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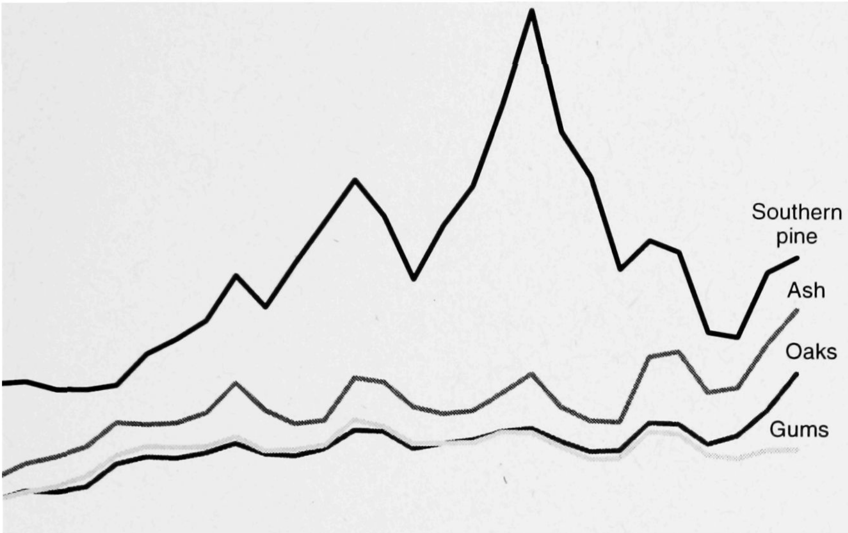

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Forestry Investments and Option Values: Theory and Estimation

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I. INTRODUCTION

Benefit-cost analysis has long been a standard tool for assessing the value of investment projects. If the future stream of benefits and costs related to a project are known, benefit-cost analysis involves the calculation and evaluation of the project's net present value, equal to discounted benefits less discounted costs. If the net present value (*NPV*) is positive and costs do not exceed the available budget, the project is economical and should be undertaken unless another project yields a higher *NPV*. Of course, future benefits and costs are never known with certainty; rather, investors hold probabilistic beliefs about these values. Standard practice in this case has been to assume risk-neutrality, replace random variables with their expected values, and evaluate expected net present values (*ENPV*) in a manner identical to the deterministic case (e.g., Nickell 1978). As above, project i is worth undertaking if $ENPV_i \geq 0$ and $ENPV_i \geq ENPV_j$ for all $j \neq i$.

Most investments are characterized, at least to some degree, by irreversibility. For instance, if a dam is built today, the decision cannot be reversed tomorrow, except at great expense. Recent research on investment under uncertainty has shown that the *ENPV* criterion is invalid when irreversibilities are present (e.g., McDonald and Seigel 1986; Dixit and Pindyck 1994). Uncertainty and irreversibility give rise to a value to delay the investment decision in order to acquire new information about the project's profitability. This value, termed an *option value*[1], lowers the expected value of investing today. Consequently, investments must meet a stricter standard than traditionally applied: ENPV must be greater than or equal to the option value. In general, option values are greater the larger is the variance in the value of the investment.

Forestry investments are characterized, to a great extent, by irreversibilities and uncertainties. Since trees take decades to grow to maturity, harvesting a stand is, for practical purposes, irreversible. Uncertainty is an especially important consideration for forestry investments due to long growing cycles of trees and effects of weather, pests, and fire. In addition, prices for raw material inputs such as timber tend to be volatile due to linkages with end product markets.

[1]There are two interpretations of the option value in the economics literature (Fisher and Hanemann 1986). In the first, the option value is the difference between a consumer's willingness to pay to preserve a future option (referred to as option price) and expected consumer surplus (Cicchetti and Freeman 1971; Bishop 1982). In the second, the option value is the difference between the expected value of an irreversible investment project that accounts for forthcoming information on the project's profitability and the value of the project when this information is ignored (Arrow and Fisher 1974; Dixit and Pindyck 1994). This bulletin is concerned with the second definition of option value.

These features of forestry investments suggest that option values may be substantial and therefore an important consideration in the evaluation of forestry projects.

This bulletin considers option values related to a principal problem for forestry investors, the timing of harvests. The purpose is to present a general theory of the rotation problem under uncertainty and irreversibility (Section II) and provide a methodology for empirically estimating option values (Section III). Modifications of the framework for analyzing options values related to other aspects of forestry investments are also discussed (Section IV).

