Fall 2004

Adaptive Synchronization of Chaos for Secure Communication

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By
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A Thesis Submitted to the
Graduates Studies Office
in partial Fulfillment of the Requirements for the Degree of
Master of Science in Aerospace Engineering

Embry Riddle Aeronautical University
Daytona Beach, Florida
Fall, 2004

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This thesis was prepared under the direction of the candidate's thesis committee chairman Dr. Yechiel Crispin, Department of Aerospace Engineering, and has been approved by the members of this thesis committee. It was submitted to the Office of Graduation Studies and was accepted in partial fulfillment of the requirements for the degree of Masters of Science in Aerospace Engineering.

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Acknowledge

I would like to express my sincere appreciation to my thesis advisor Dr. Y Crispin for his guidance, knowledge and insight. Dr. Crispin was very supportive of this work and has provided me with valuables advices from his academic and professional experience in the chaos theory. Thanks to Dr. H Islami and Dr. L Narayanaswami for being the member of this thesis committee. I would like to thank Prof. John Novy and Prof. Richard Skovholt for letting my use the Electrical Engineering Lab to conduct my experimental part. I would also like to thank Mr. Mike Potash for his technical support on finding the right electrical components and helping me troubleshoot the electrical circuit design. Lastly I would like to thank all the faculty member and staff at Embry-riddle Aeronautical University, Aerospace Engineering Department for providing us with the opportunity and continues support for the completion of my graduate studies.

I would like to dedicate this thesis to my parent, my wife Juliyana and my late brother Capt. Ashish Panth, who were there for me in times of trouble and hardship, without their support and blessings I would not have completed this work.

ABSTRACT

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Title: Adaptive Synchronization of Chaos for Secure Communication

Institution: Embry Riddle Aeronautical University, Daytona Beach, FL

Degree: Master of Science in Aerospace Engineering

Year: 2004

The purpose of this thesis is to study the use of adaptive synchronization of chaos for secure communication. Several non-linear models were studied, the one-dimensional logistic map, two-dimensional coupled logistic map, three dimensional Lorenz systems with and without time delay. Numerical simulation using MATLAB was conducted. The Lorenz system with time delay was studied numerically as well as by testing an analog circuit and comparisons was made between them. It has been pointed out in many research papers that for low-dimensional chaotic processes, once intercepted, the information can be readily extracted, so the interest has been directed to higher dimensional chaotic system synchronization. The analog circuit gives a more accurate precise real time simulation.

The approach for secure communication utilizes the driver and driven synchronization of two identical chaotic systems. At the transmitter, information is added as a time varying parameter of the Lorenz system, and one of the state variables is transmitted through the public channel. This state variable is used to drive the receiver causing the two systems to synchronize in time. The driver system is used as the model reference for the adaptive controller to synchronize both the transmitter and the receiver system. The results of the Lorenz system are very conclusive both from the computer simulation and the analog circuits.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Chaos is a class of complex behaviors that emerges from nonlinear dynamical systems, and is ever present both in the technological and natural world. The new science of chaos is defined as aperiodic behavior occurring in a deterministic system. A nonlinear system is described as a system whose time evolution equation is nonlinear, that is, the dynamical variables describing the properties of the system is in a nonlinear form [Hillborn, 1994]. The main three properties of the deterministic systems are its time-evolution equations, parameters that describe the system, and the initial conditions. According to one definition, "Chaos theory is the qualitative study of unstable aperiodic behavior in deterministic nonlinear dynamical systems." [Strogatz, 2000].

With the definition given above, we can draw several conclusions about the characteristics of chaos. First, we can say that the system is a dynamical system, meaning that its properties change over time. Second, that the behavior of the system is aperiodic and unstable thus the system behavior does not repeat itself. A non-periodic behavior does not follow a set pattern. If there is periodic behavior in a system, then future behavior may be determined. Non-periodicity is an outcome of apparent randomness, a sign of a chaotic system. Third, deterministic chaotic system has the property of sensitivity with respect to initial conditions, thus the trajectories of two perfectly identical chaotic systems starting with nearby initial conditions diverge from each other exponentially. This basic definition derived from the discovery of chaos in 1961, by Edward Lorenz [Lorenz, 1996]; while working on a system of equations that is now called the Lorenz system, he ran the computer modeling program twice with the same initial conditions, except that one had the accuracy of six digits, while the other had an accuracy of three digits. At first the two behaved identically, but after a short while they acted drastically different.

From a practical standpoint, biological systems such as the human heart, prediction of weather and stock markets exhibit chaotic behavior and therefore been of great interest to researchers. As stated earlier, a chaotic system is aperiodic, and does which do not settle down to a fixed points, periodic orbits or quasi-periodic orbits as time approaches infinity. These characteristics have been extensively researched recently in various applications such as chemical and biological systems. The aim of the work reported here is to investigate the use of chaos in secure communication.

Synchronization of chaotic systems plays an important role in many applications. If two chaotic oscillators are uncoupled, then knowing the dynamics of one oscillator does not tell anything about the state of the other oscillator at that moment. It turns out that if we couple both the chaotic systems in a way, then they tend to oscillate exactly the same way. Since both chaotic oscillators have the same dynamical characteristics, the trajectories of two chaotic systems are locked to each other and are said to be in identical synchronization. This means that when the transmitter has a certain amplitude at a given moment, the receiver at that moment is exactly of the same amplitude. The characteristic of the identical synchronization after coupling the oscillators is a diagonal line of the individual state variable of the receiver and the transmitter side. The diagonal line appears because we plot a point for each moment in time for the whole. If at some moment the two amplitudes were not equal, a deviation from the diagonal line can be seen.

In a real situation, time delay is inevitable, since the propagation speed of an information signal is finite. Chaotic attractors of time-delay systems can have a much higher dimension, and this suggests other possible way to improve the security for the use in secure communication. In this report we have utilized Lorenz systems to create the time delay chaotic system. The time delay unit is added to one of the three state variables of the Lorenz system. Both the transmitter and receiver circuit are made to synchronize, using the same time delay and the same initial condition of the state variables. Despite a small number of system variables, the embedding dimension and number of positive Lyapunov exponent increases as the delay time increases, and the system eventually transits to hyperchaos. The knowledge of Lyapunov exponents gives the indication about the predictability of the system. The analysis of Lyapunov exponents has not been investigated in this research. The time delay on a chaotic system has been further investigated in this report.

1.2 Chaos and its Application to Communications:

Recently a lot of work on secure communication has been implemented on the research phase, which all started in early 1960's when Lorenz observed that in the set of nonlinear equations, making very small changes of the parameters had a large effect on their solutions. The work of Pecora and Carroll [Pecora, 1990, 1991] on synchronization of chaotic system had recommended the possibility of secure communication using chaos synchronization. Also in their work of synchronization of chaotic systems such as Lorenz and Rossler system were divided into two identical systems, so-called the response or transmitter system, while the other drive or receiver system.

Following the process of synchronization, a number of methods have been proposed for secure communication [Cuomo, 1993], [Kocarev, 1995], [Boccaletti 1997], [Liu, 2000] with the help of the chaotic signals. Cuomo and Oppemheim [Cuomo, 1993] had shown that a small difference between the initial conditions between the drive and the response system would uncouple and distort the synchronization process. In 1999, Lin, Peng, and Wang [Lin, 1999] considered synchronized chaos in discrete systems, more precisely they studied the synchronized chaotic behavior of the popular model in coupled map lattices. However, Perez [Perez, 1995] in his paper pointed out that message masked with low-dimensional chaotic processes, once intercepted, can be sometimes readily extracted, so that the interest has been directed to higher dimensional chaotic system. For more secure communication purposes hyper-chaotic systems are used for more reliability and security, which can be easily generated by a time delay approach.

Two factors that are important to secure communications with chaotic systems are the dimensionality of the chaos and the effort required to obtain the necessary parameters to match that of the driven systems [Roy, 1999]. One way to avoid this extraction of information from chaotic signal is to implement higher dimension with the time-delay chaotic system, which has been studied in this work. The other process proposed by Palaniyandi and Lakshmanan [Palaniyandi, 2001] to overcome by transmitting via a multi-step parameter modulation combined with alternative driving of different drive system variables, which makes that attractor reconstruction impossible. It has researched, that without the receiver circuit, one can unmask the message from the modulation drive signal from the return map. In their work the changes were made in the mode of transmission of the digital message by the multi-step parameter modulation instead of single step in-order to complicate the attractor in the return map.

1.3 Overview of the Present Work

In this thesis, the problem of control and synchronization of chaos in systems with time varying parameter is treated. There are two ways of signal modulation in the chaotic system.

1. Modulation of state variable signal
2. Modulation of system parameter signal

Much of the previous work in this area has focused on modulating the state variable signal. In our research the time varying system parameter has been modulated. Since the message is encoded in a time varying parameter, synchronization with the response system must be first established, and the recovery of the message is not ensured unless the variables parameter is identified at the receiver side. Therefore the proposed method improves the security of communication even when the parameters are initially substantially different [Crispin, 2002].

The goal of this work is to utilize chaos as the means of transmitting messages in a communication system. The primary goal is to hide information on a chaotic carrier signal, which is sent from the drive system (transmitter) to the driven system (receiver) through public channels. In this work, the extraction of information has been accomplished by synchronization of drive and the response system and adaptive control method. On the second part of this thesis, a nonlinear system with a time-delayed feedback approach has been utilized. The method of synchronization and parameter identification has been extensively used in order to achieve our goal [Crispin, 2004]. This work consists of building an analog chaotic circuit, and also analyzing and comparing it with the numerical solutions. It can be seen that, chaotic signals of higher dimension can be used as an encryption technique for private communication because it's hard to reconstruct its attractor.

In order to understand the method completely in the three-dimensional model, first the characteristics of the one-dimensional logistic map and the coupled lattice logistic map are investigated in this thesis. The results from the computer simulation show that the use of synchronization of the two chaotic systems and the adaptive method could be used for the purpose of the secure and private communication.

In Chapter II and III, we present extensive analysis of one, two and three-dimensional model of chaotic systems. In Chapter III numerical model as well as practical implementation with the analog circuit of the Lorenz system is studied. The Lorenz system displays trajectories a trajectory, plotted in three dimensions, that winds around and around, occupying a region known as its attractors. The process of synchronization is implemented with linking the trajectories of one system to the same value in the other so that they remain in the step with each other, through the transmission of a signal [Boccaletti, 1997]. The synchronization of two identical systems is established with different initial condition between the drive and response system. In all the cases the information of very low frequency and amplitude information is added to the parameters of the system.

In Chapter IV, study on nonlinear systems with a time-delayed feedback approach on Lorenz equation is studied. The dynamics of nonlinear feedback systems cover a wide range up to high-dimensional chaotic behavior [Voss, 2001]. Nonlinear delay differential systems, a peculiar class of infinite-dimensional dynamical systems, are frequently used in several areas of science [Piccardi, 2001]. It has been known that very simple time-delay system is able to exhibit hyperchaos. Therefore, a time-delay system provides alternative simple and efficient tools for secure communication with low detectability. The main aim of chapter IV is to show the synchronization process extended to the delay differential systems, which has already been applied to the finite dimensional system.

The analog circuit in this research allowed us to monitor the real time simulation of the chaotic signals and implement our goal. The computer simulation can only give us the approximate difference equation of the system and the model is analyzed in a finite time domain. Thus to analyze the differential equation, the analog circuit gives an infinite time dimension model and a real time simulation of our practical problem. Since the integrators, summing amplifiers used in the analog circuit would represent the true and perfect operation and thus avoid the truncation error. The other advantage of the analog circuit analysis over digital system is the removal of an imposed sampling frequency. The Fourier transform converts a signal from the time domain, without loss of information, into the frequency domain. The frequency domain representation is exactly the same signal, in a different format. The inverse Fourier transform takes the frequency series of complex values and maps them back into the original time series. The perfect invertability of the Fourier transform is an important property for building filters, which remove noise or particular components of a signals spectrum. When the signal is converted to digital form, the precision is limited by the number of bits available. The errors that occur in the digital system are the artifacts and RMS error, which corrupt the actual information while processing the signal. All analog systems operate in real time, digital systems that depend on a computer to perform system computations may or may not work in real time. Only standard commercially available electronic components have been used and the observed signals are quiet robust. In chapter V the conclusion and the future development on the work has been stated.

CHAPTER 2

ONE & TWO DIMENSIONAL MAPS

2.1. General Analysis: One Dimensional Map

Recently various methods have been investigated to use chaos in secure communication system. The basic idea is to hide a message signal on a chaotic carrier. The first approach in our report is to utilize this principle with the coupled logistic maps for the application to secure communication. Also in this chapter we have compared the one and two-dimensional models and simulate a two-dimensional coupled logistic map communication scheme.

This chapter describes the use of chaos in one-dimensional map and two-dimensional coupled logistic map in private communication system. Generally the nonlinear dynamics of physical systems are analyzed by obtaining discrete models, mathematically known as maps. In the discrete time domain we can say that a map is simply a function on the phase space that gives the next state, of the system given its current state. The dimension of the phase space is dependent on the number of variables in the system. One-dimensional system has only one state variable *x*, and the dynamical equation is generally represented by $\dot{x} = f(x)$, where f is a smooth function from the real line in the x-axis.

$$x_n = f(x_{n-1}) \tag{2.1}$$

The iterative scheme begins with the initial value of x_0 and generates a trajectory by successive application of the map function. The one-dimensional iterative maps show a much greater range of dynamical behavior than the one dimensional differential equation system because the iterated maps are free from the constraints of continuity [Hilborn, 1994].

The fixed points in state space play an important role in understanding the dynamics of the system. A fixed point xn = x(n-1) for a one dimensional map is given by $x = f(x)$. If the trajectory happens to get the fixed point, then the trajectory will remain at that particular point. [Strogatz, 1994].

2.2 Coupled Logistic Maps (2D Map).

The remarkable feature of the logistic map is in the simplicity of its form and the complexity of its dynamics and is one of the simplest forms of a chaotic process. In this thesis we have used two dimensional coupled logistic maps as a means to create a secure communication system. The system of *N* globally coupled one dimensional maps is described as

$$x_{n+1}(i) = (1-\varepsilon)f[x_n(i)] + \frac{\varepsilon}{N}\sum_{j=1}^{N} f[x_n(j)] \qquad (2.2)$$

Where *i* and *j* are indices of maps with the lattice, *n* and *n+1* are iteration indices, the coupling strength is ε, which varies between zero and one and f(x) is the map function. If the coupling strength approaches unity, we expect the lattice to fully-synchronize. The $x_n(i)$ is the nth iteration of the *ith* map. The function is the logistic equation when $f(x_n) = p\, x_n (1-x_n)$ where p is the control parameter.

For the simple uncoupled one dimesional logistic map can be represented as:

$$x_{n+1} = f(x_n) = p\, x_n (1-x_n) \qquad \text{for} \quad 0 \le p \le 4 \qquad (2.3)$$

It represents the discrete-time model of the population growth in an ecological stystem and p being the control parameter, taken to be positive and real value. The value of x must lie between some finite interval along the x-axis and between $0 \le x \le 1$.

The fixed points are given by:

$$x_f = p\, x_f(1-x_f) \qquad \text{or} \qquad x_f\,[\, p\, x_f + (1-p)] = 0 \tag{2.4}$$

$$x_{f1} = 0 \text{ and } x_{f2} = 1-1/p \tag{2.5}$$

This gives the fundamental of the logistic map which is utilized on our first part of this research for the case of the secure cummunication system.

Considering two dynamical systems that are coupled in a unidirectional way, where one system, the drive, produces a scalar signal that is added to the response system. The transmitter system is assumed to be a discrete time autonomous dynamical system. In this case we are particularly interested in complete synchronization of the two dynamical systems.

If we consider two logistic equation functions f(x) and f(y), the coupled logistic map is represented as a two-dimensional logistic map as shown below.

$$x_{n+1}(i) = (1-\varepsilon)f[x_n(i)] + \varepsilon f[y_n(j)] \tag{2.6}$$

$$y_{n+1}(j) = \epsilon\, f\,[x_n(i)] + (1-\epsilon)\, f\,[y_n(j)] \tag{2.7}$$

where $f(x_n) = p\, x_n\, (1-x_n)$ and $f(y_n) = p\, y_n\, (1-y_n)$ and ϵ is the coupling strength

Increasing the number of dimensions (variables) for iterative maps increases the range of possible behaviors. This type of coupling between two dimensional maps is also called the dissipative coupling. This work utilizes the same type of coupling procedure for the receiver and the transmitter unit in our work.

2.3 Synchronization and Parameter Identification.

We know that synchronization can be achieved by forcing one chaotic system to behave same as the other chaotic system. A drive system and a response system should operate in the same dynamics inorder to synchronize. The drive system creates the chaotic signal, which is coupled with the response system with one of the state variable. The response side then reconstructs the chaotic signal, which is exactly the same as the drive system.

The one-dimensional iterative map can be expressed as:

$$x(n+1) = f\left(x(n); p(n)\right) \tag{2.8}$$

$$y(n+1) = f\left(y(n); q(n)\right) \tag{2.9}$$

Where $f : R^m \rightarrow R^m$, $x, y \in R^m$, $p(n) \in R^K$, $q(n) \in R^K$

where n is the time, x(n) is the state variable and p(n) is the independent time varying parameter of the drive system. Similarly y(n) is the state variable and q(n) the time varying parameter of the response system. The initial conditions of the state variable x(n)= x(0) and y(n) = y(0) for n = 0 are not necessarily the same. Similarly initial values of the parameters p(0) = p0 and q(0) = q0 of both the systems are different.

Since the chaotic systems are sensitive to the initial conditions, the drive and the response system will not synchronize unless the response system is controlled and forced to synchronize with the drive system using a transmitted signal [Crispin, 2002]. Thus a scalar signal s(n), which is a function of x(n) can be transmitted to the response system.

$$s(n) = h(x(n)) \tag{2.10}$$

In order to achieve our goal of synchronization, as also shown in Fig 2.5 and to transmit the information from the transmitter side to the receiver side, the two dynamics of the equations on the drive and the response side are represented as:

$$x(n+1) = f\left(x(n); p(n) + I(n)\right) \quad (2.11)$$

$$y(n+1) = f\left(y(n); q(n)\right) \quad (2.12)$$

Where I(n) is the information to be transmitted and is added to the time varying parameter. Thus the synchronization can also be achieved even when the parameters p(n) and q(n) are initially substantially different [Crispin, 2002].

$$\lim_{n\to\infty} |p(n) - q(n)| = 0 \quad (2.13)$$

The synchronization can be achieved if the dynamics of the response system parameter q(n) is determined. In other words, the differential equations governing the evolution of the response parameters q(n) need to be derived systematically for dynamical system of the form of Eqs 2.7-2.8 [Crispin, 2002].

For the case of two dimensional coupled maps in the discrete time domain, the drive side can be expressed in the following way.

The drive system:

$$xt(n+1) = (1-\varepsilon)\, fxt(n) + \varepsilon\, fyt(n) \quad (2.14)$$

$$yt(n+1) = (1-\varepsilon)\, fyt(n) + \varepsilon\, fxt(n) \quad (2.15)$$

where $fxt(n) = p1(n)\; xt\,(n)\; (1-xt(n))$ and $fyt(n) = q1(n)\; yt\,(n)\; (1-yt(n))$

At the response side:

$$xr(n+1) = (1-\varepsilon)\, fxt(n) + \varepsilon\, fyr(n) \quad (2.16)$$

$$yr(n+1) = (1-\varepsilon)\, fyr(n) + \varepsilon\, fxr(n) \quad (2.17)$$

where $fxr(n) = p2(n)\ xr(n)\ (1-xr(n))$ and $fyr(n) = q2(n)\ yr(n)\ (1-yr(n))$ and $xt(n), yt(n)$ are the state variable of the drive and $xr(n), yr(n)$ are the state variable of the response side with different initial conditions. The time varying parameters are p1(n), q1(n) and p2(n) and q2(n) for the transmitter and the receiver side. As stated in the synchronization algorithm one of the transmitter variable xt(n) has to be added to the response system as shown in Eqs. 2.15. Later in the chapter the possibility of encoding a message from the chaotic dynamics through the parameter identification and adaptive process has been extensively explained.

In the adaptive control and parameter identification of the one-dimensional logistic map given in equation 2.7 and 2.8, the sensitivity method is used to design the adaptive law so that the estimated parameters are adjusted in a direction that minimizes the error function. Given the key logistic function and the transmitted signal xt(n), the problem is to identify the parameter p(n) and extract masked information [Crispin].

The error e(n) represents the deviation of the receiver signal from that of the transmitted signal:

$$e(n) = yr(n) - yt(n) \qquad (2.18)$$

Thus the adaptive process is derived by minimizing the objective function for error $e(n)$, which is represented by:

$$J(n+1) = \frac{1}{2}(yr(n)\text{-}yt(n))^2 = \frac{1}{2}e^2(n+1) \qquad (2.19)$$

J is based on the Euclidean distance between the vectors x and y. As an example let us consider the design of an adaptive law for updating the parameter vector θ.

$$\theta(n) = [p1(n), p2(n), \ldots pn(n)]^T \tag{2.20}$$

$$\theta^*(n) = [q1(n), q2(n), \ldots qn(n)]^T \tag{2.21}$$

where θ^* is the unknown parameter vector. Thus when $\theta = \theta^*$ implies that error e(n+1) = 0, a non-zero value of error $e_{(i)}$ is represented by $\theta \neq \theta^*$. Since the output is dependent on the vector θ^* and the error vector is also dependent on theθ^*. Inorder to reduce the error e(n+1) to zero is to adjust θ^* in a certain way that minimizes the objective function.

$$J(\theta) = e^2(\theta) \tag{2.22}$$

The simple method to minimize is to use steepest decent method that decreases the objective function, a possible adaptation process is given by:

$$\theta^*(n+1) - \theta^*(n) = -\gamma \; \nabla J(\theta^*) = -\gamma \; e(n+1) \; \nabla e(\theta^*) \tag{2.23}$$

$$\text{where } \nabla e(\theta^*) = \left[\frac{\partial e}{\partial \theta_1^*}, \frac{\partial e}{\partial \theta_2^*}, \ldots\ldots\ldots \frac{\partial e}{\partial \theta_n^*}. \right]^T \tag{2.24}$$

is the gradient of $e_{(i+1)}$ with respect to

$$\theta^* = [\theta_1^*, \theta_2^* \ldots\ldots \theta_n^*] \tag{2.25}$$

As we know that x(n+1) does not depend on $\theta^*_{(i)}$. Then we have:

$$\theta^*(n+1) - \theta^*(n) = -\gamma \; e(n+1) \; \nabla y(\theta^*) \text{ or } \theta^*(n+1) = \theta^*(n) - \gamma \; e(n+1) \nabla y(\theta^*) \tag{2.26}$$

where $\gamma > 0$ is the arbitrary design constant referred as the adaptive gain. Thus the adaptive process reduces to:

$$q(n+1) = q(n) - G \;\; e(n+1) \;\; \nabla y(\theta^*) \tag{2.27}$$

Where q(n+1) is the output or the recovered messages, where the initial condition of the parameter is given by q(0) some constant and the error e(n+1) is the difference between the transmitter and the receiver state variable.

