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Creation, Coordination, and Activation of Resources in Physics and Mathematics Learning

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Principal Investigator: Wittmann, Michael C. **Award ID:** 0633951

Organization: University of Maine

Submitted By:

Wittmann, Michael - Principal Investigator

Title:

Creation, Coordination, and Activation of Resources in Physics and Mathematics Learning

Project Participants

Senior Personnel

Name: Wittmann, Michael

Worked for more than 160 Hours: Yes

Contribution to Project:

Name: Donovan, John

Worked for more than 160 Hours: Yes

Contribution to Project:

In year 1 of the grant, co-PI Donovan was active in mentoring and paper writing. In year 2 of the grant, he left the University of Maine and started a position at a new institution. He has, as a result, left the grant work behind. He is still working on the paper that was originally planned to be submitted nearly a year ago, but that was held up with his move and several other personal and professional problems that arose. We plan to submit that paper in year 3 of the grant, as a result.

Name: Thompson, John

Worked for more than 160 Hours: Yes

Contribution to Project:

Post-doc

Name: Bucy, Brandon

Worked for more than 160 Hours: Yes

Contribution to Project:

While PI Wittmann was on sabbatical, Brandon Bucy was mentoring graduate students on their research work, running a biweekly seminar on literature relevant to the grant, and in general assisting with the daily functioning of the research group as its activities included this grant work.

Name: Frank, Brian

Worked for more than 160 Hours: Yes

Contribution to Project:

Guiding research work by graduate students as well as research on priming and cuing of resources in kinematics and optics.

Graduate Student

Name: Black, Katrina

Worked for more than 160 Hours: Yes

Contribution to Project:

PhD research on student reasoning about integration methods when dealing with separable differential equations.

Name: McCann, Kate

Worked for more than 160 Hours: Yes

Contribution to Project:

Masters student, research on both the use of minus signs when writing physically meaningful differential equations and on the 'a-ha' moment when conceptual change occurs on a relatively fast time scale.

Name: McIntyre, Zachary

Worked for more than 160 Hours: Yes

Contribution to Project:

Masters student, research on misconceptions about variables, including context and population dependence.

Name: Sayre, Eleanor

Worked for more than 160 Hours: Yes

Contribution to Project:

PhD student, research on the creation of resources, developing an understanding of 'plasticity' and 'solidity' of ideas that are used in conjunction with other ideas when reasoning about physics.

Name: Smith, Trevor

Worked for more than 160 Hours: Yes

Contribution to Project:

Masters student, research using standardized tests in reform instruction courses, analyzing content clusters, using a resource model to analyze false positives on accepted tests.

Name: Springuel, R. Padraic

Worked for more than 160 Hours: Yes

Contribution to Project:

PhD student, research on student understanding of 2-dimensional kinematics and vectors, using cluster analysis to find non-a priori groupings of student responses and use methods to analyze context dependence of responses.

Name: Van Deventer, Joel

Worked for more than 160 Hours: Yes

Contribution to Project:

Masters student, research on student responses to isomorphic math and physics questions dealing with vector topics, such as addition, subtraction, dot products, and cross products, including development of a survey, interviews, and more.

Name: Pollock, Evan

Worked for more than 160 Hours: Yes

Contribution to Project:

Evan's work took the project in unexpected directions in advanced physics, namely the use of mathematical formalism in thermodynamics. The ideas used there are extensions of those introduced in the mechanics course which had been our primary area of study, and are a welcome addition to the project.

Name: Reed, Daniel

Worked for more than 160 Hours: Yes

Contribution to Project:

Studying the intersection of mathematics and physics conceptual learning in a population of engineering technology students - weaker in mathematics than engineers - taking a physics course. Not paid in the grant, but mentored by PI Wittmann

Name: Nagpure, Bhupendra

Worked for more than 160 Hours: Yes

Contribution to Project:

Investigation of student reasoning of vector use in two dimensional kinematics, comparing student learning when using one method rather than another. Not paid in the grant, but mentored by co-PI?Thompson.

Name: Kaczynski, Adam

Worked for more than 160 Hours: Yes

Contribution to Project:

Research on student interactions and behaviors when learning thermal physics, specifically areas in which mathematical and physical knowledge might be in conflict.

Name: Hawkins, Jeff

Worked for more than 160 Hours: Yes

Contribution to Project:

Research on student understanding of two dimensional vector addition. Survey design and analysis, interviews on student understanding. Research builds on previous work done by a Masters student at UMaine.

Name: Anderson, Mindi

Worked for more than 160 Hours: Yes

Contribution to Project:

Investigation of effectiveness of three different curricula for teaching Newton's Second Law. Each curriculum can be described as emphasizing different reasoning resources as it establishes the basic physics. The study was carried out using research tools developed by another graduate student in the project.

Name: Murphy, Casey

Worked for more than 160 Hours: Yes

Contribution to Project:

Masters thesis work on students epistemological framing of a laboratory activity.

Name: Bajracharya, Rabindra

Worked for more than 160 Hours: Yes

Contribution to Project:

Studies of integration, cuing of responses based on graphical form of the question, comparison of physics and physics-less (math) versions of identical questions.

Undergraduate Student

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

Other Collaborators or Contacts

Though there were no direct collaborations with other researchers on elements of this project, there were lengthy interactions with colleagues that advanced our work. In general, the areas of our interactions lay in areas of mathematics use in physics, cognitive models within the ?knowledge-in-pieces? tradition of research (including the resources framework as we used it, as well as phenomenological primitives as described by diSessa and symbolic forms as described by Sherin), and cognitive modeling using the conceptual blending framework.

Year 1: We have worked at times with Noah Finkelstein (Colorado University), E.F. 'Joe' Redish (University of Maryland), Rachel E. Scherr (University of Maryland), and Michelle Zandieh (Arizona State University), on issues of resource development, linking of resources, activation in networks, and blending theory.

Year 2: We have discussed ideas in detail with: Joseph Perner (University of Salzburg), about metacognition and executive function; Andrea diSessa (UC Berkeley), about resource activation and linking, specifically when mixing procedural and conceptual resources; Bruce Sherin (Northwestern U), about cluster analysis, social issues in resource activation, the nature of cognitive objects such as resources, and the difference between his nodes and graphs and our resource graphs; David Hammer (University of Maryland), about resource linking and activation; and Andy Elby (University of Maryland), about conceptual blending and its role in resource development. David Hammer served as external reader on PhD candidate Ellie Sayre?s dissertation on the plasticity of resources (and resource coordination), as well.

Year 3: Discussions have continued with diSessa (UC Berkeley), Sherin (Northwestern), Hammer (Maryland) and Elby (Maryland). Rachel Scherr (visiting at Seattle Pacific U) has been instrumental in advancing theoretical work on the application of blending theory to modeling resources in physics. Collaborators on the thermodynamics and statistical mechanics elements of this project (looking at math use in upper-division physics courses) included Mike Loverude (Cal State - Fullerton) and David Meltzer (Arizona). For issues related to transfer (including a mindset of ?transfer in pieces,? consist with our knowledge-in-pieces resources framework), we have interacted closely with Joe Wagner (Xavier).

Year 4: Collaborations have focused on ongoing work with Rachel Scherr and Hunter Close (Seattle Pacific University) on the topic of embodied cognition (as linked to resource activation and creation). Valuable input came from Zandieh (Arizona) and Chris Rasmussen (San Diego State U) on issues of blending theory and gesture analysis when observing students? problem solving. In addition, new post doc Brian Frank kept in close contact with members of his former research group at the University of Maryland (Ayush Gupta, Luke Conlin) and Tufts (David Hammer). Collaborations continued with Loverude and Meltzer on thermodynamics and Wagner on transfer. Bruce Sherin served as external reader on PhD candidate Padraic Springuel?s dissertation.

Activities and Findings

Research and Education Activities: (See PDF version submitted by PI at the end of the report)

Findings: (See PDF version submitted by PI at the end of the report)

Training and Development:

As originally planned, the project was supposed to fund only 3 students part-time. Because of an interest by many others in the work done on this project, many more students have been funded. This development of physics and math education researchers has been an unexpected but very welcome part of our grant. As students graduate, others fill their spots in our funding structure (where none are fully funded, but nearly all are partially funded for their research work).We have found new connections in our data, analyzed more detailed and specific questions with our data, and investigate topics we would not have otherwise investigated.

New research skills have been developed by nearly all project members. These skills include:

1. Individual and student group interviews. Many of the students on the project had never carried out an interview, and are now highly experienced. Skills include learning to listen without biasing student reasoning, asking leading questions that don?t guide one?s choice of models (if possible), and learning to help groups continue to interact with each other in a way that lets the students? ideas be seen in situ.

2. Analysis of classroom video. Using discourse analysis to analyze group interactions

requires that we attend to language far more than we used to. Skills developed during interviews are useful for analysis of classroom video where no instructor is present.

3. Model analysis. A method developed by Lei Bao at Ohio State University, it has helped us understand performance on standardized tests, looking at individual clusters of responses (as determined by a content analysis of the physics).

4. Cluster analysis. We are leading the use of cluster analysis in the US PER community. It helps us analyze free response data using highly descriptive coding that is then clustered into common groups of responses. New computer programming was carried out in the process.

Outreach Activities:

Our goal in this project was to develop new insights into research creation, coordination, and activation, and share these with a larger research community. We have given extensive contributed and invited presentations, as well as publishing at a rate equal to the most prestigious and active physics education research groups in the US.

The mindset behind this project, that student reasoning consists of resources that are slowly created, linked together, and activated based on appropriate cues, has been used to guide teaching in two physics courses at UMaine. One was a sophomore level mechanics course, with only a small population already interested in physics. The other was a ?general education? course in which non-science majors are fulfilling a university core curriculum requirement to study science in a laboratory setting. In that course, we have used many of the tools for understanding differential equations, but tuned to a population that is often afraid of the math. Thus, we have used graphical representations where possible, rather than mathematical and algebraic analysis. We have helped students build an entirely new network of ideas (in a topic they have, by their own account, never studied in this fashion, namely quantum physics). In the process, we have shown students how science functions as a connection of small ideas, individually developed, and built into a larger whole. In year 2, the both courses were taught by Katrina Black, a graduate student in this project, using previously established methods. In the process, she developed skills necessary for a planned future faculty job.

In addition to teaching students based on the ideas in this project, the PI has taught a course on educational psychology for future teachers. This course has as a major component the idea of reasoning resources, their linking together, and changes to the linked structure of the resources as a way of describing conceptual change and learning in general. In year 1, 13 students took this course ? all are in-service teachers or planning to be teachers or college instructors. We expect that their understanding of the guiding theoretical constructs studied in this project will affect their teaching and research work (thesis work is required of students who are taking this course as part of a Master of Science in Teaching). This course was not taught in year 2, while the PI was on sabbatical. In year 3, the course was completed by 15 students. In year 4, 7 students completed the course.

Journal Publications

Michael C. Wittmann, "Using resource graphs to represent conceptual change", Physical Review Special Topics Physics Education Research, p. 020105, vol. 2, (2006). Published, 10.1103/PhysRevSTPER.2.020105

Trevor I. Smith, Michael C. Wittmann, "Comparing three methods of teaching Newton's Third Law", Physical Review Special Topics Physics Education Research, p. 020105, vol. 3, (2007). Published, 10.1103/PhysRevSTPER.3.020105

Eleanor C. Sayre, Michael C. Wittmann, "The plasticity of intermediate mechanics students' coordinate system choice", Physical Review Special Topics Physics Education Research, p. 020105, vol. 4, (2008). Published, 10.1103/PhysRevSTPER.4.020105

R. Padraic Springuel, Michael C. Wittmann, John R. Thompson, "Applying clustering to statistical analysis of student reasoning about two-dimensional kinematics", Physical Review Special Topics Physics Education Research, p. 020107, vol. 3, (2007). Published, 10.1103/PhysRevSTPER.3.020107

J.E. Donovan II, "The Importance of the Concept of Function for Developing Understanding of First-Order Differential Equations in Multiple Representations", Proceedings of the Conference on Research in Undergraduate Mathematics Education, http://cresmet.asu.edu/crume2007/eproc.html, p. , vol. , (2007). Published,

Eleanor C. Sayre, Michael C. Wittmann, "Intermediate mechanics students' coordinate system choice", Proceedings of the Conference on Research in Undergraduate Mathematics Education, http://cresmet.asu.edu/crume2007/eproc.html, p. , vol. , (2007). Published,

Sayre, E.C.; Wittmann, M.C., "Plasticity of intermediate mechanics students' coordinate system choice", Physical Review Special Topics - Physics Education Research, p. 020105, vol. 4, (2008). Published, 10.1103/PhysRevSTPER.4.020105

Smith, T.I.; Wittmann, M.C., "Applying a resources framework to analysis of the Force and Motion Conceptual Evaluation", Physical Review Special Topics - Physics Education Research, p. 020101, vol. 4, (2008). Published, 10.1103/PhysRevSTPER.4.020101

Wittmann, Michael C. and Thompson, John R., "Integrated approaches in physics education: A graduate level course in physics, pedagogy, and education research", American Journal of Physics, p. 677, vol. 7, (2008). Published, 10.1119/1.2897287

Hayes, K.M., and Wittmann, M.C., "The role of sign in students' modeling signs of scalar equations", The Physics Teacher, p. 246, vol. 48, (2010). Published, 10.1119/1.3361994

Springuel, R.P., Thompson, J.R., and Wittmann, M.C., "How different is 'not the same'", Physical Review Special Topics - Physics Education Research, p. , vol. , (2009). Submitted,

Springuel, R.P., Thompson, J.R., and Wittmann, M.C., "Erratum: Applying clustering to statistical analysis of student reasoning about two-dimensional kinematics", Physical Review Special Topics - Physics Education Research, p. 029902(E), vol. 5, (2009). Published, 10.1103/PhysRevSTPER.5.029902

Black, K.E. and Wittmann, M.C., "Visualizing changes in student responses using consistency plots", Physical Review Special Topics - Physics Education Research, p. , vol. , (2009). Submitted,

Black, K.E.; Wittmann, M.C., "Understanding the use of two integration methods on separable first order differential equations", Physical Review Special Topics - Physics Education Research, p. , vol. , (2009). Submitted,

Wittmann, Michael C. and Black, Katrina E., "Emergent Meaning in Conceptual Blends of Gesture and Language", The Journal of the Learning Sciences, p. , vol. , (2010). Submitted,

Thompson, John R., Christensen, Warren M., and Wittmann, Michael C., "Preparing future teachers to anticipate student difficulties in physics in a graduate-level course in physics, pedagogy, and education research", Physical Review Special Topics - Physics Education Research, p. , vol. , (2010). Submitted,

Books or Other One-time Publications

E.C. Sayre, M.C. Wittmann, J.E. Donovan, "Resource Plasticity: Detailing a Common
Chain of Reasoning with Damped
Harmonic Motion", (2007). Refereed conference proceedings, Published
Editor(s): P. Heron, L. McCullough, J. Marx

Collection: Physics Education Research Conference
Proceedings 2006, AIP Conference
Proceedings 883
Bibliography: p. 85-88

Black, Katrina E. and Wittmann, Michael
C., "Epistemic Games in Integration: Modeling
Resource Choice", (2007). Peer-reviewed proceedings article, Published
Editor(s): L. Hsu, L. McCullough, P. Heron
Collection: AIP Conference Proceedings 951, 2007
Physics Education Research Conference
Proceedings
Bibliography: p.53-56

Van Deventer, Joel, and Wittmann,
Michael C., "Comparing Student Use of Mathematical
and Physical Vector Representations", (2007). Peer-reviewed proceedings article, Published
Editor(s): L. Hsu, L. McCullough, P. Heron
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Wittmann, Michael C. and Black, Katrina
E., "Describing the Conceptual and
Procedural Resources Used in Two
Epistemic Games of Integration", (2008). Peer-reviewed proceedings article, Published
Editor(s): Jonker, Vincent; Lazonder, Ard; and
Hoadley, Christopher
Collection: Proceedings of the 2008 International
Conference on the Learning Sciences
Bibliography: electronic publication

Mountcastle, Donald B., Bucy, Brandon
R., and Thompson, John R., "Student estimates of probability and
uncertainty in advanced laboratory and
statistical physics courses", (2007). peer-reviewed conference proceedings, Published
Editor(s): L. Hsu, C. Henderson, L. McCullough
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Bibliography: p.152-155

Pollock, Evan B., Thompson, John R., and
Mountcastle, Donald B., "Student understanding of the physics and
mathematics of process variables in P-V
diagrams", (2007). peer-reviewed conference proceedings, Published
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Collection: AIP Conference Proceedings 951 2007
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Black, K.E.; Wittmann, M.C., "Procedural Resource Creation in
Intermediate Mechanics", (2009). Refereed conference proceedings, Published
Editor(s): C. Henderson, M. Sabella, C. Singh
Collection: AIP Conference Proceedings 2009 Physics

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Proceedings
Bibliography: AIP Conf. Proc. Volume 1179, 2009 Physics Education Research Conference, p. 97-101, DOI: 10.1063/1.3290980

Anderson, M.K.; Wittmann, M.C., "Comparing Three Methods of Teaching Newton's Second Law", (2009). Refereed conference proceedings, Published
Editor(s): Henderson, C.; Sabella, M.; Singh, C.
Collection: AIP Conference Proceedings 2009 Physics Education Research Conference Proceedings
Bibliography: AIP Conf. Proc. Volume 1179, pp. 301-304, DOI: 10.1063/1.3266742

Hawkins, Jeffrey M., Thompson, John R., and Wittmann, Michael C., "Students Consistency of Graphical Vector Addition Method on 2-D Vector Addition Tasks", (2010). refereed conference proceedings, Published
Editor(s): Sabella, Mel, Henderson, Charles, and Singh, Chandralekha
Collection: AIP Conference Proceedings Volume 1179, 2009 Physics Education Research Conference
Bibliography: AIP Conference Proceedings Volume 1179, 2009 Physics Education Research Conference, p. 161-164, DOI: 10.1063/1.3266704

Hawkins, Jeffrey M., Thompson, John R., Wittmann, Michael C., Sayre, Eleanor C., and Frank, Brian W., "Students' Responses To Different Representations Of A Vector Addition Question", (2010). Refereed conference proceedings, Published
Editor(s): Chandralekha Singh, Mel Sabella, Sanjay Rebello
Collection: AIP Conf. Proc. Volume 1289, 2010 Physics Education Research Conference
Bibliography: AIP Conf. Proc. Volume 1289, 2010 Physics Education Research Conference, p. 165-168, DOI: 10.1063/1.3515188

Smith, Trevor I., Thompson, John R., and Mountcastle, Donald B., "Addressing Student Difficulties with Statistical Mechanics: The Boltzmann Factor", (2010). Refereed conference proceedings, Published
Editor(s): Chandralekha Singh, Mel Sabella, and Sanjay Rebello
Collection: AIP Conf. Proc. Volume 1289, 2010 Physics Education Research Conference
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Smith, Trevor I., Christensen, Warren M., and Thompson, John. R., "Addressing Student Difficulties with Concepts Related to Entropy, Heat Engines and the Carnot Cycle", (2009). Refereed conference proceedings, Published
Editor(s): Sabella, Mel, Henderson, Charles, and

Singh, Chandralekha
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Research Conference, p. 277-280,
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Wittmann, Michael C. and Black, Katrina E., "Using conceptual blending to describe
emergent meaning in wave propagation", (2010). Refereed conference proceedings, Published
Editor(s): Kimberly Gomez, Leilah Lyons, and
Joshua Radinsky
Collection: ICLS '10 Proceedings of the 9th
International Conference of the Learning
Sciences
Bibliography: ICLS '10 Proceedings of the 9th
International Conference of the Learning
Sciences, Volume 1, p. 659-666

Frank, Brian W., "Multiple Conceptual Coherences in the
Speed Tutorial: Micro-processes of Local
Stability", (2010). Refereed conference proceedings, Published
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R. Padraic Springuel, "Applying Cluster Analysis to Physics
Education Research Data", (2010). Thesis, Unpublished thesis, available online
Bibliography: Unpublished Ph.D. dissertation, University
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Methods for Air Resistance Problems", (2010). Thesis, Unpublished thesis, available online
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Casey Murphy, "Answer-Seeking and Idea-Constructing
During Collaborative Active-Learning
Activities in a Physics Laboratory", ( ). Thesis, Unpublished thesis, available online
Bibliography: Unpublished MST thesis, University of
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Kate M. Hayes, "A qualititative analysis of student
behavior and language during group

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Mindi Kvaal Anderson, "Comparing the Effectiveness of Three
Unique Research Based Tutorials for
Introducing Newton?s Second Law", (2009). Thesis, Unpublished thesis, available online
Bibliography: Unpublished MST thesis, University of
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Bhupendra Nagpure, "The effects of reasoning about vector
components on student understanding of
two-dimensional acceleration", (2008). Thesis, Unpublished thesis, available online
Bibliography: Unpublished MST thesis, University of
Maine, 2008

Joel Van Deventer, "Comparing student performance on
isomorphic math and physics vector
representations", (2008). Thesis, Unpublished thesis, available online
Bibliography: Unpublished MST thesis, University of
Maine, 2008

Glen Davenport, "The reliability of the force and motion
conceptual evaluation", (2008). Thesis, Unpublished thesis, available online
Bibliography: Unpublished MST thesis, University of
Maine, 2008

Web/Internet Site

URL(s):

http://perlnet.umephy.maine.edu/materials/

Description:

A link on this page takes one to the modified FMCE analysis template created by T.I. Smith as a way of making the analysis of the Force and Motion Conceptual Evaluation consistent with the analysis of the survey based on resource activation. This template has been made public and advertised on the leading mailing lists of the PER community, PhysLrnr and YoungPER.

