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An Optimization Technique of Airlines' Seat Inventory Management

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AN OPTIMIZATION TECHNIQUE OF AIRLINES' SEAT INVENTORY MANAGEMENT

by

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This thesis was prepared under the direction of the candidate's thesis committee chair, Dr. Bijan Vasigh, Department of Aviation Business Administration, and has been approved by the members of this thesis committee. It was submitted to the Aviation Business Administration Department and was accepted in partial fulfillment of the requirements for the degree of Master of Business Administration in Aviation.

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ABSTRACT

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This study consisted of a simulation to maximize an airline's Origin-Destination revenues. It has been hypothesized that Linear Programming is capable of maximizing airlines' networks revenues. Simulation was performed with Linear Interactive Discrete Optimizer (LINDO). This project was designed to consider different fares and its classes along with multi flight connections. The seats' allocation process came out with the maximum possible revenues, and enough flexibility in terms of changing such allocation to work around competition and trends. Results came supportive to the hypothesis. Sensitivity analysis provided our model with a tool to modify and change different variables, like fares, without affecting the reached goal (maximized network revenues). Although, the utilized network is a portion of a real world one, this study should inspire revenue management departments build their own simulation based on this model. Conclusion and recommendations are submitted.

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Chapter One
INTRODUCTION

In spite of twenty years of having airline deregulation, air transportation is still looking for a commendable future. Commendable future, here, is defined as safe, reliable, low-risk, and profitable air transportation. Airlines these days, more so than in the past, are looking at their revenue management departments as saviors. High cost seats and firm competition among airlines created a real dilemma for those deciding to keep running.

In 1938, the Civil Aeronautics Act (CAA) established a policy to regulate the airline industry. This Act created the Civil Aeronautics Board (CAB) and gave it a broad responsibility in regulating air fares, routes, and entry and exit from the market. For an airline to enter the market, a Certificate of Public Convenience and Necessity had to be issued by the CAB. This certificate determined several conditions including competition with other rivals. A proposed airline should comply with all items listed in that certificate. Routes and markets were granted and assigned. Regulations also included fares, and fares decisions were based on the Industry Rate of Return (ROR) and always guaranteed to be higher than the average cost (AC)[1] . Airlines were allowed to mark up fares by 5% maximum, and down by 50% maximum. Any other changes would call for a CAB approval, then competitors would be notified by notice via tariff.

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[1] Total cost of industry seats divided by total seats.

Deregulation of the airline industry in 1979 gave airlines full control in terms of scheduling and pricing. Also more freedom has been given to airlines in areas like policy making, market entry, mergers and control, intercarrier agreements, subsidy, operating equipment exemption, and small community service. Competition increased and passengers enjoyed a wider choice of service and fares. Along with the significant increase in discounted fares, airlines invested more and more in new technologies and solutions to generate incremental revenues[2].

New comers, like ValueJet[3] , reduced profit margins forcing existing airlines to examine their high cost structure and expand their yield management efforts to optimize revenue and magnify their load factor. Researchers concentrated on predicting the behavior of demand, competitors' next move, substitute methods of transportation, customer sensitivity to price and time, service levels, and customer's degree of perception. In order for revenue management to be gratified, aircraft not only should leave with full boarding but also with the most they can get from the traveler's pocket. From that bench mark, level of on-board service started to decline.

This thesis extends efforts searching for a method to allocate full and discounted fares in a way that maximizes revenue.

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[2] Vinod, B., "Origin Destination Class Yield Management," Presented at IATA-The Sixth International Airline Yield Management Conference, Barcelona, Spain, October 1994.

[3] ValueJet concentrates on the East Coast of the United States.

Problem Statement

After airline deregulation in 1978, managing air transportation has become troublesome. When scheduling became the airlines' responsibility, they started to invade every possible market. Hub-and-spoke systems for major carriers aggravated the problem. Entry barriers almost vanished, starting an airline became easier, and the market became larger, intensifying competition.

Air transportation suffered from the economic recession in the eighties and early nineties. Air transportation depends on the level of income to induce passengers to fly rather than drive. But in most of long trips, air transport has the advantage. From that point of view, overseas traveling is dominated by the airline industry.

Prior to deregulation, the government guaranteed each operator a piece of the market, then regulated fares on all routes. To distinguish itself from other rivals, an airline had to provide superior on-board service. After deregulation, airlines introduced a wide range of fares in order to enhance their market share. Before 1978 discounted seats were 50% of total seats sold and it rose to 90% by 1990[4]. Full fares grew at a higher rate than discounted ones.

Airlines offer discounted fares with different restrictions. Restrictions are installed to prevent full fare holders from switching to discounted ones. Such restrictions are framed with time and trip configurations. For example, airlines may require the purchase of discounted seats at least one week in advance. Also, markup decisions result in various fares' amount even among same fare class seats. Such fare variety is called price discrimination due to the fact of charging

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[4] Pfeifer, Phillip E, "The Airline Discount Fare Allocation problem" University of Virginia, Charlottesville, Virginia

similar passengers different prices reflecting no cost discrepancies[5]. Price discrimination takes place when airlines' potentials to enhance profits coexist with passengers willingness of paying different prices. Therefore, airlines will do their best to enhance profits as long as passengers' potentials of pay vary. Passengers' elasticity of time and money plays a significant role in such variety. Motive for airfare discrimination can be better understood by introducing the concept of passengers' surplus[6]. Passengers' surplus is the value of a given fare above the market equilibrium fare (F^*) as shown in Figure (1). Passengers' total fare value is the area under the demand curve, or $OABQ^*$ area while surplus is represented with the F^*AB area. For example, if a given passenger is willing to pay $600 for a certain seat but able to obtain it for $450, he or she enjoys $150 worth of passengers' surplus.

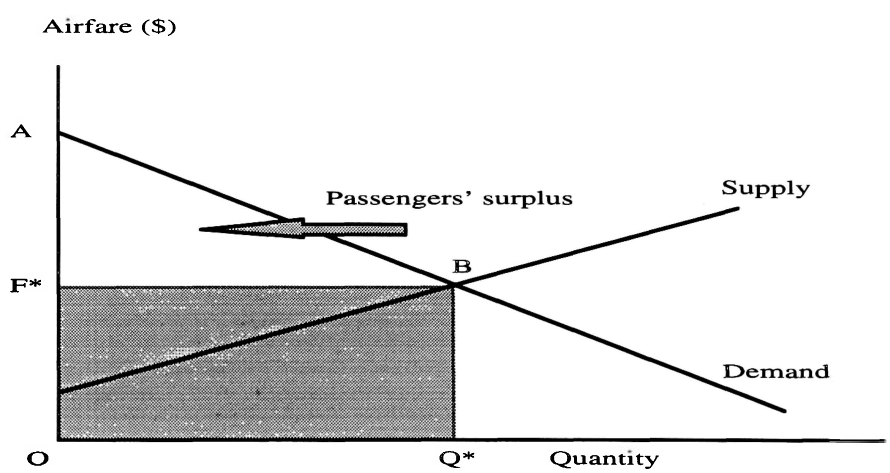

Figure 1. An Illustration of Passengers' Surplus[6]

[5] Frank, Robert H. "When Are Price Differentials Discriminatory?" Journal of Policy Analysis and Management, Vol. 2, No. 2, 238-255 , 1983.
[6] Hirschey, M., and Pappas, J. Managerial Economics. Orlando, FL. : Dryden Press, 1993.

Thus when airfare value differs among travelers, a motive for fare discrimination is created. And airlines will always seek the maximum fare each passenger is willing to pay. The motive for this fare discrimination increases when demand gets stronger. In this case, airlines revenue will be the entire area under the demand curve (Figure 2), due to the fact of charging travelers the highest possible fare amount.

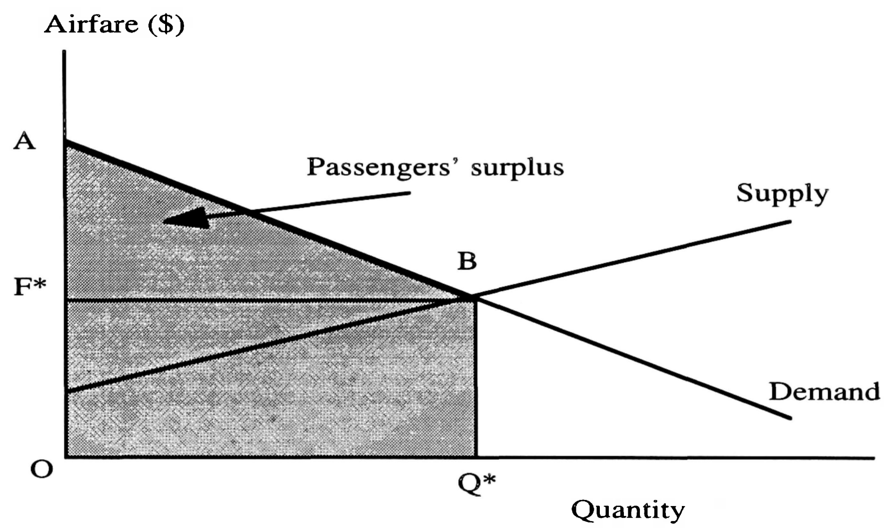

Figure 2. Passengers' Surplus and Maximum Fare Discrimination

Therefore this discrimination happens even in the same class where passengers obtain seats for different fares. Fare amount is also affected by circumstances surrounded every purchase process. In other words, airline fare is like a commodity for sale at different stores. Price of this commodity varies in accordance with the purchase source and date of purchase. Passengers who are concerned about these differences are price-sensitive (Figure 3). An example of those passengers would be leisure travelers who don't care about when to travel as much as they care about how much such fares will cost them.

