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# A Drag Estimate for Concept-Stage Ship Design Optimization

Douglas Read

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**A DRAG ESTIMATE FOR CONCEPT-STAGE  
SHIP DESIGN OPTIMIZATION**

By

Douglas Read

B.S. Webb Institute, 1997

S.M. Massachusetts Institute of Technology, 2001

A THESIS

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

(Interdisciplinary in Ocean Engineering)

The Graduate School

The University of Maine

August, 2009

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## DISSERTATION ACCEPTANCE STATEMENT

On behalf of the Graduate Committee for Douglas Read, I affirm that this manuscript is the final and accepted dissertation. Signatures of all committee members are on file with the Graduate School at the University of Maine, 42 Stodder Hall, Orono Maine.

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# A DRAG ESTIMATE FOR CONCEPT-STAGE SHIP DESIGN OPTIMIZATION

By Douglas Read

Thesis Advisor: Dr. Michael Peterson

An Abstract of the Thesis Presented  
in Partial Fulfillment of the Requirements for the  
Degree of Doctor of Philosophy  
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August, 2009

During the initial phases of ship design, the naval architect would like to have as much information as possible about the design space. This information not only helps determine a good set of initial characteristics, it allows for informed design changes when reacting to evolving requirements. One of the most difficult performance measures to evaluate is the ship wave drag. This estimate is important in an optimization, because wetted surface and wave drag must be balanced.

Multi-parameter optimization algorithms exist, but need a very fast and inexpensive fitness evaluation for them to be effective. Even though linear theory does capture some of the physics of the problem, it has long been out of favor due to its tendency to grossly over-estimate the wave drag. The other options available are parametric drag estimates and state-of-the-art boundary element codes. Here we present an intermediate method that makes a parametric correction to the linear theory using an artificial neural network.

The method starts with a training set consisting of a large number of panel code evaluations for a systematic hull series, and then uses two approaches to the parametric correction. The first method uses the ratio of linear theory to panel code data as targets for an artificial neural network with parametric inputs. In the second method,

we re-derive the linear theory with a new boundary condition, leading to a waterline integral term with unknown coefficients. The linear theory error is then used in a constrained minimization problem to solve for the unknown coefficients, which again provides targets for a neural network.

Coupled with a mathematical hull form that can approximate realistic hull shapes, the results show promise for an intermediate wave drag estimation method that is fast enough to be used as a fitness evaluation for a multi-parameter optimization routine such as a genetic algorithm.

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# 1. Introduction

## 1.1 Background

Ship wave drag has traditionally been one of the most difficult hull characteristics to predict, especially early in the design process when decisions about global hull parameters are being made. Model test data are typically not available at this point, and if the hull varies significantly from systematic series tests the confidence in the drag estimate can be low. Historically, the only other tools available to predict the wave drag have been parametric methods developed from regression analysis of the systematic experiments and a linearized potential flow solution. In the past 15 to 20 years, advances in numerical methods have made wave resistance much more accessible before the model test. Iterative boundary element codes, in particular, have become effective design tools.

One of the main goals of understanding the wave drag at the concept design stage is to balance the proportions of wave and frictional components of the total drag. The relative contribution of each must be estimated correctly in order to select length, beam, and draft ratios and to pick hull shape coefficients that result in a design with balanced wave drag behavior and minimal wetted surface. The complex relationship between the hull and various drag components leads to a multi-parameter optimization problem.

In the context of the optimization problem, each method has certain limitations. The parametric methods do not use the exact hull shape, the linear methods exaggerate the wave drag and are unable to correctly balance the friction for realistic ship forms, and the boundary element methods are expensive and take too long to set up and run for a very large optimization space. At the early design stage, one would like to have a method that combines the best features of these methods into a fast estimate of the total drag. As the design matures, detailed hull shaping should then be done using state-of-the-art potential flow and viscous flow analysis.

This work presents an intermediate method that combines the parametric and linearized analytical methods to estimate the wave drag quickly without exaggerating its magnitude.

## 1.2 Motivation

The difficulty with early stage optimization lies not only with the performance prediction. Another problem present in concept stage design is changes in the design requirements. Even if the naval architect can perform optimization early in the design process, changes to the design requirements may invalidate the results. We want to move the optimization to an earlier point, but we must, in that case, be able to adapt when the requirements change. If a revision to the requirements changes the length or displacement, for example, how should the other parameters change such that the drag components are still balanced?

The motivation behind this work is to find a fast wave drag estimate that will allow optimization to be moved to an earlier design stage, such that the design can be appropriately modified as design requirements change. This method will allow the naval architect to see the performance cost impact of extreme design pressures from

<b>Parameter</b>	<b>Symbol</b>	<b>Units</b>
Length of Waterline	L	m
Beam on Waterline	B	m
Draft	T	m
Waterplane Area	$A_{WP}$	$m^2$
Maximum Section Area	$A_X$	$m^2$
Wetted Surface Area	S	$m^2$
Displaced Volume	$\nabla$	$m^3$

**Table 1.1.** Basic hull parameters.

speed, payload, range, or topside shaping requirements. The method will use both the hull offsets and the fineness coefficients to estimate the wave drag. Such a method will be a global optimization, showing good combinations of hull length ratios and shape coefficients, while also providing guidance on section shape, waterline shape, and sectional area curve. Further local optimization, using more advanced methods, should be used later to fine tune the hull, but the goal is to find the best starting point possible for a given set of requirements.

