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An Analysis of Middle School Physical Science Teachers' Understanding of Accelerated Motion

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AN ANALYSIS OF INSERVICE PHYSICAL SCIENCE TEACHERS' UNDERSTANDING OF ACCELERATED MOTION

By

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Abstract

Continued observation of teachers within the University of Maine Physical Sciences Partnership showed persistence over many iterations of professional development (PD) the use of an inconsistent model of accelerated motion. This model identified acceleration in the same direction as velocity as positive (speeding up is defined as positive acceleration) and acceleration opposed to velocity as negative; we will call this the speed model. We found use of this model in middle school physical science teachers in a survey and through interviews. A PD activity was also observed to study the teachers' use of vectors and coordinate systems to solve kinematics problems. The "speed model" is used in place of the coordinate-based formalism of physics – termed the "direction model" in this paper – even though the speed model is insufficient to describe all physical situations. After careful identification of teacher resources, we see that they have the mathematical skills, and ability to use vectors within a coordinate system, which should allow them to arrive at the direction model; however, when faced with acceleration questions, many revert to using the speed model. The speed model may come from minus sign confusion in calculating changes in velocity, or it may be a velocity-dependent coordinate system; either way its persistence in the teacher population needs to be addressed.

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I. Introduction

The purpose of our study was to assess teachers' use of an incomplete model of accelerated motion which was identified over the course of a few years of observation in a population of middle school physical science teachers. We study teachers' content knowledge as a means of improving instruction of physics at the middle school level. Teachers' content knowledge and knowledge of curriculum both affect the efficacy of instruction (Hill, Ball, & Schilling, 2008), and therefore the study of teachers' content knowledge is a step towards improving instruction for this difficult topic. Our focus on middle school physical science allows the improvement of physics teaching at an early stage. We target a specific standard in the NGSS related o defining kinematics in arbitrary coordinate systems and use vectors to accomplish this goal so that students are better prepared for High School physics. By improving middle school teaching, student will have more to take with them to high school and undergraduate physics classes.

The teachers were members of the Maine Physical Sciences Partnership (PSP), a collaboration of teachers brought together through the Research in STEM Education Center at the University of Maine. The model that was found labeled acceleration in a way that only sometimes agrees with the accepted physics formalism. This model says that when speed is increasing, the acceleration is positive, and when speed is decreasing, acceleration is negative. Because this model relies on the colloquial terms "speeding up" and "slowing down," it will be referred to as the "speed model." Acceleration is a vector quantity, meaning that its direction is dependent on the coordinate system and is defined by the direction of the change in velocity; for simplicity this will be called the "direction model." The speed model aligns with the direction model of acceleration in cases in which an object's velocity is positive (Table 1), which may be partially the cause of the issue for solving accelerated motion problems.

This project was conducted in collaboration with a Master's Thesis project by Peter Colesworthy, who was designing a new module to address the speed model in the 8th grade physical science unit and give students new tools to incorporate the direction model in an grade-appropriate way. Students at that age are not expected to deal with kinematics in an algebraic way (NGSS Lead States, 2013). Teachers with the PSP use the Project Based Inquiry Science curriculum for 8th grade science. Kinematics is covered by the Vehicles in Motion text (Kolodner, 2010). Colesworthy's proposal was to introduce vectors to the middle school Vehicles in Motion Module as a tool for interpreting kinematic quantities in reference to a fixed coordinate system. When vectors are used, students see the connection between displacement, velocity and acceleration as arrows which represent directed quantities with a magnitude and direction.

Table 1: Comparison of Speed Model and Direction Model

Direction of Velocity (+ or -)	**Speeding up or slowing down?**	**Speed model interpretation**	**Direction Model interpretation**	**Agreement of both Models**
+	Speeding up	Positive **a**	Positive **a**	Yes
+	Slowing down	Negative **a**	Negative **a**	Yes
-	Speeding up	Positive **a**	Negative **a**	No
-	Slowing down	Negative **a**	Positive **a**	No

The proposal is that by understanding kinematics through vector quantities, and representing these vectors within fixed coordinate systems, students will learn the direction model in a simple and grade-appropriate way that will simplify future physics classes.

Due to the nature of curricular change, when working on curricula we need not only consider the difficulties of learning physics and the possible solutions. We must also show that our changes target the standards for learning set by the state department of education. In Maine, mathematics is governed by the Common Core State Standards (CCSS) (National Governors Association, 2010) and physics is structured through the Next Generation Science Standards in most schools involved in the PSP (NGSS Lead States, 2013). NGSS is organized by grade level and science field; we address the MS.PS.2.2 which states students should develop the skills to “Plan an investigation to provide evidence that the change in an object’s motion depends on the sum of the forces on the object and the mass of the object” (NGSS Lead States, 2013). This does not mention vectors, but the *Core Ideas* section relating to PS.2.2 does not exclude vectors, which the 3rd grade standard prohibits. The standard reads, “All positions of objects and the directions of forces and motions must be described in an arbitrarily chosen reference frame” (NGSS, 2013). In the NGSS, eighth graders are limited to kinematics in one dimension, *i.e.,* points on a number line. Vectors on a number line have a direction and a magnitude with direction encoded in the sign of the vector. The CCSS states that 6th grade students must “Understand that positive and negative numbers are used together to describe quantities having opposite directions” (National Governors Association, 2010). This justifies the conceptual capability of students in eighth grade to be introduced to vectors in one dimension and treat them both pictorially and mathematically.

To determine the teachers' commitment to the speed model, we must assess their interpretations of a few different accelerated motion situations and see how they use the speed model, and where it fits into their existing conceptual frameworks. Teachers were asked to complete a survey, were interviewed, and attended a PD activity. The purpose of these assessments was to see how teachers use the speed model in the context of their mental ecologies (Redish, 1994) and see how they solve acceleration problems. Mental Ecology is the natural system of a person's mental frameworks and conceptual pieces (Redish, 1994). If the speed model is used even though other resources are being applied correctly, the goal is to know what it is about the speed model that attracts the teachers to use that incomplete model. We wish to highlight the ability of teachers to solve physics problems by identifying productive knowledge pieces. We know that they deal with physics content every year and have a good set of resources for solving problems. We wish to determine what pieces the teachers have that apply to accelerated motion, and where the speed model fits into those teachers' mental ecology.

