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## Implementation of a Low Cost Micro Electro Mechanical Systems Inertial Navigation Solution

Charles Njenga

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**Implementation of a low cost Micro Electro Mechanical Systems  
Inertial Navigation Solution**

By

Charles Njenga

A Thesis Submitted to the  
Graduate Studies Office in Partial Fulfillment of the Requirements for the  
Degree of Master of Science in Aerospace Engineering

Embry-Riddle Aeronautical University  
Daytona Beach, Florida  
Spring 2008

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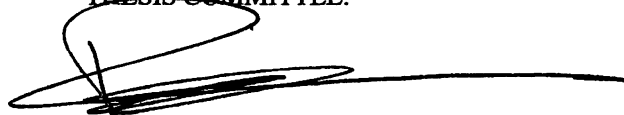
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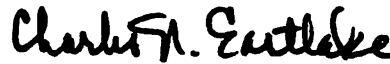
Charles Ndungu Njenga

This Thesis was prepared under the direction of the candidate's thesis committee chairmen, Dr Richard Anderson Department of Aerospace Engineering and Professor Charles Eastlake, Department of Aerospace Engineering and has been approved by the members of the thesis committee. It was submitted to the Aerospace Engineering Department and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

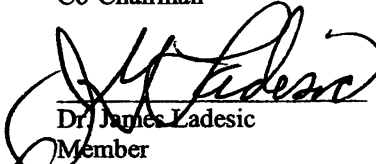
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# ABSTRACT

Author: Charles N. Njenga  
Title: Implementation of a low cost Micro Electro Mechanical Systems Inertial Navigation Solution  
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The purpose of this study is to develop a low cost navigation solution for General Aviation (GA) aircraft and flight testing aircraft. This low cost solution uses a Global Positioning System (GPS) coupled with an Inertial Navigation Solution (INS) to provide a more accurate, stable navigation solution that can be acceptable for aircraft navigation. LabVIEW was utilized to perform data acquisition and computation. LabVIEW was the backbone software to perform real time computation. MATLAB ® Simulink ® was used to plot graphs for final analysis of the code. A software Kalman Filter was utilized to provide signal stability and signal continuity. This study will be utilized as a preliminary step towards the development of inexpensive alternate methods of aircraft navigation for GA and flight testing.

# Table of Contents

ABSTRACT.....	v
Table of Contents.....	vi
List of Figures.....	viii
List of Tables.....	ix
List of Acronyms.....	x
List of Variables.....	xii
Chapter 1: Introduction.....	1
Problem statement.....	4
Statement of Hypothesis.....	5
Chapter 2: Literature Review.....	7
Global Positioning System.....	8
SBAS GPS.....	10
Wide Area Augmentation System.....	12
GPS Navigation Conclusion.....	12
Inertial Navigation Systems.....	13
Bias.....	14
Scale Factor.....	16
Noise.....	17
Non-Linearity.....	18
INS/GPS Aiding.....	18
INS Kalman Filtering.....	19
Conclusion.....	20
Chapter 3: Methodology.....	21
GPS Navigation Solution.....	23
GPS Variance.....	23
AHRS.....	24
Inertial Navigation.....	25
Inertial Navigation Solution Computation.....	26
General Kalman Filter Equations.....	34
Acceleration Kalman Filter.....	37
Prediction.....	38
Kalman Gain.....	39
Measurement Update.....	40
Position Kalman Filter.....	41
Prediction.....	41
Kalman Gain.....	42
Measurement Update.....	43
Process Noise Matrix.....	44
Tuning the Kalman Filter.....	46
LabVIEW Code.....	47
GPS Code.....	48
Inertial Navigation Solution.....	52
Kalman Filter Code.....	56
Conclusion.....	57



Chapter 4: Results and Conclusion .....	59
Results .....	60
Input data reduction .....	60
Stationary GPS error analysis .....	60
Stationary AHRS error analysis .....	62
Navigation algorithm results .....	63
Process Noise matrix .....	65
Inertial Navigation Solution results .....	69
Integrated Solution .....	70
Conclusion .....	74
Current Hardware .....	74
GPS .....	74
AHRS .....	75
Problems encountered .....	76
Possibility of future research: .....	77
References .....	78

# List of Figures

Figure 1: NAVSTAR Satellite orbit arrangement .....	9
Figure 2: Illustration of Receiver Height Measurements.....	10
Figure 3: Bias error between current signal and anticipated signal .....	15
Figure 4: AHRS bias in the acceleration due to gravity in the Z direction ECEF frame..	16
Figure 5: Scale Factor .....	17
Figure 6: AHRS Output .....	26
Figure 7: Coordinate transform for acceleration and gravity compensation .....	27
Figure 8: Inertial solution obtained using the AHRS solely .....	29
Figure 9: GPS acceleration computation and acceleration error analysis.....	31
Figure 10: AHRS accelerations corrected for signal bias using GPS .....	33
Figure 11: Position error computation .....	33
Figure 12: Compensated aircraft position solution.....	34
Figure 13: GPS Initialization code.....	49
Figure 14: Reading the GPS NMEA sentence .....	49
Figure 15: Breaking down the ARINC-429 sentence to its various components .....	50
Figure 16: Breaking down Latitude and Longitude to Degrees and Minutes.....	51
Figure 17: Conversion of degrees to feet .....	51
Figure 18: AHRS initialization code.....	52
Figure 19: AHRS conversion from two's complement to decimal values .....	53
Figure 20: Separation and conversion of Euler angles and accelerations.....	54
Figure 21: Inertial Navigation Solution .....	55
Figure 22: Kalman filter algorithm in LabVIEW .....	56
Figure 23: Top Level Block Diagram.....	58
Figure 24: GPS standard deviation over a period of one hour.....	61
Figure 25: Acceleration in the X direction (Filtered and Unfiltered) .....	62
Figure 26: Acceleration Bias in the Z direction.....	63
Figure 27: Difference between GPS acceleration and AHRS acceleration (X-Direction)	66
Figure 28: Acceleration Kalman Filter output for the X-direction (Acceleration Bias)...	67
Figure 29: Difference between GPS position and Inertial position (X-direction) .....	68
Figure 30: Position Kalman Filter output for the X-direction .....	69
Figure 31: Inertial Solution.....	70
Figure 32: Integrated solution V.S. GPS solution V.S. Inertial solution.....	71
Figure 33: Integrated Aircraft path into the Eastern Iowa Airport .....	72
Figure 34: GPS and Integrated Aircraft Altitude.....	73

## List of Tables

Table 1: Comparison of Gyroscope Performance.....	14
Table 2: Summary of stationary GPS variance.....	61
Table 3: Summary of stationary AHRS error .....	63
Table 4: Kalman Filter Inputs .....	64
Table 5: Process noise matrix values for the Acceleration Kalman Filter.....	65
Table 6: Process noise matrix values for the Position Kalman Filter .....	66

## List of Acronyms

ABC	Aircraft Body Coordinate system
AHRS	Attitude Heading and Reference System
ARINC	Aeronautical Radio INCorporated
ASL	Above Sea Level
C/A	Coarse/Acquisition
DCM	Direction Cosine Matrix
DGPS	Differential Global Positioning System
DLL	Dynamic Link Library
AHRS	Dynamic Measuring Unit
ECEF	Earth Centered Earth Fixed coordinate system
FAA	Federal Aviation Administration
FOG	Fiber Optic Gyroscope
GA	General Aviation
GEO	Geostationary Satellites
GPS	Global Positioning System
INS	Inertial Navigation Solution
INSY	Integrated Navigation SYstem
LAAS	Local Area Augmentation System
MEMS	Micro Electro-Mechanical Systems
NAS	National Airspace System
NAVSTAR	Navigation Satellite Timing and Ranging
NED	North East Down coordinate system

NMEA	National Marine Electronics Association
PRN	Pseudorandom code Number
RLG	Ring Laser Gyroscope
SBAS	Satellite-Based Augmentation System
STC	Supplemental Type Certificate
SVN	Space Vehicle Number
TSO	Technical Standard Order
WAAS	Wide Area Augmentation System
WGS84	World Geodetic System of 1984

## List of Variables

$R_{\oplus}$  Earth Radius in feet

$\lambda$  Latitude in degrees

$\lambda_{ft}$  Latitude in feet

$l$  Longitude in degrees

$l_{ft}$  Longitude in feet

$h_{ft}$  Altitude in feet

$h_m$  Altitude in meters

## **Chapter 1: Introduction**

Navigation is the primary means by which aircraft get from one point to the next. Traditionally, Aircraft have relied on of navigational information from the ground, satellites, and Independent sources of navigation such as Inertial Navigation Systems (INS) [1]. Independent sources of navigation have become highly accurate in recent years and are being used in numerous passenger aircraft for long range navigational purposes. These navigation solutions using independent sources however, are quite costly and have been effectively restricted to only commercial and military aviation. The high cost of airplane navigation equipment has restricted their availability in the General Aviation (GA) and flight testing aircraft industry. Recently, Inexpensive alternatives utilizing Micro-Electro-Mechanical Systems (MEMS) technology have to be found for use in these crucial aviation markets.

The issue addressed in this thesis is to implement a low cost navigation solution that may be used by GA and flight testing aircraft that will aid in precision navigation and data acquisition. This solution will use existing equipment; namely GPS and MEMS based Attitude Heading and Reference System (AHRS), in conjunction with a Kalman Filter algorithm to perform aircraft navigation. This system must also be cost effective in order for it to be viable in small GA and flight testing aircraft [2]. A solution that relies solely on Global Positioning System (GPS) will be vulnerable to spoofing and signal loss. Using a Wide Area Augmentation System (WAAS) capable GPS, can provide an aircraft with more precise navigation guidance. This means of navigation, though augmented, is not capable of self monitoring without an aiding source to improve error detection.



The necessary aiding of the GPS will come from an inexpensive inertial solution using MEMS technology. The internal nature of the accelerations and rates ensures signal integrity as there is no spoofing or signal corruption that may occur via radio signal interference. The MEMS based INS is composed of an AHRS that makes inertial measurements and an algorithm that utilizes those measurements and computes the INS solution. The MEMS technology reduces the cost of the INS solution dramatically due to ease of manufacture and a high production rate, ensuring that GPS-MEMS/INS combination is a viable solution. MEMS however, has larger noise and bias error that cause the INS solution to have high drift rates leading to the INS solution being accurate for only short periods of time. The high drift rates are as a result of misalignment, high noise, and temperature varying biases.

As in any aircraft system, redundancy ensures that there is always navigation information coming from multiple sources. If for any reason, any one system should fail, then the Kalman Filter algorithm is used to predict the next position of the aircraft, keeping the aircraft on track. In this solution, the GPS navigational information is backed up by using the independent INS system as a second source since it is invulnerable to jamming.

## **Problem statement**

The main problem is to develop real-time code that integrates both a low cost AHRS, used for the purpose of deriving an Inertial Navigation Solution, with an aiding GPS for use in GA precision navigation and flight test instrumentation purposes. The code must perform the following tasks:

1. The code must compute the GPS Feet North, Feet East and Altitude
2. The code must compute the Inertial Navigation Solution
3. The code must compute the Integrated Navigation Solution
4. The code must provide a solution more accurate than either GPS or INS alone
5. The code must be able to quantify the accuracy of the navigation solution ( there must be a quantifiable coast phase)

## Statement of Hypothesis

1. The code must compute the GPS Feet North, Feet East and Altitude:

The GPS receiver acquires data in the form of degrees of latitude, degrees of longitude and altitude in meters. The GPS code will convert the degrees of latitude to feet.

2. The code will compute the Inertial Navigation Solution:

The code will convert the AHRS accelerations from the body navigation coordinates to the Earth Centered Earth Fixed (ECEF) navigation coordinates. The code will utilize a Direction Cosine Matrix (DCM) to perform the coordinate transform from Body to ECEF. The code will compute the acceleration error by obtaining the difference between the GPS accelerations and the Attitude Heading and Reference System (AHRS) accelerations.

The code will use the Kalman filter to compute the acceleration bias, using the acceleration error as the input to the Kalman filter. The code will compute the stable accelerations by obtaining the difference between the AHRS accelerations and the acceleration bias. The code will use the stable accelerations to compute the velocity and position at the next time step. This will be the Inertial Navigation Solution.

3. The code must compute the Integrated Navigation Solution:

The code will compute the position error by taking the difference between the GPS position and the Inertial Navigation Solution. The code will use the Kalman filter to compute the position bias by using the position error as the input to the Kalman filter. The code will compute the stable position by obtaining the difference between the Inertial Navigation Solution and the position bias. This will be the Integrated Navigation Solution.

4. The code must provide a solution more accurate than the GPS or INS alone:

The code will update the randomness in the GPS solution with the more accurate INS. The code will update the drift in the INS with the more long duration GPS. The code will eliminate the bias in the GPS and INS solution.

5. The code must be able to quantify the accuracy of the navigation solution ( there must be a quantifiable coast phase):

The code will compute the GPS frequency. The code will compute the INS frequency. The code will compute the combined GPS and INS frequency. The code will compute the lag time between the GPS and INS.

## **Chapter 2: Literature Review**

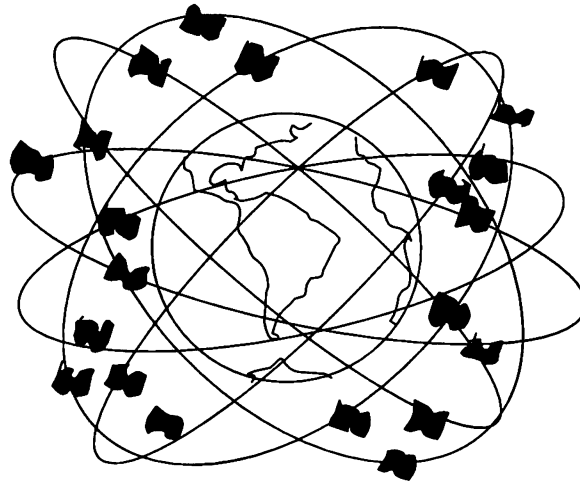
Since the introduction of aircraft, there has been a need for navigation equipment to aid in safe flight. This need has become more emphasized as the airspace has become more crowded with general aviation, commercial and military aircraft. Several types of navigation solutions have been developed over the course of time to assist in more precise navigation. A versatile form of navigation that is completely self-contained is called inertial navigation. Inertial navigation uses gyros and accelerometers (inertial sensors) to measure the state of motion of the vehicle by noting changes in that state caused by accelerations. Another form of navigation is radio based systems such as GPS. GPS relies on satellites strategically positioned around the earth's orbit sending data down to receiving equipment located in an aircraft. GPS quality has been improving over the past decades and has been touted to replace other means of navigation [2].

Studies conducted to aid in this navigation transition have shown considerable vulnerabilities of GPS to intentional and unintentional signal outages. GPS systems are vulnerable to spoofing and signal drop, and this makes them unreliable as the single source of aircraft navigation [3]. These problems necessitate the use of another aiding navigation solution to reduce the vulnerabilities of the GPS system.

## **Global Positioning System**

The Global Positioning System (GPS) is a satellite navigation system capable of providing a highly accurate continuous global navigation service independent of other positioning aids. The GPS system uses the NAVSTAR (NAVigation Satellite Timing And Ranging) satellites which consist of 24 operational satellites to provide a GPS

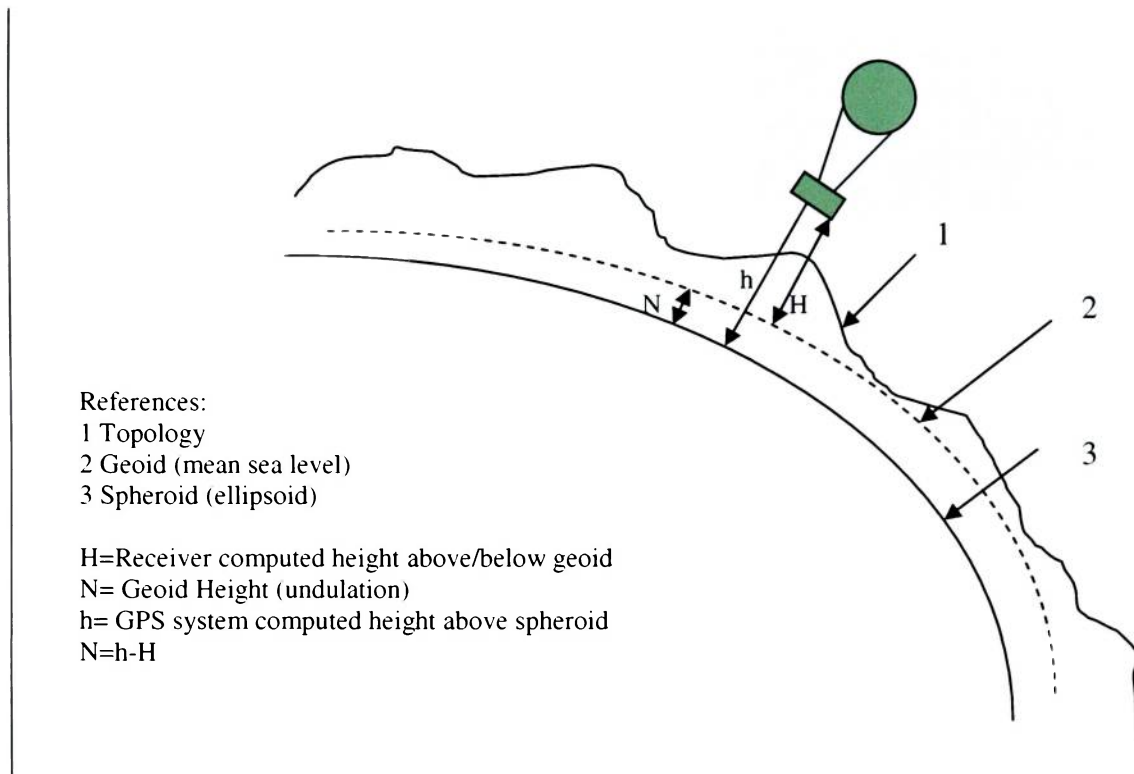
receiver with at least six satellites in view at all times as shown in Figure 1. A minimum of four satellites in view are needed to allow the receiver to compute its current latitude, longitude, altitude with reference to mean sea level and the GPS system time [4].



