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Data Reduction Procedure for an Experimental Method of Measuring the Velocity-Coupled Response Function of Solid Propellants

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DATA REDUCTION PROCEDURE FOR AN EXPERIMENTAL METHOD OF MEASURING THE VELOCITY-COUPLED RESPONSE FUNCTION OF SOLID PROPELLANTS

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by

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This thesis was prepared under the direction of the candidate's thesis committee chairman, Dr. L.L. Narayanaswami, Department of Aeronautical Engineering, and has been approved by the members of his thesis committee It was submitted to the School of Graduate Studies and Research and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aeronautical Engineering.

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Abstract

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A computer program has been developed for calculation of the velocity-coupled response function of solid-propellants from experimentally measured pressure data. The proposed velocity-coupled response function measurement apparatus consists of an end-burner combustor where the propellant sample is oscillated in a direction normal to the flow in the presence of a standing acoustic wave within the combustion chamber. The pressure measurements are made at select points along the length of the chamber.

The data reduction program consists of a Runge-Kutta routine driven by a BFGS multivariable search routine. The Runge-Kutta routine determines the pressure distribution within the chamber of the proposed apparatus for a specific velocity-coupled response function (Rv). The BFGS optimization routine searches for the Rv which minimizes the difference between the calculated and measured pressure distributions.

The data reduction program has demonstrated convergence to the proper Rv value for pressure data that was generated by sampling from gaussian distributions.

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Symbols

u_n - velocity of flow normal to the propellant

p - pressure

ρ - density

T - temperature

a - acoustic velocity

u_t' - average relative velocity of propellant sample

$(\bar{\ })$ - mean flow quantity

$(\ )'$ - perturbation flow quantity

R_v - velocity-coupled response function

R_p - pressure-coupled response function

i - square root of negative one

ω - angular frequency

α - growth constant, parameter in linear equation

p_m' - measured perturbation pressure

p_n' - numerically calculated perturbation pressure

E - least squares error function

k_1 - conductivity of combustion product

k_2 - conductivity of chamber wall

r - radius of combustion chamber

L - length of combustion chamber

T_∞ - ambient temperature

Δt - thickness of chamber wall

$\bar{M}$ - mean flow Mach number

λ - equal to $2 \frac{k_2}{k_1} \frac{1}{r \Delta t}$

x - coordinate direction along axis of combustion chamber

1.0 Introduction

From the beginning of solid propellant rocket development, the rocket engine designer has been troubled by the phenomenon broadly described as "unstable combustion". Normal combustion of a solid propellant is defined as the regular predictable process associated with steady conditions. Under steady conditions the burning rate of the propellant depends on the local mean pressure and the local mean mass flow moving parallel to the burning surface, and the initial propellant temperature. With the exceptions of detonations and explosions, unstable combustion in any combustion process is undesirable.

It is important to distinguish between two aspects of unstable combustion: irregular combustion refers to combustion associated with large deviations in the mean burning rate from that predicted under steady conditions while oscillatory combustion refers to combustion with periodic variations in flow properties about their mean values.

Oscillatory combustion instabilities arise from the coupling of pressure and velocity perturbations that are associated with the presence of acoustic modes within the combustion chamber, with the combustion processes in the burning zone at the solid propellant surface.

It is customary to assume that the fluctuation in the mass flow can be separated into a pressure-coupled part and a velocity-coupled part which are independent of each other.

Accordingly, the effects of pressure and velocity perturbations on the burn rate can be represented by the following equation (Ref. 1).

$$\frac{m_b'}{\bar{m}_b} = R_p \frac{p'}{\gamma \bar{p}} + R_v \frac{\overline{u_t'}}{\bar{a}} \quad (1.1)$$

Since the solid propellant combustion responses, Rp and Rv, are the sources of acoustic amplification, their accurate determination is required for a complete stability analysis. Due to the complicated nature of the combustion process, it is difficult to predict the burn rate of a solid propellant on the basis of its chemical properties alone. For this reason, researchers have relied heavily on experimental approaches to combustion response function prediction. At the present time, methods to measure the combustion responses are being developed.

This investigation will focus on the data reduction procedure for a new method of experimentally measuring the velocity-coupled response function.

2.0 Historical Background

At the present time, a theoretical analysis of combustion instability containing all relevant parameters does not exist. This is largely because of an incomplete understanding of the response of the propellant burning rate to oscillations in the pressure and velocity of the flow adjacent to the propellant surface. Satisfactory models and reliable experimental techniques to predict the velocity-coupled response functions of solid propellants have remained particularly elusive. Until further progress is made in our understanding of the mechanistic basis for velocity coupling, research must be directed towards the development of experimental techniques for the measurement of the velocity-coupled response function. Solution to the problem of velocity-coupled response function measurement will be adequate only when there is quantitative agreement between the various experimental methods and realistic analytical models.

The measurement of combustion response functions has been attempted by a variety of methods, each having its advantages and shortcomings. The methods are best categorized by the type of burner used in the measurement and by the nature of the exciting disturbance. Burner configurations which have been used in the past include the center-vented burner, the modulated exhaust flow burner, the impedance tube burner and the bulk-mode self-excited burner. Acoustic oscillations in the burners are either self-excited or generated by external means. The center-vented burner, commonly referred to as the T-burner, is an example of a self-excited burner. The rotating-valve method is an example of a burner where the oscillations are generated by external means. The T-burner and the rotating-valve methods deserve particular attention not only because of their extensive use and evaluation, but also because they are helpful in understanding the basic approach to experimental combustion response measurement.

The simplest form of the T-burner (Ref. 2) for pressure-coupled response measurements consists of a circular tube with propellant disks mounted at both ends. The products of combustion are exhausted through a vent at the center of the tube. This configuration allows maximum excitation of the fundamental longitudinal acoustic mode in the tube, thereby generating maximum acoustic pressures at the burning propellant surfaces. By placing the exhaust vent at the center of the tube where the pressure oscillations have the lowest amplitude, energy losses from the acoustic field are minimized. The exhaust vent is usually connected to a surge tank which is pressurized with nitrogen to the desired mean pressure. When the propellant disks are ignited, the propellant reaction flushes out the inert nitrogen, and oscillations develop and grow exponentially until losses limit the amplitude. When the propellant burns out, the oscillations decay in a roughly exponential manner. In its simplest use, it is assumed that the damping is the same during the period of growing oscillations as during the period of decaying oscillations. The initial growth constant of the acoustic field (α) can then be regarded as the difference between the energy gains and the measured damping contribution, hence

$$\alpha = \alpha_{gain} - \alpha_{damp} \quad (2.1)$$

Using linear acoustic theory, the rate at which energy is transferred to the acoustic field can be related to the response function. For end-burners the relation between and the pressure-coupled response is given by

$$\alpha_{gain} = 4f(\bar{M}_b R_p) \quad (2.2)$$

where f is the frequency of the oscillating field and $\bar{M}_b$ is the mean flow Mach number. Using equations (2-1) and (2-2), the response function can be inferred from measurements of the growth constant and the damping constant.

Measurement of the velocity-coupled response function is much more difficult than measurement of the pressure-coupled response function. However, methods for adapting the T-burner for velocity-coupled measurements have been considered. In order to induce velocity-coupled response, the propellant test sample is exposed to a parallel velocity flow in locations which maximize velocity-coupled acoustic excitations. One means of accomplishing this is to mount the test charges on the side walls and the driver propellant at the ends. The difficulty with this arrangement is that test samples respond to both pressure and velocity oscillations. Hence, independent information about the pressure-coupled response is required before the velocity-coupled contribution can be extracted from the measured pressure data. Variable-area methods with the propellant samples located at the L/4 and 3L/4 positions along the tube have also been considered. The basis for the variable-area method becomes clear if we extend equation (2.1) to the case where the propellant surface area is not equal to the cross-sectional area of the burner tube. Equation (2.1) becomes

$$\alpha = \alpha_{gain}\left(\frac{S_b}{S_c}\right) - \alpha_{loss} \qquad (2.3)$$

which is the equation of a straight line for α with the area ratio as the dependent variable. The energy constant (α_{gain}) is the slope of this line. From the energy constant the real part of the response function can be determined. The primary disadvantage of variable-area methods is that several tests must be conducted to obtain the response function at a specific frequency and pressure. Other disadvantages include the theoretical uncertainties that are not reflected in the simplified expression for α in equation (2.1).

The basic approach to rotating-valve methods (Ref. 3) is to use exhaust modulation to control the acoustic oscillations in the combustion chamber. The usual burner configuration is a rocket motor with a cylindrical mounted charge and a conventional nozzle to control the mean combustion pressure. A secondary orifice is operated periodically to produce oscillations at a desired frequency. For velocity-coupled response measurements, two rotating valves are required. With one valve positioned at either end of the combustion chamber and oscillated directly out of phase, controlled velocity oscillations can be generated. Analytical solutions of the mass, momentum and energy equations for the rotating-valve burner configuration along with measured acoustic response data are then used to predict the velocity-coupled response function. Unlike the T-burner method, the rotating valve approach determines both the real and imaginary components of the response function.

Direct measurement of the oscillatory burning surface regression using microwaves has also been considered. This approach provides both magnitude and phase information about the combustion response functions without having to indirectly calculate them from measured acoustic data. However, the difficulty of isolating pressure-coupled contributions from the velocity-coupled contributions has not been resolved.

At present there is a lack of qualitative agreement among the different approaches, and it is difficult to ascertain which method is the best. Accuracy, cost and repeatability are criteria for the evaluation of their relative merits.

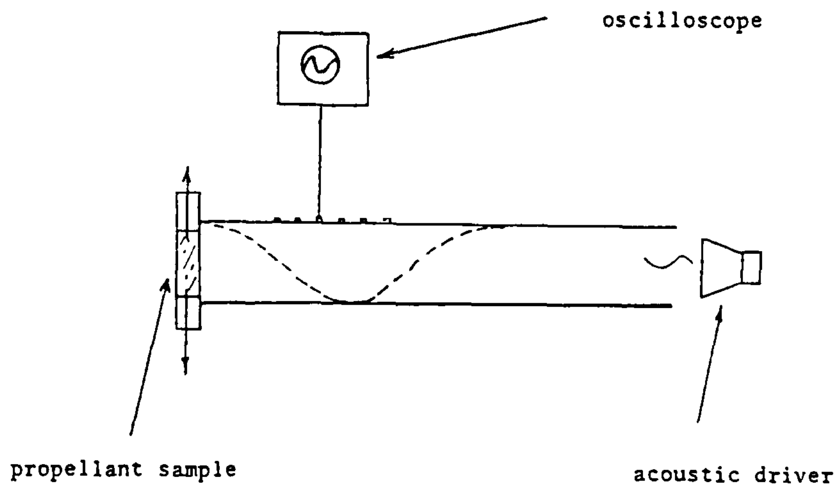
3.0 New Method

In a new attempt to determine the velocity-coupled response function from pressure oscillation data, L.L. Narayanaswami has suggested an end-burner test configuration where the propellant sample is oscillated normal to the flow in the presence of a standing acoustic wave in the combustion chamber (see Fig. 3.1).

Pressure coupling between the propellant burn rate and the acoustic field is generated by the oscillating pressure at the propellant surface. Velocity coupling is generated by relative flow oscillating tangential to the oscillating propellant surface.

The configuration of the standing wave within the combustion chamber is that of a forced standing wave in a closed-end tube modified slightly by the existence of pressure and velocity coupling between the propellant burn rate and the acoustic field. For given propellant sample, environmental conditions, and acoustic frequency, the configuration of the standing wave is uniquely determined by the combustion response functions Rp and Rv. The pressure-coupled response function (Rp) will be known from prior experiment and the velocity-coupled response function will be determined from pressure measurements along the length of the experimental device. This investigation will focus on the calculation of Rv from the pressure measurement data.

Figure 3.1 Schematic of Rv-Measurement Apparatus



4.0 Data Reduction Procedure

The data reduction procedure that was developed for the calculation of the velocity-coupled response function from experimentally measured pressure data is presented here. The prime motivation for the data reduction procedure is the lack of an accurate means for direct measurement of the perturbation velocity at the propellant surface.

An expression for the velocity-coupled response function is obtained from equation (1.1) with modifications. Using the continuity relation

$$\frac{m_b'}{\bar{m}_b} = \frac{\rho'}{\bar{\rho}} + \frac{u_n'}{\bar{u}_n} \quad (4.1)$$

and the isentropic relation

$$\frac{\rho'}{\bar{\rho}} = \frac{p'}{\gamma \bar{p}} \quad (4.2)$$

equation (1.1) may be rewritten as

$$R_v = \left(\frac{u_n'}{\bar{u}_n} - \frac{p'}{\gamma \bar{p}} (R_p - 1) \right) \frac{\bar{u}_b'}{\bar{a}} \quad (4.3)$$

The conditions at the propellant surface are not isentropic: however, for lack of an alternative, the above substitution has been used to eliminate the density ratio from the expression for Rv.

All quantities on the right-hand side of equation (4.3), except for the perturbation velocity (u_n') are known at the propellant surface from either measurements downstream of the propellant or by simple computation (see Appendix 1).

Because of the hostile environment near the burning propellant, the perturbation velocity at the propellant surface cannot easily be measured directly. The perturbation velocity at the propellant surface is, however, a boundary condition to the system of differential equations which governs the flow within the combustion chamber.

In principle, the general solution of the governing equations along with the experimental pressure measurements should allow easy computation of the boundary condition u_r'. The correct boundary condition u_r' would force the general solution for the pressure distribution to match the measured pressure distribution at the points along the length of the Rv-measurement apparatus where the pressure measurements were taken. The velocity-coupled response function (Rv) would then be known from equation (4.3).

Unfortunately, the governing equations do not lend themselves to a closed form solution. In light of this, the following numerical data reduction procedure has been developed.

Step (1) Assume a value for Rv.

Step (2) Calculate the boundary condition u_r' using equation (4.3).

Step (3) Use u_r' along with the remaining boundary conditions to numerically solve for the pressure distribution within the experimental device.

Step (4) Compare the calculated pressure distribution to the experimentally measured pressure distribution.

Step (5) Choose a new Rv on the basis of the results of the comparison.

Step (6) Repeat steps (2) through (5) until the measured and computed values agree to within prescribed limits.

Steps (3), (4) and (5) will be discussed in greater detail in the following sections.

Since the velocity-coupled response function is a physical property of solid propellants, it is expected that the iterative procedure presented here will converge to a unique value. The validity of the final result can be checked by comparing the experimentally measured pressure distribution to the pressure distribution that would result from the Rv value found.

5.0 Numerical Solution of the Governing Equations

Step (3) of the data reduction procedure requires the computation of the perturbation pressure p' as a function of the axial distance from the propellant sample. A fourth-order Runge-Kutta numerical technique has been used. The discussion here will show how the governing equations have been reduced to the form required by the RK-4 numerical integration algorithm.

5.1 Governing Equations

The equations which govern the fluid flow within the experimental device are

(1) conservation of mass

(2) conservation of momentum

(3) conservation of energy, and

(4) ideal equation of state.

Written in differential form these equations are

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0 \qquad (5.1.1)$$

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = 0 \qquad (5.1.2)$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \gamma p\frac{\partial u}{\partial x} = 0 \qquad (5.1.3)$$

$$p = \rho R T \qquad (5.1.4)$$

The energy equation (5.1.3) incorporates the ideal equation of state, and is approximately valid for negligible heat conductivity of the combustion product gases. Furthermore, convective and radiative processes are considered to make negligible contributions to the energy equation.

Since the flow within the combustion chamber involves both mean and perturbation flows, it is convenient to separate the two as follows

$$\begin{aligned} p &= \bar{p} + p' \\ \rho &= \bar{\rho} + \rho' \\ u &= \bar{u} + u' \\ T &= \bar{T} + T' \end{aligned} \qquad (5.5.5)$$

Furthermore, the perturbations are periodic so that

$$\begin{aligned} p' &= P_o e^{i\omega t} \\ \rho' &= \rho_o e^{i\omega t} \\ u' &= u_o e^{i\omega t} \end{aligned} \qquad (5.5.6)$$

For periodic functions, the time-derivative operator reduces to $i\omega$, enabling the governing equations to be treated as ordinary differential equations.

Neglecting second-order effects the governing equations reduce to the following when substitutions (5.1.5) and (5.1.6) are made.

Continuity

$$\bar\rho \frac{\partial \bar u}{\partial x} + \bar u \frac{\partial \bar\rho}{\partial x} = 0 \qquad (5.1.7)$$

$$i\omega\rho' + u'\frac{\partial \bar\rho}{\partial x} + \bar\rho\frac{\partial u'}{\partial x} + \rho'\frac{\partial \bar u}{\partial x} + \bar u\frac{\partial \rho'}{\partial x} = 0 \qquad (5.1.8)$$

Momentum

$$\bar\rho\bar u\frac{\partial \bar u}{\partial x} + \frac{\partial \bar p}{\partial x} = 0 \qquad (5.1.9)$$

$$\rho' i\omega u' + \bar\rho\bar u\frac{\partial u'}{\partial x} + \bar\rho u'\frac{\partial \bar u}{\partial x} + \rho'\bar u\frac{\partial \bar u}{\partial x} + \frac{\partial p'}{\partial x} = 0 \qquad (5.1.10)$$

Energy

$$\bar\rho\bar u C_v\frac{\partial \bar T}{\partial x} + \bar p\frac{\partial \bar u}{\partial x} = 0 \qquad (5.1.11)$$

$$\bar\rho C_v i\omega T' + \bar\rho\bar u C_v\frac{\partial T'}{\partial x} + \bar\rho u' C_v\frac{\partial \bar T}{\partial x} + \rho'\bar u C_v\frac{\partial \bar T}{\partial x} + p'\frac{\partial \bar u}{\partial x} + \bar p\frac{\partial u'}{\partial x} = 0 \qquad (5.1.12)$$

Equation of State

$$\bar p = \bar\rho R\bar T \qquad (5.1.13)$$

$$p' = \bar\rho R T' + \rho' R\bar T \qquad (5.1.14)$$

Assuming that the mean flow Mach number of the flow normal to the propellant is much less than 1, terms containing $\bar{u}^2$ can be neglected. Rearrangement of the previous equations under the assumption of negligible mean flow Mach number yields the following equations.

$$\frac{\partial \bar{p}}{\partial x} = 0 \qquad (5,1,15)$$

$$\frac{\partial \bar{\rho}}{\partial x} = -\frac{\bar{\rho}}{\bar{T}} \frac{\partial \bar{T}}{\partial x} \qquad (5.1.16)$$

$$\frac{\partial \bar{u}}{\partial x} = \frac{\bar{u}}{\bar{T}} \frac{\partial \bar{T}}{\partial x} \qquad (5,1,17)$$

$$\frac{\partial u'}{\partial x} = -\frac{1}{\partial \bar{p}} i\omega p' - \frac{\bar{u}}{\partial \bar{p}} \bar{\rho} i\omega u' + \frac{1}{\gamma \bar{p}} (\bar{\rho} u' \bar{u} - \gamma p') \frac{\partial \bar{u}}{\partial x} \qquad (5,1,18)$$

$$\frac{\partial p'}{\partial x} = -\bar{\rho} i\omega u' - \bar{\rho} \bar{u} \frac{\partial u'}{\partial x} - \bar{\rho} u' \frac{\partial \bar{u}}{\partial x} \qquad (5,1,19)$$

These equations are in the form required by the Runge-Kutta numerical algorithm for integration. If the temperature distribution T(x) is known, these equations along with the boundary conditions at the propellant surface, are sufficient to calculate the pressure distribution (p') along the length of the experimental device.

5.2 Boundary Conditions

The boundary conditions required at the propellant surface are listed below

mean pressure

mean density

mean velocity

perturbation pressure

perturbation velocity

For the purpose of this investigation, typical values for the above quantities will be used. The values used, along with other environmental parameters are given in Appendix 1.

5.3 Temperature Distribution

Since the heat flux through the measurement apparatus wall is unknown the mean flow temperature distribution T(x) cannot be found directly. However, the following approximate expression for the temperature distribution using the one-dimensional steady-state heat diffusion equation has been used.

$$T(x) = (T_0 - T_\infty)\, e^{\lambda x} + T_\infty \qquad (5.3.1)$$

where

$$\lambda = 2 \frac{k_2}{k_1} \frac{1}{r \Delta t}$$

and, and are the thermal conductivities of the combustion gas and the apparatus wall respectively.

In the future, it may be necessary to refine the above temperature distribution to incorporate three-dimensional effects.

6.0 <u>Error Function</u>

In order to compare the numerical solution for the pressure distribution to the measured pressure distribution, the following least-squares error function has been considered

$$E(R_v) = \sum_{i=1}^{N} (p'_{m_i} - p'_{n_i})^2 \quad (6.1)$$

where N is the number of pressure measurements taken at selected points along the length of the experimental Rv-measurement device. The quantities p'_m and p'_n are the measured and calculated pressures at the i'th selected pressure port.