II. Motivation

The field of physics education research aims to improve instruction at all levels and all physics content areas. Our project focuses on kinematics and, more precisely, interpretations of accelerated motion in 8th grade physical science. We need to improve instruction of acceleration to meet the NGSS standards for middle school physical science. However, acceleration has been shown to be difficult at any level of instruction (Knight, 1995; Reif & Allen, 1992; Shaffer & McDermott, 2005). The PSP uses the *Project Based Inquiry Science: Vehicles in Motion* (VIM) as the 8th grade curriculum module for the force and motion unit, which contains the speed model in the text (Kolodner, 2010), and the speed model has been shown to be used persistently by teachers in research done through the RiSE center (Kranich, Wittmann, & Alvarado, 2015; Kranich, 2016). At the eighth grade level, standards limit students to kinematics in one dimension without algebraic reasoning (NGSS Lead States, 2013), so we wish to help teachers understand the correct model in terms of the standards within which they are expected to teach. P. Colesworthy's thesis project involved creating a course module that introduced vectors to the VIM unit to provide another tool that students can use to understand kinematics in an intuitive way, and see how all vector quantities are labeled with directions which depend on the coordinate system.

A. Research on Acceleration

The VIM text contains the very misconception we wish to dispel with the introduction of the vector unit. "When an object is speeding up it has positive acceleration. When an object is slowing down it has negative acceleration" (Kolodner, 2010). This is not likely to be the root cause of our issue however; as we know that if people learned and accepted everything in textbooks, teaching would be much easier. Analysis of the teachers' responses to surveys administered by the RiSE center, after the implementation of VIM, showed need for improvement in a number of areas. Following that discovery, teachers had the opportunity to attend a PD to address some of the issues that were coming up in the first implementation of VIM. After the focused PD session, improvements were still not being made in a few crucial areas of the VIM content areas (Kranich, 2016). Finally in 2014, the VIM was overhauled to target misconceptions that were persistent through instruction with the original materials. After the overhaul there were improvements in overall performance and understanding seen in yearly surveys administered by the RiSE center, but at a small group PD there was evidence of the speed model of accelerated motion (Kranich, Wittmann, & Alvarado, 2015). At this PD, the competing models were discussed by two teachers representing the competing models for a problem involving acceleration. They found that their models agreed in certain circumstances (Table 1). The proposal by Colesworthy is that by introducing vectors into the beginning of the VIM curriculum we suggest an increased understanding of coordinate system use and vector quantities in kinematics.

Research on teaching acceleration shows that successfully imparting conceptual understanding is difficult. Accelerated motion is something that people experience in everyday life and those experiences interfere with instruction (Freudenthal, 1993). The literature also shows that visualizing acceleration is more difficult than other kinematic concepts, and that the same difficulties that exist in secondary students and undergraduates, are also present in graduate TA's (Shaffer & McDermott, 2005). To construct knowledge of the accurate physical formalism within teachers who have a naïve belief in the speed model, we need to use the existing pieces of knowledge that the teachers possess and build up to the accepted formalism (Smith III, diSessa, & Roschelle, 1994). In the literature there are a few different perspectives describing how learners arrive at incorrect conclusions about physics. Two of these perspectives are knowledge in pieces, and misconceptions; both of which are described in a paper in the context of teaching special relativity (Scherr, 2014). Knowledge in pieces reasons that concepts are malleable and subject to change slowly over a period of time, whereas the misconception reasoning states that ideas are rigid and difficult to change, but change can be sudden and fundamental. The misconceptions point of view is no longer widely found in the literature but it is common in the foundational literature of the field. We will highlight teacher resources and use the knowledge in pieces framework to describe their understanding of acceleration.

B. Learning Theories

By introducing vectors in the context of kinematics, we may be able to bring other concepts in that teachers are more comfortable with to make kinematics problems easier. Accommodation theory, where new information is synthesized with old knowledge by replacing an inadequate model, requires that learners feel that their existing model is inadequate. Learners must also find a new model which is shown to be more comprehensive and be reasonable and believable (Posner, Strike, Hewson, & Gertzog, 1982). Knowledge in pieces describes learners incorrect answers as the result as the misapplication of resources; by creating discomfort in their wrong answers, we can use accommodation theory to describe how the learner incorporates new pieces and uses them to solve problems in a way they did not do previously. Teacher belief in the speed model is preventing use of the direction model; to overcome this we wish to work through examples which challenge the old belief and provide evidence for the direction model (Hammer & Elby, 2003). In a situation where we have a misunderstanding of a physical situation, we must take into account the teachers' previously held beliefs and construct the correct model from pieces they already possess. This project aimed to elicit the speed model, understand where it shows up when teachers solve problem, create a moment where they will realize that the model is not sufficient, and supply them with a new model which works for all situations and matches with the rest of the kinematic vector quantities. We will show the existing pieces, and analyze the use of the speed model within this framework.

C. Vectors

The idea that vectors are needed as tool for understanding kinematics comes from research at the collegiate level, as many secondary teachers do not use vectors. To succeed in introductory college physics students need to understand vectors (Thornton & Sokoloff, 1998). It is also well known that vector understanding is not easy (Nguyen & Meltzer, 2003; Barniol & Zavala, 2014; Flores, Kanim, & Kautz, 2004). At the middle school level, mathematics is not at the level expected of college freshman; however the idea of using a pictorial representation for kinematics can be extremely beneficial. We know that the inclusion of a vector unit prior to the teaching of force and motion was shown to increase high school students understanding of 1D vector concepts (Harada, Morgan, & Prause, 2006). Vectors can be made as simple as arrows, but to describe them accurately one also needs to understand coordinate systems. A 2010 paper by Hayes showed how students run into problems when coordinate systems are not used consistently in problem solving (Hayes & Wittmann, 2010). Hayes found that use of minus signs incorrectly leads to confusion about the direction of vectors quantities. In the context of accelerated motion, this could lead to misinterpreting equations implying the wrong sign for acceleration vectors.

