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Investigating Student Learning of Analog Electronics

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INVESTIGATING STUDENT LEARNING OF ANALOG ELECTRONICS

By

Kevin L. Van De Bogart

B.S. University of Idaho, 2008

A DISSERTATION

Submitted in in Partial Fulfillment of the
Requirements for the Degree of
Doctor of Philosophy
(in Physics)

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May 2017

Advisory Committee:

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INVESTIGATING STUDENT LEARNING OF ANALOG ELECTRONICS

By Kevin L. Van De Bogart

Dissertation Advisor: Dr. MacKenzie R. Stetzer

An Abstract of the Dissertation Presented
in Partial Fulfillment of the Requirements for the
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Instruction in analog electronics is an integral component of many physics and engineering programs, and is typically covered in courses beyond the first year. While extensive research has been conducted on student understanding of introductory electric circuits, to date there has been relatively little research on student learning of analog electronics in either physics or engineering courses. Given the significant overlap in content of courses offered in both disciplines, this study seeks to strengthen the research base on the learning and teaching of electric circuits and analog electronics via a single, coherent investigation spanning both physics and engineering courses.

This dissertation has three distinct components, each of which serves to clarify ways in which students think about and analyze electronic circuits. The first component is a broad investigation of student learning of specific classes of analog circuits (*e.g.,* loaded voltage dividers, diode circuits, and operational amplifier circuits) across courses in both physics and engineering. The second component of this dissertation is an in-depth study of student understanding of bipolar junction transistors and transistor circuits, which employed the systematic, research-based development of a suite of research tasks to pinpoint the specific aspects of transistor circuit behavior that students struggle with the most after instruction. The third component of this dissertation focuses more on the experimental components of electronics instruction by examining in detail the practical laboratory skill of troubleshooting.

Due to the systematic, cross-disciplinary nature of the research documented in this dissertation, this work will strengthen the research base on the learning and teaching of electronics and will contribute to improvements in electronics instruction in both physics and engineering departments. In general, students did not appear to have developed a coherent, functional understanding of many key circuits after all instruction. Students also seemed to struggle with the application of foundational circuits concepts in new contexts, which is consistent with existing research on other topics. However, students did frequently use individual elements of productive reasoning when thinking about electric circuits. Recommendations, both general and specific, for future research and for electronics instruction are discussed.

DEDICATION

To my wife, Sylvia

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Throughout my last six years at the University of Maine, there are numerous people who have made a positive impact on my life, in both professional and personal capacities. I would like to thank those who have helped make this work possible, and to apologize in advance if I have left anyone out.

I would like to start by acknowledging my advisor, MacKenzie Stetzer. With his guidance, I have learned a great deal about research, teaching, and electronics. Before coming to the University of Maine, I had essentially no practical experience with circuits beyond what is taught in introductory physics; now I strive to better understand electronics myself and to spread such knowledge to students in ways that enrich their lives. Mac has also helped provide thoughtful input into the process of refining my rough ideas into targeted questions or well-structured arguments. Together we have shared in the struggles of running and changing a course, and I feel prepared to step into the role of a professional instructor and researcher thanks to his guidance.

I would also like to thank all of the members of Physics Education Research Laboratory whom I've had the pleasure of working with during my time at Maine. Their friendship and guidance have been invaluable in supporting my development as a scientist, and I hope that I can continue to collaborate with my peers from UMaine throughout my career. I would also like to thank Nuri Emanetoglu and Duane Hanselman in the Department of Electrical and Computer Engineering for their cooperation and help throughout this project, as well as my committee members John Thompson and James McClymer and my external reader Christian Kautz for their thoughtful feedback on this dissertation.

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Throughout the years in graduate school, I have had to devote the bulk of my time to my schooling or my research. However, I would like to thank those who have helped me stay connected to the other things in life I love. Foremost, I must thank my wife Sylvia, without whom I would be incomplete. The quiet joy we share in simply being together has helped me through many trying times. I would like to thank my parents Lee and Mary, who have gone to ridiculous lengths to support me, even if it meant mailing gardening supplies first-class. I also have to thank my brother, Jeremy, as a sounding board for any number of eccentric schemes throughout the years. And thank you to one particularly Cynical Brit.

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Chapter 1

INTRODUCTION

Physics education research (PER) has systematically explored the nature of students' understanding and abilities in a multitude of physics contexts with many important results. Numerous studies at the undergraduate level have found that students often complete introductory courses with a relatively poor conceptual understanding of physics when taught by traditional means (see [1] for an overview). Thus, in-depth investigations of specific, focused topics (*e.g.*, waves [2], work-energy and impulse-momentum theorems [3], and angular momentum [4]) have been performed to better understand the nature of student difficulties. Such studies have been key in informing the development of research-validated assessment tools, such as the Brief Electricity and Magnetism Assessment (BEMA) [5] and the Determining and Interpreting Resistive Electric Circuit Concepts Test (DIRECT) [6]. Data from PER investigations have also been key in the development of new, research-based instructional materials (such as the Tutorials in Introductory Physics [7]). While the majority of PER has been conducted in the context of introductory courses, in the last fifteen years a growing number of researchers have taken interest in student understanding of physics content beyond the introductory level such as: junior-level mechanics [8], electricity and magnetism [9], quantum mechanics [10], thermodynamics [11], and upper-division laboratory courses [12–14].

Among the numerous topics in introductory physics explored by researchers in physics education, a rich and robust body of work has focused on student understanding of electric circuits, dating back to the foundations of PER (for example, see [15–18]). Of particular note is McDermott and Shaffer's investigation of students' conceptual understanding of basic DC circuits [19], and subsequent development and testing of tutorials aimed at addressing the difficulties identified by their research [20]. McDermott and Shaffer found that after traditional instruction, many students did not demonstrate a coherent conceptual model for simple DC circuits. Furthermore, other studies have shown that many students struggle with ideas about what constitutes a complete circuit well past their first year [21], and that students are often unable to relate the microscopic physical phenomena in a circuit to the transient behavior of voltages within circuits [22].

Despite the ubiquity of electronics courses as part of the contemporary physics curriculum, there are few researchers in physics education who are currently studying either upper-division electronics topics (*e.g.*, instruction on topics beyond RLC circuits) or student understanding of fundamental circuits concepts in electronics courses. For example, in interviews that investigated the selection of resources in nearly-novel situations, students were tasked with designing a vacuum diode [5]. Here electronics served primarily to provide context, and there was not an in-depth investigation of student understanding of diodes themselves. Getty used the DIRECT (designed for introductory contexts) as a metric to assess the effectiveness of changes to instruction in his upper-division electronics laboratory course [23]. In a different project, Stetzer *et al.* [21] examined student understanding of complete circuits and Kirchhoff's junction rule in both upper-division and introductory courses. Thus, while there is some literature bridging research on student understanding between introductory circuits courses and their upper-division counterparts, there remains a lack of focused research on student understanding of electronics within PER.

As a laboratory course, there are additional instructional goals for most physics electronics classes beyond mastering content. To date, relatively little research has focused on students' activities within the instructional laboratory environment and the development of skills unique to experimental physics [12]. The American Association of Physics Teachers (AAPT) has recently issued a new set of guidelines for the undergraduate laboratory curriculum, identifying the development of experimental design skills (including troubleshooting) as well as technical and laboratory skills (such as understanding the limitations of measurement devices) as two of six critical focus areas [24]. Other nation-wide efforts have called for both improving [25] and studying [26] laboratory instruction in science courses, with a particular emphasis on the need for research supporting the creation of instruments to assess learning outcomes in the instructional lab setting and to measure both metacognitive and problem-solving skills [14].

While seeking to establish and enhance the research base on the learning and teaching of analog electronics, it is critical to recognize that instruction on electronic circuits is also a required component of many undergraduate engineering programs, thus making circuits and electronics common topics across physics and engineering curricula. Indeed, electrical engineering departments typically offer a sequence of circuits courses that cover a much wider variety of topics than what is typically taught in a physics department. In addition to understanding the behavior of individual components, electrical engineers may also need to understand in detail the function and implementation of larger-scale circuits in order to be able to design such networks. Thus, there is a greater emphasis on both detailed models that more precisely describe device behavior (*e.g.*, hybrid-pi models of transistors) and the physics pertaining to semiconductor properties.

Since there is a considerable degree of overlap in content taught to both physics and engineering students, it is unsurprising that there is also a corresponding overlap in the discipline-based education research efforts associated with both fields. Indeed, engineering education research (EER) is a more recently established but quickly growing field, with modern efforts driven by changes in the Accreditation Board for Engineering and Technology (ABET) criteria in the mid-1990s. (See [27] and [28] for a more complete history.) The basic research tools used in EER are similar to those used in PER, and the goals of the research (such as investigating student understanding of core concepts [29] and improving instruction) are likewise typically aligned between both fields. However, many engineering degrees require a year of basic science and mathematics courses prior to the core engineering course sequence, and thus EER has naturally tended to focus on non-introductory topics from its inception.

While still limited, there are a number of relevant studies from EER on circuits and electronics topics beyond what is typically covered in an introductory physics course. Mazzolini *et al.* have reported on the impact of replacing a subset of traditional lectures on operational amplifiers [30] and resonance [31] with interactive lecture demonstrations. Andreatos and Kliros [32] have published on teaching methods for identifying the roles of bipolar-junction transistor (BJT) within circuits, demonstrating how to convert groups of multiple transistors into single logical groups denoted by their particular function. However, none of these studies focused on student understanding of the behavior of circuits containing particular circuit elements.

Despite the overlap of numerous subject areas (such as mechanics, thermodynamics, and electronics) between physics and engineering programs, the learning outcomes targeted by the corresponding courses in both programs may in fact depend on the disciplines in which they are taught. As a result, the instructional sequences, learning tools, and mathematical formalisms emphasized may vary between electronics courses depending on the discipline in which a subject is taught, as has been reported in cross-disciplinary research conducted in the context of thermodynamics [33]. While it is plausible that such variations in instruction may lead to differences in student conceptual understanding, to date little work has systematically probed for disciplinary differences in the context of electronics.

Thus far, EER and PER studies of topics that cross disciplines, including electronics, have been conducted independently for the most part; work in one field typically has not built upon work in the other. It is even rarer for instructional materials or research instruments developed in one field to be used in the other, although several concept inventories developed for physics students have seen use in engineering courses [34]. There is a growing recognition that researchers in engineering education and physics education could benefit from increased collaboration, considering the extensive overlap in research focus. Indeed, there is already interest in collaboration between the EER and PER communities, as demonstrated by an article in a special 2008 edition of the *Journal of Engineering Education*, which was co-authored by a leading EER researcher and a leading PER researcher [35]. This paper served as a joint effort aimed at identifying what subjects are most important for engineering students to understand and what is known about student learning, with a focus on results from PER. The scope of this paper was limited to providing a survey of current research relevant to engineers, and to date there have been relatively few significant efforts to study student performance, understanding, and difficulties across disciplines (for example, see [36]).

For the aforementioned reasons, a primary goal of this dissertation is the documentation and characterization of the reasoning employed by students while analyzing electronic circuits. In particular, this dissertation serves to document the kinds and relative prevalence of specific student difficulties (discussed in the following chapter) with electronics across a spectrum of courses from both physics and engineering. To this end, a number of free-response questions were used to probe student reasoning across a wide range of subject material, from voltage dividers (which may be analyzed using only introductory physics knowledge) to bipolar junction transistor amplifier circuits (which rely on sophisticated inferential reasoning chains to model their behavior). While this research includes circuits incorporating a variety of different passive and active elements, student understanding of several foundational ideas (*e.g.*, Kirchhoff's laws) that are critical for properly interpreting the behavior of all steady-state circuits was also explored.

In general, the research on analog electronics described in this dissertation was guided by the following two broad research questions:

I. To what extent do students develop a functional understanding of certain types of circuits (*e.g.,* bipolar junction transistor circuits) after relevant instruction? In particular:
    a. What ideas and approaches, both correct and incorrect, do students employ when analyzing these circuits?

    b. What specific conceptual difficulties do students exhibit?

II. To what extent are identified difficulties dependent upon disciplinary context, and to what extent do they transcend disciplines?

In addition, a portion of this dissertation is devoted to an investigation of student troubleshooting in the context of the electronics laboratory. Specifically, Chapter 8 focuses on pivotal decision-making episodes occurring in interviews in which pairs of students were troubleshooting an operational amplifier circuit. These interviews were then systematically analyzed in order to better understand the processes through which students select and enact testing strategies. For this particular investigation, the framework of socially mediated metacognition is introduced as a tool to better understand the collaborative nature of the interactions that occurred during troubleshooting. This chapter is based upon a paper submitted to *Physical Review Letters – Physics Education Research,* and is presented as a self-contained investigation due to the somewhat different focus of the research. The work on troubleshooting addresses the following research questions:

III. To what extent are student groups engaging in metacognitive behaviors while troubleshooting a pre-assembled operational amplifier circuit?

IV. What role does metacognition play in the process of decision-making while troubleshooting?

The unifying goal of all investigations discussed in this dissertation is the documentation and characterization of the nature of thinking students use when reasoning about electronics. To this end, free-response questions covering a variety of topics and range of difficulty were administered to students in a number of different courses. This approach allowed for the investigation of how students were incorporating ideas about new electronic devices into their thinking as well as how students applied foundational circuits concepts in novel circuit contexts. Furthermore, the analysis of student responses was conducted in sufficient detail to inform the development of research-based instructional materials. In addition, the study on troubleshooting provided insight into what approaches students employed in a laboratory setting when working on an ill-defined electronics problem.

Chapter 2 presents a review of the relevant background research on circuits and electronics, which informed the current investigation. Chapter 3 includes an overview of the courses in which data were gathered, in order to provide a sense for the breadth of content coverage across all courses. More specific details about course instruction on individual devices and circuit configurations are introduced later in the relevant sections of Chapters 4-8. Chapter 3 also includes a discussion of the particular research methodologies employed as well as the statistical analysis tools used.

Chapters 4-6 describe cross-disciplinary investigations of student understanding of topics that are common to courses in both engineering and physics, with each chapter focusing on a single class of circuits. Chapter 4 discusses student understanding of voltage division and circuit loading. Chapter 5 describes an investigation of student understanding of basic diode circuits, and Chapter 6 focuses on student understanding of operational amplifier circuits. These chapters present both quantitative and qualitative evidence from written student responses in order to examine student understanding after relevant instruction and to identify student difficulties associated with each class of circuits. These chapters also serve to examine potential differences in performance or the relative prevalence of specific difficulties in different populations (such as students enrolled in physics and engineering courses or students enrolled in introductory and upper-division courses.)

Chapter 7 focuses on an in-depth investigation of student understanding of bipolar junction transistor (BJT) circuits. This chapter also discusses the development of a suite of free-response questions, each targeting different aspects of basic transistor circuits. This complementary set of tasks was used to gain better insight into those aspects of transistor circuits that students generally understand and those with which students struggle.

Chapter 8 presents a self-contained investigation of student troubleshooting of an operational amplifier circuit. The framework of socially mediated metacognition is used in order to characterize pivotal decision-making episodes during the task of troubleshooting. In particular, the chapter explores socially mediated metacognition as an explanatory mechanism for how students come to further substantiate their ideas when working in groups.

Chapter 9 summarizes the findings from the previous chapters, and highlights commonalities across multiple circuit contexts. Due to the broad data corpus associated with the investigations described in this dissertation, conclusions drawn from these studies may be used to support the generalizability (or lack thereof) of the observed conceptual difficulties across a wide variety of circuit contexts and instructional environments. Furthermore, Chapter 9 reflects on the extent to which large-scale performance differences between engineering and physics courses were observed. Possible explanations for documented phenomena, as well as suggestions for future research tasks, are discussed along with the implications for instruction originating from this research.

Chapter 2

PRIOR RESEARCH

The field of physics education research (PER) comprises a broad spectrum of work, ranging from investigations of K-12 instructors' knowledge for teaching to the creation and validation of assessment tools for undergraduate courses. However, the unifying theme is the focus on the teaching, learning, and understanding of physical phenomena. To this end, a large body of the research within PER has been devoted to discovering what ideas students have about physics both before and after instruction. In practice, this has been done by analyzing classroom data, video interviews, and written responses in order to discover underlying themes in student answers and reasoning [1]. This research project is thus well aligned with the goals of PER but also explores a niche that has received little attention thus far, thereby extending the breadth of the literature.

As mentioned in the introduction, engineering education research (EER) has a significant overlap with PER due to commonalities in course content. EER is a more recently established but quickly growing field, with the modern effort tracing its roots to changes in the ABET accreditation made in the mid-1990s. (See [27] and [28] for a more complete history.) EER has been focused on non-introductory topics from its inception, as many engineering degrees require a year of basic science and mathematics courses prior to the core engineering course sequence. Despite differences in the disciplines, the basic research tools used in EER are similar to those used in PER, and several of the research goals (such as investigating student understanding of core concepts [29]) are likewise aligned in both fields.

The research documented in this dissertation draws on elements from a number of disparate fields. In part, this is due to the fact that it covers investigations of topics that are beyond the scope of what is well documented in the existing PER literature. However, the research methodologies employed have been adapted from previous studies in introductory physics, particularly work on dc circuits. Thus, whenever applicable, the findings from this dissertation will be related to difficulties observed in introductory physics. While the literature from EER is somewhat more limited, it is extremely relevant, as it is often the only source of applicable research on student understanding for many topics in electronics. Thus, this chapter presents a targeted overview of work from both PER and EER on electric circuits and analog electronics in order to better contextualize the research documented in this dissertation.

2.1 Physics Education Research

Research on student understanding of electric circuits in introductory physics courses was among some of the earliest work conducted in PER [15,16]. Student understanding of introductory circuits concepts has been an area of continued study, and such work has typically focused on identifying and investigating the scope of student difficulties or examining the effectiveness of specific instructional interventions designed to address known difficulties. (Mulhall *et al.* [37] contains an overview of ongoing research.) Perhaps the most influential work is a pair of papers published by McDermott and Shaffer in 1992 describing a systematic investigation of conceptual understanding of basic DC circuits among introductory university students and K-12 teachers [19], and the subsequent development and testing of curricular materials aimed at addressing the difficulties identified in the first study [20]. While a prior study of student understanding of circuits did characterize some common difficulties students held with regard to potential difference [16], it lacked detail in terms of identifying the prevalence of difficulties and neither designed nor tested specific instructional interventions.

McDermott and Shaffer found that after traditional instruction, many students did not demonstrate a coherent conceptual model for simple DC circuits. In particular, they noted that students frequently treated batteries as sources of constant current, and that students often expressed the idea that current was "used up" by circuit elements. Based on such observed difficulties, they designed and tested a series of tutorials, which have since been published as part of a larger collection in *Tutorials in Introductory Physics* [7]. In the tutorials, students are typically asked to make predictions that will result in a logical inconsistency or contradictory observation if an incorrect model (*e.g.*, that a battery is a constant current source) is used. These tutorials have been shown to lead to marked improvements in student reasoning about simple DC circuits [20]. In addition, it has been reported that students who had tutorial instruction on topics in introductory electricity and magnetism (including DC circuits) in introductory physics courses had higher gains in conceptual understanding in later courses (as measured by the Brief Electricity and Magnetism Assessment, or BEMA) than those who did not [9].

Another example of research-based instructional materials is the body of Physics Education Technology (PhET) simulations developed at the University of Colorado Boulder [38]. These educational programs were carefully designed to help students develop appropriate models of a number of physical phenomena, including the behavior of electrical circuits. Indeed, it has been shown that under appropriate conditions, such simulations may be as effective of a learning tool as the physical laboratory [39]. At the time of writing this dissertation, the available simulations pertaining to circuits or electronics were focused almost exclusively on introductory circuits topics, with a single simulation on the microscopic behavior of electrons in diode circuits [38]. Thus, while development of PhET simulations has continued, content for upper-division electronics topics has not yet been created.

In addition to focusing on the development of instructional materials, a number of research efforts have been directed toward creating research-validated assessment tools. The Determining and Interpreting Resistive Electric Circuits Concepts Test (DIRECT), developed by Engelhardt and Beichner is one such assessment which has been systematically tested and validated [6]. It was found that both high school and university students frequently experienced difficulties with understanding the effects of multiple batteries in series or parallel, and in translating from a symbolic representation of circuits to a realistic one. Since its development, the DIRECT has often been adopted as a standard instrument for measuring the effect of instructional interventions, such as new instructional sequences for capacitive circuits [40] or multiple bulb circuits [41]; the instrument has also been used for ongoing assessment of student understanding of circuits concepts [42]. Additionally, the DIRECT has become relatively well-known in other education research disciplines, and has been used in the context of science education [43], artificial intelligence tutoring programs [44], and engineering education [45,46]. While a number of similar concept inventories exist (such as the Circuits Concept Inventory, AC/DC Concepts Test, and Electric Circuits Concept Evaluation), all target a similar spread of topics and the DIRECT is still one of the most widely used [34]. It should also be noted that the Brief Electricity and Magnetism

Assessment (BEMA) [5] is a widely used assessment in introductory courses, and includes several items on dc circuits.

There are a number of ongoing efforts investigating student understanding of introductory circuits content. For example work by Smith and van Kampen [41] discussed the development and testing of an extension of the *Physics by Inquiry* [47] curriculum on the treatment of circuits with multiple batteries in multiple loops. Through instruction, students became more adept at making accurate predictions for such circuits, but the authors noted that students often found it difficult to transfer their understanding to new contexts. The same authors also worked to create another extension to the *Physics by Inquiry* curriculum to include RC circuits, which resulted in substantial improvements in student qualitative reasoning [48]. Other relatively recent work includes a study of the effect of context in student understanding of open circuits, where John and Allie created eight different variations a single base task [49]. Variants included either a resistor, a heater, or a light bulb, and students were asked if there would be current through the element, if charge would flow through the element, or if the bulb would light/if the heater would heat up. Despite none of the circuits consisting of a closed loop, only 15% of students gave correct responses across all eight variations, highlighting the importance of context on student reasoning.

There exist several studies that used upper-division courses as a context for probing student ideas about foundational circuits concepts. For instance, Getty used the DIRECT as a metric to assess the effectiveness of changes to instruction in his upper-division electronics laboratory course [23]. After modifying his course to use inquiry-style methods to teach basic circuits concepts of voltage and current, the results from the

DIRECT suggested that his instructional modifications may have led to improved scores and improved understanding. However, it should be noted that this result was not statistically significant due to a small sample size. This particular work is notable in that it serves as a small bridge between PER and EER; the course was taught to upper-division physics students and the research is clearly based upon only literature in PER, but the instructor was in the department of engineering and the results were published in an EER conference proceedings paper. Unfortunately, since the work solely focused on physics students, it did not serve as a cross-disciplinary study of the learning and teaching of circuits and electronics.

In another instance of investigating student understanding of foundational circuits concepts in upper-division courses, Stetzer *et al.* [21] examined student understanding of complete circuits and Kirchhoff's junction rule in both upper-division and introductory courses. They found that between one-third and one-half of introductory students completed a calculus-based introductory physics course on electromagnetism without developing a functional understanding of complete circuits. More importantly, such difficulties were persistent; over half of the students in upper-division electronics-related courses demonstrated similar difficulties with the application of Kirchhoff's junction rule to a single loop circuit, and over half of graduate TAs were unable to answer a question about a two-battery open circuit correctly. Such research suggests that student difficulties with foundational circuits concepts may very well impact student performance on more advanced topics in electronics.

There are relatively few PER studies on upper-division electronics topics. As part of a study that examined student selection of resources in nearly novel situations, Sayre *et* *al.* [13] asked students to rank the currents through resistors in six differently configured diode circuits. Fewer than half of the 11 students in the interviews were able to correctly predict the behavior of the circuits presented to them. The authors suggested that students who were incorrect were typically treating the diodes as if they were ohmic elements (*i.e.,* obeyed Ohm’s law) when forward biased. Unfortunately, the data were extremely limited in scope, and the investigation of student understanding of diode behavior was secondary to the author’s research task. An in-depth, multi-institutional investigation of student understanding of operational amplifier circuits by Papanikolaou *et al.* was published in 2015 [36]. The article, which was co-authored by the writer of this dissertation, primarily focused on physics students enrolled in electronics courses at three different institutions. A relatively short comparison of student performance in physics and engineering courses was also included, which was drawn from the work reported in Chapter 6 of this dissertation. Research on the role of modeling in student troubleshooting of operational amplifier circuits was reported in an article co-authored by the writer of this dissertation, and was published in 2016 [51]. Research on the role of socially mediated metacognition in student troubleshooting of operational amplifier circuits, which emerged from a companion analysis of the same data corpus, is the focus of Chapter 8 of this dissertation.

2.2 Engineering Education Research

While it was noted previously that engineering education research has a history of research in advanced courses, there are relatively few EER studies that have been conducted in upper-division courses on electronics. In addition to more traditional research articles, however, there are several relevant didactic papers, which typically outline teaching approaches that are either supported by student opinion surveys or observations by an instructor. While such papers do not contribute to a broader understanding of the difficulties students encounter during instruction, they do indicate interest in specific content areas, suggesting that further research would be well received. In the sections that follow, relevant EER literature will be discussed

Streveler *et al.* [52] conducted a Delphi study among other engineers in teaching positions in 2006, with the aim of identifying those concepts that were considered to be the most important and those that were considered to be the most difficult. From the ten experienced electrical engineering faculty they recruited, the concepts they considered most important converged to the following: AC steady state circuit analysis, Kirchhoff's Laws, Thévenin/Norton equivalence, and the five fundamental electrical quantities (charge, current, voltage, power, and energy). However, in subsequent interviews with students, they found a mismatch between student performance and some predictions from the Delphi study; namely, students often demonstrated poor performance on questions targeting concepts that were rated by professors to be both important and well understood. This suggests that a priori expectations of difficulties may not align well with what occurs in practice. As will be discussed later, this motivates the use of a modified grounded theory approach [53,54] to guide the analysis of written student data throughout the investigations of student understanding documented in this dissertation.

2.2.1 Research on Circuits

In several instances, EER researchers have applied concept inventories from PER in engineering courses. For example, the DIRECT has been used in EER as an instrument to assess student understanding in introductory circuits courses at Purdue [55], the

University of Auckland [56], and the Dublin Institute of Technology [46,57]. While some of these studies do include comparisons of engineering student performance to the reported results for introductory physics students, the causes for any differences (or even whether or not the differences were statistically significant) were not a focus of these works. The DIRECT has also been used as an assessment tool to measure the impact of novel teaching methods [58] and to test for differences in the prevalence of specific "misconceptions" among freshman, sophomore, and senior Electrical Engineering Tech (EET) students [59].

In a project on classroom interventions, Timmermann *et al*. reported on the development [60] and subsequent refinement [61] of a tutorial-style activity designed to aid students in connecting ideas about electric potential and voltage as a potential difference. Even after instruction, it was common for approximately half of students to state that there would be no potential difference across an open switch. A subsequent paper [62] noted that this difficulty has been commonly observed in various PER investigations of circuits [6,63]. Thus, while this work was conducted in introductory circuits courses, it is an example of research building ties between both disciplines. Indeed, it should be noted that one of the investigators received his Ph.D. for work in physics education research.

A series of interviews were conducted by Timmermann and Kautz in order to investigate student understanding of circuit theory as a model [64]. They noted that students had trouble recognizing valid circuits (*i.e.*, configurations that did not violate Kirchhoff's laws) and that students struggled to relate circuit diagrams to their mathematical models. As the sample size for the interviews was quite small (ten students), there is still need for a broader investigation of the scale and nature of these difficulties.

One of the topics that has received a substantial amount of attention from EER researchers is student understanding of the behavior of ac circuits involving basic resistive, capacitive, and inductive elements. For example, Kautz [65] conducted investigations into student understanding of phase relationships in AC circuits among first-year engineering students and junior physics students, and subsequently developed tutorial worksheets informed by his findings. In particular, he noted that students had difficulty with a number of important ideas, including: (a) that voltages across parallel elements and currents through elements in series must be the same regardless of context; (b) that there are characteristic phase properties associated with resistors, inductors, and capacitors; and (c) that current and voltage are not necessarily in phase in an ideal ac voltage source. Based on Kautz's questions, Bernhard, Carstensen, and Holmberg conducted an independent investigation of student understanding of phase [66]. They also observed that the majority of students (>70%) tended to ignore phase entirely when summing voltages or currents.

In order to better understand what ideas students had about RC filters, Coppens, De Cock, and Kautz [13] conducted a series of interviews with students who had completed a 2nd year electronics engineering course. They documented a number of specific difficulties exhibited by students and, whenever possible, related them to previous relevant literature. For example, when asked to analyze a high-pass filter, three out of the four interviewees used reasoning primarily based on current, and were subsequently unable to correctly predict the behavior of the circuit. In addition, the authors also documented student difficulties with understanding potential as well as a general lack of conceptual understanding regarding the behavior of capacitors in ac circuits. Furthermore, they reported difficulties with frequency representation, recognizing phase shifts, and understanding of real-life signals. Since their results were based on an extremely limited number of interviews, a follow-up study was designed to investigate the prevalence of student difficulties utilizing free-response questions [67]. Among the key findings were that many students could recognize and construct signals when asked, but students struggled to interpret signals provided to them, particularly in the frequency domain.

It should also be noted that Mazzolini, Daniel, and Edwards reported on an assessment of Interactive Lecture Demonstrations (ILDs) on resonance in *LRC* circuits [31]. A separate paper details two of the most common difficulties in the activity: misinterpreting what it means for a phase to lead or lag, and inappropriately summing root mean square (RMS) voltages in LRC circuits [68].

2.2.2 Research on Electronics

The most relevant work on operational amplifier (op-amp) circuits was performed by Mazzolini *et al,.* where the impact of replacing a subset of traditional lectures on operational amplifiers with ILDs was examined [30]. The researchers developed a seven-question instrument to assess the impact of the demonstrations on student understanding of op-amps. From a combination of data from written questions, student surveys, and focus group discussions, they concluded that there was an improvement in student understanding resulting from the implementation of ILDs in the course. While the authors noted improvements in understanding, they did not discuss in detail what specific ideas about op-amps students struggled with either before or after instruction. Thus, as they do not report specific difficulties, it is not possible to make detailed comparisons between their results and those reported in this dissertation. Furthermore, it is important to note that of the seven assessment questions posed to students in class, two of the most difficult questions (on which students showed no improvement after implementation of the ILDs) had variations in the circuit that had not been seen by students previously, suggesting that student understanding after ILD instruction was not as robust as the instructor had expected. Furthermore, the authors noted that students tended to employ ‘shallow learning’ approaches in which they memorized standard op-amp circuit configurations and gain formulas, which could lead to students having difficulties when circuits were drawn in a non-traditional manner or labeled in unorthodox ways.

As discussed previously, concept inventories have been used widely within the EER community (see [69] for an overview). A number of new concept inventories have been developed for upper-division subjects including thermodynamics [70], fluid mechanics [71], and systems and signals [72]. The most relevant to this dissertation is the Systems and Signals Concept Inventory (SSCI), which was developed for electrical engineering courses typically taken by third-year students. Questions in the SSCI were informed, in part, by interviews with students that were designed to uncover prevalent incorrect lines of reasoning on topics related to systems and signals [73]. Furthermore, the validity of the concept inventory has been tested with comparisons to interviews and test questions [74]. However, the concepts surveyed by this assessment (*e.g.,* convolution of signals) were beyond the scope of what is typically taught in the physics electronics course offered at UMaine and were therefore beyond the scope of the research described in this dissertation.

J.M. Oliveira and J.P. Estima de Oliveira reported that the use of more open-ended, qualitative problems was productive in improving students' conceptual understanding in an electronics course [75]. During the class period, students were asked to work through a variety of problems on various analog electronics topics, including operational amplifier circuits and transistor circuits. From their classroom observations and written student responses, they informally concluded that the activities "seemed to have an impact on the students' conceptual reasoning." No specific difficulties with content were noted, but they did report that the engineering students considered purely qualitative questions to be "too theoretical," and that they were less engaged in such tasks. Ultimately, this work demonstrated that there is perceived value in the use of qualitative, conceptual questions in EER research, but it did not probe the specifics of how effectively such questions supplemented learning, nor what difficulties were being addressed.

Among the didactic papers, one is of particular relevance to this dissertation. Andreatos and Kliros [32] published their methods for identifying the roles of bipolar-junction transistor (BJT) within circuits. Their intent was to demonstrate how to convert groups of multiple transistors into single logical groups denoted by their particular function (*e.g.*, constant current source, active load, or emitter follower). They identified ten such configurations and also noted that similar groupings could be constructed for field effect transistors (FETs), but have yet to publish on the use of any of these groupings in a classroom setting. The existence of this paper, however, indicates that there is interest in improving instruction on more advanced electronics topics.

2.3 Summary

Overall, educational research on circuits and analog electronics has focused primarily on introductory topics in either physics or engineering, where significant progress has been made in addressing student difficulties. Some research has extended to the contexts of upper-division courses, but there are relatively few studies on student understanding of topics exclusive to electronics courses, and fewer still that could inform research-based instructional improvements. Furthermore, most of the reported investigations have been constrained to courses within a single educational discipline. This dissertation thus serves to fill several notable gaps in the existing literature base, by describing a cross-disciplinary investigation of student understanding of analog electronics.

Chapter 3

RESEARCH CONTEXTS AND METHODS

This chapter provides a detailed overview of the instructional environments in which data were collected as well as the research methods that were employed. In order to conduct a detailed investigation of student understanding of analog electronics, it was most appropriate to perform research across a variety of different courses. This chapter begins with an overview of the relevant circuits or electronics content covered in each course investigated, and includes general information about the nature of instruction. This chapter continues with a summary of what data were gathered as well as the justification for why particular data were collected. The theoretical framework guiding the overall investigation is discussed, and relevant analysis frameworks used in the interpretation of written student responses are presented. While statistical comparisons were not the primary focus of this work, there were several occasions in which they were used to examine performance differences between courses or the effectiveness of an instructional intervention. As a result, this chapter concludes with a discussion of the relevant statistical tests employed as well as how the results of such tests were interpreted.

3.1 Courses Studied

Data for this dissertation were collected in a total of seven courses at three separate universities: the University of Maine (UM), the University of Washington (UW), and the University of Colorado, Boulder (CU). Research was conducted across three engineering courses and four physics courses, as shown in Table 3.1. While all but one of the courses primarily focused on topics in circuits/electronics beyond what is taught in an introductory physics course, the specifics of both the topics covered and the mathematical tools used varied considerably across the courses. Thus, many research tasks on more sophisticated topics (such as diode circuits) were only applicable to a smaller subset of courses, while tasks on other topics (such as voltage division) were applicable to all. In order to provide the reader with sufficient context both for understanding why particular research tasks were or were not administered in a given course and for interpreting the subsequent results, each course is characterized below in terms of expected outcomes, typical activities, and the nature of lecture and laboratory instruction.

3.1.1 University of Maine

The investigation was conducted across five courses at the University of Maine, consisting of two courses in the Department of Physics and Astronomy and three courses in the Department of Electrical and Computer Engineering.

3.1.1.1 Introductory Physics II

Due to the focus on upper-division electronics topics, only data pertaining to a single task were collected in the Physics 122 course (Physics for Engineers and Physical

	UM					UW	CU
	Physics		Engineering			Physics	Physics
Course Description	Electronics	Physics II	Circuits, Majors	Circuits, Non-majors	Electronics	Electronics	Electronics
Course Number	PHY 441	PHY 122	ECE 210	ECE 209	ECE 342	PHYS 334	PHYS 3330
Year Taken	Junior	Freshman	Sophomore	Sophomore+	Junior	Junior	Junior
Textbook	Diefenderfer, Galvez, or Lawless	Knight	Nilsson and Riedel	Nilsson and Riedel	Sedra and Smith	Horowitz and Hill	Horowitz and Hill
Typical Enrollment	10-20	200+	40-60	20-90	25-45	30-80	30-60
Laboratory Time	2 Hours	2 Hours	In-class	None	3 Hours	3 Hours	3 Hours
Lecture Time	2 Hours	2 Hours + 2 Hours Recitation	5 Hours	3 Hours	3 Hours + 1.5 Hours Recitation	2 Hours	2 Hours

Table 3.1. Summary of courses studied

Scientists II), which is the second semester of the introductory calculus-based physics sequence at the University of Maine. This course is required for all physics (and engineering physics) majors and all engineering majors. The on-sequence variant of the course is typically taken during the second semester of the 1st year of undergraduate study, with over 200 students enrolled across two different lecture sessions. The relevant instruction on electric circuits takes place over 3-4 weeks and includes an introduction to resistance, voltage, and current, as well as coverage of Ohm's law, Kirchhoff's voltage law and current law, multiple-battery circuits, equivalent resistance/capacitance of elements in series and parallel, and time-domain behavior of RC circuits.

The course has two 50-minute lecture sessions per week with the professor, as well as an additional pair of 50-minute recitation sections held with a teaching assistant (TA) and a Maine Learning Assistant (MLA) wherein students work through a mixture of qualitative and quantitative problems as well as tutorials from *Tutorials in Introductory Physics* in small groups. The laboratory portion of the course consisted of weekly 2-hour sessions, typically with three or four laboratory activities on circuits. Laboratory sections consist of 20–25 students overseen by a teaching assistant, and experiments are expected to be finished entirely within the lab session. Short written reports or worksheets are typically due a week after the completion of the lab.

Content on circuits is typically covered on one of three midterm exams as well as on the cumulative final. Students have weekly homework assignments in the course, and thus students complete several assignments on circuit analysis.

3.1.1.2 Physical Electronics Laboratory

The physics electronics course at the University of Maine (PHY 441 – Physical Electronics Laboratory) was the primary focus of this study. This one-semester course is required for physics majors, and is typically taken in the first semester of their junior year in the degree program, with approximately 10–20 students enrolled each year. The only prerequisite circuits instruction for this course is that included in the second semester of the introductory calculus-based physics courses (Physics II). The course begins with a review of introductory circuits topics such as Kirchhoff's laws and Ohm's law, and then covers voltage division, Thévenin equivalent circuits, impedance, ac circuits, filters, operational-amplifier circuits, diode circuits, and bipolar junction transistor circuits.

The course has two 50-minute lecture sessions per week, with time in class divided between lecture, clicker questions (with peer discussions), and guided problem-solving activities or tutorials. During most years of the investigation, the latter two activities were facilitated by the course instructor as well as a pair of undergraduate MLAs and a single graduate student (the author). In addition to the lecture, the course includes a weekly 2-hour laboratory session, with students divided into two sections. Students are expected to complete their experiments within this time frame, although exceptions are made on a case-by-case basis. Students work through guided lab activities in pairs, with the instructor, a single MLA, and a graduate student available for assistance. The course is designated as "writing intensive," and students are therefore required to complete formal written lab reports for approximately half of their experiments; these reports are critiqued and graded by the course instructor as well as a technical writing instructor.

The course culminates with a two-week project in which groups of three or four students work together to design, construct, and test analog temperature controllers.

The course typically includes a final exam and a single midterm exam, focusing on the formal analysis (both quantitative and qualitative) of analog circuits. No homework is assigned apart from the laboratory reports and several pre-post surveys.

3.1.1.3 Electric Circuits

The engineering circuits course for ECE majors (ECE 210) at the University of Maine contains a significant amount of content that overlaps with the junior-level physics electronics course. This one-semester course is required for electrical and computer engineering majors, and is typically taken by students during the first semester of their sophomore year, with approximately 40–60 students enrolled at a time. Physics II (PHY 122) is a co-requisite, and in practice many of the students in the engineering circuits course for majors were concurrently enrolled in the off-sequence version of Physics II. Content includes all circuits topics in the introductory physics course, with the addition of: passive sign conventions, mesh and nodal analysis, Thévenin and Norton equivalent circuits, operational amplifier circuits, transient analysis of RC/RL/RLC circuits, ac behavior of RC/RL/RLC circuits, power analysis in ac and dc circuits, and two-port networks. It is important to note that this is a four-credit-hour course, and that prior to 2011 the content was split between a pair of three-credit courses.

The course has five 50-minute lecture sessions per week, with class time divided between lectures introducing new content, review of homework questions, and in-class laboratory activities. Although the course does not include an official laboratory component, students purchase a multimeter and a set of basic components for use in in-class activities. Such activities were generally completed alone or in small groups, following a guided worksheet format.

The course has a final written exam as well as three in-class midterms, focusing mostly on quantitative circuit analysis. Homework is assigned weekly, and the instructor collects and grades only a subset each the assignment.

3.1.1.4 Fundamentals of Electric Circuits

The engineering circuits course for non-majors (ECE 209) at the University of Maine is 3 credits. This one-semester course is an elective for engineering students who are not majoring in either electrical engineering or computer engineering. While it is possible for students to take the course as soon as the first semester of the sophomore year, it is not a prerequisite for any additional courses and in practice many students are enrolled during their junior or senior years. Topic coverage is similar to that in ECE 210, with some later topics covered in less depth or omitted.

Approximately 20–90 students are enrolled in the course each semester, with considerably fewer students in the off-sequence spring offering. As is the case for the circuits course for ECE majors, Physics II is a co-requisite, although in practice many students had completed Physics II in a prior semester. The non-major circuits course consists of three 50-minute lecture sessions per week, and class time is divided between instruction on new content, example problems, and homework review. The course does not include a laboratory component.

Homework for the course is assigned, collected, and graded on a weekly basis. The course has a final written exam as well four in-class midterms exam, focusing mostly on quantitative circuit analysis.

3.1.1.5 Electronics I

The engineering electronics course (ECE 342 – Electronics I) at the University of Maine has an extensive degree of content overlap with the physics electronics course. This course is the first part of a two-semester sequence that is required for electrical and computer engineering majors, and is typically taken in the first semester of their junior year in the degree program; approximately 25–40 students are enrolled in the course each year. Both the calculus-based introductory physics sequence (including Physics II) and the engineering circuits course (as well as an additional circuits laboratory course) are prerequisites. The course begins with a review of operational amplifiers followed by a discussion of the non-ideal properties of real op-amps, and then provides an overview of the electrical properties of semiconductors before covering diode circuits, field effect transistor circuits, and bipolar junction transistor circuits.

The course has three 50-minute lecture sessions per week, and class time is divided between lecture, clicker questions (in later years), and weekly quizzes. A weekly 90-minute recitation session provides students with additional practice on a variety of topics, with assistance from the instructor. The course also includes weekly 3-hour laboratory sessions with students divided into multiple laboratory sections. There are a total of nine laboratory activities for the course (some covering multiple weeks) as well as a laboratory practical examination. Students have access to the laboratory space at other times as well, and are expected to spend additional time to complete their experiments and projects if needed. Students work through guided lab activities in pairs, with both a TA and MLA facilitating. Similar to the physics electronics course, the engineering counterpart is also designated as “writing intensive,” and students are required to complete several extensive formal laboratory reports on their experiments. However, students in the engineering course are also required to take a one-credit writing seminar (ECP 342). In addition, students purchase and work with an Analog Discovery device, which is a portable multi-function USB oscilloscope that can be used essentially anywhere.

Homework is assigned biweekly, with a subset of assigned problems graded for credit. The course includes a final written exam and three midterm exams, focusing on the formal analysis (both quantitative and qualitative) of analog circuits with a mix of multiple choice and open-ended questions.

3.1.2 External Institutions

In addition to the courses surveyed at the University of Maine, data were collected from two physics electronics courses at the University of Washington (UW) and the University of Colorado Boulder (CU).

3.1.2.1 Electric Circuits Laboratory I

The physics electronics course at the University of Washington, Physics 334 (Electric Circuits Laboratory I), is comparable to its Maine counterpart. It is a one-quarter (~10 week) course that is required for physics majors, and is typically taken in the second quarter of the junior year, although it is also offered in the summer. Roughly 30-80 students are enrolled at a time. The calculus-based introductory physics sequence (Physics 121,122, and 123) is a prerequisite, which introduces students to the general behavior of electric circuits. The course content is similar to that covered in the physics electronics course at the University of Maine, with the addition of field effect transistor circuits and select topics on digital circuits at the end of the quarter.

The course has 2 50-minute lecture sessions per week. In addition to lectures, the course includes a weekly 3-hour laboratory session. In the laboratory, students work through guided lab activities in pairs, modified from the *Student Manual for the Art of Electronics* by Hayes and Horowitz [76]. Students are expected to complete the lab within the scheduled time and short lab reports are submitted at the end of the period.

The course typically includes a final written exam and one midterm exam. Homework is also assigned and collected weekly.

3.1.2.2 Electronics for the Physical Sciences

The physics electronics course (PHYS 3330) at the University of Colorado Boulder is also comparable to its Maine counterpart. It is a one-semester course that is required for all physics majors, and is typically taken in the first semester of the junior year. Approximately 30–60 students are enrolled in the course at a given time. The calculus-based introductory physics sequence is the sole prerequisite, and this sequence includes instruction on basic circuits. The course content is similar to that covered by the physics electronics course at the University of Maine, with additional coverage of field effect transistors and digital topics at the end of the course.

The course has 2 50-minute lecture sessions per week. In addition, the course included a weekly 3-hour laboratory session. Students have free access to the laboratory space outside of laboratory time, and are often expected to complete labs outside of the allotted time. In a typical lab, students work through guided experimental activities in pairs. Students are graded on laboratory notebooks, but the course does not share UM's formal writing requirements. The course culminates with a 5-week project where students work alone or in small groups to design and build a device of their choice.

The course typically includes a final written exam as well as several midterm exams. A pre-laboratory activity is due before each of the formal labs, and students are required to give a presentation on their group project at the end of the course.

3.2 Methodology

This work has two distinct components: an in-depth investigation of student understanding of key topics in electronics, and a study of the strategic decision-making processes that occur as students engage in the practical laboratory skill of troubleshooting electronic circuits. As such, the theoretical frameworks underlying the two broad investigations as well as the associated approaches to gathering, analyzing, and interpreting data differed substantially. A thorough overview of both the theoretical frameworks and the research methods employed in the troubleshooting investigation is presented in detail in Chapter 8 (which has been submitted to *Physical Review – Physics Education Research*). The discussion in the rest of this chapter is thus focused on the investigation of student understanding of analog electronics.