The quantities Rv, p'_m and p'_n in equation (6.1) are complex. The velocity-coupled response function is complex since the velocity-coupled response is not generally in phase with the tangential velocity perturbations. Also, the perturbation pressure is complex having both magnitude and phase. Consequently, the error function E is a complex function. It is meaningless, therefore, to consider minimizing E as defined by equation (6.1). In light of this, the following approaches have been considered.

(1) Minimize both the real and imaginary components of E simultaneously.

(2) Minimize the magnitude of E only.

(3) Minimize the phase angle of E only.

All three approaches are valid since both the magnitude and phase of the perturbation pressure distribution are unique for a given velocity coupled response function (Rv). As will be shown in Appendix 5, the use of the phase distribution provides the smoothest error function (E), and consequently, equation (6.1) has been modified through the use of the phase angles of the perturbation pressures. Thus the error function, E(Rv), given by

$$E(R_v) = \sum_{i=1}^{N} \left(\angle p_{m_i}' - \angle p_{n_i}' \right) \qquad (6.2)$$

is a real valued function of two independent variables. The symbol $\angle$ denotes the angular component of the perturbation pressure. The independent variables are the real and imaginary components of Rv.

The desired value of Rv is that which minimizes the above error-function. When E is minimized, the pressure distribution which has been calculated by numerically integrating the governing equations will match the measured pressure distribution as closely as possible at the N selected points.

7.0 Location of Pressure Ports

As mentioned in the previous section, the numerical solution for the perturbation pressure distribution can be fitted to the measured distribution at selected points along the length of the experimental device. The experimental apparatus has provisions for measurement of pressure at numerous locations along its length. The positions of the pressure measurement will affect the performance of the curve-fit procedure. The optimum location of these pressure measurements is at the part of the curve where variation is greatest since the shape of the curve will be most sensitive to slight variations in admittance value. Perturbation pressure phase distributions are provided in Appendix 5 for a wide range of admittance values and a driver frequency of 600 hertz. From these figures it can be seen that maximum variation of the curves occurs in the region between 0.100 meters and about 0.150 meters along the length of the Rv-measurement apparatus. The locations of the pressure measurements have been chosen to be at ten equally spaced locations from 0.100 meters to 0.145 meters along the length of the apparatus.

8.0 Optimization Procedure

8.1 BFGS Optimization Algorithm

The BFGS (Broyden, Fletcher, Goldfarb, Shanno) numerical optimization algorithm (Ref.4) has been used to minimize the error function (equation 6.2). Since the velocity-coupled response function (Rv) is complex, the domain has been considered to be the R^2 plane with the X1-coordinate representing the real part of Rv and the X2-coordinate representing the imaginary part of Rv.

Most search algorithms involve a line search given by the following equation

$$x_{k+1} = x_k - \alpha H_k \nabla y(x_k) \qquad (8.1.1)$$

where y is the function of interest. For a gradient search, H_k is the unit matrix (I) and α is the parameter of the gradient line. for Newton's method H_k is the inverse of the Hessian matrix, H_k^{-1}, and α is 1. The Hessian (H_k) is the matrix of second partial derivatives evaluated at the point x_k. For quasi-Newton methods, H_k is a series of matrices beginning with the identity matrix, I, and ending with the inverse of the Hessian matrix (H_k^{-1}). The quasi-Newton algorithm that employs the BFGS formula for updating the Hessian matrix is considered to be the most effective of the unconstrained multivariable search techniques, according to Fletcher (Ref. 5).

The minus sign in equation (7.1) indicates the direction of steepest descent and a positive sign in the equation would give the direction of steepest ascent. However, equation (7.1) is really steep descent rather than steepest descent. Only if the function of interest,the error function, is scaled such that a unit change in each of the independent variables reduces the same unit change in the error function will the gradient move in the direction of steepest descent. Scaling is a problem that has been encountered during this investigation. It was resolved by multiplying the real and imaginary components of the admittance by 1000.

The BFGS algorithm is outlined in what follows.

(1) Choose starting point Rv - x_k. ($k \quad 1$)

(2) Compute gradient E(x_k) using a forward difference approximation.

(3) Form the Hessian matrix given below.

$$H_{k+1} \quad H_k - \left[\frac{H_k \gamma_k \delta_k^T + \delta_k \gamma_k^T H_k}{\delta_k^T \gamma_k} \right] + \left[\frac{\gamma_k^T H_k \gamma_k}{\delta_k \gamma_k} \right] \left[\frac{\delta_k \delta_k^T}{\delta_k^T \gamma_k} \right] \quad (8.1.2)$$

where $\delta_k \quad x_{k+1} \quad x_k$ and $\gamma_k - \nabla E(x_{k+1}) - \nabla E(x_k)$

(4) Form the gradient line given below.

$$x_{k+1} \quad x_k \quad \alpha_{k+1} H_k \nabla E(x_k) \quad (8.1.3)$$

(5) Search for parameter α_{k+1} which minimizes the error function (E) along the gradient line. This is described in detail in the next section.

(6) Compute x_{k+1} .

(7) Repeat steps (2) through (6) until convergence.

As mentioned previously, the initial value for the Hessian matrix is the identity matrix.

The algorithm generates a sequence of values x_k that move rapidly from the starting point x_k to the neighborhood of the optimum x^* and terminate when the difference between successive iterations is less than some prescribed value.

The value of the parameter α which minimizes the error function along the gradient line is found by a Fibonacci line search. The Fibonacci line search algorithm is discussed in the following in the following section.

8.2 Fibonacci Line Search

To find the optimum of a search problem, a search plan is required. A search plan is a set of instructions for performing N sets of "experiments" $x_1, x_2, \ldots, x_n$ (values of the independent variables). In this discussion, an "experiment" will consist of specifying the value of the independent variable (α) and determining the value of the function E(α) for that specified value.

The Fibonacci search (Ref. 4) will be used to search for the parameter α which minimizes the error function (E) along the gradient line (equation 8.1.1). Values for range from 0 to 1 generating points along the line from x_k to x_{k+1}.

The first step of the Fibonacci search requires the calculation of the number of experiments required to reduce the final interval of uncertainty in α to within tolerable limits. This desired final interval of uncertainty

(I_n) is given by the following equation.

$$I_n = \frac{I_0}{A_{n+1}} \qquad (8.2.1)$$

where I_0 is the initial interval, and A_{n+1} is the (n+1)'th Fibonacci number. The initial interval (I_0) is 1 since the parameter α has possible values ranging from 0 to 1.

The Fibonacci numbers are generated as follows

$A_1 \quad 1$

$A_2 - 2$

$A_i \quad A_{i-1} + A_{i-2} \qquad$ for i > 2

The number of experiments required to reduce the interval of uncertainty from I_0 1 to I_n is the smallest value n such that

$$A_{n+1} \geq \frac{I_0}{I_n} \qquad (8.2.3)$$

The values of α for the first two experiments are the following

$$\alpha_1 = A_{n-1} I_n = A_{n-1} \left\{ \frac{I_0}{A_{n+1}} \right\} \qquad (8.2.4)$$

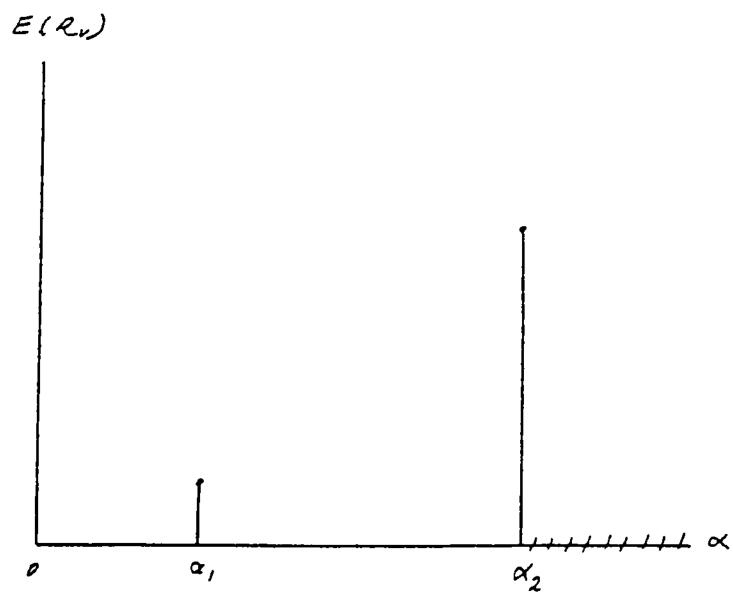
$$\alpha_2 = A_n I_n = A_n \left\{ \frac{I_0}{A_{n+1}} \right\} \qquad (8.2.5)$$

The position of α_1 is symmetrically to α_2 , a distance I_2 from the upper bound of the interval.

The procedure continues after the first two experiments are evaluated by discarding the interval that does not contain the minimum. It is important to realize that the function is assumed to be unimodal along the gradient line. The error function (equation 6.2) is not unimodal. However, it has been found that the distances between local minimums is large with respect to typical gradient line lengths.

The third experiment is placed in the remaining interval symmetrically to the one which was not in the discarded interval. The procedure is continued by placing the remaining experiments symmetrically to the previous one with the best value until the final experiment is placed.

The procedure is best illustrated by the following figures.

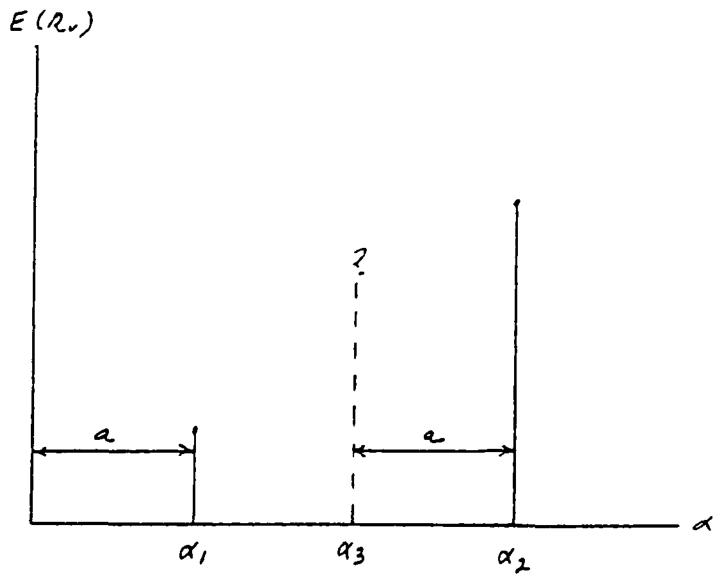


Error Function along Gradient Line

Figure 8.2.1

Suppose $E(\alpha_2) > E(\alpha_1)$ then by unimodality the interval $[\alpha_1, 1]$ would not contain the minimum. Therefore, it is discarded and the second interval of uncertainty is then $[0, \alpha_2]$.

The third experiment is placed symmetrically to α_1 in the remaining interval as illustrated below.



Error Function along Gradient Line

Figure 8.2.2

The remaining experiments are placed in a similar manner until the desired final interval of uncertainty I_n is obtained. The optimal value for α is then taken to be either the upper bound or the lower bound the final interval of uncertainty depending on whether the previously discarded interval was to the right or left. Since the final interval is chosen to be small, any point in that interval could be used as the optimal value.

9.0 Description of Computer Program

The data reduction program consists of a Runge-Kutta routine driven by a BFGS multivariable search routine. The Runge-Kutta solves the governing equation for the perturbation pressure distribution within the experimental device for a specified admittance value of the solid propellant. The following sections will discuss the admittance to velocity-coupled response function conversion, the Runge-Kutta algorithm and the BFGS multivariable search algorithm in greater detail.

9.1 Admittance to Velocity-Coupled Response Function Conversion

The admittance (Y) of a solid propellant is defined by the following equation.

$$\frac{u_n'}{a_n} = Y \frac{p'}{\gamma \bar{p}} \quad (9.1.1)$$

An expression for the perturbation velocity is obtained by rearranging equation (4.3).

$$u_n' = \bar{u}_n \frac{p'}{\gamma \bar{p}} (R_p - 1) + R_v \bar{u}_n \frac{\overline{u_t'}}{\bar{a}} \quad (9.1.2)$$

Upon elimination of the normal perturbation velocity from the above equations the following relationship between the admittance and the velocity-coupled response function is obtained.

$$Y = \frac{\gamma \bar{u}_n}{\bar{a}} (R_p - 1) + \frac{\gamma \bar{u}_n}{\bar{a}_n} \frac{\bar{p}}{p'} \overline{u_t'} R_v \quad (9.1.3)$$

Program 2 (Appendix 3) has been written to perform the above conversion.

9.2 Runge-Kutta Algorithm

A fourth-order Runge-Kutta algorithm (Ref. 6) is used to determine the perturbation pressure distribution by integration of the governing equations (5.1.15) through (5.1.19). The input to the algorithm consists of the environmental parameters and the boundary conditions at the propellant surface. Typical values for the environmental parameters and all boundary conditions except for the perturbation velocity (u_{κ}') at the propellant surface are given in (Appendix 1). For calculation of the velocity-coupled response function for a particular propellant sample in a specified environment all input quantities except for u_{κ}' are fixed. This boundary condition is determined by admittance values which are generated according to the multivariable search procedure. For a particular admittance value, the perturbation velocity (u_{κ}') is given by

$$u_{\kappa}' = Y \bar{a} \frac{p'}{\partial \bar{p}} \quad (9.2.1)$$

The step size (H) governs the locations at which the perturbation pressures are calculated. It is important that the perturbation pressure be calculated at points along the length of the experimental device where pressure measurements are made. Improved accuracy can be obtained by decreasing the step size (H). Alternatively, the step size can be adjusted in the middle of the calculation to accommodate a pressure distribution that is rapidly varying.

9.3 BFGS Optimization Algorithm

The purpose of this algorithm is to search for the particular admittance value which provides the boundary condition (u_{π}') which, when used as input to the Runge-Kutta routine, yields the pressure distribution matching the measured pressure distribution.

The BFGS algorithm has been encoded into a main program, two subroutines and three function subroutines.

The three function subroutines are as follows. The function FUNCT contains the function to be minimized (equation 6.2). The function F uses FUNCT for the value of the error function in the line search. The function FIBON uses the values of F in the Fibonacci line search. The two subroutines are slope, which evaluates the partial derivatives using a forward difference approximation, and PRINT, which prints the results of the computations.

The input variables for the BFGS routine are the starting point (Y_0) and the stopping criterion. The program will terminate when the difference between the error function values of two successive iterations is less than or equal to the stopping criterion. The output results are the iteration number, the value of the error function, and the admittance value.

The main portion of the BFGS algorithm proceeds from iteration zero, the starting point, and generates successive points until the stopping criterion is met. Initially, the Hessian matrix (H) is the identity matrix. The gradient is computed using a forward difference approximation of the partial derivatives (using subroutine SLOPE). The Fibonacci search function, FIBON, is used to locate the minimum along the gradient line from x_k to x_{k+1}. Then the stopping criterion is checked, and the Hessian matrix is stored in ERROLD for future comparisons. The search direction to the next point is calculated and stored in the vector S. The value of the parameter of the line in the search direction, K, is calculated using FIBON to locate the next point. The value of the error function at the new point is calculated and stored in ERR. The values of the iteration counter, the function at the new point, and the new point are printed using PRINT. The values of the gradient at the current point are computed and stored in the vector GRAD. The Hessian matrix is updated, and the program returns to repeat the calculation until the error criterion is satisfied.

10.0 Simulation of Experiment

For the purpose of testing the data reduction program, experimental pressure phase angle data have been simulated by introducing random fluctuations in calculated pressure phase angles for specific values of admittance. Experimental data was simulated for two admittance values: (0.04,-0.02) and (0.00,0.02). Program 3 (Appendix 4) was used to obtain random fluctuations from gaussian distributions having standard deviations ranging from zero through five degrees.

Using the simulated data, the program was tested to see whether or not convergence to the above admittance values from an arbitrary starting point was attained. The results are presented in the following section.

11.0 Simulation Results

Results are presented here for two sample runs of the data reduction program. Using admittance values of (0.04,-0.02) and (0.00,0.,02) the perturbation pressure distribution was calculated. The following computer printouts show the results for random fluctuations in input pressure data ranging from zero to five degrees. The performance of the data reduction program has proven to be satisfactory for perturbation pressure phase measurement data having random fluctuations of less than five degrees.

The velocity-coupled response function values corresponding to the admittance values were found using Program 2 (Appendix 3) to be (0.15,0.48) and (-0.19,0.13) for the respective admittance values (0.04,-0.02) and (0.00,0.02). It was necessary to assume a value for the pressure-coupled response function to perform the above conversion. The value used for Rp was (0.1,-0.1). Since the pressure-coupled response of solid propellants was not investigated, the admittance to Rv conversion was not included in the main data reduction program.

The test results are summarized at the end of this section. The summary table shows the admittance values which were recovered when pressure phase data corresponding to the above two admittance values were input. It is clear from these results that performance of the data reduction program is generally good when random fluctuations in the input pressure phase data does not exceed four degrees.

Standard Deviation in Random Error of Input Data = 0 degrees

```
BF[illegible]-FIBONACCI SEARCH FOR ADMITTANCE
---------------------------------------------

INPUT DATA:

Stopping Criterion       .5000

Starting Point (Yo)    5.00  10.00

Experimental Pressure Phase Values (degrees)

        P( 1) =   93.962
        P( 2) =   95.161
        P( 3) =   97.294
        P( 4) =  102.203
        P( 5) =  124.158
        P( 6) =-120.902
        P( 7) =-101.671
        P( 8) = -97.054
        P( 9) = -95.016
        P(10) = -93.853

RESULTS:

Iteration          Error Function          Y x 1000

    0                 145.357          5.000  10.000
    1                   2.703          -.551  13.331
    2                    .253           .143  15.569
    3                    .253           .143  15.569

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 1 degree

BFGS-FIBONACCI SEARCH FOR ADMITTANCE

INPUT DATA:

Stopping Criterion 5000

Starting Point (Yo) 5.00 10.00

Experimental Pressure Phase Values (degrees)

```
F( 1) =  94.364
F( 2) =  95.040
F( 3) =  98.703
F( 4) = 101.274
F( 5) = 124.953
F( 6) =-121.193
F( 7) =-100.645
F( 8) = -94.231
F( 9)    -95.469
F(10) = -93.295
```

RESULTS:

Iteration	Error Function	Y X 1000	
0	144.550	5.000	10.000
1	14.559	.336	18.661
2	13.053	.291	13.176
3	13.053	.291	19.176

NORMAL TERMINATION OF PROGRAM

Standard Deviation in Random Error of Input Data = 2 degrees

```
GAUSS-FIBONACCI SEARCH FOR ADMITTANCE
-------------------------------------

INPUT DATA:

Stopping Criterion       5000

Starting Point (x):   10.00   15.00

Experimental Pressure Phase Values (degrees)

        P( 1) =   37.113
        P( 2) =   97.797
        P( 3) =   35.975
        P( 4) =  104.801
        P( 5) =   13.862
        P( 6) =-121.313
        P( 7) =-101.966
        P( 8) = -68.061
        P( 9) = -33.738
        P(10) = -81.33[illegible]

RESULTS:

Iteration            Error Function           X x 1000

     0                 101.328              0.000   15.000
     1                 [illegible]          [illegible]   [illegible]
     2                 42.[illegible]       [illegible]   [illegible]
     3                 44.001               2.350   22.715
     4                 44.002               2.350   22.718

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 3 degrees

```
BFGS-FIBONACCI SEARCH FOR ADMITTANCE
------------------------------------

INPUT DATA:

Stopping Criterion        5000

Starting Point (Yo)   10.00   15.00

Experimental Pressure Phase Values (degrees)

        P( 1) =   90.158
        P( 2) =   92.878
        P( 3) =  100.965
        P( 4) =  102.085
        P( 5)    123.445
        P( 6) =-120.909
        P( 7)    -96.136
        P( 8) =  -93.569
        P( 9)    -94.053
        P(10)    -97.027

RESULTS:

Iteration              Error Function              Y x 1000

    0                      520.142             10.000   15.000
    1                       96.429               .645   13.441
    2                       26.600               .170   16.029
    3                       18.591             - .182   15.131

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 4 degrees

```
BFGS-FIBONACCI SEARCH FOR ADMITTANCE
------------------------------------

INPUT DATA:

Stopping Criterion        0100

Starting Point (Yo)    5.00   10.00

Experimental Pressure Phase Values (degrees)

        P( 1) =   93.064
        P( 2) =   98.413
        P( 3)     94.366
        P( 4) = 101.513
        P( 5) = 123.687
        P( 6) =-116.263
        P( 7) = -99.870
        P( 8)    -92.986
        P( 9) = -93.959
        P(10)    -97.452

RESULTS:

Iteration            Error Function           Y X 1000

    0                    119.865            5.000   10.000
    1                     67.970            3.130   17.323
    2                     64.350            2.211   17.424
    3                     64.350            2.211   17.424

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 5 degrees

```
BFGS-FIBONACCI SEARCH FOR ADMITTANCE
------------------------------------

INPUT DATA:

Stopping Criterion       .0100

Starting Point (Yo)    5.00   15.00

Experimental Pressure Phase Values (degrees)

        P( 1) =   80.007
        P( 2) =   94.544
        P( 3) =   98.881
        P( 4) =  101.513
        P( 5) =  114.159
        P( 6) =-128.118
        P( 7) =-103.380
        P( 8) - -95.704
        P( 9) =-101 160
        P(10)    -85.622

RESULTS:

Iteration              Error Function              Y X 1000

     0                    450.159             5.000   15.000
     1                    424 472             2 434   15.000
     2                    139.223             -.143   29.756
     3                    138.226             - 143   29.755

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 0 degrees

```
BFGS-FIBONACCI SEARCH FOR ADMITTANCE
------------------------------------

INPUT DATA:

Stopping Criterion       .5000

Starting Point (Yo)  30.00 -25.00

Experimental Pressure Phase Values (degrees)

        P( 1) =  86.505
        P( 2) =  85.244
        P( 3) =  82.700
        P( 4)    74.818
        P( 5) = -21.187
        P( 6) = -77.341
        P( 7)   -83.329
        P( 8) = -85.506
        P( 9) = -86.637
        P(10) = -87.335

RESULTS:

Iteration            Error Function           Y X 1000

     0                  1506.887           30.000 -25.000
     1                      .237           40.350 -20.175
     2                      .179           40.306 -20.079

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 1 degree

```
BFGS-FIBONACCI SEARCH FOR ADMITTANCE
------------------------------------

INPUT DATA:

Stopping Criterion      .5000

Starting Point (Yo)  30.00 -25.00

Experimental Pressure Phase Values (degrees)

        P( 1) =  86.907
        P( 2)    85.123
        P( 3) =  84.109
        P( 4) =  73.887
        P( 5) = -20.497
        P( 6) = -77.632
        P( 7) = -82.303
        P( 8) = -82.673
        P( 9) = -87.110
        P(10) = -86.777

RESULTS:

Iteration           Error Function           Y X 1000

     0                1592.050           30.000 -25.000
     1                  12.126           40.660 -20.062
     2                  12.097           40.629 -19.993

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 2 degrees

```
BFGS-FIBONACCI SEARCH FOR ADMITTANCE
------------------------------------

INPUT DATA:

Stopping Criterion        5000

Starting Point (Yo)   30.00 -25.00

Experimental Pressure Phase Values (degrees)

        P( 1) =  36.981
        P( 2)    87.880
        P( 3) =  61.391
        P( 4) =  77 414
        P( 5) = -25.663
        P( 6)   -77 757
        P( 7) = -93.524
        P( 8) = -64.533
        P( 9) = -35.417
        P(10) = -85.313

RESULTS:

Iteration            Error Function            Y X 1000

     0                  1183.286           30.000 -25.000
     1                    29.460           38.670 -21.032
     2                    29.302           38.578 -20.647

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 3 degrees

```
BFGS-FIBONACCI SEARCH FOR ADMITTANCE
------------------------------------

INPUT DATA:

Stopping Criterion      .3000

Starting Point (Yo)  30.00 -25.00

Experimental Pressure Phase Values (degrees)

        F( 1) =  83.309
        F( 2) =  82.959
        F( 3) =  86.371
        F( 4) =  74.678
        F( 5) = -16.900
        F( 6) = -77.346
        F( 7) = -77.797
        F( 8) = -82.011
        F( 9) = -85.674
        F(10) = -90.509

RESULTS:

Iteration            Error Function            Y x 1000

    0                  1973.404            30.000 -25.000
    1                    79.720            42.249 -19.447
    2                    79.721            42.249 -19.447

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 4 degrees

```
BFGS-FIBONACCI SEARCH FOR ADMITTANCE
------------------------------------

INPUT DATA:

Stopping Criterion      .5000

Starting Point (Yo)  30.00 -25.00

Experimental Pressure Phase Valuee (degrees)

        F( 1)     82.550
        F( 2) =   84.827
        F( 3) =   82.287
        F( 4)     73.282
        F( 5) =  -31.186
        F( 6) =  -84.557
        F( 7) =  -85.038
        F( 8) =  -84.146
        F( 9) =  -82.781
        F(10) =  -79.104

RESULTS:

Iteration            Error Function          Y X 1000

     0                  991.810          30.000 -25.000
     1                  173.712          37.075 -21.803
     2                  170.172          37.551 -22.737
     3                  170.172          37.551 -22.737

NORMAL TERMINATION OF PROGRAM
```

Standard Deviation in Random Error of Input Data = 5 degrees

BFGS-FIBONACCI SEARCH FOR ADMITTANCE

INPUT DATA:

Stopping Criterion .5000

Starting Point (Yo) 30.00 -25.00

Experimental Pressure Phase Values (degrees)

F(1) = 90.607
F(2) = 88.496
F(3) = 80.372
F(4) = 74.126
F(5) = -21.658
F(6) = -72.702
F(7) = -81.528
F(8) = -81.428
F(9) = -85.580
F(10) = -90.934

RESULTS:

Iteration	Error Function	Y x 1000
0	1655.979	30.000 -25.000
1	33.488	41.029 -20.053
2	61.326	41.301 -20.680
3	61.326	41.301 -20.680

NORMAL TERMINATION OF PROGRAM

SUMMARY OF TEST RESULTS

Simulated Input Data for an Admittance Value of (0.00,20).

Standard Deviation of Random Error	Value of Error Function	Starting Point	Value Recovered
0	0.253	(5,10)	(.143,19.6)
1	13.1	(5,10)	(.201,19.2)
2	44.0	(10,15)	(2.05,22.7)
3	88.6	(10,15)	(-.182,16.1)
4	64.4	(5,10)	(2.21,17.4)
5	138	(5,15)	(-.143,29.8)

Simulated Input Data for an Admittance Value of (40,-20).

Standard Deviation of Random Error	Value of Error Function	Starting Point	Value Attained
0	0.179	(30, -25)	(40.3, -20.1)
1	12.1	(30, -25)	(40.6, -20.0)
2	29.3	(30, -25)	(38.6, -20.8)
3	79.7	(30, -25)	(42.2, -19.4)
4	81.3	(30,-25)	(41.3, -20.7)
5	170	(30, -25)	(37.6, -22.7)

(note: all admittance values are multiplied by 1000)

12.0 Conclusion

A data reduction program has been written for the calculation of the velocity-coupled response function of solid propellants from experimentally measured pressure data. The program has been written for use with a new velocity-coupled response function measurement apparatus that is currently being developed at Embry-Riddle Aeronautical University. The data reduction procedure has proven satisfactory for simulated experimental data which have random fluctuations with standard deviations up to four degrees.

13.0 Recommendations for Future Work

The following recommendations for future investigation which are outlined here are based on the possible need for improvements in the performance and accuracy of the data reduction program.

(1) Study the heat transfer between the combustion gases and the surroundings. Revise the approximation for the temperature distribution given by equation (5.3.1).

(2) Investigate the shape of the error function (equation 6.2). Proper scaling of this function will improve the performance of the BFGS optimization routine. The method of steepest descent works best when shape of the error function is parabolic. Procedures for scaling are described in detail by Wilde (Ref. 6) and Wilde and Beighter (Ref. 7).

(3) Investigate other types of error functions for the possibility of obtaining one which is unimodal for the domain of realistic Rv values. For unimodal error functions, the global convergence property assures convergence to the correct solution from any starting point.

References

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7. Wilde, D.J., Optimum Seeking Methods, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1964.

8. Wilde, D.J., and Beighter C.S., _Foundations of Optimization_, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1967.

Appendix 1: Input Data

Typical values for environmental parameters are given below. The variable names used are the same as those used in the computer program.

ratio of specific heats: GAM - 1.4

universal gas constant: R .. 287 J/kgK

specific heat at constant volume: CV = 717 J/kgK

conductivity of combustion gases: K1 30 W/mK

conductivity of apparatus wall: K2 - 40 000 W/mK

radius of apparatus: RADIUS 0.05 m

length of apparatus: LENGTH 0.01 m

ambient temperature: TINF 273 K

The following are boundary conditions for the variables at the propellant surface.

mean pressure: P0 = 101320 Pa

perturbation pressure: PP0 = (0,1200) Pa

density: RHO0 - 1.226 kg/m^3

temperature: TNOT = 3500 K

The mean flow velocity ($\bar{u}_\lambda$) at the propellant surface can be calculated from continuity and the propellant mass burn rate. The mass flow ($\dot{m}$) is related to the mass burn rate by

$$\dot{m} - r_b \rho_b A_b \quad (A.1)$$

The continuity relation is

$$\dot{m} - \bar{u}_n \bar{\rho} A_b \quad (A.2)$$

Equating (A.1) and (A.2), the following expression for the mean flow velocity is obtained

$$\overline{u_n} = \frac{r_b \rho_b}{\bar{\rho}} \qquad (A.3)$$

Typical values for the quantities in equation (A.3) are given below

mass burn rate: r_b = 0.0063 kg/s

propellant density: ρ_b 1200 kg/m^3

exhaust gas density: $\bar{\rho}$ - 1.226 kg/m^3

The mean flow normal velocity at the propellant surface is then

mean flow velocity: u_o = 6 m/s

The frequency of the standing acoustic wave was given a value of 600 hertz. This value is within the frequency range at which the Rv-measurement apparatus is expected to be operated. The angular frequency is then 2 x 3.14 x 600 hz.

angular frequency of acoustic wave: ω 3770 rad/s

It is necessary that the mass flow perturbation frequency be identical to the acoustic frequency. This requirement is met if the propellant sample is oscillated at one-half the acoustic frequency since the mass flow response is not sensitive to the direction of the tangential perturbation velocity.

The average tangential velocity obtained by integrating over one period is

$$\overline{u_t'} = \frac{2}{\pi} \overline{u_{t_o}'} \quad (A.4)$$

where $\overline{u_{t_o}'}$ is the maximum value obtained by the tangential perturbation velocity. A typical value for $\overline{u_{t_o}'}$ is 30 m/s. The mean tangential perturbation velocity from the previous equation is 19 m/s.

Appendix 2: Data Reduction Program

This appendix contains a listing of the data reduction program. This program determines the admittance value of the propellant. The admittance to velocity-coupled response function conversion is performed by Program 2 (Appendix 3).

```
FIN7X
$FILES 2,20
C
C      PROGRAM BFGS MULTIVARIABLE OPTIMIZATION
C
C      WILLIAM H  JARVIS
C      EMBRY RIDDLE AERONAUTICAL UNIVERSITY
C      DAYTONA BEACH  FLORIDA
C      APRIL, 1989
C

       INTEGER ITER
       DOUBLE PRECISION TOLER, FUNCT  FIBON,
      & HESS(20,20), GRAD(20), GRAD1(20), GAMMA(20), DELTA(20 ,
      & HS(20), K, ERF, ERFOLD, EPS, GFHC, DFG  X(20), S(20),F

       COMMON X, S, NTERM

       INTEGER STEP,COUNT
       REAL GAM R,CV W H,K1,F3,RADIUS,LENGTH,TNOT,TINF
       REAL P0 RHO0,U0
       COMPLEX FF0,RV FP,UI
       REAL PA(10)
       COMMON/BLOCK/STEP,COUNT,GAM,R,CV,W,H,K1 F2 RADIUS LENGTH TNOT
      $TINF,P0,RHO0 U0 FF0,PA,RV FP,UI

       CALL INPUT



       ITER=0
       K=0

C
C      READ AND ECHO INPUT DATA
C

       WRITE(1,*) 'BFGS-FIBONACCI SEARCH FOR ADMITTANCE'
       WRITE(1,*) '-------------------------------------'
       WRITE(1,*)   '
       NTERM = 2
       WRITE (1,*) 'INPUT STOPPING CRITERION '
       READ(1,*) EPS
       WRITE (1,*) 'INPUT EXPERIMENTAL PRESSURE PHASE MEASUREMENTS '
       READ(1 *) (PA(I), I=1,10)
       WRITE (1 *) 'INPUT STARTING VALUE FOR ADMITTANCE (Yo)'
       READ(1,*) (X(I)  I=1,NTERM)
       WRITE(1,*)
       WRITE(1 *     '
       WRITE(1,*)  BFGS-FIBONACCI SEARCH FOR ADMITTANCE
       WRITE 1 *)   --- ------ ---- ---- ---   ----------- -
       WRITE(1 *) '
       WRITE(1,*)  INPUT DATA
       WRITE (1, )
       WRITE(1 100) EPS
 100   FORMAT(  Stopping Criterion ,F1[illegible]
       WRITE (1 *)     '
       WRITE(1,[illegible] X(1),X(2)
 201   FORMAT(' Starting Point (Yo) , (1X F[illegible]
       WRITE (1,*)
       WRITE (1,*)  Experimental Pressure Phase Values (degrees)
       WRITE (1,*)
       DO 11 I = 1,10
       WRITE (1 110) I PA(I)
```

```
 11    CONTINUE
 110   FORMAT ('          P(',I2,') =',F8.3)
       WRITE(1,601)
 601   FORMAT(/,/,'RESULTS ',
      &    /, /,'Iteration',10X,'Error Function',10X,'X X 1000 ',/)

C
C      BFGS SEARCH
C

       ERR= FUNCT( X )
       CALL PRINT( ITER, NTERM, ERR, X, K )
       CALL SLOPE( GRAD, ERR )

C
C      FORM THE IDENTITY MATRIX
C

       DO 40 I=1,NTERM
          DO 40 J=1,NTERM
             IF (I.NE.J) HESS(I,J)= 0.0
             IF (I.EQ.J) HESS(I,J)= 1.0
 30    CONTINUE
 40    CONTINUE

       ERROLD= ERR
       ITER= ITER + 1

C
C      S(I) = HESSIAN * GRADIENT
C

       DO 50 I=1,NTERM
          S(I)= 0.0
          DO 50 J=1,NTERM
             S(I)= S(I) + HESS(I,J) * GRAD(J)

 50          CONTINUE

C
C      K = ALPHA IN SECTION 9.3
C

       K= FIBON( DUMMY )

C
C      DETERMINE NEXT X
C
C      DELTA = ALPHA * HESSIAN * GRADIENT
C

       DO 60 I=1,NTERM
          DELTA(I)= K * S(I)
          X(I)= X(I) - DELTA(I)
          CONTINUE
       ERR = FUNCT(X)
       CALL SLOPE( GRAD1, ERR )

C
C      DETERMINE NEW BFGS MATRIX
C

       DPG= 0.0
       DO 70 I=1,NTERM
          GAMMA(I)= GRAD1(I) - GRAD(I)
          DPG= DPG + GAMMA(I) * DELTA(I)
```

```
 70    CONTINUE
       DO 80 I=1,NTERM
          GRAD(I)= GRAD1(I)
 80    CONTINUE
       GPHG= 0 0
       DO 90 I=1,NTERM
          HG(I)= 0 0
          DO 90 J=1,NTERM
             HG(I)= HG(I) + HESS(I,J) * GAMMA(J)
             GPHG= GPHG+ HESS(I,J) * GAMMA(I) * GAMMA(J)
 90    CONTINUE
       DO 100 I=1,NTERM
          DO 100 J=1,NTERM
             HESS(I,J)= HESS(I,J) - (HG(I) * DELTA(J) /DPG)
     &                   - (DELTA(I) * HG(J) / DPG)
     &                   + (1 + (GPHG / DPG)) * (DELTA(I)
     &                   * DELTA(J) / DPG)
 100   CONTINUE
       TOLER = DABS(ERR-ERROLD)
       IF (TOLER  GE. EPS) CALL PRINT( ITER, NTERM, ERR, X, K)
       IF (TOLER  GE  EPS) GO TO 30
       CALL PRINT(ITER,NTERM,ERR,X,K)
       WRITE (1,*)' '
       WRITE(1,*)  NORMAL TERMINATION OF PROGRAM'
       STOP
       END

C
C      COMPUTATION OF PARTIAL DERIVATIVES
C

       SUBROUTINE SLOPE( DERIV, E)
       DOUBLE PRECISION DERIV(30), E, DELTA,TEMPX, Y X(20) S(20 , FUNCT
       COMMON X, S  NTERM

       DO 30 I=1,NTERM
          DELTA= 1 0E-04
          TEMPX= X(I)
          X(I)= X(I) + DELTA
          Y= FUNCT( X )
          DERIV(I)= (Y   E)/DELTA
          X(I)= TEMPX
 30       CONTINUE
       RETURN
       END

C
C      PRINT RESULTS
C

       SUBROUTINE PRINT( I, N, VAL,  [illegible]
       DOUBLE PRECISION X(20),VAL,K
       WRITE (1,500) I,VAL,(X(J),J=1,N)
 500   FORMAT(1X I3,12X,F10 3,8X,10(1X [illegible]
       RETURN
       END

C
C      FIBONACCI SEARCH FUNCTION
C

       DOUBLE PRECISION FUNCTION FIBON( DUMMY )
       DOUBLE PRECISION RATIO, FIBO(50) ,
     & LBOUND, HBOUND, INTER, FINTER, DELTA, TESTLB
     & TESTHB, TLBV, THBV, TEST, FACT, TLB ,F
       INTEGER EXPCNT, EXPNO, FLAG
```

```
      LBOUND = 0.0
      TEST   = 1.0
      HBOUND = 1.0
      FINTER = 0.0001
      FACT   = 1.618031

C
C     DETERMINE THE INTERVALS OF THE FIBONACCI SEARCH
C

 10   CONTINUE
      TLBV = F ( TEST )
      THBV = F ( HBOUND )
      IF (TLBV.GT.THBV) GO TO 20
         TLB = TEST
         TEST= HBOUND
         HBOUND= HBOUND * FACT
         GO TO 10
 20   CONTINUE

C
C     DERMINE BOUNDS AND DELTA FOR FIBONACCI SEARCH
C

      IF(TEST.NE. 1 ) LBOUND = TLB
      INTER= HBOUND- LBOUND
      DELTA = TEST  LBOUND
      TESTLB= TEST
      TESTHB= HBOUND - DELTA
      IF (TESTLB.LT. TESTHB) GOTO 38
      TLB = TESTLB
      TESTLB = TESTHB
      TESTHB = TLB
      DELTA = TESTLB - LBOUND
      TESTHB = HBOUND -DELTA
 38   CONTINUE
      INTER  = HBOUND - LBOUND
      RATIO  = INTER/FINTER

C
C     DETERMINE THE NUMBER OF EXPERIMENTS REQUIRED
C

      FIBO(1) = 1
      FIBO(2) = 2
      DO 39 I =3,50
         FIBO(I) = FIBO(I-1) + FIBO(I-2)
         IF (FIBO(I).LT. RATIO) EXPNO  I + 1
 39   CONTINUE

C
C     START CLOSED BOUND FIBONACCI SEARCH


      DO 40 EXPCNT=1, EXPNO
      TLBV= F (TESTLB)
      THBV= F (TESTHB)
      IF (TLBV.GE.THBV) GO TO 30
         LBOUND= TESTLB
         INTER= HBOUND - LBOUND
         DELTA= INTER- DELTA
         TESTLB= TESTHB
         TESTHB= HBOUND   DELTA
         FLAG  = 1
         GO TO 40
 30   CONTINUE
```

```
          HBOUND= TESTHB
          INTER= HBOUND - LBOUND
          DELTA= INTER - DELTA
          TESTHB= TESTLB
          TESTLB= LBOUND + DELTA
          FLAG = 0
 40    CONTINUE
       IF (FLAG  EQ  1) FIBON = TESTLB
       IF (FLAG  EQ  0) FIBON = TESTHB
       RETURN
       END

C
C      FUNCTION EVALUATION FOR FIBONACCI SEARCH
C

       DOUBLE PRECISION FUNCTION F(K)
       DOUBLE PRECISION K, TEST(20), X(20), S(20), FUNCT
       COMMON X, S, NTERM
       DO 10 I=1, NTERM
          TEST(I)=X(I) - K * S(I)
 10    CONTINUE
       F= -FUNCT( TEST )
       RETURN
       END

C
C      CALCULATE VALUE OF ERROR FUNCTION
C

       DOUBLE PRECISION FUNCTION FUNCT (X)
       DOUBLE PRECISION X(20)
       INTEGER STEP,COUNT
       REAL GAM,P,CV,W,H,K1,K2,RADIUS,LENGTH,TNOT,TINF
       REAL P0,RHO0,U0
       COMPLEX PP0,RV,PP,UT
       REAL PA(10),MAG
       COMMON/BLOCK/STEP,COUNT,GAM,R,CV,W,H,K1,K2,RADIUS,LENGTH,TNOT,
      ATINF,P0,RHO0,U0,PP0,PA,RV,RP,UT
       REAL P(100 ),RHO(100 ),U(100 ),A0
       COMPLEX UP(100 ),PP(100 ),UP0
       REAL RZ, IP ,PHASE

