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Collaborative Project: Developing a tutorial approach to enhance student learning of intermediate mechanics

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Principal Investigator: Wittmann, Michael C. **Award ID:** 0442388
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Collaborative Project: Developing a tutorial approach to enhance student learning of intermediate mechanics

Project Participants

Senior Personnel

Name: Wittmann, Michael
Worked for more than 160 Hours: Yes
Contribution to Project:

Post-doc

Graduate Student

Name: McCann, Kate
Worked for more than 160 Hours: Yes
Contribution to Project:
Classroom observations, study of student reasoning about mathematics when learning physics.

Name: Black, Katrina
Worked for more than 160 Hours: Yes
Contribution to Project:
Curriculum development, study of student reasoning about integration methods.

Name: Sayre, Eleanor
Worked for more than 160 Hours: Yes
Contribution to Project:
Curriculum development, study of student reasoning about applications of coordinate systems.

Name: Van Deventer, Joel
Worked for more than 160 Hours: Yes
Contribution to Project:
Survey design, study of students' uses of vectors in mathematics and physics.

Undergraduate Student

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

University of New Hampshire

Prof. Dawn Meredith of the Physics Department at the University of New Hampshire has collaborated with the co-PIs on both the testing of existing instructional materials in the suite of 'Intermediate Mechanics Tutorials' ('IMT') as well as the development of new materials. She has provided multiple sets of pretest and post-test data from classes in which she used selected IMT materials.

Drury University

Prof. Brant Hinrichs of the Department of Physics at Drury University has collaborated closely with the co-PIs in the process of testing and assessing selected tutorial materials in upper division classes that he has taught, both in mechanics and electromagnetism. He has also shared several sets of pretest and post-test data from his classes.

West Chester University of Pennsylvania

Prof. Carolyn Sealfon of the Department of Physics at West Chester University of Pennsylvania has recently begun collaborating with the co-PIs in the process of pilot-testing selected IMT materials. At the time of the submission of this report, she is currently teaching intermediate mechanics for the first time, and she plans to collect pretest and post-test data in order to help measure the effectiveness of the tutorials.

University of Michigan Dearborn

Prof. Carrie Swift, Assistant Professor of Physics and Astronomy at the University of Michigan-Dearborn, has been an enthusiastic collaborator with the co-PIs. She reported at a regional AAPT meeting on findings from an initial test run of IMT materials, and her observations have led to significant refinements of the tutorials that she used.

Pacific University

Prof. Juliet Brosing of the Department of Physics at Pacific University has collaborated with the co-PIs on testing and refining several IMT tutorials. She provided detailed suggestions for the tutorials--including typographical errors and unclear questions that have since been addressed--that have helped improve their effectiveness.

Seattle Pacific University

With the invitation and cooperation of Profs. Stamatis Vokos and John Lindberg of Seattle Pacific University (SPU), one of the co-PIs (Ambrose) conducted project-related activities while spending a sabbatical leave in residence with the Department of Physics at SPU. He taught an upper division physics/engineering course in mechanics, which allowed the opportunity to test existing IMT materials and to develop and pilot-test new tutorials recently incorporated into the collection.

Bowling Green State University

Prof. Stephen Van Hook of the Department of Physics and Astronomy at Bowling Green State University invited one of the co-PIs (Ambrose) to attend and present workshops at the annual Northwest Ohio Symposium on Science, Math and Engineering Teaching. He has led workshops at the 2005 and 2006 symposia in order to publicize and disseminate results from the IMT project to college faculty as well as K-12 teachers.

University of Washington

Lillian McDermott, director of the Physics Education Group at the University of Washington (UWPEG), was the Ph.D. advisor for one of the co-PIs (Ambrose). The research that contributed to the development of 'Tutorials in Introductory Physics,' published by McDermott, Peter Shaffer, and UWPEG (Prentice-Hall, 2001), was also instrumental in guiding the proof-of-concept stages of the work supported by this grant. Members of UWPEG also invited Ambrose to present a seminar presenting results from the IMT project.

Tarleton State University

Prof. Daniel Marble of the Department of Mathematics and Physics at Tarleton State University has collaborated with the co-PIs in testing and assessing the effectiveness of selected tutorials from IMT. His feedback led to modifications of some tutorial worksheets. He has also provided detailed observations of students who used tutorials as part of distance-learning courses on intermediate mechanics.

Western Michigan University

Prof. Charles Henderson of the Physics Department at Western Michigan University, a fellow researcher in physics education, invited one of the co-PIs (Ambrose) to present a departmental colloquium on the research and curriculum development underlying the IMT project.

Grandville High School

Michael Evele, a master high school physics teacher at Grandville High School in Grandville, MI, obtained his M.Ed. degree with Physics Emphasis at Grand Valley State University, the home institution of one of the co-PIs (Ambrose). Evele worked with Ambrose extensively for many M.Ed. courses in physics, and he incorporates innovative active-learning techniques with his students. He has tested carefully chosen materials from IMT, specifically a tutorial on angular momentum and Kepler's second law, with a class of honors physics students. He shared the (surprisingly successful) results of that tutorial and observations of his students as they worked through the materials. (Please note, however, that this implementation of IMT tutorials was with a highly selective student population and an extremely talented teacher. IMT has been designed for upper division undergraduate physics students, not introductory level or secondary level students.)

Seoul National University

Prof. Gyonghou Lee of Seoul National University has been an enthusiastic (though informal) collaborator with one of the co-PIs (Ambrose) in probing student understanding of conservative forces and applications of vector calculus to force fields. He has frequently discussed the adaptation and implementation of specific research tasks for use in his research. That collaboration has helped Lee design materials for use at his home institution to help enhance student learning.

Other Collaborators or Contacts

Obviously, on a collaborative grant, I've worked closely with Bradley S. Ambrose of Grand Valley State University. This report is a mirror of the report that he submitted for his grant, since our work was done in close collaboration and the results are shared between the two institutions.

Activities and Findings

Research and Education Activities: (See PDF version submitted by PI at the end of the report)

Please see the attached file at the end of this document.

Findings: (See PDF version submitted by PI at the end of the report)

Please see the attached file at the end of this document.

Training and Development:

With this project the co-PIs (Ambrose and Wittmann) have taken one of the first systematic steps into probing student thinking and learning in upper level mechanics courses. The work of the co-PIs has benefited greatly from prior research at the introductory level, and at the same time this work has expanded (and continues to expand) that research base. This research has contributed to the training of physics education research graduate students under the supervision of one of the co-PIs (Wittmann) at the University of Maine, including: Katrina E. Black, T. Carter, K. McCann, Eleanor C. Sayre, Trevor I. Smith, R. Padraic Springuel, and Joel Van Deventer. The project has also provided unique teaching and professional development experiences for those who have helped facilitate faculty workshops on Intermediate Mechanics Tutorials. These include Katrina Black of the University of Maine and Natalie Beyer of Grand Valley State University (home institution for co-PI Ambrose).

A multitude of new research questions have emerged pertaining to: students' conceptual understanding of the relevant physics; students' functional understanding of the mathematics that describes the physics; and students' ability to describe physics using the relevant mathematics. This research has also led to the expansion of conventional research methods used in empirical physics education research, notably at the University of Maine in conducting 'mini-interviews' immediately following class meetings and in videotaping homework help sessions. These methods, along with the analysis of student responses to written research tasks on pretests (ungraded quizzes), homework problems, and exam questions, have been utilized effectively and extensively by the co-PIs and by research assistants involved in the project.

Outreach Activities:

The research and curriculum development underlying the Intermediate Mechanics Tutorials project afforded the opportunity for several outreach activities going beyond the physics departments of the co-PIs. The events listed below are examples of presentations given by co-PI Ambrose to faculty and students outside his home department at Grand Valley State University (GVSU). Two events were sponsored by GVSU and another was part of a senior capstone for natural science majors at Seattle Pacific University (SPU).

Journal Publications

E.C. Sayre and M.C. Wittmann, "The plasticity of intermediate mechanics students' coordinate system choice", Physical Review Special Topics Physics Education Research, p. , vol. , (2007). Submitted,

Books or Other One-time Publications

E.C. Sayre, M.C. Wittmann, J.E. Donovan, "Resource Plasticity: Detailing a Common
Chain of Reasoning with Damped
Harmonic Motion", (2007). Book, Published
Editor(s): P. Heron, L. McCullough, J. Marx
Collection: Physics Education Research Conference
Proceedings 2006, AIP Conference
Proceedings 883
Bibliography: p.85-88

K.E. Black and M.C. Wittmann, "Epistemic Games in Integration: Modeling
Resource Choice", ( ). Refereed Conference Proceedings, Submitted
Editor(s): L. Hsu, L. McCullough, P. Heron
Collection: Physics Education Research Conference
Proceedings 2007, AIP Conference
Proceedings
Bibliography: submitted for publication

K. McCann and M.C. Wittmann, "Students creating mathematical meaning
in Mechanics: Signs in scalar equation", (2007). Refereed Conference Proceedings, Submitted
Editor(s): L. Hsu, L. McCullough, P. Heron
Collection: Physics Education Research Conference
Proceedings 2007, AIP Conference
Proceedings
Bibliography: submitted for publication

J. Van Deventer and M.C. Wittmann, "Comparing Student Use of Mathematical
and Physical Vector Representations", (2007). Refereed Conference Proceedings, Submitted
Editor(s): L. Hsu, L. McCullough, P. Heron
Collection: Physics Education Research Conference
Proceedings 2007, AIP Conference
Proceedings
Bibliography: submitted for publication

E.C. Sayre, "Plasticity: Resource Justification and
Development", (   ). Thesis, Unpublished dissertation, soon to be published online after final revisions.
Bibliography: Unpublished Ph.D. dissertation,
University of Maine, 2007

E.C. Sayre and M.C. Wittmann, "Intermediate mechanics students'
coordinate system choice", (2007). Conference Proceedings, Published
Collection: Conference on Research in
Undergraduate Mathematics Education
Bibliography: http://cresmet.asu.edu/crume2007/
eproc.html

Web/Internet Site

URL(s):

http://perlnet.umaine.edu/imt/

Description:

This web site provides small-group learning materials for teaching intermediate mechanics. The physics is Newtonian with very little Lagrangian formalism. Materials are a mix of conceptual, mathematical, and problem solving University of Washington-style tutorials, as well as related research results, examination questions, and suggested course outlines. Materials are freely posted under a Creative Commons 2.5 license (share-alike, derivative products, attribution). Materials are also linked to at http://www.compadre.org/PER/items/detail.cfm?ID=5522, where ComPADRE is the central repository of PER materials on the internet.

