUAV, the Hermes 450 (see Figure 11-3), manufactured by the Israeli company Elbit Systems Ltd.


Figure 11-2: A vortex-lattice model of the Shadow Observer. The model was used to estimate the static and dynamic stability derivatives for flight simulation.


Figure 11-3: The Elbit Systems Hermes 450 (here operated by the US Customs and Border Protection) is comparable to the Sparrow Hawk in number of areas.
(Photo by Gerald L. Nino)

As evident, the two aircraft feature similar configuration and properties, including wing span, fuselage length, weight, and engine power. The aerodynamic properties of the Shadow Observer were based on a limited amount of information available in the public domain about the Hermes 450. For instance, the drag polar was generated by reverse engineering its performance using the methods presented in

Gudmundsson [1]. The resulting model has a similar top speed and maximum rate-of-climb as the Hermes 450. It is expected its glide performance should be similar as well, although no data on this was found in the public domain. Additionally, optimized airspeeds, such as $V_{x}, V_{y}$, and $V_{b g}$, are estimated and, thus, may or may not be close the same airspeeds of the Hermes 450. The aerodynamic model was completed using the Vortex Lattice Method, as implemented in the aircraft design software SURFACES. The inertia model was developed using the tools for that purpose offered in SURFACES. Important aircraft properties for the Shadow Observer are shown in Table 11-1 and the aerodynamic model is shown in Appendix C-1. It is important to state that even though the Shadow Observer is based on the Hermes 450, it is not the same aircraft. The idea is only to demonstrate what benefits are in store for an airplane of the same size, weight, and power.

Table 11-1: Aircraft Properties for the Shadow Observer

| Description | Metric | UK-System |
| :---: | :---: | :---: |
| Engine | UAV Engines Limited R802/902(W) Wankel |  |
| Rated max power at S-L | 38.8 kW | 52 BHP |
| Idle engine power at S-L (estimated) | 3.7 kW | 5 BHP |
| Specific Fuel Consumption | $0.18 \mathrm{~kg} / \mathrm{kW} / \mathrm{hr}$ | $0.8 \mathrm{lb}_{\mathrm{f}} / \mathrm{BHP} / \mathrm{hr}$ |
| Mission take-off weight | 449 kg | $992 \mathrm{lb}_{\mathrm{f}}$ |
| Fuel weight (token value) | 22.7 kg | $50 \mathrm{lb}_{\mathrm{f}}$ |
| Moment of inertia, $I_{x x}$ | $474.8 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ | 350.2 slugs $\cdot \mathrm{ft}^{2}$ |
| Moment of inertia, $I_{y y}$ | $1209 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ | 892.0 slugs $\cdot \mathrm{ft}^{2}$ |
| Moment of inertia, $I_{z z}$ | $1629 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ | 1202 slugs $\cdot \mathrm{ft}^{2}$ |
| Moment of inertia, $I_{x z}$ | $48.5 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ | 35.8 slugs $\cdot \mathrm{ft}^{2}$ |
| Current CG-location (wrt wing LE) | 0.3048 m | 1.000 ft |
| Stick-fixed neutral point | 0.5485 m | 1.211 ft |
| Wing reference chord | 0.79 m | 2.60 ft |
| Wing span | 10.5 m | 34.5 ft |
| Wing area | $8.30 \mathrm{~m}^{2}$ | $89.3 \mathrm{ft}^{2}$ |
| Aspect Ratio | 13.3 |  |
| Initial wing loading (W/S) | $54.1 \mathrm{~kg} / \mathrm{m}^{2}$ | $11.1 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$ |
| Minimum drag coefficient | 0.0378 |  |
| Best lift-to-drag ratio | 14.55 |  |
| Stalling speed, $V_{s}$ | $89 \mathrm{~km} / \mathrm{h} \mathrm{CAS}$ | 48 KCAS |
| Best glide speed, $V_{b g}$ | $109 \mathrm{~km} / \mathrm{h}$ CAS | 59 KCAS |
| Best angle of climb, $V_{x}$ | $98 \mathrm{~km} / \mathrm{h}$ CAS | 53 KCAS |
| Best rate of climb, $V_{y}$ | $106 \mathrm{~km} / \mathrm{h}$ CAS | 57 KCAS |
| Best rate of climb, $R O C_{\text {max }}$ | $\approx 3.0 \mathrm{~m} / \mathrm{s}$ | $\approx 600 \mathrm{fpm}$ |
| Typical cruising speed, $V_{c}$ | $130 \mathrm{~km} / \mathrm{h}$ CAS | 70 KCAS |

### 11.1.2. sUAV in the $\mathbf{2 5 0} \mathbf{~ k g ~ C l a s s ~ - ~ T h e ~ S p a r r o w ~ H a w k ~}$

The Sparrow Hawk (see Figure 11-4) is named after the renowned raptor, which uses soaring and gliding on long distance flights, besides taking advantage of thermals when available. The Sparrow Hawk is a piston-powered aircraft of conventional T-tail, low-drag sailplane-style configuration, with a mass of 227 $\mathrm{kg}\left(500 \mathrm{lb}_{\mathrm{f}}\right)$. It utilizes a parasol wing to keep the distribution of section lift coefficients as uniform as
possible to reduce lift-induced drag. It represents a capable and efficient multi-role surveillance sUAV. Important aircraft properties for the Sparrow Hawk are shown in Table 11-2. The aerodynamic model for the Sparrow Hawk is shown in Appendix C-2.

The Sparrow Hawk is powered by a small 20 BHP piston engine intended for cruise flight sustenance only (requiring mechanical launch), whose altitude performance conforms to the Gagg and Ferrar model (see Gudmundsson [1]). The propeller is a 2.5 ft diameter fixed pitch propeller with a maximum propeller efficiency of 0.65 . It is assumed the engine is never shut off en route (to avoid restart problems) and that its idle thrust is $3 \mathrm{lb}_{f}$ and idle power is assumed 2 BHP .


Figure 11-4: A vortex-lattice model of the Sparrow Hawk.

Table 11-2: Aircraft Properties for the Sparrow Hawk

| Description | Metric | UK-System |
| :---: | :---: | :---: |
| Rated max power at S-L | 14.9 kW | 20 BHP |
| Idle engine power at S-L | 1.5 kW | 2 BHP |
| Specific Fuel Consumption | $0.27 \mathrm{~kg} / \mathrm{kW} / \mathrm{hr}$ | $0.8 \mathrm{lb}_{\mathrm{f}} / \mathrm{BHP} / \mathrm{hr}$ |
| Mission take-off weight | 227 kg | $500 \mathrm{lb}_{\mathrm{f}}$ |
| Fuel weight | 22.7 kg | $50 \mathrm{lb}_{\mathrm{f}}$ |
| Moment of inertia, $I_{x x}$ | $310.8 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ | 229.3 slugs. $\mathrm{ft}^{2}$ |
| Moment of inertia, $I_{y y}$ | $290.5 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ | 214.3 slugs $\cdot \mathrm{ft}^{2}$ |
| Moment of inertia, $I_{z z}$ | $556.6 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ | 410.6 slugs $\cdot \mathrm{ft}^{2}$ |
| Moment of inertia, $I_{x z}$ | $38.9 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ | 28.7 slugs $\cdot \mathrm{ft}^{2}$ |
| Current CG-location | 0.50 m | 1.64 ft |
| Stick-fixed neutral point | 0.60 m | 1.97 ft |
| Wing reference chord | 1.15 m | 3.78 ft |
| Wing span | 9.14 m | 30 ft |
| Wing span | 9.14 m | 30 ft |
| Wing area | $9.29 \mathrm{~m}^{2}$ | $100 \mathrm{ft}^{2}$ |
| Aspect Ratio | 9 |  |
| Initial wing loading (W/S) | $24.4 \mathrm{~kg} / \mathrm{m}^{2}$ | $5 \mathrm{lb} / \mathrm{ft}^{2}$ |
| Minimum drag coefficient | 0.0300 |  |
| Best lift-to-drag ratio | 16.76 |  |
| Stalling speed, $V_{s}$ | $59 \mathrm{~km} / \mathrm{h}$ CAS | 32.0 KCAS |
| Best glide speed, $V_{b g}$ | $78 \mathrm{~km} / \mathrm{h}$ CAS | 42.3 KCAS |
| Best angle of climb, $V_{x}$ | $68 \mathrm{~km} / \mathrm{h}$ CAS | 36.5 KCAS |
| Best rate of climb, $V_{y}$ | $84 \mathrm{~km} / \mathrm{h}$ CAS | 45.2 KCAS |
| Best rate of climb, $R O C_{\text {max }}$ | $\approx 2.5 \mathrm{~m} / \mathrm{s}$ | $\approx 480 \mathrm{fpm}$ |
| Typical cruising speed, $V_{c}$ | 93-111 km/h CAS | 50-60 KCAS |

### 11.2 Demonstration Mission - A Complex 170 km Surveillance Operation

The evaluation mission consists of 13 segments and is designed to test the LiSSA for multitude of wind condition (see Figure 11-6 and Table 11-3). Note that in the interest of readers familiar with the world of aviation, from here on, all units are presented in format familiar to US aviation. Thus, distance, speed, and weight will be in terms of nautical miles ( nm ), knots or $\mathrm{ft} / \mathrm{s}$, and $\mathrm{lb}_{\mathrm{f}}$, respectively. The mission begins near the center of the figure, requiring an initial SW-bound heading ( N is up). Then, the airplane turns W and shortly thereafter N . This heading is maintained for 18 nm at 5000 ft , after which it turns E and begins to reduce altitude to 2000 ft , as it flies S -bound through the canyon (see Figure 11-5). At the end of the canyon, a climb on a N -bound heading is commanded. The target altitude here is 7000 ft as the east part of the topography has several tall mountains. The mission is completed near the center of the figure, not far from where it began. This mission will be optimized using both PFM and BPS methods, through a special tool provided in the SURFACES Flight Simulator, called LiSSA Trip Optimizer (see Figure 11-7).

The LiSSA utilizes this mission as a template for its trajectory optimization. It ensures the entire set of waypoints specified by the user will always be visited, as it is assumed these play an imperative role in the planning; perhaps serving as surveillance positions. The trajectories between the waypoints that were defined by the user are straight lines. However, the trajectories generated by the PFM or BPS algorithms between those waypoints will differ greatly, because both insert a number of additional waypoints to permit the aircraft to harvest atmospheric energy.


Figure 11-5: Simulating flight through the canyon - cockpit view. The messages near the top center indicate the autopilot is maintaining 50 KCAS at 2000 ft on a heading of $174^{\circ}$ en route to the $\mathbf{7}^{\text {th }}$ of 14 waypoints.

Thus, when a mission segment calls for an increase or reduction in altitude the user can select an energy efficient airspeed. As discussed in Chapter 10, The Generic Intelligent Control Algorithm, the problem of selecting the appropriate airspeed can be delegated to the LISSA Trip Optimizer (see Figure 11-7), which selects the proper airspeed automatically. This is done by checking the "Allow GICA to dictate airspeeds". The importance of this feature is studied in this chapter.


Figure 11-6: A top view of the evaluation surveillance mission (as defined by the user) shows the perimeter of the region and the water-filled canyon is explored. The width and height of the regions is about $30 \times 30 \mathrm{~km}$.


Figure 11-7: The LiSSA Trip Optimizer tool allows the user to optimize the user-mission to take advantage of atmospheric convection.

Table 11-3: Definition of the User Mission

| Waypoint | North <br> (ft) | East <br> (ft) | Altitude <br> (ft) | Airspeed <br> (KCAS) | Remarks |
| :---: | :---: | :---: | :---: | :---: | :--- |
| 1 | 1000 | 0 | 2000 | 50 | First waypoint. Climb to 4000 ft. |
| 2 | -45000 | -25000 | 4000 | 50 | Climb to 5000 ft. |
| 3 | -45000 | -45000 | 5000 | 50 | Long distance patrol at $5000 \mathrm{ft}$. |
| 4 | 45000 | -45000 | 5000 | 65 | Start of descent segment. Increase speed. |
| 5 | 45000 | -12857 | 3000 | 60 | Continued of descent segment. Reduce speed. |
| 6 | 25000 | -24286 | 2000 | 50 | Level flight in canyon. |
| 7 | -22000 | -16095 | 2000 | 50 | Level flight in canyon. |
| 8 | -31500 | -10952 | 3000 | 50 | Climb out of canyon. |
| 9 | -25000 | -5000 | 4000 | 50 | Long distance patrol. Climb to 7000 ft. |
| 10 | 40000 | -5000 | 7000 | 50 | Long distance patrol. |
| 11 | 45000 | 45000 | 7000 | 60 | Long distance patrol. Descend to $2000 \mathrm{ft}$. |
| 12 | -45000 | 45000 | 2000 | 50 | Begin return to base. Maintain 2000 ft. |
| 13 | -20000 | 10000 | 2000 | 55 | Descent to 1500 ft. |
| 14 | 1000 | 10000 | 1500 | 55 | Final waypoint. |

### 11.3 Simulation Results for the Shadow Observer

The simulation begins by the submittal of the user-mission of Table 11-3 to the LiSSA. This yields four additional missions; two PFM and two BPS missions. Then, all five are simulated, allowing the energy consumption to be compared (e.g. comparison of mass of fuel or Ah of battery capacity consumed). This is repeated for the eight general wind directions of $0^{\circ}, 45^{\circ}, 90^{\circ}, 135^{\circ}, 180^{\circ}, 225^{\circ}, 270^{\circ}$, and $315^{\circ}$ specified earlier. This section present simulations results for the Shadow Observer only. The results of these comparisons are listed in Tables 11-4 and 11-5. Simulation results for the Sparrow Hawk are presented in Section 11.3, Simulation Results for the Sparrow Hawk, produced using identical approach.

Of course, the simulation results presented are only hypothetical until the LiSSA can be validated in real experiments. Should the correlation between experiment and simulation build confidence in the methodology, the latter becomes a tool to evaluate the effectiveness for other atmospheric conditions. The primary advantage of flight simulation is its resemblance to real experiments by permitting a full control of variables. In fact, assuming experiment validates the modeling, we are likely to learn much more than possible in real experiments, because the control of atmospheric conditions is beyond our means: It impractical, if not impossible, to gather data for all wind directions, in turn, making pattern identification harder. For instance, the wind-topography effects presented later would have remained undiscovered had the simulation not been conducted for the full circle of wind directions.

### 11.3.1. Basic Comparison in No Convection Conditions

For reference, a no-wind simulation case is provided in the top row of Table 11-4. It shows that the usermission on a calm day is predicted to take 1 hour, 22 minutes, and 10 seconds using performance theory. This is denoted using shorthand notation as $1 \mathrm{~h}: 22 \mathrm{~m}: 10 \mathrm{~s}$. The simulated mission took some 26 seconds longer, or $1 \mathrm{~h}: 22 \mathrm{~m}: 36 \mathrm{~s}$. This disparity can be attributed to the inherent dynamic response of the aircraft model that occur in the simulator and are omitted from the theoretical predictions using performance theory. For instance, the airplane is subject to dynamic transients that affect the instantaneous drag (can be called transient drag). This is not accounted for in the performance theory. Also, the throttle action is assumed instantaneous in the performance theory, but in the simulation it takes time. Using performance theory, the airplane changes airspeed instantly when clearing waypoints, but in the simulation the airplane behaves much like it would in the real world. Additionally, performance theory does not account for turning radius, but rather the aircraft changes its heading instantaneously with a zero turning radius.

In spite of these disparities, a difference of flight time of 26 seconds between the two approaches for a flight lasting $1 \mathrm{hr}, 22$ minutes, and 36 seconds ( 4956 sec ) shows the two methods are indeed in close agreement. Additionally, the baseline mission entry in Table 11-4 shows that performance theory predicts the mission will consume $17.41 \mathrm{lb}_{\mathrm{f}}$ of fuel, while the simulation resulted in $16.64 \mathrm{lb}_{\mathrm{f}}$. This is a $4.4 \%$ difference. However, while this also shows good agreement between performance theory and flight mechanics, the difference can be expected to vary more for the PFM and BPS generated trajectories due to the larger number of waypoints and, thus, more frequent maneuvering. It is vitally important for the predictions made by performance theory to match the simulation as closely as
possible, because in real applications the LISSA selects the trajectory based on the lowest fuel consumption (i.e. greatest energy recovery). The resulting fuel savings are detailed in Table 11-5.

Table 11-4: Comparison of User and LiSSA Trajectories - Raw Data (Shadow Observer)

|  |  |  | Trajectory Distance, nm | Total Time |  | $\mathbf{W}_{\text {fuel, }}$ lbf |  | COST, nm/gal |  | Simulation <br> Rank (5 best) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Description | Wind | Trip/Simulation | Predicted | Simulation | Predicted | Simulation | Predicted | Simulation |  |
| ぃ | Baseline | No wind | 91.18 | 1h:22m:10s | 1h:22m:36s | 17.41 | 16.64 | 31.42 | 32.88 |  |
| 1 | Baseline 1 | $30 \mathrm{ft} / \mathrm{s} 0^{\circ}$ | 91.18 | 1h:30m:10s | 1h:30m:31s | 19.63 | 18.61 | 27.87 | 29.40 | 3 |
|  | PFM 1 |  | 99.97 | 1h:38m:14s | 1h:38m:12s | 19.63 | 18.73 | 27.87 | 29.21 | 2 |
|  | PFM Speed 1 |  | 99.97 | 1h:42m:58s | 1h:43m:07s | 19.74 | 18.81 | 27.71 | 29.08 | 1 |
|  | BPS 1 |  | 126.88 | 2h:03m:16s | 2h:07m:03s | 17.68 | 18.33 | 30.94 | 29.85 | 4 |
|  | BPS Speed 1 |  | 126.88 | 2h:08m:38s | 2h:12m:51s | 17.52 | 18.19 | 31.23 | 30.08 | 5 |
| 2 | Baseline 2 | $30 \mathrm{ft} / \mathrm{s} 45^{\circ}$ | 91.18 | 1h:28m:44s | 1h:29m:08s | 19.01 | 18.04 | 28.78 | 30.33 | 1 |
|  | PFM 2 |  | 95.51 | 1h:32m:31s | 1h:32m:33s | 15.50 | 15.25 | 35.30 | 35.87 | 2 |
|  | PFM Speed 2 |  | 95.51 | 1h:37m:05s | 1h:37m:22s | 15.38 | 15.13 | 35.57 | 36.16 | 3 |
|  | BPS 2 |  | 107.38 | 1h:43m:33s | 1h:45m:10s | 14.70 | 14.88 | 37.22 | 36.77 | 4 |
|  | BPS Speed 2 |  | 107.38 | 1h:49m:06s | 1h:50m:52s | 14.08 | 14.35 | 38.86 | 38.12 | 5 |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:26m:48s | 1h:27m:10s | 18.35 | 17.61 | 29.81 | 31.07 | 1 |
|  | PFM 3 |  | 95.80 | 1h:30m:17s | 1h:31m:36s | 11.80 | 12.34 | 46.36 | 44.33 | 4 |
|  | PFM Speed 3 |  | 95.80 | 1h:35m:18s | 1h:36m:24s | 11.74 | 12.01 | 46.60 | 45.55 | 5 |
|  | BPS 3 |  | 101.12 | 1h:36m:49s | 1h:37m:51s | 16.34 | 16.36 | 33.48 | 33.44 | 2 |
|  | BPS Speed 3 |  | 101.12 | 1h:41m:45s | 1h:42m:38s | 16.36 | 16.05 | 33.44 | 34.09 | 3 |
| 4 | Baseline 4 | $30 \mathrm{ft} / \mathrm{s} 135^{\circ}$ | 91.18 | 1h:26m:43s | 1h:27m:13s | 17.89 | 17.18 | 30.58 | 31.84 | 1 |
|  | PFM 4 |  | 96.08 | 1h:31m:15s | 1h:31m:59s | 13.27 | 13.57 | 41.23 | 40.32 | 4 |
|  | PFM Speed 4 |  | 96.08 | 1h:37m:34s | 1h:38m:19s | 13.01 | 13.39 | 42.05 | 40.86 | 5 |
|  | BPS 4 |  | 109.81 | 1h:42m:45s | 1h:44m:41s | 15.10 | 15.64 | 36.23 | 34.98 | 2 |
|  | BPS Speed 4 |  | 109.81 | 1h:51m:01s | 1h:52m:47s | 14.86 | 15.38 | 36.82 | 35.57 | 3 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:27m:15s | 1h:27m:54s | 17.63 | 17.01 | 31.03 | 32.16 | 3 |
|  | PFM 5 |  | 96.06 | 1h:31m:46s | 1h:32m:06s | 17.07 | 16.71 | 32.05 | 32.74 | 5 |
|  | PFM Speed 5 |  | 96.06 | 1h:39m:40s | 1h:40m:05s | 17.32 | 16.92 | 31.59 | 32.33 | 4 |
|  | BPS 5 |  | 115.66 | 1h:50m:27s | 1h:53m:17s | 16.14 | 17.20 | 33.90 | 31.81 | 1 |
|  | BPS Speed 5 |  | 115.66 | 2h:00m:18s | 2h:02m:46s | 16.21 | 17.13 | 33.75 | 31.94 | 2 |
| 6 | Baseline 6 | $30 \mathrm{ft} / \mathrm{s} 225^{\circ}$ | 91.18 | 1h:27m:03s | 1h:27m:41s | 17.88 | 17.18 | 30.60 | 31.84 | 1 |
|  | PFM 6 |  | 96.93 | 1h:32m:27s | 1h:32m:56s | 16.03 | 15.74 | 34.13 | 34.76 | 3 |
|  | PFM Speed 6 |  | 96.93 | 1h:39m:48s | 1h:40m:29s | 16.36 | 15.89 | 33.44 | 34.43 | 2 |
|  | BPS 6 |  | 104.93 | 1h:37m:14s | 1h:37m:52s | 14.44 | 14.38 | 37.89 | 38.04 | 4 |
|  | BPS Speed 6 |  | 104.93 | 1h:47m:31s | 1h:48m:08s | 14.42 | 14.03 | 37.94 | 38.99 | 5 |
| 7 | Baseline 7 | $30 \mathrm{ft} / \mathrm{s} 270^{\circ}$ | 91.18 | 1h:27m:12s | 1h:27m:42s | 18.68 | 17.79 | 29.29 | 30.75 | 1 |
|  | PFM 7 |  | 98.46 | 1h:34m:02s | 1h:34m:50s | 15.98 | 15.90 | 34.24 | 34.41 | 3 |
|  | PFM Speed 7 |  | 98.46 | 1h:40m:00s | 1h:41m:07s | 16.65 | 16.03 | 32.86 | 34.13 | 2 |
|  | BPS 7 |  | 106.73 | 1h:39m:15s | 1h:42m:33s | 14.63 | 14.74 | 37.39 | 37.12 | 4 |
|  | BPS Speed 7 |  | 106.73 | 1h:47m:19s | 1h:50m:08s | 14.55 | 14.24 | 37.60 | 38.42 | 5 |
| 8 | Baseline 8 | $30 \mathrm{ft} / \mathrm{s} 315^{\circ}$ | 91.18 | 1h:28m:59s | 1h:29m:43s | 19.46 | 18.74 | 28.11 | 29.19 | 1 |
|  | PFM 8 |  | 98.59 | 1h:35m:56s | 1h:36m:43s | 14.85 | 14.92 | 36.84 | 36.67 | 4 |
|  | PFM Speed 8 |  | 98.59 | 1h:40m:35s | 1h:41m:18s | 14.82 | 14.93 | 36.91 | 36.64 | 3 |
|  | BPS 8 |  | 102.57 | 1h:39m:02s | 1h:39m:58s | 15.22 | 15.06 | 35.94 | 36.33 | 2 |
|  | BPS Speed 8 |  | 102.57 | 1h:46m:22s | 1h:46m:58s | 14.93 | 14.72 | 36.64 | 37.17 | 5 |

