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B802: Base-Age Invariant Polymorphic Site Index Curves for Even-Aged Spruce-Fir Stands in Maine

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BASE-AGE INVARIANT POLYMORPHIC SITE INDEX CURVES FOR EVEN-AGED SPRUCE-FIR STANDS IN MAINE

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ABSTRACT

Stem analysis data previously collected from 198 circular plots in even-aged spruce-fir stands throughout western, northern and eastern Maine were used in developing site index curves. Regression of a height model where Height = $b_1(1-e^{b_2(Age)})$ provided the best fit of linear and nonlinear models tested. Subsequent transformation resulted in base-age invariant polymorphic site index equations for spruce and balsam fir. Statistical tests and graphical comparisons showed no significant effects of either stand density or location on height growth patterns. A method was developed for estimating the sample size necessary to attain confidence intervals of a specified width about site index curves.

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BASE-AGE INVARIANT POLYMORPHIC SITE INDEX CURVES FOR EVEN-AGED STANDS OF SPRUCE AND BALSAM FIR IN MAINE

By

Bret P. Vicary, Thomas B. Brann, and Ralph H. Griffin*

INTRODUCTION

The spruce-fir forest cover type, occupying nearly 8 million acres in Maine, accounts for approximately 50 percent of the growing stock volume in the State. A similar portion of Maine's commercial forest land is owned and managed by forest industry, with spruce and fir being the mainstay of the industry (10).

Analyses in recent years of Maine's timber supply have shown softwood removals to exceed growth. The dramatic effects of the spruce budworm (Choristoneura fumiferana (Clemens)) on the spruce-fir forest of Maine has heightened concern over the timber supply. A greater emphasis on management is necessary if timber growth is to keep pace with demand. With increasing demand for timber, and the increasing value of timber products, intensive management is becoming economically feasible. High labor costs have led to an increase in mechanized harvesting. Consequently, the stage has been set for a greater emphasis on even-aged management of the spruce-fir forest type in Maine. As red spruce (Picea rubens Sarg.)[1], black spruce (Picea mariana (Mill.) B.S.P.), white spruce (Picea glauca (Moench) Voss), and balsam fir (Abies balsamea (L.) Mill.) are the backbone of Maine's forest industry, it is desirable to

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[1]Scientific names according to Little, E.L., Jr. 1979. Checklist of United States Trees (Native and Naturalized). USDA Agriculture Handbook No. 541. For. Serv., U.S. Department of Agriculture. 375pp.

identify those sites best suited to the growth of these species.

An easily attained and sufficiently accurate method of estimating the relative quality of a particular site is essential to sound forest management. Site index, defined as being the height attained by the dominant stand at an arbitrarily chosen age, commonly 50 years in the northeastern United States, has been the most widely used measure of site quality (17). In addition to being an easily measured indicator of relative site quality, site index provides a crucial parameter in the estimation of the ultimate capability of forest land to produce wood volume (7).

A graph of site indices for a given species consists of a series of height-over-age curves. Early anamorphic (harmonized, proportional) curve construction described by Bruce (4) and Osborne and Schumacher(23) was based on the assumptions that the shape of the height growth curve was the same (proportional) for all sites, and the average site quality was the same for each age class. A polymorphic approach was introduced by Bull in 1931 for red pine (Pinus resinosa Ait.) plantations (5). The assumption of the polymorphic approach is that height curves for a species may differ in shape depending upon the site. Trees situated on favorable sites grow in height more rapidly and taper off in growth more abruptly than those growing on less favorable sites (5, 7).

Today, the polymorphic approach is considered to be superior to the anamorphic approach in the construction of site index curves.

Heger (13) found site index curve shapes to differ markedly depending upon the choice of the base age, even though the curves are adjusted to pass through the same points at the index age. Because site index, as a variable, enters conventional models prior to regression, the estimates of general parameters are unique to the preselected base age. Bailey and Clutter (2) addressed this problem in developing base-age invariant site index curves for Monterey pine (Pinus radiata D. Don).

Three prior studies (3, 18, 29) developed individual site index curves for spruce and balsam fir growing in western, northern, and eastern Maine, respectively. Data collected in those studies were used in this study to (1) test for regional differences in the height growth patterns of spruce and balsam fir, and (2) formulate base-age invariant polymorphic site index curves for each species.

STUDY AREA

The eastern region lies in the downeast portion of Maine, and includes parts of Aroostook, Penobscot, Hancock, and Washington Counties. The northern region includes Aroostook County and the northern portion of Piscataquis County, and the western region encompasses parts of Piscataquis, Somerset, Franklin, and Oxford Counties (Figure 1). The choice of 46 degrees north latitude as the division between the northern region and the other two study areas arose out of the desire to divide Maine's spruce-fir forest land into convenient sample units, and to minimize the effects of climate on tree growth within a study area.

The three natural climatological divisions in Maine defined by Lautzenheiser (19) are depicted in Figure 1. The eastern region, with plots falling within the Southern Interior Division, experiences 40 to 44 inches of precipitation per year, with 60 to 90 inches of snow. The Northern Division, which encompasses both the western and northern regions, averages near 40 inches of precipitation yearly. Snowfall in the Northern Division ranges from an average of 90 to 110 inches, with greatest annual snowfall in the area between Moosehead Lake and the Canadian border.