The methodology presented in this bulletin may be applied to a number of problems with relevance for forest policy. The first relates to the behavior of nonindustrial private forest landowners (NIPFs). NIPFs own more than one-half of the forest land in the United States and therefore significantly influence forest products markets through their decisions to manage and sell timber. Public agencies in the business of selling timber have undertaken a substantial research effort towards understanding the management objectives of NIPFs (e.g., Royer and Risbrudt 1983). A common view is that NIPFs are reluctant to harvest timber and, in general, do not manage their lands according to economic criteria (USDA Forest Service 1990). The analysis presented in this bulletin suggests that the apparent unwillingness of NIPFs to harvest timber may in fact indicate optimizing behavior. In the presence of irreversibilities and uncertainties, the harvesting decision involves a fundamentally different process compared with traditional benefit-cost analyses. Investors monitor the value of their stand over time and harvest only when the value is sufficiently high.[2] The benefits of harvesting must be greater than those required in standard deterministic models due to the value of postponing the irreversible harvesting decision.

The investment model developed here also has implications for public purchases of, or acquisition of easements to, forest land for development, recreation, preservation, and other uses. In most cases, land acquisition will take place over a period of years as funds are allocated to the program. For instance, the Conservation Reserve Program, an agricultural land set-aside program, has been operating for the last ten years. When a public agency purchases forest land from a private interest, the sale is irreversible, implying that a private owner cannot buy back a parcel at a later date if, for instance, timber

[2]Surveys of NIPFs often include questions like, "Do you plan to harvest timber in the next five years, the next five to ten years, etc.?" (e.g., Kingsley and Birch 1980; Kingsley 1976). Negative or uncertain responses are interpreted as a reluctance to harvest timber; however, such responses may be consistent with rotation decisions that take account of option values.

prices increase. In theory, the private owner must be compensated for foregoing the option to hold or sell the parcel in the future. This implies that the costs to a public agency of acquiring a given amount of land are likely to be higher than those suggested by standard benefit-cost calculations or sale prices in competitive land markets. In the latter case, land sales are not necessarily irreversible.

II. THE OPTIMAL FOREST ROTATION AND OPTION VALUES: THEORY

This section presents a general description of the harvesting problem and the option values related to delaying the irreversible harvesting decision. Several authors have considered the rotation problem in a stochastic environment (Reed 1993; Thomson 1992; Clarke and Reed 1989; Morck et al. 1989). In most studies, timber prices, or the value of the stand, are assumed to follow geometric Brownian motion (GBM). GBM provides a reasonable representation of historical price trends for some timber species and, in many cases, permits the derivation of tractable analytical models. However, GBM has a strong implication for option values associated with the harvesting decision. Specifically, GBM prices imply that option values arise only from the possibility of suspending management and harvesting activities if prices fall too low. Realizations of prices above this minimum level contain no information that may potentially influence the harvesting decision. Thus, the stochastic prices do not introduce any asymmetries into the problem such that harvesting takes place for some price realizations but not for others.

To illustrate this point, the rotation problem as posed in Thomson is considered. The current timber price is P. In the next period, it increases to uP with probability p and declines to dP with probability $1 - \pi$. The parameters are defined as $u = \exp(\sigma\sqrt{\Delta t})$, $d = u^{-1}$, and $\pi = [\exp(\mu\Delta t) - d]/(u - d)$. As $\Delta t \rightarrow 0$, the model, referred to as the two-state option pricing model, converges to GBM with drift rate μ and standard deviation σ. With no management costs or alternative land uses, Thomson's equation for the value of the investment is

$$V[P,Q] = \max\left\{ P_t Q(a_t) + V[P_t, Q(0)], \frac{\pi V[uP_t, Q(a_t+1)] + (1-\pi)V[dP_t, Q(a_t+1)]}{1+r} \right\} \quad (1)$$

where $Q(a_t)$ is the (deterministic) timber volume of a stand of age a_t and subscripts denote the period. The first term in the braces is the value of the investment if harvesting takes place in period t where

$V[P_t, Q(0)]$ is the value of bare land. The second term is the value of the investment if harvesting is delayed.

In the last period T, the stand must be harvested. Thus, if the timber price increased between periods T-2 and T-1, equation (1) in period T-1 is

$$V[P,Q] = \max\left\{ uP_{T-2}Q(a_{T-1}) + \frac{P_{T-2}Q(1)[\pi u^2 + (1-\pi)ud]}{1+r}, \frac{P_{T-2}Q(a_{T-1}+1)[\pi u^2 + (1-\pi)ud]}{1+r} \right\} \quad (2)$$

and

$$V[P,Q] = \max\left\{ dP_{T-2}Q(a_{T-1}) + \frac{P_{T-2}Q(1)[\pi ud + (1-\pi)d^2]}{1+r}, \frac{P_{T-2}Q(a_{T-1}+1)[\pi ud + (1-\pi)d^2]}{1+r} \right\} \quad (3)$$

if the price declined. It can be shown that the solutions to the problems in (2) and (3) must be the same. That is, the optimal harvest period following an increase in price (equation 2) must be the optimal harvest period following a decrease in price (equation 3). This implies that from the standpoint of period T-2, delaying harvest to period T-1 yields no information that influences the harvesting decision. In particular, the harvesting decision is not affected by the realization of the price in period T-1 and the option value equals zero. This result may be extended to T-period and continuous-time models.