**Figure 1: NAVSTAR Satellite orbit arrangement**

The GPS satellite signal identifies the satellite and provides the positioning, timing, ranging data, satellite status and the corrected ephemerides (orbit parameters) of the satellite to the end users. The satellites can be identified either by the Space Vehicle Number (SVN) or the Pseudorandom Code Number (PRN). The user receiver consists of equipment which tracks and receives the satellite signals. The user equipment must be capable of simultaneously processing the signals from a minimum of four satellites to obtain accurate position, velocity and timing measurements. An ellipsoid (also spheroid) is a mathematical representation of the earth's surface. GPS positions are referred to a position known as the WGS84 (World Geodetic System of 1984). The difference between the ellipsoid and the geoid (The equipotential surface of the Earth gravity field

that most closely approximates the mean sea surface.) helps in converting the GPS heights to a more usable mean sea level height as shown in Figure 2 [4].



**Figure 2: Illustration of Receiver Height Measurements**

GPS signal accuracy is distorted by ionospheric errors and wrong satellite clock times. Accuracy is enhanced through the use of wide area corrections for GPS satellite orbits and ionospheric errors. Wide Area Augmentation System (WAAS) is a method of Satellite-Based Augmentation System (SBAS) that reduces these GPS errors.

## **SBAS GPS**

A (SBAS) is a type of geo-stationary satellite system that improves the accuracy, integrity, and availability of the basic GPS signals. Accuracy is enhanced through the use of wide area corrections for GPS satellite orbits and ionospheric errors. Integrity is enhanced by the SBAS network that can quickly detect satellite signal errors and send



alerts to receivers so the failed satellites are not used. Availability is improved by providing an additional ranging signal to each SBAS geostationary satellite. The primary functions of SBAS are:

- Data collection
- Determining ionospheric corrections
- Determining satellite orbits
- Determining satellite clock corrections
- Determining satellite integrity
- Independent data verification
- SBAS message broadcast and ranging
- System operations & maintenance

The SBAS is made up of a series of Reference Stations, Master Stations, Ground Uplink Stations and GEO satellites. The Reference Stations, which are geographically distributed, pick up GPS satellite data and route it to the Master Stations where wide area corrections are generated. These corrections are sent to the Ground Uplink Stations which up-link them to the GEOs for re-transmission on the GPS L1 frequency [4]. These GEOs transmit signals which carry accuracy and integrity messages, and also provide additional ranging signals for added availability, continuity and accuracy. These GEO signals are available over a wide area. GPS user receivers are thus able to receive SBAS data in-band and use not only differential corrections, but also integrity, residual errors and ionospheric information for each monitored satellite. The signal broadcast via the SBAS GEOs to the SBAS users is designed to minimize modifications to standard GPS receivers. As such, the GPS L1 frequency (1575.42 MHz) is used, together with GPS-

type modulation – for example a Coarse/Acquisition (C/A) pseudorandom (PRN) code. In addition, the code phase timing is maintained close to GPS time to provide a ranging capability [5].

## **Wide Area Augmentation System**

WAAS is an extremely accurate navigation system developed for civil aviation. WAAS provides integrity data of the GPS GEO satellites with wide area differential corrections. This differential signal is determined by ground stations and uplinked to the satellites. The satellites then rebroadcast the signal to receivers [2]. With the WAAS, the U.S. National Airspace System (NAS) now has the ability to provide horizontal and vertical navigation for precision approach operations for all users at all locations. WAAS provides service for all classes of aircraft in all flight operations including en route navigation, airport departures, and airport arrivals. This includes precision landing approaches in all weather conditions at all locations throughout the NAS [5].

## **GPS Navigation Conclusion**

Even with the improved way of correcting timing and ionospheric errors, the GPS is still vulnerable to system failures. The GPS signal is distorted by the ionosphere as it makes its way down to the receiving unit. In addition to that, the GPS signal can be obscured by tall buildings and mountains hence resulting in error prone navigation solution. It is important to compare the GPS navigation solution with an independent navigation solution [2]. The inertial system is a perfect choice for this role. The GPS is excellent for long term navigation whereas the INS is great for short term navigation as it drifts with

increased time. The INS has an added advantage of not requiring external signals and is therefore not susceptible to radio frequency problems.

## **Inertial Navigation Systems**

For inertial navigation to work, it requires six Degree Of Freedom (DOF) gyro package to perform a double integration in the navigation frame to arrive at a position and velocity [2]. The most common types of INS systems are the Ring Laser Gyros (RLG) and Fiber-Optic Gyros (FOG) and more recently, vibration gyros. Ring Laser Gyros require a laser in a closed-path cavity with mirrors at each intermediate corner in the path. The operation of the RLG is based on optical and electronic phenomena, rather than the mechanical phenomena for the rate/rate gyros. As a result, the RLG exhibit little error associated with acceleration, as with mechanical gyros. FOG is similar to RLG except that with the FOG, light is bounced through a wound optical cable. The RLG and FOG rely on laser light that travel in a fiber optic in case of the FOG or through a quartz crystal in the case of the RLG. The RLG measures pulses sampled periodically and are then converted to aircraft attitude data. The vibrating beam measures oscillations induced on a cantilever beam by Coriolis effect. It combines linear accelerometers, rotational sensors and magnetometers [6]. The MEMS/INS uses angular rate sensors and linear acceleration sensors that are micro-machined devices. The three angular rate sensors consist of vibrating ceramic plates that utilize Coriolis force to output the angular rate independent of acceleration. The three MEMS accelerometers are surface micro-machined silicone devices that use differential capacitance to sense acceleration. The magnetic sensors are state of the art miniature fluxgate sensors. Fluxgate sensors make the AHRS sensitive and responsive.

With the advent of microchip design, the vibration based INS use this easy to manufacture small scale design to reduce the size of the INS while holding down the cost.

These devices have varied performance. Table 1 shows the performance of the RLG the FOG and the MEMS based gyro. The vibration gyro will be represented by MEMS based gyro. In this thesis, MEMS gyros will be used interchangeably with vibration gyros.

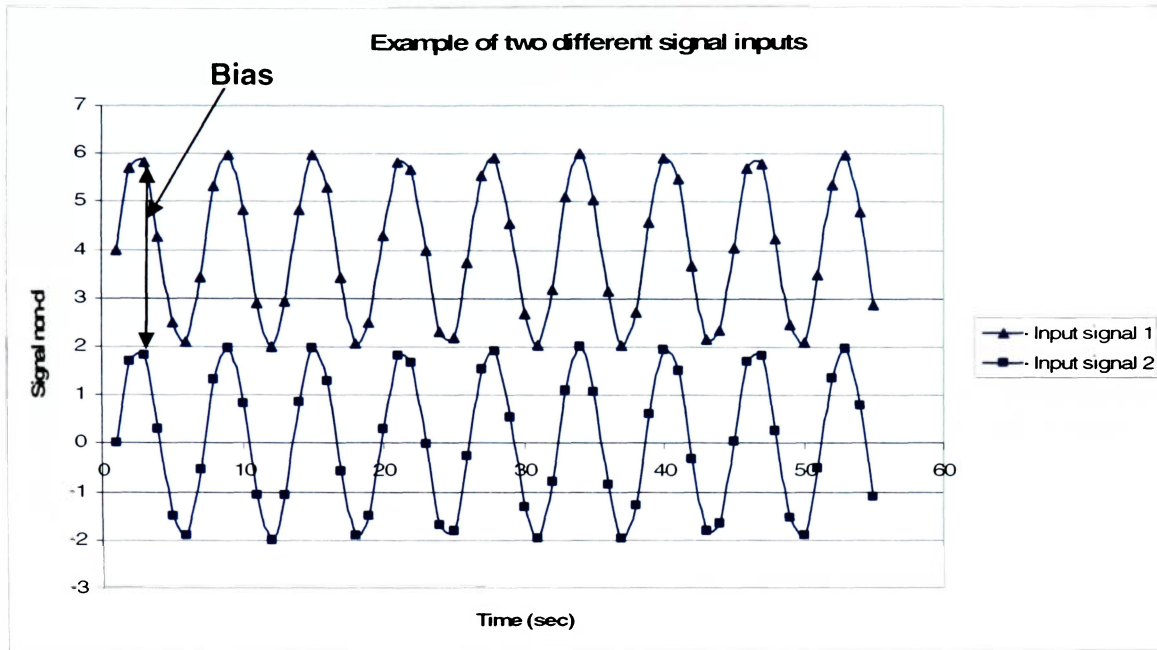
**Table 1: Comparison of Gyroscope Performance**

<i>Characteristic</i>	<i>RLG</i>	<i>FOG</i>	<i>MEMS</i>
g-independent bias (deg/hour)	0.001-10	0.5-50	360-1800
g-dependent bias	0	<1	18
Scale Factor non-linearity (%)	5-100	0.05-0.5	0.2-0.3

The MEMS gyro is less accurate than the Ring Laser Gyro or the Fiber optic Gyro due to the mechanical nature of its components. The errors from MEMS gyro are more than for the RLG or FOG as shown in Table 1. The errors experienced by the MEMS/INS computations are bias (g-independent), scale factor, noise, and non linearity.

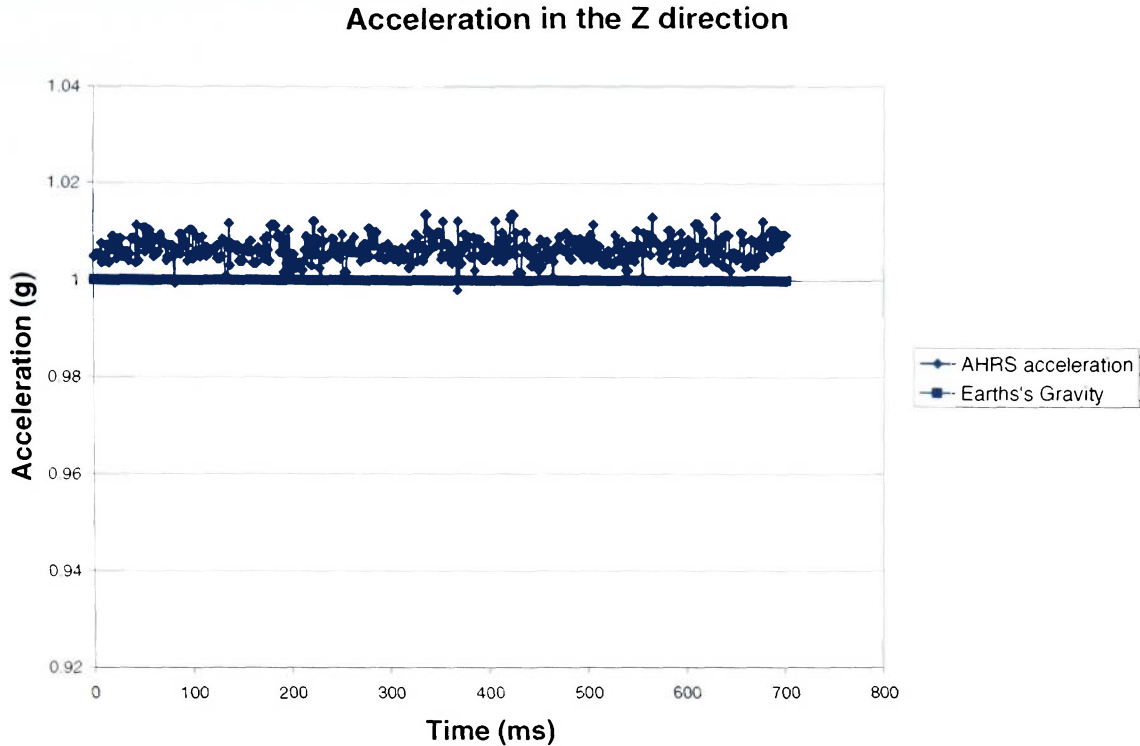
## **Bias**

This is the permanent offset from the expected value due to external and internal factors of the measuring instrument. This is usually the largest error in the measuring system. In Table 1 the bias is represented by the g-independent bias. To explain further what the bias is, Figure 3 displays an example of a biased signal.



**Figure 3: Bias error between current signal and anticipated signal**

In Figure 3, the expected input signal is shown with data points indicated using squares. The bias in that signal pushes the input signal to reflect the signal with data points indicated using triangles. In this example, the bias is approximately 4 units. In this next example, the Crossbow AHRS has a bias error as is shown in the Figure 4. In Figure 4, the acceleration due to gravity as measured by the AHRS is displayed. The bias in this case is 0.0062g. The straight line displays the actual average gravity that should be experienced by the AHRS in the Z direction in the ECEF frame. When the AHRS is tilted, the normal acceleration value changes and so does the bias. This change is compensated by transforming the normal acceleration



**Figure 4: AHRS bias in the acceleration due to gravity in the Z direction ECEF frame**

To reduce the bias, the Kalman filter estimates the bias by measuring the difference between the GPS measured accelerations and the INS computed accelerations [2].

## Scale Factor

Today's inertial sensors are controlled by digital computers which, unlike earlier sensors, require Analog to Digital converters to digitize the signals. Scale factor is the ratio between a change in the output signal and a change in input signal. Scale factor is the ratio of the input signal to the output signal. Figure 5 shows the scale factor, where the output signal has been linearized. The input and output signals are varied over a specified range and curve fitted using a method of least squares [9]. The curve fit ends up being a single number defined as the slope of a straight line. Since most sensors provide an

output signal directly proportional to the input, scale factor is not considered in this thesis.

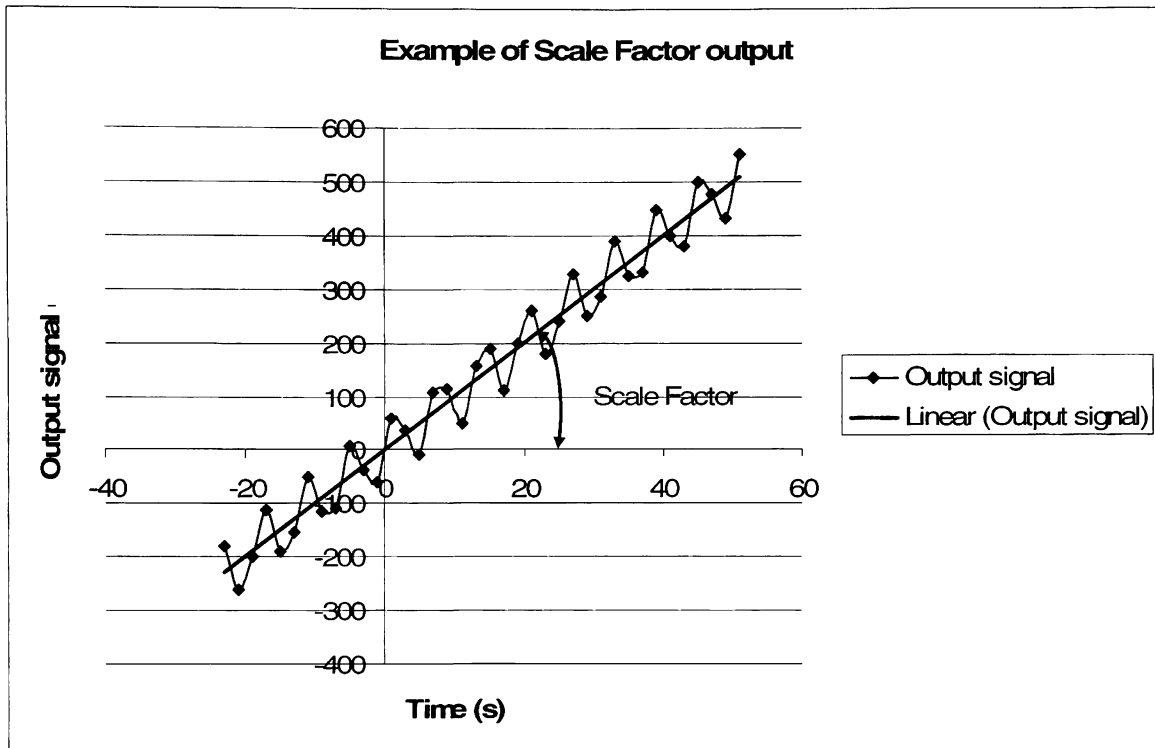


Figure 5: Scale Factor

## Noise

Noise is another error that is usually associated with the measurement error. This comes about as thermal noise and random variations in electrical circuits in the computer and the instruments. [10] This error is usually small compared to the bias in the system. This error is amplified every time the measured signal is integrated. The final integrated output of acceleration is position and for angular rates is Euler angles. The error is propagated as well and magnified after integration, sometimes masking the real value. This thesis uses the Kalman filter algorithm to minimize the noise in the system. The Kalman filter

algorithm takes the error between the GPS and INS accelerations and tracks this error during flight.

