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Sensitivity Analysis of Optimum Tutor Staffing Schedule Using Discrete Event Simulation

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by

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SENSITIVITY ANALYSIS OF OPTIMUM TUTOR STAFFING SCHEDULE USING DISCRETE EVENT SIMULATION

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This thesis was prepared under the direction of the candidate's thesis committee chair, Dahai Liu, Ph.D., Department of Human Factors and Systems, and has been approved by the members of the thesis committee. It was submitted to the Department of Human Factors and Systems and has been accepted in partial fulfillment of the requirements for the degree of Master of Science in Human Factors and Systems.

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Abstract

Author: Tara Michelle Allen

Title: A Sensitivity Analysis Using Discrete Event Simulation to Determine an Optimum Tutoring Center Staffing Schedule

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This study utilized the discrete-event simulation (DES) approach to optimize the daily tutor cost for the Unified Tutoring Center, located at the Daytona Beach campus of Embry-Riddle Aeronautical University. The simulation model was built using Arena version 12, and the objective of the study was to determine an optimum tutor staffing schedule to minimize operating costs, while also servicing as many students as possible and with the students experiencing a minimal wait time. Data was collected by videotaping four weeks (or 20 nights) worth of evening tutoring sessions, held Sunday through Thursday between 6-9pm. Two models, the General Study Room and the Physics & Chemistry Lab, were validated by comparing the observed tutor utilization with the model's output utilization. A sensitivity analysis was conducted on both models using constrained optimization, based on average wait time, maximum wait time and maximum tutor utilization. A more practical tutor staffing schedule was found for the General Study Room and the Physics & Chemistry Lab, yielding an overall decrease in the weekly operational cost of tutors by $204.00. These results are discussed and conclusions are given at the end of this paper.

Introduction

The Unified Tutoring Center

The Unified Tutoring Center (UTC) was established in the Fall of 2009, at the Daytona Beach campus of Embry-Riddle Aeronautical University. Several distinct campus tutoring programs were combined into one university wide and centrally located tutoring program. Some of the these tutoring programs include the Athletic Department's Braddock Education Success Team (BEST), First Year Programs, the Naval ROTC program, the Writing Center, the Math department's MATRIX Lab and the Physical Science department's Physics and Chemistry Lab. Figure 1 shows the hierarchical structure of the Unified Tutoring Center.

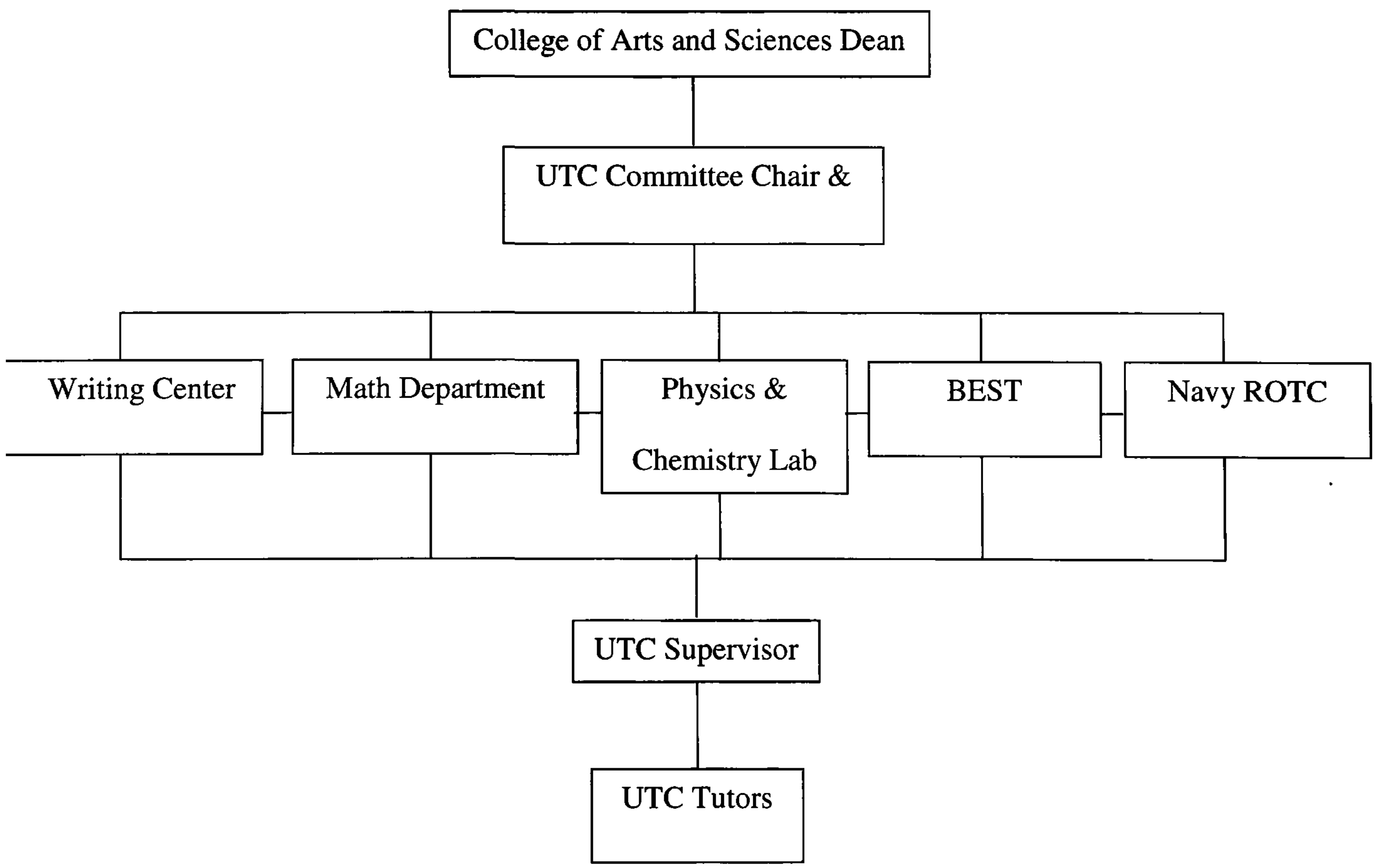


Figure 1: Hierarchical Structure of UTC

These different programs were combined into one unified tutoring center to make more efficient use of funds and resources. Previously, each program conducted their tutoring in different locations, with different hours of operation and each program employed different tutors, which could lead to an inefficient use of resources. In addition to these tutoring programs not efficiently utilizing resources, it was confusing and difficult for students to find the help they needed, especially since all of these tutoring programs were not open to the entire student body. By creating the Unified Tutoring Center, tutoring is now available to all students and is easier for students to attain the help needed without going to multiple locations on campus. Additionally, the hours of operation and subjects offered are reliable and consistent; the UTC is open Sunday through Thursday, from 6-9 pm.

The tutoring subjects offered each evening include Math, Physics, Chemistry, Engineering Science, Aeronautical Science (AS), Business and Computer Programming (CP). Corporate Finance (CF) and Aviation Maintenance Science (AMS) are being offered as an experimental trial for the Spring 2010 semester, to see how many students need help with these courses. These subjects are being offered three nights and two nights a week, respectively. The Unified Tutoring Center utilizes nine different rooms each evening, with each room offering tutoring in a different course. The room break downs are shown in Table 1, with Figure 2 showing the physical layout of these rooms at the Unified Tutoring Center.

Table 1:

UTC Room Allocation

Room	Subject
A-101	MATRIX Lab (General Math)
A-105	Math Supplemental Instruction
A-106	Writing Lab
A-107	Math Supplemental Instruction
A-108	Computer Lab/Math Supplemental Instruction
A-109	Silent Room (Independent Study)
A-111	General Study Room (AS, ES, CS, Business, AMS, Corporate
A-115	Group Study
A-117	Physics and Chemistry Lab

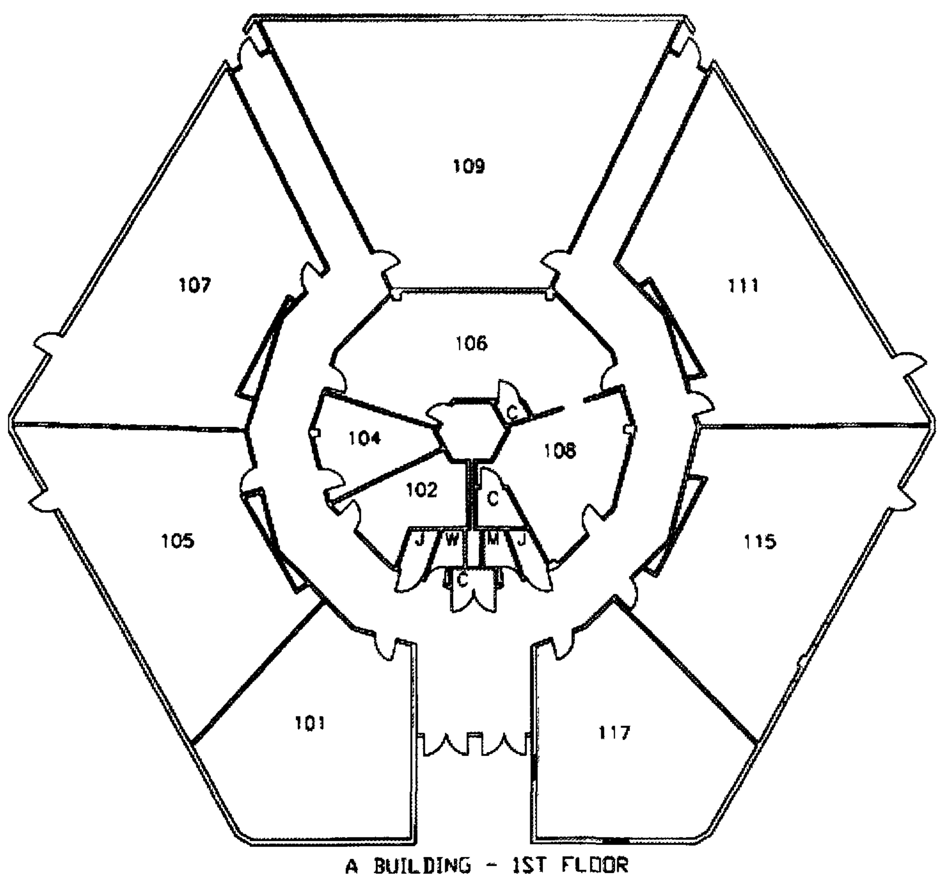


Figure 2: Layout of the Unified Tutoring Center

The utilization of the UTC by the general student population increased in the spring 2010 semester, when compared to the Fall 2009 semester. Given that the UTC is still in its development stages, the advertisement of its resources, benefits and hours of operation are still spreading to the students. As professors and current users of the UTC continue to promote the resources and benefits offered, it is expected that more students will start attending and utilizing the tutors. However, if the number of students utilizing tutors at the UTC continues to grow, the current staffing schedule may become inefficient and obsolete. By studying the UTC process, it is desired to obtain an optimal staffing schedule for the center's current usage and to prepare the UTC for future student demands. Given new demands in the future, the model should be able to determine a new schedule that reflects the usage at that time. It is believed that a computer based simulation model of the UTC is the most effective method to address the tutor staffing schedule, since there is a large degree of uncertainty involved.

This section will first review the concepts of simulation, followed by the types of simulation studies and the advantages and disadvantages of simulation. The elements of simulation studies and a review of previous simulation literature will be covered last.

Simulation

A simulation refers to a broad collection of methods and applications used to mimic the behavior of real systems, usually on a computer with the appropriate software. From a practical perspective, simulation is the process of designing and creating a computerized model of a real system to conduct numerical experiments and give the user a better understanding of the systems behavior, under a given set of conditions (Kelton, Sadowski & Sturrock, 2007).

There are two types of simulation models: physical and logical simulation models (Kelton, Sadowski & Sturrock, 2007). A physical model typically consists of a physical system replica or scaled model of the system. Physical models are primarily used to evaluate the effects of layout changes in the transportation or manufacturing industry and to train employees who work in highly complex environments. Models of complex environments might include a flight simulator for pilot training or a simulated control room of a nuclear power plant to help train its operators (Kelton, Sadowski & Sturrock, 2007). A logical model is a set of approximations and assumptions about the way a system works. Complex systems with many questions to be answered are generally modeled using computer simulation. By using a logical model to experiment and understand the system, the investigation process is usually cheaper, easier and faster. Many questions can be answered through the simple manipulation of the programs inputs. The use of logical modeling allows for mistakes on the computer, where they don't waste time and money rather than for real (physical modeling) where they do (Kelton, Sadowski & Sturrock, 2007).

Computer simulation has been applied for cost analysis and business research since the late 1950's. In the early years it was a very expensive and specialized tool, so it was generally used by only large corporations. Many of the applications used to run the simulations cost between $600-$1000 per hour. The use of computer simulation broadened during the 1970 to late 1980s, when simulations were run to problem solve why catastrophic events occurred. This was also the time when computer simulation began working its way into academic research and business applications. In the late 1980s, some businesses started requesting simulations be run before production began or before the approval was given for major capital investments. Although many managers admit that simulation can add value to their projects and insight to important business decisions, simulation software has not become a standard business tool yet (Kelton, Sadowski & Sturrock, 2007).

Early versions of computer simulation were based on command (Computer Programming) language, which required a long time to learn and master, due to the complexity. With vast improvements in technology over the last several decades though, simulation software companies have been able to utilize graphical user interface technology (GUI) to produce simulation software packages that require little programming knowledge and are more user-friendly. Computer technology advancements and simplified programming software have removed the need for large complex hardware and have reduced the cost associated with creating and running simulations. Software like the Arena simulation software package, developed and produced by Rockwell Automation, is one of the many user-friendly software packages available. Users can now begin creating realistic and useful simulation models with little to no programming experience. The combination of a simulation model's ability to handle very complicated systems and its user-friendliness has made computer simulation a very popular, versatile and powerful tool (Kelton, Sadowski & Sturrock, 2007).

Types of Simulation Studies

There are many different types and classifications of computer simulation, as explained by Kelton, Sadowski and Sturrock (2007) and Banks, Carson and Nelson (1996). Static models are independent of time, whereas dynamic models depict the time varying behavior of a system. There are discrete and continuous models. Discrete models simulate changes at different and specific points in time, such as parts in a manufacturing system arriving and leaving at specific times or workers taking breaks. In a continuous model though, the state of the system can change continuously over time. For instance, the level of a reservoir as water flows in and is let out, and as precipitation and evaporation occur. There are also deterministic and stochastic models. Deterministic models contain no random input, similar to an appointment book, with fixed service times. Stochastic models are not completely random, but at least some inputs are random, such as a bank with randomly arriving customers who require varied service times. In addition to the aforementioned types of simulation studies, there are also terminating and non-terminating simulation models. A terminating simulation has a natural event that ends the simulation run, which specifies the length of each run. A non-terminating or steady-state simulation has no distinct event that ends the simulation, which theoretically means the simulation could run forever. For most realistic applications, dynamic stochastic models are used to mimic the real world dynamics of system behavior. Due to the same reason, the Unified Tutoring Center model uses a discrete-event simulation model with dynamic, stochastic and terminating characteristics.

Advantages and Disadvantages of Simulation

Computer simulation brings many benefits that are unique to system modeling and assessments. Law and Kelton (1991) identified several advantages that simulation and modeling have to offer. Most real-world systems have stochastic elements (random input) that are very difficult and sometimes impossible to be analytically evaluated with mathematical models. Moreover, modeling provides the flexibility and ease of evaluating a system under different operating conditions, to predict alternative performance measures and/or find a better solution. Simulation also provides better control over experimental conditions, compared to testing a change through physical system changes. Take the UTC as an example. Experimentation with the staffing schedule using the real system would be a waste of resources and could negatively impact student performance if the schedule is not verified. In addition to evaluating a system under different conditions, numerous system designs or layouts can be simulated, with the different results being compared to determine the best design or layout, without disturbing the actual system operations. Another one of the greatest advantages to computer simulation is the ability to study a long period of "simulated time" for a model in a relatively short amount of actual time. This provides a quick assessment, which a real system cannot provide. For example, a 24 hour period of simulated time could be run in a matter of minutes and for dozens of replications. Multiple replications are important for stochastic models with a large sample size and random inputs, because a true range of behavior needs to be seen. Take the Unified Tutoring Center for example, the UTC is only open for three hours a night, five nights a week. Multiple replications help show a true range of behavior for the model, which in turn, provides a more accurate measurement for random behavior of the system.

In addition to the many benefits and advantages that simulation has to offer, there are also some disadvantages to simulation. One of the biggest deterrents for computer simulation is the amount of time and expense involved in creating and developing a functionally realistic and accurate system model (Banks, Carson & Nelson, 1996; Law & Kelton, 2000). If management elects to skimp on resources or reduce the level of detail for the model, the end result may be a model that is insufficient for the task or analysis.

A disadvantage of modeling and simulating complex real-world systems, is that stochastic simulation only produces estimates of how the system will perform, not guaranteed or factual information (Law & Kelton, 2000). Furthermore, because stochastic models contain random input, it means that the output data is typically random in one way or another (Kelton, Sadowski & Sturrock, 2007; Law & Kelton, 2000). Another issue with stochastic models is the number of replications placed on the system model. If a stochastic model (with random input), is only run one time before it is abandoned, it would be the same as administering a random physical experiment once, the output will likely be different (Kelton, Sadowski & Sturrock, 2007). That is why multiple replications are necessary.

Although animation can be included in the simulation model to help "show" how the system works together, too much confidence tends to be placed in animation (Law & Kelton, 2000).

An additional disadvantage is the difficulty in determining the initialization bias and warm-up period for a simulation run, as every project is different (Kelton, Sadowski & Sturrock, 2007). A further disadvantage for simulation is the inability to run a simulation for long periods of time to calm the output. This is inappropriate if the system has operational constraints that allow it to only be open during certain times, which is the case for the UTC (Kelton, Sadowski & Sturrock, 2007).