We have seen ample motivation for our vector modifications to VIM from the physics literature. In addition, the topic of vectors is in the intersection of science and mathematics (Megowan, 2005). Mathematics education research has added a great deal to the pedagogy of vectors at the secondary and post-secondary levels. A 2003 paper by Watson and colleagues described mathematics learning as consisting of three levels representing the journey to attaining mastery of a concept (Watson, Spyrou, & Tall, 2003). These levels, or worlds, are the embodied, the proceptual – a combination of procedure and concept – and the formal. In terms of kinematics, the day to day experience of motion informs students understanding, this is followed by the use of symbols to encode information (proceptual) and finally the use of accepted formalism to allow easy communication of ideas with others. Vectors act as a procept to bridge the embodied experience of motion to the abstraction of formal kinematics. Vectors are a useful tool to access the physics we are studying and their position as a mathematical object allows us to add a new tool to teacher and student problem solving toolboxes.

D. Acceleration in Everyday Language

One other thing to consider when analyzing teachers' understanding of accelerated motion is the use of physics words in everyday language. Acceleration is difficult because of the everyday experience of driving in a car and stepping on the "accelerator". If a student is being driven in a car, the direction the car is pointing creates a natural positive direction for a self-centered coordinate system. In this experience, acceleration almost always aligns with the speed model. The terms "speeding up" and "slowing down" are used to determine the sign of the acceleration, and if a car always points in the positive direction, then one is used to experiencing acceleration aligned with the speed model. The direction model says that our coordinate system should be fixed, such as the compass directions, and not one moving with us in the car. If we fail to make this distinction then positive acceleration is simply an increase in speed, without consideration of a fixed frame. In this way, addition to our speed can easily be confused for an increase in positive velocity, leading to the speed model. For example, imagine that we have an object moving with initial velocity of -3 m/s, and the velocity is changed by -1m/s (equation 1). The final velocity is found by vector addition of the initial velocity and the change in velocity. This comes up when discussing the mathematics behind vector addition, which is the conceptual base of using vectors to describe motion.

eq. 1 $$\vec{v_i} + \Delta\vec{v}\ can\ be\ written\ two\ ways$$

$$(-3)\left[\frac{m}{s}\right] + (-1)\left[\frac{m}{s}\right] = -[3+1]\left[\frac{m}{s}\right]$$

Both terms in the equality are mathematically equivalent, but lead to different interpretations for the change in velocity. On the left the interpretation is that change in velocity is a negative value being added to a negative initial velocity. On the right the speed increased by a positive amount but the direction of the final velocity is negative. In the first case the change in velocity, and therefore the acceleration, is negative, but in the second case the speed increases and therefore acceleration is positive. The mathematical confusion shown by the different interpretations of equivalent mathematical statements is another motivation for using vectors to simplify problems. Vectors allow learners to see arrows drawn within a coordinate system so that the direction is always visually apparent.

The benefits of introducing vectors to this point in instruction are greater than just increasing a student's proficiency as measured by the NGSS. By starting to construct a formal definition of a vector, they will be better prepared to use the concept of vectors when they get to more advanced physics and mathematics in high school or college. The vector instruction maps directly to a scaffolded development of skills with vectors in two dimensions. Once students are comfortable with vector addition, it follows that addition in two dimensions will benefit as well. This type of addition has been identified as a difficulty for students at the introductory physics level (Aguirre & Rankin, 1989). By addressing the difficulty of students at the 8th grade level, preparing them for future topics in physics, and targeting the standards set forth by the Maine Department of Education. Our project aims to improve the quality of learning in the physical sciences at the eighth grade level by understanding the teachers' resources and building new instructional skills.

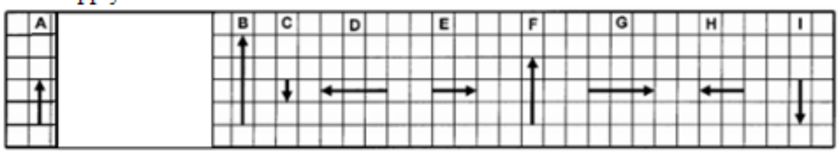
III. Methods

Data were collected from three sources: A pre-survey, one-on-one interviews, and PD sessions. Audio data were collected from the interviews and PD sessions, and the PD sessions were also video-taped. Surveys were completed on paper to allow for the drawing of vectors. All three targeted understanding of kinematics and the use of vectors in problem solving. The goal was to see what pieces are used in displacement and velocity problems that can be applied to acceleration. As the standards for middle school physics do not allow for algebraic solutions, we designed the survey to observe how teachers use vectors in kinematic problem solving.

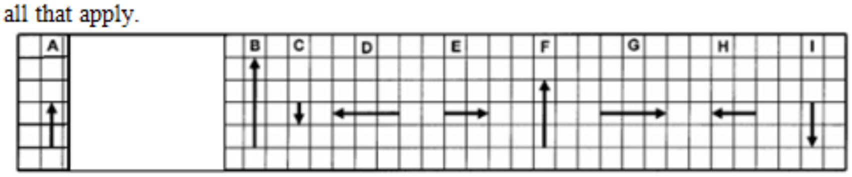
The survey was divided into two sections. First, questions were asked to identify teachers' understanding of vector direction and magnitude. The pictures were adapted from the Force Concept Inventory (Figure 1, Hestenes, Wells, & Swackhamer, 1992). Following this was a series of questions about a ball that had an initial velocity towards a fan while experiencing a constant force. The survey gives positions and times in a "stop motion" series of figures (Figure 2). In this situation, there is a constant negative acceleration, meaning that in the first four seconds shown, the ball is slowing down with negative acceleration, and the last four seconds show speeding up with negative acceleration.

Figure 1: Vector Questions

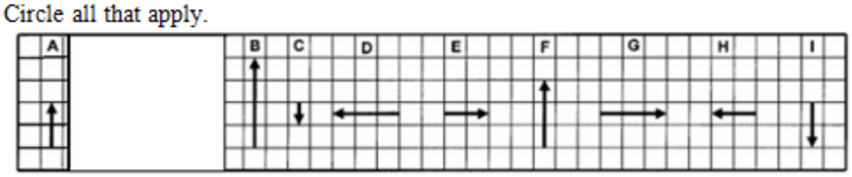
1. Which of the following vector(s) have the same *magnitude* as the vector on the left (vector **A**). Circle all that apply.



2. Which of the following vector(s) have the same *direction* as the vector on the left (vector **A**). Circle all that apply.