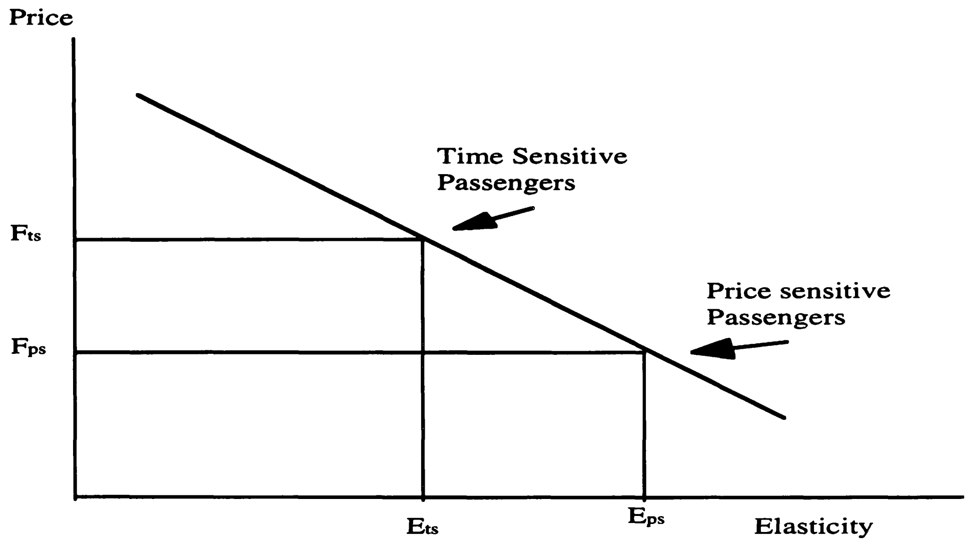

Figure 3. Passengers' Time and Price Elasticity

On the other hand, businessmen traveling on their companies' expense, do not care about fares as much as about when to fly. Because of incremental revenues provided by such a discrimination, an airline may be able to offer routes that could not be supported by revenues from one fare amount alone. Therefore, watching passengers' price and time sensitivity added more responsibility on the yield management operations.

Demand also is one of yield management's major concerns. Demand for air transportation changes from day to day, season to season, and year to year. Thus, major carriers rely on information gathered and distributed by the Department of Transportation and the Federal Aviation Administration. Airlines do their forecasts based on the historic behavior of demand assuming that trend will continue with the same behavior (time series forecast). Time series forecasting can be accurate if all factors affecting demand, except time, are fixed and not interrupted by natural or national effects. Factors affecting demand are :

- Air fare.
- Income level.
- Price of other modes of transportation.
- Time of departure and arrival, and trip purpose.

Accurate forecasting can help airlines to:

- Size aircraft to the market.
- Make the first move (gain competitive edge).
- Locate full and discounted fares (maximize revenue), and
- Cast precise future strategies.

Due to lack of flexibility in the cost structure, yield management is experiencing difficulty determining and allocating air fares. This process utilizes historical data supported by individual market segmentation and the projection of forecasting, in an effort to maximize revenue from a time-limited inventory.

Literature Review

Williamson and Belobaba[7] (1988), introduced their paper "Optimization Techniques for Seat Inventory Control". They discussed different concepts of maximizing revenue from single flight legs, and from the entire network. They stated that revenue maximization has become more complicated for many reasons, for example :

- The need to cover the entire network (Origin-Destination) rather than single flight legs.
- Price became a variable, even in same class.
- The introduction of the hub-and-spoke system where the network became more complicated.

Airlines espouse the easiest way of maximizing revenue, which is the leg-based method. This method maximizes revenue on a single flight leg regardless of passengers' ultimate destination. Other airlines take into account the ultimate passenger destination to maximize their entire network revenue. Airlines taking into account the entire network revenue face obstacles such as :

- Demand uncertainty due to cyclical and stochastic variations.
- Demand cancellation.

These two factors require close attention to demand behavior.

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[7] Williamson, E, and Belobaba, P, 1988, "Optimization Techniques for Seat Inventory Control," AGIFORS 28th Annual Symposium Proceedings, pp 153-170

Regarding inventory control, airlines utilize one of the following methods[2] :

- Leg Class Controls; where inventory controls are established by flight leg for each fare class. Airlines define between five and 26 fare classes to control such inventory. Flight leg-based inventory controls are defined as nested and non-nested. In both cases overbooking is not applied. Nesting method is divided into serial and parallel themes. Serial nesting allows controllers move seats to the next higher fare class, while parallel nesting method moves seats to the highest fare class. A third method is called distinct where airlines can't do the move and may result in unsold seats.
- Segment Class Controls; where inventory controls are established by segment for each fare class. Origin-Destination demand is disregarded. Segment Class Indicator (SCI) is used to restrict sales for selected segment classes that are lower-valued when demand for higher-valued segment class is observed.
- Origin-Destination Inventory Control; where all flight connecting points are honored. This control method is really complicated but rewarding. Controlling seats is based on different factors such as itinerary, departure date, fare class, and published fares. such considerations will end up selling seats to high-valued passengers.
- Virtual Nesting Controls. This method is accomplished by considering the various Origin-Destination fare classes that flow over a flight leg into a number of buckets, based on passengers' value. A dynamic programming model is usually established to minimize the variance of customer values within each bucket. Then buckets are serially nested to build up sales for each flight. Virtual nesting was developed at American Airlines in 1986.

When airlines run the nested booking method, they move the seat from low fare class to upper fare depending on what is called the Expected Marginal Seat Revenue (EMSR). Simply, booking system allocates the seat to the fare class that has the highest demand probability. The expected marginal revenue of the i-eth seat is :

$$\text{EMSR}(S_i) = f_i \,.\, P(S_i) + 0 \,.\, [1 - P(S_i)] \quad , \textit{or}$$

$$\text{EMSR}(S_i) = f_i \,.\, P(S_i)$$

EMSR takes into account demand history and current booking. One of its disadvantages is lack of consideration among flight legs. But, one of the advantages is the compatibility with all current reservation systems, and consideration of probability of demand.

Few airlines ponder the passenger's ultimate destination before considering booking limits. This process is called "virtual nesting". This process is no different from the leg based process. It locks seats and reserves them for those passengers with the destination requirements. EMSR method can be applied here appreciating the dollar range of the entire flight. The next step would be dispatching the booking limits to travel agents.

Williamson and Belobaba concluded their paper with recommendations of utilizing Linear Programming, with a process of nesting the seats in accordance with shadow prices.

Cross[8] (1988), addressed the airlines' cost issue as a key factor in offering discounted fares. Major airlines have massive fixed costs while "new comers"

[8] Cross, Robert G., President, Aeronautics Incorporated, " The Employment Of Yield Management Methodologies To Overcome Cost Disadvantages." Airline Group Of The International Federation Of Operational Research Societies, AGIFORS Annual Symposium, 1988, 203-216.

started with almost half the cost. Average trip distance increased which yielded lower cost per available seat mile, but still giving the advantage to the low cost carriers. Effective yield management systems maximize allocation efficiency by:

- Stimulating traffic by offering deep discounts.
- Shifting passengers from high demand flights to low demand by using price incentives to win the chance of selling seats with high fares on those high demand flights.

Passengers can be classified into two categories: leisure and business. Leisure passengers are price sensitive while business passengers are time · sensitive.

Fromholzer[9] (1988), addressed the departments of yield management. He stated that there are two departments, inventory management and pricing. The inventory department deals with allocating the discounted fares and overbooking limits. The pricing department deals with the process of evaluating the price range for each class. They depend on each other under the mother department, yield management. Airlines support yield management for the critical role it plays in generating and maximizing revenues. Resources and support are shaped into four systems :

- Dynamic update system to review and update capacity.
- Space planning system.
- Dynamic system for forecasting booking demand, and
- A system for optimizing capacity planning decisions.

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[9] Fromholzer, Dennis, Director, Corporate R&D, United Airlines, "Yield Management Variables." Airline Group Of International Federation Of Operational Research Societies, AGIFORS Annual Symposium, 1988, 83-88.

United Airlines' yield management system was a full-fledged discussion in Fromholzer's paper. Their system consisted of four major components :

- Software package for managing flights.
- An artificial information model.
- An analysis, and
- A reservation system.

Carrington[10] (1988), addressed in his paper the importance of understanding the public's reactions to the airlines' actions. Decisions made by yield management can be risky, and airlines should closely monitor public's reaction (like/dislike). Areas to be monitored should include :

- Overbooking.
- Class mix.
- Segment revenue.
- Sales, and
- Average inflation.

Spry[11] (1988), stated that the main task of a yield management system is to watch the fares mix on flight segments. Fares are defined as:

- Super discounted fares.
- Discounted fares, and
- Full fares.

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[10] Carrington, David, Head, Marketing Tactics Department, British Airways, "Determining Yield Management Payoff" Airline Group Of International Federation Of Operational Research Societies, AGIFORS Annual Symposium, April 6-8, 1988, 63-68

[11] Spry, E C, Senior Director, Industry Automation and Finance Services, International Air Transportation Association "What Is Yield Management" Airline Group Of International Federation Of Operational Research Societies, AGIFORS Annual Symposium, April 6-8, 1988, 43-62

Pricing can be classified into two types, segmental/market pricing which is used by scheduled carriers, and uniform pricing which is utilized by charters. Uniform pricing, helps charter carriers initiate flights when sufficient demand is perceived.

James[12] (1988), addressed the effect of deregulation on airlines' fares. The way he addressed that was in the contour of statistics. During the ten years followed the Deregulation Act of 1978:

- Full fares increased by 156 percent.
- Discount fares increased by 52 percent, and

The overall average increased by 31 percent as a result of traffic slide from high to low fares.

Wysong[13] (1988), approached the value of origin-destination based yield management. Wysong referred to virtual classes in his research. Most airlines prefer an optimization technique that considers the entire itineraries of the demanded seats. A major gain from such consideration would be winning a more valued customer when demand exceeds capacity. In this case, available seats would be sold to passengers willing to pay more.