Elements of Simulation

Banks, Carson and Nelson (1996) defined a system as a group of objects that are joined together in some regular interaction toward the accomplishment of some purpose. An example may be a bank with different kinds of customers and services, like teller windows, automated teller machines, loan desks and safety deposit boxes (Kelton, Sadowski & Sturrock, 2007). Systems are generally studied to measure its performance, improve its operation or to design it if the system doesn't exist yet. According to Gordon, a system is often affected by changes that occur outside the system, called the system environment (Gordon, 1978). When modeling a system, it is important to determine and define the boundary between the system and its environment, which varies depending on the purpose of a study.

Within each system model, there are some key elements that must be considered and included (Banks, Carson & Nelson, 1996). These include entities, attributes, activities, state variables and events. An entity is an object of interest in the system, an attribute is a property of an entity and an activity represents a time period of a specific length. A state variable is the collection of data to describe the system at any point in time and an event is an instantaneous occurrence that may change the state of the system.

Applications of Discrete-Event Simulation on Staffing and Scheduling Models

Many simulation studies have been conducted on a variety of system types, investigating the balance between resource utilization, wait time and operating cost, three characteristics that are of an interest to this study. Simulation studies that try to specifically determine a better staffing or scheduling plan can be found in several domains. While there were no simulation studies found in the tutoring domain or with a system model similar to the Unified Tutoring Center, several simulation studies relating to service centers, hospitals, airports and restaurants were found in the literature, which possess various elements that are similar to this study.

A study conducted by Tateno and Shimizu (2007) involved the development and simulation of maintenance repair work for large products. The researchers concluded that the decision support program helped managers assign work to employees based on their skill level, continued training plan and the repair works' due date, through a more optimized staff assignment schedule. The application of the decision support system can help improve the turnaround time for repair work, thus reducing cost and improving business.

A simulation model was constructed of Northwest Airlines' Terminal A at the Detroit/Wayne County Airport to investigate the best policies for managing aircraft during extreme delay situations (Kontoyiannakis, Serrano, Tse, Lapp & Cohn, 2009). There were three specific policies examined: inbound aircraft remain on the tarmac, commanding an outbound aircraft to increase gate availability and commanding an outbound aircraft after optimal swap time. The results indicated that holding inbound aircraft on the tarmac until a gate is available during an extreme delay is suboptimal. Inbound passengers who were held on the tarmac became unhappy much quicker than those passengers delayed inside the terminal. The swap time policy performed better than the other two policies, although it is rarely practiced in the real world. The swap time policy required that after a specified "swap time," an outbound aircraft would depart from the gate to allow the inbound aircraft to disembark. If the outbound flight was scheduled to depart before the inbound flight and they had not boarded yet, then the outbound flight would return to the gate to pick up passengers after the inbound flight had disembarked. This study shows how effective simulation and scheduling can be applied in another domain.

In another article, a group of researchers expanded on a previous study that investigated the scheduling and allocation of ticketing agents for Air Canada. Chong, Grewal, Loo and Oh (2003) created a decision support system (DSS) in Arena to model a typical Friday for Air Canada at the Calgary International Airport. Investigators collected and analyzed the arrival rate, queue wait time and service time of all passengers on Friday's. Ticket agent staffing schedules were adjusted for the Transborder, East and West check-in counters for Air Canada based on the simulation output. It was also noted that minor changes in flight scheduling resulted in drastic differences in passenger arrival rates and subsequently their queue wait time also.

Cao, Nsakanda and Pressman (2003) examined the passenger check-in system at the Ottawa International Airport. Their study focused on the check-in counter queue structure and the ticketing agent work schedule. The primary objective was to provide an alternate agent schedule to minimize cost while still meeting the demand of varying passenger loads throughout the day. Through the use of linear programming and the Arena simulation software, the authors developed a more scientifically based and optimized ticket agent staffing schedule. Sensitivity analyses were performed on the changes in passenger loads and service rates. The sensitivity analyses showed that the revised schedule still outperformed the original schedule, and was therefore the schedule recommended to the Ottawa International Airport.

Simulation for scheduling purposes can also be applied in the medical domain. Sickinger and Kolisch (2009) conducted a study considering the problem of scheduling a given number of outpatients to a medical service facility with two resources servicing outpatients, inpatients and emergency patients. Each class of patients had specific arrival processes and cost figures, which were used to maximize the total expected reward. The total expected reward was based on the revenue of served patients, the cost of waiting patients and the cost of denied patients.

In another study, a simulation model and an integer linear program were combined and used to help management at hospitals determine adequate staffing for emergency room (ER) operations (Centeno, Giachetti, Linn & Ismail, 2003). Staffing schedules for the ER were determined using scientific management tools, not arbitrarily created. Hancock and Chan reported in 1988 that over 60% of a hospital's operating cost is associated to staffing. Centeno et al. successfully created a user-friendly program based on the Arena and LINGO software packages that aided hospital management with ER staffing.

Bieger et al. (2009) investigated solutions to reduce the heavy workload and mental strain on medics at a Brazilian emergency response call center. With limited funding from the government to operate 24 hours a day, it is difficult for the call center to train new employees and retain their experienced employees. It is imperative that the call center operate as efficiently as possible, while staying within their budget.

A model of the call center was constructed using the Arena simulation software, with call center data from September of 2008 entered into the model. The simulation results indicated two simple solutions to reducing medic workload and mental strain. It was found that an improved scheduling system could drastically reduce mental workload by up to 3.3% while also improving response time. This could be achieved by increasing the number of scheduled medics by one or two during peak hours. It was also discovered that streamlining the task flow for medics could reduce mental workload and increase response time.

A study conducted by Brann and Kulick (2002) showed that computer simulation like Arena can also be used to analyze and determine more efficient restaurant designs. In this study, the Restaurant Modeling Studio (RMS) was simulated using the Arena software. The investigators ran the simulation model using actual point-of-sale data from the restaurant to help ensure the models validity.

The three designs analyzed in this study were first compared at high sale volumes to observe the breakpoint of the designs and see where the layouts start to show differences in performance. Running the simulation with nine employees staffed eliminated one of the three designs because it was clearly behind in its throughput of transactions when compared to the other configurations. Another simulation run with ten employees staffed eliminated an additional design, due to its underperformance in transaction throughput. The conclusion was that one kitchen design had noticeably better potential for producing transaction throughput than the other kitchen designs. This kitchen design was recommended to management, who accepted and implemented the new kitchen design.

Numerous simulation studies have been conducted, with optimized resource scheduling as the objective. Although a simulation study similar to the Unified Tutoring Center was not found, many of the studies listed above, in some way, directly relate to this study: to determine an optimum tutor staffing schedule based on tutor utilization and student wait time. This investigation determines a better staffing schedule for the Unified Tutoring Center, while also adding to the body of scientific knowledge.

Method

In this section, the methods are described, starting with the Arena Simulation Modeling Software, followed by a high level model structure of the Unified Tutoring Center. Model input, decision independent variables, decision dependent variables and the evaluation output performance measures are discussed following the model structure.

Conducting a Simulation Study

Generally speaking, there are seven essential steps that need to be completed to conduct a successful simulation study (Law & McComas, 2001). These seven steps are adopted in this study and are further defined below with a process flow chart, as depicted in Figure 3.

The first step is to formulate the problem. This is often accomplished by conducting a kickoff meeting with the major decision makers to determine the overall objectives of the study, specific questions to be answered by the study, the system configurations that are to be modeled and the time frame for the study. The objective of this study is to determine an optimal tutor staffing schedule, by modeling the Physics & Chemistry Lab and the General Study room through discrete event simulation.

The collection of information/data to construct a conceptual model is the second step. Collecting information on the system layout and operating procedures are essential to constructing a realistic and valid model. It is also important to collect data to specify model parameters and probability distributions. Model assumptions should also be documented in this step. The data for this study will be collected through the reviewing/observation of video recordings for each room under investigation. The student arrival rate, subject percentage, as well as the time spent between a tutor and student was carefully analyzed and then entered into the Arena simulation model.

Determining if the conceptual model is valid is the third step for model development. Performing a walk-through of the conceptual model with the major decision makers is a good way to ensure that major system components are captured and to validate the conceptual model. The conceptual model of the UTC was shared during a committee meeting for approval.

The fourth step is constructing the model based on the understanding of the previous steps. The conceptual model is programmed using a GUI based commercial simulation software or a general purpose programming language. The model for the UTC was programmed using the Arena simulation software version 12, by Rockwell Automation®. Arena offers a powerful and time efficient method to analyze how a system works, along with predicting optimized performance measures.

After the model has been constructed, it must be validated, which is the fifth step for model development. Validation is a two step process, with face validation being first and the model's performance measures being the second step. Face validation can be achieved by sharing the model with the major decision makers (the UTC committee) during a meeting and ensuring that the decision makers agree with the model logic. The second step involves the comparison of the model's performance measures with data collected from the actual system. If the performance measures match and the major decision makers agree with the model logic and the performance output under a statistical confidence, then the model is considered valid.

Once the model is validated, the sixth step is to design, make and analyze simulation experiments. The experimenter determines the models run length, warm up period and the number of replications to be conducted on each system configuration. After these decisions have been made, each system configuration is conducted appropriately. The UTC model utilized a termination condition, rather than a steady-state simulation, due to the fact that the tutoring center strictly runs from 6 – 9pm, Sunday through Thursday, excluding holidays. The model was run on a daily basis with an hourly schedule.

The final step is to document and present the simulation results. The conceptual model, programmed model and simulation results are included in the final document. Animation and a discussion of the model building and validation process are included in the final presentation to increase the credibility of the study. Recommendations and limitations of the study are also discussed.

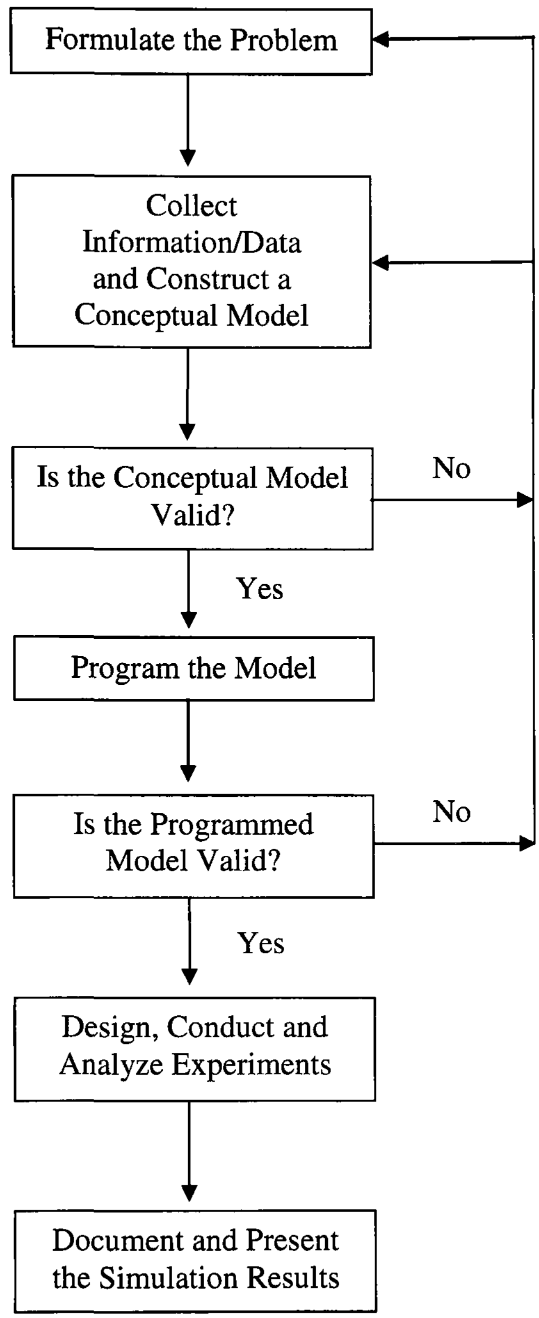


Figure 3: Model Development
(Adopted from Law & McComas, 2001)

UTC Rooms under Investigation

Although there are a total of nine rooms available to students at the Unified Tutoring Center, only two are focused on for this study, the Physics & Chemistry Lab and the General Study Room. These two rooms were chosen because the current staffing schedules in these rooms seem to be inadequate and the student demand varies from day to day. Since the general operating procedures and service provided by the Physics & Chemistry Lab and the Matrix Lab are very similar, only the Physics & Chemistry Lab is examined in this study. The model logic and process for the Physics & Chemistry Lab could be extended for the Matrix Lab, with new input data and the number of tutors staffed during data collection. The General Study Room was selected in addition to the Physics & Chemistry Lab because of its complexity. The General Study Room offers tutoring in more than one subject, which means there are multiple tutors located in this room, with each of them being qualified to tutor only one subject. Carefully reviewing and documenting the arrival and service time distributions for each subject in this room is extremely important. The subjects in this room (Engineering Science, Aeronautical Science, Business, Computer Programming, Corporate Finance and Aviation Maintenance Science) are not core class requirements and are therefore not as popular as Math, Physics and Chemistry, and are consequently grouped together in one room, called the General Study Room.

Arena Simulation Software

The model of the Unified Tutoring Center was developed using the modeling software Arena version 12, developed and distributed by Rockwell Automation®. The Arena simulation software is designed to "help demonstrate, predict, and measure system strategies for effective, efficient and optimized performance," (Rockwell Automation, 2010). This simple but powerful tool is based on the common SIMAN simulation language (Kelton, Sadowski & Sturrock, 2007).

Its user-friendly GUI interface allows for the easy creation of system models by simply dragging and dropping various modules and processes to the work area. The model is completed by adding resources and process times, adding variables, connecting the different modules and running the simulation. The final system model is essentially a process flow diagram, but with details on how different parts of the system interact with one another.

The Arena software package offers several other add on features in addition to its modeling capabilities, which include an Input Analyzer and OptQuest. The Input Analyzer is designed to fit a given set of probability distributions to observed real-world data, for specifying model inputs (Rossetti, 2010; Kelton, Sadowski & Sturrock, 2007). OptQuest is an add-on application based on the Tabu search algorithm that can decide how to change model inputs to optimize a specified output performance measure empirically. The optimized input combination is determined through several simulation runs, being careful not to violate any constraints that may be defined (Rossetti, 2010; Kelton, Sadowski & Sturrock, 2007). The applications using these features are explained in later sections.

Model Assumptions

The following assumptions were made when developing the Unified Tutoring Center model, to help keep the model as simple as possible without losing any significant factors:

Tutor breaks were ignored from the model. Food breaks are not allowed at the Unified Tutoring Center and the amount of time needed for a bathroom break, which rarely happens, is negligible for the purpose of this study.

Tutoring can only happen between the hours of 6-9pm, as these are the hours of the tutoring center. Students may not arrive before 6pm and cannot arrive after 9pm. All tutoring sessions will end at 9pm, as that is when the UTC shuts down. The limited operating hours of the UTC makes a steady-state analysis unpractical, which is why a terminating condition was chosen.

- Priority was given to students in the order in which it is requested. This service priority was set to a first-in first-out (FIFO) rule.

  While there have been occasions where a tutor assisted more than one student at a time, the percentage of time this has occurred is negligible and therefore was not modeled in this investigation. Specifically, tutors were only allowed to help one student at a time in the simulation model of the UTC.

  A tutor can only assist in the one subject area he/she was hired to tutor. If a Physics tutor is helping a student, it is assumed the tutor is tutoring only Physics, not any other subjects. This makes the resources unchangeable between subjects.

- Only students who enter with a backpack, books or school materials and sit down at a desk and study were documented on the data collection forms and included as input data. Students and/or staff who briefly enter the room to speak with a tutor or chat with another student and did not study or seek help were excluded from the input data.
- A student who raises their hand or verbally asks a tutor for help was considered to be "seeking" help from a tutor. This helps to identify the request for help by a student from the video recordings.
- A student who has a book or school material on the desk and is speaking with a tutor was assumed to be "receiving tutoring" in the subject the respective tutor has been hired for. Service time is defined as the total time difference between the start of the tutoring session to when the tutoring session ends. This includes any gap in tutoring of less than one minute for tutoring sessions with more than one question.

- A student is considered to be "waiting" if all tutors are busy when the student initially "seeks" help. The total wait time was computed from the time the student starts waiting until the time the tutor begins the tutoring session.

UTC Simulation Model Structure

Figure 4 illustrates the high level logic of the model used to simulate the Unified Tutoring Center. Students first enter the system and their arrival time is recorded. Upon arrival, each student can either immediately seek help from a tutor or they can start studying on their own, based on their own individual needs. For the small percentage of students who never seek help from a tutor, they will simply leave the system, without any interaction time with the tutor. For those students who utilize the tutors, their tutoring session will immediately begin if the tutor is available, otherwise the student must wait. The student will continually check for an available tutor, at which point the student's total wait time will be recorded and the tutoring session will begin. If a student is being tutored, the student and tutor are held for the duration of the tutoring session, as a tutor can truly help only one student a time. When the tutoring session is complete, the total tutoring time is recorded and the student can continue studying or seek help again.