This investigation of student understanding was designed and conducted through the lens of the specific difficulties empirical framework [77–79], with the goal of identifying common incorrect responses given by students and subsequently characterizing the associated incorrect lines of reasoning (*i.e.*, specific difficulties) in sufficient detail to guide the development of instructional interventions. Thus, the focus is primarily on identifying difficulties that are the most prevalent, as the results from such an approach would be the most impactful for informing instructional improvements. However, for some tasks, there is sufficient data to comment on the relative prevalence of difficulties between different student populations, which may help in identifying if there are substantial differences in learning outcomes from differing instruction.

The specific difficulties framework focuses on the identification of those conceptual difficulties that students typically encounter during instruction. The term "specific difficulties" refers to incorrect or inappropriate ideas expressed by students, as well as flawed patterns of reasoning [77]. In order to better elicit such ideas from students, written free-response tasks were administered with explicit prompts for students to explain their reasoning.

3.2.1 Data Collection

Data on student understanding of circuits were collected primarily in the form of student written responses to free-response tasks administered as ungraded conceptual questions. Students were typically given ten to twenty minutes to complete the tasks during class. Several tasks were administered to students either on midterm or final exams; such instances are noted in each individual chapter. On a few occasions, research tasks were incorporated into homework assignments, usually due to time constraints. All such instances are noted, as it is plausible that students would respond differently when asked to complete a given task in class under time constraints and without notes versus out of class with essentially no time constraints and access to additional educational resources (*e.g.*, course notes). Supplemental classroom observations were performed and field notes were taken to better interpret student responses in terms of the methods and language introduced in class.

3.2.2 Data Sources

A number of topics are discussed in this dissertation, with the overall theme being subjects relevant to instruction in upper-division electronics courses. In particular, this dissertation explores student understanding of introductory circuits principles in new contexts as well as circuits and components from the electronics curriculum. Tasks related to the first category (understanding of basic circuit principles) are important for probing if and how students' reasoning changes as they acquire experience working with circuits. Additionally, student responses to such questions are more readily tied to the body of research on student understanding of basic circuits (*e.g.*, students employing local [19] or current-based [13] reasoning). Tasks based on circuits and devices that are first introduced in the electronics course provide critical information about how students are incorporating the ideas they have been taught into practice. Furthermore, such questions help explore the coherence of student ideas about fundamental principles (*e.g.*, Kirchhoff's laws).

Detailed discussions of all research tasks, along with the associated lines of correct and complete reasoning, are presented in Chapters 4-7. Furthermore, the timing of tasks, along with information about the courses and years in which they were administered, are presented in each sub-section. Most questions did not require extensive numerical calculations, and frequently students were asked to make comparisons between the behavior of similar circuits. To facilitate such comparisons, quantities affecting the answers were typically selected such that the calculated values would typically be integer quantities. Furthermore, circuits chosen for tasks were often slight modifications from forms of common circuits, in order to ensure that student responses were more reflective of their reasoning using relevant principles rather than memorized responses. Common to all questions was an explicit prompt for students to explain their reasoning. These explanations provided useful information about students' thought processes, and also provided insight into what issues should be targeted for instructional interventions. Indeed, such responses allow for the identification of both difficulties with circuits containing new devices as well as difficulties in applying foundational circuits concepts in new contexts.

Data were collected in a total of seven courses at three different institutions. It is important to note that only one research task was administered pre-instruction, as all of the other free-response questions contained new content that most students would likely not be able to attempt in the absence of relevant instruction. For introductory physics topics, students are more likely to have some relevant prior experience from everyday life or prior schooling, and thus conceptual questions or concept inventories (such as the FMCE [80]) are typically administered both before and after instruction in order to ascertain the impact of teaching. For the subject of electronics, students are less likely to have coherent naïve ideas about devices and circuits with which they have no familiarity (*e.g.*, operational amplifiers). As this study of student understanding was primarily designed to probe what difficulties exist after instruction rather than changes occurring due to instructional interventions, the lack of pre-test information is not a major constraint.

3.2.3 Analysis Methodologies

In order to interpret responses to questions and identify student difficulties, a qualitative analysis was performed by the author. While the answers given by students were typically unambiguous and usually fell within a small spread of explanations, the wording for their reasoning was often unique to each student. Thus, it was necessary to perform further analysis of students' reasoning in order to generalize responses sufficiently to be useful for informing instruction.

Specifically, a grounded theory approach [53,81] was employed to identify the general lines of reasoning used from the specific responses provided by students. This is in contrast to other possible, theory-driven approaches where a priori categories would be established based on the particular theoretical framework employed. As there is an insufficient body of literature (effectively none) that could predict what student ideas about most electronic devices might be, grounded theory provided the most suitable methodology for making sense of student work.

After the initial categorization, the lines of reasoning identified were refined into broader, more inclusive categories, which were based on difficulties noted in prior research when applicable. This was done in order to achieve a balance between uncovering new difficulties and recognizing existing trends across context (*e.g.,* tendencies to inappropriately treat circuit elements as ohmic). When creating new categories, care was taken to ensure that they captured the central features of an explanation, rather than extraneous information. The incorrect lines of reasoning emerging from this analysis therefore represent specific difficulties that students were observed to encounter within the task. When applicable, difficulties observed in only a subset of courses are related to the nature of the instruction students received, when it may assist the reader in interpreting student approaches.

In addition to identifying the prevalence of difficulties, there were instances when it was desirable to determine if there were differences between populations of students (*e.g.,* are there differences in responses between physics and engineering courses?) or before and after instruction. In addition, there were instances when, for the sake of clarity, it was desirable to combine data from multiple question administrations into a single dataset. In order to more objectively determine the answers to such questions, appropriate statistical tests were employed. As all of the data collected were categorical in nature (as opposed to continuum data such as duration of work), the relevant statistical tests used in this research were the χ^2 test and Fisher's exact test. Both the χ^2 test and Fisher's exact test can be used to determine if a statistically significant difference exists between portions of a contingency table (*i.e.*, a table relating of the frequency of student responses to another variable such course), however they are applicable in different circumstances, as discussed below.

The χ^2 test is best suited for dealing with relatively large sets of data, and is considered unsuitable to use if the expected value a cell within a contingency table is less than 5 (although this may be overly conservative [82]). Additionally, the test statistic assumes a continuous probability distribution, which may cause quantization errors when there are few possible outcomes. Despite this limitation, there are no upper limits on the sample size, and it is possible to calculate both p-values (determining if differences exist) as well as an effect size in the form of either ϕ or Cramer's V (both of which characterize the magnitude of any differences). Effect sizes are critical for making informed decisions about the utility of instructional interventions, and the American Statistical Association warns that p-values alone provide only weak evidence for or against a null hypothesis [83].

Fisher's exact test is suitable for small data sets, as it properly accounts for quantization effects that would lead χ^2 to estimate inappropriately small p-values, which would result in a subsequent over-estimate of significance. However, it is computationally intensive, and is unsuitable for overly large sample sizes. In addition, the only test statistic found from Fisher's exact test is a p-value, and thus information about effect size is lost. Thus, χ^2 was tested for suitability first and used wherever possible, as it provides additional useful information compared to Fisher's exact test.

When testing populations for statistical differences, the threshold of $\alpha = .05$ is used as the point of comparison for determining differences. This means that p-values lower than .05 will be accepted as evidence towards rejecting the null hypothesis (*i.e.*, that populations are identical). This is generally interpreted as indicating that there is a 1/p chance that the null hypothesis will be rejected when it was unwarranted to do so. When running multiple statistical tests on the same data set, it is important to be mindful of the increased probability of obtaining a false positive. The most basic way of accounting for this is to use the Bonferroni method of dividing the threshold of significance by the number of tests performed (*e.g.*, using $\alpha = .025$ if two tests are performed on the same dataset). In almost all instances, statistical tests were not repeated within the same data sets and thus adjustments were typically not warranted; the appropriate corrective factors are explicitly noted in the subsequent chapters of this dissertation when relevant.

Chapter 4

INVESTIGATING STUDENT UNDERSTANDING OF VOLTAGE DIVIDERS & LOADING IN PHYSICS AND ENGINEERING COURSES

Voltage division is a particularly ubiquitous and foundational concept in analog electronics. The most basic voltage divider circuit consists of two resistors (or, more generally, elements with impedance) in series and is used to produce an output voltage (V_{out}) that is a fraction of the circuit's input voltage (V_{in}), as shown in Fig. 4.1A In practice, many sub-circuits with two or more components in series may be treated as voltage dividers in order to quickly evaluate their behavior. However, students need to be able to determine the extent to which the addition of a circuit element across the output of such a divider circuit perturbs the output, a phenomenon known as "loading." Some relatively simple cases, involving purely resistive elements and ideal voltage sources, may be introduced in introductory physics courses, often in the context of real battery behavior. However, at UMaine formal instruction on loading typically occurs later in the instructional sequence, such as in sophomore-level engineering circuits courses or in junior-level electronics courses.

Several investigations of student understanding of introductory circuits have discussed student ideas related to equivalent resistance and voltage in simple DC circuits (for example, see [6,19]). However, to date, there have been no studies specifically focused on the ideas of circuit loading and voltage division, particularly as they pertain to upper-division physics and engineering courses on analog electronics. Thus, due to a lack of existing research on student understanding of these topics, this study serves to probe both general trends in student performance and key difficulties related to circuit loading. As such, it contributes to the existing research base on both introductory circuits and electronics across disciplines.

4.1 Research Questions

This project was designed to characterize student thinking on topics in analog electronics in sufficient detail to inform instructional interventions. As this study was performed in an explicitly interdisciplinary context, a secondary goal was to determine if there were differences in outcomes between similar courses in physics and engineering, and, if so, to attempt to attribute differences to instructional approach where possible. Furthermore, due to the ubiquity and broad applicability of voltage division, it was possible to collect significant data on how student understanding changes across course sequences. As mentioned previously, the introductory physics course on electricity and magnetism (which includes several weeks of instruction on circuits) is a pre-requisite for the junior-level electronics course in physics. As such, it might be expected that student performance at the end of the introductory course might be similar to student performance at the beginning of the electronics course. A similar conjecture may be made for student performance at the end of the introductory circuits course for engineering majors and the beginning of the engineering electronics course. Given the broad project goals as well as the specific course offerings in which voltage division was covered, the following research questions were developed to guide this investigation:

1. To what extent do students develop a functional understanding of voltage division and circuit loading throughout the instructional sequences on electronics in physics and engineering? In particular:

    1.1. Are there differences in learning outcomes from comparable courses offered in the two different disciplines?
    1.2. Are there differences in student performance at the end of the prerequisite course and the beginning of the more advanced course in each sequence?
    1.3. Does student performance differ before and after instruction in the junior-level electronics courses?
2. What specific difficulties emerge from student responses to written questions, and does the prevalence of difficulties vary between courses?

In order to answer these research questions, several free-response written questions were developed to probe student understanding of voltage division and loading. In this chapter, discussion is limited to a single task that was administered multiple times to a broad variety of different student populations.

4.2 Context for Research

Students enrolled in five different courses, all at the University of Maine, participated in this study. In the Department of Physics and Astronomy, both the introductory, on-sequence physics II course on electricity and magnetism and the junior-level physics electronics laboratory were surveyed. In the Department of Electrical and Computer Engineering (ECE), data were gathered from the junior-level electronics course as well as two variants of the introductory engineering circuits course either for ECE majors or for all other non-ECE engineering majors.

4.3 Overview of Instruction on Voltage Division and Loading

As voltage division is used as a basis for describing the behavior of a wide variety of circuits, it is typically one of the first topics covered in courses beyond introductory circuits in physics (*i.e.*, in the engineering circuits courses and the physics electronics course). The simplest voltage divider circuit consists of a single voltage source (V_{in}) in series with two resistors (R_1 and R_2), where the output voltage (V_{out}) is taken across R_2, as shown in Fig. 4.1.A. Since the circuit elements are all in series, there is a single current through all of them, which will result in a voltage drop from V_{in} to V_{out} due to R_1. This relationship may be expressed as $V_{out} = V_{in}(R_2/(R_1 + R_2))$. Thus, the voltage divider circuit serves to produce an output voltage that is strictly smaller than the input voltage.

Throughout both the engineering circuits course and physics electronics course, voltage division is used to varying degrees when introducing several new circuits. For example, basic filters built from resistors, capacitors, and inductors may be treated as voltage dividers by generalizing the treatment of voltage dividers to use impedances rather than resistances. Voltage division is also used in the context of operational amplifier circuits, where it is applied to divider chains that include the feedback loop (*e.g.*, in a non-inverting amplifier circuit, the divider chain connecting the op-amp output, the inverting op-amp input, and ground).

After students learn about the basic voltage division circuit, they are introduced to the idea of loading a circuit. Loading is a general term for attaching a new circuit element

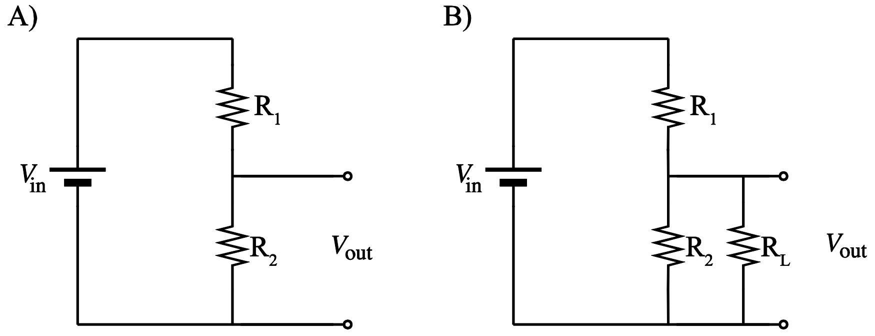


Fig. 4.1. Canonical voltage divider circuits A) Base voltage divider circuit. B) Loaded voltage divider circuit.

(or collection of circuit elements) across the output terminals, and is used frequently when discussing the behavior of interconnected circuits. A typical example of a loaded circuit is depicted in Fig. 4.1.B, where a load resistor R_L is added across V_{out}. Since R_2 and R_L are in parallel in such a circuit, they may be combined into an equivalent resistor, reducing the circuit once again to a simple voltage divider. For load resistances significantly larger than R_2, the equivalent parallel resistance will be essentially equal to R_2, and thus V_{out} will remain virtually unchanged by the added resistor (*i.e.*, the circuit is unloaded).

Voltage division and loading were taught in the first month of both of the engineering circuits courses as well as the physics electronics course. In the engineering electronics course, there was no additional explicit instruction on either topic, although students were expected to be familiar with both. In the portion of Physics II involving circuits, neither topic was explicitly introduced to students. However, from the basic circuits concepts taught (e.g., Ohm's Law, Kirchhoff's Voltage Law, and Kirchhoff's Current Law) students in the course would have the means to analyze voltage dividers and the effects of adding a load from first principles.

4.4 Data Collection

Table 4.1 summarizes the number of students participating in the study as well as when the data were collected. In most courses, the research task was usually given as an in-class conceptual problem with approximately 10 minutes of time allocated to it. However, it should be noted that the task was administered as an online, extra credit assignment in Physics II.

As stated previously, in the electronics courses in both physics and engineering, the task was administered to students twice in the same semester, both before and after all course instruction. This is indicated in the pretest and post-test columns, respectively, of Table 4.1. It is important to note that the physics electronics course included explicit instruction on voltage division between the pretest and post-test; this was not the case in the engineering electronics course, although the ideas of voltage division were applied. For the other courses (Physics II and both introductory engineering circuits courses), pretests were not administered due to the fact that students would not be expected to have sufficient understanding of circuits to attempt the task before instruction.

4.5 Basic Loading Task

In the basic loading task, students are presented with a pair of voltage divider circuits A and B which consist of only batteries and resistors, as shown in Fig. 4.2. Students are asked to compare the voltages across the 20-kΩ resistors in the two circuits. In both instances, the *unloaded* circuit (similar to Fig. 4.1A) contains the same 2:1 ratio of resistors, and hence would produce the same output voltage of $V_{out} = V_{in} \cdot 2/3 = 4\ V$ across the lower resistor. This task was designed to examine how students would ascertain the impact of the 20-kΩ load resistor on each circuit's output voltage, as a correct treatment requires consideration of both resistors in the original circuit.

	Physics			Engineering			
	Physics II	Electronics		Circuits, Non-majors	Circuits, Majors	Electronics	
Year		Pretest	Post-test			Pretest	Post-test
2013		17	16		33	33	19
2014		12	11		34	41	34
2015	98	13	11	100		38	
Total	**98**	**42**	**37**	**100**	**67**	**112**	**53**

Table 4.1. Overview of the number of respondents for the basic loading task by year and course. The question is shown in Fig. 4.1.

4.5.1 Correct Response

The correct response is that the absolute value of the voltage across the 20-kΩ resistor in circuit A (V_A) is greater than that across the 20-kΩ resistor in circuit B (V_B), or more compactly, $V_A > V_B$. Even though the unloaded voltage dividers in both circuits have a 1:2 ratio between the resistors, the addition of the 20-kΩ resistor impacts both circuits differently. In circuit A, since 20 kΩ is much greater than 200 Ω, the 20-kΩ resistor has a negligible impact on the equivalent resistance of the lower branch in comparison to the upper resistor, and therefore the output voltage (*i.e.*, the voltage across the 20-kΩ resistor) is essentially unchanged. In circuit B, since 20 kΩ is much less than 2 MΩ, the 20-kΩ resistor greatly reduces the equivalent resistance of the lower branch in comparison to the resistance of the 1-MΩ upper resistor, thereby decreasing the output voltage across the lower branch significantly (*i.e.*, the added resistor loads the circuit). Students could also reach this conclusion by explicitly calculating the equivalent resistances of the lower branches in both circuits and subsequently using Kirchhoff's voltage law and Ohm's law to determine the currents though and voltages across the relevant circuit elements.

4.5.2 Overview of Student Performance on the Basic Loading Task

In this section, trends across all administrations of the basic loading task are discussed. Between 43% and 81% of students in a given course indicated in their responses that $V_A > V_B$, as shown in Table 4.2. Furthermore, between 5% and 63% of students in a given course (roughly 30% of all students) gave correct answers that were supported with correct and complete reasoning. For example, one student wrote, *"It is greater in circuit A because the 20 kΩ and 200 Ω resistors in parallel are a more* *significant number compared to the 100 Ω resistor than the 2 MΩ and 20 kΩ resistor compared to the 1MΩ resistor."* Note that in order to be considered completely correct, an explanation had to discuss all three resistors in each circuit; otherwise there would not be sufficient justification for the correct answer.

A sizeable portion (10%) of students provided the correct answer, but supported it with incomplete reasoning, such as the following student's response: *"the voltage across the 20 kΩ resistor in circuit A is greater than across circuit B. With voltage division, the voltage is used up getting across the 1 MΩ resistor. In the first circuit, the majority of the voltage gets through and goes through the 20 kΩ and 200 Ω resistors."* Responses such as these highlight the effect of the different upper resistors (*i.e.*, those closest in the diagram to the battery's positive terminal) without accounting for the effect of the other resistances. Such reasoning may also implicitly assume that the 100 Ω and 1-MΩ resistor have the same current through them (which is incorrect) and as a result students would

Consider the circuits at right, and assume all components are ideal. Is the absolute value of the voltage across the 20-kΩ resistor in circuit A *greater than, less than,* or *equal to* that across the 20-kΩ resistor in circuit B? Explain.

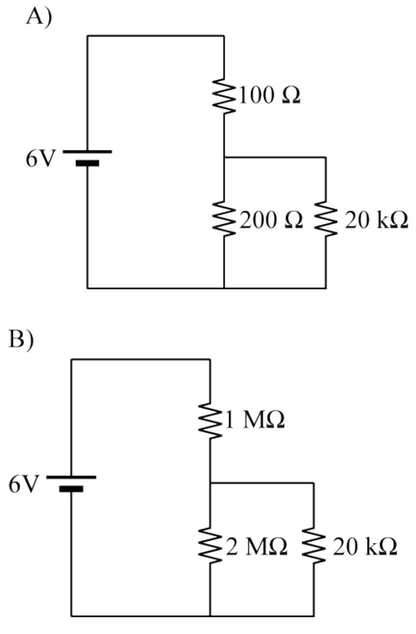


Fig. 4.2. Basic loading task, in which students are asked to compare voltages across 20-kΩ resistors for circuits A and B.

predict a larger voltage drop across the larger resistor. It is possible that similar responses may be related to tendencies of students to consider current as a quantity that is “used up” in a circuit [19], but such a model was not explicitly stated in the context of this question.

Between 5% and 33% of students in a given course (roughly 20% of all students) stated that $V_A = V_B$, and the majority of explanations supporting this response were similar. As one student noted, *“Based on voltage division, we know the proportions are the same, so the voltages are same.”* Indeed, the reasoning supplied by between 15% to 65% of the students in a given course indicated that $V_A = V_B$ focused on the fact that the ratio of the leftmost resistors (*i.e.,* those in the unloaded voltage divider) was the same. Taken together, this suggests that roughly 10% of all students failed to recognize that the addition of the load *can* impact the voltage division and that the resistance of the load must be compared to the resistances in the divider chain to ascertain the load’s potential for impacting the voltage division.

	Physics			*Engineering*				
		Electronics				Electronics		
	Physics II (N = 98)	Pretest (N = 42)	Post-test (N = 37)	Circuits, Non-Majors (N = 100)	Circuits, Majors (N = 67)	Pretest (N = 112)	Post-test (N = 53)	Total (N = 509)
$V_A > V_B$ *(Correct)*	**43%**	**61%**	**81%**	**59%**	**72%**	**63%**	**77%**	**61%**
Correct Reasoning	*5%*	*10%*	*49%*	*24%*	*43%*	*42%*	*63%*	*31%*
Compare Upper Resistors	*15%*	*22%*	*14%*	*5%*	*13%*	*7%*	*6%*	*11%*
$V_A = V_B$	33%	15%	5%	21%	12%	23%	9%	20%
Compare Ratio of Leftmost Resistors	*5%*	*7%*	*5%*	*10%*	*3%*	*15%*	*4%*	*8%*
$V_A < V_B$	24%	24%	14%	20%	13%	14%	13%	18%
Compare Parallel Resistors	*13%*	*10%*	*3%*	*9%*	*4%*	*2%*	*6%*	*7%*

Table 4.2. Student responses to the basic loading task by course

The remaining 20% of students incorrectly answered that $V_A < V_B$. Of these students, a single common line of reasoning once again emerged. For example, one student wrote, *"I always thought the current took the path of <u>least</u> resistance. So, in A we have two paths, one 200 Ω, the other 20×10^3 Ω, so more would travel on the 200 Ω side. But, in B we have 2×10^6 Ω and 20×10^3 Ω, so more goes through the kΩ side. So, B has more net voltage across the 20×10^3 Ω side than A."* Here, the student began by comparing the resistance of the 20-kΩ resistor to the resistance of the resistor in parallel with it for each circuit. The student then correctly argued that less of the total current in circuit A will pass through the 20-kΩ resistor and that more of the total current in circuit B will pass through the 20-kΩ resistor. However, the student incorrectly concluded that the 20-kΩ resistor in circuit B will have therefore have a larger current and a correspondingly larger voltage across it than the 20-kΩ resistor in circuit A. Similar reasoning was given by approximately 10% of all students. Students using this approach failed to recognize that the battery currents in circuits A and B would not be the same. While these students' local comparison of how currents would divide was correct, more information was needed to reach a proper conclusion.

Overall, many students had more difficulty than might be expected in answering a straightforward question with only resistive elements and dc voltages, considering that all students would have been taught the basic analysis techniques required beforehand. Furthermore, in all of the datasets except that from Physics II and the pretest data from the physics electronics course, students would have had some prior explicit instruction on voltage division. Nevertheless, less than a third of all students gave correct answers supported by complete reasoning. Indeed, the tendency of students to reason based on local, rather than global, considerations has been observed in prior research [19].

4.5.3 Basic Loading Task: Specific Difficulties Identified

From the analysis of all data from the basic loading task, one overarching difficulty is evident across all common incorrect lines of reasoning used by students: a tendency to form conclusions based on partial information using local comparisons. Furthermore, as the common incorrect responses all persisted both throughout the semester, and indeed from year to year of instruction, it is evident that additional, targeted instructional interventions may be beneficial for students in all of the courses studied.

Tendency to reason based on local comparisons. In all three common lines of incorrect reasoning, students made comparisons between only a subset of the components in the circuit; such local reasoning has been noted in previous research on circuits [19]. As such, each comparison included implicit assumptions that were unfounded. For students comparing the upper resistors in the two circuits, the assumption was that the battery currents and thus the currents through the 100 Ω and 1-MΩ resistors were the same, and therefore students incorrectly concluded that the voltage drops across the upper resistors would be directly proportional to their resistances. In practice, there would be substantially less current from the voltage source in circuit B, which students did not address. When students compared the ratio of the resistances of the leftmost resistors in both circuits, they implicitly assumed that attaching the 20-kΩ load would not significantly alter the equivalent resistance of the lower portion of each circuit, which is not the case for circuit B. Finally, students who considered only the effect of the new resistor on the parallel resistance of the lower branch did correctly identify that a larger fraction of the current from the upper resistor would pass through the 20-kΩ branch of circuit B than in circuit A. However, they incorrectly assumed that the same current enters the parallel branch. Taken together, these incorrect lines of reasoning indicate that students at all levels of instruction on circuits and electronics may not be systematic in considering the behavior of circuits, as has been observed in other research on resistive circuits [19].

4.5.4 Comparisons Between Courses

As mentioned previously, data were collected both before and after instruction in the upper-division electronics courses. In the physics course, voltage division was heavily emphasized throughout instruction, though loading was less frequently considered. The engineering course occasionally utilized ideas about voltage division, but students were more typically asked to calculate the detailed behavior of relatively complex circuits (*e.g.*, using nodal or mesh analysis), including considerations such as the exact, exponential behavior of the diode IV-curve and small-signal models of transistor behavior. Thus, while the students were expected to be familiar with ideas about voltage

	Physics Electronics (N = 36)		*Engineering Electronics (N = 46)*		*Total (N = 82)*	
	Pretest	Post-test	Pretest	Post-test	Pretest	Post-test
$V_A > V_B$ (Correct)	**56%**	**80%**	**67%**	**78%**	**62%**	**79%**
Correct Reasoning	*8%*	*50%*	*46%*	*63%*	*29%*	*57%*
Compare Top	*16%*	*14%*	*9%*	*7%*	*11%*	*10%*
$V_A = V_B$	17%	6%	26%	9%	22%	9%
Compare Ratio	*8%*	*6%*	*22%*	*4%*	*16%*	*5%*
$V_A < V_B$	28%	14%	7%	13%	16%	13%
Compare Parallel	*11%*	*3%*	*4%*	*4%*	*7%*	*4%*

Table 4.3. Matched student post-test responses to the basic loading task.

division, they were not actively practicing this skill to the same degree as their counterparts in the physics electronics course.

In order to account for the fact that not all students were present for both the pretest and the post-test, only matched data were considered when comparing performance between the beginning and the end of the courses, shown in Table 4.3. As demonstrated in later sub-sections, the use of matched data alone did not alter the resulting conclusions.

In this section, it is most relevant to compare learning outcomes from the electronics courses in physics and engineering (research question 1.1). Comparing introductory courses across disciplines is less likely to yield insight, as the physics course is a co-requisite for its engineering counterparts and furthermore does not focus exclusively on circuits. As might be suspected from Table 4.3, there is indeed a statistically significant difference in students giving correct responses with correct reasoning on the pretest between the physics (8% correct) and engineering (46% correct) electronics courses ($p = .0005$ $\chi^2 = 12$) with a moderate effect size ($\phi = .32$). Given that students in the physics course would likely not have had any instruction on circuits in more than a year, whereas engineering students typically would have at least two more recent courses on circuits, this is not an unexpected outcome.

At first glance, performance appears to have increased overall in both electronics courses at the end of the semester. After all instruction in either electronics course, approximately 80% of all students gave correct comparisons, and approximately half of students explicitly supported their answers with correct reasoning. In addition, the common incorrect lines of reasoning were not only present, but also remained the most prevalent incorrect responses; no new difficulties were observed in the post-test data.

After instruction, it appears that students in the engineering course might be providing correct answers supported with correct reasoning slightly more often than their peers in physics (63% correct in engineering versus 50% correct in physics). However, the difference is slightly above the typical threshold of significance ($p = .11$ $\chi^2 = 2.48$ with Yates correction) with a small to moderate effect size ($\phi = .20$). Thus, after explicit instruction on voltage division, students in the physics electronics course have mostly closed the gap in performance.

4.5.4.1 Comparison Between Electronics Courses and Introductory Courses

With this dataset, it is possible to determine if there are differences in responses between students finishing relevant instruction in a prerequisite introductory course (*i.e.*, Physics II or the engineering circuits course) and the beginning of the corresponding electronics course, addressing research question 1.2. From such comparisons, it can be better determined if the student populations are similar enough to treat post-test responses in introductory courses as equivalent to pretest responses in electronics courses, which would potentially increase the scope of claims that can be made.

A chi-squared test was used to test for differences in the rates at which students gave the three possible answers (greater than, less than, or equal to) between the Physics II course and the physics electronics course. The result was slightly above the threshold of significance ($p = 0.07$, $\chi^2 = 5.4$) with a small to moderate effect size (Cramer's V = .20). As it is unlikely that students have learned more about circuits in the time between the introductory course and electronics course, it is most plausible that any difference might be due to differences in the student populations; the introductory course is required for students pursuing engineering or physical science degrees, whereas the electronics course is solely for physics and engineering physics majors. However, further data are needed to determine if this is the case.

In order to determine if students responded similarly at the end of the engineering circuits course and the start of the electronics course, a chi-squared test was used to test for differences in the rates at which students gave the three possible answers (greater than, less than, or equal to). There was a statistically significant difference between courses ($p = 0.009$, $\chi^2 = 9.5$) with a moderate effect size (Cramer's V = .25). From the responses shown in Table 4.2, the difference is likely due to the larger number of students in the electronics course stating that $V_A = V_B$. This may be due to an expectation that well-designed circuits should not be impacted by the addition of a suitable load; specific, targeted student interviews could serve to explore this hypothesis in future work. It should also be noted students typically would have some additional electronics instruction between these courses in the form of a sophomore-level "Electrical Circuits Laboratory" course; further investigation of this intermediate course might help better pinpoint possible causes of changes in responses.

4.5.5 Changes in Student Responses

In addition to noting general trends that occur from the start to the end of a semester, a more detailed analysis of student responses on an individual level was performed. This was made possible by having matched data from a sizable number of students in both physics and engineering electronics courses. Using such matched individual data, the following questions may be addressed:

- Did the responses for these students resemble their class as a whole?
- How did students' answers change after a semester of instruction?

- Did students' reasoning change if their answer remained the same?

4.5.5.1 Changes in Student Responses: Physics Electronics

Via the use of statistical tests, it was concluded that students in the physics electronics course who provided matched answers did not answer differently from the body of all students on either the pretest (p = .86) or post-test (p = 1, no un-matched data). Thus, it is unlikely that any of the following results could be explained by either high or low performing students being excluded from the matched data. Table 4.4 shows how responses changed over the course of a semester. Note that not only were more students correct at the end of the course than at the beginning (81% vs. 56%), but the difference is significant ($\chi^2 = 4.09$, p = .04) with a small to moderate effect size (V= 0.27). Thus, there is evidence that students have acquired a better understanding of voltage division as a result of course instruction. Furthermore, few students changed from correct to incorrect responses (12% of total), which suggests that students were answering at least somewhat thoughtfully and consistently in their responses.

In addition to students changing their answers, their reasoning changed over the course of the semester as well. For example, of those students (N = 15) correctly answering that $V_A > V_B$ on both pretest and post-test, many more students initially compared the upper resistors (40%) than had correct & complete reasoning (14%), as

Physics Electronics (N = 35)	**A>B Post**	A<B Post	A=B Post	Pre Total
A>B Pre	**44%**	**6%**	**6%**	**56%**
A<B Pre	**25%**	3%	0%	28%
A=B Pre	**11%**	6%	0%	17%
Post Total	**81%**	14%	6%	100%

Table 4.4. Matched pre-post responses in the physics electronics course, as a percentage of total answers.

shown in Table 4.2. However, as can be seen in Table 4.3, 60% of these students responded with correct and complete reasoning on the post-test, with only 20% comparing the upper resistors. While there are few students in this population, the difference is still statistically significant (p = .033). From these data, it can be concluded that not only are students more frequently correct, but they are shifting to more complete reasoning as well.

4.5.5.2 Changes in Student Responses: Engineering Electronics

It should be noted that students with matched data (2013 & 2014) were representative of the course as a whole for both pretest (p = .71) and post-test (p = 1) answers in the engineering electronics course. Table 4.5 shows how student responses changed over the semester. While more students were correct by the end of the semester (78% vs. 67%), the difference is not significant statistically ($\chi^2 = 0.88$, p = .35), nor is the effect size large (V = 0.12). This is perhaps not unexpected, as teaching voltage division is not a primary goal of the course.

The changes in reasoning between the start and end of the course may also be compared. In this case, 68% of those students who were correct at the beginning of the course (N = 19) supported their answer with correct and complete reasoning. By the end of the course, this had increased to 85% of those students. However, this difference is

Engineering Electronics (N = 46)	**A>B Post**	A<B Post	A=B Post	Pre Total
A>B Pre	**59%**	**7%**	**2%**	**67%**
A<B Pre	**4%**	2%	0%	7%
A=B Pre	**15%**	4%	7%	26%
Post Total	**78%**	12%	12%	100%

Table 4.5. Matched pre-post responses in the engineering electronics course, as a percentage of total answers.

once again not statistically significant ($p = .45$). One possible explanation for the lack of significance is that there may be a ceiling effect, due to the fact that a much larger percentage of the engineering students were initially supporting their correct answers with correct reasoning than was the case in the electronics course in physics.

4.6 Summary

The basic loading task proved difficult for students, with anywhere between one quarter and one half of each population failing to make a proper comparison between the two loaded voltage divider circuits. However, in the analysis of student responses, it was shown that most students did use productive ideas about circuits as the basis for their (incomplete) reasoning. Thus, while the majority of students likely possessed either all or some of the requisite knowledge, they did not access and apply their knowledge in a systematic way, as evidenced by the local reasoning used in support of the prevalent incorrect responses.

Longitudinal data on the basic loading task were only collected in two courses, both upper-division courses (physics and engineering) at the University of Maine. While students generally performed better at the end of the semester, the effect was less pronounced in the engineering course, where students' instruction was predominantly on other topics. Nevertheless, this study suggests that a significant percentage of students struggled with the foundational concepts of voltage division and loading, both after instruction in circuits (where between 35% and 55% of students in a given course were incorrect) or after instruction in electronics (with approximately 20% of students incorrect) in either physics or electrical engineering. Three separate incomplete lines of reasoning were identified, each strongly associated with a single answer. Furthermore, all common incorrect (and correct) lines of reasoning were observed across five separate courses, with similar prevalence between disciplines. These findings suggest that the specific difficulties students encounter in reasoning about loaded circuits may be universal, rather than strongly dependent on the educational discipline in which they are taught. This in turn implies that loading is an appropriate subject for further development of instructional interventions, which could be beneficial for students in a wide variety of courses across disciplines.

Chapter 5

INVESTIGATING STUDENT UNDERSTANDING OF DIODE CIRCUITS IN PHYSICS AND ENGINEERING COURSES

Semiconductor diodes are a key part of the electronics curriculum, as a thorough understanding of their functional behavior is critical for successfully understanding the operation of other semiconductor devices such as bipolar-junction transistors and field-effect transistors. This section focuses in particular on pn junction diodes, which are formed by combining a semiconductor material with an abundance of electrons (n-type) with a material that has an abundance of holes (p-type). The result is a depletion region at the junction, which leads to an asymmetric I-V characteristic in which there can be significant current through the device in only a single direction. Diodes are typically the first polar two-terminal device (*i.e.*, the behavior of the device depends on its orientation) that students encounter in electronics courses, and are one of the first non-ohmic (*i.e.,* the current-voltage characteristic of the device cannot be modeled as a straight line going through the origin) elements introduced. Although students often work with real light bulbs in introductory physics courses prior to taking upper-division electronics courses, the non-ohmic characteristics of the bulb are typically downplayed in instruction.

Because of the unique current-voltage characteristics of semiconductor diodes, discrete diodes are commonly used in a number of practical applications, such as rectifying ac signals and over-voltage protection. Furthermore, many pn semiconductor junctions will exhibit diode-like properties, which makes understanding diodes critical for understanding the behavior and limitations of discrete devices (such as transistors) as well as integrated circuits in general. Lastly, light emitting diodes (LEDs) have become ubiquitous in modern electronics, but follow the same essential principles as pn diodes.

Despite the utility and ubiquity of semiconductor diodes in both practical electronics applications and in the undergraduate physics curriculum, there has been little published research on student understanding of diode circuits. To date, the only work reporting on student understanding of diodes was primarily focused on the selection of resources by students in a "nearly novel" situation in which students were tasked with designing a vacuum diode [50]. As part of this investigation, Sayre *et al*. reported on student performance (for N = 11 participants) on a current ranking task involving six simple diode circuits. In this article, electronics was simply a context for studying a more general phenomenon and the purpose of the current ranking task were primarily used to investigate ties between conceptual understanding and resource selection. It should also be noted that there is a growing interest from physics educators in introducing LEDs in introductory courses, particularly in instructional laboratory sequences [84–87].

5.1 Context for Research and Overview of Diode Coverage

Diode circuits were covered only in the upper-division electronics courses investigated in this study. In both courses, diodes were a significant part of the curriculum, discussed extensively in lectures and used in one or more laboratories. It is important to note that the coverage and models used varied somewhat between physics and engineering, and an overview of relevant instruction is described below.

Since the semiconductor diode served as the basis for understanding the behavior of other subsequent semiconductor devices in both courses surveyed, instruction on diode circuits necessarily preceded coverage of transistor circuits. Diodes were introduced after filtering circuits in the physics electronics course (after approximately two-thirds of the course instruction was completed). In the engineering course, diodes were introduced after coverage of real operational amplifier behavior (after approximately the first third of the course instruction was completed). Students in the engineering course had also gained some practical experience using diodes in a sophomore laboratory course, but instruction focused on their utility in circuits (*e.g.*, using diodes for rectification) rather than on the details of the behavior. In the both of the junior-level courses in physics and engineering, students were introduced to a simplified diode model that could be characterized by the *I-V* behavior depicted in Fig. 5.1.B. In this model, the voltage across the diode is considered to be exactly 0.6 V when there is current through the diode, and thus it is referred to here as the constant voltage drop model.

As diodes are polar devices, it is necessary to define voltages across them in an unambiguous manner. The voltage across a diode, V_d, is thus defined as the difference between the electric potential at the anode V_a, and the electric potential at the cathode V_c. (See Fig. 5.1.A). When V_d is negative, the diode is said to be reverse-biased, and there is

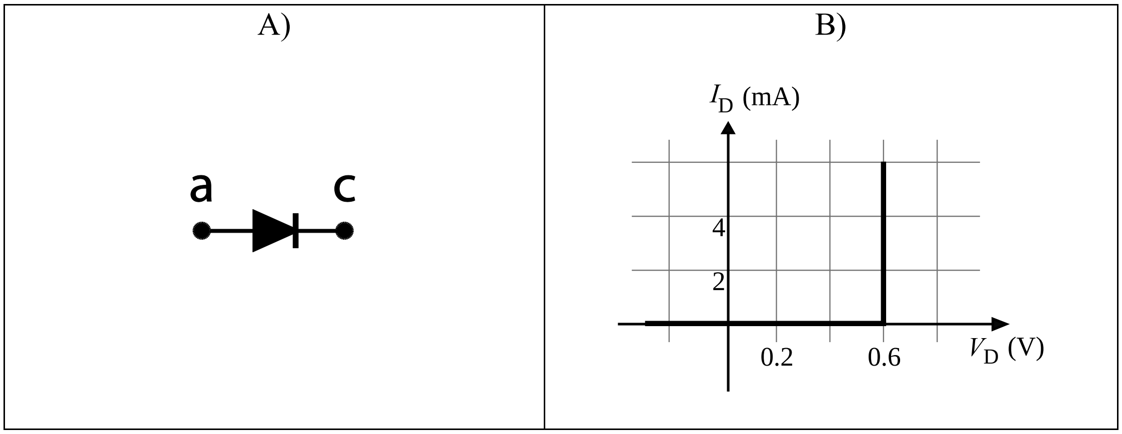


Fig. 5.1. Diode schematic and IV characteristics. A) Schematic symbol for diodes, with anode (a) and cathode (c) junctions labeled. B) Characteristic IV behavior for an ideal diode with a knee voltage of 0.6 V.

no current through an ideal diode in such cases. When V_d is positive, there will still be no current through an ideal device until a characteristic threshold voltage (often denoted as the “knee voltage,” “diode voltage,” or “forward voltage drop”) is reached. When this voltage is reached, the current through the diode is effectively determined by the configuration of the circuit in which it is located, and the voltage across the diode will not increase further. The knee voltage for a Si diode is typically between 0.6 and 0.7 V at room temperature; for clarity, 0.6 V will be used throughout this chapter. However, a voltage of 0.7 V was occasionally used in the engineering electronics course, and either voltage was considered correct in analyzed student work.

In both the physics and engineering courses, students learned about basic diode behavior, as represented by a constant voltage drop diode model (Fig. 5.1.B), and constructed multiple circuits (in the laboratory) that exploited the diode’s unique I-V characteristics. In the engineering course, students also discussed how semiconductor properties give rise to a diode’s behavior. They subsequently learned a more precise exponential model (for example, see [88]) of the diode’s I-V characteristic behavior in which the current through the diode (I) and voltage across it (V) are related by $I = I_S(e^{V/nV_T}-1)$, where I_S and n depend on the diode’s construction and material properties, and V_T is a function of temperature. It should be noted that in-depth knowledge of solid-state semiconductor physics is crucial when designing integrated circuits, which is a common career path for students enrolled in the engineering electronics course. In contrast, students in the physics course are most likely to need to understand how to incorporate diodes into simple discrete circuits for use in experimental apparatus. Thus, the difference in treatment between the two courses is both reasonable and practical.

5.2 Research Questions

Given the lack of empirical work exploring student understanding of the behavior of diode circuits, a primary goal of this investigation was to explore student thinking about such circuits in sufficient detail, in both physics and engineering courses, to inform both instruction on the topic in general and the development of targeted research-based instructional materials on diode circuits. Broadly speaking, this chapter seeks to answer the following research questions:

1. To what extent did students develop a functional understanding of diode behavior? In particular:
    1.1. Did students recognize when diodes would be either forward or reverse biased?
    1.2. Did students apply an appropriate model for describing the diode's behavior?
    1.3. Were students coherent in their treatment of circuits containing multiple diodes?
    1.4. Were there differences between outcomes from different educational disciplines for comparable courses?
2. What specific difficulties emerged from the responses provided by students, and did the prevalence of difficulties vary between courses?

In this chapter, two different research tasks are discussed. In the first, the reverse-biased diode task, students were asked to determine the direction of current in a circuit containing a single reverse-biased diode, as well as to rank voltages across several elements and to rank currents at several relevant points. In the second, the three-diode network task, students were effectively asked to find the voltages across three diodes in a circuit containing multiple loops, where a single diode is forward biased and the other two are reverse biased. Both of these tasks will serve to address these research questions and provide significant insight into student understanding of diode circuits.

5.3 Reverse-Biased Diode Task

The diode circuit discussed in this section is one in which the diode is reverse-biased (meaning that the voltage at the anode is lower than the cathode voltage), as shown in Fig. 5.2. This task was expressly designed to elicit ideas about the current through and voltage across a diode under reverse-bias conditions, as this behavior represents a significant departure from that of ohmic devices such as resistors.

5.3.1 Task Overview

In the reverse-biased diode task, students are shown a circuit containing a diode and two resistors in series. It is stated that the diode is ideal and that the two resistors are identical. Care is taken to indicate that no load is attached to the circuit's output (*i.e.*, V_{out} is not connected to any additional elements) and that the input V_{in} is a constant, dc voltage of + 8V from an ideal source. The diode is oriented such that the anode is connected to ground through R_2, while the cathode is connected to V_{in} through R_1. In the first part of the task, students are asked if the direction of the current through point *a* would be to the left, to the right, or if there will be no current. For the second part of this task, students are asked to rank the magnitudes of the currents through the points labeled *a*-*d* on the diagram, and to explicitly state if any currents are equal to zero. In the third and final part of the task, students are asked to rank the absolute values of voltages across the three different circuit elements, and again to state explicitly if any voltages are equal to one another or are equal to zero. Students are prompted to explain their reasoning in each part of the task.

5.3.2 Correct Response

To begin, students should first recognize that since the diode's anode connects (through R_2) to 0V while the cathode connects to +8.0 V through R_1, it will be reverse biased, and hence there will be no current through the diode (and points *c* and *d*). Since the output terminal is unloaded, there can be no current through point *b*, and thus by applying Kirchhoff's current law to the three-way junction in the circuit it holds that there can be no current through point *a* either. As a result, there is no current anywhere in the circuit, and the absolute values of the currents through all four points (*a*, *b*, *c*, and *d*) are zero (parts 1 and 2). Since there is no current through either resistor, Ohm's law implies that there can be no voltage drop across either one ($V_{R1} = V_{R2} = 0$ V). In order to satisfy Kirchhoff's voltage law, the entirety of the input voltage must be dropped across the reverse-biased diode ($V_{D1} = -8$V). It should be noted that this is commensurate with the constant voltage drop model of diode behavior, as shown in Fig. 5.1.B; alternatively students could treat the diode as behaving like an open switch to arrive at the same result. Thus, the final voltage ranking is $|V_{D1}| > |V_{R1}| = |V_{R2}| = 0$.

5.3.3 Overview of Student Performance on the Reverse-Biased Diode Task.

Data for this task were collected from a total of N=148 students in both physics and engineering electronics courses at UM over the course of five years (2011-2015). It is notable that the instructor for the physics electronics course differed by year, with one instructor teaching the 2011 course and another teaching later courses. However, there were no statistically significant differences (in fact, $p > 0.50$) in students' answers to any of the three parts of the question. Hence, it is reasonable to combine all five years of physics data. Similarly, there were no statistically significant differences between the three years of responses from the engineering course. This is perhaps unsurprising, as the instructor remained the same for all three years. As such, it is justifiable to present the data for this question for all years collectively, with division by educational discipline.