The average annual temperature ranges from near 40° F in the Northern Division to 44°F in the Southern Interior Division. Within the Northern Division there is a range of from near 37°F to 43°F from north to south. The average date of last freezing temperature in the spring varies from early May in the Southern Interior Division to the end of May in extreme northern Maine. The freeze-free season over the two divisions usually ends in September, with the mean number of frost-free days ranging from 120 to 140 in the Southern Interior Division, 100 to 120 over most of the Northern Division, and less than 100 in extreme northern and northwestern Maine.

The amount of sunshine averages from 50 to 60 percent of the potential over most of the southern half of Maine, and drops to near 45 percent at high elevations and over much of northern Maine.

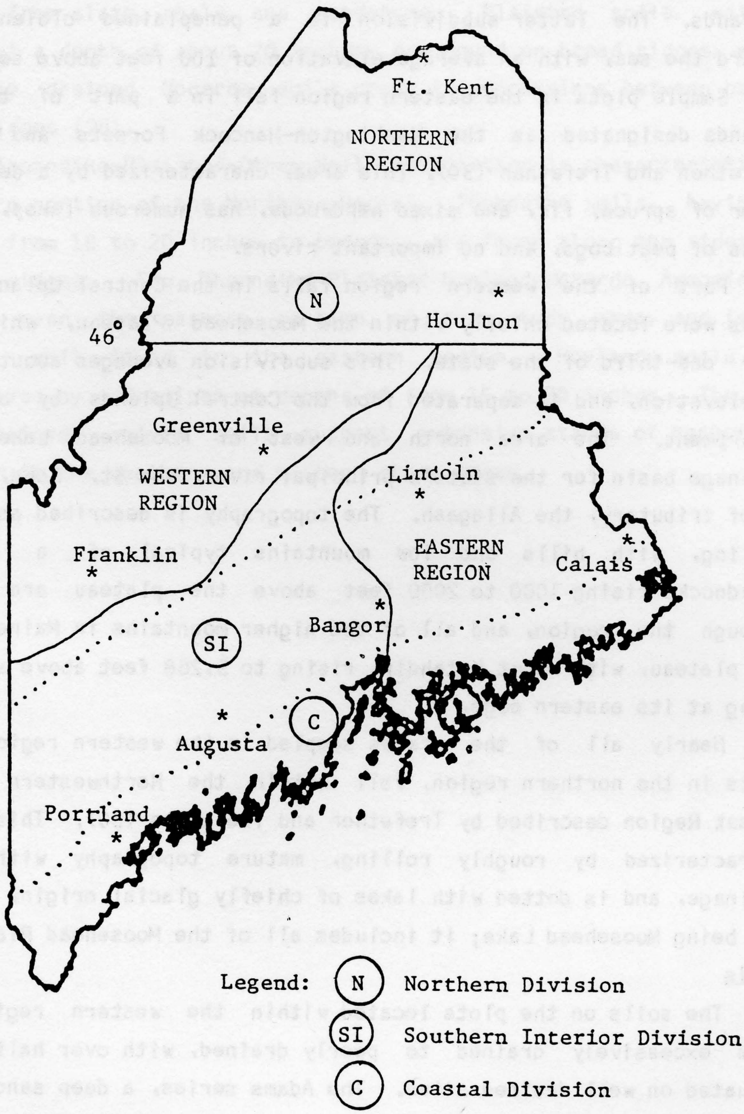

Figure 1. Location of Study Regions (delineated by solid lines) and Climatological Divisions as Described by Lautzenheiser (delineated by dotted lines).

The eastern region occurs in both the Central Uplands and Coastal Lowlands. The latter subdivision is a peneplained oldland sloping toward the sea, with an average elevation of 100 feet above sea level.

Sample plots in the eastern region fell in a part of the Central Uplands designated as the Washington-Hancock Forests and Barrens by Trefethen and Trefethan (30). This area, characterized by a dense forest cover of spruce, fir, and mixed hardwoods, has numerous lakes, extensive areas of peat bogs, and no important rivers.

Part of the western region falls in the Central Uplands, though plots were located chiefly within the Moosehead Plateau, which covers over one-third of the state. This subdivision averages about 1000 feet in elevation, and is separated from the Central Uplands by a distinct escarpment. The area north and west of Moosehead Lake forms the drainage basin for the state's principal river, the St. John, and its chief tributary, the Allagash. The topography is described as generally rolling, with hills and low mountains typical of a peneplain. Monadnocks rising 1000 to 2000 feet above the plateau are scattered through the region, and all of the higher mountains in Maine rise from the plateau, with Mount Katahdin, rising to 5,268 feet above sea level, lying at its eastern edge.

Nearly all of the stands sampled in the western region, and all plots in the northern region, fell within the Northwestern Lake and Forest Region described by Trefethen and Trefethen (30). This region is characterized by roughly rolling, mature topography with immature drainage, and is dotted with lakes of chiefly glacial origin, the largest being Moosehead Lake; it includes all of the Moosehead Plateau.