### 1.3 Nomenclature

The following sections use certain definitions extensively. These definitions are divided into ship drag terms and ship hull shape terms below, based on the following basic parameters shown in Table 1.1.

### 1.3.1 Ship Drag Analysis

The two primary non-dimensional terms used to analyze ship drag are the Froude number and Reynolds number. The Froude number relates the gravitational forces to the inertial forces and is the essential non-dimensional speed value for wave drag.

$$Fr = \frac{U}{\sqrt{gL}} \quad (1.1)$$

Here  $U$  is the ship velocity,  $g$  is the acceleration due to gravity, and  $L$  is the length of the waterline, all in consistent units. The Reynolds number relates the ratio of inertial forces to viscous forces and is the important relation for determination of frictional drag and viscous flow characteristics.

$$Re = \frac{UL}{\nu} \quad (1.2)$$

Here  $\nu$  is the kinematic viscosity of the water. These two numbers are central to the classic ship model testing problem. Both numbers cannot be simultaneously matched at model scale, and matching Reynolds number is usually impractical due to the high speeds required. Froude number must be matched while assuring that the Reynolds number is above a critical value such that turbulent flow is achieved. A model-ship correlation line is then used to scale the Reynolds number dependent component of the drag.

Two such correlation lines are the Schoenherr line and the ITTC 1957 line.

$$\text{Schoenherr} : \frac{0.242}{\sqrt{C_F}} = \log_{10}(Re C_F) \quad (1.3)$$

$$\text{ITTC 1957} : C_F = \frac{0.075}{(\log_{10} Re - 2)^2} \quad (1.4)$$

The ITTC 1957 method is used in this work, but the Schoenherr line is given since it was used in the Taylor systematic hull series on which the experimental comparison is based.

The ITTC 1957 method, in its simplest form, then defines an equation for the components of the total ship drag.

$$C_T = C_W + (1 + k)C_F \quad (1.5)$$

Here  $k$  is a form factor representing the viscous pressure drag as some fraction of the frictional resistance coefficient. Subscripts define the drag components such that  $C_T$  is the total drag,  $C_W$  is the wave drag, and  $C_F$  is the frictional drag. This simple equation leaves out air drag and some other small drag components, but is suited for the level of detail necessary for the present estimate.

All hull drag coefficients are defined as

$$C_D = \frac{R}{\frac{1}{2}\rho S U^2}, \quad (1.6)$$

where  $R$  is the resistance due to a particular component,  $\rho$  is the water density, and  $S$  is the at-rest hull wetted surface.

Detailed information on ship drag and model testing can be found in [7] and [16].

### 1.3.2 Hull Form Coefficients and Ratios

The hull form coefficients, also known as fineness coefficients, are shown in table 1.2. The hull proportions are also described by the ratios shown in table 1.3. These parameters are used extensively to describe the data set in Chapter 4, and as inputs

Coefficient	Equation
Prismatic	$C_P = \frac{\nabla}{A_X L}$
Volumetric	$C_V = \frac{\nabla}{L^3}$
Maximum Section	$C_X = \frac{A_X}{BT}$
Block	$C_B = \frac{\nabla}{L B T}$
Vertical Prismatic	$C_{PV} = \frac{\nabla}{A_{WP} T}$
Waterplane	$C_{WP} = \frac{A_{WP}}{L B}$

**Table 1.2.** Definition of hull shape coefficients.

to the artificial neural networks in Chapter 5. Some notable relations include:

$$C_B = C_X C_P \quad (1.7)$$

$$C_V = \frac{1}{\text{Slenderness}^3} \quad (1.8)$$

$$\text{Slenderness} = \frac{1}{\sqrt[3]{C_V}} \quad (1.9)$$

## 1.4 Characteristics of Ship Wave Resistance

To understand the effect of parametric variation on drag, the resistance curves for several carefully varied hulls are presented here. Figure 1.1 illustrates the complexity of the ship resistance problem. Each subplot represents variation in wave drag for one of four hull parameters, with Froude number and wave resistance coefficient on