In the circuit at right, both resistors (R_1 and R_2) are identical. Assume that diode D_1 is ideal. Assume that the power supply is ideal and that *no load* is connected to the output of the circuit. Both V_{in} and V_{out} are measured with respect to ground. V_{in} is constant and is equal to +8.0 V.

1. Is the current at **point *a*** *to the right, to the left,* or *equal to zero?* Explain.
2. Rank, from largest to smallest, the absolute values of the currents at points *a, b, c,* and *d.* If any of the currents are equal in absolute value or are equal to zero, state so explicitly. Explain.
3. Rank, from largest to smallest, the absolute values of the voltages across resistor R_1, resistor R_2, and diode D_1. If any of the voltages are equal in absolute value or are equal to zero, state so explicitly. Explain.

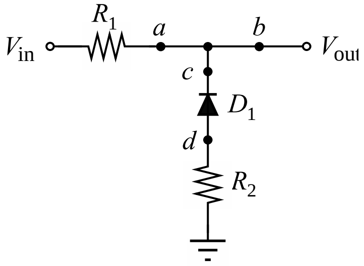


Fig. 5.2. Reverse-biased diode task. Students were asked to characterize the currents in the circuit as well as rank the voltages across elements.

5.3.3.1 Part 1: Current Direction at Point *a*

In the first part of the task, students were asked if the direction of the current at point *a* would be to the left, to the right, or if there would be no current. As can be seen in Table 5.1, approximately half of all students correctly stated that there would be no current through point *a*. Nearly all of these students (>90%) supported their answer with correct reasoning, such as the following: "*Zero, diode is reverse biased so no current can pass through it, and no current will go through the output branch as there is no load applied to it.*" All explanations characterized as correct reasoning indicated that the diode would prevent any current within the circuit, although approximately half of the explanations did not explicitly mention the unloaded output. For example, one student wrote, "*Zero, the diode is reverse bias so there is nowhere for the current to go.*" While a response addressing both the diode's behavior and the unloaded output explicitly would be more thorough, many students (and indeed instructors) would not feel the need to explicitly state that there would be no current through an unconnected terminal; such explanations were also considered correct.

Most of the remaining responses (38% - 46%) indicated that the current through point *a* would be to the right. The majority of these students (62%) further supported this answer with reasoning indicating that current is directed from high to low potential, or from the input to the output of the circuit. An example of the former reasoning is the following: "V_{in} *has a positive voltage, the current will flow from positive to negative*". The latter reasoning is illustrated by the following example: "*The current at point a is to the right. Since* D_1 *is reverse biased (since* $V_c > V_d$*) so no current can flow through* D_1*. Therefore all current will flow to* V_{out}*.*"

It is important to note that even those students indicating that current would be to the right through point *a* frequently (>75% of such responses) indicated that there would be no current through the diode, either in their reasoning to this portion (part 1) of the task or in their response to part 2. Thus, it is possible that this difficulty may have been less related to the behavior of the diode itself, and may have instead stemmed primarily from the way in which students were interpreting output connections in the context of these more advanced and increasingly abstract circuit diagrams. Such representations are more compact and are particularly useful when depicting circuits that are to be connected together; however, they do not explicitly depict complete loops and therefore represent a significant departure from the representations of circuits first introduced in introductory

	Engineering Electronics (N=92)	Physics Electronics (N=56)	Total (N=148)
Part 1. Direction of Current: Point *a*			
Zero (Correct)	**48%**	**63%**	**53%**
No current due to diode	*45%*	*59%*	*50%*
Right	46%	38%	43%
Current from V_{in}	*28%*	*23%*	*26%*
Left	4%	0%	3%
Part 2. Current Ranking			
$\boldsymbol{I_A = I_B = I_C = I_D = 0}$ **(Correct)**	**46%**	**59%**	**51%**
Correct Reasoning	*35%*	*54%*	*42%*
$I_A = I_B > I_C = I_D = 0$	26%	25%	26%
No current through diode	*24%*	*18%*	*22%*
All $I_C = I_D = 0$	76%	86%	80%
Part 3. Voltage Ranking			
$\boldsymbol{V_D > V_1 = V_2 = 0}$ **(Correct)**	**25%**	**41%**	**31%**
No resistor current, no voltage (Correct)	*24%*	*31%*	*23%*
$V_1 > V_2 = V_D = 0$	13%	18%	15%
No R_2 *current, no voltage*	*10%*	*15%*	*12%*
$V_1 = V_2 = V_D = 0$	9%	9%	9%
No current, no voltage	*5%*	*5%*	*5%*

Table 5.1. Overview of student performance on the reverse-biased diode task across physics and engineering courses. The question is shown in Fig. 5.2.

physics courses. Furthermore, out of all responses to the task, only a single student explained that they expected a current from V_{out} to V_{in}; this suggests that students are indeed overgeneralizing the input and output labels to apply to current as well.

A small number of students (<5%) in the engineering course indicated that current would flow to the left, or that it would be non-zero without indicating a particular direction. Such responses were infrequent enough that there were no discernable patterns to the provided reasoning. Thus, there were essentially two common lines of reasoning with corresponding answer commonly observed in the first portion of this task. Furthermore, there was no statistically significant difference ($p = .11$) in student answers between educational disciplines.

5.3.3.2 Part 2: Current Ranking

For the second part of this task, students were asked to rank the currents through the points labeled *a*-*d* on the diagram, and to explicitly state if any currents were zero. Between 46% to 59% of students in a given course correctly indicated that the currents through all four points were zero, as shown in Table 5.1. Most of these students further supported their responses with correct reasoning, such as one student who stated, "*All equal to zero. With diode reverse biased we cannot have any current flow*". All of the students with correct reasoning similarly indicated that the diode would prevent any current from flowing in the circuit, and correct reasoning was provided by essentially all students who provided any support for their correct ranking. Thus, students were not using incorrect lines of reasoning in order to arrive at a correct answer to part 2 of the reverse-biased diode task.

The most common incorrect response, given by approximately 25% of students in either course, was that there would be no current through points *c* and *d*, but that there would be current through points *a* and *b* ($I_a = I_b > I_c = I_d = 0$). It is noteworthy that all of the responses that provided any reasoning (between 71% to 92% of students in a given course providing this ranking) supported their answer with responses similar to that given by the following student: "*The diode is reverse biased, so it doesn't let current down that branch of the circuit. That makes it like an open switch, which means the branch doesn't affect the rest of the circuit.*" These students were correctly applying the idea that the diode would prevent current from flowing in its branch of the circuit, but were either implicitly or explicitly treating the circuit's output as a viable path for current. No other incorrect rankings were given by more than 5% of all students, and thus there were too few responses to make meaningful generalizations in those cases.

5.3.3.3 Part 3: Voltage Ranking

In the third part of the task, students were asked to rank voltages across the three different circuit elements. Here, students had significantly more trouble in comparing the voltages in the circuit, with only approximately one third of students (25% to 41% of students in a given course) correctly predicting that the diode would have the entirety of the input voltage V_{in} across it and there would be no voltage across either resistor. An example of typical reasoning in support of the correct ranking is the following: "$V_{D1} > V_{R1} = V_{R2} = 0$, *All the voltage is dropped across the diode and no current flows so* R_1 *&* R_2 *drop no voltage.*" Such explanations typically used the fact that since there would be no current through the resistors, there would be no voltage across them. These responses account for between 75% and 95% of reasoning provided in support of the correct answer. However, as seen in the example, students frequently did not provide a specific reason for why the diode would have the entire input voltage across it (*e.g.*, by referring to Kirchhoff's voltage rule). Even so these responses were categorized as correct reasoning, as they captured the key element of reasoning required to answer the question correctly (namely, that there can be no voltage drop across a resistor through which there is no current), even though they were slightly incomplete.

The most common incorrect response was to indicate that the voltage across R_1 would be the largest, with no voltage across R_2 or the diode ($V_{R1} > V_{R2} = V_D = 0$). This response was given by approximately 15% of students, with typical reasoning such as the following: *"Reverse biased diode allows no current flow, providing no voltage drop across diode or resistor in series."* While many students did not provide reasoning, approximately 65% of those students giving this ranking similarly indicated that there would be no voltage drop across R_2 because there would be no current through that resistor. It should be noted that all but one of these students (95%) indicated in part 1 that they expected a non-zero current at point a. Thus, these students were still applying relevant information about the behavior of the diode (*e.g.,* that it prevents current in R_2) while simultaneously failing to recognize that V_{out} is not a valid path for current in an unloaded circuit. In addition, all of these students indicated in part 2 of the task that there would be no current through point *d*, and roughly two-thirds (58% - 70%) gave the most common incorrect ranking ($I_a = I_b > I_c = I_d = 0$) for currents. Taken together, these students were answering consistently with an assumption that the output terminal is a viable path for current, even though it is explicitly stated that the circuit is unloaded in the problem description.

The next most common ranking for voltages was that all three voltages were equal to zero ($V_{R1} = V_{R2} = V_D = 0$), given by 9% of all students (in both courses). While many of these students did not provide reasoning, those who did (~50%) used lines of reasoning similar to the following: *"Current is not flowing. There will be no voltage drops."* All such responses indicated that because there was no current, there would be no voltage across any element. These students may have been inappropriately attributing ohmic behavior to the diode (referred to in the literature as "current-based" reasoning [13]), despite the fact that diodes may have a voltage across them with no current present due to their non-ohmic *I-V* characteristic (shown in Fig. 5.1b). Such responses are not consistent with Kirchhoff's voltage law, as no voltage drop is attributed to any circuit element even though there is a potential difference of V_{in} = +8V across the series network of the diode and two resistors.

As can be seen from the voltage rankings, students were frequently unsure of how to treat the reverse-biased diode. Indeed, approximately one quarter of all students incorrectly indicated that the diode would have no voltage across it. This difficulty persists in related tasks, which will be discussed in the sections that follow.

5.3.4 Comparisons Across Task Components and Discussion

As summarized in Table 5.2, approximately 30% of students correctly answered all three parts of the question, and nearly 80% of those students supported all of their answers with correct reasoning, with the remainder neglecting to justify their answers in one or more parts. Thus, these students appeared to be applying appropriate reasoning about diodes and open circuits throughout their responses. When considering patterns of responses across all three parts of the task, there is one particular incorrect combination chosen by a sizable fraction of students. It was observed that approximately 10% of students responded to the three prompts by indicating that current would be to the right at point *a*, that currents through points *a* and *b* would be equal while points *c* and *d* would have zero current, and that the absolute values of the voltages would be ranked $V_{R1} > V_{R2} = V_D = 0$. As described previously, such responses are consistent with the idea that there is a valid path for current through the output of the circuit.

One way to further determine if students were employing useful elements of reasoning about diodes was to examine how many students correctly indicated that the currents through both points *c* and *d* were equal to zero. One could argue that such students were at least recognizing the proper current behavior associated with a reverse-

	Engineering Electronics (N=92)	Physics Electronics (N=56)	Total (N=148)
All Parts Correct	**25%**	**41%**	**31%**
With Correct Reasoning	*18%*	*36%*	*25%*
(Implied) current to V_{out}	*7%*	*13%*	*9%*
$I_C = I_D = 0A$	76%	86%	80%
$I_D = 0$ and $V_{R2} \neq 0$	16%	6%	12%

Table 5.2. Overview of overall responses to reverse-biased diode task and comparison across parts.

biased diode (*i.e.*, that there will be no current into or out of the diode in such circumstances). Based on this analysis, approximately 80% (76% - 86%) of students correctly recognized that the currents through both points must be zero. All of the remaining 20% of students who incorrectly predicted the behavior of current through the diode were subsequently unable to correctly rank voltages in the third part of this task. This finding further supports the idea that understanding a reverse-biased diode's impact on current may be required in order to correctly determine the voltage across it within a circuit.

Another method for investigating the consistency of student reasoning is to compare student treatment of elements across the second and third parts of the task. In particular, it can be determined if students were self-consistent in their treatment of R_2 in stating that it would have neither voltage across it nor current through it. While most (87%) students did recognize that there was no current through point *d* (from resistor R_2 to ground), 16% of these students did not state that the voltage across R_2 was zero. In half of these instances, V_{R2} was ranked as either the smallest or tied for the smallest voltage. Thus, it is possible that such responses could be due to a failure to explicitly indicate that the second resistor's voltage is zero. However, in the other half of the responses, students unambiguously ranked V_{R2} as larger than either V_{R1} or V_D while simultaneously indicating that there would be no current through R_2, which explicitly violates Ohm's law. It should also be noted that only a single student gave a response that instead violated Ohm's law by implying current through R_2 with no voltage across it. While limited in number, the aforementioned responses highlight that students may not be utilizing consistency checking strategies to evaluate their answers, even in upper-division courses.

For this task, differences in performance were observed between engineering and physics students. Physics students provided more correct answers for all three parts of the task than their peers in engineering ($\chi^2 = 3.48$, $p = 0.06$), with a small to medium effect size ($\phi = .17$). This performance difference primarily stems from the fact that physics students were typically correct more often on the voltage ranking part of the task ($\chi^2 = 3.48$, $p = 0.06$, $\phi = .17$). Indeed, there were no significant differences in responses for the direction of current ($p = .11$) or current rankings ($p = .13$) between courses. However, when examining the reasoning used by students, the same common difficulties were observed in both physics and engineering courses, typically with similar prevalence. Thus, this suggests that there is not a large-scale, systematic difference in instruction that could account for the moderate difference in outcome.

5.3.5 Difficulties Identified from the Reverse-Biased Diode Task.

In each part of the task, there was at least one common incorrect response given by students with a strongly associated line of reasoning. Furthermore, these lines of reasoning (specific difficulties) were not unique to either discipline, and occurred with roughly similar prevalence in both physics and engineering courses.

Tendency to associate the absence of current in a reverse-biased diode with the absence of voltage. In the two most common incorrect responses to part 3 of the reverse-biased diode task, over a quarter of students indicated that there would be no voltage drop across the diode because there was no current through it. While such reasoning does not follow from either the constant voltage drop model of diode behavior (which has no current for a range of voltages) or an exponential model, it is commensurate with applying Ohm's law to the diode (*i.e.*, students may be using current-based reasoning), as no voltage would imply no current *regardless* of resistance for an ohmic device [13]. It is worth noting that in at least some contexts, an analogy between open switches and reverse-biased diodes was used by students, and encouraged by instructors. Thus, it should come as unsurprising that a similar difficulty has been observed when students are asked to reason about the behavior of open switches [62].

Tendency to treat the unloaded output of a circuit as a path for current. While not strictly limited to diode circuits, many students appeared to incorrectly treat the output terminal of the circuit as a viable path for current, despite the fact that no load is attached. Thus, even when the diode itself was analyzed correctly, students still struggled with the interpretation of what "input" and "output" indicate. Indeed, such behavior has been observed in the context of operational amplifier circuits as well (see Chapter 6 of this dissertation and [36]).

Tendency to assume current always comes from V_{in} or always goes from V_{in} to V_{out}. In addition to the previous difficulty, many students not only assumed that V_{out} was a valid path for current, but reasoned that the current through the circuit would be from V_{in} to V_{out}. Such reasoning is likely unrelated to the diode circuit specifically, but is being revealed in this context because students are assuming that V_{in} supplies current to the circuit. Interviews with students, similar to what was done in the broader study on op-amp circuits [36], could provide valuable insight into if this is indeed how students are treating the circuit's input connection.

5.4 Three-diode Network Task

While the previous task was useful in eliciting student thinking about reverse-biased diodes in single-loop circuits, it did not probe student ability to reason about more complex circuits containing diodes under a variety of operating conditions. Indeed, diode behavior under forward-biased conditions is particularly relevant for electronics instruction as most practical application of diodes involve at least some scenarios in which there is current through the devices. Thus, another task was needed to gain insight into the extent to which students could productively reason about circuits containing forward-biased diodes. Furthermore, some common diode circuits (*e.g.*, full-wave rectifiers) require students to simultaneously analyze multiple circuit branches with diodes under various biasing conditions. Therefore, it was also appropriate to investigate how students approach diode circuits containing multiple branches. However, using a standard circuit such as the full-wave rectifier could cue memorized responses instead of reasoning from basic principles. Ultimately, an additional task, developed by a faculty member in Electrical and Computer Engineering, was adopted for this project and administered to students in both courses to elicit further ideas about diode behavior.

5.4.1 Task Overview

The three-diode network task, shown in Fig. 5.3, was used to probe student understanding of multi-loop circuits containing diodes in both forward and reverse-biasing conditions. In this task, students are asked to find the voltages (with respect to ground) at three different points in a network of three diodes and three resistors. Minor modifications in component values were made before it was administered to students in the physics electronics course. In addition, the physics version of task states that the diodes were ideal, but the engineering version includes a prompt stating that "the only known fact about the diodes is that I_D = 1mA at V_D = 0.6V." Such information is a common specification given on commercial datasheets for diodes, and enabled students to use either a constant voltage drop model (*i.e.,* the "ideal diode" model used in the physics course) or the more accurate exponential *I-V* relationship.

It is also important to note that this task differs from some of the other tasks employed in this investigation of student understanding of analog electronics in that students were asked to find numerical values for voltages rather than to make qualitative comparisons between values at different points or from different circuits. As a result of this more open-ended design, there were a wider variety of responses given by students, which in turn afforded different insights into student thinking.

5.4.2 Correct Response

To form a correct response to this task, students would need to draw upon their knowledge of diode I-V characteristics, Kirchhoff's laws, and voltage division. Since there is only a single voltage source and each of the diodes has one terminal directly connected to ground, it can be determined visually which diodes have the potential to be forward biased (D_2) and which must be reverse biased (D_1 and D_3). In the absence of the two rightmost loops, consideration of the left loop containing the battery, the 1.2-kΩ resistor, and D_1 results in D_1 being reverse biased, and hence there is no current through D_1. Adding the second loop with the 1.5-kΩ resistor and D_2 to the circuit provides a viable path for current through the forward-biased D_2. Finally, the addition of rightmost

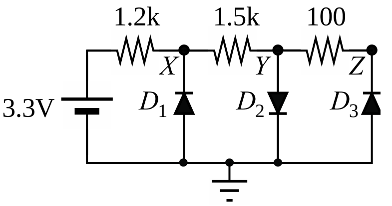
Three identical, ideal diodes are used in the circuit at right. Assume the battery is ideal 1. Calculate the voltages at points *X, Y,* and *Z* (V_X, V_Y, and V_Z, respectively). Show your work and explain your reasoning.	

Fig. 5.3. Three-diode network task. Students are (effectively) asked to evaluate the voltages across two reverse-biased diodes (D_1 and D_3) and one forward-biased diode (D_2).

loop containing the 100 Ω resistor and D_3 does not result in an additional path for current as D_3 will be reverse biased.

After determining the biasing of all three diodes, it is also necessary to determine if the diode D_2 has sufficient voltage across it to conduct current (according to the constant voltage drop model). Since there is a single path for current in the circuit (from the high end of the 3.3 V source, through the 1.2-kΩ resistor, the 1.5-kΩ resistor, and D_2 before reaching ground) and the source voltage is greater than 0.6 V, students may conclude that D_2 is operating at its knee voltage and there will in turn be some current through the aforementioned loop.

Next, it follows from the loop rule that, if 0.6 V is dropped across D_2, then 2.7 V must be dropped across the 1.2-kΩ and 1.5-kΩ resistors. Using voltage division, it can be shown that the 1.2-kΩ resistor has a voltage drop of 1.2 V ($\Delta V_{1.2k\Omega}=2.7\ V\cdot(1.2\ k\Omega/(1.2\ k\Omega+1.5\ k\Omega))$). Similarly, the 1.5-kΩ resistor will have a voltage drop of 1.5 V. Thus, students should conclude that the voltage at point *X* is 2.1 V by subtracting the 1.2-kΩ resistor's voltage from the source (3.3 V – 1.2 V) and the voltage at point *Y* is 0.6 V as expected (3.3 V – 1.2 V – 1.5 V). Since there is no current through the reverse-biased D_3, there can be no current through the adjacent 100 Ω resistor. This in turn implies that there is no voltage drop across the resistor and that the voltage at point *Z* is equal to that at point *Y*. Thus, a student giving a completely correct response would indicate that $V_X = 2.1$ V and $V_Y = V_Z = 0.6$ V. (In the version of the task administered in the 2013 engineering electronics course, the same circuit topology was used with different source and component values, resulting in different numerical responses. For the analysis that follows, all responses from this alternate version were mapped to the analogous responses on the standard version.)

5.4.3 Overview of Student Performance on the Three-Diode Network Task

It should be noted that as an open-ended task, student responses across all three portions were widely varied, with few commonalities appearing when considering the entire task. Indeed, as shown in Table 5.7, only 15% of all students provided correct voltages for all three parts of the task, and 10% of all students supplied both correct answers as well as correct reasoning. Thus, this analysis begins with a consideration of each of the three voltages in the circuit before discussing the tenuous general trends. This three-diode network task was given to students over three years of the UM physics electronics course and three years of the UM engineering electronics course, for a total of N = 136 responses.

5.4.3.1 Part 1: Voltage at Point *X*

From Table 5.3, it can be seen that approximately one quarter of students correctly determined the voltage at point *X*. Most of these students supported their answer with appropriate reasoning, as in the following example:

"*D_1 is reversed biased $I_{D1} = 0$.*
D_3 is reversed biased $I_{D3} = 0$.
Knee voltage of D_2 is .6V.
By loop rule,
$\Delta V_{Batt} - \Delta V_{1.2k} - \Delta V_{1.5k} - \Delta V_{D2} = 0.$
$3.3\ V - \Delta V_{1.2k} - \Delta V_{1.5k} - .6V = 0.$
$2.7\ V = \Delta V_{1.2k} + \Delta V_{1.5k}$
$I_{1.2k} = I_{1.5k}$
$\Delta V = IR$
$2.7\ V = (1.2\ k\Omega) + 1.5\ k\Omega)\ I$
$2.7\ k\ \Omega\ I$
$I = 1mA$

$X = 3.3 \text{ V} - 1\text{mA } (1.2 \text{ k}\Omega) = 2.1 \text{ V}$.”

Students using such reasoning correctly calculated the current through the resistors, and subsequently used this current to find the voltage at point X.

Some students used a slightly different approach, as illustrated by the following student response:

> *“One can consider the reverse-bias diodes, D_1 and D_3 to be open thus we have: [circuit is redrawn without diodes 1 and 3]. $V_{D2} = .6 \rightarrow V_Y = .6 \text{ V} \rightarrow V_Z = .6 \text{ V}$*
> *$V_X = (3.3\text{V} - .6\text{V}) * 1500 / (1200+1500) + .6 \text{ V} = 2.1 \text{ V}$”*

As shown above, this student correctly applied ideas about voltage division, explicitly subtracting the diode voltage from the source voltage in their calculations for the resistors, and subsequently adding the voltages across both the 1.5-k**Ω** resistor and D_2 in order to determine the voltage at point *X.*

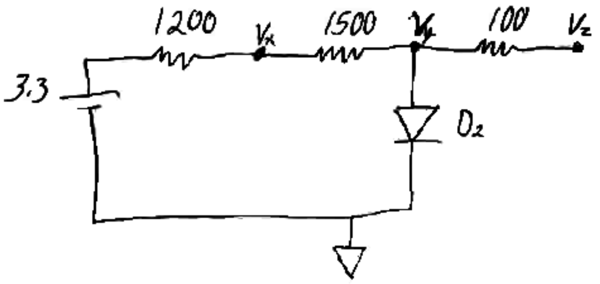
As noted previously, the prompt given to students in the engineering course included a statement that there would be 1 mA of current through a diode when the voltage across it was 0.6 V. Of the engineering students who determined the correct voltage at point *X,* approximately half started from an assumption of a 1mA diode current instead of

	Engineering Electronics (N = 97)	Physics Electronics (N = 39)	Total (N = 136)
V_X = 2.1 V (Correct)	**26%**	**36%**	**29%**
Correct & Complete Reasoning	*9%*	*33%*	*16%*
Assumption of I = 1 mA	*14%*	*0%*	*10%*
V_X = 1.5 V	13%	13%	13%
Voltage division and $V_X = \Delta V_{1.5k\Omega}$	*11%*	*13%*	*11%*
V_X = 1.2 V	6%	15%	9%
Voltage division and $V_X = \Delta V_{1.2k\Omega}$	*4%*	*15%*	*8%*
V_X = 3.3 V (Source voltage)	12%	10%	12%
$V_X = V_{source}$ because D_1 reverse biased	*10%*	*5%*	*9%*
V_X = 0.6 V (Forward biased diode voltage)	11%	0%	8%

Table 5.3. Student responses for the voltage at point *X.*

determining the current through D_2 based on the configuration of the circuit elements. For example, one engineering student wrote the following:

"D_1, D_3 reverse bias



$V_z = V_y$ *no current* $V_y \rightarrow V_z$

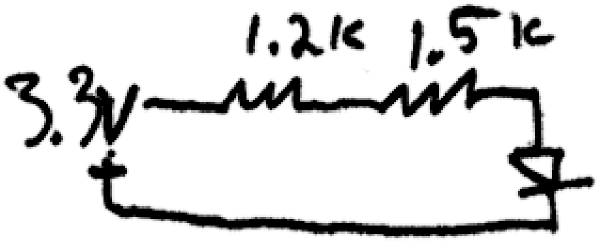
1 mA

$V_{R1} = 1mA\ 1200 = 1.2\ V$

$V_{R2} = 1.5\ V$*"*

This student correctly determined that there was a single loop in the circuit that would have current through it, and redrew the circuit with only the most relevant components. Since the current through diode D_2 in this configuration was 1 mA, the a priori assumption of a 1 mA current (likely primed by the statement at the beginning of the engineering version of the task) led to a correct answer.

The most common incorrect responses for the voltage at point *X* stemmed from errors occurring while students were performing voltage division. For example, one student redrew the circuit (as shown below) and gave the following response:



"The reverse bias diodes behave like open switches, so they can be ignored, just like the 100 Ω resistor because it goes to a dead end. This leaves us with the above circuit. We

know .6 volts will drop across the forward bias diode, so to solve for V_X using voltage division.

$V_X = (1.5 / 1.2 + 1.5) * (3.3 - .6)$

$[V_X] = 1.5$"

Approximately 10% of students gave such responses leading to a conclusion that $V_X = 1.5$ V, which is actually the potential difference *across* the 1.5-kΩ resistor rather than the potential difference *between* point *X* and ground. Students giving such responses failed to account for the voltage drop across D_2 when calculating the voltage at point *X*. This may be due to a lack of distinction on the student's part between the voltage at a point and the voltage across an element.

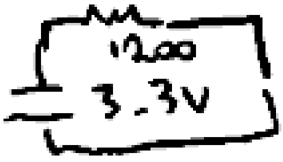
Another common incorrect response is illustrated by the following example:

"D_2 has a .6V difference because it is in forward bias mode
D_1 & D_3 are in reverse bias mode
D_3 open
$V_X = 2.7 / 2700 * 1200$ [=1.2 V]"

Although the calculation was left unfinished, the result of 1.2 V was indicated in the student's final answer. Thus, it is evident that they were performing voltage division to find the voltage across the 1.2-kΩ resistor. Approximately 10% of students used similar approaches; however, it is not clear if they were attempting to calculate the voltage across the 1.2-kΩ resistor in order to subtract it from the source voltage (which would yield a correct response) or if students were unsure of how to use the information about the voltage across an element ($\Delta V_{1.2k\Omega}$) in order to find the voltage at a point (V_X).

Approximately 10% of all students stated that the voltage at point *X* was equal to that of the source, namely 3.3 V. As a specific example of the associated reasoning, one student wrote,

"So there is no current flowing through D_1 so D_1 is off."



Implicit in such responses are the ideas that diode D_1 is reverse biased, and that because there is no current through the diode there can be no voltage drop across the 1.2-kΩ resistor. Furthermore, these students appear to be treating the first loop in isolation, as evidenced in this example by the redrawn circuit. A total of 75% of students who stated that point *X* would have a potential of 3.3 V likewise focused solely on D_1's biasing as justification.

Approximately 10% of students in the engineering course (and none in the physics course) found the voltage at point *X* to be 0.6 V, commensurate with assuming diode D_1 is forward biased. No explicit line of reasoning was common across all of these responses, but they are consistent with treating D_1 as being forward biased.

5.4.3.2 Part 2: Voltage at Point *Y*

As shown in Table 5.4, a sizable fraction (approximately 55%) of all students correctly determined that the voltage at point *Y* would be higher than ground by the voltage drop associated with a forward-biased, current-conducting diode (0.6 V). Essentially all of these students were correctly treating the diode as being forward biased, and students typically did not provide any explanation beyond mentioning the diode biasing or otherwise indicating the voltage or the direction of current on their diagrams.

No incorrect lines of reasoning were particularly common across all populations (*e.g.* > 10% prevalence) for this part of the task, but there was one approach documented that was interesting from a pedagogical perspective. The most common response, given by 8% of students, was to indicate that the voltage at point *Y* would be equal to the potential difference across the 1.5-kΩ resistor. It should be noted that half of these students had previously used voltage division to conclude that the voltage at point *X* was equal to that across the 1.2-kΩ resistor. Thus, these students were at least implicitly attending to the *biasing* of the three diodes in terms of which paths are available for current, but neglected to account for D_2's voltage after performing voltage division, which may indicate confusion between the voltage at a point and the voltage across an element.

5.4.3.3 Part 3: Voltage at Point *Z*

Students struggled to determine the voltage at point *Z*, with only approximately 40% correctly predicting that the voltage would be equal to that of a forward-biased diode (shown in Table 5.5). An example of one student's justification is as follows: "*$I_Z = 0$ because D_3 reverse-biased $\Rightarrow V_Z = V_Y = .6$ V.*" Approximately three-quarters of students who indicated that point Z would have a potential of 0.6 V similarly justified their answer by indicating that diode D_3 would be reverse biased, function as an open circuit, or otherwise have no current through it. Students varied in the degree of detail provided,

	Engineering Electronics (N = 97)	Physics Electronics (N = 39)	Total (N = 136)
V_Y = 0.6 V (Correct)	**57%**	**54%**	**56%**
Forward-biased diode voltage	*37%*	*54%*	*42%*
V_Y = 1.5 V (ΔV_{R2})	5%	15%	8%
Voltage division and $V_Y = \Delta V_{1.5k\Omega}$	*3%*	*13%*	*6%*

Table 5.4. Student responses for the voltage at point *Y*.

with some supplying very explicit responses similar to the previous example, and others simply indicating via the diagram that there would be no current in the last branch of the circuit.

The most common incorrect response, given by approximately one-third of the students, was to state that the voltage at point *Z* would be 0 V. As a specific example, one student wrote, "*V_Z = 0 V because diode is reverse bias and therefore open-circuited.*" Indeed, three quarters of students indicating that $V_Z = 0$ supported their answer by reasoning that the diode D_3 would be reverse biased, function as an open circuit, or have no current through it. These lines of reasoning were similar to those used by students giving the correct response, but students ultimately came to a very different conclusion. These students were correctly reasoning that the diode would act as an open circuit, but incorrectly assumed that such a configuration implies no voltage drop across the reverse-biased diode. Thus, from this part of the task, there is considerable evidence that most students understand the behavior of the reverse-biased diode in terms of current, but they do not know how to relate that behavior to the voltage across the element. A similar difficulty has been noted in previous literature on open switches [16,62] and it is likely that those students who stated that there would be no current and thus no voltage at point *Z* are using the similar, current-based reasoning [13].

	Engineering Electronics (N = 97)	Physics Electronics (N = 39)	Total (N = 136)
V_Z = 0.6 V (Correct)	**36%**	**44%**	**38%**
D_3 is reverse biased	*24%*	*41%*	*29%*
V_Z = 0 V	35%	26%	32%
D_3 is reverse biased	*23%*	*26%*	*24%*

Table 5.5. Student responses for the voltage at point *Z*

5.4.4 Consistency of Student Responses Across Task Components

While the discussion thus far has focused on student responses for voltages at specific points, it is also important to discuss consistency between responses across multiple points as well. Those students who correctly recognized that the diode in the rightmost branch was reverse biased should have subsequently concluded that the voltages at points *Y* and *Z* were the same, since this would imply no current through the resistor between the two points and thus no voltage difference. In practice, only approximately half of all students concluded that the voltages at points *Y* and *Z* were the same, as shown in Table 5.6. Of these students, approximately 60% correctly recognized that both points would be at 0.6 V. However, the remaining 40% (20% of all students) successfully recognized that the voltages V_Y and V_Z should be equal, but were unable to determine the actual value they would be equal to correctly.

When considering student responses for all three voltages requested simultaneously, this task proved quite difficult, with only between 10% and 26% of students in a given course able to determine all three voltages correctly (Table 5.7). Furthermore, approximately 40% of students did not find correct voltages at any of the three points. Together, this indicates that students may have difficulty analyzing multiple diodes together in a single network, even after instruction in canonical multi-diode circuits such as the full-wave rectifier in junior-level electronics courses.

	Engineering Electronics (N = 97)	Physics Electronics (N = 39)	Total (N = 136)
$V_Y = V_Z$	49%	56%	51%
$V_Y = V_Z = 0.6\ V$	28%	36%	30%

Table 5.6. Consistency of student responses between points *Y* and *Z*.

While students were not asked to state which diodes were forward or reverse biased, many did so spontaneously, or indicated their assumptions clearly in their work (*e.g.,* by redrawing the circuit with only D_2 present). From student responses, it was observed that approximately two-thirds of all students unambiguously indicated that D_2 would be forward biased and that both D_1 and D_3 would be reverse biased (see Table 5.7). An additional 8% of students in the engineering course gave answers consistent with correct assumptions for the diode biasing in all three parts, but they did not indicate in any explicit manner whether each diode was forward or reverse biased. Nearly a third of students either made incorrect assumptions about the orientation of one or more diodes or did not communicate their assumptions (*e.g.*, some students indicated that $V_Y = V_Z$ without providing reasoning). This is notable because some of these students gave responses consistent with a circuit in which the orientations of all three diodes were reversed; such students therefore demonstrated their understanding of the general behavior of diodes without the appropriate conditional observations to correctly map it to the circuit diagram. Such difficulties in reasoning about the polarity of elements do not

	Engineering Electronics (N = 97)	Physics Electronics (N = 39)	Total (N = 136)
All three voltages correct	10%	26%	15%
With correct reasoning	4%	23%	10%
Two voltages correct	33%	23%	30%
V_X & V_Y correct, $V_Z = 0$ V	11%	8%	10%
$V_X = 1.5$ V, V_Y & V_Z correct	5%	8%	6%
One voltage correct	22%	10%	18%
$V_X = 1.5$ V, V_Y correct, $V_Z = 0$ V	4%	5%	4%
No voltages correct	35%	41%	37%
All biasing correct	58%	77%	63%

Table 5.7. Summary of the percentage of correct responses given by students in the three-diode network task.

typically arise in an introductory circuits course, as orientation does not affect the behavior of the basic circuit elements (resistors, capacitors, or inductors) typically covered in such courses.

There are three notable patterns that occur when considering all responses simultaneously, as noted in Table 5.7. The most common incorrect pattern of responses, given by 10% of students, was to correctly indicate the voltages at *X* and *Y*, but indicate that point *Z* was at 0 V; such responses (2.1 V, 0.6 V, 0V for points *X*, *Y*, and *Z*, respectively) are inconsistent in their treatments of diodes 1 and 3. Another 10% of students neglected the diode's contribution to the voltage at point *X* but correctly determined the voltage at *Y*; such responses were then split between providing the correct response for *Z*, given by 6% of all students (1.5V, 0.6 V, 0.6 V), and indicating no voltage at *Z*, given by 4% of students (1.5 V, 0.6 V, 0 V). Taken together, correct and common incorrect responses across the entire task only account for approximately a third of all students. However, as shown in the analysis of student responses for each point, a piecewise analysis provided substantial insight into the difficulties students encountered in reasoning about more complex diode circuits.

5.4.5 Three-Diode Network Task: Specific Difficulties Encountered

Student performance across the pair of diode tasks suggests that most students struggled with the application of basic circuits concepts as well as diode *I-V* relationships to predict the behavior of diode circuits under both forward and reverse biasing conditions. Several specific difficulties are discussed, the first of which was also observed in the reverse-biased diode task.

Tendency to associate the absence of current in a reverse-biased diode with the absence of voltage. As with the first task, many students did not recognize that there would be a voltage drop across a reverse-biased diode. Indeed, in the third portion of this task, over half of all students correctly reasoned that diode D_3 was reverse-biased. However, approximately one-third of engineering students and one-quarter of physics students concluded that the voltage at point Z was zero, thereby indicating that the voltage drop across the reverse-biased diode was zero. For comparison, a quarter of students exhibited the same difficulty in the reverse-biased diode task. Thus, this difficulty is ubiquitous enough to be observed across multiple tasks. It should also be noted that this line of reasoning was rarely applied to diode D_1 despite also being reverse-biased, and essentially no students (<5%) consistently exhibited this difficulty for both D_1 and D_3 in this task; contextual information may thus determine if this line of reasoning is attractive for a given circuit.

Failure to correctly interpret diode orientation and biasing from circuit diagrams. In this task, approximately one third of students made incorrect assumptions about the biasing of one or more diodes. However, only 2% of all students indicated the opposite biasing from the correct response for all three diodes. Thus, a sizable fraction of students were having difficulties in the initial stage of recognizing which loops in this circuit would be viable paths for current. This is concerning from an instructional standpoint as incorrect assumptions about a diode's directionality undermine all further analysis of a circuit. Furthermore, this difficulty may be context-dependent, as less than 5% of students responding to the reverse-biased diode task indicated that the diode would be forward-biased in their voltage rankings. As noted earlier, diodes are the first truly polar circuit element many of these students have encountered, so it is possible that the students were not as attentive to the correct mapping of the symbol to the locations of the diode's anode and cathode.

Tendency to confuse the voltage at a point with the voltages across elements connected to that point. Even when students correctly chose to apply voltage division to ascertain some voltage in the circuit, many gave responses that corresponded to the potential difference across a single resistor in their response for point X (approximately 20% of students) or point Y (approximately 10% of students). Such difficulties may stem from students neglecting to consider the voltage differences of multiple relevant circuit elements when coming to their final answer, or from misinterpreting the meaning of the prompt when asked for voltages at points. It should be noted that similar difficulties may not be observed in introductory contexts where it is uncommon for instructors to use the idea of a voltage at a point, which typically occurs after the introduction of ground as a reference point.

Tendency to determine the currents and voltages in one loop of a multi-loop diode circuit independently of the other loops. As noted in Table 5.7, only 15% of all students correctly found all three voltages in the circuit. Furthermore, the most common incorrect combination of responses (correctly finding V_X and V_Y but incorrectly concluding that $V_Z = 0$ V) only accounted for 10% of all students. Yet, when considering responses for each point individually, both the correct and incorrect responses were strongly associated with individual lines of reasoning. In particular, as illustrated by responses in which students indicated that $V_X = V_{Source}$, students may be systematically considering only subsets of the circuit when forming their responses. This lack of consistency between portions of the task suggests that students may either lack (or not apply) consistency-checking strategies such as verifying that their responses satisfy Kirchhoff's laws.

5.5 Discussion and Conclusions

As seen in this section, student difficulties with diode circuits consist of a combination of known difficulties in new contexts (*e.g.*, assuming no voltage implies no current) and difficulties new to this circuit element (*e.g.*, incorrectly interpreting the diode's orientation). Even on the straightforward exercise of determining currents in the reverse-biased diode task, over 40% of all students in either physics or engineering courses provided incorrect answers. Furthermore, on the three-diode network task, many students selected an appropriate strategy for a task (*e.g.*, voltage division) but made errors in the execution (*e.g.*, by reporting the voltage across a nearby resistor.) This suggests that students could benefit from additional, targeted instruction on diode circuits as well as on more fundamental circuits concepts. Indeed, there has been ongoing development of a tutorial designed to introduce students to diode behavior that may be modified in accordance with findings from this work. A key feature of the difficulties observed in this section is that, in many cases, responses stemming from them violate either Kirchhoff's voltage law or Kirchhoff's current law. This suggests that instructional interventions utilizing such fundamental rules as a form of consistency checking might prove beneficial in helping students assimilate diodes into their previous understanding of circuits.

Chapter 6

INVESTIGATING STUDENT UNDERSTANDING OF OPERATIONAL AMPLIFIER CIRCUITS IN PHYSICS AND ENGINEERING COURSES

Operational amplifiers (op-amps) are typically the first integrated circuits introduced in courses on circuits and electronics. Op-amps are high gain differential amplifiers, which produce an output voltage that is proportional to the difference between two voltage inputs. In order to achieve such voltage amplification, op-amps are powered by a pair of connections to an external power supply. While relatively simple transistor circuits can be used to create amplifier circuits (see Chapter 7), op-amps typically achieve much higher gain than simpler transistor circuits, which in turn may be used to improve amplifier stability via negative feedback. Op-amps can be used as the basis for a wide variety of circuits, such as voltage amplifiers, constant current sources, log-amplifiers, active filters, and oscillators. As such, they may be considered "the main building block of analog circuits [89]" and are thus ideal for continued discussion throughout electronics courses.

This chapter is based in part on work published in an article that appeared in the *American Journal of Physics* [36]; as such, some of the text and narrative is drawn directly from that manuscript, on which the dissertation writer was a co-author.

6.1 Overview of Op-amp Coverage

At UMaine, students are first introduced to op-amps in either the physics electronics course, or one of the engineering circuits courses. In these courses, students learn that an operational amplifier (depicted in Fig. 6.1) is a high-gain differential amplifier with five different terminals: a non-inverting input (V_+), an inverting input (V_-), an output (V_{out}), and power connections to positive and negative rails (typically ±15 V). Both inputs are characterized by extremely large input impedances (modeled as infinite for an ideal op-amp) and therefore they typically draw negligible currents. The output is characterized by a relatively low output impedance (typically 50 Ω, but modeled as zero for an ideal device). When functioning as intended, the output of the op-amp is equal to the difference in the input voltages times the open-loop gain G (typically G > 10,000 for real devices, and is infinite for ideal devices), which can be expressed succinctly as $V_{out} = \mathrm{G}(V_+ - V_-)$. In addition, the output is further constrained by the voltages of the power connections, and may not exceed them. It should be noted that many texts omit the power connections (also referred to as "power rails" or "rails") and treat op-amps as three-terminal devices immediately after their introduction. Indeed, classroom instruction on op-amps often omits treatment of the power rails in circuit analyses when they do not constrain the circuit's behavior.

When an op-amp is placed in a circuit with negative feedback (in general, this occurs when the output is coupled to the inverting input), its ideal behavior may be described by two "Golden Rules," which Horowitz and Hill articulate as follows: "*I. The output attempts to do whatever is necessary to make the voltage difference between the inputs*

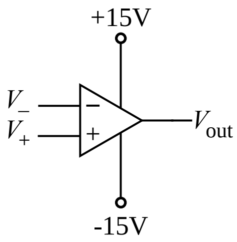


Fig. 6.1. Standard schematic of an operational amplifier or op-amp. The op-amp has two input terminals (the non-inverting input indicated by a "+" and the inverting input indicated by a "–") and one output terminal. The +15 V and -15V supplies are connected to the positive and negative power rails, respectively**.**

zero…. II. The inputs draw no current" [90]. In the physics electronics course, these Golden Rules are covered explicitly and referred to by name; in the introductory engineering courses, the same ideas are motivated and discussed in instruction but slightly different terminology is used. In the courses introducing op-amps, students typically spend 2-3 weeks discussing op-amps and their typical applications in circuits, with periodic discussions of additional circuits throughout the semester. Op-amps are also revisited in the junior-level engineering electronics course, with the expectation that students already understand ideal op-amp behavior from their sophomore course. The engineering electronics course therefore focuses on deviations from ideal behavior that occur when using real op-amps.

6.2 Prior Research

To date, relatively little work has been conducted on student understanding of operational amplifiers by researchers in either physics education or engineering education. Of the most relevance is the work of Mazzolini *et al.*, which discussed the implementation and assessment of a series of interactive lecture demonstrations for teaching operational amplifier circuits in an electronics course. While the identification of specific difficulties was not a primary goal of their research, it was found that students encountered difficulties when they were asked to analyze standard op-amp circuits drawn in non-traditional manners [30]. Their work suggested that memorization of specific circuits, gain formulas, and specific key results may play a substantive role in student ability to solve canonical op-amp circuits successfully.

6.3 Op-amp Specific Research Questions

Based on findings from the aforementioned study, two op-amp tasks were designed to better probe student understanding by including portions that could not be answered with a memorized formula and would instead require a robust understanding of fundamental op-amp behavior. This chapter seeks to answer the following research questions:

1. To what extent did students develop a functional understanding of operational amplifier behavior? In particular:
    1.1. Did students recognize when it was appropriate to apply the op-amp's gain formula?
    1.2. Did students correctly apply and interpret the op-amp Golden Rules?
    1.3. Were there differences between outcomes from different educational disciplines for comparable courses?
2. What specific difficulties emerged from the responses provided by students, and did the prevalence of difficulties vary between courses?

In this chapter, two different research tasks are discussed. In the first, the three amplifier task, students were asked to compare outputs between a non-inverting amplifier circuit and two slightly modified versions of the non-inverting amplifier circuit. In the second, the inverting amplifier task, students were asked to find the output of a canonical inverting amplifier, as well as to characterize and compare several currents in the circuit. Both of these tasks served to provide insight into the research questions posed here, in particular by asking students to reason about circuits in which using gain equations alone would be unproductive.

6.4 Three Amplifier Task

This section focuses on the first of two tasks administered to probe student understanding of op-amp circuits in the physics and engineering electronics courses. In this task, students are asked to compare the output voltages of three non-inverting amplifier circuits, two of which have been modified slightly from the canonical non-inverting amplifier circuit discussed in the courses.