Other Specific Products

Product Type:

Software (or netware)

Product Description:

Software to carry out cluster analysis using Python libraries. Lets us take multi-dimensional data sets and cluster most similar results into tree graphs which show relations between most similar responses.

Sharing Information:

Two methods:
1. publication on arxiv.org to accompany a publication in a journal
2. publication on our own web site at the University of Maine

Contributions

Contributions within Discipline:

The principal disciplinary field is physics education research, in which theoretical modeling is only slowly taking hold. Developing and extending resource theory is of great importance when building a foundation on which to ask better experimental questions and understand experimental data better. We highlight several elements of our work.

First, there is the methodological issue of measuring the plasticity of resources. By bringing in ideas from RBC theory (recognize, build-with, construct), we are able to observe how students make sense of ideas in real time ? and how their sensemaking changes on both long and short time scales. Researchers can use our tools to better understand when a resource is in play and what resources are being used in a given setting. Resource plasticity can more easily be observed and understood, allowing us to understand the in-between stages as students move from not knowing a topic to being at least locally expert in their understanding.

Second, on the topic of resource coordination and activation, there is a methodological issue of cluster analysis. This has only rarely been used in PER, and has great value. By evaluating student responses (on free response questions) based on what students do, rather than what they are interpreted to be doing, we can group responses and find common themes that are at times surprising and unexpected. Items that (on a macroscopic analysis, typical of PER) seem closely related are in reality much less related than thought. Methodologically, this tool allows us to find emergent connections in the data, rather than seeking for existing categories. Such work is helpful in modeling resource coordination (which ideas are used at the same time) and activation (which questions elicit which kinds of ideas).

Third, we have greatly extended the range of studies carried out on the use of mathematics in advanced physics classes. This has included the methodological work of asking isomorphic questions in physics and non-physics forms. By comparing student responses on each, we are able to study the context-dependence of questions, allowing better recognition of which resources are being activated where.

Fourth, we have extended our application of the resources framework to include the analysis of standardized tests which were not originally designed with this cognitive model in mind (and were, as a matter of fact, designed with a completely different and partially contradictory model of learning). Our example is the Force and Motion Conceptual Evaluation. We find that our analysis accounts for experimental data while also making claims about previously unpublished false positives on the test. Having predicted the false positive, we find evidence for it in our data. Our results call into question some assumptions previously made about this standardized test. We note that subsequent results from a cluster analysis of student data on the FMCE are closely aligned with the resources-based analysis of the test, but suggest that there is greater coherence in student responses than is assumed when splitting the test into several different groups of questions.

Fourth, we have applied the resources framework to areas in which it was not previously discussed. So, for example, we have used resources to analyze the Force and Motion Conceptual Evaluation, showing that question context is consistent with a resources interpretation of the most common student responses; this work modified the original grouping of questions, as given by the survey authors, and found a false positive result that was previously undocumented. Similarly, we have looked at the first application of epistemic games to physics have deepened our understanding of epistemological framing by giving a more detailed analysis of epistemic games within the resources framework.

Finally, we have applied the resources framework to epistemological issues, looking at the persistence of students' activated epistemological resources in the context of a conceptually oriented laboratory activity. The persistence of a given activation (be it 'knowledge as invented stuff' or 'knowledge from authority') through a series of activities designed to promote the former and dissuade students from the latter suggests that some laboratory activities are not as effective at promoting effective learning behaviors as was originally assumed.

These contributions are all part of masters theses or doctoral dissertations, and we are in the process of sharing those results which have not already been published.

Contributions to Other Disciplines:

As we study the role of mathematics in physics, we have found many different areas in which the physics affects our use of mathematics and vice versa. So, for example, we have studied student responses on isomorphic math and physics vector tasks, looked at the use of coordinate systems in physics, and considered how physics reasoning affects mathematical modeling as vectors are translated into scalar quantities within a coordinate system. These contributions are of interest and importance to mathematics education researchers.

Our cluster analysis software and analysis is of interest to those working in the learning sciences who are doing similar work on bringing cluster analysis to bear on understanding student reasoning about the physical world.

Contributions to Human Resource Development:

As stated earlier, there are many more graduate students involved in this project than originally planned.

Three students have completed their PhD work while supported in part by this project. One spent time in a post doctoral position at the Ohio State University and has recently been offered a position as a tenure track faculty position at Kansas State University. Another has moved on to post doctoral work as part of St Anselm monastery and plans to teach at the college level at a Benedictine school. The third is a post doc at the University of Maine as part of a new NSF funded project.

The additional students are all Masters students, either for a Master of Science in physics or in the Master of Science in Teaching (MST) program at the University of Maine. The MST students have moved on to a variety of careers: some are teachers in the physical sciences at the high school level or at the university level. Others have continued in graduate school with a focus on educational psychology, physics education research, or other STEM-related area. Some have joined industry, primarily in the field of education studies and evaluation.

In year 1, students on this project have received 1 Ph.D. and 2 Master of Science in Teaching (M.S.T.) degrees. The Ph.D. student was hired as a post doc at the Ohio State University (and has since been offered a tenure track faculty position after spending several years as a visiting faculty member at Wabash College). One M.S.T. student joined us as a Ph.D. student. The other is now teaching.

In year 2, we had 1 recently graduated student from another UMaine project join us as a post doc, 1 student receive an M.S. degree, and 3 students receive their M.S.T. degree. At the end of year 2, the post doc was hired, partially based on experience in this grant, to be a visiting faculty at Randolph Macon Academy in Richmond, VA. The M.S. student has moved as a Ph.D. student to another research area. One M.S.T. student is now teaching, one is moving into educational consulting and analysis, and the third moved on to a Ph.D. program in educational psychology.

In year 3. we have graduated 1 M.S.T. student who is doing consulting work.

In year 4, we graduated 1 M.S.T. student and 2 Ph.D. students, whose career paths were described above. We also hired a new post doc, Brian Frank, who has since been hired to be part of new NSF funded work at UMaine.

In year 1, the PI was promoted to associate professor with tenure, with the strength of this project being a major sign of success for the PI. Co-PI Donovan left the project at the end of year 1. In year 2, one co-PI, John Thompson, was also promoted to associate professor. PI Wittmann spent year 2 abroad on sabbatical. In year 3, co-PI Thompson spent the year abroad on sabbatical as part of a Fulbright Fellowship.

Contributions to Resources for Research and Education:

We have developed several tools of use to researchers in physics education. Especially in the analysis of standardized test data for commonly used tests in PER, like the Force and Motion Conceptual Evaluation, our use of model analysis, and cluster analysis provide a toolbox for researchers.

1. To carry out model analysis, one should focus on questions that are all on the same topic. Defining these groups of questions is of great importance when trying to understand consistency of reasoning, for example. The groups defined by Trevor Smith in his work allow us to understand in finer detail what kind of learning is happening in our physics classes. He has revised analysis tools created by the project PI (while at the University of Maryland as a post doc) and has published these electronically.

2. The cluster analysis methods learned by Padraic Springuel have been coded using common programming languages. These have been shared as part of a publication under review, letting others quickly adopt the tool for their own research. They have also been published (with full documentation) as required by the open-source software license under which they were developed.

3. To help analyze interview or classroom data as students struggle with new ideas, we have developed coding schemes that allow us to analyze the plasticity of resources as they are developing in students' minds.

4. To help analyze classroom video, we have provided detailed analysis of students' interactions with each other, with the space around them, and with sources of authority as a way of analyzing their epistemological framing of a situation. In the process, we have come up with methods to convert observations of behavior into descriptions of resource activation.

Contributions Beyond Science and Engineering:

See those listed under Outreach Activities, specifically dealing with future teachers.

Conference Proceedings

Categories for which nothing is reported:

Organizational Partners

Any Conference

Annual Project Report for Award 0633951 "Creation, Coordination, and Activation of Resources for Learning Undergraduate Physics"

In the past 4 years, researchers at the University of Maine, including 3 faculty, 2 post doctoral research associates, and 15 graduate masters and doctoral research assistants, have combined to complete 11 dissertations or masters theses, publish 9 papers in refereed journals, 17 in peer-reviewed conference proceedings, and submit 5 more (while more than 5 are in advanced stages of preparation). In addition, grant personnel have presented 23 invited talks, 38 contributed talks, and 74 posters at local, national, and international conferences. The PI and co-PI have also been lead organizers of conferences (Foundations and Frontiers of Physics Education Research and the Transforming Research in Undergraduate STEM Education) at which results were discussed at length. Students graduating from our research group have moved on to tenure track research faculty positions at leading physics education research groups, become middle or high school teachers, taken on post doctoral positions, moved into educational research, development, or testing firms, or continued their studies as part of PhD program, medical school, or seminary. In no small way, the project funded by the National Science Foundation has laid the foundation for ongoing, high-quality investigations of student learning by the Physics Education Research Laboratory at the University of Maine, helping us carry out work in intermediate and upper-division physics courses, studying the intersection of mathematics and physics reasoning.

These results have affected not just the studied courses, but also courses for non-science majors and future teachers. In the Integrated Approaches to Physics Education, graduate students (be they Ph.D. students in physics or candidates for the Master of Science in Teaching) learn about the applications of physics education research to the classroom. In an Educational Psychology course for scientists and mathematicians, the cognitive framework that forms the basis of this project is used to connect common issues in educational psychology, including studies of students' misconceptions, conceptual change theories, framing in discourse, and sociocultural issues in learning. Thus, the effect of the research project extends beyond the participants, going deeply into the classroom.

In this final report, we repeat the results presented in the online project report. The order of this document follows that of the online report system.

I. Participants

A. Project Participants

There were originally 3 faculty involved in this grant. With co-PI Donovan leaving the university of Maine, funds were freed up to hire a full time (rather than half time) post doc for the project in year 4, as well as support far more graduate students than had been originally planned. This greatly increased the scope of our project beyond the original submission.

Faculty:

1. Michael C. Wittmann -- Principal Investigator
2. John Donovan -- CoPrincipal Investigator (until 2007)
3. John R. Thompson -- CoPrincipal Investigator

Post doctoral research associates:

1. Brandon R. Bucy (2007-2008)
2. Brian W. Frank (2009-2010)

Graduate students (in reverse order of completing their thesis work):

1. Jeff Hawkings (Ph.D.)
2. Adam Kaczynski (Ph.D.)
3. Rabindra Bajracharya s (M.S.T.)
4. Katrina E. Black (Ph.D. 2010)
5. R. Padraic Springuel (Ph.D. 2010)
6. Casey Murphy (M.S.T. 2010)
7. Kate McCann Hayes (M.S.T. 2009)
8. Mindi Kvaal Anderson (M.S.T. 2009)
9. Bhupendra Nagpure (M.S.T. 2008)
10. Joel Van Deventer (M.S.T. 2008)
11. Evan B. Pollock (M.S. 2008)
12. Zachary S. McIntyre (M.S.T. 2007)
13. Daniel Reed (M.S.T. 2007)
14. Eleanor C. Sayre (PhD. 2007)
15. Trevor I. Smith (M.S.T. 2007, started Ph.D. 2007)

Details on these researchers are given in the online data entry system.

B. Partner Organizations

None.

C. Other Collaborations or Contacts

Though there were no direct collaborations with other researchers on elements of this project, there were lengthy interactions with colleagues that advanced our work. In general, the areas of our interactions lay in areas of mathematics use in physics, cognitive models within the "knowledge-in-pieces" tradition of research (including the resources framework as we used it, as well as phenomenological primitives as described by diSessa and symbolic forms as described by Sherin), and cognitive modeling using the conceptual blending framework.

Year 1: We have worked at times with Noah Finkelstein (Colorado University), E.F. 'Joe' Redish (University of Maryland), Rachel E. Scherr (University of Maryland), and Michelle Zandieh (Arizona State University), on issues of resource development, linking of resources, activation in networks, and blending theory.

Year 2: We have discussed ideas in detail with: Joseph Perner (University of Salzburg), about metacognition and executive function; Andrea diSessa (UC Berkeley), about resource activation and linking, specifically when mixing procedural and conceptual resources; Bruce Sherin (Northwestern U), about cluster analysis, social issues in resource activation, the nature of cognitive objects such as resources, and the difference between his nodes and graphs and our resource graphs; David Hammer (University of Maryland), about resource linking and activation; and Andy Elby (University of Maryland), about conceptual blending and its role in resource development. David Hammer served as external reader on PhD candidate Ellie Sayre's dissertation on the plasticity of resources (and resource coordination), as well.

Year 3: Discussions have continued with diSessa (UC Berkeley), Sherin (Northwestern), Hammer (Maryland) and Elby (Maryland). Rachel Scherr (visiting at Seattle Pacific U) has been instrumental in advancing theoretical work on the application of blending theory to modeling resources in physics. Collaborators on the thermodynamics and statistical mechanics elements of this project (looking at math use in upper-division physics courses) included Mike Loverude (Cal State - Fullerton) and David Meltzer (Arizona). For issues related to transfer (including a mindset of "transfer in pieces," consist with our knowledge-in-pieces resources framework), we have interacted closely with Joe Wagner (Xavier).

Year 4: Collaborations have focused on ongoing work with Rachel Scherr and Hunter Close (Seattle Pacific University) on the topic of embodied cognition (as linked to resource activation and creation). Valuable input came from Zandieh (Arizona) and Chris Rasmussen (San Diego State U) on issues of blending theory and gesture analysis when observing students' problem solving. In addition, new post doc Brian Frank kept in close contact with members of his former research group at the University of Maryland (Ayush Gupta, Luke Conlin) and Tufts (David Hammer). Collaborations continued with Loverude and Meltzer on thermodynamics and Wagner on transfer. Bruce Sherin served as external reader on PhD candidate Padraic Springuel's dissertation.

II. Activities and Findings

This project had two major strands which were interwoven throughout. On the one hand, there was theory-building work in understanding the resources framework and the creation, coordination, and activation of resources. On the other hand, the area of research was primarily in advanced physics topics (waves, quantum physics, mechanics, thermodynamics, statistical mechanics) in which mathematical reasoning plays a core role in one's conceptual understanding. Thus, the research activities and findings described here touch on both these areas. An additional strand of activity developed over time, namely the study of interactions and the role of communication within group learning activities. Especially in those areas where work involved video-based data gathering techniques, data came from the analysis of social interactions. Thus, methods of interaction analysis and discourse analysis became more important in years 3 and 4 of the project.

A. Research and Education Activities

Resources are basic building blocks of our thinking and have been shown to be effective as elements of a model of reasoning in physics. We wish to understand how reasoning resources in physics come to be, how they are linked to each other and coordinate to build larger ideas, and how one set of ideas gets chosen over another set of ideas in a given context.

- Resource creation: Looking at how students build new ideas into usable "chunks" (which we call resources), allowing for more concise and higher speed reasoning about physics and math.
- Resource activation: Understanding how resources get activated in a given context, particularly in situations where math and physics ideas must come together for a full understanding, or where representations seem to affect student reasoning.
- Resource coordination: Studying how individual resources are used in conjunction with each other to develop more advanced ideas.

A major point is to understand the way in which physics and math ideas merge to create a conceptually and mathematically coherent and physically rich models of the world around us. Relevant issues include the observing the creation of new concepts, understanding the methodological issues of finding connections between resources, and describing ways in which representations and contexts affect the activation and coordination of resources, some of which are still weakly built and only little understood by their users.

1. Data gathered and methods of analysis

Data were gathered from a variety of settings:

1. Weekly group interviews with students over a whole semester ("group mini-views")
2. Videotaped homework help sessions
3. Individual student interviews
4. Classroom video observations, either of group learning activities or of group quizzes
5. Surveys, ungraded free response quizzes, and other written work such as exams and homework problems

Results were analyzed using common tools common to each form of data. Video data were transcribed, annotated, and analyzed where appropriate. Annotations include information connected to gesture, discourse, and interaction. Our analysis methods built off or are consistent with the discussion of interaction analysis given by Jordan and Henderson (1995) and the discussion of video analysis given in Derry et al. (2010) in the *Journal of the Learning Sciences*.

Survey and free response data were analyzed using a variety of methods (including content, textual, cluster, and model analysis). A major result of our work was to analyze the Force and Motion Conceptual evaluation in terms of the resources framework and then build analysis tools to help others use our analysis. We have then used this analysis to define the mental models that can be used when carrying out Bao's method of model analysis.

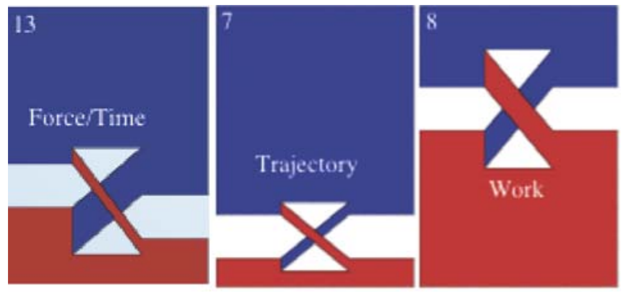


Figure 1: Kanim's escalator diagram, showing different movement of students between pre- and post-instruction testing. The vertical axis indicates the number of correct (blue) and incorrect (red) respones. In the Force/Time diagram, more students go from incorrect to correct than go from correct to incorrect. In the Trajectory diagram, equal numbers change. In the Work diagram, a different equal number change.

We developed a specific kind of survey in which students answered isomorphic physics and "physics-less" questions on vectors. These questions had the same graphic, but different descriptions, so that in some cases one merely added vectors, while in others one had to find, for example, the forces acting on an object, or, in another example, the change in velocity for an object traveling on a curved path. In comparing results on these tests, we extended Kanim's idea of "escalator diagrams" (Figure 1) in which the shift in students' responses before and after instruction are represented graphically. By including information about incorrect responses, as well, we are able to compare resource activation in different contexts more easily (Figure 2).

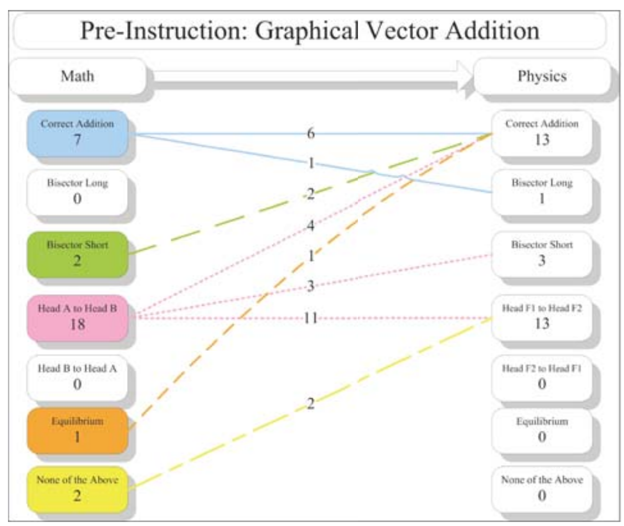


Figure 2: Van Deventer's extension of Kanim's escalator diagram includes information not only about the correct answer on a given question, but also the kinds of incorrect answers that students were giving. Different answers are indicative of different kinds of resource activation.

The mindset behind this analysis was continued in two different projects. In the one, Black looked at how students answered identical questions in the middle of and at the end of a semester. She added a second dimension to Van Deventer's plot (Figure 2) to create a consistency plot (Figure 3). This plot shows the remarkable fluidity of students' methods for answering an integration problem in a sophomore level mechanics class. The details of this plot (including issues of *circulation*, *attraction*, and *starbursts*) are described in more detail in a paper that has been submitted for publication and is being revised after reviewer comments.