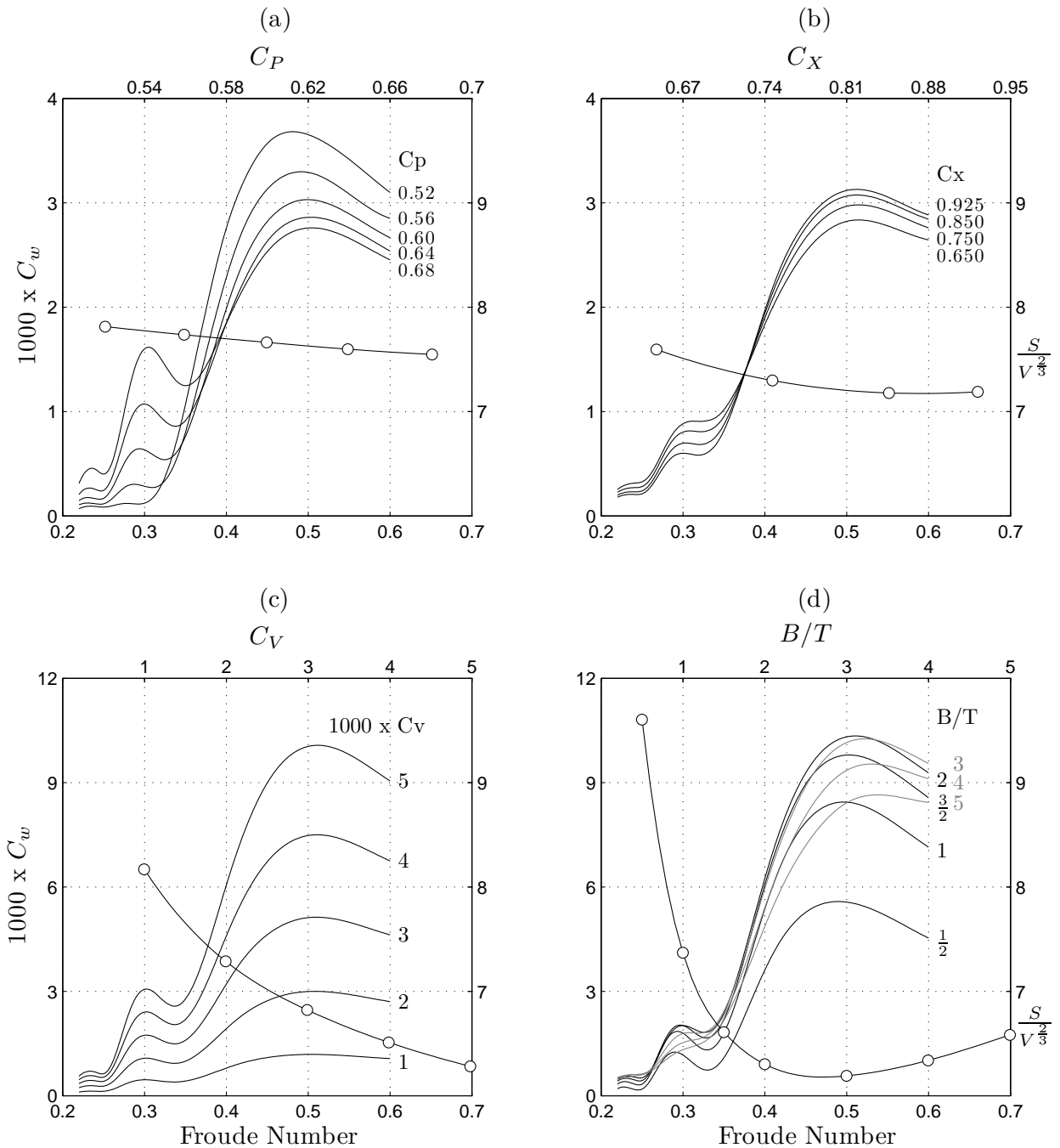
<b>Ratio</b>	<b>Equation</b>
Beam to Draft	$\frac{B}{T}$
Length to Beam	$\frac{L}{B}$
Wetted Surface to Volume	$\frac{S}{\nabla^{2/3}}$
Slenderness	$\frac{L}{\nabla^{1/3}}$

**Table 1.3.** Definition of hull form ratios.

the primary X and Y axes. The variation in wetted surface to volume ratio is then plotted against that hull parameter on the secondary X and Y axes. For example, Figure 1.1a shows variation in wave drag for five prismatic coefficients ranging from 0.52 to 0.68. The surface area to volume ratio for each of those prismatic coefficients is then plotted as open circles on the secondary axes. In each example only one of the four parameters varies, but selecting a different set of hulls could reveal different trends.

Looking at each subplot in turn, Figure 1.1a shows that the desired prismatic coefficient is highly dependent on the design speed. For very low Froude number, low prismatic coefficient gives the lowest drag. At high speed, high prismatic coefficient gives lower drag. In the range of typical ship designs, say Froude number 0.30 to 0.40, selection of prismatic coefficient is important. Lower prismatic coefficient hulls are slightly less efficient in terms of wetted surface to volume ratio, but the impact is small in the range shown.

Figure 1.1b shows the effect of maximum (usually the same as midship) section coefficient. Note that the wave resistance lines cross around  $Fr = 0.38$  such that high



**Figure 1.1.** Effect of hull shape parameters on drag. Primary X and Y axes show wave resistance coefficient vs. Froude number for several values of the parameter. Secondary X and Y axes show surface to volume ratio vs the value of (a)  $C_P$  (b)  $C_X$  (c)  $C_V$  (d)  $B/T$ .



midship coefficient results in lower wave resistance coefficient at low Froude number. The ship with low midship coefficient is less efficient at enclosing the hull volume, with slightly more impact than the prismatic coefficient.

Figure 1.1c shows the effect of volumetric coefficient (related to slenderness by Equations 1.8 and 1.9). The trends in volumetric coefficient are much more clear. Long slender ships have less wave drag than short blocky ships. The lines do not cross. However, the surface to volume ratio shows large variation with slenderness, with low volumetric coefficient ships having more wetted surface.