       RV   = (1,0)* X(1) + (0,1) * X(2)
       A0   SQRT(GAM*P0/RHO0)
       UP0 = RV*A0*PP0/GAM/P0/1000
       CALL RFFOUR(GAM,P,CV,COUNT,H,W,K1,K2,RADIUS,LENGTH,P0,RHO0 U0,
      AUP0,PP0,P,RHO U,UP,PP,TNOT,TINF)

       OPTIM = 0 0
       DO 747 N=1,10
             RZ = REAL (PP(N+19))
             IP = AIMAG(PP(N+19))
             PHASE = ATAN (IP/RZ)*180/3 14159265
             IF ((RZ EQ 0 )  AND  (IP  EQ  0 )) PHASE = 0
             IF ((RZ LT 0 )  AND  (IP  EQ  0 )) PHASE = 180
             IF ((RZ EQ 0 )  AND  (IP LT 0 )) PHASE = -90
             IF ((RZ LT 0 )  AND  (IP GT 0 )) PHASE = PHASE+180
             IF ((RZ LT 0 )  AND  (IP LT 0 )) PHASE   PHASE-180
              MAG - PHASE
              OPTIM = OPTIM + (PA(N)-MAG)**2
747    CONTINUE

       FUNCT = OPTIM
       RETURN
       END
```

```
C
C     READ RV-MEASUREMENT APPARATUS AND ENVIROMENTAL DATA
C

      SUBROUTINE INPUT

      INTEGER STEP,COUNT
      REAL GAM,R,CV,W,H,K1,K2,RADIUS,LENGTH,TNOT,TINF
      REAL P0,RHO0,U0
      COMPLEX PP0,RV,FP,UT
      REAL PA(10)
      COMMON/BLOCK/STEP,COUNT,GAM,R,CV,W,H,K1,K2,RADIUS,LENGTH,TNOT,
     &TINF,P0,RHO0,U0,PP0,PA,RV,RP,UT

      OPEN (UNIT = 3,FILE='INDAT  ST',STATUS = 'OLD')

      READ(3,*) GAM,R,CV,STEP,COUNT,H,W,K1,K2,RADIUS,LENGTH,P0,RHO0,PP0
     & RV,FP,UT  ,U0,TNOT,TINF,PA(1),PA(2),PA(3),PA(4),PA(5)


      RETURN
      END



      SUBROUTINE RKFOUR (GAM,R,CV,COUNT,H,W,K1,K2,RADIUS,LENGTH,P0,
     $RHO0,U0,UP0,PP0,P,RHO,U,UP,PP,TNOT,TINF)
      INTEGER COUNT
      REAL GAM,R,CV,W,H,K1,K2,RADIUS,LENGTH,T1,T2
      REAL P(100 ),RHO(100 ),U(100 ),P0,RHO0,U0
      COMPLEX UP(100 ),PP(100 ),UP0,PP0
      COMPLEX A1,A2,A3,A4,A5,B1,B2,B3,B4,B5,C1,C2,C3,C4,C5,D1,D2,D3,
     $D4,D5
      COMPLEX FUNC1,FUNC2,FUNC3,FUNC4,FUNC5
      REAL TDIST,TDERIV

C
C     BOUNDARY CONDITIONS AT PROPELLANT SURFACE
C

      P(1)     = P0
      RHO(1)   = RHO0
      U(1)     = U0
      UP(1)    = UP0
      PP(1)    = PP0

C
C     RUNGE-KUTTA FOURTH-ORDER INTEGRATION ALGORITHM
C

      DO 1 N = 1,COUNT

      T1 = TDIST (N,LENGTH,TNOT,TINF,K1,K2,RADIUS,LENGTH)
      T2 = TDERIV (N,LENGTH,TNOT,TINF,K1,K2,RADIUS,LENGTH)

      A1 = FUNC1 (P(N),RHO(N),U(N),UP(N),PP(N),T1,T2,GAM,R,CV,W) *H
      A2 = FUNC2 (P(N),RHO(N),U(N),UP(N),PP(N),T1,T2,GAM,R,CV,W) *H
      A3 = FUNC3 (P(N),RHO(N),U(N),UP(N),PP(N),T1,T2,GAM,R,CV,W) *H
      A4 = FUNC4 (P(N),RHO(N),U(N),UP(N),PP(N),T1,T2,GAM,R,CV,W) *H
      A5 = FUNC5 (P(N),RHO(N),U(N),UP(N),PP(N),T1,T2,GAM,R,CV,W) *H

      B1 = FUNC1 (P(N) + A1/2,RHO(N) + A2/2,U(N) + A3/2,UP(N) + A4
     $/2,PP(N) + A5/2,T1,T2,GAM,R,CV,W) *H
      B2 = FUNC2 (P(N) + A1/2,RHO(N) + A2/2,U(N) + A3/2,UP(N) + A4
     $/2,PP(N) + A5/2,T1,T2,GAM,R,CV,W) *H
```

```
      B3 = FUNC3 (P(N) + A1/2,RHO(N) + A2/2,U(N) + A3/2,UP(N) + A4
     $/2,PP(N) + A5/2,T1,T2,GAM,R,CV,W) *H
      B4 = FUNC4 (P(N) + A1/2,RHO(N) + A2/2,U(N) + A3/2,UP(N) + A4
     $/2,PP(N) + A5/2,T1,T2,GAM,R,CV,W) *H
      B5 = FUNC5 (P(N) + A1/2,RHO(N) + A2/2,U(N) + A3/2,UP(N) + A4
     $/2,PP(N) + A5/2,T1,T2,GAM,R,CV,W) *H

      C1 = FUNC1 (P(N) + B1/2,RHO(N) + B2/2,U(N) + B3/2,UP(N) + B4
     $/2,PP(N) + B5/2,T1,T2,GAM,R,CV,W) *H
      C2 = FUNC2 (P(N) + B1/2,RHO(N) + B2/2,U(N) + B3/2,UP(N) + B4
     $/2,PP(N) + B5/2,T1,T2,GAM,R,CV,W) *H
      C3 = FUNC3 (P(N) + B1/2,RHO(N) + B2/2,U(N) + B3/2,UP(N) + B4
     $/2,PP(N) + B5/2,T1,T2,GAM,R,CV,W) *H
      C4 = FUNC4 (P(N) + B1/2,RHO(N) + B2/2,U(N) + B3/2,UP(N) + B4
     $/2,PP(N) + B5/2,T1,T2,GAM,R,CV,W) *H
      C5 = FUNC5 (P(N) + B1/2,RHO(N) + B2/2,U(N) + B3/2,UP(N) + B4
     $/2,PP(N) + B5/2,T1,T2,GAM,R,CV,W) *H

      D1 = FUNC1 (P(N) + C1,RHO(N) + C2,U(N) + C3,UP(N) + C4,PP(N)
     $ + C5,T1,T2,GAM,R,CV,W) *H
      D2 = FUNC2 (P(N) + C1,RHO(N) + C2,U(N) + C3,UP(N) + C4,PP(N)
     $ + C5,T1,T2,GAM,R,CV,W) *H
      D3 = FUNC3 (P(N) + C1,RHO(N) + C2,U(N) + C3,UP(N) + C4,PP(N)
     $ + C5,T1,T2,GAM,R,CV,W) *H
      D4 = FUNC4 (P(N) + C1,RHO(N) + C2,U(N) + C3,UP(N) + C4,PP(N)
     $ + C5,T1,T2,GAM,R,CV,W) *H
      D5 = FUNC5 (P(N) + C1,RHO(N) + C2,U(N) + C3,UP(N) + C4,PP(N)
     $ + C5,T1,T2,GAM,R,CV,W) *H

      P   (N+1) = P   (N) + 1 /6 *(A1 + 2*B1 + 2*C1 + D1)
      RHO (N+1) = RHO (N) + 1 /6 *(A2 + 2*B2 + 2*C2 + D2)
      U   (N+1) = U   (N) + 1 /6 *(A3 + 2*B3 + 2*C3 + D3)
      UP  (N+1) = UP  (N) + 1 /6 *(A4 + 2*B4 + 2*C4 + D4)
      PP  (N+1) = PP  (N) + 1 /6 *(A5 + 2*B5 + 2*C5 + D5)
 1    CONTINUE
      RETURN
      END

C
C     FUNCTION FUNC 1 THRU 5 ARE GOVERNING EQUATIONS IN RK FORM
C

      COMPLEX FUNCTION FUNC1 (A,B,C,D,E,F,G,H,I,J,K)
      REAL A,B,C,F,G,H,I,J,K
      COMPLEX D,E
      FUNC1 = 0
      RETURN
      END

      COMPLEX FUNCTION FUNC2 (A,B,C,D,E,F,G,H,I,J,K)
      REAL A,B,C,F,G,H,I,J,K
      COMPLEX D,E
      FUNC2 = -D/E*G
      RETURN
      END

      COMPLEX FUNCTION FUNC3 (A,B,C,D,E,F,G,H,I,J,K)
      REAL A,B,C,F,G,H,I,J,K
      COMPLEX D,E
      FUNC3 = +C/E*G
      RETURN
      END

      COMPLEX FUNCTION FUNC4 (A,B,C,D,E,F,G,H,I,J,K)
      REAL A,B,C,F,G,H,I,J,K
      COMPLEX D,E
```

```
      COMPLEX FUNC3,L
      L = FUNC3 (A,B,C,D,E,F,G,H,I,J,K)
      FUNC4 = (1 /(B*C*C - H*A))*(0 ,1.)*K*E + (C/(B*C*C - H*A))*B*
     $(0 ,1 )*K*D + (1 /(B*C*C - H*A))*(H*E - B*D*C)*L
      RETURN
      END

      COMPLEX FUNCTION FUNC5 (A,B,C,D,E,F,G,H,I,J,K)
      REAL A,B,C,F,G,H,I,J,K
      COMPLEX D,E
      COMPLEX FUNC3,L,FUNC4,M
      L = FUNC3 (A,B,C,D,E,F,G,H,I,J,K)
      M = FUNC4 (A,B,C,D,E,F,G,H,I,J,K)
      FUNC5 = -B*(0 ,1 )*K*D   P*C*M   B*D*L
      RETURN
      END

C
C     FUNCTION TDIST CALCULATES TEMPERATURE ALONG AXIS
C

      REAL FUNCTION TDIST (N,LENGTH,TNOT,K1,K2 RADIUS)
      INTEGER N
      REAL LENGTH,TNOT,K1,K2,RADIUS,LAMBDA,DIST VALUE
      LAMBDA = 2*K2/K1/RADIUS/LENGTH
      DIST = N/100*LENGTH
      VALUE = (TNOT - TINF)*EXP(-LAMBDA*DIST)+TINF
      TDIST   VALUE
      RETURN
      END

C
C     FUNCTION TDERIV CALCULATES TEMP  GRADIENT ALONG AXIS
C

      REAL FUNCTION TDERIV (N,LENGTH,TNOT,K1,K2,RADIUS)
      INTEGER N
      REAL LENGTH,TNOT,K1,K2,RADIUS,LAMBDA,DIST,VALUE
      LAMBDA = 2*K2/K1/RADIUS/LENGTH
      DIST = N/100*LENGTH
      VALUE = -LAMBDA*(TNOT - TINF)*EXP(-LAMBDA*DIST)
      TDERIV = VALUE
      RETURN
      END
```

Appendix 3: Admittance to Rv Conversion

This appendix contains a listing of the program which is used to convert admittance values to velocity-coupled response function values.

Program 2

```
FIN7X

C
C     ADMITTANCE TO VELOCITY-COUPLED RESPONSE FUNCTION CONVERSION
C

      PROGRAM ADRV
      REAL A,U,GAM,P0,UT
      COMPLEX AB,PP,RP,RV
      GAM = 1.4
      P0 = 101300
      U = 6.0
      A = SQRT(GAM*P0/1 224)
      UT=19
      WRITE (1,*)' '
      WRITE (1,*) 'ADMITTANCE-RV CONVERSION'
      WRITE (1,*) '--------------------------'
      WRITE (1,*) ' '
      WRITE (1,*) 'INPUT PRESSURE-COUPLED RESPONSE FUNCTION VALUE'
      READ (1.*) RP
      WRITE (1,*)' '
      WRITE (1,*)'INPUT ADMITTANCE VALUE
      READ (1,*) AB
      PP = (0 ,1200.)
      RV   A/U/UT * (AI*A*PP/GAM/P0 - U*PP/GAM/P0 *(RP-1))
      WRITE (1,*) ' '
      WRITE (1,*) 'RV VALUE IS ',RV
      END
```

```
CI.42> ADRV

 ADMITTANCE-RV CONVERSION
 ------------------------

 INPUT PRESSURE-COUPLED RESPONSE FUNCTION VALUE
(0.1,-0.1)

 INPUT ADMITTANCE VALUE
(0.00,0.02)

 RV VALUE IS  (- 18716,.13643)
```

```
CI.42> ADRV

 ADMITTANCE-RV CONVERSION
 ------------------------

 INPUT PRESSURE-COUPLED RESPONSE FUNCTION VALUE
(0.1,-0.1)

 INPUT ADMITTANCE VALUE
(0.04,-0.02)

 RV VALUE IS  (.15684,.48043)
```

Appendix 4: Gaussian Random Number Generator

This program was used to generate the random numbers that were used to simulate experimental data. A listing of the random numbers that were used in the testing of the program is included.

Program 3

```
FTN7X

C
C      RANDOM NUMBER GENERATOR - SAMPLING FROM GAUSSIAN DIST
C

       PROGRAM GAUSS

       REAL R

       WRITE (6,*) 'GAUSSIAN RANDOM NUMBER GENERATOR'
       WRITE (6,*)
       DO 100 I=1,5
          WRITE (6,*) 'STANDARD DEVIATION = ',I
          DO 200 N = 1,10
             R = I*GRAN (1 )
             WRITE (6,300) R
 300   FORMAT ('    R = ',F6.3)
 200   CONTINUE
       WRITE (6,*)
 100   CONTINUE
       END
```

```
GAUSSIAN RANDOM NUMBER GENERATOR

STANDARD DEVIATION = 1
  R        402
  R  =     121
  R  -   1 409
  R        929
  R  =     700
  R  -     291
  R  =   1 026
  R  =   2 833
  R  =     473
  R  =     558

STANDARD DEVIATION = 2
  R  =   3 156
  R  =   2 636
  R  =  -1 319
  R  =   2 598
  R     -4 476
  R  =     416
  R  =     195
  R  =     973
  R  =   1 230
  R  =   2 022

STANDARD DEVIATION   3
  R  =  -3 196
  R     -2 285
  R  =   3 671
  R  =     138
  R  =   4 287
  R  =     007
  R  =   5 532
  R  =   3 495
  R  =     963
  R     -3 174

STANDARD DEVIATION = 4
  R  =   4 102
  R  =   3 252
  R  .  -2 328
  R  =   - 690
  R        471
  R  =   4 639
  R      1 001
  R  =   4 078
  R  =   1 057
  R  =  -3 559

