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PLANNING FOR FLIGHT SYSTEM AVAILABILITY UNDER UNCERTAINTY

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The prime problem of planning for aircraft availability is to economically evaluate and determine if system availabilities are continuously being achieved, as well as to identify availability problems and pinpoint where they exist.[1] The risk to human life and property is far too great to warrant the use of structural components which may cause vital equipment to fail when most needed.

MATHEMATICAL MODELS

One of the principal functions of the engineering sciences is to develop mathematical models to represent the processes of natural or expected occurrences, thereby reducing great quantities of experimental data to simple relations between variables and permitting a unification of apparently diverse phenomena. A good model will include the most important features of the process, be mathematically simple (if possible), involve a minimum of assumptions, and be fruitful for purposes of prediction control and theoretical speculation.

The acid test of any model is its approximation to reality, subject to empirical observations. Unless the model corresponds reasonably well to physical observations, it can serve no practical purpose.

Occasionally, more than one model may be developed that accounts equally well for the observational data obtained from a system's test. In this case, and until the experimental results are sufficiently refined to favor one hypothesis over the others, the choice of models can be a matter of personal taste. Usually, preference is given to the simplest of several alternative hypotheses.

To illustrate the development and use of a relatively simple mathematical model consider a fleet of exactly m aircraft in which malfunctioning equipments are instrument monitored. All other preventive maintenance activities are performed outside the scheduled flight operating times of the aircraft and do not interfere with availability. Suppose the mean arrival rate for repair for each aircraft in operation is λ, and the mean repair rate is μ. Repair priorities are such that immediate repair service is initiated for any fleet aircraft that is temporarily out of operation. It is assumed that aircraft beyond repair are immediately replaced to keep the combined total number that are in operation and in repair to the number m.

Let $P_n(t)$ be the probability of n aircraft out of operation at time t. Observe that since $0 \leq P_n(t) \leq 1$ and

$$\sum_{n=0}^{m} P_n(t) = 1,$$

relatively simple relationships can be subsequently developed,[2] i.e., equations (1,a,b,c):

$$P_0(t+\Delta t) = P_0(t)[1-m\lambda\Delta t]+P_1(t)\mu\Delta t \quad (1,a)$$

$$P_n(t+\Delta t) = P_n(t)[1-(m-n)\lambda\Delta t](1-n\mu\Delta t)$$
$$+ P_{n-1}(t)(m-n+1)\lambda\Delta t$$
$$+ P_{n+1}(t)(n+1)\mu\Delta t, m>n>0, \quad (1,b)$$

$$P_m(t+\Delta t)=P_m(t)(1-m\mu\Delta t)+P_{m-1}(t)\lambda\Delta t, \quad (1,c)$$

where $(m-n)\lambda\Delta t$ and $n\mu\Delta t$ are the probabilities of exactly one aircraft in need of repair and one service completion respectively, during the increment of time Δt in the interval $[t,t+\Delta t]$; where n is the number in repair at time t.

If in each of the equations (1,a,b,c) the appropriate $P_n(t)(n=0,1,\ldots,m)$ is subtracted from both sides and the equation is divided by Δt (in the limit as $\Delta t \to 0$ the left side is the derivative, $P_n'(t)$) the equations become:

$$P_0'(t) = -m\lambda P_0(t)+\mu P_1(t) \quad (2,a)$$

$$P_n'(t) = -[(m-n)\lambda+n\mu]P_n(t)$$
$$+(m-n+1)\lambda P_{n-1}(t)+(n+1)\mu P_{n+1}(t), \quad (2,b)$$
$$0<n<m,$$

$$P_m'(t) = -m\mu P_m(t)+\lambda P_{m-1}(t). \quad (2,c)$$

These linear differential equations which are functions of t, are first order difference equations with respect to n; hence are called linear differential difference equations. For the initial conditions $P_0(t=0)=1$, and $P_n(t=0)=0$, $0<n\leq m$, the Laplace transformation[3] yields

$$(s+m\lambda)P_0(s) = 1+\mu P_1(s) \quad (3,a)$$

$$[s+(m-n)\lambda+n\mu]P_n(s) = (m-n+1)\lambda P_{n-1}(s)$$
$$+(n+1)\mu P_{n+1}(s) \quad (3,b)$$

$$(s+m\mu)P_m(s) = \lambda P_{m-1}(s) \quad (3,c)$$

Since steady state conditions are of greater interest than short operating time, only steady state relationships will be pursued here.