*Note: 1 US gallon of AvGas weighs 6 lb $\mathrm{b}_{\mathrm{f}}$.

### 11.3.2. Comparing Missions for a Range of Atmospheric Convection Conditions

The next step of the evaluation involves simulating the mission for a range of wind conditions, all which feature an average wind speed at altitude of $9 \mathrm{~m} / \mathrm{s}(30 \mathrm{ft} / \mathrm{s})$ for eight different wind directions: $0^{\circ}(\mathrm{N})$, $45^{\circ}$ (NE), $90^{\circ}$ (E), $135^{\circ}$ (SE), $180^{\circ}$ (S), $225^{\circ}(\mathrm{SW}), 270^{\circ}(\mathrm{W})$, and $315^{\circ}$ (NW). The results of this simulation are presented in Table 11-4 and consist of the time required to complete the mission and the required fuel-burn (for both the predicted and simulated cases). Note that each wind direction has a set of five
results. The first is always the user-mission, planned without any concern for atmospheric convection over the topography. The next two missions are PFM missions with and without LISSA selected airspeed. The last two missions are BPS missions with and without LISSA selected airspeeds. In Table 11-4, PFM stands for Potential Flow Method and BPS for Best Path Search. The term "Speed" refers to missions for which the LiSSA Trip Optimizer selected the airspeed as described in bullet (5) of Section 11.3.3. Note that the simulated missions in Table 11-4 do not take advantage of the capability of the vehicle to loiter in an updraft field and acquire greater energy savings than otherwise. Thus, they represent a "worst case" energy savings.

Table 11-5: Comparison of User and LiSSA Trajectories - Savings in Fuel Consumption (Shadow Observer)

|  |  |  | Trajectory |  |  |  | ngs, $\mathrm{lb}_{\mathrm{f}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 芯 | Description | Wind | Simulation | $\Delta W_{\text {fuel }}$ | $\Delta$ | \% $\Delta$ | $\Delta_{\text {max }}$ | Method |
|  | Baseline | No wind | 91.18 | -0.77 |  |  |  |  |
| 1 | Baseline 1 | $30 \mathrm{ft} / \mathrm{s} 0^{\circ}$ | 91.18 | -1.02 | 0.00 | 0.0 | 0.42 | BPS Speed |
|  | PFM 1 |  | 99.97 | -0.90 | -0.12 | -0.6 |  |  |
|  | PFM Speed 1 |  | 99.97 | -0.93 | -0.20 | -1.1 |  |  |
|  | BPS 1 |  | 126.88 | 0.65 | 0.28 | 1.5 |  |  |
|  | BPS Speed 1 |  | 126.88 | 0.67 | 0.42 | 2.3 |  |  |
| 2 | Baseline 2 | $30 \mathrm{ft} / \mathrm{s} 45^{\circ}$ | 91.18 | -0.97 | 0.00 | 0.0 | 3.69 | BPS Speed |
|  | PFM 2 |  | 95.51 | -0.25 | 2.79 | 15.5 |  |  |
|  | PFM Speed 2 |  | 95.51 | -0.25 | 2.91 | 16.1 |  |  |
|  | BPS 2 |  | 107.38 | 0.18 | 3.16 | 17.5 |  |  |
|  | BPS Speed 2 |  | 107.38 | 0.27 | 3.69 | 20.5 |  |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | -0.74 | 0.00 | 0.0 | 5.60 | PFM Speed |
|  | PFM 3 |  | 95.80 | 0.54 | 5.27 | 29.9 |  |  |
|  | PFM Speed 3 |  | 95.80 | 0.27 | 5.60 | 31.8 |  |  |
|  | BPS 3 |  | 101.12 | 0.02 | 1.25 | 7.1 |  |  |
|  | BPS Speed 3 |  | 101.12 | -0.31 | 1.56 | 8.9 |  |  |
| 4 | Baseline 4 | $30 \mathrm{ft} / \mathrm{s} 135^{\circ}$ | 91.18 | -0.71 | 0.00 | 0.0 | 3.79 | PFM Speed |
|  | PFM 4 |  | 96.08 | 0.30 | 3.61 | 21.0 |  |  |
|  | PFM Speed 4 |  | 96.08 | 0.38 | 3.79 | 22.1 |  |  |
|  | BPS 4 |  | 109.81 | 0.54 | 1.54 | 9.0 |  |  |
|  | BPS Speed 4 |  | 109.81 | 0.52 | 1.80 | 10.5 |  |  |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | -0.62 | 0.00 | 0.0 | 0.30 | PFM |
|  | PFM 5 |  | 96.06 | -0.36 | 0.30 | 1.8 |  |  |
|  | PFM Speed 5 |  | 96.06 | -0.40 | 0.09 | 0.5 |  |  |
|  | BPS 5 |  | 115.66 | 1.06 | -0.19 | -1.1 |  |  |
|  | BPS Speed 5 |  | 115.66 | 0.92 | -0.12 | -0.7 |  |  |
| 6 | Baseline 6 | $30 \mathrm{ft} / \mathrm{s} 225^{\circ}$ | 91.18 | -0.70 | 0.00 | 0.0 | 3.15 | BPS Speed |
|  | PFM 6 |  | 96.93 | -0.29 | 1.44 | 8.4 |  |  |
|  | PFM Speed 6 |  | 96.93 | -0.47 | 1.29 | 7.5 |  |  |
|  | BPS 6 |  | 104.93 | -0.06 | 2.80 | 16.3 |  |  |
|  | BPS Speed 6 |  | 104.93 | -0.39 | 3.15 | 18.3 |  |  |
| 7 | Baseline 7 | $30 \mathrm{ft} / \mathrm{s} 270^{\circ}$ | 91.18 | -0.89 | 0.00 | 0.0 | 3.55 | BPS Speed |
|  | PFM 7 |  | 98.46 | -0.08 | 1.89 | 10.6 |  |  |
|  | PFM Speed 7 |  | 98.46 | -0.62 | 1.76 | 9.9 |  |  |
|  | BPS 7 |  | 106.73 | 0.11 | 3.05 | 17.1 |  |  |
|  | BPS Speed 7 |  | 106.73 | -0.31 | 3.55 | 20.0 |  |  |
| 8 | Baseline 8 | $30 \mathrm{ft} / \mathrm{s} 315^{\circ}$ | 91.18 | -0.72 | 0.00 | 0.0 | 4.02 | BPS Speed |
|  | PFM 8 |  | 98.59 | 0.07 | 3.82 | 20.4 |  |  |
|  | PFM Speed 8 |  | 98.59 | 0.11 | 3.81 | 20.3 |  |  |
|  | BPS 8 |  | 102.57 | -0.16 | 3.68 | 19.6 |  |  |
|  | BPS Speed 8 |  | 102.57 | -0.21 | 4.02 | 21.5 |  |  |

Columns labeled "Predicted" show results obtained using performance theory to estimate the mission flight time and take into account the wind conditions along each segment (horizontal and vertical wind). The COST column shows the "fuel mileage" using the Baseline-mission of 91.18 nm . The actual PFM and

BPS missions always require greater range. The ranking column shows which mission the most efficient for each set is of wind directions. It is evident that in each case, the user-mission is the least efficient. Table 11-5 provides a further insight into the energy recovery. The column labeled $\Delta W_{\text {fuel }}$ is the difference between the simulated and predicted fuel consumptions. The average difference between the two is $0.20 \mathrm{lb}_{\mathrm{f}}$ with a standard deviation of $0.58 \mathrm{lb}_{\mathrm{f}}$. The first fuel savings column shows the difference between the PFM- and BPS fuel consumptions and the user mission. This columns shows that for the first set, the most efficient mission is the BPS Speed 1 with $0.42 \mathrm{lb}_{f}$ less fuel consumption than the user-mission. The next column shows this amounts to $2.3 \%$ fuel savings and the third column shows this is the largest such value in the set. The column labeled "Method" indicates that the BPS Speed mission was the most efficient one.

### 11.3.3. Important Observations for the Shadow Observer

The results from the eight sets of simulations, shown in Tables 11-4 and 11-5, provide an important insight into energy savings for winds from all directions. It is to be expected that the atmospheric convection will always reduce the efficiency of the user mission over the no-convection case. In contrast, the efficiency of the LiSSA missions, which takes advantage of the atmospheric convection, always increases. The reduction in the efficiency of the user mission is caused by the headwind increasing the flight time along a segment more than tailwind reduces it. This can be seen from the following simple example: Consider a trip, some 5000 ft long, during which an airplane's true airspeed $\left(V_{T A S}\right)$ is accurately maintained at $100 \mathrm{ft} / \mathrm{s}$ in no wind conditions. The time to traverse will be $t=$ $5000 / 100=50$ seconds. If this mission is repeated in a $30 \mathrm{ft} / \mathrm{s}$ tailwind, the same distance will be covered in $t=5000 /(100+30)=38.5 \mathrm{~s}$, or 11.5 sec less. If repeated in a $30 \mathrm{ft} / \mathrm{s}$ headwind, the time required is $t=$ $5000 /(100-30)=71.4$ seconds, i.e. 21.4 sec longer. Thus, if the airplane flies back and forth in wind condition (headwind one way tailwind the other), it will always consume greater amount of fuel than flying back and forth in no wind conditions, because the total flight time will be longer. We can now make the following observations regarding Tables 11-4 and 11-5:
(1) Eight sets of five missions are compared in Table 11-4. As stated earlier, each set corresponds to a specific wind direction and consists of the original user-mission, two PFM missions, and two BPS missions, as described in Section 10.6.2, Description of Individual Blocks Comprising the GICA. Missions denoted with the "Speed" label use optimized airspeeds selected by the LiSSA, as described below in bullet (3) rather than the user prescribed airspeed.
(2) Fuel consumption cost is given in $\mathrm{nm} / \mathrm{gal}$ (see Table 11-4). It is compared using the range covered by the user-mission and not the range of the corresponding optimized mission. As an example, the fuel consumption during the Baseline 1 mission (total range is 91.18 nm ) amounted to $18.61 \mathrm{lb}_{\mathrm{f}}$. This results in a "fuel mileage" of $91.18 /(18.61 / 6)=29.40 \mathrm{~nm}$ per gallon of fuel (which assumes 1 gallon of AvGas weighs $6 \mathrm{lb}_{\mathrm{f}}$ ). In comparison, the PFM 1 mission has a range of 99.97 nm and consumed $18.73 \mathrm{lb}_{\mathrm{f}}$ of fuel, rendering a $99.97 /(18.73 / 6)=$ $32.02 \mathrm{~nm} / \mathrm{gal}$. Note that while the PFM mission resulted in better fuel mileage than the user mission (as expected), in this particular case the fuel weight consumed was greater (18.73 versus $18.61 \mathrm{lb}_{\mathrm{f}}$ ). Thus, it is more appropriate to calculate fuel mileage using the baseline
range of 91.18 nm , as this is the distance the aircraft would have covered had we followed the intent of the user. Using this approach, the cost of the PFM 1 mission reduces to $91.18 /(14.50 / 6)=29.21 \mathrm{~nm} /$ gal. This is the approach in Tables 11-4 and 11-5.
(3) The LiSSA Trip Optimizer selects airspeed based on the following: (1) If the mission segment requires a climb to a higher altitude, then either the best rate-of-climb ( $V_{y}$ ) or best angle-ofclimb $\left(V_{x}\right)$ is selected. If the required climb gradient is greater than the maximum gradient the airplane is capable of at $V_{y}$, then the $V_{x}$ is selected. If less, then $V_{y}$ is selected. (2) If the mission segment requires level flight, then the user-specified cruising speed is selected. (3) If the mission segment requires a descent, then the best glide speed, $V_{b g}$, is selected.
(4) As shown in Table 11-5, the PFM or BPS missions always consumed the least amount of fuel for all wind directions simulated. Of these, the PFM or BPS missions with LISSA selected airspeeds consumed the least amount of fuel seven times out of eight. The best case savings was for a wind direction of $90^{\circ}$ in the topography provided. It amounted to $5.60 \mathrm{lb}_{\mathrm{f}}$ or $31.8 \%$ for the PFM with LISSA selected airspeeds. This is considerable savings considering the relatively high wing loading of the Shadow Observer ( $54.1 \mathrm{~kg} / \mathrm{m}^{2}$ or $11.1 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$ ). This would be even greater for less wing loading. The top savings using the PBS method for the same mission amounted to $1.56 \mathrm{lb}_{\mathrm{f}}$ or $8.9 \%$. Table $11-4$ shows that sometimes the PFM beats the BPS and vice versa. The least fuel savings was for a wind direction of $180^{\circ}$ and amounted to merely $0.30 \mathrm{lb}_{\mathrm{f}}$ or $1.8 \%$ using the PFM, barely beating the user-mission. The reason for the reduced savings can be attributed to the topography and wind direction, which resulted in smaller regions of updrafts for energy harvesting. There are also instances (Set 1 and 5), where the PFM and BPS missions (one or the other) consumed more fuel than the user mission (albeit not by much). This is primarily attributed to the engine continuing to consume fuel at idle power coupled with the increased flight time.
(5) The PFM and BPS methods both lead to longer flight time to complete a mission since they inevitably call for longer distances to be flown. The PFM and BPS methods with the LISSA selected airspeeds take even longer to complete. In this simulation, the longest flight duration occurred for BPS Speed 1 ( $0^{\circ}$ wind direction) of $2 \mathrm{~h}: 12 \mathrm{~m}: 51 \mathrm{~s}$. The same mission flown as a usermission took $1 \mathrm{~h}: 30 \mathrm{~m}: 31 \mathrm{~s}$, some 42 minutes longer. However, in spite of this difference, the BPS mission consumed $0.42 \mathrm{lb}_{f}$ less fuel, or $18.19 \mathrm{lb}_{f}$ versus $18.61 \mathrm{lb}_{f}$ for the user-mission. It is possible to incorporate logic into the LiSSA that recommends user-mission over energy harvesting trajectories only when recovery exceeds a certain value, although this is not done here.
(6) The application of the PFM and BPS methods changes the original trajectory substantially, as can be seen for selected wind directions in Figure 11-8. It may be said, as was stated in Chapter 1, that the resulting flight reflects biomimetic behavior - it would not be imminently obvious to an unwary observer where the vehicle is headed.

Wind $30 \mathrm{ft} / \mathrm{s}$ at $0^{\circ}$


Wind $30 \mathrm{ft} / \mathrm{s}$ at $90^{\circ}$


Wind $30 \mathrm{ft} / \mathrm{s}$ at $180^{\circ}$


Figure 11-8: A top view of how the PFM- and BPS-algorithms modify the surveillance mission based on wind directions.
(7) While the PFM and BPS methods always consumed less fuel than the corresponding usermissions, the amount of fuel savings varies greatly with wind direction. There is clear correlation between the topography and wind direction; each topography has its own characteristic ideal wind directions. In this simulation, the east- and westward directions yielded the greatest energy recovery. This shows the fuel savings are strongly dependent on wind and topography. This is reflected in Figure 11-9 and Figure 11-10, which show that the E and NW wind directions provide greater energy recovery in the example topography. It also shows good agreement between the prediction and simulation, indicating the prediction is an effective tool to select the right trajectory.


Figure 11-9: Weight of fuel consumed between the predicted and simulated trips, when using the Potential Flow Method (Shadow Observer).


Figure 11-10: Weight of fuel consumed between the predicted and simulated trips, when using the Best Path earch Method (Shadow Observer).
(8) Figure 11-11 shows the effect of wind direction (assuming an average wind speed of $30 \mathrm{ft} / \mathrm{s}$ ) on the mission flight time and fuel consumption. It can be seen that the user missions always take the least amount of time to complete, while consuming the greatest amount of fuel. It can also be seen that the wind direction leads to a variation in the fuel consumption, also evident in Figure 11-9 and Figure 11-10, an example of the topographical influence, which here features mountains that are predominantly aligned with the N-S directions.



Figure 11-11: Mission flight time and fuel consumption as function of wind direction (simulation for the Shadow Observer).
(9) Figure $11-12$ shows difference between the predicted and simulated mission flight time and fuel consumption. As stated elsewhere, it is vital that the predictions conducted using performance theory is as precise as possible to gain confidence in that LiSSA module will reliably select the optimal trajectory. The left graph shows that worst flight time accuracy was about 4.22 minutes for a mission that lasted some 132.9 minutes. This corresponds to about $3.17 \%$ difference. Overall, the simulated missions tend to last a couple of minutes, or so, longer than predicted. The reasons for these discrepancies were discussed earlier.
(10)The right graph of Figure $11-12$ shows the difference between predicted and simulated fuel consumption. There were roughly equal amount of instances for which simulated mission consumed slightly more fuel than predicted and vice versa. The maximum difference in this capacity amounted to $1.060 \mathrm{lb}_{\mathrm{f}}$, the minimum $0.020 \mathrm{lb}_{\mathrm{f}}$, and the average was $0.514 \mathrm{lb}_{\mathrm{f}}$. Regardless, the most important question to answer is would the LiSSA module have picked the most energy efficient trajectory based on the prediction results? The answer is: Yes, in all but one case. This can be seen by comparing the lowest predicted fuel consumption values in Table 11-4 to the simulated optimum trajectories in Table 11-5. They match for all the wind directions simulated. It is essential that LiSSA picks the best trajectory, as this gives confidence in the fidelity of this approach.


Figure 11-12: Difference between predicted and simulated mission flight time and fuel consumption as function of wind direction (Shadow Observer).