A second approach to Van Deventer's work came when Springuel sought ways to "assume less" about students' responses to questions, and allow group sorting methods find those common responses which required further analysis. We looked to cluster analysis, rather than factor analysis, because the method of agglomerative hierarchical cluster analysis more generally pulls out common themes in student responses without making assumptions about the kinds of differences we are going to find. Our application of cluster analysis is essentially new to the field of physics education research. Other researchers at Ohio State University and Northwestern have done some work, but not in as much detail as we have carried out. Our goal in applying cluster analysis to PER data was to avoid using *a priori* assumptions about how students are answering questions and find patterns of responses that both gave evidence of resource coordination and context-dependent activation of resources. Springuel's PhD dissertation is being prepared for publication. Three articles are planned. The first is on the details of cluster analysis and rigorous definitions of the data that one analyzes - previous researchers have defined similarity of data inappropriately, leading to an incorrect analysis of results. The second describes the application of cluster analysis to physics education research data, including the heuristics one uses to manage issues of noise, consistency of results, and pedagogical meaning when creating cluster dendograms (Figure 4). These first two papers serve as primers on the application of the method to PER. The final paper will include examples from an analysis of data from the Force and Motion Conceptual Evaluation.

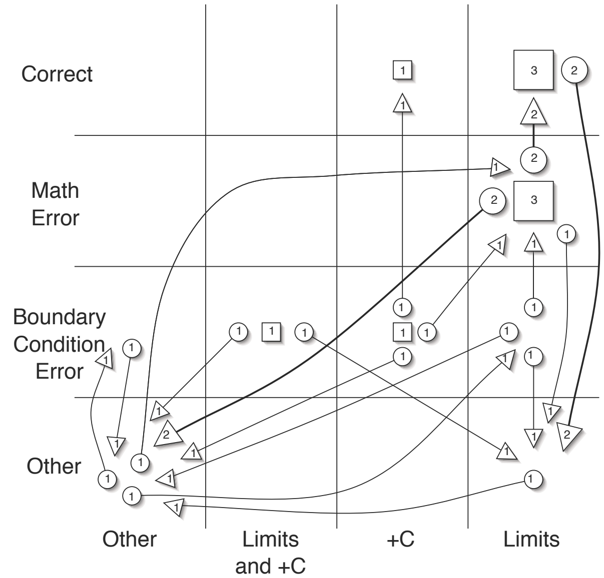


Figure 3: Black's consistency plot. Student mid-term responses are given by circles, final exam responses by triangles. Students who stay in the same location on the plot are indicated by a square.

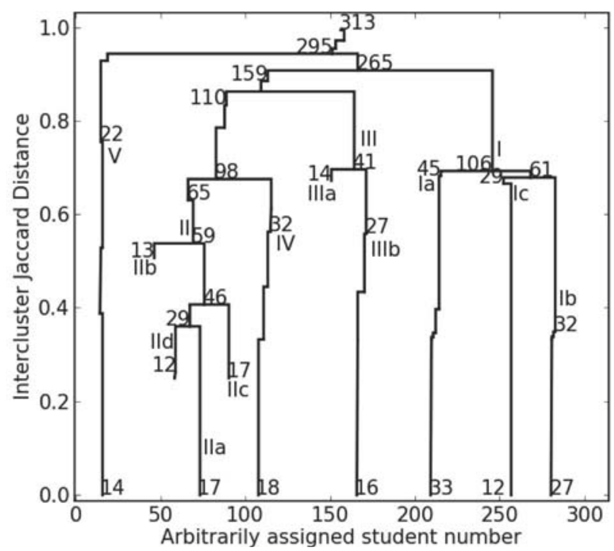


Figure 4: Springuel's application of cluster analysis to vector questions describing motion in 2 dimensions. Numbers indicate the size of a given group, while Roman numerals indicate meaningful groups of students.

2. Specific Projects by Graduate Students and Post Docs

Due to the richness of work carried out by the graduate students and post docs involved in this project, we summarize each of their individual projects in the space below. Many build off each other, which will be noted in the summaries. Later, we connect these projects into larger themes of work. Projects are listed alphabetically by student. Students who graduated with a Ph.D., Master of Science in Teaching (M.S.T.) or Master of Science (M.S. in physics) are noted. Those who worked on grant related topics while receiving funding from other sources (primarily M.S.T. students supported through teaching assistantships related to their plan of study) are also noted.

1. M.K. Anderson: Comparing three methods for teaching Newton's Second Law. Investigating the effectiveness of three separate small-group teaching curricula, each of which introduces Newton's Second Law in slightly different forms, using tools developed by T.I. Smith (#13). Results were published in peer-reviewed conference proceedings. M.S.T. received in 2009. (Not funded by this grant, but mentored by PI Michael C. Wittmann on grant-related work.)
2. R. Bajracharya: Investigating cuing in understanding mathematics and physics versions of a typical integral problem. The research questions are a continuation of work done by Pollock (#10). In studying how students carry out integrals, the work also builds on Black's results (#3). Questions of resource activation are looked at in terms of shapes of integrals, the interaction between value, slope, and area in students' reasoning, and the question of how mathematical notation is applied and modified in a physics classroom. (Not funded by this grant, but mentored by co-PI John R. Thompson on grant-related work.)
3. K.E. Black: This multi-faceted work formed a core element of the project. Work took place in the context of studying students' choices of integration methods when solving separable differential equations. In terms of the resources framework, we first extended the definition of resources to include procedural resources (scripts) that are carried out while solving problems. We discussed the creation of resources through the reification of laboriously carried out scripts into tightly compiled actions. In the process, we connected the work to epistemic games and issues of epistemological framing. The different activation of resources was represented through new methods, including a "consistency plot," which represents the shift of answers to identical questions after a period of time. Finally, resource coordination was modeled through a process of conceptual blending of gestures and discourse in the context of carrying out mathematical manipulation of equations. Papers on many of these topics have been published in peer-reviewed conference proceedings, and papers for journals are either under review, being revised, or being prepared. Ph.D. received in 2010.
4. J. Hawkins: Understanding student reasoning about two dimensional vector addition. Building off of work by J. Van Deventer (described below, #15), a study to investigate how minor changes in visual representation can affect student responses to simple graphical 2-d vector addition questions. Results show that students are cued to give certain answers based on procedural, visual, or conceptual cues, and that they persist in the solution method with which they began their work when answering a series of vector

addition questions. Results have been published in peer-reviewed conference proceedings.

5. K. Hayes (formerly McCann): Understanding the use of signs in differential equations. Using both individual student interviews and classroom video during small group "tutorial" exercises, we can observe students creating the appropriate differential equations to mathematically model physical situations. Using discourse analysis, we can observe linguistic clues which alert us to violations of expectations in how they frame the activities they carry out. In particular, we find that students are inconsistent, using both mathematical reasoning and physical reasoning to arrive at contradictory results. Results have been published in a peer-reviewed journal. M.S.T. received in 2009. (Not funded by this grant, but mentored by PI Michael C. Wittmann on grant-related work.)
6. A. Kaczynski: Analyzing conceptual learning in small-group situations. Since most classroom data involves group interactions, often with no facilitator present, we are curious as to who "owns" the resources being discussed at the table. Building on work by K. Hayes (#5), we can investigate how groups come to build an idea, and individuals come to make it their own. This work is taking place in the same course studied by K.E. Black (#2) and E.C. Sayre (#12), and builds off their results in analyzing resources, this time in the context of simple and damped harmonic motion.
7. Z.S. McIntyre: Analyzing student misconceptions about variables in different mathematics settings. We developed a survey which allowed us to pre- and post-test students' understanding of variables in algebraic equations. This work is related to K. Hayes's results (#5) and also unpublished work by K.E. Black (#2) on the different ways that letters are used in mathematical sentences (constants, variables, functions, parameters, place-holders, etc.). M.S.T. completed 2007.
8. C. Murphy: Interaction analysis of students' use of epistemological resources and the ways they frame a conceptual laboratory activity on light and shadow that has been modified to promote epistemological thinking. Her results show that students enter an epistemological mode and persist in it across a series of activities; one case study describes an idea-constructing group while another describes an answer-seeking group. M.S.T. completed 2010.
9. B. Nagpure: Studying student learning when using two different ways of thinking about vector equations in 2-d kinematics situations involving acceleration both with changes in speed and changes in direction. M.S.T. completed 2008. (Not funded by this grant, but mentored by co-PI John R. Thompson on grant-related work.)
10. E.B. Pollock: Student use of mathematics in a thermodynamics context, specifically in the context of partial differential equations. This project was the first of the grant-related work in upper-division thermodynamics and statistical mechanics, an area of research on the interplay between mathematics and physics reasoning that became increasingly important as the grant progressed. Results were published in peer-reviewed conference proceedings. M.S. completed 2008. (Not funded by this grant, but mentored by co-PI John R. Thompson on grant-related work.)
11. D. Reed: Comparing student knowledge of mathematics and physics in an engineering technology class using a series of standardized tests, examination questions, and

interview. M.S.T. completed 2007. (Not funded by this grant, but mentored by PI Michael C. Wittmann on grant-related work.)

12. E.C. Sayre: Studying resource plasticity in the context of learning about coordinate systems. Resource creation is defined in terms of the plasticity (or solidity) of connections between different resources that students use when solving problems. Coordination of resources is a primary activity in learning. Her theoretical work brought together ideas from physics education research, mathematics education research, and cognitive science. Results are described in more detail below. Publication of this work comes in peer-reviewed conference proceedings, refereed journals, and is still ongoing. Ph.D. completed in 2007.
13. T.I. Smith: Using model analysis to understand changes in student learning in reform physics courses. Major work was done on understanding the resources and facets of reasoning used by students as they answer questions on a commonly used standardized test, the Force and Motion Conceptual Evaluation. We have continued to publish these results in peer-reviewed journals, with 1 manuscript under revision. We have also published a modified analysis tool to make it more consistent with the theoretical model developed during thesis work. M.S.T. completed in 2007.
14. R.P. Springuel: Using cluster analysis to uncover hidden patterns in student responses. This analysis looked at free response (including graphical) questions about 2-dimensional vector kinematics and survey responses to the Force and Motion Conceptual Evaluation (FMCE). On the vector questions, we coded free response (graphical and verbal) data using basic descriptors (about arrow direction, for example), building a table of hundreds of descriptions of a single student's response from which we built a vector of a student response. On the FMCE questions, we built student answer vectors from their responses, regardless of the correctness. Using cluster analysis, we clustered common responses and look for characteristic responses within these clusters. Results were then interpreted based on full-test responses, rather the targeted analysis that has been carried out in the past. Thus, rather than using a resources-based analysis of individual questions (as was done with Smith, #12), we could investigate if other grain-sizes of analysis were appropriate. Results show that we can use cluster analysis to uncover the resources that students use at scales different from what is typically discussed in the literature. Three manuscripts are in preparation, under review, or being revised for peer-reviewed journals. Ph.D. was completed in 2010.
15. J. Van Deventer: Understanding student performance on isomorphic mathematics and physics vector questions. We have used interviews to guide the development of questions for a survey which asks identical questions in different contexts. This work formed the basis for much of Nagpure's (#8) and Hawkins's M.S.T. completed 2008. Results have been published in peer-reviewed conference proceedings. (Not funded by this grant, but mentored by PI Michael C. Wittmann on grant-related work.)

In addition to the work done by the many graduate students involved in this project, we have had 2 post docs involved. The first, B. Bucy was active in the project when the PI was on sabbatical. Working closely with the co-PI, Thompson, Bucy advised students and studied the role of mathematical reasoning in upper-division courses. The second, B. Frank, joined the project as part of its 1 year no-cost extension. He was instrumental in mentoring graduate students, first and foremost working with Black on procedural resources, while also being involved in a myriad of his own projects. For these, he studied resource activation in the context of polysemous words (those that have two meanings, such as "faster" meaning that something takes less time or has a higher velocity. Projects included:

- the study of kinematics, and how different resources are activated when comparing two balls being thrown, and
- light and optics, and the various meanings of the word "straight" as it applies to light passing through a hole and incident on a surface.

Further work was done by Frank in the context of students' use of epistemological resources. This work, undertaken with Murphy, looked at how students rules of argumentation based on their activation of epistemological resources in a conceptual-based lab for non-science majors.

Finally, Frank has introduced new methodological tools into the research group, including the use of a "PER Lab" environment in which we can study tipping phenomena – ways in which question phrasing cues one or the other idea. Hawkins has worked closely with Frank on this project.

Several papers are under preparation based on these different elements of his work. Frank, who was at one point not sure if he would pursue an academic career, has chosen to continue in academia.

3. Common themes in project activities

Several strands of research have established themselves throughout this project:

1. *Resource coordination in the context of mathematics.* Black, Bucy, Hayes, Pollock, Sayre, and Smith have looked at the use of analytical mathematical tools in intermediate and upper-division classes. In each case, the use of differentials played a role. Also, the issue of variables was of great importance. Describing this work in terms of resource coordination has helped us analyze learning as a process of reification of coordinated resources. This builds off of work introduced to the project by original co-PI Donovan. In particular, the following areas have been studied in details:
    a. *Integration*. With work done by Bajracharya, Black, Bucy, Pollock, we have greatly extended our understanding of how integration is used in physics. Our results touch on the role of graphical representations in integration; the meaning of end points, integration limits, and integration constants; and the mechanics of actually carrying out the integral.
    b. *Differentials*.
2. *Resource activation in the context of Newton's Laws.* Anderson carried out a study on student learning of Newton's Second Law. This was patterned off of published work begun by Smith as part of his undergraduate senior thesis and extended in his M.S.T. thesis.

3. *Vector Analysis.* Issues raised in Nagpure's and Van Deventer's work have raised concerns that we do not understand the ideas students use when they carry out simple 2-d vector addition. Hawkins has begun to investigate this issue, showing the strong dependence of student responses on "hidden" triggers in the visual cues used when asking the questions. Cues include the arrangement of vectors relative to each other, the use (or not) of a grid in the problem, and the alignment (or not) of vectors relative to any coordinate system. Using methods introduced to our group by Springuel in a different study, Hawkins has carried out a series of interviews which include distractor tasks to observe students' consistencies when answering vector questions. Results indicate that we must analyze their responses in terms of their use of procedural resources, visual cues, and conceptual understanding. This is ongoing work.

In addition, we have applied several methodological tools to our work:

1. *New methods for analyzing standardized tests.* While Smith used theoretical ideas about resource activation to group questions on a common physics standardized test, Springuel has used cluster analysis to see if common groupings might be discovered with no *a priori* assumptions about the questions being answered. Results show that we can use common student responses to look for consistencies across question groups in ways that Smith's analysis was incapable of doing. This work allows us to connect student responses across question groups and allows us to analyze thinking across several topics in kinematics and dynamics. More details are given in the discussion of Figures 1–4, above.
2. *Interaction analysis*. Throughout the project, data has been gathered using video analysis of small group learning environments, homework help sessions, and interviews. We have used methods discussed by Jordan and Henderson (1995) and Derry et al. (2010) to analyze the video. Black, Murphy, and Sayre have been the primary video analysts. As expected, the nature of the data informs the analysis, such that gesture and discourse analysis play a major role in interpreting students' actions.
3. *New approaches to doing control studies in large lecture classes*. Building off ideas by Dan Schwartz and his "Preparation for Future Learning" tasks, as well as using tools from psychology experiments, we have stepped away from the more common pre- and post-instruction assessments. Instead, we have focused on slightly different questions asked in quick succession, a few days apart, to see how students' responses might change with time. We have introduced distractor tasks in the middle of interviews. We have used a PER Laboratory environment where students get different versions of similar questions but cannot compare their work to each other. These are all common methods in other education research fields, but were new to our research group during the time of the grant.

The findings of these activities have been published and presented extensively. As shown below, this project has supported the final theses of 3 Ph.D.s and 8 Masters degrees. In addition, there are 5 papers under review, 9 published in peer-reviewed journals, and 17 published in peer-reviewed conference proceedings. Finally, there were 23 invited, 38 contributed, and 74 poster presentations supported in part by this project. In all, this dissemination of our work has been extensive, ongoing, and is not complete. In addition to the papers under review, another 5 are actively in preparation, and there are plans for several more.

B. Major findings

One major point is that the issues of creation, coordination, and activation cannot easily be separated from each other. Thus, while the topics studied are nominally separate, each project underway contains some overlap of results. This has been touched on in the previous section.

As best possible, we put results on the use of mathematics in advanced physics courses into the discussion of resource creation, coordination, and activation. We also have a separate section on our findings in this area, where the work is better described on its own.

1. Resource plasticity and coordination

Since resources act as "chunks" in student reasoning, it is important to understand how solid these chunks are - when activated, how large is the activated structure? We have constructed a new measure, called plasticity, to help describe how ideas that are weakly linked and must be constructed for every use eventually become more solid, meaning well linked and strongly compiled. This work used theories from math education, including Process/Object theory and RBC (Recognize, Build-With, Construct) to help develop appropriate measures for observations of resource plasticity.

Because resources may contain within their structure other resources, one can study how a resource such as coordinate system comes to be by looking at the mix of procedural, epistemological, and reasoning resources that comprise it. For example, we can use a variety of tasks which allow for more than one coordinate system to be used to observe the interplay of resources which occur to create a larger resource. In another example, we can observe students' increasingly compiled use of resources when manipulating equations algebraically. Students shift from explanations with many connected parts to shorter explanations in which the several parts of been compiled or reified into a new resource.

Evidence for resource creation came from students' shifts in reasoning across several modalities. As students develop a more compiled resource for dealing with some set of procedural steps while solving a math problem in physics, their language, math formalism, and gestures all change in concert with each other. They start with many formal mathematical steps and use gestures to help isolate terms in the equation, indicating a formal, analytical description of the algebra. They move to more informal mathematical language and use different gestures to describe moving terms about, indicating an informal, embodied description.

To analyze these results, we have reached toward the theory of conceptual blending. In particular, we have looked at how the theory suggests mechanisms which not only describe but also explain why certain kinds of reification happen. These results have been published in the proceedings of the International Conference on the Learning Sciences, and further publications are under preparation or submitted for review. Examples are shown in Figures 5 and 6. In Figure 5, we explain the work done by students as a seemingly simple mathematical step (of grouping objects separated by a minus sign) is observed to be quite difficult for students, often requiring the use of a grouping gesture as one moves to treating the sign-separated terms as a single term. In Figure 6, we show how students' gestures are indicative of moves of pieces on a gameboard -

but constrained by the mathematical rules of the situation. Notably, the circling of one term in the Gameboard Algebra blend is consistent with the Reified Math Object blend. Black has described examples in which this connection does not occur, showing that we are observing distinct procedures.

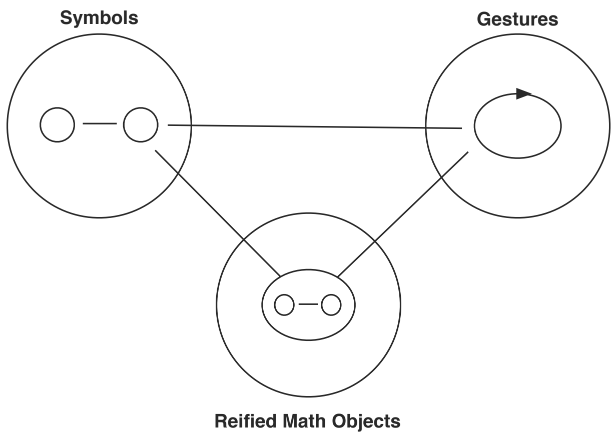


Figure 5: A simple blend in which a gesture is used to group independent mathematical terms into a single, refied math object. The *gathering* gesture indicates students grouping terms into one; the new math object can be used as a single object, even though it contains multiple pieces.

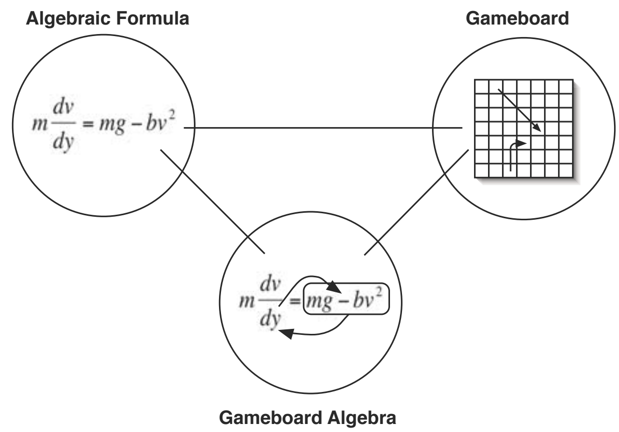


Figure 6: A blend indicating connections between mathematical manipulation to separate variables and moves on a gameboard. Data to support the blends in this and the previous figure come from multiple classroom observations of students working on a group quiz.

Black has extended this analysis to include not only blends, but symbolic forms, epistemological resources, and more. A resource graph of the slowly-created procedural resource of "separating variables" is shown in figure 7. It only includes procedures, but not epistemological, symbolic, or conceptual resources that are also involved in this mathematical activity.