By means of the z-generating transformation[4] of (3,a,b,c) and application of the Laplace transform final value theorem allows the reduction of equations (3,a,b,c) to

$$\lim_{s \to 0} s\,dP(s,z)/dz = dP(z)/dz = m\lambda P(z)/(\mu+\lambda z)$$

hence integration for P(z=1)=1 yields

$$P(z) = [(\mu+\lambda z)/(\mu+\lambda)]^m. \tag{4}$$

The inverse z-generating transform of equation (4), i.e.,

$$P_n = \frac{1}{2\Pi j}\oint_C P(z)z^{-n-1}dz$$

yields

$$P_n = \binom{m}{n}\left(\frac{\lambda}{\lambda+\mu}\right)^n\left(\frac{\mu}{\lambda+\mu}\right)^{m-n} \tag{5}$$

The expected number of aircraft out of operation at any one time can be readily obtained by differentiating the z-generating transform P(z) with respect to z for $z \to 1$; yielding

$$P'(z=1) = \sum_{n=0}^{m} nP_n = E(n)$$

$$\therefore E(n) = m\lambda/(\mu+\lambda). \tag{6}$$

The mean time between repairs for the aircraft can be shown to be

$$E(t_r) = E(n)/\lambda = m/(\mu+\lambda). \tag{7}$$

Similar to equation (6), the variance of the number of aircraft out of operation, σ_n^2, can be obtained by the following:

$$\sigma_n^2 = P''(z=1)+P'(z=1)-[P'(z=1)]^2$$

hence

$$\sigma_n^2 = m\lambda\mu/(\mu+\lambda)^2 \tag{8}$$

From the mean number of failures E(n) and the variance in the number of failures σ_n^2 given by equations (6) and (8), control limited can be established so as to form a control limit. If the level of significance is set to correspond to three standard deviations, the upper control limit $UCL_n=E(n)+3\sigma_n$ and the lower control limit $LCL_n=E(n)-3\sigma_n$, hence

$$UCL_n = [m\lambda+3\sqrt{m\lambda\mu}]/(\lambda+\mu) \tag{9,a}$$

and

$$LCL_n = [m\lambda-3\sqrt{m\lambda\mu}]/(\lambda+\mu) \tag{9,b}$$

The ideal flight system maintenance and rework program would have all of its relevant resources focused to yield the maximum availability of all pertinent aircraft at minimal cost. The development and implementation of such a program requires that all resources and their interactions be clearly defined in order to specify their functional relationships. Since any aircraft with structural failures or those scheduled for regular maintenance action will be restored to flight operation in a finite span of time, the best figure of merit in general is the flight system's "availability." Any system's availability is defined as the probability that the system is in an acceptable state at any time t, given that the system was fully operating at time zero.

Since the availability of the fleet of aircraft is based upon, say, having k or less aircraft out of operation, then the availability is simply

$$\text{Availability} = \sum_{n=0}^{k} P_n$$

$$= \sum_{n=0}^{k}\binom{m}{n}\left(\frac{\lambda}{\mu+\lambda}\right)^n\left(\frac{\mu}{\mu+\lambda}\right)^{m-n} \tag{10}$$

Even though equation (10) was developed for steady state conditions, this definition still holds, because of the long term averaging effect.

SYSTEM AVAILABILITY

The availability of a flight system may be defined as the fraction of time that it is able or available to perform assigned flight duties, even though the availability of a complex system may be a function of several variables. These include such things as circuit reliability, total system maintainability, environmental stability, and fail-safe capability. All of these are important and must be carefully weighed to determine a system's feasibility and the economics of its performance. It is sometimes possible to have excellent reliability and yet have very poor maintainability, resulting in poor availability. Another example of limited availability is where a high reliability figure is evident, but pertains only to very narrow and restrictive environmental conditions. Thus one must carefully evaluate each performance factor in estimating the availability as a figure of merit.

Of all other considerations, costs of availability may be of prime concern in determining the optimum combination of availability levels necessary to achieve a particular objective.