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[12] Dr James, George, President, Airline Economics, Inc, "The Critical Importance Of Airline Revenue Enhancement A U S View" Airline Group Of The International Federation Of Operational Research Societies, AGIFORS Annual Symposium, 1988, 83-88

[13] Wysong, Richard, "A Simplified Method For Including Network Effects In Capacity Control" Airline Group Of International Federation Of Operational Research Societies, AGIFORS, 28th Annual Symposium, New Seabury, Massachusetts, October 16-21, 1988, 113-121

Smith[14] (1988), addressed four different market control strategies and five alternative yield management strategies. Market strategies are :

- Segment/class.
- Leg/class.
- Virtual nesting, and
- Full origin-destination.

The yield management strategies are :

- No control.
- Class code controls.
- "Greed" controls.
- Load displacement adjustments, and
- Network flow displacement.

Smith approached the impact of each of these strategies based on a typical American Airlines connecting hub. He found out that the "no control" strategy granted the highest load factor while the "greedy controls" provided the lowest load factor with higher benefit. Network adjustments recorded improvements on both sides; load factor and revenue.

In 1990, group of researchers[15] introduced a paper called "Putting Pleasure Into Yield Management". That paper was written around Marketing

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[14]Smith, B C , and Penn, C W , "Analysis Of Alternate Origin-Destination Control Strategies " Airline Group Of International Federation Of Operational Research Societies, AGIFORS, 28th Annual Symposium, New Seabury, Massachusetts, October 16-21, 1988, 123-144

[15] Curry, R E , and M Jaul, and A Storey, 1990, " Putting Pleasure Into Yield Management," Thirtieth AGIFORS Symposium, Macau

Inventory Data Analysis System (MIDAS), the yield management system utilized by QANTAS Airlines of Australia. In the following paragraphs we will address the main points discussed in that paper.

Yield management has four major functions :

- Limiting seat inventory.
- Controlling overbooking and compensating for no-shows.
- Compensating for cancellations.
- Forecasting demand, cancellations, and no-shows.

Factors affecting the process of limiting inventory :

- Inventory control mechanism.
- Optimization methodology, and
- “Computational Algorithms”.

Control mechanisms include nesting, discreting, or a mix between both. These mechanisms are adjusted, when utilized, in accordance with flight configuration. In leg-based cases, considerations are different from those in segmented cases. In the optimization methodology circle, real world simulation is required to minimize deviations from the real world. A real world simulation simulates all daily life aspects of seat booking process. Real world simulation entails :

1. Leg based; nested in serial, nested in parallel, discrete, or a mixture (Hybrid).

2. Segment; classes are nested within segments. QANTAS prefers using this method since it proved to increase revenue more than the a leg based mechanism .
3. Origin and destination; the entire flight is considered as one unit. Classes are nested serially in each fare class.

Hypothetical mechanisms include :

1. Origin-Destination/segment; classes are nested serially within segments. Linear programming is hired to maximize the entire system revenue. Allocations recommended by the LP are based on nesting classes serially.
2. Origin-Destination/segment, discrete.
3. Origin-Destination/segment, parallel within segments.
4. Virtual classes nested in parallel and serial. Leg-based approaches where fare class and O/D combinations are associated with a hidden seat inventory class.

QANTAS Airlines had problems counting on their reservation control system QANTAM. Because of these problems, QANTAS launched an extensive research. The main concern was to join QANTAM and MIDAS (Marketing Inventory Data Analysis System) in one package that will put pleasure into yield management. The result was Enhanced Revenue Optimization System (EROS). EROS is a personal/mainframe computer optimization system with the capability of controlling seat inventory in the reservation system. Under this system, QANTAM limited the class booking. Classes do not share inventory, and can not book beyond their limits. QANTAS called for simulation to give a real picture of the outcome. Unfortunately when segments were controlled by legs and Segment Closed Indicator, no optimal solution was found.

EROS optimization technique is performed in two stages :

1. Maximum Path Length (MPL). Class allocations are determined in a discrete way. No shows and overbooking are considered. If overbooking was not considered, then MPL will be a linear programming case. MPL determines the inventory pool for the second stage.
2. Defines the seat inventory, determines the booking limits, then nests classes in a parallel/serial way.

Inventory pools consist of :

1. Virtual/leg inventory pool, where seats are used efficiently.
2. Segment inventory pool, where interference between segments' seats is prevented.

Therefore, EROS controls seat inventory by discrete classes considering cancellations and no shows. Optimal booking limits are then determined depending on the nesting method. The next step would be figuring out the dollar amount expected from harvesting the boarded seats. Dollar amounts, or the "net expected revenue", is calculated by hiring a formula. This formula, in brief, adds revenues from all segments then subtracts the cost of canceled seats and no-shows. Another way would be figuring out the difference between marginal revenue and marginal cost. An advantage of this formula is the ability to determine the maximum flow and the minimum cost associated with each seat. Seats are observed based on the difference between these figures. Seats can be tested for a minimum marginal cost by moving them across classes. Then, optimum discrete class allocation is determined by increasing the flow by one seat. The result will be an optimal allocation of seats in each segment and inventory pool. Having the inventory pool, optimization methods are readily applicable. QANTAS Airlines realized, after this extensive research, that problems with yield management declined, but did not depart.

Sugitani[16] (1990), in his paper "Development of Yield Management Support System" claimed that yield management requires two computer systems :

1. A computer reservation system to control inventory by selling class and segment, and
2. A decision support system to provide the first system with the right decisions based on accurate data.

The yield management system has three functions :

1. Overviews seat inventory status with different approaches.
2. Forecasts future seat inventories, and
3. Makes recommendations for those flights under a control need.

Smith[17] (1990), in his paper "A Group Decision Support Model" stated that most of the yield management systems deal with individual demand rather than group demand. He claimed that a group demand approach provides a better

[16] Sugitani, Yukio, "Development Of A Yield Management Support System In JAL" Airline Group Of International Federation Of Operational Research Societies, AGIFORS, 30th Annual Symposium, Taipa Island, Macau, September 3, 1990, 9-26

[17] Smith, Barry C, "A Group Decision Support Model" Airline Group Of International Federation Of Operational Research Societies, AGIFORS, Thirtieth Annual Symposium, Taipa Island, Macau, September 3, 1990, 27-39

chance of increasing revenue than an individual one. Margined seats would be a function of leg seats available (LSA= capacity sold seats). Smith framed his model with a group demand approach, and pronounced it as a good sales assistant when :

- Negotiating group contracts, and
- Evaluating the performance of groups' revenue.

Ward[18] (1992), remarked that revenue is the responsibility of yield management departments, not the distributors. Yield management is "a combination of science and art aimed at gaining the optimum revenue for each product or service for which a variety of tariffs are in the market place".

Airlines should support their inventory and distributors as much as possible. Supporting inventory stems from being responsible for it. Supporting distributors comes from the fact of having changeable regulations that might affect the search for distributing techniques.

Belobaba[19] (1992), came back to the stage with a paper called "Yield Management Optimization and Forecasting Techniques Made Simple". In his paper, Belobaba addressed two critical issues when maximizing revenues. The first issue is the use of the appropriate optimization model. The second one is accuracy in forecasting demand.

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[18]Ward, John, Business Development Manager Galileo International, "Distribution Network Constraints And Compliance' An IATA Conference & Exhibition The Fourth International Yield Management Conference, Hotel-Intercontinental, Miami 13-14 October, 1992, 120-300

[19] Belobaba, Peter, Assistant Professor, MIT Flight Transportation Laboratory, "Yield Management Optimization And Forecasting Techniques Made Simple' An IATA Conference & Exhibition The Fourth International Yield Management Conference, Hotel-Intercontinental, Miami 13-14 October, 1992, 120-300

Then he discussed the three roles of forecasting :

- Provide the optimization model with necessary data.
- Increase revenue by using accurate inventory decisions, and
- Estimate future fare class demand.

Forecasting methods are:

- Causal, where demand is affected by different variables.
- Time series, where demand is continuos with its historic behavior.
- Combined.

Hypothesis

Linear Programming method can be used to maximize airlines' entire network revenues. Entire flight network revenues can be maximized by allocating full and discounted fares in a way that :

- Takes into account the highest demand probability.
- Considers fair amount and class priority, and
- Considers competition with other airlines.

Once demand and market share are forecasted, yield management can propose fares then utilizes Linear Programming for allocating those fairs.

If this hypothesis is verified, then a computer version of Linear Programming will be recommended for industry utilization.

Objectives of Research

The main objectives of this research were :

- Aware yield management of factors affecting demand for air transportation.
- To discuss the latest future outlook of the aviation industry in the eyes of the Federal Aviation Administration.
- Expand the Linear Programming technique to include the entire network rather than flight legs to maximize revenue.
- Facilitate the process of maximizing airlines' revenue with appropriate computer aid, and
- Submit recommendations.

Chapter Two

DEMAND FOR AIR TRANSPORTATION

Estimating demand for air transportation is the most critical variable in the process of maximizing revenues. Variables include demand, passenger revenue, air fare, level of economic activity, and price of other modes of transportation. Predicting demand is a challenge for several reasons but it is possible to write the demand function for air transportation in the following general form :

Demand = $f(f_i, Y, f_o, A, R, t, E)$ where:

Y: Household income.

R: Changes in industry regulations.

A: Availability of other modes of transportation.

f_i: Airfare.

f_o: Fares offered by other airlines.

t: Time of the trip.

E: Change in level of economic activity.

Sudden fare drops such as fares offered by airlines filing for bankruptcy, or airlines recovering from a strike[20]. These reasons form the backbone of the process of forecasting demand. Airlines deal with demand regardless whether the trend will continue or not . Trend is the demand behavior over a period of time (Time-Series).

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[20] American Airlines, pilots' attempt to strike in February 1997.