Other Specific Products

Contributions

Contributions within Discipline:

The co-PIs on this project (Ambrose and Wittmann) have developed and refined a set of research-based teaching materials for intermediate mechanics designed by best practices in physics education research. As mentioned previously, this work has served to extend the research base of student understanding in introductory physics. Previous research has shown that the traditional mode of lecturing effects very little conceptual change among physics students. The research has also guided the design of innovative, highly effective, and highly interactive instructional strategies for teaching physics.

Contributions to Other Disciplines:

Contributions to math education research: When students take intermediate mechanics they are expected not only to deepen their conceptual understanding of mechanics but to learn how to use a wider array of mathematical tools to be able to solve problems. Past and current research efforts by the co-PIs of this project have therefore sought to probe student conceptual understanding of upper level mechanics as well as to characterize the ability of students to understand and apply the necessary mathematics. As a result, much of the research and curriculum development undertaken during the project can be used to inform discipline-based education research in mathematics and engineering, particularly in regard to difficulties students may encounter when: setting up or solving differential equations, using or interpreting complex numbers and complex exponentials, calculating or interpreting the meaning of gradients and curls, calculating or interpreting the meaning of vectors in different coordinate systems (e.g., translating from Cartesian to polar, or vice versa). (For examples of these types of difficulties documented during the project, see Section 3 of the Findings portion of this report, as well as those papers referenced in this report that will soon be published in the 2007 Physics Education Research Conference proceedings.) These findings may provide valuable insight for other discipline-based education researchers in mathematics and engineering.

Contributions to Human Resource Development:

Contributions that have provided opportunities for research and teaching in science: The research supported by this grant has opened new avenues for both empirical and theoretical studies in physics education research (PER). The various research questions that have been motivated by this project have guided not only the work of the co-PIs but that of PER graduate students who have been supervised by one of the co-PIs (Wittmann). In addition, through numerous formal and informal interactions with other colleagues in PER, our work has influenced other research and curriculum development efforts in other areas of physics, including upper division electromagnetism and introductory mechanics.

Contributions to Resources for Research and Education:

Contributions to resources for research and education: Please see the attached Activities section of this report for a detailed description of the instructional materials ('Intermediate Mechanics Tutorials') that have been developed with support by this grant.

Contributions Beyond Science and Engineering:

Categories for which nothing is reported:

Any Product

Contributions: To Any Beyond Science and Engineering

1. Overview of findings

In this section we, the co-Principal Investigators (co-PIs) of this collaborative project, provide a general discussion of the major findings of our project. We describe specific examples of the research that was conducted during the scope of the project and how the results from that research were used to inform the development, refinement, and assessment of the *Intermediate Mechanics Tutorials (IMT)*. To reflect the strongly collaborative nature in which our work was designed and executed, this report on our research findings has been filed identically by each co-PI.

The student populations of primary interest included several groups of students taking junior-level intermediate mechanics at Grand Valley State University (GVSU, 2 classes), the University of Maine (UMaine, 2 classes), and Seattle Pacific University (SPU, 1 class). These classes were taught by either one or the other of the co-PIs (Ambrose at GVSU and SPU, Wittmann at UMaine). Although the details of the courses vary somewhat, all courses cover a common core of topics including air resistance, linear oscillations (simple harmonic, damped, driven), conservative forces, non-inertial reference frames, and central forces. The results presented here were taken mostly from the analysis of responses to written questions on pretests (ungraded quizzes administered before tutorial instruction but usually after lecture instruction) and examinations (after tutorial instruction). Results were also obtained from informal observations of students during class. In addition, Wittmann and his colleagues at UMaine videotaped a variety of student settings—ranging from short interviews, lectures, and homework help sessions—from which were obtained rich observations of student interactions with each other, the instructor(s), and the curriculum. These methods, commonly employed in empirical physics education research (PER), were used to track the evolution of student thinking before, during, and after tutorial instruction.

It is not possible for us in this limited space to provide a comprehensive discussion of all of the research conducted during the time frame of the grant. Several publications are either published, submitted, or in preparation, giving more detail about these and other findings. Here, we select particularly illustrative examples of our efforts in identifying obstacles to student learning, designing appropriate tutorial activities to address those obstacles, and assessing the effectiveness of the tutorials. We focus in particular on two topics: harmonic oscillations and conservative forces.

2. Identifying obstacles to student learning: difficulties in developing conceptual understanding

In this section we restrict our discussion to selected results from our research on student learning of oscillations. This research has been guided by the following specific questions:

(a) How well do students understand the factors that affect the *frequency* of different types of linear oscillations?

(b) How well do students interpret and understand *formal representations* of oscillatory motion, such as x *vs.* t graphs of 1-D oscillators and x-y trajectories of 2-D oscillators?

(c) To what extent do students answer qualitative questions by bringing to bear their knowledge of *general principles* relevant to the physical situation at hand?

Example: Simple harmonic oscillations in 1-D

On the basis of their work in introductory mechanics, students might be expected to apply (in 1-D) and extend (to 2-D) the idea that the frequency $\omega_o = [k/m]^{1/2}$ of a simple harmonic oscillator is determined solely by the spring constant and mass. In order to measure the ability of students to understand this relationship, two sets of tasks were developed for use on pretests.

On the first pretest students are shown a strobe picture illustrating a block connected to an ideal spring that is released from rest on a level, horizontal surface. They are asked how the period would be affected by: (i) changing the release point of the block from 0.5 m to 0.7 m from equilibrium, (ii) replacing the original spring with one that is stiffer, and (iii) replacing the original block with one having four times the mass. The students were expected to recognize that the period will not change in case (i), decrease in case (ii), and increase (double) in case (iii).

Although most students gave correct responses (ignoring reasoning) for each case, case (i) yielded the lowest percentage of complete and correct explanations (~10%). Many correct responses were supported by "compensation arguments" relating amplitude, average speed, and period. As one student explained, "It may seem that the block is moving faster, but it is also moving farther to compensate." While such justifications make it *plausible* that the period is unaffected by changing the amplitude, they show no evidence of understanding that *only* the spring constant and mass affect the period. Even more telling, the most common *incorrect* explanation (from ~25% of the students) was based on the incorrect intuition that the greater initial displacement from equilibrium, and hence the larger amplitude, would cause the period to increase because, for example, "the block travels farther during each period."

The above results are interesting because they suggest recurring, incorrect intuitions that may lead to confusion in the context of 2-D oscillations. These results also motivated the design of the tutorial *Simple harmonic oscillations,* in which students show by direct substitution that sinusoidal functions such as $x(t) = A \sin(\omega_o t + \phi_o)$ solve the differential equation $m\ddot{x} = -kx$ for the simple harmonic oscillator only if $\omega_o = (k/m)^{1/2}$. Despite the increased performance by students on tutorial homework and exams (not discussed here), the specific reasoning patterns elicited by the 1-D simple harmonic oscillator pretest students suggested the need to explore how students proceed from 1-D to the 2-D case.

Example: Simple harmonic oscillations in 2-D

Students often were introduced to 2-D oscillations as an application of conservative forces, several weeks after covering 1-D oscillations. The following pretest was designed to probe student understanding of the relative frequencies along the *x*- and *y*-axes of a 2-D oscillator.

Students are asked to consider an undamped 2-D oscillator with $U(x, y) = ½k_1x^2 + ½k_2y^2$. They are also reminded about the relationship $k_i = m\omega_i^2$ for each force constant. For each *x-y* trajectory shown in Fig. 1, students are asked whether that trajectory is possible for such an oscillator and, if so, whether ω_1 is greater than, less than, or equal to ω_2. (A previous version of the pretest asked instead for a comparison of the force constants k_1 and k_2.)

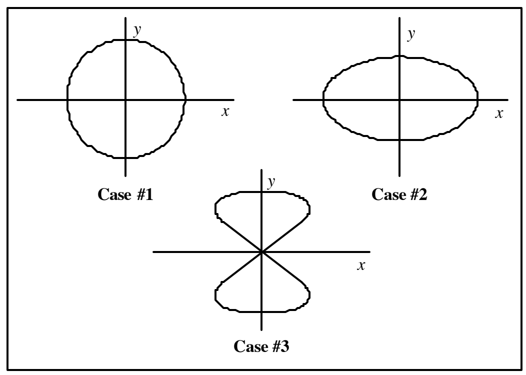


Figure 1. Example *x-y* trajectories from a pretest on 2-D oscillators. For each case, students were asked to compare the frequencies (or, alternatively, the force constants) along the *x*- and *y*-directions.