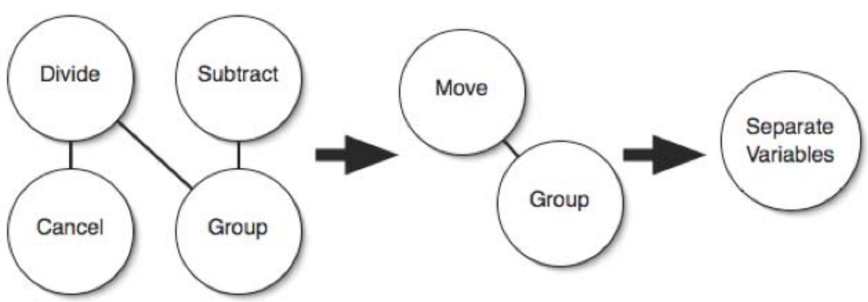


Figure 7: A possible conceptual pathway for the reification of the "Separate Variables" procedural resource.

2. Resource activation

We have studied the process of resource activation in several different physical and mathematical contexts. Primarily, this work has been done in introductory physics classes (looking at the use of vectors), sophomore level mechanics classes (where equations require a closer connection between physical and mathematical reasoning, and differential equations first become common in physics learning), and senior level thermodynamics and statistical mechanics courses. Results have been published extensively in peer-reviewed conference proceedings, and additional papers are under preparation or submitted and under review.

In the context of writing physics equations, we find that students have a hard time using minus signs appropriately. On the one hand, they will use physically standard formats (F=ma) to determine an equation. On the other hand, given that they have some arbitrarily chosen coordinate system, they will then force minus signs into the problem to ensure that the outcome is as they desire (writing, for example, F= –ma, incorrectly). Depending on the choice of their solution, different and contradictory ideas are used. A resources model easily describes students' activation of different ideas in different settings.

Similar inconsistency was found in the context of students' use of integration methods when solving separable differential equations. There are two methods, the use of indefinite constants commonly taught in mathematics classes, and the use of integration limits, commonly used in physics. Depending on cues, students use the more mathematical or the more physics-oriented solution method, where one typically leads to incomplete solutions and the other leads to complete physical descriptions of the situation. To account for the differences in student performance, we have developed the idea of procedural resources and talked about their context-dependent activation. This work has connected us to Collins and Ferguson's epistemic games, as well as to ideas about epistemological framing. (Notably, questions about epistemological framing and the effects of activating certain epistemological resources have been explored further by Hayes and Murphy in both non-science and mechanics courses.) In addition, we have developed the idea of consistency plots, in which we can map students' responses to identical questions at different times of the semester. Finally, we have addressed questions about resource coordination by using conceptual blending to explain how resources come to be coordinated. Papers on each of these areas, procedural resources, epistemic games and epistemological framing, the use of consistency plots, and explanations via conceptual blending, are submitted and under review or in preparation. Preliminary results were published in peer-reviewed conference proceedings.

In another area of our work, we have looked at how students make sense of vectors. These representations (arrows showing direction and magnitude, but with an arbitrary location, typically defined by the physical situation) are commonly used in physics but rarely used in math classes in the same way. Students learn about them first in our physics courses, yet apply many commonly used mathematical tools (such as addition and subtraction) to these new constructs. In a series of studies carried out primarily by Van Deventer and Hawkins, we have found that students answer questions differently if given identical questions with a physics context and a non-physics (more math-like) context. This context dependence is appropriately analyzed in terms of resource activation in the different question formats. We have followed up on this work by investigating how representations affect student reasoning. So, for example, giving a tail-to-tail orientation of vectors looks much like a typical problem using free body diagrams, and students are perhaps more likely to add vectors correctly while thinking about forces and not just plain vectors. Similarly, a head-to-tail orientation of vectors is more common for displacement type problems, where the angle between the vectors plays a role in how students give their answers. We have found procedural cues (that one or another orientation suggests the first step of a solution), physical cues (that one or another orientation suggests a kind of physics thinking that is helpful), and visual cues (that one or another representation activates different kinds of solutions), and are actively pursuing the question of which cues dominate when asking different kinds of vector addition problems. Since there is a complicated interaction between resource activation and coordination, the answer is far more difficult than first expected. A further surprising result, found by Hawkins, has been that students typically pick a vector addition method and then stick to it, even as questions change and cues change. The issue of resource activation and the persistence of the activation require further study in this context.

In other work on vectors, we have also looked at how a vector description of accelerating motion in two dimensions depends on the choice of representation and the physical situation being described. We asked questions in which students were to draw acceleration and velocity vectors describing different paths of motion for travel along open and closed shapes, symmetric and asymmetric shapes, and with constantly increasing or variable motion. Nagpure did a primarily qualitative analysis of this work, looking at the use of vector components compared to the full vectors.

Springuel took this work much further, using cluster analysis to evaluate which questions activated which commonly given answer. It turns out that direction of travel and the path along which one travels activate different reasoning about the velocity and acceleration vectors one should draw in a given situation - even when the shapes are nearly identical. The use of cluster analysis (described in more detail below) has helped us find groupings of students whom we otherwise would have missed, and allowed us to do so without making *a priori* assumptions about the kind of reasoning we expected to see. Early results of this work have been published in peer-reviewed journals, but the results on question-specific activation have not yet been published. The use of cluster analysis, meaning its application to physics education research and the heuristics for making it a useful tool in PER, is among the most important results of this grant. An example of his cluster dendogram was given in the previous section, on research activities.

3. Resource linking

Issues of resource coordination have already been given in the examples of the previous 2 sections. Further examples are given here that have not yet been discussed.

The work on analyzing standardized tests arose out of a desire to answer questions which are not commonly discussed in the literature. For example, the Force and Motion Conceptual Evaluation (FMCE) is either analyzed as a whole test (looking at scores before and after instruction, then calculating gains or normalized gains), or with subgroups of questions being scored (such as questions about kinematics, reversing directions, or Newton's Third Law). The two rarely intersect, with a full-test analysis of all subgroups of questions. One could use a pivot table to explore this (given answers of X on the 3rd Law questions, how does one answer reversing directions questions), but we chose another route. Wittmann and Smith carried out a resource-based analysis of the FMCE, revising a previously designed analysis tool in the process, and used this fine-grain analysis to uncover results that the FMCE authors had not previously discovered. In particular, a false positive was found in an unexpected situation.

Further work on the FMCE made use of cluster analysis to find common student responses. Heuristics had to be developed to determine the relevance of groups, how common a given response had to be within a group for it to be defined as a group, and how much noise one was willing to accept within the cluster analysis groupings. Once groups of common results were found, these were interpreted not in terms of previously determined question groups, but were analyzed across questions, looking for larger-grain common responses. This work by Springuel is being prepared for publication.

A similar analysis, crossing questions groups, was carried out by Wittmann and Anderson when looking at students' thinking after instruction on Newton's Second Law. As with Springuel's work, her analysis looked at questions across contexts, and found that certain questions, outside of the commonly accepted groups (including those defined by Smith and Wittmann) show a kind of coordination of resources which is often lost in a more traditional analysis of student data. The results of Springuel, Smith, and Anderson suggest that the FMCE is a far more complicated test to understand than has been assume, and that one's choice of analysis affects the resources one is likely to observe students using. These results are consistent with the idea of resources being scalable structures (akin to schemas possibly nested in other schemas, or scripts which include other scripts).

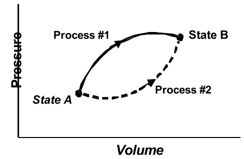
The idea of resource coordination was explored further by Black and Wittmann in the context of algebraic manipulation of separable differential equations, helping to explain how networks of resources are pulled together and how new resources emerge. Work in this area required an analysis of the semiotic function of gestures, in particular the way that a circling gesture was used to group mathematical terms before a dragging gesture was used to indicate division across the equals sign. Using conceptual blending, we analyzed students' thinking to show how new ideas emerge in the context of problem solving. Examples and figures are given in Figures 5–7. This very promising work will be explored further in the future.

4. The use of mathematics in advanced physics

While studying resource creation, coordination, and activation, we have also looked in great deal at areas of mathematics use in physics where the individual resources being used are not elucidated enough for us to discuss issues of resource coordination and activation. Instead, we have focused more generally on activation and cuing of ideas in problem solving.

In the area of integration, we have investigated students' understanding of path integration, their use of anti-derivatives when solving integrals, and whether they think of integration in terms of Riemann sums or not. We have investigated the mathematical underpinnings of student responses to questions comparing ("thermodynamic") work done by identical ideal gas samples that start at the same state and end at the same state, but have different thermodynamic processes, shown as different paths on a pressure-volume (P-V) diagram (see Figure 5). Students were asked several questions regarding first law quantities along with similar mathematical questions devoid of all physical context. We compared student responses to physics questions involving interpretation of ideal gas processes on P-V diagrams and analogous mathematical qualitative questions about the signs and comparisons of magnitudes of various integrals. Overall results coupled with individual student performance on the paired questions shows evidence of isolated understanding of the physics and/or the math. This context-dependent response is consist with our other work in vector problems, for example.

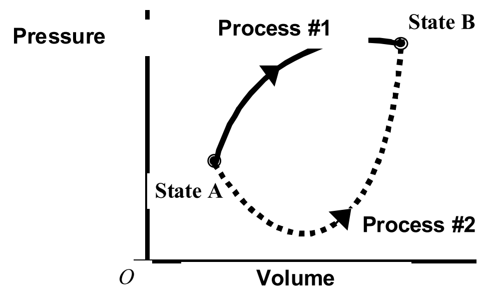
Analysis of students' difficulties shows that some students use the symmetry of the paths on the P-V diagram to justify their response (that the works/integrals were equal). To reduce the visual distraction of the symmetrical paths, we have modified the graphs to be asymmetric, that is, the "lower" path (one with less area under its curve) is now longer than the "upper" curve (Figure 8). This allows for student responses to be distinct between area-based and length-based reasoning. This research and analysis is ongoing, with Bajracharya and co-PI Thompson building on work by Evan Pollock (M.S. 2008) and Brandon Bucy (Ph.D. 2007).



[In these questions, ***W*** represents the work done ***by*** the system during a process; ***Q*** represents the heat transfer ***to*** the system during a process.]

P1. Is *W* for Process #1 ***greater than, less than,*** or ***equal to*** that for Process #2? Explain.

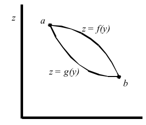
P2. Which would produce the largest change in total energy (kinetic plus potential) of all the atoms in th e system: ***Process #1, Process #2***, or ***both produce the same change***?



Two functions have been graphed on the *z-y* graph shown at right, and are labeled *f(y)* and *g(y)*. Both functions start at point *a* and end at point *b*.

Consider the integrals

$$I_1 \equiv \int_a^b f(y)\,dy \qquad I_2 \equiv \int_a^b g(y)dy$$



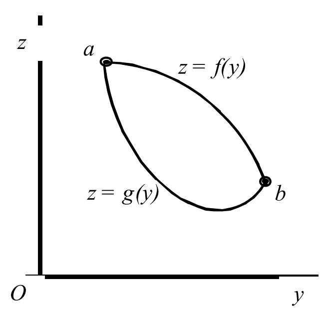


Figure 8: Integration questions looking at students' activation of resources in physics (top) and physics-less (bottom) versions of very similar questions.

In the area of differential equations, including multivariable functions, partial derivatives, and mixed second-order partial derivatives, we have studied the different ways in which mathematics and physics notation is used, how it affects student problem solving, and what sense students make of the various shorthands in use. This work has taken place primarily in the context of studying the thermodynamic concept of state function. A state function is a function whose integral is independent of path (or, in this particular context, thermodynamic process). Textbooks provide several examples and sometimes even a mathematical appendix designed to teach students the distinction between exact and inexact differentials. In spite of explicit time and effort in the classroom, students often apply state function reasoning to inexact differentials as well as exact ones, and fail to notice the distinction made by the textbook authors.

To study whether students' difficulties are related to the physics or to the mathematics, we developed a six-question math diagnostic quiz, which we administered to 5 sections of UMaine's undergraduate Calculus III course, taught by the mathematics department. The diagnostic quiz contains three questions asked in our thermal physics courses, including the integral questions and partial derivative questions described above, as well as other questions dealing with the complementary concept of differentiation. Importantly, the questions were asked in a completely mathematical context, without any reference to physical situations. Drafts were provided to the math faculty involved, and none of these instructors identified any of the questions as being inappropriate for their students (one question required minor revision to a mathematical expression to make the terminology consistent with that used in the course). Survey results were gathered from over 180 students. Many of the data gathered overall are remarkably consistent with that observed in our thermal physics courses. On the Calculus I integral question, about 55% of students correctly determined that I_1 was greater than I_2 (see Figure 8). 27% of students stated that the integrals would be identical, using some form of path-independent reasoning. On a loop integral question, less than 30% identified the loop integral of the quantity *dH* (made up of the loop from a to be and back again, first along the upper, then along the lower path) as being equal to zero. A unique response from calculus students (about 20% gave this response) was "negative," according to the reasoning that "the path is clockwise." This is a convention used in mathematics to evaluate path integrals, not regular loop integrals. One more interesting finding is that for the same loop, we asked students, in separate questions, to decide the sign (positive, negative, zero or not enough information) of both integrals of *zdy* and of *dH*. Almost 45% of the students gave identical responses for both questions (i.e., said that both integrals were zero, or both were positive). More than half did not. This suggests that students' difficulties in physics may arise from unfamiliarity with certain issues in integration, or are mathematics and not physics difficulties.

C. Training and Development

As originally planned, the project was supposed to fund only 3 students part-time. Because of an interest by many others in the work done on this project, many more students have been funded. This development of physics and math education researchers has been an unexpected but very welcome part of our grant. As students graduate, others fill their spots in our funding structure (where none are fully funded, but nearly all are partially funded for their research work).We have found new connections in our data, analyzed more detailed and specific questions with our data, and investigate topics we would not have otherwise investigated.

New research skills have been developed by nearly all project members. These skills include:

1. Individual and student group interviews. Many of the students on the project had never carried out an interview, and are now highly experienced. Skills include learning to listen without biasing student reasoning, asking leading questions that don't guide one's choice of models (if possible), and learning to help groups continue to interact with each other in a way that lets the students' ideas be seen *in situ*.

2. Analysis of classroom video. Using discourse analysis to analyze group interactions requires that we attend to language far more than we used to. Skills developed during interviews are useful for analysis of classroom video where no instructor is present.
3. Model analysis. A method developed by Lei Bao at Ohio State University, it has helped us understand performance on standardized tests, looking at individual clusters of responses (as determined by a content analysis of the physics).
4. Cluster analysis. We are leading the use of cluster analysis in the US PER community. It helps us analyze free response data using highly descriptive coding that is then clustered into common groups of responses. New computer programming was carried out in the process.

D. Outreach Activities

Our goal in this project was to develop new insights into research creation, coordination, and activation, and share these with a larger research community. We have given extensive contributed and invited presentations, as well as publishing at a rate equal to the most prestigious and active physics education research groups in the US.

The mindset behind this project, that student reasoning consists of resources that are slowly created, linked together, and activated based on appropriate cues, has been used to guide teaching in two physics courses at UMaine. One was a sophomore level mechanics course, with only a small population already interested in physics. The other was a "general education" course in which non-science majors are fulfilling a university core curriculum requirement to study science in a laboratory setting. In that course, we have used many of the tools for understanding differential equations, but tuned to a population that is often afraid of the math. Thus, we have used graphical representations where possible, rather than mathematical and algebraic analysis. We have helped students build an entirely new network of ideas (in a topic they have, by their own account, never studied in this fashion, namely quantum physics). In the process, we have shown students how science functions as a connection of small ideas, individually developed, and built into a larger whole. In year 2, the both courses were taught by Katrina Black, a graduate student in this project, using previously established methods. In the process, she developed skills necessary for a planned future faculty job.

In addition to teaching students based on the ideas in this project, the PI has taught a course on educational psychology for future teachers. This course has as a major component the idea of reasoning resources, their linking together, and changes to the linked structure of the resources as a way of describing conceptual change and learning in general. In year 1, 13 students took this course – all are in-service teachers or planning to be teachers or college instructors. We expect that their understanding of the guiding theoretical constructs studied in this project will affect their teaching and research work (thesis work is required of students who are taking this course as part of a Master of Science in Teaching). This course was not taught in year 2, while the PI was on sabbatical. In year 3, the course was completed by 15 students. In year 4, 7 students completed the course.

III. Publications and Products

Listed, where possible, within the database online and here as well:

A. Dissertations and Theses

1. Katrina E. Black, “Multiple Perspectives on Student Solution Methods for Air Resistance Problems,” Unpublished Ph.D. dissertation, University of Maine, 2010
2. R. Padraic Springuel, “Applying Cluster Analysis to Physics Education Research Data,” Unpublished Ph.D. dissertation, University of Maine, 2010
3. Eleanor C. Sayre, “Plasticity: Resource Justification and Development,” Unpublished Ph.D. dissertation, University of Maine, 2007
4. Casey Murphy, “Answer-Seeking and Idea-Constructing During Collaborative Active-Learning Activities in a Physics Laboratory,” Unpublished MST thesis, University of Maine, 2010
5. Kate McCann Hayes, “A qualititative analysis of student behavior and language during group problem solving,” Unpublished MST thesis, University of Maine, August, 2009.
6. Mindi Kvaal Anderson, "Comparing the Effectiveness of Three Unique Research Based Tutorials for Introducing Newton’s Second Law," Unpublished MST thesis, University of Maine, August, 2009
7. Bhupendra Nagpure, “The effects of reasoning about vector components on student understanding of two-dimensional acceleration,” Unpublished MST thesis, University of Maine, August, 2008.
8. Joel Van Deventer, "Comparing student performance on isomorphic math and physics vector representations," Unpublished MST thesis, University of Maine, August, 2008.
9. Glen Davenport, “The reliability of the force and motion conceptual evaluation,” Unpublished MST thesis, University of Maine, August, 2008.
10. Dan Reed, "Evaluating Factors Contributing to Engineering Technology Students’ Introductory Physics Experience," Unpublished MST thesis, University of Maine, August, 2007.
11. Trevor I. Smith, "Comparing the Effectiveness of Research-Based Curricula for Teaching Introductory Mechanics," Unpublished MST thesis, University of Maine, May, 2007.

B. Papers under review

1. Wittmann, M.C. and Black, K.E. “Emergent Meaning in Conceptual Blends of Gesture and Language,” under review at *The Journal of the Learning Sciences*.
2. Thompson, J.R, Christensen, W.M., and Wittmann, M.C. “Preparing future teachers to anticipate student difficulties in physics in a graduate-level course in physics, pedagogy, and education research,” under review at *Physical Review Special Topics Physics Education Research*.

3. Black, K.E., and Wittmann, M.C. "Understanding the use of two integration methods on separable first order differential equations," under review at *Physical Review Special Topics Physics Education Research*. Pre-print available online at http://arxiv.org/abs/0902.0748.
4. Black, K.E., and Wittmann, M.C. "Visualizing changes in student responses using consistency plots," under review at *Physical Review Special Topics Physics Education Research*. Pre-print available online at http://arxiv.org/abs/0812.3136.
5. Springuel, R.P., Thompson, J.R., and Wittmann, M.C., "How different is 'not the same'?" under review at *Physical Review Special Topics Physics Education Research*.

C. Published Papers

1. Hayes, K., and Wittmann, M.C. (2010) "The role of sign in students' modeling signs of scalar equations," *The Physics Teacher*. 48(4), 246-249. Pre-print available online at http://arxiv.org/abs/0901.4912.
2. Springuel, R.P., Wittmann, M.C., and Thompson, J.R. (2009) "Erratum: Applying clustering to statistical analysis of student reasoning about two-dimensional kinematics [Phys Rev. ST Phys. Educ. Res. 3, 020107 (2007)]" *Physical Review Special Topics Physics Education Research* 5, 029902(E). Available online at http://prst-per.aps.org/abstract/PRSTPER/v5/i2/e029902.
3. O'Brien, M.J. and Thompson, J.R. (2009) "Effectiveness of ninth-grade physics in Maine: Conceptual understanding," *The Physics Teacher* 47(4), 234-239.
4. Sayre, E.C. and Wittmann, M.C. (2008) "The plasticity of intermediate mechanics students' coordinate system choice," *Physical Review Special Topics Physics Education Research* 4 020105. Available at http://prst-per.aps.org/abstract/PRSTPER/v4/i2/e020105.
5. Smith, T.I. and Wittmann, M.C. (2008) "Applying a resources framework to analysis of the Force and Motion Conceptual Evaluation," *Physical Review Special Topics Physics Education Research* 4, 020101. Available at http://prst-per.aps.org/abstract/PRSTPER/v4/i2/e020101.
6. Wittmann, M.C. and Thompson, J.R. (2008) "Integrated approaches in physics education: A graduate level course in physics, pedagogy, and education research," *American Journal of Physics* 76:7, 677-683. Draft version available online at http://www.arxiv.org/abs/physics/0608240.
7. Smith, T.I. and Wittmann, M.C. (2007) "Comparing three methods of teaching Newton's Third Law," *Physical Review Special Topics Physics Education Research* 3, 020105. Available at http://prst-per.aps.org/abstract/PRSTPER/v3/i2/e020105.
8. Springuel, R.P., Wittmann, M.C., and Thompson, J.R. (2007) "Applying clustering to statistical analysis of student reasoning about two-dimensional kinematics," *Physical Review Special Topics Physics Education Research*, 3, 020107.
9. Wittmann, M.C. (2006) "Using resource graphs to represent conceptual change," *Physical Review Special Topics Physics Education Research* 2, 020105. Available online at http://prst-per.aps.org/abstract/PRSTPER/v2/i2/e020105.