As a simple illustration, consider the case of two independent variables, when it is desired to find the maximum availability combination for a fixed total expenditure by calculating a number of feasible input combinations that would yield the same total cost. It is conceivable in general to express the availability function mathematically and then to find an optimum by some mathematical procedure.

For simplicity, consider the case in which the availability is a function of two inputs, design time T and operating time t, which have unit cost rates C_1 and C_2 respectively. Suppose that there is also a fixed cost C_0 for all the other input factors which are fixed. Consider, i.e., the availability function which is simply the reliability for a non-maintained system and is given by

$$A(T,t) = e^{-kt/T} \qquad (11)$$

where k is a proportionality constant. (Note that the maintenance policy in this case does not interfere with the operating times in which the system is available.) The total cost C(T,t) could be, e.g.,

$$C(T,t) = C_0+C_1e^{VT}+C_2t. \qquad (12)$$

where V is a time constant.

The mathematical objective is to maximize the availability A(T,t) for a fixed total budget, i.e., for a constant value of C(T,t) = K where K is a constant.

The maximum availability can be found if the constant cost function is rearranged so that

$$C_0+C_1e^{VT}+C_2t-K = 0$$

This permits the reliability function to be written as

$$A(T,t,\lambda) = e^{-kt/T} + \lambda(C_0+C_1e^{VT}+C_2t-K) \qquad (13)$$

simply by tacking on the parametric form of the constant cost function.

Now to find the maximum availability, given the cost restriction, proceed with the standard method of differentiating partially with respect to each variable T, t, and λ, and set the partial derivatives equal to zero. The results are:

$$\partial A/\partial T = \frac{kte^{-kt/T}}{T^2} + \lambda C_1 Ve^{VT} = 0 \qquad (14,a)$$

$$\partial A/\partial t = -ke^{-kt/T} + \lambda C_2 = 0 \qquad (14,b)$$

$$\partial A/\partial \lambda = C_0+C_1e^{VT}+C_2t-K = 0 \qquad (14,c)$$

This gives a system of three equations, (14,a,b,c), which can be solved for the three unknowns T, t and λ. This method is due to Lagrange, and the parameter λ is generally called a Lagrangian Multiplier. This method can be extended to include a great many variables which may be relevant to the equation and its constraints, assuming that the total cost function is continuous and differentiable and that an optimum actually exists. However, as the number of independent variables increases, the number of equations to be solved increases. Moreover, the equations are generally nonlinear, as is our case in this simple example.

Hence, the computation may become extremely difficult, and for a large number of variables and/or constraints, practically impossible.

It is for this reason steady state models are used for a first approximation.

Another important modeling consideration, especially during the design or modification phase of an aircraft, is the reliability of its structural components.

STEADY STATE STRUCTURAL RELIABILITY

In almost all practical situations, the load applied to a specific aircraft structural component or number varies, sometimes over very wide ranges. For example, the mission of most military aircraft involves landing and taking off, weapons being fired, the variation in conditions of aerodynamic loading, etc. Suppose experimentally an expression is found for the probability that a particular structural component has a strength s_0 (psi) or greater. NOTE: these could be expressed as total stress values, i.e. in g's (ft per sec^2.) If the probability density function (p.d.f.) for the component's strength is denoted by $f_C(s)$, the probability of failure for a stress larger than s_0 will be

$$P(s \geq s_0) = \int_{s_0}^{\infty} f_C(s)\,ds \qquad (15)$$

If $f_L(s)$ represents the p.d.f. for environmental load stresses, the probability of encountering a stress in the immediate neighborhood of s_0 is given by $f_L(s_0)\Delta s$, where Δs is a small interval about the point s_0. Therefore, if the stress is in the infinitesimal neighborhood of s_0, the reliability of the component is given by the product

$$[f_L(s_0)\Delta s]\int_{s_0}^{\infty} f_C(s)\,ds$$

Finally, if s_0 is allowed to range over all possible (i.e., positive) values, we have

$$\text{reliability} = \int_0^{\infty}\left[\int_{s_0}^{\infty} f_C(s)\,ds\right] f_L(s_0)\,ds_0 \qquad (16)$$

which can be expressed in several alternative forms:

$$\text{reliability} = \int_0^{\infty}\left[1-\int_0^{s_0} f_C(s)\,ds\right] f_L(s_0)\,ds_0 \qquad (17,a)$$

and

$$\text{unreliability} = \int_0^{\infty}\left[\int_0^{s} f_L(s_0)\,ds_0\right] f_C(s)\,ds \qquad (17,b)$$