### 11.4 Mission Sensitivities for the Shadow Observer

This section considers the effect of modifying selected parameters in the simulation study and considers the Shadow Observer only. Topics of interest range from the effect of using controls such as a yaw damper (the Sparrow Hawk has poor Dutch roll damping) or permitting a reliable piston engine shutdown and restart to the impact of gusts and thermals. Other effect of interest is the presence of thermals and the size of the topographical panels. Only two missions, each from two sets of the eight wind directions will be evaluated; the ones with the highest and lowest energy recoveries. These missions involve Set 3 ( $90^{\circ}$ wind direction) and Set 5 ( $180^{\circ}$ wind direction). Note that there is limit to the amount of data that can be included in this section, but many of the following studies should be considered initial in nature and worthy of further study. For this reason, the average calculated below each table should be treated with care - it is intended for comparison discussions only. Also note that in the tables below, a negative value in the column labeled $\Delta W_{\text {fuel }}$ means less fuel was consumed than during the reference simulation: in other words, a negative $\Delta W_{\text {fuel }}$ means improvement in energy recovery.

### 11.4.1. Effect of Piston Engine In-Flight Shutdowns

The simulation results in Table 11-4 were obtained assuming a piston engine that is never shut-off completely en route, but rather, when not needed, is operated at idle-power (which corresponds to 5 BHP at S-L). It is of interest to consider how much additional energy could be recovered if engine shutoff were permitted. To answer this question, simulations were run assuming such an engine. The results for the test missions are listed in Table 11-6. It is evident that substantial additional energy-savings are realized by operating such an engine. For the four missions considered, the average fuel savings were $1.44 \mathrm{lb}_{\mathrm{f}}$. First consider the mission with the lowest fuel savings ( $30 \mathrm{ft} / \mathrm{s} @ 180^{\circ}$ wind), denoted as Baseline 5 and PFM 5. Baseline 5 is improved from 17.01 to $15.72 \mathrm{lb}_{f}$ by $1.29 \mathrm{lb}_{\mathrm{f}}$ or $7.6 \%$. PFM 5 improved from 16.71 to $15.26 \mathrm{lb}_{f}$ by $1.16 \mathrm{lb}_{\mathrm{f}}$ or $7.1 \%$. Next consider the mission with the highest fuel savings ( $30 \mathrm{ft} / \mathrm{s} @ 90^{\circ}$ wind), denoted as Baseline 3 and PFM Speed 3. Baseline 3 is improved from 17.61
to $16.42 \mathrm{lb}_{\mathrm{f}}$ by $1.19 \mathrm{lb}_{\mathrm{f}}$ or $6.8 \%$. PFM 5 improved from 12.01 to $10.17 \mathrm{lb}_{\mathrm{f}}$ by $1.84 \mathrm{lb}_{\mathrm{f}}$ or $15.3 \%$. From an aircraft design standpoints, engine shut-down capability of this nature would call for the use of either a fully-feathering or foldable propeller. However, the added cost of such a propeller would arguably be justified in the long run.

Table 11-6: Best (3) and Worst (5) Missions with and without In-Flight Engine Shutdowns (Shadow Observer)

| ü | Description | Wind | Trajectory | Total Time |  | $\mathbf{W}_{\text {fuel }}$, lbf |  | COST, nm/gal |  | $\begin{gathered} \Delta W_{\text {fuel }} \\ \mathrm{lb}_{\mathrm{f}} \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Trip/Simulation | Simulation | Eng Shutoff | Simulation | Eng Shutoff | Simulation | Eng Shutoff |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:27m:10s | 1h:27m:12s | 17.61 | 16.42 | 31.07 | 33.32 | -1.19 |
|  | PFM Speed 3 |  | 95.80 | 1h:36m:24s | 1h:36m:28s | 12.01 | 10.17 | 45.55 | 53.79 | -1.84 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:27m:54s | 1h:27m:56s | 17.01 | 15.72 | 32.16 | 34.80 | -1.29 |
|  | PFM 5 |  | 96.06 | 1h:32m:06s | 1h:32m:08s | 16.71 | 15.26 | 31.94 | 35.85 | -1.45 |
|  |  |  |  |  |  |  |  |  | Average $=$ | -1.44 |

### 11.4.2. Effect of Thermals

The primary simulation results in Table 11-4 were also obtained in the absence of thermals. As stated in Chapter 5, Atmospheric Modeling, thermals result in intermittent supplemental lift, with the associated (and lesser in magnitude) sink in between. It is of interest to study the effect this has on the overall energy recovery (if any). This is accomplished using the SURFACES Flight Simulator's built-in thermal generator tool. The thermal distribution used in the investigation is shown in Figure 11-13 and was generated using the settings in the form shown in Figure 11-14. The meaning of the entries is given in Chapter 5. The aircraft will intermittently penetrate thermals of random diameters and strength and experience subsequent sink outside of it, here, as far as 3-radii from the thermal center. While the flight simulator can command a position hold that would permit the thermal energy to be exploited for altitude increase, this is not practical for an airplane of the wing loading and drag characteristics of the Shadow Observer. Thus, this is not implemented here and only a thermal fly-through is simulated. The randomness in the position, size, and strength of the thermals means that any new thermal field yields different results, although, due to their random nature, they can be characterized using mathematical expectation and bounds (which could result in reduced, unchanged, or improved energy recovery). The results of this simulation for the test cases are listed in Table 11-7. These results show, that, on the average, the presence of thermals appears to improve energy recovery in spite of the larger sink regions (whose sink strength is smaller than the updraft strength). On the average, this improvement amounts to $1.52 \mathrm{lb}_{f}$ for the missions selected for the comparison or about $8.6 \%$ savings of the baseline values.

Table 11-7: Best (3) and Worst (5) Missions including Thermals (Shadow Observer)

| 苂 |  | Wind | Trajectory Distance, nm | Total Time |  | $\mathbf{W}_{\text {fuel }} \mathrm{lbf}$ |  | COST, nm/gal |  | $\begin{gathered} \Delta \mathbf{W}_{\text {fuel }} \\ \mathbf{I b}_{f} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Description |  | Trip/Simulation | Simulation | Thermals | Simulation | Thermals | Simulation | Thermals |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} \mathrm{90}$ | 91.18 | 1h:27m:10s | 1h:27m:05s | 17.61 | 15.86 | 31.07 | 34.49 | -1.75 |
|  | PFM Speed 3 |  | 95.80 | 1h:36m:24s | 1h:36m:17s | 12.01 | 10.80 | 45.55 | 50.66 | -1.21 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:27m:54s | 1h:27m:47s | 17.01 | 15.30 | 32.16 | 35.76 | -1.71 |
| 5 | PFM 5 |  | 96.06 | 1h:32m:06s | 1h:32m:05s | 16.71 | 15.30 | 31.94 | 35.76 | -1.41 |
|  |  |  |  |  |  |  |  |  | Average $=$ | -1.52 |



Figure 11-13: Thermal distribution is generated using the Voronoi methodology presented in Chapter 5.


Figure 11-14: Form and settings used to create the thermal field.

### 11.4.3. Effect of Gusts

The simulation results in Table 11-4 did not include gusts. Gusts introduce a new dimension to the operation of the aircraft in the form of excursions from both the average wind speed and direction simulated earlier and, thus, its effect on fuel consumption is of great interest. The introduction of gusts causes the head- or tailwind and up- or downdraft to vary randomly around the average values. This is set up using the SURFACES Flight Simulator's built-in wind field generator, shown in Figure 11-15. Once the wind field has been generated, the wind simulator routine automatically generates appropriate gusts in 6-DOF. As an example, the setting shown in Figure 11-15 will generate random wind speed of $30 \pm 15 \mathrm{ft} / \mathrm{s}$ at $90^{\circ} \pm 30^{\circ}$. The results of applying such settings to simulation sets 3 and 5 are listed in Table $11-8$. It can be seen that the gusts result in small energy recovery, some $0.02 \mathrm{lb}_{\mathrm{f}}$. This author expects there to be minor reduction in the energy recovery, despite the opposite resulted for the cases tested here. In either case, it is not hard to argue it is most likely negligible.

### 11.4.4. Effect of Gusts and Thermals

A realistic wind field includes thermals and gusts and is further modified by hilly and mountainous topography. This complex interaction is simulated in the SURFACES Flight Simulator, which provides a tool to observe the state of atmospheric convection in real time. This tool is known to sailplane pilots as variometer and is shown in Figure 11-16. It shows the stochastic effects generated by the wind simulator due to $a$ combination of orographic and thermal lift (or sink) and gusts. The results for the combination of thermals, and gusts are listed in Table 11-9. It can be seen that the improvement in energy recovery, which amounts to $1.51 \mathrm{lb}_{\mathrm{f}}$, is similar to that of the thermals alone (see Table 11-7) and, again, indicates the influence of gusts is insignificant.


Figure 11-16: Variometer display in the presence of orographic and thermal lift combined with atmospheric turbulence (gusts).


Figure 11-15: Form and settings used to create the wind field, including gust levels.

Table 11-8: Best (3) and Worst (5) Missions including Gusts (Shadow Observer)

| 芯 |  |  | Trajectory Distance, nm | Tota | ime |  |  | COS |  | $\Delta W_{\text {fuel }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Description | Wind | Trip/Simulation | Simulation | Gusts | Simulation | Gusts | Simulation | Gusts |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:27m:10s | 1h:27m:01s | 17.61 | 17.57 | 31.07 | 31.14 | -0.04 |
|  | PFM Speed 3 |  | 95.80 | 1h:36m:24s | 1h:36m:15s | 12.01 | 12.02 | 45.55 | 45.51 | 0.01 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:27m:54s | 1h:27m:50s | 17.01 | 16.96 | 32.16 | 32.26 | -0.05 |
|  | PFM 5 |  | 96.06 | 1h:32m:06s | 1h:32m:04s | 16.71 | 16.70 | 31.94 | 32.76 | -0.01 |
|  |  |  |  |  |  |  |  |  | Average $=$ | -0.02 |

Table 11-9: Best (3) and Worst (5) Missions including Gusts and Thermals (Shadow Observer)

| " |  |  | Trajectory Distance, nm | Tota | ime |  |  | COST | /gal | $\Delta \mathbf{W}_{\text {fuel }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Description | Wind | Trip/Simulation | Simulation | Gst+Thrml | Simulation | Gst+Thrml | Simulation | Gst+Thrml |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:27m:10s | 1h:26m:55s | 17.61 | 15.91 | 31.07 | 34.39 | -1.7 |
|  | PFM Speed 3 |  | 95.80 | 1h:36m:24s | 1h:36m:07s | 12.01 | 10.82 | 45.55 | 50.56 | -1.19 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:27m:54s | 1h:27m:38s | 17.01 | 15.29 | 32.16 | 35.78 | -1.72 |
|  | PFM 5 |  | 96.06 | 1h:32m:06s | 1h:31m:55s | 16.71 | 15.28 | 31.94 | 35.80 | -1.43 |
|  |  |  |  |  |  |  |  |  | Average $=$ | -1.51 |

### 11.4.5. Effect of Reduced Average Wind Speed

It is of interest to study how reduced wind energy affects the energy recovery. In particular reduced wind speed. To evaluate this effect, the wind speed was reduced to 10 and $20 \mathrm{ft} / \mathrm{s}$ for test sets 3 and 5 (with $0 \mathrm{ft} / \mathrm{s}$ wind speed already determined in Table 11-4). Resulting flight time (in hours) and fuel consumption (in $\mathrm{lb}_{\mathrm{f}}$ ) are shown in Figure 11-17 and in Table 11-10. The left graph of Figure 11-17 shows that, using the airspeed constraints implemented in the simulation, the user missions (the baselines) always take the least amount of time to complete.

The right graph shows how both the PFM and BPS methods depend on atmospheric convection. For instance, the PFM 5 mission recovers less energy than the Baseline 5 mission when wind speed is low (here $10 \mathrm{ft} / \mathrm{s}$ and $20 \mathrm{ft} / \mathrm{s}$ ). Recall that this deviation is affected by fuel consumption associated with the engine idling. Of course, the LiSSA module always selects the best energy recovery missions. This implies there may be an average wind speed below which the user mission may be the best trajectory.


Figure 11-17: The effect of wind speed on the mission flight time and fuel consumption (Shadow Observer).

### 11.4.6. Maximum Energy Recovery

Finally, it is of interest to ask what would have been the best energy recovery considering the foregoing sensitivities. This would call for a somewhat idealized version of the mission with the best recovery of the eight missions simulated in Table 11-4 (PFM Speed 3 with $30 \mathrm{ft} / \mathrm{s}$ at $90^{\circ}$ ). For this idealization, the
vehicle features an engine that permits in-flight shut-downs and the atmospheric convection assumes the previous thermal distribution. Implementing this approach yielded a simulated flight time of $1 \mathrm{~h}: 34 \mathrm{~m}: 30 \mathrm{~s}$ and $8.66 \mathrm{lb}_{f}$ of fuel consumed. This compares to $1 \mathrm{~h}: 22 \mathrm{~m}: 53 \mathrm{~s}$ and $16.48 \mathrm{lb}_{f}$ for the user mission (Baseline 3). This represents a $50.8 \%$ improvement in fuel consumption over the user mission. Comparing to the original PFM Speed 3 mission, whose flight time and fuel consumption amounted to $1 \mathrm{~h}: 36 \mathrm{~m}: 24 \mathrm{~s}$ and $12.01 \mathrm{lb}_{\mathrm{f}}$, this represents a $28.9 \%$ improvement.

Table 11-10: Best (3) and Worst (5) Missions assuming 10 and $20 \mathrm{ft} / \mathrm{s}$ Average Wind (Shadow Observer)

| \# | Wind Speed $10 \mathrm{ft} / \mathrm{s}$ |  | Trajectory | Total Time |  | $\mathbf{W}_{\text {fuel }}$, Ibf |  | COST, nm/gal |  | $\Delta W_{\text {fuel }}$ $\mathrm{lb}_{\mathrm{f}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Description | Wind | Trip/Simulation | Simulation | $10 \mathrm{ft} / \mathrm{s}$ | Simulation | $10 \mathrm{ft} / \mathrm{s}$ | Simulation | $10 \mathrm{ft} / \mathrm{s}$ |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:27m:10s | 1h:23m:01s | 17.61 | 16.66 | 31.07 | 32.84 | -0.95 |
|  | PFM Speed 3 |  | 95.80 | 1h:36m:24s | 1h:30m:12s | 12.01 | 15.26 | 45.55 | 35.85 | 3.25 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:27m:54s | 1h:22m:53s | 17.01 | 16.48 | 32.16 | 33.20 | -0.53 |
|  | PFM 5 |  | 96.06 | 1h:32m:06s | 1h:24m:17s | 16.71 | 16.77 | 31.94 | 32.62 | 0.06 |
|  |  |  |  |  |  |  |  |  | Average $=$ | 0.46 |


| \# | Wind Speed $20 \mathrm{ft} / \mathrm{s}$ |  | Trajectory | Total Time |  | $\mathbf{W}_{\text {fuel, }}$ l lbf |  | COST, nm/gal |  | $\Delta W_{\text {fuel }}$ $\mathrm{lb}_{\mathrm{f}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Description | Wind | Trip/Simulation | Simulation | $20 \mathrm{ft} / \mathrm{s}$ | Simulation | $20 \mathrm{ft} / \mathrm{s}$ | Simulation | $20 \mathrm{ft} / \mathrm{s}$ |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:27m:10s | 1h:24m:30s | 17.61 | 17.06 | 31.07 | 32.07 | -0.55 |
|  | PFM Speed 3 |  | 95.80 | 1h:36m:24s | 1h:32m:22s | 12.01 | 13.22 | 45.55 | 41.38 | 1.21 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} \mathrm{180}{ }^{\circ}$ | 91.18 | 1h:27m:54s | 1h:24m:36s | 17.01 | 16.55 | 32.16 | 33.06 | -0.46 |
|  | PFM 5 |  | 96.06 | 1h:32m:06s | 1h:28m:20s | 16.71 | 17.07 | 31.94 | 32.05 | 0.36 |
|  |  |  |  |  |  |  |  |  | Average $=$ | 0.14 |

### 11.5 Simulation Results for the Sparrow Hawk

This section present simulations results for the Sparrow Hawk only. The results of these comparisons are listed in Tables 11-11 and 11-12.

### 11.5.1. Important Observations for the Sparrow Hawk

The simulation results in Tables 11-11 and 11-12, again, permit the following observations to be noted.
(1) Eight sets of five missions are compared in Table 11-11, with each set corresponding to a specific wind direction and presenting the original user-mission, two PFM missions, and two BPS missions.
(2) Fuel consumption cost in Table 11-11 is given in nm/gal. The fuel consumption during the Baseline 1 mission (total range is 91.18 nm ) amounted to $14.67 \mathrm{lb}_{\mathrm{f}}$ (this compares to $18.61 \mathrm{lb}_{\mathrm{f}}$ for the Shadow Observer). The resulting "fuel mileage" is $91.18 /(14.67 / 6)=37.29 \mathrm{~nm} / \mathrm{gal}$. In comparison, the PFM 1 mission has a range of 99.97 nm and consumed $14.50 \mathrm{lb}_{\mathrm{f}}$ of fuel, rendering a 99.97/(14.50/6) $=41.37 \mathrm{~nm} / \mathrm{gal}$.
(3) For all wind directions simulated, the user missions always consumed the largest amount of fuel. The PFM or BPS mission with GICA selected airspeeds always consumed the least amount of fuel. The best case savings was for a wind direction of $90^{\circ}$ in the topography provided. It amounted to $6.07 \mathrm{lb}_{f}$ or $44.1 \%$ for the PFM with GICA selected airspeeds. The top savings using
the PBS method for the same mission amounted to $1.57 \mathrm{lb}_{\mathrm{f}}$ or $11.4 \%$. The worst case savings was for a wind direction of $180^{\circ}$ and amounted to $1.49 \mathrm{lb}_{\mathrm{f}}$ or $11.3 \%$ using the BPS with GICA selected airspeeds. It was the best savings for that wind condition.