Figure 1.1d shows the last parameter, beam to draft ratio.  $B/T$  values from 3 to 5 are shown as gray lines. For low speed, higher  $B/T$  values show a much less pronounced peak in wave resistance around  $Fr = 0.30$ . At high speed  $B/T$  of 2 and 3 represent the highest drag. Note that  $B/T$  is essentially the aspect ratio of the ship sections, and can have a large effect on surface to volume ratio. For typical monohull values, however, the effect is no more than that of prismatic or midship section. Multi-hulls, however, which can be high slenderness and low  $B/T$  to reduce wave drag, may have a large wetted surface for their volume.

Constrained or coupled optimization may prevent the ship from following some of these trends. Very high speed ships could end up with high midship coefficients because a length and draft restriction coupled to ship weight forces them to higher midship section. Ships with low design speed are typically trying to carry as much cargo as possible, so that low prismatic coefficients shown here are not practical. Beam to draft ratio is coupled to stability, so  $B/T$  values below about 2 appear only in multi-hulls.

The point of this discussion is simply to illustrate the nonlinear, sometimes counterintuitive behavior of the wave drag and its coupling to the friction resistance through the wetted surface to volume ratio.

Note that for a ship designed around hull speed ( $Fr = 0.40$ ), the goal would be to make the volumetric coefficient as low as possible while tending to a high prismatic coefficient. Beam to draft and midship section coefficient have little effect.

## 1.5 Current Tools

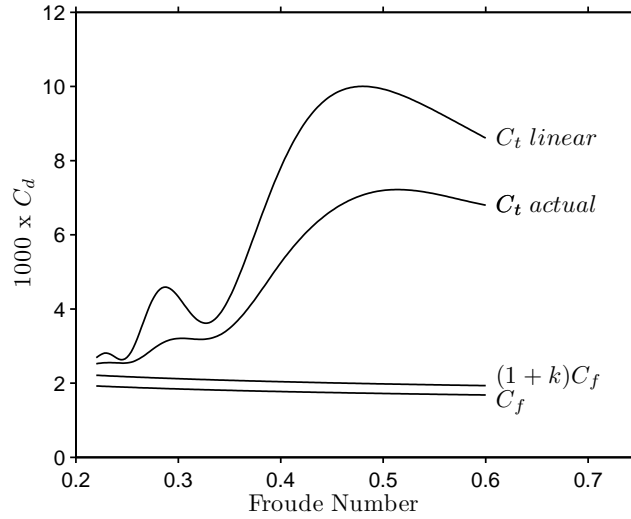
As discussed above, the current design tools fall in to three categories. These methods are discussed in more detail here. Research tools, such as viscous free surface codes and 2D + time methods are not included. Such tools are still under development and are not currently available to most designers.

### 1.5.1 Parametric Methods

Parametric methods are those that use only global hull form characteristics to predict drag based on empirical data. These methods do not use the hull offsets themselves, but integrated properties and length scale ratios. Typically a systematically varied hull form series is evaluated experimentally, and regression analysis of the data provides an equation for the drag based on these parameters. One such method is that of Holtrop [11, 12], which uses 15 geometric parameters and speed as input to the resistance regression equation. If the lines for the ship in question are similar to those of the ships tested, this method yields good results.

### 1.5.2 Michell Integral or Linear Theory

The linearized theory of ship wave resistance was first published by J.H. Michell in 1898 [17]. This method results in an analytical solution for certain simple hull shapes, with a numerical solution based on a center plane source distribution available for



**Figure 1.2.** Typical linear theory performance. Typical proportion of frictional and form resistance components are also shown.

arbitrary hull offsets. The solution is based on the assumption that the ship is thin, i.e. the beam is much less than the length. While the hull offsets themselves are used, the Michell boundary condition does not necessarily hold for ships of typical proportions. As a result, the method over predicts the drag and exaggerates the peaks and troughs in the wave resistance curve [27].

Figure 1.2 shows the typical behavior of the linear theory. The wave drag is represented by the distance between the  $(1+k)C_f$  line and either of the  $C_t$  lines. The linear theory is capturing the physics of the problem, but it over predicts the wave drag, especially at the peaks in the curve. Just below  $Fr = 0.30$ , for example, the drag is exaggerated by over 100%. The location of the peaks and troughs is well predicted however. This plot also shows the components of the total drag that need to be balanced. The friction and viscous drag are represented by  $(1+k)C_f$ , as discussed in Section 1.3.1. If linear theory is used to optimize a hull near  $Fr = 0.30$ , reduction in wave drag would be far more heavily weighted than wetted surface, which is not accurate.