STANDARD DEVIATION
  R  =  -3 955
  R  =     617
  R      - 413
  R  =  -1 554
  R  =  -9 959
  R  =  -7 216
  R  =  -1 709
  R  -   1 760
  R  =  -6 144
  R  =   8 231
```

Appendix 5: Parametric Studies

The following figures show how the pressure phase angle distribution within the Rv-measurement apparatus varies with the real and imaginary components of the admittance value for a solid propellant for conditions specified in appendix 1. Figures A.1 through A.5 show the phase angle distribution dependence on the real component of the admittance when the imaginary component is held fixed. Figures A.6 through A.10 show a similar dependence when the real component is held fixed. From the figures presented here it can be seen that the phase angle distribution is slightly more sensitive to variations in the real component of the admittance. Consequently, better performance of the curve-fit procedure can be expected for the real component of the admittance than for the imaginary component.

The apparent discontinuity in the curves of figures A.2 through A.10 results from the choice of scale. The curves cross the 180 degree line which is equivalent to the -180 line. These curves have the same form as the curve in figure A.1.

Whereas, the figures show the phase angle distributions at the location nearest the propellant sample where variation is greatest, the tabulated results are provided to show how both magnitude and phase angle components of the pressure vary throughout the length of the Rv-measurement apparatus. The magnitude of the pressure is provided in decibel units as well as Pascals units for convenience.

A typical plot of the magnitude component of the perturbation pressure distribution is provided in figure A.11. This plot reveals the severe non-linearity at very low pressures which prevents effective use of magnitude information in numerical curve-fit procedures.

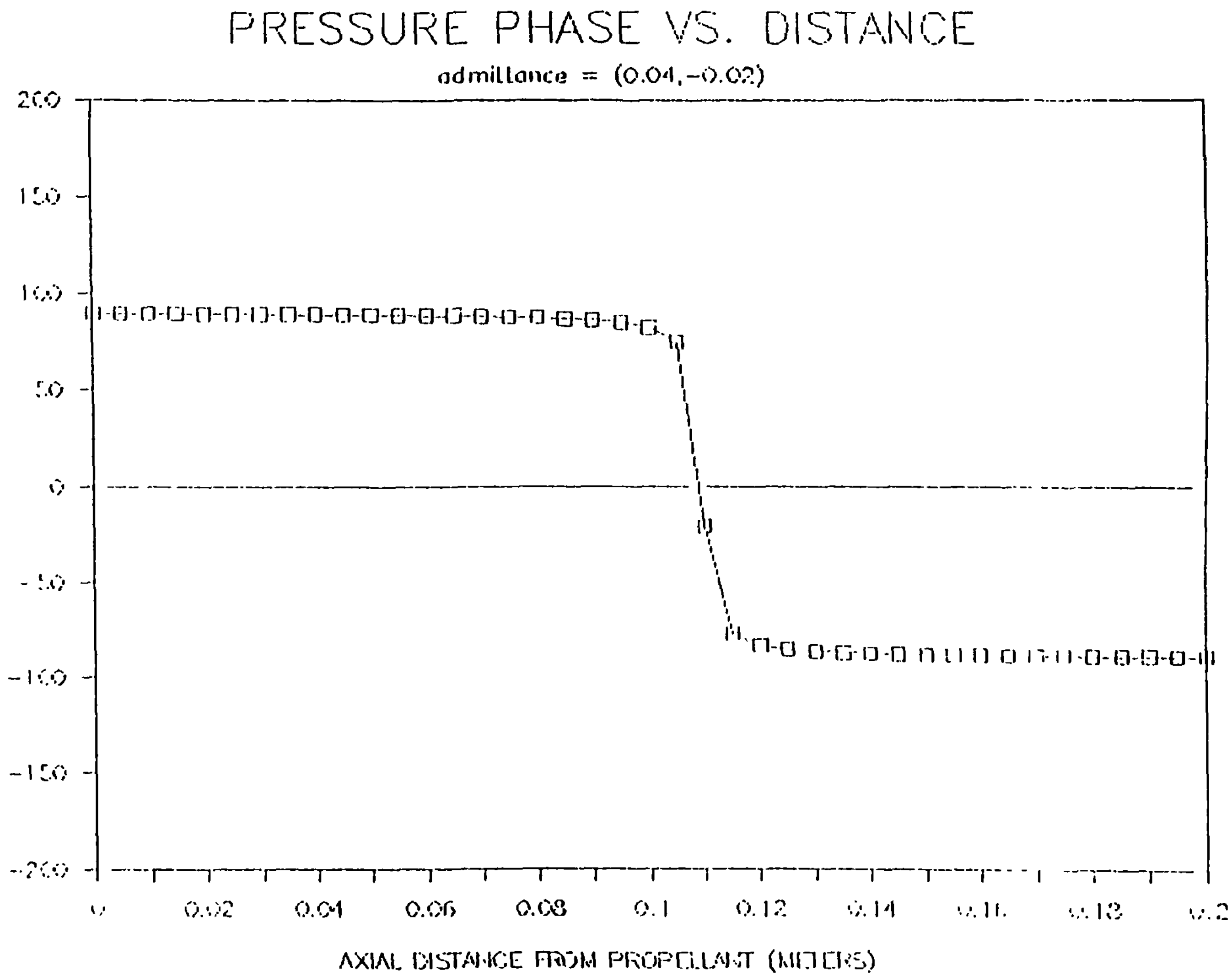

PRESSURE PHASE VS. DISTANCE
admittance = (0.04,-0.02)
20.0
15.0
10.0
5.0
0
-5.0
-10.0
-15.0
-20.0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
AXIAL DISTANCE FROM PROPELLANT (METERS)

Figure A.1

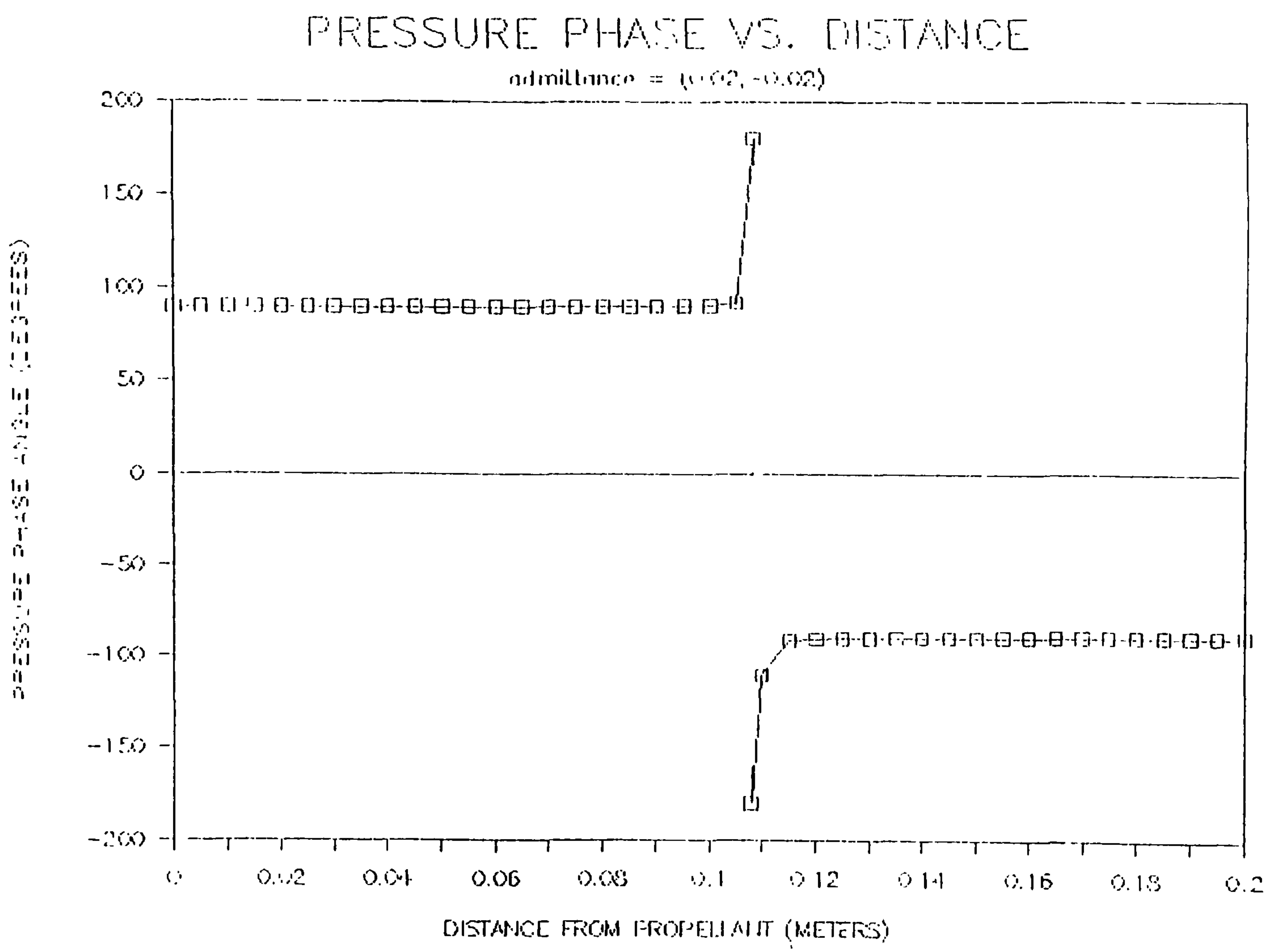

PRESSURE PHASE VS. DISTANCE
200
150
100
50
0
-50
-100
-150
-200
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
DISTANCE FROM PROPELLANT (METERS)

Figure A.2

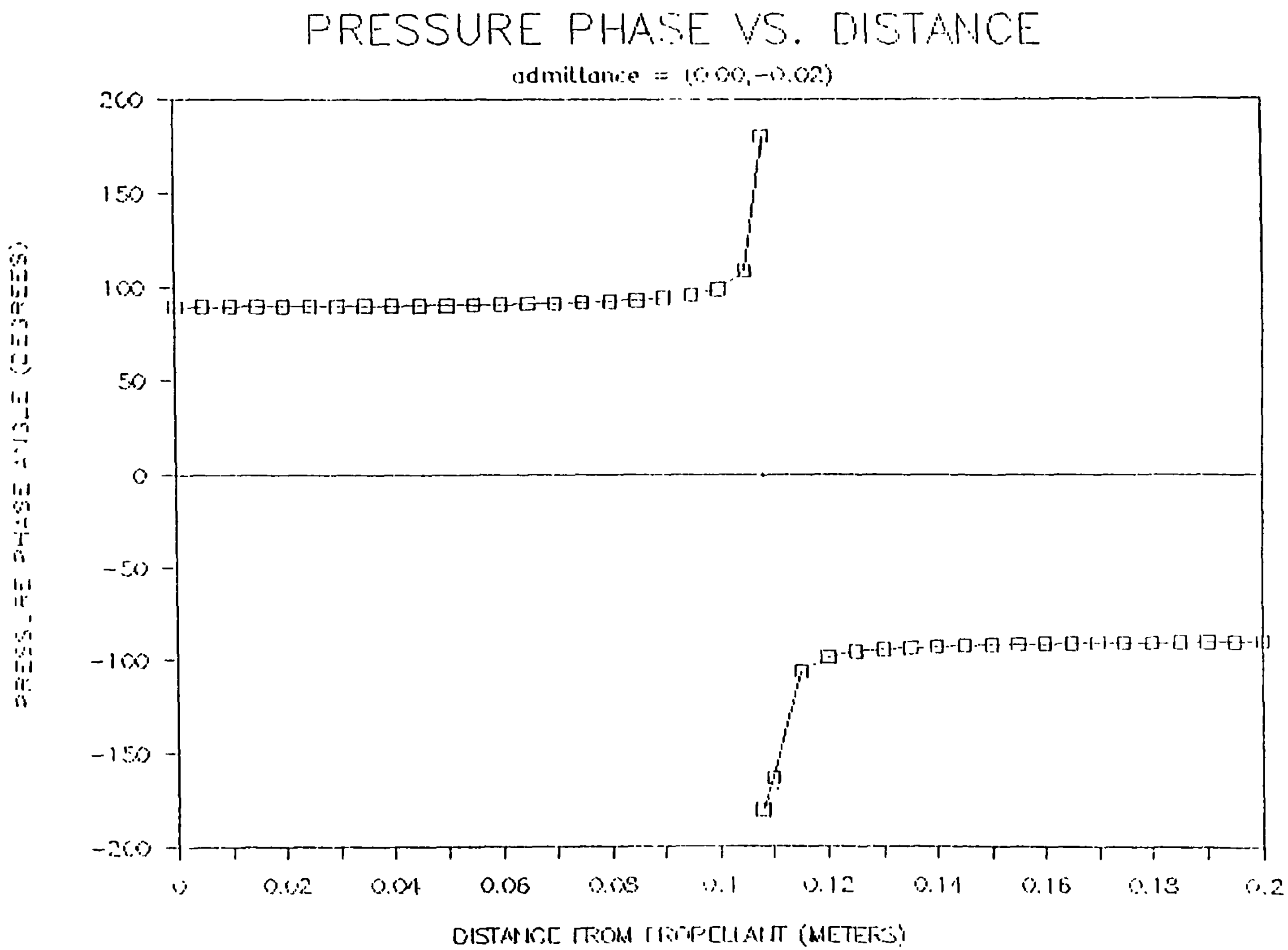

PRESSURE PHASE VS. DISTANCE
admittance = (0.00,-0.02)
200
150
100
50
0
-50
-100
-150
-200
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
DISTANCE FROM PROPELLANT (METERS)

Figure A.3

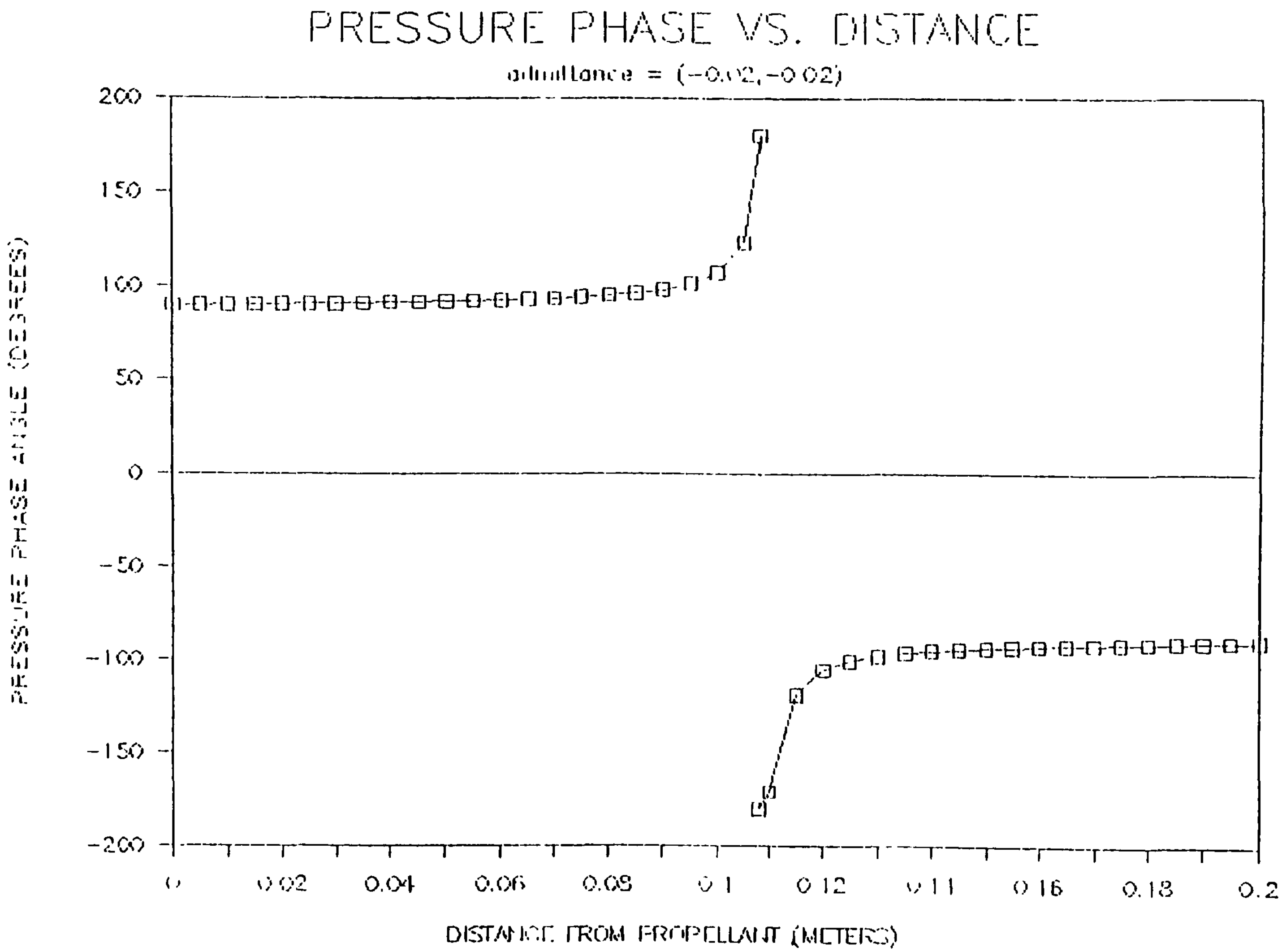

PRESSURE PHASE VS. DISTANCE
admittance = (-0.02,-0.02)
PRESSURE PHASE ANGLE (DEGREES)
200
150
100
50
0
-50
-100
-150
-200
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
DISTANCE FROM PROPELLANT (METERS)

Figure A.4

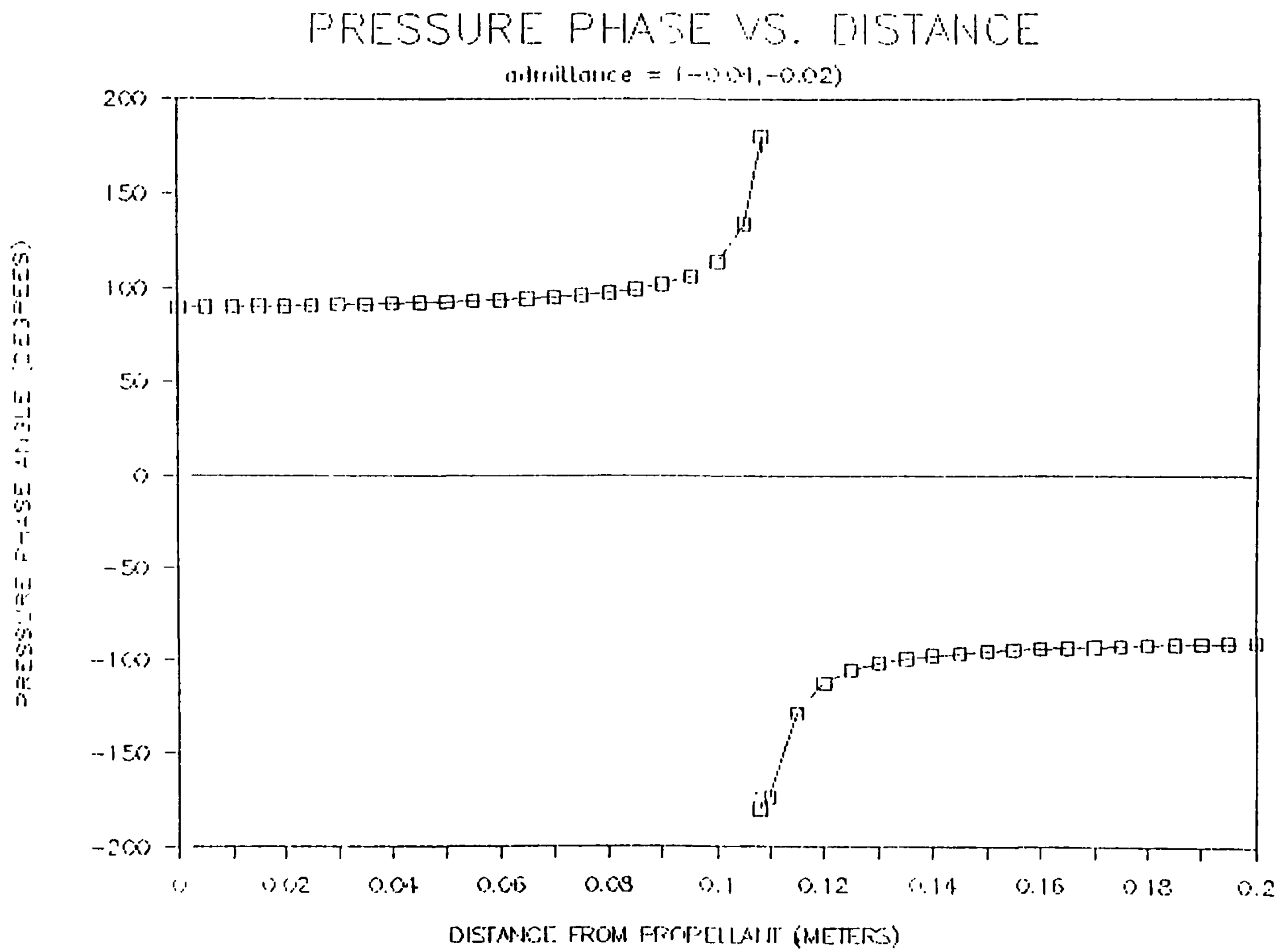

PRESSURE PHASE VS. DISTANCE
admittance = (-0.04,-0.02)
PRESSURE PHASE ANGLE (DEGREES)
200
150
100
50
0
-50
-100
-150
-200
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
DISTANCE FROM PROPELLANT (METERS)

Figure A.5

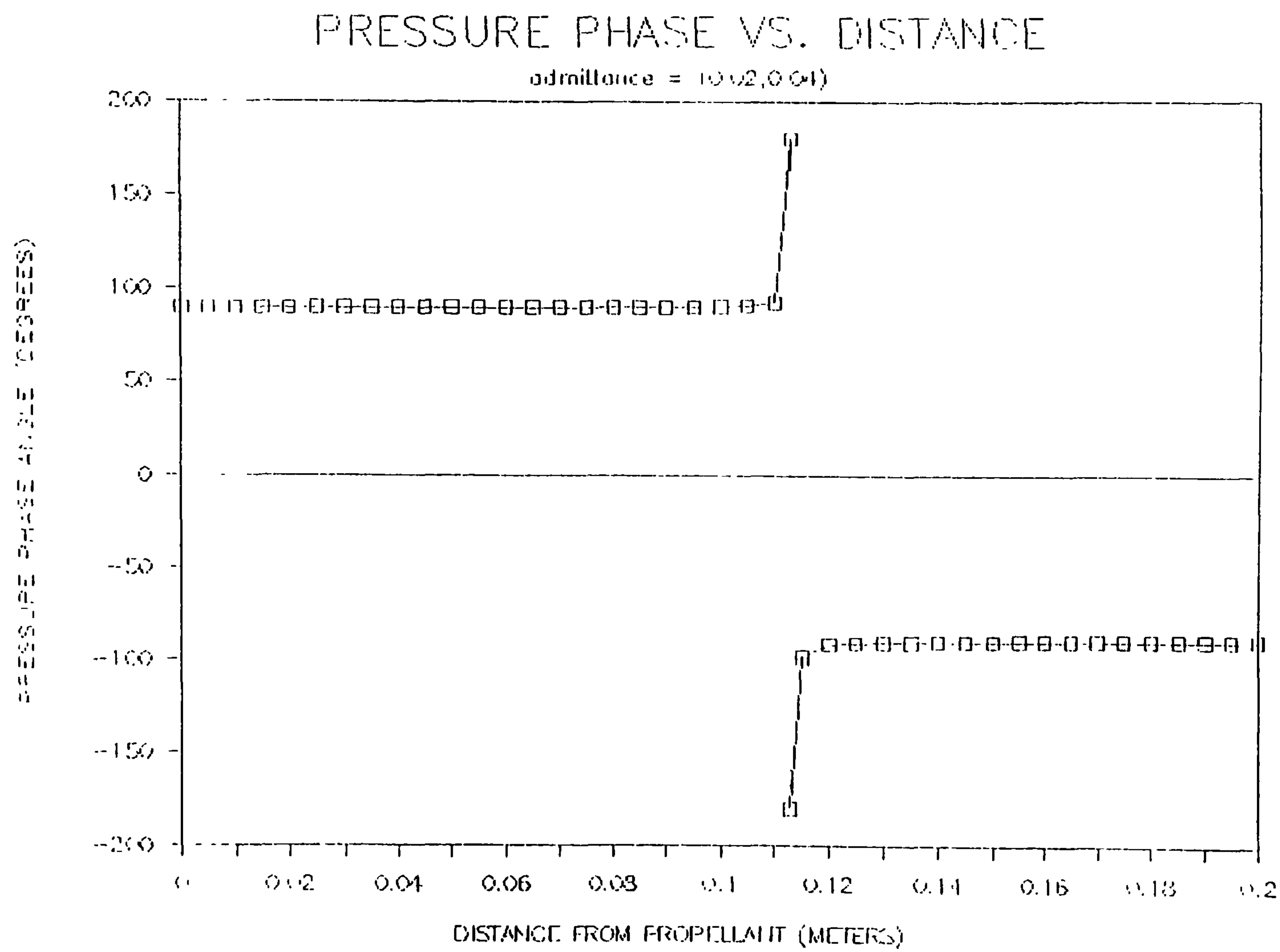

PRESSURE PHASE VS. DISTANCE
admittance = (0.02,0.04)
DISTANCE FROM PROPELLANT (METERS)
200
150
100
50
0
-50
-100
-150
-200
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

Figure A.6

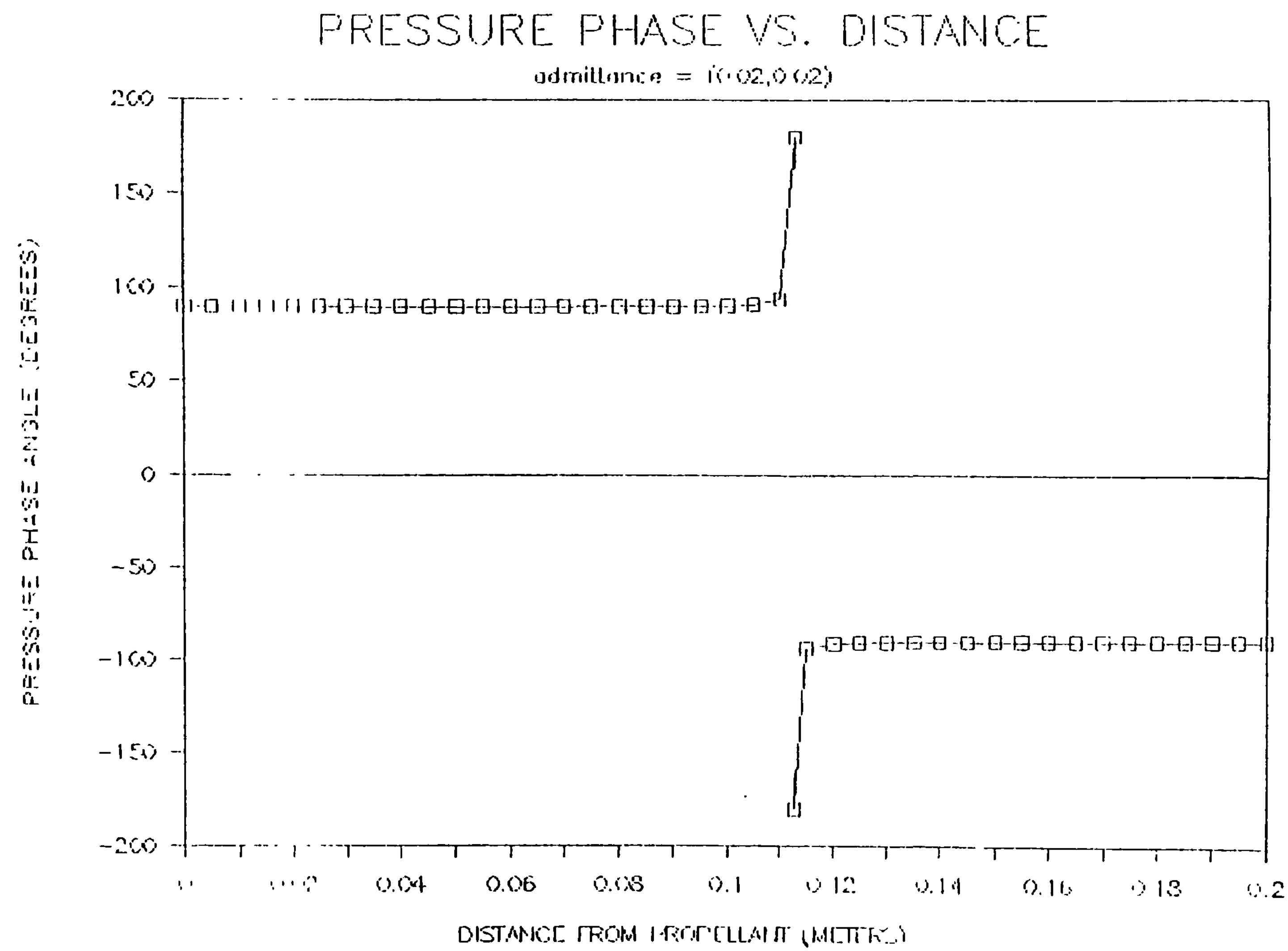
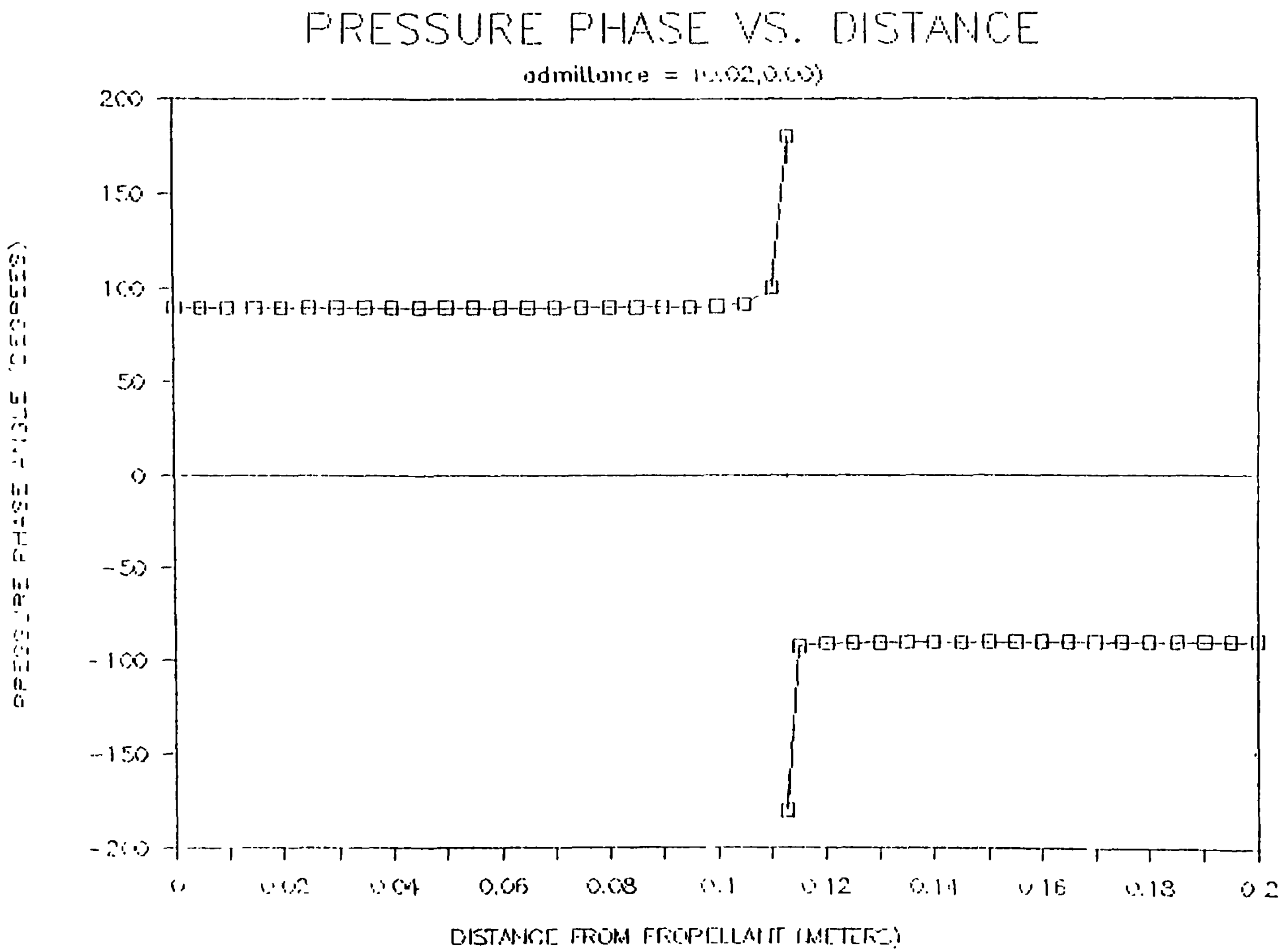
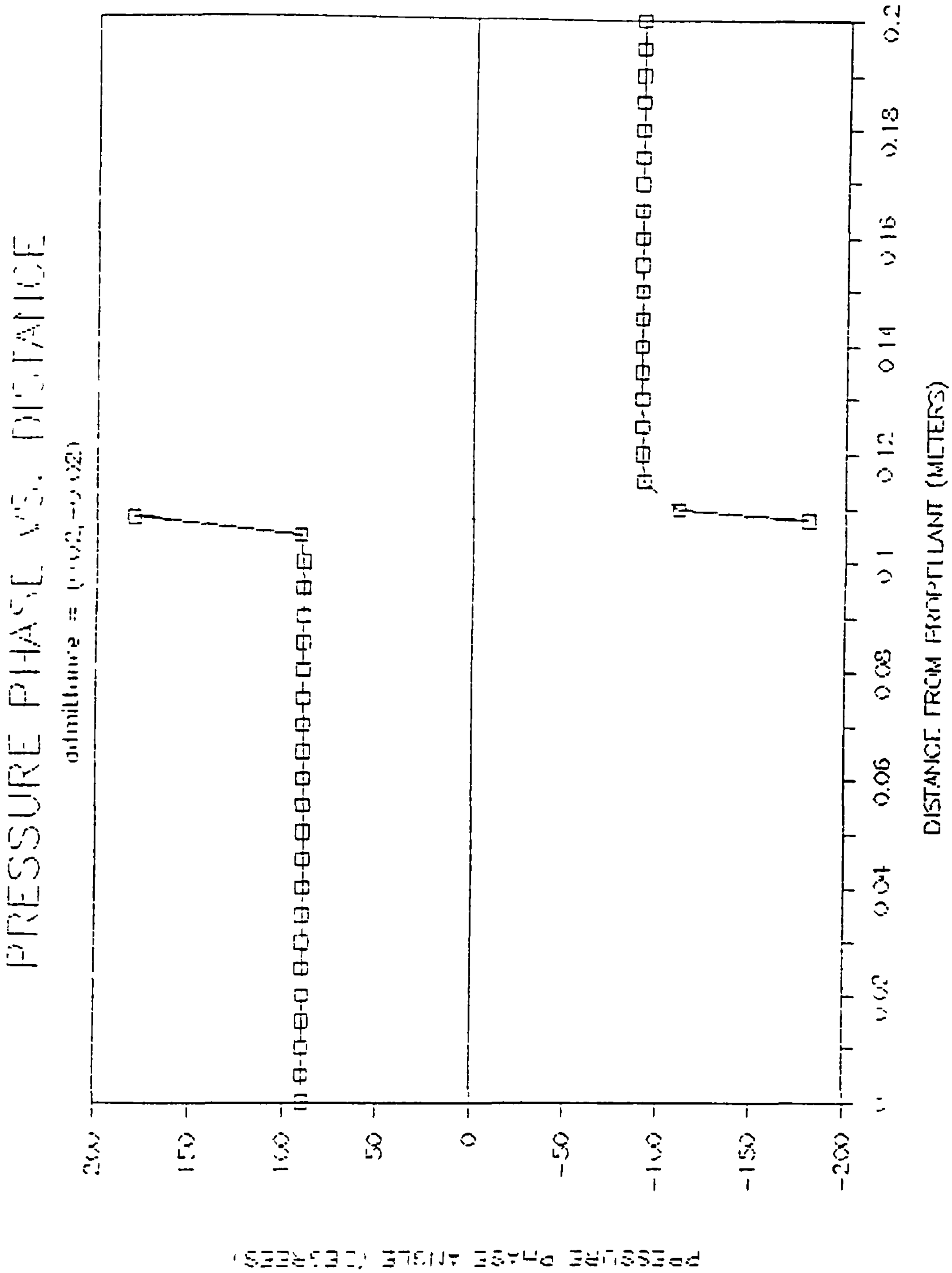

PRESSURE PHASE VS. DISTANCE
admittance = (0.02,0.02)
PRESSURE PHASE ANGLE (DEGREES)
200
150
100
50
0
-50
-100
-150
-200
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
DISTANCE FROM PROPELLANT (METERS)

Figure A.7



PRESSURE PHASE VS. DISTANCE
admittance = (0.02,0.00)
200
150
100
50
0
-50
-100
-150
-200
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
DISTANCE FROM PROPELLANT (METERS)

Figure A.8



PRESSURE PHASE VS. DISTANCE
DISTANCE FROM PROPELLANT (METERS)
PRESSURE PHASE ANGLE (DEGREES)

Figure A.9

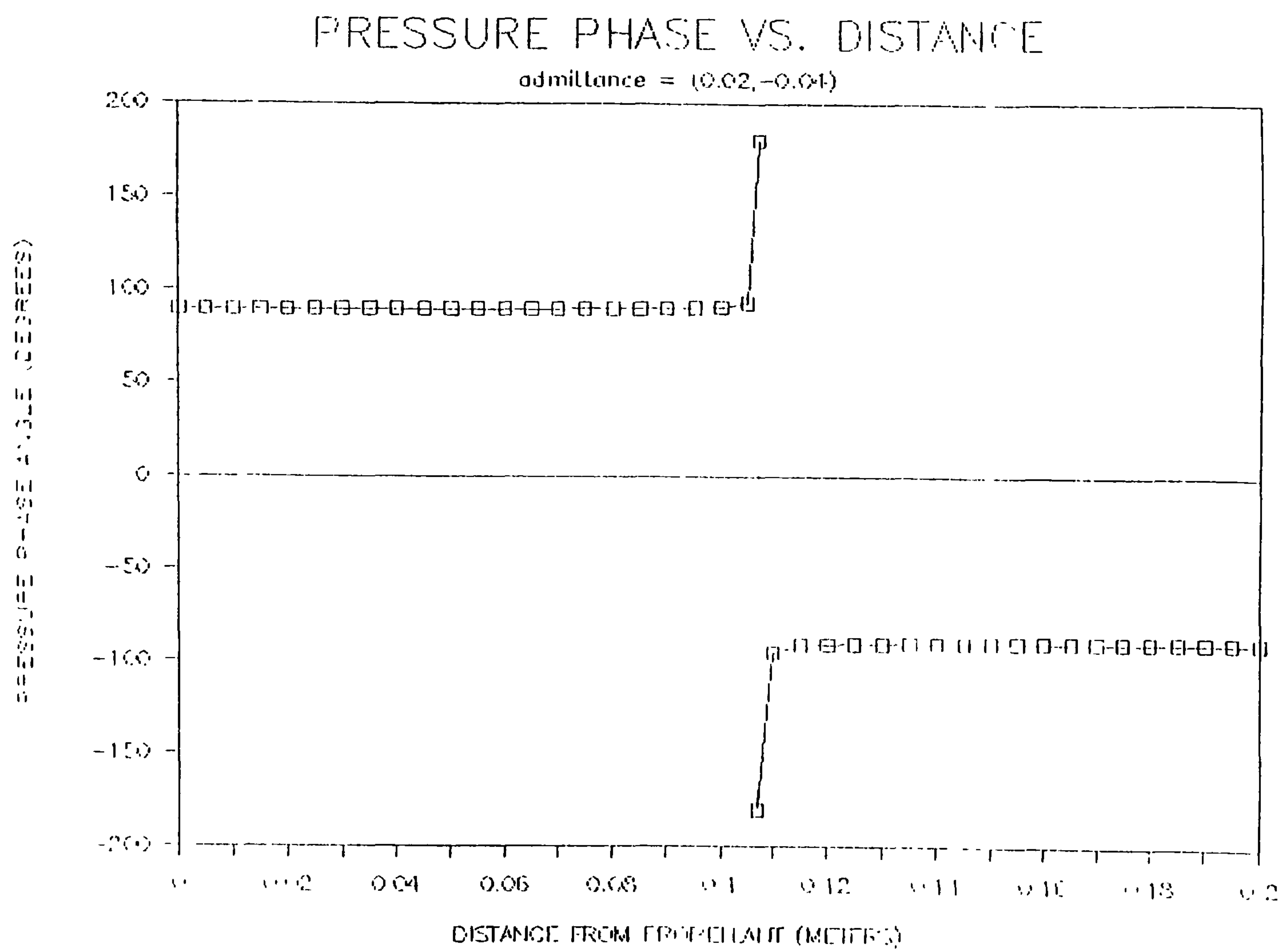

PRESSURE PHASE VS. DISTANCE
admittance = (0.02,-0.04)
200
150
100
50
0
-50
-100
-150
-200
0.04
0.06
0.08
0.12
DISTANCE FROM PROPELLANT (METERS)

Figure A.10

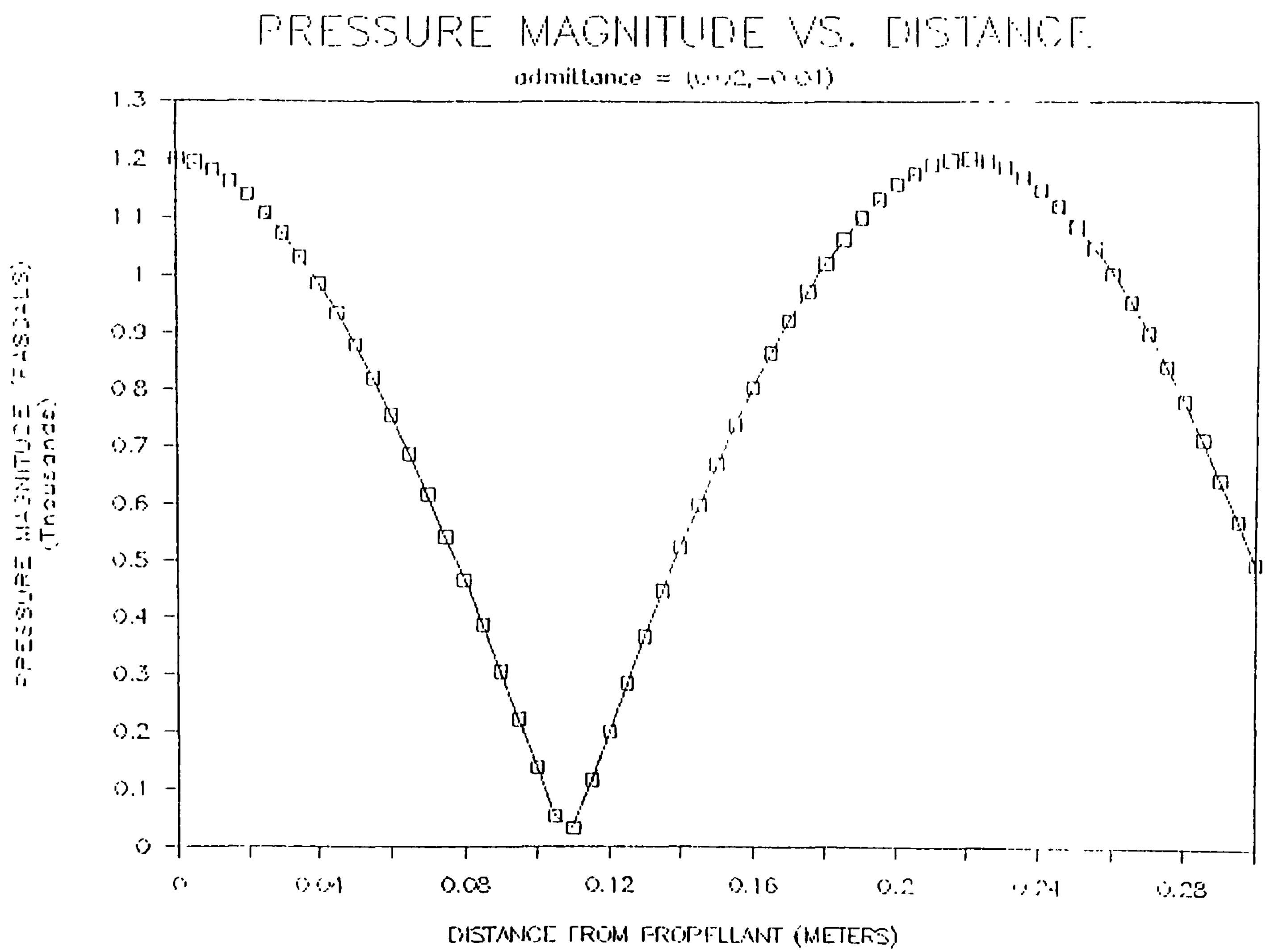

PRESSURE MAGNITUDE VS. DISTANCE
admittance = (0.02,-0.01)
PRESSURE MAGNITUDE (PASCALS)
(Thousands)
DISTANCE FROM PROPELLANT (METERS)

Figure A.11

ADMITTANCE = (4 E-2,-2 E-2)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152 467	90 000	1200 000
1	005	152 433	89 929	1195 298
2	010	152 355	89 857	1184 609
3	015	152 232	89 784	1167 987
4	020	152 063	89 708	1145 515
5	025	151 847	89 628	1117 306
6	030	151 580	89 544	1083 500
7	035	151 259	89 454	1044 269
8	040	150 881	89 357	999 808
9	045	150 441	89 250	950 340
10	050	149 930	89 130	896 115
11	055	149 342	88 994	837 405
12	060	148 664	88 838	774 504
13	065	147 889	88 652	707 730
14	070	146 972	88 426	637 418
15	075	145 908	88 143	563 925
16	080	144 645	87 773	487 625
17	085	143 116	87 263	408 910
18	090	141 206	86 505	328 194
19	095	138 699	85 244	245 928
20	100	135 109	82 700	162 671
21	105	128 901	74 816	79 593
22	110	117 902	-21 187	22 436
23	115	130 466	-77 341	95 307
24	120	135 922	-83 329	178 616
25	125	139 240	-85 506	261 739
26	130	141 608	-86 637	343 746
27	135	143 433	-87 335	424 113
28	140	144 904	-87 813	502 399
29	145	146 125	-88 163	578 193
30	150	147 156	-88 433	651 106
31	155	148 039	-88 650	720 769
32	160	148 801	-88 830	786 829
33	165	149 461	-88 982	848 953
34	170	150 034	-89 114	906 839
35	175	150 530	-89 331	960 165
36	180	150 958	-89 336	1008 695
37	185	151 325	-89 432	1052 173
38	190	151 675	-89 531	1090 383
39	195	151 892	-89 604	1123 132
40	200	152 099	-89 682	1150 255
41	205	152 259	-89 758	1171 618
42	210	152 373	-89 831	1187 112
43	215	152 443	-89 903	1196 661
44	220	152 468	-89 974	1200 215
45	225	152 450	-90 044	1197 758
46	230	152 389	-90 116	1189 303
47	235	152 263	-90 189	1174 888
48	240	152 172	-90 264	1154 590
49	245	151 933	-90 343	1128 509
50	250	151 685	-90 424	1096 776
51	255	151 386	-90 511	1059 550
52	260	151 070	-90 606	1017 019
53	265	150 613	-90 709	969 394
54	270	150 130	-90 824	916 916
55	275	149 572	-90 953	859 848
56	280	148 928	-91 101	798 477
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

57	.285	148.186	-91.275	733 111
58	.290	147 328	-91.484	664 081
59	295	146.326	-91 743	591 734
60	.300	145.143	-92 076	516 438
61	305	143.724	-92 525	438 577
62	310	141 974	-93 172	358 559
63	315	139.727	-94 197	276 815
64	320	136 632	-96 094	193 849
65	325	131 746	-100 852	110 443
66	330	120.865	-131 487	31 557
67	335	127 111	108 834	64 776
68	.340	134 354	98 116	147 424
69	345	128 147	95 134	230 779
70	350	140 802	93 716	313 272
71	355	142 800	92 892	394 307
72	360	144 289	92 347	473 419
73	365	145 693	91 957	550 190
74	370	146 790	91 660	624 224
75	.375	147.725	91 426	695 144
76	.380	148 529	91 234	762 590
77	385	149 225	91 072	826 223
78	390	149 629	90 933	825 721
79	395	150 253	90 811	940 786
80	400	150 806	90 702	991 142
81	405	151 195	90 603	1036 534
82	410	151 525	90 512	1076 737
83	415	151 802	90 427	1111 546
84	420	152 027	90 347	1140 789
85	425	152 205	90 270	1164 318
86	430	152 336	90.196	1182 016
87	425	152.422	90 124	1193 793
88	440	152 464	90 053	1199 551
89	445	152 462	89 982	1199 380
90	450	152.417	89 911	1193 162
91	455	152 328	89.839	1180 967
92	460	152 194	89 765	1162 858
93	465	152 013	89 688	1138 923
94	.470	151 784	89 607	1109 285
95	475	151 504	89 522	1074 091
96	480	151.169	89 430	1033 518
97	485	150 776	89 331	987.770
98	.490	150 319	89 221	937 076
99	495	149 789	89 058	881 590
100	500	14[illegible]	88 957	821 [illegible]

ADMITTANCE (2.E-2,-2 E-2)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152.467	90 000	1200.000
1	005	152 433	90 010	1195 297
2	010	152 355	90 021	1184 606
3	015	152 232	90 032	1167 979
4	020	152 063	90 043	1145 500
5	025	151 846	90 054	1117 282
6	030	151 579	90 067	1083 467
7	035	151 259	90 080	1044 222
8	040	150 881	90 094	999 746
9	045	150 440	90 110	950.261
10	050	149 929	90 127	896 014
11	055	149 341	90 147	837 279
12	060	148 662	90 170	774 348
13	065	147 878	90 197	707.538
14	070	146 968	90 230	637 183
15	075	145 903	90 272	563 636
16	080	144 638	90 326	487 265
17	085	143 106	90 401	408 453
18	090	141 190	90 512	327 596
19	095	138 670	90 697	245 100
20	100	135 040	91 074	161 380
21	105	128 590	92 274	76 875
22	110	109 640	-110 678	8 667
23	115	130 257	-91 882	93 041
24	.120	135 864	-90 980	177 433
25	125	139 214	-90 659	260 952
26	130	141.593	-90 493	343 167
27	135	143 424	-90 390	423 665
28	140	144 898	-90 320	502 041
29	145	146 120	-90 269	577 902
30	150	147 153	-90 229	650 868
31	155	148 037	-90 198	720 573
32	160	148 799	-90 171	786 668
33	165	149 459	-90 149	848 822
34	170	150.033	-90 130	906 723
35	175	150 529	-90 113	960 081
36	180	150 958	-90 097	1008 629
37	185	151 324	-90 083	1052 123
38	190	151 634	-90 070	1090 345
39	195	151 892	-90 058	1123 105
40	200	152 099	-90 046	1150 228
41	205	152 259	-90 035	1171 607
42	210	152 373	-90 025	1187 107
43	215	152 443	-90 014	1196 658
44	220	152.468	-90 004	1200 215
45	225	152 450	-89 994	1197 757
46	230	152 389	-89 983	1189 299
47	235	152 283	-89 972	1174 882
48	240	152 132	-89 961	1154 578
49	245	151 933	-89 950	1128 489
50	250	151 685	-89 938	1096 746