In each class very few students (between 0% and 15%) gave correct responses for all cases. Most students incorrectly compared the relative frequencies (or relative force constants) by using inappropriate "compensation arguments" involving the relative amplitudes along the *x*- and *y*-axes. For example, for case #2 most incorrectly predicted that $\omega_1 < \omega_2$ (or $k_1 < k_2$) for reasons such as: "the spring goes farther in the *x*-direction, so [the] spring must be less stiff in that direction," or "since we now have an oval curve with the *x*-axis longer, ω_2 must be greater to compensate." The prevalence of this type of reasoning or intuition suggested that most students failed to recognize that *x-y* trajectories like those from the pretest yield frequency information about the 2-D oscillator. In addition, the tendency for students to link amplitudes with frequencies (or force constants) appeared to be analogous to the most common incorrect mode of reasoning used on the 1-D oscillator pretest.

In order to determine to what extent the design of the pretest question triggered (inadvertently) the amplitude-frequency (or amplitude-force constant) connection among students, an alternate version of the pretest was recently developed. In the new pretest, students consider several different cases of 2-D oscillators, and for each case the students are told both (1) the ratio k_y/k_x of the force constants and (2) the initial conditions of motion of the oscillator. One of those cases was non-isotropic, with $k_y/k_x = 4$, and with the oscillator beginning from a location along the +*x* axis with initial velocity in the +*y* direction.

This particular case elicited the similar "compensation" reasoning from students, who predicted incorrectly that the trajectory would form an ellipse with its major axis aligned with the *x*-axis (see Fig. 2). However, a new idea was also elicited by many of the students, namely that the role of the each spring is to "bring the oscillator to equilibrium." Students articulating this type of intuition drew *x*-*y* trajectories that were not closed but instead spiraled inward toward the origin (see Fig. 3, which shows the sketch from a student who held both of these incorrect ideas).

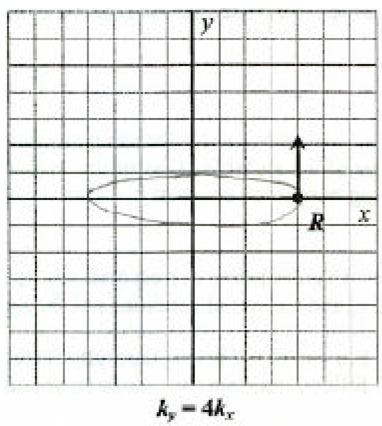


Figure 2. Incorrect trajectory of a non-isotropic 2-D oscillator drawn by a student who believed that the path would be "an ellipse instead of a circle because the force constants are different."

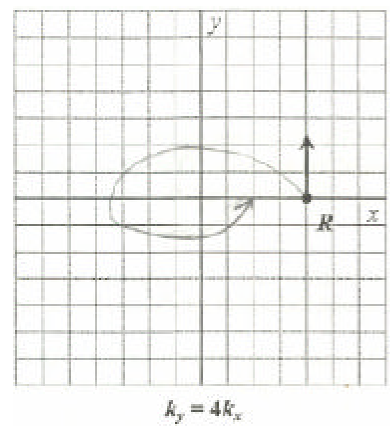


Figure 3. Another incorrect trajectory of a non-isotropic oscillator sketched by a student who thought that the oscillator "travels less in the *y*-direction because of the stiffer spring." The student also believed that the path would spiral inward because "the springs attempt to return the object to equilibrium."

The modified pretest described above has been administered thus far to only one class (7 students), so the results from that pretest are to be treated as preliminary at best. More research is needed to explore how students progress in their thinking from the idea that "the springs attempt to return the object to equilibrium" (a natural idea from the point of view of Hooke's law) to an inward spiraling trajectory, which would suggest that energy is not conserved! It is also striking that the change in format to the pretest task—asking students to draw their own *x*-*y* trajectories rather than interpret several trajectories already drawn —elicited the same incorrect intuition connecting relative amplitudes to force constants.

Example: Motion of underdamped oscillators

When students encounter underdamped oscillators, instructors expect them to recognize that the frequency of oscillations will depend upon three factors, namely the spring constant, the mass, and the damping constant γ, such that: $\omega_d = (\omega_o^2 - \gamma^2)^{1/2}$, where $\omega_o = (k/m)^{1/2}$. Given the types of reasoning difficulties we had seen previously among the upper level students, a pretest was devised to probe how well students could recognize and apply this relationship after lecture instruction. The pretest began by showing students the *x vs. t* graph of a simple harmonic oscillator (no damping) released from rest (see solid curve in Fig. 4). They were then told to assume that a linear damping force is applied, causing the oscillator to become underdamped. In part A of the pretest, students were asked to sketch a qualitatively correct graph of the underdamped oscillator having the same initial conditions as the original (undamped) one. In part B, they were asked to consider the instant it first passes $x = 0$: at that instant is the oscillator speeding up, slowing down, or moving with maximum speed?

Few students (~25% or fewer) in each class answered part A correctly. Any curve like the dashed curve shown in Fig. 4 would have been acceptable. However, most students (60% to 70%) drew graphs like the one shown in Fig. 5, showing a gradually decreasing amplitude (which is correct) but a frequency that is equal to that for the undamped case. Most student explanations—for example, "the amplitude shrinks in time but the period shouldn't change since they are independent of each other"—suggest an overgeneralization from simple harmonic motion.

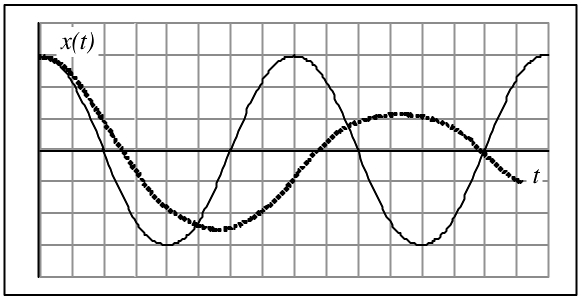


Figure 4. Motion graph from part A of the pretest on underdamped oscillators. The solid curve represents the motion of a simple harmonic oscillator. The dashed curve (not shown to students) illustrates a qualitatively correct graph for an underdamped oscillator.

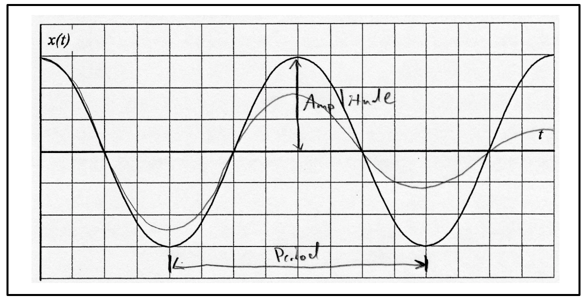


Figure 5. Example of a typical incorrect student graph elicited by the pretest on underdamped oscillations. Most students drew graphs like this one, showing equal frequencies for the undamped and underdamped cases.

Other errors arose on part A, including the tendency to show *both* the amplitude *and* period as gradually decreasing. These responses could be interpreted as recurrences of the belief that amplitude and frequency are connected, a belief that was detected on each of the two pretests described previously. More research is needed, though, to tell for certain.

Part B of the pretest was equally difficult for students. Students could answer part B by taking the differential equation of motion, $m\ddot{x} = -c\dot{x} - kx$, setting $x = 0$, recognizing that acceleration and velocity must be opposite in direction, and concluding that the oscillator must be slowing down at $x = 0$. Students could get the same answer by drawing a free-body diagram and finding that the net force opposes the velocity.

Only 20% to 30% of the students answered correctly. The most common incorrect answer was to state that the oscillator experienced its maximum speed upon passing $x = 0$. Some did not seem to take the damping into account, saying that there was no acceleration because the spring was neither pushing nor pulling. Others did not at all invoke forces or Newton's laws, saying simply that the slope of the *x vs. t* graph would be at a maximum at $x = 0$. Both modes of reasoning strongly suggest a tendency to overgeneralize from the case of simple harmonic motion.

This particular pretest has also been given to over 30 college and university faculty members attending *IMT* workshops facilitated by one or both co-PIs. Both parts of the pretest were found to be challenging to instructors: just over half (17/32) answered correctly on part A (correct qualitative behavior of amplitude and period) and about 40% (13/32) gave correct responses on part B (motion of oscillator upon passing $x = 0$). The most prevalent errors given by students, discussed previously, also arose among the faculty members. A few instructors attempted to answer the pretest questions by solving "from scratch" the differential equation of motion for the damped oscillator (before running out of time), rather than bring Newton's laws to bear. These types of responses suggested to us that many faculty members simply had not applied (or attempted to apply) a conceptual approach to understanding basic aspects of underdamped motion.

3. Identifying obstacles to student learning: difficulties connecting the mathematics with the physics

In addition to building a qualitative foundation of intermediate mechanics, students are expected to develop facility in using mathematics both to describe physical situations and to derive new physics. One important topic in mathematics that is emphasized heavily in most intermediate mechanics courses is differential equations—translating a physical situation into an equation, solving the resulting equation, and testing the limits in which that equation is valid. Another important set of mathematical tools for mechanics comes from vector calculus—the del operator, gradient of a scalar field, curl of a vector field, and the applications of these tools to the study of conservative forces.