6.4.1 Task Overview

In the three amplifiers task (Fig. 6.2), students are shown three circuits that all act as non-inverting amplifiers. Circuit B corresponded to a canonical non-inverting amplifier. In circuit A, a single 10-kΩ resistor is inserted between V_{in} and the non-inverting input of the op-amp. In circuit C, a 10-kΩ resistor is instead placed between the output of the op-amp and the output of the circuit V_C. All op-amps are assumed to be identical and ideal, and all three circuits have identical and unchanging positive input voltages V_{in} (from ideal voltage sources). Students are told to assume that no loads are connected to the outputs of the circuits. Students are asked (1) to compare the absolute values of the output voltages V_{B} and V_{A}, and (2) to compare the absolute values of the output voltages V_{C} and V_{B}. By setting up the questions in this manner, students have to compare the behavior of each perturbed circuit to that of the canonical non-inverting amplifier.

6.4.2 Correct Response

A correct response to the task does not necessarily require explicit determination of output voltage for each circuit; rather, students could make a careful analysis of whether or not each modification to the canonical inverting amplifier (circuit B) would impact the output voltage. There are many approaches that students could use to determine V_{B}. For example, students could simply apply the gain formula for the non-inverting amplifier ($G_B = 1 + R_2/R_1$, where R_1 corresponds to the 5-kΩ resistor and R_2 corresponds to the 20-kΩ resistor in circuit B) and correctly determine that $V_{\mathrm{B}} = 5V_{\mathrm{in}}$. Alternatively, students might apply Golden Rule I to conclude that because the voltage at the inverting input should be V_{in}, there would in turn be a voltage drop of V_{in} across the 5-kΩ resistor. Since the current through the 5-kΩ resistor is equal to that through the 20-kΩ resistor

Shown at right are three op-amp circuits (A – C). All op-amps are identical and ideal, and all three circuits have identical and unchanging positive input voltages V_{in} (from ideal voltage sources). Assume that no loads are connected to the outputs of the circuits.

1. Is the absolute value of the output voltage V_{B} *greater than, less than,* or *equal to* the absolute value of the output voltage V_{A}? Explain.

2. Is the absolute value of the output voltage V_{C} *greater than, less than,* or *equal to* the absolute value of the output voltage V_{B}? Explain.

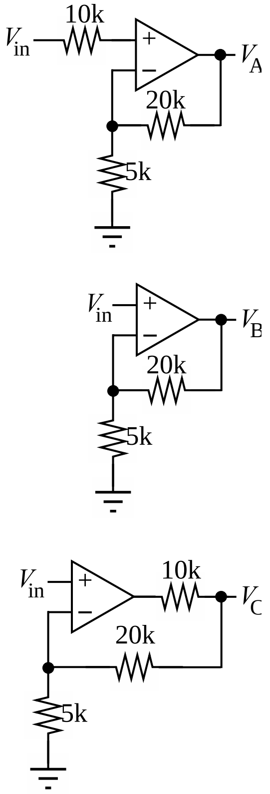


Fig. 6.2. Three amplifiers task in which students are asked to make two pairwise comparisons between the absolute values of the output voltages from three non-inverting amplifier circuits with identical positive input voltages V_{in}.

(due to Golden Rule II and Kirchhoff's junction rule), there must have been a drop of $4V_{\text{in}}$ across the 20-kΩ resistor from the output and thus students would arrive at the same conclusion that $V_B = 5V_{\text{in}}$.

In circuit A, since there can be no current through (and thus no voltage drop across) the 10-kΩ input resistor due to Golden Rule II combined with Ohm's law, the addition of the resistor does not change the circuit's behavior, and thus $V_A = V_B = 5V_{\text{in}}$. In circuit C, the voltage at the inverting input is again equal to V_{in} (from Golden Rule I), and the voltage across and the current through the 5-kΩ resistor are necessarily the same as in circuit B. The subsequent analysis is therefore identical, so $V_C = V_B = 5V_{\text{in}}$. All three output voltages are thus equal in absolute value and non-zero. Note that the output voltage of the op-amp in circuit C, $7V_{\text{in}}$ in this case, must be larger than in circuit B since there is a single current through all three resistors and thus there is a voltage drop across the newly added 10-kΩ output resistor. While it is possible that the op-amp would be unable to produce such an output if it were constrained by the power rails (due to an overly large input voltage), such concerns were beyond the scope of this question and in practice students did not use such arguments in their reasoning.

6.4.3 Overview of Student Performance on Three Amplifiers Task

Data from the three amplifiers task has been gathered from the junior-level physics electronics course (N = 49), from both introductory circuits (N = 97) and junior-level electronics (N = 59) engineering courses for ECE majors, as well as from introductory circuits courses for non-ECE majors (N = 63). Results are summarized in Table 6.1 and discussed in detail below. It should be noted that this work was part of a larger project conducted in collaboration with the University of Athens and the University of

Washington, and these results along with those from the collaborating institutions are reported by Papanikolau *et. al.* [36].

Between 15% and 35% of students in a given course at the University of Maine correctly ranked the absolute values of all three circuits ($|V_A| = |V_B| = |V_C|$), as shown in Table 6.1. The percentages of students who supported their correct ranking with correct reasoning ranged from 0% to nearly 75% between the four different courses, with a statistically significant difference between the physics course and any of the engineering courses ($p < .0001$, $\chi^2 = 23.7$) with a moderate effect size (Cramer's V = .32). It should be noted that detailed written explanations were commonly expected of students in the physics electronics course, whereas symbolic proofs were accepted (and in some cases preferred) in the engineering courses. However, as will be shown in the following analysis, the difference is not solely due to differences in how many students provided

	Engineering			Physics	
	Circuits, non-majors (N = 63)	Circuits, majors (N = 97)	Electronics (N = 59)	Electronics (N = 49)	Total (N = 268)
$\lvert V_A\rvert = \lvert V_B\rvert = \lvert V_C\rvert$ (Correct)	**24%**	**19%**	**14%**	**33%**	**21%**
Correct Reasoning	5%	1%	0%	29%	7%
$\lvert V_A\rvert = \lvert V_B\rvert$ (Correct)	**44%**	**46%**	**53%**	**49%**	**48%**
Correct Reasoning	25%	25%	32%	45%	30%
Input resistor doesn't matter	5%	6%	2%	2%	4%
Resistor not part of equations	6%	8%	3%	0%	5%
$\lvert V_B\rvert > \lvert V_A\rvert$	43%	48%	42%	41%	44%
Resistor lowers voltage	29%	34%	24%	35%	31%
$\lvert V_B\rvert = \lvert V_C\rvert$ (Correct)	**40%**	**29%**	**22%**	**61%**	**36%**
Correct Reasoning	10%	2%	2%	51%	13%
Resistor doesn't matter	8%	10%	8%	4%	8%
$\lvert V_B\rvert > \lvert V_C\rvert$	38%	57%	61%	31%	49%
Resistor lowers voltage	*33%*	*44%*	*29%*	*27%*	*35%*
$\lvert V_B\rvert < \lvert V_C\rvert$	18%	12%	9%	8%	12%
Resistor increases gain	*0%*	*3%*	*4%*	*6%*	*3%*

Table 6.1. Overview of student performance on the three amplifiers task in both physics and engineering courses. The question is shown in Fig. 6.2.

explicit written reasoning. Indeed, anywhere between 15% to 30% of students (depending on the course) supported their correct ranking with correct yet incomplete reasoning.

For the first part of the task alone (comparing circuit B to A, where A differs from B only through the addition of a resistor between V_{in} and the op-amp's non-inverting input), approximately 50% of students correctly recognized that that $|V_B| = |V_A|$. Of those students, nearly 60% provided correct and complete reasoning; for example one student wrote, *"equal to, no current in positive terminal in either, meaning 10k resistor has no affect[sic]."* In this response, the student justified the lack of impact of the added resistor by noting that there would be no current into the positive terminal (and implicitly, no current through the resistor). Approximately 20% of students who reasoned that $V_A = V_B$ gave arguments that did not sufficiently justify why the two would be equal. For instance, one student wrote, *"Equal, 10k has no effect on* V_A*."* Such responses are incomplete in that they do not provide a causal mechanism for why the added resistor would not affect the circuit's function, and were given by ~10% of students answering 'equal'. A related line of reasoning is typified in the following student's explanation: *"Equal to each other, 10k at the* V_{in} *plays no roll[sic] in the equations."* Such responses, given by a similar proportion of students, stemmed from the argument that the added 10-kΩ resistor is not used in the gain equation for the non-inverting amplifier. While these last two categories of explanations were not categorized as correct and complete, it is likely that some of the students giving these responses knew the physical justification but did not state so in their response.

Slightly fewer than half of students (45%) indicated that $|V_B| > |V_A|$. Between 25% to 35% of students in a given course justified this incorrect comparison by explicitly focusing on a voltage drop across the input resistor. For example, one student wrote: *"V_B is greater than V_A because V_{in} for V_A must pass through a resistor which causes a voltage drop."* In this response, there is no mention of any current through the input 10-kΩ resistor. Upon examining all 118 responses given by students supporting this comparison, none explicitly attribute the voltage drop to an input current, and only four responses included an implicit current through the application of Ohm's law. This response pattern suggests that students may in fact be automatically (and possibly subconsciously) ascribing a voltage drop to the resistor without analyzing the situation through the more formal lens of Ohm's law. Such behavior is consistent with a "knowledge in pieces" [91] or resource [92] model of student thinking in which, for example, a student might draw upon a more informal notion that "increased resistance leads to less result." This informal notion is included in diSessa's Ohm's p-prim [91]. This is in contrast to the responses from the larger study, where approximately 30% of students at UA explicitly attributed a current to the 10-kΩ resistor in circuit A [36].

Very few (< 10%) students stated that the output of circuit A would be greater than that of circuit B. Furthermore, student reasoning supporting such responses did not follow any notable trends. This suggests that there are no clear or intuitive reasoning paths that could lead to such an answer, and there are no straightforward accidents in calculation that could result in such a response.

Comparisons between circuits C and B showed more variation, both in terms of responses to the question and in terms of responses chosen by course. Indeed, the percentage of students making correct comparisons ranged from 22% (in the engineering electronics course) to 60% (in the physics electronics course), with an average of 36%. It should be noted that while most physics students supported their answers with correct and complete reasoning, few (>25%) engineering students did so. As an example of a typical correct justification, one student wrote, *"the voltage drop across the 5k resistor must be V_{in}, so the current in both circuits will be the same, with that I can say both currents drop the same voltage across the 20k so $V_C = V_B$."* In this response, the student implicitly (and correctly) used both golden rules to justify that the voltage change from ground would have to be the same in both circuits B and C. The most common line of incomplete reasoning, provided by up to a third of students stating $V_B = V_C$, is typified in the following student's response: *"10k has no effect."* While it is true that the added 10-kΩ resistor does not change the voltage at the point in question, such responses stating that the resistor does not matter did not provide any justification for why this is the case.

From the spread of answers given, it is appropriate to test for differences between courses. The responses from the physics electronics course are statistically distinguishable from the non-majors engineering circuits course ($p = .0016$, $\chi^2 = 9.95$) with a moderate effect size ($V = .29$). However, there was not a difference between the engineering courses for majors and non-majors ($p = .8$), nor between the circuits and electronics courses for ECE majors ($p = .27$). Thus, it is plausible that the differences in treatment across disciplines are responsible for such a result. This may be due to more time spent on analyzing atypical op-amp circuits in the physics course, whereas the engineering courses typically spent additional time introducing practical applications.

Between 30% and 60% of students in a given course stated that $|V_C| < |V_B|$. Nearly all of these students supported their answers with reasoning such as the following: *"$V_C < V_B$ because the voltage drop is less than that of V_B past the op-amp (where the 10k is located)."* In such responses, students focused on the voltage drop due to the output resistor and appeared to be implicitly assuming that the outputs of the op-amps in circuits B and C were identical. Relatively few of the written responses explicitly provided reasoning supporting this assumption. In one response, however, a student wrote:

> *"V_C is less than V_B because there is a resistor on the output of the op-amp which creates a voltage drop before the output of V_C. Both V_B and V_C have the same gain so there is no difference there that would change anything."*

This student argues that the gain of both B and C was the same, implicitly determined by the 5k and 20k resistors, and thus both op-amps should have the same output. Such responses seem to draw on a combination of localized and sequential reasoning, arguing that any change *after* the op-amp shouldn't impact its output. This line of reasoning, however, is inconsistent with the notion of negative feedback, which is critical for many op-amp circuits and a key conditional requirement for the first golden rule.

Approximately 10% of all students incorrectly claimed that $|V_C| > |V_B|$. The most prevalent line of incorrect reasoning supporting this comparison (given by roughly 0-5% of all four populations) involved the erroneous claim that the additional output resistor increased the gain of the circuit. For example, one student wrote:

> *"$V_C > V_B$. This is because of the gain formula which is R_{out} / R_{in} where R_{in} is the resistor to ground and R_{out} is before that, and R_{out} for C is greater than B."*

If the output of the circuit were taken from the output of the op-amp (as is the case for circuit B), this reasoning would be correct. For circuit C, however, such responses suggest a failure to differentiate between the output of the *circuit* and the output of the *op-amp*. While this particular line of reasoning was not explicitly prevalent in any of the courses at the University of Maine, approximately one quarter of students at UW and a third of students at UA supported a $V_B < V_C$ ranking with such reasoning [36]. This suggests that such a tendency to consider the added resistor's impact on the gain may be more related to the particular instruction employed rather than the discipline in which the course is taught.

6.4.4 Specific Difficulties Noted Across Disciplines

Student performance on the three amplifiers task suggests that all students struggled with the application of basic circuits concepts and op-amp rules to circuits that differ only slightly from canonical op-amp circuits. In practice, all of the specific difficulties identified in the larger study [36] were present in both physics and engineering courses as well.

Lack of a functional understanding of Golden Rule II. Roughly one third of the students across all four courses provided reasoning when comparing circuits B and A that would only be appropriate if there were a current into the non-inverting input of the op-amp. This was fairly consistent across all courses, but lowest in the engineering electronics course. The reasoning given by all such students is inconsistent with Golden Rule II, and calls into question the extent to which students have developed a truly functional understanding of the implications of the op-amp inputs have very high input impedances. At the very least, many students are not verifying that their responses obeyed relevant circuits principles, possibly reflecting a lack of familiarity or practice with consistency checking strategies. As noted in other tasks discussed in this work, students frequently did not appear to recognize when their answers were not consistent with either fundamental principles or the behavior of specific devices.

Tendency to ascribe a voltage drop to a resistor regardless of current. From Golden Rule II, there cannot be a voltage drop across the input resistor since there is no current into the non-inverting input. However, over 40% of students in every course incorrectly stated that $|V_B| > |V_A|$. When examining *all* explanations given in support of such responses (N = 118) from this task, no students explicitly mentioned a current through the input resistor in their written responses. While this does not preclude the possibility that many of the other students may have thought there was a current through the resistor (and into the non-inverting terminal), it suggests that some students may in fact be automatically (and possibly subconsciously) ascribing a voltage drop to the resistor without analyzing the situation through the more formal lens of Ohm's law. Indeed, it was observed that a few students (4 in total) attempted to use Ohm's law to imply that there *must* be a current through the added 10k resistor. For the majority of students, it is likely that a significant percentage simply attributed a voltage drop to the resistor without even considering the presence or absence of current.

Lack of a functional understanding of Golden Rule I. Approximately half of all students incorrectly claimed that $|V_C| < |V_B|$. Furthermore, roughly a third of all students indicated either implicitly or explicitly in their reasoning that the output voltages of both op-amps would be the same. If this were indeed the case, then it would imply either the potential at the inverting input (V_-) would necessarily be less than $V_+ = V_{in}$ (due to the "same" output voltage being divided over three resistors), or that somehow the newly added resistor had no voltage across it. Thus, both the reasoning and answers provided by a sizable portion of all students are inconsistent with either Golden Rule I or Ohm's law. Students did not appear to draw on the Golden Rules in order to test the viability of their responses, but rather focused on explaining the local effect of the perturbing resistor. It should be noted that the engineering electronics course had both the most students making this comparison (~60%) but the fewest providing the justification that the resistor was responsible (accounting for approximately half of the responses). However, the difference is mostly accounted for in that many students in the course provided no written reasoning on this portion of the task.

Tendency to reason locally and sequentially about the behavior of op-amp circuits. As noted previously, over one-third of students argued that altering the circuit by adding a resistor to the op-amp's feedback network would result in a smaller voltage drop over the existing resistors, implying that the voltage from the op-amp would not change. Reasoning that a change "downstream" (*i.e.*, between the op-amp and 20-kΩ resistor) will not affect the "upstream" behavior of the circuit (*i.e.*, the output of the op-amp itself) is typically referred to as local or sequential reasoning, and it is well documented in the literature on introductory circuits [20]. Although this particular instantiation is a relatively clear-cut example of local reasoning, it is somewhat more surprising given that all courses in which students were first introduced to op-amps emphasized the importance of negative feedback and feedback loops for op-amp operation. When students were presented with less familiar situations in this task, they appeared to be relying on local reasoning, accounting for only a subset of circuit elements when making their analysis. Similar behavior was also observed when students reasoned about voltage dividers in Chapter 4. Such a phenomenon has also been identified when probing the reasoning used by upper-division and graduate physics students about the behavior of open circuits [21] as well.

6.5 Inverting Amplifier Task

The findings from the three amplifiers task suggested that many students likely did not possess a robust understanding of the behavior of the non-inverting amplifier circuit itself, even after all instruction on basic op-amp circuits. In order to better understand the extent to which students understood the details of how operational-amplifier circuits worked to produce voltage amplification, a second task was developed at the University of Washington in which students would be forced to consider in detail the currents and voltages in another canonical op-amp circuit – the inverting amplifier. The purpose of this task was to determine the extent to which students possessed the level of understanding required to derive the inverting amplifier's gain formula from first principles, as well as to determine their functional understanding of op-amp circuit behavior including that specified by the two Golden Rules.

While several different versions of the inverting amplifier task have been administered at various institutions [36], only one form was administered at the University of Maine. This task was specifically designed to probe students' understanding of the op-amp as a device that must satisfy Kirchhoff's Junction Law.

6.5.1 Task Overview

In this task, students are shown the inverting amplifier circuit in Fig. 6.3, in which seven points *(A–G)* are labeled on the diagram. Students are told that the op-amp is ideal and that there is no load connected to the output of the circuit. The input voltage V_{in} is constant and equal to –5 V. In part 1, students are asked to find the value of the circuit's output voltage V_{out}. In part 2, students are asked to indicate the direction of the current through point *A* or to state explicitly if there is no current through that point. In part 3, students are asked to compare the absolute values of the currents through points *F* and *G* (*i.e.,* into the two inputs) and to indicate explicitly if any currents are equal to zero. Finally, in part 4, students are asked to rank, from largest to smallest, the absolute values of the currents through points *A–D.* (In a slightly modified version of the question administered in the physics course the first time the task was used, where students were

Consider the op-amp circuit shown at right. Assume that the op-amp is ideal and that there is no load connected to the circuit's output. The constant input voltage V_{in} is –5 V.

1. What is the value of circuit's output voltage, V_{out}? Briefly explain.

2. Is the current through point *A to the right, to the left,* or *equal to zero?* Briefly explain.

3. Is the absolute value of the current through point *F greater than, less than,* or *equal to* the absolute value of the current through point *G?* If any currents are equal to zero, state so explicitly. Explain.

4. Rank, from largest to smallest, the absolute values of the currents through points *A, B, C* and *D.* If any currents are equal to each other in absolute value or are equal to zero, state so explicitly. Explain.

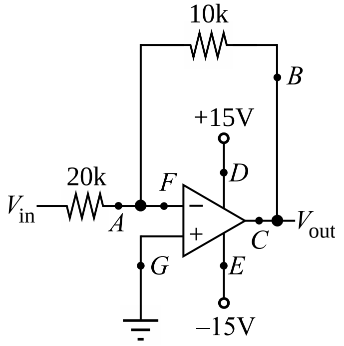


Fig. 6.3. The inverting amplifier task, in which students were asked predict the circuit's voltage output and characterize the behavior of currents critical for the circuit's operation.

only asked about points *A*–*C*). If any of the currents were equal in absolute value or equal to zero, students are prompted to indicate that explicitly. For all parts of the task, students are required to explain their reasoning. While the first part of this task may be answered using the circuit's gain formula, the rest require students to apply their understanding of Kirchhoff's laws and the Golden Rules to be completed successfully.

6.5.2 Correct Response

In order to clearly outline the reasoning required for all parts of this task, an analysis of the entire circuit is presented. From Golden Rule II, it is known that the currents through points *F* and *G* are both equal to zero (part 3). Because the circuit is connected such that there is negative feedback between the output and inverting input, the first Golden Rule applies, and the electric potential at point *F* must be equal to that at point *G* (0 V). This in turn implies that the current through point *A* is to the left because the potential at point *F* is higher than V_{in}, which is at -5 V (part 2). Applying Kirchhoff's junction rule to the node between points *A* and *F*, the current through the 20-kΩ resistor is equal to that through the 10-kΩ resistor (since there is no current through point *F*), so the current through point *B* is up the page. Since there is a single current through both resistors, a voltage drop of 5 V across the 20-kΩ resistor implies that there is half as much voltage (*i.e.*, 2.5 V) dropped across the 10-kΩ resistor (from Ohm's law). Since point F is at ground, V_{out} is therefore +2.5 V (part 1). It should be noted that students in all classes had been explicitly taught the more general inverting amplifier gain formula $G = -R_F/R_{in}$, in which R_{in} corresponds to the resistance of the input resistor (20 kΩ for this circuit) and R_F corresponds to the resistance of the feedback resistor (10 kΩ); this expression can also be used to determine the output voltage.

Because no load is attached to the output of the circuit, there is no viable path for current through V_{out}, and thus the current through *B* must equal that through *C* via Kirchhoff's junction rule. Thus, $|I_A| = |I_B| = |I_C|$. Since the direction of current is from high to low potential, the currents through points *D* and *E* are both oriented down the page (into and out of the op-amp, respectively). Furthermore, as a powered device, there will be currents through the op-amp rails even when there is none through the output terminal ($|I_D|$ & $|I_E| > 0$). By recognizing that the total current into the op-amp must equal the total current out of the op-amp (again by applying Kirchhoff's junction rule) and that the currents through points *F* and *G* are both zero (from Golden Rule II), the current into the op-amp through point *D* must split into the current down through point *E* to the negative rail and the current to the right through the op-amp's output and point *C*. Thus, $|I_D| > |I_A| = |I_B| = |I_C| > 0$ (part 4) and $|I_D| > |I_E| > 0$.

6.5.3 Overview of Student Performance on the Inverting Amplifier Task

Versions of the inverting amplifier task have been administered at UMaine in the physics electronics course (N = 59), engineering circuits course for ECE majors (N = 101), and the engineering circuits course for non-ECE majors (N = 76) after all relevant instruction. This task was also given to students in the engineering electronics course (N = 68) early in the semester, during instruction on properties of real operational amplifiers. Data for this task were collected in close proximity to the amplifier comparison task, typically with less than a week between the two sets of questions. Student performance on the task is described in this section, and the results are summarized in Table 6.2.

For part 1 of the task, between 55% and 80% of students in a given course gave correct values or expressions for V_{out}. An additional 10–15% of students in a given course made a sign error, indicating that the output voltage would be negative. Nearly all of these explanations supporting the correct answer (or the one with a sign error) included either a correct derivation of the op-amp's behavior from first principles (most prevalent in the physics electronics course and the engineering non-majors circuits course) or the inverting amplifier gain equation (most prevalent in the engineering circuits and electronics courses for ECE majors). Nevertheless, on what is arguably one of the most standard questions that can be posed about an op-amp circuit, an appreciable population

	Engineering			Physics	
	Circuits, non-majors (N=76)	Circuits, Majors (N=101)	Electronics (N=68)	Electronics (N=59)	Total (N=290)
V_{out} = +2.5 V (Correct)	**54%**	**68%**	**82%**	**69%**	**68%**
V_{out} = -2.5 V (Sign Error)	9%	12%	13%	13%	12%
Derived from KCL (Correct)	37%	2%	7%	54%	22%
Gain Equations (Correct)	26%	77%	84%	20%	55%
Left (Correct)	**29%**	**46%**	**62%**	**69%**	**49%**
Correct Reasoning	14%	22%	50%	60%	32%
Right	53%	44%	28%	29%	40%
Current from V_{in} or from V_{in} to V_{out}	22%	17%	6%	9%	14%
Zero	13%	10%	7%	2%	9%
Golden rule II	3%	6%	1%	2%	3%
Virtual ground at A	0%	1%	0%	0%	0%
$\lvert I_F \rvert = \lvert I_G \rvert = 0$ (Correct)	**67%**	**70%**	**79%**	**84%**	**74%**
Correct Reasoning	46%	48%	54%	73%	53%
$V_F = V_G = 0$	1%	5%	4%	2%	3%
$\lvert I_A \rvert = \lvert I_B \rvert = \lvert I_C \rvert > 0$	**11%**	**9%**	**33%**	**53%**	**22%**
Correct Reasoning	7%	8%	9%	36%	12%
$\lvert I_A \rvert = \lvert I_B \rvert > \lvert I_C \rvert = 0$	21%	7%	12%	11%	12%
$\lvert I_A \rvert = \lvert I_B \rvert = \lvert I_C \rvert = 0$	0%	1%	0%	0%	0%
$\lvert I_C \rvert = 0$ *justified by Overgeneralizing Golden Rules*	8%	6%	6%	9%	7%
$\lvert I_C \rvert > \lvert I_A \rvert = \lvert I_B \rvert > 0$	4%	13%	3%	16%	9%
$\lvert I_A \rvert = \lvert I_B \rvert > \lvert I_C \rvert > 0$	22%	10%	33%	0%	17%
$\lvert I_D \rvert > \lvert I_A \rvert = \lvert I_B \rvert = \lvert I_C \rvert > 0$ (Completely Correct)	**0%**	**2%**	**3%**	**16%**	**4%**

Table 6.2. Student responses to the non-inverting amplifier task.

(accounting for between 3–30% of students in individual courses) gave fundamentally incorrect responses.

From the range of responses (seen in Table 6.2), it appears that students in the engineering electronics might have been correct somewhat more often than the students in the equivalent physics course. However, the difference is not statistically distinguishable ($p = 0.15$, $\chi^2 = 2.06$); furthermore, the difference between the two engineering circuits courses was slightly above the threshold of significance ($p = 0.07$, $\chi^2 = 3.22$) with a small to moderate effect size ($V = 0.15$). These results suggest that there were not meaningful differences in outcomes between comparable courses. It should also be noted that the difference between the engineering circuits and electronics courses for majors was not statistically significant either ($\chi^2 = 1.1$, $p = .29$), which is unsurprising as students would not have had substantial additional instruction on op-amps in the ECE course sequence.

On part 2, many students correctly recognized that current is to the left through point *A*, with considerable variation (30–75%) between courses. At least half (50–80%) of these students in a given course supported the correct answer with correct reasoning. For example, one student wrote, "*Left,* V_{in} *is negative and virtual ground on the other side*." While it was unstated that traditional current is directed from high to low voltage, and some students did state this explicitly, it was not expected that students would use such fine-grained reasoning in responses to this task.

Between 30–50% of students in a given course incorrectly indicated that the current through point *A* is to the right. Of these incorrect responses, approximately 20–40% were supported by statements indicating that current either would come from V_{in} into the circuit or from V_{in} to V_{out}. For example, one student wrote, *"To the right. It cannot be 0 because there is a potential difference between V_{in} and A. It is to the right because current flows through the circuit from V_{in} to V_{out}."* Another simply stated that *"V_{in} causes current to flow to V_{out}."* This idea that current comes from the voltage source was the most prevalent incorrect explanation offered for a current to the right through point *A*. Between 5% and 40% of these incorrect responses from a given course were supported by *correct* reasoning (*e.g.*, they stated that the direction of the current was from high to low potential) which suggests that some students may have been treating V_{in} as a positive voltage. It is also conceivable, however, that some of these students were trying unsuccessfully to reconcile correct formal reasoning with a perhaps more intuitive sense that current should come from the voltage source.

Between 2% and 13% of students claimed that there was no current through point *A*. Nearly all of the reasoning justifying these responses (when given) was based on Golden Rule II, suggesting that many students either failed to recognize that point *A* is located to the left of the junction or did not realize that it is possible to have current through the feedback loop. For example, one student wrote, *"0, Golden Rule II states there is no flow of current at point A."* Such responses, along with those observed in the first op-amp task, further support the idea that some students are unaware of the nuances of the second Golden Rule in that it applies only to the inputs of the op-amp itself.

On part 3, the majority of students (67–85%) correctly indicated that the currents through points *F* and *G* were both zero, with an additional 6–12% indicating that the currents were equal (but not explicitly zero). Roughly 45% to 75% of students in a given course supported correct answers with correct reasoning. However, it is important to note that a small portion of students (roughly 3%) incorrectly argued that both currents are zero because both points are grounded. For example, one student wrote:

> *"The current at F and G are equal because it's an ideal op-amp. G is grounded so the volts and amps at G equal 0. This means the voltage at F is also zero which means no current is flowing through F."*

Such a line of reasoning may be due to students attributing ohmic behavior to the op-amp's input terminals. It should be noted that some textbooks, when describing the properties of real op-amps, depict a large but finite impedance between the two inputs [93]; however, it was not clear if students were using such a model as the basis of their responses.

On part 4, few students (between 0% and 16% of those in a given course) were able to correctly rank all four currents A–D, and the reasoning used to justify the current through D was nearly always incorrect. Furthermore, instruction on op-amps in most classes typically did not discuss rail currents, although more emphasis was added in later years in the physics electronics course. Thus, it is more productive here to focus solely on students' treatment of the currents through points A–C, which students would have encountered in derivations of the inverting amplifier's gain.

More students were able to successfully determine the relationship between the currents at points A–C, with between 10% and 55% of students in a given course correctly ranking the three currents: $|I_A| = |I_B| = |I_C| > 0$. The most prevalent incorrect ranking, given by between 2% and 32% of student in a given course, was $|I_A| = |I_B| > |I_C| \neq 0$. This is of particular interest, as nearly no students (2%) in the UMaine physics course gave this ranking, and indeed it was less prevalent in physics courses at UW and UA [36]. In this dataset, the next most prevalent incorrect response was similar to the former response but explicitly indicated that there would be no current at point C: $|I_A| = |I_B| > |I_C| = 0$; this response was given by between 7% and 21% of students in a given course. Finally, between 3% and 16% of students in a given course indicated that $|I_C| > |I_A| = |I_B| > 0$, but no single line of reasoning was particularly common to this response.

Despite the ranking $|I_A| = |I_B| > |I_C|$ being the most prevalent, few students (<25%) provided any sort of reasoning in support of this response. Nevertheless, from the limited reasoning provided, there was a tendency for students to assume that there would be current through V_{out}. For example, one student wrote, *"A = B because no current goes through the amp, the current from B is split between* V_{out} *and C…"* This tendency to ascribe a current to the unloaded output terminal of the circuit, previously discussed in the context of diode circuits, was thus also present, though far less prevalent, in the context of operational amplifier circuits. When examining responses in support of $|I_A| = |I_B| > |I_C| = 0$, the explanations tended to focus on why the current through point *C* must be zero. In the broader, cross-institutional study [36], two distinct categories of reasoning emerged which were used to justify why $I_C = 0$: a tendency to generalize Golden Rule II inappropriately, and a failure to account for the correct behavior of the rails when applying Kirchhoff's junction rule to the op-amp.

Of those two lines of reasoning, only the former (*i.e.*, an assumption that there is no current into or out of any terminal of the op-amp) was observed at UMaine as justification in support of student answers. For example, one student wrote, *"A, D, and C are zero, no current enters or exits the rails or the terminals of the op-amp."* This student appears to have incorrectly generalized Golden Rule II to include the op-amp's *output* in addition to its inverting and non-inverting inputs. Other student explanations were considerably more specific, with one student noting, *"Op amp output gives no current because it has infinite output impedance."* Although students are typically taught that an extremely *low* output impedance is an important and useful characteristic of op-amps, this student appears to have applied the idea of infinite input impedance to the output of the op-amp instead. Between 5–10% of responses in a given course explanations fell into this category.

6.5.4 Additional Difficulties Noted Across Disciplines

In addition to the difficulties highlighted in response to the three amplifiers task, several additional difficulties have been identified associated with the inverting amplifier task.

Tendency to apply Kirchhoff's junction rule inconsistently in op-amp circuits. A significant percentage (approximately 80%) of all students at UMaine gave rankings in which the currents through points *A*–*C* were not equal. When doing so, the junction rule was often applied to certain junctions but not others. This tendency varied widely, ranging from 50% of the physics electronics course to 90% of the engineering circuits course. A focus on student rules about op-amps (*e.g.*, an overgeneralized Golden Rule II) often seemed to preclude the application of the junction rule to the node joining point *B, point C,* and the circuit's output, V_{out}. While most students in all courses investigated had a basic understanding of the junction rule, the salience of specific features in these advanced circuits appeared to trigger alternative lines of reasoning, making it more difficult for students to recognize the need to apply the junction rule in such cases. Indeed, Kautz reported similar phenomena in the context of ac circuits [65].

Tendency to assume current always comes from V_{in} or always goes from V_{in} to V_{out}. A significant fraction students expressed the idea that current always comes from the power supply, apparently ignoring the sign of V_{in} and treating the supply as though it is only able to output current. A similar difficulty was noted in the reverse-biased diode task in Chapter 5, where approximately a quarter of students made similar assumptions. This is reminiscent of and may be related to the tendency of introductory students to think of the battery as a constant current source, a prevalent difficulty that has been documented in the literature [19]. Moreover, for some students, the voltage input and output of an op-amp circuit seemed to correspond to the input and output of current, respectively; such responses may be related to tendencies of students to use current-based reasoning [13]. As a result, these students struggled to analyze the circuit's currents in a productive manner and typically failed to draw on relevant fundamental circuits concepts.

6.6 Discussion, Conclusions, and Ongoing Work

In this multi-course study, student conceptual understanding of basic operational-amplifier circuits was investigated in the context of an upper-division physics course on analog electronics as well as electrical engineering courses on introductory circuits and analog electronics. It was found that students in all populations struggled to analyze basic op-amp circuits after relevant instruction; in particular, tasks requiring predictions of the behavior of "perturbed" op-amp circuits or detailed examinations of the currents and voltages in a canonical circuit served to highlight which difficulties were most prevalent. As discussed previously, students often gave reasoning and drew conclusions that were inconsistent with the Golden Rules. In addition, students largely failed to demonstrate a basic understanding of the role of the op-amp's power rails, and many students did not apply fundamental circuits concepts consistently and systematically. It should be noted that while students struggled with the same difficulties across physics and engineering courses, in some cases there were large differences between responses in the four courses. In particular, students in the physics electronics course performed somewhat better than students in the equivalent engineering courses; this may in part be explained by the fact that preliminary results from this project have been used to inform instruction in that course. However, these findings still suggest a need for increased emphasis on certain relevant topics (*e.g.,* the power rails) and for research-based and research-validated instructional materials that address the difficulties identified.

In response to the difficulties observed in this study, a short tutorial that targeted student understanding of op-amp currents was developed, shown in Appendix A. Through this activity, students were guided to first reason through a full derivation of the gain of a non-inverting amplifier circuit. Then, after concluding that (under the circumstances presented) there would be a current from the op-amp's output but none into either input, the op-amp rail currents were introduced as a way of reconciling the circuit's behavior with Kirchhoff's current law. Associated with the tutorial is a short, in-class activity that uses a trio of ammeters to visually demonstrate that the op-amp indeed follows Kirchhoff's laws for any reasonable input voltage. Despite variations in implementation for the post-assessment, the preliminary results from the physics electronics course and the engineering circuits course for ECE majors were promising, and in the future this activity will be further refined as more data are gathered from additional courses and institutions.

Chapter 7

INVESTIGATING STUDENT UNDERSTANDING OF TRANSISTOR CIRCUITS

Bipolar-junction transistors (BJTs) are semiconductor components that were foundational in the development of modern electronics, and although field effect transistors have superseded them for some applications, BJTs still see frequent usage in modern circuits and devices. In particular, the npn common emitter amplifier circuit is a fundamental building block of other, more sophisticated circuits. This amplifier (shown in Fig. 7.1) is typically designed such that it has both a high input impedance (meaning that the input voltage will not be impacted by the addition of the amplifier) and a low output impedance. Discrete versions of such amplifiers are well suited for use as part of audio amplifier circuits and may be combined with other transistor circuits for interfacing control circuits with high-power loads (*e.g.*, motors and resistive heaters). As such, bipolar-junction transistors are useful for many practical experimental applications.

As with diodes, BJT circuits were taught only in upper-division courses in either physics or engineering. This is unsurprising, as understanding diode functionality is a prerequisite for understanding the behavior of BJTs. BJTs are three-terminal devices that, in most typical operating modes, act as current amplifiers. Their ability to provide a larger output current than input current is due to the fact that the device is effectively "powered" via the connection of a third terminal to a power supply. Furthermore, it should be noted that biasing is of particular importance in transistor circuits; not only do input signals into canonical BJT amplifiers need biasing, but the output of many transistor circuits is biased around a constant, non-zero dc offset. This is in contrast to typical operational amplifier circuits, such as the inverting and the non-inverting amplifiers discussed in the previous chapter. For such circuits, the input and output voltage signals are centered around ground when used as ac amplifiers.

To date, there has been no published work specifically focused on student conceptual understanding of the behavior of bipolar-junction transistor circuits. While there has been some work reported on possible approaches for teaching about transistor circuits in engineering education journals, it has primarily centered around the pedagogical tool of combining transistors into functional groups [32]; this publication does not provide any data on the efficacy of such an approach, nor does it provide insight into what problems students might encounter with basic circuits involving a single transistor.

In order to better understand which aspects of transistor behavior are well understood by students after instruction as well as which aspects students struggled with, an in-depth, multi-year investigation across multiple institutions was performed. Unlike the investigations reported in previous chapters, this study was more focused on the development of a deeper and nuanced understanding of what features of transistor circuits were challenging for students across a variety of institutions, rather than a comparison between physics and engineering courses contexts at a single institution. In addition, the iterative process of developing additional targeted conceptual questions on the basis of emerging findings from research will be highlighted. By establishing a more coherent description of student understanding of transistor circuits, this work lays the foundation for guiding the development of targeted research-based instructional materials.

7.1 Context for Research and Overview of BJT Coverage

This study was conducted in upper-division courses on analog electronics in physics at the University of Washington (UW), the University of Maine (UM), and the University of Colorado Boulder (CU), as well as in upper-division courses on analog electronics for Electrical and Computer Engineering (ECE) students at UM. Transistors were typically covered during the second half of all courses investigated, after students had studied diode circuits (as mentioned previously). Many published instructional sequences in physics (see, for example, those in Horowitz and Hill [90] or Galvez [89]), introduce transistor circuits before operational amplifiers, and in such sequences transistor amplifiers would be the first amplification circuit students encounter. However, it is more common in engineering curricula and texts to instead sequence instruction on op-amps first [88]. In the courses surveyed, a transistor-first approach was used at UW and during the first three years of UM physics course, and an op-amp first approach was used at CU, the UM engineering courses, and during the last two years of the UM physics course.

As labeled in Fig. 7.1, the three terminals of a BJT transistor are the collector, base and emitter, whose voltages are denoted V_C, V_B, and V_E, respectively. In a first-order model of the bipolar-junction transistor as a current amplifier, the relationship between

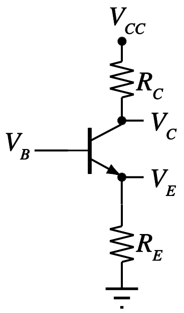


Fig. 7.1. The canonical BJT common-emitter amplifier circuit. The base, collector, and emitter voltages have been labeled for clarity.

these three voltages determines the operational mode of the transistor, which in turn determines the relationships between currents at each junction. The requisite conditions and subsequent behavior for an npn transistor to be forward-active, as articulated by Horowitz and Hill [90], are paraphrased as follows:

1. The collector (V_C) must be at a higher potential than the emitter (V_E).
2. The base-emitter (V_{BE}) and base-collector (V_{BC}) connections consist of p-n semiconductor junctions, and thus behave like diodes. Normally the base-emitter diode is conducting and the base-collector diode is reverse-biased. (Note that there may still be current from collector to emitter in this situation).
3. Any given transistor has maximum values for the collector current (I_C), base current (I_B), and collector-emitter junction voltage (V_{CE}) that cannot be exceeded without damaging the device.
4. When rules 1-3 are obeyed, I_C is roughly proportional to I_B and can be written as $I_C = \beta I_B$, where β is typically about 100. The collector and base currents are into the device, and the current at the emitter is out of the transistor.

While this is a relatively informal treatment of transistors, it is sufficient for the description of the operation of transistors in the forward-active regime (stipulated by the first three rules). In all courses, students were expected to be able to use such a model to explain the basic behavior of emitter follower circuits (*i.e.*, circuits with an output at V_E in Fig. 7.1) as well as common emitter amplifier circuits (*i.e.*, circuits with an output at V_C in Fig. 7.1). It should be noted that while some courses may have included more thorough/sophisticated treatments of transistor circuits (*e.g.*, a small-signal model was introduced in the UM engineering course), the basic model described here is sufficient to reason about all tasks presented in this chapter.

7.2 Research Questions

This chapter seeks to answer the following research questions:

1. To what extent did students develop a functional understanding of bipolar junction transistor behavior? In particular:
    1.1. Did students recognize those circuits in which it was appropriate to apply a gain formula, and did they apply the correct gain?
    1.2. Did students productively apply diode-like reasoning to the transistor's BE junction?
    1.3. Did students correctly apply and interpret the BJT's current gain relationship?
    1.4. Are there differences in learning outcomes from comparable courses?
2. What specific difficulties emerged from the responses provided by students, and did the prevalence of difficulties vary between courses?

In order to address these questions, a total of five tasks are discussed, each focused on a different aspect of transistor circuits. The first of these, the three amplifier comparison task, was an open-ended task that was designed to probe if students had a functional understanding of common-emitter amplifier and follower circuits. It was also recognized that there was a need to explicitly focus on student understanding of foundational transistor behavior, which motivated the design of the second and third tasks (follower currents and follower graphing). After analyzing data from the first three tasks, it became evident that further targeted questions were needed to better determine which facets of transistor behavior students struggled with most, and thus the transistor supply voltage variation task and the revised amplifier comparison task were created. In addition to these, a sixth task regarding the behavior of ac biasing networks (which frequently accompany transistor circuits) is discussed at the end of the chapter.

7.3 Three Amplifier Comparison Task

Fig. 7.2 illustrates the basic circuit that was permuted by changing the values of resistors R_E and R_C or by selecting the output voltage from either V_E or V_C. Such emitter amplifier circuits are primarily used for their ac amplification characteristics, and are part of students' introduction to the topic of signal processing (after filtering circuits). Indeed, the leftmost three components in this circuit (the 56-kΩ and 5.6-kΩ resistors, as well as the capacitor C) may be thought of as equivalent to a biased high-pass filter, which will be discussed in detail in section 7.8.

While there are three possible BJT amplifier topologies (common base, common emitter, and common collector), the common emitter amplifier (which uses V_C as the output) is the most broadly used, and thus it is the first (and sometimes only) amplifier configuration introduced in all textbooks used in courses investigated in this study. This canonical circuit is used extensively for small-signal voltage amplification, and represents an important building block for use in more complex transistor circuits. Given

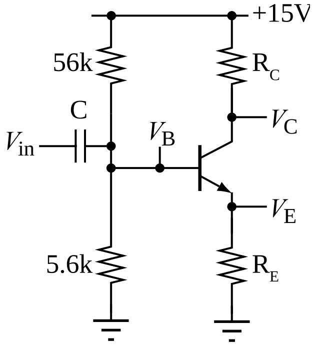


Fig. 7.2. Canonical BJT common-emitter amplifier circuit used in the three amplifier comparison task. The base, collector, and emitter voltages have been labeled for clarity.

the ubiquity of the circuit in both instruction and in electronics applications, the first task was designed to probe student understanding of this relatively complex but important circuit by introducing slight modifications to a base circuit. Performance on this task would provide information on the extent to which students were developing a functional understanding transistor circuits.

7.3.1 Task Overview

In the three amplifier comparison task, shown in Fig. 7.3, students must compare the small signal ac behavior of three properly and identically biased transistor circuits. Circuit B is the basic common-emitter amplifier circuit, and the other two are slight modifications of circuit B. In circuit A, the collector and emitter resistors are switched, which affects the amplifier's gain. In circuit C, the output voltage is taken at the transistor's emitter, and thus circuit C is an emitter follower configuration. Students are then asked to rank the peak-to-peak amplitudes of the output voltages of all three circuits ($V_{out,A}$, $V_{out,B}$, and $V_{out,C}$) from largest to smallest and to explain their reasoning. In order to answer this question correctly, students need a sufficiently robust understanding of the circuit's behavior in order to ascertain the impact of switching the resistors (the comparison between circuits A and B) and switching the location of the output terminal (the comparison between circuits B and C).

Consider the following three circuits (A – C) shown at right. All three *npn* bipolar-junction transistors are identical, and the input voltage V_{in} for each circuit is a 1 kHz sinusoidal signal with a 1V peak-to-peak amplitude. Note that the portion of each circuit to the *left* of the transistor is *identical* in all three cases.

Rank, from largest to smallest, the peak-to-peak amplitudes of the output voltages from the three amplifier circuits ($V_{out,\,A}$, $V_{out,\,B}$, and $V_{out,\,C}$). If any of the peak-to-peak amplitudes are equal in magnitude or are equal to zero, state so explicitly. If there is not enough information to rank the output voltages, state so explicitly and indicate what additional information is necessary. In all cases, explain.