## **Non-Linearity**

Non-linear errors are also associated with the measurement error. Non-linearity is the behavior of a circuit in which the input excitation signal does not vary directly with the output signal [11]. This non-linearity causes distortion in the output signal thus causing repeatable non-linear readings. The best way to minimize the effects of non-linearity on signal reading is to use digital signals that are fully on or fully off. Non-linearity errors are much smaller than the bias errors. For the purposes of this thesis, the error will be ignored.

## **INS/GPS Aiding**

GPS provide precise location accuracy and thus can be used to align an inexpensive Inertial Navigation System (INS). The GPS could be used to provide an initialization of the INS and the INS could be used to provide navigation when the GPS experiences signal dropouts. This GPS/INS system is thus more useful and precise for navigation as compared to either system by itself.

The notion of INS being used for primary navigation in aircraft is not a new one. Commercial aircraft have used INS for guidance and navigation. For some time these systems however, have been quite expensive. This prohibitive cost of procurement has resulted in INS systems not being extensively utilized in General Aviation and flight



testing aircraft. INS use a combination of gyros and accelerometers hence reducing the need for external aiding [2]. A general rule of thumb is the longer the INS can navigate independently the more expensive (and more accurate) the INS is. For INS that are used for independent long range navigation, special attention is paid to the drift of the system. If the INS is used for aiding GPS navigation, the system need not be accurate over long periods of time [2]. The MEMS based technology has provided a low cost alternative for attitude reference systems. The MEMS based INS has much higher drift rates than the other types of INS and will thus have to be tightly coupled with the GPS.

### **INS Kalman Filtering**

The accelerations from the INS in the navigation frame will have errors as discussed earlier. These errors resulting from the measurement devices themselves and computation round off, affect the output of the INS navigation and contribute to drift in the system [2]. The Kalman filter is used to track these measurement errors and minimize them. The INS navigation solution is subtracted from an external navigation aid, such as the GPS to determine the difference in the navigation solution. The errors are then estimated and subtracted from subsequent INS solutions. The errors vary slowly with time, or with the position of the vehicle with respect to the earth. The six errors that need to be computed are three acceleration errors and three position errors. These errors vary slowly with time and thus will be easy to track using a Kalman filter.

## **Conclusion**

The complete system will consist of INS, GPS and a Kalman filter algorithm. The INS/GPS aiding is not new and has been used in the past. This thesis attempts to address the cost of this system by using commercially available off the shelf equipment and existing error computations schemes easy to implement in general aviation and flight testing aircraft.

## **Chapter 3: Methodology**

To obtain a navigation solution, the differential GPS latitude, longitude, and altitude are used as inputs to the computation. An INS solution has been added to the navigation solution to provide independent monitoring of the GPS signal [2]. The INS provides an independent reference system since it is invulnerable to external jamming or spoofing. To increase the accuracy of the signal, a Kalman filter is utilized.

The difference in the lag between the GPS and the INS needs to be known for navigation solution correction purposes. This is achieved by computing the difference between the GPS and INS update rates. The system monitors the variance in the GPS signal, variance in the MEMS based INS signal, and the correlation between the INS and GPS.

Good and consistent correlation between the INS and GPS is an indicator of a healthy GPS signal. Variation of the INS is measured in real time and is used to calculate the expectation of the minimum random walk [2].

An algorithm has been developed that performs the navigation solution computation by performing the following tasks:

1. Determines the GPS Position, velocity and acceleration.
2. Determines the INS navigation solution, position, velocity and acceleration
3. Determines the Integrated Navigation Solution position

## GPS Navigation Solution

The GPS receiver obtains latitude, longitude, and altitude from the satellites. The latitude and longitude are provided in degrees and minutes while the altitude is provided in meters. The latitude and longitude are converted to feet using Equation 1 and Equation 2:

$$\lambda_{ft} = (\lambda_2 - \lambda_1) * R_{\oplus}$$

**Equation 1**

$$l_{ft} = (l_2 - l_1) * R_{\oplus} * \cos(\lambda)$$

**Equation 2**

Given the starting point has the point set at 0 feet, the change in distance and altitude from the starting point will yield a present position distance and altitude. The GPS variance is an important factor essential to the navigation solution.

## GPS Variance

The GPS variance is determined by averaging a series of measurement errors for each particular measurement. The variance is obtained by measuring the GPS output for a period of one hour. The GPS variance can be obtained by measuring for a shorter period of time. The GPS is making measurements in the navigation frame. This differs from the MEMS based AHRS that is a strap down system whose measurements are in the body's frame of reference. The variances in the GPS are computed independently for all three axes. The GPS signal is vulnerable to spoofing and jamming. Another independent source that may be utilized to provide an integrity signal that is invulnerable to spoofing is the

MEMS based AHRS. The MEMS based AHRS is used to provide an integrity signal to the GPS navigation solution.

## **AHRS**

The AHRS provides raw accelerations and angular rate measurements to the INS solution. The accelerations provided are aligned with the body axis of the aircraft, hence a strapdown system [2]. The AHRS measurements have errors that may be due to external factors such as wind turbulence as well as internal factors due to manufacturing processes. These factors have to be separated in order for the pure measurement to be obtained. Usually internal factors have high frequencies.

External atmospheric effects that impinge on the aircraft are usually low frequency events. Any interference that is less than 50Hz will be filtered out by the aircraft. This is due to the inherent inertia and mass of the aircraft, allowing the aircraft to act as a low pass filter [2]. The measurement errors experienced must therefore be due to the random nature of the AHRS itself. The AHRS must be shielded from high frequency output otherwise the measurements will be affected. The raw measurements are used as inputs to the INS computation

## Inertial Navigation

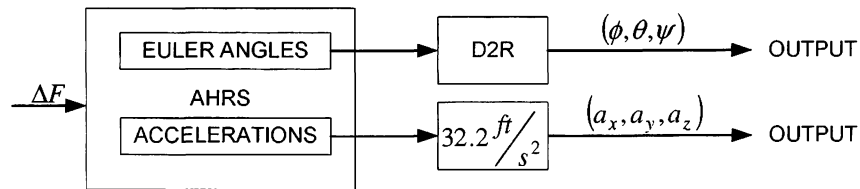
Inertial navigation integrates accelerations and angles from an AHRS to form a navigation solution in terms of present position and altitude. The accelerations are mapped through the Euler angles from the body axis to the navigation frame.

The strapdown AHRS measures the accelerations in the body axis frame. The accelerations are integrated once to obtain the vehicle's velocity. The double integration of the accelerations yields a vehicle's position. The random noise in the acceleration measurements is converted to random walk, which is known as drift in the navigation solution. The MEMS based INS has higher drift rates as compared to traditional INS systems. Traditional systems, once given location of initial position, do not need to be updated in regular flight. The MEMS based INS however has to be updated in regular flight due to the high drift rates experienced. This update is performed by the Kalman filter error model. The AHRS integrated output is compared to the GPS position based navigation solution. The difference is estimated as a slowly time varying signal known as the offset. The offset is subtracted from the INS accelerations solution and acts as a constant of integration in the solution. The drift in the INS accelerations is estimated in the Kalman filter. Process noise is added to the estimate to allow the signal to vary with time. The accuracy of the response of the estimate can be tuned by adjustment of the process noise,  $Q_k$ , of the Kalman filter estimate, which acts as a filter time constant.

## Inertial Navigation Solution Computation

The main basis for this whole system is coupling an Inertial Navigation Solution (INS) with a GPS to obtain error corrected aircraft attitude information. The AHRS is the mainstay of the INS solution. The AHRS measures raw Euler angles  $(\phi, \theta, \psi)$ , rates  $(\dot{\phi}, \dot{\theta}, \dot{\psi})$  and accelerations  $(a_x, a_y, a_z)$  in the ABC (body) frame. The attitude of the aircraft must be determined in order to transform the linear acceleration from the body frame to the navigation frame.

The AHRS outputs that are used in the computations are the Euler angles and the accelerations. The Euler angles, which are in degrees, are converted to radians as shown in Figure 6. The accelerations are in g and thus they have to be multiplied by  $32.2 \text{ ft} / \text{s}^2$  in order to be used in this computation as shown in Figure 6.



**Figure 6: AHRS Output**

In mathematical Matrix format,

$$S_{\text{deg}} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

**Equation 3**



In radians,

$$S_{rad} = \frac{1}{57.3} \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

**Equation 4**

Accelerations;

$$a_g = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

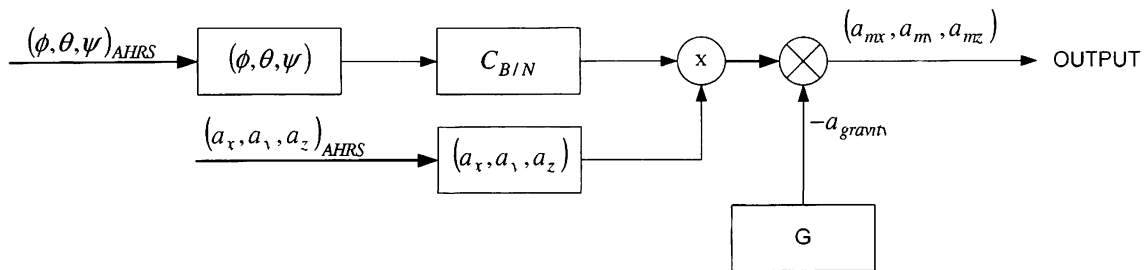
**Equation 5**

In ft/s<sup>2</sup>

$$a_{fps} = 32.2 \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

**Equation 6**

The Euler angles are then used as the inputs to the Direction Cosine Matrix (DCM) which transforms the AHRS accelerations to the navigation or NED (North-East-Down) coordinate frame to allow for ease of comparison of the AHRS accelerations with the GPS accelerations as shown in Figure 7.



**Figure 7: Coordinate transform for acceleration and gravity compensation**

In mathematical matrix format, the DCM is as follows in Equation 7:

$$C_{N/B} = \begin{bmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) & (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) & (\sin \phi \cos \theta) \\ (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) & (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) & (\cos \phi \cos \theta) \end{bmatrix}$$

**Equation 7**

The inputs to the DCM are the Euler angles in radians. The DCM is then transposed so that the accelerations can be transformed from the Body frame to the Navigation frame coordinate system.

$$C_{B/N} = \begin{bmatrix} \cos \theta \cos \phi & (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) & (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ \cos \theta \sin \phi & (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) & (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ -\sin \theta & (\sin \phi \cos \theta) & (\cos \phi \cos \theta) \end{bmatrix}$$

**Equation 8**

Equation 8 simply states that in going from body frame coordinate system to navigation frame coordinate system the matrix is used. The accelerations are then multiplied with the DCM as shown in Equation 9

$$\begin{bmatrix} a_{nx} \\ a_{ny} \\ a_{nz} \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \phi & (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) & (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ \cos \theta \sin \phi & (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) & (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ -\sin \theta & (\sin \phi \cos \theta) & (\cos \phi \cos \theta) \end{bmatrix} * \begin{bmatrix} a_{fps} \\ a_{fps} \\ a_{fps} \end{bmatrix}$$

**Equation 9**

The accelerations still have gravity effects and this has to be compensated for as follows in Equation 10 and Equation 11:

$$\begin{bmatrix} a_{mx} \\ a_{my} \\ a_{mz} \end{bmatrix} = \begin{bmatrix} a_{nx} \\ a_{ny} \\ a_{nz} \end{bmatrix} - \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

**Equation 10**

where

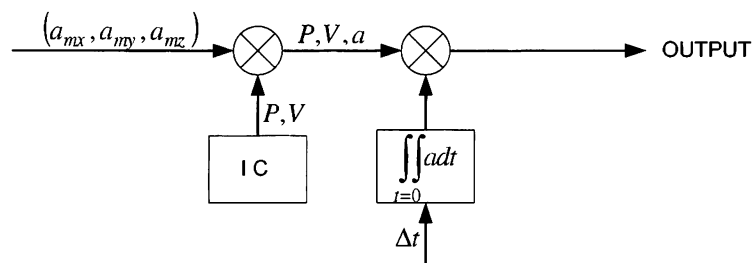
$a_m$  is the acceleration whose gravity effect has been removed

$a_n$  is the acceleration that still has the gravity effects been removed

$$\text{where } \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 32.2 \end{bmatrix}$$

**Equation 11**

Initial conditions pertaining to the speed and velocity are inputs specified by the user. (In this case the initial values are all zeros since the assumption is that the point of reference is the starting point.) The user updates using the accelerations from the AHRS and Newton's equations of motion (phi) as shown in Figure 8 below.



**Figure 8: Inertial solution obtained using the AHRS solely**

After the gravity adjustment, the acceleration is combined with the larger initial attitude matrix as follows in Equation 12:

$$att_{init} = \begin{bmatrix} s_x \\ v_x \\ a_{mx} \\ s_y \\ v_y \\ a_{my} \\ s_z \\ v_z \\ a_{mz} \end{bmatrix}$$

**Equation 12**

The phi matrix is the integration of the accelerations to produce velocity, and the integration of the velocity to produce position. The phi matrix is as follows in Equation 13:

$$\Phi = \begin{bmatrix} 1 & \Delta T & 0.5\Delta T^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \Delta T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta T & 0.5\Delta T^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta T & 0.5\Delta T^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

**Equation 13**

The new attitude is obtained by matrix multiplying the two matrices together as follows in Equation 14:

$$\begin{bmatrix} s_x \\ v_x \\ a_x \\ s_y \\ v_y \\ a_y \\ s_z \\ v_z \\ a_z \end{bmatrix}_{new} = \begin{bmatrix} 1 & \Delta T & 0.5\Delta T^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \Delta T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta T & 0.5\Delta T^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta T & 0.5\Delta T^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x \\ v_x \\ a_{mx} \\ s_y \\ v_y \\ a_{my} \\ s_z \\ v_z \\ a_{mz} \end{bmatrix}$$

Equation 14

It is important to remember that the AHRS measurement errors will affect the final inertial solution. The GPS is used to correct the AHRS drift by using the more constant accelerations that are derived from the position data by time differentiation as shown in Figure 9 below:

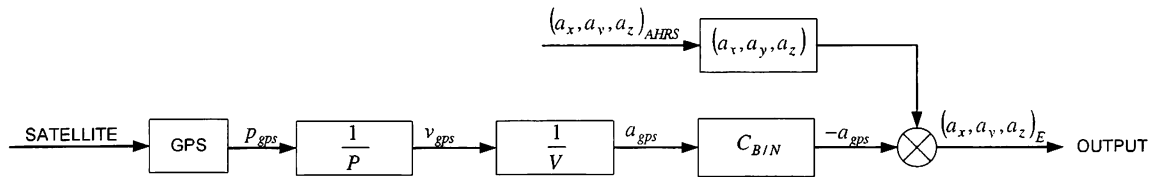


Figure 9: GPS acceleration computation and acceleration error analysis

The GPS velocity and acceleration computation is as follows in Equation 15 and Equation 16:

$$\frac{d}{dt} \begin{bmatrix} s_{gx} \\ s_{gy} \\ s_{gz} \end{bmatrix} = \begin{bmatrix} v_{gx} \\ v_{gy} \\ v_{gz} \end{bmatrix}$$

**Equation 15**

$$\frac{d}{dt} \begin{bmatrix} v_{gx} \\ v_{gy} \\ v_{gz} \end{bmatrix} = \begin{bmatrix} a_{gx} \\ a_{gy} \\ a_{gz} \end{bmatrix}$$

**Equation 16**

The GPS accelerations which are in NED frame are transformed to the body frame so that they can be compared with the AHRS accelerations.

$$\begin{bmatrix} a_{gx} \\ a_{gy} \\ a_{gz} \end{bmatrix}_t = \begin{bmatrix} \cos \theta \cos \phi & (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) & (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ \cos \theta \sin \phi & (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) & (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ -\sin \theta & (\sin \phi \cos \theta) & (\cos \phi \cos \theta) \end{bmatrix} \begin{bmatrix} a_{gx} \\ a_{gy} \\ a_{gz} \end{bmatrix}$$

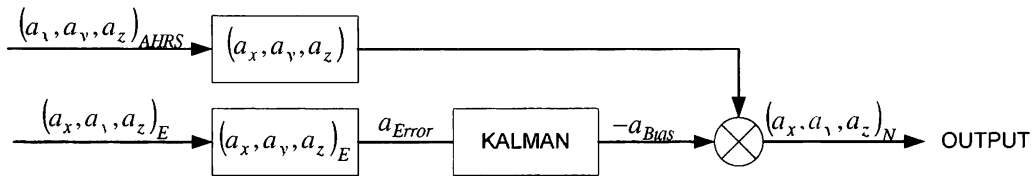
**Equation 17**

The AHRS accelerations are subtracted from the GPS accelerations as shown in Equation 18.

