



---

The Space Congress® Proceedings

1965 (2nd) New Dimensions in Space  
Technology

---

Apr 5th, 8:00 AM

## Missile Attitude Sensing With Polarized Laser Beams

John L. Dailey

*Missile and Surface Radar Division, Radio Corporation of America*

Follow this and additional works at: <https://commons.erau.edu/space-congress-proceedings>

---

### Scholarly Commons Citation

Dailey, John L., "Missile Attitude Sensing With Polarized Laser Beams" (1965). *The Space Congress® Proceedings*. 4.

<https://commons.erau.edu/space-congress-proceedings/proceedings-1965-2nd/session-6/4>

This Event is brought to you for free and open access by the Conferences at Scholarly Commons. It has been accepted for inclusion in The Space Congress® Proceedings by an authorized administrator of Scholarly Commons. For more information, please contact [commons@erau.edu](mailto:commons@erau.edu).

**EMBRY-RIDDLE**  
Aeronautical University™  
SCHOLARLY COMMONS

# MISSILE ATTITUDE SENSING WITH POLARIZED LASER BEAMS

John L. Dailey

Missile and Surface Radar Division  
Radio Corporation of America  
Moorestown, New Jersey

## Summary

An optical system has been designed to monitor the attitude of a missile during early launch phase. The system utilizes passive reflective components mounted on the missile to return a pair of laser beams transmitted from a ground station. The beams have their polarization state modulated by the reflective elements such that polarization is a function of missile attitude. The returned beams are passed through a polarization analyzing system at the ground station and missile attitude computed from the measured polarization parameters.

## Introduction

A design study has been completed for an optical system intended to monitor the absolute pitch, roll and yaw of a climbing rocket, from lift-off to 50,000 feet of altitude, and to report this in real time at a rate of 10 readings per second. The system uses pulsed laser beams, transmitted from a single ground station, to illuminate a retroreflector package on the missile. The package contains optical cube corners faced with polarization modulating components. The light returned to the ground station by the cube corners must pass through these components, which alter the polarization state from a linearly polarized reference state to some other state which is determined by the attitude of the missile relative to the beam and the station local vertical.

When the reflected light reaches the ground station, it is passed through a polarization analyzing system which determines its polarization state and passes this information to a computer. The computer inserts the polarization parameters into a system of simultaneous equations, which it solves to find the attitude of the missile relative to the beam and the local vertical. Then, with a set of transforms involving the azimuth and elevation of the beam and survey information of the station relative to the launch pad, it transforms its results to pitch, roll and yaw in launch pad coordinates. To these, it adds the time of day at which the measurement was made, as taken from the range clock, and passes the final results to the real time users as well as to a recording system which stores it for post-flight analysis.

The philosophy which underlies this system may be summarized as follows.

The pitch, roll and yaw which are to be measured constitute three independent variables, and, since all three are to be measured simultaneously, the sensing

system must have three independent variables which can be made functions of the missile's attitude. It will be shown in a later section that the polarization state of light reflected from optical cube corners mounted on the missile can be made a function of, and only of, the three-dimensional rotation of the missile with respect to the incident beam direction. Therefore, the polarization state of a completely polarized laser beam may be used to carry information from the missile to the ground station. But, since the polarization state of a beam of light is completely defined by the azimuth and eccentricity of its polarization ellipse, a beam can carry only two pieces of information in its polarization state. Since three pieces of information are needed, it will be necessary to use two beams of light, separated in wavelength so that they may be isolated from one another by spectral filters.

Two beams of light, with four independent variables in their polarization ellipses, contain a redundancy. In the present case, this is useful, since, as the eccentricity of an ellipse approaches zero, that is, when the ellipse is nearly circular, the azimuth becomes difficult to determine accurately, and, in the limiting case, circularly polarized light, azimuth is not defined. There are therefore some polarization states for which azimuth cannot be determined accurately and one for which it cannot be determined at all.

To evade this problem in part, the system can be set up so that both polarization ellipses have a common azimuth. This reduces the number of independent variables to three, one of which occurs twice. The azimuth of each beam serves as a backup for the azimuth of the other, so that the number of cases in which a reading is unobtainable is minimized. (In the proposed system, this is found to be 1 case in 900.)

Once the variables which are to convey the attitude information are selected, the next step is to find a method of making them functions of the missile attitude. The method which was selected is to pass each reflected beam through a missile-borne sheet polarizer to give it a fixed reference state in missile coordinates and then to pass it through a special form of Savart plate, as will be described.