In this section we present examples of the challenges that students typically face when they attempt to bridge the mathematics with the physics. As is the case with the conceptual and reasoning difficulties identified in the research, these difficulties with the mathematics gave us valuable insight that informed the design of instructional materials.

Example: Understanding signs in mathematical equations

In another important area, we have looked at how students understand the simplest of differential equations needed in intermediate mechanics: first-order linear differential equations describing air resistance. Many different aspects of this topic have been studied already, including how students choose coordinate systems to relate the mathematics to the physics and how students carry out integrals using different but equivalent methods, each of which can lead to the correct answer. (Analyzing students' methods of solution when two correct answers or reasoning pathways are available to them continues to be an interesting area of study at the University of Maine.) Where the data described in other parts of this report are gathered from pretests, quizzes, and exams, the data described in this section are gathered primarily from interviews, classroom observations (videotaped for later study), and examinations.

Students need to understand how an arbitrarily chosen coordinate system (say, whether positive is up or down in the case of one-dimensional vertical motion) is irrelevant when finding consistent and coherent descriptions of the physics. For example, from the equation $F = ma = mg - cv$ students should read that the coordinate system chosen for the equation has positive in the downward direction (because the downward weight force, mg, is positive), and they should also recognize that they cannot determine the direction in which the object is moving (because the v in the $-cv$ term can have both positive and negative values, and the minus sign in front of cv is needed to ensure that the force of air resistance always opposes the motion of the object). Note that the mathematical operation of subtracting the cv term has an interpretation as if one were multiplying the cv term by negative one and adding it to the system. This mathematically isomorphic statement actually has quite important conceptual meaning when interpreting mathematics equations. So, were the coordinate system reversed, then the equation would be $F = ma = -mg - cv$, where only the sign of the weight force changes. These equations (depending on choice of coordinate system) describe all motion of the object, up and down, and starting from above or below terminal velocity. It is that generality of the mathematics that we wish to convey to our students as part of the intermediate mechanics course.

We find that students have a very hard time recognizing the consistency of the mathematics available to them and the physical situation being described. In one study, we regularly found that students were unable to make the right changes to mathematical equations when the physical situation changed. For example, when throwing a ball downward with initial speed greater than (instead of less than) terminal velocity, students regularly changed the sign in front of the "ma" term in Newton's Second Law to match what they *thought* was the appropriate direction. Thus, using downward as the positive direction, they wrote "$mg - cv = -ma$" rather than "$mg - cv = ma$." Half the students in one class made this error at some point in their reasoning. (It is important to note that many fixed the error on their own, but that classroom observation allowed us to document their reasoning in great detail as it occurred.) One student was interviewed after class and described that the correct equation worked when "reasoning forward," using v to get a, the acceleration, but that the incorrect equation was necessary when "reasoning backward," using a to get v. And, since one knew that net force on the thrown ball was negative (because the ball had to slow down to reach terminal velocity), the "ma" term had to be negative as well. This student quite vehemently expressed that he assumed a was positive and therefore a minus sign was needed in his equation.

Similar issues with minus signs have been observed when reversing the direction of positive in the coordinate system, changing whether the object starts its motion in the positive or negative direction, or changing the object's initial velocity from above to below the terminal velocity of the system. We have also found that the central issue of understanding how to use a variable quantity that can have negative value in an equation shows up in other areas. Students need help in learning how to use negative signs appropriately in equations. These results have been submitted for publication in the *Physics Education Research Conference Proceedings* for 2007.

Example: Solving differential equations with physical meaning

In a series of problems related to describing the motion of an object experiencing air resistance, we have found that students (appropriately) use two different methods for integrating separable equations. Students were asked the questions shown in Figures 6a and 6b, showing isomorphic mathematics and physics problems. The mathematics is identical in each, but the content of the physics question is very different in that additional information (not least of which, the physical setting) is given. In a series of mini-interviews, we observed how students answered these questions, and in examination questions we asked a similar question, shown in Fig. 7.

In each case, we found that students use two different but correct methods for solving the problem. One can use the method of indeterminate integration, in which one adds an undefined integration constant after taking the antiderivative of the integrand. We refer to this as the "+*C*" method, for obvious reasons. Or, one can use the method of choosing limits of integration appropriate to the problem and plugging those limits into the equation immediately after finding the antiderivative of the integrand. We refer to this as the "limits" method. The two are equivalent, and each has its own advantages depending on the equation one is considering. The advantage to a physicist of using the "limits" method is that the dependence of the solution on initial conditions is quickly apparent: one can immediately explore the parameter space to see how changes to initial conditions affect the solution.

We find that students begin our courses familiar with the "+*C*" method and generally unfamiliar with the "limits" method. They learn quickly. They learn attendant skills such as choosing the right boundary conditions when specific information is not given. (Fig. 6a has mathematically specific information, the problem in Fig. 6b requires interpretation of the physical information, and Fig. 7 gives no information at all, for example). Students must also learn in the "limits" method that the choices of limits must be consistent across both sides of a separable equation (the lower limit of the time integral must match the lower limit of the velocity integral, in Fig. 6b). These

$$\frac{dy}{dx} + y = 0, \quad y(0) = A_0$$

Can you find an expression for y?

Figure 6a: Math-like miniview question.

A group is working on the following problem.

A bullet fired horizontally has a muzzle velocity of 366 m/s and experiences a $-cv^2$ air resistance. Find an equation that describes the horizontal velocity of the bullet with respect to time.

A student writes: $\sum F_x = -cv^2 = m\frac{dv}{dt}$

$$-\frac{c}{m}dt = \frac{dv}{v^2}$$

What would you do next?

Figure 6b: Physics-like miniview, a differential equation couched as an air resistance problem.

You are at the top of the building with a beach ball. It is of a size that only the quadratic velocity air resistance term exists, $F = -cv^2$.
You throw the ball vertically downward. Because it's convenient, you choose down to be positive (so the ball is traveling in the positive direction). You observe that the ball *slows down* as it travels downward, eventually seeming to move at a constant speed until it hits the ground.
a. Write the equation of motion of the ball before it hits the ground.
b. What is the ball's speed when you observe it moving at a constant speed?
c. Find the equation for the velocity of the ball as a function of height.

Figure 7: The exam problem, presented on both the first and final exams.

and other issues can be difficult to learn, and we have developed new tutorial materials to assist students.

Our major result comes when looking at how students use the two methods to come to physically meaningful equations. If one does not find the constant *C* in the "+*C*" method, one has not fully solved the equation and arrived at a complete description of the physics. What seems like a minor mathematical problem can have serious consequences, especially if *C* depends upon many parameters in the system. We evaluated the completeness of students' solutions to differential equations that were of the form shown in Fig. 7, giving the question both on a midterm and (unannounced) on the final examination in the same class. We created a consistency plot, showing how students' responses changed from the midterm to the final (see Fig. 8). On this plot, circles represent student responses on the midterm; triangles, on the final. Squares represent those students whose solution method and correctness did not change from the midterm to the final. Four populations stand out.

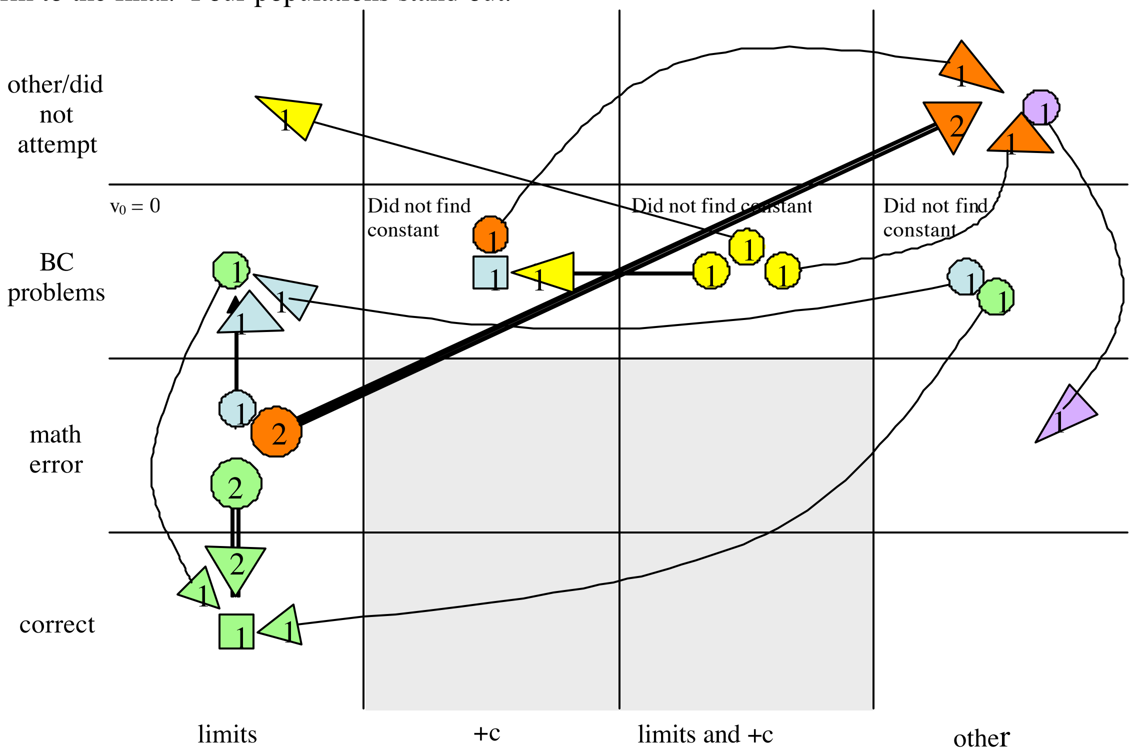


Figure 8: A consistency plot of midterm – final response pairs, color coded to indicate particular groups. Blue response pairs do not belong to any particular group.