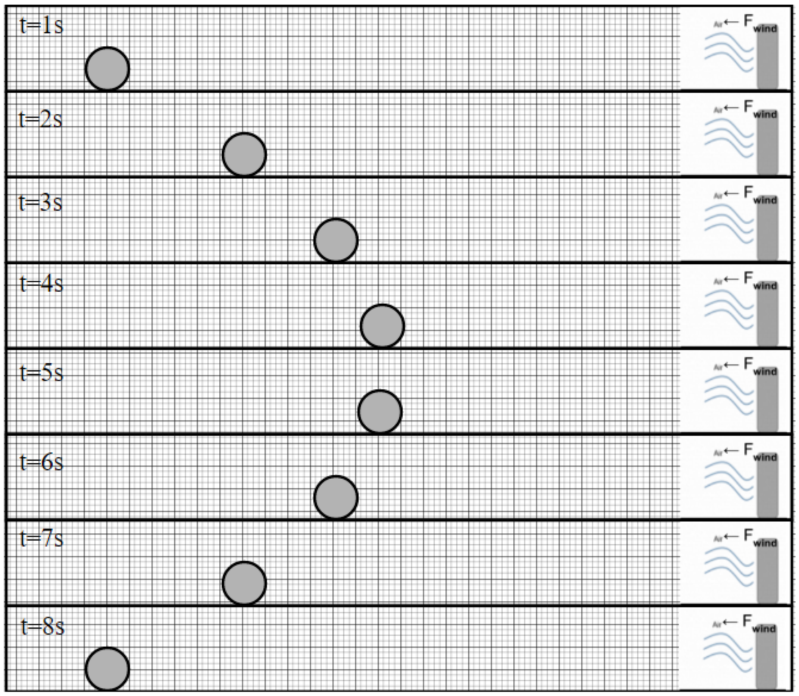


3. Which of the following vector(s) have the *opposite direction* as the vector on the left (vector **A**). Circle all that apply.



We asked teachers to draw displacement vectors, and draw the change in the displacement vectors. Because the time intervals are all equal (and by design also equal to one second), the displacements over 1s intervals are also average velocities, and the changes in average velocity are average accelerations.

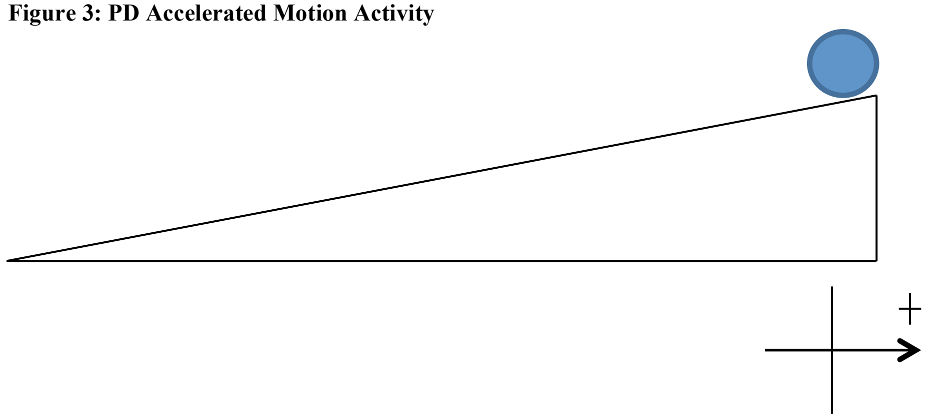
Figure 2: Survey Accelerated Motion Scenario



Once all the vectors were constructed, we asked them to identify regions in which the velocity was positive and negative, and where the acceleration was positive and negative. (Note: the population of the survey is larger and contains teachers who do not teach middle school physical science because it was administered at a large meeting of STEM teachers who were gathered for a collaborative; for our analysis, we looked only at those teachers who regularly teach 8th grade physical sciences. See Appendix A for full text of survey). We asked questions on displacement and velocity to see what pieces teachers use to solve those problems, and to see if they use those pieces correctly when talking about accelerated motion. By using vectors to talk about displacement and velocity, teachers must show how they think about vectors within a coordinate system so that when asked about acceleration, the use of the speed model will be apparent.

The interviews were conducted one-on-one with the researcher, and audio recorded. The audio recordings were then analyzed and transcribed. There were three tasks during the interview. First was a card sorting task to prime think-a-loud behavior and begin thinking about kinematics. The cards were: Mass, Force, Displacement, Speed, Charge, and Acceleration. The goal was to get teachers to think about scalar and vector quantities, as well as talk about their thought processes out loud. This task is very open and so our anticipated result was not realized, but it did accomplish the goal of getting the teachers to think aloud and think about kinematics. The next section was two stop-motion pictures of uniform motion shown one after the other. We hoped to get the teachers thinking about vectors to describe motion, identifying uniform motion with vectors, and identifying a difference in direction as a difference in velocity. The final section was the most interesting and got at the heart of the issue of describing accelerated motion. We ask them to analyze the reverse of the survey question (the mirror image of Figure 2). That is, an object with an initial velocity to the left (in a positive to the right coordinate system) and a constant force to the right so that the ball slows down with positive acceleration, and then speeds up with positive acceleration. This task illuminated the teachers' ability to construct vectors that describe the motion of objects, as well as show the use of the speed model of accelerated motion.

Figure 3: PD Accelerated Motion Activity



Finally, the PD activities focused on a new course module that introduced vectors to the vehicles in motion materials. The first PD session focused on uniform motion and displacements and using vectors. The second PD session reinforced uniform motion and using vectors to describe displacements and velocities, then gave a task covering accelerated motion. This task was an inclined track problem which was given as a physical set-up for the teachers to work through as an experiment. This incline was set up so that the ball always has a negative velocity, and was speeding up (Figure 3). Displacements were measured along the table, and not along the track.

Teachers were instructed to take data on the displacement over equal time intervals on the horizontal (the angle of inclination is exaggerated here for effect; the real slope was approximately 5 inches of rise for 7 feet of run). The locations of the ball at 1 second intervals were connected to make vectors; the displacement vectors were then abstracted to average velocity vectors. We then asked them for change in average velocity vectors. After all the vectors were constructed, we asked the teachers to interpret the motion and give their description of the acceleration. Data from the PD came from this activity and the discussion around it, as teachers tied all concepts together here and realized the discrepancy between the two models.