### The Retroreflector Package

The optical components mounted on the missile are sketched in Figure 1. The package contains two sets of reflectors, filters, and polarization components. Except for adjustments for the different wavelengths they are designed to pass, the two sets are identical. For reasons to be shown, they are rotated  $90^\circ$  with respect to one another about an axis normal to the face plate.

Reading from left to right in the side view, the first elements are an array of cube corners. These, of course, are selected because they have the property of returning a beam of incident light in the direction from which it came. As shown in the front view, they have hexagonal pupils for maximum efficiency. An array is used instead of a single large hex reflector for considerations of size, weight and cost.

Next, cemented to the cube corners, are gelatin filters, such as Wratten filters. Their function is to provide isolation of one set of components from its unwanted laser wavelength. That is, each filter will pass one of the laser wavelengths, but

stop the other. If they are to accomplish this, the wavelengths must be widely separated. The lasers selected are ruby, with output at  $.69\mu$ , and neodymium, with output at  $1.06\mu$ . The  $3000 \text{ \AA}$  separation of these two is sufficient that two gelatin filters are available which will, in the two passes in and out, pass  $10^4$  more of the wanted wavelength than of the other. This is adequate "channel separation" for the purpose at hand.

Cemented to the gelatin filters are sheet polarizers of the Polaroid type. When the lasers leave the transmitter, they pass through a pseudo-depolarizer (of the type, for instance, described by Peters<sup>1</sup> so that a constant fraction of the beam is transmitted through the missile-borne polarizer, regardless of missile attitude. The beam strikes the cube corners linearly polarized, and its polarization state is altered by the reflections within the cube. On being reflected back through the polarizer, it loses some of its intensity because of this alteration. When the reflected beam emerges from the polarizer, it has about 14% of its incident intensity. It is linearly polarized at  $+45^\circ$  to the vertical axis of the missile and it is directed toward the ground station.

The last component through which the beam passes as it leaves the missile is the polarization modulation plate. The purpose of this plate is to alter the eccentricity of the polarization ellipse without changing its azimuth, and to alter it so that it is a first order function of the beam direction relative to the plate axes. The plate designed to do this is a modification of the Savart plate.

#### The Modified Savart Plate

The standard Savart plate, which has long been used in interferometry and polarimetry, consists of two plates, cut from a uniaxial crystal at  $45^\circ$  to the optic axis, superposed and rotated  $90^\circ$  with respect to one another, so that the projections of the optic axes of the two plates upon a common surface are orthogonal to one another. When viewed between crossed polarizers, this double plate presents an interference pattern of dark and light lines which are almost straight. Figure 2 is a photograph of such an interference pattern. (The reason for the unevenness of the image is that the surfaces were sawed surfaces, not polished, but simply immersed in an index matching oil to prevent diffusion. To obtain the wide line separation, the crystal had to be sliced so thin that it was too fragile to survive polishing).

The defects of the plate are two-fold. First, it is non-linear. The curvature of the interference lines is quite pronounced at high angles of incidence and the mathematical expression for the phase shift consists of a combination of first and second order terms creating ambiguities in its solution. And secondly, if the interference lines are to have enough angular separation to be easily resolved, the plate must be impractically thin.

A detailed mathematical analysis of the Savart plate would be too long for inclusion here, but it can be shown that both the non-linearity and the angular line spacing are a minimum for a standard plate when it is cut at  $45^\circ$  from the optic axis. The line separation can be increased by cutting the plates at a shallower angle, but when this is done, the line curvature increases sharply.

However, it can be shown that when two standard Savart plates are combined by



superposing them after a  $90^\circ$  rotation of one with respect to the other, the interference pattern is perfectly linear; the lines are mathematically straight. Moreover, this linearity holds regardless of the angle from which the plates were cut from the crystal. It is possible therefore to make a plate with widely separated lines by cutting at a shallow angle and to generate a purely first order interference pattern.

To make the modified plate, four plates of equal thickness are cut from a uniaxial crystal at some general angle  $\phi$  to the optic axis as in Figure 3 and the four plates are superposed, as in Figure 4, such that the projection of their optic axes on their top surfaces are at angles  $0^\circ$ ,  $90^\circ$ ,  $-90^\circ$  and  $0^\circ$  with respect to a vertical axis. The plates are cemented together to form a single plate and a polarizer is cemented to the top surface with its transmission axis at  $45^\circ$ .