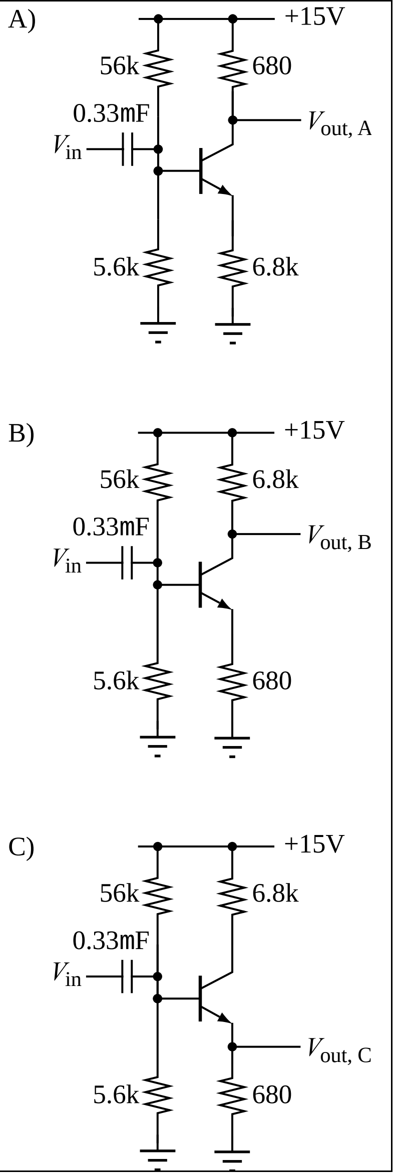


Fig. 7.3. Three amplifier comparison task, in which students were asked to rank the ac outputs of three pertubations of a BJT amplifier circuit.

7.3.2 Correct Response

In this section, the steps required to arrive at a correct ranking of all three peak-to-peak voltages are presented. In order to ensure that this solution is accessible to readers who may be somewhat less familiar with transistor circuits, additional background information will be provided as needed. In addition, since there is a need to differentiate between ac and dc voltages, a prefix of a lower case delta is used (δ) when discussing periodic (ac) variations in voltages with respect to time whereas no prefix indicates a dc quantity.

7.3.2.1 AC Biasing Network

As mentioned previously, transistor circuits typically require input signals to be biased around a constant, non-zero dc voltage. In order to create such a signal for an arbitrary input, the left three components in all three circuits (the "blocking" capacitor *C,* the 56-kΩ resistor, and the 5.6-kΩ resistor) form a biasing network that serves both to remove any existing offset from V_{in} (the primary function of the capacitor) as well as to introduce a constant offset. This offset value is determined by the voltage divider formed by the two resistors, and for this circuit the resulting offset is +1.36 V. It should be noted that the biasing network also acts as a filter, but for this task, the component values have been chosen such that the biasing network would not cause any attenuation of the input (*i.e.*, $\delta V_{in} = \delta V_B$). Since the biasing networks are identical and appropriate for all three circuits, students who operate under the assumption that all inputs are properly biased (without verifying so explicitly) can still arrive at a correct ranking. The question of student treatment of such ac biasing networks is specifically addressed later in section 7.8.

7.3.2.2 Emitter Follower Circuit

For the given 1 V peak-to-peak amplitude of the input (δV_{in} = 1 V) and the 1.36 V dc offset, the transistor will be properly biased for all values of the V_{in}. Since the BE junction has diode-like behavior (*i.e.*, the voltage across the junction V_{BE} remains approximately constant at 0.6 V), any variation in the voltage at the transistor's base will cause a corresponding change in the emitter voltage ($\delta V_B = \delta V_E$). Thus, the peak-to-peak amplitudes at the input, base, and emitter will all be the same ($\delta V_{in} = \delta V_B = \delta V_E$). For the follower circuit (circuit C in Fig 5.2), where the output is taken from the emitter, this implies that the ac gain of the circuit is 1 (*i.e.*, $\delta V_{out,C} = \delta V_{in} = 1\ \mathrm{V_{pk\,pk}}$).

7.3.2.3 Common-Emitter Amplifier Circuit

To find the outputs of the other circuits (A and B) in Fig. 7.3, further analysis is required. In particular, it is necessary to determine the voltage at the collector. This is related to the collector current, which in turn possesses a known relationship with the emitter current. Since the emitter voltage is known, a variational form of Ohm's law may be used to relate the voltage and current at the emitter: $\delta I_E = \delta V_E / R_E$. For the collector resistor, note that the same equation is true, but the point of measurement is not *across* the resistor but rather from the collector terminal to ground. As a result, *increasing* the voltage across the collector resistor will result in a *decrease* in the voltage at the transistor's collector (point V_C). This 180° phase shift yields $\delta I_C = - \delta V_c / R_C$.

Next, because the currents through the collector and emitter may be treated as essentially equal ($I_E \approx I_C$), any variations in emitter current and collector current are likewise approximately equal ($\delta I_E \approx \delta I_C$). This may in turn be substituted into the variational form of Ohm's law to relate the collector and emitter voltages:

$\delta V_C / R_C = -\delta V_E / R_E$. Furthermore, the variation of the input voltage may be substituted in place of emitter voltage and terms rearranged to arrive at the gain expression for the common-emitter amplifier: $\delta V_C = -\delta V_{in} (R_C / R_E)$.

This result, derived and covered in all courses studied, implies that the magnitude of the variation in the output, δV_C is scaled by the ratio of collector to emitter resistors. It should be noted that common emitter amplifiers are typically designed to increase the magnitude of the output voltage relative to the input of circuit, and thus students typically encounter designs where R_C is larger than R_E.

Using appropriate component values and simplifying for circuits A and B, it can be shown that $|\delta V_{out,A}| = 1/10\ \delta V_{in}$ and $|\delta V_{out,B}| = 10\delta V_{in}$. Together with the result for the follower circuit, this implies that $V_{out,B} > V_{out,C} > V_{out,A}$. While this section demonstrated the derivations of the small-signal gain for both emitter follower and common-emitter amplifier circuits, students could also come to a correct answer by recalling the appropriate gain equations and applying them to the circuits in Fig. 7.3.

7.3.3 Overview of Student Performance on the Three Amplifier Comparison Task

Data for this task were collected in three classes each in the engineering electronics courses at UM (N = 57) as well as physics electronics courses at UM (N = 42), CU (N = 142), and UW (N = 169) for a total of 410 responses, as summarized in Table 7.1. The question was administered either as an ungraded conceptual question, which students were asked to complete in approximately 10-15 minutes, or as part of a course's final exam.

As shown in Table 7.1, the distribution of students giving a correct ranking of the peak-to-peak output voltages of all three circuits ($V_{out,B} > V_{out,C} > V_{out,A}$) varied widely across the different institutions, with students at CU appearing to be much more successful on the task (42% correct) than their peers at UM or UW (ranging from 7% and 14% correct in a given course). In terms of the percentage of students giving correct responses, the courses at UM and UW were indistinguishable from one another ($p = 0.39$, $\chi^2 = 1.89$). However, when comparing responses from these courses to those from CU, there was both a statistically significant difference ($p < .0001$, $\chi^2 = 26.94$) as well as a small to moderate effect size (Cramer's V = 0.2367). Thus, it is possible that there were systematic instructional differences between the coverage of relevant BJT circuits at CU and that at the other institution.

An example of one student's correct and complete responses is given below:

> *"The DC voltage at the base is determined by the voltage divider:*
> $V_B = 5.6\ k\Omega / [5.6\ k\Omega + 56\ k\Omega] \cdot 15\ V$
> $V_B = 1.36\ V$
> $V_E = V_B - 0.6 = 0.76\ V$*, but this doesn't matter.*
> *Since C is an emitter follower, it has unity gain, thus* $V_{out,C} = 1 V$ *p-p*
> *In cases A and B, the gain of a common emitter is given by* $G = -R_C / R_E$

	CU Physics (N = 142)	UM Engineering (N = 57)	 Physics (N = 42)	UW Physics (N = 169)	Total (N = 410)
$V_B > V_C > V_A$ (Correct)	**42%**	**7%**	**14%**	**14%**	**23%**
Correct & Complete Reasoning	*37%*	*0%*	*5%*	*6%*	*16%*
$V_A > V_B > V_C$	32%	33%	38%	30%	32%
Closest Resistor and Drop	*9%*	*14%*	*12%*	*18%*	*14%*
Rank Bias Voltages	*20%*	*2%*	*21%*	*2%*	*10%*
$V_B > V_A > V_C$	9%	30%	10%	12%	13%
Closest Resistor	*2%*	*4%*	*2%*	*7%*	*4%*
Gain/Follower Error	*5%*	*5%*	*2%*	*2%*	*3%*
$V_A > V_C > V_B$	2%	4%	7%	11%	7%
Closest Resistor	*0%*	*2%*	*0%*	*7%*	*3%*

Table 7.1. Overview of student performance on the transistor amplifier comparison task in electronics courses at three different institutions. The question is shown in Fig. 7.3.

$$\text{For A: } R_C = 680\ \Omega,\ R_E = 6800\ \Omega,\ |G| = 1/10$$
$$V_{out,A} = 0.1\ V\ p\text{-}p$$
$$\text{For B: } R_C = 6800\ \Omega,\ R_E = 680\ \Omega,\ |G| = 10$$
$$V_{out,B} = 10\ V\ p\text{-}p$$
$$V_{out,B} > V_{out,C} > V_{out,A}\text{”}$$

A total of 37% of students at CU and between 0% and 5% of students at either UM and UW similarly supported their answers with correct reasoning involving the gain expression of the common-emitter amplifier. Thus, while the majority of students who gave a correct response at CU supported their answer with correct reasoning, students at UM and UW who arrived at a correct ranking typically used incorrect or incomplete lines of reasoning to do so. However, there was not any single common line of incorrect reasoning used by a significant number of students to support a correct ranking. This is perhaps unsurprising, since if students had to reconstruct the gain relationship from first principles, they would have had to reconstruct a relatively complex chain of reasoning.

The most common incorrect ranking, given by approximately one third of all students, was that $V_{out,A} > V_{out,B} > V_{out,C}$. One student supported this ranking in the following manner:

> *“Because in circuit A, there won't be as much of a voltage drop across the 680 Ω resistor as there will be across the 6.8 kΩ resistor. As circuits B and C have the voltage divider switched, A will be greater. And $V_B > V_C$ due to the voltage drop across the transistor."*

Here the student made the comparison between circuits A and B by considering the collector and emitter resistors, and ranked the voltage in circuit A as being higher due to the smaller resistor causing a smaller voltage drop with respect to the +15V supply. The comparison between B and C was made with the knowledge that they are two points measured across otherwise identical circuits, and that there was a decrease in voltage from the collector to the emitter of the transistor, and thus the output of circuit B (V_B)

must be at a higher voltage than the output of circuit C (V_C). Approximately 15% of all students gave similar reasoning, with many students explicitly using the nomenclature of a “diode drop” due to the transistor when justifying a comparison of the emitter and collector voltages. It should be noted that such behavior is not guaranteed for a forward active transistor; the voltage difference would be nearly V_{CC} in situations with little current through the transistor.

This line of reasoning produces the correct ranking for the bias (dc) voltages in the circuit, but even for that purpose it is incomplete. There is an implicit assumption made by the students when using the resistances to compare voltages between circuits A and B that the collector currents in both circuits are the same, which is untrue. In practice, increasing the resistance at the emitter would decrease the current through the collector, also resulting in a smaller voltage across the collector resistor. Regardless of intention or completeness, this line of reasoning is fundamentally unsuitable for analyzing the ac behavior of the common emitter amplifier.

An example of another common incorrect line of reasoning leading to the same ranking is the following:

“Consider A. The current through the emitter is approximately the same as the current through the collector, $I_E \approx I_C$. *Furthermore, the emitter also acts as a diode (shifts the voltage down* $\approx 0.6V$ *at every point so amplitude remains the same). Current through 6.8k =* $V / R = 1V / 6.8k = .14$ *mA. Then since* $I_E = I_C$, $V_{out,A} = (.14mA)(680)=.1V$ *with reference to 15 V, we have 14.9 V. For circuit B, current through 680 resistor =* $V / R = 1V / 680 = 1.4$ *mA. Again, through the 6.8k, the voltage is ((1.4mA)(6.8k) =10V), with respect to 15 V we have 5 V. For C we have the voltage across the 680 resistor = 1 V.”*

As in the previous response, this student also made a comparison of the bias voltages in the circuit. In this case, the student explicitly used the currents through the resistors to make a comparison between the output voltages of circuits A and B. Approximately 10%

of students used similar reasoning based on calculated currents through the resistors in the circuit. This reasoning is an example of a correct method for finding the *bias* voltages, although the prompt asked for the peak-to-peak signal voltages. Furthermore, this student used the ac peak-to-peak amplitude of 1V instead of the appropriate dc voltage of 1.36 V from the biasing network for the calculations.

The next most prevalent answer was the ranking $V_{out,B} > V_{out,A} > V_{out,C}$, given by between 10% and 30% of students. In support of this ranking, one student wrote:

> *"I think that because $V_{out,C}$ is on the right hand side of the diode type emitter, that it will have the smallest drop because of the -0.65 V. Then, the voltage drop across $V_{out,B}$ will be the largest because there is a larger resistance between it and the +15 V. So, ratings go $V_{out,C} < V_{out,A} < V_{out,B}$."*

Another student reasoned as follows:

> *"At B the peak to peak will be very high b/c of the large resistance & low current. A is a lot like B except the resistance is smaller, therefore the $V_{pk\text{-}pk}$ is smaller. At C, the resistance is really low but the current is very high so the chop is smaller."*

These responses differ greatly in the justification used for ranking circuit C, but they both used the same sort of reasoning to compare A and B: that the larger collector resistor in circuit B results in a larger voltage. Neither of these students discussed a difference from the 15V source in their reasoning, so their comparisons are of the voltages *across* the collector resistors for circuits A and B (explicitly so in the former case). Thus, neglecting the specific reasoning associated with C, between 10% and 60% of the reasoning provided for this ranking by students in a given course was based on the relative resistances of the collector resistors. Justification for circuit C's output being the smallest varied greatly, and no explanations were common to enough to warrant extensive discussion here.

A major difference between the former answer and most common incorrect response of $V_{out,A} > V_{out,B} > V_{out,C}$ was that students were not incorporating the role of the +15 V supply into their argument. This line of reasoning may be a result of students not differentiating between the voltage across the collector resistor (ΔV_C) and the output voltages (at a point) $V_{out,A}$ and $V_{out,B}$, which are measured with respect to ground.

It is important to note that there was an additional common line of reasoning supporting the ranking $V_{out,B} > V_{out,A} > V_{out,C}$, as demonstrated by the following student response: *"Gain is based on ratio of (R_C / R_E) (mostly), so, $V_{out,B} > V_{out,A}$, and $V_{out,C} = V_B - 0.6V$, which is a very small voltage."* This student was able to correctly use the gain formula to compare the voltages between circuits A and B, but did not correctly analyze the output voltage of the follower circuit. Between 15% and 55% of students in a given course who ranked $V_{out,B} > V_{out,A} > V_{out,C}$ gave similar responses, with a substantial amount of variation in the reasoning behind the follower circuit's ranking. Thus, a small subset of students (<5% of the total population) were recognizing the common emitter amplifier circuits and applying the gain formula correctly, but struggled to use appropriate reasoning to justify the behavior of the follower circuit. Similar to what was observed in the context of op-amps, these students may have a fragmented understanding of the circuits in question, recalling the results for the outputs of canonical circuits without fully understanding how such results arise from device properties and circuit configurations.

The final ranking that occurs with any reasonable frequency is $V_{out,A} > V_{out,C} > V_{out,B}$, given by between 2% and 11% of students in a given course. An example of one student's reasoning supporting this response is the following: *"680 Ω is much smaller* *than 6.8 [k]Ω, thus there would be a greater* V_{drop} *over the 6.8 kΩ resistor and [therefore]* V_{out} *would be the highest…"* This student was using the closest resistor to argue that there would be a smaller voltage drop across the collector resistor in circuit A than B. Indeed, similar reasoning was used by approximately half of all students giving this answer, with widely varied reasoning for why C would be the intermediate voltage. Thus, for this ranking, the most prevalent reasoning for comparing circuits A and B is the same as in the most common incorrect response ($V_{out,A} > V_{out,B} > V_{out,C}$). However, these students did not explicitly attribute diode-like behavior to the BE junction.

Out of all the tasks discussed in this dissertation, the transistor amplifier comparison task was administered to the broadest range of students, with data across three institutions and across disciplines. However, the four rankings discussed above account for at least two-thirds of all responses; indeed, no other answers were seen with more than 10% prevalence in any individual course. This suggests that the difficulties associated with such responses are likely to be relevant to most electronics courses, rather than being specific to instruction at a single institution.

7.3.4 Overview of Student Performance: Pairwise Comparisons

While the overall ranking and reasoning used provide valuable insight into students' thinking about transistor circuits, it is evident that most students struggled to correctly compare the outputs of all three circuits. Thus, it is also useful to examine how students treated the relevant modifications from a canonical base circuit, namely either changing the output location (circuit C versus B) or reversing the collector and emitter resistor values (circuit A versus B). A summary of the breakdown of student responses for these two comparisons is presented in Table 7.2.

Of students responding with any ranking (*i.e.*, comparing at least two voltages), approximately 70% correctly found that $V_{out,C} < V_{out,B}$, with a spread between 48% and 88% of students correct in a given course. However, this particular comparison supports both ranking by peak-to-peak (ac) amplitudes or by (dc) bias voltages. Nevertheless, both lines of reasoning are consistent with the application of some productive reasoning about transistor circuits. Few students (approximately 5%) indicated that the modification of output position would have no impact on the voltages ($V_{out,C} = V_{out,B}$), but approximately 20% of students ranked the output voltage for $V_{out,C}$ as being greater than $V_{out,B}$. Common to these latter responses is that students did not apply fundamental ideas about transistor behavior (*e.g.*, that the collector voltage will always be higher than the emitter voltage when the transistor is in the forward active regime) when comparing the two circuits.

There was more variation in students' comparisons of circuits A and B, with the correct ranking ($V_{out,A} < V_{out,B}$) given by between 23% and 67% of students in a given course. A similar number of students responded that $V_{out,A} > V_{out,B}$, accounting for between 33% and 63% of responses from a given course. The majority (80%) of these incorrect responses stemmed from two of the common incorrect rankings

	CU Physics (N = 142)	UM Engineering (N = 57)	 Physics (N = 42)	UW Physics (N = 169)	Total (N = 410)
$V_C < V_B$	**85%**	**70%**	**66%**	**63%**	**72%**
$V_C = V_B$	4%	8%	7%	8%	6%
$V_C > V_B$	10%	13%	18%	25%	17%
$V_A < V_B$	**58%**	**46%**	**31%**	**38%**	**45%**
$V_A = V_B$	39%	51%	60%	53%	49%
$V_A > V_B$	3%	4%	10%	9%	6%

Table 7.2. Specific comparisons made by students in the transistor amplifier comparison task.

($V_{out,A} > V_{out,B} > V_{out,C}$ and $V_{out,A} > V_{out,C} > V_{out,B}$). It should also be noted that students could also conclude that the output of circuit A would be greater than circuit B if they accidentally reversed the resistors used in the gain formula; in practice few of the students providing this answer (< 5%) made such an error. In contrast, a single line of reasoning (comparing voltages by using the drop from +15V across collector resistors) accounts for nearly half (45%) of these responses, with another quarter of students explicitly discussing bias voltages. Thus, this individual comparison ($V_{out,B} < V_{out,A}$) is associated with two clear and distinct lines of reasoning.

7.3.5 Specific Difficulties Identified

Failure to differentiate between ac signal and dc bias. After all instruction on transistors, the majority of students did not appear to be addressing the ac behavior of the amplifier circuit, but rather employed reasoning appropriate only for the dc or bias voltages in the circuit. While, in some cases, this may be due to students misreading the question prompt (and thus responding to a different question), it suggests nonetheless that many students failed to differentiate between the ac and dc behavior of the circuit.

Tendency to use local features to make comparisons. The most common incorrect line of reasoning students used was to make comparisons based on the resistors adjacent to the output, as was found in approximately a quarter of all students' responses. This is unsurprising, as similar tendencies of using local reasoning were observed in the context of loading, diode circuits, and operational-amplifier circuits in the previous chapters. Such a lack of systematic analysis also supports the hypothesis that students may not have coherent models of circuits [19].

Lack of a functional understanding of the gain formula. As a stronger extension of the previous difficulty, many students did not attend to the ac behavior of the circuit at all. Only one quarter of student responses attempted to apply the common-emitter amplifier gain formula (or to derive said formula from foundational principles) when analyzing circuits A and B, and this was heavily skewed by the responses from CU. Furthermore, one third of these students made some error either in the application of the formula (~3/4 of errors) or only in the subsequent analysis of the behavior of the follower circuit (~1/4 of errors). This indicates that even when students did recognize the gain formula as a relevant feature, a significant proportion either did not know how to utilize it properly or could not rederive the behavior of the follower circuit. As reasoning through the follower circuit is an intermediate step in deriving the behavior of the common emitter amplifier, this may indicate that the later population does not have a robust understanding of transistor behavior.

Tendency to treat collector current as being independent of emitter resistor. In comparing the outputs of circuits A and B, students frequently made the assumption that the currents in both circuits would be the same. Indeed, such an assumption was implicit in all responses that relied on a comparison of the relative resistances of the exchanged resistors. Such responses are inconsistent with the very derivation of the common emitter amplifier's behavior, which relies on the fact that the emitter resistor (in combination with the base voltage) is used for determining the emitter current as well as the collector current; the same reasoning is also applicable to dc currents in the circuit. Thus, many students were not even recognizing fragments of the correct reasoning chain as being appropriate for this particular task.

The majority of students were unable to correctly rank the peak-to-peak output voltages of the follower and common emitter amplifier circuits in this task, despite having explicit instruction on both circuits in their respective courses. While a wide variation in the prevalence of specific responses from course to course was observed, student responses were mostly captured by four distinct rankings, each supported by either one or two lines of reasoning. These lines of reasoning in turn served to foreground specific aspects of the circuit analysis with which students struggled (*e.g.*, comparing bias voltages rather than signal voltages). The overall difficulties identified were also present in responses given by students at CU, even though those students were generally more successful on the task. Thus, the findings from the three amplifier comparison task provided considerable insight into the ways in which students were thinking about transistors even after all relevant instruction. Such information, in turn, may be used to inform the development of additional research tasks, the interpretation of data from additional research tasks, and the development of suitable instructional interventions.

7.4 Follower Current Ranking Task

On the three amplifier comparison task, the majority of the students struggled in their efforts to rank all three circuits according to the magnitudes of the peak-to-peak output voltages, and they frequently adopted reasoning that did not explicitly draw upon any properties of the transistor itself. Thus, it is difficult to tell if the difficulties students encountered stemmed from a lack of understanding of basic transistor properties, or if they possessed such knowledge but either did not draw upon it or failed to articulate it in their responses. Therefore another, more focused task was designed to probe the extent to which students understood the fundamental relations between the three terminal currents when the transistor is forward-active (which is applicable to both follower and common-emitter amplifier circuits).

7.4.1 Task Overview

In the follower current ranking task (see Fig. 7.4), students are shown a simple BJT emitter follower circuit, consisting of a single 3.3-kΩ resistor and a single transistor. The input voltage (at the base of the transistor) is +3 V, the collector is connected to a +15 V supply, and the emitter is connected to ground via the 3.3-kΩ resistor. Although the prompt did not explicitly state that all components are ideal, in practice the students treated them as such. Students were asked to rank the currents at the base, collector, and emitter terminals of the transistor (labeled *X*, *W*, and *Y*, respectively) and to state explicitly if any currents were equal or equal to zero. Students were also asked to explain their reasoning.

7.4.2 Correct Response

To answer this task correctly, students should first note that the collector voltage is higher than the base voltage and that the emitter is connected to ground via a resistor,

An NPN transistor is incorporated into a circuit as shown at right.

Suppose the input voltage V_{in} is constant at +3 V. Rank the absolute values of the currents through points *W, X,* and *Y* from largest to smallest. If any currents are equal in magnitude or are equal to zero, state so explicitly. Explain.

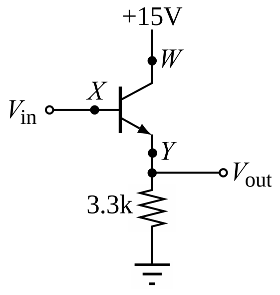


Fig. 7.4. Follower currents task in which students were asked to compare the currents in a transistor follower circuit

which results in the transistor being in the forward-active regime. As a result, the transistor current gain equation may be applied ($I_C \approx 100\ I_B$) to compare the collector and base currents ($I_C > I_B$). Furthermore, as a consequence of Kirchhoff's current law, it is also known that the currents entering the collector and the base must leave through the emitter ($I_E = I_C + I_B$), and thus the emitter current is necessarily the largest ($I_E > I_C$). Thus, in terms of the variables used in the prompt, a correct response may be written as $I_Y > I_W > I_X$. It should be noted that as the base current is typically about 1% of the collector current, it was common in instruction to assume that the emitter and collector currents were approximately equal (*i.e.*, $I_E \approx I_C$), and thus the ranking $I_Y = I_W > I_X$ would also be considered correct.

7.4.3 Overview of Student Performance on the Follower Current Ranking Task

The task was administered to a smaller cohort of students than the amplifier comparison task, corresponding to a single class at UM and three at UW. Overall, students were considerably more successful in the follower current ranking task than they were in the three amplifier comparison task, as can be seen in Table 7.3. Indeed, between 25% and 64% of students in a given course indicated the correct ranking of currents ($I_Y > I_W > I_X$). An additional 11% to 25% of students indicated that the collector and emitter currents would be the same *(i.e.,* $I_Y = I_W > I_X$). Thus, at least half of students in a either course successfully indicated that they were familiar this aspect of transistor behavior.

There were two predominant lines of reasoning used to support students' correct answers. The most common reasoning is exemplified by the following student response:

> "*A small current flowing through X will cause a much larger current to flow through W. At point Y, the current is the combined current from W and X*"

Such reasoning, in which students used the transistor gain relationship to compare currents through *X* and *Y*, along with either Kirchhoff's junction rule (informally stated in this case) or the approximation that *W* and *Y* were equal, was given by 50% of students with correct rankings.

Another common line of reasoning, given by between 8% and 14% of students in a given course in support of with correct rankings, is illustrated by the following student response:

> *"X is smallest, W is next biggest, Y is largest. Y and W are close, because Y is pretty much W + X, and X is much smaller than W (by design)"*

These students were using the assumption that the current through *X* is small rather than either explicitly or implicitly using the transistor's current gain. This assumption is usually true for many of applications of BJTs, but it is violated in configurations in which the transistor is saturated. Thus, while appropriate for the context of this problem, it is unclear if these students understood the limitations of this approximation.

For this task, no individual incorrect ranking accounted for more than 10% of the total number of responses given by students. However, it was observed that similar reasoning was used by approximately 13% of all students to justify a current ranking at

	UM (N = 12)	UW (N = 155)	Total (N = 167)
Correct Ranking	**50%**	**75%**	**73%**
$I_Y > I_W > I_X$ (Correct)	25%	64%	61%
$I_Y = I_W > I_X$ (Approximation)	25%	11%	12%
Transistor gain	25%	36%	35%
Small base current	8%	14%	13%
Incorrect rankings	50%	25%	26%
All current rankings (correct or incorrect) based on voltage	17%	13%	13%

Table 7.3. Responses to follower current comparison task. Note that the reasoning for transistor gain and small base current apply to both rankings that are fully correct and rankings that are approximations.

least in part by making a comparison of the voltages at the indicated points. For example, one student wrote: *"$I_Y > I_W > I_X$. X + W = Y, Closed loop and V_{in} is less than +15 V, so W > X."* In this response, the student used the fact that the voltage at *W* was higher than the voltage at *X* to justify the current at *W* being larger than that at *X*. The specific ranking resulting from such reasoning depended on what additional information students brought to bear on the task, but the majority (approximately 2/3) of students who used similar reasoning ultimately arrived at the correct answer for fundamentally incorrect reasons. It is likely that this difficulty is related to previously observed tendencies of students to confuse voltages and currents, or compare currents through specified points using the voltages at those points.

7.4.4 Summary of Findings

After relevant instruction, between half and three quarters of students in a given course were able to correctly rank all three currents through a follower circuit when the transistor was in the forward-active operational state. Thus, it is apparent that the majority of students do have an understanding of the functional relationships among the currents in such circuits under these conditions. Furthermore, most students were also able to support their answer with correct reasoning, and only a relatively small percentage of students (~10%) arrived at a correct ranking by incorrectly comparing voltages. Due to the small number of responses from UM, it was not possible to make meaningful comparisons of differences between student performance in courses at UM and UW.

7.5 Follower Graphing Task

While the follower current ranking task probed the extent to which students understood the functional relationships among currents for a forward-active transistor, it remained to be tested if students could productively apply ideas about the base to emitter voltage in transistor circuits. To first order, the BE junction of an npn transistor (in isolation) may be treated as a diode: allowing no current before a 0.6 V threshold is reached, and thereafter the current is determined by the circuit configuration. Indeed, some practical circuit designs exploit this behavior and intentionally use discrete transistors as diodes. Thus, to better understand how students treat the BE junction specifically, the follower graphing task (shown in Fig. 7.5) was created and administered.

7.5.1 Task Overview

In the follower graphing task, students are presented with the same BJT follower circuit used in the follower current ranking task. However, in this case, students are told that the input voltage increases linearly from -2 V to 2 V over a time interval of 8 seconds, as depicted graphically in Fig. 7.5. Students are asked to produce a quantitatively correct graph of the circuit's output in the space provided and to explain their reasoning.

7.5.2 Correct Response

To give a correct response to this task, a student must identify the time interval in which the voltage at the base is at least 0.6 V greater than ground, and the interval in which it is not. When $V_B > 0.6$ V, the transistor will be in the forward-active regime, and the voltage at the emitter will be (approximately) 0.6 V lower than the base (*i.e.*, $V_E = V_B - 0.6$ V). When the base voltage is lower than 0.6 V, the diode-like base-emitter junction will not be forward biased, and thus there can be no current through the emitter. This in turn implies that V_{out} will be 0 V, as there can be no current through the 3.3-kΩ resistor, and thus the potential difference across the resistor must be 0 V. Therefore, a quantitatively correct graph of V_{out} remains at 0 V until V_{in} is 0.6 V (at $t \approx 5.2$ s), and then increases linearly and with the same slope as V_{in} after that time (as depicted in Fig. 7.6A). It should be noted that, when considering only the relationship between the input and output voltages during the time interval shown, the circuit essentially behaves identically to a circuit in which the transistor is removed and a semiconductor diode replaces the transistor's BE junction.

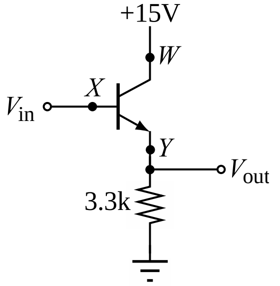
7.5.3 Overview of Student Performance on the Follower Graphing Task

The task was administered to students in four classes at UM and four classes at UW, either at the same time as the follower current task or as an independent question. As seen in Table 7.4, between 25% and 60% of students in a given course were able to produce a graph with the requisite quantitative features (*i.e.,* $V_{out} = V_{in} - 0.6$ V when $V_{in} > 0.6$ V, and $V_{out} = 0$ V otherwise). In addition, 10% of students produced graphs that had *qualitatively* correct features. To be considered qualitatively correct, the graphical response would depict an output that was zero below some threshold voltage (possibly not 0.6 V), and indicate that the output increased linearly (but possibly with an incorrect slope) for inputs above the threshold. Such responses could have incorrect thresholds, incorrect slopes, or both features incorrect. In either case, almost all students (>85%) supported a quantitatively or qualitatively correct graph with correct reasoning. For example, one student explained: *"The voltage V_{out} is equal to I_yR, I_y varies w/ V_{in} and further, $Vy - .6\ V = V_{out}$ but only when V_{in} is above .6 V, thus the one follows from the other, staggered by .6 V."* While the language used was informal, this student correctly recognized the diode-like limitations on the transistor's base to emitter junction. This also highlights that students may have difficulty in translating their correct reasoning into graphical form. Indeed, difficulties related to graphical interpretation have been studied extensively in the context of kinematics [94].

The most common incorrect response, given by approximately 20% of students, was to depict a linear output that was offset by a constant, negative amount, as shown in Fig. 7.6B. These students typically focused on the diode-like voltage drop of the transistor alone; for example, one student wrote, *“V_{out} is equal to the emitter voltage. The emitter voltage is 0.6 volts less than the base voltage... which is V_{in}. $V_{out} = V_{in}$ -0.6 volts”.*

An NPN transistor is incorporated into a circuit as shown at right.

Suppose the input voltage V_{in} increases linearly from –2 V to +2 V during an 8 s period as show below at right. In the space provided, make a *quantitatively* correct plot of the output voltage V_{out} as a function of time. Explain.



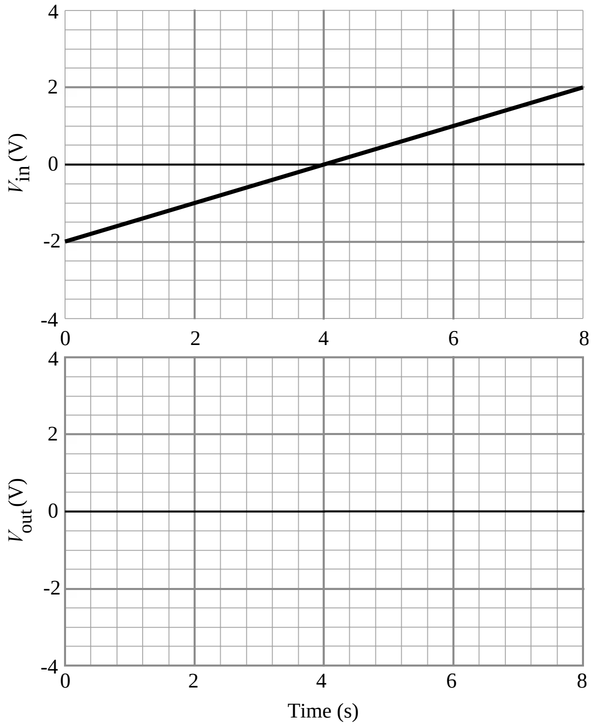


Fig. 7.5. The follower graphing task, in which students were asked to predict the output of a BJT follower circuit for a given input.

Nearly all (>80%) of the students who drew such graphs provided similar justifications for their responses. These responses may stem from a failure to recognize that a negative voltage at the emitter would imply incorrectly that current is somehow directed into the semiconductor junction at the emitter. As noted previously, the relevant transistor property for this task is the diode-like behavior of the BE junction, which would never allow (significant) current from the emitter to the base due to the orientation of the *pn* junction. Thus, these responses likely stem from difficulty in understanding the extent to which typical semiconductor diode behavior may be mapped to the BE junction (which may be exacerbated by the fact that the BC junction does not typically exhibit such behavior) as well as difficulty in correctly identifying the directionality of the junction (which was identified as a difficulty with diode circuits in Chapter 5).

The next most common incorrect response was for students to create a graph of the output that was identical to the input at all times, which was given by approximately 10% of students and depicted in Fig. 7.6C. Such responses were frequently supported with reasoning similar to that articulated by the following student: *“Since V_{in} increases linearly, V_{out} has to increase linearly as well because there is nothing changing in the circuit. Also, the equation concerning V_{in} and V_{out} is a linear equation.”* Over half of students producing linear V_{out} graphs without an offset from V_{in} provided similar

	UM (N = 57)	UW (N = 157)	Total (N = 214)
Quantitatively correct graph	**61%**	**25%**	**35%**
Correct reasoning	***58%***	***19%***	***30%***
Qualitatively (not quantitatively) correct graph	12%	10%	10%
Correct reasoning	10%	8%	8%
Linear with offset	12%	22%	20%
Transistor acts as a diode	12%	18%	16%
Linear without offset	2%	12%	9%
Configuration is a follower	2%	6%	5%
No offset	0%	8%	6%
No negative output	0%	5%	4%

Table 7.4. Student responses to the follower graphing task.

reasoning. While these students may have correctly recognized the function of the circuit (the output voltage "follows" the input voltage), none of them used such terminology in their explanations, nor did they apply any constraints to the circuit's output voltage. It should be noted that, for the canonical op-amp version of the follower circuit, there is no voltage threshold required for V_{out} to follow V_{in}, but almost none of these students would have had instruction on op-amps at the time this question was administered.

One additional response is presented here despite its relatively low prevalence, as it is particularly noteworthy from a pedagogical perspective. Approximately 5% of students produced a graph in which the voltage was constant (and zero) for input voltages less than 0 V, and linear above that threshold, as shown in Fig. 7.6D. As a specific example of reasoning in support of this graph, one student wrote, *"Since current must flow through B*

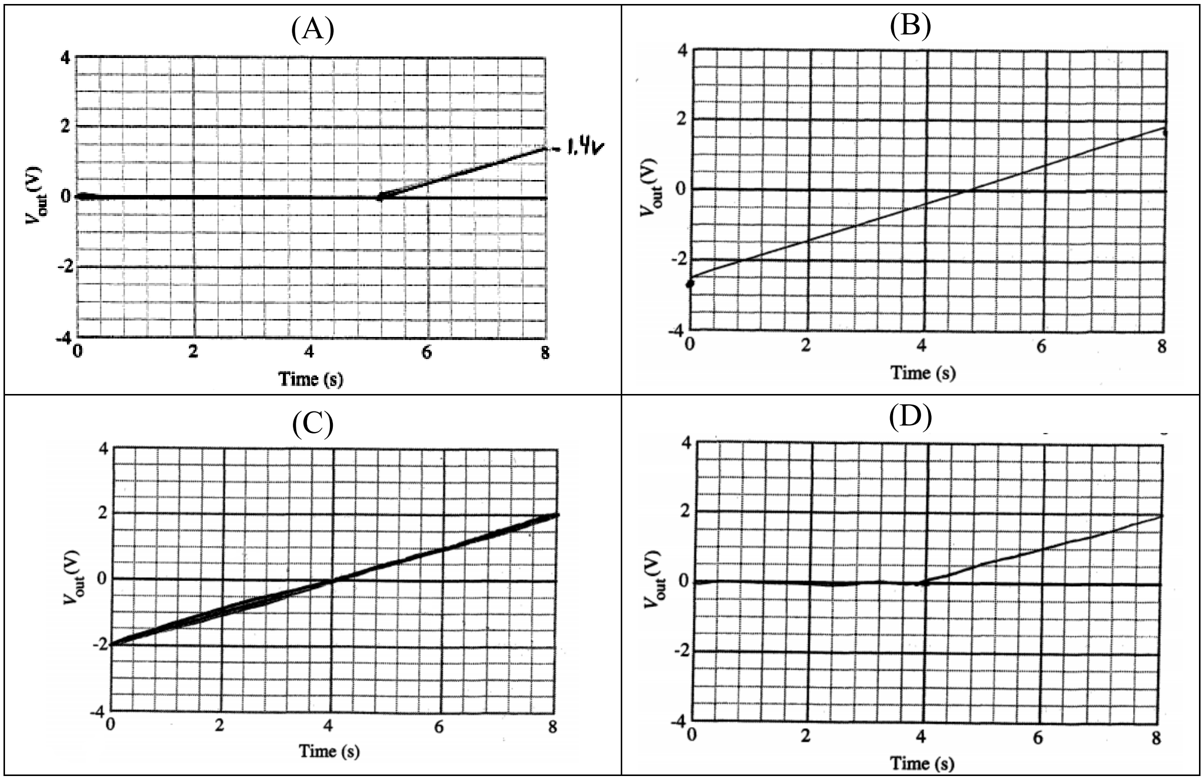


Fig. 7.6. Common responses to the graphing task. These responses represent: (A) correct response, (B) linear with offset, (C) linear without offset, and (D) no offset.

in order for current to flow through E, and the transistor acts like a diode. If the voltage is lower than ground, no current flows, hence no voltage." Similar reasoning, in which students used ground as the threshold that V_{in} must reach to bias the transistor appropriately, was given by approximately 60% of students drawing such graphs. These students were mostly correct in that they recognized that there would be no current through the transistor when the BE junction is reverse biased, but they did not take into account that a finite voltage is needed before a significant current may pass through the junction.

The remaining 20% of responses varied greatly, with no other answers accounting for more than 3% of students. Thus, the four categories of answers presented (as well as the associated lines of reasoning) fully characterize over two-thirds of all responses given by students. It is particularly notable that each graphical response was primarily supported by a distinct line of reasoning, and likewise each line of reasoning was primarily accompanied by one graphical response. In addition, most students were applying productive ideas about the behavior of transistors to the circuit, even if they did not include all necessary elements to come to the proper conclusion.

7.5.4 Summary of Findings

Many students struggled to correctly identify the behavior of the output voltage of the transistor follower for a linearly increasing input voltage, and only approximately one third of all students were able to produce a quantitatively correct graph, with an additional 10% having qualitatively (but not quantitatively) correct features. Nearly all of these students supported their answers with correct reasoning, and there were no incorrect lines of reasoning leading to a correct response. However, slightly over a third of students produced graphs that had elements of the correct response, recognizing some aspect of diode-like behavior such as maintaining the same slope, or recognizing that there should be no output voltage for negative input voltages.

7.5.5 Specific Difficulties Identified

In this task, there were three common incorrect responses given by students, each of which had a strongly associated line of reasoning. Some difficulties were less prevalent or absent at UM, but others (such as a tendency use a constant offset for V_{out}) were common to students in both courses.

Tendency of students to account for BE junction via a fixed diode drop between V_{out} and V_{in} for all V_{in}. Approximately one fifth of students treated the BE junction as having a fixed 0.6 V drop for the entire range of input voltages. Such responses did not attend to the biasing requirements of the transistor's behavior. Thus, even if students were considering the transistor to act as a diode, these responses did not capture the fact that semiconductor diodes must be forward biased by 0.6 V in order to allow current through the junction.

Tendency of students to treat the BJT follower as producing $V_{out} = V_{in}$ even under dc conditions. Accounting for approximately 10% of all responses were students who indicated that the output would be equal to the input, regardless of the value of V_{in}. Similar to students with the previous difficulty, these students did not address the biasing of the transistor in any way. It is possible that these students are overgeneralizing follower behavior, and it should be noted that this behavior is true of operational amplifier follower circuits.

Failure to consider BE junction biasing conditions. While most students correctly recognized that the circuit's output would follow the input over some range of voltages, over 15% did not include any offset in their output voltage. This was true for responses from both those students who recognized the transistor's cutoff conditions and those who did not. This difficulty is especially interesting, in that not accounting for a voltage change across the BE junction would primarily affect predictions about the dc behavior of the circuit but not the ac behavior.

7.6 Transistor Supply Voltage Modification Task

In order to better gauge student understanding of the functional relationship between biasing voltages and the resulting currents in transistor circuits, a new task was created, as shown in Fig. 7.7. The primary goal of this task was to answer the two following questions:

- To what extent do students recognize that the collector current is independent of the collector voltage (for the simplified model of a forward-active BJT presented in the physics electronics course)?
- To what extent are students able to correctly predict the impact of changes made to the emitter biasing voltage on currents in the transistor?

By design, this task would require students to use some of the same elements of reasoning required in the amplifier comparison task. However, by addressing single, specific parts of the required reasoning, this task may provide additional insight into student understanding of fundamental transistor behavior that was not seen in the amplifier comparison task.

7.6.1 Task Overview

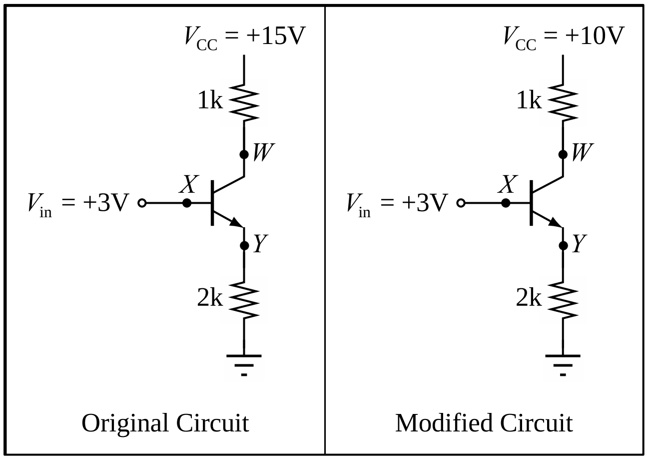
In the transistor supply voltage modification task, students are first presented with two pairs of common emitter amplifier circuits using identical component values, as shown in Fig. 7.7. In the first part of the task, students are told that the collector voltage V_{CC} is decreased from 15 V to 10 V. In the second part of the task, students are told that the emitter voltages V_{EE} is increased from 0 V to +1 V). For each part of the task, students are asked to determine how, if at all, the specified change in supply voltage will impact the (collector) current through point *W*, and to explain their reasoning. It is important to note that such modifications were not an explicit part of instruction in the course and are not typically discussed in detail in most texts. Thus, it would be unexpected for students to draw upon a memorized response associated with these changes, and thus they would have to reason from basic principles about transistor circuits. Furthermore, by asking students specifically about dc inputs and variations, it was expected that the subsequent interpretation of data would be more straightforward than what was seen in the ac amplifier comparison task.

7.6.2 Correct Response

In order to arrive at a correct response to part 1 of the task, students must recognize that the collector current is determined by the emitter current (since $I_C \sim I_E$), which is in turn set by the voltage drop across the resistor R_E. As long as the collector voltage (V_W) is at least ~0.1 V higher than the base voltage (V_X), the BJT will remain in the forward-active regime. In part 1 of the task, since V_{in}, V_{EE}, and the emitter resistor remain unchanged, then both the emitter current (through point *Y*) and the collector current (through point *W*) remain the same. Thus, in this instance, a change that is local to the point in question (point *W*) does not result in a change in current through that point.

A circuit containing an *npn* transistor is shown at right. V_{in}, from an ideal source, is constant and equal to +3 V.

1. If the original circuit were modified such that V_{CC} were lowered to +10 V, as shown, would the absolute value of the current through point *W* *increase, decrease,* or *remain the same?* Explain.



2. If, instead, the original circuit were modified such that V_{EE} were increased from 0 V (ground) to +1 V, would the absolute value of the current through point *W increase, decrease,* or *remain the same?* Explain.