Variables Affecting Revenues

Flight schedule

Airlines publish flight schedules based on the existing passenger load data. Observing a satisfactory demand is the principal motivation for establishing any flight schedule. The next steps would be :

- Determining the departure time, and
- Determining the frequency.

Schedule changes stem from changes in :

- Demand
- Competition
- Cost structure
- Revenue per seat/mile
- Airports regulations, and
- Market priority.

Price, time, and passenger's elasticity

Passengers can be classified into two categories in accordance with their elasticity. The first category is price sensitive where passengers are less flexible with changes in the air fare. Price elasticity can be defined as the change in demand due to price change :

Price Elasticity = *Percentage change in loading / Percentage change in fare*[21]

[21] Nikulainen, Mikko, "A Simple Mathematical Model to Define Demand for Schedule Planning", AGIFORS, 32nd Annual Symposium, Budapest, Hungary, October 4, 1993

or: $$Epd = \frac{ds}{df} * \frac{f}{s}$$

The second category is time sensitive where passengers are less flexible with changes in departure or arrival time. Time elasticity is defined as the change in demand due to a departure or arrival time change:

Time Elasticity[22] *= Percentage change in demand / percentage change in time*

Network design

Financial capabilities determine the frame of the airline's network. Being a newcomer with low cost structure, ValueJet has the ability to offer low fares and enter small markets. On the other hand, high cost carriers, like USAir finds it hard to expand into small markets due to the airline's high cost of structure. Whether the reason behind considering a new market is demand or competition, only the capability of paying the route's bill determines the reach of any airline.

Aircraft allocation

Having a variety of aircraft size, sizing aircraft to market is important. In other words, maximize Return On Assets (ROA). In 1994, USAir started evaluating and reengineering the relationship between aircraft and existing market. On of its proposals was to switch B767s with B757s on a certain flight, a move which saved them more than $1.23 million in 76 days.

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[22] Price and time elasticity has an absolute value between zero and infinity.

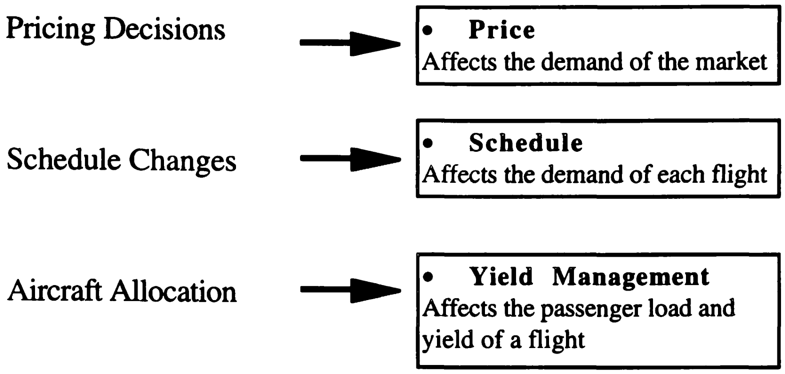

Figure 4. Three Components in Yield Management Affect Demand.[21]

Thus in order to fulfill future demand, airlines should be careful in pricing decisions, schedule changes, aircraft allocation, and network design.

Mathematical Demand Approach

Although demand is unpredictable, many attempts tried to approach it mathematically. One of those approaches is Nikulainen's formula. With this formula, demand for a flight departing at an arbitrary time-of-a day (Tj) can be expressed as follows :

$$PAX(T_j) = f_i\,(t)\;(2 - e^{[\lambda_1(T_j - T_i)]} - e^{[\lambda_2(T_e - T_j]})$$

Where :

$f_i\,(t)$ = Market demand.

λ_1, λ_2 = Parameters used to define passengers' behavior.

T_i = Time of last departure.

T_e = Time of next departure.

With strong market knowledge, this model can be used to estimate changes' effects on load factor.

Overbooking Flights and "No-Show"[23]

Most airlines overbook their scheduled flights to compensate for "no-shows". Passengers are sometimes left behind or "bumped" as a result. When overbooking occurs, the U. S. Department of Transportation requires airlines to ask passengers who aren't in a hurry to give up their seats voluntarily, in exchange for compensation.

Voluntary bumping

Passengers' groups include some people with urgent travel needs and others who may be more concerned about the cost of their tickets than about getting to their destination on time. In accordance with the rules, airlines seek out people who are willing to give up their seats for some compensation before bumping anyone involuntarily. Airline employees will look for volunteers when it appears that the flight has been oversold. If you're not in a rush to arrive at your next destination, you can give your reservation back to the airline in exchange for compensation and a later flight. The Department of Transportation has not decided how much the airlines have to compensate volunteers with. Airlines may negotiate with their passengers for a mutually acceptable amount of money-or may be a free trip or other benefits. If the airline offers you a free ticket, ask about restrictions. How long is the ticket good for? Is it "blacked out" during holiday

[23] U.S. Department of Transportation. ISBN 0-16-045193-0 September, 1994. pgs 15-16

periods when you might want to use it? Most importantly, can you make a reservation, and if so, how far before departure are you permitted to make it?

Involuntary bumping

Airlines should give all passengers who are bumped involuntarily a written statement describing their rights and explaining how the carrier decides who gets on an oversold flight and who doesn't. Those travelers who don't get to fly are frequently entitled to an on-the-spot payment of denied boarding compensation. The amount depends on the price of their ticket and the length of the delay.

In order to reduce the risk of being bumped, get to the airport early. On oversold flights the last passengers to check in are usually the first to be bumped, even if they have met the check-in deadline. Allow extra time: assume that the airport access road is backed up, the parking lot is full, and there is a long line at the check-in counter. However, if you arrive so early that your airline has another flight to your destination leaving before the one that you are booked on, either switch to the earlier flight or don't check your bag until after the first flight leaves. If you check your bag early, it might get put on the earlier flight and remain unattended at your destination airport for hours.

Airlines may offer free future flights in place of a check for denied boarding compensation. However, involuntarily bumped passengers have the right to insist on a check if that is their preference. However, if being bumped costs you more money than the airline will pay you at the airport, you can try to negotiate a higher settlement with their complaint department with 30 days from the date on the check to decide if you want to accept the amount of the check. Passengers are always free to decline the check and take the airline to court to try to obtain more compensation. Finally, don't be a “no-show”. If you are holding confirmed reservations you don't plan to use, notify the airline. If you don't, they will cancel all onward or return reservations on your trip.

Chapter Three

AIRLINE INDUSTRY AND A FUTURE OUTLOOK[24]

In 1997, Federal Aviation Administration and U.S. Department of Transportation released a new version of aviation forecast studies. Our interest, in this thesis, will be around economy, demand, aircraft capacity, load factor, and yields.

Economic Environment

The U.S. economy is witnessing its longest period of expansion since World War II. But, in the last two quarters of 1996, consumer expenditures and exports slipped behind. Inflation rate, measured by the consumer price index, rose at 2.8 percent from the previous year. In 1996, unemployment rates fell from 5.6 to 5.2 percent. Gross Domestic Product (GDP) is forecasted to keep growing at 2.2 percent for the next 10 years. Forecasts for the next 10 years show favorable economic conditions: low interest rates, increasing rate of returns along with accelerating technology. Aviation economic forecasts are also favorable, but with a number of uncertainties that may slow or limit such a growth. Some of those uncertainties include corporate downsizing, automation, elimination of middle management, which will affect the number of business travels. Other factors expected to affect aviation economy include personal bankruptcies which means less traveling expenditures, and middle-class income stagnation along with inequality in income distribution.

[24] Federal Aviation Administration, March 1997.

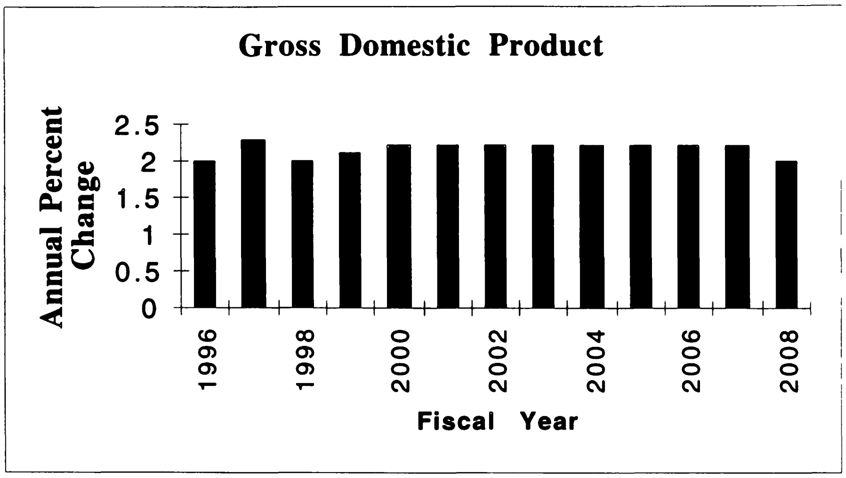

Figure 5. Gross Domestic Product[24]

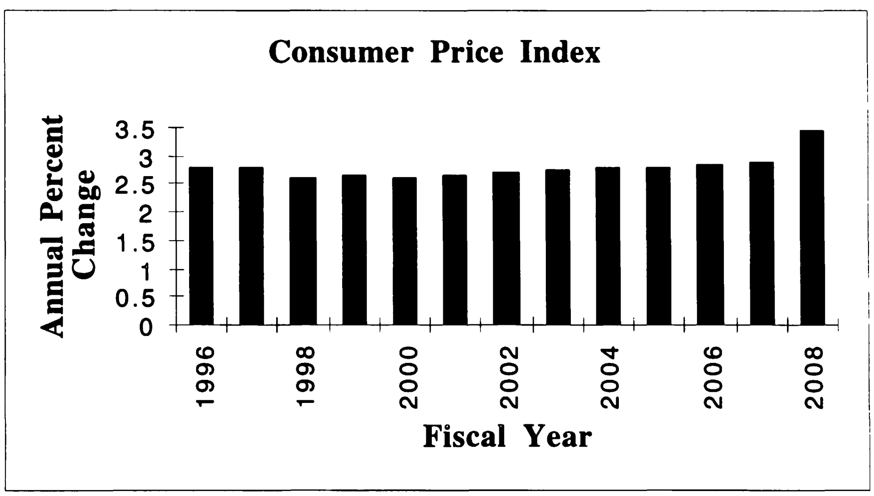

Figure 6. Consumer Price Index[24]

Commercial Air Carriers

Demand for transportation

Passenger enplanements rose by 5.3 percent in 1996. A strong traffic growth is expected in 1997 and 1998. After 1998, slower growth is expected through 2008. The overall annual growth average for the next 10 years is expected to be around 4.1 percent.