D. Published, Refereed Conference Proceedings

1. Brian W. Frank (2010) "Multiple Conceptual Coherences in the Speed Tutorial: Micro-processes of Local Stability," Proceedings of the 9th International Conference of the Learning Sciences (ICLS 2010) - Volume 1, Full Papers, p.873-881. Published on line at arXiv:1008.3258v1
2. Michael C. Wittmann, (2010) "Using conceptual blending to describe emergent meaning in wave propagation," Proceedings of the 9th International Conference of the Learning Sciences (ICLS 2010) - Volume 1, Full Papers, p.659-666. Published on line at http://arxiv.org/abs/1008.0216
3. Trevor I. Smith, John R. Thompson, and Donald B. Mountcastle, (2010) "Addressing Student Difficulties with Statistical Mechanics: The Boltzmann Factor," AIP Conf. Proc. 1289, *2010 Physics Education Research Conference*, p. 305-308. doi:10.1063/1.3515230
4. Jeffrey M. Hawkins, J.M., Thompson, J.R., Wittmann, M.C., Sayre, E.C., and Frank, B.W. (2010) "Students' Responses To Different Representations Of A Vector Addition Question," AIP Conf. Proc. 1289, *2010 Physics Education Research Conference*, p. 165-168. doi: 10.1063/1.3515188
5. W.M. Christensen and J.R. Thompson, "Investigating Student Understanding of Physics Concepts and the Underlying Calculus Concepts in Thermodynamics," in *Proceedings of the 13th Annual Conference on Research in Undergraduate Mathematics Education* (Mathematical Association of America, 2010).
6. Black, K.E. and Wittmann, M.C. (2009) "Procedural Resource Creation in Intermediate Mechanics," in C. Henderson, M. Sabella, C. Singh (Eds.) AIP Conference Proceedings 1179 *2009 Physics Education Research Conference Proceedings*, p.97-101. http://link.aip.org/link/?APCPCS/1179/97/1
7. Hawkins, J., Thompson, J.R., and Wittmann, M.C. (2009) "Students' Consistency on 2-D Vector Addition Tasks," in C. Henderson, M. Sabella, C. Singh (Eds.) AIP Conference Proceedings 1179 *2009 Physics Education Research Conference Proceedings*, p.161-164. http://dx.doi.org/10.1063/1.3266704
8. Anderson, M.K. and Wittmann, M.C. (2009) "Comparing Three Methods of Teaching Newton's Second Law," in C. Henderson, M. Sabella, C. Singh (Eds.) AIP Conference Proceedings 1179 *2009 Physics Education Research Conference Proceedings*, p. 301-304. http://dx.doi.org/10.1063/1.3266742
9. J.R. Thompson, W.M. Christensen, E.B. Pollock, B.R. Bucy, and D.B. Mountcastle, "Student understanding of thermal physics concepts and the underlying mathematics in the upper division," in proceedings of Frontiers in Science Education Research, 22-24 March 2009, Famagusta, North Cyprus, 177-186 (2009).
10. Wittmann, M.C. and Black, K.E. (2008) "Describing the Conceptual and Procedural Resources Used in Two Epistemic Games of Integration," *Proceedings of the 2008 International Conference on the Learning Sciences*, electronic publication (2008).
11. D.B. Mountcastle, B.R. Bucy, and J.R. Thompson, "Student estimates of probability and uncertainty in advanced laboratory and statistical physics courses," in *2007*

Physics Education Research Conference, L. Hsu, C. Henderson, L. McCullough, eds., AIP Conference Proceedings 951, 152-155 (2007).
12. E.B. Pollock, J.R. Thompson, and D.B. Mountcastle, "Student understanding of the physics and mathematics of process variables in P-V diagrams," in *2007 Physics Education Research Conference*, L. Hsu, C. Henderson, L. McCullough, eds., AIP Conference Proceedings 951, 168-171 (2007).
13. J. Van Deventer and M.C. Wittmann, "Comparing Student Use of Mathematical and Physical Vector Representations," *2007 Physics Education Research Conference Proceedings*, L. Hsu, C. Henderson, L. McCullough, eds., AIP Conference Proceedings 951, 208–211 (2007).
14. Black, K.E. and Wittmann, M.C. (2007) "Epistemic Games in Integration: Modeling Resource Choice," *2007 Physics Education Research Conference Proceedings*, L. Hsu, C. Henderson, L. McCullough, eds., AIP Conference Proceedings 951, 53–56.
15. Donovan, J.E., II. (2007). The Importance of the Concept of Function for Developing Understanding of First-Order Differential Equations in Multiple Representations [Electronic Version]. Conference on Research in Undergraduate Mathematics Education, from http://cresmet.asu.edu/crume2007/eproc.html.
16. Sayre, E.C. and Wittmann, M.C. (2007) "Intermediate mechanics students' coordinate system choice," Conference on Research in Undergraduate Mathematics Education, from http://cresmet.asu.edu/crume2007/eproc.html.
17. Sayre, E.C., Wittmann, M.C., and Donovan, J.E. (2007) "Resource Plasticity: Detailing a Common Chain of Reasoning with Damped Harmonic Motion," in P. Heron, L. McCullough, J. Marx (Eds.) AIP Conference Proceedings 883 2006 Physics Education Research Conference Proceedings, 85–88.

E. Invited presentations

1. "Thinking about the 'function' in 'state function': Investigating student understanding of the math behind the physics of state functions," J. R. Thompson, UMaine RiSE Center colloquium, April 2010.
2. "Investigating Student Understanding of Integrals in Upper-Division Thermodynamics," J.R. Thompson, AAPT national meeting, Portland OR, July 2010.
3. "How Systems, Dependencies, and Constraints Affect Our Physics Education Research," M.C. Wittmann, AAPT national meeting, Ann Arbor MI, 2009 July.
4. "Including diversity in Physics Education Research: A report from the 2008 PER Conference," M.S. Sabella, J.R. Thompson, N.M. Gillespie, AAPT national meeting, Ann Arbor MI, 2009 July.
5. "Student understanding of thermal physics and the associated mathematics: Challenging assumptions in physics education." J.R. Thompson. Plenary speaker, Foundations and Frontiers in Physics Education Research 2009 Conference (International), Bar Harbor, ME, June 15, 2009.

6. “Investigations of student understanding of thermal physics and the associated mathematics.” J.R. Thompson. Department of Physics and Materials Science, Uppsala University, Uppsala, Sweden, June 3, 2009.
7. “Investigations of student understanding of thermal physics beyond the first year.” J.R. Thompson. Department of Physics and Materials Science, Uppsala University, Uppsala, Sweden, June 2, 2009.
8. “Physics education and physics education research: What do we know about teaching and learning in physics?” J.R. Thompson. School of Physics, Dublin Institute of Technology, Dublin, Ireland, May 28, 2009.
9. “Investigations of student understanding of thermal physics beyond the first year.” J.R. Thompson. School of Physics, Dublin Institute of Technology, Dublin, Ireland, April 2, 2009.
10. “Student understanding of thermal physics and associated mathematics concepts beyond the first year.” J.R. Thompson. School of Physics, University College Dublin, Dublin, Ireland, February 25, 2009.
11. “Connecting Physics and Mathematics: Probing Student Learning in Intermediate Mechanics,” B. Ambrose, M.C. Wittmann, 2009 AAPT Winter Meeting, Chicago IL, 2009 January.
12. “Investigating student understanding of thermal physics and associated mathematics concepts beyond the first year.” J.R. Thompson. Centre for the Advancement of Science Teaching and Learning, Dublin City University, Dublin, Ireland, Nov. 25, 2008.
13. “Investigating student understanding of physics and mathematics concepts in upper-level thermal physics,” J.R. Thompson, Rochester Institute of Technology Physics Department Colloquium, Rochester, NY, April 11, 2008.
14. “Comparing Student Performance on Mathematical and Physical Isomorphic Vector Tasks,” J. Van Deventer, M.C. Wittmann, 2008 AAPT Winter Meeting: Baltimore, MD, 2008 January.
15. “When duality isn’t,” M.C. Witmann, invited physics department seminar, University of Vienna, 2007 November.
16. “Using resources to understand duality,” M.C. Wittmann, invited presentation, Foundations and Frontiers of Physics Education Research conference, 2007 August.
17. “Student understanding of relationships between physics and mathematics concepts in upper-level thermodynamics.” J.R. Thompson, University of Maine Physics & Astronomy Colloquium, U. Maine, Orono, ME, 2007 April.
18. Thompson, J.R., “Student understanding of relationships between physics and mathematics concepts in upper-level thermodynamics,” Kansas State University Physics Department Colloquium, Manahattan, KS, 2007 April.
19. “Modeling mathematical reasoning in physics,” M.C. Wittmann, invited physics department seminar, Rutgers University, 2007 April.
20. “Using resource graphs to model learning in physics,” M.C. Wittmann, American Physical Society, National Meeting, 2007 April.

21. "Conceptions of functions: A model, a methodology, and their importance in calculus," Donovan, J.E. II. Mathematics Department, Plymouth State University, 2007 February.
22. "Investigating student connections between mathematics and thermal physics," J.R. Thompson, Winter National Meeting of the AAPT, Seattle WA, 2007 January.
23. "The Implications of an "Aha!" moment: Some examples from reasoning about first-order differential equations." Donovan, J.E. II. Department of Physics, University of Maine, Orono, ME, 2006 December

F. Published Abstracts for contributed presentations

1. J.R. Thompson, B.R. Bucy, and D.B. Mountcastle, "Identifying student difficulties with aspects of partial differentiation in upper-level thermodynamics," *Bulletin of the American Physical Society* 53(5), 192 (2008). (Abstract)
2. B.R. Bucy, J.R. Thompson, and D.B. Mountcastle, "Addressing student difficulties with aspects of partial differentiation in upper-level thermodynamics," *Bulletin of the American Physical Society* 53(5), 192 (2008). (Abstract)

G. Contributed presentations

1. "Understanding the Nature of Missed Learning Opportunities during Tutorial Instruction," B.W. Frank, A.C. Kaczynski, M.C. Wittmann, AAPT national meeting, Portland OR, July 2010.
2. "Analysis of Student Modes of Communication in Intermediate Mechanics Tutorials," A.C. Kaczynski,M.C. Wittmann, B.W. Frank, AAPT national meeting, Portland OR, July 2010.
3. "Student Ideas Relating to the Boltzmann Factor and Its Derivation," T.I. Smith, J.R. Thompson, D.B. Mountcastle, AAPT national meeting, Portland OR, July 2010.
4. "Students' Responses to Different Representations of a Vector Addition Question," J.M. Hawkins, J.R. Thompson, M.C. Wittmann, E.C. Sayre, J.W. Clark, AAPT national meeting, Portland OR, July 2010.
5. "Investigating student understanding of physics concepts and the underlying calculus concepts in thermodynamics," J.R. Thompson, W.M. Christensen, and D.B. Mountcastle, APS national meeting, Portland, OR, March 2010.
6. "Student understanding of calculus within physics and mathematics classrooms," W.M. Christensen and J.R. Thompson, APS national meeting, Portland, OR, March 2010.
7. "Investigating student understanding of physics concepts and the underlying calculus concepts in thermodynamics," J.R. Thompson and W.M. Christensen, Thirteenth Conference on Research in Undergraduate Mathematics Education, Raleigh, NC, February 2010.
8. "Research on Student Learning of Upper-Level Thermal and Statistical Physics," J.R. Thompson, AAPT national meeting, Washington, DC, February 2010.

9. “Curriculum adaptation in upper-level thermodynamics: entropy and the second law,” W.M. Christensen, T.I. Smith, J.R. Thompson, D.B. Mountcastle, AAPT national meeting, Ann Arbor MI, 2009 July.
10. “Exploring Student Difficulties with Multiplicity and Probability in Statistical Physics,” D.B. Mountcastle, J.R. Thompson, T.I. Smith, AAPT national meeting, Ann Arbor MI, 2009 July.
11. “Preliminary results of curriculum development in upper-level thermodynamics: heat engines,” T.I. Smith, W.M. Christensen, J.R. Thompson, D.B. Mountcastle, AAPT national meeting, Ann Arbor MI, 2009 July.
12. “Applying Cluster Analysis to Standardized Tests: Analyzing the FMCE,” R.P. Springuel, M.C. Wittmann, AAPT national meeting, Ann Arbor MI, 2009 July.
13. “Exploring the Effect of Presentation on Student Vector Addition Methods,” J. Hawkins, J.R. Thompson, M.C. Wittmann, AAPT national meeting, Ann Arbor MI, 2009 July.
14. “Observing Different Levels of Resource Coordination in Differential Equations Problems,” K.E. Black, M.C. Wittmann, AAPT national meeting, Ann Arbor MI, 2009 July.
15. “Using Shifts in Student Language and Behavior to Identify ”A-ha“ Moments,” K. Hayes, M.C. Wittmann, B.R. Bucy, AAPT national meeting, Ann Arbor MI, 2009 July.
16. “Tutorials in Intermediate Mechanics,” M.C. Wittmann , C. Swift, D. Meredith, AAPT national meeting, Ann Arbor MI, 2009 July.
17. “Student understanding of thermal physics concepts and the underlying mathematics in the upper division,” J.R. Thompson, W.M. Christensen, E.B. Pollock, B.R. Bucy, and D.B. Mountcastle. Frontiers in Science Education Research 2009 (FISER’09), Famagusta, North Cyprus, 22-24 March 2009.
18. “Student thinking regarding derivative and slope concepts in multivariable calculus,” W.M. Christensen and J.R. Thompson. American Physical Society March Meeting 2009, Pittsburgh, PA, March 2009.
19. “How Different is ‘Not the Same’?” R.P. Springuel, J.R. Thompson, M.C. Wittmann, AAPT Summer Meeting, Edmonton, Alberta, Canada, July 2008.
20. “Research on the learning and teaching of the first law of thermodynamics and the associated mathematics,” E.B. Pollock and J.R. Thompson, University of Maine Graduate Student Government Research Exposition, Orono, ME, April 2008.
21. “Identifying student difficulties with aspects of partial differentiation in upper-level thermodynamics,” J.R. Thompson, B.R. Bucy, and D.B. Mountcastle, American Physical Society April Meeting 2008, St. Louis, MO, April 2008.
22. “Addressing student difficulties with aspects of partial differentiation in upper-level thermodynamics,” B.R. Bucy, J.R. Thompson, and D.B. Mountcastle, American Physical Society April Meeting 2008, St. Louis, MO, April 2008.
23. “Research on learning and teaching of thermal and statistical physics,” J.R. Thompson, 2008 Winter National Meeting of the American Association of Physics Teachers (AAPT), Baltimore, MD, January 2008.

24. “Interpretations of entropy among advanced undergraduates across disciplines,” B.R. Bucy, J.R. Thompson, D.B. Mountcastle, 2007 Summer National Meeting of the American Association of Physics Teachers (AAPT), Greensboro, NC, July/August 2007.
25. “Investigating mathematical fluency among upper-division physics students,” D.B. Mountcastle, B.R. Bucy, and J.R. Thompson, 2007 Summer National Meeting of the AAPT, Greensboro, NC, July/August 2007.
26. “Revised methods for analyzing the Force and Motion Conceptual Evaluation,” T.I. Smith, M.C.Wittmann, T. Carter, 135th National AAPT Meeting, Greensboro, NC, July 2007.
27. “Comparing cluster analysis and traditional analysis methods in PER,” R.P. Springuel, M.C.Wittmann, J.R. Thompson, 135th National AAPT Meeting, Greensboro, NC, July 2007.
28. “Comparing student use of mathematical and physical vector representations,” J.V. Deventer, M.C. Wittmann, 2007 Joint Spring Meeting NES APS/AAPT, Orono ME, 2007 April.
29. “Intermediate mechanics students' coordinate system choices for simple pendula,” E.C. Sayre, M.C. Wittmann, 2007 Joint Spring Meeting NES APS/AAPT, Orono ME, 2007 April.
30. “Reconsidering the analysis of the Force and Motion Conceptual Evaluation,” T.I. Smith, M.C. Wittmann, T. Carter, 2007 Joint Spring Meeting NES APS/AAPT, Orono ME, 2007 April.
31. “Students' interpretations of signs in scalar differential equations,” K. McCann, M.C. Wittmann, 2007 Joint Spring Meeting NES APS/AAPT, Orono ME, 2007 April.
32. “Students' reasoning toward solutions of first-order differential equations,” K.E. Black, M.C. Wittmann, 2007 Joint Spring Meeting NES APS/AAPT, Orono ME, 2007 April.
33. “Using cluster analysis on written responses to 2-D kinematics questions,” R.P. Springuel, M.C. Wittmann, J.R. Thompson, 2007 Joint Spring Meeting NES APS/AAPT, Orono ME, 2007 April.
34. “Investigating the reliability of the Force and Motion Conceptual Evaluation,” G.A. Davenport and J.R. Thompson, Joint Meeting of the New England Sections of the AAPT and APS (regional), University of Maine, Orono ME, 2007 April.
35. “Student application of integration when considering P-V diagrams,” J.R. Thompson, B.R. Bucy, D.B. Mountcastle, E.B. Pollock, American Physical Society Meeting, Denver CO, 2007 March.
36. “The importance of the concept of function for developing understanding of first-order differential equations in multiple representations.” Donovan, J.E., II. Tenth Conference on Research in Undergraduate Mathematics Education. San Diego, CA, 2007 February.

37. “Intermediate mechanics students' coordinate system choice,” Sayre, E.C., & Donovan, J.E., II. Tenth Conference on Research in Undergraduate Mathematics Education. San Diego, CA, 2007 February.
38. “Student Solutions to First-Order Differential Equations in Intermediate Mechanics,” M.C. Wittmann, K.E. Black, 134th AAPT National Meeting, Seattle WA, 2007 January.

H. Posters

1. “Investigating student understanding of thermodynamics concepts and underlying integration concepts,” J.R. Thompson, D.B. Mountcastle, PERC 2010, Portland OR, July 2010.
2. “Characterizing Participation in and around the Physics Classroom,” Targeted poster session organized by B.W. Frank, PERC 2010, Portland OR, July 2010.
3. “How Students Structure Argument through the Interplay of Claims Made about Phenomena and Instruction,” B.W. Frank, PERC 2010, Portland OR, July 2010.
4. “Addressing Student Difficulties with Statistical Mechanics: The Boltzmann Factor,” T.I. Smith, J.R. Thompson, D.B. Mountcastle, PERC 2010, Portland OR, July 2010.
5. “Curriculum Development Addressing Multiplicity, Probability and Density of States in Statistical Physics,” D.B. Mountcastle, J.R. Thompson, PERC 2010, Portland OR, July 2010.
6. “Exploring the Transition Between Quantum and Classical Physics Using Compelling Graphical Representations,” D.B. Mountcastle, PERC 2010, Portland OR, July 2010.
7. “Students’ responses to different representations of a vector addition question,” J.M. Hawkins, J.R. Thompson, M.C. Wittmann, E.C. Sayre, J.W. Clark, PERC 2010, Portland OR, July 2010.
8. “Describing Collaborative Activity in Terms of Substantive and Interactional Constraints,” B.W. Frank, A.C. Kaczynski, B. Harrer, M.C. Wittmann, AAPT national meeting, Portland OR, July 2010.
9. “Curriculum Development Addressing Multiplicity and Probability in Statistical Physics,” D.B. Mountcastle, J.R. Thompson, T.I. Smith, AAPT national meeting, Portland OR, July 2010.
10. “Addressing Student Difficulties with the Boltzmann Factor: Preliminary Results,” T.I. Smith, J.R. Thompson, D.B. Mountcastle, AAPT national meeting, Portland OR, July 2010.
11. “Students' Responses to Different Representations of a Vector Addition Question,” J.M. Hawkins, J.R. Thompson, M.C. Wittmann, E.C. Sayre, J.W. Clark, AAPT national meeting, Portland OR, July 2010.
12. “Students' Responses to Different Representations of a Vector Addition Question,” J.M. Hawkins, J.R. Thompson, M.C. Wittmann, E.C. Sayre, J.W. Clark, Transforming Research in Undergraduate STEM Education (TRUSE), Orono ME, June 2010.