Soils

The soils on the plots located within the western region ranged from excessively drained to poorly drained, with over half the plots situated on well drained sites. The Adams series, a deep sandy soil of outwashed granitic material, characterizes many of the sites sampled in the southern portion of the area (27). Soils of the Plaisted series were found in the northern portion of the study region, and the Monarda series was well represented throughout the area. The Plaisted and

Monarda soil series are strong, acidic, glacial till soils, derived primarily from slate, shale, and sandstone. Plaisted soils, with a fragipan at a depth of about 20 inches, are found on broad ridges, while the poorly drained Monarda soils occur in depressions between better drained ridges (28).

The Thorndike-Plaisted-Other Soils Association is characteristic of the western portion of the Northern Region. Thorndike soils, having a depth of from 18 to 20 inches to bedrock, are found along the sides and tops of ridges. The Thorndike-Plaisted-Howland-Monarda Association dominates over the eastern portion of the study area, and is the principal soil found in the eastern region. Howland soils are characterized by a fragipan at depths of from 15 to 20 inches. The more poorly drained, acidic soils support extensive stands of spruce and balsam fir in the northern and eastern study areas.

COLLECTION OF FIELD DATA

Sample plots were located in the three study areas over a period of nine years. A total of 87 plots was measured in western Maine during the summers of 1969 and 1970 (3, 28), 65 plots in Northern Maine during the summers of 1974 and 1975 (18,1), and 55 plots in eastern Maine during the summers of 1976 and 1977 (29, 12), for a total of 207 plots.

Company maps, cutting records, and photographs were used in locating suitable even-aged spruce-fir stands. A sample plot had to contain only spruce or balsam fir trees in both the overstory and understory. Initially, a plot was to be considered even-aged if the range in breast-height age of its overstory trees was no greater than ten years. However, the age-range criterion was expanded to 15 years when difficulty was encountered in locating enough plots that satisfied the initial prerequisite. In sparsely stocked stands, a tree was considered to be in the overstory if its height was at least 75 percent of the height of the tallest tree on the plot. Overstory trees had to be at least five years of age at breast height.

The stand had to have developed naturally on mineral soils, without any apparent serious injury from insects, disease, or other elements. Stands having developed on organic soils, commonly poorly drained peat, were not sampled, as height growth relationships were not thought to be consistent with those in stands growing on mineral soils. Heger and Lowry (14) found that black spruce site index curves differed significantly in shape between wet and dry sites.

Each sample plot was circular and large enough to encompass ten overstory trees. The plot center was defined as being one foot to the northeast of a suitable overstory tree (usually the tallest tree in the stand). The ten overstory trees were marked and numbered in increasing distance from the plot center to the far side of each tree. The plot radius was established as the distance from the plot center to a point midway between the far side of the tenth overstory tree and the far side of the next closest overstory tree outside the plot.

Measurements taken on each plot and used in this study include the following:

Overstory Trees

1. Tally by species
2. Diameter at breast height (d.b.h.) to the 0.05 inch
3. Total height to the nearest 0.1 foot
4. Total age at the root collar, at breast height, and at the top of each successive four-foot section of the bole.

Understory Trees

1. Tally by species
2. D.b.h. to the 0.05 inch.

Mean values for the stand parameters measured are presented in Table 1.

TABLE 1
Mean Values For Sample Plot Stand Parameters

	West		North		East	
Variable	Spruce	Fir	Spruce	Fir	Spruce	Fir
D.B.H.[1] (inches)	5.08	4.76	5.56	4.65	5.23	4.53
Total Height[1] (feet)	40.1	39.6	43.7	43.3	40.8	38.2
Total Age[1]	59.2	50.5	63.7	64.2	66.8	53.7
Breast Height Age[1]	42.9	38.9	48.2	50.8	51.6	41.2
Basal Area[2]	213		200		199	
Stand Density Index[2] (Reineke, 1933)	550		495		509	
Trees Per Acre[2]	2931		1895		2393	

1 Overstory trees only.

2 Both overstory and understory trees included, spruce and fir combined.

ANALYSIS OF DATA

The first stage in the development of site index equations involved the regression of height equations, relating the heights and ages recorded from stem analysis. The pooled data from all three regions were stratified into 20-year age classes, and from each age class a random sample of 25 percent of the height-age pairs was selected. This test sample was useful later in ascertaining which parameters estimated from the larger portion of the data were contributing sustantially to the prediction of the dependent variable, and which parameters were serving merely to enhance the equation's fit to the larger data set.

Because shade-tolerant species such as the northeastern spruces and balsam fir are able to withstand extended periods of suppression early in life and then respond to natural or artificial release, poor correlation between height and age is observed prior to release. Hoar and Young (15) found red and white spruce to generally be free to grow once breast height had been attained. Similarly, Husch (16) claimed that most species do not begin normal height growth until a height of 4.5 feet has been attained. Therefore, the variable (age) was taken to be age at breast height in the study. Because a tree should be zero feet tall when zero years of age, the variable (height) was taken to be height above breast height. Although the average time to attain breast height was approximately 15 years, it ranged from less than five to over 50 years.

When trees are sectioned for stem analysis data, the section points do not necessarily coincide with the tip of the annual leader (6, 20,). Carmean, assuming the sectioning point occurred midway in the annual leader and annual height growth within a bolt was equal, developed the following formula for adjusting height-age data:

$$\text{Adjusted height} = \text{Unadjusted height} + \frac{(\text{bolt length/age difference})}{2}$$

Carmean's method was used to adjust all height-age data in the study.