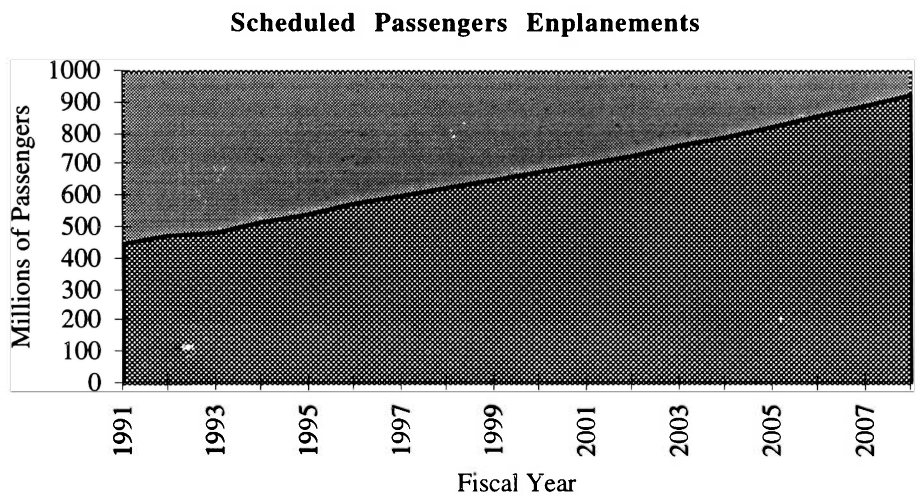

Figure 7. Scheduled Passenger Enplanements[24]

Average seats per aircraft

Between 1983 and 1992, the average seats per aircraft remained at 152 seats. From 1993 through 1996, the domestic average fell to 140 seats due to the large increase in utilization rates of small jets like the B737 and MD88. It is forecasted that for the next 10 years, this average will grow at 2 seats/aircraft/year.

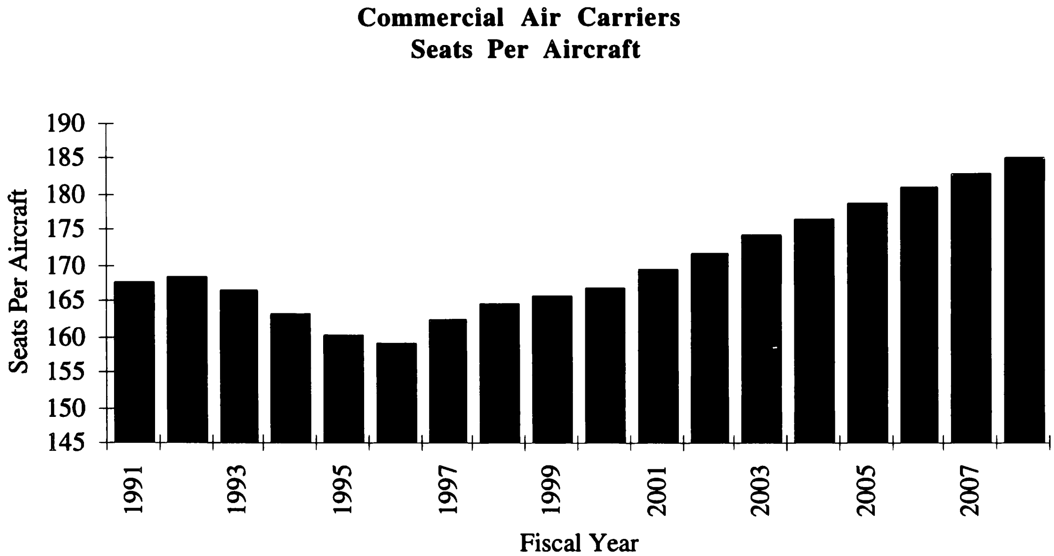

Figure 8. Commercial Air Carriers, Seats Per Aircraft[24]

Passenger load factors

In 1996, load factors increased by 2.3 percent from 1995. On the other hand, passenger capacity increased by 3.1 percent. It is expected that the load factor will hit 68.5 percent by 1997. Capacity will increase at a slower rate than traffic. Forecasts show a 0.5 percent drop by the year 2000, and for eight years beyond.

Passenger Load Factor

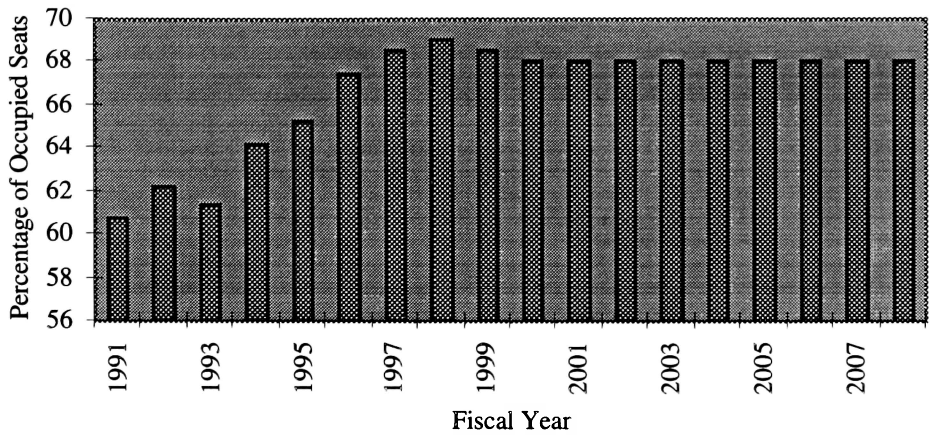

Figure 9. Passenger Load Factor[24]

Table 1.

Carriers Load Factor for 1996

Airline	Load Factor (%)
Delta	66.16
Southwest	64.5
Continental	74.3
American	74
Trans World	62.8
America West	67.6
Western Pacific	60.1
Mesa	56.5

Passenger yields

After 1978, real yield[25] dropped sharply due to the drop in fares. Yield continued to fall till it hit a drop of 13.08 cents per mile. In the 1990s, this drop was a result of the introduction of low-cost carriers like ValueJet. Strong competition is expected to result in a continuous yield decline for the next 10 years.

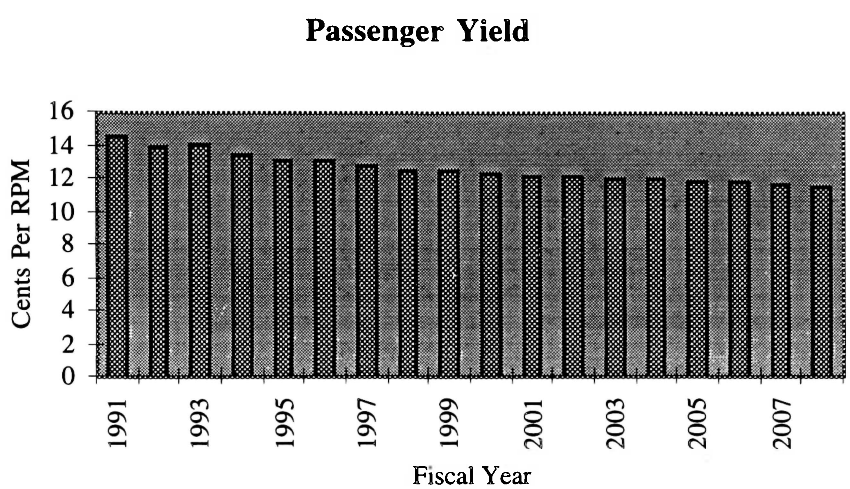

Figure 10. Passenger Yield[24]

[25] Yield is the dollar amount(mostly in cents) paid by passenger to fly him or her one mile.

Chapter Four

SIMULATION DESIGN AND PROCEDURES

Introduction to Linear Programming

Linear Programming (LP) is a method for solving optimization problems. Optimization can be either maximizing or minimizing. A simplex method was developed by George Dantzig in 1947 to solve linear programming problems. LP is used in different industries. "In a survey of 500 firms, 85% of those responding said they had used linear programming"[26].

Linear Programming consists of three parts :

1. Objective Function.
2. Constraints : to restrict values used by the linear function variables.
3. Sign restrictions.

Having the three parts ready, LP searches for an optimal solution. Optimal solutions may not use all of the available inventory. Any left over inventory is called "slack".

Linear Programming As a Computer Package

The need for solving complicated linear functions drove Linus Scharge[27] in 1986 to develop a computer program called LINDO (Linear INteractive and Discrete

[26] Winston, W. *Operations Research*. Belmont, CA. : Wadsworth Publishing Company, 1994.

[27] Scharge, L. LINDO : An Optimization Modeling System. San Francisco, Ca. The Scientific Press, 1991.

Optimizer) which can solve linear, integer, and quadratic programming problems in a short time.

Simulation/Research Outline

Seat inventory techniques used by airlines are often employed to operate on flight leg bases rather than origin-destination cases. Although leg-based models are consistent with current booking systems, they are outdated since needs moved to maximize the entire network revenues (Origin-Destination). Linear Programming can be utilized to solve entire network revenues under certain conditions :

- Demand, as input, will be considered deterministic. Market experience supported with mathematical forecasting techniques can get demand represented in numbers.
- Seats between fares' classes will be nested in accordance with their shadow price, and
- Origin-destination techniques will determine legs booking limits.