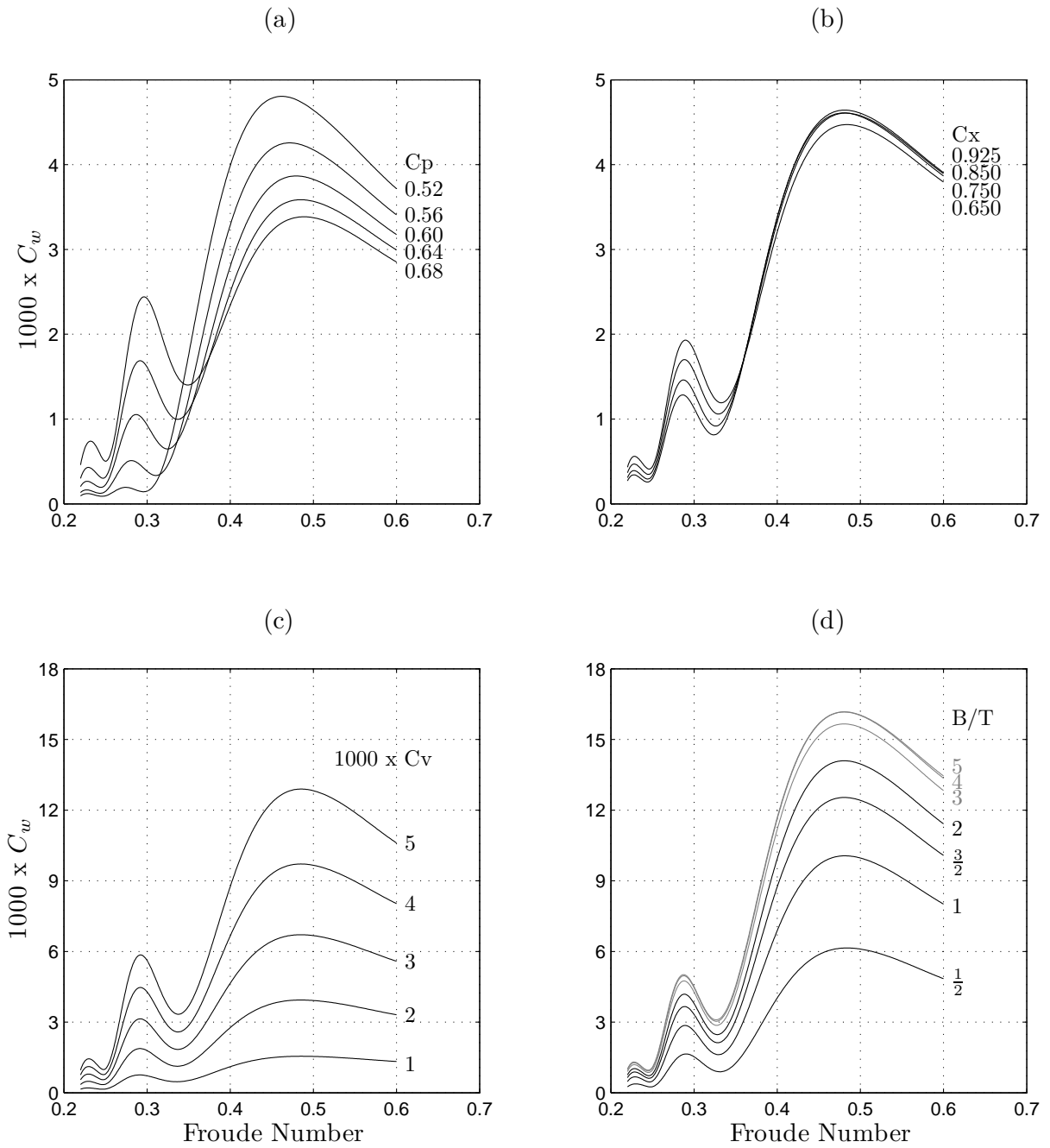
Figure 1.3 is a reproduction of Figure 1.1 but linear theory is used to evaluate the same hulls in each case. As noted above, the magnitude of wave resistance, especially at the peaks in the curve are exaggerated (note the difference in scales). Comparing trends though, linear theory accurately ranks prismatic and volumetric coefficient. Trends due to midship section coefficient are also well predicted, though not as clearly at high speed. Linear theory does not do well in ranking drag of ships with  $B/T$  greater than 2, which is exactly where many monohull designs fall. This failure is not surprising, since linear theory relies on a centerplane source distribution. If we can extend the theory into higher  $B/T$  values and reduce the exaggeration of the peaks in the curve, linear theory might be better suited to early stage hull optimization.

The linear theory has been studied extensively, most notably by Havelock [9], Noblesse [21, 19, 20, 22], Tuck [29, 27], Wehausen [30], and many others. The method is derived in detail in Chapter 2.

### 1.5.3 Boundary Element Methods

Boundary element methods represent the current state-of-the-art in ship wave resistance calculations. These methods discretize the hull surface and free surface near the ship into panels. They solve for the unknown source strengths on the panels (instead of the center plane) and iteratively apply the boundary condition on the free surface. One of the difficulties in the ship wave problem is that the free surface boundary condition is known, but the free surface shape is initially unknown.

The boundary element methods provide an accurate estimate of the ship wave drag, especially on a comparative basis. While they are useful design tools, they do have some drawbacks. They require training to set up, run, and interpret, and the



**Figure 1.3.** Further comparison of linear theory results. Geometric parameters are (a)  $C_P$  (b)  $C_X$  (c)  $C_V$  (d)  $B/T$ . Linear theory captures much of the wave drag physics. Compare to Fig. 1.1.