51	255	151 383	-89 925	1059 507
52	260	151 029	-89 911	1016 562
53	265	150 612	-89 896	969 321
54	270	150 139	-89 880	916 823
55	275	149 570	-89 861	859 731
56	280	148 927	-89 839	798 332

57	285	148 184	-89 813	732 933
58	290	147 325	-89 783	663 862
59	295	146 322	-89 745	591 466
60	300	145 138	-89 696	516 106
61	305	143 716	-89 630	438 160
62	310	141 961	-89 535	358 020
63	315	139 704	-89 385	276 088
64	320	136 584	-89 105	192 777
65	325	131 592	-88 393	108 510
66	330	118 428	-82 628	23 836
67	335	126 644	87 143	61 384
68	340	134 169	88 805	145 980
69	345	138 113	89 248	229 877
70	350	140 754	89 456	312 628
71	355	142 789	89 576	392 815
72	360	144 381	89 656	473 031
73	365	145 686	89 714	549 677
74	370	146 786	89 757	623 968
75	375	147 722	89 791	694 933
76	380	148 527	89 819	762 417
77	385	149 224	89 843	826 081
78	390	149 826	89 863	865 606
79	395	150 352	89 881	940 694
80	400	150 805	89 897	991 069
81	405	151 194	89 912	1036 478
82	410	151 525	89 925	1076 654
83	415	151 801	89 938	1111 516
84	420	152 027	89 949	1140 768
85	425	152 204	89 961	1164 305
86	430	152 335	89 971	1182 009
87	435	152 422	89 982	1193 790
88	440	152 464	89 992	1199 590
89	445	152 462	90 003	1199 379
90	450	152 417	90 013	1193 160
91	455	152 328	90 024	1180 962
92	460	152 194	90 034	1162 847
93	465	152 013	90 046	1138 906
94	470	151 784	90 057	1109 259
95	475	151 504	90 070	1074 054
96	480	151 169	90 083	1033 468
97	485	150 776	90 098	987 703
98	490	150 318	90 114	936 990
99	495	149 788	90 132	881 583
100	500	149 178	90 153	821 778

ADMITTANCE = (0 E+0,-2 E-2)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152 467	90 000	1200 000
1	005	152 433	90 092	1195 299
2	010	152 355	90 185	1184 612
3	015	152 232	90 280	1167 793
4	020	152 063	90 378	1145 525
5	025	151 847	90 481	1117 321
6	030	151 580	90 589	1083 523
7	035	151 260	90 706	1044 300
8	040	150 882	90 832	999 650
9	045	150 441	90 970	950 395
10	050	149 931	91 124	896 185
11	055	149 343	91 300	837 491
12	060	148 665	91 502	774 611
13	065	147 882	91 742	707 861
14	070	146 974	92 034	637 579
15	075	145 911	92.399	564 124
16	080	144 649	92 877	487 872
17	085	143 122	93 536	409 222
18	090	141 217	94 514	328 603
19	095	138 719	96 138	246 495
20	100	135 156	99 403	163 549
21	105	129 096	109 331	81 404
22	110	119 897	-163 308	28.230
23	115	130 603	-106 189	96 830
24	120	135 961	-98 597	179 423
25	125	139 258	-95 801	262 277
26	130	141 618	-94 343	344 142
27	135	143 439	-93 443	424 421
28	140	144 908	-92 826	502 644
29	145	146 128	-92.374	578 392
30	150	147 159	-92 025	651 269
31	155	148 041	-91 745	720 903
32	160	148 802	-91 513	786 939
33	165	149 462	-91.316	849 043
34	170	150 034	-91 145	906 901
35	175	150 531	-90 994	960 223
36	180	150 959	-90 858	1008 740
37	185	151 325	-90 734	1052 208
38	190	151 635	-90 620	1090 408
39	195	151 892	-90 512	1123 149
40	200	152 097	-90 410	1150 267
41	205	152 259	-90 313	1171 625
42	210	152 373	-90 218	1187 116
43	215	152.443	-90 126	1196 662
44	220	152 468	-90 034	1200 215
45	225	152 450	-89 943	1197 758
46	230	152 389	-89 850	1189 303
47	235	152 283	-89 754	1174 892
48	240	151 132	-89 659	1154 598
49	245	151 933	-89 558	1128 522
50	250	151 636	-89 452	1096 756
51	255	151 286	-89 337	1059 579
52	260	151 020	-89 217	1017 056
53	265	150 514	-89 083	969 444
54	270	150 130	-88 935	916 575
55	275	149 572	-88 769	859 928
56	280	148 929	-88 577	798 576

57	.285	148 188	-68 353	733 233
58	290	147 329	-88 082	664.230
59	.295	146 328	-87.748	591.717
60	300	145 147	-87 318	516 665
61	305	143 730	-86 738	438 863
62	310	141 983	-85 903	358 926
63	315	139 742	-84 581	277 312
64	320	136 665	-82 143	194 580
65	325	131 848	-76 083	111 749
66	330	121 986	-41 181	35 902
67	335	127 405	66 208	67 001
68	340	134 312	79 556	149 416
69	345	138 170	83 389	231 395
70	350	140 614	85 201	313 713
71	355	142 807	86 264	394 643
72	360	144 393	86 967	473 685
73	365	145 697	87 472	550 405
74	370	146 792	87 834	634 400
75	375	147 736	88 157	695 262
76	380	148 530	88 405	762 708
77	385	149 226	88 614	826 319
78	390	149 830	88 794	885 799
79	395	150 353	88 952	940 649
80	400	150 806	89 093	991 191
81	405	151 195	89 221	1036 572
82	410	151 526	89 338	1076 764
83	415	151 803	89 448	1111 566
84	420	152 027	89 552	1140 802
85	425	152 205	89 651	1164 336
86	430	152 336	89 747	1182 030
87	435	152 422	89 840	1193 794
88	440	152 464	89 933	1199 550
89	445	152 462	90 023	1199 379
90	450	152 417	90 115	1193 162
91	455	152 326	90 208	1180 969
92	460	152 194	90 304	1162 863
93	465	152 013	90 403	1138 934
94	470	151 784	90 508	1109 302
95	475	151 504	90 618	1074 115
96	480	151 170	90 736	1033 552
97	485	150 777	90 865	987 814
98	490	150 319	91 007	937 133
99	495	149 790	91 166	881 763
100	500	149 320	91 348	821 962

ADMITTANCE (-2.E-2,-2 E-2)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152 467	90 000	1200 000
1	005	152 433	90 173	1195 303
2	010	152 355	90 348	1184 628
3	015	152 232	90 528	1168 029
4	020	152 064	90 713	1145 589
5	025	151 847	90 907	1117 422
6	030	151 581	91 112	1083 670
7	035	151 261	91 331	1044 503
8	040	150 884	91 569	1000 120
9	045	150 444	91 830	950 744
10	050	149 935	92 121	896 676
11	055	149 348	92 451	838 043
12	060	148 672	92 833	775 292
13	065	147 892	93 285	708 692
14	070	146 988	93 833	638 606
15	075	145 930	94 520	565 388
16	080	144 677	95 417	489 443
17	085	143 164	96 650	411 210
18	090	141 285	98 473	331 198
19	095	138 845	101 470	250 076
20	100	135 443	107 351	169 045
21	105	130 170	123 501	92 117
22	110	125 146	-170 970	51 660
23	115	131 392	-118 714	106 029
24	120	136 202	-105 921	184 483
25	125	139 370	-100 851	265 684
26	130	141 681	-98 195	346 660
27	135	143 479	-96 476	426 375
28	140	144 935	-95 322	504 206
29	145	146 147	-94 472	579 661
30	150	147 172	-93 816	652 309
31	155	148 051	-93 289	721 758
32	160	148 810	-92 853	787 641
33	165	149 468	-92 482	849 616
34	170	150 039	-92 160	907 365
35	175	150 534	-91 875	960 593
36	180	150 961	-91 619	1009 030
37	185	151 327	-91 385	1052 429
38	190	151 636	-91 169	1090 572
39	195	151 893	-90 967	1123 265
40	200	152 100	-90 774	1150 343
41	205	152 259	-90 590	1171 670
42	210	152 373	-90 412	1187 138
43	215	152 443	-90 237	1196 669
44	220	152 468	-90 064	1200 216
45	225	152 451	-89 892	1197 760
46	230	152 399	-89 718	1189 314
47	235	152 283	-89 540	1174 920
48	240	152 132	-89 357	1154 251
49	245	151 934	-89 167	1128 806
50	250	151 687	-88 966	1096 925
51	255	151 387	-88 753	1059 759
52	260	151 032	-88 523	1017 300
53	265	150 616	-88 271	969 761
54	270	150 134	-87 992	917 385
55	275	149 577	-87 678	860 436
56	280	148 936	-87 317	799 206
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

57	285	148 197	-86.894	734 008
58	290	147 342	-86.385	665 182
59	.295	146 345	-85 756	593 086
60	300	145 172	-84 949	518 111
61	305	143 766	-83 862	440.673
62	310	142 040	-82 303	361 364
63	315	139 841	-79 853	280.460
64	320	136 868	-75 406	199 180
65	325	132 448	-64 942	119 738
66	.330	125.878	-24 876	56 198
67	335	128 917	50 244	79 747
68	340	134 663	70.823	154 522
69	345	138 315	77 665	235 388
70	350	140 891	80 959	316 511
71	.355	142 854	82 975	396 782
72	360	144 424	84 291	475 379
73	365	145 718	85 237	551 775
74	370	146 808	85.556	625 519
75	.375	147 738	86.526	696 308
76	380	148 539	86 993	763 464
77	385	149.233	87 386	826 928
78	390	149 835	87 725	886 302
79	395	150 357	88 022	941 352
80	400	150 809	88 286	991 509
81	405	151 197	88 530	1036.918
82	410	151 537	88 752	1076 948
83	415	151 803	88 959	1111 898
84	420	152 028	89.155	1140 852
85	425	152 205	89 342	1164 382
86	430	152 336	89 522	1182 050
87	435	152 423	89 698	1193 806
88	440	152 464	89 871	1199 553
89	445	152 462	90 044	1199 379
90	450	152 417	90 217	1193 168
91	455	152 328	90 393	1180 990
92	460	152 194	90 574	1162 906
93	465	152 014	90 761	1139 006
94	470	151 785	90 958	1109 413
95	475	151 505	91 166	1074.275
96	480	151 172	91 389	1033 770
97	485	150 779	91 632	988 103
98	490	150 323	91 900	937 504
99	495	149 795	92 200	882 331
100	500	149 186	92 542	822 365

ADMITTANCE (-4 E-2,-2 E-2)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152 467	90 000	1200 000
1	005	152 433	90 254	1195 309
2	010	152 355	90 512	1184 653
3	015	152 233	90 776	1168 086
4	020	152 064	91 048	1145 692
5	025	151 849	91 333	1117 584
6	030	151 583	91 634	1083 917
7	035	151 264	91 956	1044 330
8	040	150 888	92 305	1000 555
9	045	150 449	92 689	951 306
10	050	149 942	93 116	897 339
11	055	149 358	93 601	838 932
12	060	148 685	94 161	776 391
13	065	147 909	94 822	710 047
14	070	147 010	95 625	640 261
15	075	145 961	96 629	567 423
16	080	144 722	97 936	491 569
17	085	143 231	99 726	414 399
18	090	141 393	102 352	335 346
19	095	139 039	106 609	255 753
20	100	135 870	114 669	177 557
21	105	131 485	134 216	107 176
22	110	128 435	-173 830	75 436
23	115	132 423	-128 843	119 392
24	120	136 566	-112 749	192 372
25	125	139 546	-105 736	271 094
26	130	141 781	-101 896	350 686
27	135	143 543	-99 473	429 512
28	140	144 978	-97 797	506 717
29	145	146 177	-96 559	581 703
30	150	147 194	-95 600	653 984
31	155	148 068	-94 829	723 135
32	160	148 822	-94 189	788 772
33	165	149 477	-93 645	850 540
34	170	150.046	-93 173	908 113
35	175	150 539	-92 755	961 190
36	180	150 965	-92 379	1009 497
37	185	151 330	-92 036	1052 786
38	190	151 638	-91 718	1090 835
39	195	151 891	-91 421	1123 450