13. “Qualitative analysis of student difficulties with damped harmonic motion,” A.C. Kaczynski, B.W. Frank, M.C. Wittmann, Transforming Research in Undergraduate STEM Education (TRUSE), Orono ME, June 2010.
14. “Embodied Mathematics: Gestures and Language as Signs of Emergent Meaning,” M.C. Wittmann, K.E. Black, Transforming Research in Undergraduate STEM Education (TRUSE), Orono ME, June 2010.
15. “Student understanding of slope and derivative after multivarible calculus,” W.C. Christensen, J.R. Thompson, Transforming Research in Undergraduate STEM Education (TRUSE), Orono ME, June 2010.
16. “Curriculum Development Addressing Multiplicity, Probability and Density of States in Statistical Physics,” D.B. Mountcastle, J.R. Thompson, Transforming Research in Undergraduate STEM Education (TRUSE), Orono ME, June 2010.
17. “Addressing student difficulties in statistical mechanics: The Boltzmann factor,” T.I. Smith, J.R. Thompson, D.B. Mountcastle, Transforming Research in Undergraduate STEM Education (TRUSE), Orono ME, June 2010.
18. “Investigating student understanding of thermodynamics concepts and underlying integration concepts,” J.R. Thompson, D.B. Mountcastle, Transforming Research in Undergraduate STEM Education (TRUSE), Orono ME, June 2010.
19. “Embodied Physics: Gesture and Language as Signs of Emergent Meaning,” M.C. Wittmann, K.E. Black, AAPT National Meeting, Washington, DC, 2010 February.
20. “Three Methods of Comparing Newton’s Second Law,” M.K. Anderson, M.C. Wittmann, 2009 Physics Education Research Conference, Ann Arbor MI, 2009 July.
21. “Addressing student difficulties considering entropy and heat engines,” T.I. Smith, W.M. Christensen, J.R. Thompson, D.B. Mountcastle, 2009 Physics Education Research Conference, Ann Arbor MI, 2009 July.
22. “Conceptual Difficulties with Binomial Distributions in Statistical Physics,” D.B. Mountcastle, J.R. Thompson, T.I. Smith, 2009 Physics Education Research Conference, Ann Arbor MI, 2009 July.
23. “Using Cluster Analysis to Group Student Responses on the FMCE,” R.P. Springuel, M.C. Wittmann, 2009 Physics Education Research Conference, Ann Arbor MI, 2009 July.
24. “Comparing Cluster Analysis and Traditional Analysis Methods in PER,” R.P. Springuel, A. Kaczynski, M.C. Wittmann, J.R. Thompson, 2009 Physics Education Research Conference, Ann Arbor MI, 2009 July.
25. “Exploring Student Consistency in Vector Addition Method Choices.” J. Hawkins, M.C. Wittmann, J.R. Thompson, 2009 Physics Education Research Conference, Ann Arbor MI, 2009 July.
26. “Identifying ’A-ha’ Moments in Group Problem Solving,” K. Hayes, M.C. Wittmann, B.R. Bucy, 2009 Physics Education Research Conference, Ann Arbor MI, 2009 July.
27. “Conceptual Difficulties with Binomial Distributions in Statistical Physics,” D.B. Mountcastle, J.R. Thompson, T.I. Smith, AAPT national meeting, Ann Arbor MI, 2009 July.

28. “Addressing student difficulties considering entropy and heat engines,” T.I. Smith, W.M. Christensen, J.R. Thompson, D.B. Mountcastle, AAPT national meeting, Ann Arbor MI, 2009 July.
29. “Using Cluster Analysis to Group Student Responses on the FMCE,” R.P. Springuel, M.C. Wittmann, AAPT national meeting, Ann Arbor MI, 2009 July.
30. “Comparing Cluster Analysis and Traditional Analysis Methods in PER,” R.P. Springuel, A. Kaczynski, M.C. Wittmann, J.R. Thompson, AAPT national meeting, Ann Arbor MI, 2009 July.
31. “Exploring Student Consistency in Vector Addition Method Choices.” J. Hawkins, M.C. Wittmann, J.R. Thompson, AAPT national meeting, Ann Arbor MI, 2009 July
32. “Identifying ’A-ha’ Moments in Group Problem Solving,” K. Hayes, M.C. Wittmann, B.R. Bucy, AAPT national meeting, Ann Arbor MI, 2009 July.
33. “Exploring Student Consistency in Vector Addition Method Choices.” J. Hawkins, M.C. Wittmann, J.R. Thompson, Foundations and Frontiers of Physics Education Research, 2009 June.
34. “Conceptual Difficulties with Binomial Distributions in Statistical Physics,” D.B. Mountcastle, J.R. Thompson, T.I. Smith, Foundations and Frontiers of Physics Education Research, 2009 June.
35. “Addressing student difficulties considering entropy and heat engines,” T.I. Smith, W.M. Christensen, J.R. Thompson, D.B. Mountcastle, Foundations and Frontiers of Physics Education Research, 2009 June.
36. “Student thinking regarding derivative and slope in multivariable calculus,” W.M. Christensen and J.R. Thompson, Foundations and Frontiers of Physics Education Research, 2009 June.
37. “Graduate student ideas about common student thinking concerning force and motion,” W.M. Christensen, J.R. Thompson, and M.C. Wittmann. 2009 Conference on the Preparation of Physics and Physical Science Teachers (Physics Teacher Education Coalition), Pittsburgh, PA, March 2009.
38. “Student understanding of P-V diagrams and related conceptions about integration,” J.R. Thompson, E.B. Pollock, B.R. Bucy, and D.B. Mountcastle. Science and Mathematics Education Conference (SMEC) 2008, Dublin City University, Dublin, Ireland, 11-12 Sept. 2008.
39. “Addressing student difficulties with aspects of partial differentiation in upper-level thermodynamics,” B.R. Bucy, J.R. Thompson, and D.B. Mountcastle, Integrating Science and Mathematics Education Research into Teaching: Resources and Tools for Improved Learning, University of Maine, Orono, ME, June 2008.
40. “Assessing the evolution of content knowledge and pedagogical content knowledge in a graduate course in physics, pedagogy, and education research,” W.M. Christensen, J.R. Thompson, and M.C. Wittmann, Integrating Science and Mathematics Education Research into Teaching: Resources and Tools for Improved Learning, University of Maine, Orono, ME, June 2008.
41. “The consistency of student answers on the force and motion conceptual evaluation,” G.A. Davenport and J.R. Thompson, Integrating Science and

Mathematics Education Research into Teaching: Resources and Tools for Improved Learning, University of Maine, Orono, ME, June 2008.

42. "Identifying student concepts of gravity," R.E. Feeley and J.R. Thompson, Integrating Science and Mathematics Education Research into Teaching: Resources and Tools for Improved Learning, University of Maine, Orono, ME, June 2008.
43. “The effect of reasoning about vector components on student understanding of two-dimensional acceleration,” B. Nagpure and J.R. Thompson, Integrating Science and Mathematics Education Research into Teaching: Resources and Tools for Improved Learning, University of Maine, Orono, ME, June 2008.
44. “Relating student understanding of thermodynamic work and of integration,” E.B. Pollock, B.R. Bucy, J.R. Thompson, and D.B. Mountcastle, Integrating Science and Mathematics Education Research into Teaching: Resources and Tools for Improved Learning, University of Maine, Orono, ME, June 2008.
45. "The difficulties in turning students into numbers," R.P. Springuel, J.R. Thompson, and M.C. Wittmann, Integrating Science and Mathematics Education Research into Teaching: Resources and Tools for Improved Learning, University of Maine, Orono, ME, June 2008.
46. “Comparing student use of mathematical and physical vector representations,” J. Van Deventer, J.R. Thompson, and M.C. Wittmann, Integrating Science and Mathematics Education Research into Teaching: Resources and Tools for Improved Learning, University of Maine, Orono, ME, June 2008.
47. “The effect of reasoning about vector components on student understanding of two-dimensional acceleration,” B. Nagpure and J.R. Thompson, University of Maine Graduate Student Government Research Exposition, Orono, ME, April 2008. (Poster)
48. “Assessing the evolution of content knowledge and pedagogical content knowledge in a graduate course in physics, pedagogy, and education research,” J.R. Thompson, W.M. Christensen, and M.C. Wittmann, 2008 Conference on the Preparation of Physics and Physical Science Teachers (Physics Teacher Education Coalition), Austin, TX, February-March 2008.
49. “Comparing advanced undergraduate reasoning about entropy across disciplines,” B.R. Bucy, J.R. Thompson, D.B. Mountcastle, Foundations and Frontiers in Physics Education Research 2007 Conference, College of the Atlantic, Bar Harbor, ME, August 2007. (Poster)
50. “Student estimates of probability and uncertainty in advanced laboratory and statistical physics courses,” D.B. Mountcastle, B.R. Bucy, and J.R. Thompson, Foundations and Frontiers in Physics Education Research 2007 Conference, College of the Atlantic, Bar Harbor, ME, August 2007. (Poster)
51. “Comparing student understanding of physics and mathematics in P-V diagrams,” E.B. Pollock, J.R. Thompson, B.R. Bucy, D.B. Mountcastle, Foundations and Frontiers in Physics Education Research 2007 Conference, College of the Atlantic, Bar Harbor, ME, August 2007. (Poster)

52. “Comparing cluster analysis and traditional analysis,” R.P. Springuel, J.R. Thompson, and M.C. Wittmann, Foundations and Frontiers in Physics Education Research 2007 Conference, College of the Atlantic, Bar Harbor, ME, August 2007. (Poster)
53. “PER Lemonade, Maine style, ” J.R. Thompson, Foundations and Frontiers in Physics Education Research 2007 Conference, College of the Atlantic, Bar Harbor, ME, August 2007.
54. “Comparing advanced undergraduate reasoning about entropy across disciplines,” B.R. Bucy, J.R. Thompson, D.B. Mountcastle, 2007 Summer National Meeting of the AAPT, Greensboro, NC, July/August 2007.
55. “Student estimates of probability and uncertainty in advanced laboratory and statistical physics courses,” D.B. Mountcastle, B.R. Bucy, and J.R. Thompson, 2007 Summer National Meeting of the AAPT, Greensboro, NC, July/August 2007.
56. “Comparing student understanding of physics and mathematics in P-V diagrams,” E.B. Pollock, J.R. Thompson, B.R. Bucy, D.B. Mountcastle, 2007 Summer National Meeting of the AAPT, Greensboro, NC, July/August 2007.
57. “Comparing advanced undergraduate reasoning about entropy across disciplines,” B.R. Bucy, J.R. Thompson, D.B. Mountcastle, 2007 Physics Education Research Conference, Greensboro, NC, August 2007.
58. “Student estimates of probability and uncertainty in advanced laboratory and statistical physics courses,” D.B. Mountcastle, B.R. Bucy, and J.R. Thompson, 2007 Physics Education Research Conference, Greensboro, NC, August 2007.
59. “Comparing student understanding of physics and mathematics in P-V diagrams,” E.B. Pollock, J.R. Thompson, B.R. Bucy, D.B. Mountcastle, 2007 Physics Education Research Conference, Greensboro, NC, August 2007.
60. “Mapping student reasoning about math- and physics-oriented differential equation solutions” K.E. Black, M.C. Wittmann, Physics Education Research Conference 2007, Greensboro NC, 2007 August.
61. “Students creating mathematical meaning in mechanics: Signs in scalar equations,” K. McCann, M.C. Wittmann, Physics Education Research Conference 2007, Greensboro NC, 2007 August.
62. “Analyzing the Force and Motion Conceptual Evaluation using Model Analysis,” T.I. Smith, M.C. Wittmann, T. Carter, Physics Education Research Conference 2007, Greensboro NC, 2007 August.
63. “Comparing cluster analysis and traditional analysis methods in PER,” R.P. Springuel, M.C. Wittmann, J.R. Thompson, Physics Education Research Conference 2007, Greensboro NC, 2007 August.
64. “Comparing Student Use of Mathematical and Physical Vector Representations,” J. Van Deventer, M.C. Wittmann, Physics Education Research Conference 2007, Greensboro NC, 2007 August.
65. “Mapping student reasoning about math- and physics-oriented differential equation solutions,” K.E. Black, M.C. Wittmann, Physics Education Research Conference 2007, Greensboro NC, 2007 August.

66. “Comparing Student Use of Mathematical and Physical Vector Representations,” J. Van Deventer, M.C. Wittmann, 135th National AAPT Meeting, Greensboro NC, 2007 July.
67. “Students creating mathematical meaning in mechanics: Signs in scalar equations,” K. McCann, M.C. Wittmann, 135th National AAPT Meeting, Greensboro NC, 2007 July.
68. “Intermediate mechanics students' coordinate system choices for simple pendula,” E.C. Sayre, M.C. Wittmann, 135th National AAPT Meeting, Greensboro NC, 2007 July.
69. “Mapping student reasoning about math- and physics-oriented differential equation solutions,” K.E. Black, M.C. Wittmann, 135th National AAPT Meeting, Greensboro NC, 2007 July.
70. “Effect of Instructional Method Changes on an Introductory Physics Class at a Two-Year College,” T. Carter, T.I. Smith, M.C. Wittmann, 135th National AAPT Meeting, Greensboro NC, 2007 July.
71. “Analyzing the Force and Motion Conceptual Evaluation using Model Analysis,” T.I. Smith, M.C. Wittmann, T. Carter, 135th National AAPT Meeting, Greensboro NC, 2007 July.
72. “Comparing student understanding of physics and mathematics in P-V diagrams,” E.B. Pollock, J.R. Thompson, B.R. Bucy, D.B. Mountcastle, University of Maine Graduate Student Government Research Exposition, Orono, ME, 2007 April.
73. “Comparing student understanding of physics and mathematics in P-V diagrams,” E.B. Pollock, J.R. Thompson, B.R. Bucy, D.B. Mountcastle, Joint Meeting of the New England Sections of the AAPT and APS (regional), University of Maine, Orono, ME, 2007 April.
74. “Student Estimates of Probability and Uncertainty in Statistical Physics,” D.B. Mountcastle, B.R. Bucy, and John R. Thompson, 134th AAPT National Meeting, Seattle WA, 2007 January.

IV. Contributions

A. Within Discipline

The principal disciplinary field is physics education research, in which theoretical modeling is only slowly taking hold. Developing and extending resource theory is of great importance when building a foundation on which to ask better experimental questions and understand experimental data better. We highlight several elements of our work.

First, there is the methodological issue of measuring the plasticity of resources. By bringing in ideas from RBC theory (recognize, build-with, construct), we are able to observe how students make sense of ideas in real time – and how their sensemaking changes on both long and short time scales. Researchers can use our tools to better understand when a resource is in play and what resources are being used in a given setting. Resource plasticity can more easily be observed and understood, allowing us to understand the in-between stages as students move from not knowing a topic to being at least locally expert in their understanding.

Second, on the topic of resource coordination and activation, there is a methodological issue of cluster analysis. This has only rarely been used in PER, and has great value. By evaluating student responses (on free response questions) based on what students do, rather than what they are interpreted to be doing, we can group responses and find common themes that are at times surprising and unexpected. Items that (on a macroscopic analysis, typical of PER) seem closely related are in reality much less related than thought. Methodologically, this tool allows us to find emergent connections in the data, rather than seeking for existing categories. Such work is helpful in modeling resource coordination (which ideas are used at the same time) and activation (which questions elicit which kinds of ideas).

Third, we have greatly extended the range of studies carried out on the use of mathematics in advanced physics classes. This has included the methodological work of asking isomorphic questions in physics and non-physics forms. By comparing student responses on each, we are able to study the context-dependence of questions, allowing better recognition of which resources are being activated where.

Fourth, we have extended our application of the resources framework to include the analysis of standardized tests which were not originally designed with this cognitive model in mind (and were, as a matter of fact, designed with a completely different and partially contradictory model of learning). Our example is the Force and Motion Conceptual Evaluation. We find that our analysis accounts for experimental data while also making claims about previously unpublished false positives on the test. Having predicted the false positive, we find evidence for it in our data. Our results call into question some assumptions previously made about this standardized test. We note that subsequent results from a cluster analysis of student data on the FMCE are closely aligned with the resources-based analysis of the test, but suggest that there is greater coherence in student responses than is assumed when splitting the test into several different groups of questions.

Fifth, we have applied the resources framework to areas in which it was not previously discussed. So, for example, we have used resources to analyze the Force and Motion Conceptual Evaluation, showing that question context is consistent with a resources interpretation of the most common student responses; this work modified the original grouping of questions, as given by the survey authors, and found a false positive result that was previously undocumented. Similarly, we have looked at the first application of epistemic games to physics have deepened our understanding of epistemological framing by giving a more detailed analysis of epistemic games within the resources framework.

Finally, we have applied the resources framework to epistemological issues, looking at the persistence of students' activated epistemological resources in the context of a conceptually oriented laboratory activity. The persistence of a given activation (be it 'knowledge as invented stuff' or 'knowledge from authority') through a series of activities designed to promote the former and dissuade students from the latter suggests that some laboratory activities are not as effective at promoting effective learning behaviors as was originally assumed.

These contributions are all part of masters theses or doctoral dissertations, and we are in the process of sharing those results which have not already been published.

B. To Other Disciplines

As we study the role of mathematics in physics, we have found many different areas in which the physics affects our use of mathematics and vice versa. So, for example, we have studied student responses on isomorphic math and physics vector tasks, looked at the use of coordinate systems in physics, and considered how physics reasoning affects mathematical modeling as vectors are translated into scalar quantities within a coordinate system. These contributions are of interest and importance to mathematics education researchers.

Our cluster analysis software and analysis is of interest to those working in the learning sciences who are doing similar work on bringing cluster analysis to bear on understanding student reasoning about the physical world.

C. Contributions to Human Resource Development

As stated earlier, there are many more graduate students involved in this project than originally planned. Three students have completed their PhD work while supported in part by this project. One spent time in a post doctoral position at the Ohio State University and has recently been offered a position as a tenure track faculty position at Kansas State University. Another has moved on to post doctoral work as part of St Anselm monastery and plans to teach at the college level at a Benedictine school. The third is a post doc at the University of Maine as part of a new NSF funded project.

The additional students are all Masters students, either for a Master of Science in physics or in the Master of Science in Teaching (MST) program at the University of Maine. The MST students have moved on to a variety of careers: some are teachers in the physical sciences at the high school level or at the university level. Others have continued in graduate school with a focus on educational psychology, physics education research, or other STEM-related area. Some have joined industry, primarily in the field of education studies and evaluation.

In year 1, students on this project have received 1 Ph.D. and 2 Master of Science in Teaching (M.S.T.) degrees. The Ph.D. student was hired as a post doc at the Ohio State University (and has since been offered a tenure track faculty position after spending several years as a visiting faculty member at Wabash College). One M.S.T. student joined us as a Ph.D. student. The other is now teaching.

In year 2, we had 1 recently graduated student from another UMaine project join us as a post doc, 1 student receive an M.S. degree, and 3 students receive their M.S.T. degree. At the end of year 2, the post doc was hired, partially based on experience in this grant, to be a visiting faculty at Randolph Macon Academy in Richmond, VA. The M.S. student has moved as a Ph.D. student to another research area. One M.S.T. student is now teaching, one is moving into educational consulting and analysis, and the third moved on to a Ph.D. program in educational psychology.

In year 3. we have graduated 1 M.S.T. student who is doing consulting work.

In year 4, we graduated 1 M.S.T. student and 2 Ph.D. students, whose career paths were described above. We also hired a new post doc, Brian Frank, who has since been hired to be part of new NSF funded work at UMaine.

In year 1, the PI was promoted to associate professor with tenure, with the strength of this project being a major sign of success for the PI. Co-PI Donovan left the project at the end of year 1. In year 2, one co-PI, John Thompson, was also promoted to associate professor. PI Wittmann spent year 2 abroad on sabbatical. In year 3, co-PI Thompson spent the year abroad on sabbatical as part of a Fulbright Fellowship.

D. Contributions to Resources for Research and Education

We have developed several tools of use to researchers in physics education. Especially in the analysis of standardized test data for commonly used tests in PER, like the Force and Motion Conceptual Evaluation, our use of model analysis, and cluster analysis provide a toolbox for researchers.