Table 11-11: Comparison of User and LiSSA Trajectories - Raw Data (Sparrow Hawk)

|  |  |  | Trajectory | Tota | Time |  |  | COST | m/gal | Simulation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 芯 | Description | Wind | Trip/Simulation | Predicted | Simulation | Predicted | Simulation | Predicted | Simulation | (5 best) |
|  | Baseline | No wind | 91.18 | 1h:37m:42s | 1h:38m:06s | 12.67 | 12.53 | 43.18 | 43.66 |  |
| 1 | Baseline 1 | $30 \mathrm{ft} / \mathrm{s} 0^{\circ}$ | 91.18 | 1h:51m:52s | 1h:52m:11s | 14.84 | 14.67 | 36.87 | 37.29 | 1 |
|  | PFM 1 |  | 99.97 | 2h:01m:52s | 2h:01m:42s | 14.58 | 14.50 | 37.52 | 37.73 | 3 |
|  | PFM Speed 1 |  | 99.97 | 2h:23m:00s | 2h:22m:56s | 14.80 | 14.54 | 36.96 | 37.63 | 2 |
|  | BPS 1 |  | 117.74 | 2h:23m:55s | 2h:25m:31s | 11.53 | 12.30 | 47.45 | 44.48 | 4 |
|  | BPS Speed 1 |  | 117.74 | 2h:48m:08s | 2h:50m:22s | 10.50 | 11.35 | 52.10 | 48.20 | 5 |
| 2 | Baseline 2 | $30 \mathrm{ft} / \mathrm{s} 45^{\circ}$ | 91.18 | 1h:49m:19s | 1h:48m:37s | 14.26 | 14.12 | 38.36 | 38.75 | 1 |
|  | PFM 2 |  | 95.51 | 1h:54m:01s | 1h:54m:04s | 11.09 | 11.49 | 49.33 | 47.61 | 4 |
|  | PFM Speed 2 |  | 95.51 | 2h:15m:00s | 2h:15m:19s | 10.67 | 11.02 | 51.27 | 49.64 | 5 |
|  | BPS 2 |  | 107.21 | 2h:09m:28s | 2h:10m:44s | 11.98 | 12.65 | 45.67 | 43.25 | 2 |
|  | BPS Speed 2 |  | 107.21 | 2h:31m:46s | 2h:34m:19s | 11.58 | 11.72 | 47.24 | 46.68 | 3 |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:45m:50s | 1h:46m:.09s | 13.73 | 13.75 | 39.85 | 39.79 | 1 |
|  | PFM 3 |  | 95.80 | 1h:49m:26s | 1h:51m:04s | 8.04 | 8.84 | 68.04 | 61.89 | 4 |
|  | PFM Speed 3 |  | 95.80 | 2h:09m:49s | 2h:12m:48s | 6.53 | 7.68 | 83.78 | 71.23 | 5 |
|  | BPS 3 |  | 106.34 | 2h:05m:21s | 2h:05m:46s | 12.28 | 12.46 | 44.55 | 43.91 | 2 |
|  | BPS Speed 3 |  | 106.34 | 2h:27m:35s | 2h:28m:47s | 12.68 | 12.08 | 43.15 | 45.29 | 3 |
| 4 | Baseline 4 | $30 \mathrm{ft} / \mathrm{s} 135^{\circ}$ | 91.18 | 1h:45m:41s | 1h:46m:13s | 13.35 | 13.41 | 40.98 | 40.80 | 1 |
|  | PFM 4 |  | 96.08 | 1h:51m:04s | 1h:51m:40s | 9.24 | 9.83 | 59.21 | 55.65 | 3 |
|  | PFM Speed 4 |  | 96.08 | 2h:16m:55s | 2h:18m:15s | 8.00 | 8.90 | 68.39 | 61.47 | 5 |
|  | BPS 4 |  | 109.94 | 2h:07m:18s | 2h:09m:36s | 10.12 | 10.93 | 54.06 | 50.05 | 2 |
|  | BPS Speed 4 |  | 109.94 | 2h:41m:24s | 2h:44m:30s | 8.47 | 9.66 | 64.59 | 56.63 | 4 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:46m:38s | 1h:47m:23s | 13.11 | 13.22 | 41.73 | 41.38 | 1 |
|  | PFM 5 |  | 96.06 | 1h:52m:22s | 1h:52m:28s | 12.39 | 12.66 | 44.15 | 43.21 | 2 |
|  | PFM Speed 5 |  | 96.06 | 2h:25m:29s | 2h:25m:34s | 12.31 | 12.61 | 44.44 | 43.38 | 3 |
|  | BPS 5 |  | 116.56 | 2h:14m:29s | 2h:16m:28s | 12.17 | 12.02 | 44.95 | 45.51 | 4 |
|  | BPS Speed 5 |  | 116.56 | 2h:50m:50s | 2h:53m:20s | 11.51 | 11.78 | 47.53 | 46.44 | 5 |
| 6 | Baseline 6 | $30 \mathrm{ft} / \mathrm{s} 225^{\circ}$ | 91.18 | 1h:46m:23s | 1h:47m:05s | 13.38 | 13.34 | 40.89 | 41.01 | 1 |
|  | PFM 6 |  | 96.93 | 1h:53m:01s | 1h:53m:09s | 11.68 | 11.76 | 46.84 | 46.52 | 2 |
|  | PFM Speed 6 |  | 96.93 | 2h:22m:09s | 2h:21m:56s | 11.61 | 11.54 | 47.12 | 47.41 | 3 |
|  | BPS 6 |  | 104.93 | 1h:57m:51s | 1h:58m:08s | 10.56 | 10.69 | 51.81 | 51.18 | 4 |
|  | BPS Speed 6 |  | 104.93 | 2h:34m:44s | 2h:34m:40s | 10.56 | 9.87 | 51.81 | 55.43 | 5 |
| 7 | Baseline 7 | $30 \mathrm{ft} / \mathrm{s} 270^{\circ}$ | 91.18 | 1h:46m:31s | 1h:47m:09s | 13.92 | 13.74 | 39.30 | 39.82 | 1 |
|  | PFM 7 |  | 98.46 | 1h:54m:59s | 1h:55m:32s | 11.83 | 11.90 | 46.25 | 45.97 | 2 |
|  | PFM Speed 7 |  | 98.46 | 2h:19m:50s | 2h:20m:36s | 11.73 | 11.70 | 46.64 | 46.76 | 3 |
|  | BPS 7 |  | 107.10 | 2h:00m:35s | 2h:05m:17s | 9.74 | 10.63 | 56.17 | 51.47 | 4 |
|  | BPS Speed 7 |  | 107.10 | 2h:24m:46s | 2h:28m:04s | 8.57 | 9.08 | 63.84 | 60.25 | 5 |
| 8 | Baseline 8 | $30 \mathrm{ft} / \mathrm{s} 315^{\circ}$ | 91.18 | 1h:49m:53s | 1h:50m:12s | 14.71 | 14.45 | 37.19 | 37.86 | 1 |
|  | PFM 8 |  | 98.59 | 1h:58m:09s | 1h:58m:38s | 10.13 | 10.87 | 54.01 | 50.33 | 4 |
|  | PFM Speed 8 |  | 98.59 | 2h:18m:19s | 2h:18m:55s | 9.62 | 10.29 | 56.87 | 53.17 | 5 |
|  | BPS 8 |  | 107.21 | 2h:06m:29s | 2h:07m:34s | 12.90 | 12.67 | 42.41 | 43.18 | 2 |
|  | BPS Speed 8 |  | 107.21 | 2h:37m:12s | 2h:38m:06s | 12.05 | 12.08 | 45.40 | 45.29 | 3 |

(4) The PFM and BPS methods both lead to longer flight time to complete a mission. The longest flight duration occurred for BPS Speed 5 ( $180^{\circ}$ wind direction) of $2 \mathrm{~h}: 56 \mathrm{~m}: 41 \mathrm{~s}$. The same mission as a user-mission took $1 \mathrm{~h}: 47: 23 \mathrm{~s}$. However, in spite of this difference, the former consumed $11: 73 \mathrm{lb}_{\mathrm{f}}$ versus $13.22 \mathrm{lb}_{\mathrm{f}}$ for the user-mission.
(5) As for the Shadow Observer, the PFM and BPS methods always consumed less fuel than the corresponding user-missions and added savings were always obtained using the GICA selected airspeeds. As before, the fuel savings are affected by wind and topography (see Figure 11-18 and Figure 11-19). It also shows good agreement between the prediction and simulation, indicating the prediction is an effective tool to select the right trajectory.

Table 11-12: Comparison of User and LiSSA Trajectories - Savings in Fuel Consumption (Sparrow Hawk)

|  |  |  | Trajectory |  |  |  | ings, lb |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| を | Description | Wind | Simulation | $\Delta \mathbf{W}_{\text {fuel }}$ | $\Delta$ | \% $\Delta$ | $\Delta_{\text {max }}$ | Method |
|  | Baseline | No wind | 91.18 | -0.14 |  |  |  |  |
| 1 | Baseline 1 | $30 \mathrm{ft} / \mathrm{s} 0^{\circ}$ | 91.18 | -0.17 | 0.00 | 0.0 | 3.32 | BPS Speed |
|  | PFM 1 |  | 99.97 | -0.08 | 0.17 | 1.2 |  |  |
|  | PFM Speed 1 |  | 99.97 | -0.26 | 0.13 | 0.9 |  |  |
|  | BPS 1 |  | 117.74 | 0.77 | 2.37 | 16.2 |  |  |
|  | BPS Speed 1 |  | 117.74 | 0.85 | 3.32 | 22.6 |  |  |
| 2 | Baseline 2 | $30 \mathrm{ft} / \mathrm{s} 45^{\circ}$ | 91.18 | -0.14 | 0.00 | 0.0 | 3.10 | PFM Speed |
|  | PFM 2 |  | 95.51 | 0.40 | 2.63 | 18.6 |  |  |
|  | PFM Speed 2 |  | 95.51 | 0.35 | 3.10 | 22.0 |  |  |
|  | BPS 2 |  | 107.21 | 0.67 | 1.47 | 10.4 |  |  |
|  | BPS Speed 2 |  | 107.21 | 0.14 | 2.40 | 17.0 |  |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 0.02 | 0.00 | 0.0 | 6.07 | PFM Speed |
|  | PFM 3 |  | 95.80 | 0.80 | 4.91 | 35.7 |  |  |
|  | PFM Speed 3 |  | 95.80 | 1.15 | 6.07 | 44.1 |  |  |
|  | BPS 3 |  | 106.34 | 0.18 | 1.29 | 9.4 |  |  |
|  | BPS Speed 3 |  | 106.34 | -0.60 | 1.67 | 12.1 |  |  |
| 4 | Baseline 4 | $30 \mathrm{ft} / \mathrm{s} 135^{\circ}$ | 91.18 | 0.06 | 0.00 | 0.0 | 4.51 | PFM Speed |
|  | PFM 4 |  | 96.08 | 0.59 | 3.58 | 26.7 |  |  |
|  | PFM Speed 4 |  | 96.08 | 0.90 | 4.51 | 33.6 |  |  |
|  | BPS 4 |  | 109.94 | 0.81 | 2.48 | 18.5 |  |  |
|  | BPS Speed 4 |  | 109.94 | 1.19 | 3.75 | 28.0 |  |  |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 0.11 | 0.00 | 0.0 | 1.44 | BPS Speed |
|  | PFM 5 |  | 96.06 | 0.27 | 0.56 | 4.2 |  |  |
|  | PFM Speed 5 |  | 96.06 | 0.30 | 0.61 | 4.6 |  |  |
|  | BPS 5 |  | 116.56 | -0.15 | 1.20 | 9.1 |  |  |
|  | BPS Speed 5 |  | 116.56 | 0.27 | 1.44 | 10.9 |  |  |
| 6 | Baseline 6 | $30 \mathrm{ft} / \mathrm{s} 225^{\circ}$ | 91.18 | -0.04 | 0.00 | 0.0 | 3.47 | BPS Speed |
|  | PFM 6 |  | 96.93 | 0.08 | 1.58 | 11.8 |  |  |
|  | PFM Speed 6 |  | 96.93 | -0.07 | 1.80 | 13.5 |  |  |
|  | BPS 6 |  | 104.93 | 0.13 | 2.65 | 19.9 |  |  |
|  | BPS Speed 6 |  | 104.93 | -0.69 | 3.47 | 26.0 |  |  |
|  | Baseline 7 | $30 \mathrm{ft} / \mathrm{s} 270^{\circ}$ | 91.18 | -0.18 | 0.00 | 0.0 | 4.66 | BPS Speed |
|  | PFM 7 |  | 98.46 | 0.07 | 1.84 | 13.4 |  |  |
| 7 | PFM Speed 7 |  | 98.46 | -0.03 | 2.04 | 14.8 |  |  |
|  | BPS 7 |  | 107.10 | 0.89 | 3.11 | 22.6 |  |  |
|  | BPS Speed 7 |  | 107.10 | 0.51 | 4.66 | 33.9 |  |  |
| 8 | Baseline 8 | $30 \mathrm{ft} / \mathrm{s} 315^{\circ}$ | 91.18 | -0.26 | 0.00 | 0.0 | 4.16 | PFM Speed |
|  | PFM 8 |  | 98.59 | 0.74 | 3.58 | 24.8 |  |  |
|  | PFM Speed 8 |  | 98.59 | 0.67 | 4.16 | 28.8 |  |  |
|  | BPS 8 |  | 107.21 | -0.23 | 1.78 | 12.3 |  |  |
|  | BPS Speed 8 |  | 107.21 | 0.03 | 2.37 | 16.4 |  |  |



Figure 11-18: Weight of fuel consumed between the predicted and simulated trips, when using the Potential Flow Method (Sparrow Hawk).


Figure 11-19: Weight of fuel consumed between the predicted and simulated trips, when using the Best Path Search Method (Sparrow Hawk).
(6) Figure 11-20 shows the effect of wind direction on the mission flight time and fuel consumption. It can be seen that the user missions always take the least amount of time and the greatest amount of fuel.
(7) Figure 11-21 shows difference between the predicted and simulated mission flight time and fuel consumption. As stated elsewhere, it is important the predictions lead to a correct selection of the best trajectory. The left graph shows that worst flight time accuracy was about 4.7 minutes for a mission that lasted some 125.3 minutes. This corresponds to about $3.75 \%$ difference.

Overall, the simulated missions tend to last a couple of minutes, or so, longer than predicted. The reasons for these discrepancies were discussed earlier.
(8) The right graph of Figure 11-21 shows the difference between predicted and simulated fuel consumptions with a worst case being $1.19 \mathrm{lb}_{\mathrm{f}}$ (greater consumption than predicted). This corresponds to about $12.3 \%$ difference. In this case, the simulation of the Sparrow Hawk demonstrated that the LiSSA always selected the least energy costly trajectory. This can be seen by comparing the lowest predicted fuel consumption values in Table 11-11 to the simulated optimum trajectories in Table 11-12, which match for all the wind directions simulated, giving further confidence in the fidelity of the LiSSA.


Figure 11-20: Mission flight time and fuel consumption as function of wind direction (Sparrow Hawk).


Figure 11-21: Difference between predicted and simulated mission flight time and fuel consumption as function of wind direction (Sparrow Hawk).

### 11.6 Mission Sensitivities for the Sparrow Hawk

This section considers the effect of modifying selected parameters in the simulation study. Topics of interest range from the effect of using controls such as a yaw damper (the Sparrow Hawk has poor Dutch roll damping) or permitting a reliable piston engine shut-down and restart to the impact of gusts and thermals. Other effect of interest is the presence of thermals and the size of the topographical panels. Only two missions, each from two sets of the eight wind directions will be evaluated; the ones with the highest and lowest energy recoveries. These missions involve Set 3 ( $90^{\circ}$ wind direction) and Set 5 ( $180^{\circ}$ wind direction). Note that there is limit to the amount of data that can be included in this section, but many of the following studies should be considered initial in nature and worthy of further study. For this reason, the average calculated below each table should be treated with care - it is intended for comparison discussions only. Also note that in the tables below, a negative value in the column labeled $\Delta W_{\text {fuel }}$ means less fuel was consumed than during the reference simulation: in other words, a negative $\Delta W_{\text {fuel }}$ means improvement in energy recovery.

### 11.6.1. Effect of a Yaw Damper

Since the aircraft model used for the simulation was (purposely) designed with low lateral-directional oscillation convergence (Dutch roll damping), it is of interest to evaluate if enabling the autopilot to serve as a yaw damper too will affect fuel conservation. To do this, the simulation sets 3 and 5 were repeated with the Yaw Hold (or Yaw Damper) autopilot function active. The results of this simulation are listed in Table 11-13. The corrective rudder deflection was minor and varied between $\pm 0.2^{\circ}$ maximum. It can be seen that the difference between the yaw damper on and off is a minor improvement in energy recovery of the order of $0.12 \mathrm{lb}_{\mathrm{f}}$ using the data. While this is not particularly large in scale, it supports aircraft design philosophy in which energy recovery is improved in the proper range of wing loading (as shown in Chapter 6), but also dynamic response that either naturally damped or features control laws that make it appear it is so.

Table 11-13: Best (3) and Worst (5) Missions with and without Yaw Damper (Sparrow Hawk)

|  | Description | Wind | Trajectory Distance, nm | Total Time |  | $\mathbf{W}_{\text {fuel }}$, lbf |  | COST, $\mathrm{nm} / \mathrm{gal}$ |  | $\begin{gathered} \Delta W_{\text {fuel }} \\ \mathrm{Ib}_{\mathrm{f}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Trip/Simulation | Simulation | Yaw Damper | Simulation | Yaw Damper | Simulation | Yaw Damper |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:46m:.09s | 1h:46n:10s | 13.75 | 13.66 | 39.79 | 40.05 | -0.09 |
|  | PFM Speed 3 |  | 95.80 | 2h:12m:48s | 2h:12m:47s | 7.68 | 7.63 | 71.23 | 71.70 | -0.05 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:47m:23s | 1h:47m:23s | 13.22 | 13.19 | 41.38 | 41.48 | -0.03 |
|  | BPS Speed 5 |  | 116.56 | 2h:53m:20s | 2h:54m:07s | 11.78 | 11.49 | 46.44 | 47.61 | -0.29 |
|  |  |  |  |  |  |  |  |  | Average $=$ | -0.12 |

### 11.6.2. Effect of Piston Engine In-Flight Shutdowns

As for the Shadow Observer, the simulation results in Table 11-11 assume the piston engine is never shut-off completely and idle-power of 2 BHP at S-L. Again, simulations were run assuming engine shutoff was permitted. The results are listed in Table 11-14. Again, substantial energy-savings are to be had by this, with average fuel savings close to $2.00 \mathrm{lb}_{\mathrm{f}}$. The worst case fuel consumption is improved by $9.4 \%$ to $13.75 \mathrm{lb}_{f}$ and the best case fuel consumption of $7.68 \mathrm{lb}_{f}$ by a whopping $31.5 \%$.

Table 11-14: Best (3) and Worst (5) Missions with and without In-Flight Engine Shutdowns (Sparrow Hawk)

| ~ | Description | Wind | Trajectory <br> Distance, nm$\|$ | Total Time |  | $\mathbf{W}_{\text {fuel, }} \mathrm{lbf}$ |  | COST, nm/gal |  | $\begin{gathered} \Delta W_{\text {fuel }} \\ \mathbf{I b}_{\mathrm{f}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Simulation | Eng Shutoff | Simulation | Eng Shutoff | Simulation | Eng Shutoff |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:46m:.09s | 1h:46m:10s | 13.75 | 12.46 | 39.79 | 43.91 | -1.29 |
|  | PFM Speed 3 |  | 95.80 | 2h:12m:48s | 2h:12m:53s | 7.68 | 5.26 | 71.23 | 104.01 | -2.42 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:47m:23s | 1h:47m:24s | 13.22 | 11.85 | 41.38 | 46.17 | -1.37 |
|  | BPS Speed 5 |  | 116.56 | 2h:53m:20s | 3h:01m:41s | 11.78 | 8.88 | 46.44 | 61.61 | -2.9 |
|  |  |  |  |  |  |  |  |  | Average $=$ | -2.00 |

### 11.6.3. Effect of Thermals

Simulations in the presence of thermals were also run, with the results shown in Table 11-15. Again, on the average, the presence of thermals appears to improve energy recovery. On the average, this improvement amounts to $1.44 \mathrm{lb}_{\mathrm{f}}$ for the missions selected for the comparison or about $10 \%$ savings of the baseline values.

Table 11-15: Best (3) and Worst (5) Missions including Thermals (Sparrow Hawk)

| 華 | Description | Wind | Trajectory Distance, nm | Total Time |  | $\mathbf{W}_{\text {fuel, }}$ lbf |  | COST, nm/gal |  | $\begin{gathered} \Delta W_{\text {fuel }} \\ \mathbf{l b}_{\mathrm{f}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Trip/Simulation | Simulation | Thermals | Simulation | Thermals | Simulation | Thermals |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:46m:.09s | 1h:46m:05s | 13.75 | 12.21 | 39.79 | 44.81 | -1.54 |
|  | PFM Speed 3 |  | 95.80 | 2h:12m:48s | 2h:11m:59s | 7.68 | 6.29 | 71.23 | 86.98 | -1.39 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:47m:23s | 1h:47m:20s | 13.22 | 11.66 | 41.38 | 46.92 | -1.56 |
|  | BPS Speed 5 |  | 116.56 | 2h:53m:20s | 2h:52m:25s | 11.78 | 10.50 | 46.44 | 52.10 | -1.28 |
|  |  |  |  |  |  |  |  |  | Average $=$ | -1.44 |

### 11.6.4. Effect of Gusts

Simulation sets 3 and 5 run in the presence of gusts are listed in Table 11-16. It can be seen that the gusts result in negligible energy recovery, some $0.06 \mathrm{lb}_{\mathrm{f}}$.

Table 11-16: Best (3) and Worst (5) Missions including Gusts (Sparrow Hawk)

| ~ | Description | Wind | Trajectory Distance, nm | Total Time |  | $\mathbf{W}_{\text {fuel }}$, lbf |  | COST, nm/gal |  | $\begin{gathered} \Delta \mathbf{W}_{\text {fuel }} \\ \mathbf{l b}_{\mathbf{f}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Trip/Simulation | Simulation | Gusts | Simulation | Gusts | Simulation | Gusts |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:46m:.09s | 1h:46m:09s | 13.75 | 13.68 | 39.79 | 39.99 | -0.07 |
|  | PFM Speed 3 |  | 95.80 | 2h:12m:48s | 2h:12m:29s | 7.68 | 7.64 | 71.23 | 71.61 | -0.04 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:47m:23s | 1h:47m:16s | 13.22 | 13.17 | 41.38 | 41.54 | -0.05 |
|  | BPS Speed 5 |  | 116.56 | 2h:53m:20s | 2h:52m:05s | 11.78 | 11.70 | 46.44 | 46.76 | -0.08 |
|  |  |  |  |  |  |  |  |  | Average $=$ | -0.06 |

### 11.6.5. Effect of Gusts and Thermals

The results for the combination of orographic lift, thermals, and gusts are listed in Table 11-17. It can be seen that the improvement in energy recovery, which amounts to $1.43 \mathrm{lb}_{\mathrm{f}}$, is similar to that of the thermals alone and indicates the influence of gusts is insignificant. This parallels that observed for the Shadow Observer.