If complications such as management costs and alternative land uses are included in models of form (1), there are non-zero option values related to the suspension of management activities or conversion to another use if timber prices fall too low (Thomson 1992; Morck et al. 1989). The focus here is on option values that arise from movements in the timber price. Specifically, delaying the harvest allows new prices to be observed which may contain information about future price trends. As demonstrated above, GBM models do not capture these effects. For this reason, the approach here departs from option value models using GBM or related stochastic processes (see Dixit and Pindyck 1994). Instead, a model is developed along the lines of Fisher and Hanemann (1986, 1990). In Section III, an alternative stochastic process is considered, which models information arising from timber price movements.

The decision for the investor is whether to harvest in the current period t, delay the harvest to a future period $T > t$, or never harvest.[3]

[3]For simplicity, only a single rotation is considered. In most analyses of the rotation problem, additional rotations shorten the first rotation.

If the harvesting decision is made in the current period, or rather the investor ignores any information forthcoming in future periods (for instance, future price information), the maximization problem is

$$W_t^1 = \max\{0, V_t, E_t[V_{t+1}](1+r)^{-1}, E_t[V_{t+2}](1+r)^{-2}, \ldots, E_t[V_T](1+r)^{-T}\} \quad (4)$$

where 0 indicates no harvest, V_i is the value of the timber in period i, $E_i[\cdot]$ is the expectation with respect to information available in period i and r is the interest rate.[4] Equation (4) is the investment problem consistent with the *ENPV* criterion. Risk-neutrality is assumed since it isolates the effects of irreversibility and uncertainty on the harvesting decision and demonstrates that the existence of option values does not depend on particular risk preferences. If the investor recognizes the irreversibility of the harvesting decision and the possibility of acquiring new information in the future, the maximization problem is

$$W^2_t = \max\{0, V_t, E_t[\max\{V_{t+1}(1+r)^{-1}, E_{t+1}[W^2_{t+2}](1+r)^{-2}\}]\} \quad (5)$$

where W_{t+2}^2 is the continuation value in period $t+2$.[5] The investor takes into account the information gained by delaying to period $t+1$ and harvests in period $t+1$ if $V_{t+1} \geq E_{t+1}[W_{t+2}^2](1+r)^{-1}$ and delays harvest to period $t+2$ if $V_{t+1} < E_{t+1}[W_{t+2}^2](1+r)^{-1}$ The option value in period t is defined as $OV_t = W_t^2 - W_t^1$

To aid in the elaboration of the option value OV_t and to provide a clear contrast to the *ENPV* approach, it is assumed in what follows that $V_t > 0$ and $V_t = E_t[V_s](1+r)^{-(s-t)}$ for all $t < s \leq T$ This restriction implies that the expected value of the stand is growing at the rate of interest and, according to the *ENPV* criterion, that the investor is indifferent to harvesting the stand in the current period and delaying the harvest to a future period. Below, conditions are derived for

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[4]The problem is formulated as an open-loop control problem. Fisher and Hanemann (1990) recognize another possibility, the open-loop feedback formulation. In the present context, equation (4) would remain the same, yet if the harvesting decision is delayed to period $t+1$, the maximization problem becomes

$$W_{t+1}^1 = \max\{0, V_{t+1}, E_{t+1}[V_{t+2}](1+r)^{-1}, \ldots, E_{t+1}[V_T](1+r)^{-(T-1)}\}$$

rather than

$$W_{t+1}^1 = \max\{0, E_t[V_{t+1}], E_t[V_{t+2}](1+r)^{-1}, \ldots, E_t[V_T](1+r)^{-(T-1)}\}$$

Since it is assumed below that the stand is financially mature according to (4), this distinction is unimportant.

[5] The no-harvest choice (0) is reflected in W_{t+2}^2. In all periods $s < T$, not harvesting is equivalent to delaying the harvest decision to the next period. Only in the final period is the decision made to harvest or never harvest (see Section III).

$OV_t \geq 0$ and $OV_t > 0$. In the second instance, it is optimal to delay the harvest past period t, implying harvesting is optimal when the expected growth rate in the value of the stand is strictly less than the interest rate. Thus, the presence of non-zero option values requires a departure from the standard Faustmann harvesting rule (Faustmann 1995). At the end of the subsection, OV_t is determined under the less restrictive and more realistic assumption $V_t \geq E_t[V_s](1+r)^{-(s-t)}$ for all $t < s \leq T$

Case I. $OV_t \geq 0$.