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} - \begin{bmatrix} a_{gx} \\ a_{gy} \\ a_{gz} \end{bmatrix}_{trans} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}_{ERROR}$$

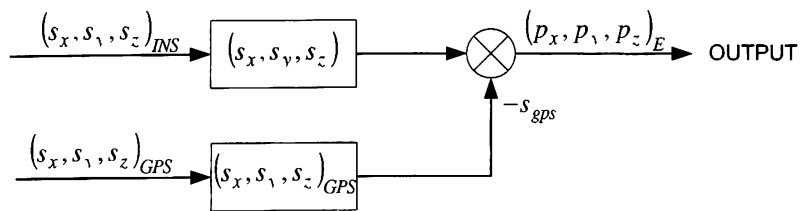
**Equation 18**

The error thus becomes a Kalman Filter input known as the Measurement, ( $Z_k$ ). The Kalman filter computes the new state estimate ( $X_{k+1}$ ) which is subtracted from the AHRS accelerations. The new state estimate in this case is also known as the signal bias. The accelerations from the AHRS are offset from the true value by a constant state estimate and by subtracting the estimate; the true value is obtained as shown in Figure 10. It should be noted however, that the signal still varies “around” the mean true value as it still has other errors.



**Figure 10: AHRS accelerations corrected for signal bias using GPS**

With this correction, the accelerations in the Inertial Navigation Solution are more constant and predictable. However, the Inertial Solution now only relies on accelerations for stability. This reduces data redundancy as it provides the system a single point of failure, namely the accelerations. To increase data redundancy and data integrity, the other two parts (velocity and position) of the Inertial Navigation Solution may be used. In this paper, only one part (the position) of the Inertial Navigation Solution was considered.



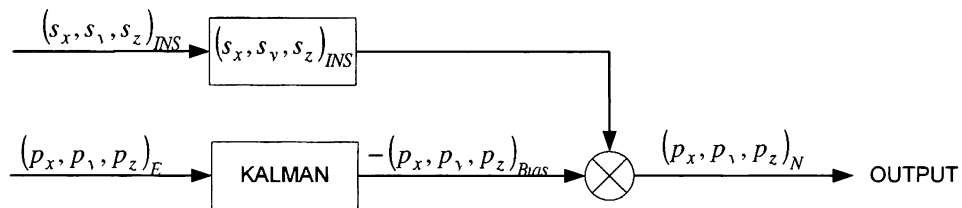
**Figure 11: Position error computation**

In matrix math terms;

$$\begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}_{new} - \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}_{GPS} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}_{ERROR}$$

**Equation 19**

This error is then used as the Measurement ( $Z_k$ ) and fed into the Position Kalman Filter as one of the inputs. The output of the Position Kalman Filter is a steady state solution bias known as the state estimate. This bias is then subtracted from the INS position hence providing more stable real values of the current aircraft position.



**Figure 12: Compensated aircraft position solution**

## General Kalman Filter Equations

The navigation algorithm implements a Kalman filter to estimate constants and to blend redundant measurement. The states of the filter are the three accelerations and position components of the aircraft. The navigation algorithm implements a Kalman Filter to estimate constants and blend redundant measurements.

The standard equations for the extended Kalman Filter are:



$$x_{k+1} = \phi_k x_k + w_k$$

**Equation 20**

and

$$z_k = H_k x_k + v_k$$

**Equation 21**

where

$x_{k+1}$  is the updated state,

$\phi_k$  is the discrete state transition matrix

$x_k$  is the present state,

$w_k$  is the random process noise

$z_k$  is the measurement

$H_k$  is the connection matrix between the state and the measurement,

$v_k$  is the random measurement noise

The solution of these equations takes the form of the matrix Riccati equations. The Kalman gain is computed as shown in Equation 22:

$$K_k = P_k^- H_k^T \left( H_k P_k^- H_k^T + R_k \right)^{-1}$$

**Equation 22**

where

$K_k$  is the Kalman gain,

$P_k^-$  is the covariance matrix of the current estimate,

$R_k$  is the variance in the measurement noise vector,  $v_k$

The states and covariance matrices are updated by the following sequence of Riccati equations

$$\hat{X}_k = \hat{X}_k^- + K_k \left( Z_k - H_k \hat{X}_k^- \right)$$

**Equation 23**

$$P_k = (I - K_k H_k) P_k^-$$

**Equation 24**

$$\hat{X}_{k+1}^- = \phi_k \hat{X}_k$$

**Equation 25**

$$P_{k+1}^- = \phi_k P_k \phi_k^T + Q_k$$

**Equation 26**

where

$\hat{X}_k$  is the transitional state estimate,

$\hat{X}_k^-$  is the current state estimate,

$P_k$  is the updated covariance matrix

$\hat{X}_{k+1}$  is the updated state estimate

$Q_k$  is the process noise of the state transition matrix

The general Kalman Filter equations can now be combined to be more specific for this application of independent aircraft navigation. The Riccati Equations that comprise the Kalman Filter are computed in Equation 27, Equation 28, Equation 29 and, Equation 30:

$$M_k = \phi_k P_k \phi_k^T + Q_k$$

**Equation 27**

$$K_k = M_k H_k \left( H M_k H^T + R_k \right)^{-1}$$

**Equation 28**

$$P_k = \left( I - K_k H \right) M_k$$

**Equation 29**

$$\hat{X}_k = \phi_k \hat{X}_{k-1} + K_k \left( Z_k - H \phi_k \hat{X}_{k-1} \right)$$

**Equation 30**

## **Acceleration Kalman Filter**

The Acceleration Kalman Filter is used to reduce the error occurrence between the INS and the GPS computed accelerations. The Acceleration Kalman Filter has the following steps to perform

- Prediction
- Kalman Gain
- Measurement update

## Prediction

In the prediction step, there are no measurements available. To calculate the values of the

covariances  $M_k$  and predicted state estimates  $\hat{X}_k$  at time  $t_k$ , the system model is used

for propagation:

$$M_k = \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} P_{k-1} & 0 & 0 \\ 0 & P_{k-1} & 0 \\ 0 & 0 & P_{k-1} \end{bmatrix} \begin{bmatrix} \phi_k^T & 0 & 0 \\ 0 & \phi_k^T & 0 \\ 0 & 0 & \phi_k^T \end{bmatrix} + \begin{bmatrix} Q_k & 0 & 0 \\ 0 & Q_k & 0 \\ 0 & 0 & Q_k \end{bmatrix}$$

Equation 31

$$\hat{X}_k = \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} \hat{X}_{k-1} \\ \hat{X}_{k-1} \\ \hat{X}_{k-1} \end{bmatrix} + \begin{bmatrix} K_k & 0 & 0 \\ 0 & K_k & 0 \\ 0 & 0 & K_k \end{bmatrix} \begin{bmatrix} a_{xbias} \\ a_{ybias} \\ a_{zbias} \end{bmatrix} - \begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} \hat{X}_{k-1} \\ \hat{X}_{k-1} \\ \hat{X}_{k-1} \end{bmatrix}$$

Equation 32

Here, the  $M_k$  and  $\hat{X}_k$  are the covariance and state estimates respectively. The values of

$\phi_k^T$  are initially set to a large enough number indicating the reduced confidence in the

initial estimates. The 3x3 matrices indicate the three axes of rotation that are considered in this thesis; the X, Y, and Z planes.

The difference,  $\left( \begin{array}{c} \begin{bmatrix} a_{xbias} \\ a_{ybias} \\ a_{zbias} \end{bmatrix} - \begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \begin{bmatrix} \hat{X}_k^- \\ \hat{X}_k^- \\ \hat{X}_k^- \end{bmatrix} \end{array} \right)$  is called the measurement

innovation, or the residual. The residual reflects the discrepancy between the predicted

measurement  $\begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \begin{bmatrix} \hat{X}_k^- \\ \hat{X}_k^- \\ \hat{X}_k^- \end{bmatrix}$  and the actual measurement  $\begin{bmatrix} a_{xbias} \\ a_{ybias} \\ a_{zbias} \end{bmatrix}$ . A residual

of zero means that the two are in complete agreement.

## Kalman Gain

The Kalman gain  $K_k$  reflects the optimal trade-off between trusting the measurements (which gives noisy output) and trusting the propagated state estimate from (which gives bad tracking possibility).

$$\begin{pmatrix} K_k \\ K_k \\ K_k \end{pmatrix} = \begin{bmatrix} M_k & 0 & 0 \\ 0 & M_k & 0 \\ 0 & 0 & M_k \end{bmatrix} \begin{bmatrix} H_k^T & 0 & 0 \\ 0 & H_k^T & 0 \\ 0 & 0 & H_k^T \end{bmatrix} \left( \begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \begin{bmatrix} M_k & 0 & 0 \\ 0 & M_k & 0 \\ 0 & 0 & M_k \end{bmatrix} \begin{bmatrix} H_k^T & 0 & 0 \\ 0 & H_k^T & 0 \\ 0 & 0 & H_k^T \end{bmatrix} \right)^{-1} + \begin{bmatrix} R_k & 0 & 0 \\ 0 & R_k & 0 \\ 0 & 0 & R_k \end{bmatrix}$$

**Equation 33**

where  $R_k$  is the covariance of the measurement noise  $v_k$ .

## Measurement Update

When an observation is available, the Kalman filter has more information and therefore the previously predicted states are no longer the optimal ones. The updated covariances and states are then:

$$\begin{pmatrix} P_k \\ P_k \\ P_k \end{pmatrix} = \left( \begin{bmatrix} I_k & 0 & 0 \\ 0 & I_k & 0 \\ 0 & 0 & I_k \end{bmatrix} - \begin{bmatrix} K_k & 0 & 0 \\ 0 & K_k & 0 \\ 0 & 0 & K_k \end{bmatrix} \begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \right) \begin{bmatrix} M_k & 0 & 0 \\ 0 & M_k & 0 \\ 0 & 0 & M_k \end{bmatrix}$$

**Equation 34**

$$\begin{pmatrix} \hat{X}_{k+1} \\ \hat{X}_{k+1} \\ \hat{X}_{k+1} \end{pmatrix} = \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} \hat{X}_k \\ \hat{X}_k \\ \hat{X}_k \end{bmatrix} + \begin{bmatrix} K_k & 0 & 0 \\ 0 & K_k & 0 \\ 0 & 0 & K_k \end{bmatrix} \begin{bmatrix} a_{xbias} \\ a_{ybias} \\ a_{zbias} \end{bmatrix} - \begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} \hat{X}_k \\ \hat{X}_k \\ \hat{X}_k \end{bmatrix}$$

**Equation 35**

The above filter computes the error in the acceleration bias. The acceleration error ( $Z_k$ ) is given by:

$$a_{error} = a_{INS} - a_{GPS}$$

**Equation 36**

The acceleration error is the input into the Kalman Filter equation. The output is called the bias. This bias is more constant with time and hence will be subtracted from the AHRS accelerations.

## **Position Kalman Filter**

The Position Kalman Filter performs similar function as the Acceleration Kalman Filter. As a matter of fact, the only difference is in the position bias. The Position Kalman Filter has inputs as shown below.

### **Prediction**

In the prediction step, there are no measurements available. To calculate the values of the covariances  $M_k$  and predicted state estimates  $\hat{X}_k$  at time  $t_k$ , the system model is used for propagation:

$$M_k = \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} P_{k-1} & 0 & 0 \\ 0 & P_{k-1} & 0 \\ 0 & 0 & P_{k-1} \end{bmatrix} \begin{bmatrix} \phi_k^T & 0 & 0 \\ 0 & \phi_k^T & 0 \\ 0 & 0 & \phi_k^T \end{bmatrix} + \begin{bmatrix} Q_k & 0 & 0 \\ 0 & Q_k & 0 \\ 0 & 0 & Q_k \end{bmatrix}$$

**Equation 37**

$$\hat{X}_k = \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} \hat{X}_{k-1} \\ \hat{X}_{k-1} \\ \hat{X}_{k-1} \end{bmatrix} + \begin{bmatrix} K_k & 0 & 0 \\ 0 & K_k & 0 \\ 0 & 0 & K_k \end{bmatrix} \left( \begin{bmatrix} p_{xbias} \\ p_{ybias} \\ p_{zbias} \end{bmatrix} - \begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} \hat{X}_{k-1} \\ \hat{X}_{k-1} \\ \hat{X}_{k-1} \end{bmatrix} \right)$$

**Equation 38**

Here, the  $M_k$  and  $\hat{X}_k$  are the covariance and state estimates respectively. The values of  $\phi_k^T$  are initially set to a large enough number indicating the reduced confidence in the initial estimates. The 3x3 matrices indicate the three axes of rotation that are considered in this thesis; the X, Y, and Z planes.

## Kalman Gain

The Kalman gain  $K_k$  reflects the optimal trade-off between trusting the measurements (which gives noisy output) and trusting the propagated state estimate from (which gives bad tracking possibility).



$$\begin{pmatrix} K_k \\ K_k \\ K_k \end{pmatrix} = \begin{bmatrix} M_k & 0 & 0 \\ 0 & M_k & 0 \\ 0 & 0 & M_k \end{bmatrix} \begin{bmatrix} H_k^T & 0 & 0 \\ 0 & H_k^T & 0 \\ 0 & 0 & H_k^T \end{bmatrix} \left( \begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \begin{bmatrix} M_k & 0 & 0 \\ 0 & M_k & 0 \\ 0 & 0 & M_k \end{bmatrix} \begin{bmatrix} H_k^T & 0 & 0 \\ 0 & H_k^T & 0 \\ 0 & 0 & H_k^T \end{bmatrix} + \begin{bmatrix} R_k & 0 & 0 \\ 0 & R_k & 0 \\ 0 & 0 & R_k \end{bmatrix} \right)^{-1}$$

**Equation 39**

where  $R_k$  is the covariance of the measurement noise  $v_k$ .

## Measurement Update

When an observation is available, the Kalman filter has more information and therefore the previously predicted states are no longer the optimal ones. The updated covariances and states are then:

$$\begin{pmatrix} P_k \\ P_k \\ P_k \end{pmatrix} = \left( \begin{bmatrix} I_k & 0 & 0 \\ 0 & I_k & 0 \\ 0 & 0 & I_k \end{bmatrix} - \begin{bmatrix} K_k & 0 & 0 \\ 0 & K_k & 0 \\ 0 & 0 & K_k \end{bmatrix} \begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \right) \begin{bmatrix} M_k & 0 & 0 \\ 0 & M_k & 0 \\ 0 & 0 & M_k \end{bmatrix}$$

**Equation 40**

$$\begin{pmatrix} \hat{X}_{k+1} \\ \hat{X}_{k+1} \\ \hat{X}_{k+1} \end{pmatrix} = \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} \hat{X}_k \\ \hat{X}_k \\ \hat{X}_k \end{bmatrix} + \begin{bmatrix} K_k & 0 & 0 \\ 0 & K_k & 0 \\ 0 & 0 & K_k \end{bmatrix} \begin{bmatrix} p_{xbias} \\ p_{ybias} \\ p_{zbias} \end{bmatrix} - \begin{bmatrix} H_k & 0 & 0 \\ 0 & H_k & 0 \\ 0 & 0 & H_k \end{bmatrix} \begin{bmatrix} \phi_k & 0 & 0 \\ 0 & \phi_k & 0 \\ 0 & 0 & \phi_k \end{bmatrix} \begin{bmatrix} \hat{X}_k \\ \hat{X}_k \\ \hat{X}_k \end{bmatrix}$$

**Equation 41**

The above filter computes the error in the position bias. The position error ( $Z_k$ ) is given by:

$$P_{error} = P_{INS} - P_{GPS}$$

Equation 42

## Process Noise Matrix

In a perfect world, there is no noise in a system. In the real world however, every system has some measure of noise within it (due to the nature of how it is built) and the environment [4]. The covariance matrix is a best guess estimate of the signal broken down into two separate parts as shown in Equation 27. The two parts are the bias and the random process noise. The bias, as stated earlier can be predicted by an equation since it is a constant. The random process noise is very random and hence cannot be accurately represented by any equation. If the random process noise is zero it means that we believe our Kalman filter model of the real world is excellent [4]. Thus the code will mirror the motion of the INS. This is the simplest form of the Kalman Filter equations.

When the process noise matrix ( $Q_k$ ) is added, the Kalman Filter equation attempts to quantify the noise that is in the system. Noise is not quantifiable with any degree of accuracy, hence the name noise. It is erratic and unpredictable, and thus, one cannot come up with a deterministic equation for the noise. Large values of the random process noise means that we do not believe that the Kalman filter has a good model of the real world and the filter must be prepared for a great deal of uncertainty [4]. In this case the code will mirror the GPS's motion.