Table 11-17: Best (3) and Worst (5) Missions including Gusts and Thermals (Sparrow Hawk)

| 華 |  |  | Trajectory Distance, nm | Total Time |  | $\mathbf{W}_{\text {fuel, }} \mathrm{lbf}$ |  | COST, nm/gal |  | $\begin{gathered} \Delta W_{\text {fuel }} \\ \mathbf{I b}_{\mathrm{f}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Description | Wind | Trip/Simulation | Simulation | Gst+Thrml | Simulation | Gst+Thrml | Simulation | Gst+Thrml |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:46m:.09s | 1h:45m:58s | 13.75 | 12.15 | 39.79 | 45.03 | -1.6 |
|  | PFM Speed 3 |  | 95.80 | 2h:12m:48s | 2h:11m:42s | 7.68 | 6.29 | 71.23 | 86.98 | -1.39 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:47m:23s | 1h:47m:18s | 13.22 | 11.69 | 41.38 | 46.80 | -1.53 |
|  | BPS Speed 5 |  | 116.56 | 2h:53m:20s | 2h:52m:25s | 11.78 | 10.57 | 46.44 | 51.76 | -1.21 |
|  |  |  |  |  |  |  |  |  | Average = | -1.43 |

### 11.6.6. Effect of Reduced Average Wind Speed

The effect of reduced wind speed (10 and $20 \mathrm{ft} / \mathrm{s}$ for the test sets 3 and 5) was included, with the resulting mission flight time (in hours) and fuel consumption (in $\mathrm{lb}_{\mathrm{f}}$ ) shown in Figure 11-22 and in Table 11-18, revealing similar trends as for the Shadow Observer.


Figure 11-22: The effect of wind speed on the mission flight time and fuel consumption (Sparrow Hawk).

Table 11-18: Best (3) and Worst (5) Missions assuming 10 and $20 \mathrm{ft} / \mathrm{s}$ Average Wind (Sparrow Hawk)

|  | Wind Speed $10 \mathrm{ft} / \mathrm{s}$ |  | Trajectory Distance, nm | Total Time |  | $\mathbf{W}_{\text {fuel, }}$ Ibf |  | COST, nm/gal |  | $\begin{gathered} \Delta \mathbf{W}_{\text {fuel }} \\ \mathbf{l b}_{\mathrm{f}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Description | Wind | Trip/Simulation | Simulation | $10 \mathrm{ft} / \mathrm{s}$ | Simulation | $10 \mathrm{ft} / \mathrm{s}$ | Simulation | $10 \mathrm{ft} / \mathrm{s}$ |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:46m:.09s | 1h:38m:56s | 13.75 | 12.62 | 39.79 | 43.35 | -1.13 |
|  | PFM Speed 3 |  | 95.80 | 2h:12m:48s | 2h:00m:07s | 7.68 | 10.57 | 71.23 | 51.76 | 2.89 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:47m:23s | 1h:38m:43s | 13.22 | 12.44 | 41.38 | 43.98 | -0.78 |
|  | BPS Speed 5 |  | 116.56 | 2h:53m:20s | 2h:25m:48s | 11.78 | 13.92 | 46.44 | 39.30 | 2.14 |
|  |  |  |  |  |  |  |  |  | Average $=$ | 0.78 |
| ~ّ | Wind Speed $20 \mathrm{ft} / \mathrm{s}$ |  | Trajectory Distance, nm | Total Time |  |  |  | COST, nm/gal |  | $\Delta W_{\text {fuel }}$ |
| i | Description | Wind | Trip/Simulation | Simulation | $20 \mathrm{ft} / \mathrm{s}$ | Simulation | $20 \mathrm{ft} / \mathrm{s}$ | Simulation | $20 \mathrm{ft} / \mathrm{s}$ |  |
| 3 | Baseline 3 | $30 \mathrm{ft} / \mathrm{s} 90^{\circ}$ | 91.18 | 1h:46m:.09s | 1h:41m:28s | 13.75 | 12.98 | 39.79 | 42.15 | -0.77 |
|  | PFM Speed 3 |  | 95.80 | 2h:12m:48s | 2h:03m:42s | 7.68 | 8.69 | 71.23 | 62.96 | 1.01 |
| 5 | Baseline 5 | $30 \mathrm{ft} / \mathrm{s} 180^{\circ}$ | 91.18 | 1h:47m:23s | 1h:41m:37s | 13.22 | 12.67 | 41.38 | 43.18 | -0.55 |
|  | BPS Speed 5 |  | 116.56 | 2h:53m:20s | 2h:33m:33s | 11.78 | 12.84 | 46.44 | 42.61 | 1.06 |
|  |  |  |  |  |  |  |  |  | Average = | 0.19 |

## REFERENCES

${ }^{1}$ Gudmundsson, Snorri, General Aviation Aircraft Design - Applied Methods and Procedures, ButterworthHeineman (of Elsevier), 2013.

## 12. Conclusions

This final chapter brings to a close the development of the Generic Intelligent Control Algorithm, GICA. Foregoing chapters have presented a description of the most important elements of the multidisciplinary approach its operation requires, as well as its synergistic power. With respect to this work, the demonstration of the energy recovery potential is of greatest importance and this was demonstrated in Chapter 11, Simulation Samples. The basic conclusion of this effort is that it supports the notion the GICA has the potential to provide substantial energy conservation for fixed-wing aircraft of the appropriate wing loading, through the energy harvesting of atmospheric convection. This chapter presents this and other concluding remarks regarding further research.

### 12.1 Summary of Conclusions

This section summarizes conclusions that can be drawn from the results produced in Chapter 11, Simulation Samples. The capability of the GICA was simulated using two "hypothetical" sUAVs, called the Shadow Observer and Sparrow Hawk (which has one-half the wing loading of the Shadow Observer). The aircraft were operated above topography that represents a typical mountainous region and were subjected to eight wind directions ranging from $0^{\circ}$ to $315^{\circ}$ (i.e. $0^{\circ}, 45^{\circ}, \ldots, 315^{\circ}$ ). The user-mission is justifiably described as one of multi-heading, multi-elevation complexity and can be considered an example of a "frequently operated surveillance mission," executed on a regular basis, subjecting the sUAV to contrasting meteorological conditions.

Besides the user-mission, four additional flights were produced for each wind direction by modifying the user mission using the Lift-Seeking Sink-Avoidance (LiSSA) algorithm in accordance with the Potential Flow Method (PFM) and Best Path Search (BPS) algorithms. These algorithms maintain the waypoints originally specified by the user and only work on the trajectories between them. This guarantees the user-waypoints are treated as vital geographic positions that must be visited, no matter the cost. The LiSSA can also be directed to specify optimum airspeed at which the airplane will operate as it traverses the PFM and BPS trajectories.

The following conclusions can be drawn from the simulations detailed in Chapter 11.
(1) The energy recovery, in part, depends on the characteristics and performance capabilities of the aircraft. The ideal aircraft design from energy conservation and harvesting standpoint is one with a high maximum lift-to-drag ratio, low wing loading ( $1-10 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}$ ), and, thus, low minimum rate-of-descent. The drawback of low wing loading is reduced airspeed for optimum speeds (i.e. $V_{b g}, V_{\min } R O D, V_{x}, V_{y}$, etc.), which reduces headwind penetration capability. However, energy extraction at low wind speeds is greatly improved.
(2) The energy recovery, in part, depends on the topography over which the aircraft operates. In this context, a mountainous region is ripe with vertical convection, both orographic and thermal, while flatlands primarily offer thermals. Furthermore, how the airplane is operated in
mountainous regions plays a major role in the effectiveness of the convective energy extraction.
(3) The energy recovery, in part, depends on wind direction for any given topography. This is evident from Figures 11-9 and 11-10 (Shadow Observer) and Figures 11-18 and 11-19 (Sparrow Hawk), which show greater energy recovery for specific wind directions. For the topography shown, the greatest energy recovery was made for wind direction of $90^{\circ}$ (wind out of East). The least was for wind direction of $180^{\circ}$ (wind out of South). Which directions turn out to yield the greatest energy recovery depends on the topography.
(4) The energy recovery is improved by operating the aircraft at optimum airspeeds. The LiSSA selects airspeeds in accordance with whether a mission segment (trajectory between adjacent waypoints) calls for climb, level altitude cruise, or descent. The details of this selection are described in Bullet (3) of Section 11.3.3, Important Observations for the Shadow Observer (same approach is used for the Sparrow Hawk). This is evident from Table 11-5, which shows seven of eight optimal missions took advantage of the LiSSA airspeed optimization. Similar results were found from Table 11-12 for the Sparrow Hawk.
(5) It should be noted that, ultimately, the energy recovery depends, in part, on which airspeeds the user selects for each mission segment. The user-mission was designed with some "reasonable" airspeed in mind and was based on known aircraft performance characteristics. These could have been considered "suitable" airspeeds by a mission planner who does not consider (or "worry about") the effect of atmospheric convection. For instance, the climb is implemented at 50 KCAS and level flight at 60 KCAS, both which are "easy, round numbers." In fact, the 50 KCAS for the climb segments is actually close to $V_{y}$ and the 60 KCAS for the level segment is close to the best range airspeed for this aircraft. The point is that, had higher operational airspeeds been selected (for instance a 70 KCAS typical cruising speed), the fuel savings obtained by the LiSSA would have been even greater than reported.
(6) The presence of atmospheric thermals improved fuel savings in this simulation by an average of $1.52 \mathrm{lb}_{\mathrm{f}}$ of fuel (see Table 11-7 and 11-15). It should be kept in mind that thermal field has random thermals with maximum core strength of $5-20 \mathrm{ft} / \mathrm{s}$ within a radius of 300-1500 ft, surrounded by downdraft regions extending $3 X$ the core radius away from the center.
(7) The presence of gusts was found to have negligible effect in the simulation (see Table 11-8 and 11-16), other than perhaps ride quality.
(8) The presence of gusts and thermals improved fuel savings by an average of $1.51 \mathrm{lb}_{\mathrm{f}}$ of fuel in this simulation (see Table 11-9 and 11-17). Based on Bullets (6) and (7) above, this can almost solely be attributed to the thermals.
(9) The most efficient trajectories were always obtained using either the PFM or BPS trajectories. In the case of the Shadow Observer sUAV (see Chapter 11), seven out of eight times these included the LiSSA selected airspeeds. In the case of the Sparrow Hawk sUAV, the LiSSA selected airspeed were always the best.
(10) Operating an engine that can be shut-off completely, while harvesting atmospheric energy or when exchanging potential to kinetic energy in glide, improved the fuel savings by additional $1.44 \mathrm{lb}_{\mathrm{f}}$ (see Table 11-6 and 11-14). Such a feature should be considered a design requirement in a GICA controlled aircraft. Another feature is responsive engine throttling, as this reduces airspeed transitioning time (i.e. time to accelerate or decelerate between target airspeeds). This is important advantage for battery powered aircraft.
(11) As discussed in Bullet (10) of Section 11.3.3, Important Observations for the Shadow Observer, it is vital for the LiSSA to select the most energy efficient trajectory based on the prediction results. In the case of the Shadow Observer, it did so in all but one case. In the case of the Sparrow Hawk (half the wing loading of the Shadow Observer), it always selected the correct trajectory. It appears that this may happen when the wind direction/topography results in low energy recovery and may be improved by higher fidelity performance prediction. From a computational standpoint, the performance prediction is a simplified and substantially faster method than the simulation. However, there is room for improvements (see later).
(12) A general conclusion can be summarized as follows. The atmospheric energy recovery for an airplane depends on many factors; aircraft characteristics, wind speed and direction, topography, airspeed selection, presence or absence of thermals, to name a few. Successful harvesting requires all to be considered.

### 12.2 Further Research

The state of development of the GICA is far from being complete and this work reveals a number of areas for further research. These can be put into two categories; theoretical and experimental. These will now be briefly discussed.

### 12.2.1. Theoretical Areas for Further Research

One of the primary requirements for the GICA is a correct selection of the optimized trajectory. This places a requirement for a high fidelity theoretical performance prediction. While the current model is largely adequate, it can still be improved. For instance, accounting for turn radii and the time required to accelerate and decelerate between airspeeds will improve prediction accuracy. The turn radius depends on the airspeed, but, in part, on the wind speed and direction, which may cause a significant overshooting in tailwind, which, in turn, requires additional distance to be flown. In spite of that, the difference between the predicted and simulated fuel burns is generally not large. As can be seen in

Table 11-5, the average of the $\Delta \mathrm{W}_{\text {fuel }}$ column is $-0.16 \mathrm{lb}_{f}$ (negative means that the simulated fuel burn is less than predicted) with a standard deviation of $0.55 \mathrm{lb}_{\mathrm{f}}$.

Another area of improvement is the incorporation of existing topography using Digital Elevation Maps (DEM). This will allow simulation to be performed over existing geographical areas, something of great importance when validating the GICA. DEMs can be downloaded from various websites, for instance US Geological Survey (www.usgs.gov). This will largely require the selected DEM to be converted into NURBS, which is used by the flight simulator software. The development of such a conversion code would represent a worthy topic for a graduate student.

Additional work must be done to utilize data from analyses performed using some of the methods discussed in Section 2.3, Atmospheric Modeling, such as the Mass-Consistent models, the Jackson-Hunt method, Computational Fluid Dynamics (CFD), and Mesoscale Numerical Weather Prediction (Mesoscale NWP) and by the use of mesoscale weather prediction software called Weather Research and Forecasting Model (WRF). The last one is frequently used for local weather forecasting, providing a 24 hour outlook. Currently, this tool is typically applied to topographical mesh size of $4 \times 4 \mathrm{~km}$ and is subject to stability issues for meshes smaller than $1 \times 1 \mathrm{~km}$. The mesh sizes suitable for the GICA are typically of the order of $0.1 \times 0.1$ to $0.15 \times 0.15 \mathrm{~km}$. However, vertical mesh size limits permitted by WRF are much smaller and this is used to capture boundary layer shapes such as those described in Section 5.2.1, Modeling the Planetary (Atmospheric) Boundary Layer. On the other hand, NWP software is capable of predicting meteorological phenomenon such as mountain blocking and even dynamics of mountain waves. While software interfacing may pose some challenges in terms of data conversion and longrange Wi-Fi transmission, NWP software has a great potential to supply reliable forecast that can be used by the GICA. For instance, the data conversion would return the average wind vector on a given topographical panel with 500 or 1000 ft elevation separation. Then, the forecast at each time interval would be evaluated through interpolation. Thus, if a WRF prediction is made over a 24 hour interval (typical), this would allow the LiSSA to account for changes in wind direction that inevitable take place during a long endurance flight.

Further research should be done in trajectory planning by incorporating the $A^{*}$ heuristic algorithm. This topic, too, represents a suitable topic for a graduate student. The A* algorithm permits terrain and downdraft regions to be combined to form a single obstruction and the resulting path will, thus, automatically bypass terrain without requiring climb. However, there is a potential for the range of the resulting trajectory to be much greater, in particular if the trajectory requires the airplane to fly around a mountain range, rather than flying above it. Alternatively, en expansion of the $A^{*}$ algorithm to 3dimensions should be considered as well. Thus, the resulting trajectory would have to be compared to the PFM and BPS missions before selection.

### 12.2.2. Experimental Areas for Further Research

Further development of the GICA calls for experimental validation. Such an experiment should involve the fabrication of a proof-of-concept aircraft, equipped with an Automated Flight Management System
(AFMS) capable of running the algorithm and performing the rapid trajectory evaluation in flight. Ideally, two such experimental aircraft should be acquired to allow comparison through dual launching (i.e. both are launched at the same time and fly two trajectories; a user-mission versus a PFM or BPS mission). Such experimentation can be accomplished without the acquisition of an expensive test vehicle. It just has to be large enough to accommodate the necessary flight computer and other necessary equipment in its fuselage. Such a vehicle would likely have a 3 m wingspan, wing loading of approximately $5 \mathrm{~kg} / \mathrm{m}^{2}$ $\left(1 \mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}\right)$ and should also feature solar cells and AFMS to allow it to operate using GICA. For instance, it is certainly feasible to purchase a commercially available electric sailplane, such as the E-flite Radian XL, which has a 2.6 m wingspan, and with some modifications, install the necessary equipment. The AFMS could be based on the well known Arduino flight control system, which permits programming through codes such as Matlab/Simulink. This project is large enough to utilize one or more graduate assistants and, thus, would call for some funding.

Furthermore, the flight time of a vehicle such as the Radian XL could be enhanced using solar energy using commercially available solar cells with $21.5 \%$ efficiency and assuming a solar constant of 1000 W/m² (see Section 9.4.2, Active Means for Energy Replenishing - Solar Energy). Assuming that 90\% of its $0.610 \mathrm{~m}^{2}\left(6.57 \mathrm{ft}^{2}\right)$ wing area could be used for power generation, the total power generated could be of the order of $0.90 \times\left(0.610 \mathrm{~m}^{2}\right) \times 0.215 \times\left(1000 \mathrm{~W} / \mathrm{m}^{2}\right)=118 \mathrm{~W}$. If it is further assumed that the curvature of the wing surface reduced efficiency by, say, $30 \%$, the resulting power generation would be of the order of 83 W . If this was used to replace battery power from a 3 S LiPo at 12 V , the resulting current would be $83 / 12=6.89$ A, which is sufficient to maintain level flight over a range of airspeeds. Since the operation of the GICA involves long periods of engine shut-off in pure glide, this system should be sufficient to allow the aircraft to, at least, partially recharge the LiPo and remain airborne for hours during daytime operations. It should be added, that powering electric sailplanes in this fashion is not new, as discussed in Section 9.4.2.

Experiments of this nature would help further development of the GICA through validation. Such experiments are also likely to encourage development of other features, yet undiscovered. It is certain there is great value in the development of a GICA for fully autonomous UAVs. Their role and utility is bound to expand in the coming decades, calling for ongoing research and development efforts.

## Appendix A - Code Snippets

This appendix contains several subroutines and functions that were used in the making of the flight simulator. All code snippets are written using Visual Basic.