Five students (denoted by the green symbols) out of a total of 14 ended up with the correct solution that was physically complete. Of these, three used the limits method on the midterm already. Four students (orange) ended the semester with serious difficulties in solving the problem, using incomplete reasoning or inappropriate mathematical tools. Three students on the midterm (yellow) used *both* the "limits" and "+*C*" method, indicating that they did not understand that the methods are mathematically identical. Finally, no students (the grey shaded area) used the "+*C*" method and found a physically complete solution to the problem! Further analysis of these results, including detailed interview transcripts, are being prepared for submission to the *Physical Review – Special Topics in Physics Education Research*.

Example: Relationship between force and potential energy

Students are led to build upon their previous experience with gravitational and electric fields by being shown that any conservative force **F** has a corresponding potential energy function *U* such that $\mathbf{F} = -\nabla U$. The following pretest task has proven highly effective in probing how students interpret this relationship between force and potential energy. In the pretest students were given an equipotential diagram (see Fig. 8). One of the questions posed on the pretest asks students to rank the three labeled locations *(A, B,* and *C)* according to the magnitude of the force exerted on a test charge placed at that location. On the basis of their prior experience with electric field and electric potential energy, the students were to recognize that the closer the equipotential lines were to one another at a given location, the greater the magnitude of the force at that location, thus: $F_B > F_A > F_C$.

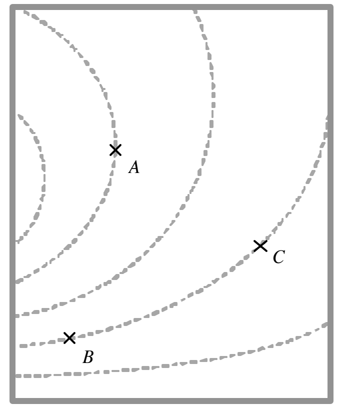


Figure 8. Equipotential map from a pretest question that tests student understanding of the relationship between force and potential energy.

At both co-PIs' institutions only about 20% of the students answered correctly with correct reasoning. The most common incorrect line of reasoning used by the students was to rank the locations in a way that would be correct for the potential energy rather than force. As one student responded, "$F_A > F_B = F_C$. Since *F* is proportional to *V,* higher *V* means higher *F*." In the early stages of this project (before the start of the grant period), this particular difficulty remained one of the most prevalent and persistent; on exam questions similar to the pretest some students explicitly referred to the gradient in supporting their incorrect answers, *e.g.*: "The greater the force, the higher [the] potential energy. $\mathbf{F} = -\nabla U$." Thus it was clear to us that students needed careful guidance in interpreting the meaning of the gradient and, in so doing, recognizing the difference between potential energy and its spatial rate of change (*i.e.,* force).

4. Internal assessment: measuring the effectiveness of *IMT* materials in enhancing student learning

The research results discussed in the preceding section serve to highlight particular types of conceptual and reasoning difficulties that the co-PIs have diagnosed and attempted to address through the incorporation of tutorials. Those results are intended to be illustrative (not comprehensive) of the obstacles that hinder meaningful learning of the physics as well as the mathematics of intermediate mechanics. In this section we share examples of our work in measuring the effectiveness of the tutorial materials in addressing these difficulties. As before, we do not have sufficient space to give a complete discussion of our assessment efforts, but the examples we have chosen to discuss illustrate the extent to which our research and curriculum development work have been iterative in nature.

Results from preliminary versions of two tutorials

During the proof-of-concept stage of this project, preliminary versions of the tutorials *Harmonic motion in two dimensions (H2D)* and *Conservative forces and equipotential diagrams (CFP)* were written so as to help students develop a robust understanding of the correct relationships between frequency and spring constant (in the former tutorial) and between force and potential energy (in the latter). In *H2D* students are guided through the reasoning necessary to "read" *x-y* trajectories such as those shown in Fig. 1 and in so doing to glean information about the relative frequencies (and hence the relative spring constant values) along the *x*- and *y*-axes. And in *CFP* students make an analogy between topographic maps and equipotential diagrams, using a "flat earth" approximation for gravitational potential energy and ignoring all frictional effects, so that the students would connect the direction and magnitude of the force (slope) at a given location to the gradient of potential energy (as indicated by the orientation and relative proximity of elevation lines).

Although the teaching strategies used in the tutorial seemed effective, results on post-tests (exams) for both tutorials were lackluster, usually around 40% to 50% correct. Compared to student performance on the corresponding pretests (around 20% correct, at best) such results reflected a modest but unsatisfying level of improvement. Detailed analysis of the students' responses showed that specific difficulties elicited by the pretests—prevalent intuitions connecting amplitude and frequency, or the belief that force and potential energy are proportional to one another—were not always being addressed by the tutorials. (One striking example from a post-test for *CFP* was already mentioned in the preceding section, namely the student who responded, "The greater the force, the higher [the] potential energy. $\mathbf{F} = -\nabla U$.")

Results from modified versions of the same two tutorials

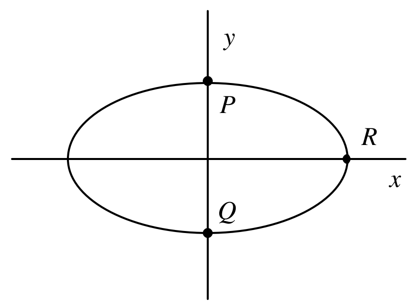
Rather than make sweeping changes to the tutorial worksheets themselves, it was decided instead to add homework questions specially designed to reinforce the results from tutorial by helping students redirect the tenacious but unproductive (incorrect) intuitions elicited by the pretest. For example, the homework question from the *H2D* tutorial shown in Fig. 9 is intentionally posed in two parts. In part A students critique an incorrect statement based on the most common mode of reasoning ("smaller amplitude implies stiffer spring"). In part B they are then guided to recognize that different amplitudes imply not different force constants but different potential energies (and hence different speeds).

Shown at right is the *x*-*y* trajectory for a 2-D oscillator.

A. Consider the following *incorrect* statement:

"The oscillator goes farther in the *x*-direction than in the *y*-direction. That means the spring in the *y*-direction must be stiffer than the spring in the x-direction."

Identify the error in the above statement, and state how you would modify it to make it correct.



B. Rank the labeled points *(P, Q,* and *R)* according to: (i) total energy, (ii) potential energy, (iii) kinetic energy. Explain. In particular, explain how the difference in amplitudes in the *x*- and *y*-directions, used *incorrectly* in the statement in part A, can be used to justify a *correct* conclusion in this part of the problem.

Figure 9. Two-part problem added to the tutorial homework for *Harmonic motion in two dimensions*.

A similar addition was made to the homework for the *CFP* tutorial. Two short problems each force students to confront a naïve intuition about the relationship between force and potential energy. These intuitions, phrased much the same as actual students have articulated them on pretests, are explicitly pointed out as being *incorrect:*

> "For a conservative force, the magnitude of the force is related to potential energy. The larger the potential energy, the larger the magnitude of the force."

> "For a conservative force, the magnitude of the force is related to potential energy. For any equipotential contour line, the magnitude of the force must be the same at every point along that contour."

The student's task is to give one or more specific examples that they have encountered previously (whether on the tutorial worksheet or on previous problems in the tutorial homework) that help refute the ideas being expressed in the statements. In this way, students must reflect on what they learned in tutorial—which includes what the gradient of potential energy *means*—and solidify their thinking by understanding what the gradient does *not* mean.

Although these changes are certainly not the only ones made recently to the *H2D* and *CFP* tutorials, the results obtained from comparable exam questions (post-tests) after tutorial instruction have improved markedly. On *H2D* post-tests the percentage of correct and complete responses increased to approximately 80% over the past two years, including on questions that asked students to evaluate the ratio k_y/k_x of force constants (not just determine which force constant, if either, was larger). Similarly, *CFP* exam questions that were similar to (but not identical to) the pretest yielded correct response rates of about 75%. Again, we cannot claim that the modifications described above are solely responsible for the increased performance of students, but the added homework problems seem to have contributed to the enhanced effectiveness of the tutorials. We also do not claim that these tutorials have achieved their optimal level of effectiveness; we plan to continue investigating student thinking and reasoning patterns and to continue refining and testing the tutorial materials appropriately.

5. External assessment: gathering feedback from pilot-testers

The quality of the existing set of *IMT* materials has benefited from those instructors who have pilot-tested tutorials in their classes and given their observations regarding their effectiveness. In turn, many of those instructors have offered their reflections on how incorporating a tutorial approach in their course has changed the way they think of teaching.