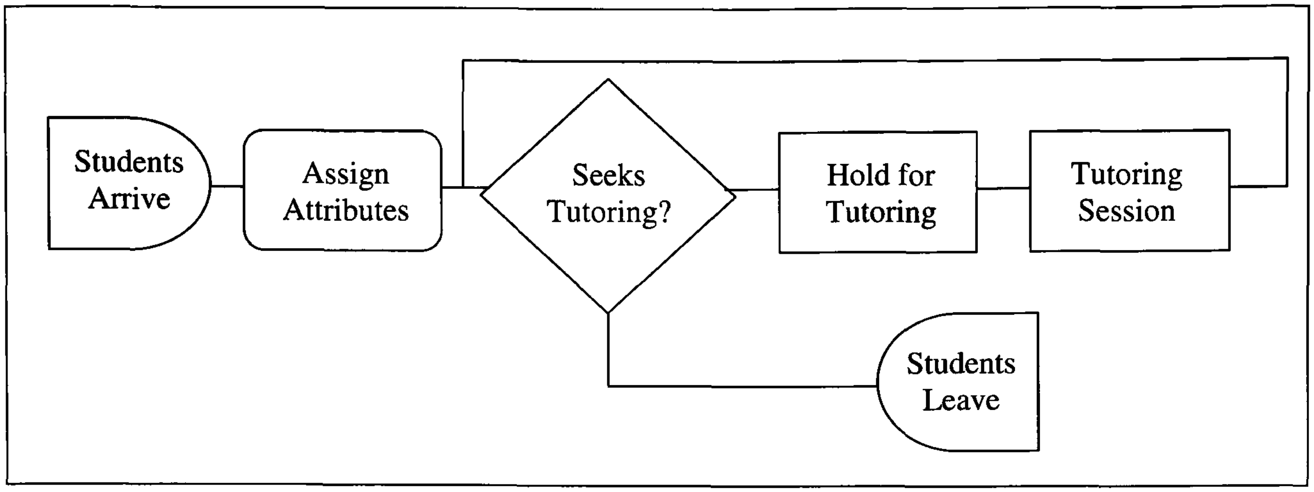


Figure 4: High Level Model Structure

Input Data Analysis

The input data used to build the Unified Tutoring Center model was collected through the observation of video recordings in the Physics & Chemistry Lab and General Study room, over a four week period. The specific weeks of data collection included March 7 to 11, 2010, March 21, 2010 to April 1, 2010 and April 11 to 15, 2010 with only Sunday through Thursday from 6-9pm being recorded. There were a total of three video cameras used throughout the video taping process, including a Sony HandyCam DCR-SR68, a Sony HandyCam DCR-HC96, and a Canon PowerShot S5 IS. Each student's arrival time, wait time, subject tutored, service time and exit time were carefully extracted by reviewing the video recordings and documenting the data on the Data Collection form, shown in Appendix A. Figures 5 & 6 show the layout of the General Study Room and the Physics & Chemistry Lab, respectively. The yellow shading on the Physics & Chemistry Lab layout are areas of the lab that were not filmed or used for data collection purposes. These areas contain computers, where very little tutoring is conducted. Due to the limited amount of tutoring found in these areas and the configuration of the room, these shaded areas are not reflected in the Arena simulation model of the UTC.

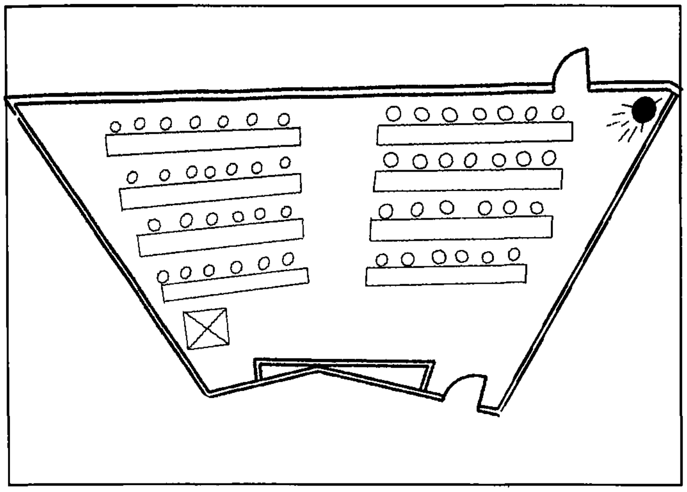


Figure 5: General Study Room Layout
(The red dot indicates the location of the video camera)

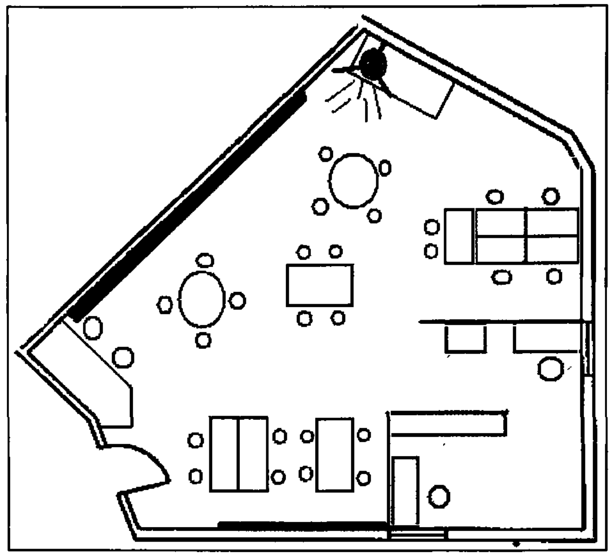


Figure 6: Physics & Chemistry Lab Layout
(The orange dot indicates the location of the video camera)

From the documented data, the independent variable information was extracted and analyzed, which describe the system behavior characteristics of the UTC. These input data include the student demands and service information provided by tutors at the Unified Tutoring Center. A complete list of this information is shown in Table 2.

Table 2:

Independent Variables

Input Data Name	Description	Type/Nature
Student Inter-Arrival Time	The time each student enters the system (room).	Continuous, Random Variable, In Minutes
Subject Tutored	The subject a student seeks help in.	Category, Percentage (%)
Student Service Time	The time spent for one session between a student and tutor.	Continuous, Random Variable, In Minutes
Tutor Hourly Wage	The total cost of staffed tutors per hour.	Deterministic; $8.50 per hour

The decision dependent (or control) variable for this study is the tutor staffing schedule. The number of resources (tutors) scheduled by subject each evening was systematically manipulated to determine an optimal staffing schedule, while meeting the defined needs and constraints (maximum student wait time and tutor utilization threshold). Table 3 provides detailed information on the control variable used in this study. Also, a copy of the survey used to determine and define the constraints used in this study can be found in Appendix B.

Table 3:

Control Variable

Variable Name	Description	Type/Nature
Tutor Resource	The number of tutors staffed each hour, by subject.	Category, Capacity, Schedule

A more detailed representation of the current tutor staffing schedule for the Physics & Chemistry Lab and the General Study room are shown in Tables 4 & 5, respectively.

Table 4:

Current Physics & Chemistry Lab Tutor Schedule

Subject	Sunday	Monday	Tuesday	Wednesday	Thursday
Physics & Chemistry	3	2	3	2	3

Table 5:

Current General Study Room Tutor Schedule

Subject	Sunday	Monday	Tuesday	Wednesday	Thursday
Business	1	1	1	1	1
Corporate Finance	1	0	1	0	1
Engineering Science	3	1	2	1	3
Aeronautical Science	1	1	1	1	1
Computer Programming	1	1	1	1	1
Aviation Maintenance Science	1	0	0	1	0

Before the input data was entered into the simulation model, the data was analyzed to find a distribution that fit the data, since the data were stochastic. The Input Analyzer is specifically designed to fit probability distributions to observed real-world data and measure how well the data fits the estimated parameters. There are two main types of distributions: theoretical and empirical (Kelton, Sadowski & Sturrok, 2004; Law & Kelton, 1991). Theoretical distributions generate samples based on mathematical formulas, giving a stationary probability distribution function, such as the Gaussian distribution. Empirical distributions divide the data into groups and calculate the proportion of values in each group, producing a histogram for the specified data range (Kelton, Sadowski & Sturrok, 2004; Law & Kelton, 1991).

The Input Analyzer in Arena provides the user an option to fit all of the applicable distributions to the data at once. The analyzer calculates the statistics for each distribution and presents the best-fit distribution, or the distribution with the minimum square error value (Altiok & Melamed, 2007; Kelton, Sadowski & Sturrok, 2004).

To critically analyze a sample's best-fit distribution, a goodness-of-fit test was utilized in Input Analyzer. There are primarily two goodness-of-fit tests used, the chi-square and the

Kolmogorov-Smirnov (K-S) tests. These tests are standard statistical hypothesis tests that evaluate if a theoretical distribution is a good fit to the data (Kelton, Sadowski & Sturrok, 2004). The chi-square test is best utilized with large samples of data to generate a smooth histogram, while the K-S test can be conducted with relatively small amounts of data, since it does not require a histogram (Altiok & Melamed, 2007). If the results of the chi-square and K-S tests indicate one or more distributions with a high p - value (0.10 or greater), then a theoretical distribution may be used, with a fair degree of confidence that it is a good representation of the data. However, if the p - values are low, an empirical distribution may better represent the data (Kelton, Sadowski & Sturrok, 2004).

The statistics for calculating the chi-square goodness of fit test are shown below (Howell, 2007). The observed frequencies (Fo) refer to the actual observed data that was recorded and the expected frequencies (Fe) refers to the frequencies you would expect from the experiment, if the null hypothesis were true. The null hypothesis (H_O) refers to the conclusion that there is no association between the rows and columns in a contingency table. A small chi-square value indicates that the expected and observed frequencies are close enough that the null hypothesis could be true. The hypothesis (H_A) refers to the expected correlation of the data, relative to the study's objective (Howell, 2007; Law & Kelton 1991).

$$X^2 = \sum \frac{(\text{Observed Frequencies} - \text{Expected Frequencies})^2}{\text{Expected Frequencies}}$$

$$= \sum \frac{(Fo - Fe)^2}{Fe}$$

The arrival rate and service time data for students utilizing the Physics & Chemistry Lab and the General Study room at the Unified Tutoring Center were analyzed using the goodness-of-fit testing in the Input Analyzer. The theoretical distributions were initially selected for the data and the goodness of fit tests were computed. For data that did not have adequate goodness of fit p - values, the empirical distribution was used. Based on the original observed data, the continuous piecewise-linear empirical distribution was calculated using the following formula (Law & Kelton, 1991).

$$F(x) = \begin{cases} 0 & \text{if } x < X_{(1)} \\ \dfrac{i-1}{n-1} + \dfrac{x - X(i)}{(n-1)(X_{(i+1)} - X_{(i)})} & \text{if } X_{(i)} \le x < X_{(i+1)} \\ & \text{for } i = 1, 2, \ldots\ldots, n\text{-}1 \\ 1 & \text{if } X(n) \le x \end{cases}$$

Where F(x) = cumulative probability, $X(i)$ is the discrete value possible for the random variable, and n is the number of values.

Evaluation Output Performance Measures

The input for a system controls the system's behavior and the output is the observable behavior response (Fishwick, 1995). A system's output is a function of the system state and input data. For the Unified Tutoring Center, there are three evaluation output performance measures. These include average student wait time, the daily cost of tutors and the utilization of tutors, with all of these performance measures being broken down by subject.

Student wait time is defined as the amount of elapsed time from when a student seeks help to when the tutor is free and the tutoring session begins. The average wait time is tallied using the following equation (Kelton, Sadowski & Sturrock, 2007).

$$\frac{\sum_{i=1}^{N} WQi}{N}$$

Where WQ_i is the waiting time for the *i*th entity (student) and N is the total number of entities (students) who finished tutoring sessions.

The daily cost of tutors was calculated by multiplying the number of tutors staffed by the tutor wage ($8.50), which was broken down by subject.

Tutor utilization is defined as the proportion of time a tutor is busy during the simulation run (Kelton, Sadowski & Sturrock, 2007). The equation to determine tutor utilization is shown below, where T is the total simulation time, three hours in this study (Kelton, Sadowski & Sturrock, 2007).

$$B(t) = \begin{cases} 1 \text{ if the tutor is busy at time } t \\ 0 \text{ if the tutor is idle at time } t \end{cases} \qquad Utilization = \frac{\int_0^t B(t)\,dt}{T}$$

Statistical Analysis

Removal of Initialization Bias. Since the Unified Tutoring Center has specific start and stop time periods every day and there are no unfinished customers in the system when closing or opening back up the following day, the UTC model utilized a termination simulation, so there is no need for the removal of initialization bias. The removal of initialization bias is for eliminating the statistics for the warm-up period for steady-state simulation models, thus not necessary for this study.

Validation of the Model. The input data used to build and run the simulation model was from actual observed data at the UTC between March 7 to 11, 2010 and March 21, 2010 to April 8, 2010, with only Sunday through Thursday from 6-9pm being recorded. This provided 20 nights of relevant data for determining an optimized staffing schedule for the UTC. The actual observed tutor utilization was compared against the model's output of tutor utilization. As per the recommendation of Law and Kelton (2000), the Welch's t test was used to compare the actual observed data to the model's output data. The Welch's t test is preferred over the standard t test because the two independent samples have unequal variances (Howell, 2007).

Experimentation. Once the model is validated, experimentation was conducted in order to determine an optimum tutor staffing schedule. A theoretical approach to determining an optimum tutor staffing schedule would utilize a non-linear mathematical programming model illustrated below (Blanchard & Fabrycky, 2006).

Objective Minimum Daily Cost $E = \sum_i \sum_j Cx_{ij}$

Subject to

1. Waiting Time Constraints

$$\frac{\sum_{k=1}^{N} WQ_k}{N} \leq WQ_{average}$$

or

$$\max_{k} WQ_k \leq WQ_{max}$$

2. Utilization Constraints

$$Scheduled\ Utilization_j \leq Utilization\ Threshold$$

Where

$x_{ij} = Tutor\ Capacity\ for\ i^{th} hour, j^{th} subject$

$i = 1, 2, 3\ Hour$

$j = 1, 2, \cdots 5\ Subject$

$N = Number\ of\ students\ finished\ Tutoring$

$C = \$8.50, Hourly\ Wages$

Due to the complexity and non-linearity nature of the system, it would be impossible to obtain an analytical mathematical programming model for the above optimization problem. An empirical solution was applied as an alternative to approximate numerical solutions.

The empirical solution to solving the optimized tutor staffing problem was employed through the use of OptQuest, an add on package to the Arena simulation software, that is produced by OptTek Systems Incorporated. OptQuest methodically searches through the model for potential controls and responses, allowing the investigator to specify what OptQuest should focus on. After the controls and constraints have been specified, OptQuest intelligently manipulates possible scenario combinations in an iterative fashion to converge quickly and reliably on an optimal combination of input control values (Kelton, Sadowski & Sturrock, 2007).

Specifically, OptQuest determines the optimal or near optimal combination of input control values by using the Tabu search method. This search method uses memory structures to remember the combinations that have been explored and marked. Those combinations that fail to meet the specified criterion are marked "tabu" so they are not visited again, and the search can continue looking for possible input control combinations that meet the specified criterion. The Tabu search will continue until some stopping criterion is met (Cvijovic & Klinowski, 1995).

A sensitivity analysis was then performed on the final results from OptQuest to determine the relationship between the cost and the varying constraints. A sensitivity analysis was used because it allows for experimentation with key variables to see how sensitive the recommended results are to changes in the constraints. The simulations were run under various tutor staffing schedules to get a finer picture of this relationship and determine the minimum number of tutors needed.

Results

This section presents the Arena simulation model developed for this study, the input data analysis process and results, followed by the model validation and the optimization results from the experimentation through OptQuest.

Model Development

There were two separate models developed for this study: the General Study Room model and the Physics & Chemistry Lab model. Both models were simulated for a specified run-time length of 180 minutes (three hours) to represent the daily operation of the Unified Tutoring Center (6 – 9pm). Each model was built using two separate sections. The main section, shown in Figures 7 and 8, contains the main model logic and processes. This section takes care of generating the arrival of students each evening based on an arrival schedule, assigning student attributes (subject types) and then routing each student based on their attributes. Some students leave without ever getting help, while other students will go through a continuous cycle of studying and seeking tutoring before the night ends. These students progress through the decide modules to the studying and tutoring sessions and then back to the second decide module.

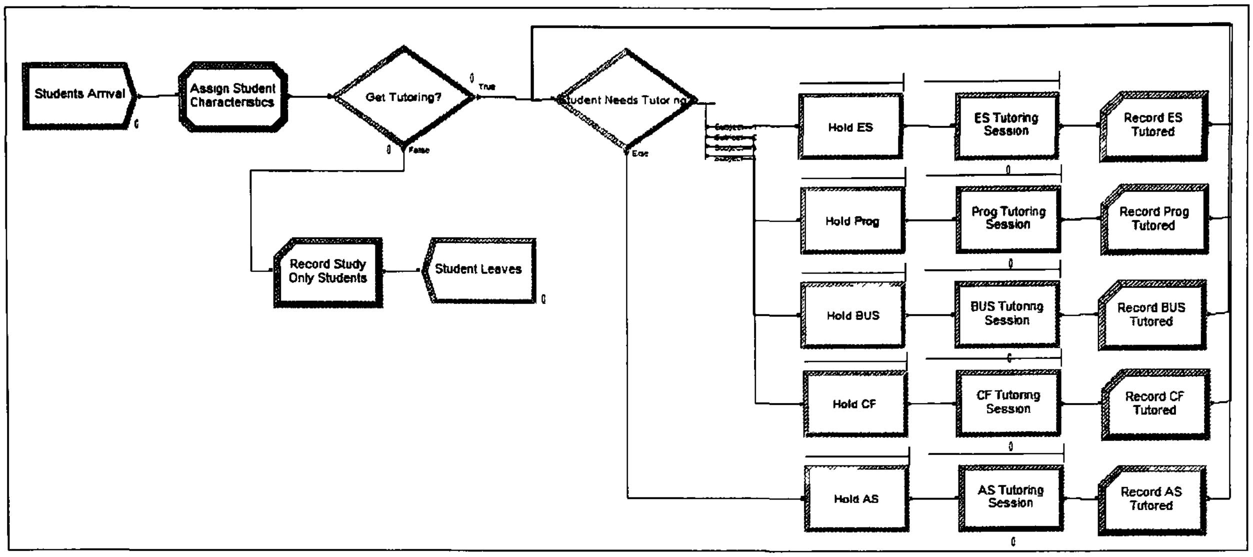


Figure 5: Create, Assign and Route Students through the General Study Room

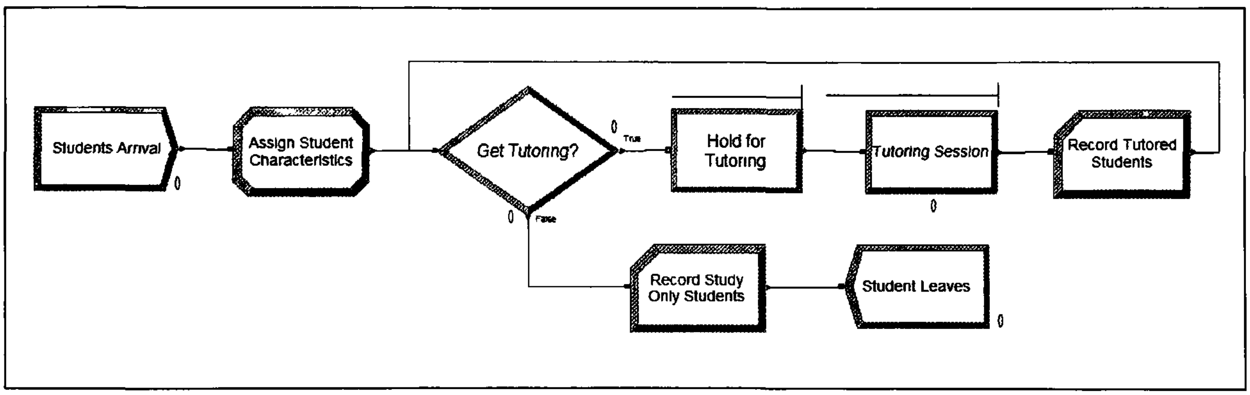


Figure 6: Create, Assign and Route Students through the Physics and Chemistry Lab

The second section contains model logic that creates and signals a student to request tutoring help. This signal is sent to a student in the hold for tutoring block. When this signal is received, one student is released from the hold block and seizes a tutor (resource), at which point that tutoring session will begin, if a tutor is available. This section is shown in Figure 9 for the General Study Room and in Figure 10 for the Physics & Chemistry Lab, respectively.