2.4 Using the Coupled logistic map for secure communication:

Considering the two coupled logistic map at the drive system.

$$x_1(n+1) = (1-\varepsilon)\, fx1(n) + \varepsilon\, fx2(n) \quad (2.28)$$

$$x2(n+1) = (1-\varepsilon)\, fx2(n) + \varepsilon\, fx1(n) \quad (2.29)$$

where the functions are one-dimensional logistic maps and $\varepsilon = 0.7$ is the coupling strength.

$$fx1(n) = p_1(n)\, x1(n)\, (1-x1(n)) \quad (2.30)$$

$$fx2(n) = p2(n)\, x2(n)\, (1-x2(n)) \quad (2.31)$$

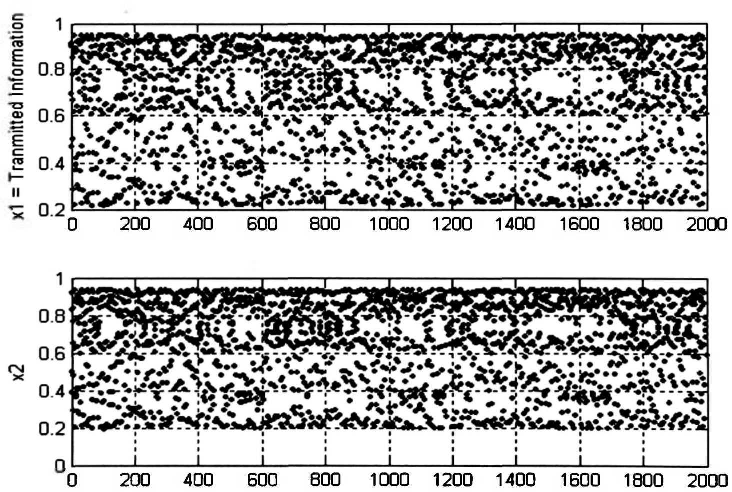


Fig 2.2: Coupled logistic maps x1(n+1) and x2(n+1) at the drive (transmitter) side

where p1(n) and p2(n) are the time varying parameters for the drive. Two information I1(n) and I2(n) are added to the parameters; where $b_1=3.7$; $b_2=3.823$;

$$p1(n) = p10 + I1(n)\ ; \ \text{where } p10 = b_1(\text{parameter}) \quad (2.32)$$

$$p2(n) = p20 + I2(n);\ \text{where } p20 = b_2\,(\text{parameter}) \quad (2.33)$$

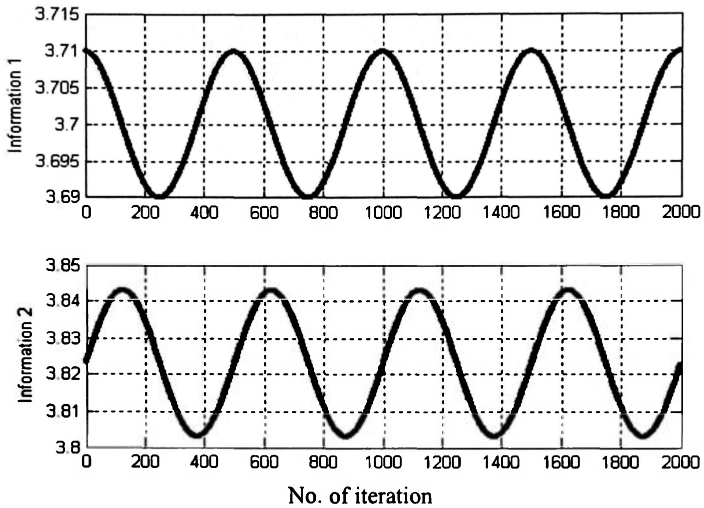


Fig 2.3: Two Information signals I1(n) and I2(n) to be transmitted which are added to the time varying parameters

Given the key logistic function and the transmitted signal x(n+1), the problem is to identify the parameters p1(n) and p2(n) and extract the masked information. In the response side has the same dynamical equation with the initial conditions being different from that of the drive side.

$$y_1(n+1) = (1-\varepsilon)\ fy1(n) + \varepsilon\ fy2(n) \tag{2.34}$$

$$y2(n+1) = (1-\varepsilon)\ fy2(n) + \varepsilon\ fx1(n) \tag{2.35}$$

where

$$fy1(n) = q11(n)\ y1(n)\ (1-y1(n)) \tag{2.36}$$

$$fy2(n) = q22(n)\ y2(n)\ (1-y2(n)) \tag{2.37}$$

where $q11(i) = b_1$ and $q22(n) = b_2$ are the parameters of the response side.

One of the state variables from the response side fx1 is added to the receiver coupled equation 2.27.

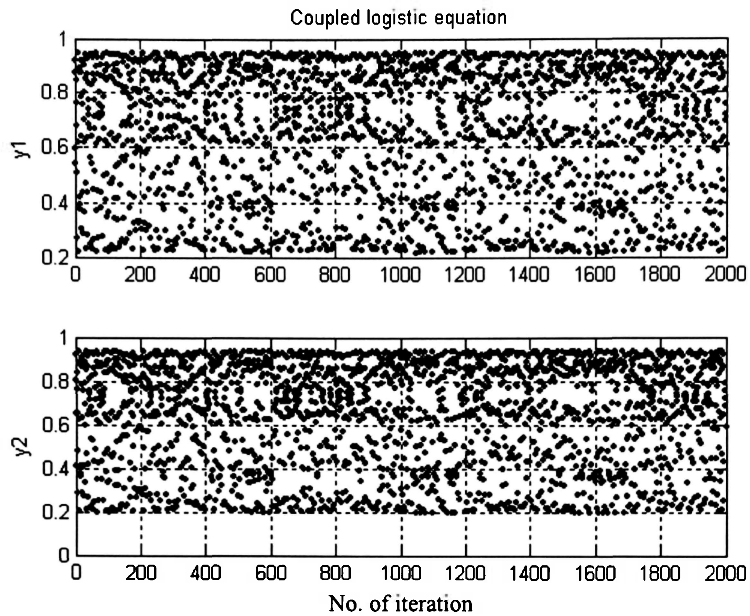


Fig 2.4: Coupled logistic maps y1(n+1) and y2(n+1) at the driven side

Using the adaptive process extraction of the first information I1(n) shown in Fig 2.6 (a).

$$q11(n+1) = q11(n) + G\ e_1(n+1)\ \frac{dfy_{1(n+1)}}{dq_{11(n+1)}} \tag{2.38}$$

where $\frac{dfy_{1(n+1)}}{dq_{11(n+1)}} = (1-\varepsilon)\ y_{1(n)}\ (1-y_{1(n)})$;

Extraction of the second information I2(n) which is shown in the Fig 2.10 (b) and is compared with the actual information.

$$q22_{(n+1)} = q22_{(n)} + G\ e_{2(n+1)}\ \frac{dfy_{2(n+1)}}{dq_{22(n+1)}} \tag{2.39}$$

where $\frac{dfy_{2(n+1)}}{dq_{22(n+1)}} = (1-\varepsilon)\ y_{2(n)}\ (1-y_{2(n)})$ and $e_{1(n+1)} = y_{1(n+1)} - x_{1(n+1)}$ and $e_{2(n+1)} = y_{2(n+1)} - x_{2(n+1)}$

and the G = 50.

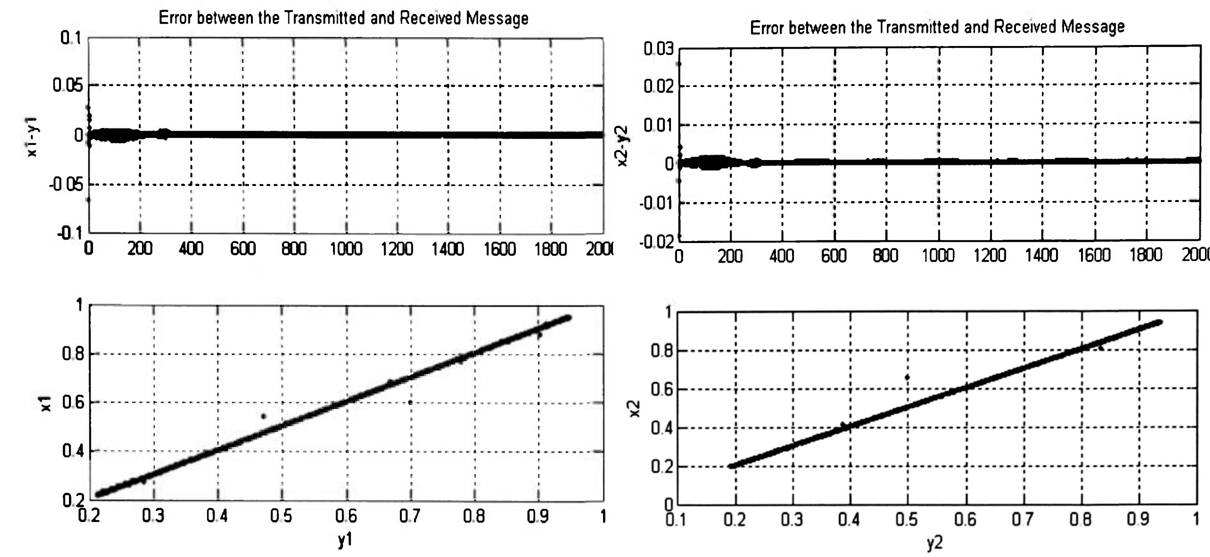
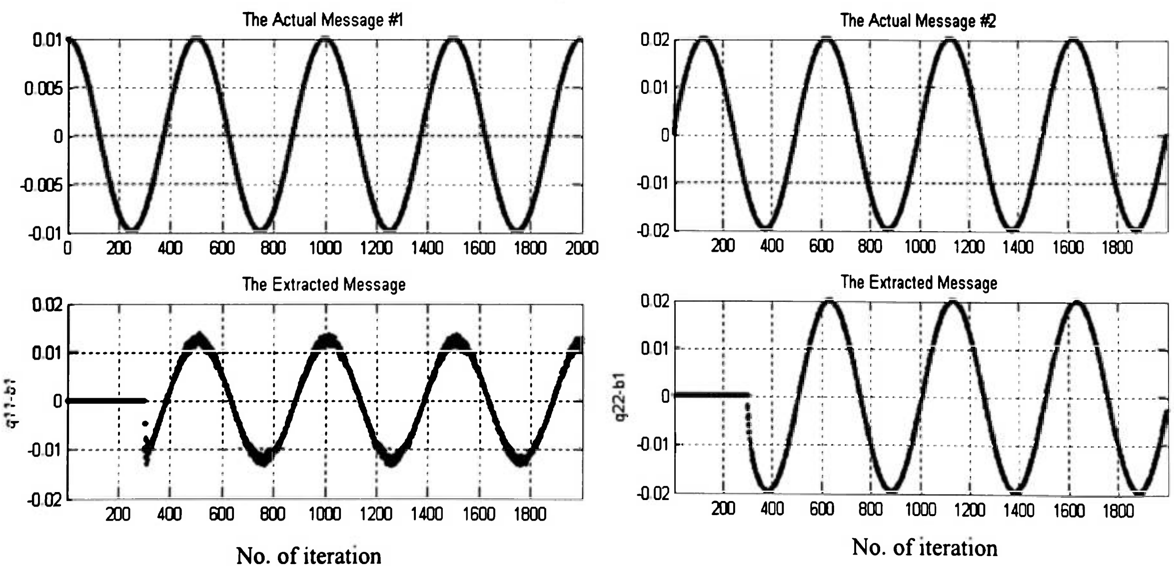


Fig 2.5: Error and synchronization characteristics between the drive and the response signal.



(a) (b)

Fig 2.6: The comparison between the actual information and the extracted information using the synchronization process and parameter identification: (a)1st information. (b) 2nd information. The G = 0 for n $\leq$300 and G = 50 for n $\geq$300

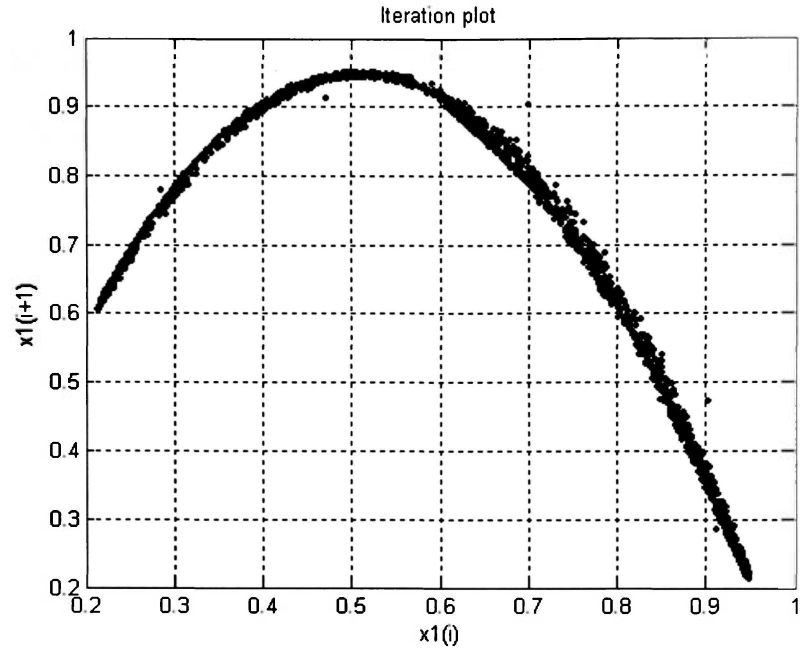


Fig 2.7: The iterative plot of the transmitted variable $x_1(n)$ vs. $x_1(n+1)$.

2.5 Conclusion:

As stated above, the purpose of this chapter was to propose a generalized method of control, synchronization and parameter identification of chaotic systems with the time varying parameter. The proposed synchronization and parameter identification scheme demonstrates good signal and parameters identification as well as the information extraction.

From the results we can see that the results were recovered perfectly. As seen on the Fig 2.5 the synchronization between the same transmitter and the receiver variable has a diagonal line and the error between them is very small. The recovered messages may contain a high frequency noise as seen on Fig 2.6(a) which can be eliminated by using low pass filter.

The reason for applying the gain $G = 50$ after 300 iteration was to let the system settle down in the synchronization process. As seen for the iteration $n < 300$ the error in Fig 2.5 is more compared to the error after $n > 300$ iteration. The results obtained from this computer simulation was satisfactory and this same approach has been implemented on the three dimensional Lorenz system in the next chapter.

CHAPTER III

THREE DIMENSIONAL FLOW: LORENZ SYSTEM

3.1. Lorenz system of Equations.

Chaotic systems provide a rich mechanism for signal design, generation and processing. An interesting aspect of modern chaos study is the emerging notion of secure communication via synchronized chaos. A numerical model of the Lorenz system has been implemented to demonstrate how the chaotic signal can be used in various contexts to mask information-bearing waveforms. The information is added to one of the time varying parameters of the Lorenz system. An adaptive process and parameter identification is used for the recovery of the information at the receiver side. Since only one of the state variables of the Lorenz transmitter system is transmitted, the eavesdropper will not be able to reconstruct the whole dynamics of the system to extract the information.

The application of the Lorenz systems to weather prediction has led to a popular metaphor known as the butterfly effect and its many practical applications such as secure communication, monitoring stock market and more. The Lorenz system is defined by the very simple, coupled, three nonlinear first-order ordinary differential equations.

$$\frac{dx}{dt} = \sigma(y - x) \tag{3.1}$$

$$\frac{dy}{dt} = rx - y - xz \tag{3.2}$$

$$\frac{dz}{dt} = xy - bz \tag{3.3}$$

where σ, r, b > 0 and σ is the Prandtl number, r is the Rayleigh number and b is a constant. The system has only two nonlinearities xy and xz. The solution of these ordinary differential equations describe a trajectory, which when plotted in three dimensional phase plane is known as the Lorenz attractor.

3.2 Synchronization of two Lorenz Systems.

As stated earlier chaotic systems provide signals for design, generation and processing for the use of secure and private communication via synchronized chaos. In this report numerical model as well as analog circuit of the Lorenz system has been implemented to demonstrate the use of chaotic signal to mask the information-bearing waveforms.

In this chapter control and synchronization of chaos with time varying parameter is treated using the adaptive method and parameter identification. The driven system is controlled by varying its parameters using an adaptation process. Multiple messages of secure information can be carried using the single scalar transmitted signal [Crispin, 2002], thus in this setup only one information signal is transmitted.

Consider the two similar chaotic systems, the first is a Lorenz system serving as the drive system and the second is the driven system with one common state variable which are shown in equations 3.4 to 3.9. The parameters in the drive system are p_1, p_2, p_3, σ, r and b, where p_2 and p_3 =1. Similarly the parameters in the driven system are q_1, q_2, q_3, σ, r and b where q_2 and q_3 =1. The scalar signal x_2 is the only transmitted signal from the drive side, which enables to synchronize the two systems. The problem is to identify the parameter p_1 and extract the masked information. This can be achieved only if the two systems synchronize with the common signal x_2.

Consider the Lorenz system at the transmitter side:

$$\frac{dx_1}{dt} = \sigma(x_2 - x_1) \tag{3.4}$$

$$\frac{dx_2}{dt} = p_1x_1 - p_2x_2 - p_3x_1x_3 \tag{3.5}$$

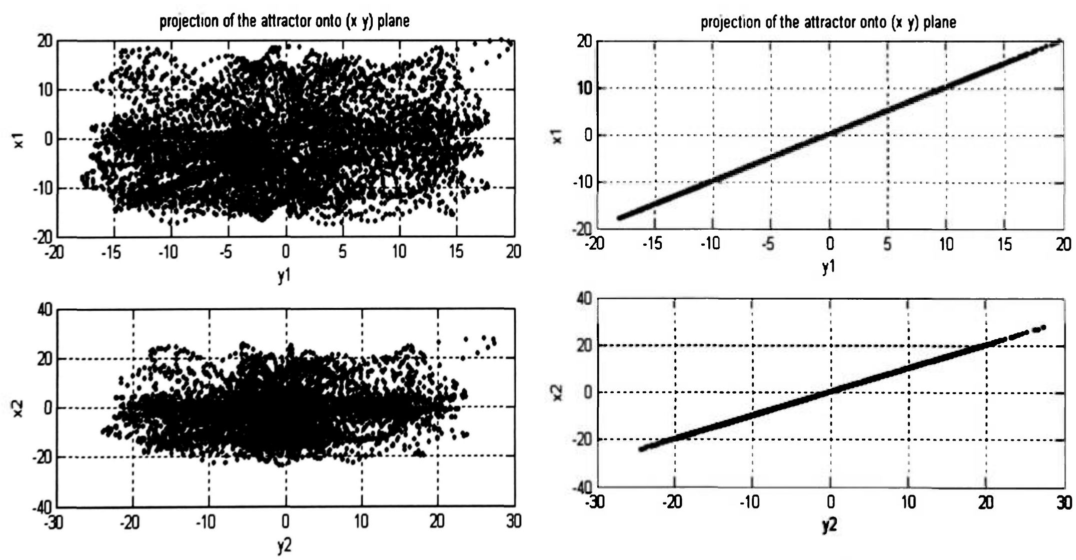
$$\frac{dx_3}{dt} = x_1x_2 - bx_3 \tag{3.6}$$

In the receiver or the response system:

$$\frac{dy_1}{dt} = \sigma(x_2 - y_1) \tag{3.7}$$

$$\frac{dy_2}{dt} = q_1y_1 - q_2y_2 - q_3y_1y_3 \tag{3.8}$$

$$\frac{dy_3}{dt} = y_1x_2 - by_3 \tag{3.9}$$



(a) (b)

Fig 3.1: Synchronization properties of the Lorenz system between the transmitter and the receiver system with different initial condition: (a) without coupling the transmitter and the receiver side of the Lorenz system (b) with coupling the transmitter and the receiver side of the Lorenz system with one of the system variable.