It has been argued that since site index is the variable ultimately being estimated, it should enter the regression model as the dependent variable (8). However, this would mean parameter estimates are a function of some preselected base age, and any height-age pairs from trees younger than the preselected base age would be unavailable for analysis.

It was desirable to define, as accurately as possible, the relationship between height and age across all the data to reduce age bias in the analysis. By regressing height on age, all height-age pairs, regardless of the age of the tree, were used in developing height equations. By substituting base age for age and site index for height, the height equation became a site index equation. Subsequent combination of the two equations resulted in a base-age invariant equation.

Thirteen linear and nonlinear height equations were explored, with coefficients of multiple determination (R^2) ranging from .69 to .94 (31). The variables basal area, stand density index (25), and trees per acre were introduced, but found to make no significant contribution to the estimation of height-age data in the study.

Two basic nonlinear height models were tested and found to result in higher R^2 values and lower standard errors of the regression estimates (SE_r) than the linear models. A model proposed by Richards (26) and later developed for site index by Lundgren and Dolid (21) of the following form was tested:

$$\text{Height} = b_1(1-e^{b_2(\text{Age})})^{b_3} \qquad (1)$$

where: e = base of the Naperian logarithm

The following variation on Equation 1, which was examined by Carmean (6), was also tested:

$$\text{Height} = b_1(1-e^{b_2(\text{Age})}) \qquad (2)$$

A second basic height equation tested, which was developed by

Bailey and Clutter (2) and used successfully on Monterey pine, was also tested:

$$\text{Log(Height)} = b_1 + b_2(1/\text{Age})^{b_3} \quad (3)$$

Equation 1 was found to be inconsistent over the range of data for northern Maine (18,11). Equation 3 was used successfully by Schiltz (29) on eastern Maine height-age data.

Regressions of Equations 1, 2, and 3 on the pooled 75 percent base data resulted in R^2 values of from .9574 to .9707, and values for SE_r ranging from 5.3 to 6.1 feet.

Equations 1, 2, and 3 were then used to predict individual tree heights from age observations in the 25 percent residual test data. Standard errors ranged from 6.8 to 7.4 feet, with no one equation standing out as being superior. Equation 1 did not predict height any more accurately than Equation 2. The b_3 parameter in Equation 1 did not contribute to the accuracy of height prediction, so the simpler form of Equation 2 was preferred. While statistical comparisons of Equations 2 and 3 resulted in little apparent differences, graphical comparison showed a tendency for Equation 3 to overpredict height at advanced ages.

As a final comparison between Equations 2 and 3, site index values were predicted using height and age observations from the residual test data. Equation 2 was converted to a base-age invariant site index equation through several steps. First, Equation 2 was expressed in terms of site index as follows:

$$SI = b_1(1-e^{b_2(A_b)}) \quad (4)$$

where: SI = site index

A_b = base age

To achieve polymorphism, SI must be a function of the b_2 parameter in Equation 4, resulting in the following: $b_2 = \frac{Ln(1 - \frac{SI}{b_1})}{A_b}$ (5)

Substituting the expression for b_2 into Equation 2 results in the following expression:

$$Height = b_1(1-e^{(\frac{Ln(1-(SI/b_1))}{A_b})(Age)}) \quad (6)$$

Equation 6 is used to build site index curves by predicting height, and may be rewritten to predict site index as follows:

$$SI = b_1(1-e^{(A_b/Age)(Ln(\frac{Height - b_1}{-b_1}))}) \quad (7)$$

A similar procedure was followed in transforming Equation 3 into a base-age invariant polymorphic site index equation, resulting in the following two equations:

$$Height = 10^{b_1}(SI/10^{b_1})^{(A_b/Age)^{b_3}} \quad (8)$$

$$SI = 10^{b_1}(Height/10^{b_1})^{(1/(A_b/Age)^{b_3})} \quad (9)$$

Equations 7 and 9 were used to predict site index of the residual test data trees at base age 50. Standard errors of estimate were 6.2 feet for spruce and 6.7 feet for fir using Equation 7, compared with 7.3 feet for spruce and 7.2 feet for fir using Equation 9. The same comparisons were made at base age 25 to increase the number of observations[2]. With the younger trees and base age, little difference

[2]Only trees of at least base age can be used in this test; about twice as many trees were at least 25 years old at breast height, compared with the number at least 50 years old.

was found between Equations 7 and 9 in the accuracy of site index prediction, as all standard errors ranged from 3.4 to 3.7 feet.

Plotting site index curves from Equations 6 and 8 highlighted further differences in the nature of the curves. The b_1 parameter in Equation 6 is the asymptotic or ultimate height attained for a species, so all curves eventually converge on this height. High site index curves exhibited rapid initial height growth and subsequent rapid decline in height growth, while low site index curves showed slow initial height growth with a gradually decreasing slope throughout the life of a tree.

Site index curves from Equation 8 followed those of Equation 6 quite closely in early years, but overpredicted site index at advanced ages. In fact, the parameter estimates showed increasing slope, or rate of height growth, ad infinitum.

On the basis of the above comparisons, the root height equation, Equation 2, and its transformed height and site index equations, Equations 6 and 7, were selected to describe height-age relationships in the study.