The electric vector of light transmitted through the polarizer will be resolved by the first plate into two components, one parallel to the optic axis (the vertical component) and one perpendicular to the optic axis (the horizontal component). These propagate through the plate at different velocities. The velocity of the component parallel to the optic axis, called an extraordinary ray, propagates at a velocity which varies with direction, while the orthogonal component, called an ordinary ray, propagates at constant velocity. At the interface between the first and second plates, and again at the interface between the third and fourth plates, the horizontal and vertical components exchange roles as ordinary and extraordinary rays. As a result, they follow paths through the crystal plates like those shown in Figure 5. When they emerge from the plate, one component lags behind the other by a distance  $d$ , as shown in the figure. This creates a phase shift,  $\delta$ , between them given by

$$\delta = \frac{2 \pi d}{\lambda} \quad (1)$$

By geometric ray tracing techniques, the two optical path lengths may be found and subtracted one from the other, to yield an expression for  $\delta$ , which is

$$\delta = \frac{4 \pi T}{\lambda} \frac{(n_o^2 - n_e^2) \sin \phi \cos \phi}{(n_e^2 \cos^2 \phi + n_o^2 \sin^2 \phi)} \sin i \sin a \quad (2)$$

where  $T$  is the thickness of an individual plate (e.g., one fourth the total thickness)

$\lambda$  is the wavelength of the transmitted light

$n_o$  is the ordinary index of refraction of the crystal

$n_e$  is the extraordinary index of refraction

- $\phi$  is the cutting angle  
 $i$  is the angle of incidence of the beam  
 $a$  is the azimuth of the beam with respect to the horizontal axis

The significance of the angles  $i$  and  $a$  is shown in Figure 6, which is simply a set of spherical coordinates without the vector length shown. The  $x$  and  $y$  axes are parallel to the plate edges and the  $z$  is parallel to the missile vertical axis. One can see from Figure 6 that if the beam direction is one axis of a coordinate system and the projection of the local vertical at the station upon a plane normal to the beam is another, with their mutual normal as the third, the attitude of the missile coordinates is defined in beam coordinates by  $i$  and  $a$  plus the rotation of the missile about the beam. These, then, are the three parameters that the system intends to measure.

Equation (2) may be abbreviated to

$$\delta_1 = k_1 \sin i \sin a \quad (3)$$

with the subscripts referring to plate number one in the missile-borne package. The second plate is like plate number one, but rotated  $90^\circ$  from it about a normal to its surface. For the second plate, the angle  $a$  has become  $(a + 90^\circ)$ . Making this change in equation (3) gives the phase shift equation of the second plate;

$$\delta_2 = k_2 \sin i \cos a \quad (4)$$

and since the angles  $i$  and  $a$  are the same for both plates, equations (3) and (4) form a simultaneous system, and if  $k_1$  and  $k_2$  are known, measuring  $\delta_1$  and  $\delta_2$  will yield  $i$  and  $a$ . Therefore, it is possible to find two of the three attitude parameters of the plate by measuring the phase shift angles of the two beams reflected through them.

It is not possible to measure the phase angle between two vectorial components of a beam of light unless the directions of the two components are known. It is necessary to find, in ground coordinates, the directions of the horizontal and vertical components of the polarized beams as they are defined in missile coordinates. To see that this can be done, consider the general equation of the polarization ellipse.

$$\frac{E_x^2}{a_x^2} + \frac{E_y^2}{a_y^2} - \frac{2E_x E_y}{a_x a_y} \cos \delta = \sin^2 \delta \quad (5)$$

in which

$E_x$  or  $y$  is the instantaneous component of the electric vector in the  $x$  or  $y$  direction

$a_x$  or  $y$  is its maximum amplitude

$\delta$  is the phase angle between the named vectors

It has been stipulated that, in missile coordinates, the incident vector is at  $45^\circ$  to the  $x$  axis, and therefore  $a_x = a_y$  in the system under consideration. Noting that  $a^2 = I$ , the intensity, one may cast equation (5) into polar form as

$$\rho^2(1 - 2 \sin \psi \cos \psi \cos \delta) = I \sin^2 \delta \quad (6)$$

This is the equation of an ellipse whose azimuth is at  $\psi_{\rho \max}$ , the angle at which  $\rho$  is a maximum and where  $\frac{d\rho}{d\psi} = 0$ . But,

$$\frac{d\rho}{d\psi} = \frac{I \sin \delta \cos \delta \cos 2\psi}{(1 - \cos \delta \sin 2\psi)^{3/2}} \quad (7)$$

this has a zero value when (ignoring the trivial solution  $I = 0$ )