IV. The Direction Model

We ask, both in the interview and the survey, a question of accelerated motion in a fan-ball system (Figure 2). In the situation shown in the figure, positive is to the right and acceleration is always to the left (negative). The way that we wanted to see a correct answer constructed would be as follows. First, displacement vectors can be constructed between consecutive points and their magnitudes can be found with signs indicating direction. The displacement vectors are then used to obtain average velocity vectors because each represents the displacement over an equal time interval, each 1 second in duration. These findings can be represented in a table (table 2).

Table 2: Displacements and Average Velocities Over 1s Time Intervals

Time Interval	**Displacement**	**Average Velocity**
[1,2]s	6 [units]	6 [units / s]
[2,3]s	4 [units]	4 [units / s]
[3,4]s	2 [units]	2 [units / s]
[4,5]s	0 [units]	0 [units / s]
[5,6]s	-2 [units]	-2 [units / s]
[6,7]s	-4 [units]	-4 [units / s]
[7,8]s	-6 [units]	-6 [units / s]

After constructing average velocity vectors with magnitudes and directions denoted by the sign, change in velocity is found by subtracting one average velocity from the previous.

Eq. 2 $$\Delta\vec{v} = \vec{v}_{i+1} - \vec{v}_i$$

These changes in velocity can be abstracted to acceleration vectors when divided by a 1 second interval. Because we have velocities for every integer second interval, the accelerations will be the average value of acceleration on half-integer, one-second intervals (table 3).

Table 3: Displacements and Average Velocities Over 1s Time Intervals

Time Interval	**Δv**	**Average Acceleration**
[1.5,2.5]s	-2 [units / s]	-2 [(units / s)/s]
[2.5,3.5]s	-2 [units / s]	-2 [(units / s)/s]
[3.5,4.5]s	-2 [units / s]	-2 [(units / s)/s]
[4.5,5.5]s	-2 [units / s]	-2 [(units / s)/s]
[5.5,6.5]s	-2 [units / s]	-2 [(units / s)/s]
[6.5,7.5]s	-2 [units / s]	-2 [(units / s)/s]
[7.5,8.5]s	-2 [units / s]	-2 [(units / s)/s]

To an expert these results are obvious; we assume the fan is exerting a constant force on a ball which changes the velocity by the same amount, in the same direction, every second. This is evidence of constant acceleration for the whole situation shown. The subtle point here is that we used average velocities and accelerations to approximate what is happening when we only have data taken every second. It is possible (yet unlikely) that the ball travels way beyond the fan, dances around, and then neatly shows up in a parabolic approach of the fan right when we took our measurements. This subtlety is something that can be ignored for the level of education we are aimed at, but the idea is that we are taking averages of quantities instead of using a calculus-like approach which would be inappropriate for the grade level. Everything done here could be within reach of 8^{th} grade students.

V. Results

From Kranich *et al.* 2014, we expected middle school teachers to have some level of belief in the speed model, and through surveys, interviews and teacher PD meetings, we saw some situations in which the model was used. From listening to teachers work through accelerated motion problems, we can see the use of pieces of knowledge which are productive and necessary to solve kinematics problems. However, the teachers reverted to using the speed model when confronted with acceleration problems. For all data were are looking at 4 teachers for whom we have all three data sources.

A. Teachers' Productive Resources for Kinematics Problems

1. Vectors

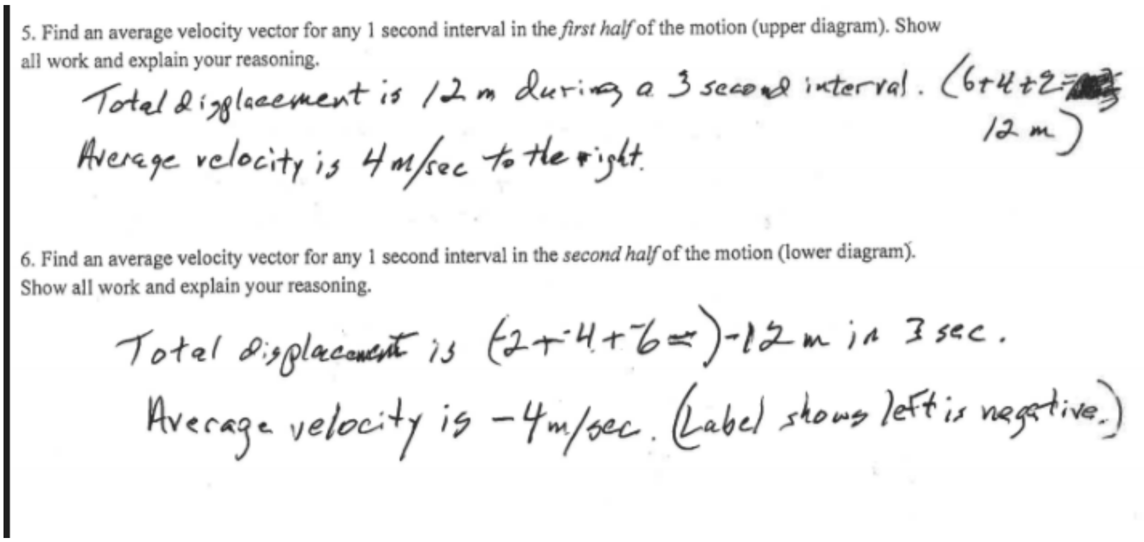
The first piece in a successful understanding of vector quantities is the qualitative comparison of vector magnitudes. The survey showed that all teachers who responded understood the concept of magnitude in terms of vectors. We also found that magnitude comparisons were easy for the teachers in terms of displacements and average velocities. For example, teachers were able to use stop-motion diagrams to compare speeds at different time intervals, and abstract displacements to average velocities. In the context of vectors, we see from the interviews that magnitude comparisons are done easily. All teachers interviewed identified the two uniform motion diagrams and could see that the same displacement in equal time intervals implied equivalent speeds. This shows they have knowledge of how to use vectors to compare speeds at different times.

<u>2. Coordinate Systems</u>

The next aspect is the importance of direction to the comparison of vector quantities, and how the sign of the quantity associated with direction is relative to a coordinate system. Teachers showed understanding of the directed nature of the quantities displacement and velocity. We see in the survey how teachers labeled velocities with directions – either using positive and negative aligning with the given coordinate system, or “to the right” and “to the left” – which is an important piece of using the direction model for solving accelerated motion problems. In the interviews we also see evidence of productive language use surrounding velocity. Teachers use a variety of coordinate systems such as positive/negative, left/right, and east/west. All three are valid and were used correctly to compare velocities of equal magnitudes but opposite directions.