This chapter uses the principle that was published in the research work by Crispin, [Crispin, 2002]. In order to achieve the synchronization, we have to determine the dynamics of the parameter q(t). An analogy from the hydrodynamics, the Langrangian approach for describing the evolution of a scalar quantity convected in a flow field [Milne-Thomson, 1968] is used for this purpose. This approach used the equation of motion of two marker particles advected in the fluid flow described by the vector fields is given, $w(x,p) = f(x,p)$ and $w(y,q) = f(y,q)$ at any given point $x \varepsilon R^n$ and $y \varepsilon R^n$ and the right hand sides are to be interpreted as the local velocity vector as [Crispin 2002].

$$\frac{dx}{dt} = w(x,p) = f(x,p) \tag{3.10}$$

$$\frac{dy}{dt} = w(y,q) = f(y,q) \tag{3.11}$$

Considering the time variation of a scalar property J(x, y) of the flow along a trajectory of the response system as it evolves in the phase $y \varepsilon R^n$ .The total rate of change is given by the substantial derivative of the scalar property, the derivatives following the flow [Crispin, 2002]:

$$\frac{DJ}{Dt} = \frac{\partial J}{\partial t} + F(y(t),q)\nabla J \tag{3.12}$$

where $\nabla = \left(\frac{\partial}{\partial y_1}, \frac{\partial}{\partial y_2}, \ldots\ldots\ldots \frac{\partial}{\partial y_n} \right)$

The difference or error between the drive and the driven state variable $e_2 = y_2 - x_2$ in the Lorenz system shown in Fig 3.18. The parameter p_1 contains the information, the purpose of the receiver is to adapt to the dynamics of the transmitter by varying the parameter q_1 [Crispin, 2002]. The adaptive process is derived by minimizing the objective function for error e(i) is represented by:

$$J_{(t)} = \frac{1}{2}\left|\vec{y}-\vec{x}\right|^2 = \frac{1}{2}\sum_{i=1}^{n}(y_i - x_i)^2 = \sum_{i=1}^{n} e_i^2 \tag{3.13}$$

where J is the Euclidean distance between the state vectors $\vec{x}$ and $\vec{y}$. The derivative of J with respect to time is given by.

$$\frac{\partial J}{\partial t} = \sum_{i=1}^{n} e_i \frac{de_i}{dt} \text{ or } \frac{\partial J}{\partial t} = e_i \tag{3.14}$$

Substituting the values to equation 3.13

$$\frac{DJ}{Dt} = \sum_{i=1}^{n} e_i [\frac{de_i}{dt} + f_i(y(t), q)] \tag{3.15}$$

since $e = y - x$; thus $\frac{de}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = f(y, q) - f(x, p)$

The equation 3.16 reduces to

$$\frac{DJ}{Dt} = \sum_{i=1}^{n} e_i [2f_i(y(t), q) - f_i(x(t), p)] \tag{3.16}$$

which defines the rate of change of positive scalar property J in terms of the state variable x and y of the drive and the response system, the parameters p and q. The synchronization can be achieved even when the parameters p(t) and q(t) are initially substantially different and thus the dynamics of the response system parameters q(t) need to be determined. This can be accomplished by controlling the response system y such that the parameter p(t) of the drive system x are eventually identified, this is [Crispin, 2002].

$$\lim_{t\to\infty} |p(t) - q(t)| = 0 \tag{3.17}$$

In order to achieve synchronization, the deviation between the drive and the response system should be continuously decreasing, thus for the perfect synchronization, errors between the two systems $e = y - x = 0$.

A possible adaptation process is given by.

$$\frac{\partial q}{\partial t} = -G\nabla q\left[\sum_{i=1}^{n} e_i f_i\big(y(t), q\big)\right] \quad (3.18)$$

3. 3 Lorenz system and its use in secure communication.

Synchronization of coupled chaotic oscillators has become the subject of much discussion in the past decade [Fujisaka, 1983]. A useful property of the Lorenz system is that it possesses a self-synchronization property. A chaotic system is self-synchronization if it can be decomposed into at least two subsystems; a drive system or the transmitter and the receiver or the response that synchronizes when coupled with a common signal or in this case a common state variable. The synchronization considered in this report is similar to the one proposed by Pecora and Carroll [Pecora, 1990,1991] by introducing into consideration the drive-response scheme for the condition of synchronization.

Consider the Lorenz system at the transmitter side:

$$\frac{dx_1}{dt} = \sigma(x_2 - x_1) \quad (3.19)$$

$$\frac{dx_2}{dt} = p_1x_1 - p_2x_2 - p_3x_1x_3 \quad (3.20)$$

$$\frac{dx_3}{dt} = x_1x_2 - bx_3 \quad (3.21)$$

where x_1, x_2 and x_3 are the state variables with the initial condition of $[x_1^*, x_2^*, x_3^*]$ = [0.1; 0.12; 0.45] and the p_1, p_2 and p_3 are the time varying parameters of the system.

$p_1 = r_o + I_n$ where $r_o = 28$ and I_n = the information added to the parameter = A Sin (ωt)) were amplitude A = 1. The scalar signal x_2 is transmitted through the public to drive the system and achieve synchronization.

$$s(t) = h(x(t)) = x_2(t) \quad (3.22)$$

In the receiver or the response system:

$$\frac{dy_1}{dt} = \sigma(s(t) - y_1) \tag{3.23}$$

$$\frac{dy_2}{dt} = q_1y_1 - q_2y_2 - q_3y_1y_3 \tag{3.24}$$

$$\frac{dy_3}{dt} = y_1s(t) - by_3 \tag{3.25}$$

where y_1, y_2 and y_3 are the state variables with the initial condition of [0.12; 0.17; 0.48] and q_1, q_2 and q_3 are the parameters of the receiver side. As the response system evolves and synchronizes with the drive system, the parameters q_1, q_2 and q_3 will follow the original parameter p_1, p_2 and p_3 of the drive system [Crispin 2002].

According to adaptive process equation 3.19 the three dimensional Lorenz system is:

$$\sum_{i=1}^{3} e_i f(y,q) = (y_1 - x_1)[\sigma\, (s(t)-y_1)] + (y_2 \quad x_2)\, [q_1(t)\, y_1 - q_2(t)\, y_2 - q_3(t)\, y_1y_3] + (y_3 - x_3)\, (y_1\, s(t) - b\, y_3) \tag{3.26}$$

The gradient with respect to the parameters q is given by:

$$\Delta q[\sum_{i=1}^{3} e_i f_i(y,q)] = [(y_2 - x_2)\, y_1),\ - (y_2 - x_2)\, y_2,\ - (y_2 - x_2)\, y_1y_3)]^t \tag{3.27}$$

The differential equation governing the controlled parameter is

$$\frac{dq_1}{dt} = - G_{11}\, (y_2-s(t))\, y_1 + G_{12}\, (y_2-s(t))\, y_2 + G_{13}\, (y_2-s(t))\, y_1y_3$$

$$\frac{dq_2}{dt} = - G_{21}\, (y_2-s(t))\, y_1 + G_{22}\, (y_2-s(t))\, y_2 + G_{23}\, (y_2-s(t))\, y_1y_3 \tag{3.28}$$

$$\frac{dq_3}{dt} = - G_{31}\, (y_2-s(t))\, y_1 + G_{32}\, (y_2-s(t))\, y_2 + G_{33}\, (y_2-s(t))\, y_1y_3$$

where G_{11}, G_{12}…G_{33} are the gain of the system. Considering the case where only one of the parameter p_1 is to be identified and the other two parameters p_2 and p_3 are some constant and known. Thus the equations 3.27 & 3.29 can be reduced to [Crispin, 2002]: where q_2 and q_3 are some constants.

$$\Delta q[\sum_{i=1}^{3} e_i f_i(y, q)] = [(y2 - x2)\, y1, 0, 0]^t \tag{3.29}$$

$$\frac{dq_1}{dt} = -G_{11}(y_2 - s(t))\, y_1 + G_{12}(y_2 - s(t))\, y_2 + G_{13}(y_2 - s(t))\, y_1 y_3$$

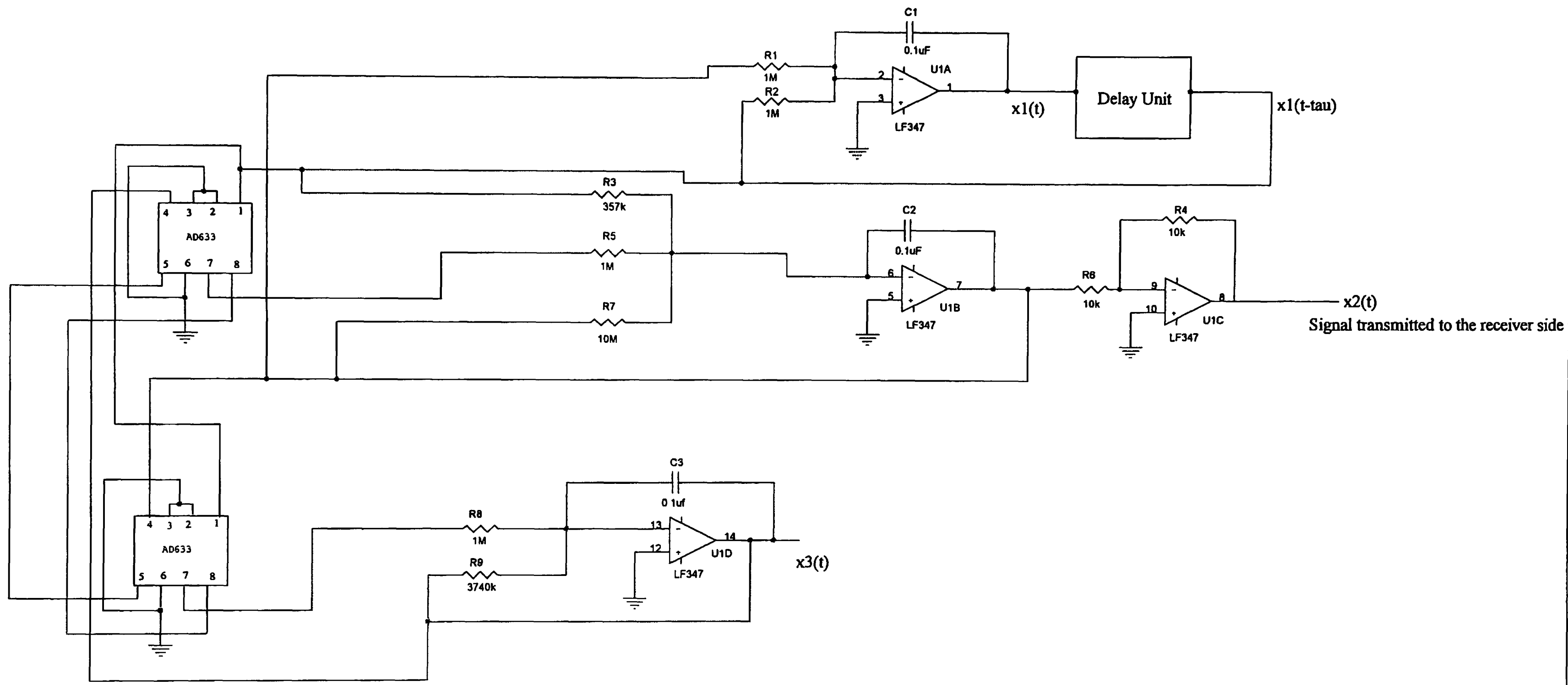
$$\frac{dq_2}{dt} = 0; \frac{dq_3}{dt} = 0; \tag{3.30}$$

$\frac{dq_1}{dt} = -G_{11}(y_2 - s(t))y_1$ or $= -G_{11}(y_2 - x_2)y_1$ where a scalar signal s(t) is a function of the state x(t) and $\frac{dq_2}{dt} = 0$ and $\frac{dq_3}{dt} = 0$

3.4 Experimental Setup.

In this section the circuit implementation of the chaotic Lorenz system has been described. The behavior of the numerical simulation matches that of the results from the analog circuit. The dynamics of the receiver side Lorenz circuit is exactly the same as transmitter circuit in order to synchronize the two systems. Information to be transmitted is added to the time varying parameters at the transmitter side. The basic addition at the receiver side is the adaptive process to recover the actual information using the parameter identification process.

For the implementation of the Lorenz equations in an analog electronic circuit was done with op-amps (LF347BN) using it as the summing amplifier, inverter and an integrator and analog multipliers (AD633) for the nonlinear function of xy and xz. Only standard commercial available electronic components have been used and the observed signal is quiet robust. The tolerances of the components were at the maximum of 2 % on the op-amp, 0.2% on the multiplier and 1% on the values of the resistance. Each circuit, the driver and the driven system, was constructed independently, and then tested for proper operation. To be certain that both circuits were identical, the same types of electronic components were used. Once both circuits were completed, they were coupled together and all the connections made and the waveforms were observed by Tektronix TDS310 oscilloscope. Typical data were sampled at a rate of 100 scans/second.



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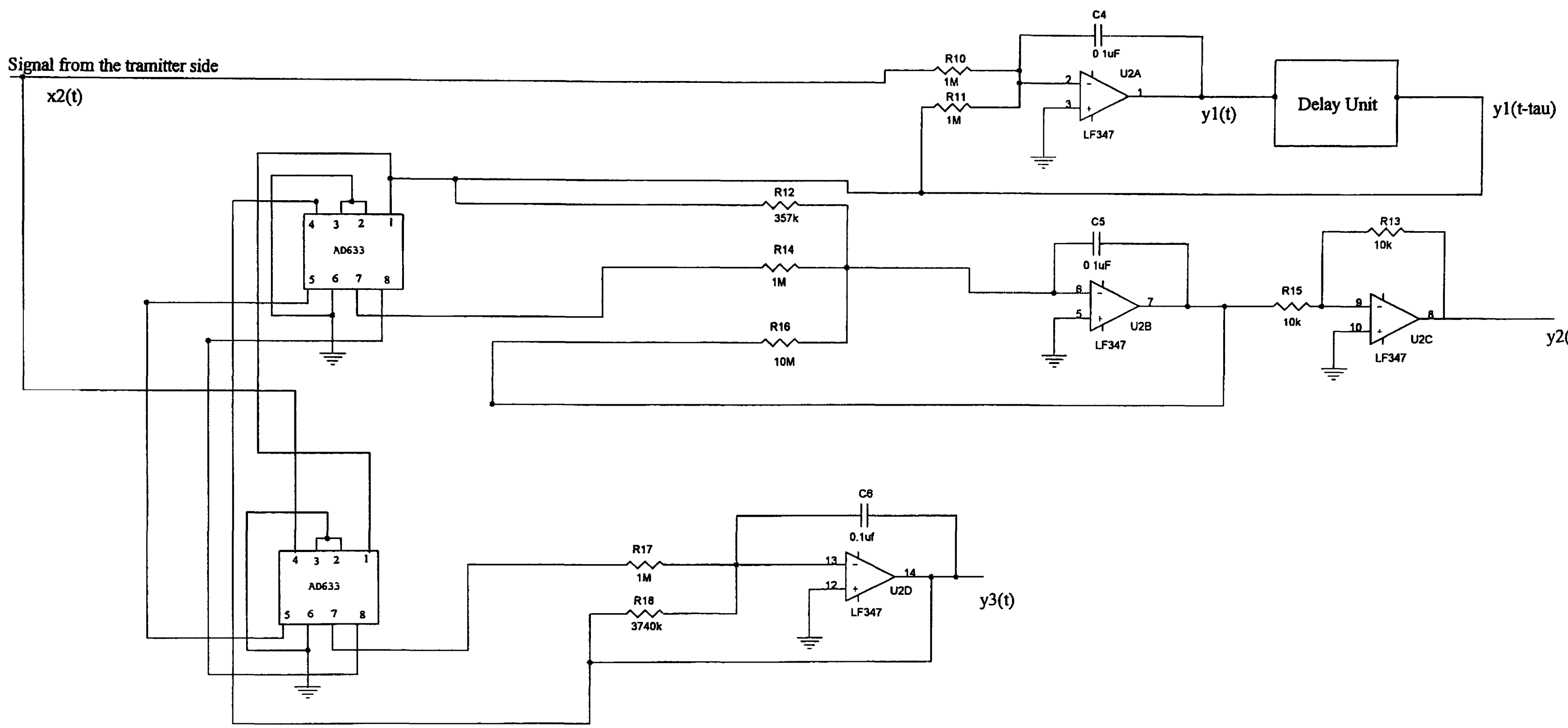

Title: Lorenz Receiver with delay

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Date: Thursday, May 27, 2004 | Sheet 2 of 2

3.4.1 Circuit Setup.

For the 1st Lorenz system on the transmitter side:

$$\frac{dx_1}{dt} = \sigma(x_2 - x_1) \text{ where sigma =10. The output from the integrator}$$

$$\frac{dq_1}{dt} = -\frac{1}{C_4}\left(x_2\frac{R_{16}}{R_{15}} + y_2\left(1 + \frac{R_{16}}{R_{15}}\right)\left(\frac{R_{21}}{R_{12}+R_{21}}\right)\right) = \sigma(x2 - x1) \tag{3.31}$$

where $C_1 = 0.1\mu F$, $R_5 = R_6$

For the Lorenz 2nd equation

$$\frac{dx_2}{dt} = p_1x_1 - p_2x_2 - p_3x_1x_3 \text{ where } p_2 = p_3 = 1 \text{ and } p_1 = r_o + \text{Information} \tag{3.32}$$

$$Vout_1 = -\left(\frac{I(t)}{R_4} + \frac{10V}{R_3}\right)R_{14} = -(r_o + I(t)) \tag{3.33}$$

Inverting the $Vout_1$ signal with the inverter

$$Vout_2 = -(r_o + I(t))\left(-\frac{R_2}{R_1}\right) = (r_o + I(t)) = p_1(t) \tag{3.34}$$

The output from the integrator:

$$\frac{dx_2}{dt} = -\frac{1}{C_2}\left(-p_1(t)x_1\left(\frac{1}{R_{17}}\right) + x_1x_3\left(\frac{1}{R_9}\right) + x_2\left(\frac{1}{R_{11}}\right)\right) \tag{3.35}$$

Substituting the values will be equal to 2nd Lorenz system

For the Lorenz 3rd equation:

$$\frac{dx_3}{dt} = x_1x_2 - bx_3 \text{ where b=8/3} \tag{3.36}$$

The summing integrator:

$$\frac{dx_3}{dt} = \frac{1}{C_3}\left(-\frac{x_1x_2}{R_{12}} + \frac{x_3}{R_{13}}\right) = \frac{dx_3}{dt} = (x_1x_2 - \frac{8}{3}x_3) \tag{3.37}$$

The values of the components are:

R_1, R2, R3, R4, R8, $R_{10} = 10k\Omega$ R_5, R_6, R9, R17, R12 =$1M\Omega$ R_7, $R_{11} = 10M\Omega$, $R_{13} =$ 3740 kΩ. The capacitors C_1, C2, C3 = $0.1\mu f$ and the multipliers AD633.

In the receiver side:

For the 1st Lorenz equation at the receiver side x_2 is the variable that transmitted information from the transmitter side.

$\frac{dy_1}{dt} = \sigma(x_2 - y_1)$ where σ=10. The output from the integrator for the first variable:

$$\frac{dy_1}{dt} = -\frac{1}{C_1}\left(-x_2\left(\frac{1}{R_5}\right) + y_1\left(\frac{1}{R_6}\right)\right) \tag{3.38}$$

For the Lorenz 2nd equation at the receiver side;

$$\frac{dy_2}{dt} = q_1 y_1 - q_2 y_2 - q_3 y_1 y_3 \text{ where } q_1, q_2 \text{ and } q_3 \text{ are constants} \tag{3.39}$$

Summing of the parameters:

$$V_{out1} = -\left(\frac{5V}{R_3} + \frac{10V}{R_4}\right) R_{21} = -(5 + r_o) = -q_1 \tag{3.40}$$

Inverting the V_{out1} signal with an inverter:

$$V_{out2} = -q_1\left(-\frac{R_1}{R_2}\right) = q_1 \tag{3.41}$$

The output of the integrator

$$\frac{dy_2}{dt} = -\frac{1}{C_2}\left(-q_1 y_1\left(\frac{1}{R_7}\right) + y_1 y_3\left(\frac{1}{R_{10}}\right) + y_2\left(\frac{1}{R_{14}}\right)\right) \tag{3.42}$$

For the Lorenz 3rd equation on the receiver side

$$\frac{dy_3}{dt} = y_1 x_2 - b y_3 \text{ where } b = 8/3 \tag{3.43}$$

$$\frac{dy_3}{dt}=\frac{1}{C_3}\left(-\frac{y_1x_2}{R_{17}}+\frac{y_3}{R_{18}}\right) \tag{3.44}$$

The Adaptation process:

$$\frac{dq_1}{dt}=-G_{11}(y_2-x_2)y_1 \tag{3.45}$$

$$\text{for } e_2=y_2-x_2 \tag{3.46}$$

The summing amplifier:

$$e_2=-\left(x_2\frac{1}{R_{28}}-y_2\frac{1}{R_{27}}\right)R_{26}=(y_2-x_2) \tag{3.47}$$

The final output

$$\frac{dq_1}{dt}=-G(y_2-x_2)y_1 \quad \text{where } G=\text{Gain}$$

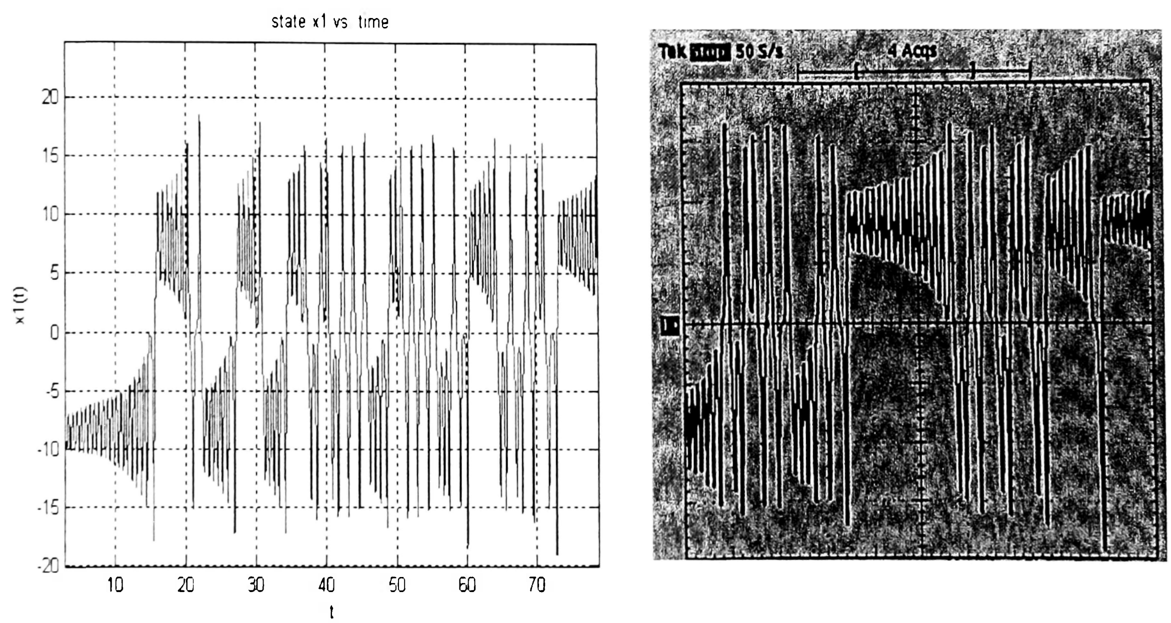
$$\frac{dq_1}{dt}=-\frac{1}{C_4}\left(-\frac{(y_2-x_2)y_1}{10R_{19}}R_{21}\right)R_{20} \tag{3.48}$$

The values of the components are:

R_1, R_2, R_8, R_9, R_{10}, R_{11}, R_{12}, R_{13}, R_{15}, R_{16}, R_{19}, R_{20}, R_{22}, R_{23}, R_{24}, R_{25}, R_{26}, R_{27} = 10kΩ, R_4= 357kΩ, R_3, R_5 , R_6, R_7, R_{17}, R_{21} = 1MΩ , = 35.7kΩ, R_{14} = 10MΩ, R_{18} = 3740 kΩ .The capacitors C_1, C_2, C_3, C_4 = 0.1uf multipliers AD633.