RESULTS

The base data and residual test data were pooled for final regression analysis, including observations from 82 plots in western Maine, 62 plots in northern Maine, and 54 plots in eastern Maine, for a total of 198 plots. These data included 24 plots which were uneven-aged (i.e., with greater than 15 years difference between the youngest and oldest breast height ages of the overstory trees). The following height equations resulted:

Spruce

$$\text{Height} = 110.9866(1-e^{-.0086(\text{Age})}) \quad (10)$$

$R^2 = .9524$

$SE_r = 5.7$ feet

Balsam Fir

$$\text{Height} = 112.3188(1-e^{-.0093(\text{Age})}) \quad (11)$$

$R^2 = .9440$

$SE_r = 6.2$ feet

Nonlinear regression of the pooled data excluding the 24 uneven-aged plots resulted in coefficients and statistical measures nearly identical to the above results. Because of the difficulty in ascertaining whether a spruce or balsam fir stand that appears even-aged is actually so under normal field sampling situations, the uneven-aged plots were left in the analysis. Equations 10 and 11 represent the final height equations. The asymptotic heights represented by the b_1 estimates are 110.9866 feet for spruce and 112.3188 feet for balsam fir.

To test for differences in regional height growth patterns, Equation 2 was regressed on data from each study region. Neither statistical tests nor graphical comparison showed significant differences among regional height growth curves. Figure 2 shows pooled and regional height curves for balsam fir plotted together.

Site index equations derived from regional and pooled height equations also showed little difference between height estimates along the site curves. The maximum difference in predicted height between regional and pooled site index curves at breast height ages from 10 to 70 years along the site index 50, base age 50 curves, was 3.7 feet.

Differences between regional and pooled heights and site index curves were well within the standard error of regression estimates for all equations, so there is little support for segregating the data and equations by region. The pooled equations are sufficient to describe height-age relationships for even-aged stands of spruce and balsam fir in Maine.

The final height and site index equations, derived from Equations 10 and 11, are as follows:

Spruce

$$\text{Height} = 110.9866(1-e^{(\frac{\ln(1-(SI/110.9866))}{A_b})(\text{Age})}) \quad (12)$$

$$SI = 110.9866(1-e^{(\frac{A_b}{\text{Age}})(\ln(\frac{\text{Height} - 110.9866}{-110.9866}))}) \quad (13)$$

Balsam Fir

$$\text{Height} = 112.3188(1-e^{(\frac{\ln(1-(SI/112.3188))}{A_b})(\text{Age})}) \quad (14)$$

$$SI = 112.3188(1-e^{(\frac{A_b}{\text{Age}})(\ln(\frac{\text{Height} - 112.3188}{-112.3188}))}) \quad (15)$$

The final site index curves for spruce and balsam fir are shown for base age 50 in Figures 3 and 4. Tabular values are presented in Tables 2 and 3, respectively.

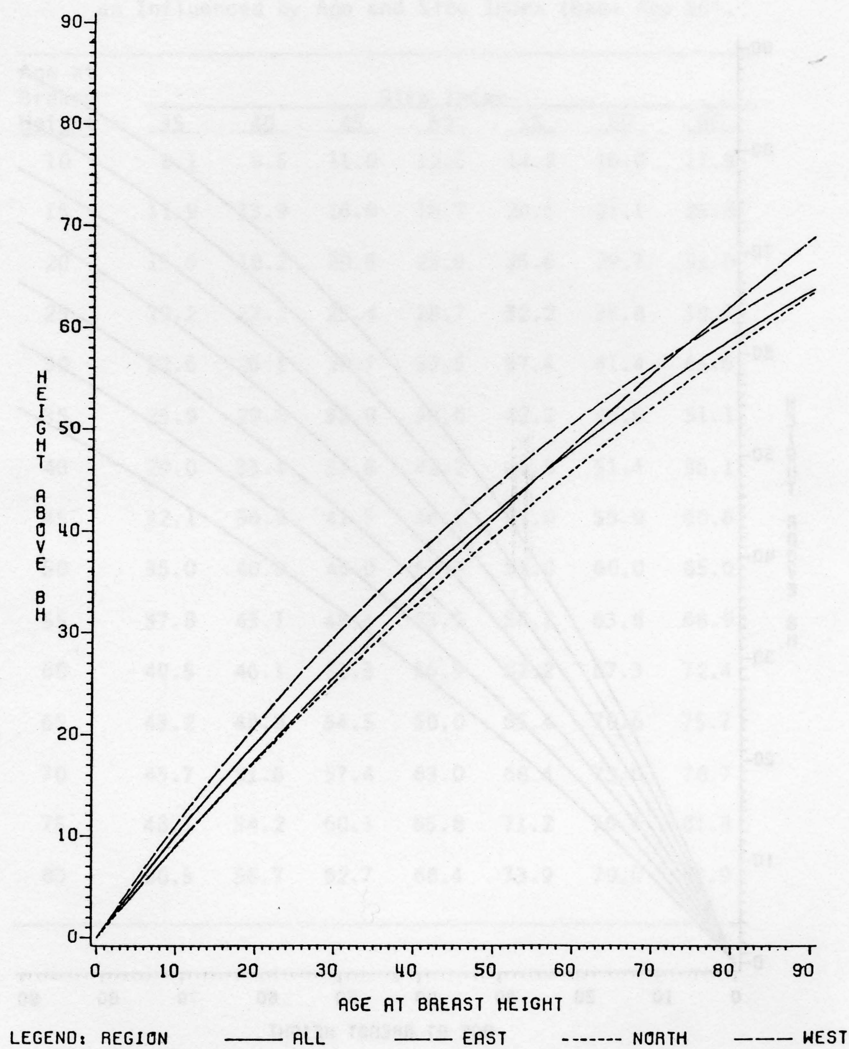

Figure 2. Regional Height Curves as Compared with the Pooled Height Curve for Balsam Fir.