Teachers were not familiar with the use of vectors to solve these types of problems, so our results look at a concept that is familiar to the teachers, but approached with a new method of solution. We see that velocity vectors were more natural for them to construct than acceleration vectors. In the survey, teachers found average velocity vectors but did so for different intervals than they were asked to find. Most teachers found average velocity for two halves of the motion, one with positive velocity and one with negative velocity. Figure 4 shows teacher 1's calculation of average velocity for the two different halves of motion.

Figure 5: Teacher 2 Survey Velocity Answers



5. Find an average velocity vector for any 1 second interval in the *first half* of the motion (upper diagram). Show all work and explain your reasoning.

Total displacement is 12 m during a 3 second interval. (6+4+2= 12 m)
Average velocity is 4 m/sec to the right.

6. Find an average velocity vector for any 1 second interval in the *second half* of the motion (lower diagram). Show all work and explain your reasoning.

Total displacement is (-2+-4+-6=)-12 m in 3 sec.
Average velocity is -4m/sec. (Label shows left is negative.)

<u>3. Mathematical Pieces</u>

Mathematically, teachers showed the capacity to find average velocity, but lack of familiarity with the type of questions we asked resulted in our not seeing how they handled shorter average velocities. In the interviews and PD session, however, they had no problem generating the vectors for 1s displacements, and on prompting abstracted to average velocities. For example, teacher 2 worked through the fan-ball problem, "We have a displacement of (counting) 1, 2, 3, 4 units, over an interval of 1s, so that's a speed of 4 units per second… and that is a velocity of 4 units/s to the west." This teacher created the vectors, counted the magnitude and then used an east/west coordinate system to describe direction of velocity, which she knows is important when distinguishing velocity from speed. The PD problem of accelerated motion in one direction required teachers to construct velocity vectors, and when asked, they all identified the velocity vectors as implying a negative sign for the velocity throughout the whole motion. They made this conclusion by pointing in the direction which was defined as positive and saying "that way is positive so the velocity is negative." This shows that teachers understand that velocity is given a sign based on the fixed coordinate system of the problem. So, we see that teachers recognize the importance of coordinate systems for kinematics problems, and they have the mathematical tools necessary to go from displacements to velocities to accelerations by dividing by time and finding averages. We also see that, though unfamiliar with using vectors to solve problems, they are capable of setting up vectors in kinematic situations, and using the visual cues of the arrows to determine relative magnitude, and putting direction in terms of a fixed coordinate system.

B. Emergence of the Speed Model

1. Evidence

With all of the tools discussed above in place, where does the speed model come in? In the survey, the final question asked teachers to "identify regions of the motion where the ball has positive acceleration and where it has negative acceleration." Results showed that even though teachers had all the resources to answer the question correctly, the speed model is in their minds the correct formalism for describing the situation. Teacher 1 said, "From t_1 through t_4 the acceleration is negative since it's in the opposite direction of the motion… and from t_5 through t_8 there is positive acceleration in the same direction as motion." Teacher 2 answered very similarly, stating "Positive acceleration → speeding up, negative acceleration → slowing down." So before intervention we see that 8th grade physics teachers have this model for describing accelerated motion, which is not sufficient to describe all physical situations.

In the interviews we saw the same emergence of the speed model as soon as we ask acceleration-related questions. Teacher 2, when answering the reversed fan-ball situation, had successfully constructed change in velocity arrows which were identical for any 1s time interval, however, when asked about the acceleration she reverted to using the speed model. “Well here the ball is slowing down so that is negative acceleration. Right?” Teacher 3 was even more assured of the speed model. When asked about acceleration, he said, “So in the first half of the motion the ball is slowing down, so that’s negative acceleration, and after the turn around it’s speeding up so that’s positive acceleration.” His declarative language shows that he knows and has confidence in the speed model and is an “expert” at using this model to describe physical situations. This plays out further at the PD session where he stated that the ball which is speeding up in the negative direction must have positive acceleration because the speed model tells him that, even though the arrows are all pointing him in the right direction. Once confronted with the direction model, he was able to abstract the situation to another disagreement between the two models, saying with a tone of incredulousness “If we had it the other way around, and were slowing down, would that be positive acceleration? (Followed by a chuckle of incredulousness).” He had made a correct point without realizing it; the direction model does say that slowing down with a negative velocity is in fact positive acceleration. For him this was impossible; how could it be that a ball slowing down with negative velocity had a positive acceleration? Teacher 4 also had a very telling reaction to the direction model once it was revealed that it was the correct physical formalism, which was told to the teachers after the discussion of acceleration. Teacher 4 stopped and paused for a significant amount of time before questioning what we were saying. She said “It doesn’t matter because we’re not talking about displacement; we’re talking about change in velocity. So the speed is getting larger then the acceleration is positive.” After this she paused for 15 seconds before continuing. The level of discomfort created by learning that she had an incorrect model showed how this new model was being accommodated and the old model was no longer satisfactory for describing accelerated motion using the framework of accommodation (Posner *et al.*, 1982).

2. Possible Origins of the Speed Model

The speed model results in teachers describing accelerated motion in an incomplete way. As one teacher said, this may be because this model is consistent for the types of questions they ask their students. Teacher 3 said, “we always ask it the other way [velocity in the positive direction] so this model works for what we always deal with.” So the teachers’ familiarity with the types of questions they ask their students may have led to them using a simplified model which is always correct. That is, if you modify the speed model to say “if velocity is in the positive direction then speeding up is always positive acceleration and vice versa” then the two models agree by definition, but the speed model only describes half of accelerated motion situations. The speed model, as used in the observations, acts like a velocity dependent coordinate system. In this coordinate system within a system, the acceleration is given a sign relative to the velocity. Teacher 4 said during the discussion about the sign of acceleration at the PD session, “It doesn’t matter the sign [of the arrow] because we’re talking about change in velocity.” So this teacher sees the sign of acceleration as determined by the change in velocity vector, relative to the direction of velocity itself. Even though she was looking at the change in velocity vector, constructed in the formally correct way, she abandons the formal solution and uses the speed model instead.