To carry out model analysis, one should focus on questions that are all on the same topic. Defining these groups of questions is of great importance when trying to understand consistency of reasoning, for example. The groups defined by Trevor Smith in his work allow us to understand in finer detail what kind of learning is happening in our physics classes. He has revised analysis tools created by the project PI (while at the University of Maryland as a post doc) and has published these electronically.

The cluster analysis methods learned by Padraic Springuel have been coded using common programming languages. These have been shared as part of a publication under review, letting others quickly adopt the tool for their own research. They have also been published (with full documentation) as required by the open-source software license under which they were developed.

To help analyze interview or classroom data as students struggle with new ideas, we have developed coding schemes that allow us to analyze the plasticity of resources as they are developing in students' minds.

To help analyze classroom video, we have provided detailed analysis of students' interactions with each other, with the space around them, and with sources of authority as a way of analyzing their epistemological framing of a situation. In the process, we have come up with methods to convert observations of behavior into descriptions of resource activation.

E. Contributions Beyond Science and Engineering

See those listed under Outreach Activities, specifically dealing with future teachers.

B. Major findings

One major point is that the issues of creation, coordination, and activation cannot easily be separated from each other. Thus, while the topics studied are nominally separate, each project underway contains some overlap of results. This has been touched on in the previous section.

As best possible, we put results on the use of mathematics in advanced physics courses into the discussion of resource creation, coordination, and activation. We also have a separate section on our findings in this area, where the work is better described on its own.

1. Resource plasticity and coordination

Since resources act as "chunks" in student reasoning, it is important to understand how solid these chunks are - when activated, how large is the activated structure? We have constructed a new measure, called plasticity, to help describe how ideas that are weakly linked and must be constructed for every use eventually become more solid, meaning well linked and strongly compiled. This work used theories from math education, including Process/Object theory and RBC (Recognize, Build-With, Construct) to help develop appropriate measures for observations of resource plasticity.

Because resources may contain within their structure other resources, one can study how a resource such as coordinate system comes to be by looking at the mix of procedural, epistemological, and reasoning resources that comprise it. For example, we can use a variety of tasks which allow for more than one coordinate system to be used to observe the interplay of resources which occur to create a larger resource. In another example, we can observe students' increasingly compiled use of resources when manipulating equations algebraically. Students shift from explanations with many connected parts to shorter explanations in which the several parts of been compiled or reified into a new resource.

Evidence for resource creation came from students' shifts in reasoning across several modalities. As students develop a more compiled resource for dealing with some set of procedural steps while solving a math problem in physics, their language, math formalism, and gestures all change in concert with each other. They start with many formal mathematical steps and use gestures to help isolate terms in the equation, indicating a formal, analytical description of the algebra. They move to more informal mathematical language and use different gestures to describe moving terms about, indicating an informal, embodied description.

To analyze these results, we have reached toward the theory of conceptual blending. In particular, we have looked at how the theory suggests mechanisms which not only describe but also explain why certain kinds of reification happen. These results have been published in the proceedings of the International Conference on the Learning Sciences, and further publications are under preparation or submitted for review. Examples are shown in Figures 5 and 6. In Figure 5, we explain the work done by students as a seemingly simple mathematical step (of grouping objects separated by a minus sign) is observed to be quite difficult for students, often requiring the use of a grouping gesture as one moves to treating the sign-separated terms as a single term. In Figure 6, we show how students' gestures are indicative of moves of pieces on a gameboard - but constrained by the mathematical rules of the situation. Notably, the circling of one term in the Gameboard Algebra blend is consistent with the Reified Math Object blend. Black has described examples in which this connection does not occur, showing that we are observing distinct procedures.

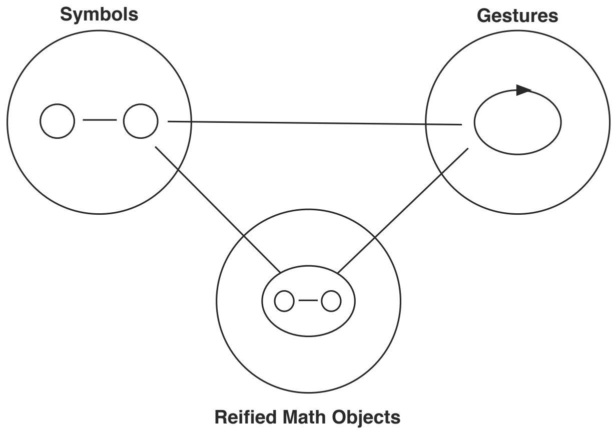


Figure 5: A simple blend in which a gesture is used to group independent mathematical terms into a single, refied math object. The *gathering* gesture indicates students grouping terms into one; the new math object can be used as a single object, even though it contains multiple pieces.

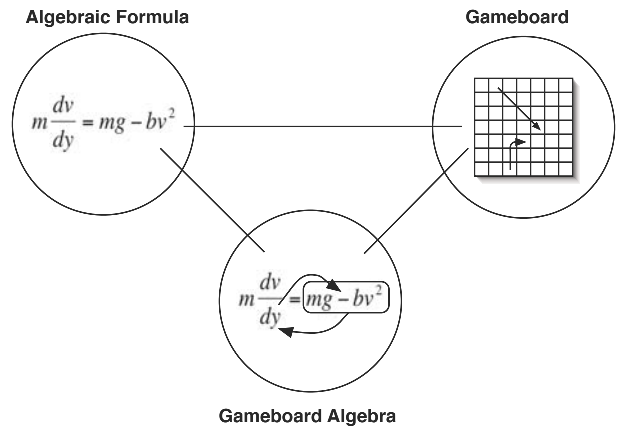


Figure 6: A blend indicating connections between mathematical manipulation to separate variables and moves on a gameboard. Data to support the blends in this and the previous figure come from multiple classroom observations of students working on a group quiz.

Black has extended this analysis to include not only blends, but symbolic forms, epistemological resources, and more. A resource graph of the slowly-created procedural resource of "separating variables" is shown in figure 7. It only includes procedures, but not epistemological, symbolic, or conceptual resources that are also involved in this mathematical activity.

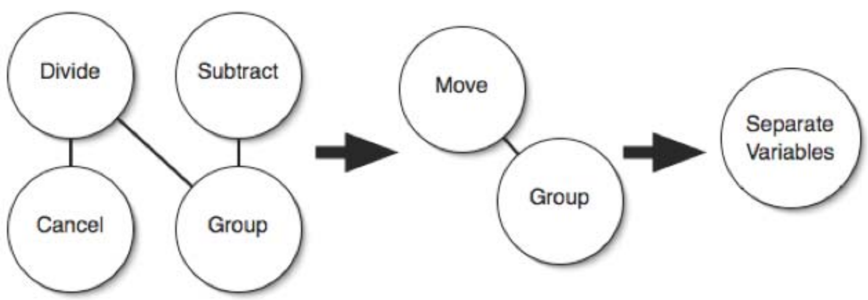


Figure 7: A possible conceptual pathway for the reification of the "Separate Variables" procedural resource.

2. Resource activation

We have studied the process of resource activation in several different physical and mathematical contexts. Primarily, this work has been done in introductory physics classes (looking at the use of vectors), sophomore level mechanics classes (where equations require a closer connection between physical and mathematical reasoning, and differential equations first become common in physics learning), and senior level thermodynamics and statistical mechanics courses. Results have been published extensively in peer-reviewed conference proceedings, and additional papers are under preparation or submitted and under review.

In the context of writing physics equations, we find that students have a hard time using minus signs appropriately. On the one hand, they will use physically standard formats (F=ma) to determine an equation. On the other hand, given that they have some arbitrarily chosen coordinate system, they will then force minus signs into the problem to ensure that the outcome is as they desire (writing, for example, F= –ma, incorrectly). Depending on the choice of their solution, different and contradictory ideas are used. A resources model easily describes students' activation of different ideas in different settings.

Similar inconsistency was found in the context of students' use of integration methods when solving separable differential equations. There are two methods, the use of indefinite constants commonly taught in mathematics classes, and the use of integration limits, commonly used in physics. Depending on cues, students use the more mathematical or the more physics-oriented solution method, where one typically leads to incomplete solutions and the other leads to complete physical descriptions of the situation. To account for the differences in student performance, we have developed the idea of procedural resources and talked about their context-dependent activation. This work has connected us to Collins and Ferguson's epistemic games, as well as to ideas about epistemological framing. (Notably, questions about epistemological framing and the effects of activating certain epistemological resources have been explored further by Hayes and Murphy in both non-science and mechanics courses.) In addition, we have developed the idea of consistency plots, in which we can map students' responses to identical questions at different times of the semester. Finally, we have addressed questions about resource coordination by using conceptual blending to explain how resources come to be coordinated. Papers on each of these areas, procedural resources, epistemic games and epistemological framing, the use of consistency plots, and explanations via conceptual blending, are submitted and under review or in preparation. Preliminary results were published in peer-reviewed conference proceedings.

In another area of our work, we have looked at how students make sense of vectors. These representations (arrows showing direction and magnitude, but with an arbitrary location, typically defined by the physical situation) are commonly used in physics but rarely used in math classes in the same way. Students learn about them first in our physics courses, yet apply many commonly used mathematical tools (such as addition and subtraction) to these new constructs. In a series of studies carried out primarily by Van Deventer and Hawkins, we have found that students answer questions differently if given identical questions with a physics context and a non-physics (more math-like) context. This context dependence is appropriately analyzed in terms of resource activation in the different question formats. We have followed up on this work by investigating how representations affect student reasoning. So, for example, giving a tail-to-tail orientation of vectors looks much like a typical problem using free body diagrams, and students are perhaps more likely to add vectors correctly while thinking about forces and not just plain vectors. Similarly, a head-to-tail orientation of vectors is more common for displacement type problems, where the angle between the vectors plays a role in how students give their answers. We have found procedural cues (that one or another orientation suggests the first step of a solution), physical cues (that one or another orientation suggests a kind of physics thinking that is helpful), and visual cues (that one or another representation activates different kinds of solutions), and are actively pursuing the question of which cues dominate when asking different kinds of vector addition problems. Since there is a complicated interaction between resource activation and coordination, the answer is far more difficult than first expected. A further surprising result, found by Hawkins, has been that students typically pick a vector addition method and then stick to it, even as questions change and cues change. The issue of resource activation and the persistence of the activation require further study in this context.

In other work on vectors, we have also looked at how a vector description of accelerating motion in two dimensions depends on the choice of representation and the physical situation being described. We asked questions in which students were to draw acceleration and velocity vectors describing different paths of motion for travel along open and closed shapes, symmetric and asymmetric shapes, and with constantly increasing or variable motion. Nagpure did a primarily qualitative analysis of this work, looking at the use of vector components compared to the full vectors.

Springuel took this work much further, using cluster analysis to evaluate which questions activated which commonly given answer. It turns out that direction of travel and the path along which one travels activate different reasoning about the velocity and acceleration vectors one should draw in a given situation - even when the shapes are nearly identical. The use of cluster analysis (described in more detail below) has helped us find groupings of students whom we otherwise would have missed, and allowed us to do so without making *a priori* assumptions about the kind of reasoning we expected to see. Early results of this work have been published in peer-reviewed journals, but the results on question-specific activation have not yet been published. The use of cluster analysis, meaning its application to physics education research and the heuristics for making it a useful tool in PER, is among the most important results of this grant. An example of his cluster dendogram was given in the previous section, on research activities.

3. Resource linking

Issues of resource coordination have already been given in the examples of the previous 2 sections. Further examples are given here that have not yet been discussed.

The work on analyzing standardized tests arose out of a desire to answer questions which are not commonly discussed in the literature. For example, the Force and Motion Conceptual Evaluation (FMCE) is either analyzed as a whole test (looking at scores before and after instruction, then calculating gains or normalized gains), or with subgroups of questions being scored (such as questions about kinematics, reversing directions, or Newton's Third Law). The two rarely intersect, with a full-test analysis of all subgroups of questions. One could use a pivot table to explore this (given answers of X on the 3rd Law questions, how does one answer reversing directions questions), but we chose another route. Wittmann and Smith carried out a resource-based analysis of the FMCE, revising a previously designed analysis tool in the process, and used this fine-grain analysis to uncover results that the FMCE authors had not previously discovered. In particular, a false positive was found in an unexpected situation.

Further work on the FMCE made use of cluster analysis to find common student responses. Heuristics had to be developed to determine the relevance of groups, how common a given response had to be within a group for it to be defined as a group, and how much noise one was willing to accept within the cluster analysis groupings. Once groups of common results were found, these were interpreted not in terms of previously determined question groups, but were analyzed across questions, looking for larger-grain common responses. This work by Springuel is being prepared for publication.

A similar analysis, crossing questions groups, was carried out by Wittmann and Anderson when looking at students' thinking after instruction on Newton's Second Law. As with Springuel's work, her analysis looked at questions across contexts, and found that certain questions, outside of the commonly accepted groups (including those defined by Smith and Wittmann) show a kind of coordination of resources which is often lost in a more traditional analysis of student data. The results of Springuel, Smith, and Anderson suggest that the FMCE is a far more complicated test to understand than has been assume, and that one's choice of analysis affects the resources one is likely to observe students using. These results are consistent with the idea of resources being scalable structures (akin to schemas possibly nested in other schemas, or scripts which include other scripts).

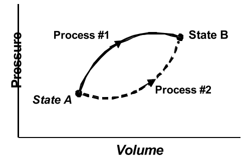
The idea of resource coordination was explored further by Black and Wittmann in the context of algebraic manipulation of separable differential equations, helping to explain how networks of resources are pulled together and how new resources emerge. Work in this area required an analysis of the semiotic function of gestures, in particular the way that a circling gesture was used to group mathematical terms before a dragging gesture was used to indicate division across the equals sign. Using conceptual blending, we analyzed students' thinking to show how new ideas emerge in the context of problem solving. Examples and figures are given in Figures 5–7. This very promising work will be explored further in the future.

4. The use of mathematics in advanced physics

While studying resource creation, coordination, and activation, we have also looked in great deal at areas of mathematics use in physics where the individual resources being used are not elucidated enough for us to discuss issues of resource coordination and activation. Instead, we have focused more generally on activation and cuing of ideas in problem solving.

In the area of integration, we have investigated students' understanding of path integration, their use of anti-derivatives when solving integrals, and whether they think of integration in terms of Riemann sums or not. We have investigated the mathematical underpinnings of student responses to questions comparing ("thermodynamic") work done by identical ideal gas samples that start at the same state and end at the same state, but have different thermodynamic processes, shown as different paths on a pressure-volume (P-V) diagram (see Figure 5). Students were asked several questions regarding first law quantities along with similar mathematical questions devoid of all physical context. We compared student responses to physics questions involving interpretation of ideal gas processes on P-V diagrams and analogous mathematical qualitative questions about the signs and comparisons of magnitudes of various integrals. Overall results coupled with individual student performance on the paired questions shows evidence of isolated understanding of the physics and/or the math. This context-dependent response is consist with our other work in vector problems, for example.

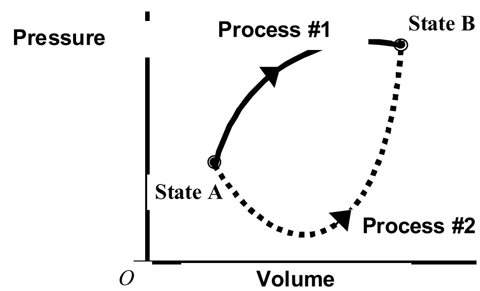
Analysis of students' difficulties shows that some students use the symmetry of the paths on the P-V diagram to justify their response (that the works/integrals were equal). To reduce the visual distraction of the symmetrical paths, we have modified the graphs to be asymmetric, that is, the "lower" path (one with less area under its curve) is now longer than the "upper" curve (Figure 8). This allows for student responses to be distinct between area-based and length-based reasoning. This research and analysis is ongoing, with Bajracharya and co-PI Thompson building on work by Evan Pollock (M.S. 2008) and Brandon Bucy (Ph.D. 2007).



[In these questions, ***W*** represents the work done ***by*** the system during a process; ***Q*** represents the heat transfer ***to*** the system during a process.]

P1. Is *W* for Process #1 ***greater than, less than,*** or ***equal to*** that for Process #2? Explain.

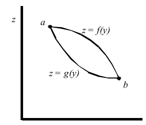
P2. Which would produce the largest change in total energy (kinetic plus potential) of all the atoms in the system: ***Process #1, Process #2***, or ***both produce the same change***?



Two functions have been graphed on the *z-y* graph shown at right, and are labeled *f(y)* and *g(y)*. Both functions start at point *a* and end at point *b*.

Consider the integrals

$$I_1 \equiv \int_a^b f(y)\,dy \qquad I_2 \equiv \int_a^b g(y)dy$$



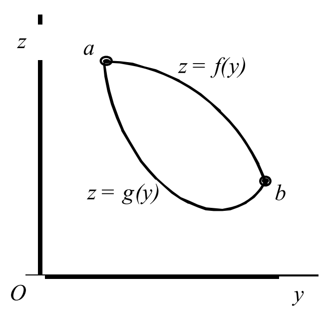


Figure 8: Integration questions looking at students' activation of resources in physics (top) and physics-less (bottom) versions of very similar questions.

In the area of differential equations, including multivariable functions, partial derivatives, and mixed second-order partial derivatives, we have studied the different ways in which mathematics and physics notation is used, how it affects student problem solving, and what sense students make of the various shorthands in use. This work has taken place primarily in the context of studying the thermodynamic concept of state function. A state function is a function whose integral is independent of path (or, in this particular context, thermodynamic process). Textbooks provide several examples and sometimes even a mathematical appendix designed to teach students the distinction between exact and inexact differentials. In spite of explicit time and effort in the classroom, students often apply state function reasoning to inexact differentials as well as exact ones, and fail to notice the distinction made by the textbook authors.

To study whether students' difficulties are related to the physics or to the mathematics, we developed a six-question math diagnostic quiz, which we administered to 5 sections of

UMaine's undergraduate Calculus III course, taught by the mathematics department. The diagnostic quiz contains three questions asked in our thermal physics courses, including the integral questions and partial derivative questions described above, as well as other questions dealing with the complementary concept of differentiation. Importantly, the questions were asked in a completely mathematical context, without any reference to physical situations. Drafts were provided to the math faculty involved, and none of these instructors identified any of the questions as being inappropriate for their students (one question required minor revision to a mathematical expression to make the terminology consistent with that used in the course). Survey results were gathered from over 180 students. Many of the data gathered overall are remarkably consistent with that observed in our thermal physics courses. On the Calculus I integral question, about 55% of students correctly determined that I_1 was greater than I_2 (see Figure 8). 27% of students stated that the integrals would be identical, using some form of path-independent reasoning. On a loop integral question, less than 30% identified the loop integral of the quantity *dH* (made up of the loop from a to be and back again, first along the upper, then along the lower path) as being equal to zero. A unique response from calculus students (about 20% gave this response) was "negative," according to the reasoning that "the path is clockwise." This is a convention used in mathematics to evaluate path integrals, not regular loop integrals. One more interesting finding is that for the same loop, we asked students, in separate questions, to decide the sign (positive, negative, zero or not enough information) of both integrals of *zdy* and of *dH*. Almost 45% of the students gave identical responses for both questions (i.e., said that both integrals were zero, or both were positive). More than half did not. This suggests that students' difficulties in physics may arise from unfamiliarity with certain issues in integration, or are mathematics and not physics difficulties.

II. Activities and Findings

This project had two major strands which were interwoven throughout. On the one hand, there was theory-building work in understanding the resources framework and the creation, coordination, and activation of resources. On the other hand, the area of research was primarily in advanced physics topics (waves, quantum physics, mechanics, thermodynamics, statistical mechanics) in which mathematical reasoning plays a core role in one's conceptual understanding. Thus, the research activities and findings described here touch on both these areas. An additional strand of activity developed over time, namely the study of interactions and the role of communication within group learning activities. Especially in those areas where work involved video-based data gathering techniques, data came from the analysis of social interactions. Thus, methods of interaction analysis and discourse analysis became more important in years 3 and 4 of the project.

A. Research and Education Activities

Resources are basic building blocks of our thinking and have been shown to be effective as elements of a model of reasoning in physics. We wish to understand how reasoning resources in physics come to be, how they are linked to each other and coordinate to build larger ideas, and how one set of ideas gets chosen over another set of ideas in a given context.

- Resource creation: Looking at how students build new ideas into usable "chunks" (which we call resources), allowing for more concise and higher speed reasoning about physics and math.
- Resource activation: Understanding how resources get activated in a given context, particularly in situations where math and physics ideas must come together for a full understanding, or where representations seem to affect student reasoning.
- Resource coordination: Studying how individual resources are used in conjunction with each other to develop more advanced ideas.