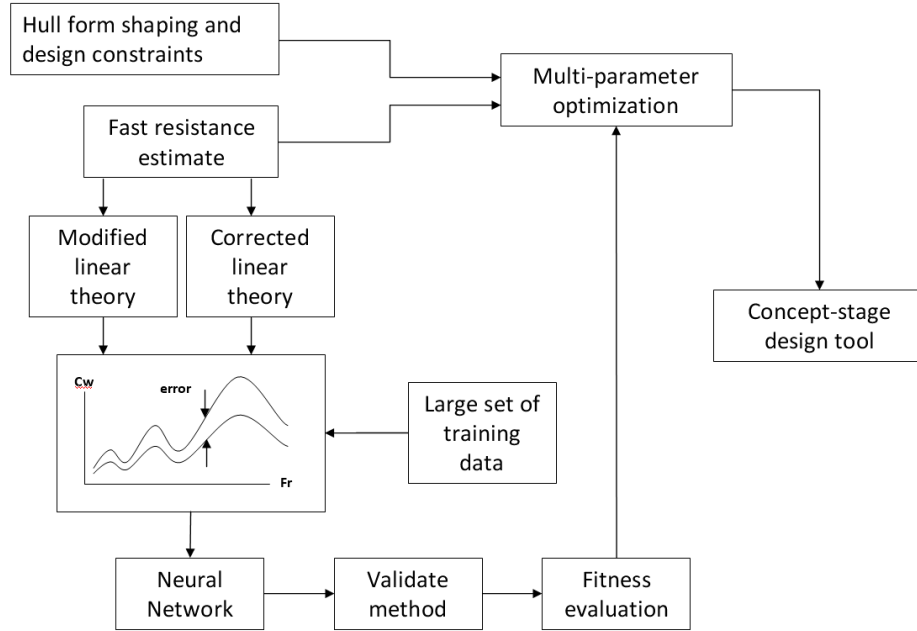
result can sometimes “blow up” due to numerical instability. These tools are also priced out of reach of many design firms. While they are relatively fast, they are not fast enough to be used for an extensive multi-parameter optimization program for early stage design. These methods are, however, indispensable for detailed hull refinement at later stages in the design process.

## 1.6 Description of Present Method

The present method combines a constrained hull form shaping algorithm with a fast resistance estimate that can serve as a fitness evaluation to a multi-parameter optimization algorithm. Figure 1.4 shows a flow chart describing the method. The optimization algorithm is envisioned to be a genetic algorithm or other tool that can handle a large parameter space without converging to a solution that is a local minima. the hull form shaping method can be any type suited to the particular ship type. A 12 parameter method is presented in Appendix C.

The heart of the method lies in a neural network based correction to the linear theory, that serves as the fitness evaluation for the optimization routine. A neural network is trained with targets that minimize the error between the modified linear theories and a large training set of panel code data. Given a set of hull parameters, the network then provides values of the unknown coefficients as output.

As shown in Figure 1.4, two alterations to the linear theory are used. Corrected linear theory refers to a simple multiplier coefficient that acts as a correction to linear theory. This method is further broken down into two approaches. The first and simplest approach determines the correction at only a single Froude number and applies this correction to a certain speed range. By careful selection of the target Froude number, a desirable correction can be achieved over a small range of speed.



**Figure 1.4.** Flow chart of present method.

The second approach uses Froude number as input to the artificial neural network and finds a new correction for each speed. Note that the corrected linear theory still uses the actual hull offsets, so it is not the same as applying a regression analysis correction.

Modified linear theory refers to a correction made with additional terms derived from an extended boundary condition. The new condition results in a waterline integral term with unknown coefficients. By using the training data, values for these unknown coefficients are determined using a constrained error minimization. These coefficients then become the new targets for the artificial neural network. This method reduces the number of inputs to the network by removing Froude number but requires two coefficients as output. It is similar in spirit to the quasi-linear theory of Amromin [1, 2], but alters the wave energy based on a waterline integral instead of enforcing a maximum wave energy limit.

## 2. Derivation and Numerical Treatment of the Michell Integral

The linear theory of ship wave resistance is the basis for the methods that follow. In order to provide an understanding of the analytical problem, the equations are derived in detail using a perturbation method [6]. The numerical methods used to evaluate the integrals for arbitrary shape are also treated in detail, since convergence and speed of computation are particularly important for an optimization scheme. Subsequent variations in boundary conditions, directed at finding additional terms to train the neural network, follow the same analytical sequence and employ the same numerical methods.

The goal of this section is to derive an expression for the wave resistance of a ship subject to linearized boundary conditions. The equations should be sufficiently general to be valid for hulls having a transom stern and for ships with multiple hulls. Hull asymmetry is not treated, since local viscous effects are not part of this analysis. The change in hull orientation with speed, known as sinkage and trim, can be implemented separately since those effects essentially represent an alteration in the hull geometry. The impact of additional terms is included in sinkage and trim calculations, since those terms alter the wave component of the hull dynamic pressure.