## A. 1 Routine to Determine Atmospheric Properties

The following function uses the material presented in Section 5.1.8, Atmospheric Properties from S-L to Upper Mesosphere, to estimate temperature, pressure, density, and the associated ratios. The user calls the function by passing two arguments, H and PropertyId. The former contains the true altitude in ft and the latter indicates the result the function should return. For instance, if PropertyID $=2$, then the density ratio at altitude H is returned. Note that this function only uses the UK-system of units. Thus, the temperature is returned as ${ }^{\circ} \mathrm{R}$, pressure is returned as $\mathrm{Ib}_{\mathrm{f}} / \mathrm{ft}^{2}$, and density as slugs $/ \mathrm{ft}^{3}$.

```
Function AtmosProperty(H As Single, PropertyID As Byte) As Single
'This function calculates an atmospheric property based on the variable
'PropertyID at the given altitude H in ft, where:
' If PropertyID = 0 then return Temperature ratio
' If PropertyID = 1 then return Pressure ratio
' If PropertyID = 2 then return Density ratio
' If PropertyID = 10 then return Temperature
' If PropertyID = 11 then return Pressure
' If PropertyID = 12 then return Density
'Initialize
    Dim TempRatio As Single, R As Single
    Dim PressRatio As Single
    Dim DensRatio As Single
'Select altitude
    If H < 36089 Then
        R = 1 - 0.0000068756 * H
        TempRatio = R
        PressRatio = R ^ 5.2561
        DensRatio = R ^ 4.2561
    ElseIf H >= 36089 And H < 65671 Then
        R = -(H - 36089) / 20806
        TempRatio = 0.751865
        PressRatio = 0.223361 * Exp (R)
        DensRatio = 0.297176 * Exp(R)
    ElseIf H >= 65671 And H < 104987 Then
        TempRatio = 0.682457 + H / 945374
        PressRatio = (0.988626 + H / 652600) ^ -34.1632
        DensRatio = (0.978261 + H / 659515) ^ -35.1632
    ElseIf H >= 104987 And H < 154199 Then
        TempRatio = 0.482561 + H / 337634
        PressRatio = (0.898309 + H / 181373) ^ -12.20114
        DensRatio = (0.857003 + H / 190115) ^ -13.20114
    ElseIf H >= 154199 And H < 167323 Then
        R = - (H - 154199) / 25992
        TempRatio = 0.939268
        PressRatio = 0.00109456 * Exp(R)
        DensRatio = 0.00116533 * Exp(R)
    ElseIf H >= 167323 And H < 232940 Then
            TempRatio = 1.434843 - H / 337634
```

```
    PressRatio = (0.838263 - H / 577922) ^ 12.20114
    DensRatio = (0.79899 - H / 606330) ^ 11.20114
    ElseIf H >= 232940 And H < 278386 Then
        TempRatio = 1.237723 - H / 472687
        PressRatio = (0.917131 - H / 637919) ^ 17.0816
        DensRatio = (0.900194 - H / 649922) ^ 16.0816
    End If
'Output
    Select Case PropertyID
    Case 0
        AtmosProperty = TempRatio
    Case 1
        AtmosProperty = PressRatio
    Case 2
        AtmosProperty = DensRatio
    Case 10
        AtmosProperty = TempRatio * 518.67
    Case 11
        AtmosProperty = PressRatio * 2116
    Case 12
        AtmosProperty = DensRatio * 0.002378
    End Select
End Function
```


## A. 2 Rapid Interpolation of 2D Lookup Tables

The following Visual Basic routine implements the 2-dimensional interpolation presented above:

```
Function MATH_LookupXYTable(inXY() As Single, inX As Single) As Single
'This function determines the Y that corresponds to the inX in a table of Xs and Ys.
'The vector inXY() has }x\mathrm{ in the first column and y in the second one and assumes
'a sorted vector wrt x. Thus, the vector looks as shown below:
'
' inXY(i,1) = x
' inXY(i,2) = y
'
'Initialize
    Dim i As Integer, N As Integer
    Dim X1 As Single, X2 As Single, Parameter As Single
'Presets
    N = UBound(inXY, 1)
'Search
    For i = 1 To N
        'X is less than the first X, so give Y the first Y-value
        If i = 1 And inX <= inXY(1, 1) Then
            MATH_LookupXYTable = inXY(1, 2)
            Exit For
        'X is greater than the last X, so give Y the last Y-value
        ElseIf i = N And inX >= inXY(N, 1) Then
            MATH_LookupXYTable = inXY(N, 2)
            Exit For
        ElseIf inX <= inXY(i, 1) Then
            X1 = inXY(i - 1, 1)
            X2 = inXY(i, 1)
            'Perform interpolation
            Parameter = (inX - X1) / (X2 - X1)
            MATH_LookupXYTable = inXY(i - 1, 2) * (1 - Parameter) + inXY(i, 2) * Parameter
```

```
            Exit For
        End If
    Next i
End Function
```