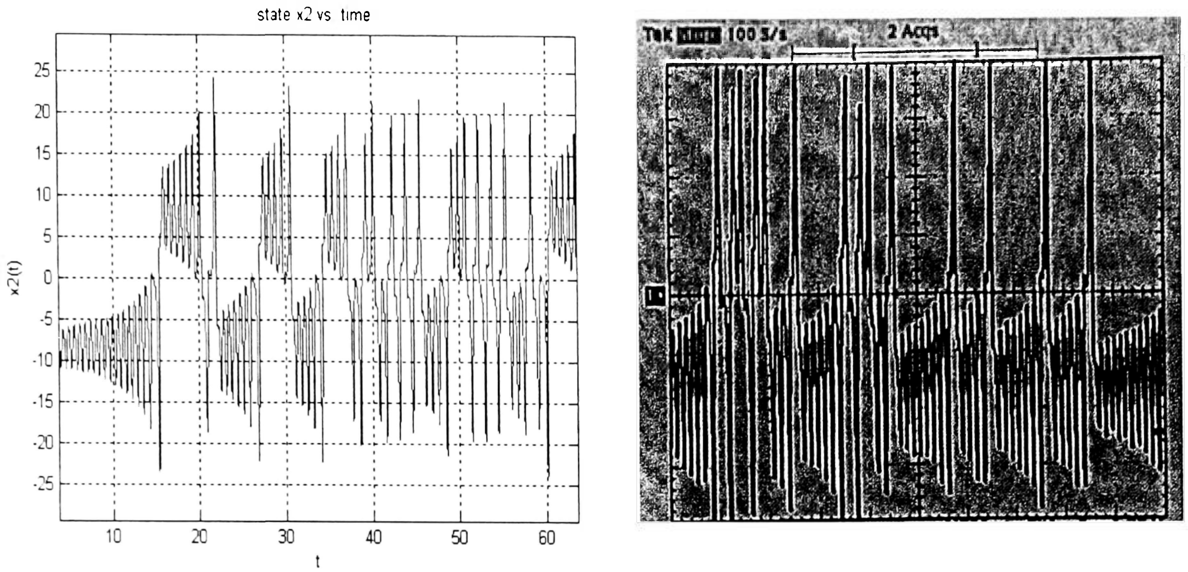
3.4 Results and Conclusion

In this chapter we have been able to characterize perfect synchronization of the Lorenz oscillators in an electronic (analog) circuit. The comparison of the results obtained from the numerical analysis and the analog circuit were very similar. Below are the list of figures between the numerical analysis and the actual analog circuit. The results obtained were comparatively satisfactory but many other factors like band limitation, noise interference of the signal on the system has not been taken into consideration.



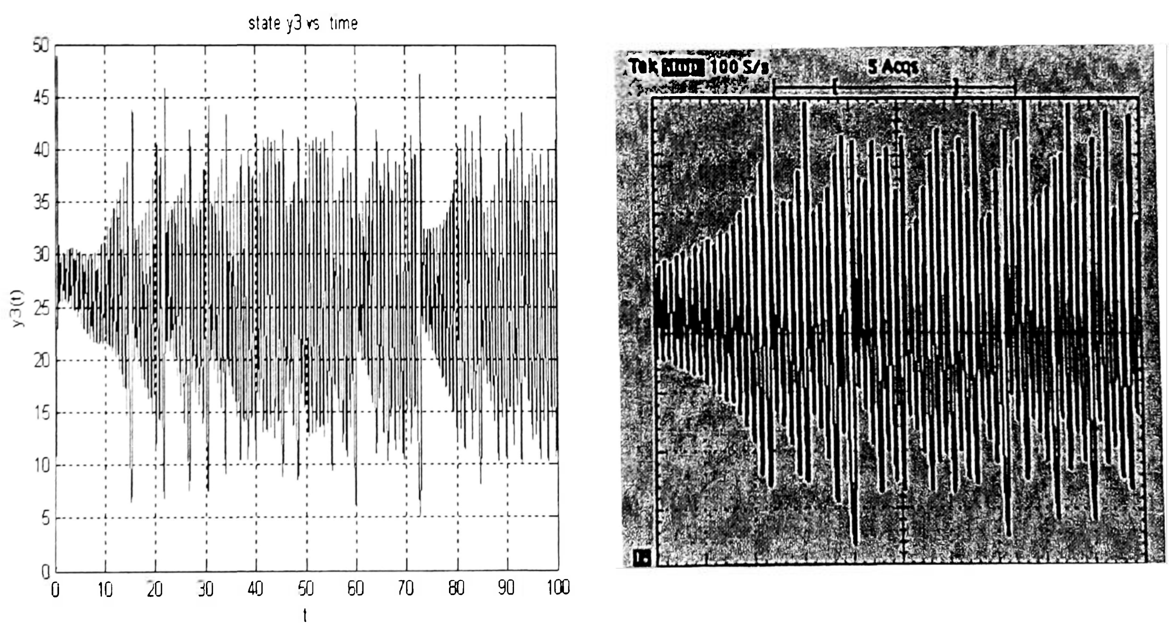
(a) (b)

Fig 3.4 The time series of the $x_{1(t)}$ vs. time of the Transmitter side: (a) computer simulation (b) analog circuit output



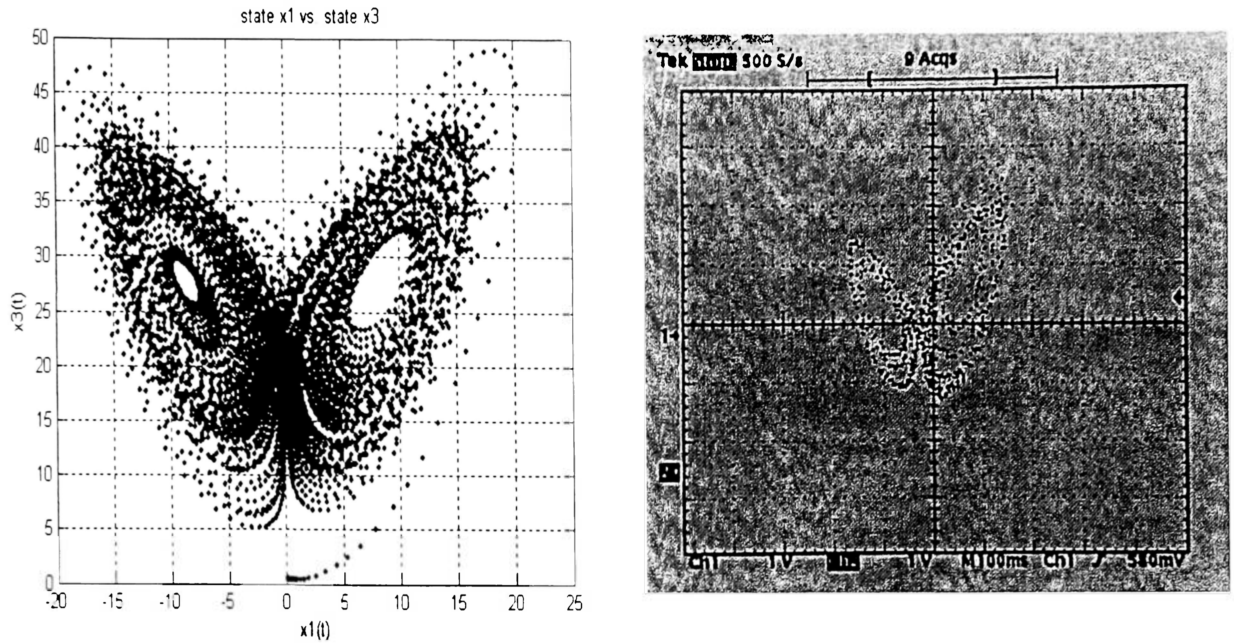
(a) (b)

Fig 3.5 The time series of the $x_{2(t)}$ vs. time of the Transmitter side (a) numerical solution (b) analog circuit output



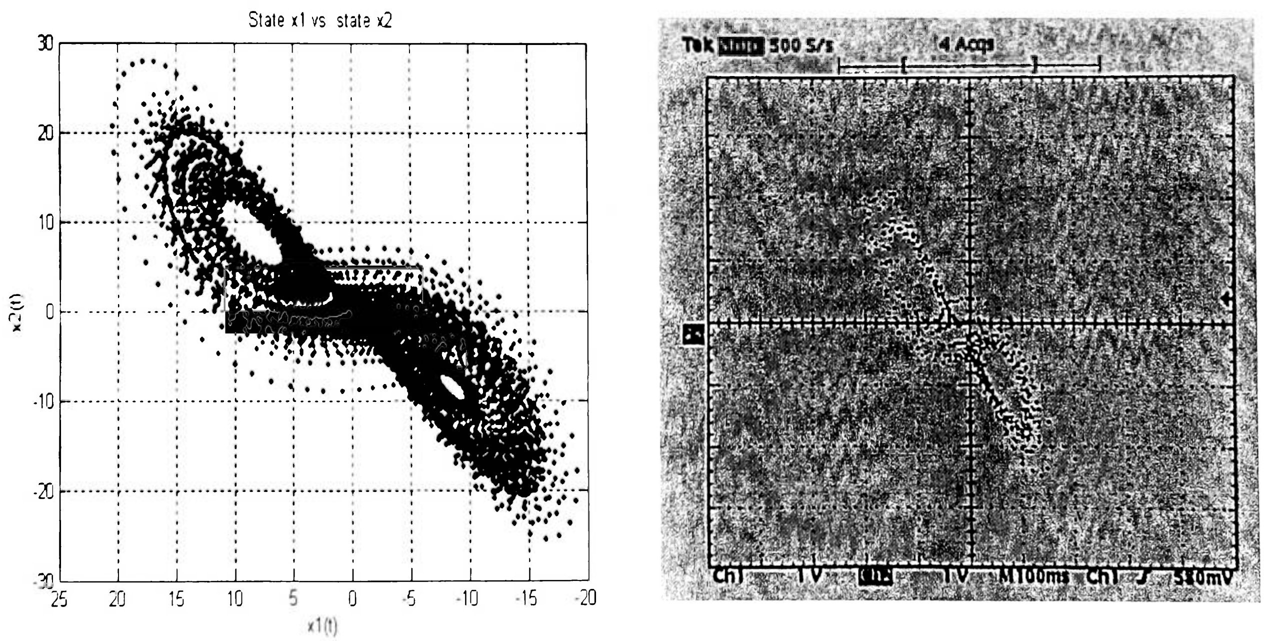
(a) (b)

Fig 3.6 The time series of the $x_{3(t)}$ vs. time of the Transmitter side: (a) numerical solution (b) analog circuit output



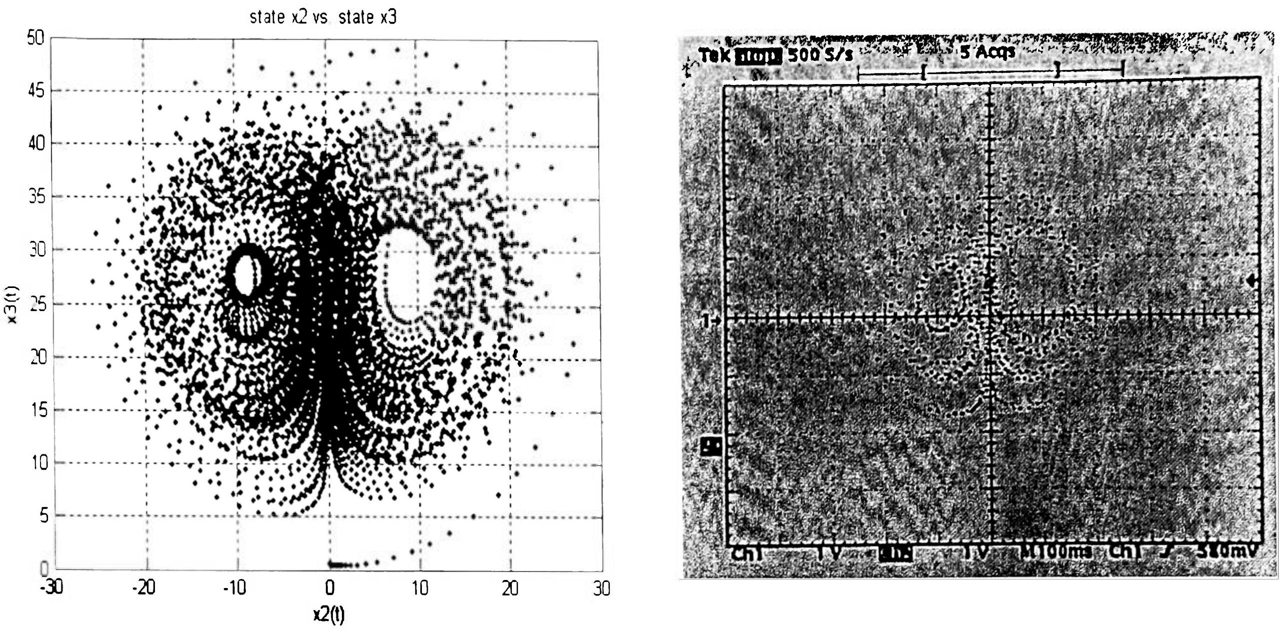
(a) (b)

Fig 3.7 Attractor on the $x_{3(t)}$ and $x_{1(t)}$ co-ordinate (2D) (a) numerical solutions (b) analog circuit output



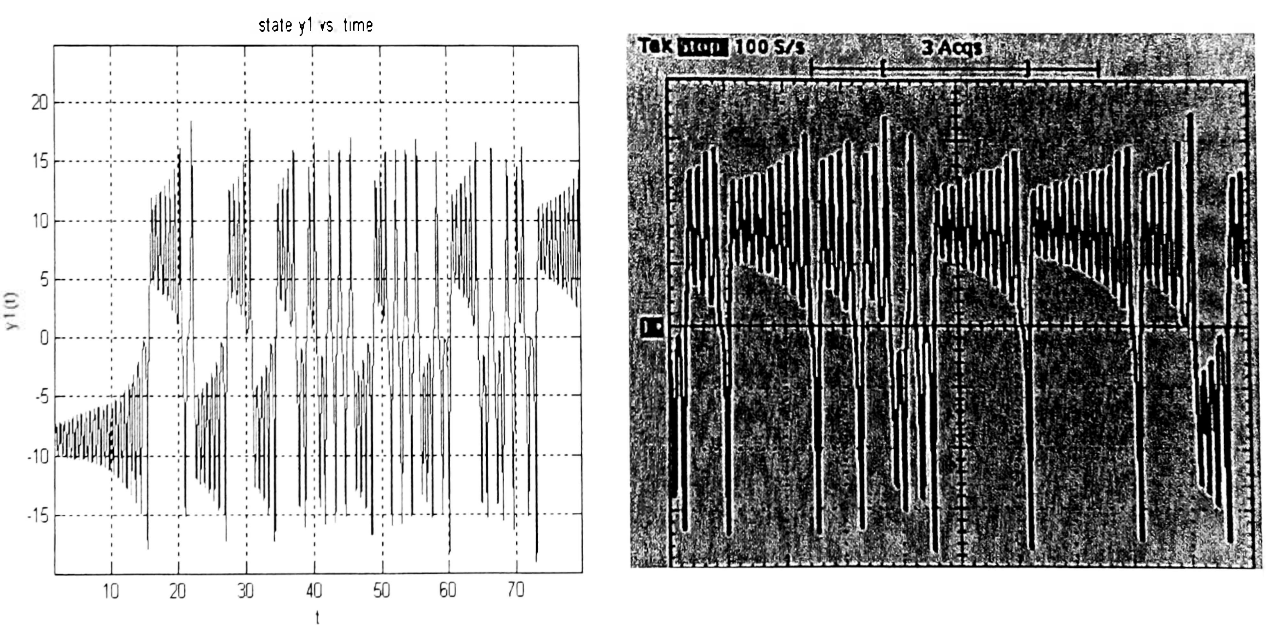
(a) (b)

Fig 3.8 Attractor on the $x_{2(t)}$ and $x_{1(t)}$ co-ordinate (2D): a) numerical solutions (b) analog circuit output



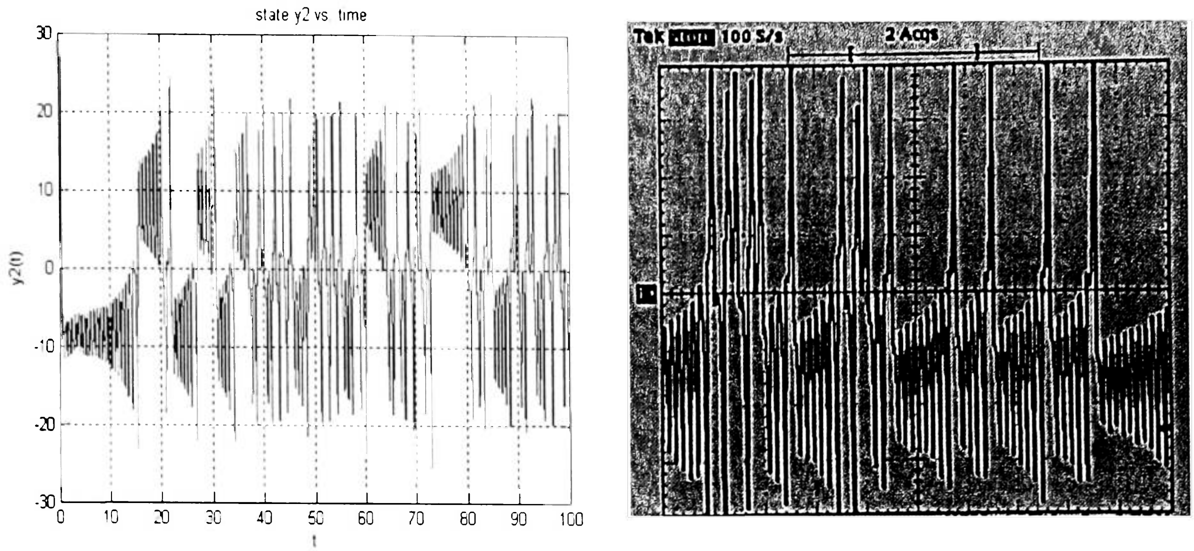
(a) (b)

Fig 3.9 Attractor on the $x_{3(t)}$ and $x_{2(t)}$ coordinates (2D): (a) numerical solutions (b) analog circuit output



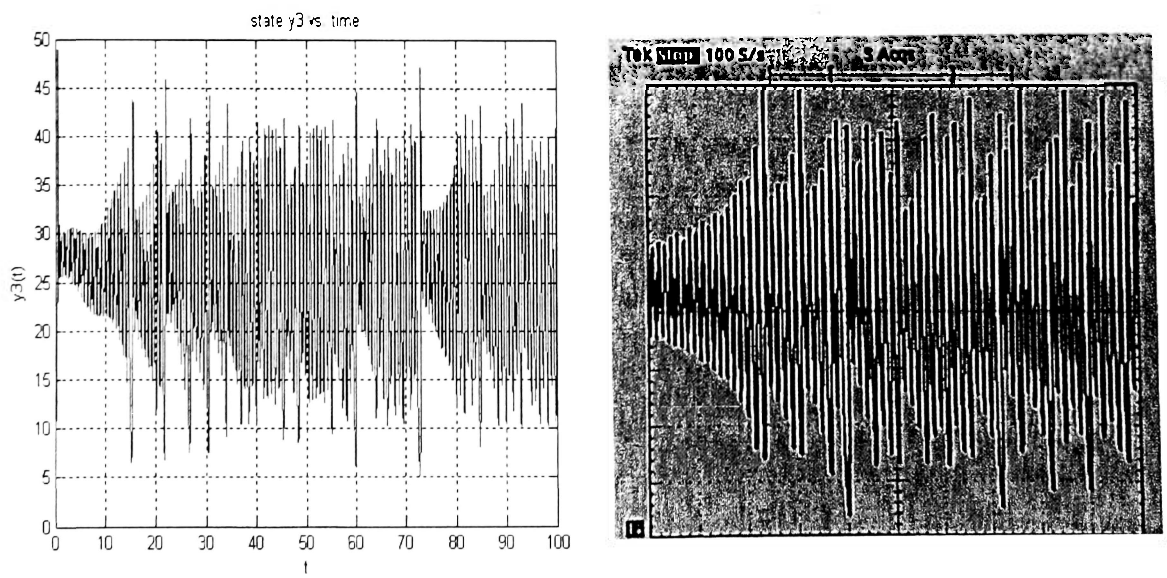
(a) (b)

Fig 3.10 The time series of the $y_{1(t)}$ vs. time of the Receiver side: a) numerical solutions (b) analog circuit output