Without imposing any restrictions on (5), it is possible to show that $OV_t \geq 0$. First, note that $E_t[W_{t+2}^2] = E_t[E_{t+1}[W_{t+2}^2]]$ by the Total Probability Theorem. It then follows from the convexity of the maximum operator and Jensen's Inequality that

$$E_t[\max\{V_{t+1}(1+r)^{-1}, E_{t+1}[W_{t+2}^2](1+r)^{-2}\}] \geq \max\{E_t[V_{t+1}](1+r)^{-1}, E_t[E_{t+1}[W_{t+2}^2]](1+r)^{-2}\} \tag{6}$$

Further

$$E_t[E_{t+1}[W_{t+2}^2]] \geq \max\{E_t[V_{t+2}], E_t[V_{t+3}](1+r)^{-1}, \ldots, E_t[V_T](1+r)^{-(T-2)}\} \tag{7}$$

since W_s^2 accounts for the possibility of acquiring new information whereas W_s^1 does not (Fisher and Hanemann 1987). Together, (6) and (7) imply

$$E_t[\max\{V_{t+1}(1+r)^{-1}, E_{t+1}[W_{t+2}^2](1+r)^{-2}\}] \geq \max\{E_t[V_{t+1}](1+r)^{-1}, E_t[V_{t+2}](1+r)^{-2}, \ldots, E_t[V_T](1+r)^{-T}\} \tag{8}$$

Since the right-hand side (rhs) of (8) equals V_t by assumption, it follows that

$$OV_t = E_t[\max\{V_{t+1}(1+r)^{-1}, E_{t+1}[W_{t+2}^2](1+r)^{-2}\}] - V_t \geq 0 \tag{9}$$

Now consider (5). Equation (9) indicates that delaying the harvest to period $t+1$ may be optimal even though the stand is financially mature according to (4).

Case II. $OV_t > 0$.

The period t option value is strictly positive with the following restrictions on (5): $V_{t+1} > E_{t+1}[W_{t+2}^2](1+r)^{-1}$ for $V_{t+1} \in \Omega_1$, $V_{t+1} < E_{t+1}[W_{t+2}^2](1+r)^{-1}$ for $V_{t+1} \in \Omega_2$, and $V_{t+1} = E_{t+1}[W_{t+2}^2](1+r)^{-1}$ for $V_{t+1} \in \Omega_3$, where Ω is defined as the set of possible realizations of V_{t+1},

$\Omega_1 \cup \Omega_2 \cup \Omega_3 = \Omega$, $\Omega_1 \neq \varnothing$, and $\Omega_2 \neq \varnothing$. As discussed in section III, these restrictions hold if timber prices are mean-reverting, indicating prices above (below) the historical average have a tendency to decline (increase) in subsequent periods. This implies that $V_{t+1} \in \Omega_1$ corresponds to large stand values associated with high timber prices and $V_{t+1} \in \Omega_2$ corresponds to small stand values associated with low timber prices. Timber prices may be realistically modeled as mean-reverting (Dixit and Pindyck 1994). In the short term, timber prices may fluctuate due to unanticipated shocks to forest products markets; however, in the long term the price may tend toward the long-run marginal cost of producing timber.

Under the above restrictions, equation (8) becomes

$$\begin{aligned} &E_t[V_{t+1}|V_{t+1} \in \Omega_1](1+r)^{-1} + E_t[E_{t+1}[W_{t+2}^2]|V_{t+1} \in \Omega_2](1+r)^{-2} + \\ &E_t[V_{t+1}|V_{t+1} \in \Omega_3](1+r)^{-1} > \\ &\max\{E_t[V_{t+1}](1+r)^{-1}, E_t[V_{t+2}](1+r)^{-2}, \ldots, E_t[V_T](1+r)^{-T}\} \end{aligned} \quad (10)$$

The strict inequality in (10) is established in two steps. First, assume that the rhs of (10) equals $E_t[V_{t+1}](1+r)^{-1}$ Then (10) can be rewritten as

$$E_t[E_{t+1}[W_{t+2}^2]|V_{t+1} \in \Omega_2](1+r)^{-2} > E_t[V_{t+1}|V_{t+1} \in \Omega_2](1+r)^{-1} \quad (11)$$

which holds by the above restrictions. Next, suppose that the rhs of (10) equals

$$\max\{E_t[V_{t+2}](1+r)^{-2}, E_t[V_{t+3}](1+r)^{-3}, \ldots, E_t[V_T](1+r)^{-T}\} \quad (12)$$

Then the strict inequality follows from (7) and the above restrictions. Now, equation (9) becomes

$$OV_t = E_t[\max\{V_{t+1}(1+r)^{-1}, E_{t+1}[W_{t+2}^2](1+r)^{-2}\}] - V_t > 0 \quad (13)$$

indicating it is optimal to delay harvest in period *t* in order to receive period *t*+1 information about the value of the stand.