Another conjecture goes back to the mathematical interpretations of the change in velocity equation (Eq. 1), where the sign of the change in velocity is associated with the operation. If you subtract when the velocity is getting smaller (in magnitude) and add when the velocity is getting larger, one could interpret the operation as determining the sign of the acceleration. By doing this you arrive at the speed model even though your velocities and changes in velocity are all correct for the fixed coordinate system. We do not yet have enough evidence to distinguish between the two models and need further exploration of teacher reasoning to determine which cause, if either is behind the teacher use of the speed model. We would need to have them explain where they learned the speed model, and why they use it for solving accelerated motion problems.

VI. Conclusions

Our results make evident that teachers have productive resources for solving kinematics problems. They understand the difference between the quantities of speed and velocity which is a result of understanding the vector nature of kinematic quantities. Teachers also understand the relationship between displacement, velocity and acceleration in a qualitative sense as seen by their ability to go from one to the next with minimal prompting. By forcing the use of vectors in our data collection, we saw that teachers use coordinate systems to determine the sign of displacements and velocities, and thus we see that they have all the necessary resources for solving acceleration problems; yet here is where the speed model shows up.

We have a few conjectures as to the origin of the speed model. In the future, teacher reasoning should be probed so that we can understand why they use the speed model for accelerated motion, and through this we may find the best way to confront this model with the accepted formalism, and allow accommodation of the direction model. Velocity-dependent coordinate systems are something that may stem from peoples' everyday experience, and would make sense that the speed model results from their experience. On the other hand, we are talking about teachers who teach VIM every year and they show ability to solve problems closely related to acceleration. It is possible that mathematical skills of teachers, when combined with years of simplifying problems for the 8^{th} grade level, lead to misinterpretations of equations to arrive at the speed model.

We see that teachers have the required tools to understand the direction model and fit that model within their existing mental ecology. The mathematical level of 8^{th} graders means that the teachers need to be able to explain the use of vectors to go from displacement to velocity to acceleration without differentiation. Teachers have the mathematical ability to teach this way, and by becoming more familiar with using vectors, they will be able to explain this procedure to their students. With the goal of getting teachers to accommodate the direction model into their conceptual frameworks and teach the correct model to their students, we wish to continue the use of vectors as an instructional tool. Vectors bridge the gap from reality to formalism and as such are good for physics novices to learn as a problem solving method. Acceleration is difficult, but with improved instruction and teacher understanding, achievement will be easier.

Regardless of the reason behind the prevalence of the speed model, we have highlighted that teachers have pieces of knowledge which are productive for answering questions. In physics education research, this is an important point to make; we not only need to pay attention to difficulties, but also productive resources to be able to properly assess the causes of misconceptions, and address them to improve instruction.

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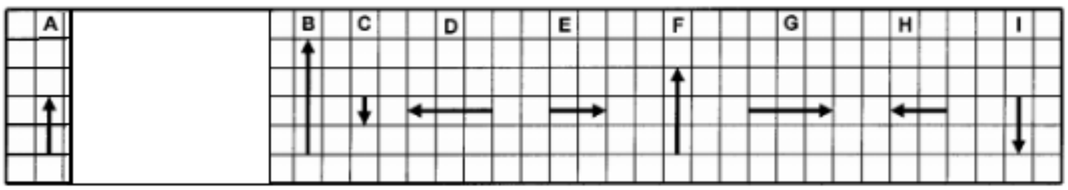
Appendix

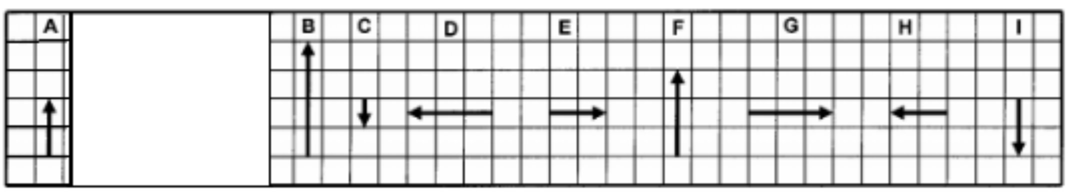
Teacher survey:

Page 1- Vectors

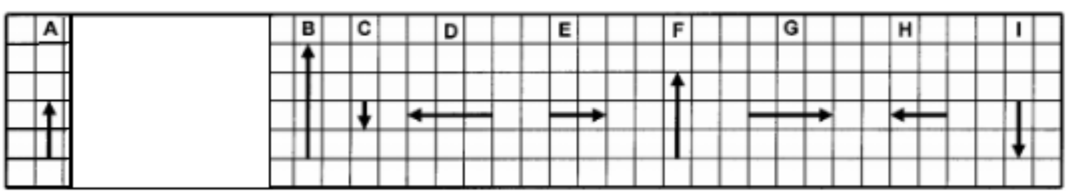
1. Which of the following vector(s) have the same *magnitude* as the vector on the left (vector **A**). Circle all that apply.



2. Which of the following vector(s) have the same *direction* as the vector on the left (vector **A**). Circle all that apply.