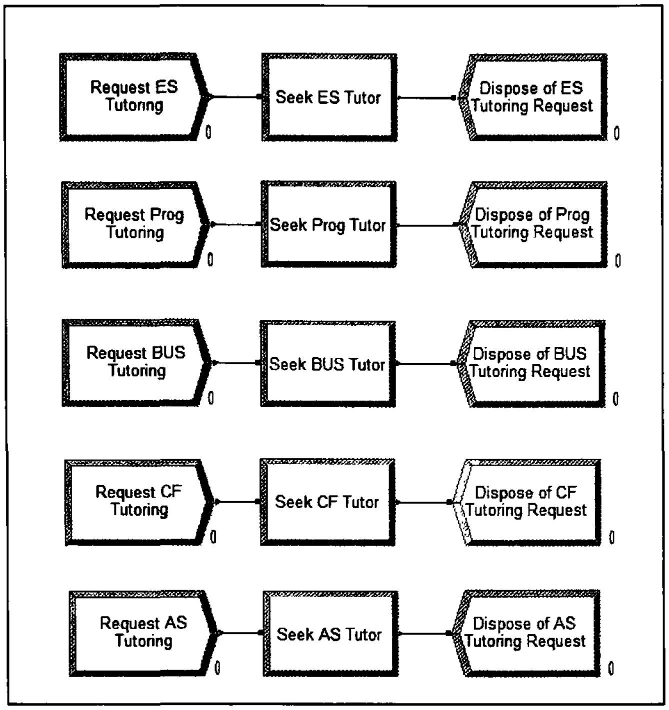
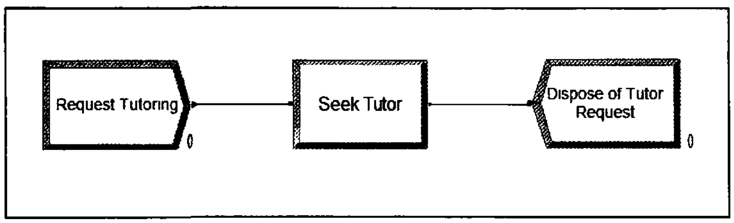


Figure 7: Request Tutoring Signals for the General Study Room



·*Figure 8:* Request for Tutoring Signal for the Physics & Chemistry Lab

The third section of the model was used for validation purposes. This section instructs the Arena software to write the tutor utilization results from the simulation runs to a specified Excel file. Figures 11 & 12 show how Arena was instructed to write these results to an Excel file. How these utilization results were specifically used to validate the model will be discussed later.

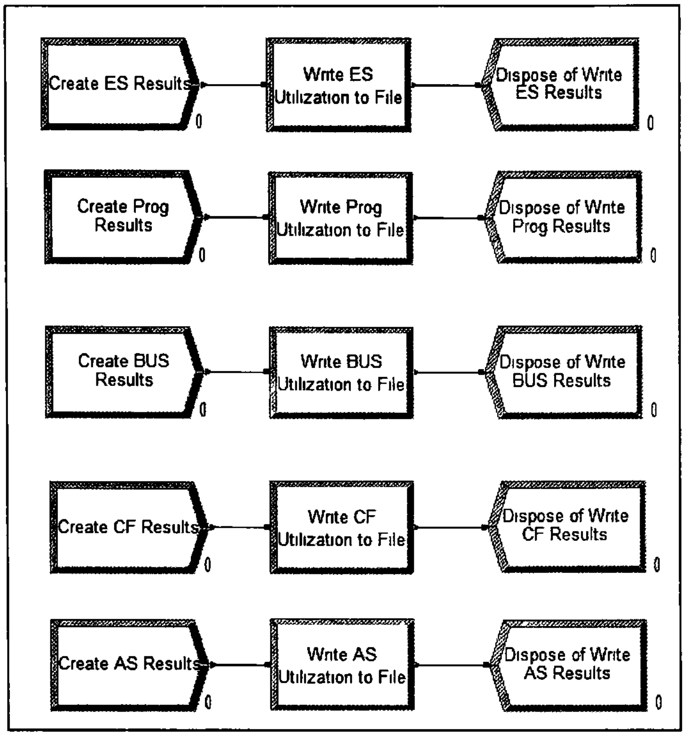


Figure 9: Write Utilization Results for the General Study Room

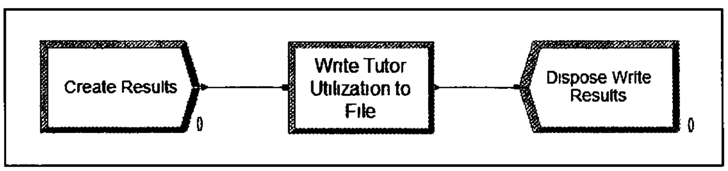


Figure 10: Write Utilization Results for the Physics & Chemistry Lab

Input Data Analysis

Although video cameras captured five weeks worth of data collection, only four weeks of data were used in this investigation. The fourth week of data collection experienced some technical difficulties that resulted in several gaps of data loss. Therefore weeks one (March 7-11), two (March 21-25), three (March 28 – April 1) and five (April 11-15) were used.

For the student arrival time, there were two different student arrival schedules used, one for the General Study Room model and the other for the Physics & Chemistry Lab model. Table

6 and Figure 13 show the aggregated arrival rate used in the simulation model for each half-hour time block for both the General Study Room and the Physics & Chemistry Lab. Please note that the arrival rate for the General Study Room was not broken down by subject.

Table 6:

Student Arrival Rate Schedule for the General Study Room and the Physics & Chemistry Lab (Students Per Hour)

	6:00-6:30	6:30-7:00	7:00-7:30	7:30-8:00	8:00-8:30	8:30-9:00
General Study Room	8.9	3.0	3.3	2.5	1.9	0.6
Physics & Chemistry Lab	9.9	5.5	3.9	2.5	1.5	0.1

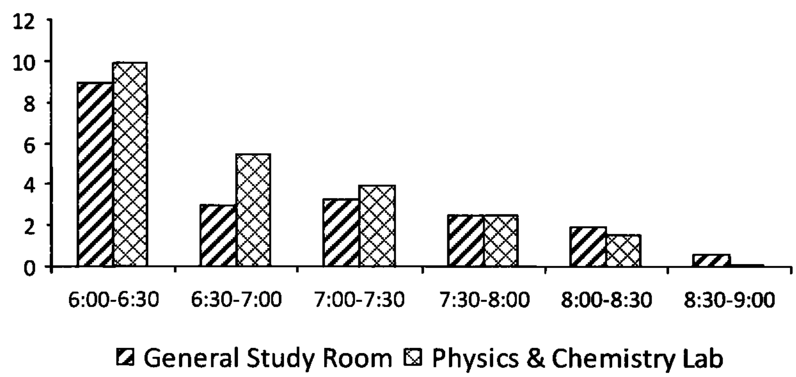


Figure 11: Student Arrival Rate for the General Study Room and the Physics and Chemistry Lab

From Figure 13, it is easily seen that the arrival rate is not stationary over the three hour period, with the peak occurring in the beginning, thus an arrival schedule based on observed data was chosen over an arrival distribution.

The breakdown percentage of students seeking help in each subject or not seeking help in the General Study Room was carefully recorded through observation. Based on actual observed data, the discrete distribution of DISC(0.22,1,0.34,2,0.50,3,0.55,4,0.58,5,1,6) was used in the simulation model to assign subject characteristics to students upon arrival. The discrete distribution adds the total percentage of students tutored for each subject (numerical reference number) in the General Study Room. Forty two percent of students who used the General Study room did not seek any tutoring. A more detailed breakdown of these percentages are shown in Table 7 and in Figure 14.

Table 7:

Percentage of Students Seeking Tutoring by Subject

	Engineering Science	Programming	Business	Corporate Finance	Aeronautical Science	No Tutoring
Percentage	0.22	0.12	0.16	0.05	0.03	0.42
Numerical Reference	1	2	3	4	5	6

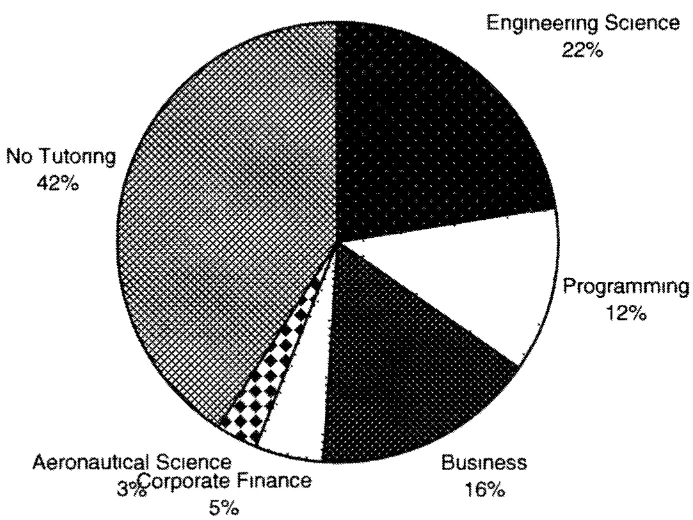


Figure 12: Percentage of Students Seeking Tutoring by Subject

The Physics & Chemistry Lab followed a different percentage distribution of students seeking tutoring versus students studying and then leaving. A discrete distribution of DISC(0.65,1,1,2) was used to assign tutoring characteristics to students upon arrival. Specifically, 65% of students who attended the Physics & Chemistry Lab sought help from tutors before leaving, while the remaining 35% of students simply left the tutoring lab without ever seeking help. The value of numerical reference number two was derived by combining the 65% of students who sought help with the 35% who didn't seek help, to give a 100% representation of the data.

The Input Analyzer was used to perform the goodness-of-fit test to find the best fitting distribution for the inter-arrival rate of tutoring requests (independent variable) and for the tutoring service times (independent variable) for each subject studied at the Unified Tutoring Center.

The inter-arrival request rate for Engineering Science tutoring was found to follow an exponential distribution (EXPO) of 0.5 + EXPO(16.5) with a corresponding Chi-Square goodness-of-fit p-value of 0.18. The service rate for ES followed a lognormal distribution (LOGN) of 0.5 + LOGN(9.79, 17.2) and a high corresponding Chi-Square goodness-of-fit p – value ($p = 0.558$). Both p – values above 0.15 indicate that the theoretical distributions are representative of the data, thus they can be used in the simulation to generate the arrival and service times. The inter-arrival request distribution and the service time distribution are shown in Figures 15 & 16, respectively.

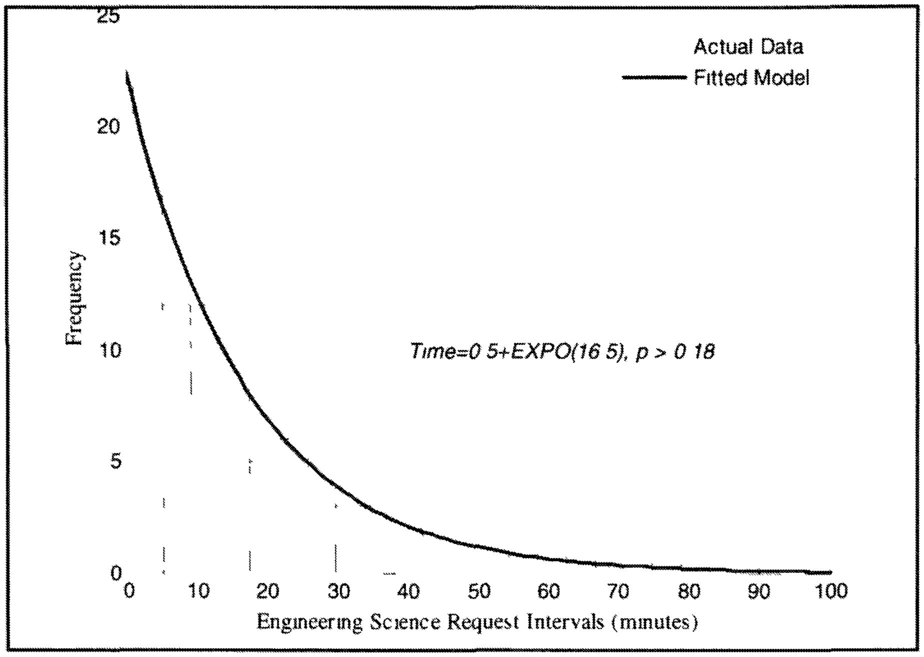


Figure 13: Engineering Science Inter-Arrival Request Distribution

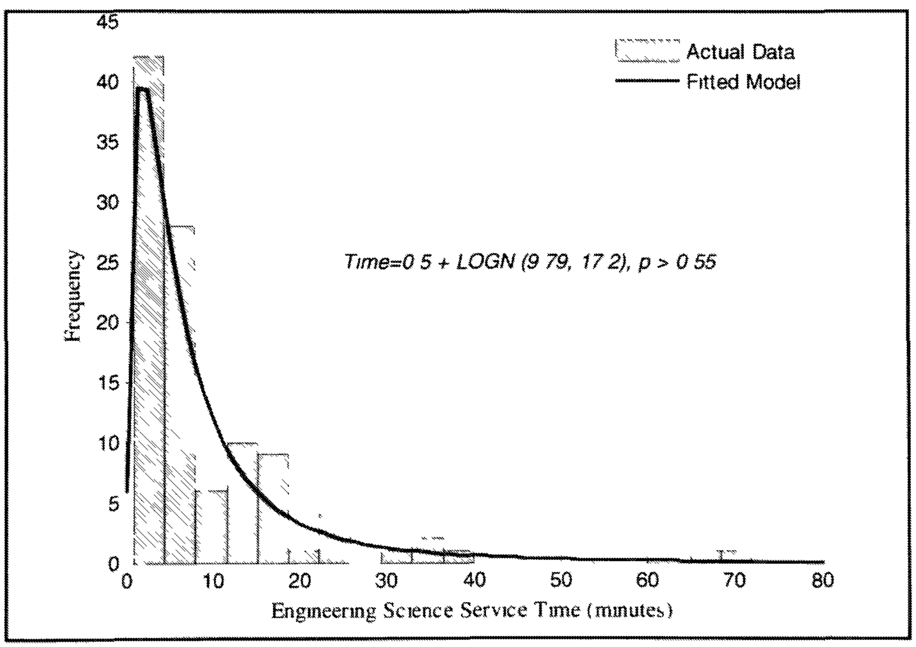


Figure 14: Engineering Science Service Time Distribution

Programming found a high corresponding p-value of $p > 0.75$ for the lognormal distribution of 1.5 + LOGN(14.2, 25) for an inter-arrival tutor request rate, as shown in Figure

17. The service time distribution for programming also had a lognormal distribution of 0.5 + LOGN(8.32, 15.2) and a corresponding p-value of $p = 0.368$, shown in Figure 18. Both distributions were considered a good fit for the data because the p values were above 0.10.

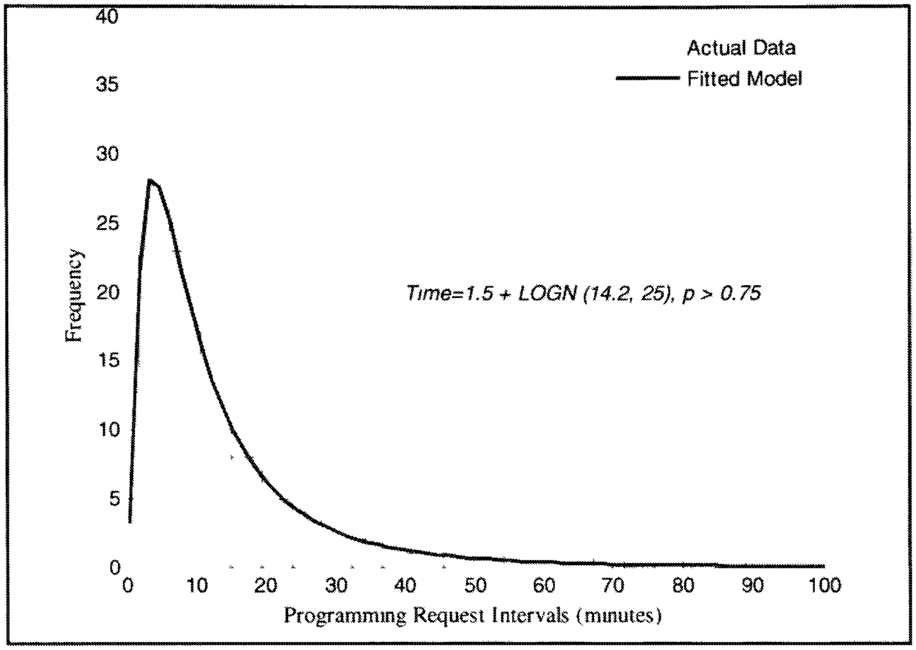


Figure 15: Programming Inter-Arrival Request Distribution

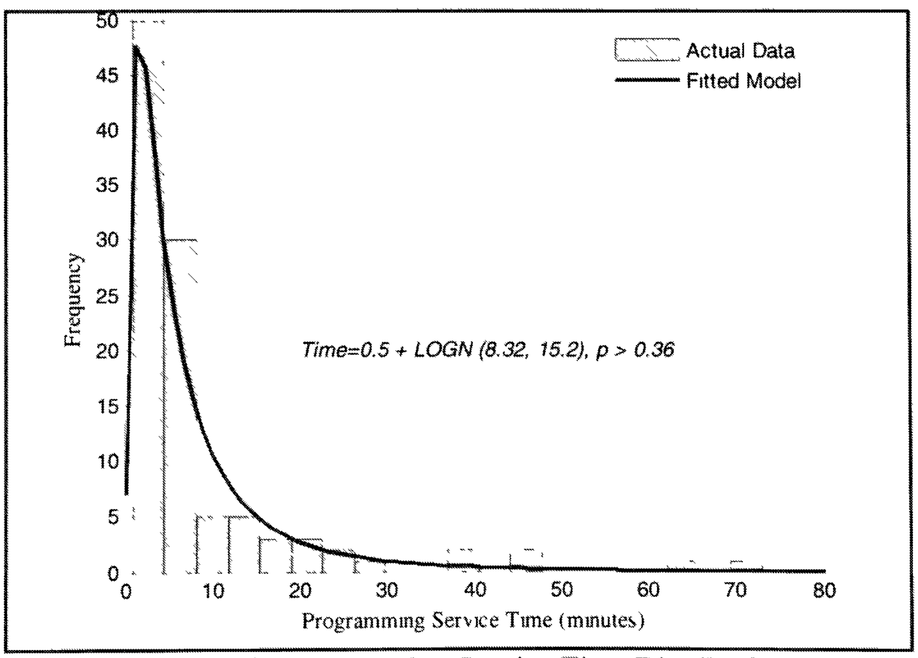


Figure 16: Programming Service Time Distribution

The Business inter-arrival tutoring request rate and the service time both failed to follow the lognormal distribution. For the inter-arrival request: 2.5 + LOGN (13.4, 20.4), with $p = 0.216$. The service time followed 0.5 + LOGN(5.88, 9.36) with $p = 0.425$. Again, high p – values fail to reject the null hypotheses that both are good fits for the data, thus can be used in the model. *Figures* 19 & 20 show the inter-arrival and service time distributions, respectively.