Cases I and II establish the conditions for $OV_t \geq 0$ and $OV_t > 0$. However, the results rely on the strong assumption that $V_t = E_t[V_s](1+r)^{-(s-t)}$ for all $t < s \leq T$ While it is convenient to assume the stand is financially mature by (4), it is more realistic to assume that $V_t \geq E_t[V_s](1+r)^{-(s-t)}$ for all $t < s \leq T$, thereby allowing the present value of the stand to decline after it has reached the "optimal" rotation age. The adoption of this assumption introduces the possibility that V_t is greater than or equal to the left-hand sides of (8) and (10) which by (5) indicates $OV_t = 0$. Thus, there is a value to delaying the harvest to period $t+1$, but it is outweighed by the value of harvesting in the current period. If V_t is less than the left-hand sides of (8) and (10), the above results carry through. Note that the option value cannot be less than zero (Fisher and Hanemann 1987). This is an implication of (7): additional information can only increase the expected value of the stand.

III. THE OPTIMAL FOREST ROTATION AND OPTION VALUES: ESTIMATION

The above analysis indicates that option values are strictly positive when delaying the harvest yields information that asymmetrically influences harvesting decisions. In case II, the realization of V_{t+1} indicates whether harvesting or delaying is optimal in period $t+1$. The ARIMA is a stochastic process consistent with the restrictions in case II.[6] In what follows, the net value of the timber stand in period t is assumed to be $V_t = P_t Q(a_t)$ where $Q(a_t)$ is the deterministic volume dependent on stand age a_t and the stumpage (standing volume) price P_t follows the ARIMA(p,d,q) process

$$w_t = \phi_1 w_{t-1} + \ldots + \phi_p w_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q} + \delta \tag{14}$$

where $w_t = \Delta^d P_t$, d indicates the number of times the price is differenced, the ϕ_i are the autoregressive parameters, the θ_i are the moving average parameters, δ is a constant, and ε_t is a normal random variable with $E[\varepsilon_t] = 0$, $E[\varepsilon_t^2] = \sigma^2$, and $E[\varepsilon_i \varepsilon_j] = 0$, $i \neq j$.[7]

ARIMA processes are mean-reverting in the limit. Thus, if current and past values of w_t are above (below) the mean, long-term declines (increases) are expected. The exact pattern of expected price movements depends on the order of the process (i.e., the value of p, d, and q) and the levels of current and past prices. For simple processes the patterns are readily determined. For instance, consider the AR(1) process given by

$$P_t = \phi P_{t-1} + \delta + \varepsilon_t \tag{15}$$

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[6]See Pindyck and Rubinfeld (1981) for an introduction to time-series modeling using the ARIMA.

[7]An investor can observe a cross-section of similar stands and thereby make an accurate assessment of stand volume changes over time. Prices, in contrast, are the outcome of complex market interactions and therefore, are modeled as random variables from the perspective of the investor.

The process has mean $\delta/(1-\phi)$ and is stationary or mean-reverting if $|\phi|<1$. Assuming the current price $P_t = \delta/(1-\phi)$, $Q(a_t) = Q(a_{t+1})(1+r)^{-1} = Q(a_{t+2})(1+r)^{-2}$, and $T = t+2$, it follows that

$$V_t = E_t[V_{t+1}](1+r)^{-1} = E_t[V_{t+2}](1+r)^{-2} \tag{16}$$

and

$$V_{t+1} \mathop{=}\limits_{<}^{>} E_{t+1}[V_{t+2}](1+r)^{-1} \quad \text{as} \quad P_{t+1} \mathop{=}\limits_{<}^{>} \delta/(1-\phi) \tag{17}$$

(17) satisfies the conditions stated in Case II above and therefore, $OV_t > 0$. Qualitatively similar results may be obtained for different values of P_t, $Q(a_s)$, and T.

More complicated ARIMA processes are difficult to analyze in general terms; however, with appropriate data it is possible to estimate OV_t for specific cases. A time series on stumpage prices and timber yield data for the corresponding species are required. The first step is to solve the problem in (4), thereby satisfying the condition $V_t \geq E_t[V_s](1+r)^{-(s-t)}$ for all $t < s \leq T$ and establishing the current age of the stand, a_t, as the "optimal" rotation age. The ARIMA model is estimated on the stumpage price data and future prices are forecast from the last observations of the series ($P_t, P_{t-1},\ldots$). This yields the forecasts $\hat{w}_t(t+1), \hat{w}_t(t+2), \ldots, \hat{w}_t(T)$ where t indicates the period from which the forecast is made and T is the last period of the analysis. The expected value of the stand in period $t+i$, $i = 1,2,\ldots,T-t$, conditional on period t information, is then $\hat{P}_t(t+i)Q(a_{t+i})$ where $Q(a_{t+i})$ is derived from the yield data. The age of the stand in period t can be varied until the value of the stand is just growing at the rate of interest r, indicating the stand is financially mature in period t according to (4).