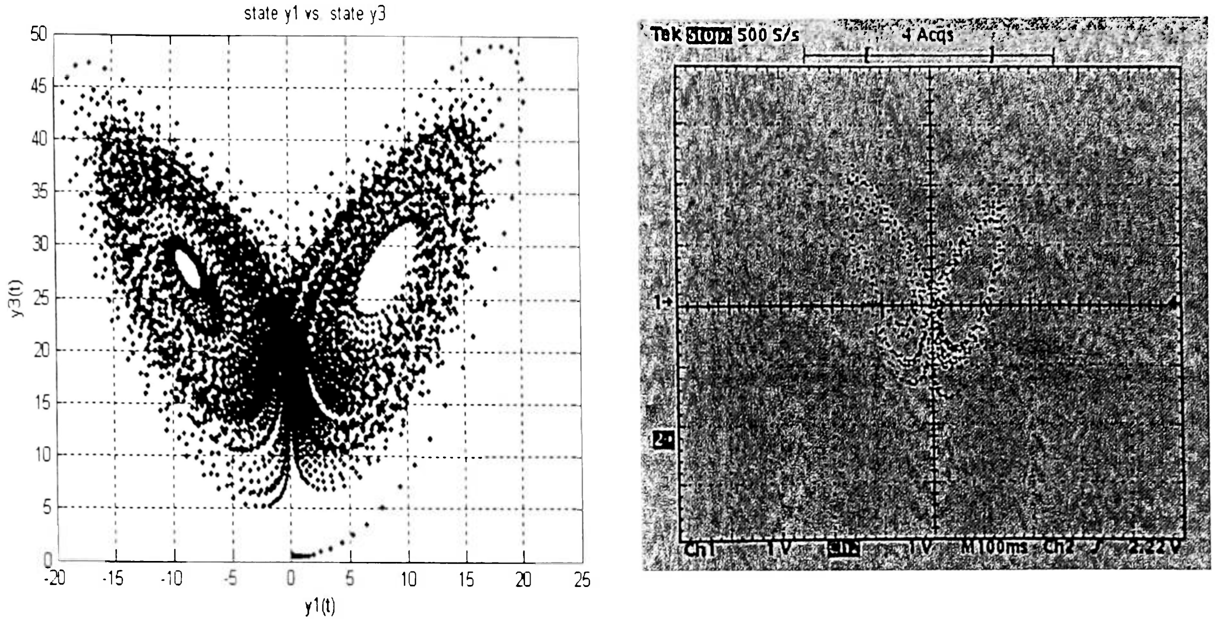
(a) (b)

Fig 3.11 The time series of the $y_{2(t)}$ vs. time of the Receiver side: a) Numnerical solutions (b) analog circuit output



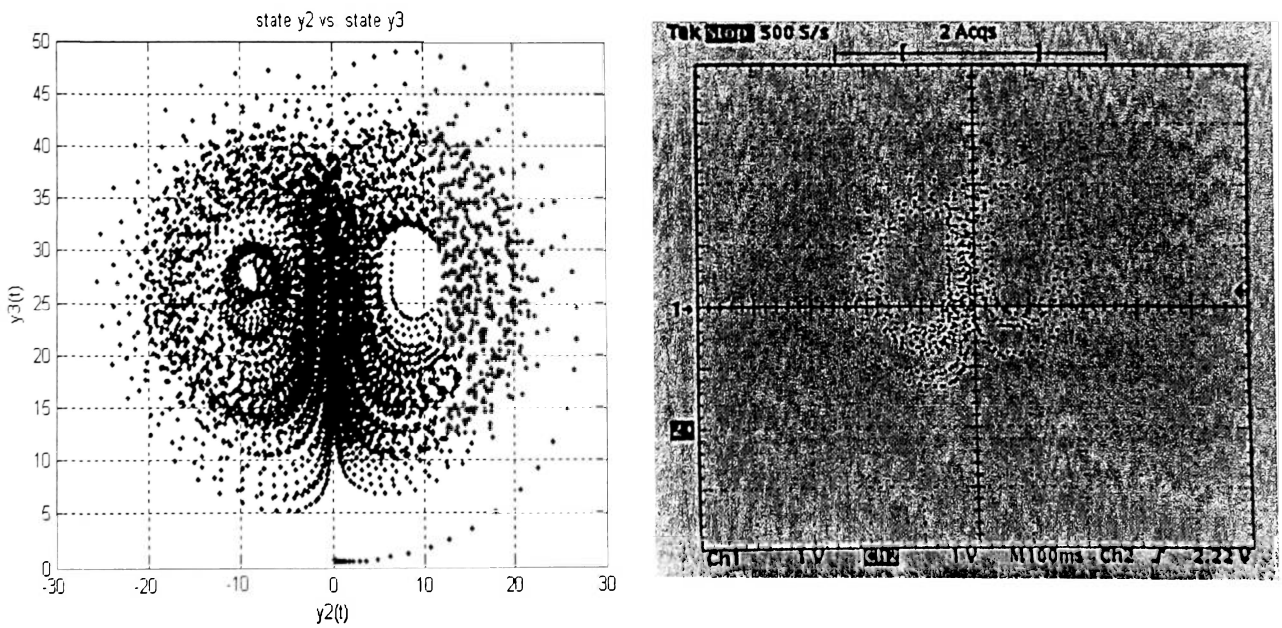
(a) (b)

Fig 3.12 The time series of the $y_{3(t)}$ vs. time of the Receiver side: a) Numerical solutions (b) analog circuit output



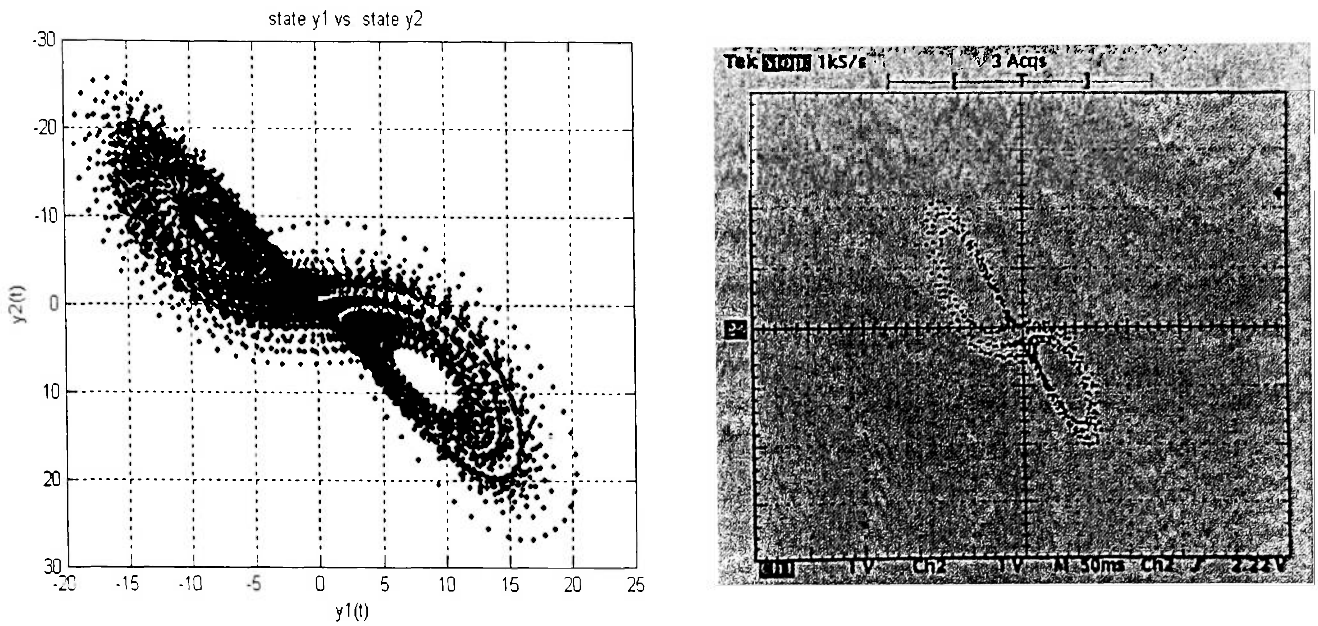
(a) (b)

Fig 3.13 Attractor on the $y_{3(t)}$ and $y_{1(t)}$ co-ordinate (2D): a) Numnerical solutions (b) analog circuit output



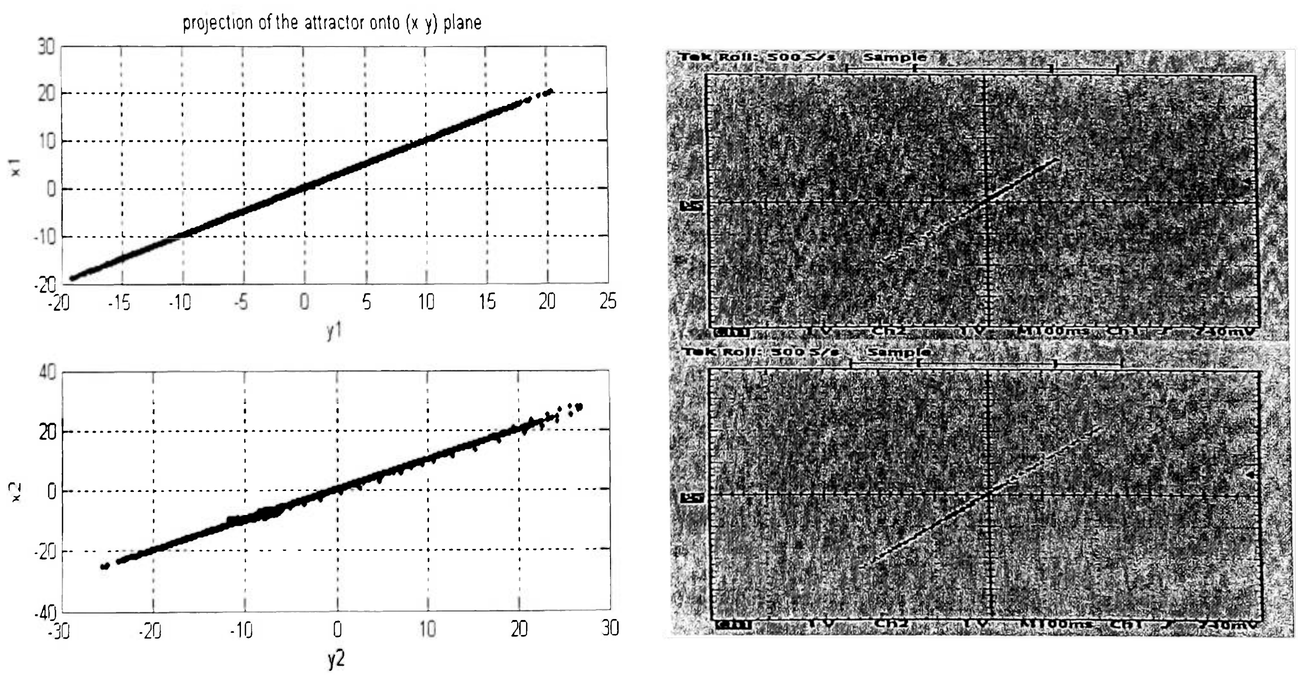
(a) (b)

Fig 3.14 Attractor on the $y_{3(t)}$ and $y_{1(t)}$ co-ordinate (2D): a) Numnerical solutions (b) analog circuit output



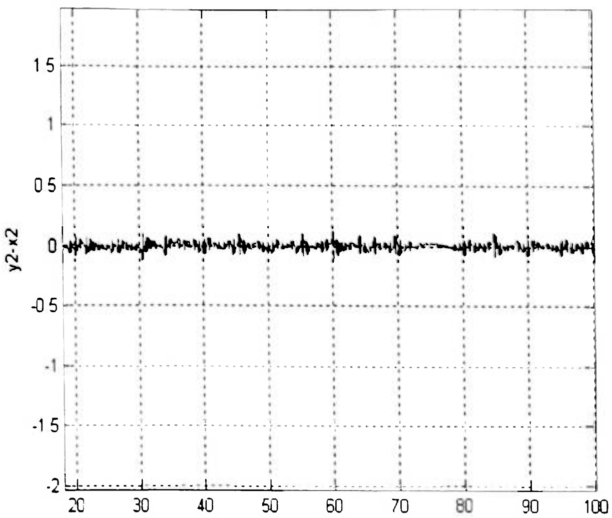
(a) (b)

Fig 3.15 Attractor on the $y_{2(t)}$ and $y_{1(t)}$ co-ordinate (2D): a) Numnerical solutions (b) analog circuit output

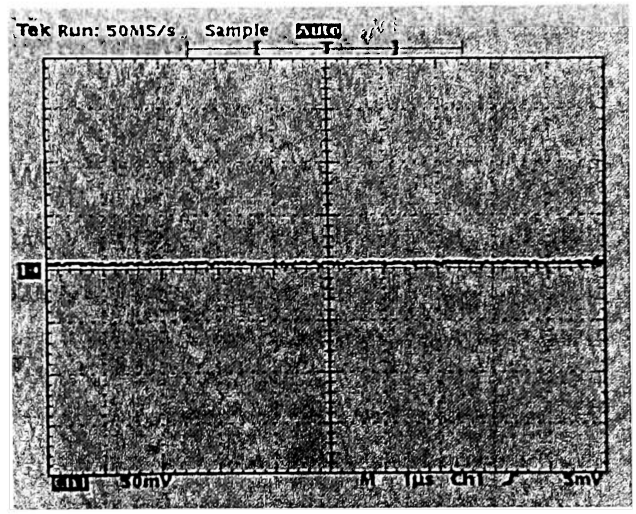


(a) (b)

Fig 3.16 Properties of the synchronization between the receiver and the transmitter state variables: (a) computer simulation (b) analog circuit output

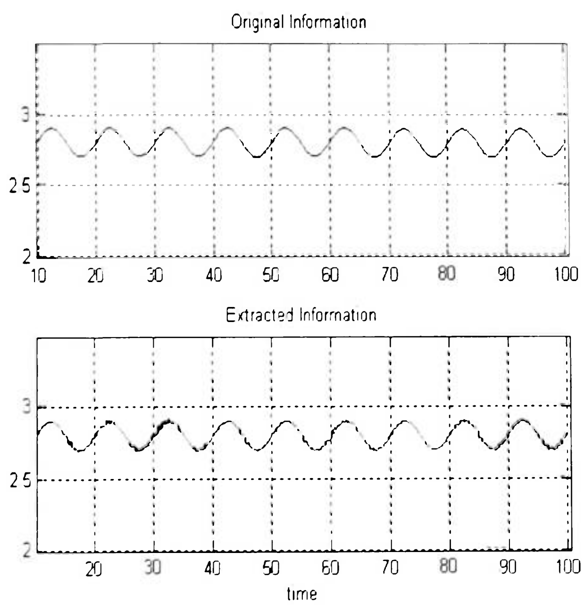


(a)

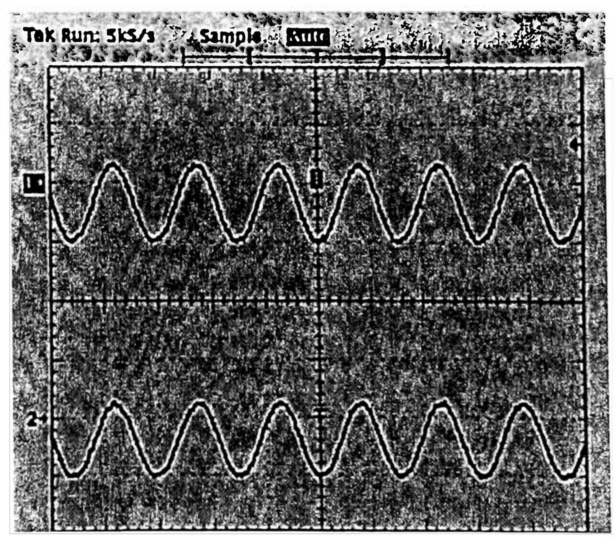


(b)

Fig 3.17 Difference between the receiver state variable (y_2) and the transmitter variable (x_2). (a) computer simulation (b) analog circuit output



(a)



(b)

Fig3.18 The original information and the extracted information: (a) computer simulation (b) analog circuit output

CHAPTER 4

DELAY DIFFERENTIAL LORENZ EQUATION:

4.1 Introduction:

Nonlinear systems with a time-delayed feedback recently have attracted much research due to the wide abundance of time delays in nature and technologies [Voss, 2001]. In this chapter, we have addressed a practical problem of synchronization of high dimensional system by introducing the time delay feedback. It has been studied that message masked with low-dimensional chaotic processes, once intercepted, can be sometimes readily extracted, so that the interest has been directed to higher dimensional chaotic system. Thus with this new approach security of a communications scheme can be enhanced by making the transmitted signal more complex.

The proposed technique, based on the nonlinear control method, has several advantages over the existing methods Gassi, Mascolo [Gassi, 1998].

- It enables synchronization to be achieved in a systematic way.
- It can be successfully applied to a wide class of hyperchaotic systems.
- It does not require the computational of any Lyapunov exponents.
- It does not require initial conditions belonging to the same basin of attraction.

As we know that synchronization of two systems occurs when the trajectories of one of the systems converges to the same values as the other and they will remain in step with each other. For the chaotic systems synchronization is performed by the linking of chaotic systems with a common signal or signals (the so-called drivers).

Our objective is to implementation the Pecora-Carroll approach of chaos synchronization in hyperchaos, which takes advantage of increased randomness and unpredictability of the Lorenz systems of equation. The schematic setup of the synchronization experiment is shown in Fig 4.1, where one of the parameter of the transmitter's chaotic output state variable is coupled to the driven system.

4.2 Hyperchaos Synchronization

Considering a simple map representation of the time delayed system Kye, Choi, Kwon, Kim, Park [Kye, 2004)]

$$\frac{dx(t)}{dt} = f(x(t), x(t-\tau)) \tag{4.1}$$

where $\tau > 0$ is the time delay, x (t) $\in$ R is the state variables. The above system is of an infinite dimension system, because the state at time t is an element in the space of the continuous function x (•): [t-τ, t] → R. [Piccardi 2001].

The delay differential equation represented by the discrete form:

$$x_{n+1} = f(x_n, x_{n-\tau}) \text{ where the function f: } R \times R \to R \tag{4.2}$$

x_n is the value of x at nth time step such that x_n = x(nΔt) and $x_{n-\tau}$ = x(nΔt − τΔt).

$$f(x_n, x_{n-\tau}) = x(n\Delta t) + F(x(n\Delta t), x(n\Delta t - \tau\Delta t)\Delta t \tag{4.3}$$

It is known that the above system is equivalent to the $(\tau + 1)$ dimensional coupled maps.

$$x_{n=1} = f(x_n, x_n^\tau) \tag{4.4}$$

where the variables $x_n^1, \ldots. x_n^\tau$ constructs the feedback loop. The above representation plays an important role in analyzing the time-delayed system with fixed delay time.

The Lorenz system was chosen to implement the above described scheme of time delay. Consider the 3D delay-differential equations for the Lorenz system, the drive system is represented by:

$$\frac{dx_1}{dt} = \sigma(x_2(t) - x_1(t-\tau)) \tag{4.5}$$

$$\frac{dx_2}{dt} = rx_1(t-\tau) - x_2(t) - x_1(t-\tau)x_3(t) \tag{4.6}$$

$$\frac{dx_3}{dt} = x_1(t-\tau)x_2(t) - bx_3(t) \tag{4.7}$$

where σ, r ,b > 0 and the time delay τ >0. The transmitted signal for synchronization is chose as the single variable.

$$s(t) = h(x(t)) = x_2(t) \tag{4.8}$$

The response system is given by

$$\frac{dy_1}{dt} = \sigma(s(t) - y_1(t-\tau)) \tag{4.9}$$

$$\frac{dy_2}{dt} = ry_1(t-\tau) - y_2(t) - y_1(t-\tau)y_3(t) \tag{4.10}$$

$$\frac{dy_3}{dt} = y_1(t-\tau)s(t) - by_3(t) \tag{4.11}$$

where σ, r ,b > 0 and the time delay τ >0 and state variable $x_2(t)$ from the drive system is injected to the driven system for both the system to synchronize. In the driven system the forcing delayed function is the same as the driver system in order the both the system to achieve synchronization. To achieve synchronization, the parameters of the two systems need to match. Two dynamical systems are termed synchronized if the difference between their states converges to zero for $t \rightarrow \infty$.

$$\dot{e} = \frac{dy_2}{dt} - \frac{dx_2}{dt} = 0 \tag{4.12}$$

4.3 Experimental Setup

In order to understand the result of the experiment, first coupled nonlinear time-delayed system is considered. Here, only unidirectional coupling between a 3D drive and the driven system with state $x_2(t)$ is taken into account. The signal from the drive system $x_2(t)$ is injected into the response system. The time delay between the drive and driven system is varied between 10 ms to 100 ms range. In this particular test the time delay of 85ms was used with the state variable $x_1(t)$. The two systems are identical but differ in their structure and tolerance. The difference between the parameters deviation of the Lorenz drive and the driven system are kept to a very small value as possible.

The drive and the driven system consist of nonlinear Lorenz system, built purely in analog domain from commercially available operational amplifiers and multipliers. A commercially available bucket bridge delay line is used in both the system. As anticipated the nonlinearities of the two oscillators are identical and since the bucket delay line induced some distortion of the signal, a long term run reveals that the synchronizing was achieved between the two systems. The time varying information signal is added one of the parameter.

4.3.1 Circuit Setup.

To model the equations 4.4 - 4.10 consisting of the drive and the driven system are as follows. The drive system is given by:

The Lorenz first delay differential equation:

$$\frac{dx_1}{dt} = \sigma(x_2(t) - x_1(t-\tau)) \text{ where } \sigma = 10 \text{ and } \tau > 0 \tag{4.13}$$

$$-C_1\frac{dx_1}{dt} = \left(-\frac{x_2(t)}{R_1} + \frac{x_1(t-\tau)}{R_2}\right) \tag{4.14}$$

$$\frac{dx_1}{dt} = 10\left(x_2(t) - x_1(t-\tau)\right) \tag{4.15}$$

where $R_1 = R_2$ and $C_1 = 10\ \mu f$

The Lorenz 2nd delay differential equation:

$$\frac{dx_2}{dt} = rx_1(t-\tau) - x_2(t) - x_1(t-\tau)x_3(t) \text{ were } r = 28 \tag{4.16}$$

$$-C_2\frac{dx_2}{dt} = \left(\frac{x_1(t-\tau)}{R_3} - \frac{x_1(t-\tau)x_3(t)}{R_5} - \frac{x_2(t)}{R_7}\right) \tag{4.17}$$

$$\frac{dx_2}{dt} = -\left(rx_1(t-\tau) - x_1(t-\tau)x_3(t) - x_3(t)\right) \tag{4.18}$$

Inverting the signal in Eqs 4.18 by an inverter

$$\frac{dx_2}{dt} = -\left(rx_1(t-\tau) - x_1(t-\tau)x_3(t) - x_3(t)\right)\left(-\frac{R_4}{R_6}\right) = \left(rx_1(t-\tau) - x_1(t-\tau)x_3(t) - x_3(t)\right)$$

3rd Lorenz delay differential equation:

$$\frac{dx_3}{dt} = x_1(t-\tau)x_2(t) - bx_3(t) \text{ where } b = 8/3 \tag{4.19}$$

$$-C_3\frac{dx_3}{dt} = -\frac{x_1(t-\tau)x_2(t)}{R_8} + \frac{x_3}{R_9} \tag{4.20}$$

$$\frac{dx_3}{dt} = x_1(t-\tau)x_2(t) - bx_3(t) \tag{4.21}$$

The components values are given as:

R_1, R_2, R_5, R_8 = 1MΩ, R_3 = 357kΩ, R_4, R_6 = 10kΩ , R_7 = 10MΩ, R_9 = 3740KΩ

C_1, C_2, C_3 = 0.1uf and the time delay unit.