As a measure of the importance of LINDO in solving optimization problems, the author decided to utilize it for the problem of maximizing revenues from an airline's seat inventory. In order to provide LINDO with the required inputs, the author chose one of USAir's main hubs : Charlotte Douglas International Airport (CLT) in North Carolina. Simulation studied flights between (CLT) and four other airports :

- Washington National Airport (DCA)
- Los Angeles International Airport (LAX)

- Miami International Airport (MIA), and
- Frankfurt International Airport (FRA) in Germany.

Simulation inputs are listed in Table (2).

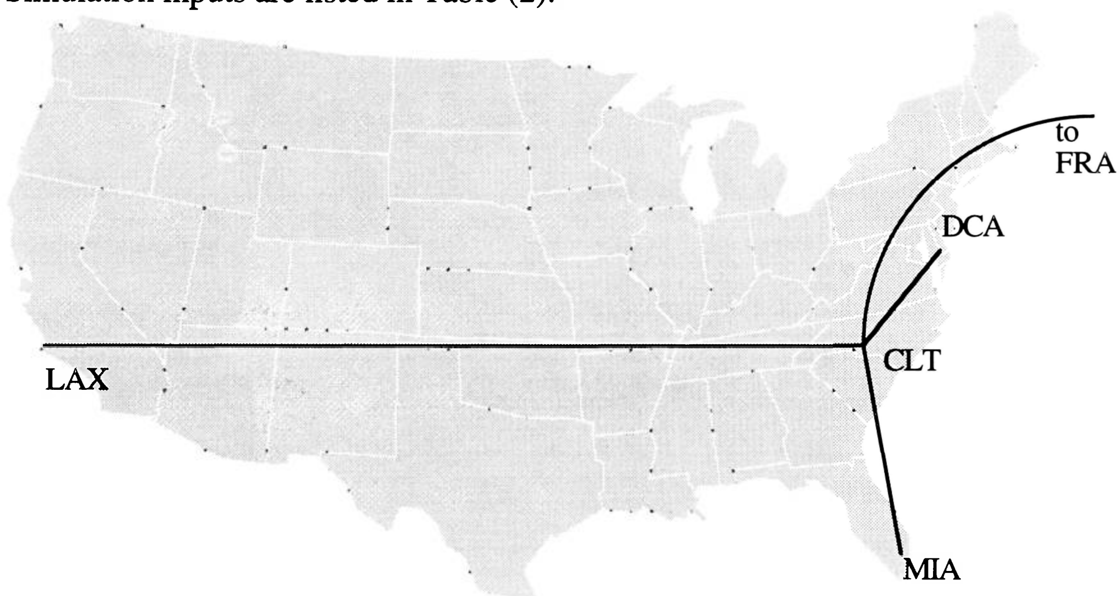

Figure 11. Simulation's Network Model.

After simulation assigns seats, booking limits can be determined depending on inventory's slack and shadow price.

Industry's current booking systems behave as shown in Figure (12) below.

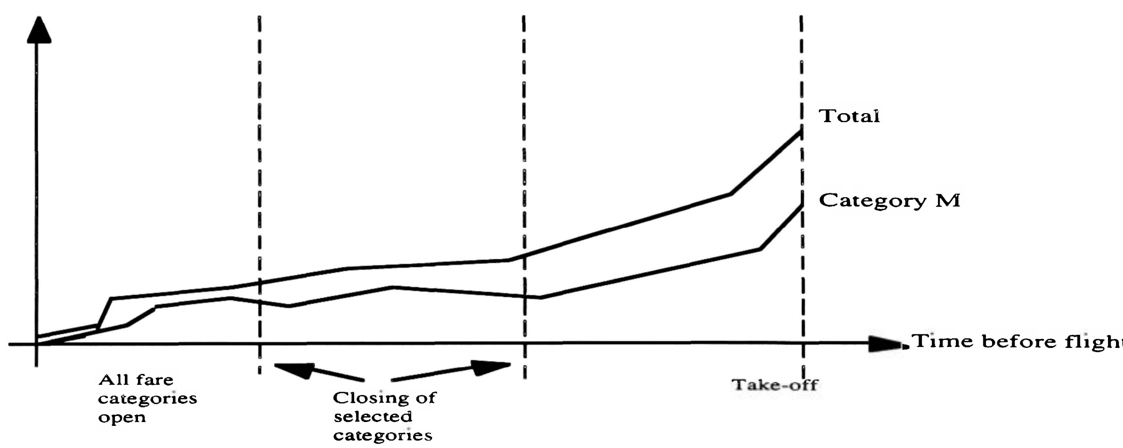

Figure 12. Booking Process[21]

Simulation Inputs

Table 2.

Network Design Data.

Aircraft	Boeing 767, 210 Seats.
Fare Classes Seats	140 Coach, 40 Bus., 30 First Class.
Simulation's Seats	140 Coach.
Simulation Fare Classes	Y, B, M, Q
Number of Destinations	5
Possible City Pairs	5 x 2 = 10
Directional Itineraries	10 x 2 = 20
Total Network Fare Classes	20 x 4 = 80

Therefore there are four flights in and out CLT Airport :

DCA-CLT

LAX-CLT

FRA-CLT

MIA-CLT

CLT-DCA

CLT-LAX

CLT-FRA, and

CLT-MIA.

The following tables illustrates fare amount, travel demand, and forecast demand variation from actual demand on each flight leg.

Table 3

Data for Flights Out-Bounding from DCA.

FLIGHT LEG		Y	B	M	Q
			FARE CLASSES		
DCA-MIA		$812	$556	$422	$309
	Demand	12	9	18	22
	S. Dev.	3	2	4	7
DCA-LAX		$841	$576	$437	$321
	Demand	10	8	16	18
	S. Dev.	2	1	3	4
DCA-FRA		$1,191	$816	$619	$454
	Demand	8	12	14	18
	S. Dev.	1	3	4	5
DCA-CLT		$446	$306	$232	$170
	Demand	6	9	14	23
	S. Dev.	1	1	3	5

Table 4.

Data for Flights Out-Bounding from MIA.

FLIGHT LEG		Y	B	M	Q
			FARE CLASSES		
MIA-DCA		$782	$536	$407	$298
	Demand	7	11	14	20
	S. Dev.	1	2	3	5
MIA-LAX		$940	$644	$489	$358
	Demand	10	6	8	28
	S. Dev.	2	1	1	6
MIA-FRA		$1,152	$789	$599	$439
	Demand	8	6	18	20
	S. Dev.	1	1	4	5
MIA-CLT		$709	$486	$369	$270
	Demand	8	6	16	22
	S. Dev.	1	1	2	5

Table 5.

Data for Flights Out-Bounding from LAX.

FLIGHT LEG		Y	B	M	Q
		FARE CLASSES			
LAX-DCA		$811	$556	$422	$309
	Demand	6	9	15	22
	S. Dev.	1	2	4	5
LAX-FRA		$1,321	$905	$687	$503
	Demand	11	6	14	21
	S. Dev.	2	1	3	5
LAX-CLT		$748	$512	$389	$285
	Demand	8	10	15	19
	S. Dev.	1	2	4	5
LAX-MIA		$961	$658	$500	$366
	Demand	9	6	16	21
	S. Dev.	2	1	3	4

Table 6.

Data for Flights Out-Bounding from FRA.

FLIGHT LEG		Y	B	M	Q
		FARE CLASSES			
FRA-DCA		$1,087	$745	$565	$414
	Demand	9	7	17	19
	S. Dev.	2	1	4	5
FRA-MIA		$1,017	$697	$529	$412
	Demand	11	8	14	19
	S. Dev.	3	2	2	4
FRA-LAX		$1,214	$832	$631	$463
	Demand	7	11	16	18
	S. Dev.	1	2	4	4
FRA-CLT		$714	$489	$371	$301
	Demand	8	11	14	19
	S. Dev.	1	2	4	3

Table7.

Data for Flights Out-Bounding from CLT.

FLIGHT LEG		*FARE CLASSES* Y	B	M	Q
CLT-DCA		$410	$281	$213	$156
	Demand	9	6	14	23
	S. Dev.	1	1	3	5
CLT-MIA		$720	$493	$374	$274
	Demand	10	7	16	19
	S. Dev.	2	1	3	4
CLT-LAX		$720	$493	$374	$274
	Demand	7	9	16	20
	S. Dev.	1	2	4	5
CLT-FRA		$763	$523	$411	$350
	Demand	8	11	14	19
	S. Dev.	1	2	4	5

Formulating The Model

The process of formulating our model consists of the following :

Define variables

Variables are the allocated seats in every fare class. Variables for modeling purposes can be expressed as in the following example :

782MDY

where:

M	=	Miami International Airport, as flight's origin
D	=	Washington National, as flight's destination
Y	=	Fare class
782	=	Coefficient, fare amount for a flight between Miami and Washington National, class (Y).

Objective function

The object of this simulation is to maximize the network's revenues. Therefore, each flight leg will contribute with its yield as follows :

$$\textit{Network General Revenues (Z)} = \sum_{j=11}^{n} \sum_{i=1}^{m} f_{ij} S_{ij}$$

where :

j	=	Flight legs (j=1,...,n).
i	=	Fare classes : Y, B, M, and Q (i=1,....,m)
S_i	=	Seats allocated for fare class "i".

In other words, objective function is defined as revenues from all assigned seats, each multiplied by its fare amount. Therefore, our objective function, as an input for LINDO, will be expressed as :

Table 8.