Improvements to the tutorials by pilot-site instructors

To be sure, the many faculty members who have served to pilot-test tutorials have also provided more pairs of eyes with which to inspect and critique the materials. We have received positive feedback from practically everyone regarding the content, teaching strategies, and format of the tutorials. We have also fielded constructive criticism from pilot-testers, has led to significant improvements in some of the tutorials:

- Typographical errors have been spotted and corrected.
- Some questions needed to be more precisely worded in order to clarify the intent of the question or the expected level of detail the students were to provide in their response—and hence the level of detail the instructors were to listen for. (However, we have also frequently heard comments from faculty who say that the sequencing of questions generally succeed in "telegraphing" to the instructor the essential points or "punch lines" of the tutorial.)
- Other suggestions have helped us identify topics for future tutorials. One faculty member in particular, Prof. Dawn Meredith (U. of New Hampshire), has begun developing tutorial materials on new topics such as rigid-body dynamics. Initial discussions are underway for Meredith to collaborate with both co-PIs of this project on tutorial development and dissemination.

Benefits perceived by the instructors from using the tutorials

In addition to the improvements to the tutorials that have been made possible by pilot-site instructors, we have learned from several such instructors how the process of incorporating and implementing tutorials in their classes have helped them improve as teachers. By having a research-based resource such as the *IMT* materials with which to try teaching-by-questioning techniques, many instructors have seemed to appreciate the value of these techniques over traditional lecturing. Although it was difficult for many pilot-testers to supply adequate pretest and post-test data that would be deemed statistically useful (*e.g.*, many had classes with as few as 3 – 5 students), the informal observations they have relayed to us strongly suggest that the tutorial approach provided deeper intellectual engagement by the students and deeper understanding of the material. Most of the instructors' comments (examples of which are quoted or paraphrased below) could be loosely categorized as follows:

- Many faculty members recognized the value of both the pretests and the discussions with students during tutorial in characterizing the initial state of student thinking about a particular topic. As some instructors mentioned:
  - *"They [the tutorials] seemed to accomplish the stated goal of increasing student understanding, and they also increased the instructor's understanding of how little the students really did understand!"*
  - *[An instructor describes student difficulties on damped oscillations.] "They [the students] were just SURE(!) that the period didn't change when the [simple harmonic oscillator] was damped and that the velocity remained max when x was zero. Any arguments [by me] to the contrary just confused them. It anything, it was educational for me because it showed how deeply that concept was buried in their heads."*

- Some instructors, in reflecting upon the previous background of the students in their intermediate mechanics classes, recognized the extent to which the introductory course could fail to provide adequate instruction of fundamentals:
  - *"I found that a significant number of students didn't know when the kinematic equations were valid, how to draw proper free body diagrams, the system requirements for linear momentum or mechanical energy conservation, basic Calculus skills, etc. This variability has a lot to do with how the Physics I class is taught (rote memory, less is more, active, passive, etc.)."*

- Instructors were sometimes surprised to discover how deeply-rooted student ideas can be—and how difficult it can be to help students trust their own reasoning, especially when that reasoning leads to unfamiliar (but correct!) ideas:
  - *[One instructor describes a conversation with a group of tutorial students on the motion of an underdamped oscillator.] "I tried to walk them thru the logic. The sharpest student in the group saw the logic of where it was heading but then said something like, 'So it would look the velocity is not maximum at x = 0. But that can't be true which is okay because physics is always counter-intuitive at first.' In other words, the one thing he _KNEW_ was true was that v was max when x was zero."*

- Some instructors have pointed out that, apart from pretest and post-test data in written form, the growth of students has been evident to them in the questions that they ask (whether during class or outside class). Near the beginning of the course the students' questions reflect a lack of understanding of the basic terminology, but as the course progresses the students are able to ask deeper, more meaningful, and more substantive questions about the material.

Thus, to complement pretest and post-test data collected by the co-PIs' at their home institutions, the instructors who have served (and continue to serve) as pilot-testers of the tutorials have offered some evidence suggesting that similar gains in student performance are possible on a wider scale. What is also clear is that *IMT* has become a valuable resource for instructors interested in learning about and utilizing PER-based curricular materials. The fact that intermediate mechanics courses tend to be small and require far less logistical support than introductory courses has helped instructors observe more directly the effect on their students of implementing tutorials. In a small class they are able to witness firsthand the obstacles that students often encounter when making sense of the physics or the mathematics (or both), and they can more easily recognize the benefits that can be reaped by deviating from a standard lecture approach. We expect that current and future pilot-testers will continue to provide us valuable information—and additional "hard" data from pretests and post-tests—that will make possible further refinement and dissemination of the *IMT* materials.

1. Overview of activities

The co-Principal Investigators (co-PIs) have found the need for increased conceptual and mathematical understanding when teaching upper-level mechanics courses in physics. Using the past success of a proof-of-concept investigation by of one of the co-PIs (Ambrose), we have endeavored in this collaborative project to design innovative curricular materials by which future scientists, future secondary science teachers, and future engineers can create an effective bridge between the mathematical reasoning emphasized in intermediate mechanics and the physical sense-making that will guide their future work and life-long-learning. The following report serves as the final report for this collaborative project. To reflect the strongly collaborative nature in which our work was carried out, this activity report has been filed identically by each co-PI.

We have created a series of small-group tutorials, entitled *Intermediate Mechanics Tutorials (IMT),* in which students work together to develop their own understanding of the physics while instructors act as facilitators rather than lecturers. These materials act as supplements to (rather than replacements of) regular lecture instruction. Our materials—based on *Tutorials in Introductory Physics (TiIP)* by the University of Washington Physics Education Group (UW-PEG) and *Activity-Based Tutorials (ABT)* by members of the University of Maryland Physics Education Research Group (UM-PERG) —are designed to address specific difficulties students have when learning the physics and allow sufficient flexibility to be implemented in a variety of instructional settings, including lecture-based courses, studio or seminar courses, and upper division laboratories.

We have enacted a two-tiered evaluation process. Internally, we have evaluated student learning using these materials by comparing results from specially designed tasks given to students before and after tutorial instruction. Wherever possible, we also have compared the performance of students post-instruction with tutorials to that of previous classes that did not use the tutorials. Externally, we have gathered feedback from researchers in physics education who have experience developing advanced physics instructional materials. We have started to disseminate materials locally and nationally through existing channels such as workshops at meetings of the *American Association of Physics Teachers* and a web site.

2. Experiments conducted and data collected

During the grant period (May 2005 – June 2007) each co-PI conducted major research and curriculum development efforts on *IMT* while serving as instructor of record for intermediate mechanics. During the grant period one co-PI (Ambrose) taught the course three times (twice at his home institution of Grand Valley State University [GVSU] and once while on sabbatical leave at Seattle Pacific University [SPU]) and the other co-PI (Wittmann) taught the course twice at his home institution (University of Maine). The co-PI's also facilitated several regional and national workshops for college and university instructors [UME] interested in learning about and implementing *IMT* materials in their mechanics courses. These settings served as the primary context for the research and development supported by the award.

Research activities have included the following: (1) using tutorial pretests to measure the prevalence of various patterns of student thinking before modification to instruction, (2) using exams (post-tests) to measure the prevalence of persistent conceptual and reasoning difficulties after modified instruction, (3) informal observations of students during class, particularly while working with their peers through tutorial activities, (4) "mini-interviews" done at UME with student volunteers immediately after class meetings, (5) videotaped classroom activities during a full semester of *IMT* instruction at UME, allowing us to observe what students do when not interacting with faculty, (6) videotaping of Homework Help Sessions at UME, during which we observe students working with a TA to solve textbook and tutorial homework assigned for class, (7) gathering data and feedback from faculty members who have incorporated *IMT* materials in their classes (this information has usually taken the form of items (1) – (3) listed above), (8) administering tutorial pretests to college and university faculty during *IMT* workshops. As is customarily done in discipline-based education research and curriculum development, the interpretation of results from these formal and informal research methods have guided our work in revising and refining the tutorial materials.

3. Tutorial materials developed

We had proposed to create at least 23 pencil-and-paper and computer-based tutorials to develop conceptual and mathematical understanding of the physics. Because of some fortuitous circumstances on one hand (including a sabbatical leave for one of the co-PIs during the grant period) we have developed a package comprising of 29 tutorials, not 23. On the other hand, we initially proposed a package of tutorials that included 4 computer-based tutorials and 19 pencil-and-paper tutorials (15 conceptual and 4 mathematical). One reason that all of our tutorials thus far are pencil-based is because of the predominantly lecture-based format of the classes and the lack of computers in the classrooms that served as our teaching and research setting. In addition, our research revealed such a wide array of difficulties both conceptual and procedural in nature that we believed paper-and-pencil tutorials would yield more direct and more detailed observations of student thinking about the physics and mathematics. (Selected results from this research are discussed in the "Findings" section of this report.) In summary, the set of tutorial materials developed thus far include the following:

- a total of 29 tutorials on topics in intermediate mechanics, including (as described previously):
  - 18 primarily conceptual pencil-and-paper tutorials
  - 11 mathematical pencil-and-paper tutorials
- associated pretests, used to give the instructor insight into student reasoning and to give students a preview of the topic to be covered in tutorial
- associated homework problems, for students to apply and extend results from tutorial
- associated examination questions, for post-instruction testing
- associated instructors' guides for most (but not all) of the tutorials, to help the instructor better understand the relevant student difficulties and the intent of the lines of questioning in the tutorial

All of these materials are available on a website whose URL is given to instructors at our discretion. These instructors have been chosen to participate in the project by attending a workshop facilitated by one or both of the co-PI's (see Section 4, "Major presentations") or by collaborating previously with the co-PI's (whether formally or informally) on other research or curriculum development projects. In order to provide faculty members the most flexibility in adapting and implementing the tutorials for use in their courses, the materials posted on the website (with the exception of the instructors' guides) are available in both Microsoft Word and PDF format.