The next step is to determine the value of the program under (5) and the associated option value. The solution is found by constructing a tree of future stand values and associated probabilities and then solving the problem recursively. The error term ε_{t+1} is normally distributed so the probability of reaching the price $\hat{P}_t(t+1) + \varepsilon_{t+1}$ is given by $f(\varepsilon_{t+1}) = \hat{\sigma}^{-1}(2\pi)^{-1/2}\exp(\varepsilon^2_{t+1}/2\hat{\sigma}^2)$ where $\hat{\sigma}^2$ is the variance estimate from the ARIMA estimation. To be operative, the normal distribution is partitioned and probabilities are assigned to N prices, denoted $P^i(t+1) = \hat{P}_t(t+1) + \varepsilon^i_{t+1}$, $i = 1,2,\ldots,N$. If the normal distribution is truncated at three standard deviations below and above the mean, then the width of each partition is given by $M=6/N$. The value of $P^1(t+1)$ is the mean of first partition equal to $\hat{P}_t(t+1) - (3 + M/2)\hat{\sigma}$. The probability of reaching the first price, $Pr(1)$, is approximated by the area under the normal distribution from $-3\hat{\sigma}$ to $-(3 - M)\hat{\sigma}$. Likewise, the probability, $P(2)$, of reaching the second price is the area under the normal distribution from $-(3 - M)\hat{\sigma}$ to $-(3 - 2M)\hat{\sigma}$. The prices $P^1(t+1), P^2(t+1), \ldots, P^N(t+1)$ and the associated probabilities provide the first stage of the price tree (Figure 1).

From each of the N nodes, a forecast of the mean price in period t+2 is made. The mean forecast from the ith node, $\hat{P}^i_{t+1}(t + 2)$, is based on the values $P^i(t+1), P_t, P_{t-1}, \ldots$. As indicated by the subscript on $\hat{P}^i_{t+1}(t + 2)$, the forecast takes into account period t+1 information, namely the value of $P^i(t+1)$. The forecast is then used to determine the prices $P^{ij}(t + 2) = \hat{P}^i_{t+1}(t + 2) + \varepsilon^j_{t+2}$, $j = 1,2,\ldots,N$, with associated probabilities, as above (Figure 1). There are N^2 prices or nodes for stage two, and N^k prices or nodes for stage k. The price tree is completed when prices are determined for all T-t stages. Prices are then multiplied by the volumes $Q(a_{t+k})$ where k indicates the kth

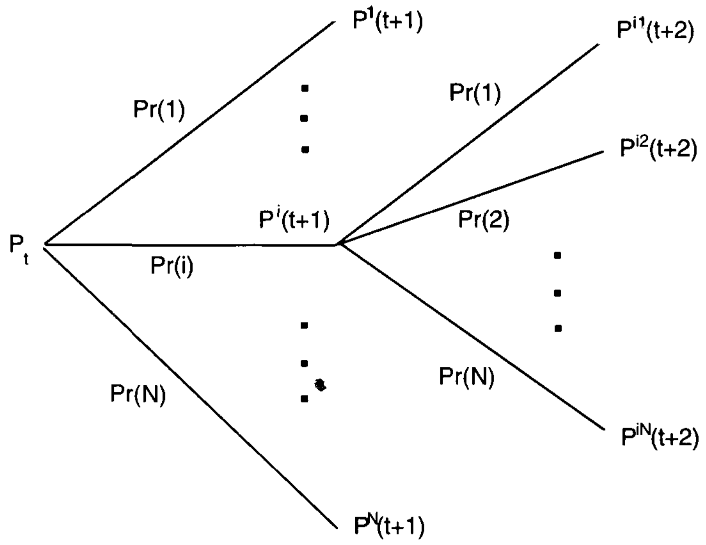

Figure 1. Construction of the price tree.

stage or the kth period past t. The volumes are the same as those used to determine the solution to (4) above. The result is a tree of stand values consisting of T-t stages.