An equation which represents a probability density function for a chain strength (identical links) and can be applied much the same as for link strengths, the p.d.f. for chain strength, which can be denoted by $f_n(s)$, is

$$f_n(s) = n\left[\int_{s_0}^{\infty} f_C(s)\,ds\right]^{n-1} f_L(s) \qquad (18)$$

Equation (16) is a special case of (18) for the case when n is unity. Equation (18), being the p.d.f. for chain strengths, can now be combined with the p.d.f. for environmental stress to obtain chain reliability just as obtained for link reliability. Therefore,

chain reliability =

$$\int_0^{\infty}\left\{\left[\int_0^{s} f_L(s)\,ds\right] n \left[\int_{s_0}^{\infty} f_C(s)\,ds\right]^{n-1} f_C(s)\right\} ds \qquad (19)$$

Since the equation derived above for chain reliability is fairly complex, it may be useful to follow a brief discussion of three limiting cases:

1. $\sigma_L \ll \sigma_C$

2. $\sigma_C \ll \sigma_L$

3. $\sigma_L \simeq \sigma_C$

where σ_L and σ_C refer to the standard deviations in environmental stress and link strength respectively. Let us consider these three cases individually.

1. In this case the environmental stresses are restricted to a narrow range of values (to one value in the limit $\sigma_L \to 0$). The reliability of the chain is thus dependent almost entirely on the variability of the links; in particular, the probability that no one link in a chain has a strength less than the given value of environmental stress. In the limit, as $\sigma_L \to 0$, the chain reliability is given by $R = r^n$, the product rule, where R and r represent the system and component reliabilities, respectively.

2. In this case the link strengths are restricted to a narrow range of values (to one value in the limit $\sigma_C \to 0$). The reliability of the chain depends almost entirely on the probability of occurrence of an environmental stress exceeding the given value of link strength. Since all links have sensibly the same strength, chain reliability is thus given (in the limit, as $\sigma_C \to 0$) by the reliability of a single link: $R = r$.

3. When $\sigma_L \simeq \sigma_C$, chain reliability will lie somewhere between the two limiting cases discussed above.

These results, although based on a somewhat idealized and over-simplified model, serve to explain a common observation: to wit, that the product rule generally tends to give a pessimistic prediction. For components manufactured under carefully controlled conditions we might expect the case $\sigma_C \gg \sigma_L$, and therefore we might expect the formula $R=r^n$ to predict, in many cases, a value of system reliability lower than that observed in actual test.

It is intuitively clear that the greater the amount of overlap between the curves $f_L(s)$ and $f_C(s)$, the larger the probability of failure will be, A rough idea of the magnitude of this effect can be obtained by assuming that both $f_L(s)$ and $f_C(s)$ are normal distributions with the same value of the standard deviation σ.*

*This assumption and the numbers that follow must be regarded with caution since it is to be expected that the dispersion of the environmental stresses (more or less uncontrolled) will be larger--perhaps much larger--than the dispersion of component strengths (carefully controlled during the manufacturing or rework process.)

Under this assumption, if the two curves coincide, the probability of success will be 50 per cent. If the means of the two distributions are separated by 4σ, the probability of success will be approximately 99.8 per cent. If the means are separated by 6σ, the probability of success will be approximately 99.998 per cent. If a piece of equipment is being designed for 95 per cent reliability, it is clear that such a large separation of means (sometimes called the safety margin) will not be necessary. Despite this, several writers on reliability persist in promulgating the view that if a safety margin of 3σ is good, a safety margin of 6σ is better, and a safety margin of 60σ is better yet, and so on, ad infinitum. Safety margins cost money, and, after a certain point, a minute improvement in reliability can be purchased only by an astronomical expenditure of funds.

Having stated the interaction between a single environmental stress and the performance of a component in mathematical terms, we can now proceed to incorporate this concept into a model for the reliability of a complex system. For this purpose we introduce the model of a chain with n links. The links are assumed to have negligible weight.