Tutorials to enhance conceptual reasoning

As revealed by ongoing research at the home institutions of both co-PIs, many students, despite being physics majors or minors, struggled to develop a robust physical and mathematical understanding of the material presented in the intermediate mechanics course. Many of their difficulties could be characterized as persistent conceptual and reasoning difficulties, and often such difficulties demonstrated a lack of understanding of fundamental kinematical and dynamical concepts. To a great extent, prevalent student errors can be attributed to critical difficulties diagnosed by many physics education researchers, such as the confusion between a quantity and its (temporal or spatial) rate of change (*e.g.,* velocity and acceleration, or potential energy and force) or the inability to interpret graphical representations (*e.g.,* graphs of position *vs.* time or velocity *vs.* time). Examples of these types of difficulties identified among intermediate mechanics students are described in the "Findings" section of this report.

On the basis of prior research as well as results collected by the co-PIs themselves, the instructional strategies implemented in the tutorials include "elicit-confront-resolve" techniques, so named by Prof. Lillian C. McDermott and her colleagues at the UWPEG and incorporated in many of the tutorial materials in *TiIP*. Based on work done at UME as part of the CCLI-funded *Intuitive Quantum Physics* tutorials (DUE 0410895), we have also used techniques related to the refinement of student resources, in which we help students identify and build upon productive intuitions, especially when results from pretests (and subsequent conversations with the students) showed few well-entrenched and useful conceptions to which the students held any strong commitment. These methods were particularly useful when designing those tutorials that bridge from the mathematics to the physics.

We list all our tutorials below, organized into six groups by physics topic. Note that many tutorials are relevant to more than one topic, so the list contains multiple references of each of those tutorials. The format of this list, however, has been helpful to instructors who wish to use our materials.

Note:
* = not yet posted online
(m) = mathematical bridge to the physics

1. Harmonic Motion:
   - Simple Harmonic Motion
   - Harmonic Motion in Two Dimensions
   - Damped Oscillations (Kinematics)
   - Damped Oscillations (Energy)
   - Graphical Interpretation of Differential Equations (m)
   - Forced Oscillations
   - Analysis of Non-sinusoidal Driving Forces *
   - Non-harmonic Oscillations * (m)
   - Phase Space (Simple Harmonic Motion)
   - Phase Space (Damped Harmonic Motion)
   - Phase Space (Self-limiting Oscillators)
   - Velocity Dependent Forces
   - Integration Error Analysis *

2. Energy and/or Conservative Forces:
   - Conservative Forces and Equipotentials
   - Damped Oscillations (Energy)
   - Separable Forces
   - Conservative Force Fields * (m)
   - Comparing Newton and Lagrangian equations (m)

3. Non-Cartesian Coordinate Systems:
   - Phase Space (Simple Harmonic Motion)
   - Phase Space (Damped Harmonic Motion)
   - Phase Space (Self-limiting Oscillators)
   - Separable Forces
   - Generalized Coordinates (m)

4. Accelerating Reference Frames:
   - Accelerating Frames (Local g)
   - Accelerating Frames (Rotation)
   - Accelerating Frames (Foucault)

5. Orbital Motion:
   - Kepler's First Law * (m)
   - Angular Momentum and Kepler's Second Law
   - Kepler's Third Law * (m)
   - Energy and Angular Momentum for Closed Orbits
   - Energy as a Constant of Orbital Motion * (m)
   - Gravity Boosts *

6. Group Problem Solving:
   - Air Resistance (m)
   - Non-harmonic Oscillations * (m)
   - Integration Error Analysis * (m)
   - Analysis of Non-sinusoidal Driving Forces * (m)

Tutorials to enhance mathematical reasoning

Tutorials designed to enhance mathematical reasoning and build a bridge between the mathematics and the physics are listed with an "(m)" in the list of tutorials, above. So far, the co-PIs have developed 11 such tutorials.

Throughout all phases of this project, both co-PIs found that one of the major weaknesses students have when learning intermediate mechanics is the inability to read a textbook and adequately learn the physics from the discussions and the mathematical derivations. Students needed help developing skills in reading technical textbooks, a skill they will need for successful life-long learning. To address this need, several tutorials were developed and tested with the goal of having students work together to better understand the physical meaning of the mathematics used in the course and to better understand the mathematical expressions of the physics.

For this purpose several kinds of mathematical tutorials were developed. In some of these tutorials students were given non-traditional problems to solve, often involving real-world situations with constraints or hidden assumptions that the students needed to consider. Other tutorials guided students through the steps necessary to derive important mathematical equation in which a new mathematical tool is applied to a situation, and a new quantity is derived.

4. Major presentations

During the grant period both co-PIs, whether jointly or separately, have had numerous opportunities to disseminate results from the research and curriculum development carried out in this project. The venues for dissemination have included the following: professional workshops, invited seminars/colloquia, invited posters, and contributed papers and posters.

The workshops took place at national and local meetings of the American Association of Physics Teachers (AAPT) as well as regional symposia on teaching math and science. These workshops were designed for college and university faculty who wished to explore new and innovative ways to teach intermediate mechanics and who were interested in learning about the format, teaching philosophy, teaching goals, and implementation of materials from *Intermediate Mechanics Tutorials*.

The other presentations (seminars/colloquia, talks, and posters) served the purpose of sharing the results from ongoing investigations of student learning. These presentations, together with the workshops, are listed below.

Workshops facilitated

- July 30, 2007, AAPT National Meeting (Greensboro, NC). Workshop/tutorial: "Tutorials in Intermediate Mechanics," B.S. Ambrose and M.C. Wittmann co-facilitating.
- March 17, 2007, Regional AAPT Meeting for Michigan Section (Grand Rapids, MI). Workshop: "Intermediate Mechanics Tutorials," B.S. Ambrose facilitating with Natalie Beyer of Grand Valley State University.
- January 6, 2007, Joint AAPT-AAS National Meeting (Seattle, WA). Workshop: "Intermediate Mechanics Tutorials," B.S. Ambrose and M.C. Wittmann co-facilitating.
- November 4, 2006, Northwest Ohio Symposium on Science, Math and Technology Teaching (Toledo, OH). Workshop G-7: "Using research to improve learning in a junior-level university mechanics course: Investigating student understanding of oscillations," B.S. Ambrose facilitating.
- March 2006, Twenty-sixth state-wide meeting of high school physics and physical science teachers (Orono, ME). Workshop: "Three ways of teaching Newton's third law," T.I. Smith and M.C. Wittmann co-facilitating.
- November 4, 2005, Northwest Ohio Symposium on Science, Math and Technology Teaching (Toledo, OH). Workshop B-7: "Because physics majors encounter conceptual difficulties too: Incorporating inquiry-based teaching in an upper level mechanics course," B.S. Ambrose facilitating.
- May 2005, Twenty-fifth state-wide meeting of high school physics and physical science teachers (Orono, ME). Workshop: "Three ways of teaching Newton's third law," T.I. Smith and M.C. Wittmann co-facilitating.

Invited papers and posters

- April 26, 2007, Physics department seminar, Rutgers University. Invited presentation, M.C. Wittmann: "Modeling mathematical reasoning in physics."
- April 17, 2007, College of Liberal Arts and Sciences Sabbatical Symposium, Grand Valley State University (Allendale, MI). Invited presentation, B.S. Ambrose: "Assessing and refining an inquiry-based approach to teach intermediate mechanics."
- April 14, 2007, American Physical Society, National Meeting. Invited presentation: M.C. Wittmann "Using resource graphs to model learning in physics."
- October 16, 2006, Physics Colloquium, Western Michigan University (Kalamazoo, MI). Invited seminar, B.S. Ambrose: "Because physics majors encounter conceptual difficulties too: Refining an inquiry-based approach to teach intermediate mechanics."
- September 22, 2006, College of Liberal Arts and Sciences Research Colloquium, Grand Valley State University (Allendale, MI). Invited paper, B.S. Ambrose: "Because physics majors encounter conceptual difficulties too: Refining an inquiry-based approach to teach intermediate mechanics."
- July 27, 2006, Physics Education Research Conference (Syracuse, NY). Invited poster TP-C1, B.S. Ambrose: "Probing student understanding of intermediate mechanics and its applications: An example with linear oscillations."

- May, 2006, Colloquium in the College of Education and Human Development, University of Maine. Invited presentation, M.C. Wittmann: "Comparing instructional modes: One element of physics education research."
- April, 2006, NES Joint Meeting of the APS and AAPT (Boston MA). Invited presentation, M.C. Wittmann: "Comparing curricula to study student learning."
- April 19, 2006, Physics Education Group Seminar, University of Washington (Seattle, WA). Invited seminar, B.S. Ambrose: "Investigating student understanding of mechanical oscillations."
- August 9, 2005, AAPT Summer National Meeting (Salt Lake City, UT). Invited paper, B.S. Ambrose: "Utilizing tutorials to investigate and enhance student learning in intermediate mechanics."