Objective Function

Maximize revenue (Z) = 812 DMY + 556 DMB + 422 DMM + 309 DMQ + 841 DLY + 576 DLB + 437 DLM + 321 DLQ + 1191 DFY + 816 DFB + 619 DFM + 454 DFQ + 446 DCY + 306 DCB + 232 DCM + 170 DCQ + 782 MDY + 536 MDB + 407 MDM + 298 MDQ + 940 MLY + 644 MLB + 489 MLM + 358 MLQ + 1152 MFY + 789 MFB + 599 MFM + 439 MFQ + 709 MCY + 486 MCB + 369 MCM + 270 MCQ + 961 LMY + 658 LMB + 500 LMM + 366 LMQ + 811 LDY + 556 LDB + 422 LDM + 309 LDQ + 1321 LFY + 905 LFB + 687 LFM + 503 LFQ + 748 LCY + 512 LCB + 389 LCM + 285 LCQ + 1087 FDY + 745 FDB + 565 FDM + 414 FDQ + 1017 FMY + 697 FMB + 529 FMM + 412 FMQ + 1214 FLY + 832 FLB + 631 FLM + 463 FLQ + 714 FCY + 489 FCB + 371 FCM + 301 FCQ + 410 CDY + 281 CDB + 213 CDM + 156 CDQ + 720 CMY + 493 CMB + 374 CMM + 274 CMQ + 720 CLY + 493 CLB + 374 CLM + 274 CLQ + 763 CFY + 523 CFB + 411 CFM + 350 CFQ

Define constraints

There are two model constraints : aircraft capacity and demand for a fare class. Aircraft capacity for the four assigned classes (Y, B, M, Q) will be 140 seats maximum. The other 70 remaining[28] seats are already been allocated for Business and First classes and will not be considered in this simulation. Therefore, total seats allocated for all fare classes will not exceed the aircraft capcity :

$$\sum_{i}^{m} S_i \leq Capacity \qquad \text{For all i's.}$$

$$S_i \leq C_i$$

[28] BOEING 767 with 210 total seats.

Coach class capacity constraints will show as :

Table 9.

Simulation's Constraints.

SUBJECT TO

2) MDY + MDB + MDM + MDQ + LDY + LDB + LDM + LDQ + FDY + FDB + FDM + FDQ + CDY + CDB + CDM + CDQ <= 140

3) DMY + DMB + DMM + DMQ + LMY + LMB + LMM + LMQ + FMY + FMB +FMM+ FMQ + CMY + CMB + CMM + CMQ <= 140

4) DLY + DLB + DLM + DLQ + MLY + MLB + MLM + MLQ + FLY + FLB + FLM + FLQ + CLY + CLB + CLM + CLQ <= 140

5) DFY + DFB + DFM + DFQ + MFY + MFB + MFM + MFQ + LFY + LFB + LFM + LFQ + CFY + CFB + CFM + CFQ <= 140

6) DMY + DMB + DMM + DMQ + DLY + DLB + DLM + DLQ + DFY + DFB + DFM + DFQ + DCY + DCB + DCM + DCQ <= 140

7) MDY + MDB + MDM + MDQ + MLY + MLB + MLM + MLQ + MFY + MFB + MFM+ MFQ + MCY + MCB + MCM + MCQ <= 140

8) LMY + LMB + LMM + LMQ + LDY + LDB + LDM + LDQ + LFY + LFB + LFM + LFQ + LCY + LCB + LCM + LCQ <= 140

9) FDY + FDB + FDM + FDQ + FMY + FMB + FMM + FMQ + FLY + FLB + FLM + FLQ + FCY + FCB + FCM + FCQ <= 140

In other words, boarding will comply with the following constraint :

$$\sum_{j=1}^{n}\sum_{i=1}^{s}(AB) \leq CAPj$$

Where :

AB	=	Flight leg
i	=	Fare classes
J	=	Network legs
CAPj	=	Coach compartment capacity (140 seats).

The second constraint , which is the class demand, is the airline's forecast of the seasonal demand for particular fare class, and will show as :

Table 10.

Fare Classes' Constraints.

10)	DMY	<=	12
11)	DMB	<=	9
12)	DMM	<=	18
13)	DMQ	<=	22
14)	DLY	<=	10
15)	DLB	<=	8
16)	DLM	<=	16
17)	DLQ	<=	18
18)	DFY	<=	8
19)	DFB	<=	12
20)	DFM	<=	14
21)	DFQ	<=	18
22)	DCY	<=	6
23)	DCB	<=	9
24)	DCM	<=	14
25)	DCQ	<=	23
26)	MDY	<=	7
27)	MDB	<=	11
28)	MDM	<=	14
29)	MDQ	<=	20
30)	MLY	<=	10
31)	MLB	<=	6
32)	MLM	<=	8
33)	MLQ	<=	28
34)	MFY	<=	8
35)	MFB	<=	6
36)	MFM	<=	18
37)	MFQ	<=	20
38)	MCY	<=	8
39)	MCB	<=	6
40)	MCM	<=	16
41)	MCQ	<=	22
42)	LDY	<=	6
43)	LDB	<=	9
44)	LDM	<=	15
45)	LDQ	<=	22
46)	LMY	<=	9
47)	LMB	<=	6
48)	LMM	<=	16
49)	LMQ	<=	21
50)	LFY	<=	11
51)	LFB	<=	6
52)	LFM	<=	14
53)	LFQ	<=	21
54)	LCY	<=	8
55)	LCB	<=	10
56)	LCM	<=	15
57)	LCQ	<=	19
58)	FDY	<=	9
59)	FDB	<=	7
60)	FDM	<=	17
61)	FDQ	<=	19
62)	FMY	<=	11
63)	FMB	<=	8
64)	FMM	<=	14
65)	FMQ	<=	19
66)	FLY	<=	7
67)	FLB	<=	11
68)	FLM	<=	16
69)	FLQ	<=	18
70)	FCY	<=	8
71)	FCB	<=	11
72)	FCM	<=	14
73)	FCQ	<=	19
74)	CDY	<=	9
75)	CDB	<=	6
76)	CDM	<=	14
77)	CDQ	<=	23
78)	CMY	<=	10
79)	CMB	<=	7
80)	CMM	<=	16
81)	CMQ	<=	19
82)	CLY	<=	7
83)	CLB	<=	9
84)	CLM	<=	16
85)	CLQ	<=	20
86)	CFY	<=	8
87)	CFB	<=	11
88)	CFM	<=	14
89)	CFQ	<=	19

END

Having all the previous inputs ready, LINDO will solve the problem even without assigning the variables' signs. The simulation's output and related analyses are found in the next chapter.

Chapter Five

RESULTS AND ANALYSIS

Objective Function Value

LINDO arrived at $411923.00 as an objective function value. This objective function value is the total seats' revenue resulted from the way LINDO allocated those seats. LINDO allocated those seats based on their fare amount. In the following table, "value" is the number of allocated seats for each variable-(fare class). "Reduced Cost" is the dollar amount required from each seat's fare to increase before being considered in the solution. For example, a seat in a DQM category needs a $95 fare increase before having this seat in LINDO's final allocation. If this seat is forced into solution without the $95 increase, then our objective function's value will drop by the same amount ($95). This gives revenue management a chance to rethink the fares' amount.

Table 11.

Simulation's Seats Assignments

VARIABLE	VALUE	REDUCED COST	VARIABLE	VALUE	REDUCED COST
DMY	12	0	LFY	11	0
DMB	9	0	LFB	6	0
DMM	18	0	LFM	14	0
DMQ	0	95	LFQ	0	113
DLY	10	0	LCY	8	0
DLB	8	0	LCB	10	0
DLM	12	0	LCM	15	0
DLQ	0	116	LCQ	19	0
DFY	8	0	FDY	9	0
DFB	12	0	FDB	7	0
DFM	14	0	FDM	17	0
DFQ	0	66	FDQ	0	37
DCY	6	0	FMY	11	0
DCB	9	0	FMB	8	0
DCM	14	0	FMM	2	0
DCQ	8	0	FMQ	0	117
MDY	7	0	FLY	7	0
MDB	11	0	FLB	11	0
MDM	14	0	FLM	16	0
MDQ	0	80	FLQ	0	99
MLY	10	0	FCY	8	0
MLB	6	0	FCB	11	0
MLM	8	0	FCM	14	0
MLQ	0	131	FCQ	19	0
MFY	8	0	CDY	9	0
MFB	6	0	CDB	6	0
MFM	18	0	CDM	14	0
MFQ	0	133	CDQ	17	0
MCY	8	0	CMY	10	0
MCB	6	0	CMB	7	0
MCM	16	0	CMM	16	0
MCQ	22	0	CMQ	19	0
LMY	9	0	CLY	7	0
LMB	6	0	CLB	9	0
LMM	13	0	CLM	16	0
LMQ	0	134	CLQ	20	0
LDY	6	0	CFY	8	0
LDB	9	0	CFB	11	0
LDM	14	0	CFM	14	0
LDQ	0	113	CFQ	10	0

Surplus Seats and Shadow Prices

Surplus or "slack" is the number of seats our solution dropped in each constraint.The following table shows that all flights are fully utilized within 140 seat category. Starting with line 10, slack indicates the surplus in each fare class.

Table 12.

Surplus and Shadow Price

ROW	SLACK OR SURPLUS	DUAL PRICES	ROW	SLACK OR SURPLUS	DUAL PRICES
2	0	156	46	0	461
3	0	234	47	0	158
4	0	267	48	3	0
5	0	350	49	21	0
6	0	170	50	0	705
7	0	222	51	0	289
8	0	266	52	0	71
9	0	295	53	21	0
10	0	408	54	0	482
11	0	152	55	0	246
12	0	18	56	0	123
13	22	0	57	0	19
14	0	404	58	0	636
15	0	139	59	0	294
16	4	0	60	0	114
17	18	0	61	19	0
18	0	671	62	0	488
19	0	296	63	0	168
20	0	99	64	12	0
21	18	0	65	19	0
22	0	276	66	0	652
23	0	136	67	0	270
24	0	62	68	0	69
25	15	0	69	18	0
26	0	404	70	0	419
27	0	158	71	0	194
28	0	29	72	0	76
29	20	0	73	0	6
30	0	451	74	0	254
31	0	155	75	0	125
32	0	0	76	0	57
33	28	0	77	6	0
34	0	580	78	0	486
35	0	217	79	0	259
36	0	27	80	0	140

Table 13.