## A. 3 Matrix Solver

The following Visual Basic routine solves any size square matrix $\mathrm{A}[N, N]$ and vector $\mathrm{B}[N]$ and returns the result as the vector $\mathrm{C}[N]$. The matrix solves the equation
using Gaussian elimination with partial pivoting.

```
Function MAT_GaussP(MatA() As Double, MatB() As Double, MatC() As Double) As Integer
'Copyright (\overline{C}) 2013 - Snorri Gudmundsson, all rights reserved.
'This subroutine is the Gaussian elimination with partial pivoting,
'to solve the following matrix equation;
'
1 A*X = B
'This subroutine checks for zero pivots, and consequently abandons the
'the computations if such a pivot is encountered.
'Initialize
    Dim Arow As Long, Acol As Long
    Dim Brow As Long, Bcol As Long
    Dim i As Long, j As Long, k As Long
    Dim N As Long, Tiny As Double
    Dim Pindex As Integer, Pk As Integer
    Dim Max As Double, Mult As Double, Sum As Double, Pivot As Double
'Checking matrix sizes
    MAT_GaussP = 0
    Arow = UBound(MatA, 1)
    Acol = UBound(MatA, 2)
    Brow = UBound(MatB, 1)
    Bcol = UBound(MatB, 2)
'Make sure that MatA isn't a square matrix
    If Arow <> Acol Then
        MAT_GaussP = -1
        Exit Function
    End If
'Make sure that MatB is a 1D vector, and of equal length to MatA
    If Brow <> Arow Or Bcol > 1 Then
        MAT_GaussP = -2
        Exit Function
    End If
'Initialize variables
    ReDim MatC(Arow, 1), P(Arow) As Long
    N = Arow
    Tiny = 1E-20
    If MATH StopProcess = 1 Then
        MAT_GaussP = -10
```

```
    Exit Function
    End If
    MATH_StopProcess = 0
'Starting the elimination
    'Giving P its initial values
    For i = 1 To N
        P(i) = i
    Next i
    For i = 1 To N - 1
        'Set solution status
        MATH_SolutionComplete = i / N
        Find the pivot
        Max = 0
        Pindex = P(i)
        For j = i To N
            If Abs(MatA(P(j), i)) > Max Then
                Max = Abs(MatA(P(j), i))
                k = j
            End If
        Next j
        'Signal if pivot is too small
        If Max <= Tiny Then
            MAT GaussP = -3
            Exi\overline{t}\mathrm{ Function}
        End If
        Otherwise swap elements in P if necessary
        If k <> i Then
            P(i) = P(k)
            P(k) = Pindex
            Pindex = P(i)
        End If
        Now, do the elimination
        Pivot = MatA(Pindex, i)
        For k = i + 1 To N
            Pk = P(k)
            Mult = MatA(Pk, i) / Pivot
            If Abs(Mult) > Tiny Then
                MatA(Pk, i) = Mult
                For j = i + 1 To N
                    MatA(Pk, j) = MatA(Pk, j) - Mult * MatA(Pindex, j)
                Next j
                MatB(Pk, 1) = MatB(Pk, 1) - Mult * MatB(Pindex, 1)
            Else
                MatA(Pk, i) = 0
            End If
        Next k
    Next i
'Perform the back-substitution
    If Abs(MatA(P(N), N)) > Tiny Then
        MatC(N, 1) = MatB(P(N), 1) / MatA(P(N), N)
        For i = N - 1 To 1 Step -1
            Pindex = P(i)
            Sum = MatB(Pindex, 1)
            For j = i + 1 To N
                sum = Sum - MatA(Pindex, j) * MatC(j, 1)
            Next j
            MatC(i, 1) = Sum / MatA(Pindex, i)
        Next i
    End If

\section*{Appendix B - Various Mathematical Tools}

This section presents selected methods that are used in various capacities in selected parts of the SURFACES Flight Simulator.

\section*{B. 1 Estimation of the Characteristics of 2 \({ }^{\text {nd }}\) Order ODE using Experimental Data}

Consider Figure B-1, which shows a representation of typical experimental data. It can be seen that this response can be represented in the following fashion
\[
\begin{equation*}
y(t)-y_{\text {avg }}=C_{1} e^{-n t} \cos \left(\omega t+\phi_{0}\right)=C_{1} e^{-n\left(t-t_{1}\right)} \cos \left(\omega\left(t-t_{1}\right)\right) \tag{B-1}
\end{equation*}
\]


Figure B-1: Typical experimental oscillation of a dynamical system.
The rightmost form, in effect, means the response is shifted to the left by the amount \(t_{1}\), justifying a representation in terms of cosine only. This is analogous to writing the solution of the \(2^{\text {nd }}\) order ODE as shown below
\[
\ddot{y}+B \dot{y}+C y=y_{\text {avg }} \Leftrightarrow y-y_{\text {avg }}=C_{1} e^{-\frac{B}{2} t} \cos \left(\frac{\sqrt{4 C-B^{2}}}{2} t\right)
\]

Since \(B \equiv 2 \zeta \omega_{n}\) and \(C \equiv \omega_{n}{ }^{2}\), this can be rewritten as
\[
\begin{equation*}
y(t)-y_{\text {avg }}=C_{1} e^{-\zeta \omega_{n}\left(t-t_{1}\right)} \cos \left(\omega_{n} \sqrt{1-\zeta^{2}}\left(t-t_{1}\right)\right) \tag{B-2}
\end{equation*}
\]

At time \(t=t_{1}\), this becomes \(\quad y\left(t_{1}\right)=y_{\text {avg }}+C_{1}\)
From which it follows that \(\quad y\left(t_{1}\right)=y_{\text {avg }}+C_{1} \Rightarrow C_{1}=y\left(t_{1}\right)-y_{\text {avg }}\)

At time \(t=t_{2}\),
\[
\begin{equation*}
y\left(t_{2}\right)=y_{a v g}+C_{1} e^{-\zeta \omega_{n}\left(t_{2}-t_{1}\right)} \cos \left(\omega_{n} \sqrt{1-\zeta^{2}}\left(t_{2}-t_{1}\right)\right) \tag{B-5}
\end{equation*}
\]

Letting \(\Delta t=t_{2}-t_{1}\) we write \(\quad y\left(t_{2}\right)=y_{\text {avg }}+C_{1} e^{-\zeta \omega_{n} \Delta t} \cos \left(\omega_{n} \sqrt{1-\zeta^{2}} \Delta t\right)\)

Also, since \(\cos \left(\omega_{n} \sqrt{1-\zeta^{2}} \Delta t\right)=1\) at \(\Delta t\) (because it is selected at a peak), we get
\[
\begin{equation*}
\cos \left(\omega_{n} \sqrt{1-\zeta^{2}} \Delta t\right)=1 \quad \Leftrightarrow \quad \omega_{n} \sqrt{1-\zeta^{2}}= \pm \frac{2 \pi k}{\Delta t}, \quad k=0,1,2, \ldots \tag{B-7}
\end{equation*}
\]

Also note that by dividing Equation (B-6) by (B-4), and taking the logarithm of both sides, we get
\[
\begin{equation*}
e^{-\zeta \omega_{n} \Delta t}=\frac{y\left(t_{2}\right)-y_{a v g}}{y\left(t_{1}\right)-y_{a v g}} \Leftrightarrow-\zeta \omega_{n}=\frac{1}{\Delta t} \ln \left(\frac{y\left(t_{2}\right)-y_{a v g}}{y\left(t_{1}\right)-y_{a v g}}\right)=\frac{A}{\Delta t} \tag{B-8}
\end{equation*}
\]

Then, by dividing Equation (B-8) by (B-7), we end up with (for \(k=+1\) )
\[
\begin{equation*}
\frac{-\zeta \omega_{n}}{\omega_{n} \sqrt{1-\zeta^{2}}}=\frac{-\zeta}{\sqrt{1-\zeta^{2}}}=\frac{A}{2 \pi} \Rightarrow \zeta= \pm \sqrt{\frac{A^{2}}{4 \pi^{2}+A^{2}}} \tag{B-9}
\end{equation*}
\]

With \(\zeta\) known, the \(\omega_{n}\) is readily obtained from Equation (B-8), i.e.
\[
\begin{equation*}
\omega_{n}=\mp \frac{\sqrt{4 \pi^{2}+A^{2}}}{\Delta t}=\mp \frac{\sqrt{4 \pi^{2}+\ln \left(\frac{y\left(t_{2}\right)-y_{a v g}}{y\left(t_{1}\right)-y_{a v g}}\right)^{2}}}{\Delta t} \tag{B-10}
\end{equation*}
\]

This methodology is particularly useful for obtaining dynamic data about a system prior to designing PID controllers. As an example, consider a scenario for which we want to design gains for a PID airspeed controller for a 250 kg UAV. Further assume we have experimental data used to determine the natural frequency, \(\omega_{n}\), and damping ratio, \(\zeta\), for an oscillatory response, like that of Figure B-1, for which the average response is \(y_{\text {avg }}=121.52\), and
\[
\begin{aligned}
& y\left(t_{1}=9.89\right)=146.24 \\
& y\left(t_{2}=29.64\right)=135.89
\end{aligned}
\]

To obtain the parameter we first, calculate \(C_{1}=y\left(t_{1}\right)-y_{\text {avg }}=146.24-121.52=24.72\) and \(\Delta t=29.64-9.89=19.75 \mathrm{~s}\). Next, calculate \(A, \zeta\), and \(\omega_{n}:\)
\[
\begin{aligned}
& A=\ln \left(\frac{y\left(t_{2}\right)-y_{\text {avg }}}{y\left(t_{1}\right)-y_{\text {avg }}}\right)=\ln \left(\frac{135.89-121.52}{146.24-121.52}\right)=-0.54254 \\
& \zeta= \pm \sqrt{\frac{A^{2}}{4 \pi^{2}+A^{2}}}=\sqrt{\frac{(-0.54254)^{2}}{4 \pi^{2}+(-0.54254)^{2}}}=0.086028 \\
& \omega_{n}=\frac{\sqrt{4 \pi^{2}+A^{2}}}{\Delta t}=0.3193
\end{aligned}
\]

\section*{B. 2 Numerical Integration Methods for Solving ODEs}

As a prologue, it is important to emphasize the importance of describing various physical processes using differential equations. It is not far-fetched to state that such equations (and their solution) made the modern world. Unfortunately, since these equations often describe highly complicated phenomena, obtaining a closed-form solution ranges from the impractical to the impossible. The solution to this predicament is offered by the use of numerical analysis methods. This section presents numerical integration schemes useful for the solution of ordinary differential equations, such as the equations of motion implemented in a flight simulator. These equations represent an initial value problem of the form
\[
\begin{equation*}
y^{\prime}=f(y, t) \quad y(0)=y_{0} \tag{B-11}
\end{equation*}
\]

Where \(y\) is a function, \(t\) is a variable, and \(y_{0}\) is the initial value. The sheer volume of methods for this purpose is large and way beyond the scope of this work. Therefore, only three methods will be presented; Euler integration and \(2^{\text {nd }}\) and \(4^{\text {th }}\) order Adams-Bashforth integration methods.

\section*{B. 3 Taylor Series}

Taylor series are of great importance in numerical integration schemes. Named after the English mathematician Brook Taylor (1685-1731, 46), they are used to represent functions as series of their derivatives at any given point. Their importance can be realized by noting that they were widely used in the early days of calculus to help build tables of standard solutions for intrinsic functions, such as \(\sin x\), \(\cosh x, \ln x\), and so on. They remain useful in computer coding to determine some intrinsic functions \({ }^{1}\). The idea can be described as follows: Consider some function whose value we want to evaluate near the point \(x=a\). One way of approximating the function is using the following polynomial \(p(x)\) of degree \(n\)
\[
\begin{equation*}
p(x)=C_{0}+C_{1}(x-a)+C_{2}(x-a)^{2}+\ldots+C_{n}(x-a)^{n} \tag{B-12}
\end{equation*}
\]

\footnotetext{
\({ }^{1}\) Another algorithm, called the CORDIC algorithm is also used, but many of its pre-programmed constants are obtained using Taylor series.
}

The only issue is the determination of the coefficients \(C_{0}, C_{1}, \ldots, C_{n}\). By differentiating this series repeatedly, it can be shown that the coefficients are given as shown below:
\[
\begin{equation*}
C_{m}=\frac{1}{m!} \frac{d^{m}}{d t^{m}} p(x) \tag{B-13}
\end{equation*}
\]

Consider a continuous single-valued function \(y=f(x)\), that both exists and is differentiable on some closed interval \(a \leq x \leq b\). If this function has \(n\) derivatives defined on said interval and whose next higher derivative exists on the open interval \(a<x<b\), then the function can be defined on the interval as
\[
\begin{equation*}
f(x)=f(a)+\frac{x-a}{1!} f^{\prime}(a)+\frac{(x-a)^{2}}{2!} f^{\prime \prime}(a)+\ldots+\frac{(x-a)^{n}}{n!} f^{(n)}(a)+R_{n+1} \tag{B-14}
\end{equation*}
\]

Where the term \(R_{n+1}\) is called the estimated reminder and is usually represented in two forms; Lagrangeand Cauchy-form, although we will only consider the former one here. Obtaining the value of \(f(x)\) at \(x=\) \(a\) is trivial (just plug \(a\) into the original function), however, knowing values of \(f(x)\) around that point and an estimate of the resulting error is the goal with this effort. The remainder in the Lagrange-form is defined as follows:
\[
R_{n+1}=\frac{f^{n+1}(\eta)}{(n+1)!}(x-a)^{n+1}
\]

Where \(\eta\) lies in the open interval \((a, x)\). \(R\) is commonly referred to as the truncation error. It shows that the deviation of the series from the exact solution is always has a factor of \((x-a)^{n+1}\). So, if we retain three terms, the error will be an order higher or to the power of four. This is central to understanding the accuracy of integration schemes.

\section*{B. 4 Euler Integration}

Euler integration is arguably the simplest numerical integration scheme available, which makes it particularly attractive for use in software. However, it suffers from instability; requiring small step sizes to remedy (see more below). Small step sizes, in turn, mean that codes will run slower, so real time simulation may be challenging. It is a first order method, which means the truncation error is of the second order. It can be derived in a multitude of ways, including using Taylor series or finite differences. The method uses the following finite difference representation. Suppose we know the function \(y(x)\) and wish to approximate its derivative using a forward stepping scheme, which involves calculating the value of the function at \(x\) and \(x+h\), where \(h\) is called the step size, we can estimate the derivative as follows:
\[
\begin{equation*}
y^{\prime}(x) \approx \frac{y(x+h)-y(x)}{h} \tag{B-15}
\end{equation*}
\]

Rearranging leads to
\[
\begin{equation*}
y(x+h) \approx y(x)+h y^{\prime}(x) \tag{B-16}
\end{equation*}
\]

This method is explicit, because it calculates the state of the system after the current state. Therefore, if we do not know a particular function, but only its derivative and the value of the function at the point \(x\), we can estimate the value of the function at a different point \(x+h\) using Equation (B-16). This is the crux of the method; predicting the value of some function while only possessing its derivate, precisely what is needed for solving the equations of motion for the flight of an aircraft. Ordinarily, the scheme would be presented as shown below, using indexes.
\[
\begin{equation*}
y_{i+1} \approx y_{i}+h y^{\prime}\left(x_{i}\right) \tag{B-17}
\end{equation*}
\]

Note that Euler integration can also be performed using a backward stepping scheme, for which Equation (B-15) would be written as follows
\[
\begin{equation*}
y^{\prime}(x) \approx \frac{y(x)-y(x-h)}{h} \tag{B-18}
\end{equation*}
\]

Or \(\quad y_{i+1} \approx y_{i}+h y^{\prime}\left(x_{i+1}\right)\)

The method is what is called implicit, because it requires the derivative to be known at the "future" point. Therefore, it is more time consuming to implement, although such schemes tend to be more stable. The error of a \(1^{\text {st }}\) order scheme is approximately
\[
\begin{equation*}
R_{2} \approx \frac{h^{2}}{2} \tag{B-20}
\end{equation*}
\]

The Euler integration method is what is called self-starting, because the solution for \(y_{i+1}\) does not require the prior knowledge of \(y(t)\) where \(t<t_{i}\) [1].

As an example of the implementation of this forward stepping Euler method, consider the following expression and its derivative
\[
y(t)=\cos t \quad \Leftrightarrow \quad y^{\prime}(t)=-\sin t
\]

The value of \(y(0)=1\) and \(y^{\prime}(0)=0\). Let's use this to evaluate the value of the function at \(t=0.25\) using Equation (B-17) and step size \(\Delta t=0.25\).
\[
y_{i+1} \approx y_{i}+\Delta t y^{\prime}\left(t_{i}\right) \Rightarrow y(0.25) \approx \overbrace{y(0)}^{=1}+(0.25) \overbrace{y^{\prime}(0)}^{0}=1
\]

The exact value is \(y(0.25)=\cos (0.25)=0.96891\). Noting that \(y^{\prime}(0.25)=-0.2474\), let's evaluate its value at \(t=0.5\) :
\[
y_{i+1} \approx y_{i}+\Delta t f^{\prime}\left(t_{i}\right) \Rightarrow y(0.5) \approx \overbrace{y(0.25)}^{=1}+(0.25) \overbrace{f^{\prime}(0.25)}^{-0.2474}=0.9382
\]

The exact value is \(y(0.5)=\cos (0.5)=0.87758\). Repeating this for \(0 \leq t \leq 2 \pi\), we generate the graph of Figure B-2. It shows that even for a relatively large step size the characteristics of the cosine function are preserved, although it is shifted to the right.


Figure B-2: The function \(y(t)=\cos t\) compared to results using Euler's method with two step sizes.

\section*{B. 5 Higher Order Methods}

When the integration of the differential equations requires faster operation, for instance, as required for real time flight simulation, the error associated with Euler's method becomes unacceptable or the method becomes unstable. The only remedy is a higher order scheme that is more stable. Recall the Taylor expansion of Equation (B-14), shown slightly modified below for convenience:
\[
\begin{equation*}
y(t+h)=y(t)+h y^{(1)}(t)+\frac{h^{2}}{2} y^{(2)}(t)+\frac{h^{3}}{6} y^{(3)}(t)+\ldots \tag{B-21}
\end{equation*}
\]

Where the superscripts inside the parentheses represent the order of the derivative of \(y\). This can be rewritten in an index form as shown below:
\[
\begin{equation*}
y_{i+1}=y_{i}+h y_{i}^{(1)}+\frac{h^{2}}{2} y_{i}^{(2)}+\frac{h^{3}}{6} y_{i}^{(3)}+\ldots \tag{B-22}
\end{equation*}
\]

As shown in Equation (B-15), the first derivative is given by
\[
\begin{equation*}
\dot{y}_{i}=y_{i}^{(1)} \approx \frac{y_{i}-y_{i-1}}{h} \tag{B-23}
\end{equation*}
\]

But this implies the second derivative is given by:
\[
\begin{equation*}
\ddot{y}=y_{i}^{(2)} \approx \frac{y_{i}^{(1)}-y_{i-1}^{(1)}}{h}=\frac{\frac{y_{i}-y_{i-1}}{h}-y_{i-1}^{(1)}}{h}=\frac{y_{i}-y_{i-1}-h y_{i-1}^{(1)}}{h^{2}} \tag{B-24}
\end{equation*}
\]

This allows us to write the first three terms of the Taylor series as follows:
\[
\begin{equation*}
y_{i+1}=y_{i}+h y_{i}^{(1)}+\frac{h^{2}}{2} y_{i}^{(2)}=y_{i}+h\left[\frac{y_{i}-y_{i-1}}{h}\right]+\frac{h^{2}}{2}\left[\frac{y_{i}-y_{i-1}-h y_{i-1}^{(1)}}{h^{2}}\right] \tag{B-25}
\end{equation*}
\]

Expanding and manipulating algebraically leads to the following expression
\[
\begin{equation*}
y_{i+1}=\frac{2}{2} y_{i}+\frac{3}{2} y_{i}-\frac{3}{2} y_{i-1}-\frac{1}{2} h y_{i-1}^{(1)}=y_{i}+\frac{h}{2}\left(3 y_{i}^{(1)}-y_{i-1}^{(1)}\right) \tag{B-26}
\end{equation*}
\]

This is known as a \(2^{\text {nd }}\) order Adams-Bashforth scheme. The error of a \(2^{\text {nd }}\) order scheme is approximately
\[
\begin{equation*}
R_{3} \approx \frac{h^{3}}{6} \tag{B-27}
\end{equation*}
\]

To implement this scheme in a computer code, it is necessary to store the derivative at the previous time step. That, in effect, is the only complication. A \(4^{\text {th }}\) order Adams-Bashforth scheme is obtained in a similar fashion and is shown below
\[
\begin{equation*}
y_{i+1}=y_{i}+\frac{h}{24}\left(55 y_{i}^{(1)}-59 y_{i-1}^{(1)}+37 y_{i-2}^{(1)}-9 y_{i-3}^{(1)}\right) \tag{B-28}
\end{equation*}
\]

The error of this scheme is approximately \(\quad R_{5} \approx \frac{h^{5}}{120}\)

To implement this scheme in a computer code, it is necessary to store the derivative at the previous three time steps. Therefore, the method is not self-starting; it requires preset values for the prior values of \(y(t)\) where \(t<t_{i}\).

At this point it is a fair question to ask; what about Runge-Kutta integration schemes - Why not employ them for flight simulation? This can certainly be done, but such schemes require derivatives at steps internal to the interval \(t_{i}\) to \(t_{i}+h\), making it more involved to implement with limited benefits. They are, thus, omitted from consideration.

\section*{B. 6 Stability Concerns}

If a numerical method with a finite region of absolute stability, applied to a system with any initial conditions, is forced to use in a certain interval of integration a step length which is excessively small in relation to the smoothness of the exact solution in that interval, then the system is said to be stiff in that interval.

\section*{REFERENCES}

\footnotetext{
\({ }^{1}\) Hornbeck, Robert W., Numerical Methods, Quantum Publishers, 1975.
}

\section*{Appendix C - Aerodynamic Properties for Simulation sUAVs}

This appendix presents the aerodynamic models for the two sUAV aircraft used for simulation, the Shadow Observer and Sparrow Hawk.

\section*{C. 1 Aerodynamic Model for the Shadow Observer (all derivatives are per radian)}
\begin{tabular}{|c|c|c|}
\hline Description & Symbol & Value \\
\hline \multicolumn{3}{|c|}{Primary Control Surface Deflections} \\
\hline Max aileron TE Up deflection & \(\delta_{a \text { min }}, \delta_{a \text { max }}\) & \(\pm 15^{\circ}\) \\
\hline Max elevator TE deflection & \(\delta_{e \text { min }}, \delta_{e \text { max }}\) & \(-20^{\circ},+15^{\circ}\) \\
\hline Max left rudder TE Lft deflection & \(\delta_{r \text { min }} \delta_{r \text { max }}\) & \(\pm 15^{\circ}\) \\
\hline \multicolumn{3}{|c|}{Drag Related Derivatives} \\
\hline Change in drag with \(\delta_{e}\) & \(C_{\text {D } \mathrm{e}}\) & 0.05834 \\
\hline \multicolumn{3}{|c|}{Lift Related Derivatives} \\
\hline Lift curve slope & \(C_{L \alpha}\) & 5.656 \\
\hline Change in lift with AOAdot & \(C_{\text {Ladot }}\) & 0.2904 \\
\hline Change in lift with de (for complete aircraft) & \(C_{L \delta e}\) & 0.6870 \\
\hline Change in lift with Q & \(C_{L q}\) & 8.033 \\
\hline \multicolumn{3}{|c|}{Side Force Derivatives} \\
\hline Change in side force with AOY & \(C_{\nu \beta}\) & -0.3502 \\
\hline Change in side force with \(P\) & \(C_{y p}\) & 0.01936 \\
\hline Change in side force with R & \(C_{y r}\) & 0.1518 \\
\hline Change in side force with \(\delta_{a}\) & \(C_{\text {y } \delta a}\) & -0.003660 \\
\hline Change in side force with \(\delta_{r}\) & \(C_{\text {ydr }}\) & -0.4361 \\
\hline \multicolumn{3}{|c|}{Rolling Moment Derivatives} \\
\hline Dihedral effect & \(C_{\text {¢ }}\) & -0.03289 \\
\hline Roll damping & \(C_{l p}\) & -0.6544 \\
\hline Roll due to yaw & \(C_{l r}\) & 0.06942 \\
\hline Aileron authority & \(C_{\text {¢о }}\) & 0.2643 \\
\hline Adverse roll & \(C_{\text {lor }}\) & -0.06526 \\
\hline \multicolumn{3}{|c|}{Pitching Moment Dericatives} \\
\hline Change in pitch with AOAdot & \(C_{\text {madot }}\) & -1.636 \\
\hline Change in pitch with \(\delta_{e}\) & \(C_{\text {m8e }}\) & -2.500 \\
\hline Change in pitch with Q & \(C_{m q}\) & -11.30 \\
\hline \multicolumn{3}{|c|}{Yawing Moment Derivatives} \\
\hline Directional stability & \(C_{n \beta}\) & 0.05045 \\
\hline Yaw due to roll & \(C_{n p}\) & -0.02065 \\
\hline Yaw damping & \(C_{n r}\) & -0.03562 \\
\hline Adverse yaw & \(C_{n \delta a}\) & -0.003440 \\
\hline Rudder authority & \(C_{n \delta \text { r }}\) & 0.09303 \\
\hline \multicolumn{3}{|c|}{PID Autopilot Gains} \\
\hline A/P: Proportional gain for roll & \(K_{P \phi}\) & -0.1 \\
\hline A/P: Proportional gain for elevator & \(K_{P V}\) & -0.002 \\
\hline A/P: Integration gain for elevator & \(K_{I V}\) & -0.0009 \\
\hline A/P: Derivative gain for elevator & \(K_{D V}\) & -0.02 \\
\hline A/P: Proportional gain for altitude hold & \(K_{\text {Palt }}\) & -0.005 \\
\hline A/P: Integration gain for elevator & \(K_{\text {Ialt }}\) & -0.001 \\
\hline A/P: Derivative gain for elevator & \(K_{\text {D alt }}\) & -0.005 \\
\hline
\end{tabular}

\section*{C. 2 Aircraft Aerodynamic Model for the Sparrow Hawk (all derivatives are per radian)}
\begin{tabular}{|c|c|c|}
\hline Description & Symbol & Value \\
\hline \multicolumn{3}{|c|}{Primary Control Surface Deflections} \\
\hline Max aileron TE Up deflection & \(\delta_{a \text { min }} \delta_{a \text { max }}\) & \(\pm 15\) \\
\hline Max elevator TE deflection & \(\delta_{e \text { min }}, \delta_{e \text { max }}\) & -20, +15 \\
\hline Max left rudder TE Lft deflection & \(\delta_{r \text { min }} \delta_{r \text { max }}\) & \(\pm 15\) \\
\hline \multicolumn{3}{|c|}{Drag Related Derivatives} \\
\hline Change in drag with \(\delta_{e}\) & \(C_{\text {D }}\) e & 0.00825 \\
\hline Additional drag due to spoilers & \(C_{\text {Dspoiler }}\) & 0.03 \\
\hline \multicolumn{3}{|c|}{Lift Related Dericatives} \\
\hline Change in lift with AOAdot & \(C_{\text {Ladot }}\) & 0.2904 \\
\hline Change in lift with de (for complete aircraft) & \(C_{L \delta e}\) & 0.612728 \\
\hline Change in lift with Q & \(C_{L q}\) & 7.766336 \\
\hline Change in lift due to spoilers & \(C_{\text {Lspoiler }}\) & -0.05 \\
\hline \multicolumn{3}{|c|}{Side Force Derivatives} \\
\hline Change in side force with AOY & \(C_{y \beta}\) & -0.28738 \\
\hline Change in side force with \(P\) & \(C_{y p}\) & -0.0387 \\
\hline Change in side force with R & \(C_{y r}\) & 0.234621 \\
\hline Change in side force with \(\delta_{a}\) & \(C_{\text {y } \delta a}\) & \(5.54 \mathrm{E}-03\) \\
\hline Change in side force with \(\delta_{r}\) & \(C_{\text {ydr }}\) & 0.202563 \\
\hline \multicolumn{3}{|c|}{Rolling Moment Derivatives} \\
\hline Dihedral effect & \(C_{\text {l }}\) & -4.43E-02 \\
\hline Roll damping & \(C_{l p}\) & -0.55104 \\
\hline Roll due to yaw & \(C_{l r}\) & 0.108814 \\
\hline Aileron authority & \(C_{\text {l }}\) a & 0.273526 \\
\hline Adverse roll & \(C_{\text {¢0 }}\) r & 2.04E-02 \\
\hline \multicolumn{3}{|c|}{Pitching Moment Dericatives} \\
\hline Change in pitch with AOAdot & \(C_{\text {madot }}\) & -1.5 \\
\hline Change in pitch with \(\delta_{e}\) & \(C_{\text {m8e }}\) & -2.15115 \\
\hline Change in pitch with Q & \(C_{m q}\) & -20.7771 \\
\hline \multicolumn{3}{|c|}{Yawing Moment Derivatives} \\
\hline Directional stability & \(C_{n \beta}\) & 9.01E-02 \\
\hline Yaw due to roll & \(C_{n p}\) & -3.17E-03 \\
\hline Yaw damping & \(C_{n r}\) & -0.08736 \\
\hline Adverse yaw & \(C_{n \delta a}\) & -1.56E-02 \\
\hline Rudder authority & \(C_{n \delta r}\) & -7.74E-02 \\
\hline \multicolumn{3}{|c|}{PID Autopilot Gains} \\
\hline A/P: Proportional gain for roll & \(K_{P \phi}\) & -0.1 \\
\hline A/P: Proportional gain for elevator & \(K_{P V}\) & -0.002 \\
\hline A/P: Integration gain for elevator & \(K_{I V}\) & -0.0009 \\
\hline A/P: Derivative gain for elevator & \(K_{D V}\) & -0.02 \\
\hline A/P: Proportional gain for altitude hold & \(K_{\text {Palt }}\) & -0.001 \\