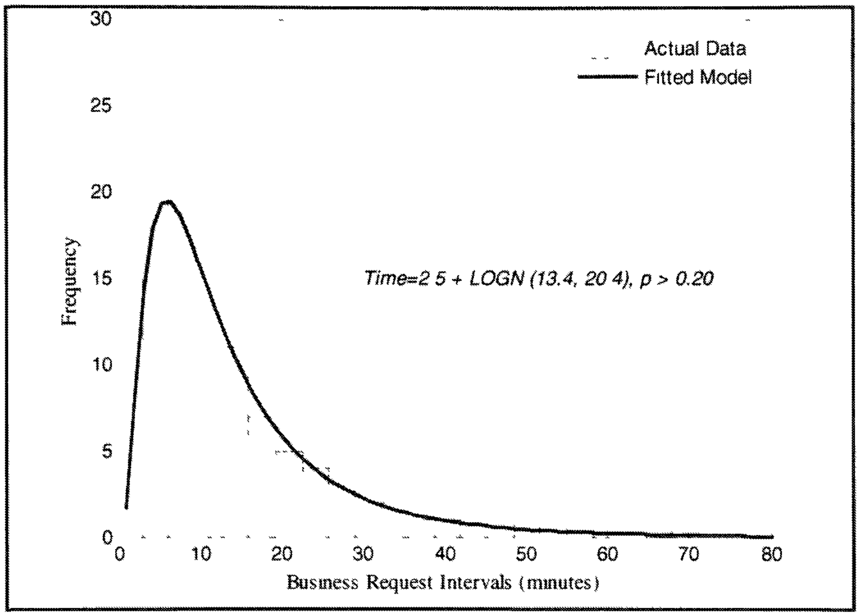


Figure 17: Business Inter-Arrival Service Request

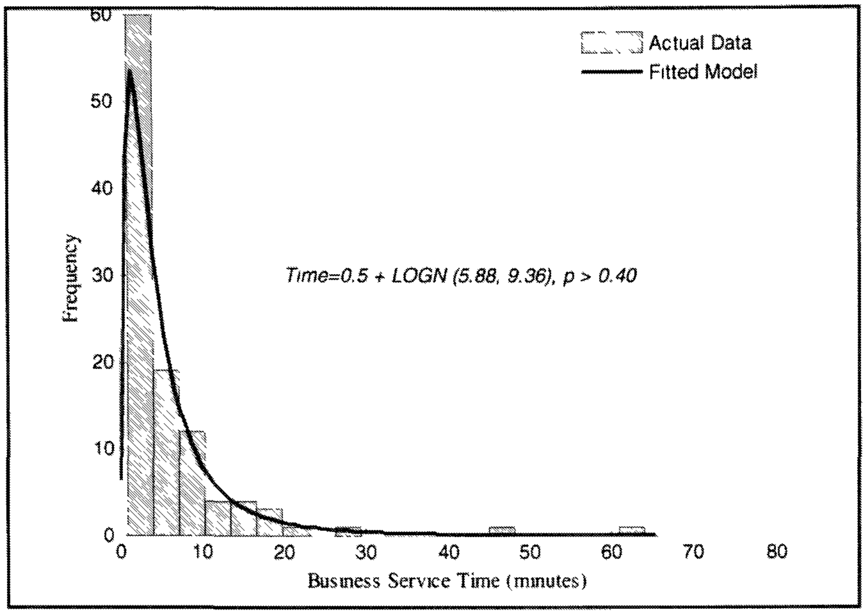


Figure 18: Business Service Time Distribution

For Corporate Finance, the lognormal distribution had the best fit for the inter-arrival tutoring request with 0.5 + LOGN(17.7, 24.5) with $p = 0.157$, shown in Figure 21. For Corporate Finance service time, the theoretical distribution could not be used due to its very low corresponding p-value of $p < 0.005$. According to Kelton, Sadowski and Sturrok (2004), an empirical distribution should be used if the corresponding p-value is low (less than 0.10). Therefore, the empirical distribution was used, which is shown in Figure 22 and can be found in Appendix C.

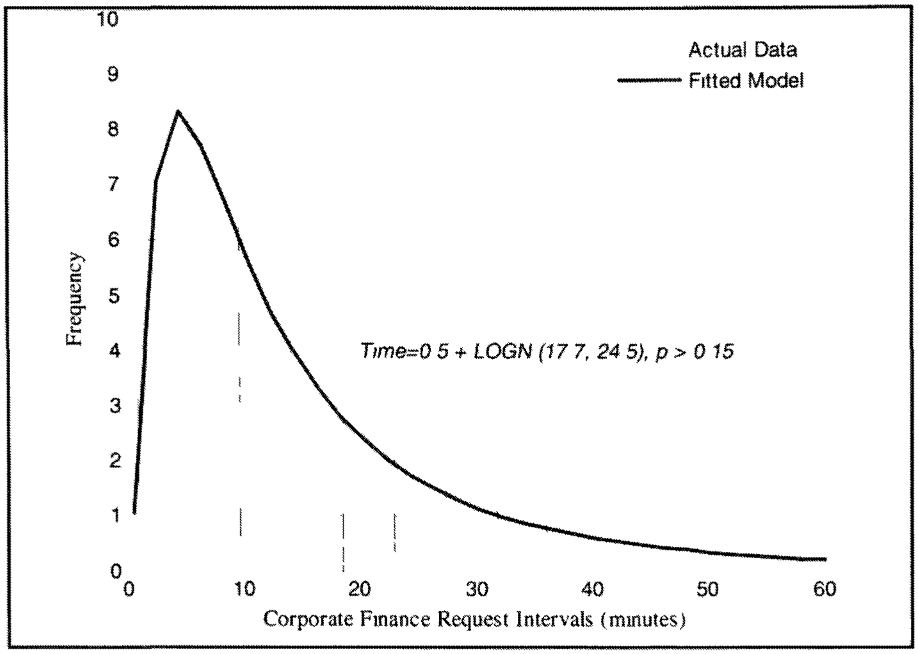


Figure 19: Corporate Finance Inter-Arrival Request Distribution

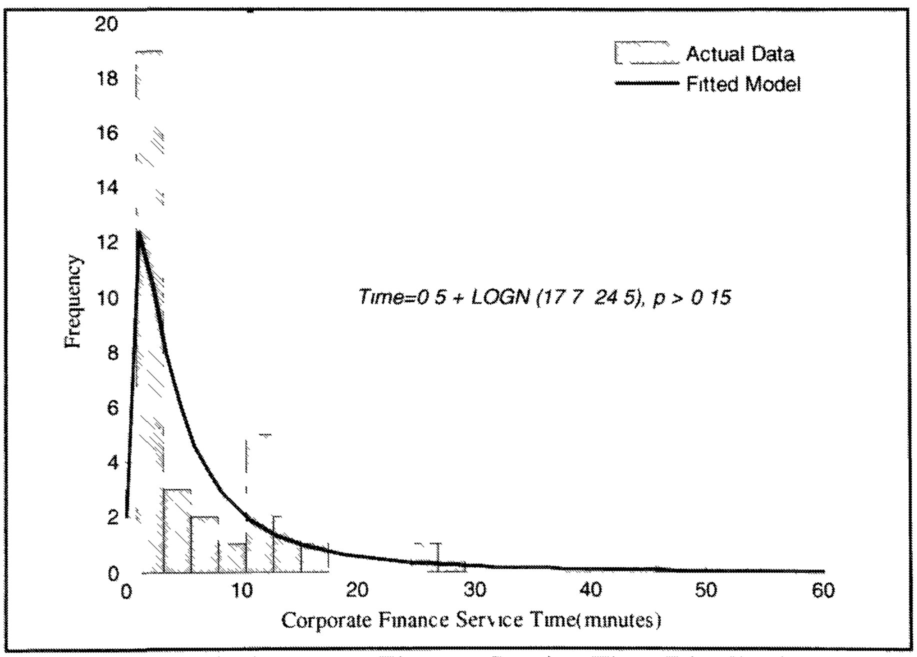


Figure 20: Corporate Finance Service Time Distribution

For the inter-arrival tutoring request rate of Aeronautical Science, the goodness-of-fit failed to find a theoretical distribution. Therefore, the empirical distribution was used to generate the inter-arrival tutoring request rate for Aeronautical Science, shown in Figure 23 and found in

Appendix C. The best-fit theoretical distribution for the service time of Aeronautical Science was found to be the Weibull distribution of 0.999 + WEIB(7.19, 0.348), which is shown in Figure 24. The Kolmogorov-Smirnov test found a good corresponding *p*-value ($p > 0.15$), indicating a good fit for the data.

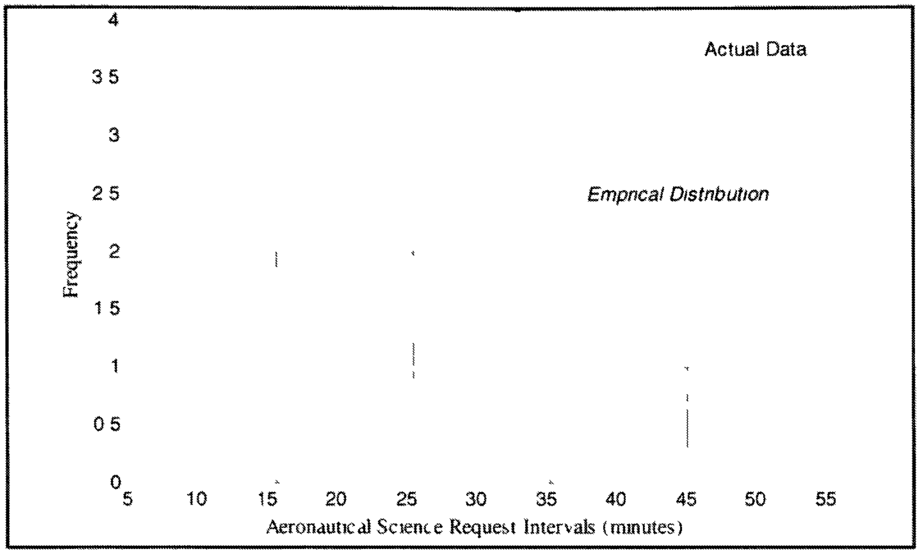


Figure 21: Aeronautical Science Inter-Arrival Request Distribution

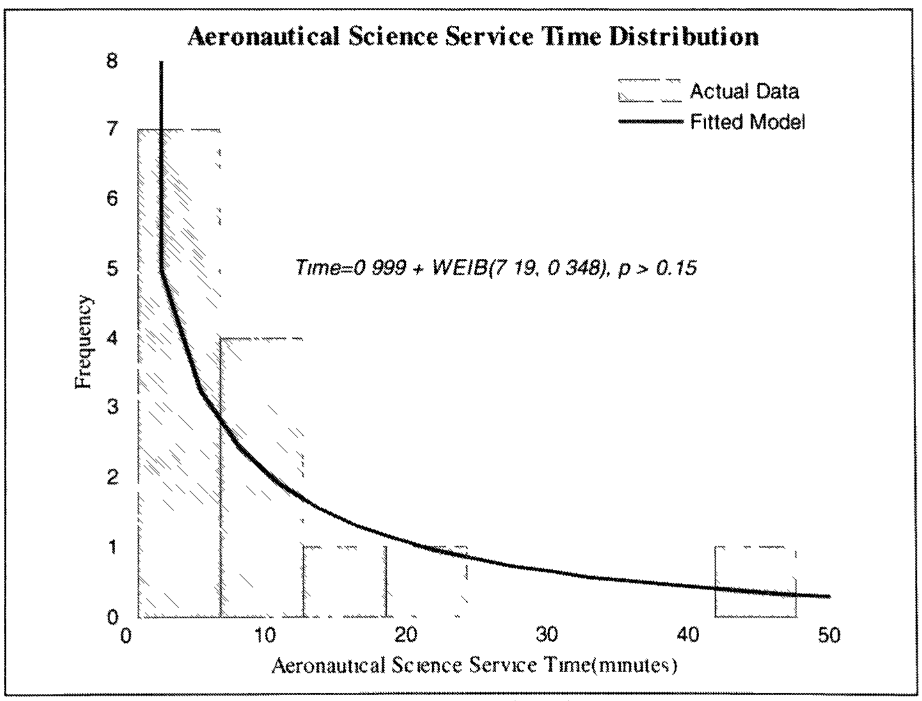


Figure 22: Aeronautical Science Service Time Distribution

Aviation Maintenance Science was eliminated from this investigation due to the lack of tutoring service requests. More specifically, over the four week period, there were no tutoring requests or service times observed. Accordingly, there is no data to suggest how many tutors to staff, based on the model. Possible reasons for the lack of tutoring requests are discussed later.

The inter-arrival tutoring request and service time for the Physics and Chemistry Lab both followed a lognormal distribution. Input Analyzer reported 0.5 + LOGN(6.7, 11.3) with a Chi-Square goodness-of-fit of $p = 0.545$ for the inter-arrival tutoring request rate. The service time distribution was: 0.5 + LOGN(7.11, 13.4) with a Chi-Square goodness-of-fit of $p = 0.302$. The inter-arrival request rate distribution and the service time distribution for the Physics & Chemistry Lab are shown in Figures 25 & 26, respectively. Both are considered a good fit for the data.

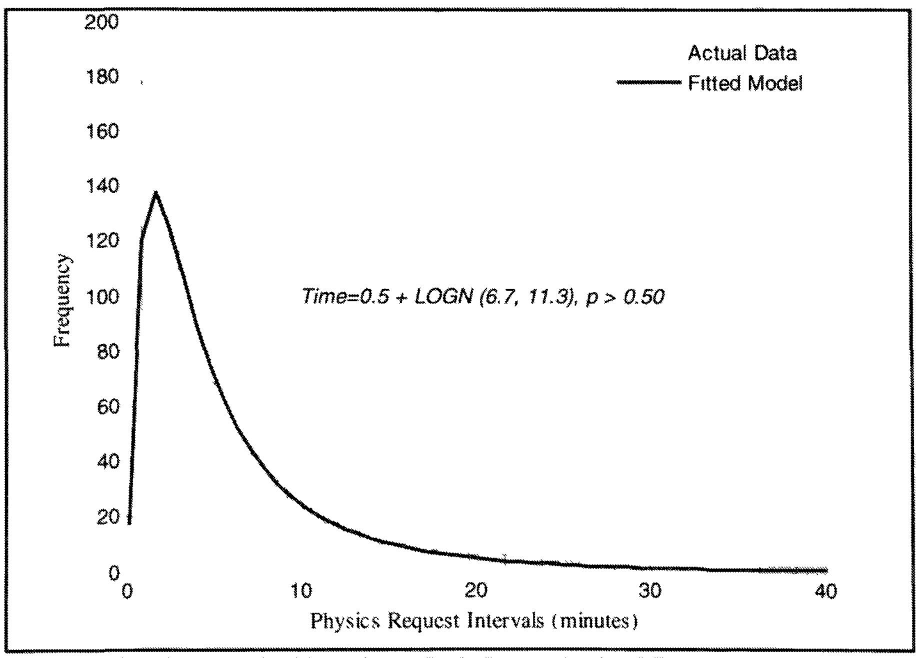


Figure 23: Physics & Chemistry Lab Inter-Arrival Request Distribution

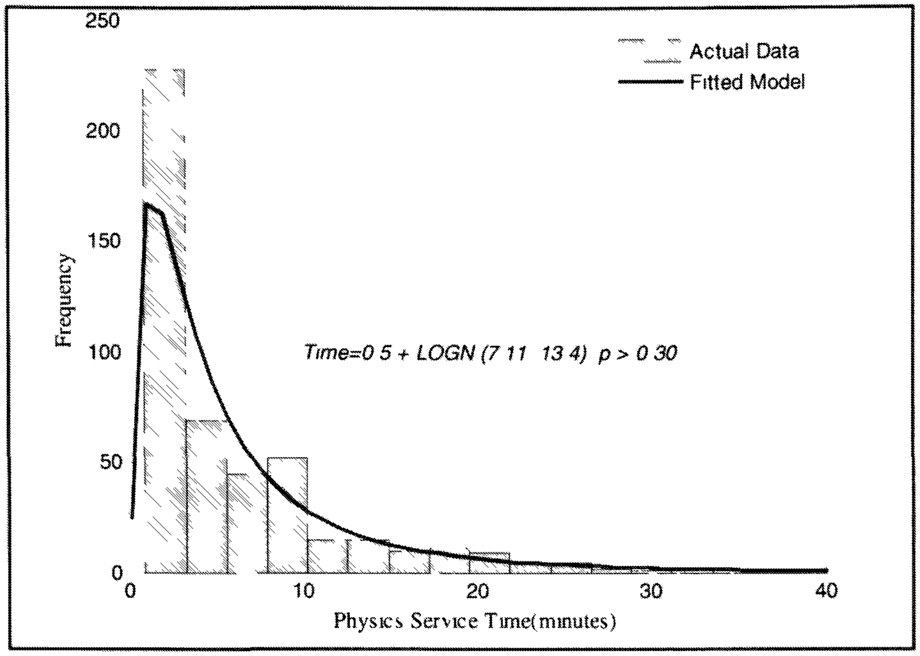


Figure 24 Physics & Chemistry Lab Service Time Distribution

Model Validation

With the fitted data, both the General Study Room and the Physics & Chemistry Lab models were run 50 times with an operational run time of three hours (180 minutes) per run, as the Unified Tutoring Center is open from 6 – 9pm Tutor utilization was used as the validation performance output measure Using the model logic previously shown in Figure 12, tutor utilization for each of the 50 runs was written into an Excel spreadsheet and compared against the actual historical data collected through observation at the Unified Tutoring Center The following table (Table 8) presents the descriptive statistics of used to validate the model

Table 8:

Descriptive Statistics for Model Validation

Subject	Actual Observed Data		95% Welch's CI	Model Output Data	
	Mean	Standard Deviation		Mean	Standard Deviation
Engineering Science	0.095	0.096	[-0.025, 0.073]	0.118	0.091
Programming	0.255	0.250	[-0.147, 0.109]	0.236	0.195
Business	0.187	0.191	[-0.059, 0.115]	0.215	0.155
Corporate Finance	0.078	0.122	[0.075, 0.059]	0.070	0.128
Aeronautical Science	0.097	0.202	[-0.099, 0.854]	0.090	0.163
Physics & Chemistry	0.377	0.161	[-0.139, 0.022]	0.318	0.111

The Statistical Package for the Social Sciences (SPSS) was employed to analyze the statistical significance in the results generated from the Arena model to the actual historical data from the UTC. A two-sample independent t-test was used in the following analyses, since the results of the Arena model are independent of the actual observed data. An alpha level of 5% was used to test the difference between the two groups. A bigger p – value is desired for validation purposes in this study, because it indicates that the two samples are not significantly different, which indicates that the Arena model and actual observed data are similar, thus failing to reject the null hypothesis.