A major point is to understand the way in which physics and math ideas merge to create a conceptually and mathematically coherent and physically rich models of the world around us. Relevant issues include the observing the creation of new concepts, understanding the methodological issues of finding connections between resources, and describing ways in which representations and contexts affect the activation and coordination of resources, some of which are still weakly built and only little understood by their users.

1. Data gathered and methods of analysis

Data were gathered from a variety of settings:

1. Weekly group interviews with students over a whole semester ("group mini-views")
2. Videotaped homework help sessions
3. Individual student interviews
4. Classroom video observations, either of group learning activities or of group quizzes
5. Surveys, ungraded free response quizzes, and other written work such as exams and homework problems

Results were analyzed using common tools common to each form of data. Video data were transcribed, annotated, and analyzed where appropriate. Annotations include information connected to gesture, discourse, and interaction. Our analysis methods built off or are consistent with the discussion of interaction analysis given by Jordan and Henderson (1995) and the discussion of video analysis given in Derry et al. (2010) in the *Journal of the Learning Sciences*.

Survey and free response data were analyzed using a variety of methods (including content, textual, cluster, and model analysis). A major result of our work was to analyze the Force and Motion Conceptual evaluation in terms of the resources framework and then build analysis tools to help others use our analysis. We have then used this analysis to define the mental models that can be used when carrying out Bao's method of model analysis.

We developed a specific kind of survey in which students answered isomorphic physics and "physics-less" questions on vectors. These questions had the same graphic, but different descriptions, so that in some cases one merely added vectors, while in others one had to find, for example, the forces acting on an object, or, in another example, the change in velocity for an object traveling on a curved path. In comparing results on these tests, we extended Kanim's idea of "escalator diagrams" (Figure 1) in which the shift in students' responses before and after instruction are represented graphically. By including information about incorrect responses, as well, we are able to compare resource activation in different contexts more easily (Figure 2).

The mindset behind this analysis was continued in two different projects. In the one, Black looked at how students answered identical questions in the middle of and at the end of a semester. She added a second dimension to Van Deventer's plot (Figure 2) to create a consistency plot (Figure 3). This plot shows the remarkable fluidity of students' methods for answering an integration problem in a sophomore level mechanics class. The details of this plot (including issues of *circulation*, *attraction*, and *starbursts*) are described in more

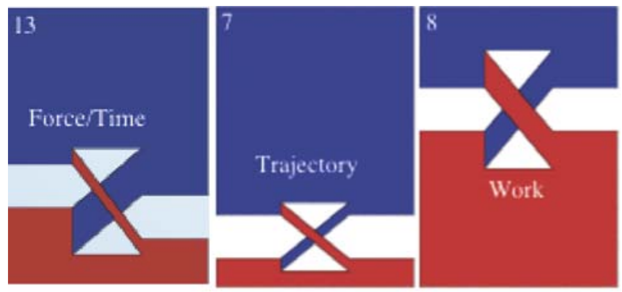


Figure 1: Kanim's escalator diagram, showing different movement of students between pre- and post-instruction testing. The vertical axis indicates the number of correct (blue) and incorrect (red) respones. In the Force/Time diagram, more students go from incorrect to correct than go from correct to incorrect. In the Trajectory diagram, equal numbers change. In the Work diagram, a different equal number change.

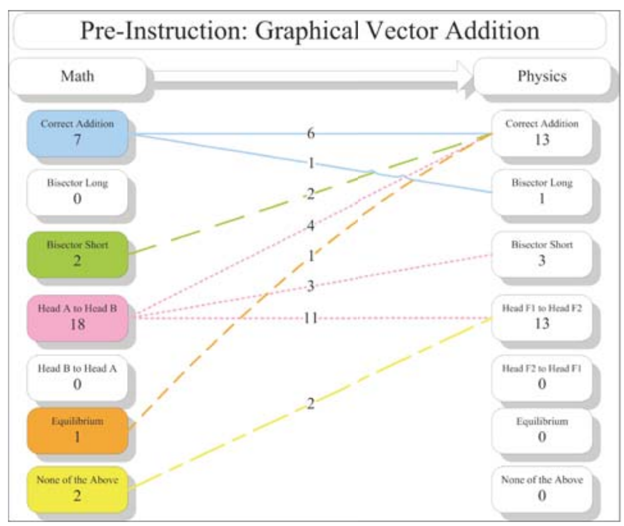


Figure 2: Van Deventer's extension of Kanim's escalator diagram includes information not only about the correct answer on a given question, but also the kinds of incorrect answers that students were giving. Different answers are indicative of different kinds of resource activation.

detail in a paper that has been submitted for publication and is being revised after reviewer comments.

A second approach to Van Deventer's work came when Springuel sought ways to "assume less" about students' responses to questions, and allow group sorting methods find those common responses which required further analysis. We looked to cluster analysis, rather than factor analysis, because the method of agglomerative hierarchical cluster analysis more generally pulls out common themes in student responses without making assumptions about the kinds of differences we are going to find. Our application of cluster analysis is essentially new to the field of physics education research. Other researchers at Ohio State University and Northwestern have done some work, but not in as much detail as we have carried out. Our goal in applying cluster analysis to PER data was to avoid using *a priori* assumptions about how students are answering questions and find patterns of responses that both gave evidence of resource coordination and context-dependent activation of resources. Springuel's PhD dissertation is being prepared for publication. Three articles are planned. The first is on the details of cluster analysis and rigorous definitions of the data that one analyzes - previous researchers have defined similarity of data inappropriately, leading to an incorrect analysis of results. The second describes the application of cluster analysis to physics education research data, including the heuristics one uses to manage issues of noise, consistency of results, and pedagogical meaning when creating cluster dendograms (Figure 4). These first two papers serve as primers on the application of the method to PER. The final paper will include examples from an analysis of data from the Force and Motion Conceptual Evaluation.

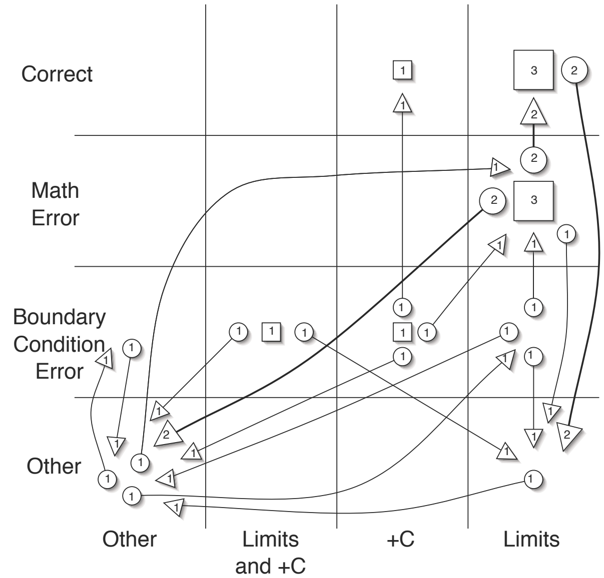


Figure 3: Black's consistency plot. Student mid-term responses are given by circles, final exam responses by triangles. Students who stay in the same location on the plot are indicated by a square.

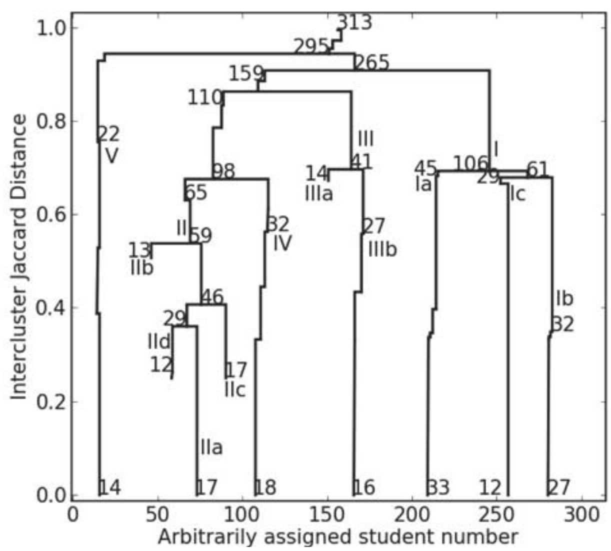


Figure 4: Springuel's application of cluster analysis to vector questions describing motion in 2 dimensions. Numbers indicate the size of a given group, while Roman numerals indicate meaningful groups of students.

2. Specific Projects by Graduate Students and Post Docs

Due to the richness of work carried out by the graduate students and post docs involved in this project, we summarize each of their individual projects in the space below. Many build off each other, which will be noted in the summaries. Later, we connect these projects into larger themes of work. Projects are listed alphabetically by student. Students who graduated with a Ph.D., Master of Science in Teaching (M.S.T.) or Master of Science (M.S. in physics) are noted. Those who worked on grant related topics while receiving funding from other sources (primarily M.S.T. students supported through teaching assistantships related to their plan of study) are also noted.

1. M.K. Anderson: Comparing three methods for teaching Newton's Second Law. Investigating the effectiveness of three separate small-group teaching curricula, each of which introduces Newton's Second Law in slightly different forms, using tools developed by T.I. Smith (#13). Results were published in peer-reviewed conference proceedings. M.S.T. received in 2009. (Not funded by this grant, but mentored by PI Michael C. Wittmann on grant-related work.)
2. R. Bajracharya: Investigating cuing in understanding mathematics and physics versions of a typical integral problem. The research questions are a continuation of work done by Pollock (#10). In studying how students carry out integrals, the work also builds on Black's results (#3). Questions of resource activation are looked at in terms of shapes of integrals, the interaction between value, slope, and area in students' reasoning, and the question of how mathematical notation is applied and modified in a physics classroom. (Not funded by this grant, but mentored by co-PI John R. Thompson on grant-related work.)
3. K.E. Black: This multi-faceted work formed a core element of the project. Work took place in the context of studying students' choices of integration methods when solving separable differential equations. In terms of the resources framework, we first extended the definition of resources to include procedural resources (scripts) that are carried out while solving problems. We discussed the creation of resources through the reification of laboriously carried out scripts into tightly compiled actions. In the process, we connected the work to epistemic games and issues of epistemological framing. The different activation of resources was represented through new methods, including a "consistency plot," which represents the shift of answers to identical questions after a period of time. Finally, resource coordination was modeled through a process of conceptual blending of gestures and discourse in the context of carrying out mathematical manipulation of equations. Papers on many of these topics have been published in peer-reviewed conference proceedings, and papers for journals are either under review, being revised, or being prepared. Ph.D. received in 2010.
4. J. Hawkins: Understanding student reasoning about two dimensional vector addition. Building off of work by J. Van Deventer (described below, #15), a study to investigate how minor changes in visual representation can affect student responses to simple graphical 2-d vector addition questions. Results show that students are cued to give certain answers based on procedural, visual, or conceptual cues, and that they persist in the solution method with which they began their work when answering a series of vector

addition questions. Results have been published in peer-reviewed conference proceedings.

5. K. Hayes (formerly McCann): Understanding the use of signs in differential equations. Using both individual student interviews and classroom video during small group "tutorial" exercises, we can observe students creating the appropriate differential equations to mathematically model physical situations. Using discourse analysis, we can observe linguistic clues which alert us to violations of expectations in how they frame the activities they carry out. In particular, we find that students are inconsistent, using both mathematical reasoning and physical reasoning to arrive at contradictory results. Results have been published in a peer-reviewed journal. M.S.T. received in 2009. (Not funded by this grant, but mentored by PI Michael C. Wittmann on grant-related work.)
6. A. Kaczynski: Analyzing conceptual learning in small-group situations. Since most classroom data involves group interactions, often with no facilitator present, we are curious as to who "owns" the resources being discussed at the table. Building on work by K. Hayes (#5), we can investigate how groups come to build an idea, and individuals come to make it their own. This work is taking place in the same course studied by K.E. Black (#2) and E.C. Sayre (#12), and builds off their results in analyzing resources, this time in the context of simple and damped harmonic motion.
7. Z.S. McIntyre: Analyzing student misconceptions about variables in different mathematics settings. We developed a survey which allowed us to pre- and post-test students' understanding of variables in algebraic equations. This work is related to K. Hayes's results (#5) and also unpublished work by K.E. Black (#2) on the different ways that letters are used in mathematical sentences (constants, variables, functions, parameters, place-holders, etc.). M.S.T. completed 2007.
8. C. Murphy: Interaction analysis of students' use of epistemological resources and the ways they frame a conceptual laboratory activity on light and shadow that has been modified to promote epistemological thinking. Her results show that students enter an epistemological mode and persist in it across a series of activities; one case study describes an idea-constructing group while another describes an answer-seeking group. M.S.T. completed 2010.
9. B. Nagpure: Studying student learning when using two different ways of thinking about vector equations in 2-d kinematics situations involving acceleration both with changes in speed and changes in direction. M.S.T. completed 2008. (Not funded by this grant, but mentored by co-PI John R. Thompson on grant-related work.)
10. E.B. Pollock: Student use of mathematics in a thermodynamics context, specifically in the context of partial differential equations. This project was the first of the grant-related work in upper-division thermodynamics and statistical mechanics, an area of research on the interplay between mathematics and physics reasoning that became increasingly important as the grant progressed. Results were published in peer-reviewed conference proceedings. M.S. completed 2008. (Not funded by this grant, but mentored by co-PI John R. Thompson on grant-related work.)
11. D. Reed: Comparing student knowledge of mathematics and physics in an engineering technology class using a series of standardized tests, examination questions, and

interview. M.S.T. completed 2007. (Not funded by this grant, but mentored by PI Michael C. Wittmann on grant-related work.)

12. E.C. Sayre: Studying resource plasticity in the context of learning about coordinate systems. Resource creation is defined in terms of the plasticity (or solidity) of connections between different resources that students use when solving problems. Coordination of resources is a primary activity in learning. Her theoretical work brought together ideas from physics education research, mathematics education research, and cognitive science. Results are described in more detail below. Publication of this work comes in peer-reviewed conference proceedings, refereed journals, and is still ongoing. Ph.D. completed in 2007.
13. T.I. Smith: Using model analysis to understand changes in student learning in reform physics courses. Major work was done on understanding the resources and facets of reasoning used by students as they answer questions on a commonly used standardized test, the Force and Motion Conceptual Evaluation. We have continued to publish these results in peer-reviewed journals, with 1 manuscript under revision. We have also published a modified analysis tool to make it more consistent with the theoretical model developed during thesis work. M.S.T. completed in 2007.
14. R.P. Springuel: Using cluster analysis to uncover hidden patterns in student responses. This analysis looked at free response (including graphical) questions about 2-dimensional vector kinematics and survey responses to the Force and Motion Conceptual Evaluation (FMCE). On the vector questions, we coded free response (graphical and verbal) data using basic descriptors (about arrow direction, for example), building a table of hundreds of descriptions of a single student's response from which we built a vector of a student response. On the FMCE questions, we built student answer vectors from their responses, regardless of the correctness. Using cluster analysis, we clustered common responses and look for characteristic responses within these clusters. Results were then interpreted based on full-test responses, rather the targeted analysis that has been carried out in the past. Thus, rather than using a resources-based analysis of individual questions (as was done with Smith, #12), we could investigate if other grain-sizes of analysis were appropriate. Results show that we can use cluster analysis to uncover the resources that students use at scales different from what is typically discussed in the literature. Three manuscripts are in preparation, under review, or being revised for peer-reviewed journals. Ph.D. was completed in 2010.
15. J. Van Deventer: Understanding student performance on isomorphic mathematics and physics vector questions. We have used interviews to guide the development of questions for a survey which asks identical questions in different contexts. This work formed the basis for much of Nagpure's (#8) and Hawkins's M.S.T. completed 2008. Results have been published in peer-reviewed conference proceedings. (Not funded by this grant, but mentored by PI Michael C. Wittmann on grant-related work.)

In addition to the work done by the many graduate students involved in this project, we have had 2 post docs involved. The first, B. Bucy was active in the project when the PI was on sabbatical. Working closely with the co-PI, Thompson, Bucy advised students and studied the role of mathematical reasoning in upper-division courses. The second, B. Frank, joined the project as part of its 1 year no-cost extension. He was instrumental in mentoring graduate students, first and foremost working with Black on procedural resources, while also being involved in a myriad of his own projects. For these, he studied resource activation in the context of polysemous words (those that have two meanings, such as "faster" meaning that something takes less time or has a higher velocity. Projects included:

- the study of kinematics, and how different resources are activated when comparing two balls being thrown, and
- light and optics, and the various meanings of the word "straight" as it applies to light passing through a hole and incident on a surface.

Further work was done by Frank in the context of students' use of epistemological resources. This work, undertaken with Murphy, looked at how students rules of argumentation based on their activation of epistemological resources in a conceptual-based lab for non-science majors.

Finally, Frank has introduced new methodological tools into the research group, including the use of a "PER Lab" environment in which we can study tipping phenomena – ways in which question phrasing cues one or the other idea. Hawkins has worked closely with Frank on this project.

Several papers are under preparation based on these different elements of his work. Frank, who was at one point not sure if he would pursue an academic career, has chosen to continue in academia.

3. Common themes in project activities

Several strands of research have established themselves throughout this project:

1. *Resource coordination in the context of mathematics.* Black, Bucy, Hayes, Pollock, Sayre, and Smith have looked at the use of analytical mathematical tools in intermediate and upper-division classes. In each case, the use of differentials played a role. Also, the issue of variables was of great importance. Describing this work in terms of resource coordination has helped us analyze learning as a process of reification of coordinated resources. This builds off of work introduced to the project by original co-PI Donovan. In particular, the following areas have been studied in details:
   a. *Integration*. With work done by Bajracharya, Black, Bucy, Pollock, we have greatly extended our understanding of how integration is used in physics. Our results touch on the role of graphical representations in integration; the meaning of end points, integration limits, and integration constants; and the mechanics of actually carrying out the integral.
   b. *Differentials*.
2. *Resource activation in the context of Newton's Laws.* Anderson carried out a study on student learning of Newton's Second Law. This was patterned off of published work begun by Smith as part of his undergraduate senior thesis and extended in his M.S.T. thesis.

3. *Vector Analysis.* Issues raised in Nagpure's and Van Deventer's work have raised concerns that we do not understand the ideas students use when they carry out simple 2-d vector addition. Hawkins has begun to investigate this issue, showing the strong dependence of student responses on "hidden" triggers in the visual cues used when asking the questions. Cues include the arrangement of vectors relative to each other, the use (or not) of a grid in the problem, and the alignment (or not) of vectors relative to any coordinate system. Using methods introduced to our group by Springuel in a different study, Hawkins has carried out a series of interviews which include distractor tasks to observe students' consistencies when answering vector questions. Results indicate that we must analyze their responses in terms of their use of procedural resources, visual cues, and conceptual understanding. This is ongoing work.

In addition, we have applied several methodological tools to our work:

1. *New methods for analyzing standardized tests.* While Smith used theoretical ideas about resource activation to group questions on a common physics standardized test, Springuel has used cluster analysis to see if common groupings might be discovered with no *a priori* assumptions about the questions being answered. Results show that we can use common student responses to look for consistencies across question groups in ways that Smith's analysis was incapable of doing. This work allows us to connect student responses across question groups and allows us to analyze thinking across several topics in kinematics and dynamics. More details are given in the discussion of Figures 1–4, above.
2. *Interaction analysis*. Throughout the project, data has been gathered using video analysis of small group learning environments, homework help sessions, and interviews. We have used methods discussed by Jordan and Henderson (1995) and Derry et al. (2010) to analyze the video. Black, Murphy, and Sayre have been the primary video analysts. As expected, the nature of the data informs the analysis, such that gesture and discourse analysis play a major role in interpreting students' actions.
3. *New approaches to doing control studies in large lecture classes*. Building off ideas by Dan Schwartz and his "Preparation for Future Learning" tasks, as well as using tools from psychology experiments, we have stepped away from the more common pre- and post-instruction assessments. Instead, we have focused on slightly different questions asked in quick succession, a few days apart, to see how students' responses might change with time. We have introduced distractor tasks in the middle of interviews. We have used a PER Laboratory environment where students get different versions of similar questions but cannot compare their work to each other. These are all common methods in other education research fields, but were new to our research group during the time of the grant.

The findings of these activities have been published and presented extensively. As shown below, this project has supported the final theses of 3 Ph.D.s and 8 Masters degrees. In addition, there are 5 papers under review, 9 published in peer-reviewed journals, and 17 published in peer-reviewed conference proceedings. Finally, there were 23 invited, 38 contributed, and 74 poster presentations supported in part by this project. In all, this dissemination of our work has been extensive, ongoing, and is not complete. In addition to the papers under review, another 5 are actively in preparation, and there are plans for several more.