In the context of our chain example the components become links in the chain, and the environmental stresses become weights that are hung from the end of the chain. The success of any particular link is judged by its ability to withstand, withour breaking, the stress of some randomly selected weight. Similar considerations apply to a chain of n links.

Since the probability that any given component link has a strength s_0 or greater is given by

$$P_C(s \geq s_0) = \int_{s_0}^{\infty} f_C(s)\,ds$$

the probability that a chain of n such links has a strength s_0 or greater is provided by the expression

$$P_{Chain}(s \geq s_0) = \left[\int_{s_0}^{\infty} f_C(s)\,ds\right]^n$$

For structural chains or any structural member on which independent failure would constitute system failure, the product of independent component reliabilities would yield the system reliability. For example, consider a system in which the reliability for each of n components or subsystems can be considered independent, the system reliability can be expressed as

$$\text{system reliability} = \prod_{i=1}^{n} R_i(t) \qquad (20)$$

where each $R_i(t)$ may be changing with time. In a similar way, where all redundant components or subsystems are utilized, the system reliability can be expressed in terms of unreliability, e.g.,

$$\text{system reliability} = 1 - \prod_{i=1}^{n} (1-R_i(s))$$

$$= 1 - [1-R(t)]^n \qquad (21)$$

In the case of the independent or chain failure such as equations (20) and (21), the reasoning is that if any one of the n links should break, the chain itself will fail. Furthermore, the links are assumed to fail independently of each other, which must be very nearly true for approximately weightless links.

In spite of benefits obtained from mathematical models which assist in designing failure free flight systems, objections may be raised, in that reliability is an expensive proposition. Hence, a cost effectiveness study may be required for any complex reliability study.

COST EFFECTIVENESS

A simple generalized total expected cost model for an aircraft system which is a function of time t, can be expressed as

$$E[C(t)]=C_0[1-R(t)]+\alpha(t)$$

where: C_0=Purchase cost

$C_0R(t)$=expected present value after time t; where R(0)=1 and R(t) is a reliability (cumulative probability) function

$\alpha(t)$=expected accumulation costs of repairs and maintenance where $\alpha(0)=0$.

Since E[C(t)] can only be expected to increase with time, its minimum value is at time zero; hence it becomes important to look at minimizing the expected cost rate, which can be expressed as

$$C_R(t) = E[C(t)]/t = [C_0(1-R(t))+\alpha(t)]/t.$$

Hence the minimum expected cost rate for the system is given for

$$C_R'(t) = 0.$$

A typical form for R(t) and $\alpha(t)$ are $R(t)=e^{-\lambda t}$ and $\alpha(t)=k(e^{\mu t}-1)$, where λ represents breakdown rate, μ represents repair rate, and k an arbitrary constant.

Hence

$$C_R(t)=[C_0(1-e^{-\lambda t})+k(e^{\mu t}-1)]/t$$

A trial and error solution might be the expedient approach to the solution for t by settling $C_R'(t)=0$. This would yield the time when the cost rate would be expected to start increasing and continue to do so regardless of availability and other important considerations.

Since certain flight systems are designed on a basis of "4-g" maximum acceleration and if monitoring accelerometer readings exceed "6-g", it seems most urgent to explore the correlation between structural failures and aircraft having excessive accelerometer readings. Likewise, for hard landings, since flight systems are designed for a maximum sinking speed, where records indicate aircraft with excessive sinking speeds in landing, all such specific aircraft should be studied for failure correlations.

If the sensitivity of accelerometers make this impractical and are locked out on landings, a more rugged counter type accelerometer should be installed on each aircraft for this purpose.

Further consideration needs to be given to implementing a number of monitoring devices or plans to test for structural failures throughout the critical components of the aircraft.

In many flight situations, the failure of system units during actual flight operation is not only costly but dangerous. If the flight system unit is characterized by a failure rate that increases with age, it may be wise to replace it before it has aged for too long a time.

Maintenance is a highly important aspect of availability. But it is another matter to keep an aircraft operating reliably by proper maintenance practices. If simulate details are considered necessary to an availability study relevant to a large number of attributes which are functions of several variables, the Monte Carlo simulation method is sometimes the only solution.