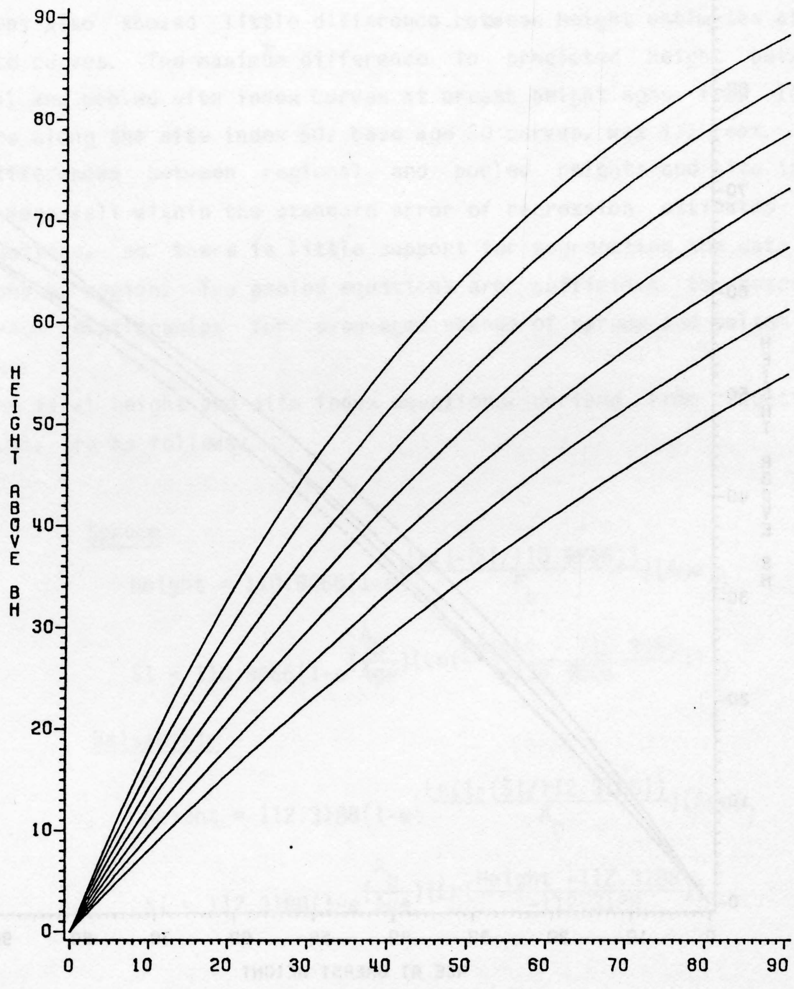

Figure 3. Final Site Index Curves (A_b=50) for Spruce from Equation 12.

TABLE 2

Height in Feet Above Breast Height of Spruce
as Influenced by Age and Site Index (Base Age 50).

Age at Breast Height	Site Index						
	35	40	45	50	55	60	65
10	8.1	9.5	11.0	12.5	14.2	16.0	17.9
15	11.9	13.9	16.0	18.2	20.6	23.1	25.8
20	15.6	18.2	20.8	23.6	26.6	29.7	33.0
25	19.2	22.2	25.4	28.7	32.2	35.8	39.5
30	22.6	26.1	29.7	33.5	37.4	41.4	45.6
35	25.9	29.8	33.9	38.0	42.2	46.6	51.1
40	29.0	33.4	37.8	42.2	46.8	51.4	56.1
45	32.1	36.8	41.5	46.2	51.0	55.9	60.8
50	35.0	40.0	45.0	50.0	55.0	60.0	65.0
55	37.8	43.1	48.3	53.5	58.7	63.8	68.9
60	40.5	46.1	51.5	56.9	62.2	67.3	72.4
65	43.2	48.9	54.5	60.0	65.4	70.6	75.7
70	45.7	51.6	57.4	63.0	68.4	73.6	78.7
75	48.1	54.2	60.1	65.8	71.2	76.4	81.4
80	50.5	56.7	62.7	68.4	73.9	79.0	83.9