Based on the results, the five subjects modeled in the General Study Room model were all found to be valid when compared against the actual historical data observed from the Unified Tutoring Center. For the utilization of the Engineering Science (ES) tutors, the Levene's test of homogeneity of variance's was not significant based on $p = 0.311$, so the data does not violate the assumption of homogeneity of variance. With equal variances assumed, the independent samples t-test was not significant based on $t(68) = 0.989$, $p = 0.326$ and a 95% Welch confidence interval for the mean difference of [-0.025, 0.073].

Levene's test of homogeneity of variance was found to be significant ($p = 0.032$) for the utilization of the programming (Prog) tutor, so equal variances were not assumed when analyzing the t-test results ($t(28.721) = -0.308$, $p = 0.761$). The 95% Welch confidence interval for the mean difference was [-0.147, 0.109]. There was no difference between the observed historical data and the model's output data.

For the utilization of Business tutors, the Levene's test of homogeneity of variance was not statistically significant ($p = 0.134$), therefore equal variances were assumed when reviewing the results of the independent samples t-test. The t-test results indicate no significant difference between the actual data and model output, based on $t(68) = 0.635$, $p = 0.528$, with a 95% Welch's confidence interval mean difference of [-0.059, 0.115].

The Levene's test of homogeneity of variance for the utilization of the Corporate Finance and Aeronautical Science tutors were not statistically significant, based on $p = 0.932$ and $p = 0.903$, respectively. With equal variances assumed, the t-test results for the utilization of CF tutors was $t(68) = -0.245$, $p = 0.807$, with a 95% Welch's confidence interval mean difference of [-0.075, 0.059]. Similarly, equal variances were assumed when assess the utilization results of the Aeronautical Science tutor, which was $t(68) = -0.152$, $p = 0.880$ and a Welch's 95% confidence interval of [-0.099, 0.854].

The results for Levene's test of homogeneity of variance for the Physics & Chemistry Lab tutors was statistically significant, which means equal variances cannot be assumed when analyzing the results of the independent samples t-test. With equal variances not assumed, the actual and simulated utilization results are not statistically different based on t(26.498) = -1.494, $p = 0.147$ and a 95% Welch's confidence interval of [-0.139, 0.022].

Table 9 presents an overview of the t-test results, whether the variances were equal, the degrees of freedom and the p – value or level of significance between the data that was used for validating the model.

Table 9:

T-test Results Summary

Subject	Equal Variance?	t-statistic/ Welch's T for Unequal Variances	Degrees of Freedom	p – value
Engineering Science	Yes	0.989	68	0.326
Programming	No	-0.308	28.7	0.761
Business	Yes	0.635	68	0.528
Corporate Finance	Yes	-0.245	68	0.807
Aeronautical Science	Yes	-0.152	68	0.880
Physics & Chemistry	No	-1.494	26.5	0.147

Sensitivity Analysis

Hourly Schedule Sensitivity Analysis. Based on the validated model, OptQuest was used to study the sensitivity of minimizing cost to determine the best tutor staffing schedules under various constraint combinations. Initial tutor staffing experimentation consisted of varying the schedule on an hourly basis (6-7pm, 7-8pm and 8-9pm). Based on the non-stationary nature of the student arrival schedule and the tutoring request rate, it was decided to use an hourly staffing schedule to try and minimize the overall operational cost of the Unified Tutoring Center. Both models minimize the total daily tutor wages under the constraints of student wait time (average and maximum) and maximum tutor utilization. The specific values used for the average wait time, maximum wait time and maximum tutor utilization were based on results from the UTC Constraints Survey and sensitivity experimentation.

OptQuest optimized the hourly tutor staffing schedule for the General Study Room by subject based on maximum tutor utilizations of 20%, 40% and 65%. For each utilization, an average wait time maximum was also defined as a constraint. Table 10 shows the best staffing schedule for each scenario, along with an overall tutor staffing cost. Figure 27 illustrates the relationship between average waiting time constraints, maximum tutor utilization and the minimum daily cost.

Table 10:

General Study Room Sensitivity Analysis (Utilization versus Average Waiting Time)

	Constraints	ES			Prog			Bus			CF	AS	Total
Utilization	Time	6-7	7-8	8-9	6-7	7-8	8-9	6-7	7-8	8-9	6-9	6-9	Cost
≤ 20%	Avg Wait ≤ 1 minute	1	3	4	2	2	2	2	2	1	1	1	$212.50
	Avg Wait ≤ 2 minutes	1	3	5	2	2	1	2	1	1	1	1	$204.00
	Avg Wait ≤ 4 minutes	1	3	5	2	2	1	2	1	1	1	1	$204.00
	Avg Wait ≤ 6 minutes	1	3	5	2	2	1	2	1	1	1	1	$204.00
	Avg Wait ≤ 10 minutes	1	3	5	2	2	1	2	1	1	1	1	$204.00
≤ 40%	Avg Wait ≤ 1 minute	1	2	2	1	1	2	1	2	1	1	1	$161.50
	Avg Wait ≤ 2 minutes	1	2	2	1	1	1	1	2	1	1	1	$153.00
	Avg Wait ≤ 4 minutes	1	2	2	1	1	1	1	1	1	1	1	$144.50
	Avg Wait ≤ 6 minutes	1	2	2	1	1	1	1	1	1	1	1	$144.50
	Avg Wait ≤ 10 minutes	1	2	2	1	1	1	1	1	1	1	1	$144.50
≤ 65%	Avg Wait ≤ 1 minute	1	2	2	1	1	2	1	2	1	1	1	$161.50
	Avg Wait ≤ 2 minutes	1	2	2	1	1	1	1	2	1	1	1	$153.00
	Avg Wait ≤ 4 minutes	1	2	1	1	1	1	1	1	1	1	1	$136.00
	Avg Wait ≤ 6 minutes	1	2	1	1	1	1	1	1	1	1	1	$136.00
	Avg Wait ≤ 10 minutes	1	1	1	1	1	1	1	1	1	1	1	$127.50

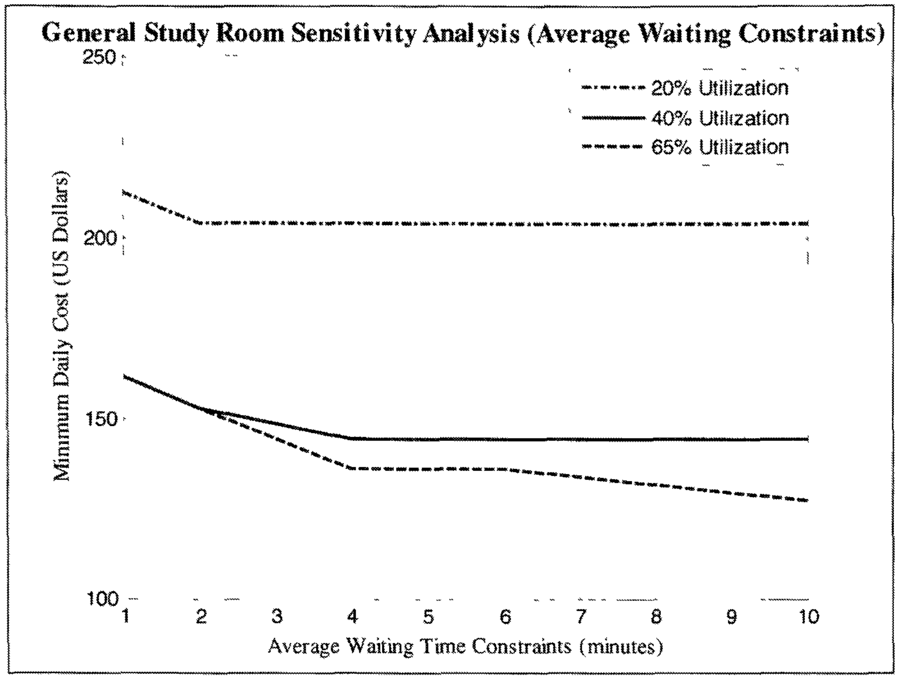


Figure 25: General Study Room Cost Sensitivity Analysis (Average Waiting Time)

For the maximum waiting time constraints, the General Study Room was to re-run OptQuest again, changing the average waiting time constraint to a maximum wait time, per tutoring session, constraint. The resulting tutor staffing schedule and overall cost is shown in Table 11, based on a maximum waiting time of 5, 10 and 20 minutes. The overall cost sensitivity is illustrated graphically in Figure 28.

Table 11:

General Study Room Sensitivity Analysis (Utilization versus Maximum Waiting Time)

	Constraints	ES			Prog			Bus			CF	AS	Total
Utilization	Time	6-7	7-8	8-9	6-7	7-8	8-9	6-7	7-8	8-9	6-9	6-9	Cost
	Max Wait ≤ 5 minutes	2	3	4	1	2	2	2	2	1	1	1	$212.50
≤ 20%	Max Wait ≤ 10 minutes	1	3	4	1	2	1	2	1	1	1	1	$187.00
	Max Wait ≤ 20 minutes	1	3	4	1	2	1	2	1	1	1	1	$187.00
	Max Wait ≤ 5 minutes	2	2	1	1	1	2	2	2	1	1	1	$170.00
≤ 40%	Max Wait ≤ 10 minutes	1	2	1	1	1	1	2	1	1	1	1	$144.50
	Max Wait ≤ 20 minutes	1	2	1	1	1	1	2	1	1	1	1	$144.50
	Max Wait ≤ 5 minutes	2	2	1	1	1	2	2	2	1	1	1	$170.00
≤ 65%	Max Wait ≤ 10 minutes	1	2	1	1	1	1	2	1	1	1	1	$144.50
	Max Wait ≤ 20 minutes	1	2	1	1	1	1	1	1	1	1	1	$136.00

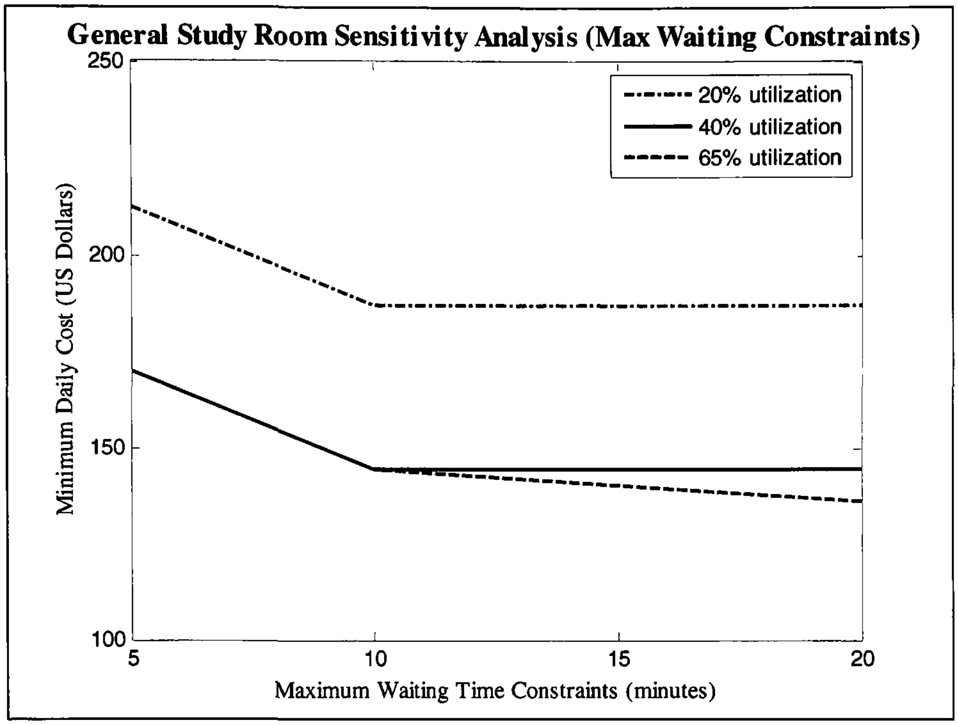


Figure 26: General Study Room Cost Sensitivity Analysis (Maximum Waiting Time)

The Physics & Chemistry Lab model was run through OptQuest very similar to the General Study Room model, with some changes to the actual constraints. The first sensitivity analysis ran on the Physics & Chemistry Lab had 30%, 35%, 40%, 50% and 60% as tutor utilization constraints. This sensitivity analysis investigated the relationship between tutor utilization maximums and student wait time maximums. The results and associated costs are shown in Table 12 below, and graphically illustrated in Figure 29.

Table 12:

Physics & Chemistry Lab Sensitivity Analysis (Utilization versus Maximum Waiting Time)

Constraints		Physics			Total Cost
Utilization	Time	6-7	7-8	8-9	
≤ 30%	Max Wait ≤ 5 minutes	6	7	6	$161.50
	Max Wait ≤ 10 minutes	6	7	6	$161.50
	Max Wait ≤ 15 minutes	8	7	2	$144.50
≤ 35%	Max Wait ≤ 5 minutes	6	6	4	$136.00
	Max Wait ≤ 10 minutes	6	6	4	$136.00
	Max Wait ≤ 15 minutes	4	8	2	$119.00
≤ 40%	Max Wait ≤ 5 minutes	5	6	5	$136.00
	Max Wait ≤ 10 minutes	5	5	4	$119.00
	Max Wait ≤ 15 minutes	5	6	2	$110.50
≤ 50%	Max Wait ≤ 5 minutes	3	5	5	$110.50
	Max Wait ≤ 10 minutes	2	4	6	$102.00
	Max Wait ≤ 15 minutes	3	5	2	$85.00
≤ 60%	Max Wait ≤ 5 minutes	3	5	5	$110.50
	Max Wait ≤ 10 minutes	2	4	4	$85.00
	Max Wait ≤ 15 minutes	3	3	3	$76.50

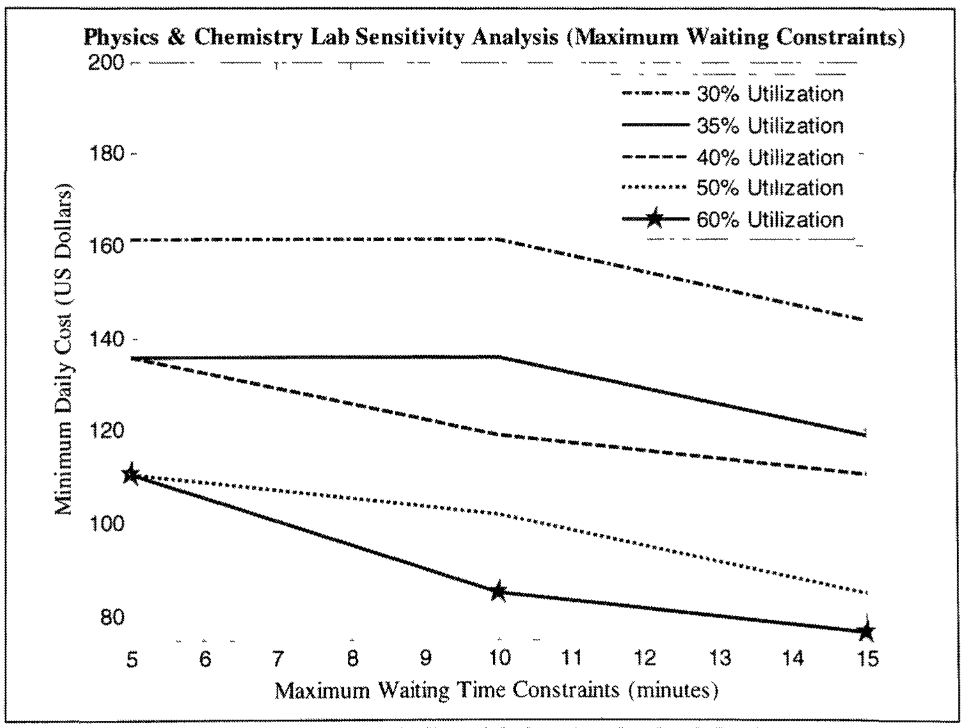


Figure 27: Physics & Chemistry Lab Sensitivity Analysis (Maximum Waiting Time)

The second sensitivity analysis on the Physics & Chemistry Lab involved the constraints of tutor utilization and the average waiting time experienced by students. More levels of utilization were included, as it was found that interaction of the average waiting time constraints and tutor utilization were more sensitive than the interaction of maximum waiting time and tutor utilization, as shown in Table 13 and Figure 30.