- 1)  $\delta = 0^\circ$  or  $180^\circ$
- 2)  $\delta = 90^\circ$  or  $270^\circ$
- 3)  $\psi = 45^\circ$  or  $135^\circ$

Inserting these values in turn into equation (5) yields

- 1) a straight line of azimuth  $45^\circ$  or  $135^\circ$
- 2) a circle without azimuth
- 3) an ellipse of azimuth  $45^\circ$  or  $135^\circ$

A more detailed analysis reveals that when  $-90^\circ < \delta < +90^\circ$ , the azimuth angle is  $45^\circ$ , and when  $+90^\circ < \delta < 270^\circ$ , the azimuth is  $135^\circ$ . Figure 7 shows a general polarization ellipse, defined in  $\theta$ , the azimuth, and  $\beta$ , the angle whose tangent is the ratio of the minor to the major axis, and is therefore an eccentricity parameter. It has just been said that, in missile coordinates,  $\theta$  is a constant over a half cycle of  $\delta$ . The directions of the horizontal and vertical components in missile coordinates are therefore known when the azimuth is found in ground coordinates, since they are always at  $+45^\circ$  to the azimuthal angle, although an ambiguity exists. It is possible therefore to measure  $\delta$  in the proper coordinate system. In practice, since the azimuth is fixed in missile coordinates, only the eccentricity of the

ellipse can vary, and therefore a relationship must exist between phase shift and eccentricity which will make it possible to find one by measuring the other. The azimuth of the ellipse is measured in ground coordinates, not to help in the determination of  $i$  and  $a$ , but because it is itself the third independent variable in the system.

The Cycle Angle. It has been pointed out that the system is unable to distinguish between a polarization ellipse whose phase shift is  $\delta$  and whose azimuth is  $\theta$  and one whose phase shift is  $(\pi - \delta)$  and whose azimuth is  $(\theta + 90^\circ)$ , since these produce the same combination of  $\theta$  and  $\beta$  in Figure 7. There are, besides these two ambiguities contained in one cycle of  $\delta$ , further ambiguities arising from the possibility of multiple cycles of  $\delta$ . While the polarization analysis system detects a phase shift between 0 and  $2\pi$ , the actual range of  $\delta$  is between 0 and  $2N\pi$ , where  $N$  is integral.

To consider this problem and the method of dealing with it, note that the direction of maximum phase variation with direction is given by equation (3) at beam azimuth  $a = 90^\circ$ , for which

$$\delta = k \sin i \quad (8)$$

In this direction,  $\delta$  completes one cycle at an incidence angle

$$i_0 = \frac{2\pi}{k} \quad (9)$$

and every integral multiple thereof. This angle is designated the cycle angle. In an operating system, the choice of cycle angle is quite important, for two reasons. First, the anticipated accuracy of the polarization analysis system is about +1%, and some simple algebra will show that the accuracy of the system as a whole is about 1% of the cycle angle, ignoring the non-linearity of the sine functions. Since the absolute accuracy of the system is determined by the cycle angle, the cycle angle is determined by the specifications.

Secondly, resolving the ambiguities arising from multiple half cycles within the angular range of the system must be done on an historical basis. That is, before launch, the measured phase shift is arbitrarily assumed to be in the first cycle. Since the angular separation of cycles is constant, this is permissible. From this point on, a careful track is kept of the number of cycles through which the missile rotates, so that by this counting method the computer knows which of several ambiguous solutions is the correct one. This is possible only if the maximum permissible rotation of the missile between measurements is very much smaller than a cycle angle. The permissible rotation rate varies from missile to missile, but is of the order of magnitude of 10 degrees per second. Since a tenth of a second elapses between measurements, the cycle angle should be at least 10 degrees so that each half cycle may be sampled about five times to permit reliable resolution of ambiguities. Ten degrees is an inordinately large cycle angle for a Savart plate and is one of the reasons that the standard plate is unsuitable. (The other reason is that it contains second order terms in its phase shift equation.)



The modified plate of four layers may be made to arbitrarily large cycle angles and 10 degrees of cycle angle is a very reasonable figure for one of these.