For the response system

The 1st Lorenz delay differential equation:

$$\frac{dy_1}{dt} = \sigma(x(t) - y_1(t\text{-}\tau)) \text{ where } \sigma = 10 \text{ and } \tau > 0 \tag{4.22}$$

$$-C_4\frac{dy_1}{dt} = \left(-\frac{x_2(t)}{R_{10}} + \frac{y_1(t-\tau)}{R_{11}}\right) \tag{4.23}$$

$$\frac{dy_1}{dt} = 10\left(x_2(t) - y_1(t-\tau)\right) \tag{4.24}$$

The Lorenz 2nd delay differential equation:

$$\frac{dy_2}{dt} = ry_1(t\text{-}\tau) - y_2(t) - y_1(t\text{-}\tau)y_3(t) \text{ where } r = 28 \tag{4.25}$$

$$-C_5\frac{dy_2}{dt} = \left(\frac{y_1(t-\tau)}{R_3} - \frac{y_1(t-\tau)y_3(t)}{R_5} - \frac{y_2(t)}{R_7}\right) \tag{4.26}$$

$$\frac{dy_2}{dt} = -\left(ry_1(t-\tau) - y_1(t-\tau)y_3(t) - y_3(t)\right) \tag{4.27}$$

Inverting the signal

$$\frac{dy_2}{dt} = -\left(ry_1(t-\tau) - y_1(t-\tau)y_3(t) - y_3(t)\right)\left(-\frac{R_{13}}{R_{15}}\right) = \left(ry_1(t-\tau) - y_1(t-\tau)y_3(t) - y_3(t)\right)$$

3^{rd} Lorenz delay differential equation:

$$\frac{dy_3}{dt} = y_1(t-\tau)x(t) - by_3(t) \text{ where } b = 8/3 \quad (4.28)$$

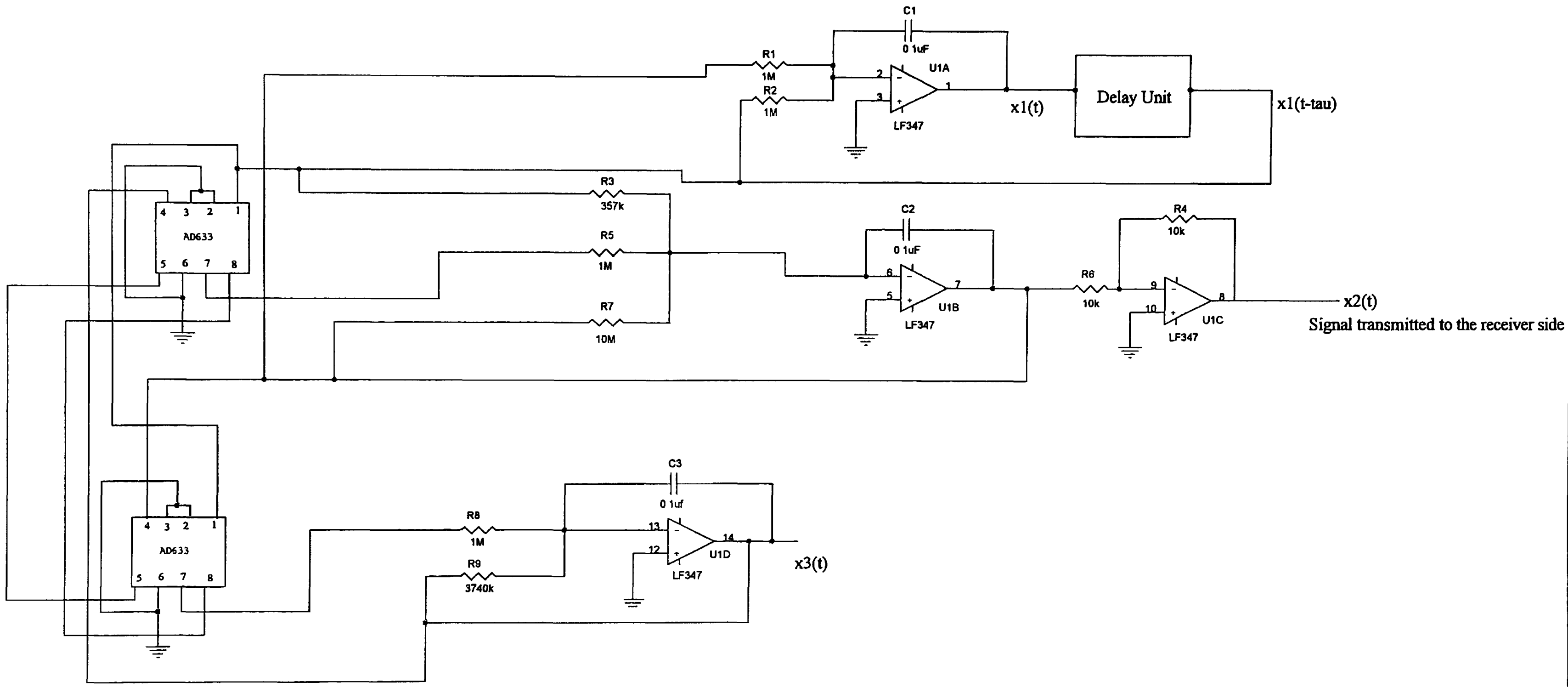
$$-C_6\frac{dy_3}{dt} = -\frac{y_1(t-\tau)x_2(t)}{R_{17}} + \frac{x_3(t)}{R_{18}} \quad (4.29)$$

$$\frac{dy_3}{dt} = y_1(t-\tau)x_2(t) - by_3(t) \quad (4.30)$$

The components values are given as:

R_{10}, R_{11}, R_{14}, R_{17} = 1MΩ, R_{12} = 357kΩ, R_{15}, R_{13} = 10kΩ, R_{16} = 10MΩ, R_{18} = 3740KΩ

C_4, C_5, C_6 = 0.1uf and the time delay unit.



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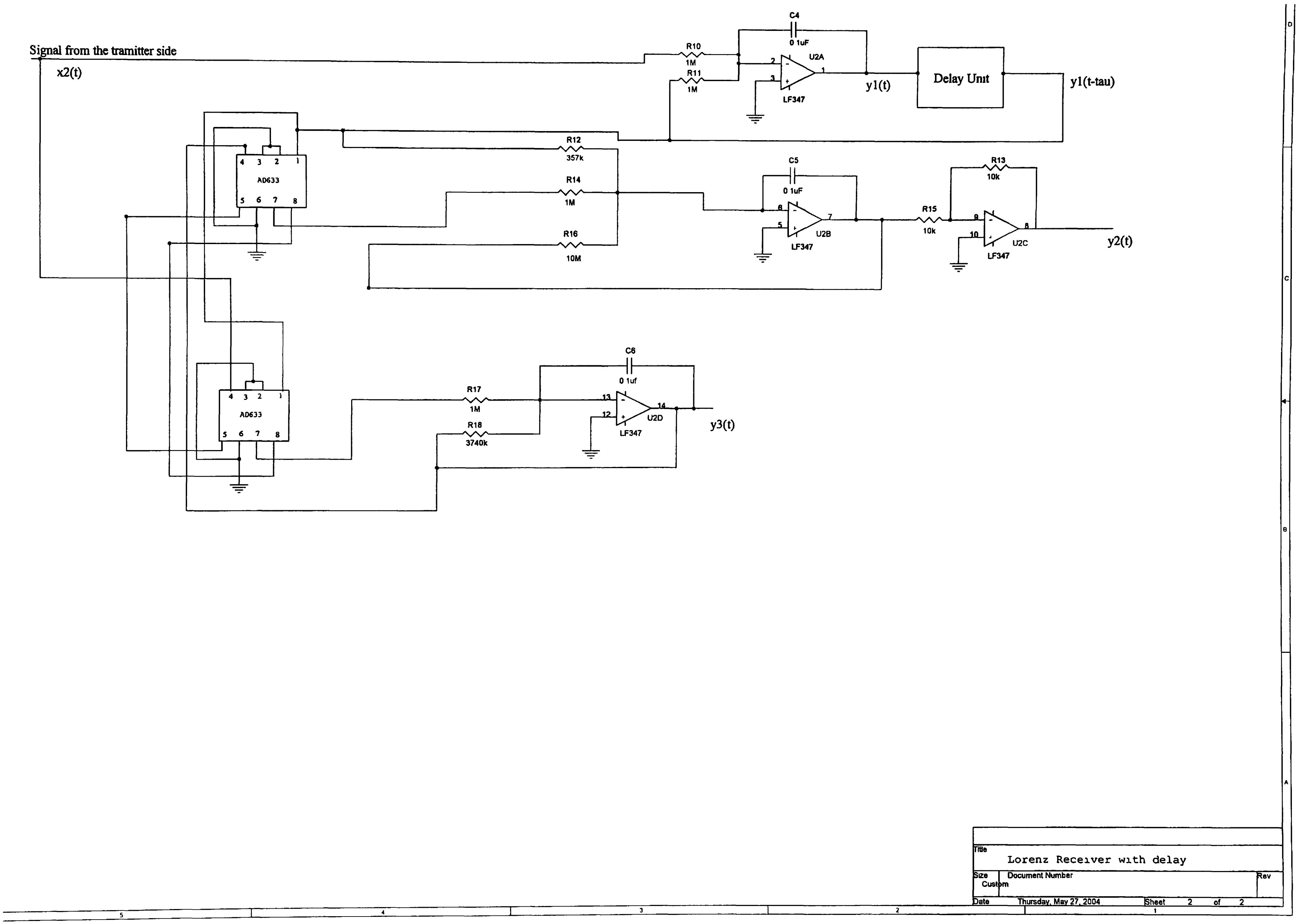
Date	Thursday, May 27, 2004	Sheet 1 of 2



Title	Lorenz Receiver with delay		
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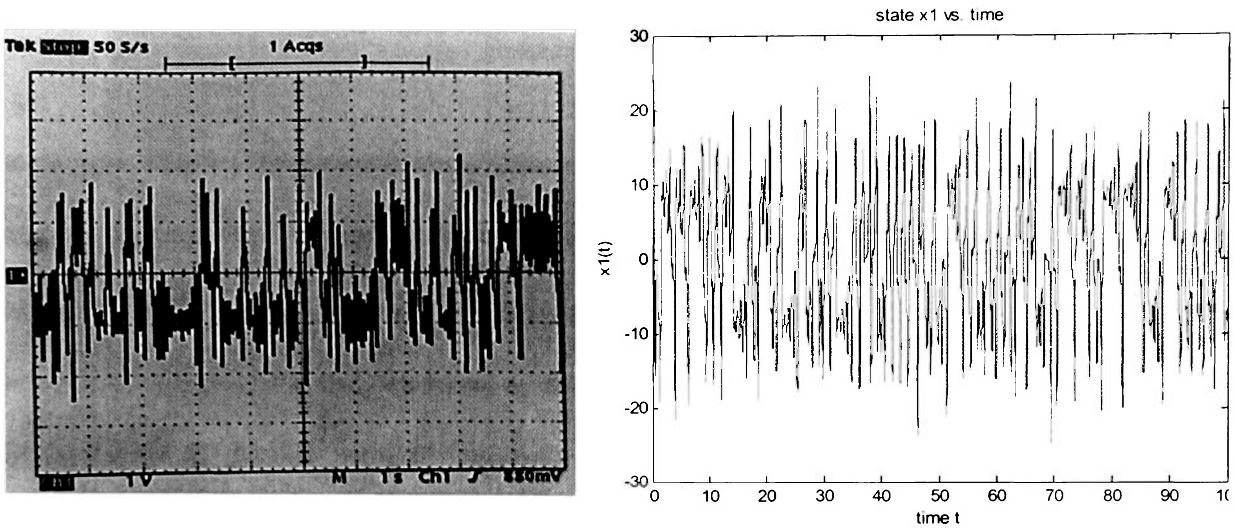
Date	Thursday, May 27, 2004	Sheet 2	of 2

4.4 Results.

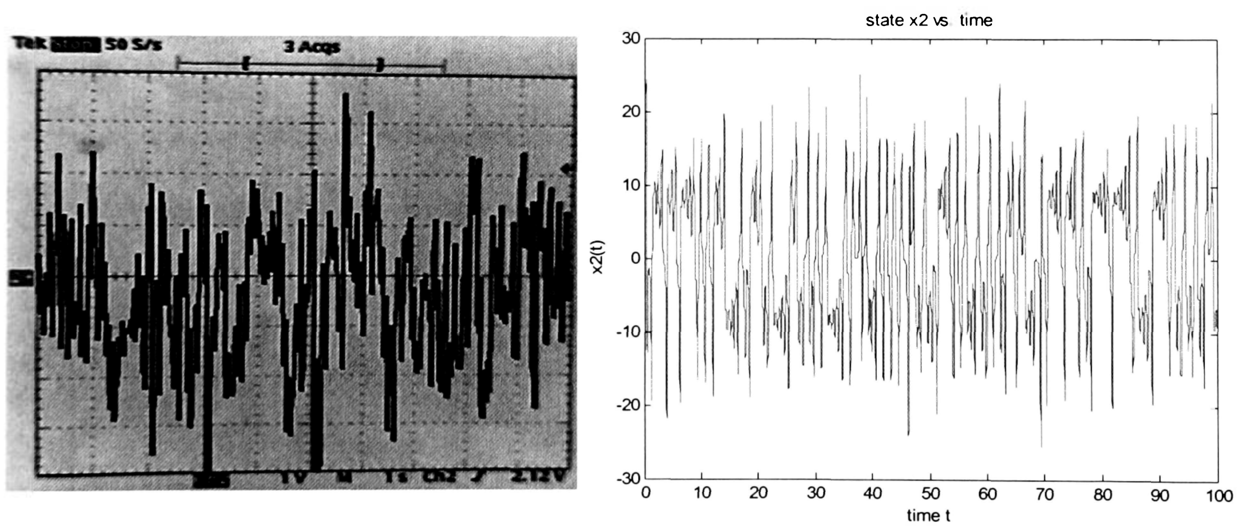
We were able to show the synchronization of the Lorenz delay differential system in an electronic (analog) circuit. The comparison of the results obtained from the numerical analysis and the analog circuit were similar. List of figures between the numerical analysis and the actual analog circuit are shown below. The results obtained were very different from the MATLAB simulation. The main reasons could be the mismatch of the time delay unit on the analog circuit, high of the tolerance level of the components. The tolerance has been kept at low as ±5% in this experiment.

For future work the tolerance level of ±1% should be used because the chaotic system is very sensitive to the initial condition and external influence could affect the whole dynamics of the output. The time delay unit used in this case was not a precise, using the digital time delay unit could improve the output, and this could also accurately match the delay time on both of the units placed on the receiver and the transmitter side.



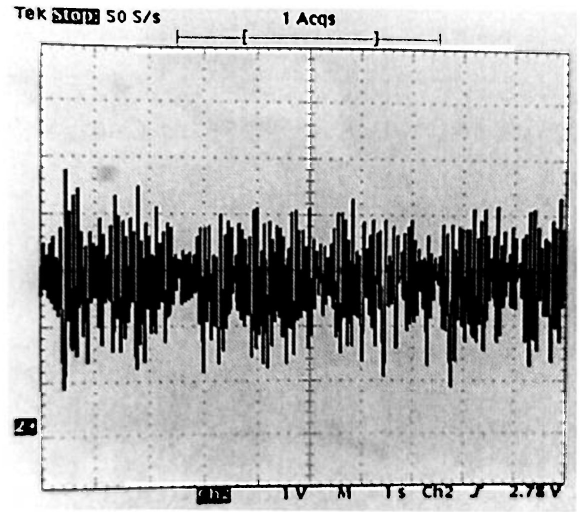
(a) (b)

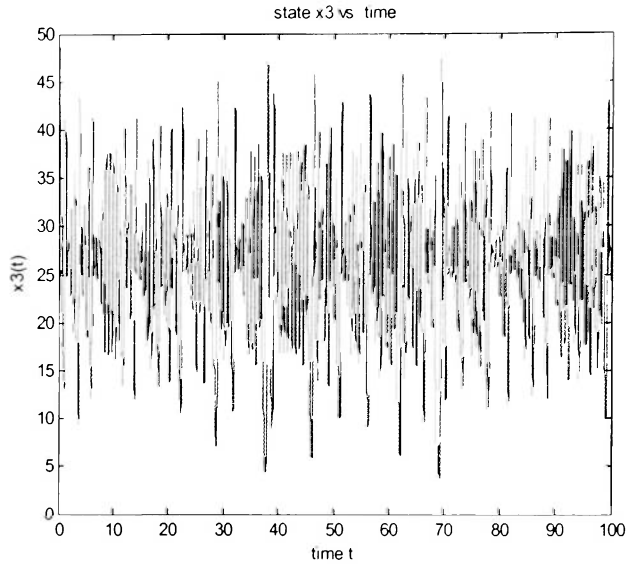
Fig 4.3 The time series of the x1(t) vs. time of the Transmitter side with time delay (a) analog circuit output (b) computer simulation



(a) (b)

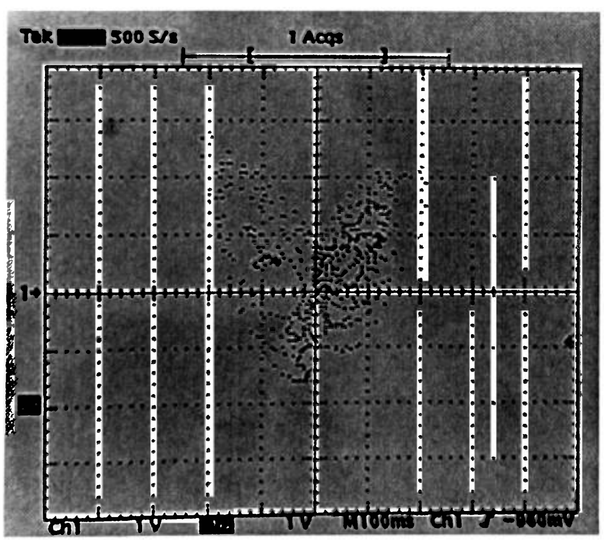
Fig 4.4 The time series of the x2(t) vs. time of the Transmitter side with time delay (a) analog circuit output (b) computer simulation

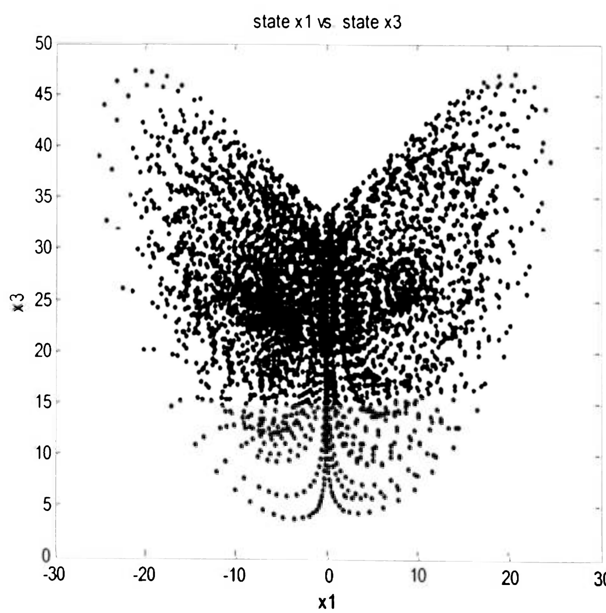




(a) (b)

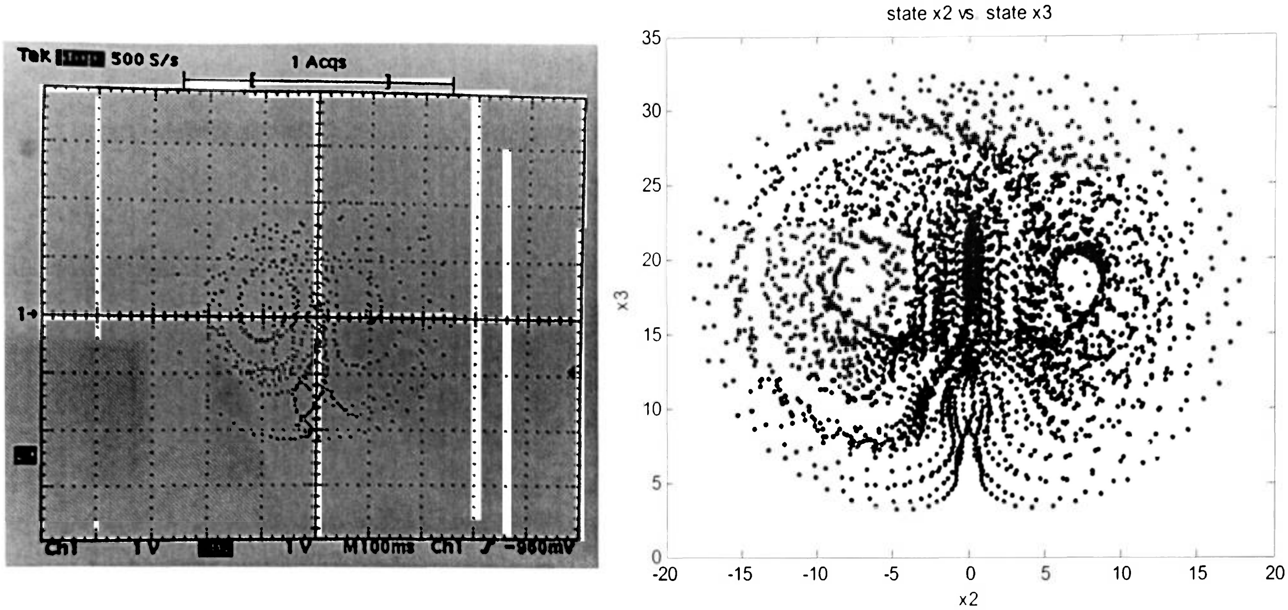
Fig 4.5 The time series of the x3(t) vs. time of the Transmitter side with time delay (a) analog circuit output (b) computer simulation