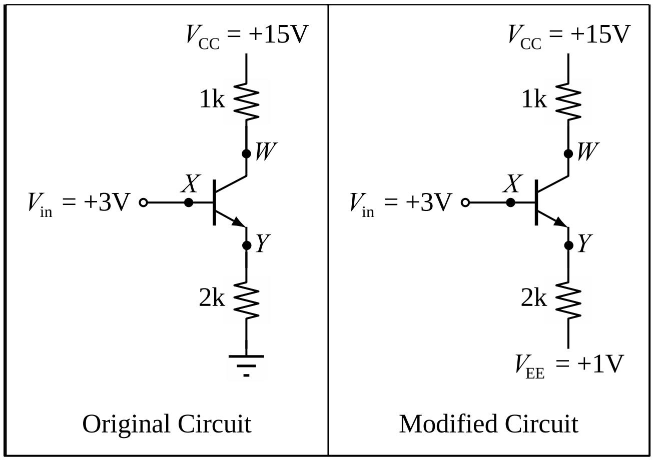


Fig. 7.7. Transistor supply voltage modification task.

In part 2 of the task, increasing V_{EE} results in a smaller potential difference across the 2-kΩ resistor, which in turn reduces the emitter current. As the collector and emitter currents are approximately equal, the current at point *W* also decreases. Note that in this instance, there were no changes in the immediate proximity of point *W*, so one might anticipate that any students applying purely local reasoning would claim (incorrectly) that the current through point *W* would not change.

7.6.3 Overview of Student Performance on the Supply Voltage Modification Task

The task was administered to students in the physics electronics course at UM across two separate years. As seen in Table 7.5, the majority of students (78%) correctly recognized that changing the collector voltage V_{CC} would not alter the current through point *W* (*i.e.,* the collector current) in part 1 of the task, and nearly all of these students (74% of total) supported their answer with correct reasoning. For example, one student noted, *"If V_{CC} were decreased to +10 V, the absolute value of the current through point W would stay the same since it is independent of V_{CC}. $I_C \approx I_E$."* The remaining 22% of students all responded that the current would decrease, with their reasoning typically stating that the reduced voltage would translate into less current through the resistors in

	UM (N = 27)
Part 1: Reduced collector voltage	
Same current (correct)	78%
Correct Reasoning	***74%***
Decreased current	22%
Ohm's law for collector resistor	*19%*
Part 2: Increased emitter voltage	
Decreased current (correct)	89%
Correct reasoning	***70%***
Increased current	7%
Same current	4%

Table 7.5. Responses to transistor supply voltage modification task.

the circuit. For instance, one student responded as follows: *"W would decrease because there is less voltage to drop across the resistors. I = V/R, so with R constant and V decreased, I must decrease. (15 > 10)."* While the student is correct in reasoning that the current through a resistor should change if the voltage across that resistor changes (due to Ohm's law), this student did not recognize that in this case, the voltage across the CE junction *(i.e.,* between *W* and *Y)* would vary in such a way that the emitter and collector currents remain essentially constant.

For the second part of the task, nearly all (~90%) students recognized that increasing the emitter voltage would subsequently decrease the collector current. In addition, 80% of these students supported their answers with correct reasoning. For example, one student wrote, *"V_Y would be the same, but voltage drop needed across 2k resistor would be smaller, so I_Y would be smaller. Since $I_Y = I_W$, current through I_W would decrease."* Thus, most students correctly recognized that the current through the emitter resistor would decrease, and furthermore that the collector current would also necessarily decrease.

7.6.4 Summary of Findings

Overall, nearly two-thirds (63%) of students gave fully correct answers with correct reasoning on both parts of the task. Thus, this task demonstrates that many students do, in fact, have the requisite understanding of the causal relationships that determine emitter and collector currents, and can use them productively in appropriate conditions. However, the most common difficulty (articulated below) identified through this task was similar to difficulties identified through earlier transistor tasks (*e.g.,* the three amplifier comparison task).

7.6.5 Specific Difficulties Identified

Tendency to reason locally or sequentially about transistor circuits. While most students were correct in their responses to this task, the most prevalent line of incorrect reasoning (accounting for approximately one fifth of student responses) stemmed from students reasoning that changing of the collector voltage would necessarily impact the collector current, likely thinking that a change in one part of the circuit should have an impact in that part of the circuit (*i.e.*, they are using local reasoning). If students were consistently using local reasoning on both parts of the task, then it would be expected that they would respond by saying that the current would decrease in the first part and remain the same in the second. In practice, only a single student did so. Indeed, students' stronger performance on the second part of the task suggests that they were better able to draw upon the non-local relationship between transistor currents in the second scenario in order to recognize that a change in one place may in fact impact a transistor current in a different location.

7.7 Revised Amplifier Comparison Task

It was noted in the first section (the three amplifier comparison task) that students typically struggled with analyzing the ac behavior of the emitter follower and common emitter amplifier circuits, and many students seemed to give responses consistent with the behavior of the same circuits under dc conditions. Indeed, unless students recalled the relevant gain expression for the common emitter amplifier, they were almost always unsuccessful in reproducing the correct line of reasoning for the circuit. However, as seen from the follower current ranking task, follower graphing task, and the supply voltage variation task, many students have a general understanding of the functional behavior of forward-active transistors. Collectively, such results suggest that the original amplifier comparison task may have been overwhelming for students, and that the complexity may have inhibited students from applying their understanding productively.

In order to better probe student understanding of common emitter amplifiers, a new task with less overhead was designed, shown in Fig. 7.8. In the new task, the circuits students must analyze have been simplified considerably compared to the circuits in the original amplifier comparison task. For instance, the leading biasing networks were omitted and new component values were chosen such that relevant voltages were either whole numbers or ratios of integers. Moreover, the new task was designed such that (a) students must only consider the impact of one modification at a time, (b) students must explicitly consider both the ac and dc behavior of the same circuits, and (c) all of the circuits compared are common emitter amplifiers. These modifications were made in an effort to eliminate several common incorrect lines of reasoning seen in the amplifier comparison task (*e.g.,* ranking bias voltages rather than signal voltages).

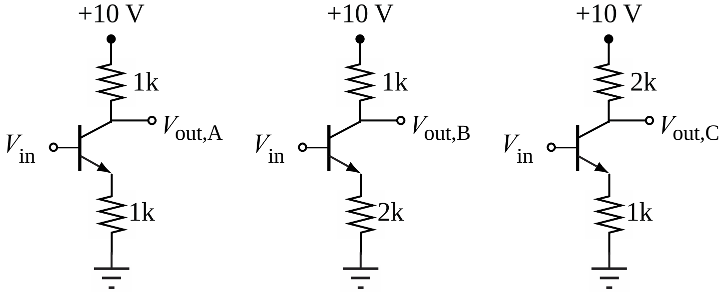
7.7.1 Task Overview

In the revised amplifier comparison task, students are presented with three common emitter amplifier circuits, labeled A, B, and C. Circuit B differs from A solely in that it has a larger emitter resistor (2 kΩ *vs*. 1 kΩ), and circuit C solely differs from A in that it has a larger collector resistor than A (2 kΩ *vs*. 1 kΩ). For the first two parts of the task, students are asked to consider the dc behavior of the circuits, and to make pairwise comparisons between the outputs of circuits B and A as well as C and A. For the last two parts of the task, students are asked instead to consider an appropriately biased ac signal, and to compare the peak-to-peak amplitudes of circuits B and A as well as C and A. By comparing student responses on parts 1 and 3 to those on parts 2 and 4, it can be determined whether or not students were consistently analyzing the circuits differently under dc and ac conditions.

7.7.2 Correct Response

In order to answer the first part of the task, students must recognize that circuit B has a larger emitter resistor than circuit A. As the voltage at the base (and thus the emitter) is identical for both circuits, the larger emitter resistance results in a smaller emitter current in circuit B via Ohm's law. In turn, the collector current in circuit B is less than that in A, as the collector current is essentially equivalent to the emitter current. The smaller collector current implies a smaller voltage drop across the 1-kΩ resistor in circuit B, and thus the voltage at the output in circuit B is higher than that in circuit A ($V_{out,B} > V_{out,A}$).

In the second portion of the dc task, circuit C has a larger collector resistor than circuit A. However, since both circuits have the same emitter resistor (and thus will have



Three circuits with *npn* transistors are shown at right. V_{in}, from an ideal source, is the same for all three circuits.

Suppose V_{in} is constant at +2.6 V

1. Is $V_{\text{out,B}}$ *greater than, less than,* or *equal to* $V_{\text{out,A}}$? Explain your reasoning.
2. Is $V_{\text{out,C}}$ *greater than, less than,* or *equal to* $V_{\text{out,A}}$? Explain.

Suppose instead that V_{in} is a sinusoidal signal with a 1V peak-to-peak amplitude and a dc offset of +2.6 V.

3. Is the peak-to-peak amplitude of $V_{\text{out,B}}$ *greater than, less than,* or *equal to* $V_{\text{out,A}}$? Explain.

4. Is the peak-to-peak amplitude of $V_{\text{out,C}}$ *greater than, less than,* or *equal to* $V_{\text{out,A}}$?

Explain.

Fig. 7.8. Revised amplifier comparison task.

the same emitter current), the currents through collector resistors in both circuits are also the same. As the collector resistor in circuit C is larger than that in circuit A, there is a larger voltage drop across the collector resistor in circuit C due to Ohm's law. Thus, the output voltage of circuit C will be lower than that of circuit A ($V_{out,C} < V_{out,A}$).

For the corresponding ac portion of the task (parts 3 and 4), the magnitude of the circuit's gain (as noted in Section 5.1) is given by the ratio of the collector resistor to the emitter resistor (*i.e.*, $\delta V_{Out} = \delta V_{in} R_C/R_E$). This results in circuits A, B, and C having gains of 1, 2, and ½, respectively. Since the peak-to-peak amplitude of the input signal is the same for all circuits, the output voltages can be compared solely via the ac circuit gains. Thus, the peak-to-peak output voltage of circuit B is less than that of circuit A ($\delta V_{Out,B} < \delta V_{Out,A}$), and the output voltage of circuit C is greater than that of circuit A ($\delta V_{Out,C} > \delta V_{Out,A}$).

7.7.3 Overview of Student Performance on the Revised Amplifier Comparison Task

Data were collected from a single semester of the UM physics electronics course. Students were substantially more successful on this task than on the amplifier comparison task, as 50% of students correctly answered all four parts, and 86% of these students also provided correct and complete reasoning, as shown in Table 7.6. The remaining students

	UM (N=14)
All parts correct	50%
With correct reasoning	**43%**
Both dc correct	86%
Both ac correct	64%
ac and dc responses differ	64%
ac and dc responses same	14%

Table 7.6. Responses to transistor ac and dc comparison.

who arrived at correct answers typically provided reasoning that contained correct elements, but was incomplete in some manner.

In responses to the four parts of the revised amplifier comparison task, 86% of students correctly answered both dc questions, whereas only 64% correctly answered both ac questions. If students were indeed reasoning more successfully about the dc behavior of the circuits, this would help explain the results from the original three amplifier comparison task. Specifically, it would support the hypothesis that students who were more proficient in reasoning about the dc behavior of transistor circuits were more likely to rely on their dc analysis strategies when presented with a complicated task (*i.e.,* the original three amplifier comparison task). Unfortunately, due to the low number of responses, there is insufficient statistical power to clearly state that these rates are different for ac versus dc. A power analysis (with significance threshold $\alpha = 0.05$ and power = 0.8) indicates that, if these responses are representative of the student population as a whole, a total of 54 additional responses would be required to confirm a significant difference between the responses to the ac and dc portions of this task. This is readily obtainable with two to three more rounds of data collection, and the findings from this task would be useful in guiding future research or instructional interventions.

Given that students tended to use dc reasoning when prompted for ac voltages in the original three amplifier comparison task, it is suitable to examine the extent to which students used different approaches when asked to compare the same two circuits under dc and ac conditions (*e.g.,* in part 1 vs. part 3 and part 2 vs. part 4). It was found that 64% of students arrived at different answers for dc and ac conditions when comparing both pairs of circuits (B *vs.* A and C *vs.* A), and the majority of these students provided correct responses. This supports the idea that many students, when explicitly asked, recognized the difference between dc and ac behavior. An additional 21% of students gave mixed responses (*i.e.*, one comparison was the same under both dc and ac conditions and the other was not). However, 14% of students indicated that the comparisons between both pairs of circuits (B *vs.* A and C *vs.* A) were the same in both dc and ac cases. Most importantly, these students employed the same reasoning across both dc and ac comparisons.

In one specific instance, in response to part 1 (a dc comparison), one student wrote: *"Since* $I_E \approx I_C$*, and there is less current in* $V_{out,B}$ *due to an increased resistance, I assume* $V_{out,B} < V_{out,A}$*."* In response to part 3 (the analogous ac comparison), the same student wrote: *"*$V_{out,B} < V_{out,A}$*. The big factor, I believe again is the resistor in the E branch of circuit 2."* This confirms the hypothesis that even after relevant instruction, some students did not distinguish between the dc and ac behavior of the circuit for the same circuit, thereby applying the same line of reasoning to both.

Care must be taken in the interpretation of these results, as they were obtained from a relatively small number of students. As such, and with students being generally successful, it is not reasonable to extrapolate generalized claims about the prevalence of any specific difficulties. Nevertheless, these data are noteworthy in that they assist in pinpointing factors that might give rise to other difficulties observed, and thus help to better interpret responses to the three amplifier comparison task.

As noted for the original three amplifier comparison task in this chapter, students had substantial difficulty in comparing the outputs of circuits B and A (in which the collector and emitter resistors were exchanged), with only 31% of students across all years of the

UM physics course correctly ranking $V_{out,B} > V_{out,A}$. However, as was noted previously, 64% of students correctly described the ac behavior of both circuits in the revised task. While there are several important differences between the two tasks, the same reasoning about how resistors affect the circuit gain is required in both cases. Thus, it is relevant to consider student responses from the UM physics electronics course for the A *vs.* B comparison in both this task (N = 14) and the original amplifier comparison task (N = 42). Using Fisher's exact test, the resulting p value is slightly above the typical threshold of significance ($p = .055$) but with a moderate effect size ($\Phi = 0.3$), which is a measure of how different the outcomes were. Although more data are needed to strengthen these findings, this analysis suggests that many students did in fact possess an understanding of the relevant ac behavior of the common emitter amplifier, but did not draw upon this relevant formal knowledge when answering the original amplifier comparison task.

7.8 AC Biasing Network Tasks

Biasing networks such as the one featured in the three amplifier comparison task are critical components in many transistor circuits, helping to ensure that the transistor remains in the proper operational mode for a wide range of inputs. However, on the three amplifier comparison task, few students addressed the network's behavior in their explanations, and such explanations typically noted that the networks were identical. Informal observations of students in the physics electronics laboratory also suggested that many students were unsure of how to properly analyze the behavior of such biasing networks.

In order to analyze a biasing network such as the one depicted in Fig. 7.9A, students were taught to first consider the Thévenin equivalent of the voltage divider circuit (*i.e.*, the +15 V source paired with two 20-kΩ resistors). The result, shown in Fig. 7.9B, is essentially a high-pass filter with an output biased around a particular dc voltage. While there has been prior research on student understanding of filters and phase relationships in ac circuits in engineering courses [13,65], this is the first investigation of student understanding of ac biasing (and filtering) networks.

In analyzing the ac biasing network, students must consider its impact on both dc and ac input voltages. To facilitate this, a set of two tasks was administered, both using the network from Fig. 7.9B. Indeed, prior research has indicated that students often experience difficulties when predicting the dc behavior of capacitors [48], and thus the first task targets dc behavior exclusively. The second task explicitly addresses the ac behavior of the circuit, as earlier in this chapter it was noted that many students struggle to reason about ac properties when given an open-ended task.

7.8.1 Overview of ac Biasing Network Tasks

For this study, students are given both ac and dc analysis tasks together on a single sheet of paper. For the dc analysis task, students are asked to determine the voltage across the capacitor, and to rank several relevant voltages in the circuit. On the target ac task, students are asked to construct a graph of the output voltage from the circuit when the input voltage is an ac signal with a frequency equal to the 3 dB frequency of the circuit. Arriving at a correct prediction of the output voltage requires the simultaneous consideration and analysis of the gain, phase shift, and dc offset associated with the output voltage signal. Given that prior research on student understanding of filters has indicated that many students encounter difficulties when analyzing canonical RC filters [13], it was expected that this task would be very challenging, even with the dc analysis tasks as scaffolding.

7.8.2 DC Analysis Task

7.8.2.1 Overview of dc Analysis Task

In the dc analysis task, students are shown a circuit (Fig. 7.9B) that is the Thévenin equivalent of a typical network used to add a dc bias to an ac signal. The topology is similar to a canonical high-pass filter, but the resistor is connected to a positive 5 V

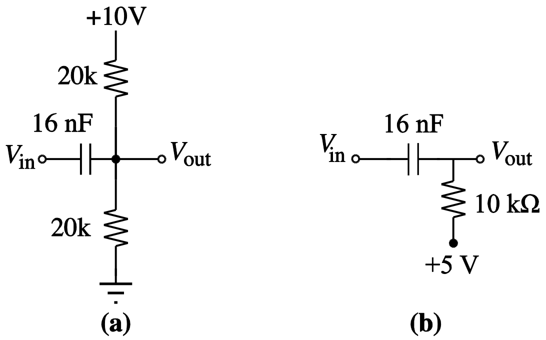


Fig. 7.9. (A) Standard schematic of a typical biasing network encountered in a transistor amplifier circuit.
(B) Thévenin equivalent circuit for the same biasing network, which is the circuit used in this assessment.

source instead of ground. Students are told that all components are ideal and that the input is connected to a +7 V source. Students are also told to assume that the circuit has been connected for a very long time *(i.e., $t >> \tau$).* Students are asked (1) to find the absolute value of the voltage across the capacitor, and (2) to rank, from largest to smallest, the absolute values of the voltage across the capacitor *(V_C)*, the voltage across the resistor *(V_R)*, and the output voltage (V_{out}). Students are also prompted to explain their reasoning for both questions, and to state explicitly if any voltages are equal to one another or are equal to zero for the ranking task.

7.8.2.2 Correct Response

To determine the voltage across the capacitor, students must recognize that there is no current through the capacitor due to the dc steady state conditions, and thus there is no current through the resistor. With no voltage drop across the resistor (via Ohm's law), the output voltage of the circuit must be equal to +5 V as set by the second dc source. The absolute value of the voltage drop across the capacitor must be 2 V, the difference between the input voltage (+7 V) and the output voltage (+5 V), in order to satisfy Kirchhoff's voltage law. For the voltage ranking task, a correct response would therefore be $|V_{\text{out}}| > |V_C| > |V_R| = 0$.

7.8.2.3 Overview of Student Performance on dc Analysis Task

Responses from the dc analysis task have been collected from students at the University of Maine in the introductory engineering circuits course (N = 45), the upper-division engineering electronics course (N = 20), and the upper-division physics electronics course (N = 29). Data from the physics course were collected over two different years and were grouped together after applying Fisher's exact test. Fisher's exact test indicated that student answers on either of the two tasks were not statistically distinguishable between the two years (p = .16 and p = .99, respectively). Results from part 1 (capacitor voltage) and part 2 (voltage ranking) of the dc analysis task are summarized in Table 7.7 and Table 7.8, respectively, and discussed in detail below.

On part 1, when asked to find the voltage across the capacitor after a long period of time had elapsed, we found that between one-third and two-thirds of students in a given course correctly stated that the voltage across the capacitor would be 2 V. The most common line of reasoning was that the capacitor would charge to the difference between the sources (7 V – 5 V). For example, one student wrote, *"The capacitor cannot pass DC current so after t >> τ, the capacitor has charged and is blocking the entire* V_{in} *-5 V."* Several other mostly correct lines of reasoning account for the remaining explanations, and these contained important correct elements such as the idea that the capacitor acts like an open switch when charged, a focus on the lack of current through the resistor, and the application of Kirchhoff's voltage law.

The most common incorrect response was that the capacitor would charge to 7 V, the input voltage, and this was given by approximately 20 - 35% of students. In the

	Engineering		Physics
	Circuits (N=45)	Electronics (N=20)	Electronics (N=29)
$\lvert V_C \rvert$ = 2 V (correct)	**53%**	**37%**	**66%**
Capacitor charges to ΔV	40%	5%	24%
Other correct reasoning	14%	16%	34%
$\lvert V_C \rvert$ = 7 V	35%	21%	21%
Capacitor charges to V_{in}	21%	11%	20%
$\lvert V_C \rvert$ = 0 V	12%	37%	3%
Capacitor is shorted	2%	16%	0%

Table 7.7. Overview of student responses to dc analysis task part 1: capacitor voltage.

engineering courses, this was most often accompanied by an argument that the capacitor would charge to the input voltage *(e.g., "7 V. The capacitor fully charges as t → ∞").* In the physics course, these responses usually included language indicating the capacitor would act as an open switch *(e.g., "After a long time the cap will be fully charged and act as an open switch making $V_C = 7$ V.").* It is important to note that any discussion of the role of the 5 V source is absent, regardless of specific arguments the students used.

Another common incorrect response, which accounts for most of the remaining student answers, was that the capacitor has no voltage across it. This result was found primarily in the engineering electronics course, but was present in all groups. Approximately one third of the responses in support of this answer argued that the capacitor would act as a short.

It is important to note that at least 85% of the students in each class provided reasoning to accompany their answers for this part of the task. Furthermore, the lines of reasoning we report characterized at least two-thirds of the responses seen in any given course. This means that the results should be expected to be representative of responses from all students, as no other lines of reasoning occurred with a prevalence greater than 10%. It is likely that the category of correct but incomplete reasoning could be broken out into more nuanced responses upon collection and analysis of more data from additional courses and institutions.

For part 2 of the dc analysis task, approximately one-quarter to one-half of students correctly ranked the absolute values of all three voltages and stated that there was no voltage across the resistor ($|V_{\mathrm{out}}| > |V_{\mathrm{C}}| > |V_{\mathrm{R}}| = 0$). However, only up to 10% of students in any course supported the correct answer with correct and complete reasoning. For example, one student wrote, *"V_C represents the entire V_{drop} across the circuit of 7 V-5 V. No current flows through the resistor $\therefore V_R = 0$. V @ V_{out} is 5 V. 5 V w.r.t ground is +5 V."* Approximately 15% to 40% of students were able to support their correct answers with reasoning that was incomplete, frequently omitting a justification for how they concluded that $V_{\text{out}} = +5$ V.

The most common incorrect ranking, accounting for about 5% to 20% of responses, was that $|V_{\text{out}}| > |V_{\text{C}}| > |V_{\text{R}}|$, without explicitly indicating that the voltage across the resistor is 0 V. While more than half of these students did not provide reasoning, those who did indicated that the voltage at the output was the sum of the other two voltages. For example, one student wrote, *"V_{out} has the voltage from the 5 V source and the 7 volt source. Vc has just the 7 volt source. V_R has just the 5 volt source."* In 75% of all instances of this ranking, the students had previously indicated incorrectly that the voltage across the capacitor was 7 V. Under the steady-state dc conditions described in the task, it is impossible for the output voltage of this RC circuit to be greater than the input voltage. This suggests that, at least among these students, this ranking may reflect a

	Percentage of total responses		
	Engineering		Physics
	Circuits (N=45)	Electronics (N=20)	Electronics (N=29)
$\|V_{\text{out}}\| > \|V_C\| > \|V_R\| = 0$ (correct)	**22%**	**30%**	**48%**
Correct and complete reasoning	4%	0%	10%
Correct but incomplete reasoning	13%	15%	38%
$\|V_{\text{out}}\| > \|V_C\| > \|V_R\|$	20%	5%	3%
V_{out} is the sum of V_C and V_R	9%	0%	3%
$\|V_{\text{out}}\| > \|V_R\| > \|V_C\| = 0$	7%	30%	0%
$V_C = 0$ and V_R determined by KVL	4%	20%	0%

Table 7.8. Overview of student responses to the dc analysis task part 2: voltage ranking.

fundamental difficulty with the treatment of voltage (*i.e.*, failing to distinguish between voltage at a point versus across elements) rather than an accidental omission of an expression indicating that the voltage across the resistor is zero.

The next most prevalent incorrect ranking, $|V_{\mathrm{out}}| > |V_{\mathrm{R}}| > |V_{\mathrm{C}}| = 0$, was given by between 0% and 30% of students, depending on the population. All of the students providing this ranking also indicated that the capacitor had no voltage across it when responding to part 1 of the dc analysis task. Students giving this ranking typically obtained the voltage across the resistor by reasoning that the output and input voltages were the same, and that the resistor's voltage was given by the difference between them. For example, one student wrote, *"Since the cap is a short ($V_C = 0$), $V_{out} = V_{in} = 7$ V and $V_R = 7 - 5 = 2$ V."* As most (> 80%) of the students who concluded that $V_{\mathrm{C}} = 0$ on the part 1 of the dc analysis task in turn provided this ranking, this ranking may represent an attempt to apply Kirchhoff's voltage law correctly to the circuit once an incorrect value for the voltage across the capacitor has been obtained.

The above three rankings account for over half of all responses, and all other rankings are sufficiently rare (< 10%) that we cannot make reasonable generalizations about them. However, another useful way to aggregate responses on the ranking task is to cluster responses into those that explicitly state that V_{out} and V_{R} are the same and those that do not. Approximately 20% of all students stated that $V_{\mathrm{out}} = V_{\mathrm{R}}$ across all three courses, with no distinguishable difference between them ($p = 0.7$). This comparison is meaningful in that students equating the two voltages may be either failing to account properly for the +5 V source (instead of ground) when analyzing the circuit or failing to recognize that V_{out} is measured with respect to ground. There is evidence of this in the explanations given, which were evenly split between either a general statement that V_{out} and V_{R} would be the same or some indication that V_{out} and V_{R} corresponded to the same node. The following student response is an example of the former case: *"If the voltage across V_C is zero, as above, the voltage must be across the resistor, which means V_R must be equal to V_{out}. $V_R = V_{out} > V_C = 0$."* This tendency to equate the voltage across the resistor (between the V_{out} connection and the +5 V connection) with the output voltage (between V_{out} and ground) may indicate that students were unsure how to handle the connection to the +5 V source. Similarly, students who stated that V_{out} and V_{R} correspond to the same node may have failed to recognize that voltages at a point (*e.g.*, V_{in} and V_{out}) are always measured with respect to ground and that the connection to the +5 V source therefore necessitates that V_{R} (measured between the output terminal and +5 V) can never be equivalent to V_{out}.

7.8.3 AC Analysis Graphing Task

7.8.3.1 Overview of ac Analysis Graphing Task

The ac analysis graphing task features the same circuit (Fig. 7.9B) used in the dc analysis task. As noted previously, both tasks are included together on the same page. Students are told that the input voltage is a 1 kHz sinusoidal signal with a 2 V peak-to-peak amplitude. A graphical representation of V_{in} is provided to the students (Fig. 7.10A). Students are asked to make a quantitatively correct plot of V_{out} under these conditions, and are provided with a grid on which to sketch their plot. Explicit instructions tell the students to scale and label the V_{out} axis appropriately, explain their reasoning, and show their work.

7.8.3.2 Correct Response

There are four specific criteria that need to be met for a response to be completely correct. From the analysis of the circuit's dc behavior, it is known that the output will be biased about +5 V. To find the ac behavior of the network, it can be shown that the 3dB frequency for the RC circuit is $f = 1/(2\pi RC) = 1.001\ kHz$. Since the input signal has a frequency of 1 kHz, the RC filter in this case is effectively operating at its 3dB point. As a result, the amplitude of the signal will be reduced by a factor of $\sqrt{2}$ and the output signal will have the same frequency as the input signal but will lead the input signal by 45° (*i.e.*, the phase shift of the output signal with respect to the input signal is +45°). Alternatively, one may derive these results from basic principles by modeling the RC network as an ac voltage divider. A (nearly) completely correct graph made by a student is shown in Fig. 7.10B; there is a slight discrepancy in the behavior of the output voltage at $t = 0$, which does not correspond to either the rest of the graph's behavior or the student's explanation.

7.8.3.3 Overview of Student Performance on ac Analysis Graphing Task

The ac analysis graphing task directly followed the dc analysis task on the written page. While the ac task was therefore administered to the same number of students (94) as the dc task, fewer students attempted to complete the ac task. There were a total of 83 responses, with 41 from the engineering circuits course, 16 from the engineering electronics course, and 26 from the physics electronics course. Responses from the physics course were once again pooled, as Fisher's test indicated no difference in the number of correct graph elements between years ($p = .54$). The majority of student responses were sinusoidal signals, all of which could be characterized by frequency, amplitude, phase shift, and dc offset. With a single exception, students indicating a sinusoidal response drew an output signal of the same frequency as the input signal. Non-sinusoidal responses were both infrequent and varied enough that they are not discussed in detail. Approximately 15% of students did not provide a vertical scale on their graphs; for the sake of comparison, these responses were treated as if the scale was identical to that provided on the input signal graph. Results for the ac analysis graphing task are summarized in Table 7.9 and discussed in detail below.

Overall, the task proved very difficult for students in all courses, with only a single student providing a (nearly) completely correct graph (Fig. 7.10B), and only two students drawing graphs with 3 of 4 correct features. Between approximately 0% and 40% of students made graphs with two correct features, which was typically (in 80% of these cases) a correct dc offset and no frequency change. 60% or more of the students in any given course provided graphs whose sole correct feature was the frequency, and between 4 and 12% of the graphical responses contained no correct features at all. Below, three of these four features are examined in detail (the dc offset, amplitude, and phase) and interesting patterns in student responses are discussed. The frequency of the output voltage is not discussed as nearly all students who indicated the output would be sinusoidal correctly indicated that the frequency would be unchanged, and few (<5%) provide any justification.

As noted previously, the second most common feature to be graphed correctly (after the frequency) was the +5 V dc offset (Fig. 7.10C), included by between 6% and 39% of students in a given course. Approximately half of these students simply asserted that there would be an offset, without explicitly addressing any other features of the output voltage (*e.g.*, phase or amplitude). For example, one student wrote, *“At high frequencies the capacitor acts as a short, allowing the wave through unaltered. Therefore the output is equal to the input plus 5 V dc.”* Between 37% and 75% of students did not include any dc offset in their graphs, and very few (<10%) made any attempt to justify why the offset was the same as that of the input voltage signal (zero).

Only up to 15% of students in a given course correctly recognized that the amplitude would be reduced by a factor of $\sqrt{2}$ (as in Fig. 7.10D). All of the students doing so used correct reasoning either by performing complex voltage division or by recognizing that the circuit is a filter and therefore comparing the signal frequency to the calculated 3dB frequency. As an example of the latter kind of reasoning, one student wrote, *“$\omega_{3dB} = 1/RC = 1/(10\ k\Omega)(15.9\ nF) = 6.3 * 10^3\ s^{-1}$. $f_{3dB} = \omega_{3dB} / 2\pi = 1001\ Hz \approx 1\ kHz$. So the*

	Percentage of total responses		
	Engineering		Physics
	Circuits (N=41)	Electronics (N=16)	Electronics (N=27)
1 kHz frequency	**85%**	**88%**	**96%**
DC offset of 5 V (correct)	**39%**	**6%**	**23%**
5 V added to V_{out}	*24%*	*0%*	*8%*
DC offset of 0 V	37%	75%	58%
Explicit justification of unchanged dc offset	*2%*	*0%*	*8%*
Amplitude of 1.4 V (correct)	**0%**	**6%**	**15%**
3dB frequency or voltage division	*0%*	*6%*	*15%*
Amplitude of 2 V	56%	50%	42%
Explicit justification of unchanged amplitude	*12%*	*6%*	*12%*
+45° phase shift (correct)	**5%**	**6%**	**4%**
Mathematical calculation	*5%*	*6%*	*4%*
No phase shift	49%	19%	50%
Explicit justification of unchanged phase	*2%*	*6%*	*4%*
Phase shift of ±90° or 180°	32%	63%	38%
Explicit justification of specified phase shift	*17%*	*31%*	*12%*
Non-sinusoidal output	15%	13%	4%

Table 7.9. Overview of graphical features in student responses to ac analysis graphing task

input voltage is attenuated by a factor of .707." The most common incorrect response, given by between 42% and 56% of students in a given course, was that the amplitude of the output voltage would be unchanged with respect to that of the input voltage. Once again, most of these responses did not contain explicit justifications for why the amplitude would remain at 2 V. The few justifications provided were quite varied; for example, one student who incorrectly used the 3dB frequency argued, *"The cutoff frequency for this filter is 1/2π(10 k)(16 n) = 994.7. Since the input voltage has frequency 1 kHz, the signal will get through."*

A total of four students (approximately 5%) correctly indicated a +45° phase shift of the output signal with respect to the input signal, all of whom supported their answer with mathematical calculations. For example, one student wrote, "$V_{out} = (10k / (10k - j (1/(2\pi k * 16\ nF)))) * 2\ V = 1.42\angle 44.8^\circ\ V$. $V_{out}(t) = 1.42 \cos(2 \pi kt + 44.8^\circ) + 5\ V$." (Note that this student erroneously used the peak-to-peak voltage instead of the amplitude in the calculation.) The most common incorrect response was the omission of the phase shift, and this was provided by between 19% and 50% of students in a given course. Again, few students explicitly justified the 0° phase shifts of output voltage with respect to the input voltage. Other common incorrect responses, accounting for between 32% and 67% of students in a given course, were those featuring a phase shift of either +90°, -90°, or 180°, with a nearly equal split among the three phase shifts. Explanations in support of any one of the three phase shifts were often incomplete or unclear and did not share a common line of reasoning. For example, one student wrote, *"The capacitor has a phase shift which shifts the phase by 90°."* Many simply asserted that a phase shift occurs due to the capacitor without further justification.

To make a more general assessment of students' approach to this task, the reasoning students used may be clustered into potentially productive ac analysis approaches to examine the filtering behavior of the circuit (*e.g.*, either using the impedances of both components or drawing upon relevant knowledge of RC filters) and approaches that either are unproductive (*e.g.*, capacitors cause phase shifts) or do not address filtering (*e.g.*, solely focusing on the dc offset). Among all students who provided reasoning along with sinusoidal output voltage graphs (N = 76), 40% used potentially productive lines of reasoning. If one were to hypothesize that students' approaches to the ac task are independent of their knowledge of the dc behavior of the circuit, one would anticipate

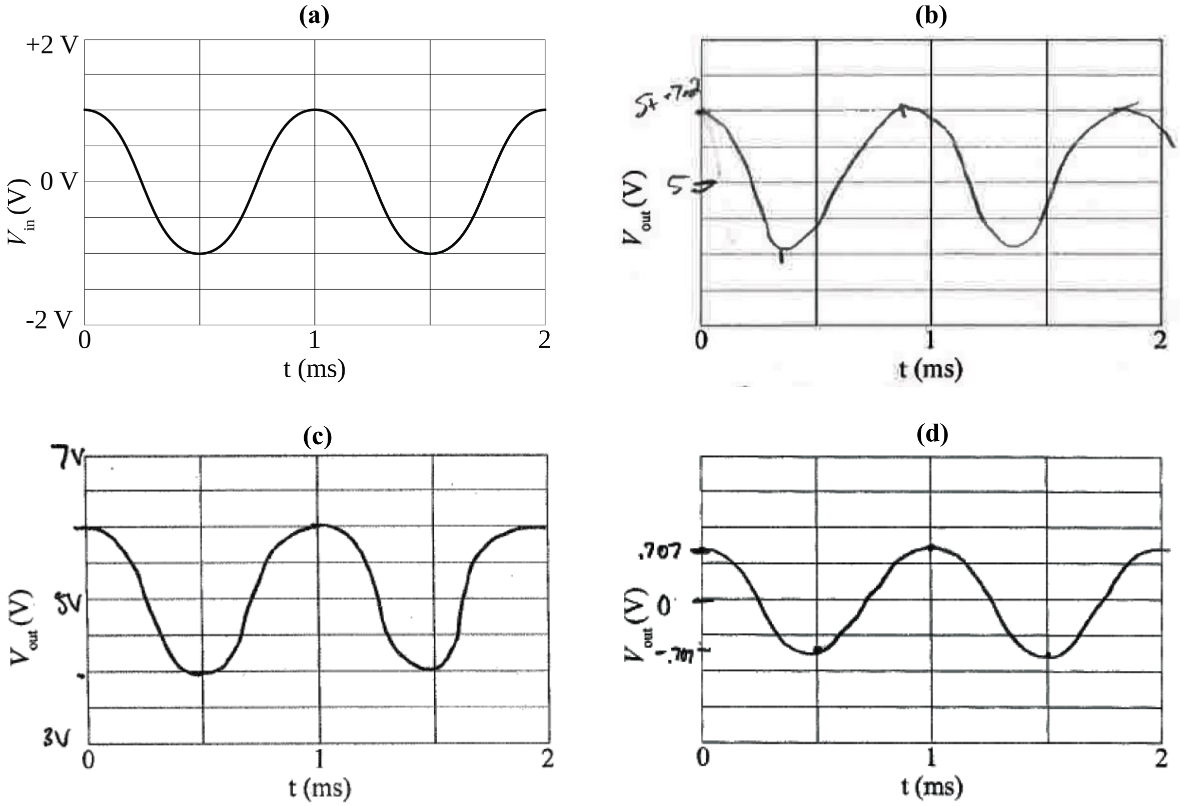


Fig. 7.10. Graphs from the ac analysis graphing task. (A) Graphical representation of the ac input voltage provided to students. (B) Completely correct student graph of the output voltage. (C) Student graph of output voltage solely characterized by the correct dc offset and frequency. (D) Student graph of output voltage solely characterized by the correct amplitude and frequency.

that this ratio would remain the same regardless of performance on the dc task. Instead, it can be shown that students who correctly analyzed the dc behavior on the dc analysis task (N = 28) used potentially productive ac analysis approaches 60% of the time in the ac task, whereas those who gave incorrect responses to both dc analysis tasks (N = 42) used potentially productive ac analysis approaches in their reasoning only 26% of the time. This difference is statistically significant ($p = .006$) and has a moderate effect size ($\phi = .35$). Thus, this analysis suggests that it is unlikely that students will apply a constructive ac analysis approach to examine the filtering behavior of the circuit if they are unable to correctly analyze the dc behavior of the circuit. Further studies are required to determine whether this relationship is causal (*e.g.*, understanding dc behavior is a prerequisite for understanding the ac behavior) or coincidental (*e.g.*, students who understand the dc behavior may simply have better general understanding of circuits).

7.8.4 Summary of Findings

These findings suggest that, after relevant instruction, a significant percentage of students lack a sufficiently robust understanding to correctly analyze the behavior of biased RC filters under both dc and ac conditions. Students struggled to analyze the biasing circuit, even in regimes in which its behavior did not differ significantly from that of a canonical high-pass RC filter. Students frequently provided reasoning that only partially justified their answers for both the dc ranking and ac graphing tasks. On the ac analysis graphing task, students also failed to provide justifications for why key features of their output voltage graphs were identical to those of the input voltage graphs; instead, they typically limited their explanations to a single feature that they predicted would change. While there may be statistically significant differences in students' responses and reasoning between the three courses, the primary goal of this study was to document and characterize prevalent difficulties observed in all populations in order to inform the development of cross-disciplinary instructional interventions that could ultimately support improved student learning of BJT circuits.

From students' approaches to the dc analysis tasks, it was found that a large percentage of students did not appear to account for presence of the +5 V source when arriving at their answers, implicitly treating the circuit as if the resistor were connected to ground. It may be that some students failed to recognize the impact of the +5 V source on the behavior of the RC filter, assuming that it behaved identically to a canonical high-pass filter. Students stating that the output voltage and the resistor voltage correspond to the same node may, in fact, have been making a similar error. Both of these behaviors also suggest underlying student difficulties with the interpretation of circuit diagrams (like Fig. 7.9B) that employ a somewhat more abstract representation than that typically used in introductory courses. Similar difficulties have been observed in the physics electronics course when canonical inverting op-amp amplifier circuits are perturbed by connecting the non-inverting terminal to a non-zero dc voltage. Further investigation, via targeted written questions and think-aloud interviews, is needed to explore student difficulties with more advanced circuit representations.

From the dc analysis ranking task, there was evidence suggesting that students who provided a mostly correct ranking but did not state that the voltage across the resistor is zero were most likely summing source voltages together to reach their answer. This approach represents a fundamentally incorrect treatment of voltage, and is incompatible with Kirchhoff's voltage law.

In the ac analysis graphing task, students tended to provide reasoning that explained only a single feature of the output voltage while not addressing the others. In particular, students frequently focused only on the dc offset resulting from the +5 V source. Students rarely provided any explicit reasoning for retaining a feature from the input signal, and did not do so frequently enough to allow for the identification of specific difficulties related to their answers. The circuit's phase behavior was particularly problematic, as even students who attempted to consider phase shifts of the output voltage frequently came to incorrect conclusions. These findings are consistent with those reported by Coppens *et al.* [13], who found that students either did not provide a meaningful explanation of a filter's phase behavior or did not account for it at all. For this ac biasing network, students used reasoning appropriate to filters less than half of the time.

Note that the circuit used in this investigation is a simplified circuit equivalent to a very common biasing network (Fig. 7.9A). Yet, even when the circuit is presented in a form that should make its filtering behavior more evident (Fig. 7.9B), the majority of students either only attended to the dc behavior or added a superficial phase shift to the output voltage. Indeed, these findings suggest that further targeted instruction may be needed for students to attain a robust understanding of biasing networks sufficient for the proper analysis of many canonical BJT amplifier circuits.

7.9 Discussion

This investigation of student understanding of transistor circuits began with a single task (the three amplifier comparison task), which demonstrated that many students struggled to reason correctly in the context of a relatively common application of BJTs. From responses to this task, it was unclear how well students understood the behavior of transistors in circuits, as the most common incorrect lines of reasoning did not involve the behavior of the transistor itself. Thus, a series of five additional tasks (related to follower currents, follower graphing, supply voltage variation, revised amplifier comparison, and biasing networks) were used to better isolate and characterize those aspects of transistor circuits which students understood well and those with which they continued to struggle even after all instruction.

In general, students experienced the least difficulty when reasoning about the behavior of the base-emitter junction in the transistor, particularly for dc input voltages. This may be due to the fact that the BE junction has diode-like voltage properties, and students had already gained considerable experience with diodes prior to transistor instruction. However, as in diode circuits, students frequently struggled with the behavior of the BE junction under reverse biasing conditions. Even among these incorrect responses, it should be noted that students frequently employed elements of productive reasoning, although they may have been used in an inappropriate context.

7.9.1 Specific Difficulties Spanning Tasks

From the responses to these six tasks, it was clear that students encountered several distinct difficulties when working with each of the individual transistor circuits. However, there emerged two overall trends that are particularly noteworthy.

Tendency to reason locally about circuit modifications. As seen on other tasks in this dissertation, students often only considered the local impact of modifications made in circuits, which could lead to incorrect assumptions about what parameters would remain constant (*e.g.*, students treated the collector currents as equal for all circuits in the three amplifier comparison task). While such of reasoning can, in some instances, be productive (such as reasoning about the emitter bias currents in Section 7.3), much of the time local reasoning leads to incorrect conclusions. Because the collector-emitter junction voltage of bipolar junction transistors is determined by external circuit constraints rather than by a particular property of the device, it is typically not possible to predict the implications of specific modifications to a given transistor circuit without a comprehensive analysis of the circuit's behavior.

Tendency to rely on dc analysis over ac analysis. In instances when students were not explicitly prompted to consider the ac behavior of a circuit, students frequently used inappropriate strategies to reason about transistor circuits. As an example from the three amplifier comparison task, most incorrect lines of reasoning centered on arguments made about dc voltages, even though students were asked about peak-to-peak values of ac voltages. Similarly, in the graphing portion of the biasing network task, most students only recognized the dc biasing behavior of the network, and did not attempt to determine whether or not the filtering behavior (which affects the magnitude and phase of the output voltage) was relevant. However, as seen in the revised amplifier comparison task, students appeared to be capable of correctly predicting the ac behavior of the transistor when asked about it explicitly and when presented with somewhat more straightforward circuits (*e.g.,* no biasing networks). Taken together, these results suggest that students may favor dc analysis over ac analysis, possibly because they either do not recognize the ac behavior as relevant or are less familiar with the appropriate procedure.

7.10 Implications for Instruction

The findings from the research described in this chapter indicate that students do not develop a robust understanding of bipolar junction transistor circuits in typical electronics courses. On the basis of student performance on multiple research tasks, the combination of lecture instruction and laboratory experience employed in these courses does not appear to be sufficient for students to gain a thorough understanding of BJT functionality in many common circuits. However, there is evidence that some aspects of BJT behavior are relatively well understood. In addition, the most common incorrect lines of reasoning given by students still drew upon productive ideas about transistors, thus suggesting that targeted instructional interventions may be warranted.

Through the suite of research tasks described in this chapter, it has been shown that, in some contexts, many students could make accurate and well-reasoned predictions about the behavior of a transistor circuit. In particular, students were relatively adept at reasoning about the base-emitter junction’s diode-like properties. Nevertheless, a number of students struggled to make correct predictions about the behavior of the base-emitter junction, exhibiting difficulties similar to those documented in diode circuits, as reported in Chapter 5. Such findings suggest that the development and refinement of additional targeted, research-based instructional materials on diode circuits might serve to strengthen student understanding of transistor circuits (and particularly the behavior of the base-emitter junction) as well.

As students often did not discriminate between the ac and dc behavior of the common emitter amplifier circuit when unprompted, it may be productive to introduce circuits with asymmetric effects under ac versus dc voltages (*e.g.,* op-amp amplification circuits with dc biases) more frequently in the curriculum. Indeed, it is noteworthy that the first inverting and non-inverting op-amp circuits that students encounter (and thus the first circuits with greater than unity gain) act identically on ac and dc voltages. Thus, it is possible that students who study op-amps before transistors (which is the case for many of the courses included in this investigation) may generalize this behavior to transistor amplifiers as well. During instruction, it may therefore be beneficial to explicitly compare and contrast the behavior of common op-amp and transistor amplifiers on identical input signals. On the basis of these findings, it is likely that research-based instructional materials focused on such comparisons might serve to strengthen student understanding of both BJT amplifier circuits as well as those constructed from op-amps.