Before leaving the subject of the plate and its functions, it should be noted that for a constant specification of thickness, flatness and surface parallelism, the phase shift accuracy and uniformity across the face of the plate increases as cycle angle increases. That is, the precision of the plate goes up as the cutting angle goes down, since the birefringence along a plate normal, which determines its performance, decreases as the plate normal approaches the optic axis of the crystal. In the field of crystal optics, it is axiomatic that the lower the birefringence of a material, the more accurate the wave plate which one may cut from it. Cutting the plates at a shallow angle of  $\phi$  as shown in Figure 3 is a way of reducing the effective birefringence of the individual plates, so that a highly accurate plate may be made without resorting to stringent specifications during fabrication.

### The Receiver System

#### The Mathematical Basis of Polarization Analysis

There are several systems of parameters which are used in the various common methods of polarimetry. The one chosen for this system is the Stokes vector, which is mathematically the simplest. The Stokes vector is treated as a four component tensor, given by

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \quad (10)$$

but it actually contains only three independent variables, since  $S_0$ , the intensity, is related to the others by the quadratic relationship.

$$S_0^2 = S_1^2 + S_2^2 + S_3^2 \quad (11)$$

These components refer to the polarization equation, given previously as equation (5) in this manner

$$\begin{aligned} S_0 &= a_x^2 + a_y^2 \\ S_1 &= a_x^2 - a_y^2 \\ S_2 &= 2a_x a_y \cos \delta \\ S_3 &= 2a_x a_y \sin \delta \end{aligned} \quad (12)$$

This set of equations may be rewritten

$$\begin{aligned}
 a_x &= \frac{1}{\sqrt{2}} \sqrt{S_0 + S_1} \\
 a_y &= \frac{1}{\sqrt{2}} \sqrt{S_0 - S_1} \\
 \sin \delta &= \frac{S_3}{\sqrt{S_2^2 + S_3^2}} \\
 \cos \delta &= \frac{S_2}{\sqrt{S_2^2 + S_3^2}}
 \end{aligned} \tag{13}$$

To understand the physical significance of the Stokes vector, the Poincare sphere is helpful. Figure 8 shows a quadrant of this sphere. Each point on its surface corresponds to a specific polarization state. The three axes of the quadrant designate specific polarization states.  $S_1$  designates linearly polarized light of azimuth  $0^\circ$ ,  $S_2$  designates linearly polarized light of azimuth  $45^\circ$  and  $S_3$  designates right circularly polarized light. Of the sphere in general, it may be said that azimuth varies with longitude, the azimuthal angle being one half the longitude, and eccentricity varies with latitude, being a maximum at the equator and zero at the poles. All right handed ellipses lie in the upper hemisphere and all left handed ellipses lie in the lower hemisphere. Moreover, every point on the surface representing a polarization ellipse is diametrically opposed to the point representing the orthogonal polarization ellipse. Thus, linearly polarized light of azimuth  $90^\circ$  is designated  $-S_1$ , of azimuth  $135^\circ$  is designated  $-S_2$  and left handed circularly polarized light is designated  $-S_3$ .

This being the case, any polarization ellipse is designated by the radius vector to its point on the sphere, which, in the manner of any vector, is defined in terms of its three orthogonal components, and polarized light may be analyzed by measuring its three Stokes parameters. If  $S_1$ ,  $S_2$  and  $S_3$  are measured, one may use equation (13) to find  $a_x$ ,  $a_y$  and  $\delta$ .

It has been stated previously that the correct value of  $\delta$  can be obtained only in the coordinate system  $a_x = a_y$ , for which the azimuth is constant at  $45^\circ$ . Referring again to Figure 7, it is obvious that if the ellipse rotates in the plane of the figure, but the indicated coordinates remain fixed, then the three parameters in equation (5),  $a_x$ ,  $a_y$  and  $\delta$ , vary, or from a more practical point of view, the two determining parameters,  $a_x/a_y$  and  $\delta$ , vary as functions of one another. But if the ellipse is expressed in terms of the parameters shown in Figure 7, only  $\theta$  varies, while  $\beta$ , the eccentricity angle, remains fixed. The angles actually available for independent measurement are therefore  $\theta$  and  $\beta$ . Since  $\theta$  is

fixed in missile coordinates,  $\theta$  in ground coordinates yields the rotation of the missile about the beam and one of the independent variables is found. What is needed now is  $\delta$  for each beam, but what is available is  $\beta$ . It was stated previously that there must be a relationship between  $\delta$  and  $\beta$  which will enable one to be found if the other is known. Before proceeding further, it is necessary to determine that relationship.