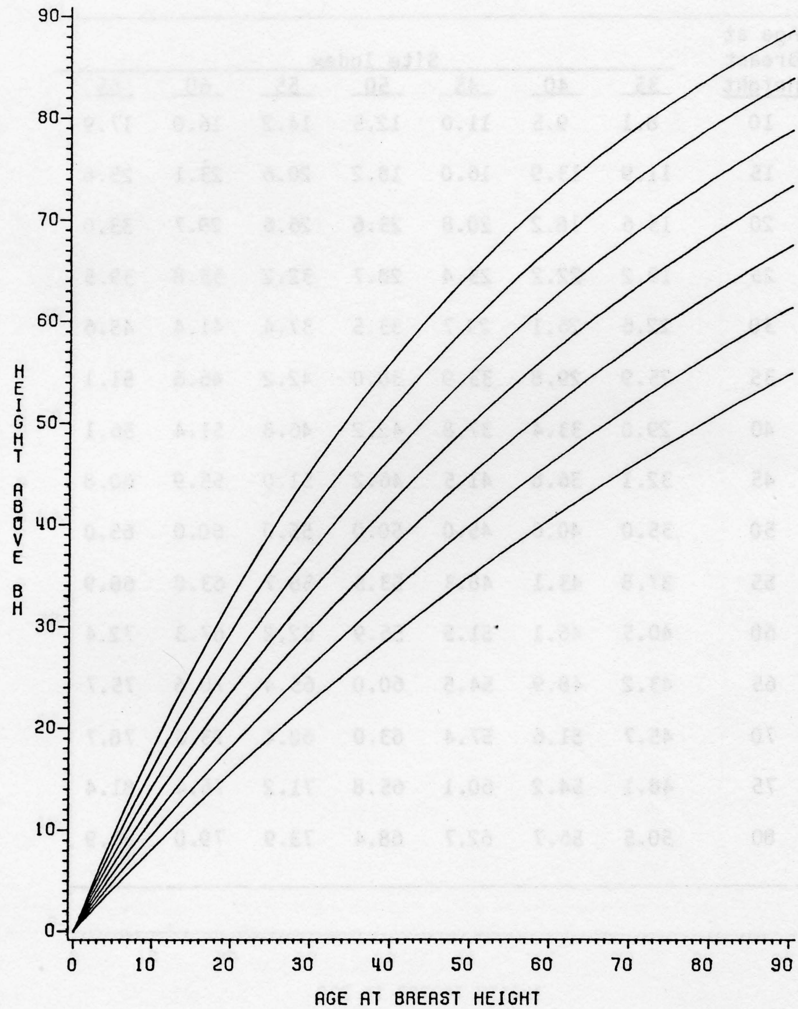

Figure 4. Final Site Index Curves (A_b=50) for Balsam Fir from Equation 14.

TABLE 3
Height in Feet Above Breast Height of Balsam Fir
as Influenced by Age and Site Index (Base Age 50).

Age at Breast Height	Site Index 35	40	45	50	55	60	65
10	8.1	9.5	10.9	12.5	14.1	15.9	17.8
15	11.9	13.9	16.0	18.2	20.5	23.0	25.7
20	15.6	18.1	20.8	23.6	26.5	29.6	32.8
25	19.1	22.2	25.4	28.7	32.1	35.7	39.4
30	22.5	26.1	29.7	33.4	37.3	41.3	45.5
35	25.8	29.8	33.8	38.0	42.2	46.5	51.0
40	29.0	33.3	37.7	42.2	46.7	51.4	56.1
45	32.1	36.7	41.5	46.2	51.0	55.8	60.7
50	35.0	40.0	45.0	50.0	55.0	60.0	65.0
55	37.8	43.1	48.4	53.6	58.7	63.8	68.9
60	40.6	46.1	51.6	56.9	62.2	67.4	72.5
65	43.2	48.9	54.6	60.1	65.5	70.7	75.8
70	45.7	51.7	57.5	63.1	68.5	73.8	78.8
75	48.2	54.3	60.2	65.9	71.4	76.6	81.6
80	50.5	56.8	62.8	68.6	74.0	79.2	84.1

Although 5-foot site index classes are shown in Figures 3 and 4, there remains the question of what class width is appropriate. McQuilken and Rogers (22) stated that confidence intervals should be included with site index estimates to avoid implications of greater precision than are statistically justifiable. The class interval should never be narrower than a confidence interval about the curve. As the distribution about a nonlinear function is unknown, it was necessary to use a linear approximation of Equations 10 and 11 by holding b_2 constant and using linear regression to solve for the b_1 parameters (31). The following formula for calculating variable-width confidence intervals about a regression curve, taken from Draper and Smith (9), was used:

$$C.I. = t_{\alpha/2}\bar{S}\sqrt{\frac{1}{n} + \frac{1}{m} + \frac{(X_k - \bar{X})^2}{SS_x}} \qquad (16)$$

where: C.I. = confidence interval

S = estimated standard deviation of population

n = sample size of population

m = future field sampling size

X_k = breast height age of future sample

$\bar{X}$ = mean breast height age of population

SS_x = corrected sums of squares of ages

In the working model of Equation 16, it was necessary to substitute the expression $(1-e^{b X_k}) - (1-e^{b \bar{X}})$ for $(X_k - \bar{X})$. Table 4 presents the parameters necessary for calculating confidence intervals for the height curves produced by Equations 10 and 11. A sample calculation of the 95 percent confidence interval about Equation 11, when m is 30 and X_k is 50, is shown below:

$$t_{\alpha/2} = t_{.05/2} = t_{.025} = 2$$

$$SE_r = 6.1787 \quad n = 8525 \quad m = 30 \quad b_2 = -.0101$$

$$X_k = 50 \quad \bar{X} = 24.0 \quad SS_x = 141.6963$$

$$C.I. = 2(6.1787)\sqrt{\frac{1}{8525} + \frac{1}{50} + \frac{((1-e^{-.0101(50)}) - (1-e^{-.0101(24)}))^2}{141.6963}}$$

$$= 2.3 \text{ feet}$$

TABLE 4

Parameters for Calculating Confidence Intervals about Site Index Curves

Species	SE_r	n	b_2	X	SS_x
Spruce	5.7085	11,328	-.0092	26.5	187.5912
Fir	6.1787	8,525	-.0101	24.0	141.6963