Chapter 8

INVESTIGATING THE ROLE OF SOCIALLY MEDIATED METACOGNITION DURING COLLABORATIVE TROUBLESHOOTING OF ELECTRIC CIRCUITS

In this chapter, a framework of socially mediated metacognition is used to explore the process of student decision-making while troubleshooting circuits in a laboratory setting. Troubleshooting is an open-ended, recursive problem-solving task that is often an implicit goal of instruction in upper-division laboratory courses in physics. However, metacognitive regulation is known to play a key role in the selection of appropriate strategies in a variety of problem-solving tasks. In this study, the framework of socially mediated metacognition was used to examine the nature and impact of interactions between students during think-aloud interviews in which eight pairs of students from two different institutions attempted to diagnose and repair a malfunctioning operational-amplifier circuit. Findings from these interviews indicate that students' metacognitive engagement in one another's ideas facilitated collaborative generation of hypotheses and testing strategies. Indeed, through their discourse, students were able to jointly identify gaps in their reasoning, which in turn led to the selection of targeted measurements and approaches. This work contributes substantively to the research base on troubleshooting by both describing how students navigate through the task of troubleshooting in electronics and by foregrounding the importance of collaborative regulation in such endeavors.

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8.1 Introduction

Students typically take multiple laboratory courses, associated with both introductory and upper-division content, as part of an undergraduate physics program. Recently, the American Association of Physics Teachers (AAPT) issued a new set of guidelines for the undergraduate laboratory curriculum, identifying the development of experimental design skills (including troubleshooting) as well as technical and practical laboratory skills (such as understanding the limitations of measurement devices) as two of six critical focus areas [24]. These guidelines indicate that students should learn to troubleshoot problems in an iterative and logical way by the completion of an undergraduate physics degree. Other national efforts have called for both improving [25] and studying [26] laboratory instruction in science courses, with a particular emphasis on creating new instruments to assess learning outcomes in the instructional lab setting and to measure both metacognitive and problem-solving skills. To date, however, relatively little research has focused on students' activities within the instructional laboratory environment and the development of skills necessary for experimental physics [12,51,95].

Throughout this article, the term troubleshooting is used to refer to the comprehensive process of identifying the existence of, cause of, and solution to a fault, as well as taking corrective action and verifying the repair [96]. Troubleshooting occurs within all branches of experimental physics and is often a significant, yet implicit, component of laboratory experiments in the undergraduate curriculum. A recent exploratory study suggests that the predominant form of explicit troubleshooting instruction among electronics instructors takes the form of apprenticeship-style interactions during laboratory activities [97]. Furthermore, within interviews many instructors of physics electronics courses have expressed an expectation that students will need to troubleshoot as “nothing works the first time [98].”

The task of troubleshooting is common to numerous professions and contexts, and research has generally focused on those areas in which the development of troubleshooting expertise is an expected outcome rather than an incidental one, such as medical diagnoses, maintenance of manufacturing equipment, and software debugging. (See [96,99] for a more comprehensive overview of the troubleshooting literature.) Existing research suggests that content knowledge is a strong predictor of successful troubleshooting [100]; however, instruction in content alone is insufficient to teach students how to successfully troubleshoot a system [96]. Prior research has focused on identifying the skills and knowledge used when troubleshooting [100], documenting differences between experts and novices [101,102], and developing instructional strategies to teach troubleshooting [103–108].

Since troubleshooting is a complex, open-ended problem-solving task, effective decision-making is critical; troubleshooters must continually monitor their progress, evaluate new information, and incorporate that information into their decisions about how to proceed. Indeed, metacognition has been shown to be an integral component of effective problem solving (see [16] for an overview). The term *metacognition* refers broadly to thinking about one’s own thinking and is often subdivided into categories of self-assessment (*e.g.*, understanding and communicating one's own thought processes), self-regulation (*e.g.*, consideration of how to perform long tasks), and knowledge from previous experience [110]. Schoenfeld's work with both expert and novice mathematicians showed that self-regulation is particularly relevant in the context of problem solving [111]. Elsewhere, it has been observed that troubleshooters tend to make ongoing assessments that are productive for selecting appropriate courses of action, and it has been suggested this may be due to differences in metacognitive knowledge [101], but no metacognitive framework was applied to test this hypothesis. Together, these works suggest that students' metacognitive skills may directly inform their decision-making processes while troubleshooting. However, to the best of our knowledge, the relationship between the two has not been explored in the undergraduate physics laboratory environment.

The educational context of electric circuits is sensible for studying troubleshooting as the behavior of basic circuits can be predicted analytically through a straightforward process. Indeed, some published work on the development of instructional strategies for teaching troubleshooting has used electric circuits as a research context [105,106]. However, these studies of Dutch high school students were focused on the impact of specific interventions on troubleshooting simple dc resistive circuits in a simulated environment; they did not actually document the process of how students went about the task of troubleshooting circuits.

Given the more complex nature of the tasks involved, upper-division electronics courses may serve as a richer context for troubleshooting than those involving only basic dc circuits. The systems students explore in upper-division electronics courses are sufficiently ordered to be analyzed in a systematic way (*e.g.*, circuits may be understood in terms of functional chunks), and multiple faults may present the same or similar symptoms, requiring functional knowledge of electronics to be able to properly investigate a malfunctioning circuit. Problems with circuits may be ill defined, as there are many potential measurements that could be made in even a moderately complex circuit. However, many flaws may be fixed with either straightforward rewiring or the replacement of components. Combined, these characteristics align well with Jonassen and Hung's criteria for what constitutes a suitable troubleshooting task [99]. Thus, we argue that the context of upper-division electronics is ideal for investigating students' troubleshooting approaches.

While there has been considerable research in PER on student understanding of introductory circuits [19,20], topics in upper-division electronics courses remain largely unstudied. Most existing upper-division work has focused primarily on student learning of circuits containing specific elements or particular functional networks, such as operational amplifier (op-amp) circuits [30,36], phase relationships in AC circuits [65], and RC filters [13]. Only recently have researchers examining the learning and teaching of upper-division electronics begun to explicitly target laboratory skills such as troubleshooting, data interpretation, and design. Indeed, while the results from a survey of electronics instructors indicated there may not be full agreement on the perceived value of developing various practical laboratory skills among instructors [14], there is evidence of a growing consensus, as reflected in the laboratory guidelines recently endorsed by the American Association of Physics Teachers [24].

For all of the reasons described above, a study examining the role of metacognition in the troubleshooting efforts of upper-division physics students in the context of laboratory instruction on analog electronics was conducted. In particular, the study reported in this chapter was designed to investigate the following research questions:

1) To what extent are student groups engaging in metacognitive behaviors while troubleshooting a pre-assembled op-amp circuit?
2) What role does metacognition play in the process of decision-making while troubleshooting?

In order to examine how students troubleshoot in the lab, we conducted think-aloud interviews during which eight pairs of physics students from two different institutions attempted to repair a malfunctioning operational amplifier circuit. Video and audio data were collected, and each interview was fully transcribed. This chapter primarily focuses on characterizing how students engaged in metacognition during the course of the troubleshooting activity, particularly when students were making strategic decisions. Previous analyses of these data focused on the role of students' model-based reasoning during the troubleshooting process [51,112]. In addition, we have previously reported a preliminary analysis of a subset of our data using the socially mediated metacognition framework [113]. This chapter aims to build upon the latter work by examining the roles that socially mediated metacognition may play in troubleshooting electronic circuits.

We begin in Section 8.2 with a brief overview of prior research that has informed and motivated this study. We then discuss the context and methodology of this investigation in Sec. 8.3; this includes an overview of the interview task, the rationale behind our design choices, an overview of how and why data were selected for analysis, and a detailed description of the framework employed. In Sec. 8.4, we discuss results from two key analyses of the nature of metacognition during the interview task, corresponding to our research questions: a broad characterization of the metacognitive discussions occurring in different phases of the troubleshooting endeavor, and an in-depth analysis of extended metacognitive discourse. Further synthesis and discussion of the repercussions of our findings is presented in Sec. 8.5. Finally, we summarize our findings and discuss their implications in Sec 8.6.

8.2 Relevant Background for Analysis Frameworks

This investigation primarily focused on students' use of metacognition during the decision-making processes that arose while troubleshooting an operational amplifier circuit. However, we found it useful to provide a broad description of students' behavior during the entire task of troubleshooting. To that end, we used a general troubleshooting framework to document the types of actions in which students engaged and in what order, which helped to both contextualize specific instances of metacognition and characterize each interview as a whole. To capture students' fine-grained metacognitive behaviors as they worked together, we employed the framework of socially mediated metacognition, which was originally developed to document metacognition that stems from group collaboration. In this section, we provide historical context and describe the development and design considerations of both frameworks. (For a more detailed overview of the published literature on troubleshooting, see [99,114]. A more extensive discussion of current research on metacognition can be found in [115].)

8.2.1 Troubleshooting

Research on troubleshooting spans multiple domains of study, including educational psychology [99], artificial intelligence [116], vocational training [117], and educational technology [107]. This diversity reflects the fact that troubleshooting as a task is rooted in the unexpected behavior of real systems. As such, numerous forms of knowledge are required of troubleshooters, including domain, system, procedural, strategic, experiential, and metacognitive knowledge [99,101,118]. Furthermore, in order to capture details about how individuals engage in the often cyclic process of troubleshooting, multiple frameworks have been developed [96,103,117].

The framework we employ is based on work by Schaafstal *et al.* [96]. Schaafstal characterized differences in the diagnostic skills of expert and novice paper mill operators, and in doing so, he noted that existing frameworks from artificial intelligence were too rigid and novice-like, but frameworks from psychology described only the local strategies for finding faults rather than capturing the entire troubleshooting process [119]. The framework Schaafstal subsequently created remedied both problems in that it was expressly designed to reflect how human experts would act (as opposed to the models from artificial intelligence research), and it incorporated important process information such as judging the seriousness of faults, the likelihood of those faults, and the outcome of repairs. While the original version consisted of eight different task categories, later work reduced this to four elements, which still capture the critical information about troubleshooting processes [96,120].

The finalized version of Schaafstal's framework subdivides troubleshooting into four sub-tasks: *formulate problem description*, *generate causes*, *test*, and *repair and evaluate*. These categories are well suited to describing students' general behavior over long periods of time, but do not capture details about how students are performing specific tasks. *Formulating problem descriptions* refers to the troubleshooter determining which portions of the systems work as expected and which do not. In this phase of the framework, the initial inspection and measurement of the apparatus take place, as well as a process known as “orienting” to the circuit; during the latter process, a troubleshooter builds mental representations of both the circuit’s structure and functions in addition to mapping these representations onto external representations such as schematics, datasheets, and equations [99,102]. *Generating causes* refers to a phase in which students are generating causal hypotheses for why the circuit is not behaving as intended, or proposing procedures to better identify and isolate faults. *Testing*, in the context of electronics, includes all tests performed with measurement devices such as oscilloscopes or multimeters, and often involves the systematic alteration of input parameters such as the frequency of the input signal. The *repair and evaluate* phase includes generating, enacting, and testing modifications to the circuit, all of which are intended to return it to a functional state. A structured approach to troubleshooting may be described as an iterative cycle involving some or all of these four tasks. A detailed description of how these codes have been applied to our interviews appears in a companion paper that examines the same data corpus with a focus on the interaction between this troubleshooting framework and modeling [51].

8.2.2 Metacognition

To capture students’ fine-grained metacognitive behaviors as they worked together, we employed the framework of socially mediated metacognition. This framework was originally developed to document metacognition that stems from group collaboration in mathematics [121] and has proven to be flexible enough to be adapted to other contexts.

It has been used, for example, to examine pairs of middle school students engaging in computer programming [122] and groups of teachers in an educational psychology course [123]. The work of both Schoenfeld and Goos also informed research by Lippmann Kung and Linder [124] on the nature of metacognition in an introductory physics laboratory. In this subsection, we discuss the research connecting metacognition with problem solving and troubleshooting. We then discuss how the socially mediated metacognition framework is related to other models of metacognition, as well as the unique ideas arising from social interaction.

Research on metacognition has been prevalent in the field of science education, and many nuanced theoretical frameworks have been used to capture particular aspects of metacognitive behavior. While the term *metacognition* refers broadly to thinking about one's own thinking, most frameworks recognize a division between *metacognitive knowledge* and *metacognitive planning and regulation* [115]. Metacognitive knowledge refers to knowledge and beliefs about cognitive matters; it may be further subdivided into knowledge of persons (*e.g.*, how to appropriately interact with a teacher), tasks (*e.g.*, how to process new information), and strategies (*e.g.*, how to solve an unfamiliar mathematics problem) [125]. Metacognitive regulation refers to planning, evaluating, or monitoring one's own cognitive activities. The frameworks that directly informed our study focus mostly on metacognitive regulation of either an individual's thinking (Schoenfeld [111]) or a group's thinking (Goos, Galbraith, and Renshaw [121]).

Schoenfeld's work examined the role of self-regulation in undergraduate mathematics problem solving [111]. He focused on the task of managing oneself during the problem solving process, including the need for verifying one's understanding of a problem, planning how to solve the problem, monitoring the effectiveness of a solution, and deciding how to allocate time. Interviews were conducted in which participants were asked to solve mathematics problems. An analysis of 100 videos of high school and college students solving unfamiliar problems showed that 60 percent of novices pursued a single solution method, with no ongoing metacognitive assessments of the appropriateness of their choices [126].

In contrast, an experienced mathematician working in an unfamiliar context spent a large portion of his time engaged in analyzing, planning, and assessing the utility of specific actions rather than immediately implementing the approaches that he considered. The expert was also found to frequently make metacognitive assessments of his progress throughout the entire task. Perhaps most importantly, students who were explicitly taught how to engage in metacognitive practices during an undergraduate mathematics class were found to exhibit more expert-like problem-solving behavior than their peers, as demonstrated by increased planning and metacognitive assessment in similar interviews. These findings indicate that targeted instructional interventions designed to support student metacognition may be beneficial in producing better problem-solving outcomes. Related studies on instructional intervention techniques suggest that metacognition may serve to improve outcomes by assisting in the selection of productive problem-solving approaches via ongoing assessments [101].

The need for the framework of socially mediated metacognition (SMM) arose from efforts to study the metacognitive strategies employed by pairs of mathematics students working on introductory physics problems. In their work, Goos initially employed a methodology similar to Schoenfeld’s, segmenting and characterizing time in interviews according to when specific behaviors were demonstrated [127]. However, it was found that while this approach captured macroscopic features of problem-solving, another level of coding was needed to describe the unique contributions students made as well as the nature of the interactions between individuals [128]. Using ideas from Vygotsky's work [129], Goos and Galbraith expected that, through collaboration, students would complement and enhance one another's knowledge and jointly establish a zone of proximal development, thus resulting in collaborative performance exceeding that of either student individually [130]. Goos and colleagues also noted that both the quality of metacognitive decision-making and the nature of the social interactions between subjects significantly influenced the outcomes of problem solving activities. To further explore the latter interaction, the secondary coding scheme was formalized and used as the basis for a more comprehensive framework of SMM [121].

The SMM framework captures the metacognition that arises in a group as a result of collaboration between participants. In applying the SMM framework to interviews, lines of dialogue are coded for their *metacognitive functions* (*e.g.*, verbalizations that may reflect their internal metacognitive processes). A second *transactive* coding scheme, modified from Kruger's work on peer collaboration [131], is used in tandem to capture how students interact with one another's ideas. Statements that are coded as both transactive and metacognitive (*i.e.*, statements about metacognitive processes directed towards one's partner) were found by Goos and colleagues to provide the greatest insight into the nature of peer interactions supporting collaborative metacognitive activity.

In addition to classifying individual lines of dialogue by metacognitive function and transactive quality, Goos *et al*. performed a supplementary analysis that captures students' engagement with one another's ideas across multiple lines of dialogue [121]. The identification of clusters of dialogue makes it possible to better characterize instances where group members are collaboratively engaged in one another's thinking, as opposed to being individually metacognitive. In the SMM framework, students' engagement with each other's ideas is described using the concepts of *metacognitive nodes* and *transactive clusters*. Metacognitive nodes describe instances in which one person's metacognitive utterance is responded to with a transactive statement. Transactive clusters refer to occasions where a single metacognitive utterance yields multiple transactive statements, which may indicate extended discussion. It is important to note that under the originally published SMM framework, a node may arise even when a person responds to his or her own statement (*e.g.*, by unprompted clarification). The frequency and nature of these transactive clusters were found to differ significantly between successful and unsuccessful problem-solving endeavors.

8.3 Context and Methodology

In order to characterize the role of socially mediated metacognition in troubleshooting, we conducted think-aloud interviews at two different institutions, the University of Colorado Boulder (CU) and the University of Maine (UM). Detailed descriptions of the institutional and course contexts for this investigation as well as the design of the think-aloud activity have been provided in Ref. [51]. In this section, we summarize the context for our investigation and the design of the think-aloud activity, emphasizing the aspects most relevant for discussing metacognition. We also provide a detailed description of our data analysis methodology and coding scheme, which has been briefly described in a manuscript documenting our pilot study [113].

8.3.1 Context for Investigation

This study was conducted with students who were either enrolled in or had recently completed upper-division physics courses on electronics at either CU or UM. The electronics courses at CU and UM are required for physics majors, and are typically taken in the third year of instruction. The courses are each one semester in length and cover a similar spectrum of topics, with an emphasis on analog components and devices such as diodes, transistors, and operational amplifiers. Consistent with the practices of other electronics instructors [97], formal instruction about troubleshooting took place almost exclusively via apprenticeship-style interactions during lab activities.

Several weeks in both courses are dedicated to introducing operational amplifiers (op-amps) and their use in a variety of practical applications. Students are taught that an op-amp is a high-gain differential amplifier with an inverting (-) input, non-inverting (+) input, a single output, and two power connections. The power connections are typically attached to positive and negative 15 V supplies, often referred to as the power rails. Students are taught a first-order model of the op-amp which describes its functional behavior via two "golden rules," articulated by Horowitz and Hill as: "I. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero… II. The inputs draw no current [90]." The golden rules are explicitly covered in both courses and are sufficient, when used in conjunction with Kirchhoff's laws, to predict the behavior of many op-amp circuits that employ negative feedback.

Instruction in both electronics courses is comparable in many ways. Both courses have two 50-minute lectures per week, which serve to familiarize students with the theoretical behavior of new circuits and circuit elements. A weekly laboratory session is an integral component of each course, and students work together in pairs to complete guided laboratory activities. Instruction is supported by undergraduate learning assistants and/or graduate teaching assistants at both institutions. Both courses include midterm and final exams, which mostly focus on the formal analysis of circuits. At the time of this study, neither course included explicit instruction on troubleshooting strategies.

The CU course has three hours of scheduled laboratory instruction per week and students have the ability to access the lab freely outside of this time. Enrollment typically consists of 30-60 students per semester, divided into two or three lab sections. In response to learning goals identified by faculty [132], the course was recently redesigned to engage students in modeling both analog circuits and standard measurement devices. The course culminates with a five-week final project that is usually done by either individual students or small groups.

The UM course has two hours of scheduled laboratory instruction per week with limited access to the lab outside of class time. Enrollment typically consists of 10-15 students per semester, divided into two lab sections. The course is designated as “writing intensive,” and students are therefore required to complete formal written lab reports for approximately half of their experiments; the reports are critiqued and graded by the course instructor as well as an external technical writing specialist. The course culminates with a two-week project in which groups of three or four students work together to design, construct, and test temperature controllers.

8.3.2 Data Collection

Critical for our investigation was the development of a research task that was both controlled (so that students would work from the same initial conditions) and authentic (so that the activity was as close to the students' electronics laboratory experience as possible). To ensure that the activity was properly controlled, we conducted clinical interviews using the same pre-assembled circuit every time. In order to enhance the authenticity of the task, students at each university were presented with a physical setup (*i.e.,* the circuit itself, associated voltage sources, and measurement equipment) that closely resembled what they had used in their respective courses. The pre-constructed circuits were assembled on breadboards identical to those used in the courses at both institutions, with care taken to ensure that the wiring was relatively easy to follow. All groups had access to multimeters, an oscilloscope, a function generator, a power supply with variable and fixed voltages, and a suite of replacement components and wires.

The students in this investigation were accustomed to working in pairs in their electronics laboratories, and were inclined to have discussions with one another with minimal interviewer intervention. As a result, we chose to conduct interviews using a think-aloud protocol with pairs of students troubleshooting a pre-constructed circuit. The use of a think-aloud protocol, in which subjects are asked to verbalize their thoughts concurrently with their actions, is relatively non-invasive in a paired setting, as students frequently clarify their thinking to their partners while justifying differing opinions, *etc.* [133].

8.3.2.1 Research Task

In the interviews, students were asked to troubleshoot an inverting cascade amplifier, shown in Fig. 8.1. The circuit can be divided into two distinct stages, each of which may be analyzed separately. Stage 1 of the circuit, consisting of the leftmost op-amp and resistors R_1 and R_2, is a non-inverting amplifier with a gain of $(1 + R_2 / R_1) = 2$. Stage 2, which consists of the rightmost op-amp and resistors R_3 and R_4, is an inverting amplifier with a nominal gain of $(-R_4 / R_3) = -10$. In a functioning circuit, the output V_{out} is equal to the product of the gains of each stage (typically referred to as the transfer function) and V_{in}, thus $V_{out} = -20\ V_{in}$. The negative sign implies that the output voltage signal is 180° out of phase with the input voltage signal. The output voltage is constrained by the voltages of the power rails such that, in practice, the output voltage must always be slightly lower than the positive rail voltage, and slightly higher than the negative rail voltage; any input voltages that would cause the output to exceed these limits will result in a saturated output voltage (*i.e.*, the output voltage will be truncated to within a volt or so of each power rail).

Two faults were intentionally introduced into the second stage of the circuit. The first fault (fault 1) was that the resistor R_3 was an order of magnitude smaller than its prescribed value. This caused the gain of the circuit to be increased by an order of magnitude, which by itself would result in saturation of the output for a relatively small input voltage. We expected fault 1 to be relatively straightforward to diagnose, as the

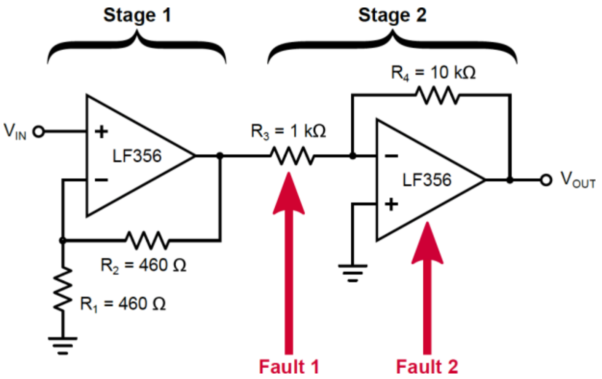


Fig. 8.1. Annotated schematic diagram for the inverting cascade amplifier, with design elements highlighted. Stage 1 of the circuit, consisting of the leftmost op-amp and resistors R_1 and R_2, formed a non-inverting amplifier with a gain of 2. Stage 2 is an inverting amplifier with a nominal gain of -10, consisting of the rightmost op-amp and resistors R_3 and R_4. The handout given to students did not include labels for stages and faults.

incorrectly colored bands on the resistor serve as a visible cue, making it possible to diagnose this fault through visual inspection of the circuit. The second fault (fault 2) was that the op-amp was damaged so that its output voltage was always a constant voltage that was close to the negative rail voltage (*i.e.*, slightly higher than -15 V); as such, the op-amp no longer obeyed the first golden rule. Similar behavior could arise from incorrectly wiring the op-amp circuit.

The malfunctioning circuit was designed to increase the likelihood that students would engage in multiple iterations of troubleshooting and employ a split-half strategy. In a split-half strategy, the troubleshooter tests the behavior of a circuit at the middle of the signal path in an attempt to localize the fault to one half of the circuit or the other. By repeating the process recursively they may isolate the fault to a single stage. Since the faults solely affected the performance of the second stage, it was ensured that students could isolate all problematic behavior to that stage alone. The split-half method is therefore a viable strategy for troubleshooting the circuit.

8.3.2.2 Think-Aloud Interviews

Participants in this study were enrolled in courses taught by two of the authors (HJL and MRS) in Fall 2014; additionally one author (KLVDB) was a teaching assistant for the course at UM. A total of 16 students were interviewed in pairs for this study, eight from CU and eight from UM, and each group will be referred to throughout the paper by a different letter from A – H. Students were invited, via email and in-person requests, to participate in interviews near the end of the course (at CU) or during the following semester (at UM). Students were allowed to select a partner if they wished, and students who did not do so were paired by the interviewers on the basis of availability.

Participants were given small monetary incentives for their time, but involvement was strictly voluntary and students did not receive any course credit for interviewing. Commensurate with student demographics in the courses and undergraduate programs at both institutions, participants were predominantly white men. A more detailed demographic breakdown is presented in [51].

The interview itself began when the interviewer presented students with a schematic diagram of the circuit and a datasheet for the op-amp. The interviewer then gave a short introductory prompt to the activity, requesting students to approach the task as if their peers had built the malfunctioning circuit in the lab. (See Appendix B for the full text of this prompt.) Students were subsequently presented with the physical circuit and tasked with diagnosing any issues with the circuit and with making the circuit work as intended. Students were asked to think aloud as they worked, and to act as though the interviewer was not present. If the students were silent for a significant length of time, the interviewer would prompt them to continue speaking; in practice, there was minimal intervention on the part of the interviewer. The activity ended either when the students had completed their repairs, or when roughly one hour had passed. The initial prompt from the interviewer was approximately two minutes in length, and students typically spent between 20 and 45 minutes on the troubleshooting activity. Seven of the eight groups were ultimately able to repair the circuit, with the remaining group running out of time prior to the completion of the task. Video and audio data were collected for all interviews, and audio data were used to generate complete transcripts.

8.3.3 Data Analysis

To characterize students' metacognitive exchanges during the troubleshooting process, we developed codes based on the SMM framework. We applied these codes to four types of episodes that occurred across multiple student groups. Specifically, we used the SMM codes to perform line-by-line analyses of the corresponding transcribed student dialogue. A detailed example of such an analysis is provided elsewhere [113]. The SMM framework was used as an *a priori* analysis scheme. We initially developed operational code definitions based on definitions from the SMM literature. Operational definitions were refined through iterative cycles of collaborative coding by two authors (D.R.D.F. and K.L.V.D.B.) and discussions with the research team as a whole. By "collaborative coding," we mean that the initial iteration of coding was performed simultaneously by the two coders. During subsequent iterations of coding, D.R.D.F. and K.L.V.D.B. first applied codes independently and then resolved all discrepancies through discussion. In this subsection, we define the four categories of episodes analyzed, discuss the rationale for selecting these episodes, and then describe how the SMM coding scheme was adapted to the context of the interviews.

8.3.3.1 Episode Definitions

In order to constrain our analysis to time intervals in which rich metacognitive dialogue was more likely to occur and to facilitate comparisons between groups of students, we selected four categories of episodes to analyze in detail: *Initial Strategizing (IS)*, *Discrepant Output (DO), Split-Half (SH)*, and *Replacement Decision (RD)* episodes. These episodes represent key decision-making moments during which students transitioned between troubleshooting subtasks. Each episode category had specific criteria that were used to select the beginning and end based on actions taken by the students.

The *initial strategizing* (IS) episodes captured how students first approached the task, beginning once the interviewer finished introducing the problem and ending when students either began checking the circuit's connectivity or began measurements of resistances or voltages. These episodes were expected to be representative of a transition from formulating a description of the problem to testing. We identified IS episodes for all eight groups, and these episodes typically lasted 1.5 minutes.

The *discrepant output* (DO) episodes captured how students responded to a mismatch between the expected output of the circuit and the measured output. These episodes began when students first observed that the output of the entire circuit was a constant dc value, and ended when students enacted a plan to make further measurements. These episodes were expected to contain a transition from generating causes for their unexpected measurement to performing additional tests. We identified DO episodes for all eight groups, and these episodes typically lasted 2.5 minutes.

The *split-half* (SH) episodes captured how students strategized after identifying a working stage in the circuit, beginning after students had eliminated the first stage of the circuit as a source of faults, and ending when students enacted a plan to make further measurements. These episodes would represent another clear transition from generating causes (necessitated by partially localizing the fault) and performing further tests. Five of the eight groups employed a split-half strategy, and these episodes typically lasted 2 minutes.

Finally, the *replacement decision* (RD) episodes captured how students came to the decision to replace the faulty op-amp, beginning with the last set of measurements made before students decided to replace the second op-amp, and ending when the replacement was made. These episodes were selected because they contained a transition from testing to repairing the circuit. Seven of the eight groups successfully replaced the faulty op-amp, and such episodes typically lasted 2.5 minutes.

The episodes in all four categories occurred in the same order, unless a category was not present. The *initial strategizing* always occurred within the first few minutes of the interview, immediately after the nature of the task had been explained. The *discrepant output* episodes tended to occur after the first third but before the second half of the interview, while the discussions following a *split-half* strategy generally occurred in the final third of the interview. *Replacement decisions* were more varied in timing, but such decisions usually were made in the final quarter of the episode.

All four episode categories were present in four of the groups. One of these groups decided to replace the faulty op-amp immediately after employing a split-half strategy, and hence a single episode was coded as both SH and RD for that group. Only three episodes were present in each of the other four groups: one group did not replace the faulty op-amp, and three groups did not employ a split-half strategy. In total, we identified 27 unique episodes across the eight participating groups. The cumulative duration of these 27 episodes was approximately one hour, accounting for roughly 20% of the aggregated interview time for all groups. For all 27 episodes, we coded the corresponding transcripts using the analysis frameworks described in the following subsections.

8.3.3.2 Socially Mediated Metacognition Coding

To characterize students' metacognitive behaviors during the troubleshooting process, we adopted the previously mentioned framework of socially mediated metacognition pioneered by Goos *et. al.* [121], in which lines of dialogue are simultaneously coded for *metacognitive function* and *transactive quality*. The codes for *metacognitive function* concern metacognitive acts in which new information is recognized or assessments are made. The codes for *transactive quality* capture the collaborative nature of the exchanges between students. Below we present our coding scheme as a hierarchical list, with operational definitions for each code and examples of sub-codes drawn from authentic student dialogue.

- **Metacognitive function:** Statements may play specific functional roles in metacognition, either by introducing new ideas or by assessing ideas.
    - **Introduction of new ideas:** A new idea is verbally expressed that is relevant to the situation. This may occur when students are:
        - **Suggesting an approach:** A new strategy for approaching the problem is suggested. *"So, I mean, I would start with just checking if the chips are working."*
        - **Suggesting an explanation:** An explanation for the circuit's behavior is suggested. *"And maybe this red one, the power is somehow touching the output?"*
        - **Making a prediction:** A prediction of the outcome of an event is articulated. *"[It's] probably going to be the second op-amp to hit rail."*

        - **Making an observation:** A piece of relevant information is observed (but not evaluated) from the circuit, measurement tools, handout, or datasheet. *"Oh hey look, it stabilized for some reason too."*
        - **Stating a relevant fact:** A piece of relevant information is recalled and stated. *"Remember, these op-amps are backwards."*
    - **Assessment:** An attempt is made to evaluate information. This may occur when students are:
        - **Assessing a result:** The reasonability of either an actual or predicted behavior of the circuit is mentioned. *"So the first one is giving us a good voltage."*
        - **Assessing a strategy:** The appropriateness or execution of a strategy is discussed. *"Yeah, I mean it will be like the brute force method of making sure it's the right chip. Pull it out and put the right one in."*
        - **Assessing their understanding:** An evaluation of the students' understanding of the problem is made. *"We have a good output for the first op amp. We are going to have, the problem is in the second one."*
- **Transactive quality:** Statements that are verbal requests for interaction with the other participant, which may in turn prompt further dialogue.
    - **Self-disclosure:** A statement is made by a student in order to clarify an idea previously expressed by that same student. *"You can't get that high of [a] voltage, you'd be hitting rail."*
    - **Feedback request:** A statement is made by one student inviting the other student to consider or critique an idea that the first student has expressed. *"So this should be inverting the signal and amplifying it, correct?"*

- **Other-monitoring:** A statement is made by one student in response to the other student with the aim of critiquing or building upon the other student's idea, or requesting further information about what the other student is thinking (monitoring ideas) or doing (monitoring actions). *"S1: Okay. So, we aren't getting anything out [of the second op-amp]. S2: We're getting something actually. It's just a DC negative voltage."*
- **Prompting for new ideas:** A statement is made by one student prompting the other student to generate and articulate a new idea or approach. *"Okay, so that's fine. Then what's next?"*

The coding scheme presented here was modified slightly from that of Goos *et al*. in order to make it better suited to the context of troubleshooting electronic circuits. In particular, different sub-types of new ideas (*e.g.*, suggesting an approach) were easily distinguished from others (*e.g.*, making an observation), and were thus tracked explicitly in our analysis. The addition of a transactive category for *prompting for new ideas* was added to the coding scheme after it was observed that such interactions occurred in interviews.

8.3.3.3 Node and Cluster Coding

After data were coded via the SMM framework, a further level of coding was applied in order to systematically capture the students' social engagement in one another's ideas. This cluster analysis, adapted from the one originally presented by Goos *et al*., identifies patterns between subsequent lines of dialogue between participants. In our modified coding scheme, we define a *node* as of a pair of statements in which one student makes an utterance that is coded as metacognitive (*i.e.*, expresses a new idea or makes an assessment) that is either prompted by or leads to a partner's transactive statement. We further define a *cluster* of dialogue as an occurrence of a series of two or more overlapping nodes and thus at least three successive turns of dialogue. These definitions differ from their original usage in that they require reciprocated verbal exchanges between both individuals, and thus explicitly capture back-and-forth interactions.

For example, consider a hypothetical exchange between two students, depicted in Fig. 8.2. In this exchange, nodes are indicated with square brackets to the left of the dialogue and given single letter labels. Lines 1 and 2 form a node (A), as S2 monitors S1's suggestion. Lines 2 and 3 also form a node (B), as S1 tries to justify his idea, but wants feedback about his assessment. Finally, lines 3 and 4 form a node (C) as S2 elaborates on S1's idea with an additional assessment. Together, the three nodes form a cluster in which the hypothetical students collaboratively clarify why it would be reasonable to measure the resistor's voltage again.

Together, the combination of the SMM coding and cluster analysis allowed us to identify and further characterize instances when students were collaboratively engaging in metacognitive activities during the interviews. In the following section, we report the results of our analyses.

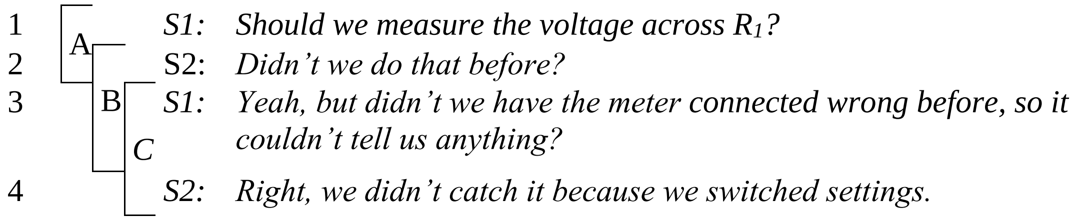
8.4 Results

We describe data and findings from two different analyses performed to examine the role of metacognition in troubleshooting. First, we provide an overview of students' metacognitive behaviors within each category of episode in order to determine the extent to which students are engaging in such behaviors while troubleshooting. Then, we investigate occurrences of clusters within the students' dialogue in order to characterize the nature and degree of students' engagement in one another's ideas as they make decisions related to troubleshooting.

8.4.1 Analysis of Episodes by Category

In the subsections that follow, we discuss and further characterize all four categories of episodes to better illuminate how students engaged in socially mediated metacognition. For each episode category, we provide a short synopsis of the notable features observed. As presenting episodes in their entirety would be cumbersome for the reader, we limit our discussion to those excerpts that contain only the most relevant dialogue. Information added to the transcripts for clarity is indicated by square brackets. Within this paper, each transcript is presented as numerically indexed list, followed by a line-by-line summary that denotes the line number and metacognitive and transactive coding in parentheses. This approach makes the reasoning process behind the coding as explicit as possible while still providing a transcript that is easily readable.

All eight groups engaged in exchanges that are well characterized by the SMM framework in at least three episodes. A summary of metacognitive code usage, grouped by episode category, is presented in Table 8.1. Across all episodes, a large (70%) fraction of conversational turns corresponded to one or more of the SMM codes. Overall, we found that the students were assessing their results more frequently than they were assessing either their own understanding or the strategies they were employing while



1 *S1: Should we measure the voltage across R_1?*
2 S2: *Didn't we do that before?*
3 *S1: Yeah, but didn't we have the meter connected wrong before, so it couldn't tell us anything?*
4 *S2: Right, we didn't catch it because we switched settings.*

Fig. 8.2. Hypothetical example of clustering

troubleshooting. This suggests that, at least during the episodes analyzed, students were not engaging in large-scale, strategic decisions as much as they were focusing on local evaluations.

8.4.1.1 Initial Strategizing

Each IS episode consisted of the first one or two minutes of the troubleshooting activity, starting just after students finished receiving instructions from the interviewer and ending when they began either making measurements or carrying out a detailed inspection of the circuit. All groups engaged in some dialogue in this stage, and every group verbalized to a varying degree an approach that they planned to take, as can be seen in Table 8.1.

The following excerpt from group E is an example of a multi-turn metacognitive exchange, occurring just after the interviewer finished introducing the task:

E1: All right. Cool. Well, how do you want to start this out? We could work out theoretically what it should do to start.
E2: They give us a pretty good transfer function right there [on the handout].
E1: Okay. Cool... That makes sense, just like inverting and not inverting smashed together.

This excerpt begins as E1 initiated a conversation about how to proceed (1: *Idea Request*) followed by his own suggestion (2: *Idea - Approach*). E2 remarked that they had been given a "pretty good" transfer function already (3: *Idea - Fact*). E1 examined the handout briefly and commented that the circuit appeared reasonable (4: *Assessment - Understanding)* and then elaborated that he viewed it as a combination of inverting and non-inverting amplifier circuits (4-5: *Idea - Fact*, *Self-Disclosure*).

Through this discussion, students in group E derived a better understanding of the circuit's functionality. After E2 indicated that the handout could be useful for making a theoretical prediction, his partner interpreted the circuit as a combination of known sub-circuits. This may have been due to either the diagrammatic representation or the mathematical form of the gain expression depicting a combination of recognizable parts. Although this information was not immediately used for making predictions, this group

			Episode Category			
Scheme	Code	Sub code	IS (N=8)	DO (N=8)	SH (N=5)	RD (N=7)
Metacognitive function	New ideas		100%	100%	100%	100%
		Suggest approach	100%	100%	100%	100%
		Make prediction	38%	50%	40%	43%
		Make observation	100%	100%	60%	83%
		State a fact	75%	100%	100%	57%
		Suggest explanation	0%	13%	40%	43%
	Assessment		38%	100%	80%	100%
		Results	38%	100%	80%	100%
		Understanding	38%	38%	40%	43%
		Strategy	13%	50%	0%	0%
Transactive quality	Other-monitoring		63%	100%	100%	100%
		Ideas	63%	100%	100%	100%
		Actions	25%	25%	20%	57%
	Self-disclosure	-	75%	100%	100%	100%
	Feedback request	-	75%	88%	60%	86%
	Idea request	-	13%	13%	0%	29%

Table 8.1. Socially mediated metacognition coding results. Shown are the percentages of groups engaging in dialogue that served one or more metacognitive functions or transactive qualities in a given episode: initial strategizing (IS), discrepant output (DO), split-half (SH), and replacement decision (RD)

later employed a split-half strategy, which relies on the identification of independently testable stages.

8.4.1.2 Discrepant Output

Each DO episode consisted of the discussions that followed immediately after students observed that the output of the circuit did not match their expectations. All groups observed that the initial output of the circuit was not what they would have expected from the input they supplied; instead, they found that the output was a constant dc voltage (between -12 V and -15 V, depending on the specific model of op-amp used).

Throughout these episodes, most groups carried out actions that would further their understanding of the malfunctioning circuit. Some, however, did not appear to use the information gained from their observations to inform and constrain the investigations immediately following the episode. Specifically, groups A and H both tested the signal with an ac input, but subsequently decided to measure resistor values. These groups did not consider that a problem with resistor values could not fully account for the faulty dc output signal they had observed; hence, one could argue that they were not making strategically sound decisions (*i.e.*, gathering more information to better formulate a description of the problem would have been a more appropriate first choice). Similarly, group D made a decision to re-investigate the circuit, but this decision was not attached to a specific hypothesis as to how their course of action would help advance their understanding.

The five remaining groups made investigations directly related to their observations, either in the form of checking the rail voltages/power connections (which were close in value to the observed output voltage) or testing the output of stage 1 (which had not been directly observed yet and could be functional). We highlight two excerpts, from two different groups, in which metacognitive discussions directly informed the groups' subsequent investigations of the malfunctioning circuit. Both groups chose to use an ac signal as a test input to the circuit.

8.4.1.2.1 Refuting a prediction

Prior to this excerpt, the students in group C had been experiencing difficulties with the probes connected to the oscilloscope. The students ended up using two separate cables as they measured the output of the second op-amp on two different channels.

1 C1: *That's getting us a dc voltage. Or is that oscillating?*
2 *That's bizarre. Why is it…?*
3 C2: *Yep, these guys [both cables] are measuring the same dc.*
4 C1: *Is something just being a voltage divider or something?*
5 *What's the value?*
6 *[The students adjust the oscilloscope to better read the signal]*
7 C1: *It's some sort of...*
8 C2: *14 volts.*
9 C1: *It's probably saturated.*
10 C2: *No, if it was saturated it would still oscillate, right?*
11 *It would just clip at the sides?*
12 *So, I mean more likely that 14 is pretty close to this guy [the power*
13 *supply]. Maybe one of the [breadboard] rails is bad underneath.*
14 *That's certainly possible.*

Here, C1 first observed that the output was a dc value, which he noted was bizarre (1-2: *Idea - Observation, Assessment - Result*) and questioned if what he observed was actually dc or oscillating (1: *Feedback Request*). C2 confirmed that they were measuring a dc value (3: *Other-Monitoring - Ideas*). C1 questioned if this could have been the result of voltage division (4: *Feedback Request*) and suggested measuring the actual value (5: *Idea - Approach*). The students adjusted the oscilloscope settings to better read the signal and C2 noted that the magnitude of the signal was 14 volts (8: *Idea - Observation*). C1 commented that this could mean that the op-amp is saturated, a term commonly used to describe an op-amp that is producing the largest (absolute value of) voltage it is capable of generating as opposed to the (even larger) voltage predicted by the gain of the circuit (9: *Idea - Explanation*). C2 countered that if the op-amp were saturated, the output voltage would still oscillate (10: *Idea - Fact, Other-Monitoring - Ideas*) and then clarified what he meant by saying that the output voltage would be limited at extreme values (11: *Self-Disclosure*). C2 then proposed that the constant output they were seeing was similar to the value of the rail voltages, and suggested that there may be an unexpected connection between one of the vertical power busses "underneath" the breadboard and other parts of the circuit, which in this context is "bad" (12-14: *Idea - Explanation*).

In this excerpt, the students jointly gathered evidence needed to substantiate a prediction of the fault they observed. First, they discerned that the output was constant and not oscillating. Next, they determined the exact voltage of the output, which was close to one of the power supply voltages. With these two pieces of evidence, D1 proposed an explanation (saturation of the op-amp) for the symptoms that they observed. However, his partner noted that their explanation could not account for a key feature that they were observing (the absence of oscillations) and that they should instead consider a different hypothesis. This exchange is an example of a student being metacognitive by monitoring the explanatory power of his partner's ideas.

8.4.1.2.2 Exploring conceptual understanding

The second excerpt occurs toward the end of group F's *discrepant output* episode. Prior to this excerpt, the students tested the input signal to the circuit and verified that it was, in fact, what they expected, which led to the following discussion of what to do next.

1 F1: Should we— we should make sure that this [op-amp 1's inverting input] is
2 zero volts.
3 F2: Um, this should not be zero volts.
4 It should be the same as V_{in} I think, right?
5 It should be zero down here [at ground].
6 F1: Okay. But, where is that coming from?
7 The feedback or something?
8 F2: It's just the golden rule of the op-amp that the inputs want to be the same.
9 F1: Yeah, but how could the negative terminal be the same as the
10 positive terminal at all times?
11 F2: I don't know how it works, it's just...

F1 began by suggesting that they check the non-inverting input of the first op-amp to make sure it is zero volts (1-2: *Idea - Approach*). F2 disagreed with his prediction (3: *Other-Monitoring - Ideas*) and suggested that that pin should instead be the same voltage as the input (4: *Idea - Prediction, Feedback Request*) and that the ground symbol on the circuit diagram ("down here") was instead the point at which one would expect to measure zero volts. (5: *Idea - Prediction*). F1 asked why that pin (the inverting input of op-amp 1) should be the same voltage as the circuit input (6: *Other-Monitoring - Ideas*) while tenuously suggesting that feedback might be the mechanism (7: *Feedback Request*). F2 told him that the (first) golden rule for op-amps is that the voltages of the non-inverting and inverting inputs will be the same (8: *Idea - Fact, Self-Disclosure*). F1 was dissatisfied with this explanation and asked for more information (9-10: *Other-Monitoring - Ideas*), which F2 admitted he could not provide (11: *Assessment - Understanding*).

This excerpt highlights how these students were exploring the limitations of their own knowledge while they were drawing upon that same knowledge to form their predictions.

There was initially a discrepancy in F1's prediction, which was subsequently refuted by his partner. In doing so, the students brought into question the mechanisms that determined the circuit's behavior. Through F1's directed probing, these students became aware that their understanding of the role of negative feedback is limited— "I don't know how it works." While op-amps characterized by the "golden rules" model require negative feedback, the specific mechanisms underlying this behavior are often unexplored in many electronics courses, as they require a nuanced discussion about the properties of real (as opposed to ideal) op-amps. The episode ends as the students measure the inverting input again, without coming to a satisfactory mechanistic explanation for the op-amp's behavior. Nevertheless, they are still able to use the golden rules to make concrete predictions later in the troubleshooting task.