Table 13:

Physics & Chemistry Lab Sensitivity Analysis (Utilization versus Average Waiting Time)

Utilization	Constraints Time	Physics 6-7	7-8	8-9	Total Cost *
≤ 10%	Avg. Wait ≤ 1 minutes	7	9	9	$212.50
	Avg. Wait ≤ 2 minutes	8	9	8	$212.50
	Avg. Wait ≤ 4 minutes	8	9	8	$212.50
	Avg. Wait ≤ 6 minutes	8	9	8	$212.50
	Avg. Wait ≤ 10 minutes	8	9	8	$212.50
≤ 15%	Avg. Wait ≤ 1 minutes	5	6	6	$144.50
	Avg. Wait ≤ 2 minutes	5	6	6	$144.50
	Avg. Wait ≤ 4 minutes	5	7	5	$144.50
	Avg. Wait ≤ 6 minutes	5	7	5	$144.50
	Avg. Wait ≤ 10 minutes	5	7	5	$144.50
≤ 20%	Avg. Wait ≤ 1 minutes	6	2	5	$110.50
	Avg. Wait ≤ 2 minutes	5	5	2	$102.00
	Avg. Wait ≤ 4 minutes	5	5	2	$102.00
	Avg. Wait ≤ 6 minutes	5	5	1	$93.50
	Avg. Wait ≤ 10 minutes	5	5	1	$93.50
≤ 25%	Avg. Wait ≤ 1 minutes	3	2	5	$85.00
	Avg. Wait ≤ 2 minutes	3	3	4	$85.00
	Avg. Wait ≤ 4 minutes	3	4	3	$85.00
	Avg. Wait ≤ 6 minutes	3	4	1	$68.00
	Avg. Wait ≤ 10 minutes	3	4	1	$68.00
≤ 30%	Avg. Wait ≤ 1 minutes	3	3	3	$76.50
	Avg. Wait ≤ 2 minutes	2	2	4	$68.00
	Avg. Wait ≤ 4 minutes	2	3	3	$68.00
	Avg. Wait ≤ 6 minutes	3	3	1	$59.50
	Avg. Wait ≤ 10 minutes	3	3	1	$59.50
≤ 35%	Avg. Wait ≤ 1 minutes	2	3	3	$68.00
	Avg. Wait ≤ 2 minutes	2	2	3	$59.50
	Avg. Wait ≤ 4 minutes	2	2	3	$59.50
	Avg. Wait ≤ 6 minutes	2	3	1	$51.00
	Avg. Wait ≤ 10 minutes	3	2	1	$51.00
≤ 40%	Avg. Wait ≤ 1 minutes	3	2	3	$68.00

	Avg. Wait ≤ 2 minutes	2	2	3	$59.50
	Avg. Wait ≤ 4 minutes	2	2	2	$51.00
	Avg. Wait ≤ 6 minutes	2	2	2	$51.00
	Avg. Wait ≤ 10 minutes	2	2	1	$42.50
≤ 60%	Avg. Wait ≤ 1 minutes	3	2	3	$68.00
	Avg. Wait ≤ 2 minutes	2	2	3	$59.50
	Avg. Wait ≤ 4 minutes	2	2	2	$51.00
	Avg. Wait ≤ 6 minutes	2	1	2	$42.50
	Avg. Wait ≤ 10 minutes	2	1	2	$42.50

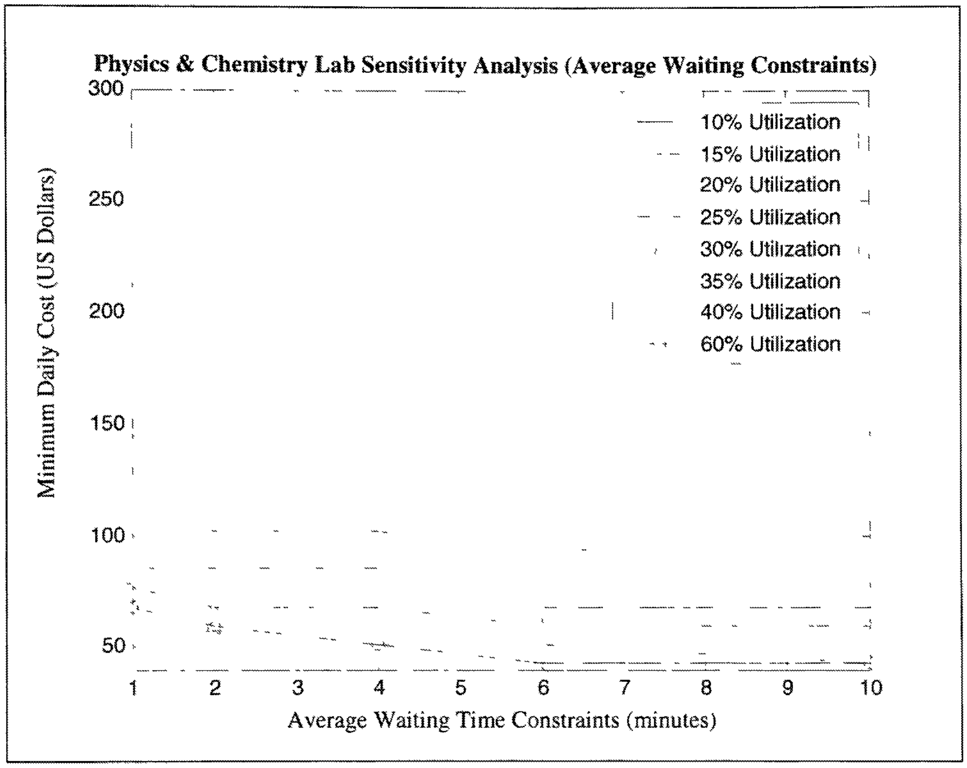


Figure 28: Physics & Chemistry Lab Sensitivity Analysis (Average Waiting Time)

Practical Consolidated Evening Schedule Sensitivity Analysis. When considered practically, a varying hourly schedule may not be applicable. Since the Unified Tutoring Center is only open from 6pm to 9pm, it would be difficult to ask some tutors to come to work for only one hour, while others may work an entire shift. Many of the tutors live off campus, which would make coming in for only one hour of work inconvenient. Therefore, a further analysis was conducted with the consolidated evening schedule, run on both the General Study Room and the Physics & Chemistry Lab models. These analyses were run to find an optimal staffing schedule on a nightly basis (from 6-9pm).

Table 14 shows the optimization results for the General Study Room based on the relationship between maximum tutor utilization and average student waiting time.

Table 14:

General Study Room Practical Sensitivity Analysis (Utilization versus Average Waiting Time)

Utilization	Constraints Time	ES 6 - 9	Prog 6 - 9	Bus 6 – 9	CF 6 – 9	AS 6 – 9	Total Cost
≤ 40%	Avg. Wait ≤ 1 minutes	2	2	1	1	2	$204.00
	Avg. Wait ≤ 2 minutes	2	1	1	1	1	$153.00
	Avg. Wait ≤ 5 minutes	1	1	1	1	1	$127.50
≤ 65%	Avg. Wait ≤ 1 minutes	2	2	1	1	2	$204.00
	Avg. Wait ≤ 2 minutes	2	1	1	1	1	$153.00
	Avg. Wait ≤ 5 minutes	1	1	1	1	1	$127.50
≤ 85%	Avg. Wait ≤ 1 minutes	2	2	1	1	2	$204.00
	Avg. Wait ≤ 2 minutes	2	1	1	1	1	$153.00
	Avg. Wait ≤ 5 minutes	1	1	1	1	1	$127.50

Similarly, for average waiting time constraints a sensitivity analysis was run on the General Study Room model for the interaction of tutor utilization and the maximum student waiting time, which is shown in Table 15.

Table 15:

General Study Room Practical Sensitivity Analysis (Utilization versus Maximum Waiting Time)

Utilization	Constraints Time	ES 6 - 9	Prog 6 - 9	Bus 6 – 9	CF 6 – 9	AS 6 – 9	Total Cost
≤ 40%	Max Wait ≤ 5 minutes	2	1	2	1	1	$178.50
	Max Wait ≤ 10 minutes	1	1	1	1	1	$127.50
	Max Wait ≤ 15 minutes	1	1	1	1	1	$127.50
≤ 65%	Max Wait ≤ 5 minutes	2	1	2	1	1	$178.50
	Max Wait ≤ 10 minutes	1	1	1	1	1	$127.50
	Max Wait ≤ 15 minutes	1	1	1	1	1	$127.50
≤ 85%	Max Wait ≤ 5 minutes	2	1	2	1	1	$178.50
	Max Wait ≤ 10 minutes	1	1	1	1	1	$127.50
	Max Wait ≤ 15 minutes	1	1	1	1	1	$127.50

The final two sensitivity analyses were conducted on the Physics & Chemistry Lab model to determine the practical tutor staffing schedule on a nightly basis. Table 16 depicts the optimized staffing recommendations based on tutor utilization and the average student wait time per tutoring session. Table 17 illustrates the results for the optimal number of tutors to staff in the Physics & Chemistry Lab, based on the interaction of tutor utilization and maximum student wait time.

Table 16:

Physics & Chemistry Lab Practical Sensitivity Analysis (Utilization versus Average Waiting Time)

Constraints		Physics	Total Cost
Utilization	Time	6 – 9	
≤ 40%	Avg. Wait ≤ 1 minutes	3	$76.50
	Avg. Wait ≤ 2 minutes	3	$76.50
	Avg. Wait ≤ 5 minutes	3	$76.50
≤ 65%	Avg. Wait ≤ 1 minutes	2	$51.00
	Avg. Wait ≤ 2 minutes	2	$51.00
	Avg. Wait ≤ 5 minutes	2	$51.00
≤ 85%	Avg. Wait ≤ 1 minutes	2	$51.00
	Avg. Wait ≤ 2 minutes	2	$51.00
	Avg. Wait ≤ 5 minutes	2	$51.00

Table 17:

Physics & Chemistry Lab Practical Sensitivity Analysis (Utilization versus Maximum Waiting Time)

Constraints: Utilization	Constraints: Time	Physics 6 – 9	Total Cost
≤ 40%	Max Wait ≤ 5 minutes	5	$127.50
	Max Wait ≤ 10 minutes	5	$127.50
	Max Wait ≤ 15 minutes	5	$127.50
	Max Wait ≤ 20 minutes	5	$127.50
≤ 65%	Max Wait ≤ 5 minutes	5	$127.50
	Max Wait ≤ 10 minutes	4	$102.00
	Max Wait ≤ 15 minutes	3	$76.50
	Max Wait ≤ 20 minutes	3	$76.50
≤ 85%	Max Wait ≤ 5 minutes	5	$127.50
	Max Wait ≤ 10 minutes	4	$102.00
	Max Wait ≤ 15 minutes	3	$76.50
	Max Wait ≤ 20 minutes	3	$76.50

Results and their implications are further discussed in the following discussion section.

Discussion

In this section the results are discussed, including the student arrival rate schedule, the service time distributions and the service request distributions, validation of the simulation model, as well as the sensitivity analysis. Limitations encountered in this study are also discussed in this section. Suggestions on areas of future research are given at the end of the section.

Student Arrival Rate

As previously mentioned in the results section, an arrival schedule was used to simulate the arrival of students to the General Study Room and to the Physics & Chemistry Lab. An arrival schedule was chosen over an arrival distribution due to the non-stationary nature of the arrival rate. By reviewing Figure 13 again, it can be noticed that the peak arrival time period is from 6:00-6:30pm for both models. This is likely due to the nature of how certain groups of students use the Unified Tutoring Center. Certain groups on campus have mandatory or required study hours for students to complete each week. Take the Athletic department's BEST Program and the Naval ROTC program as an example. These programs require their students complete anywhere from 4 to 15 hours of studying/tutoring per week. Given that the UTC is open three hours a day and five days a week, there is a maximum of 15 hours that students can log towards their requirement. Students required to complete 15 hours a week must arrive exactly at six o'clock and stay until nine o'clock, in order to fulfill this requirement. In addition to students who want to take full advantage of the free tutoring, those students logging mandatory hours might help explain such a high arrival rate from 6:00-6:30pm, for both rooms. Another explanation might be that many students would prefer tutoring be offered earlier in the day, due to other conflicts they may have. Also, many college students work part-time jobs, either in the afternoons or evenings, which may conflict with the operational times of the UTC.

Input Data Analysis

Input Analyzer was used to fit probability distributions to the historical data for the inter-arrival of tutoring requests and for the service times of all six subjects investigated. Strong goodness-of-fit indicated by large p – values were found for all but one inter-arrival request distribution and one service time distribution. The empirical distribution was used for the two theoretical distributions with inadequate goodness-of-fit p – values, which were the inter-arrival request rate for Aeronautical Science and the service time for Corporate Finance. The empirical distribution was used because there were too few historical data points to generate a smooth and fitted histogram.

When there are a sequence of tasks involved and there are a large number of random possibilities, as seen with the inter-arrival request rate and service time, the randomness often presents a lognormal distribution (Blanchard & Fabrycky, 2006). Many of the remaining distributions were found to follow the lognormal distribution, which can be explained by the nature of the inter-arrival request rate and service time. The nature of a tutoring session varies based on the number of questions asked and the complexity of the question. There are many different combinations on how the number of questions asked and question complexity interact with one another, which have an impact on the inter-arrival request rate and service time. Some students may ask lots of simple questions, which would inflate the number of inter-arrival requests and shorten the service time length. There are also students who may only ask for help with one or two questions, but may require a longer service time, due to the complexity of the question. These two extreme examples help explain the drastic variation in the inter-arrival request times and service times, that were captured by lognormal data patterns.

Model Validation

Validation is a crucial part of a simulation study. Two major and commonly used methods were applied in this study, which include face validation and statistical results validation. The model was validated by comparing the historical overall tutor utilization for each subject to tutor utilization reported by the Arena simulation model, as this was the most accurate observable event. Due to the large difference in sample size between the historical data (relatively small amount, n=20) and the simulation data (n=50), the Welch method was used for some of the statistical analysis, whenever there was an unequal variance between the two groups. Utilization results were compared for statistical differences based on Welch's t 95% confidence interval. Since there were no significant differences between the actual historical data and the simulation utilization results, the model is considered to be a valid model that represents the simulated behaviors of the General Study Room and Physics & Chemistry Lab at the Unified Tutoring Center, under the significance level of 10%.

Sensitivity Analysis

Hourly Schedule Sensitivity Analysis. Numerous sensitivity analyses based on daily cost optimization were run on both the General Study Room model and the Physics & Chemistry Lab model. Each sensitivity analysis investigated the relationship between tutor utilization thresholds, student wait time thresholds (average and maximum wait times allowed) and daily cost to determine the near optimal tutor staffing schedule per subject.

When reviewing the sensitivity analysis results for the interaction of tutor utilization and average student wait time in the General Study Room, there are drastic differences in the number of tutors to staff each hour and the overall nightly cost. For example, if the UTC committee would like to ensure that tutors are not overworked and students don't have to wait for long periods of time, the overall tutor staffing cost would be around $212.50, based on a utilization maximum of 20% and an average student wait time of one minute or less. If on the other hand the committee would like to restrict tutor utilization to 65% (allowing for bathroom breaks and such) and increase the average student wait time maximum to 10 minutes, the overall cost would reduce to $127.50 per night. The specific hourly staffing schedule with these constraints are presented in the results section.

The relationship between tutor utilization and maximum student wait time per session for the General Study Room was also investigated to give a different perspective on the student waiting time requirements. Similar to the previously mentioned sensitivity analysis, the overall minimum tutor staffing cost increases when tutor utilization and student wait time are reduced. For example, a tutor utilization maximum of 20% and a student wait time maximum of five minutes results in a nightly cost of $212.50. However, increasing the utilization maximum to 65% and the wait time maximum to 20 minutes would reduce the cost to $136.00.

Several sensitivity analyses were conducted on the staffing schedule for the Physics & Chemistry Lab. A wide variety of tutor utilization thresholds were investigated for the interaction of tutor utilization and average student wait time. The utilization thresholds ranged from 10% to 60%, with the average wait time ranging from a 1 minute threshold to a 10 minute threshold. Results show the wide range in recommended staffing levels and associated costs, based on the combination of constraints. For example, the most expensive staffing plan has a tutor utilization threshold of 10% and an average wait time of 10 minutes or less, totaling to $212.50. Likewise, the most inexpensive staffing schedule employs a utilization maximum of 60% and an average wait time of less than 6 minutes, which totals to $42.50. There are many other staffing suggestions listed, however, they may not be very practical. For example, a 35% utilization threshold and an average wait time maximum of 6 minutes suggests staffing two tutors from 6-7pm, three tutors from 7-8pm and then cutting back to only one tutor from 8-9pm. Aside from the inconvenience experienced by the tutors and students, a 35% utilization threshold is very low. Almost two thirds of their time will be spent idle or doing their own homework, which is not cost effective for the Unified Tutoring Center. For this reason, a further look at the practical aspect of the tutoring schedule was conducted.

Practical Consolidated Evening Schedule Sensitivity Analysis. While the results show the wide variety of staffing schedules to choose from based on the desire to minimize cost based on an hourly schedule, it is not very practical to staff the center each night based on an hourly schedule. Although it may be more expensive and less flexible, it is more practical to hire tutors for an entire three hour shift, rather than one hour. Many of the tutors at the UTC do not live on campus, which means they must come back to campus for work, unless they have a late afternoon class. Asking tutors to come in for only one or two hours may be inconvenient and could become a distraction to the students. For instance, if two Business tutors are helping students when the clock strikes 7pm and one of the tutors is scheduled to leave, then the student getting help from the tutor scheduled to leave will have to wait until the other tutor is available. Not only does this cause an inconvenience to the tutor, but it is also inconvenient for the student who now has to wait.