From the geometry of the ellipse and that of the Poincare sphere, it can be shown that

$$\begin{aligned}
 S_0 &= I \quad (\text{the beam intensity}) \\
 S_1 &= I \cos 2\beta \cos 2\theta \\
 S_2 &= I \cos 2\beta \sin 2\theta \\
 S_3 &= I \sin 2\beta
 \end{aligned}
 \tag{14}$$

from which

$$\begin{aligned}
 \sin 2\beta &= \frac{S_3}{S_0} \\
 \cos 2\beta &= \frac{\sqrt{S_1^2 + S_2^2}}{S_0}
 \end{aligned}
 \tag{15}$$

From the specification that  $a_x = a_y$  in missile coordinates, and from the definition  $S_1 = a_x^2 - a_y^2$ , it follows that  $S_1 = 0$  in missile coordinates, though not necessarily in ground coordinates. Therefore, in missile coordinates, one may add  $S_1$  to the equations at will without invalidating them. The last two equations in group (13) may therefore be written

$$\begin{aligned}
 \sin \delta &= \frac{S_3}{\sqrt{S_1^2 + S_2^2 + S_3^2}} = \frac{S_3}{S_0} \\
 \cos \delta &= \frac{\sqrt{S_1^2 + S_2^2}}{\sqrt{S_1^2 + S_2^2 + S_3^2}} = \frac{\sqrt{S_1^2 + S_2^2}}{S_0}
 \end{aligned}
 \tag{16}$$

Comparing (15) and (16) shows that

$$\delta = 2\beta \quad (17)$$

where  $\delta$  is measured in missile coordinates and  $\beta$  is measured in any coordinate system whatever. Therefore, it is possible to obtain a value for  $\delta$ , the phase shift in missile coordinates, independently of any rotation of the missile coordinates about the coordinate system in which the analyzer is operating, simply by measuring the Stokes parameters, which, from equation (14) will also yield the necessary azimuth.

### The Physical Components

The receiver system consists of an array of six telescopes clustered about the transmitter as shown in Figure 9. Owing to diffraction effects, the returned beam is spread to a diameter somewhat greater than that of the array, so that each receiver telescope intercepts some of the beam. Inside each telescope is a polarizing beam splitter whose function is to divide the received beam into two orthogonally polarized beams, each of a specific polarization state. In four of the telescopes, there is a Wollaston prism, which divides the beam into linearly polarized light parallel to an axis through the prism plus the component perpendicular to this. The other two prisms are Fresnel multiple prisms, made of crystalline quartz, which split a transmitted beam into right and left circularly polarized components. After passing through these prisms, the separated beams are divided spectrally by dichroic prisms, which separate the two wavelengths and pass them to separate multiplier phototubes, of which there are four in each telescope. Figure 10 shows the layout of optical components for the Wollaston prism telescopes and Figure 11 shows it for the Fresnel prism telescopes.

When light defined by a polarization vector  $S_0$  is transmitted through a polarizing beam splitting prism with its axes at  $\xi$  and  $0(\xi + 90^\circ)$ , the two emerging beams are described by

$$\begin{aligned} S_1' &= M_{\xi} S_0 \\ \text{and } S_2' &= M_{\xi + 90^\circ} S_0 \end{aligned} \quad (18)$$

where  $M$  is the Mueller matrix of the prism for the indicated beam. The Mueller matrices for Wollaston and Fresnel prisms are available from the literature<sup>2</sup>.

If a Wollaston prism splits a beam into linearly polarized components with azimuths at  $0^\circ$  and  $90^\circ$ , the two outputs may be given as

$$\begin{aligned} S_1' &= \frac{1}{2}(S_0 + S_1) \\ \text{and } S_2' &= \frac{1}{2}(S_0 - S_1) \end{aligned} \quad (19)$$



If the azimuths are at  $45^\circ$  and  $135^\circ$ , the outputs are

$$\begin{aligned} S_3' &= \frac{1}{2}(S_0 + S_2) \\ \text{and } S_4' &= \frac{1}{2}(S_0 - S_2) \end{aligned} \tag{20}$$

The outputs in the case of the Fresnel prism telescopes are

$$\begin{aligned} S_5' &= \frac{1}{2}(S_0 + S_3) \\ \text{and } S_6' &= \frac{1}{2}(S_0 - S_3) \end{aligned} \tag{21}$$

Simply by taking the differences of these equations,  $S_1$ ,  $S_2$  and  $S_3$  may be obtained. By adding them,  $S_0$  is obtained. With these measured values,  $i$ ,  $a$  and  $\delta$ , the missile rotation parameters may be computed.