It is anticipated that the noise, however, moves slowly with time. The process noise matrix always assumes that the process noise always enters the system model on the derivative of the last state (i.e. Highest derivative) [4]. The process noise matrix for the acceleration Kalman filter is as follows:

$$Q_{kA} = \begin{bmatrix} Q_{kxA} & 0 & 0 \\ 0 & Q_{kyA} & 0 \\ 0 & 0 & Q_{kzA} \end{bmatrix}$$

**Equation 43**

Where:

$Q_{kxA}$  is the process noise for the acceleration in x-direction (north)

$Q_{kyA}$  is the process noise for the acceleration in y-direction (east)

$Q_{kzA}$  is the process noise for the acceleration in z-direction (altitude)

The process noise matrix for the position Kalman filter is as follows:

$$Q_{kP} = \begin{bmatrix} Q_{kxP} & 0 & 0 \\ 0 & Q_{kyP} & 0 \\ 0 & 0 & Q_{kzP} \end{bmatrix}$$

**Equation 44**

Where:

$Q_{kx_p}$  is the process noise for the x-direction (north)

$Q_{ky_p}$  is the process noise for the y-direction (east)

$Q_{kz_p}$  is the process noise for the z-direction (altitude)

## Tuning the Kalman Filter

The process of changing  $Q_k$  matrix values to values that give a desired output is known as Tuning the Kalman Filter. Each of the two Kalman Filters used in this thesis have their own  $Q_k$  matrices. Each has to be changed on its own, beginning with the Acceleration Kalman Filter [4]. When the values do not represent the actual system noise uncertainty, it simply becomes a fudge factor that accounts for our lack of knowledge of the real world [4]. Our Kalman Filter model for the real world is excellent when  $Q_k$  is small or zero. Large values of  $Q_k$  mean that we do not believe that the Kalman Filter has a good model of the real world and the filter must be prepared for a great deal of uncertainty [4]. Usually,  $Q_k$  is determined by performing many Monte Carlo experiments under a variety of scenarios [4]. Monte Carlo method is approximate and is simply to do ensemble averaging to get the mean and standard deviation. Usually, a large number of simulation trials are required to prove confidence in the accuracy of the results [4]. Since we assumed that noise moves slowly with time, it is possible to tune the variables of the  $Q_k$  matrix so that the Kalman Filter output closely resembles the input.

The algorithm monitors the offsets and the derivatives of the offsets between the GPS and the MEMS based INS position vectors. A jump in the drift in the INS would indicate a jammed signal. In this case the GPS is assumed failed and the navigation solution uses the MEMS based INS.

## **LabVIEW Code**

LabVIEW is a software program that is used industry wide for data acquisition and analysis. Developed by National Instruments, this program has proved successful in this arena for ease of use and ease of interface with various equipments [10].

LabVIEW is a high level Graphical Development software for designing test, measurement and control systems [10]. LabVIEW has the flexibility of a programming language and is combined with built-in tools designed specifically for test, measurement, and control. For the purposes of this thesis, LabVIEW is used to perform the following functions:

1. Open interface ports with the necessary instruments (in this case GPS and AHRS)
2. Read data from the instruments
3. Filter data to obtain the desired information
4. Compute Inertial Navigation Solution
5. Compute Kalman Filter Solution
6. Compute Integrated Navigation Solution.
7. Display Navigation Solution.
8. Store solution in a tab delimited format.

For ease of programming and testing, the whole program is broken down into several parts:

1. **GPS code:** This code opens the GPS communication port, acquires GPS data from that port, filters out the unnecessary data and computes the position, velocity and acceleration. This code is only as fast as the GPS sensing unit itself.
2. **AHRS code:** This code opens the AHRS communication port, acquires AHRS data from that port, filters out the unnecessary data and transforms the accelerations from Body to Navigation frames.
3. **Acceleration Kalman Filter Code:** This code acquires the difference between the GPS and AHRS accelerations (acceleration error) and outputs a more stable, predictable acceleration bias.
4. **Inertial Navigation Solution Code:** This code computes the difference between the AHRS and Kalman filter acceleration bias to obtain starting point acceleration. The starting point is usually denoted by position and velocity vectors of zero. As the vehicle on which this is mounted moves, the position velocity and acceleration values change and this thus yields an Inertial Solution.
5. **Position Kalman Filter Code:** This code acquires the difference between the GPS position and Inertial Navigation Solution Code position (position error) and outputs a more stable, predictable position bias. This position bias is then subtracted from the Inertial Solution to obtain the Integrated Solution of position.

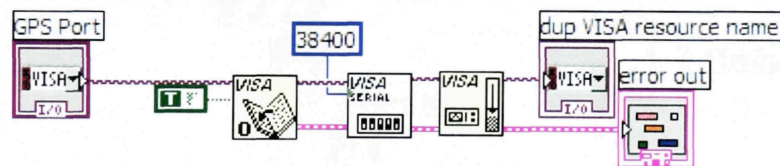
## **GPS Code**

The following code captures the GPS signals, displays and stores navigation related data.

These are used for navigation and they include the following fields:

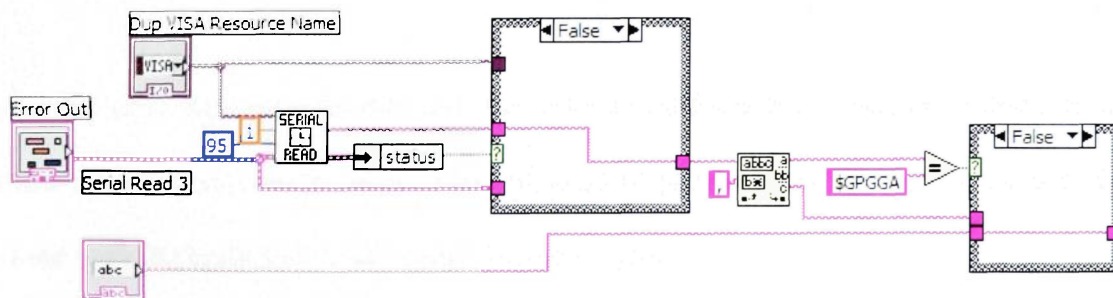
1. Latitude
2. Longitude
3. Altitude
4. Time

The first step is to initialize the respective port to receive GPS Aeronautical Radio Incorporated 429 (ARINC-429) sentences. The Baud rate- a measure of how fast data moves between two instruments connected via serial communication- varies from one receiver to the other, from as high as 38400 receiver to as slow as 4800. The code shown in Figure 13 accomplishes the task of initializing the serial port and setting the baud rate.

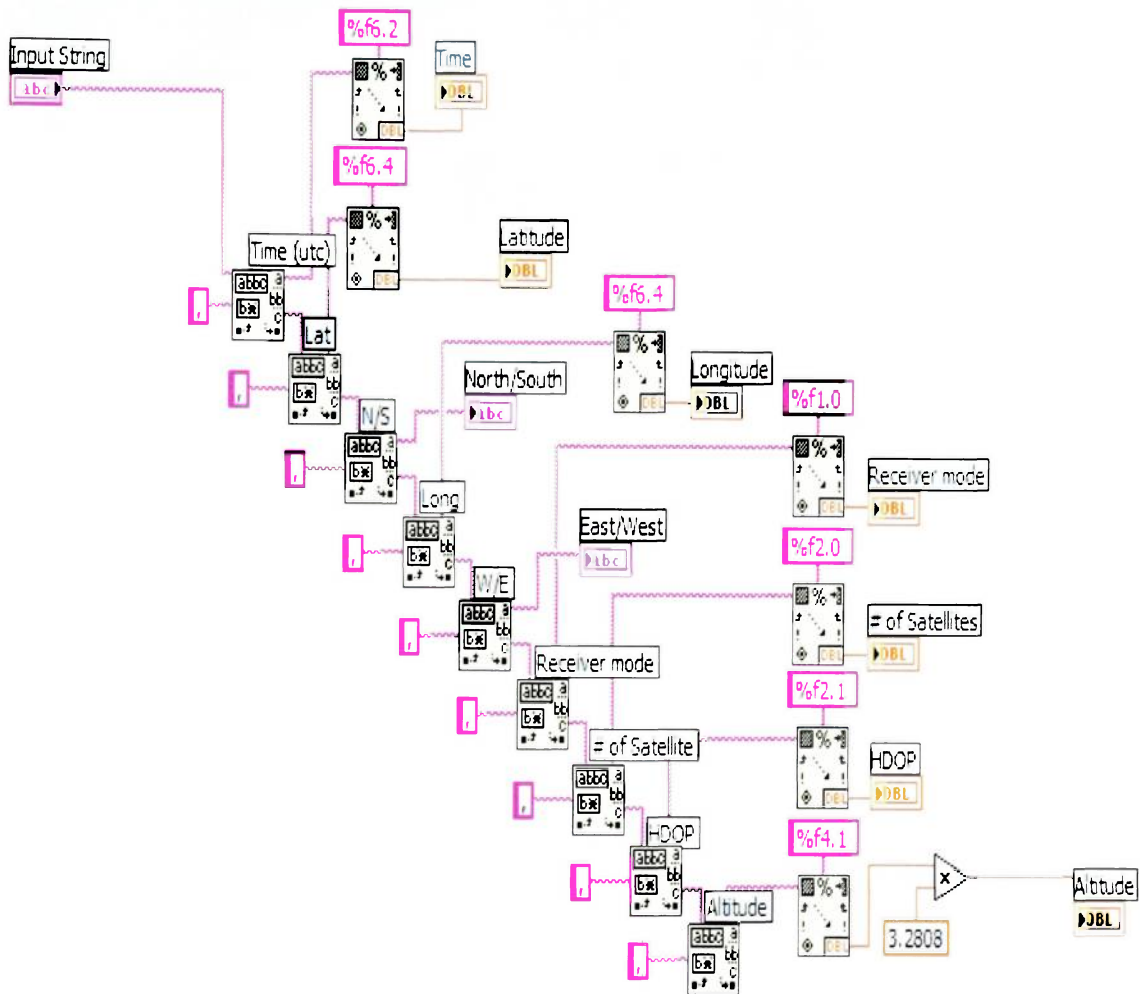


**Figure 13: GPS Initialization code**

Once the initialization has taken place, the ARINC-429 sentence has to be read by the LabVIEW code and then broken up to obtain the necessary data; latitude, longitude, and altitude. This is accomplished by the code shown in Figure 14 and Figure 15.



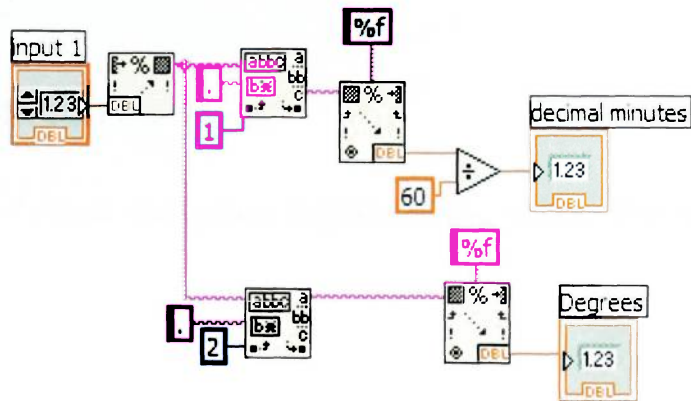
**Figure 14: Reading the GPS NMEA sentence**



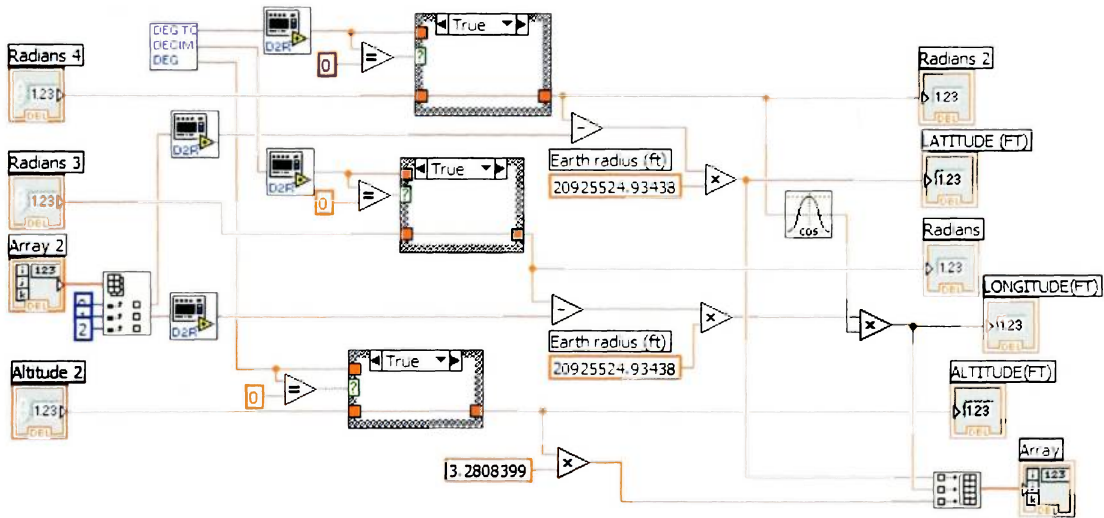
**Figure 15: Breaking down the ARINC-429 sentence to its various components**

Once the data has been broken up, the relevant information (latitude, longitude and altitude) is converted from degrees and minutes to distances on the earth's surface. This is done with the code shown in Figure 16 and Figure 17:





**Figure 16: Breaking down Latitude and Longitude to Degrees and Minutes**



**Figure 17: Conversion of degrees to feet**

It is important to note that the Altitude is transmitted in Meters and hence this value has to be converted to feet. With the conversion complete, the values are now known as Feet North, Feet East, and Altitude. These values, as will be later shown, are coupled to the INS values to provide correction for inherent deviation in the INS.

## Inertial Navigation Solution

The heart of the Inertial Navigation Solution is the AHRS. The AHRS transmits the following:

1. Roll
2. Pitch
3. Yaw
4. Roll rate
5. Pitch rate
6. Yaw rate
7. Acceleration X
8. Acceleration Y
9. Acceleration Z

The AHRS is initialized by the LabVIEW code shown in Figure 18 below:

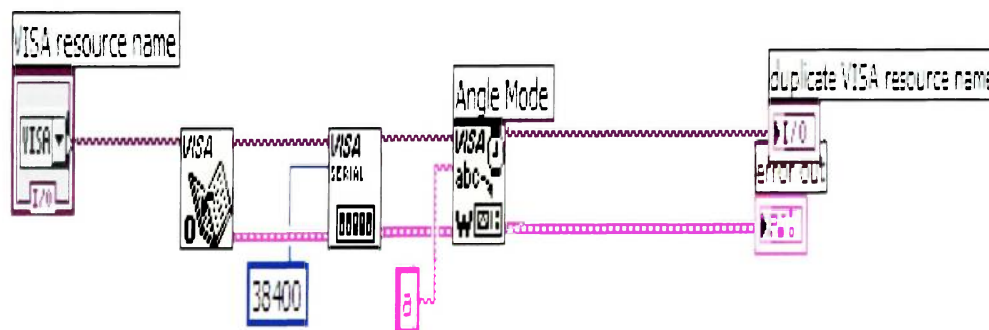


Figure 18: AHRS initialization code

Once the AHRS has been initialized, the AHRS transmits data up to a maximum frequency of 20Hz. The data is then parsed from two's complement form to decimal form by the code in Figure 19:

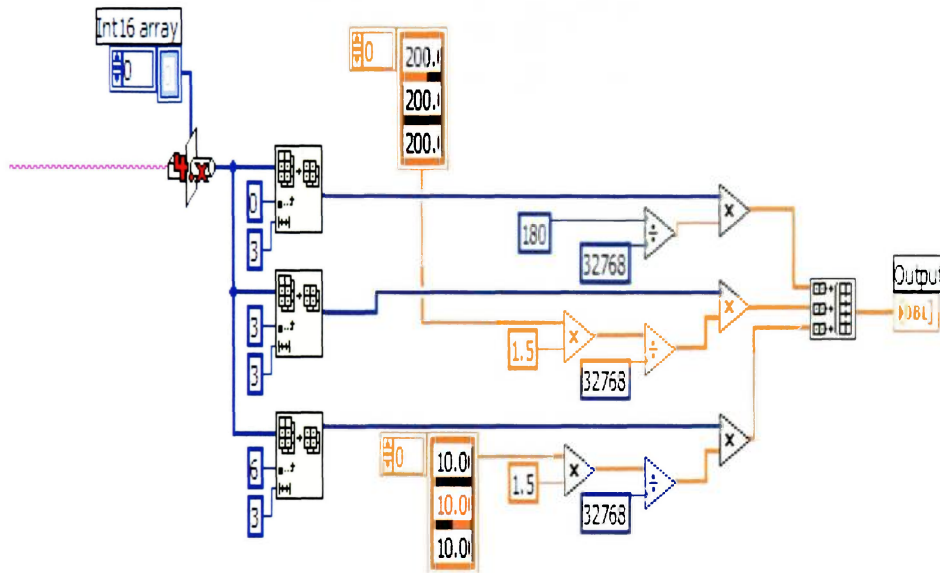
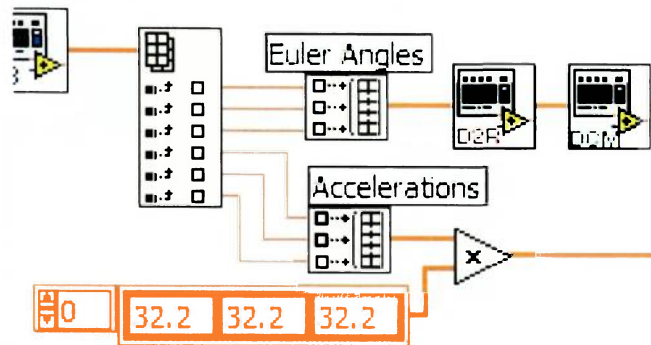


Figure 19: AHRS conversion from two's complement to decimal values

The parsed data is in the following units Euler angles (degrees), Euler rates (degrees per second), and Accelerations (g's). The Euler rates are not required for this thesis hence they are ignored. The Euler angles are converted to radians and the accelerations are converted to English units by the code shown in Figure 20:



**Figure 20: Separation and conversion of Euler angles and accelerations**

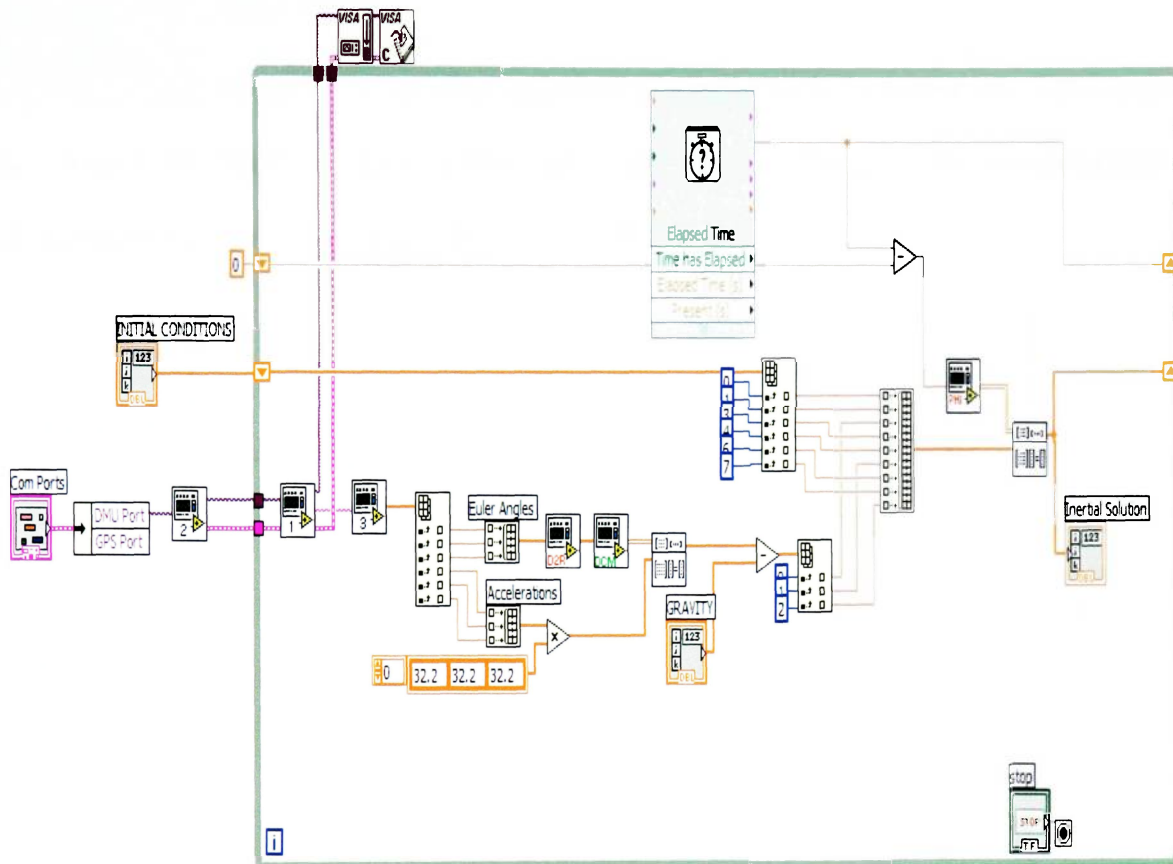
The Euler angles are the basis for the Direction Cosine Matrix which is used to transform the accelerations from the Body Coordinates to the NED Coordinates. The Initial conditions that are specified are position and velocity for a given vehicle in all three axes. These initial conditions, together with the accelerations, provide the starting point for the vehicle. The accelerations are then integrated to provide the next position and velocity for the next time step. The integration is done using Equation 45, Equation 46, and the accelerations that have already been compensated for bias.

$$s_1 = s_0 + v_0 t + \frac{1}{2} a t^2$$

**Equation 45**

$$v_1 = v_0 + a t$$

**Equation 46**



**Figure 21: Inertial Navigation Solution**

The Inertial solution, as previously discussed, tends to diverge from the true solution rather quickly. In order to counter this divergence, the INS is coupled with the GPS via a Kalman filter.

## Kalman Filter Code

The Kalman Filter code, aids the INS code by coupling it with the GPS code. The inputs to the code are signal variances and measurement vectors. The gyro and accelerometer biases represent one of the largest errors in the INS system.

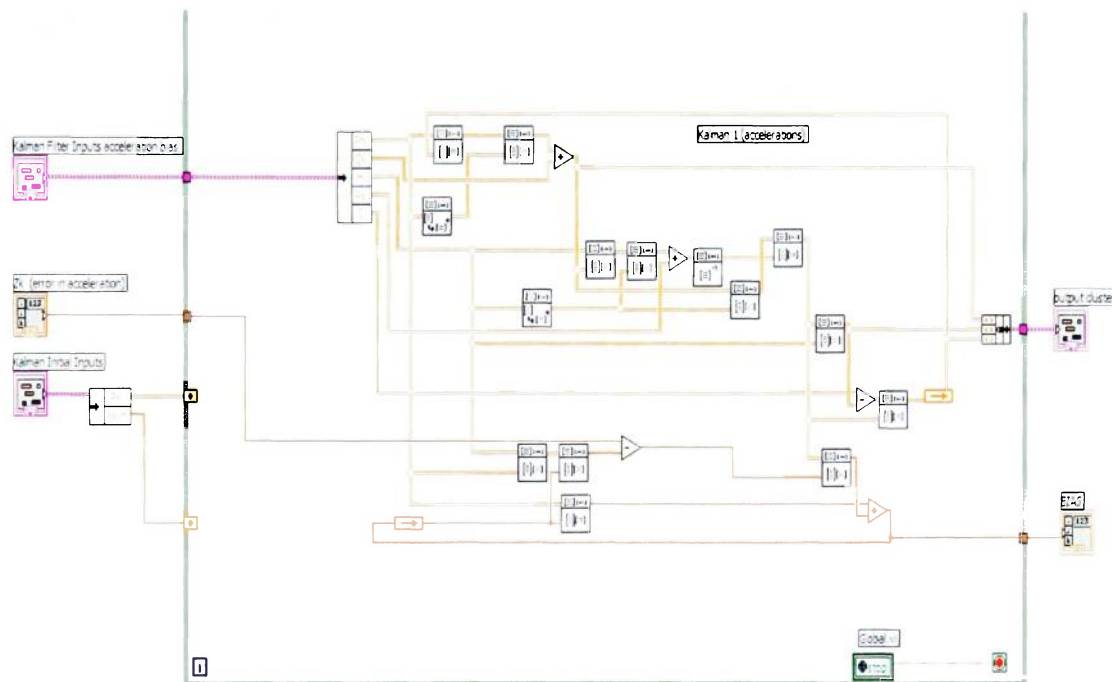


Figure 22: Kalman filter algorithm in LabVIEW

The signal error is basically the difference between the GPS navigation solution and the MEMS based AHRS solution. In this case it is the difference between the GPS and AHRS accelerations. This becomes the input to the Kalman Filter code. This is also known as the  $Z_k$  or measurement error. The Kalman Filter has to be tuned to the best possible values to minimize the error. This is a time consuming task and it is a trial and error method to obtain values as close as possible. Most of the work lies here. When all the

values are set, the Kalman filter will process the measurement noise and obtain a bias over time.

## **Conclusion**

From a top level perspective, the navigation solution looks as shown in Figure 23. Figure 23 gives an overview of the processes being performed by the navigation solution. The input is provided by an AHRS and a GPS receiver. The GPS receiver outputs go through a double derivative to obtain velocity and accelerations. These accelerations are subtracted from the AHRS accelerations providing our first error input to the Kalman filter. The bias corrected accelerations then become the basis for computing an inertial Position, Velocity and Acceleration for the vehicle whose measurements are being taken. The Inertial solution obtained, has a high drift and hence will gradually move away from the mean solution. The GPS position provides a correction factor to realign the inertial solution to the expected mean. The difference between the GPS position and the inertial position thus becomes the input to the second Kalman filter. The output is thus a more stable position solution that is more reliable and accurate than the GPS or AHRS alone.

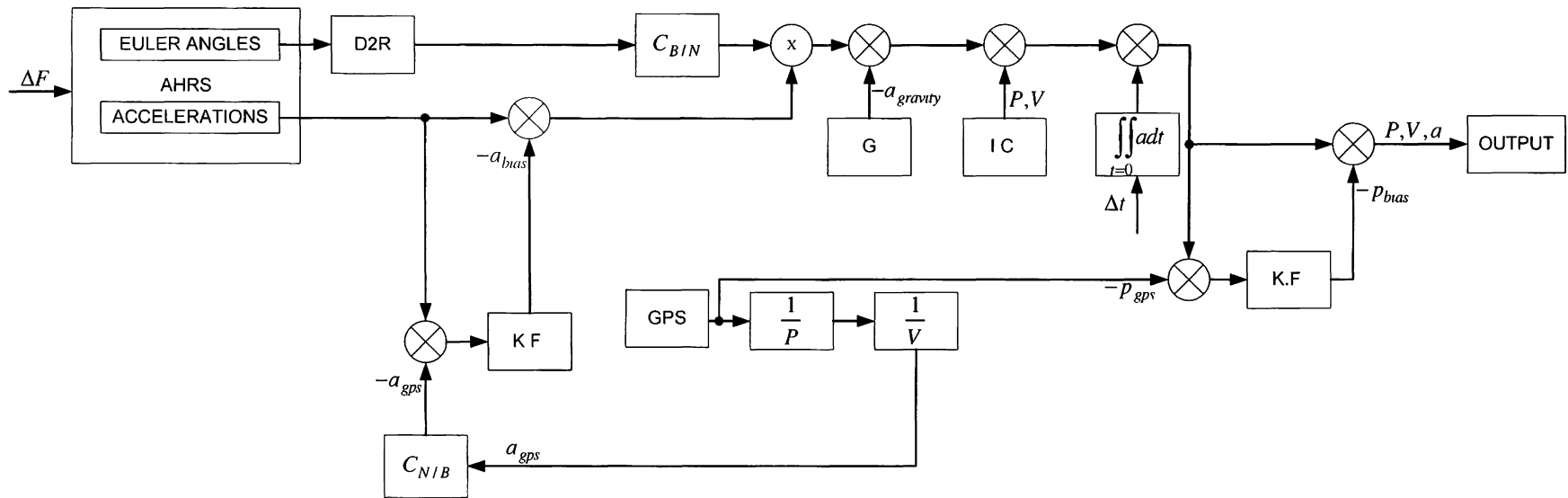


Figure 23: Top Level Block Diagram



## **Chapter 4: Results and Conclusion**

## **Results**

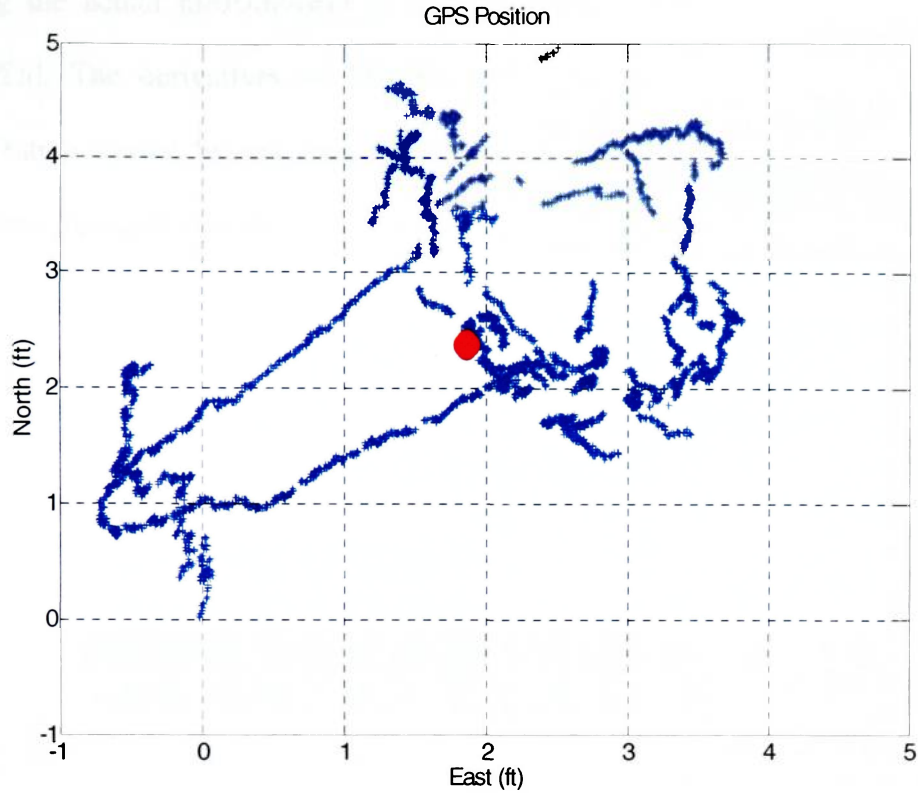
The hardware and data acquisition system is stimulated using real world navigation points to provide as accurate a picture of real aircraft results. This stimulation is based on actual flight paths and flight patterns of various real aircraft. The navigation algorithm path can then be compared to the aircraft actual path to determine its accuracy under real world scenarios.

### **Input data reduction**

The input data has errors, which, if not accounted for in the navigation algorithm, will result in drift in the navigation solution. In order to obtain these errors, a stationary analysis is performed to determine the measurement bias and errors. To determine the stationary errors, the hardware system was placed on a stationary bench. The stationary bench would allow for little interference from outside forces other than the earth's movement.

### **Stationary GPS error analysis**

Figure 24 shows signal variance of the GPS obtained by capturing data from the GPS satellites for a period of 1 hour while stationary on a bench. Figure 24 represents the horizontal position data acquired using the GPS receiver for one hour.



**Figure 24: GPS standard deviation over a period of one hour**

The bias is similar in the north and east directions. The big dot at the center represents the current position. It is clear from the graph that the GPS has quite a bit of standard deviation. The GPS acceleration had errors due to quantization caused by a low update rate of 1Hz. Table 2 shows a summary of the stationary GPS errors. The variance for the measurements of the three axes turns out to be similar in this case.

**Table 2: Summary of stationary GPS variance**

<i>Measurement</i>	<i>Units</i>	<i>Variance</i>
Altitude	Feet	1089
East direction	Feet	1089
North direction	Feet	1089

During the actual measurement of the navigation solution, an unexpected error was identified. The derivatives of velocity and acceleration yielded large errors due to quantization caused by low update rates of between 0.5Hz and 1Hz. To minimize this error, the accelerations were filtered using a fourth order, low pass Butterworth filter. Figure 25 shows the results of the filtered signal compared to the unfiltered signal.