In a 50-year old stand of fir, the confidence interval about predicted height is ± 2.3 feet with a sample size of 30 trees. Equation 16 may be rewritten to calculate the sample size required to produce a desirval as follows:

$$m = 1/CI^2/(t^2SE_r{}^2)) - \frac{1}{n} - \frac{((1-e^{b_2X_k})-(1-e^{b_2\bar{X}}))^2}{SS_x} \quad (17)$$

If the desired 95 percent confidence interval is ± 2.5 feet, the desired sample size, m, is estimated as follows:

$$m = 1/2.5^2/(2^2(6.1787^2)) - \frac{1}{8525} - \frac{((1-e^{-.0101(50)}) - (1-e^{-.0101(24)}))^2}{141.6963} = 24.6\text{, or 25 trees}$$

Because the confidence interval is the product of two basic elements, the alpha level and some function of the equation's accuracy called standard error (SE; not to be confused with SE_r), it was desirable to evaluate SE at different levels of m and X_k for a given set of site index curves, and leave the choice of the probability level to the individual. The standard errors for the final site curves for values of m from one to 50 trees are presented in Table 5.

The confidence intervals calculated by Equation 16 were derived from parameters based on the linear height model. Because the

TABLE 5

Standard Error in Feet by Sample Size (m).

Sample Size (m)	Spruce	Fir
1	5.7	6.2
2	4.0	4.4
3	3.3	3.6
4	2.9	3.1
5	2.6	2.8
6	2.3	2.5
7	2.2	2.3
8	2.0	2.2
9	1.9	2.1
10	1.8	2.0
12	1.6	1.8
15	1.4	1.5
20	1.3	1.4
25	1.1	1.2
30	1.0	1.1
40	0.9	1.0
50	0.8	1.0

equations describing final site index curves in this study predicted height with greater precision than the root height equations, it is not unreasonable to assume similar confidence bands about the site index curves.

From Table 5, a sample size of six to seven trees would be sufficient to attain a 95 percent confidence interval ($t = 2$) of $\pm$ 5 feet. This would justify a site class interval of not less than ten feet. For site class intervals of five feet ($\pm$ 2.5 feet), a sample size of between 20 and 25 trees would be required. At the 90 percent level of confidence ($t = 1.65$), a sample of from four to five trees for ten-foot site class intervals, and 15 to 20 trees for five-foot intervals, would be necessary.

CONCLUSION

Regression of height and age data collected from three regions of Maine resulted in the development of site index curves for each region through identical procedures. It was possible to compare height growth of spruce and balsam fir among the three areas and eventually justify pooling the data from all regions to arrive at a series of equations for predicting height and site index in even-aged spruce-fir stands in eastern, northern, or western Maine. Payandeh (24) concluded that biomass equations for a species may be applied over broader geographical regions than those for which they were developed, and that efforts should be concentrated on broadening the data base of existing equations, rather than trying to develop a whole new set of regional equations. The same conclusion may be extended to the final height and site index equations in this study.

FIELD APPLICATION

While each plot was centered about the tallest tree that could be located in a stand, the site occupied by each clump of ten overstory trees was probably not typical of the whole stand. In fact, the area encompassed by a stand is seldom under five acres, simply because stratifying a forest into smaller management or inventory units is not practical.[3] The height-age pairs used in the analysis represent a collection of overstory data from many different plots within a given site class. The equations may therefore be used to estimate site index or predict height on any randomly selected sample plot within an even-aged spruce-fir stand. Rather than having to measure the height and age of ten overstory trees located about the tallest tree in a stand, a valid and practical sampling scheme could entail selection of individual trees located on randomly selected plots within a stand. This would typically involve measuring the height and breast-height age of the nearest healthy dominant or codominant tree to the plot center.

Although the final height and site index equations use height above breast height as a variable, instead of total height, it is advised that total height be measured in the field; is is easier to measure height accurately from the ground level than from breast height. Total height can then be transformed to H_{bh} by subtracting 4.5 feet during data analysis. Because there is such poor correlation between height growth and age at breast height, it is reasonable to view the first 4.5 feet of merchantable bole as immaterial to the estimation of site index. Further, the site curves will intersect the indicated site index, rather than site index plus 4.5, at the base age.

Equation 17 could be used as an indication of the sample size necessary to attain a desired level of precision. The appropriate confidence interval might reasonable correspond to half the width of the site class interval in either direction for a given age class. Figures

[3]This argument is added justification for including some of the uneven-aged plots in this analysis.

3 and 4 show that the distance between site index curves varies with age; therefore, to avoid the likelihood of misclassification of site index, the confidence interval should be narrower at younger ages, and also at advanced ages where the curves begin to converge on the asymptotic height.

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