Surplus and Shadow Price (Continued)

ROW	SLACK OR SURPLUS	DUAL PRICES	ROW	SLACK OR SURPLUS	DUAL PRICES
30	0	451	74	0	254
31	0	155	75	0	125
32	0	0	76	0	57
33	28	0	77	6	0
34	0	580	78	0	486
35	0	217	79	0	259
36	0	27	80	0	140
37	20	0	81	0	40
38	0	487	82	0	453
39	0	264	83	0	226
40	0	147	84	0	107
41	0	48	85	0	7
42	0	389	86	0	413
43	0	134	87	0	173
44	1	0	88	0	61
45	22	0	89	9	0

Shadow price (referred to as "Dual Price") of a certain seat is the amount by which our objective function value will increase if one more seat is added to the right hand side (RHS) of the related constraint. For example, objective function value will improve by $451 if MLY<=10 changed to MLY<=11.

Shadow Price = New objective function value - Old objective function value[26]

Thus, shadow price can be considered as a seats' priority indicator.

Sensitivity Analysis

We are not finished when we find the solution, even if it's optimal, for several reasons :

- Uncertainty about parameters' values.
- Parameters values can change over time, and
- May want to improve on what is currently possible.

For all of these purposes, we need what is called Sensitivity Analysis. Sensitivity Analysis is a tool that indicates how changes in linear programming's parameters affect the optimal solution.

The parameters in our model are :

- Demand which is expressed as fare class capacity.
- Fares, which are changeable due to competition, and
- Total number of aircraft seats.

Also, one of the major issues is the "What If" situation. What If is applied when checking the results of changing variables' values on objective function value. Variables could be fare amount, competition movements, operating costs, and number of allocated seats.

In our model, LINDO provided the following :

Table 14.

Sensetivity Analysis

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJECTIVE COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
DMY	812	INFINITY	408
DMB	556	INFINITY	152
DMM	422	INFINITY	18
DMQ	309	95	INFINITY
DLY	841	INFINITY	404
DLB	576	INFINITY	139
DLM	437	7	27
DLQ	321	116	INFINITY
DFY	1191	INFINITY	671
DFB	816	INFINITY	296
DFM	619	INFINITY	99
DFQ	454	66	INFINITY
DCY	446	INFINITY	276
DCB	306	INFINITY	136
DCM	232	INFINITY	62
DCQ	170	18	7
MDY	782	INFINITY	404
MDB	536	INFINITY	158
MDM	407	INFINITY	29
MDQ	298	80	INFINITY
MLY	940	INFINITY	451
MLB	644	INFINITY	155
MLM	489	27	80
MLQ	358	131	INFINITY
MFY	1152	INFINITY	580
MFB	789	INFINITY	217
MFM	599	INFINITY	27
MFQ	439	133	INFINITY
MCY	709	INFINITY	487
MCB	486	INFINITY	264
MCM	369	INFINITY	147
MCQ	270	INFINITY	48
LMY	961	INFINITY	461
LMB	658	INFINITY	158
LMM	500	18	6
LMQ	366	134	INFINITY
LDY	811	INFINITY	389
LDB	556	INFINITY	134
LDM	422	6	18
LDQ	309	113	INFINITY

Table 15.

Sensitivity Analysis (Continued)

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJECTIVE COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
LFY	1321	INFINITY	705
LFB	905	INFINITY	289
LFM	687	INFINITY	71
LFQ	503	113	INFINITY
LCY	748	INFINITY	482
LCB	512	INFINITY	246
LCM	389	INFINITY	123
LCQ	285	INFINITY	19
FDY	1087	INFINITY	636
FDB	745	INFINITY	294
FDM	565	INFINITY	114
FDQ	414	37	INFINITY
FMY	1017	INFINITY	488
FMB	697	INFINITY	168
FMM	529	6	37
FMQ	412	117	INFINITY
FLY	1214	INFINITY	652
FLB	832	INFINITY	270
FLM	631	INFINITY	69
FLQ	463	99	INFINITY
FCY	714	INFINITY	419
FCB	489	INFINITY	194
FCM	371	INFINITY	76
FCQ	301	INFINITY	6
CDY	410	INFINITY	254
CDB	281	INFINITY	125
CDM	213	INFINITY	57
CDQ	156	18	6
CMY	720	INFINITY	486
CMB	493	INFINITY	259
CMM	374	INFINITY	140
CMQ	274	INFINITY	40
CLY	720	INFINITY	453
CLB	493	INFINITY	226
CLM	374	INFINITY	107
CLQ	274	INFINITY	7
CFY	763	INFINITY	413
CFB	523	INFINITY	173
CFM	411	INFINITY	61
CFQ	350	27	66

Table 16.

Sensitivity Analysis (Continued)

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	140	6*	17
3	140	3	1
4	140	4	12
5	140	9	10
6	140	15	8
7	140	0	4
8	140	1	6
9	140	1	2
10	12	1	3
11	9	1	3
12	18	1	3
13	22	INFINITY	22
14	10	12	4
15	8	12	4
16	16	INFINITY	4
17	18	INFINITY	18
18	8	8	8
19	12	8	9
20	14	8	9
21	18	INFINITY	18
22	6	8	6
23	9	8	9
24	14	8	14
25	23	INFINITY	15
26	7	4	0
27	11	4	0
28	14	4	0
29	20	INFINITY	20
30	10	8	0
31	6	8	0
32	8	INFINITY	0
33	28	INFINITY	28
34	8	4	0
35	6	4	0
36	18	4	0
37	20	INFINITY	20
38	8	4	0
39	6	4	0
40	16	4	0
41	22	4	0
42	6	14	1
43	9	14	1
44	15	INFINITY	1
45	22	INFINITY	22
46	9	13	3
47	6	13	3
48	16	INFINITY	3
49	21	INFINITY	21
50	11	6	1

* Mere recommendations for changing the aircraft or borrowing from virgin business class seats.

Table 17

Sensitivity Analysis (Continued)

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
51	6	6	1
52	14	6	1
53	21	INFINITY	21
54	8	6	1
55	10	6	1
56	15	6	1
57	19	6	1
58	9	2	1
59	7	2	1
60	17	2	1
61	19	INFINITY	19
62	11	2	11
63	8	2	8
64	14	INFINITY	12
65	19	INFINITY	19
66	7	2	1
67	11	2	1
68	16	2	1
69	18	INFINITY	18
70	8	2	1
71	11	2	1
72	14	2	1
73	19	2	1
74	9	17	6
75	6	17	6
76	14	17	6
77	23	INFINITY	6
78	10	1	3
79	7	1	3
80	16	1	3
81	19	1	3
82	7	12	4
83	9	12	4
84	16	12	4
85	20	12	4
86	8	10	8
87	11	10	9
88	14	10	9
89	19	INFINITY	9

Current coefficient represents fare amount. Allowable decrease (increase) is the allowed dollar tolerance fare can utilize without affecting the optimal solution.

For example, on DCA-LAX flight class "M", the $437 fare can be changed as follows without affecting the optimal solution :

Fare - Allowable Decrease =< Modified Fare <= Fare + Allowable Increase

437   27 =< FR <= 437 + 7

410 =< Fare Range <= 444

Such a range is beneficial when working around competition. The second portion of the table indicates the range for changing number of classes allocated seats. For example, class DMY can have the following range without affecting the optimal solution :

12   3 =< DMY <= 12 + 1

9 =< DMY <= 13

Having sensitive analysis and shadow price in hand, one can shape the desired booking limits. For a flight between two or more points, seats can be increased or decreased among classes to satisfy, for example, sudden demand changes. Such changes should be checked with both shadow price and objective function values' sensitivity to these changes. In this way, booking limits can be shaped around competition, operating costs, and demand without affecting our reached goal (objective function value).

Chapter Six

CONCLUSION AND RECOMMENDATIONS

As anticipated and hypothesized, the research indicates that Linear Programming computer package (LINDO) can find out an optimum solution for airlines entire networks' revenues. While most of airlines utilize leg-based methods to allocate seats, LINDO expanded the efforts to include the entire networks' seats. Having demand and competition forecasted, this computer program can provide not only seats allocation plans, but also seating flexibility to deal with the real world environment. The real world environment include live demand, market fare, and rivals' movements. Utilizing this program, or similar ones, guarantees fast and accurate "What If" situations. Thus, the hypothesis that LP's computer version (LINDO) can be used to maximize entire networks' revenues holds good. Studied simulation and its results stand supportive.

Advantages of The Study

There are several advantages for our simulation study :

- Cost-effective study. As in any other kind of simulation, running this computer program is all that was required. Our experiment utilized real world flights with active fares, which makes it realistic and cost effective.
- Research offers an insight into structuring airline revenue problems for the first time ever in LINDO's history.
- Study clarifies the idea and inspires future research encompassing Origin-Destination revenue management problems.

Recommendations

With great confidence in the conducted study and its results, the following recommendations are submitted :

- Small airlines' revenue management departments with insufficient yield management support, are advised to use such a simulation, structured around their airlines' nature, to establish Origin-Destination techniques instead of current Leg-Based ones.
- Airlines are advised to check their route structures based on such a simulation's "What If" scenarios.
- "What If" scenarios can help revenue management assign market fares.
- The Preston Group, in Australia is encouraged to expand their Total Aerospace and Airport Modeller (TAAM) to cover similar study areas.
- Future researchers are encouraged to take advantage of this study to :
    - Study larger networks
    - Associate LINDO with other programs to facilitate daily changes and inputs.

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