40	200	152 101	-91 138	1150 464
41	205	152 260	-90 868	1171 741
42	210	152 373	-90 605	1187 173
43	215	152 443	-90 349	1196 681
44	220	152 468	-90 095	1200 217
45	225	152 451	-89 841	1197 762
46	230	152 [illegible]	-89 585	1189 [illegible]
47	235	152 284	-89 724	1174 [illegible]
48	240	[illegible]	-89 [illegible]	[illegible]
49	245	151 [illegible]	[illegible]	1125 [illegible]
50	250	[illegible]	-88 [illegible]	[illegible]
51	255	[illegible]	[illegible]	1060 050
52	260	[illegible]	-87 [illegible]	1017 [illegible]
53	265	150 621	-87 457	[illegible]
54	270	150 140	-87 050	918 038
55	275	149 586	-86 568	861 256
56	280	148 947	-86 059	806 222

57	285	148 212	-65.439	715.258
58	290	147 362	-84 695	666 714
59	.295	146 373	-83 775	594 968
60	300	145 210	-82 597	520.437
61	305	143 823	-81 016	443 593
62	310	142 129	-78 763	365.107
63	315	139 994	-75 259	285 468
64	320	137 177	-69 056	206 390
65	325	133 269	-55 500	131 616
66	330	128 790	-17 506	78 587
67	335	130 605	39 275	96 846
68	340	135 176	63 921	163 916
69	345	138 539	72 179	241 441
70	350	141 013	76 892	320 976
71	355	142 929	79 733	400 212
72	360	144 473	81 640	478 102
73	365	145 733	83 019	553 978
74	370	146 833	84 067	627 322
75	375	147 756	84 901	697 689
76	380	148 553	85 585	764 682
77	385	149 243	86 161	827 935
78	390	149 843	86 658	887 112
79	395	150 363	87 094	941 902
80	400	150 814	87 485	992 033
81	405	151 200	87 839	1037 314
82	410	151 529	88 166	1077 249
83	415	151 804	88 470	1111 911
84	420	152 029	88 758	1141 035
85	425	152 206	89 032	1164 470
86	430	152 336	89 297	1183 097
87	435	152 422	89 556	1193 825
88	440	152 464	89 811	1199 956
89	445	152 462	90 064	1199 320
90	450	152 417	90 319	1193 177
91	455	152 328	90 578	1181 021
92	460	152 194	90 844	1162 973
93	465	152 014	91 119	1139 123
94	470	151 786	91 408	1109 993
95	475	151 507	91 713	1074 533
96	480	151 174	92 042	1034 133
97	485	150 783	92 399	988 368
98	490	150 328	92 792	938 101
99	495	149 802	93 232	882 995
100	500	149 196	93 734	823 903

ADMITTANCE (2 E-2,4 E-2)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152 467	90 000	1200 000
1	005	152 470	90 010	1200 387
2	010	152 429	90 021	1194 759
3	015	152 344	90 031	1183 146
4	020	152 214	90 042	1165 604
5	025	152 038	90 053	1142 222
6	030	151 814	90 065	1113 118
7	035	151 539	90 077	1078.436
8	040	151 210	90 091	1038 351
9	045	150 823	90 105	993 064
10	050	150 372	90 121	942 801
11	055	149 850	90 139	887 814
12	060	149 248	90 159	828.379
13	065	148.554	90 182	764 794
14	070	147 752	90 210	697 377
15	075	146 821	90 244	626 467
16	080	145 728	90 287	552 417
17	085	144 428	90 344	475 601
18	090	142 846	90 423	396 402
19	095	140 855	90 542	315 218
20	100	138 210	90 745	232 457
21	105	134 320	91 177	148 537
22	110	126 994	92 745	63 904
23	115	117 485	-98 215	21 384
24	120	131 397	-91 640	106 097
25	125	136 478	-90 903	190 435
26	130	139 633	-90 617	273 831
27	135	141 909	-90 464	355 858
28	140	143 675	-90 369	436 104
29	145	145 105	-90 302	514 166
30	150	146 295	-90 253	589 652
31	155	147 303	-90 215	662 185
32	160	148 166	-90 184	731 399
33	165	148 912	-90 159	796 950
34	170	149 558	-90 137	858 507
35	175	150 119	-90 118	915 763
36	180	150 604	-90 101	968 431
37	185	151 023	-90 086	1016 247
38	190	151 381	-90 072	1058 971
39	195	151 682	-90 059	1096 390
40	200	151 932	-90 047	1128 315
41	205	152 132	-90 036	1154 596
42	210	152 284	-90 025	1175 074
43	215	152 392	-90 014	1189 673
44	220	152 454	-90 004	1198 312
45	225	152 474	-89 994	1200 947
46	230	152 47[illegible]	-89 983	1197 565
47	235	152 381	-89 973	1198 187
48	240	152 [illegible]	-89 9[illegible]	1177 847
49	245	152 10[illegible]	-89 951	1151 [illegible]
50	250	151 [illegible]	89 946	1124 654
51	255	151 [illegible]48	-89 92[illegible]	1[illegible] [illegible]
52	260	151 34[illegible]	-89 914	1[illegible] 949
53	265	150 975	-89 900	1010 360
54	270	150 549	-89 885	963 148
55	275	150 053	-89 868	908 895
56	280	149 483	-89 849	851 089

57	285	148 825	-89 827	787 016
58	290	148 066	-89 801	722 994
59	295	147 186	-89 769	653 348
60	300	146 158	-89 730	580 429
61	305	144 942	-89 679	504 602
62	310	143 476	-89 610	426 247
63	315	141 659	-89 509	345 758
64	320	139 300	-89 345	263 537
65	325	135 929	-89 032	180 001
66	330	130 490	-88 166	95 577
67	335	111 739	-73 906	11 036
68	340	128 319	87 656	74 438
69	345	134 912	88 913	159 010
70	350	138 588	89 209	242 809
71	355	141 131	89 487	325 396
72	360	143 061	89 600	406 356
73	365	144 603	89 676	485 280
74	370	145 874	89 730	561 774
75	375	146 945	89 772	635 453
76	380	147 859	89 805	705 949
77	385	148 646	89 832	772 908
78	390	149 307	89 855	835 994
79	395	149 918	89 875	894 892
80	400	150 431	89 893	949 307
81	405	150 874	89 906	998 965
82	410	151 234	89 923	1043 618
83	415	151 576	89 936	1083 042
84	420	151 844	89 948	1117 040
85	425	152 063	89 960	1145 441
86	430	152 233	89 971	1168 103
87	435	152 357	89 982	1184 912
88	440	152 436	89 992	1195 785
89	445	152 472	90 003	1200 666
90	450	152 463	90 013	1199 532
91	455	152 411	90 023	1192 388
92	460	152 319	90 034	1179 269
93	465	152 174	90 045	1160 242
94	470	151 986	90 056	1135 402
95	475	151 749	90 068	1104 873
96	480	151 461	90 081	1068 808
97	485	151 118	90 094	1027 388
98	490	150 715	90 109	980 821
99	495	150 247	90 125	929 340
100	500	149 715	90 144	873 202

ADMITTANCE (2 E-2,2 E-2)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152 467	90 000	1200 000
1	005	152 457	90 010	1198 690
2	010	152 404	90 021	1191 375
3	015	152 307	90 031	1178 090
4	020	152 164	90 042	1158 902
5	025	151 975	90 054	1133 709
6	030	151 736	90 066	1103 234
7	035	151 447	90 078	1067 331
8	040	151 102	90 092	1025 482
9	045	150 697	90 107	978 796
10	050	150 227	90 123	927 205
11	055	149 683	90 141	870 969
12	060	149 057	90 163	810 369
13	065	148 335	90 187	745 708
14	070	147 499	90 217	677 312
15	075	146 526	90 253	605 522
16	080	145 380	90 299	530 690
17	085	144 009	90 361	453 218
18	090	142 328	90 449	373 466
19	095	140 186	90 586	291 344
20	100	137 276	90 830	208 764
21	105	132 797	91 402	124 646
22	110	122 917	94 392	39 968
23	115	123 988	-93 875	45 210
24	120	133 154	-91 339	129 874
25	125	137 489	-90 804	213 940
26	130	140 337	-90 569	296 943
27	135	142 443	-90 437	378 460
28	140	144 102	-90 351	458 083
29	145	145 457	-90 290	535 411
30	150	146 590	-90 245	610 057
31	155	147 554	-90 209	681 647
32	160	148 382	-90 180	749 822
33	165	149 098	-90 155	814 240
34	170	149 719	-90 134	874 579
35	175	150 258	-90 116	30 135
36	180	150 724	-90 100	[illegible]1 630
37	185	151 125	-90 085	1028 205
38	190	151 466	-90 07[illegible]	1069 429
39	195	151 753	-90 05[illegible]	1105 [illegible]54
40	200	151 989	-90 047	1135 622
41	205	152 174	-90 036	1160 260
42	210	152 314	-90 025	1179 084
43	215	152 409	-90 014	1192 001
44	220	152 459	-90 004	1198 946
45	225	152 466	89 [illegible]4	1199 [illegible]
46	230	152 42[illegible]	-89 [illegible]	1194 81[illegible]
47	235	[illegible] 348	-89 [illegible]	1183 [illegible]48
48	240	[illegible]	-89 [illegible]	[illegible]
49	245	[illegible] 051	-89 951	1143 [illegible]
50	250	[illegible] 831	-89 935	1115 [illegible]
51	255	151 561	-89 [illegible]	1081 [illegible]
52	260	151 227	-89 913	1041 61[illegible]
53	265	150 855	-89 899	996 336
54	270	150 410	-89 883	947 039
55	275	149 895	-89 866	892 506

56	280	149 301	-89 846	833 502
57	285	148 617	-69 823	770 322
58	290	147 826	-89 795	703 283
59	295	146 907	-89 762	632 720
60	300	145 831	-89 719	558 987
61	305	144 552	-89 664	482 454
62	310	143 000	-89 588	403 504
63	315	141 055	-89 473	322 534
64	320	138 486	-89 281	239 949
65	325	134 755	-88 864	156 169
66	330	127 985	-87 553	71 630
67	335	113 626	77 110	13 713
68	340	120 732	98 225	98 200
69	345	136 115	85 054	162 632
70	350	139 383	89 360	262 062
71	355	141 730	89 321	348 213
72	360	143 524	89 621	422 881
73	365	144 980	89 689	506 813
74	370	146 189	89 740	582 505
75	375	147 212	89 779	655 290
76	380	148 087	89 810	724 772
77	385	148 843	89 836	790 032
78	390	149 457	89 858	853 521
79	395	150 065	89 877	910 159
80	400	150 558	89 894	963 237
81	405	150 982	89 910	1011 469
82	410	151 345	89 924	1054 643
83	415	151 652	89 936	1093 533
84	420	151 906	89 949	1124 949
85	425	152 110	89 960	1151 728
86	430	152 267	89 971	1173 738
87	435	152 378	89 982	1187 871
88	440	152 445	89 992	1197 053
89	445	152 468	90 003	1210 337
90	450	152 448	90 013	1197 407
91	455	152 384	90 023	1188 579
92	460	152 275	90 034	1173 795
93	465	152 121	90 045	1153 130
94	470	151 919	90 057	1126 687
95	475	151 668	90 069	1094 559
96	480	151 365	90 081	1057 027
97	485	151 005	90 095	1014 159
98	490	150 585	90 111	966 310
99	495	150 095	90 127	913 430
100	500	149 533	90 145	855 [illegible]

ADMITTANCE (2 E-2,0 E+0)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152 467	90 000	1200 000
1	005	152 445	90 010	1196 904
2	010	152 379	90 021	1187 590
3	015	152 269	90 032	1173 035
4	020	152 114	90 043	1152 [illegible]
5	025	151 911	90 054	1125 595
6	030	151 658	90 066	1093 [illegible]
7	035	151 353	90 079	1055 625
8	040	150 992	90 093	1012 614
9	045	150 569	90 108	964 528
10	050	150 079	90 125	911 609
11	055	149 514	90 144	854 124
12	060	148 862	90 166	792 358
13	065	148 109	90 192	726 623
14	070	147 238	90 223	657 248
15	075	146 220	90 262	584 579
16	080	145 017	90 312	508 982
17	085	143 569	90 380	430 836