This description of the computation of the polarization components is rather simplified from the actual design system. A much longer derivation would show that the components which are actually wanted are  $S_1$ ,  $S_2$ ,  $S_3$  and  $U$ , where  $U$  is the unpolarized light at the laser wavelengths collected by the telescopes, making a total of four independent variables. There are twelve phototube outputs, and by selecting outputs carefully one may set up twelve equations in four unknowns, which may be solved independently three times. This permits averaging the answers to reduce error by  $\sqrt{3}$ , and also provides a measure of the unpolarized background light that has gotten into the system.

These three independent readings are obtained with little extra trouble, since four telescopes would be required as a minimum and the extra two, as Figure 9 shows, fit into space which would otherwise be left vacant and collect light that would otherwise be lost.

### Conclusion

The material just presented is the result of a design study aimed specifically at the development of a system for monitoring the attitude of a missile during early launch phase. It accomplishes this by polarization modulation of a beam of light at the missile and polarization analysis of that light at the ground station. The methods chosen to accomplish this were dictated in part by the need to use pulsed monochromatic laser beams as a carrier. Over shorter distances, and when longer integration times are permitted, white incoherent light may be used as a source and a second set of Savart plates used as the analyzer, the

plates being used as compensators in this latter case. Using Savart plates as analyzers at the detection station would eliminate ambiguities in the case of a white light source, since the phase shift equation contains wavelength as a factor, and would also permit a higher accuracy. When full scale polarization analysis must be done, the best accuracy that can be hoped for, according to a mathematical error analysis, is about 1% of the cycle angle.

An optical approach to measuring missile attitude offers some advantages besides accuracy. Chiefly, it requires no active cooperation from the missile and involves no use of the missile's power supplies. Also, virtually all of the system is at the ground station. The components on board the missile will go into the ocean after a single use, so there is a decided economic advantage in placing only inexpensive reflectors on the missile itself.

There is a large measure of convenience and reliability in the fact that the carrier involved is a beam of light. At the Cape Kennedy launch site, the available radio spectrum is crowded with telemetering bands; an optical system partially relieves this crowding and it neither interferes with other channels in the radio band nor is interfered by them.

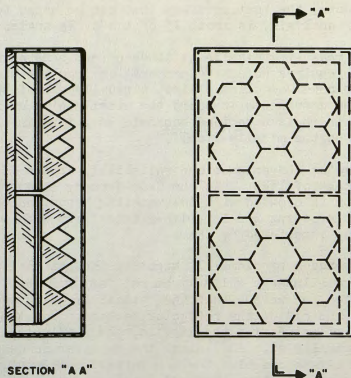
Nor is optical interference a problem. It might be thought that since the flame of the missile produces an intense white light not far from the reflectors that this would jam the system, or worse, that the optical tracking system would lock onto the missile flame instead of the reflector package. But the light from the engine flame, sunlight reflected from the missile, the skylight background, etc., are unpolarized, or virtually so. And, since the receiver is designed to measure the three polarization parameters plus the unpolarized component, it is only necessary to ground out the electrical signal corresponding to the unpolarized component to eliminate practically all of the background. Thus, the computer is deceived into thinking that the system is watching a pair of Savart plates climbing against a jet black sky.

#### References

- 1 C.J. Peters, Applied Optics, Vol. 3, p. 1502, 1964.
- 2 W.A. Shurcliff, Polarized Light, Harvard University Press, 1962.