The problem may now be solved recursively beginning in stage T-t-1. At each of the N^{T-t-1} nodes, it must be determined whether harvesting in period T-1 or delaying harvest to period T is optimal. That is, $W_{T-1}^2 = \max\{V_{T-1}, E_{T-1}[\max\{0, V_T(1+r)^{-1}\}]\}$ must be found at each node. From the standpoint of period T-1, the investor considers the expectation only over positive values of the stand in period T since, if period T is reached, the investor can elect to never harvest if the stand value is negative. The expected value of the stand in period T is an average of the positive values that may be reached from a particular period T-1 node, with the weights given by $\Pr(1), \Pr(2), \ldots, \Pr(N)$. The procedure is then repeated from period T-2: $W_{T-2}^2 = \max\{V_{T-2}, E_{T-2}[W_{T-1}^2](1+r)^{-1}\}$ is determined for each of the N^{T-t-2} period T-2 nodes where $E_{T-2}[W_{T-1}^2]$ is a weighted average of the N values of W_{T-1}^2 that may be reached from a particular node. A value for W_t^2 is determined after T-t-1 iterations and the option value is calculated as $OV_t = W_t^2 - W_t^1$

Table 1 presents estimates of the option value for two forest species. ARIMA models are estimated using times series data on average real prices of southern pine and oak stumpage sold from private lands in Louisiana (Ulrich 1988). Per-acre yields are derived from U.S. Forest Service inventories of private timberlands (Birdsey 1992). An interest rate of 5% and a planning horizon of T=t+8 is assumed.[8] Option values increase the present values of pine and oak stands \$107 and \$6, respectively, and imply that harvesting should be delayed beyond the standard Faustmann rotation. The higher option value for pine is due to greater price variance ($\hat{\sigma}^2$) which increases potential gains from new information. The solution algorithms can be used to determine the probability that harvesting takes place in periods t+1,...,t+8 or never (Figure 2). For both species, there is a considerable probability that harvesting will be delayed many years past the standard financial rotation. For instance, the probability of harvesting does not reach 50% until period t+3 for pine and period t+5 for oak. The large probability masses in periods t+5 and t+6 reflect the influence of the end of the planning horizon. Harvesting is delayed in expectation of higher prices; however, harvesting eventually occurs to avoid the possibility of never harvesting. Extending the planning horizion redistributes these probability masses across future periods.

[8]The length of the planning horizon is determined by computational limitations.

Table 1. Estimates of option values for Southern pine and oak.

Pine		Oak	
ARIMA estimation			
$P_t = \underset{(0.21)}{1.02} P_{t-1} - \underset{(0.21)}{0.26} P_{t-2} + \underset{(18.11)}{36.88}$		$w_t = \underset{(0.17)}{0.33} w_{t-1} - \underset{(0.18)}{0.67} w_{t-2} + \underset{(0.93)}{2.14}$	
$\bar{R}^2 = 0.60$	$\hat{\sigma}^2 = 645.20$	$w_t = P_t - P_{t-1}$ $\quad \bar{R}^2 = 0.30$	$\hat{\sigma}^2 = 24.45$
Option value estimation			
$W_t^1 = \$1937$		$W_t^1 = \$545$	
$W_t^2 = \$2044$		$W_t^2 = \$551$	
$OV_t = \$107$		$OV_t = \$6$	

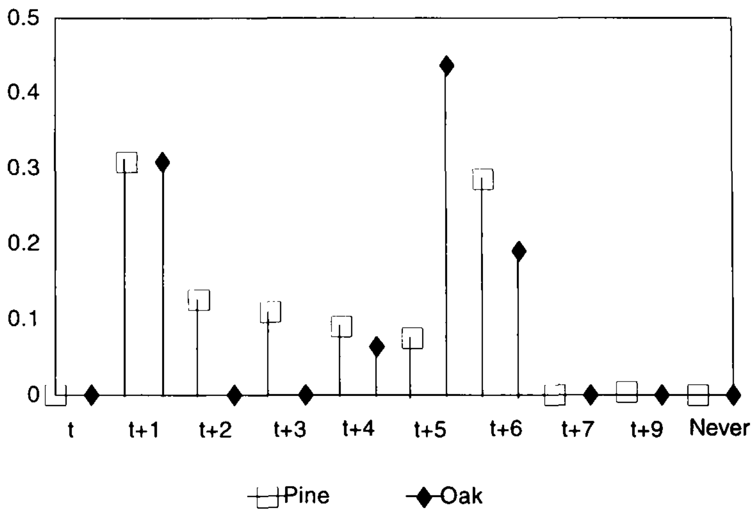

Figure 2. The probability of harvest.