## 2.1 Free Wave Spectrum

It is well known from Havelock [8] that the steady wave pattern from a moving ship can be described as the real part of the  $\theta$  integral

$$Z(x, y) = \Re \int_{-\pi/2}^{\pi/2} A(\theta) e^{-ik(\theta)[x \cos \theta + y \sin \theta]} d\theta, \quad (2.1)$$

where  $k(\theta)$  is defined by the dispersion relation for plane waves and  $A(\theta)$  is the amplitude function specific to hull shape. This amplitude function is also called the free wave spectrum and describes the far field ship waves. Based on the energy flux far from the ship, the equation for the wave resistance is [18]

$$R_w = \frac{\pi}{2} \rho U^2 \int_{-\pi/2}^{\pi/2} |A(\theta)|^2 \cos^3(\theta) d\theta. \quad (2.2)$$

The solution method therefore seeks an equation for the free wave spectrum,  $A(\theta)$ , of an arbitrary hull shape. This solution attempts to find an equation for  $Z(x, y)$  in the form of (2.1), such that  $A(\theta)$  may be factored out. Since  $A(\theta)$  is the only term dependent on hull shape, the linearized solution allows the superposition of free wave spectra from  $N$  hulls, each at position  $(x_j, y_j)$ , such that

$$A(\theta) = \sum_{j=1}^N A_j(\theta) e^{ik(\theta)[x_j \cos \theta + y_j \sin \theta]}. \quad (2.3)$$

Thus the calculation of resistance for multi-hulls such as catamarans and trimarans is also possible.

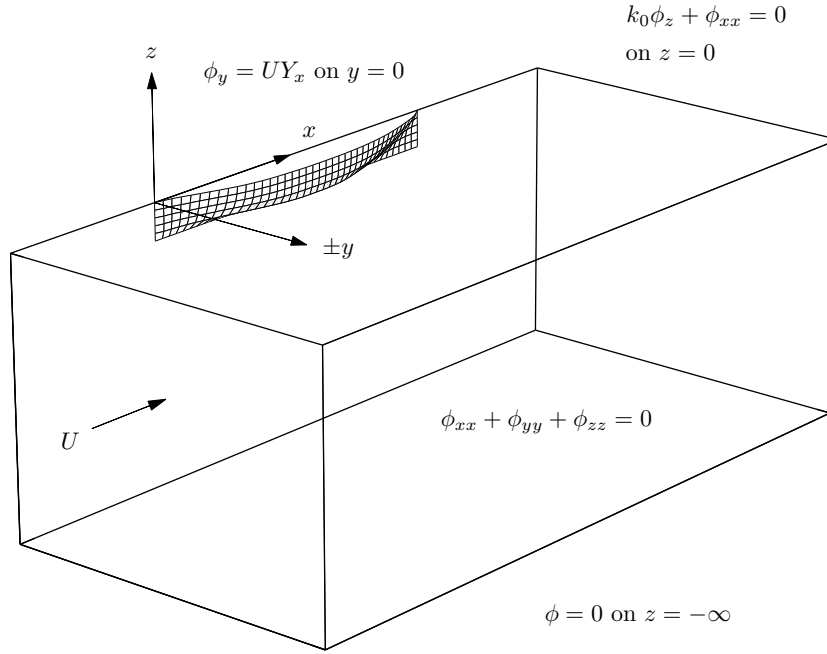


Figure 2.1. Linear theory coordinates and boundary conditions.

## 2.2 Governing Equations and Boundary Conditions

Using the coordinate system shown in Figure 2.1, the velocity potential is

$$\Phi(x, y, z) = Ux + \phi(x, y, z), \quad (2.4)$$

where  $U$  is velocity of the ship moving along the  $x$  axis and  $\phi(x, y, z)$  is the flow perturbation caused by the hull. The velocity is the gradient of the potential,

$$\nabla\Phi = \left( U + \frac{\partial\phi}{\partial x} \right) \hat{i} + \frac{\partial\phi}{\partial y} \hat{j} + \frac{\partial\phi}{\partial z} \hat{k}. \quad (2.5)$$

The flow is governed by the Laplace equation in semi-infinite space

$$\nabla^2\Phi(x, y, z) = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad (2.6)$$

*for*  $-\infty < x < \infty, y > 0, z < 0,$

and the Neumann boundary condition on the hull

$$\vec{n} \cdot \nabla\Phi = 0, \quad (2.7)$$

that specifies no flow through the hull surface  $y = Y(x, z)$ . The vector  $\vec{n}$  is for a point on the hull and is therefore

$$\vec{n} = \begin{bmatrix} \frac{\partial Y}{\partial x} \\ -1 \\ \frac{\partial Y}{\partial z} \end{bmatrix}. \quad (2.8)$$

This condition gives

$$\frac{\partial\phi}{\partial y} = \left[ U + \frac{\partial\phi}{\partial x} \right] \frac{\partial Y(x, z)}{\partial x} + \frac{\partial\phi}{\partial z} \frac{\partial Y(x, z)}{\partial z} \quad \text{on } y = Y(x, z), \quad (2.9)$$

but for a thin ship the linearized condition is

$$\frac{\partial\phi}{\partial y} = U \frac{\partial Y(x, z)}{\partial x} \quad \text{on } y = 0. \quad (2.10)$$

This equation is the Michell boundary condition, that states that the flow perturbation caused by the hull in the  $y$  direction is proportional to the velocity  $U$  times the slope of the hull offsets  $Y(x, z)$  in the  $x$  direction. The condition is applied on the plane  $y = 0$  instead of on the actual hull surface. It was first proposed by J.H. Michell in 1898 [17].