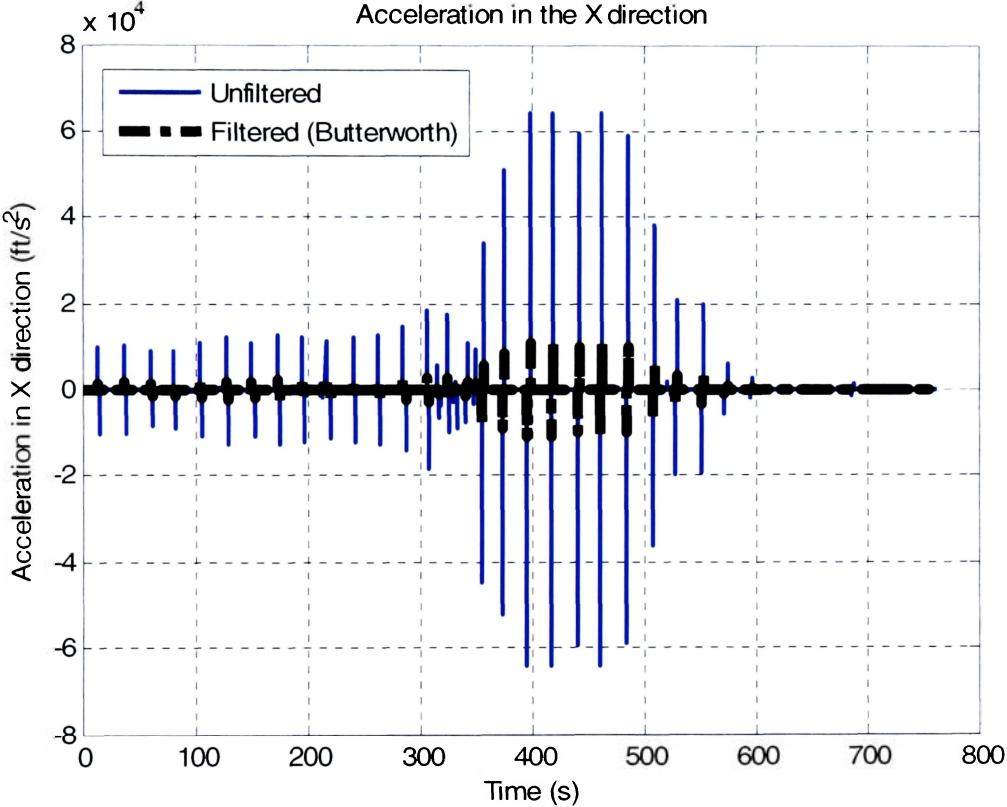
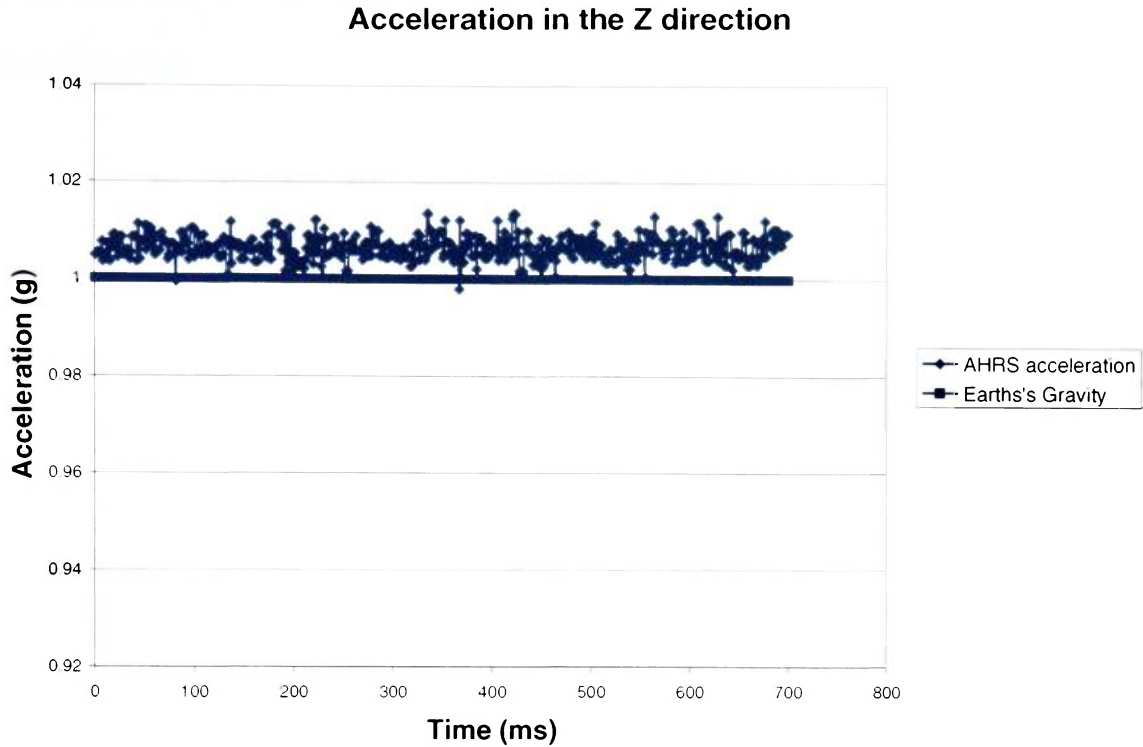


Figure 25: Acceleration in the X direction (Filtered and Unfiltered)

### Stationary AHRS error analysis

Figure 26 shows stationary AHRS data placed on a bench. The six angular quantities were recorded and their associated variances and biases can be seen in Table 3.



**Figure 26: Acceleration Bias in the Z direction**

**Table 3: Summary of stationary AHRS error**

<i>Measurement</i>	<i>Units</i>	<i>Bias</i>	<i>Variance</i>
Roll	Degrees	1.918154	0.000744
Pitch	Degrees	-0.27554	0.008022
Yaw	Degrees	101.0921	1.083531
Roll Rate	Degrees/sec	0.067538	0.103455
Pitch Rate	Degrees/sec	-0.23315	0.186614
Yaw Rate	Degrees/sec	-0.02538	0.155924
Acceleration X	G	0.003692	3.23E-6
Acceleration Y	G	0.033308	6.9E-6
Acceleration Z	G	1.006249	1.67E-5

## Navigation algorithm results

The implementation was set up for approximately 700 seconds. The latitude longitude and altitude of the aircraft were set for Eastern Iowa Airport. The initial Kalman filter

values were also set accordingly. Table 4 shows the parameters used in this navigation implementation.

**Table 4: Kalman Filter Inputs**

	<i>Name</i>	<i>Direction</i>	<i>Values (Diagonal Matrix)</i>	<i>Notes</i>
$R_k$	Signal Variance	X	1089	Variance of GPS Signal
	Signal Variance	Y	1089	Variance of GPS Signal
	Signal Variance	Z	1089	Variance of GPS Signal
$x_k$	Current state	X	1	Initial Input value
	Current state	Y	1	Initial Input value
	Current state	Z	1	Initial Input value
$P_k$	Covariance estimate	X	999999	Poor confidence in initial estimate
	Covariance estimate	Y	999999	Poor confidence in initial estimate
	Covariance estimate	Z	999999	Poor confidence in initial estimate
$\phi_k$	State transition matrix	X	1	
	State transition matrix	Y	1	
	State transition matrix	Z	1	
$I_k$	Identity Matrix	X	1	
	Identity Matrix	Y	1	
	Identity Matrix	Z	1	
$H_k$	Connection matrix	X	1	
	Connection matrix	Y	1	
	Connection matrix	Z	1	

## Process Noise matrix

The process noise matrix allows the system to model the measurement noise. This measurement noise is not quantifiable due to its random erratic nature. For this system to work, an assumption is made: the process noise changes slowly with time. This assumption allows the process noise to be quantified in a deterministic equation.

There is no scientific method of estimating to an exact degree the process noise. This is achieved by manually changing the process noise values to adjust the filter output to the most suitable value for each individual measurement. This is known as tuning the filter.

The process of tuning the acceleration Kalman filter yields the results shown in Table 5.

Similarly the process of tuning the position Kalman filter yields the results shown in Table 6.

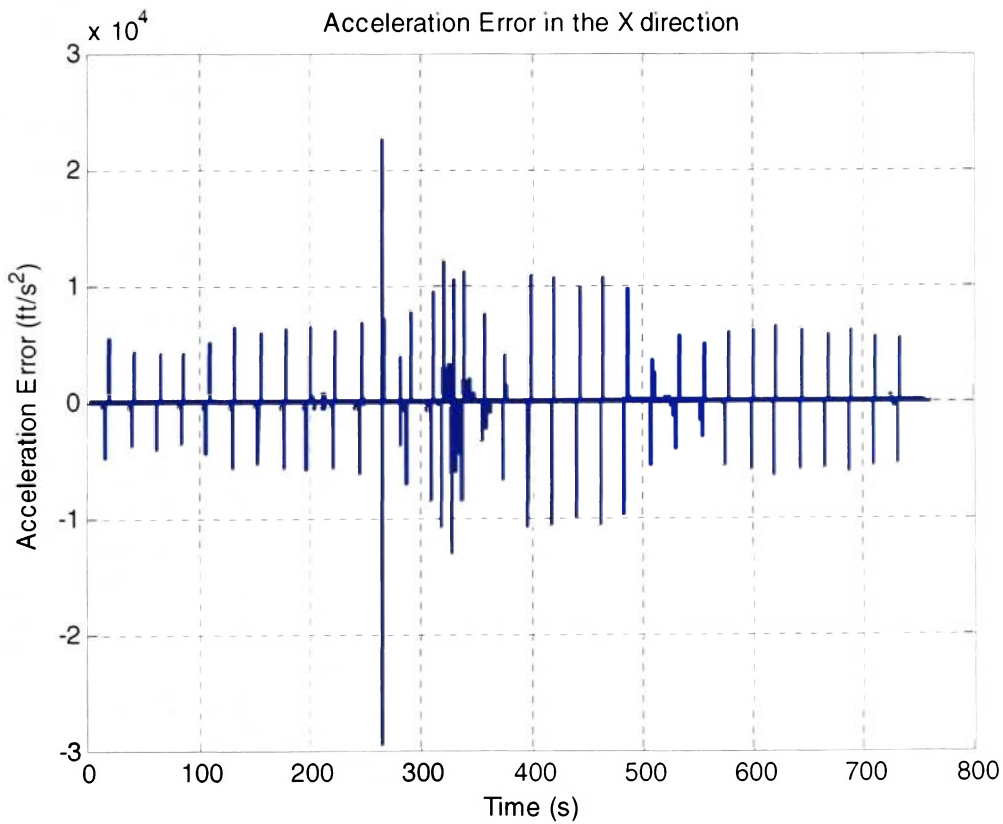
**Table 5: Process noise matrix values for the Acceleration Kalman Filter**

	<i>Axis</i>	<i>Values (Diagonal Matrix)</i>	<i>Notes</i>
$Q_{k_x}$	X	9E-9	Process Noise
$Q_{k_y}$	Y	9E-12	Process Noise
$Q_{k_z}$	Z	0.002	Process Noise

**Table 6: Process noise matrix values for the Position Kalman Filter**

	<i>Axis</i>	<i>Values (Diagonal Matrix)</i>	<i>Notes</i>
$Q_{k_x}$	X	0.009	Process Noise
$Q_{k_y}$	Y	0.009	Process Noise
$Q_{k_z}$	Z	0.9705	Process Noise

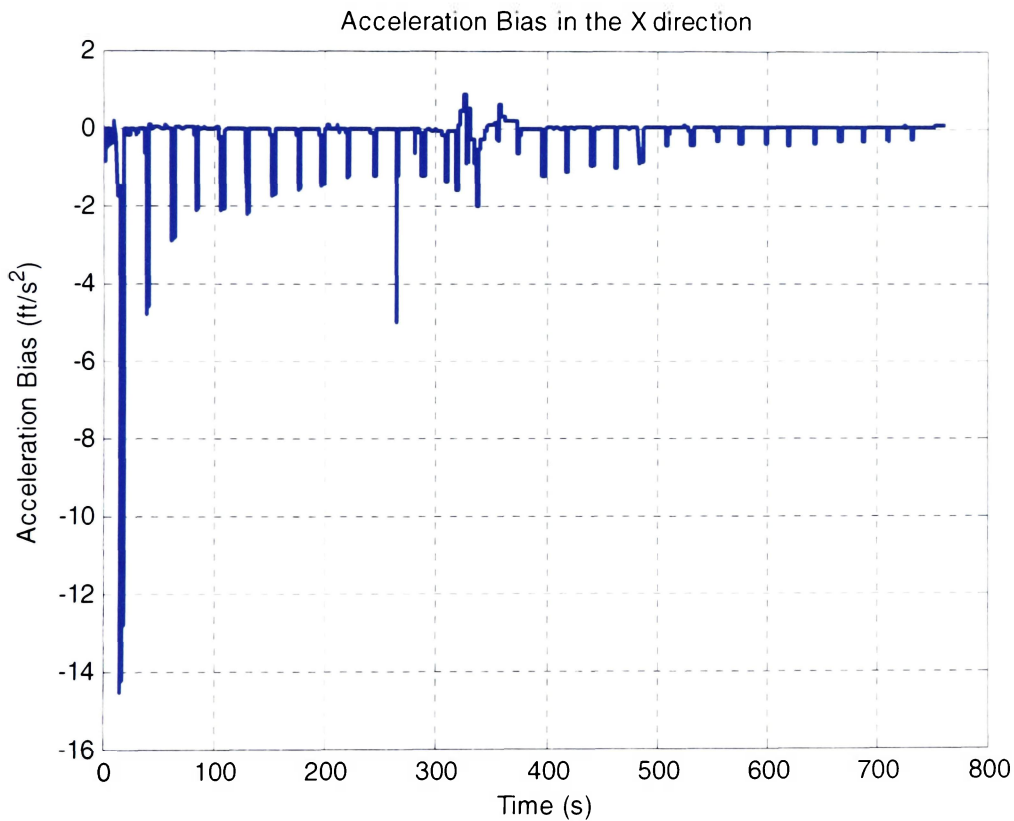
Figure 27 is the observed acceleration error in the X direction computed via the difference between the computed GPS acceleration and AHRS obtained accelerations. The error depicted in Figure 27 below shows huge spikes that are as a result of the quantization error due to the low update rates of the GPS. The Y and Z directions also exhibit similar error analyses.



**Figure 27: Difference between GPS acceleration and AHRS acceleration (X-Direction)**



With the aforementioned initial conditions, the Acceleration Kalman filter is run, tweaking the  $Q_k$  variables to obtain the set that is currently defined in Table 5. Figure 28 is the resultant of the acceleration from the Kalman filter. This is now known as the bias and can be subtracted from the AHRS acceleration parameters. Only The X- axis is depicted here, however it is noteworthy to mention that the Y and Z axes have similar curves but their  $Q_k$  values are computed independently. This independent tweaking allows for optimization of the individual  $Q_k$  values.

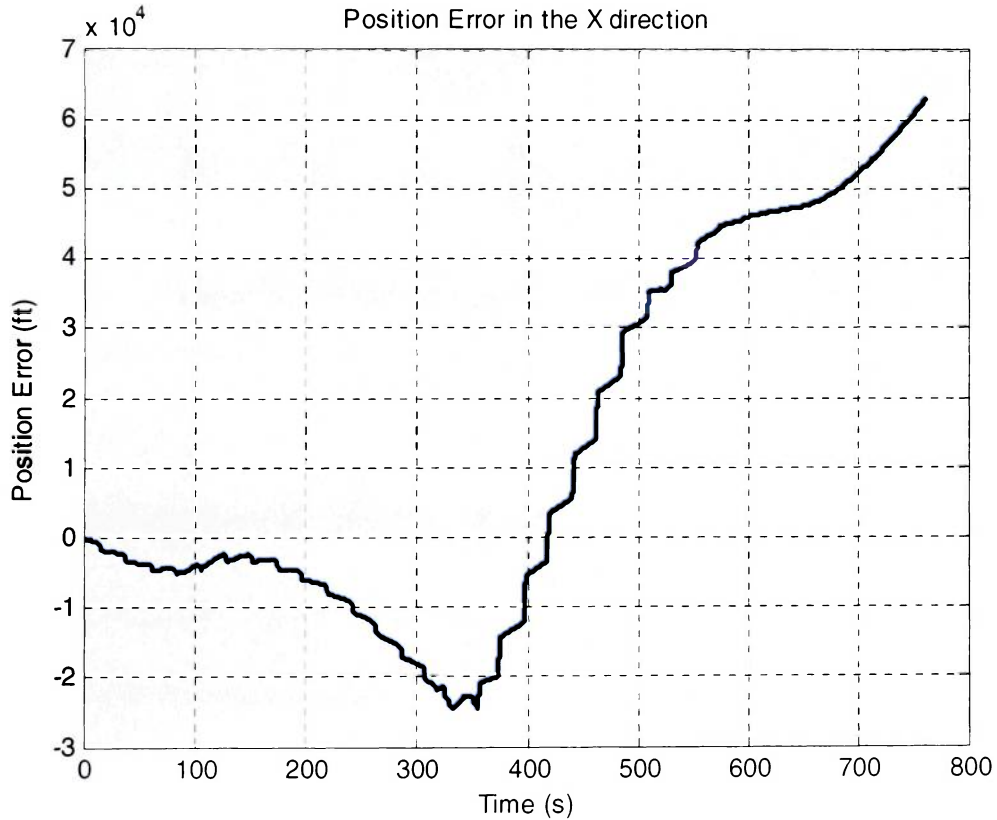


**Figure 28: Acceleration Kalman Filter output for the X-direction (Acceleration Bias)**

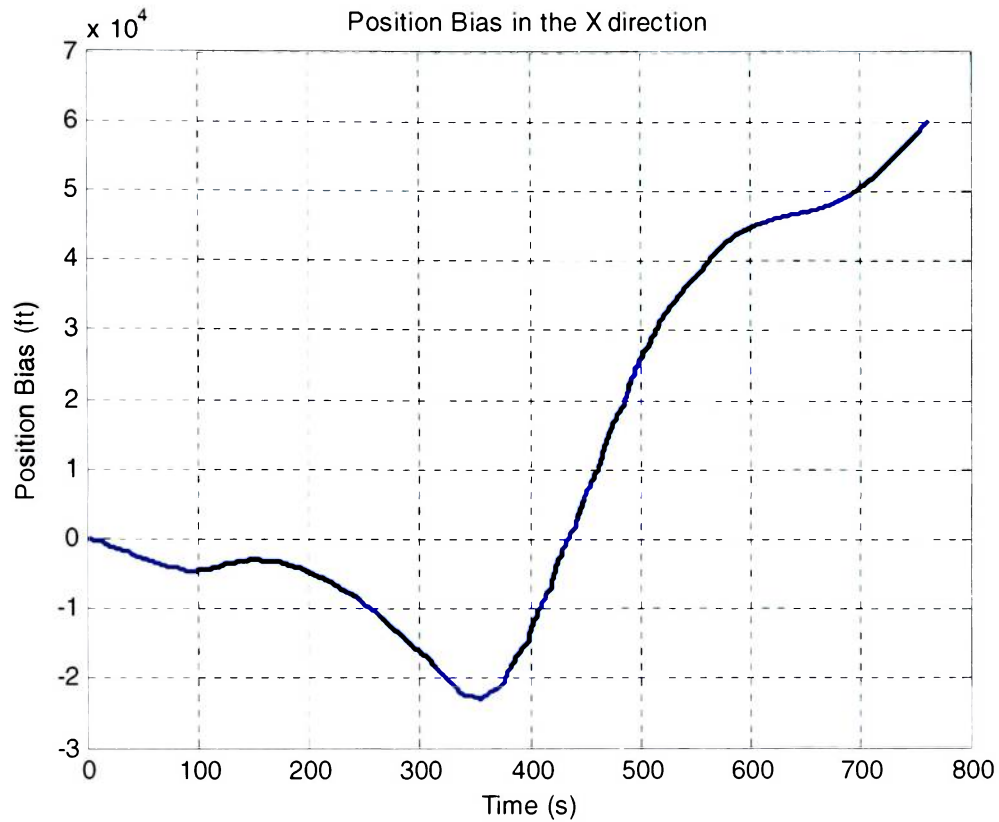
The corrected accelerations, once transformed to the navigation frame, are the basis of the Inertial Solution. The Inertial Solution requires a starting point, which can be local

coordinates of the current point or, as used for this thesis, the origin (0, 0, 0 in the x, y, and z axes).