IV EXTENSIONS OF THE MODELING FRAMEWORK

Option values are likely to arise in connection to other aspects of forestry investments. The framework in section III can be modified to estimate option values associated with forest land development and sales. The analysis presented above considers a single rotation and a finite planning horizon. The decision to develop or to sell forest land requires an assessment of a parcel's value and therefore consideration of multiple rotations over an infinite horizon. Multiple rotations are incorporated by allowing stands to be harvested, yielding $P_tQ(a_t)$, and then regrown, yielding W_t^2 as a function of P_t and $Q(0)$. The model in (1) allows for multiple rotations. Modeling an infinite time horizon is more problematic. Infinite-horizon dynamic programming problems can only be solved if they are autonomous; that is, the optimal value of the program is independent of time. However, option values arise in the model presented here precisely because of this dependence. In particular, asymmetries in harvesting behavior are explicitly linked to the past pattern of prices. An alternative approach is to approximate the infinite stream of benefits with a finite stream. The finite stream must be long enough so that benefits from additional periods are small due to discounting. The length of the stream is likely to be determined by computational limitations.

The modified procedure gives the value of the land in forestry, W_t^2. If development of a parcel is a possible choice, the problems in (4) and (5) are modified to include a development value. An option value is then associated with delaying the irreversible development to gain information about the value of the land in forestry. Development in the current period may be optimal according to the *ENPV* criterion yet suboptimal when option values are considered (Fisher and Hanemann 1986). The private landowner may also have the opportunity to sell forest land to the government for conservation purposes. As with the development problem, a sale value is included in (4) and (5). The sale value is incorporated for the periods in which the acquisition program is in effect. In this case, the option value is related to information the landowner gains about the value of the timber by delaying the sale. The landowner may need to be compensated for this option value, implying higher program costs for the agency than indicated by the *ENPV* criterion or by competitive market prices for forest land.

An extension of the modeling framework involves incorporating the value of the standing forest. Hartman (1976) examines how the optimal timber rotation changes when non-timber amenities from the standing forest are valued. If the non-timber amenities increase monotonically with the age of the stand, then the rotation is always longer than the timber only rotation. If timber prices and the value of non-timber amenities are stochastic, option values will arise from forthcoming information on both random variables (Reed 1993). In addition, the two effects will interact according to the correlation between timber prices and non-timber benefits. The stochastic process governing the benefits from non-timber amenities might be estimated from repeated sampling of forest plots. The Forest Ecosystem Research Program at the University of Maine is presently collecting plot-level information on ecosystem attributes, including plant species richness and diversity and abundance of terrestrial birds.

V. SUMMARY AND CONCLUSIONS

Forestry investments are characterized by irreversibilities and uncertainties due in large part to the length of time required to grow trees. The traditional benefit-cost method of analyzing these investments may give misleading results due to a failure to account for information gained by delaying irreversible actions. This bulletin details an approach to evaluating forestry investments under irreversibility and uncertainty. The theoretical analysis in Section II reveals that option values arise under general conditions. Specifically, the result $OV_t \geq 0$ is derived with no restrictions on (5). This result is analyzed further by exploring the implications of restrictions on (5). The restrictions imposed in case II indicate that realizations of V_{t+1} provide information on the relative magnitude of V_{t+1} and $E_{t+1}[W_{t+2}^2](1+r)^{-1}$ As a result, option values are strictly positive. The case II restrictions are consistent with a mean-reverting process for timber prices, implying that prices may flucuate in the short term due to unanticipated shocks, but in the long term may tend toward the marginal cost of producing timber. The ARIMA process embodies the mean-reverting property and therefore, is generally consistent with non-zero option values. This bulletin demonstrates how ARIMA models can be combined with dynamic programming techniques to estimate option values related to timber harvesting and other aspects of forestry investments.

Strictly positive option values are found for southern pine and oak through empirical simulations. The expected present values of pine and oak stands increase by approximately 6% and 1%, respectively, when option values are included. The higher stand values reflect the value of forthcoming information on prices and the possibility of avoiding unprofitable harvests. The higher option value for pine is due to the greater price variance, which increases potential gains from new information. Even though the option values are small relative to the timber value, there is a high probability that harvests will be delayed a number of years past the standard financial rotation. For instance, there is a greater than 60% probability that oak harvests will be delayed five or more years, even though the option value is a small fraction of the stand value.

The analysis presented here has practical applications to the evaluation of forestry investments as well as to the understanding of investment behavior. In the latter case, the model developed in this study yields insights into optimal harvesting decisions of private investors. It suggests that investors monitor the values of uncertain variables and harvest only when thresholds for these variables are reached. For instance, in the simple model presented in (15)–(17), the stand is harvested only when P_t is above the threshold $\delta / (1 - \phi)$. In contrast, models implied by traditional benefit-cost analysis suggest that investors plan harvests from the current period. The behavior of private investors has implications for timber management on public lands and the design of forest land acquisition programs. Thus, an important area of future research will be to determine if investors take option values into account. This study provides the theoretical and methodological foundation needed to explore this issue further.

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