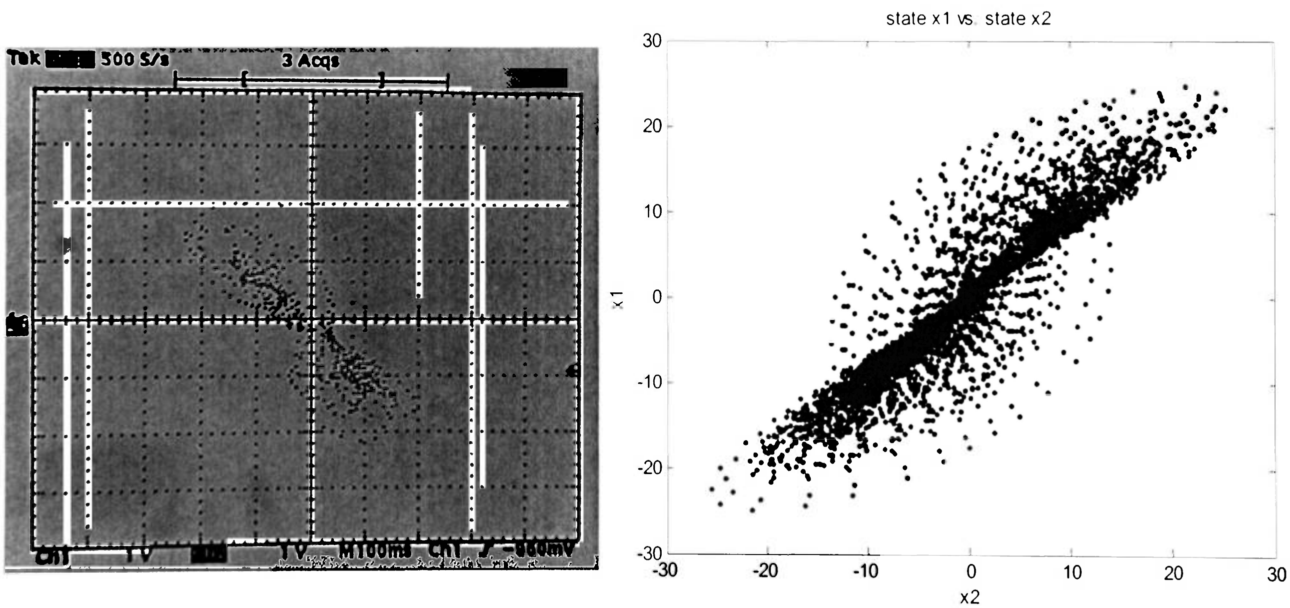
(a) (b)

Fig 4.6 Attractor on the x3(t) and x1(t) co-ordinate (2D) with time delay (a) analog circuit output (b) computer simulation



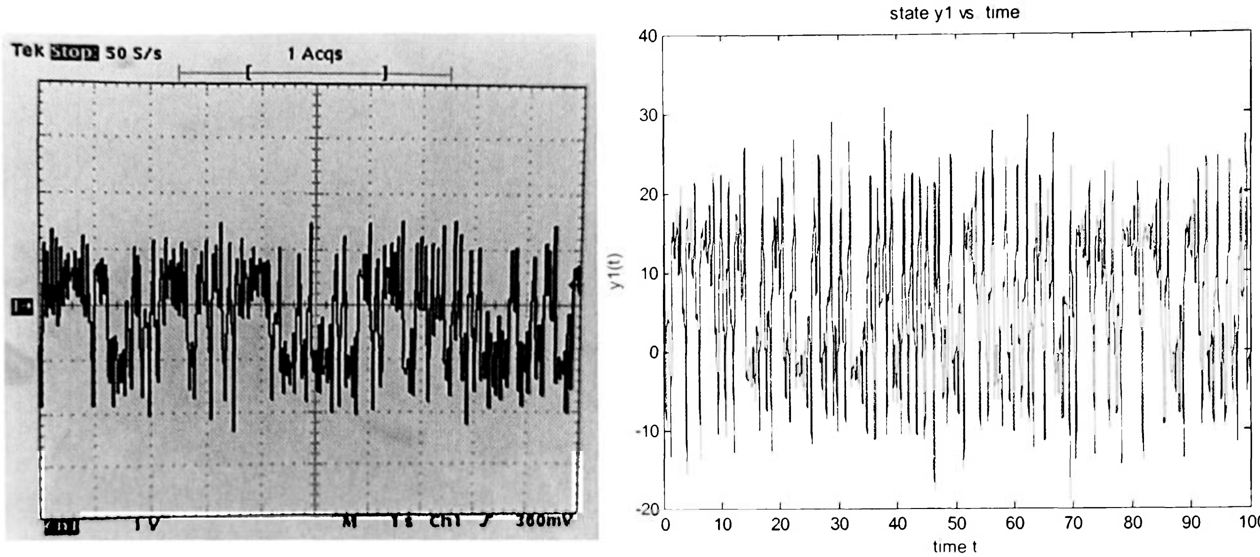
(a) (b)

Fig 4.7 Attractor on the x3(t) and x2(t) co-ordinate (2D) with time delay (a) analog circuit output (b) computer simulation



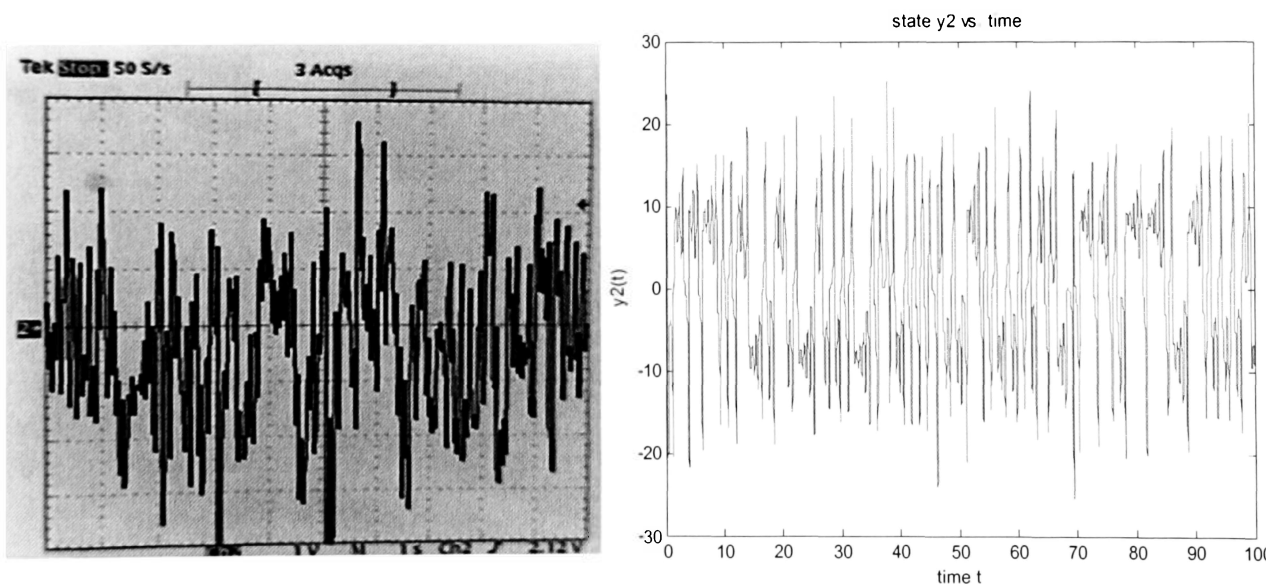
(a) (b)

Fig 4.8 Attractor on the x2(t) and x1(t) co-ordinate (2D) with time delay (a) analog circuit output (b) computer simulation



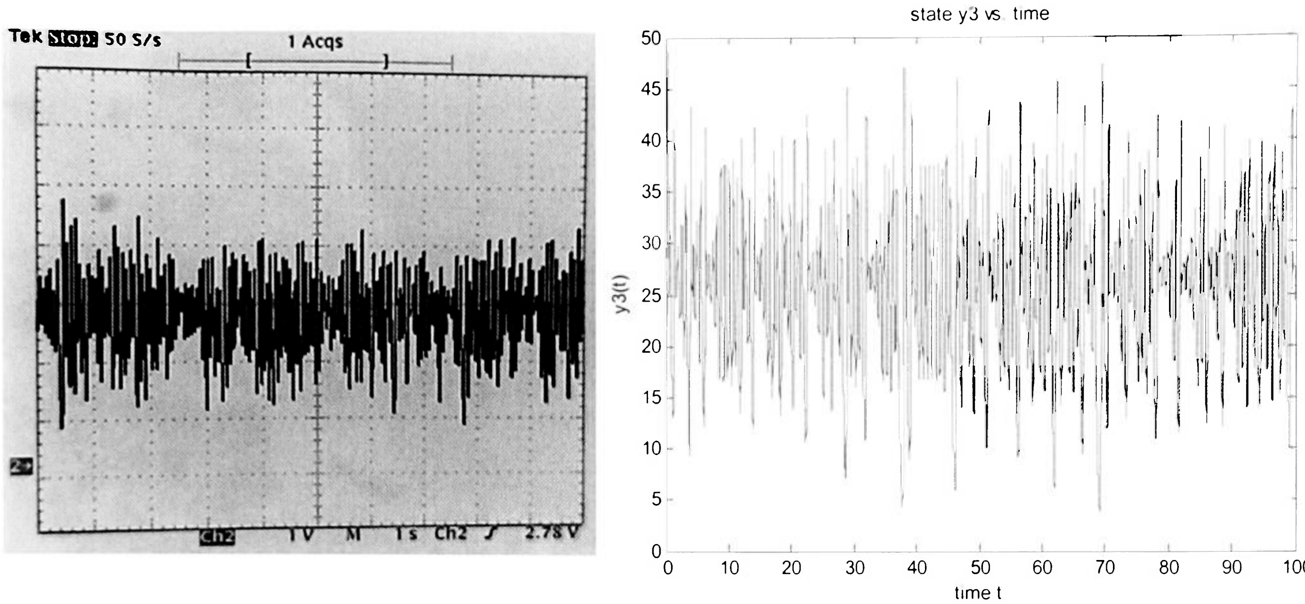
(a) (b)

Fig 4.9 The time series of the y1(t) vs. time of the Receiver side with time delay (a) analog circuit output (b) computer simulation



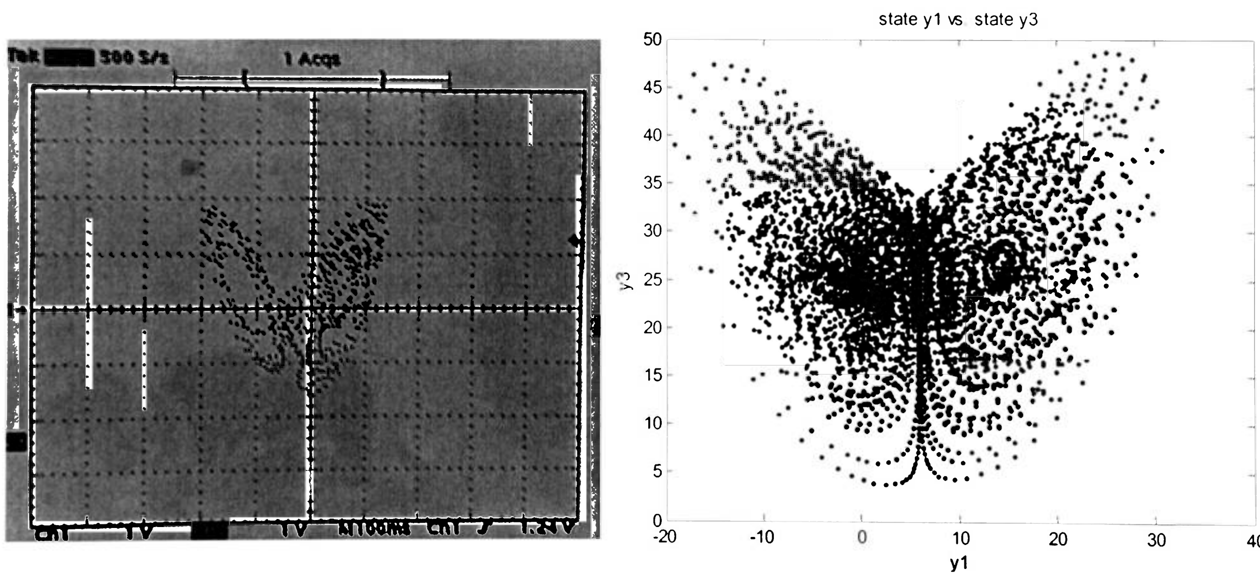
(a) (b)

Fig 4.10 The time series of the y2(t) vs. time of the Receiver side with time delay (a) analog circuit output (b) computer simulation



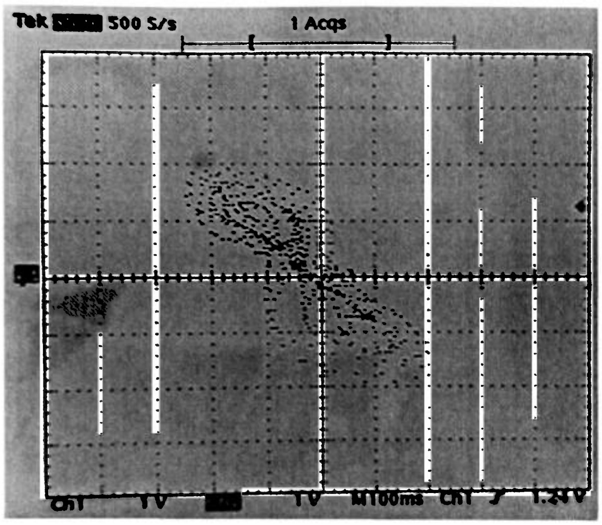
(a) (b)

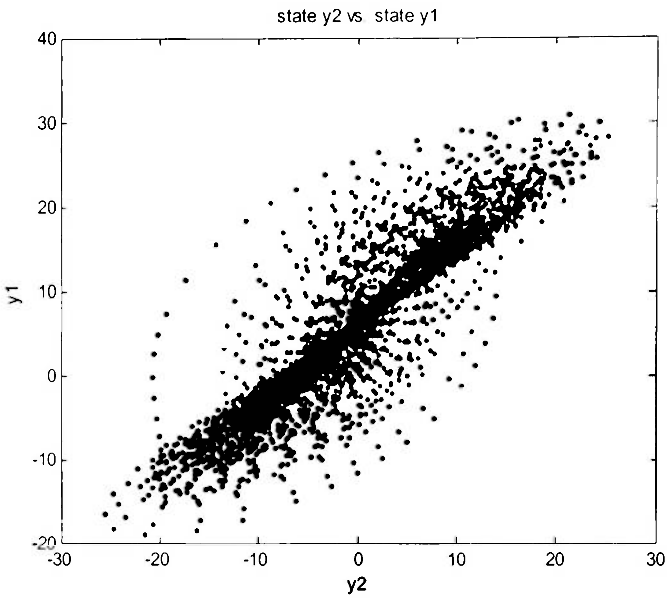
Fig 4.11 The time series of the y3(t) vs. time of the Receiver side with time delay (a) analog circuit output (b) computer simulation



(a) (b)

Fig 4.12 Attractor on the y3(t) and y1(t) co-ordinate (2D) with time delay (a) analog circuit output (b) computer simulation

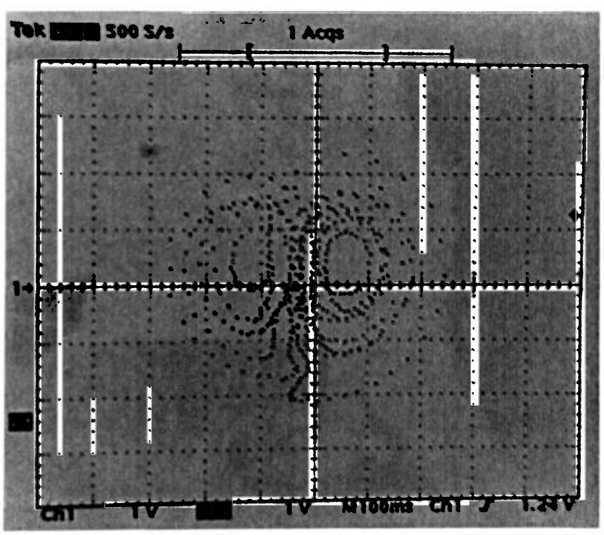


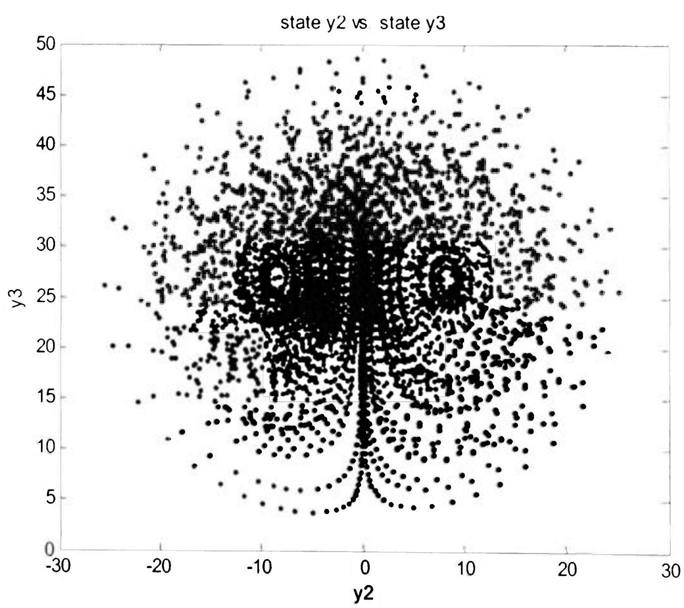


(a)

(b)

Fig 4.13 Attractor on the y2(t) and y1(t) co-ordinate (2D) with time delay (a) analog circuit output (b) computer simulation

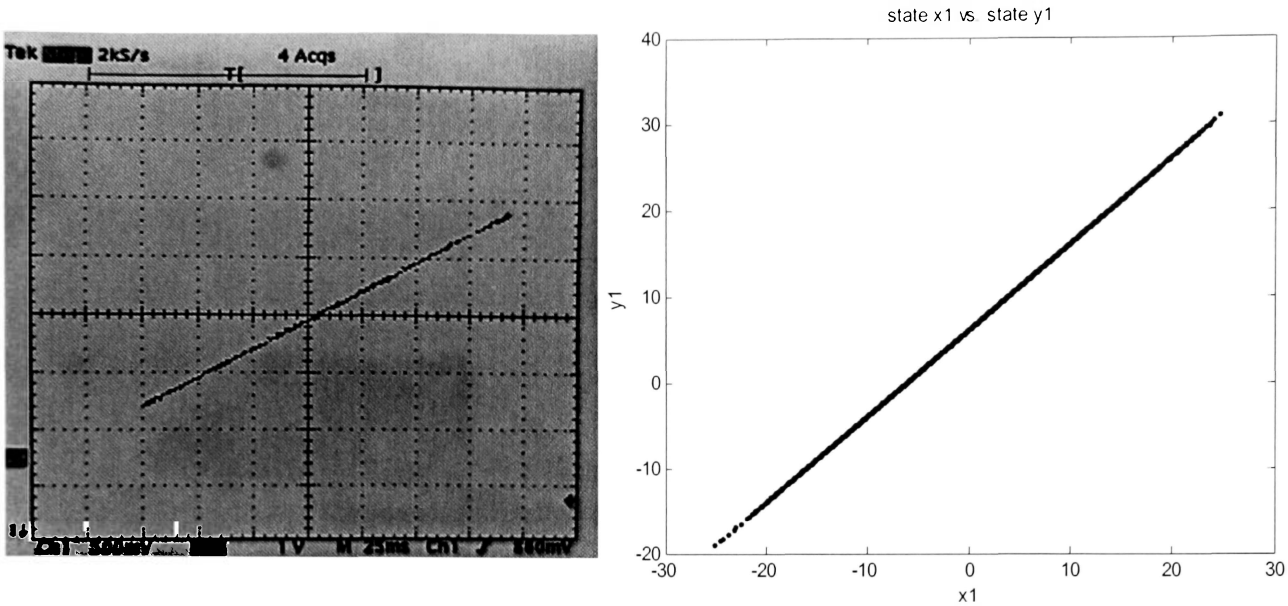




(a)