MONTE CARLO SIMULATION FOR AVAILABILITY

In the Monte Carlo technique we "play a game" with nature. If certain elements of a problem follow known probabilistic laws, we can sample randomly from the corresponding distributions to obtain, not a deterministic solution to our problem, but rather an overall distribution representing the statistical behavior of the solution we desire.

The application to availability is fairly new. The Monte Carlo technique has been applied for a longer period to problems in mathematics (solutions of certain differential-integral equations), to problems in physics (neutron diffusion), and to war games. We impose a specified input, use some random processes to select values of component parameters, and combine these according to some rule to obtain the system output. Repeating this process will yield a sample of outputs from which availability data may be deduced.

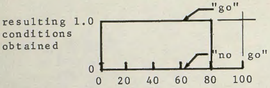
1. A suitable random process must be available. This is no problem: any good tabulation of random numbers will suffice.

2. Enough component information must be on hand so that component response distributions can be estimated with reasonable accuracy. These response distributions tell us how to weight the various probabilities of occurrence of parametric values.

3. A formula must be available giving system output as a function of system input and component parameters.

Consider a simple example, based on the generation of random numbers.[5] Associated with each number is the resulting condition obtained corresponding to this number.

Figure 1 illustrates the case where 80 per cent of the time a "go" condition exists and 20 per cent of the time a "no go" condition exists. That is, the probability of obtaining a "go" condition is .8 and the probability of a "no go" is .2.



Per cent likelihood of occurrence

Fig. 1

Suppose the random numbers vary from 1 to 1000, each having equal likelihood. Then "go" action is taken when the random numbers generated is from 1 to 800, whereas, if any number from 801 to 1000 is generated, "no go" action is taken.

By means of simulation using a high speed digital computer, it is possible to apply the Monte Carlo method in determining the average conditions which a set of probability distributions approach in the limit.

Figure 2 shows a procedure for the checkout and flight of a simple system.

Reliability estimated by the Monte Carlo method is done by taking the ratio of the number of successful simulation runs and the total number of simulation runs. It therefore becomes necessary to run the same situation many times using the same probability distribution but varying the given set of random numbers used to generate probabilities, etc. The estimated reliability which is obtained from the ratio "number of successful simulation runs/total number of simulation runs" is subject to the usual statistical error. It is therefore desirable to determine the number of simulation runs necessary based on the degree of accuracy required in the reliability estimate. According to Chestnut,[5] the number of runs may be determined from the formula:

$$N = 3.84R(1-R)/E^2$$

where

N = number of runs

R = checkout reliability

E = acceptable error range for per cent confidence in estimating reliability R. (A 95 per cent confidence level was used for this example.)

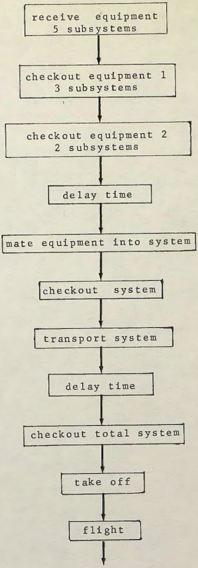


Fig. 2

In Table 1 is shown the result of 200 simulation runs using Monte Carlo methods on an example in which a total system consisted of five subsystems was tested. On comparing the simulation runs with the expected number of runs and the Monte Carlo estimated reliability with that computed analytically, it is clear that the Monte Carlo method could be used effectively to estimate a system's performance. The significance of these results could also be checked by a rerun of the simulation or by using a different set of random numbers.

RESULTS OF 200 MONTE CARLO SIMULATION RUNS

Subsystem	1	2	3	4	5	Total System
Number of runs System working	200	200	169	106	200	86
Expected number of runs	200	200	163	119	200	97
Reliability (Monte Carlo estimated)	1.000	1.000	0.845	0.530	1.000	0.430
Reliability (Analytically calculated)	1.000	1.000	0.816	0.597	1.000	0.487

Table 1

In conclusion, there is little doubt that future flight systems will be required to perform many new and complicated functions not yet conceived or devised. The advent of more complex flight and space vehicles will demand much more sophisticated systems for communication, traffic control, support equipment, i.e., ground handling and maintenance. The flight system availability problems are just now beginning; hence, many new techniques will be needed in order to cope with each new problem as it occurs. It is felt that some of the concepts and methods presented here will play a part in the designs and operations of tomorrow's systems.

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