# MISSILE MOUNTED PACKAGE

---



SECTION "A A"

FIG.1

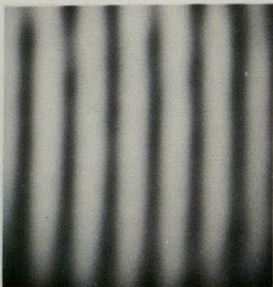


FIGURE. 2 INTERFERENCE PATTERN OF  
AN UNCOMPENSATED SAVART  
PLATE

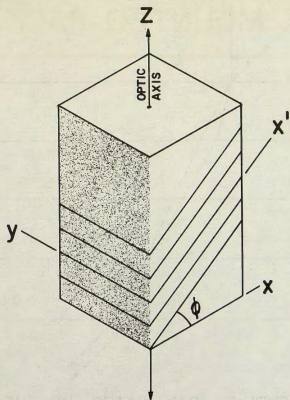


FIGURE 3. ORIENTATION OF PLATES  
BEFORE CUTTING

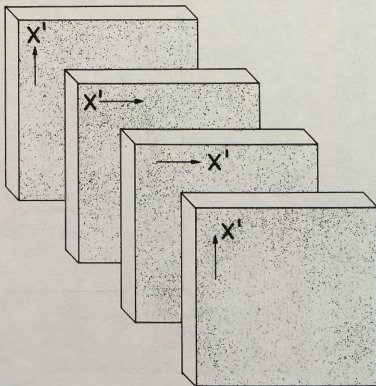


FIGURE 4. ORIENTATION OF PLATES  
RELATIVE TO EACH OTHER



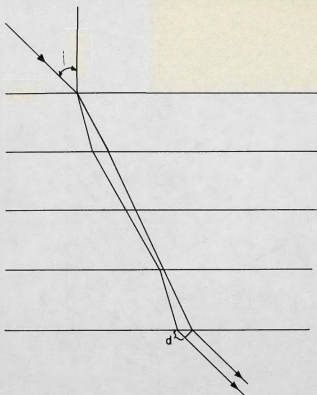


FIGURE 5. RAY PATHS IN THE COMPENSATED LINEAR SAVART PLATE

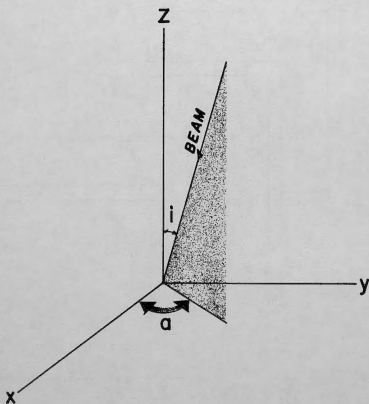
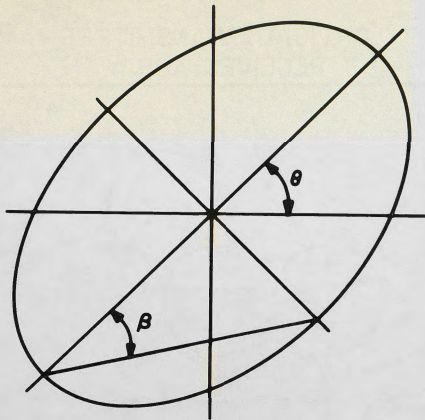


FIGURE 6. RELATIONSHIP OF BEAM DIRECTION TO MISSILE COORDINATES



POLARIZATION ELLIPSE

FIG. 7

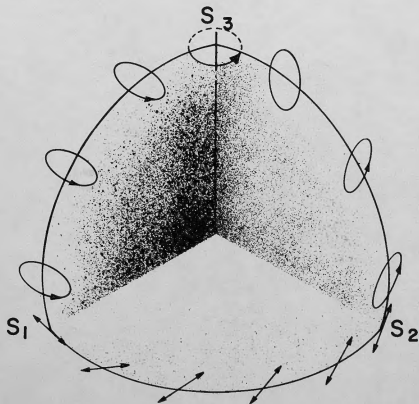


FIGURE 8. POINCARÉ SPHERE

# OPTICAL TRANSMITTER-RECEIVER ARRAY

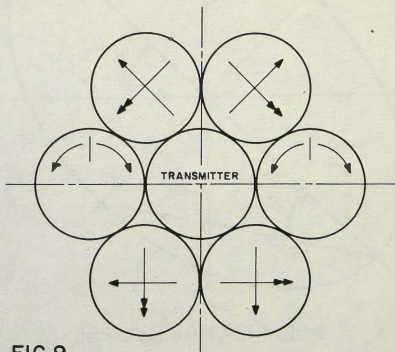


FIG. 9

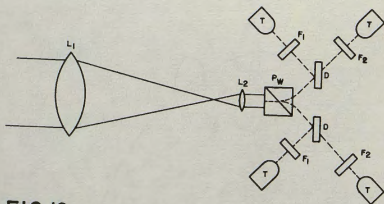


FIG. 10