Therefore additional sensitivity analyses were conducted on the two models to determine the near optimal tutor staffing schedules for each subject, based on an entire (three hour) shift. For the General Study Room model, the results for the interaction of tutor utilization and average student wait time are identical, based on tutor utilization. The staffing recommendations for 40% maximum utilization at one, two and five minute wait time maximums are identical to the staffing recommendations at 65% and 85% maximum utilization. This is likely due to the fact that the tutors are not utilized more than 40% of the time, making 40% the higher limit of the utilization, even with a minimum staff. Similarly, the recommended staffing schedule for the General Study Room (utilization versus maximum student wait time) did not change across utilization maximums. The staffing recommendation for a utilization constraint of 40% and a maximum wait time of 5, 10 and 20 minutes did not change when analyzed at 65% and 85%. Again, this is likely due to the fact that the tutors are not being utilized more than 40% of the time. Compared to the hourly staffing schedule, the consolidated practical schedule is not as sensitive to utilization and waiting time constraints.

Based on the optimized schedules from OptQuest and the feedback from the UTC Constraints Survey, there are two recommended General Study Room staffing schedules for the

UTC committee to choose from. The first schedule is based on feedback from students, who reported that they would be willing to wait up to five minutes, on average, for help. The second schedule is based on feedback from the committee who would like the tutor utilization maximum to be 85% and the maximum student wait time to be five minutes. Table 18 presents the recommended staffing schedule.

Table 18:

Recommended General Study Room Staffing Schedule

Constraints		ES	Prog	Bus	CF	AS	Total
Utilization	Time	6 – 9	6 - 9	6 – 9	6 – 9	6 - 9	Cost
≤ 65%	Avg. Wait ≤ 5 minutes	1	1	1	1	1	$127.50
≤ 85%	Max Wait ≤ 5 minutes	2	2	1	1	2	$204.00

Based on these recommendations, the most cost effective approach to staffing the General Study Room, while still providing a quality tutoring service would be to staff one tutor per subject per night. Accordingly, this would cost the UTC $127.50 per night for the General Study Room.

Results for the Physics & Chemistry Lab indicate that tutor utilization is between 40% and 65%, as the staffing recommendations change between these two thresholds, but not between 65% and 85% utilization, suggesting that 65% is the upper limit. The results show that for a maximum utilization of 40%, the staffing schedule does not change for any of the average wait time thresholds. This is likely due to the fact that the staffing schedule is more sensitive to the tutor utilization and not the average wait time maximum. For the utilization maximums of 65% and 85%, however, it appears that the average wait time threshold seems to determine the number of tutors to staff, since the tutor utilization lies somewhere between 40% and 65%.

The sensitivity results of tutor utilization and maximum student wait time for the Physics & Chemistry Lab, also show that responsive tutor utilization for minimizing daily cost lies somewhere between 40% and 65%, as this is where the staffing schedule changes. For 40% utilization, the maximum student wait time does not appear to have an effect on the staffing schedule, likely due to the utilization cap. In other words, an additional tutor must be staffed if the utilization threshold is 40%. In this case, there are so many tutors staffed, that students will not have to wait very long, indicating that tutor utilization has a dominating effect for this sensitivity analysis. However, examination of the staffing schedule for 65% tutor utilization shows that the maximum student wait time threshold varies the number of tutors needed. For instance, a maximum wait time of 5 minutes requires five tutors, a maximum wait time of 10 minutes requires four tutors and a maximum wait time of 15 or 20 minutes requires only three tutors. A maximum wait time above 20 minutes was not investigated, as this would not provide quick service to the students. Based on the constraint thresholds investigated, four recommendations are offered in Table 19 below.

Table 19:

Recommended Physics & Chemistry Lab Staffing Schedule

Constraints Utilization	 Time	Physics 6 - 9	Total Cost
≤ 65%	Avg. Wait ≤ 1, 2, 5 minutes	2	$51.00
	Max Wait ≤ 5 minutes	5	$127.50
	Max Wait ≤ 10 minutes	4	$102.00
	Max Wait ≤ 15 minutes	3	$76.50

All four recommendations are based on a 65% tutor utilization, as this provides the tutor time to take a restroom break, get something to drink or take a minute to stretch. The first recommendation is to staff two tutors, based on an average student wait time maximum of 5 minutes. When considering the maximum student wait time, the recommended number of tutors to staff ranges from three to five. Three tutors are needed if the wait time maximum is 15 minutes, but five tutors are needed if the wait time is reduced to 5 minutes or less. Based on these recommendations, the least expensive way to staff the Physics & Chemistry Lab is to hire two tutors each evening.

Limitations

There were primarily three types of limitations associated with this study. The first type of limitation relates to the assumptions defined at the beginning of the study. Specifically, the assumptions that define a student's wait time and service time. As stated earlier, a student's wait time is defined as "A student is considered to be waiting if all tutors are busy when the student initially seeks help," while the definition of a student seeking help is defined as "A student who raises their hand or verbally asks a tutor for help is considered to be seeking help from a tutor." These definitions were developed based on the method of data collection chosen, observation through video recordings. These definitions/assumptions limit the accuracy of a student's true wait time, because if a student knows the tutor is busy, they will likely wait until the tutor is available before seeking help. Since the rooms are relatively small and the tutors usually walk around asking students if they need assistance, some students may rather wait until they can seek help without having to keep their hand raised for five or more minutes. All these activities were sometimes un-observable, thus making the data collection difficult. It was observed several times when reviewing the video footage that some students would continually look up at a tutor and even stare at the tutor, waiting to ask for help until the tutor was finished helping another student. Because the student waited until the tutor was available to seek help, this wait time was not recorded, as this type of wait time was not included in the wait time definition defined in the assumptions section. There was no well defined signal to clearly indicate a request for tutoring.

Another assumption that was a limitation in this study was the true length of a tutoring session, or service time. As per the defined definition of a tutoring session, "A student who has a book or school material on the desk and is speaking with a tutor is assumed to be receiving tutoring in the subject the respective tutor has been hired for." Again, while this definition adequately covers most of the observed data, there were certain instances where it was difficult to determine if actual tutoring was taking place. For example, there were certain occasions where a student had books on the table and was talking to the tutor, however, the conversation was happening with the student and tutor leaned back in their chairs, not looking at the book or a piece of paper. If this relaxed conversation lasted more than a minute before the tutor and student appeared to be reviewing the book or homework again, the conversation was excluded as part of the service time, per the assumption of service time length. The assumption and definition of service time length was also a limitation in this study, as it was often difficult to determine a start and stop time for lengthy tutoring sessions. As per the definition, "Service time is defined as the total time difference between the start of the tutoring session to when the tutoring session ends. This includes any gap in tutoring of less than one minute for tutoring sessions with more than one question." The limitation of this assumption likely affected all inter-arrival request rates and all service time lengths, because if the same student waited more than 60 seconds before asking a second, third or fourth question, then it was treated as a new tutoring request and the service time was reset.

In some instances a student would briefly enter the Physics & Chemistry Lab with a single sheet of paper and speak with a tutor for a few moments. It appeared that the student was actually receiving help from the tutor, however, per the defined assumption of who is included and considered to have entered the room, these students were excluded from the recorded data.

The second type of limitation in this study relates to the data collection method. Observation through video recordings was the method used to collect data for this study. This method was chosen over live observation, paper sign-in and out sheets at the front door and tutoring logs completed by the tutors. Live observation was not chosen because of its limitations to double check and verify the accuracy of the documented data, any discrepancies between clocks used, the inaccuracy of short-term memory and the difficulty in finding other raters to observe and record five weeks (75 hours) of data collection per room. Voluntary paper sign-in and out sheets were also eliminated as a data collection method, as this method proved to be difficult to control in the Fall when the Unified Tutoring Center first opened. This method was initially used in the Fall to try and track how many students were utilizing various rooms and how many were seeking help from the tutors. This method captured roughly 20% of the students entering the rooms and was eventually eliminated as a usage tracking method. Tutoring logs were also eliminated as a data collection method because not all tutors are accurate in the information they record and could be a possible distraction or interruption to the learning environment. Many tutors are good about getting each student's information, however the service time is an approximation and there is no space for recording a student's wait time.

Based on the available data collection methods, observation through video recordings was chosen to be the best option, although its primary limitation is that only "observable" data can be recorded. There is no opportunity to ask a student and/or tutor if they were actually discussing school work of if they were discussing their plans for this weekend and there is no opportunity to verify a student's actual wait time. The service time and waiting times are limited to the defined assumptions/definitions and properly working technology.

The final limitation in this study was the amount of data collected. While some subjects like Engineering Science, Computer Programming and Business had adequate sample sizes (about 100 requests/sessions), other subjects did not. For example, Corporate Finance had about 35 observed data points, while Aeronautical Science only had about 13 observations. Few conclusions can be drawn about these subjects due to the inadequate sample size. The Physics & Chemistry Lab, however, had approximately 460 observations, a very nice sample size for only a four week period.

Areas of Future Research

This study laid a solid foundation for future areas of research relating to the tutoring demands on the Unified Tutoring Center, although much more data is needed. This investigation studied and analyzed four weeks of data from the middle of the semester, which were considered to be normal operational weeks. One area of future research may investigate the operational demand on the tutoring center of busy versus normal weeks. There are typically three different weeks during the semester when many teachers give tests and assign projects that are due. These three weeks could be considered busy weeks, with all other weeks being treated as normal weeks. It would be interesting to compare the different weeks and see if more tutors are needed to meet the same constraints placed on this study.

Another direction of research could examine the relationship between peak demand nights and test schedules. There were two nights in the Physics & Chemistry Lab where there were so many students that it was hard to find an open seat. Several students mentioned that many of them were taking the same professor, who happened to be giving a test the next day.

Like most college students, these students were panicking and seeking help at the last minute. If a future study were to examine the correlation between peak nights and when tests are given, an even better tutor staffing schedule could be developed at the beginning of each semester, based on the test schedules for different departments. If a correlation is found, additional tutors could be brought in on the nights before tests, providing a higher level of service to the students while also minimizing the operational cost on non-peak evenings.

These suggested studies or any other future areas of research should involve more than four weeks of data collection, especially for the subjects of Corporate Finance and Aeronautical Science. A minimum of one semester of data, preferably two semesters of data should be used in any future studies.

Conclusion

This study applied discrete event simulation (DES) through the use of Arena to simulate the General Study Room and the Physics & Chemistry Lab of the Unified Tutoring Center to determine an optimized tutor staffing schedule, based on student demands. A simulation model was built for each room, depicting the characteristics under investigation. The observed historical data was used to verify and validate the simulated models before the sensitivity analysis were run. Several different constraint combinations were run during the sensitivity analysis to determine the best number of tutors to staff based on tutor utilization and student wait time, while trying to minimize the overall cost. Results from the sensitivity analysis recommended several different tutor scheduling options to choose from.

The results from this study provide the Unified Tutoring Center with scientifically based tutor scheduling changes that can reduce operating costs while still providing a high level of service to students. Specifically, the weekly cost of tutors for the General Study Room can be reduced from $714 to $586.50 (if Corporate Finance is still only offered three nights a week), a savings of $127.50. Similarly, the weekly cost of tutors for the Physics & Chemistry Lab can be reduced from $331.50 to $255, a savings of $76.50. Given that the Unified Tutoring Center is open for 13 weeks a semester, implementing these recommended staffing changes can save the UTC $2,652.00 per semester.

While these results provide a good starting point for helping the tutoring center reduce operational costs, these models can be adapted and used again for future investigations. For instance, the constraints on tutor utilization and/or student wait time could be strengthened or relaxed, based on future student demands or on the capacity of the budget.

Using discrete event simulation (DES) for a study like the Unified Tutoring Center can provide many advantages over other methods. DES provides the advantage of being able to analyze the system under a specific set of operating conditions, propose and test alternative conditions and run the simulation for a long period of simulated time in a matter of minutes or even seconds. This investigation capitalized on all of these advantages, especially the ability to propose and test alternative conditions, the sensitivity analysis. The most beneficial advantage of DES to the Unified Tutoring Center is the ability to manipulate and test system design or staffing changes in the simulated model, before making the decision to implement the change on the real system.

References

Altoik, T. & Melamed, B. (2007). *Simulation modeling and analysis with Arena*. Massachusetts: Academic Press.

Banks, J, Carson, J.S. & Nelson, B.L. (1996). *Discrete-event system simulation: Second edition*. New Jersey: Prentice Hall.

Bieger, A., Borges, G., Kranz, S., McGowan, C., Meehan, K., Mancuso, G., Guerlain, S & Guimarães, L. (2009). Increasing the efficiency of a Brazilian emergency response call center. *Proceedings of the 2009 IEEE Systems and Information Engineering Design Symposium, 2009*, 125-130.

Blanchard, B.S. & Fabrycky, W.J. (2006). *Systems engineering and analysis: Fourth edition*. New Jersey: Pearson Prentice Hall.

Brann, D.M. & Kulick, B.C. (2002). Simulation of restaurant operations using the restaurant modeling studio. *Proceedings of the 2002 Winter Simulation Conference, 2002*, 1448-1453.

Cao, Y., Nsakanda, A.L. & Pressman, I. (2003). A simulation study of the passenger check-in system at the Ottawa International Airport. *Summer Computer Simulation Conference 2003*, 573-579.

Centeno, M.A, Giachetti, R., Linn, R. & Ismail, A.M. (2003). A simulation-ILP based tool for scheduling ER staff. *Proceedings of the 2003 Winter Simulation Conference, 2003*, 1930-1938.

Chong, K., Grewal, M., Loo, J. & Oh, S. (2003). A simulation-enabled DSS for allocating check-in agents. *Canadian Operational Research Society, Vol. 41, Iss. 3*, page 259.

Cvijovic, D. & Klinowski, J. (1995). Closed-form summation of some trigonometric series. *Mathematics of Computation, Vol. 64*, 205–210.

Fishwick, P. (1995). *Simulation model design and execution: Building digital worlds.* New Jersey: Prentice Hall.

Gordon, G. (1978). *System simulation: Second edition.* New Jersey: Prentice Hall.

Hancock, W. & Chan, T. (1988). Productivity and staffing of hospital units with uncertainty in the demand for service. *IIE Transactions, Vol. 20, Iss. 4,* 346-353.

Howell, D. (2007). *Statistical methods for psychology: Sixth edition.* California: Thomson Wadsworth.

Kelton, W.D, Sadowski, R.P. & Sturrock, D.T. (2007). *Simulation with Arena: Fourth edition.* New York: McGraw Hill.

Kontoyiannakis, K., Serrano, E., Tse, K., Lapp, M. & Cohn, A. (2009). A simulation framework to evaluate airport gate allocation policies under extreme delay conditions. *Proceedings of the 2009 Winter Simulation Conference, 2009*, 2332-2342.

Law, A.M. & Kelton, W.D. (2000). *Simulation modeling and analysis: Third edition.* New York: McGraw Hill.

Law, A.M. & McComas, M.G. (2001). How to build valid and credible simulation models. *Proceedings of the 2001 Winter Simulation Conference, 2001*, 22-29.

Rockwell Automation. (2010). *Arena simulation software.* Retrieved from http://www.arenasimulation.com. Retrieved on 3-1-10.

Rossetti, M.D. (2010). *Simulation modeling and Arena.* New Jersey: John Wiley & Sons.

Sickinger, S. & Kolisch, R. (2009). The performance of a generalized Bailey-Welch rule for outpatient appointment scheduling under inpatient and emergency demand. *Health Care Management Science, Vol. 12, Iss. 4*, page 408.

Tateno, T. & Shimizu, K. (2007). Work scheduling support in consideration of skill level and training of workers. *Transactions of the Japan Society of Mechanical Engineers, Part C, Vol. 73, Num. 10*, 2854-2862.

Appendix A: Data Collection Form

Date: ______________ Room: ______________ Evaluator: ______________

Enter Room	Student Description	Location (Left/Right + Row #)	Subject	Wait		Tutoring		Total w/ Tutor	Exit Room
				Start	Stop	Start	Stop		

Appendix B: Unified Tutoring Center Constraints Survey

Demographic Information: Please Complete the Following

Which are You? Please circle one of the following:	Student	Tutor	Committee Member
Please give your major (if applicable):	______________________		
How often have you utilized the tutors? Please circle one of the following:	0 times or n/a	1-10 times	+10 times
Which subject(s) have you received help with (if applicable)?	______________________		

Question 1: What should be the minimum and maximum number of tutors staffed on any given night? Please indicate your response in the table below.

Subject	**Minimum Staffed**	**Maximum Staffed**
Business		
Corporate Finance		
Engineering Science		
Physics & Chemistry		
Aeronautical Science		
Computer Programming		
Aviation Maintenance Science		

Question 2: On average, what should be the maximum student wait time? ________minutes

Question 3: What should be the maximum student wait time, per session? ________minutes

- -

<u>For Unified Tutoring Center Committee ONLY!!!</u>

Question 4: What should be the minimum and maximum tutor utilization boundaries (% of a tutor's time spent helping students)?

Please indicate your answer as a percentage (%) of time of the possible 180 minutes we are open in the evenings.

- Minimum Utilization
  - Answer: _______%
    - At what point are the tutors underutilized? When can we cut back on the number of staffed tutors?
- Maximum Utilization
  - Answer: _______%
    - At what point are the tutors over utilized and working too much? When do we need to hire an additional tutor?

Appendix C: Empirical Distributions

The empirical distribution for Aeronautical Science tutoring requests is:

DISC (0.091, 6.500, 0.182, 7.500, 0.364, 9.500, 0.455, 16.500, 0.545, 24.500, 0.727, 28.500, 0.818, 30.500, 1, 38.500)

The empirical distribution for Corporate Finance service time is:

CONT (0.194, 1.500, 0.333, 2.500, 0.528, 3.500, 0.611, 4.500, 0.667, 8.500, 0.694, 10.500, 0.722, 11.500, 0.833, 12.500, 0.861, 13.500, 0.889, 14.500, 0.917, 17.500, 0.944, 26.500, 0.972, 27.500)