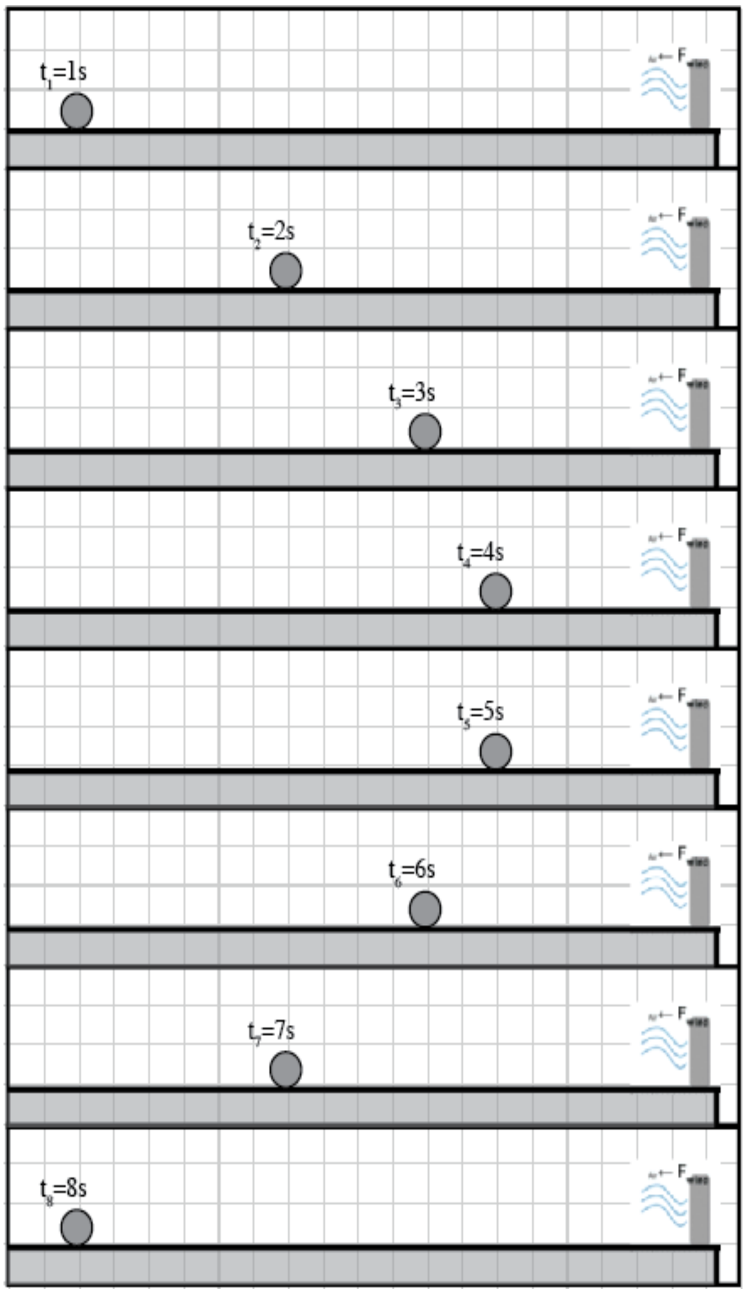
3. Which of the following vector(s) have the *opposite direction* as the vector on the left (vector **A**). Circle all that apply.



Page 2- Motion Diagrams

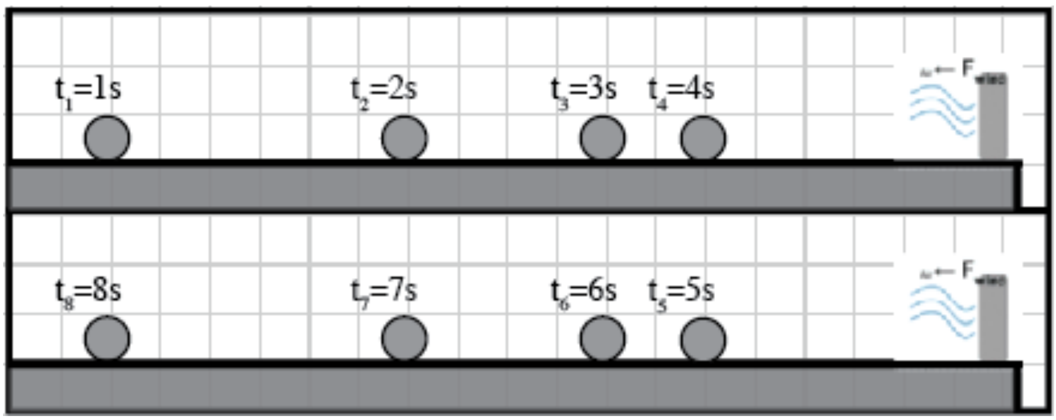
A ball on a table was given a shove to the right so that it has a velocity toward a fan which exerts a constant force (friction can be ignored). Each point shows the ball at different instants of time each 1 second apart. (*Aside:* The ball is never at rest.)

1 block = $1m^2$



Here we have combined the picture to create a single motion diagram for the ball on the table.

1 block = $1m^2$

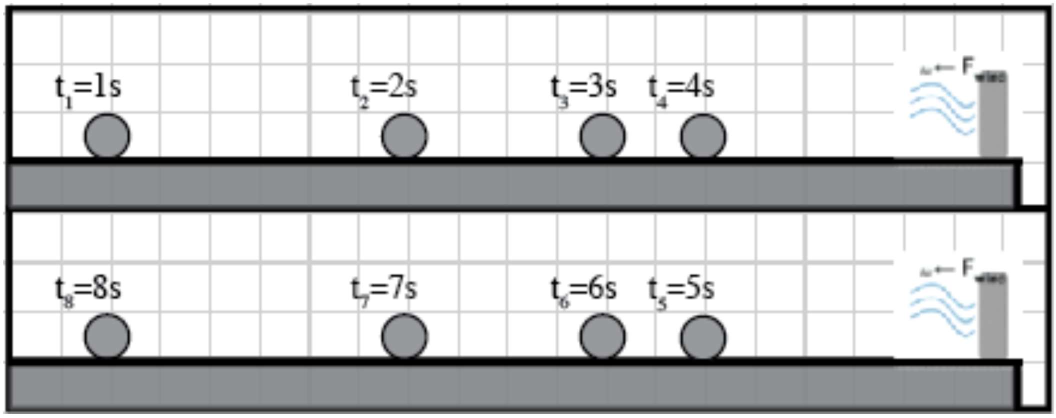


4. Draw the displacement vectors for each 1 second time interval on the motion diagram above. If there is zero displacement state so explicitly.

5. Find an average velocity vector for any 1 second interval in the *first half* of the motion, show all work and explain your reasoning.

6. Find an average velocity vector for any 1 second interval in the *second half* of the motion, show all work and explain your reasoning.

1 block = $1m^2$



7. Circle and identify the region(s) where the ball is *speeding up* and *slowing down* on the picture above.

8. Identify regions where the ball has positive acceleration and where it has negative acceleration. Explain.

9. Can the ball's motion across the table be represented by a single acceleration vector? If so, find it and explain, if not, explain why not.

Biography

Elijah Tabachnick is a Senior Physics student in the Honors College at the University of Maine. From Portland, Maine, Elijah grew up enjoying sailing, music, and a deep love of physics and the philosophy of science. He is completing a B.S. in physics, and in the Fall Elijah will attend the University of Maryland- College Park as a doctoral candidate in Physics, studying Physics Education Research.