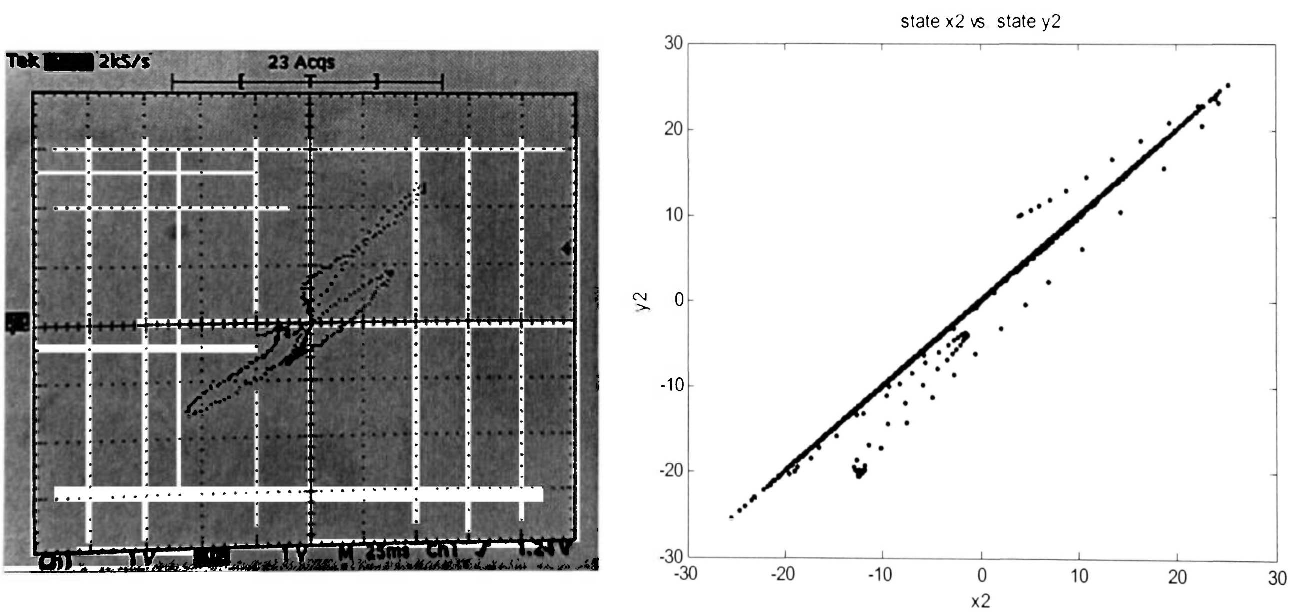
(b)

Fig 4.14 Attractor on the y3(t) and y2(t) co-ordinate (2D) with time delay (a) analog circuit output (b) computer simulation



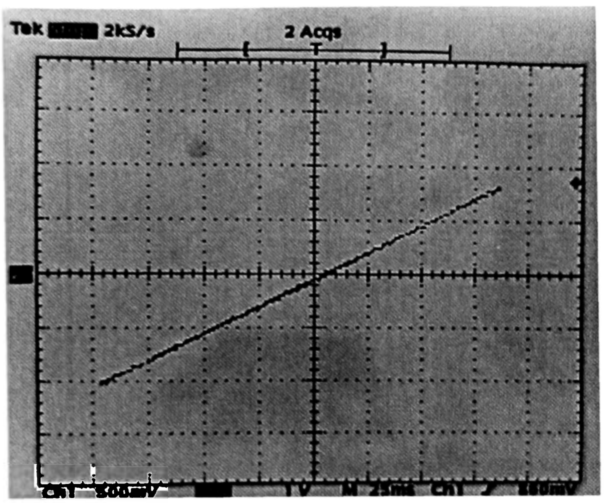
(a) (b)

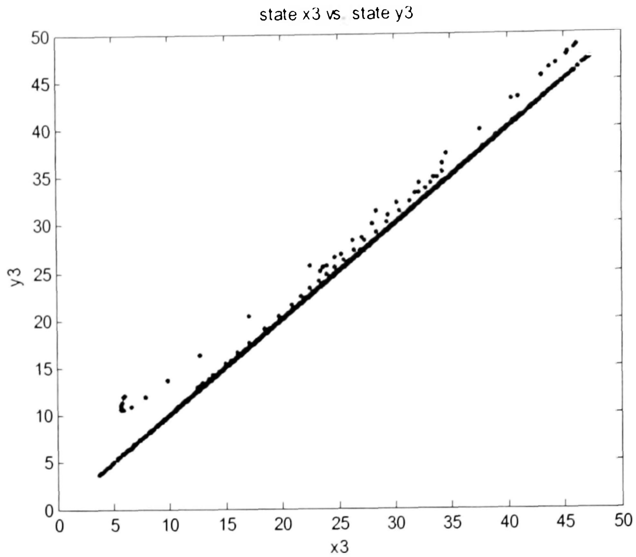
Fig 4.15 Properties of the synchronization between the receiver and the transmitter state variable with delay x1 vs. y1 (a) analog circuit output (b) computer simulation



(a) (b)

Fig 4.16 Properties of the synchronization between the receiver and the transmitter state variable with delay x2 vs. y2 (a) analog circuit output (b) computer simulation





(a) (b)

Fig 4.17 Properties of the synchronization between the receiver and the transmitter state variable with delay x3 vs. y3 (a) analog circuit output (b) computer simulation

CHAPTER IV

CONCLUSION AND RECOMMENDATIONS FOR FUTURE WORK:

It has been known that synchronization of chaotic system has motivated potential applications in secure communications. With the development of chaotic systems, many different encryption systems have been proposed, including secure communications based on synchronization of analog circuits. The use of chaotic systems has been for the purpose of encryption. This basic concept was implemented in this report, using the adaptive control process. This process combines both conventional encryption method and the synchronization of the chaotic system, so the level of security is enhanced. The two key features of chaos include a chaotic time series and sensitivity to small change in the initial conditions. These cause the chaotic transmission to have low probability of detection as an information-bearing signal is passed through the public channel.

In this thesis we studied (a) a general mathematical model for chaotic system formulated in discrete time and continuous time domain from one-dimensional model to three-dimensional Lorenz system. It uses of the chaos theory for the purpose of secure communication by adding the information to the time varying parameter. (b) It is based on an adaptive control method and parameter identification in order to an identify the information. (c) Synchronization of delay differential Lorenz equations to be further used in the implementation of secure communication.

Chapter 2 presents the synchronization phenomenon of the coupled two-dimensional logistic equation. Its application to secure communications using an adaptive control method is also demonstrated. It is known that under certain conditions, trajectories of the response system converge to exhibit a similar behavior to that of the driver system despite differences in their initial conditions. Further more in chapter 3, we propose the scheme for the three dimensional model of Lorenz system. The results from the analog circuit were comparatively similar to that of numerical solutions. In both cases the goal of synchronization was achieved. The electronic circuit described for the Lorenz system required fairly low tolerance value for the components to limit the performance.

There are several drawbacks in working with the chaotic analog systems for its practical application. The synchronization process of the signals at the transmitter and the receiver side using the analog devices. The tolerance level of each of the components should be kept to a minimum. The time delay unit between the transmitter and the receiver should match perfectly in delay time, impedance value and the tolerance level. There is also a complexity issue as the chaotic system depends on nonlinear effects that are hard to control. Thus for the future work these practical problem could be addressed.

As we know that the time delay system are present in nature and technology, due to the infinite signal transmition time, therefore the study of synchronization phenomena in such system is of great practical importance. In chapter IV we have described synchronization characteristics of time delay system. In future using the scheme of synchronization and parameter identification process of hyperchaotic system described by the delay differential equations, a communication system could be developed. Fig 5.1 gives the general layout of this secure communication scheme.

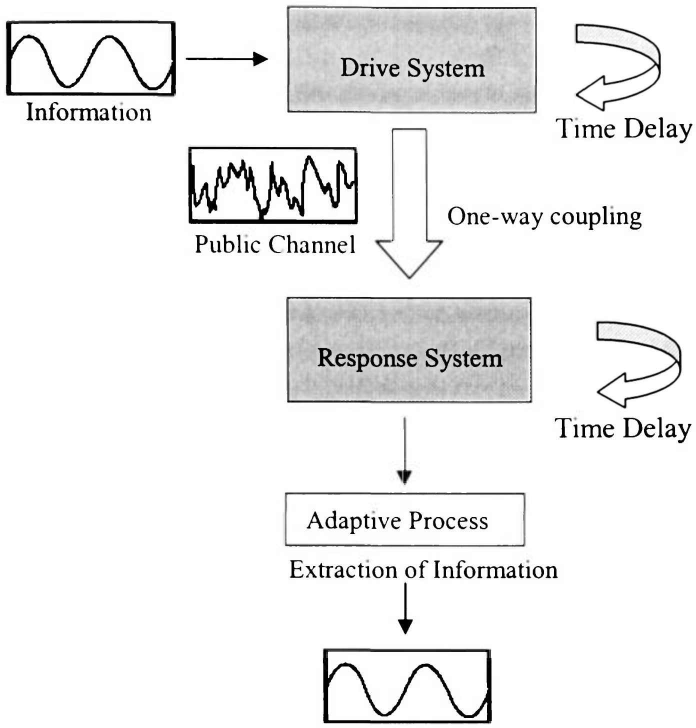


Fig 5.1: Basic set-up of communication system using chaotic dynamics with delay

In conclusion, we have presented a general approach to construct a synchronized dynamical system using an adaptive process, with and without time delay and discussed its practical application in a secure communication, where the information can be recovered without errors. One of potential applications of chaotic dynamics is secure communication in IT industry. With a rapid development of communication networks today such as Internet, telecommunications, e-business and so on, the security issue of information transmission becomes more and more important. Other areas such as military communication, banking transaction have the potential applications of chaotic communication system.

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APPENDIX A

MATLAB Program: The synchronization system described by 1-D coupled logistic map

```
clear
% Send two separate information which is added to the parameters
% Case for the constant parameter with Different Inital Condition
% Information is hidden in the parameters of the maps p1(i) and p2(i)
% The transmitter x(i) is the coupled 2D map%
%
% The receiver uses two identification models:
% y1(i)= a 2D coupled logistic map with a variable parameter q1(k)
% y2(i)= a 2D coupled logistic map with a different parameter q2(k)
% and synchronizes by adjusting the parameters of the maps
% and by using an adaptation law for the switching parameters
% B11(i) and B22(i) are constants
%
% The secret information can be recovered at the receiver only
% when the right switching sequence of the transmitter is identified
% and the right models are used.
b1=3.7; b2=3.823;                              % The parameters
                % Initial Condition            % x(1) = Transmitter, reference model
x1(1)= 0.7; x2(1)=0.5;                         % The initial condition for the Transmitter
Dp1=0.01;                                      % The amplitude of the Information signal #1
Dp2=0.02;                                      % The amplitude of the Information signal #2
                %The coupling parameter
eps=0.7; Time = 250;
                %The gain used in the adaptive process
gam1=50; gam2=50;
                %The initial condition for the error
deltaz1(1)= 0; deltaz2(1)= 0;
                % The initial condition for the Receiver
y1(1)=0.6;  y2(1)=0.65;
                %The parameters at the receiver side
q11(1)=b1; q21(1)=b2+eps; q12(1)=b1+eps; q22(1)=b2;
                % No. of Iterations
N = 1999;
for i=1:N
                % At the Transmitter Side
                % The Transmitter variable parameters p1(i) and p2(i) is added with the
        information:
p1(i) = b1 + Dp1*(cos(pi*i/Time));
p2(i) = b2 + Dp2*(sin(pi*i/Time));
                % The Logistic Equations:
fx1(i)= p1(i)*x1(i)*(1-x1(i));
fx2(i)= p2(i)*x2(i)*(1-x2(i));
                % The Coupled Logistic Equation:(2-D)
                % Tramsitted Message is x1(i) variable:
x1(i+1) = (1-eps)*fx1(i) + eps*fx2(i);
x2(i+1) = (1-eps)*fx2(i) + eps*fx1(i);
X(i)=x1(i); Y(i)=x1(i+1);
```

```
                % The Actual Message that is transmitted
MessageT1(i)= (p1(i) - b1);  MessageT2(i)= (p2(i) - b2);
                % At the Receiver Side
                % The logistic Equations: (Dynamical Equation) in which x1 is the
                % Transmitted Message
fy1(i) = q11(i)*y1(i)*(1-y1(i));
fy2(i) = q22(i)*y2(i)*(1-y2(i));
                % Coupled Logistic equation
y1(i+1) = (1-eps)*fy1(i) + eps*fy2(i);
y2(i+1) = (1-eps)*fy2(i) + eps*fx1(i);
                % Extraction of the Information
                % The difference between the Driver and Receiver Signal
deltaz1(i+1)= x1(i+1)-y1(i+1); deltaz2(i+1)= x2(i+1)-y2(i+1);
                %[B11,B22] = [1,0] B11(i)=1;
                % The adaptation Laws for the coefficients q1:
Dy1Dq11(i) = (1-eps)*y1(i)*(1-y1(i));
Dy1Dq12(i)= 0;
Dfy1Dq11(i)=B11(i)*Dy1Dq11(i);
if i <=300
   gam1= 0;
elseif i >=300
   gam1 = 50;
end
q11(i+1)= q11(i)+gam1*deltaz1(i+1)*Dfy1Dq11(i);
                % B11,B22]=[0,1]
B12(i)=0; B22(i)=1;
Dy2Dq22(i)=(1-eps)*y2(i)*(1-y2(i));
                %xr2(i+1)= B22(i) * f2r(i);
                % The adaptation Laws for the coefficients q2:
Dfy2Dq22(i)=B22(i)*Dy2Dq22(i);
                %q12(i+1)=q12(i);
if i <=300
   gam2= 0;
elseif i >=300
   gam2 = 50;
end
q22(i+1)= q22(i)+gam2*deltaz2(i+1)*Dy2Dq22(i);
end
                % Graph layout and Data
figure
subplot(2,1,1); plot(p1,'.');ylabel('Information 1');grid on;
subplot(2,1,2); plot(p2,'.');ylabel('Information 2');xlabel('time');grid on; pause;

figure
subplot(2,1,1); plot(fx1,'.');ylabel('fx1');grid on;
subplot(2,1,2); plot(fx2,'.');ylabel('fx2');xlabel('time');grid on; pause;

figure
subplot(2,1,1); plot(x1,'.');ylabel('x1 = Tranmitted Information');grid on;
subplot(2,1,2); plot(x2,'.');ylabel('x2');grid on; pause;
```

```
figure
subplot(2,1,1); plot(fy1,'.'); ylabel('fy1');title('The Receiver side Logistic Equation');grid on;
subplot(2,1,2); plot(fy2,'.'); ylabel('fy2');xlabel('time');grid on; pause;

figure
subplot(2,1,1); plot(y1,'.'); ylabel('y1');title('Coupled logistic equation');grid on;
subplot(2,1,2); plot(y2,'.'); ylabel('y2');xlabel('time');grid on; pause;

figure
subplot(2,1,1); plot(deltaz1,'.');ylabel('x1-y1');title('Error between the Transmitted and Received
Message');grid on;
subplot(2,1,2); plot(x1,y1,'.');ylabel('x1');xlabel('y1');grid on; pause;

figure
subplot(2,1,1); plot(deltaz2,'.');ylabel('x2-y2');title('Error between the Transmitted and Received
Message');grid on;
subplot(2,1,2); plot(x2,y2,'.');ylabel('x2');xlabel('y2');grid on; pause;

figure
subplot(2,1,1); plot(MessageT1,'.');title('The Actual Message #1');grid on;
subplot(2,1,2); plot(q11-b1,'.');ylabel('q11-b1'); title('The Extracted Message');grid on;
axis([1 N -0.02 0.02]); pause;

figure
subplot(2,1,1); plot(MessageT2,'.');title('The Actual Message #2');grid on; axis([1 N -0.02 0.02]);
subplot(2,1,2); plot(q22-b2,'.');ylabel('q22-b1'); title('The Extracted Message');grid on;
axis([1 N -0.02 0.02]); pause;

figure
plot(X,Y,'.'); xlabel('x1(i)');ylabel('x1(i+1)');title('Iteration plot');grid on; pause
```

APPENDIX B

MATLAB Program: To Synchronize System Described by 3-D Lorenz Equation

```
% Main Program
clear
                                          % Conditions at the Trasmitter and Receiver side:(3-D)
                                                % Costant for the Lorenz Equation
                                    % (Parameters at the Transmitter and the Receiver side is same)
global sigma r0 b p1 w Amp
                              % The parameters
sigma=11; r0=28; b=8/3;
                                                % w = omega of the 2*pi*f of the signal
                                                % Amp is the amplitude of the signal
w = 0.628; Amp = 1;
                              % The initial T0 = 0 and the TFINAL=100 is the time span
t0=0;
tfinal=100;
tspan = [t0 tfinal];
                                                % Intial Condition
x10=0.1; x20=0.12; x30=0.45;y10=0.12; y20=0.17; y30=0.48;
Xinitial = [x10 x20 x30];                       % Initial condition of x
Yinitial = [y10 y20 y30];                       % initial condition of y
                                                %
q10= r0 + 5; q20=1; q30=1;
                        % ODE45  Solve differential equations, [t,x] = ODE45('ODEFUN',TSPAN,x0)
                        % with tspan = [T0 TFINAL] integrates the system of differential
                        % equations x' = f(t,x) from time T0 to TFINAL with initial conditions x0.
X0 = [Xinitial Yinitial q10 q20 q30];
                                          % The ODE functions are Lorenz1 (subroutine)
[t,X]=ode45('Lorenz1',tspan,X0);
                                                %At the receiver side
                                                % Identifying the Individual Vector for x
x1= X(:, 1); x2= X(:, 2); x3= X(:, 3);
                                                % Identifying the Individual Vector for y
y1= X(:, 4); y2= X(:, 5); y3= X(:, 6);
q1= X(:, 7);  q2= X(:, 8);  q3= X(:, 9);
p1 = r0 + Amp*(sin(w*t));                       % The Actual signal Sent
                        % Figure and Graph
plot(t, x1);                                    % plot t on x-axis, and x1 on y-axis
title ('state x1 vs. time');                    % put the title on the figure
xlabel('t'); ylabel('x1(t)');                   % put labels on x-axis and y-axis
%axis([10 50 25 -25]); grid on; pause;

figure;
plot(t, x2);title ('state x2 vs. time');        % plot t on x-axis, and x1 on y-axis
xlabel('t'); ylabel('x2(t)');
grid on; pause;
%
figure;
plot(t, x3);title ('state x3 vs. time');xlabel('t'); ylabel('x3(t)'); grid on pause;
%
```

```
figure;
plot3(x1, x2, x3);                              % plot 3-D graph (x1, x2, x3)
title ('Lorenz Attractor chaotic master system');
xlabel('x1'); ylabel('x2'); zlabel('x3'); grid on; pause;
%
figure;
plot(t, y1);                                    % plot t on y-axis, and y1 on y-axis
title ('state y1 vs. time');                    % putting the title on the figure
xlabel('t'); ylabel('y1(t)');                   % putting labels on y-axis and y-axis
grid on
pause;
%
figure;
plot(t, y2);title ('state y2 vs. time'); xlabel('t'); ylabel('y2(t)'); grid on;pause;
%
figure;
plot(t, y3);title ('state y3 vs. time'); xlabel('t'); ylabel('y3(t)'); grid on pause;
%
figure;
plot3(y1, y2, y3);                              % plot 3-D graph (y1, y2, y3)
title ('Lorenz Attractor chaotic slave system'); xlabel('y1'); ylabel('y2'); zlabel('y3'); grid on;pause;
%
figure;
plot(x1,x3,'.');                                % plot 2-D graph
title ('state x1 vs. state x3');xlabel('x1(t)'); ylabel('x3(t)');grid on;pause;
%
figure;
plot(x2,x1,'.');                                % plot 2-D graph
title ('State x1 vs. state x2');xlabel('x2(t)'); ylabel('x1(t)');grid on;pause;
%
figure;
plot(x2,x3,'.');                                % plot 2-D graph
title ('state x2 vs. state x3');xlabel('x2(t)'); ylabel('x3(t)'); grid on pause;

figure;                                         % plot 2-D graph
plot(y1,y3,'.');title ('state y1 vs. state y3'); xlabel('y1(t)'); ylabel('y3(t)'); grid on; pause;
%
figure;                                         % plot 2-D graph
plot(y2,y1,'.');title ('state y1 vs. state y2'); xlabel('y2(t)'); ylabel('y1(t)'); grid on; pause;
%
figure;                                         % plot 2-D graph
plot(y2,y3,'.');title ('state y2 vs. state y3'); xlabel('y2(t)'); ylabel('y3(t)'); grid on; pause;
%
figure;                                         % Verifying the synchronization
subplot(211);plot(y1,x1,'.');xlabel('y1');ylabel('x1'); title('projection of the attractor onto (x y)
plane'); grid on;
subplot(212);plot(y2,x2,'.');xlabel('y2');ylabel('x2');grid on; pause;
%
figure;                                         % Verifying the synchronization
subplot(211);plot(t,y1-x1);ylabel('y1-x1');grid on; subplot(212);plot(t,y3-x3);ylabel('y3-
x3');xlabel('time');grid on; pause;
```

```
%
figure; plot(t,y2-x2);ylabel('y2-x2');grid on pause;

figure;
subplot(211); plot(t,p1); title('Original Information');grid on;
subplot(212); plot(t,q1);title('Extracted Information'); xlabel('time');grid on; pause;

                          % For the Annimation of the Lorenz 3D Map
figure
A = [ -8/3 0 0; 0 -10 10; 0 28 -1 ];
                          %initial condition
x = [10 1 4]';
                          % x value increment
delta = 0.01;
p = plot3(x(1),x(2),x(3),'EraseMode','none','MarkerSize',10);
axis([0 50 -25 25 -25 25])
hold on
for i=1:400000
   A(1,3) = x(2);
   A(3,1) = -x(2);
   xdot = A*x;
   x = x + delta*xdot;
                          % Change coordinates
   set(p,'XData',x(1),'YData',x(2),'ZData',x(3));
   xlabel('x1'); ylabel('x2');zlabel('x3')
   drawnow
   i=i+1;
end
%
%*******************************************************************%
function xdot= Lorenz1(t,X);
%Constants for the Lorenz Equation
global sigma r0 b p1 w Amp
% The driving system (transmitter): x1, x2, x3
x1=X(1); x2=X(2); x3=X(3);
p1 = r0 + Amp*(sin(w*t)); p2 = 1; p3 = 1;  % the parameters of the transmitter system
xdot(1) = sigma*(x2-x1);
xdot(2) = p1*x1 - p2*x2 - p3*x1*x3;
xdot(3) = x1*x2 - b*x3;
% The driven system (receiver): y1, y2, y3
y1=X(4); y2=X(5); y3=X(6); q1=X(7); q2=X(8); q3 = X(9);
%
xdot(4) = sigma*(x2-y1);
xdot(5) = q1*y1 - q2*y2 - q3*y1*y3;
xdot(6) = y1*x2-b*y3;
%The difference between the drive and the driven signals
error2 = y2-x2;
% Gain of the system
G=5;
xdot(7)= -G *error2*y1; xdot(8)=0; xdot(9)=0;
xdot = [xdot(1);xdot(2); xdot(3);xdot(4);xdot(5);xdot(6); xdot(7);xdot(8); xdot(9)];
```