Contributed papers and posters

- August 2007, Physics Education Research Conference 2007 (Greensboro, NC). Contributed poster, B.S. Ambrose: "New dimensions to probing student thinking about oscillations in two dimensions."
- August 2007, Physics Education Research Conference 2007 (Greensboro, NC). Contributed poster, K.E. Black and M.C. Wittmann: "Mapping student reasoning about math- and physics-oriented differential equation solutions."
- August 2007, Physics Education Research Conference 2007 (Greensboro, NC). Contributed poster, K. McCann and M.C. Wittmann: "Students creating mathematical meaning in mechanics: Signs in scalar equations."
- August 2007, Physics Education Research Conference 2007 (Greensboro, NC). Contributed poster, T.I. Smith, M.C. Wittmann, and T. Carter: "Analyzing the Force and Motion Conceptual Evaluation using model analysis."
- August 2007, Physics Education Research Conference 2007 (Greensboro, NC). Contributed poster, R.P. Springuel, M.C. Wittmann, and J.R. Thompson: "Comparing cluster analysis and traditional analysis methods in PER."
- August 2007, Physics Education Research Conference 2007 (Greensboro, NC). Contributed poster, J. Van Deventer and M.C. Wittmann: "Comparing student use of mathematical and physical vector representations."
- August 2007, Physics Education Research Conference 2007 (Greensboro, NC). Contributed poster, K.E. Black and M.C. Wittmann: "Mapping student reasoning about math- and physics-oriented differential equation solutions."
- July 2007, National AAPT Meeting (Greensboro, NC). Contributed paper, T.I. Smith, M.C.Wittmann, T. Carter: "Revised methods for analyzing the Force and Motion Conceptual Evaluation."
- July 2007, National AAPT Meeting (Greensboro, NC). Contributed poster, J. Van Deventer, M.C. Wittmann: "Comparing student use of mathematical and physical vector representations."

- July 2007, National AAPT Meeting (Greensboro, NC). Contributed poster, K. McCann, M.C. Wittmann: "Students creating mathematical meaning in mechanics: Signs in scalar equations."
- July 2007, National AAPT Meeting (Greensboro, NC). Contributed poster, E.C. Sayre, M.C. Wittmann: "Intermediate mechanics students' coordinate system choices for simple pendula."
- July 2007, National AAPT Meeting (Greensboro, NC). Contributed poster, K.E. Black, M.C. Wittmann: "Mapping student reasoning about math- and physics-oriented differential equation solutions."
- July 2007, National AAPT Meeting (Greensboro, NC). Contributed poster, T. Carter, T.I. Smith, M.C. Wittmann: "Effect of instructional method changes on an introductory physics class at a two-year college."
- July 2007, National AAPT Meeting (Greensboro, NC). Contributed poster, T.I. Smith, M.C. Wittmann, T. Carter: "Analyzing the Force and Motion Conceptual Evaluation using model analysis."
- July 2007, National AAPT Meeting (Greensboro, NC). Contributed poster, R.P. Springuel, M.C. Wittmann, J.R. Thompson: "Comparing cluster analysis and traditional analysis methods in PER."
- April 2007, Joint Spring Meeting NES APS/AAPT (Orono ME). Contributed paper, J.V. Deventer, M.C. Wittmann: "Comparing student use of mathematical and physical vector representations."
- April 2007, Joint Spring Meeting NES APS/AAPT (Orono ME). Contributed paper, E.C. Sayre, M.C. Wittmann: "Intermediate mechanics students' coordinate system choices for simple pendula."
- April 2007, Joint Spring Meeting NES APS/AAPT (Orono ME). Contributed paper, T.I. Smith, M.C. Wittmann, T. Carter: "Reconsidering the analysis of the Force and Motion Conceptual Evaluation."
- April 2007, Joint Spring Meeting NES APS/AAPT (Orono ME). Contributed paper, K. McCann, M.C. Wittmann: "Students' interpretations of signs in scalar differential equations."
- April 2007, Joint Spring Meeting NES APS/AAPT (Orono ME). Contributed paper, K.E. Black, M.C. Wittmann: "Students' reasoning toward solutions of first-order differential equations."
- April 2007, Joint Spring Meeting NES APS/AAPT (Orono ME). Contributed paper, R.P. Springuel, M.C. Wittmann, J.R. Thompson: "Using cluster analysis on written responses to 2-D kinematics questions."
- April 2007, Joint Spring Meeting NES APS/AAPT (Orono ME). Contributed paper, K.E. Black and M.C. Wittmann: "A limited toolbox when using integrals and boundary conditions."
- March 17, 2007, Regional AAPT Meeting for Michigan Section (Grand Rapids, MI). Contributed paper, B.S. Ambrose: "The frequency of conceptual difficulties with frequency: Helping students understand 1D and 2D oscillations."

- January 2007, National AAPT Meeting (Seattle, WA). Contributed paper, M.C. Wittmann, K.E. Black: "Student solutions to first-order differential equations in intermediate mechanics."
- October 14, 2006, Regional AAPT Meeting for Michigan Section (Dearborn, MI). Contributed paper, B.S. Ambrose: "Because physics majors encounter conceptual difficulties too: Refining an inquiry-based approach to teach intermediate mechanics."
- August 2006, Physics Education Research Conference 2006 (Syracuse, NY). Contributed poster, R.P. Springuel, J.R. Thompson, and M.C. Wittmann: "Effects of changing representations in 2-D motion."
- August 2006, Physics Education Research Conference 2006 (Syracuse, NY). Contributed poster, T.I. Smith and M.C. Wittmann: "Comparing three methods for teaching Newton's third law."
- August 2006, Physics Education Research Conference 2006 (Syracuse, NY). Contributed poster, K.E. Black and M.C. Wittmann: "Students' integration methods for first-order differential equations."
- August 2006, Physics Education Research Conference 2006 (Syracuse, NY). Contributed poster, E.C. Sayre and M.C. Wittmann: "Situating a common chain of reasoning in damped harmonic motion."
- August 2006, National AAPT Meeting (Syracuse, NY). Contributed paper, R.P. Springuel, J.R. Thompson, and M.C. Wittmann: "Effects of changing representations in 2-D motion."
- August 2006, National AAPT Meeting (Syracuse, NY). Contributed paper, T.I. Smith and M.C. Wittmann: "Comparing three methods for teaching Newton's third law."
- August, 2006, National AAPT Meeting (Syracuse, NY). Contributed paper, K.E. Black and M.C. Wittmann: "Students' integration methods for first-order differential equations."
- August, 2006, National AAPT Meeting (Syracuse, NY). Contributed paper, E.C. Sayre and M.C. Wittmann: "Situating a common chain of reasoning in damped harmonic motion."
- June 11–16, 2006, Gordon Research Conference on Physics Research and Education: Electromagnetism, Mt. Holyoke College (South Hadley, MA). Contributed poster, B.S. Ambrose: "Challenges in helping students understand the physics behind the formalism: An example with conservative forces."
- August 15–19, 2005, Foundations and Frontiers in Physics Education Research Conference (Bar Harbor, ME). Contributed poster, B.S. Ambrose: "Investigating student understanding of oscillatory motion in one and two dimensions."
- August 2005, Physics Education Research Conference 2005 (Salt Lake City, UT). Contributed poster, E.C. Sayre and M.C. Wittmann: "Nearly novel situations."

Publications

- Black, K.E. and Wittmann, M.C. (2007) "Using Consistency Plots to Follow Student Reasoning About Solutions to Separable Differential Equations in Intermediate Mechanics" *in preparation*.

- Black, K.E. and Wittmann, M.C. (2007) "Epistemic Games in Integration: Modeling Resource Choice" submitted to the Physics Education Research Conference 2007 Proceedings.
- McCann, K. and Wittmann, M.C. (2007) "Students creating mathematical meaning in Mechanics: Signs in scalar equation" submitted to the Physics Education Research Conference 2007 Proceedings.
- Van Deventer, J. and Wittmann, M.C. (2007) "Comparing Student Use of Mathematical and Physical Vector Representations," submitted to the Physics Education Research Conference 2007 Proceedings.
- Springuel, R.P., Thompson, J.R., and Wittmann, M.C. (2007) "Applying clustering to statistical analysis of student reasoning about two-dimensional kinematics," submitted to *Physical Review Special Topics Physics Education Research*.
- Sayre, E.C. and Wittmann, M.C. (2007) "Intermediate mechanics students' coordinate system choice," Conference on Research in Undergraduate Mathematics Education, from http://cresmet.asu.edu/crume2007/eproc.html.
- Sayre, E.C. and Wittmann, M.C. (2007) "The plasticity of intermediate mechanics students' coordinate system choice," submitted to *Physical Review Special Topics Physics Education Research*.
- Ambrose, B.S. (2006) "Probing student reasoning and intuitions in intermediate mechanics: An example with linear oscillations," *Physics Education Research Conference 2006 Proceedings,* ed. L. McCullough, L. Hsu, and P. Heron, AIP Conference Proceedings.
- Smith, T.I. and Wittmann, M.C. (2006) "Comparing three methods of teaching Newton's Third Law," accepted for publication in the *Physical Review Special Topics Physics Education Research*.
- Wittmann, M.C. (2006) "Using resource graphs to represent conceptual change," *Physical Review Special Topics Physics Education Research* 2, 020105. Available online at http://prst-per.aps.org/abstract/PRSTPER/v2/i2/e020105.