18	090	141 778	90 478	350 531
19	095	139 461	90 637	268 472
20	100	136 230	90 936	185 072
21	105	130 949	91 735	100 758
22	110	115 054	100 914	16 164
23	115	127 675	-92 534	69 116
24	120	134 614	-91 132	153 652
25	125	138 394	-90 724	237 445
26	130	140 988	-90 528	320 055
27	135	142 947	-90 412	401 062
28	140	144 509	-90 335	480 062
29	145	145 795	-90 279	556 656
30	150	146 876	-90 237	630 463
31	155	147 799	-90 203	701 110
32	160	148 593	-90 175	768 245
33	165	149 281	-90 152	831 531
34	170	149 877	-90 132	890 651
35	175	150 395	-90 114	945 308
36	180	150 842	-90 098	995 229
37	185	151 225	-90 084	1040 164
38	190	151 551	-90 071	1079 887
39	195	151 822	-90 058	1114 200
40	200	152 043	-90 047	1142 930
41	205	152 217	-90 036	1165 934
42	210	152 343	-90 025	1183 096
43	215	152 426	-90 014	1194 330
44	220	152 464	-90 004	1199 581
45	225	152 458	-89 994	1198 921
46	230	152 409	-89 983	1192 054
47	235	152 316	-89 973	1179 315
48	240	152 177	-89 962	1160 657
49	245	151 992	-89 950	1135 [illegible]
50	250	151 760	-89 938	1106 049
51	255	151 474	-89 926	1070 351
52	260	151 134	-89 912	1029 [illegible]
53	265	150 735	-89 898	983 [illegible]
54	270	150 271	-89 881	931 931
55	275	149 734	-89 863	876 119
56	280	149 116	-89 842	815 917

57	285	148 403	-89 818	751.628
58	290	147 579	-89 789	683 573
59	295	146 619	-89 754	612 093
60	300	145 491	-89 708	537 547
61	305	144 144	-89 640	460 307
62	310	142 496	-89 563	380 762
63	315	140 406	-89 433	299 311
64	320	137 587	-89 203	216.363
65	325	133 317	-88 683	132 338
66	330	124.453	-86 334	47 700
67	335	122 355	85 316	37 463
68	340	132 619	88 571	122.139
69	345	137 171	89 162	206 354
70	350	140 112	89 412	289 355
71	355	142 271	89 550	371 009
72	360	143 963	89 639	450 806
73	365	145 341	89 702	528 345
74	370	146 493	89 749	603 337
75	375	147 471	89 785	675 107
76	380	148 310	89 815	743 594
77	385	149 035	89 840	808 357
78	390	149 664	89 861	869 069
79	395	150 210	89 279	925 427
80	400	150 682	89 896	977 148
81	.405	151 089	89 911	1023 974
82	410	151 476	89 924	1065 569
83	415	151 727	89 937	1102 024
84	420	151 967	89 949	1133 859
85	425	152 157	89 960	1156 017
86	430	152 301	89 971	1177 373
87	435	152 400	89 982	1190 830
88	440	152 455	89 992	1198 321
89	445	152 465	90.003	1199 908
90	450	152.433	90 013	1195 283
91	455	152 356	90.023	1184 770
92	460	152 234	90 034	1168 331
93	465	152 067	90 045	1146 018
94	470	151 852	90 057	1117 973
95	475	151 586	90 069	1084 326
96	480	151 267	90 082	1045 248
97	485	150 891	90 097	1000 931
98	490	150 452	90 112	951 600
99	495	149 944	90 130	897 501
100	500	149 357	90 149	838 506

ADMITTANCE = (2.E-2,-2 E-2)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152 467	90 000	1200 000
1	005	152 433	90 010	1195 297
2	010	152 355	90 021	1184 606
3	015	152 232	90 032	1167 979
4	020	152 063	90 043	1145 500
5	025	151 846	90 054	1117 282
6	030	151 579	90 067	1083 467
7	035	151 259	90 080	1044 222
8	040	150 881	90 094	999 746
9	045	150 440	90 110	950 261
10	050	149 929	90 127	896 014
11	055	149 341	90 147	837 279
12	060	148 662	90 170	774 348
13	065	147 878	90 197	707 538
14	070	146 968	90.230	637 183
15	075	145.903	90 272	563 636
16	080	144 638	90 326	487 265
17	085	143 106	90 401	408.453
18	090	141 190	90 512	327 596
19	095	138 670	90 697	245 100
20	100	135 040	91 074	161 380
21	105	128 599	92 274	76 875
22	110	109 640	-110 678	8 667
23	115	130 257	-91 882	93 041
24	120	135 864	-90 980	177 433
25	125	139 214	-90 659	260 952
26	.130	141 593	-90 493	343 167
27	.135	143 424	-90 390	423 665
28	140	144 898	-90.320	502 041
29	145	146 120	-90 269	577 902
30	150	147 153	-90 229	650 868
31	155	148 037	-90 198	720 573
32	160	148 799	-90 171	786 668
33	165	149 459	-90.149	848 822
34	170	150 033	-90 130	906 723
35	175	150 529	-90 113	960 081
36	180	150 959	-90 097	1008 628
37	185	151 324	-90 083	1052 123
38	190	151 634	-90 070	1090 345
39	195	151 892	-90 058	1123 105
40	200	152 099	-90 046	1150 239
41	205	152 259	-90 035	1171 607
42	210	152 373	-90 025	1187 107
43	215	152 443	-90 014	1196 658
44	220	152 468	-90 004	1200 215
45	225	152 450	-89 994	1197 757
46	230	152 389	-89 983	1189 299
47	235	152 287	-89 972	1174 882
48	240	152 132	-89 961	1154 578
49	245	151 932	-89 950	1128 489
50	250	151 685	-89 939	1096 745
51	255	151 385	-89 925	1059 [illegible]
52	260	151 [illegible]	-89 911	1016 [illegible]
53	265	150 [illegible]	89 896	[illegible] 221
54	270	150 129	-89 880	916 823
55	275	149 570	-89 861	859 721
56	280	148 927	-89 839	798 332

57	285	148 184	-89 813	732 933
58	290	147 325	-89 783	663 862
59	295	146 322	-89 745	591 466
60	300	145 138	-89 696	516 106
61	305	143 716	-89 630	438 160
62	310	141 961	-89 535	358 030
63	315	139 704	-89 325	276 088
64	320	136 584	-89 105	192 777
65	325	131 592	-88 293	108 510
66	330	118 428	-82 628	23 836
67	335	126 644	87 143	61 384
68	340	134 169	92 805	145 980
69	345	138 113	89 248	229 277
70	350	140 784	89 456	312 628
71	355	142 789	89 576	393 815
72	360	144 381	89 656	473 031
73	365	145 689	89 714	549 977
74	370	146 785	89 757	623 969
75	375	147 722	89 791	694 933
76	380	148 527	89 819	762 417
77	385	149 224	89 843	826 081
78	390	149 828	89 863	885 906
79	395	150 352	89 881	940 654
80	400	150 805	89 897	991 069
81	405	151 194	89 912	1036 478
82	410	151 525	89 925	1076 654
83	415	151 801	89 938	1111 516
84	420	152 027	89 949	1140 768
85	425	152 204	89 961	1164 305
86	430	152 335	89 971	1182 009
87	435	152 422	89 982	1193 790
88	440	152 464	89 992	1199 590
89	445	152 462	90 003	1199 379
90	450	152 417	90 013	1193 160
91	455	152 328	90 024	1180 962
92	460	152 194	90 034	1162 847
93	465	152 013	90 046	1138 906
94	470	151 784	90 057	1109 259
95	475	151 504	90 070	1074 054
96	480	151 169	90 083	1033 468
97	485	150 776	90 098	987 703
98	490	150 318	90 114	936 990
99	495	149 788	90 133	881 583
100	500	149 178	90 153	821 758

ADMITTANCE = (2 E-2,-4 E-2)

PRESSURE DISTRIBUTION

ITERATION	DISTANCE	MAG(db)	PHASE	PRESSURE
0	0 000	152 467	90 000	1200 000
1	005	152 420	90 010	1193 601
2	010	152 330	90 021	1181 222
3	015	152 194	90 032	1162 924
4	020	152 012	90 043	1139 799
5	025	151 781	90 055	1108 969
6	030	151 500	90 067	1073 583
7	035	151 164	90 081	1032 818
8	040	150 768	90 095	986 878
9	045	150 309	90 111	935 993
10	050	149 777	90 130	880 419
11	055	149 164	90 150	820 434
12	060	148 457	90 174	756 338
13	065	147 641	90 203	688 453
14	070	146 690	90 238	617 118
15	075	145 574	90 282	542 692
16	080	144 242	90 341	465 548
17	085	142 616	90 424	386 071
18	090	140 559	90 551	304 661
19	095	137 799	90 771	221 728
20	100	133 661	91 258	137 691
21	105	125 369	93 299	53 005
22	110	121.049	-95 448	32 234
23	115	132 245	-91 497	116 972
24	120	136 956	-90 864	201 214
25	125	139 963	-90 604	284 458
26	130	142 159	-90 461	366 279
27	135	143 875	-90 370	446 267
28	140	145 270	-90 307	524 019
29	145	146 434	-90 259	599 147
30	150	147 421	-90 222	671 273
31	155	148 268	-90 192	740 036
32	160	149 000	-90 167	805 091
33	165	149 635	-90 146	866 113
34	170	150 185	-90 127	922 795
35	175	150 662	-90 111	974 853
36	180	151 072	-90 096	1022 028
37	185	151 423	-90 082	1064 082
38	190	151 717	-90 069	1100 804
39	195	151 960	-90 053	1132 011
40	200	152 154	-90 046	1157 546
41	205	152 301	-90 035	1177 282
42	210	152 402	-90 025	1191 119
43	215	152 459	-90 014	1198 998
44	220	152 473	-90 004	1200 850
45	225	152 44[illegible]	-89 993	1196 695
46	230	152 [illegible]	-89 983	1186 544
47	235	152 250	-89 972	1170 449
48	240	152 086	-89 961	1148 495
49	245	151 873	-89 950	1120 [illegible]75
50	250	151 611	-89 937	1087 445
51	255	151 296	-89 924	1048 [illegible]
52	260	150 923	-89 910	1004 635
53	265	150 488	-89 895	955 569
54	270	149 964	-89 877	901 716
55	275	149 403	-89 858	843 344

56	280	148 733	-89 835	80 748
57	285	147 960	-89 809	714 240
58	290	147 063	-89 776	644 153
59	295	146 013	-89 736	570 039
60	300	144 769	-89 683	494 666
61	305	143 265	-89 611	416 014
62	310	141 391	-89 504	335 279
63	315	138 941	-89 330	252 867
64	320	135 451	-88 980	169 193
65	325	129 439	-87 941	84 657
66	330	100 641	6 033	3 075
67	335	127 505	87 945	65 333
68	340	135 463	88 073	169 833
69	345	138 963	89 318	253 499
70	350	141 407	89 493	335 901
71	355	142 276	89 600	416 621
72	360	144 750	89 672	495 255
73	365	146 023	89 724	571 406
74	370	147 070	89 765	644 659
75	375	147 966	89 797	714 760
76	380	148 739	89 824	781 239
77	385	149 408	89 846	843 805
78	390	149 969	89 866	902 143
79	395	150 493	89 883	955 961
80	400	150 926	89 899	1004 989
81	405	151 298	89 913	1048 982
82	410	151 613	89 926	1087 719
83	415	151 875	89 938	1121 007
84	420	152 087	89 950	1148 677
85	425	152 251	89 961	1170 593
86	430	152 370	89 971	1186 644
87	435	152 447	89 982	1196 749
88	440	152 477	89 992	1200 855
89	445	152 459	90 003	1198 950
90	450	152 402	90 013	1191 035
91	455	152 300	90 024	1177 133
92	460	152 153	90 035	1157 273
93	465	151 952	90 046	1131 794
94	470	151 715	90 058	1100 544
95	475	151 430	90 071	1063 781
96	480	151 069	90 084	1021 687
97	485	150 659	90 099	974 475
98	490	150 181	90 116	922 380
99	495	149 630	90 134	865 664
100	500	149 999	90 156	804 610

Computer Hardware and Software

The data reduction program was coded in FORTRAN 77 using a HP-3000 system. The perturbation pressure graphs were done with the Lotus 123 software on an IBM clone with a Gemini 10x printer.

* Page 62 does not exist