Both excerpts demonstrate ways in which students' metacognition may, directly or indirectly, be beneficial while troubleshooting. In the first excerpt, C2 refuted his partner's idea because it was inconsistent with some of the features they were observing. This immediately prompted the pursuit of a different suggestion that could account for all of the evidence the pair had gathered. A similar refutation occurred in the second excerpt, which led to students drawing upon, and subsequently reflecting upon, the op-amp golden rules as an explanation for how a portion of the circuit should behave. Although group F was ultimately unable to find a completely satisfactory mechanism for the op-amp's behavior, the students were able to make useful predictions with their current knowledge while simultaneously recognizing the limitations of that knowledge. Common to both of the highlighted excerpts is that the groups' metacognitive exchanges directed their future inquiries.

8.4.1.3 Split-Half Strategy

Each SH episode consisted of the discussions that followed immediately after students successfully employed a split-half strategy, and ended when they began a new set of measurements. Five of the eight groups successfully employed a split-half troubleshooting strategy. When applying this strategy to the cascading amplifier circuit, the output of the first stage of the circuit must be measured in order to determine if the fault exists in the first half of the circuit or if it may be isolated to the second half. With the successful culmination of a split-half strategy, students would have concluded that the first stage functions correctly, and should subsequently investigate the second stage of the circuit to further localize the fault. We analyzed these episodes, which occurred immediately after the successful employment of a split-half strategy, through the lens of the SMM framework in order to examine the role that metacognition may play in students' formation of testable hypotheses.

In this section, we discuss a single episode in its entirety, noting that this episode was representative of most episodes within this category. The episode we discuss begins immediately after the students in group G have agreed that the first stage of the circuit functions as expected.

1 G1: *So we can isolate this part.*
2 G2: *So then this op-amp, so then, ahh let's see.*
3 *This right here [the inverting input] should be ground.*
4 G1: *Yeah, yeah, this is virtual ground—*
5 G2: *Virtual ground.*
6 G1: *—right here. No current's going through here [into the inverting input].*
7 G2: *Yeah.*
8 G1: *So, from there we can say current through here [resistor 3] is equal to*
9 *current through there [resistor 4].*
10 G2: *So this resistor right here, the* R_3*, that should have a drop of 10 volts then.*
11 *Because you have ground right here [at the inverting input].*

12 G1: *Yeah, yeah, you're right, because this [the inverting input] is zero volts,*
13 *this [stage 1's output] is 10 volts, so we should be losing—*
14 G2: *10 volts across there.*
15 G1: *—10 volts across that resistor. Okay so, I'll look at, we should be losing*
16 *10 volts across here. Alright so let's, let's check it out.*

G1 began with the idea of isolating the second op-amp (1: *Idea - Approach*). G2 examined the circuit and made the prediction that the voltage at the inverting input should be ground, which would logically follow from the first op-amp golden rule (3: *Idea - Prediction*). G1 agreed and furthermore clarified that it would be a "virtual" ground, which in this context indicates that it is not directly connected to ground (4: *Self-Disclosure, Other-Monitoring - Ideas*). G1 then drew upon the idea (from the second op-amp golden rule) that no current enters the inputs (6: *Idea - Fact*) to make the prediction that the currents through resistors R_3 and R_4 would be equal (8-9: *Idea - Prediction*). G2 subsequently predicted that R_3 would have a 10 volt drop across it (10: *Idea - Prediction*) because one end is grounded (11: *Self-Disclosure*). G1 continued with this idea (12: *Other-Monitoring - Ideas*) by indicating that there would be ground on one side of the resistor and 10 volts on the other side (12-13,15-16: *Self-Disclosure*). G1 then predicted that a measurement of the voltage across the resistor should yield a reading of 10 volts and suggested conducting such a test (15-16: *Idea - Approach*).

In the episode presented, socially mediated metacognition manifested itself in the form of a back-and-forth exchange between partners (lines 2-16) as they collaboratively constructed a hypothesis that served as the basis for a measurable prediction. They applied both op-amp golden rules to the circuit and used their knowledge of the output of the first stage to make a testable prediction for the voltage across the resistor R_3.

Similar exchanges were documented and analyzed in three of the other four groups that employed a split-half strategy, and we describe one such exchange in greater detail when discussing clustering of SMM codes in Sec. 8.6 of this paper. The only outlier was group D's episode, in which students did not form a testable hypothesis and instead retested voltages in the second op-amp circuit, despite having done so previously. We note that this group was not successful in diagnosing the problem with the circuit within the time constraints of the interview. In all of the remaining episodes, however, we found that similar back-and-forth metacognitive exchanges similarly aided in the formation of testable hypotheses, which in turn informed the measurement choices students made in service of investigating the second amplifier stage.

8.4.1.4 Replacement Decision

Seven of the eight groups ultimately repaired the circuit by replacing both the faulty resistor (R_3) and the second op-amp. This episode category, however, is exclusively tied to the replacement of the op-amp chip itself, beginning as students discuss the last measurement made before the replacement and ending as students began to replace the op-amp. In contrast with deciding to replace the incorrect resistor, which was warranted on the basis of one specific measurement (the resistance), deciding that the op-amp was faulty required inferences from multiple indirect measures of its behavior (including, for example, its output voltage). In coming to the decision to replace the chip, students moderated their investigations by implicitly asking themselves the question, "Do we have enough information to act?" Within the troubleshooting framework, this episode corresponds to a transition from testing the circuit into repairing the circuit and evaluating the repair. We discuss a single excerpt that highlights the collaborative establishment and justification of one group's decision to replace the op-amp.

Soon after observing the circuit's output, the students in group C erroneously replaced the op-amp in the first stage because they observed an unexpectedly noisy output from stage 1. Prior to this excerpt, they re-measured the input signal, the first stage's output, and the second stage's output. They noted that the first stage appeared to be functional and observed that the output of the second stage was still a large dc value. The pair had just finished measuring the inputs to the second op-amp at the beginning of the excerpt below.

1 *C2:* *Pin three [of the second op-amp] is*
2 *C1:* *Zero*
3 *C2:* *In fact zero.*
4 *However pin two [of the second op-amp] is not zero, right? And that's the*
5 *problem. That's the op-amp*
6 *C1:* *So that's saying that… We're losing our— The op-amp is wrong too?*
7 *C2:* *Yeah, it must be. That means the golden—*
8 *I mean, the first one could've been fine, in retrospect,*
9 *but certainly the second one is not working because the golden rules are*
10 *not being followed here.*
11 *C1:* *Okay, that's not it.*
12 *Want to switch that guy out?*
13 *C2:* *Yeah.*

This group had predicted that pin 2 (the inverting input) should be grounded because pin 3 (the non-inverting input) was grounded, drawing on the first golden rule of op-amps. They began by observing that pin 3 was properly grounded (2-3: *Idea - Observation*). C2 noted that pin 2 was not grounded and stated that this indicated a problem (4-5: *Assessment - Result*). C1 tentatively suggested that something was wrong with the second op-amp (6: *Feedback Request*). C2 agreed with this and commented that they may have been incorrect about their previous decision regarding the first chip (8:

Assessment - *Result*), but then finished using the golden rules to justify that the second chip was in fact faulty (9-10: *Self-Disclosure*). C1 made a comment possibly related to his previous assessment of the first op-amp, and then suggested that they replace the second chip (12: *Idea* - *Approach*).

In this excerpt, the students were making sense of a new set of voltage measurements, with some confirming, but others superseding their earlier work. They used their results to justify replacing the second op-amp, which they reasoned must have been faulty because it didn't follow the golden rules. In addition, in the course of interpreting their results, they reflected on their earlier replacement of the first op-amp, as their new measurements differed from the prior results (thereby suggesting that their original measurements may have been erroneous). Such reflection may help students in building expertise for assessing future experimental problems in the context of electronics; indeed, it has been reported that expert troubleshooters often use examples based on experience when making a diagnosis [6].

Six of the seven groups who successfully repaired the circuit justified their decision by synthesizing information from both their most recent measurements and measurements performed throughout the interview. Group D, which did not repair the circuit, spent the last quarter of the interview alternating between predictions and measurements surrounding the second op-amp, but did not integrate the evidence they collected to conclude that the chip was faulty. All groups that successfully replaced the second op-amp considered, yet subsequently rejected, problems occurring elsewhere in the circuit. This reflective synthesis of experimental results was the critical element needed in order to decide to proceed with the final repair; without this metacognitive intervention, students could potentially continue to make new measurements (*i.e.*, remain in the testing phase of troubleshooting) indefinitely while searching for a single measurement that would be sufficient to localize the fault to a single component.

8.4.1.5 Summary and Episode Discussion

In the process of answering our first research question, we found instances in which students' metacognition supported their troubleshooting practices throughout episodes from all four categories. This metacognition primarily manifested itself in building hypotheses (such as in group G's *split-half* excerpt) or collaboratively constructing understanding of an idea (such as in group F's *discrepant output* excerpt). Another less prevalent (but important) manner in which metacognition regulated student thinking was in refuting a partner's claims by demonstrating that they would lead to a contradiction, as in group C's *discrepant output* episode.

Across the episodes analyzed, we found that there were numerous occurrences of students engaging in socially mediated metacognition. The documented instances associated with metacognition tended to correspond to substantive contributions to the task of troubleshooting. Overall, the metacognitive practices in which students engaged while making decisions during episodes from these four categories primarily focused on the immediate task, such as jointly forming a new prediction to test or deciding upon an approach. Only occasionally did students make reflective assessments of strategy (such as in group C's *replacement decision* excerpt).

8.4.2 Clusters in Socially Mediated Metacognition

In this section, we codify and examine metacognitive exchanges that occur between students as they discuss one another's ideas in order to address our second research question. Motivation for this analysis stems from key findings from the literature on metacognition in mathematics and physics, discussed in Sec. 8.3.3. In particular, engagement around metacognitive statements has been shown to be an essential difference between groups that were successful and those that were not when completing a problem-solving task [37]. Thus, we aim to present a detailed analysis of the back-and-forth metacognitive exchanges (or clusters) in the transcribed episodes in order to provide greater insight into how such exchanges may support students while troubleshooting.

For all groups and episodes, we analyzed the transcripts and corresponding SMM codes in order to identify nodes and clusters within students' dialogues. We found that in the excerpts analyzed, it was useful to organize clusters into two separate categories: discussions in which students attempted to *clarify* their understanding of the circuit or discussions about (or leading to) a *suggested approach.* The percentages of episodes in each category that contained clusters of either kind are summarized in Table 8.2. It can be seen that clusters about clarification occurred throughout the interview (but not typically during the *initial strategizing* episode), whereas clusters about approaches were present only in the early stages (*i.e.*, during *initial strategizing* and *discrepant output* episodes). We present examples of clusters from both categories (*clarification* or *approach*) and characterize the nature of the dialogue students are employing in these excerpts.

		Episode Category			
Cluster code	Topic	IS (N=8)	DO (N=8)	SH (N=5)	RD (N=7)
Node	Any	88%	100%	100%	100%
Cluster	Any	38%	100%	80%	43%
	Clarification	13%	50%	80%	43%
	Approach	38%	63%	0%	0%

Table 8.2. Node and cluster coding results. Shown are the percentages of groups engaging in dialogue that had one or more nodes, and/or clusters, broken down by episode and conversational topic.

8.4.2.1 Clusters About Clarification

Eleven of the 19 identified clusters fell into the category of clarification. These incidents primarily occurred when one student was unsure of what claims were being made by a partner, or when both students were working together to better understand an aspect of the circuit. In order to demonstrate how clustering provides new insights into students' thought processes, we re-analyze an excerpt from Group G's *split-half* episode. This excerpt, previously discussed in Section 8.4.1.3, takes place after the students have localized the fault to the second stage. The excerpt begins as the students discussed their expectations for how the second op-amp should behave.

8.4.2.1.1 Building predictions

1 G1: *So we can isolate this part.*
2 *G2: So then this op-amp, so then, ahh let's see.*
3 *This right here [the inverting input] should be ground.*
4 *G1: Yeah, yeah, this is virtual ground—*
5 *G2: Virtual ground.*
6 A *G1: —right here. No current's going through here [the inverting input].*
7 *G2: Yeah.*
8 B *G1: So, from there we can say current through here [resistor 3] is*
9 *equal to current through there [resistor 4].*
10 *G2: So this resistor right here, the R_3, that should have a drop of 10*
11 *volts then. Because you have ground right here [at the inverting input].*
12 *G1: Yeah, yeah, you're right, because this [the inverting input] is zero*
13 C *volts, this [stage 1's output] is 10 volts, so we should be losing—*
14 *G2: 10 volts across there.*
15 *G1: —10 volts across that resistor. Okay so, I'll look at, we should be*
16 *losing 10 volts across here.*

When discussed previously, we noted that students were engaged in the process of jointly building a hypothesis. From the clustering analysis, it becomes clear how the students are modifying their ideas based on feedback from one another. This cluster covers three nodes in total (here labeled A-D), beginning with G2's prediction in line 3.

G1 refined his partner's prediction by incorporating the idea that the inverting input would serve as a virtual ground, in accordance with the first op-amp golden rule. G1 continued by implicitly using the other golden rule to state that no current enters the op-amp's input, and then predicted that the currents through R_3 and R_4 should be equal (3-9: A). Next, on the basis of G1's prediction, G2 was able to subsequently justify that the voltage across R_3 should be 10 volts (4-11: B). G1 agreed and added further justification by noting the voltage may be attributed to the difference between the output of the first stage and the inverting input's virtual ground (10-16: C).

In this cluster, the students began with their theoretical knowledge of ideal op-amp behavior and collaboratively built a prediction for the voltage across resistor R_3 in the second circuit. By deciding to compare the measured voltage across R_3 with their collaborative prediction, Group G was well positioned to either reveal a flaw in one of their underlying assumptions or further localize the error within the second stage. Either outcome could advance the task of troubleshooting the circuit. While the excerpt was already metacognitively rich, the clustering analysis enables us to characterize in greater depth why this was the case. As clusters are formed by the overlap of multiple nodes, each of which in turn indicates an occurrence of students engaging in one another's ideas, they represent instances where students are working to build a consensus on a single topic while working together. Such interplay between the two students' ideas may also be described in terms of establishing a collaborative zone of proximal development, as was highlighted in the original work on socially mediated metacognition [121].

Another example is from the beginning of the *replacement decision* episode for group A. In this brief excerpt, the students were interpreting a measurement of the negative power rail. First, we present an overview of the transcript as analyzed by the SMM framework, and then the episode is discussed in terms of the clusters that arose from the dialogue.

8.4.2.1.2 Interpreting measurements

1 *A1: And that's at 15 and a half.*
2 A *A2: That's at plus 15 and a half? Oh, did you measure it backwards?*
3 B *A1: Yeah.*
4 *A2: Did you have the leads flipped?*
5 *A1: Yeah yeah yeah, that's fine.*

At the beginning of this excerpt, A1 measured the negative rail voltage with the multimeter (1: *Idea* - *Observation*). A2 questioned the positive result, asking his partner if he had measured the voltage "backwards" (2: *Other-Monitoring* - *Ideas, Other-Monitoring* - *Actions*). A2 then clarified what he meant by explicitly asking if the leads were flipped, which would have explained the difference from what was expected (4: *Self-Disclosure*). A1 affirmed that he did, in fact, switch the leads, and indicated that the measurement was okay (5: *Assessment* - *Result*). After this excerpt, A2 no longer questioned that measurement and instead focused on verifying that the power rails were correctly connected to the op-amp, indicating that he had accepted the previously discrepant result as reasonable after being provided with justification.

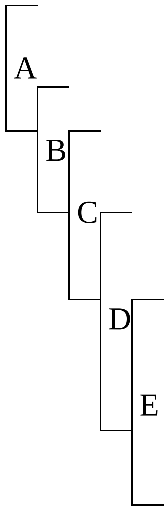
This cluster consists of A2's request for information following his partner's measurement (1-4: A) and A1's affirmation that his partner's interpretation was correct (2-5: B). The exchange served to clarify that A1's measurement was not the result of an actual flaw in the circuit, but rather stemmed from a somewhat incorrect measurement procedure. As such, it can be interpreted as a sensible result if an underlying assumption related to the measurement (in this case, the polarity of the leads) is modified to better reflect the procedure used.

This excerpt foregrounds one role that socially mediated metacognition may play while students are making comparisons between observations and their expected outcomes. Through this exchange, A2 gathered the information he needed to be able to properly interpret the data his partner had collected. Without the supplemental information from his partner, one reasonable response to A1's measurement would have been to modify the circuit by changing the connections to the power supply. Instead, the knowledge that A1 had reversed the leads factored into A2's interpretation of the measurement result, allowing him to decide that no modifications (or further tests) were required. In essence, this cluster illustrates how students may clarify their measurements in the process of experimental data interpretation, which may later inform the outcome of comparisons made while troubleshooting.

8.4.2.2 Clusters About Suggested Approaches

The other eight of the 19 clusters we identified involved discussions that either centered on modifying a previously suggested approach or led to a new suggestion for an approach. We present a single cluster from the *discrepant output* episode from group E in detail, and generalize findings from the remaining examples. In this excerpt, the students have just observed that the output of the circuit is a constant, dc voltage, and are in the process of deciding how to proceed in repairing the circuit.

8.4.2.2.1 Deciding to investigate



1 *E2: Do we even check if these are the right chips?*
2 *That would be kind of stupid.*
3 *E1: It would probably be a good call.*
4 *E2: Okay, I guess we do have-*
5 *Can we just like pull that chip out and replace it?*
6 *E1: Yeah, I mean it will be like the brute force method of making sure*
7 *it's the right chip. Pull it out and put the right one in.*
8 *E2: What we could do is get out a probe and we can just go through*
9 *the first one and measure* V_{out},
10 *and we could see if that's what we expect it to be.*
11 *E1: Yeah, for sure and then we'll measure all the power*
12 *to make sure it's doing what it should be doing.*

At the beginning of this excerpt, E2 wondered if the type of op-amp chip itself could have been the source of unexpected output, but doubted his own assessment (1-2: *Suggesting Approach, Feedback Request*). E1 agreed that checking the chips could be productive (3: *Assessing Strategy, Other-Monitoring*). E2 then elaborated by suggesting outright replacement of the chips (4-5: *Suggesting Approach, Feedback Request*), which E1 called a "brute force" approach (6: *Assessing Strategy*) and clarified what that meant (7: *Self-Disclosure*). In response, E2 suggested a different approach of measuring voltages (8-10: *Suggesting Approach*), which E1 extended by suggesting measuring the voltages powering the chip as well (11-12: *Suggesting Approach, Other-Monitoring*).

This cluster consists of five nodes in total, with the first beginning as E1 monitored E2's idea about checking the chips (1-3: A). E2 responded with an idea that they could replace the chip (3-5: B). E1 commented on E2's strategy being a brute-force approach (4-7: C). E2 decided instead to suggest an alternate approach of investigating the output of the first op-amp (6-10: D). The cluster ends after E1 agreed with the idea and furthermore built on it by suggesting that they measure the power connections as well (l8-12: E).

In this excerpt, E2 proposed three different approaches: checking if chips were correct in line 1, replacing the chips in line 5, and checking the output in line 9. In each case, his partner provided feedback on the approach. While we do know that E2 made a new suggestion following his partner's feedback in lines 6 and 7, E2 does not make his reasoning for doing so explicit. We note that, in the context of physics, a "brute force" approach is typically seen as an inelegant (and thus undesirable) method of solving problems; thus it is plausible that E2 perceived his partner's statement as negative feedback, but we cannot be certain based on our data corpus. Nonetheless, E1's assessments (directly or indirectly) appear to have prompted E2 to propose an alternative approach.

The previous excerpt demonstrates how a student may modify his or her approach in response to feedback from a partner. In this case, E2 began with the non-specific suggestion of checking the chips, then suggested chip replacement specifically, and finally proposed the less invasive method of checking specific voltages to learn more about the circuit. In the context of the information group E possessed at this point in time, investigating the first op-amp was guaranteed to yield information about the circuit, whereas replacing the second op-amp could have potentially been pointless depending on the location of the fault. In the remaining clusters about approaches, students similarly incorporated feedback from their partners as they refined their proposed approaches.

The process of selecting productive approaches is critical for the success and effectiveness of students' troubleshooting activities, as different approaches or strategies have the potential to yield information that is more or less useful in narrowing the scope of the problem. In the examples presented above, we demonstrate that the clustering analysis captures how socially mediated metacognition may enhance the decision-making process within a group as students work to establish a consensus on the best course of action.

8.4.2.3 Summary of Cluster Analysis

The cluster analysis effectively captured instances in which students engaged in one another's ideas by design. Furthermore, we observed that such back-and-forth exchanges facilitated troubleshooting in specific ways. In particular, we found that these discussions tended to occur in the process of selecting approaches and in response to the introduction of insufficiently substantiated ideas or incomplete analyses. In all cases, groups recognized that greater clarity was needed in order for the students to decide how to proceed in investigating the circuit. This realization that more information was needed prompted groups to revisit their own reasoning, and thus resulted in metacognitive regulation. By explicitly attending to clusters of metacognitive codes in student dialogues, we were therefore better able to generalize how exactly students were being metacognitive while they were making decisions, thus addressing our second research question.

8.5 Summary and Limitations

Our analysis demonstrates that students not only engaged in socially mediated metacognition to varying degrees while troubleshooting, but that extended metacognitive discussions (*i.e.*, clusters) often helped students to better support their predictions. Specifically, metacognition was a key factor in creating hypotheses, eliminating erroneous or ill-defined proposals, and making strategic decisions about further measurements to be employed throughout the troubleshooting process.

To focus the investigation on relevant discussions, key decision-making events were selected for detailed examination; our data corpus consisted of 27 episodes drawn from four different categories: *initial strategizing*, *discrepant output*, *split-half*, and *replacement decision*. Each of these categories of episodes represented a time during which students were likely to make or change plans for how to continue with their investigations, and the majority of interviews contained one episode in each category. We examined the episodes in each category to gain insight into how students' socially mediated metacognition was coupled to various troubleshooting behaviors. As noted before, we found that metacognition was a critical mechanism in the construction of hypotheses as groups considered the relevance of new information. In addition, socially mediated metacognitive exchanges served to regulate the adoption of proposed ideas as students either elaborated upon their predictions or rejected ideas with insufficient explanatory power.

We used a clustering analysis to highlight how students were engaging with one another's ideas in their discourse. This occurred most frequently during the *discrepant output* episodes, in which every group engaged in at least one such exchange, and the *split-half* episodes, in which four out of five groups exhibited a cluster of dialogue. Our cluster analysis provided insight into how students collaboratively decided on a course of action while troubleshooting by capturing the process of how students reach a consensus. This typically occurred as students built predictions collaboratively or as one individual further inquired into insufficiently substantiated predictions or analyses made by a partner. This clustering analysis effectively highlights the negotiations that occur when students jointly undertake an activity such as troubleshooting.

Throughout the current analysis, we have noted numerous occasions in which events described via the socially mediated metacognition framework informed key decision-making processes. For example, the cluster analysis investigated instances in which students were engaging with one another's thinking in a more extended manner and frequently resulted in students making revisions to their mental models as they explored the limitations of their ideas. As another example, students were engaged in numerous predictions and comparisons while they were employing a split-half strategy, ultimately deciding to focus their attention on the malfunctioning second stage. In our analysis of episodes occurring immediately afterwards, the socially mediated metacognition framework was used to describe the process of formulating testable hypotheses. These hypotheses then informed both the students' choice of measurements and the subsequent interpretation of measured results.

As described above, the findings from the current investigation clearly indicate that metacognition is important in troubleshooting endeavors, and that a detailed analysis of students' socially mediated metacognition can provide considerable insight into the decision-making processes that occur. While this work represents an important first step to better understand these processes, it is important to note that our findings may in fact be constrained by the narrow scope of the content and by the limited expertise of the participants. As such, there are several ways in which our work could be productively extended. Systematic investigations of troubleshooting in other physics content areas (*e.g.*, upper-division laboratory courses on optics or modern physics) would serve to verify that our findings are generalizable beyond the context of electronics, thereby addressing the first limitation of our study. In order to address the second limitation, investigations of collaborative troubleshooting with more experienced individuals (*e.g.*, college seniors who are completing or who have recently completed a degree specializing in electronics) could provide insight into the prevalence and frequency of collaborative strategies employed by (presumably) more skilled troubleshooters.

8.6 Conclusions

We developed a troubleshooting activity in which pairs of students were asked to repair a malfunctioning circuit. The task was designed such that two intentional faults were introduced into the same functional stage of the circuit, ensuring that a systematic troubleshooting approach would be beneficial. Audiovisual data were collected from eight pairs of students at two separate institutions, and episodes in which students were making strategic decisions were thoroughly analyzed using the framework of socially mediated metacognition. The clustering of metacognitive codes captured back-and-forth exchanges between students. Both the SMM framework and the clustering analysis were used to provide a description of how decision-making processes occurred while troubleshooting.

We demonstrated that socially mediated metacognition is a productive framework for investigating students' interactions and decision-making during troubleshooting in electronics, which is far removed from the context in which it was originally developed (high school mathematics [121]). All of the sub-codes from the framework were necessary for characterizing various actions undertaken by students, however variation in usage existed, both between groups and between episode categories. We observed that students were primarily engaging in socially mediated metacognition in ways that served to regulate their thinking about the task at hand, rather than in ways that helped to monitor their group's ongoing progress toward the goal of repairing the circuit. Clusters of dialogue highlighting students' engagement with one another's ideas were found to be a key element in such metacognitive regulation. This form of regulation was observed to help students eliminate measurements that would have been uninformative as groups worked to either reject unsubstantiated tests or jointly synthesize properly justified hypotheses. Using specific predictions grounded in relevant theory to inform experimental testing is a key component of effective troubleshooting, as students cannot make informed decisions about a circuit's functionality if the expected behavior is unknown. We anticipate that future work will draw upon the SMM framework as well as the experimental modeling framework (which has also been used to analyze this dataset [51]), both separately and in combination, to further investigate troubleshooting across varying contexts. Ultimately, findings from this ongoing research will be used to inform the development of instructional interventions for improving students' troubleshooting skills.

Chapter 9

CONCLUSIONS

This dissertation has documented an in-depth, multi-year investigation of student learning of analog electronics, in both physics and engineering courses. The majority of the investigation focused on student conceptual understanding of common classes of circuits (*e.g.,* voltage dividers, diode circuits, op-amp circuit, and transistor circuits) covered in electronics courses. A specific difficulties framework [77] informed the design of this broad investigation, which had the overarching goal of providing sufficient insight into student thinking to guide the development of targeted, research-based instructional materials on the topics investigated. Furthermore, an investigation on troubleshooting –an often unarticulated skill-based learning goal of laboratory-based electronics courses– was detailed in Chapter 8. This chapter complemented the previous work by providing insight into student interactions that occurred while working with physical implementations of circuits.

In order to probe student thinking about specific circuits, research tasks consisting of qualitative, free-response questions were administered over several years to students in a number of different courses covering a range of topics in circuits and electronics. In many of these tasks, slightly modified versions of canonical circuits were used in order to help ensure that students would need to reason from fundamental principles (rather than responding based on memorized topologies) in order to arrive at a correct response. Written data were analyzed using a grounded theory approach in order to categorize student responses in the absence of an a priori coding scheme [53,81]. Common lines of reasoning that emerged from student responses were then generalized and connected to findings from prior research whenever possible. This approach allowed for the identification of patterns in student thinking, both productive and unproductive, as well as the identification of prevalent conceptual difficulties. The most prevalent difficulties were highlighted for each task in order to inform both future research and instruction. While the actual development of instructional materials based upon these findings was not a core component of this work, some materials were designed and piloted over the course of the investigation. In particular, one of short tutorial on op-amp circuits piloted in both physics and engineering courses is briefly discussed.

Physics education research has a history of providing significant insights into student thinking, particularly for foundational topics in introductory physics. The work documented in this dissertation has served to extend such efforts into upper-division courses on circuits and electronics offered in physics and engineering programs. As discussed earlier in the dissertation, relatively little previous research has been conducted on student understanding of topics in upper-division electronics. As such, this project serves to advance the research base on the learning and teaching of electronics in both physics and engineering.

9.1 Overview of Findings from Investigation of Student Understanding of Analog Electronics Across Physics and Engineering Courses

Across all topics investigated, it was found that after all relevant instruction a significant percentage of students were unable to provide correct responses to the research tasks. Indeed, the percentage of correct responses on the tasks documented in this dissertation ranged anywhere from 7% to 80%, thereby suggesting that students had not developed a sufficiently robust conceptual understanding of many of these classes of circuits after instruction on analog electronics in both physics and engineering departments. Nevertheless, many students completed their coursework and received a passing grade. This indicates a gap between student's academic achievements and conceptual understanding, similar to what has been observed in research on introductory physics courses [3,134]. Thus, a similar process of curriculum development could be implemented to benefit students learning electronics. Below, brief overviews of major findings on each electronics topic investigated are presented.

Voltage division and loading. In the investigation described in Chapter 4, students in five different courses at the University of Maine were asked to compare the impact of adding the same resistive load to two different voltage dividers characterized by different component resistors but the same ratio of resistances. Many students struggled to arrive at a correct response, with percentages of correct answers supported by correct reasoning ranging from 5% (after instruction in an introductory physics II course) to 65% (at the end of a junior-level engineering electronics course). Three common incorrect lines of reasoning were documented, each leading to one of the three possible comparisons. These lines of reasoning all stemmed from students using reasoning based solely on specific local comparisons involving a limited subset of the three resistive elements. On this task, local reasoning was the primary factor contributing to both incorrect answers and incorrect reasoning leading to a correct answer. Similar tendencies have been reported in the literature on introductory circuits [19].

Diode circuits. In the investigation documented in Chapter 5, two complementary tasks were used to probe student understanding of diode circuits in junior-level electronics courses in physics and engineering. Students struggled on both tasks, with only approximately one-quarter providing correct answers with correct reasoning for all parts of the reverse-biased diode task, and with only 10% providing fully correct responses to the three-diode network task. In both tasks, students exhibited a tendency to treat reverse-biased diodes as ohmic, assuming that a lack of current implies that there is no voltage across it. This overgeneralization of ohmic behavior, sometimes referred to as current-based reasoning, has been reported elsewhere in the literature in other contexts. A detailed examination of student responses to the three-diode network task revealed that many students drew conclusions about both voltages and currents in the circuit on the basis of independent analyses of individual loops in the circuit. Thus, students gave contradictory responses across question parts, and such findings also suggest that students were failing to check for consistency across their different analyses.

Operational amplifier circuits. Chapter 6 reported a subset of findings from a larger, multi-institutional investigation of student understanding of op-amp circuits across the University of Maine, the University of Washington, and the University of Athens [6]. While that investigation primarily focused on electronics courses offered in physics departments, the work reported in this dissertation extended the investigation to circuits and electronics courses in engineering. It was shown students in both the engineering circuits and engineering electronics course at the University of Maine encountered similar conceptual difficulties as students in the physics electronics course. It is also worth noting that the same difficulties were also identified in physics electronics courses at other institutions [6]. While students were generally able to recognize or derive the behavior of a standard inverting amplifier circuit (with approximately 80% of all students determining the magnitude of the output correctly), they struggled with portions of tasks that were less algorithmic, with only 12 out of all 290 students (4%) able to correctly rank, according to absolute value, the currents through various points in the circuit, including the rails.

Bipolar-junction transistor circuits. Chapter 7 describes an in-depth, multi-institutional investigation of student understanding of transistor circuits. On the three amplifier comparison task, only one-quarter of all students were able to correctly rank three different transistor circuits according to peak-to-peak output voltage for identical input voltages; only 60% of these students supported their responses with correct reasoning. The poor performance on this task was somewhat unexpected, as explicit instruction on these circuits (the common-emitter amplifier and the emitter follower) was included in all courses studied; formal derivations of the behavior of these circuits was covered in lecture, and the circuits themselves were subsequently constructed and tested in the laboratory, with a corresponding laboratory report required. Upon examining student responses, it was found that many students were not attempting to use or derive an appropriate gain expression for the circuits. Instead, students tended to reason about dc (bias) voltages in the circuit, frequently considering only the impact of a local modification from one circuit to the next. While students were more successful at some institutions than others, the same difficulties were prevalent in all courses observed.

In response to student difficulties with the three amplifier comparison task, a series of additional tasks were created (both by the author and the author's advisor). These additional free-response questions were designed to probe student understanding of more fundamental aspects of transistor behavior. Students typically performed better on these more focused tasks. However, even after instruction, over one quarter of students were unable to correctly rank the terminal currents through a forward-active bipolar-junction transistor. Nevertheless, the additional tasks demonstrated that many students could indeed reason productively about transistor circuits from basic principles. Furthermore, the results from the ac biasing network task supported the idea that students struggle to consider both dc and ac behavior when analyzing transistor circuits, and may tend to perform only dc analyses even when ac analyses are required.

Trends across multiple circuits contexts. In this investigation, four broad classes of circuits were examined, and each task administered let to the identification of one or more specific student difficulties. In some cases, difficulties were primarily related to student understanding of a particular circuit element (*e.g.*, recognizing diode biasing). In other cases, more fundamental reasoning and conceptual difficulties (*e.g.*, a tendency to reason locally about circuit modifications) transcended circuit contexts.

Across multiple tasks in this dissertation, students exhibited a tendency to make comparisons between only a subset of the components in the circuit; such local reasoning has been noted in previous research on circuits [20]. In general, such comparisons included implicit assumptions that were unfounded. For example, many students made comparisons in the op-amp amplifier comparison task that assumed (for circuit C, which added a resistor between the op-amp and the circuit output) that the op-amp's output would be constant, and thus adding a resistor to the feedback loop would result in a decreased value of V_{out} for the circuit.

Among all of the conceptual difficulties exhibited by students, most contained elements of productive reasoning relevant to the circuits they were examining. For example, on the basic loading task (shown in Fig. 4.2), all of the common lines of incorrect reasoning included productive ideas about circuits (*e.g.,* students considering upper resistors recognized that, for the same current, a larger resistor implies a greater voltage difference).

Across many of the circuits contexts, there was evidence suggesting that students were struggling to interpret the more advanced diagrammatical representations used. For example, students gave responses consisted with the ideas that V_{in} represent a current input as opposed to a voltage input, and that the path of current is always from V_{in} to V_{out}. Such findings suggest that more targeted and systematic investigations of circuit representations are needed.

Perhaps most importantly, this work has demonstrated that students struggle with fundamental aspects of electronics in ways that cross disciplines. In particular, the difficulties observed in both sophomore- and junior-level engineering courses were relatively similar to those identified in the junior-level physics course. Such findings suggest that, at least for the topics investigated in this dissertation, differences in disciplinary approach or emphasis do not appear to significantly impact the nature of student understanding. The work documented in this dissertation supports the need for a single research base on the learning and teaching of analog electronics that may be leveraged by instructors and researchers in both disciplines.

9.2 Overview of Findings on the Role of Socially Mediated Metacognition in Student Troubleshooting on Analog Electronics

The research documented in Chapter 8 explored one important skill-based learning outcome of electronics instruction, namely troubleshooting a malfunctioning circuit. Specifically, interviews of pairs of students troubleshooting a malfunctioning op-amp circuit were analyzed using the framework of socially mediated metacognition in order to determine how, in the process of collaboratively repairing a circuit, students engaged in one another's ideas and how such engagement impacted the process of troubleshooting. It was observed that students did indeed engage in socially mediated metacognition while troubleshooting, and one particular finding should be reiterated. It was found not only that students did spontaneously engage in one another's ideas while troubleshooting, but that instances of such engagement often helped students to better justify their choice of action, either by building more sophisticated predictions or by rejecting insufficiently justified hypotheses. This result has numerous implications for future research and instructional improvement efforts. First, it suggests a particular way in which socially mediated metacognition may be relevant to the process of collaborative troubleshooting. In turn, this suggests that efforts to explicitly promote student engagement in such metacognitive discourse might result in more productive self-regulation while troubleshooting.

9.3 Implications for Instruction

In general, the work documented in this dissertation revealed that students struggled with many tasks after all instruction, including those involving small modifications of canonical circuits. This suggests that even after instruction on both electric circuits and analog electronics, students may not have developed a coherent conceptual model of circuit behavior. Similar inferences have been drawn about introductory physics students' model of resistive dc circuits [20]. Thus, there is considerable evidence of a gap between instructor learning goals for courses and the level of student conceptual understanding demonstrated by the end of such courses across years of instruction. Based on the findings of this investigation, several recommendations for instruction may be made.

Utilize variations of canonical circuits as instructional tools. In multiple tasks, the circuits used varied only slightly from canonical circuits, yet students struggled to reason correctly about their behavior. For example, the circuit in the reverse-biased diode task was effectively a half-wave rectifier with an additional resistor between the diode and ground; nevertheless, students struggled to analyze even the circuit's dc behavior. This suggests that explicitly introducing such modified circuits during instruction could serve to direct students away from memorized responses in favor of a first-principles approach.

Emphasize the role of consistency checking strategies. Many of the common incorrect responses given by students were not consistent with either Kirchhoff's voltage law or Kirchhoff's current law. Thus, in such instances, verifying that a response was or was not consistent with fundamental circuit behavior would serve as a quick method of identifying many incorrect predictions. However, it should be noted that such strategies may fail if students do not fully understand the properties of a particular electronic device, as was observed in the investigation of operational amplifier circuits.

Examine circuit behavior under both ac and dc conditions, regardless of typical circuit applications. For many circuits, more emphasis is placed on either the ac or dc behavior when they are introduced in the classroom, depending on its common applications. For example, while op-amp circuits are typically examined under both dc and ac conditions, the emitter follower and the common-emitter amplifier are primarily analyzed under ac conditions. As a result, students may incorrectly assume that since the functionality of op-amp circuits is the same under dc and ac conditions, this should also be true for transistor circuits, which is not the case. Moreover, as observed on the three transistor amplifier comparison task, students may not recognize which behavior (ac or dc) must be considered to determine the impact of a change in the circuit on the output voltage signal. Thus, explicit discussion of the simultaneous dc and ac behavior and associated limitations of each may be useful for helping students develop more coherent models of device behavior.

Explicitly compare categories of circuits with similar functionality but constructed from different devices. As discussed above, from responses to the transistor follower graphing task, some students seemed to be overgeneralizing the idea of a follower without considering the specific implementation (*e.g.,* the differences between an op-amp follower and a transistor follower). It should also be noted that student understanding of two different implementations of inverting amplifiers, one op-amp circuit and one transistor circuit, was examined as part of the work documented in this dissertation. Although the specific questions and prompts from the two tasks were not analogous, the kinds of reasoning used were markedly different. As voltage amplification may be achieved by a number of different means (including, for example, transformers), explicit comparison of the affordances and limitations of different amplifiers may help students build a more coherent understanding of how to choose optimal devices and circuit implementations for specific applications.

Support the development of troubleshooting skills by engaging with student ideas. As noted in the discussion of student troubleshooting, instances where students were reciprocally engaging with one another's ideas were associated with the formation of testable hypotheses. This suggests that instructors could support students in developing troubleshooting skills by engaging with the student hypotheses as research colleagues or peers, rather than as electronics experts, in order to either resolve them into testable measurements, or to help students decide if their idea has sufficient explanatory power to be useful in uncovering the circuit's fault. While such a strategy may be somewhat less efficient in helping students resolve the issue in the moment (than, for example, offering students a recommendation based on prior expertise), it is possible that it may ultimately support the development of more effective troubleshooting approaches.

9.4 Recommendations for future work

There are several ways to build upon or extend the findings from this dissertation in future research on student understanding of electronics.

Investigate student interpretation of circuits across representations. As noted in tasks involving both diode and op-amp circuits, many students indicated that there would be a current from the circuit's input to the output. This difficulty stemmed not from the particular devices, but rather the circuit representations used. This suggests that students may be less fluent in interpreting the diagrams used in upper-division courses as might otherwise be assumed. A detailed investigation into how students interpret such diagrams as well as their ability to relate them to the closed-loop representations used in introductory courses could provide valuable insight into student models of circuit behavior.

Explore student understanding of combinations of functional circuit groups. The circuits discussed in this dissertation involved six components at most, and typically were variants of a single-purpose configuration (*e.g.*, an inverting amplifier). However, in practice circuit networks consist of multiple, identifiable stages which each perform a particular function. A single, simple example of such a network was used in the BJT three amplifier task; in practice students ignored the presence of the biasing network altogether. While there is research indicating that experts may be more adept at recognizing functional circuit groups [135], it is not known what assumptions students make when reasoning about the interactions between portions of more complex circuit networks.

9.5 Summary

From this work, it was shown that many students struggle to correctly reason about electric circuits even after all instruction in a range of courses in circuits or electronics. For all tasks, the specific difficulties identified were observed almost universally among different courses, albeit with varying prevalence. Furthermore, many of the responses given by students contained elements of productive reasoning about circuits, although students frequently overgeneralized behavior or limited the scope of their analysis to local elements. Altogether, this research suggests that many students may not have a coherent understanding of fundamental circuit behavior as it pertains to analog electronics after instruction.

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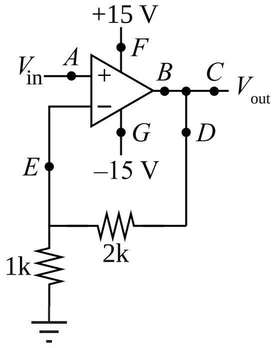
APPENDIX A – THE OPERATIONAL AMPLIFIER CURRENTS TUTORIAL

I. Deriving the output voltage of a non-inverting amplifier circuit



Consider the op-amp circuit at right. Assume that there is no load connected to the output of the circuit and that the op-amp is ideal. $V_{in} = +2$ V.

A. What, if anything, can you say about the current through and the voltage at point *A?* Briefly explain.

B. What is the voltage at point *E*?

What rule or idea are you using to answer and under what conditions does it apply?

C. Is there current through the 1-kΩ resistor? If so, determine its value and specify its direction. If not, why not? In either case, explain.

D. What is the current through point *E*? What rule or idea are you using to answer and under what conditions does it apply?

E. Is there current through the 2-kΩ resistor? If so, determine its value and specify its direction. If not, why not? In either case, explain.

F. What is V_{out}? Briefly explain.

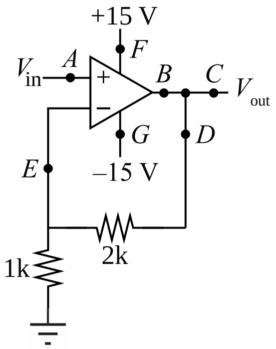
★ Stop here for a brief class discussion.

II. Investigating the currents in a non-inverting amplifier circuit

A. Is the current through point *B to the right, to the left,* or *equal to zero?* Explain.

B. How, if at all, do your predictions for the currents at points *A, B,* and *E* satisfy Kirchhoff's junction rule when applied to the operational amplifier?

C. Is the current through point *C to the right, to the left,* or *equal to zero?* Explain. (*Hint:* Is there a viable path for current?)



D. Is the absolute value of the current through point *B greater than, less than,* or *equal to* that through point *D?* Explain how you can tell.

E. Consider the following statement made by a student:

> "Because no current comes into the op-amp at points A and E, Kirchhoff's junction rule tells me that there can be no current at point B."

Do you agree or disagree with the student? Is there anything that the student is failing to consider in his or her analysis? Explain.

III. Applying Kirchhoff's junction rule to the operational amplifier

A. Experimentally, it can be shown that there are always currents through points *F* and *G.* What can you infer about the directions of those currents based on the fact that +15 V and –15 V are the highest and lowest voltages in the circuit, respectively? Explain.

B. Using Kirchhoff's junction rule as well as your response to question A, answer the following questions:

1. Is the absolute value of the current through point *F greater than, less than,* or *equal to* that through point *B?* Explain.

2. Is the absolute value of the current through point *F greater than, less than,* or *equal to* that through point *G?* Explain.

C. Now suppose $V_{in} = 0$ V. Is the absolute value of the current through point *F greater than, less than,* or *equal to* that through point *G?* Explain.

D. Now suppose $V_{in} = -2$ V. Is the absolute value of the current through point *F greater than, less than,* or *equal to* that through point *G?* Explain.

★ Stop here for a brief class discussion of and demonstration about the relationship between the rail currents and the current through the op-amp output.

APPENDIX B – INITIAL PROMPT FOR TROUBLESHOOTING INTERVIEWS

For this activity, you will be repairing a malfunctioning circuit. Specifically, you'll be working with an inverting cascade amplifier, described on this page here [Fig. 5]. For context, let's imagine that some of your peers built this circuit as part of class. They built the circuit using the same chip you've been using in class this semester. Here's the standard data sheet for that chip. Your tasks are to diagnose any issues and make the circuit work properly.

This interview is very similar to what you've been doing in class. You'll have access to much of the equipment from class, including power supplies, measurement tools, and a limited selection of electrical components. One difference from class is that you're working with a circuit someone else built. Another difference is that I'm interested in what you say to yourself as you perform this task, so I will ask you to talk aloud as you work on the circuit.

What I mean by talk aloud is that I want you to say out loud everything that comes into your mind while doing the task. Put another way, I want you say out loud what you might otherwise say to yourself silently. Of course, you should also feel free to ask each other questions and interact as you would when working together in [the electronics course]. But the more you both say out loud what you're thinking in your head, the more helpful it will be.

Act as if I am not in the room. Just keep talking. If you are silent for any length of time, I will remind you to keep talking aloud.

BIOGRAPHY OF THE AUTHOR

Kevin Van De Bogart was born and raised in Middleton, Idaho. He graduated from Middleton High School in 2004, and continued his education at the University of Idaho. In May 2008, he graduated with a Bachelor of Science in Physics as well as a Minor in Mathematics. He then began graduate studies specializing in the field of quantum computing at the University of Calgary. It was here that he was first introduced to the field of Physics Education Research, and he subsequently began his Ph.D. work at the University of Maine in 2011. Kevin is a member of the Association of Physics Teachers. He has given multiple presentations at national conferences related to his work, and has authored or co-authored multiple papers in this time [36,51,112,113,136,137].

Before coming to UMaine, Kevin had essentially no experience with designing or analyzing electronic devices. However, as a result of his research, he has become an avid hobbyist, and now will attempt to repair, modify, or just disassemble nearly anything with internal circuitry. Kevin has been deeply involved in the teaching of the physics electronics course for the past four years, culminating with him taking a role as the course's primary instructor in the fall of 2016. Kevin strives to make the world a stranger place, sometimes to the chagrin of his friends, family, and coworkers. Kevin L. Van De Bogart is a candidate for the Doctor of Philosophy degree in Physics from the University of Maine in May 2017.