Figure 29 is the observed acceleration error in the X direction computed via the difference between the computed GPS acceleration and AHRS obtained accelerations. The filter now takes the error between the Inertial Solution and the GPS receiver position and computes a bias as is shown in Figure 30. Table 6 provides the values of the process noise matrix that were used to filter the input values.



**Figure 29: Difference between GPS position and Inertial position (X-direction)**

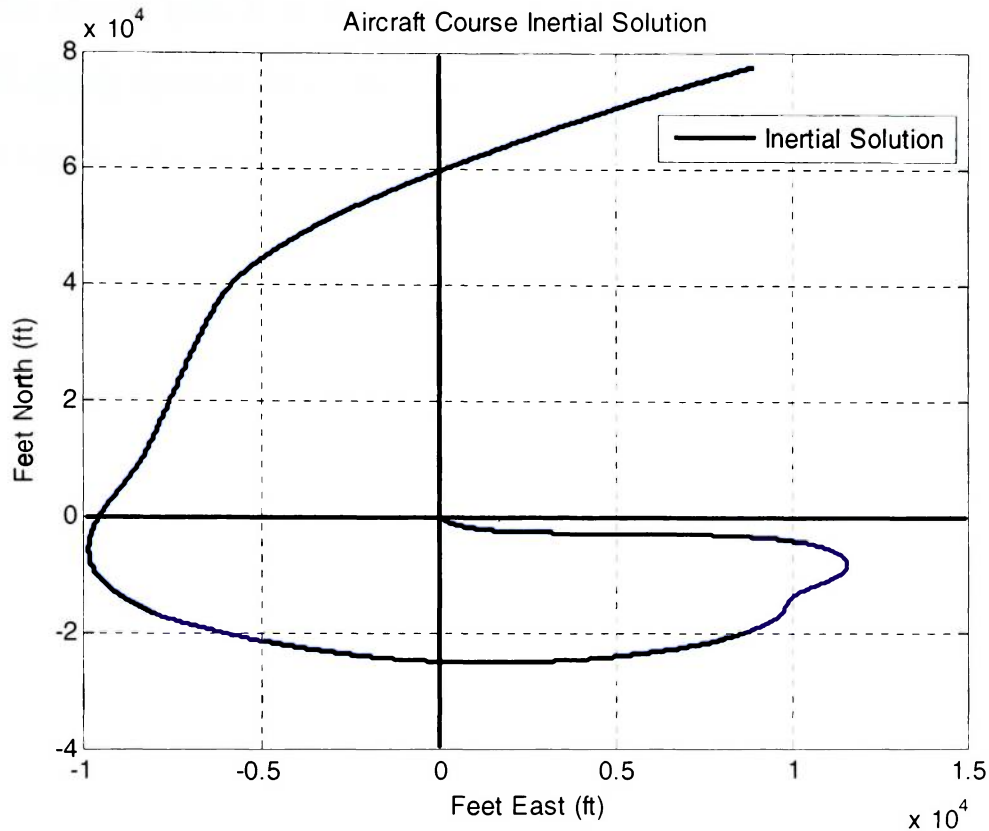


**Figure 30: Position Kalman Filter output for the X-direction**

## Inertial Navigation Solution results

The Inertial Navigation Solution diverges as the solution moves away from the “origin”.

This is reflected in Figure 31 below:



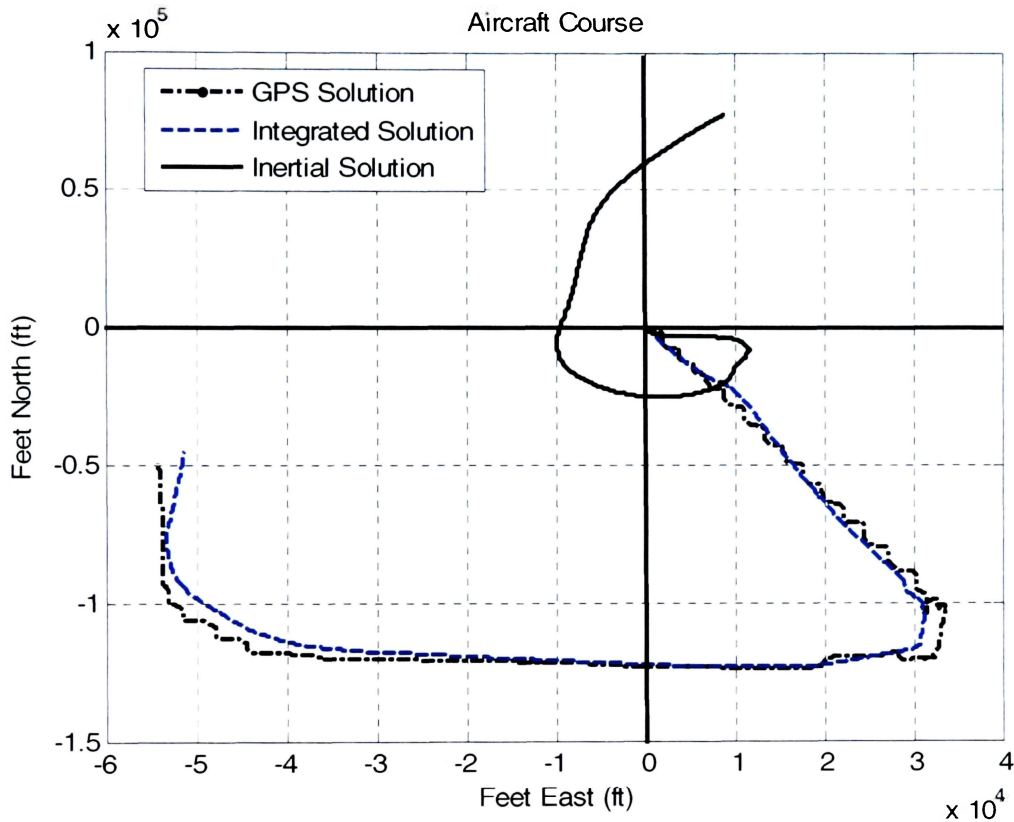
**Figure 31: Inertial Solution**

Figure 31 shows the divergence of the Inertial solution with respect to time. This is due to the inherent imperfections that exist during the manufacture of the AHRS sensor(s). Another factor that affects the Inertial solution is the environment. The external factors like vibrations, gravity effects work against the Inertial solution to diverge the solution from the correct aircraft path.

## **Integrated Solution**

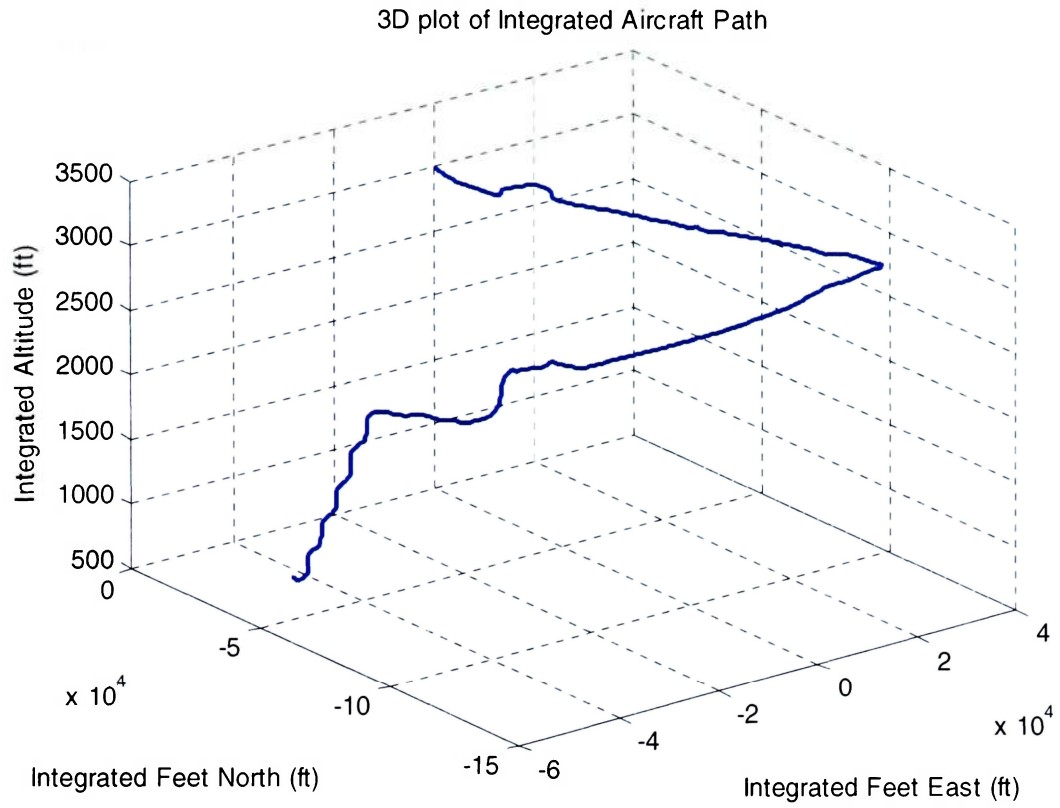
Figure 32 below shows the comparison between the Inertial solution, GPS solution and the Integrated solution. It can be clearly seen that the Inertial solution diverges quickly

from the aircraft path. It is also evident from the graph that the Integrated solution follows closely the anticipated path. The integrated solution provides a solution for the best of both worlds; the inertial solution and the GPS provided navigation solution.

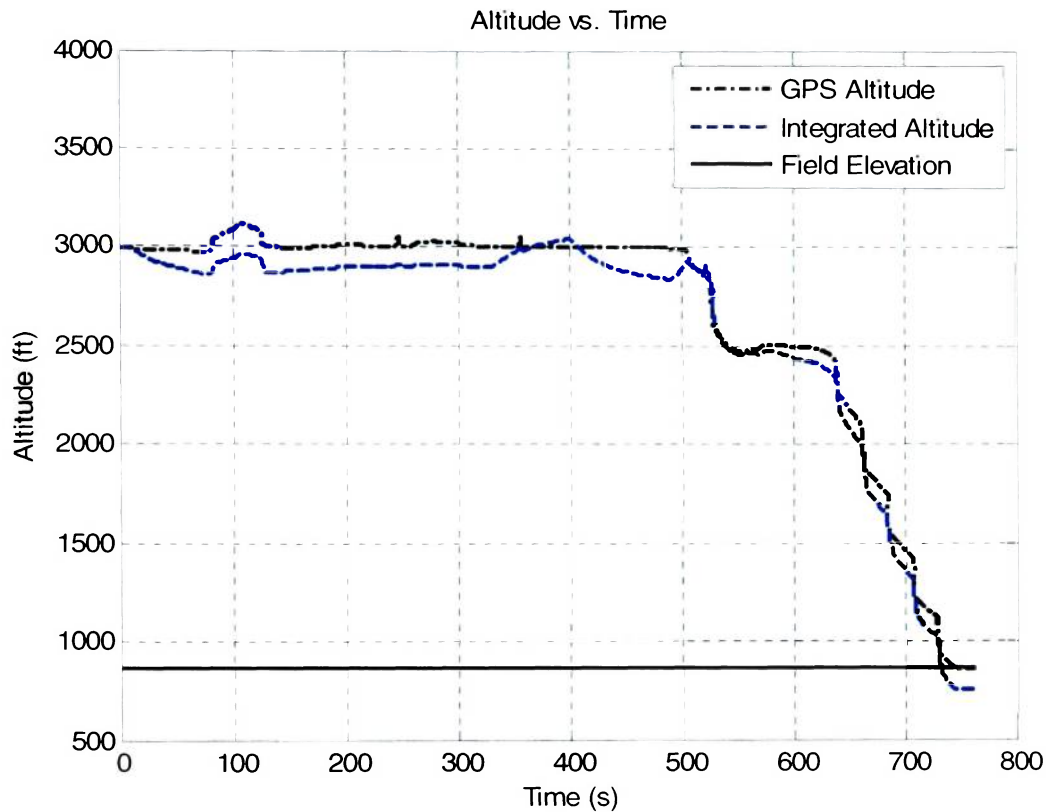


**Figure 32: Integrated solution V.S. GPS solution V.S. Inertial solution**

Figure 32 shows the Integrated solution (dashed line), which is combination of the Inertial and the GPS solution. The Integrated solution shows less quantization as compared to the GPS and more stability as compared to the Inertial solution. Figure 33 is a three dimensional plot that incorporated the GPS altitude that is shown in Figure 34.



**Figure 33: Integrated Aircraft path into the Eastern Iowa Airport**



**Figure 34: GPS and Integrated Aircraft Altitude**

Figure 33 represents a 3 dimensional view of the Integrated solution in an approach path to the Eastern Iowa Airport. Figure 32 represents the integrated altitude in comparison with the GPS altitude. The inertial altitude tracks closely the GPS altitude. The inertial altitude indicates that the aircraft lands at an altitude 50 feet below the runway altitude. The runway altitude is 869 Above Sea Level (ASL). This may be due to imperfect tuning of the process noise matrix.

In Figure 32 the solid line represents the Inertial solution. From the graph it is quite evident that the Inertial solution diverges quite rapidly from the expected solution. For the first 90 seconds, the Inertial solution dips farther away from the anticipated altitude.

This is due to the rapid deterioration of the Inertial solution before the predicted state stabilizes. At about 500 seconds, the aircraft began its descent into the airport.

## **Conclusion**

The Implementation of the navigation solution met most of its expected goals. The navigation solution can determine the error in the signal, compute the bias and provide a solution that is more accurate than the GPS or MEMS based INS solution. The MEMS based AHRS has inherent errors that reduce the accuracy of the navigation solution. The short period accuracy of the MEMS based AHRS in conjunction with a coupled long period GPS provides for better navigation accuracy.

## **Current Hardware**

The navigation solution was programmed to accept any GPS or AHRS inputs. The process noise matrix was made available to cater to the use of different hardware solutions. The components selected for this navigation solution were, Rockwell Collins GPS, and Rockwell Collins AHRS.

## **GPS**

The Rockwell Collins GPS is currently certified for use on various aircraft. The GPS has an update rate of 1Hz. An update rate of 20Hz would be desirable to improve navigational update accuracy.



The GPS sends the data, which includes a time stamp, to the computer for storage. This data will remain in the storage until the algorithm accesses them. An error that occurs due to this slow update rate leads to compounded errors in position. It is not hard to obtain GPS with higher update rates and smaller time stamps that will increase accuracy.

## **AHRS**

The Rockwell Collins AHRS uses MEMS based technology to reduce the cost of AHRS equipment in the aircraft navigation world. The self contained nature of the AHRS means that it is invulnerable to jamming or spoofing as is the GPS. The MEMS based AHRS is certainly not as accurate as the more expensive FOG or the RLG and cannot be used to navigation for long periods of time. Care has to be taken during the installation of the AHRS to avoid high frequency vibrations from affecting the AHRS output. The high frequency vibrations will be picked up by the measuring unit and mask the measurement signal.

## Problems encountered

Some of the problems encountered while working on the thesis are:

1. Difficulty during troubleshooting for errors. This was a major hindrance since it took a while to catch a majority of errors encountered early in the development cycle.
2. Poor access to hardware and equipment. This drastically increased the time spent working on changing the code to fit the equipment that was available at a particular time.
3. Difficulty in changing of code to suit the equipment at hand. Some of the equipment required a complete rewrite of the code. The AHRS from Rockwell Collins is a prime example. The data was transmitted in an unfamiliar format (ARINC). Thus, there was a need to understand the ARINC format and write code that would gather that data. Also the use of third party hardware to interface with the current available AHRS was made more cumbersome and difficult by the use of unfamiliar DLL files.
4. The signal to noise ratio of the AHRS equipment was quite high. This meant that the data collected at low speeds was masked by the noise. This meant trying to collect data while in a fast moving vehicle. The vibrations of the vehicle also hindered proper data collection since they were transmitted to the AHRS. This caused the AHRS to vibrate at ratios which made it difficult to acquire any meaningful data.

## **Possibility of future research:**

The next step in this research would be to include this navigation solution in a flight testing aircraft for data acquisition. This would need to be carried out in different aircraft to ascertain its platform independence. The second step could be to perfect the navigation solution to improve its efficiency.

With the current availability of commercially available low cost MEMS based AHRS and GPS, general aviation and flight testing aircraft will have low cost navigation solutions currently only accorded to commercial aircrafts. The MEMS based AHRS would be invulnerable to jamming or spoofing whereas the GPS would be used to initialize the navigation solution as well as continually update the correction for the navigation solution for general aviation aircraft.

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