\hline A/P: Integration gain for elevator & \(K_{\text {I alt }}\) & -0.0001 \\
\hline A/P: Derivative gain for elevator & \(K_{\text {dalt }}\) & -0.001 \\
\hline
\end{tabular}

\section*{Appendix D - Rotational Mathematics}

A proper representation of the attitude of solid bodies within a reference frame is imperative for any simulation of their dynamics. Several means to represent attitude (or orientation) is proposed in the literature, of which Euler angles, Euler rotation, and Rodrigues' parameters are the most common. In terms of Lie Groups, Euler angles are a rotation group \(\mathrm{SO}(3)\) on a 3-dimensional manifold, whose orientation can be specified using just three parameters (typically roll-pitch-yaw or a combination thereof). This leads to a \(3 \times 3\) matrix of direction cosines, constituting a 9 -parameter representation. Four parameter rotation using angle-axes parameters and Euler parameters (or unit quaternions) are also used quite frequently. This section presents a brief introduction to these methods, as they are used to keep track of the time history of the rotation of the aircraft model in the SURFACES Flight Simulator.

\section*{D. 1 Euler Angles and Euler Rotation}

Let's first present a 2-dimensional rotation matrix and then extend it to 3-dimensional space. If the point ( \(x_{\text {old }}, y_{\text {old }}\) ) is to be rotated (positive rotation is counterclockwise) through an angle \(\theta\), it moves to a new coordinate ( \(x_{\text {new }}, y_{\text {new }}\) ), given by:
\[
\left\{\begin{array}{l}
x_{\text {new }}  \tag{D-1}\\
y_{\text {new }}
\end{array}\right\}=\left[\begin{array}{cc}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{array}\right]\left\{\begin{array}{l}
x_{\text {old }} \\
y_{\text {old }}
\end{array}\right\}
\]

The square matrix of Equation ( \(D-1\) ) is called the rotation matrix. As alluded to above, rotation matrices can also be extended to 3-dimensional space. The most common method to conduct rotation in such space is using Euler Angles, attributed to the Swiss mathematician Leonhard Euler (1707-1783, 76). To solve the problem of arbitrary rotation of a body in 3-dimensional space, Euler suggested using three subsequent rotations about each axis. These rotations can be implemented in a number of ways. The most common way of implementing the rotations is the so-called 3-2-1 rotation (also called yaw-pitchroll rotation), which begins with a rotation about the original \(z\)-axis, followed by one about the resulting \(y\)-axis, and finally with rotation about the resulting \(x\)-axis.

Thus, a positive rotation of a point about the \(z\)-axis by angle \(\psi\) of the original coordinate system \(x_{\text {old }}-\) \(y_{\text {old }}-z_{\text {old }}\), results in a new coordinate system \(x^{\prime}-y^{\prime}-z^{\prime}\). This rotation is represented using the following transformation matrix:
\[
\left\{\begin{array}{l}
x^{\prime}  \tag{D-2}\\
y^{\prime} \\
z^{\prime}
\end{array}\right\}=\left[\begin{array}{ccc}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{array}\right]\left\{\begin{array}{l}
x_{\text {old }} \\
y_{\text {old }} \\
z_{\text {old }}
\end{array}\right\}
\]

A positive rotation about the \(y^{\prime}\)-axis by angle \(\theta\) of the coordinate system \(x^{\prime}-y^{\prime}-z^{\prime}\), results in a new coordinate system \(x^{\prime \prime}-y^{\prime \prime}-z^{\prime \prime}\). This rotation is represented using the following transformation matrix:
\[
\left\{\begin{array}{l}
x^{\prime \prime}  \tag{D-3}\\
y^{\prime \prime} \\
z^{\prime \prime}
\end{array}\right\}=\left[\begin{array}{ccc}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{array}\right]\left\{\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
z^{\prime}
\end{array}\right\}
\]

Finally, a positive rotation about the \(\underline{x "-a x i s ~ b y ~ a n g l e ~} \phi\) of the coordinate system \(x "-y^{\prime \prime}-z^{\prime \prime}\), results in the final coordinate system \(x-y-z\). This rotation is represented using the following transformation matrix:
\[
\left\{\begin{array}{l}
x  \tag{D-4}\\
y \\
z
\end{array}\right\}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{array}\right]\left\{\begin{array}{l}
x^{\prime \prime} \\
y^{\prime \prime} \\
z^{\prime \prime}
\end{array}\right\}
\]

It follows that the complete rotation can be represented using the total rotation matrix shown below:
\[
\left\{\begin{array}{l}
x  \tag{D-5}\\
y \\
z
\end{array}\right\}=\left[\begin{array}{ccc}
C \psi C \theta & S \psi C \theta & -S \theta \\
-S \psi C \phi+C \psi S \theta S \phi & C \psi C \phi+S \psi S \theta S \phi & C \theta S \phi \\
S \psi S \phi+C \psi S \theta C \phi & -C \psi S \phi+S \psi S \theta C \phi & C \theta C \phi
\end{array}\right]\left\{\begin{array}{l}
x_{\text {old }} \\
y_{\text {old }} \\
z_{\text {old }}
\end{array}\right\}
\]

Where \(\mathrm{C}(\cdot)=\cos (\cdot)\) and \(\mathrm{S}(\cdot)=\sin (\cdot)\). Note that this rotation matrix is known as a 3-2-1 sequence and is shown graphically in Figure D-1. Also note that the above Euler angles are limited to the angular ranges \(\phi, \psi \in[-\pi / 2, \pi / 2]\) and \(\theta \in[-\pi / 2, \pi / 2]\). When \(\theta= \pm \pi / 2\), a condition called gimbal-lock is encountered that requires the use of quaternions to avoid. This condition results in two of the three axes being parallel which eliminates 1-DOF from the initial 3-DOF rotation.


Figure D-1: The implementation of the Euler Angle 3-2-1 rotation (order of rotation is imperative).

\section*{D. 2 Rodriques' Rotation Formula}

Rodrigues' rotation formula is an alternative method to perform vector rotation. Attributed to the French mathematician Olinde Rodrigues (1795-1851, 56), the formula is used to rotate a vector in space, but is easily extended to rotate the three basis vectors of a coordinate system to yield a rotation matrix in SO(3), just like that of the Euler rotation. Thus, consider some original vector \(\mathbf{V}_{\text {old }}\) that we want to rotate through angle \(\theta\) about the unit vector \(\mathbf{n}=\left(n_{x}, n_{y}, n_{z}\right) \in \mathrm{R}^{3}\) (the rotation axis), forming a new vector, \(\mathbf{V}_{\text {new }}\). This operation is accomplished as shown below:
\[
\begin{equation*}
\mathbf{V}_{\text {new }}=\mathbf{V}_{\text {old }} \cos \theta+\left(\mathbf{n} \times \mathbf{V}_{\text {old }}\right) \sin \theta+\mathbf{n}\left(\mathbf{n} \cdot \mathbf{V}_{\text {old }}\right)(1-\cos \theta) \tag{D-6}
\end{equation*}
\]

Note that, sometimes, it is convenient to consider the unit vector \(\mathbf{n}\) as the normal to a plane established by the vectors \(\mathbf{a}\) and \(\mathbf{b}\), such that
\[
\mathbf{n}=\frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|}
\]

A matrix representation of Rodrigues' rotation formula is obtained through explicit expansion of Equation (D-6), by defining a rotation matrix \(R\) as shown below:
\[
\begin{equation*}
R=e^{\widetilde{n} \theta}=1+\widetilde{n} \sin \theta+\widetilde{n}^{2}(1-\cos \theta) \tag{D-7}
\end{equation*}
\]

Where \(\tilde{n}\) is the anti-symmetric square matrix given by
\[
\widetilde{n}=\left[\begin{array}{ccc}
0 & -n_{z} & n_{y}  \tag{D-8}\\
n_{z} & 0 & n_{x} \\
-n_{y} & n_{x} & 0
\end{array}\right]
\]

Expanded, the rotation matrix becomes
\[
R=\left[\begin{array}{ccc}
\cos \theta+n_{x}^{2}(1-\cos \theta) & n_{x} n_{y}(1-\cos \theta)-n_{z} \sin \theta & n_{y} \sin \theta+n_{x} n_{z}(1-\cos \theta)  \tag{D-9}\\
n_{z} \sin \theta+n_{x} n_{y}(1-\cos \theta) & \cos \theta+n_{y}^{2}(1-\cos \theta) & -n_{x} \sin \theta+n_{y} n_{z}(1-\cos \theta) \\
-n_{y} \sin \theta+n_{x} n_{z}(1-\cos \theta) & n_{x} \sin \theta+n_{y} n_{z}(1-\cos \theta) & \cos \theta+n_{z}^{2}(1-\cos \theta)
\end{array}\right]
\]

This allows the rotation of the original vector \(\mathbf{V}_{\text {old }}\) to be accomplished using the simple representation below.
\[
\begin{equation*}
\mathbf{V}_{\text {new }}=R \mathbf{V}_{\text {old }} \tag{D-10}
\end{equation*}
\]

Note that the advantage of this formula is that it allows a large portion of the rotation matrix to be precalculated and then, in effect, it becomes a function of the rotation angle parameter \(\cos \theta\) only. For
repeated rotations in computer algorithm, it is possible to speed up operations required for rotation. Of course, should the rotation axis vector \(\mathbf{n}\) change, the matrix must be recalculated.

\section*{D. 3 Angle-Axis Rotation}

Euler's Theorem (also called Euler's Displacement Theorem or Euler's Eigenaxis Rotation) states that the displacement of a rigid body about a fixed point can be represented as a rotation about an axis and that this rotation can be represented using three rotations about the basis vectors of the reference frame. Thus, if the axis of rotation is denoted using \(\mathbf{n}\), as defined earlier, the rotation can be described using Rodrigues' formula, using the the quadruple \(\left[\theta, n_{x}, n_{y}, n_{z}\right]\).

\section*{D. 4 Quaternions}

The term quaternion refers to a means of conducting rotation in 3-dimensional space that contrasts Euler's rotation matrices using four, rather than three parameters. Quaternions replace Eulerian rotation matrices, which are known to suffer from gimbal-lock, which occurs when a 3-axes, 3-DOF, rotational mechanism of a stabilizing gyro loses a DOF when two axes become parallel as a result of the orientation of the frame to which it is attached. While such mechanism does not actually "lock," the concept more precisely refers to the mathematical singularity that occurs and which can have serious consequences vehicle navigation that uses autopilots. Possibly the best known example of this is what is referred to as the Apollo 11 gimbal-lock incident, which took place in the Command Module during the famous Apollo 11 mission, during which man landed on the Moon for the first time. The Command Module is the section of the Apollo spacecraft that remained in orbit around the moon, while the Lunar Lander, separated and made the actual landing on the Moon. During the Apollo 11 mission, the Command Module was piloted by Major Michael Collins, while the Lunar Lander was operated by NASA's Buzz Aldrin and Neil Armstrong. The spacecraft's navigation system utilized 3-DOF gyros that left the Command Module "stuck" in this undesirable orientation as it maneuvered during its orbits around Moon (e.g. see Hoag [1]). This required the maneuvering thrusters to be fired in a specific order to remedy. This peculiar condition does not occur in the use of quaternions, rendering them more reliable for use in rotation mathematics than Euler rotation matrices.

Quaternions were discovered in 1843 by the Irish mathematician William Rowan Hamilton (1805-1865, 60). They were formally presented in 1866, after his death, in a 760+ page book titled the Elements of Quaternions [2], edited by his son William Edwin Hamilton. Early on, quaternions caused some controversy among mathematician because they abandoned commutativity and the required consequences of multiplication would only be realized in a system of two, four, or eight components. However, besides space navigation, they found practical use with the advent of the computer and computer graphics. In particular, they are practical when conducting rotation of objects in various 3dimensional drafting and simulation packages. They were introduced as early as the mid-1980s as means to generate smooth rotations in computer animations and as means to conduct vehicle rotation in flight simulators (e.g. see Shoemake [3]). Quaternions have also become the norm in the development of fast paced 3-dimensional action games (see Bobic [4]). The SURFACES Flight Simulator uses quaternions for
rotation in this fashion in lieu of the other methods presented earlier. Therefore, it is warranted to introduce the mathematics in this chapter.

Consider Euler's eigenaxis rotation of magnitude \(\theta\) of some rigid body about an axis as shown in Figure \(\mathrm{D}-2\). The eigenaxis is denoted by \(\mathbf{n}=\left(n_{x}, n_{y}, n_{z}\right) \in \mathrm{R}^{3}\) and is fixed to the body and is stationary with respect to the inertia frame denoted by \(\left(\mathbf{a}_{1}, \mathbf{a}_{2}, \mathbf{a}_{3}\right)\). After the rotation, \(\mathbf{n}\) remains unchanged, but the body fixed frame becomes \(\left(\mathbf{b}_{1}, \mathbf{b}_{2}, \mathbf{b}_{3}\right)\). This allows the rotation axis to be represented as follows:
\[
\mathbf{n}=n_{x} \mathbf{a}_{1}+n_{y} \mathbf{a}_{2}+n_{z} \mathbf{a}_{3}=n_{x} \mathbf{b}_{1}+n_{y} \mathbf{b}_{2}+n_{z} \mathbf{b}_{3}
\]

It follows that the norm of the eigenaxis (which is a unit vector) is given by
\[
\sum_{i=1}^{3} n_{i}^{2}=1
\]
where the index \(i\) represents the \(x, y\), and \(z\)-axes. Hamilton defined the quaternion as
\[
\begin{align*}
& e_{0}=\cos \frac{\theta}{2} \\
& e=\left[\begin{array}{l}
e_{1} \\
e_{2} \\
e_{3}
\end{array}\right]=\left[\begin{array}{l}
n_{x} \\
n_{y} \\
n_{z}
\end{array}\right] \sin \frac{\theta}{2} \tag{D-12}
\end{align*}
\]

Thus, the unit quaternion can now be defined in a compact form as follows:
\[
\hat{e}=\left[\begin{array}{c}
e  \tag{D-13}\\
e_{0}
\end{array}\right]
\]

Where
\[
\begin{equation*}
|\hat{e}|=e^{T} e+e_{0}^{2}=e_{0}^{2}+e_{1}^{2}+e_{2}^{2}+e_{3}^{2}=1 \tag{D-14}
\end{equation*}
\]

Equation (D-14) implies the quaternions are a point on a 4-dimensional sphere (Shoemake [3]). Next, let's rewrite the trigonometric functions cos \(\theta\) and \(\sin \theta\) as follows using the trigonometric identities
\[
\begin{gathered}
\cos \theta=1-2 \sin ^{2} \frac{\theta}{2} \quad \text { and } \\
\sin \theta=2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}
\end{gathered}
\]


Figure D-2: Rotation about an eigenaxis \(\mathbf{n}\) through eigen-angle \(\theta\).

It is now possible to substitute these into Rodrigues' rotation matrix of Equation (D-9) using the definition provided in Equation (D-12), such that the member in the first row, first column becomes
\[
R_{11}=\overbrace{\cos \theta}^{=1-2 \sin ^{2} \frac{\theta}{2}}+n_{x}^{2} \overbrace{(1-\cos \theta)}^{=1-\left(1-2 \sin ^{2} \frac{\theta}{2}\right)}=1-2\left(1-n_{x}^{2}\right) \sin ^{2} \frac{\theta}{2}=1-2\left(e_{2}^{2}+e_{3}^{2}\right)
\]

Note that this is possible because \(n_{x}^{2}+n_{y}^{2}+n_{z}^{2}=1 \Leftrightarrow n_{y}^{2}+n_{z}^{2}=1-n_{x}^{2}\); however, this leads to
\[
\left(n_{y}^{2}+n_{z}^{2}\right) \sin ^{2} \frac{\theta}{2}=e_{2}^{2}+e_{3}^{2}
\]

Similar approach for the first row, second column leads to
\[
R_{12}=n_{x} n_{y}(1-\cos \theta)-n_{z} \sin \theta=n_{x} n_{y} 2 \sin ^{2} \frac{\theta}{2}+n_{z} 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}=2 e_{1} e_{2}+2 e_{3} e_{0}
\]

Continuing in a similar fashion for the other members of the rotation matrix leads to
\[
R=\left[\begin{array}{lll}
1-2\left(e_{2}^{2}+e_{3}^{2}\right) & 2\left(e_{1} e_{2}+e_{0} e_{3}\right) & 2\left(e_{1} e_{3}-e_{0} e_{2}\right)  \tag{D-15}\\
2\left(e_{1} e_{2}-e_{0} e_{3}\right) & 1-2\left(e_{1}^{2}+e_{3}^{2}\right) & 2\left(e_{2} e_{3}+e_{0} e_{1}\right) \\
2\left(e_{1} e_{3}+e_{0} e_{2}\right) & 2\left(e_{2} e_{3}-e_{0} e_{1}\right) & 1-2\left(e_{1}^{2}+e_{2}^{2}\right)
\end{array}\right]
\]

To illustrate, consider a situation in which we want to rotate two successive rotations from \(\left(\mathbf{a}_{1}, \mathbf{a}_{2}, \mathbf{a}_{3}\right)\) to \(\left(\mathbf{b}_{1}, \mathbf{b}_{2}, \mathbf{b}_{3}\right)\) about eigenaxis \(\mathbf{n}_{1}\) through eigen-angle \(\theta_{1}\) and then about eigenaxis \(\mathbf{n}_{2}\) through eigen-angle \(\theta_{2}\) to ( \(\mathbf{c}_{1}, \mathbf{c}_{2}, \mathbf{c}_{3}\) ). This would be represented as follows:

First rotation \(\left(\mathbf{a}_{1}, \mathbf{a}_{2}, \mathbf{a}_{3}\right) \rightarrow\left(\mathbf{b}_{1}, \mathbf{b}_{2}, \mathbf{b}_{3}\right): \quad R\left(e^{\prime}, e^{\prime}{ }_{4}\right)\)
Second rotation \(\left(\mathbf{b}_{1}, \mathbf{b}_{2}, \mathbf{b}_{3}\right) \rightarrow\left(\mathbf{c}_{1}, \mathbf{c}_{2}, \mathbf{c}_{3}\right): \quad R\left(e^{\prime \prime}, e^{\prime \prime}{ }_{4}\right)\)

Therefore, the rotation \(\left(\mathbf{a}_{1}, \mathbf{a}_{2}, \mathbf{a}_{3}\right) \rightarrow\left(\mathbf{c}_{1}, \mathbf{c}_{2}, \mathbf{c}_{3}\right)\) can be represented as follows:
\[
R\left(e, e_{0}\right)=R\left(e^{\prime}, e_{0}^{\prime}\right) \cdot R\left(e^{\prime \prime}, e_{0}^{\prime \prime}\right) \Leftrightarrow R\left(\mathbf{n}, e_{4}\right)=R\left(\mathbf{n}_{1}, \theta_{1}\right) \cdot R\left(\mathbf{n}_{2}, \theta_{2}\right)
\]

From which we get \(\quad\binom{e}{e_{0}}=\binom{e^{\prime}}{e_{0}^{\prime}} \cdot\binom{e^{\prime \prime}}{e_{0}^{\prime \prime}}=\binom{e^{\prime} e_{0}^{\prime \prime}+e_{0}^{\prime} e^{\prime \prime}+e^{\prime} \times e^{\prime \prime}}{e_{0}^{\prime} e_{0}^{\prime \prime}-\left(e^{\prime}\right)^{T} e^{\prime \prime}} \leftarrow\) vector part

This establishes a method to perform dot product of quaternions. We now define the four quaternion parameters, \(e_{0}, e_{1}, e_{2}\), and \(e_{3}\) in terms of the Euler angles \(\phi, \theta\), and \(\psi\) as shown below (Shoemake [3])
\[
\begin{align*}
& e_{0}= \pm \cos \left(\frac{\phi}{2}\right) \cos \left(\frac{\theta}{2}\right) \cos \left(\frac{\psi}{2}\right)+\sin \left(\frac{\phi}{2}\right) \sin \left(\frac{\theta}{2}\right) \sin \left(\frac{\psi}{2}\right) \\
& e_{1}= \pm \sin \left(\frac{\phi}{2}\right) \cos \left(\frac{\theta}{2}\right) \cos \left(\frac{\psi}{2}\right)-\cos \left(\frac{\phi}{2}\right) \sin \left(\frac{\theta}{2}\right) \sin \left(\frac{\psi}{2}\right) \\
& e_{2}= \pm \cos \left(\frac{\phi}{2}\right) \sin \left(\frac{\theta}{2}\right) \cos \left(\frac{\psi}{2}\right)+\sin \left(\frac{\phi}{2}\right) \cos \left(\frac{\theta}{2}\right) \sin \left(\frac{\psi}{2}\right)  \tag{D-17}\\
& e_{3}= \pm \cos \left(\frac{\phi}{2}\right) \cos \left(\frac{\theta}{2}\right) \sin \left(\frac{\psi}{2}\right)-\sin \left(\frac{\phi}{2}\right) \sin \left(\frac{\theta}{2}\right) \cos \left(\frac{\psi}{2}\right)
\end{align*}
\]

The same sign (+ or -) must be chosen consistently for all the equations above. These values are typically used to initialize the orientation of aircraft in flight simulation software. The parameters depend on the body rates \(p, q\), and \(r\) as follows (see Stevens and Lewis [5]):
\[
\left[\begin{array}{c}
\dot{e}_{0}  \tag{D-18}\\
\dot{e}_{1} \\
\dot{e}_{2} \\
\dot{e}_{3}
\end{array}\right]=-\frac{1}{2}\left[\begin{array}{cccc}
0 & p & q & r \\
-p & 0 & -r & q \\
-q & r & 0 & -p \\
-r & -q & p & 0
\end{array}\right]\left[\begin{array}{l}
e_{0} \\
e_{1} \\
e_{2} \\
e_{3}
\end{array}\right\} \equiv-\frac{1}{2} \Omega_{e} \mathbf{e}
\]

Expanded, this becomes
\[
\begin{align*}
& \dot{e}_{0}=-\frac{1}{2}\left(e_{1} p+e_{2} q+e_{3} r\right) \\
& \dot{e}_{1}=\frac{1}{2}\left(e_{0} p+e_{2} q-e_{3} r\right) \\
& \dot{e}_{2}=\frac{1}{2}\left(e_{0} p+e_{3} q-e_{1} r\right)  \tag{D-19}\\
& \dot{e}_{3}=\frac{1}{2}\left(e_{0} p+e_{1} q-e_{2} r\right)
\end{align*}
\]

We can rewrite the diagonal of the rotation matrix using the identity of Equation (D-14). For instance, this will transform the term in row 1 column 1 to \(1-2\left(e_{2}^{2}+e_{3}^{2}\right)=e_{0}^{2}+e_{1}^{2}-e_{2}^{2}-e_{3}^{2}\) leading to
\[
R=\left[\begin{array}{ccc}
e_{0}^{2}+e_{1}^{2}-e_{2}^{2}-e_{3}^{2} & 2\left(e_{1} e_{2}+e_{0} e_{3}\right) & 2\left(e_{1} e_{3}-e_{0} e_{2}\right)  \tag{D-20}\\
2\left(e_{1} e_{2}-e_{0} e_{3}\right) & e_{0}^{2}-e_{1}^{2}+e_{2}^{2}-e_{3}^{2} & 2\left(e_{2} e_{3}+e_{0} e_{1}\right) \\
2\left(e_{1} e_{3}+e_{0} e_{2}\right) & 2\left(e_{2} e_{3}-e_{0} e_{1}\right) & e_{0}^{2}-e_{1}^{2}-e_{2}^{2}+e_{3}^{2}
\end{array}\right]=\left[\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}\right]
\]

It can be shown that by writing the elements of the rotation matrix in terms of the Euler angles, it is possible to relate the Euler angles \((\phi, \theta, \psi)\) to the members of the \([a]\)-matrix as shown below:
\[
\begin{align*}
& \phi=\tan ^{-1}\left(\frac{a_{32}}{a_{33}}\right) \\
& \theta=\sin ^{-1}\left(-a_{31}\right)  \tag{D-21}\\
& \psi=\tan ^{-1}\left(\frac{a_{21}}{a_{11}}\right)
\end{align*}
\]

The precision of the numerical scheme used to integrate the quaternions must be adjusted continually to ensure they satisfy Equation (D-14). This is ensured using the following scheme:
\[
\begin{align*}
& \lambda=1-\left(e_{0}^{2}+e_{1}^{2}+e_{2}^{2}+e_{3}^{2}\right) \\
& \dot{e}_{0}=-\frac{1}{2}\left(e_{1} p+e_{2} q+e_{3} r\right)+k \lambda e_{0} \\
& \dot{e}_{1}=\frac{1}{2}\left(e_{0} p+e_{2} q-e_{3} r\right)+k \lambda e_{1}  \tag{D-22}\\
& \dot{e}_{2}=\frac{1}{2}\left(e_{0} p+e_{3} q-e_{1} r\right)+k \lambda e_{2} \\
& \dot{e}_{3}=\frac{1}{2}\left(e_{0} p+e_{1} q-e_{2} r\right)+k \lambda e_{3} \quad \text { where } \quad k h \leq 1
\end{align*}
\]

Where \(h\) is the integration step (see Appendix B.2, Numerical Integration Methods for Solving ODEs). This approach is called the method of algebraic constraint (see Ref. [6]) and it requires \(\lambda\) to be calculated during each iteration. The addition corrects a possible deviation.

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\footnotetext{
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\({ }^{5}\) Stevens, Brian L. and Lewis, Frank L., Aircraft Control and Simulation, \({ }^{\text {st }}\) Ed., John Wiley and Sons, Inc., 1992.
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}```


[^0]:    ${ }^{1}$ Definition from http://dictionary.reference.com/browse/biomimetic

[^1]:    ${ }^{2}$ Forced Brownian motion refers to random-walk motion in which gradual movement in a specific direction prevails. For instance, molecules of two liquids of dissimilar density will meander such that, eventually, the heavier liquid accumulates below the lighter one, due to the force of gravity.

[^2]:    ${ }^{3}$ http://www.rcspeeds.com/

[^3]:    ${ }^{4}$ An understanding of this point is aided by studying Figures 6-23 and 6-24.

[^4]:    ${ }^{5}$ On tow refers to the cable-winch-system used to launch many sailplanes.

[^5]:    ${ }^{1}$ The term heuristic refers to the act of enabling a person (or an algorithm) to discover or learn something on their own.

[^6]:    ${ }^{2}$ Translation: "Flight of Birds as the Foundation of Flying."

[^7]:    ${ }^{1}$ Moravec, Hans P, Obstacle Avoidance and Navigation in the Real World by a Seeing Robot Rover, Stanford University, September 1980.
    ${ }^{2}$ Andrews, J. R. and Hogan, N., Impedance Control as a Framework for Implementing Obstacle Avoidance in a Manipulator, Control of Manufacturing Processes and Robotic Systems, American Society of Mechanical Engineers, Boston, 1983, pp. 243-251.
    ${ }^{3}$ Khatib, Oussama, Real-Time Obstacle Avoidance for Manipulators and Mobile Robots, International Journal of Robotics Research, Vol. 5, No. 1, pp. 90-98, 1986.
    ${ }^{4}$ Akishita, S.; Kawamura, S.; and Hayashi, K., Laplace Potential for Moving Obstacle Avoidance and Approach of a Mobile Robot, 1990 Japan-USA Symposium on Flexible Automation, A Pacific Rim Conference, pp. 139-142, Kyoto, Japan, 1990.

[^8]:    ${ }^{3}$ See http://www.wrf-model.org/index.php

[^9]:    ${ }^{1}$ Note, while these are "official numbers", they are untrustworthy. Standard aircraft of this size would be capable of $\sim 60 \mathrm{~min}$ with large LiPo packs (e.g. 10000 mAh ), but is should cover much greater distance in 60 min at 30 knots, or 30 nautical miles ( nm ).

[^10]:    ${ }^{1}$ In temperate latitudes this is approximately $0-9.7 \mathrm{~km}(6 \mathrm{mi})$. The troposphere can extend to 15 km in the tropics.

[^11]:    ${ }^{2}$ Orographic is defined by http://forecast.weather.gov/glossary.php?letter=o as "Related to, or caused by, physical geography (such as mountains or sloping terrain)."

[^12]:    ${ }^{3}$ Note that in random variables, the term process means the time-history, in this case, of $x$.

[^13]:    ${ }^{4}$ The do-loop is exited only upon user command.

[^14]:    ${ }^{5}$ Inverting a $n \times n$ matrix requires $n^{3}$ operations using Gauss-Jordan elimination.

[^15]:    ${ }^{2}$ If it helps, remember low pitch for low speed and high pitch for high speed.

[^16]:    ${ }^{1}$ This represents a typical Rotax 912 engine.

[^17]:    ${ }^{2}$ For analysis work in this dissertation, a weight of $6.0 \mathrm{lb}_{\mathrm{f}} /$ gallon is always assumed.
    ${ }^{3}$ When achieved, this ratio results in the highest temperature during combustion and is, therefore, of concern when it comes to engine durability. If the air-to-fuel mixture is less than (e.g. 14:1), it is called rich. If greater (e.g. $15: 1$ ), it is called lean. These two concepts are of great importance to pilots.

[^18]:    ${ }^{1}$ As defined by www.google.com.

[^19]:    ${ }^{2}$ A Phugoid is a classic exchange between potential and kinetic energy of an airplane, that makes its altitude oscillate approximately $\pi / 2$ out of phase with the airspeed (high-speed-low-altitude versus low-speed-highaltitude).

[^20]:    ${ }^{3}$ Irradiance refers to the flux of radiant energy per unit area normal to the direction of the light rays.