The second boundary condition applies for the assumption of infinite depth, requiring that the velocity perturbation  $\phi(x, y, z)$  approach zero as  $z$  goes to  $-\infty$ .

$$\lim_{z \rightarrow -\infty} \phi(x, y, z) = 0 \quad (2.11)$$

The third boundary condition is derived from the combination of the Neumann condition and the Bernoulli equation, both on the unknown free surface  $Z(x, y)$ . The Neumann condition specifies no flow through the free surface and is

$$\vec{n} \cdot \nabla \Phi = 0, \quad (2.12)$$

with

$$\vec{n} = \begin{bmatrix} \frac{\partial Z}{\partial x} \\ \frac{\partial Z}{\partial y} \\ -1 \end{bmatrix} \quad (2.13)$$

giving

$$\frac{\partial \phi}{\partial z} = \left[ U + \frac{\partial \phi}{\partial x} \right] \frac{\partial Z(x, y)}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial Z(x, y)}{\partial y}. \quad (2.14)$$

The Bernoulli equation for the pressure is

$$gZ(x, y) + \frac{1}{2} \left[ \left( U + \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial z} \right)^2 \right] = \frac{1}{2} U^2. \quad (2.15)$$

Rearranging (2.15) and ignoring the velocity squared terms gives

$$Z(x, y) = -\frac{U}{g} \frac{\partial \phi}{\partial x}. \quad (2.16)$$

Differentiating with respect to  $x$  and  $y$  respectively gives the terms needed in (2.14)

$$\frac{\partial Z(x, y)}{\partial x} = -\frac{U}{g} \frac{\partial^2 \phi}{\partial x^2} \quad (2.17)$$

and

$$\frac{\partial Z(x, y)}{\partial y} = -\frac{U}{g} \frac{\partial}{\partial y} \left( \frac{\partial \phi}{\partial x} \right). \quad (2.18)$$

Substituting (2.17) and (2.18) into (2.14) yields

$$\frac{\partial \phi}{\partial z} = \left[ U + \frac{\partial \phi}{\partial x} \right] \left( -\frac{U}{g} \frac{\partial^2 \phi}{\partial x^2} \right) + \frac{\partial \phi}{\partial y} \left( -\frac{U}{g} \frac{\partial^2 \phi}{\partial y \partial x} \right). \quad (2.19)$$

Now taking the only remaining linear term and moving the boundary from the unknown  $z = Z(x, y)$  to the undisturbed free surface yields the Stokes boundary condition

$$\frac{\partial \phi}{\partial z} + \frac{U^2}{g} \frac{\partial^2 \phi}{\partial x^2} = 0 \quad \text{on} \quad z = 0, \quad (2.20)$$

in which  $k_0 = g/U^2$  is usually used.

Now in summary we write the governing equations in a more compact form

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad \text{for} \quad -\infty < x < \infty, y > 0, z < 0, \quad (2.21)$$

$$\phi_y = U Y_x(x, z) \quad \text{on} \quad y = 0, \quad (2.22)$$

$$\lim_{z \rightarrow -\infty} \phi(x, y, z) = 0, \quad (2.23)$$

$$k_0 \phi_z + \phi_{xx} = 0 \quad \text{on} \quad z = 0, \quad (2.24)$$

as shown in Figure 2.1.

## 2.3 Fourier Transform Derivation of the Michell Integral

The solution is based upon a double Fourier transform, similar to the method described by Tuck [28] and Plesset & Wu [23]. The transformation to Fourier space yields an ODE that can then be solved by conventional means. Instead of inverting the transform to find the velocity potential, however, the equation for the free surface elevation is inverted to give an equation of the form of 2.1. Thus the necessary term  $A(\theta)$  can be factored out and used to estimate not only the wave drag, but the wave field and wave component of the dynamic hull pressure. A relatively high level of detail is presented here in order to understand the effect of subsequent extensions in the next chapter.

A Fourier transform pair is used to transform both the equations and boundary conditions from Section 2.2. Choosing the Fourier cosine transform [3, 6] for the  $y$  variable introduces the Michell condition directly into the equations. The transform pair is

$$\hat{\phi} = \int_{-\infty}^{\infty} \int_0^{\infty} \phi e^{i\lambda x} \cos(\mu y) dy dx \quad (2.25)$$

$$\phi = \frac{1}{\pi^2} \int_{-\infty}^{\infty} \int_0^{\infty} \hat{\phi} e^{-i\lambda x} \cos(\mu y) d\mu d\lambda. \quad (2.26)$$

Applying the transform to equation 2.21 yields

$$\hat{\phi}_{zz} - \lambda^2 \hat{\phi} - \mu^2 \hat{\phi} = \int_{-\infty}^{\infty} \phi_y(x, 0, z) e^{i\lambda x} dx. \quad (2.27)$$

Recognizing the Michell condition in the integrand of the right hand side and setting

the variable  $k^2 = \lambda^2 + \mu^2$  gives

$$\widehat{\phi}_{zz} - k^2 \widehat{\phi} = \int_{-\infty}^{\infty} U Y_x e^{i\lambda x} dx. \quad (2.28)$$

Setting the Fourier transform of  $Y_x$

$$\widehat{Y}_x = \int_{-\infty}^{\infty} Y_x e^{i\lambda x} dx \quad (2.29)$$

yields the desired ODE in Fourier transformed space as:

$$\widehat{\phi}_{zz} - k^2 \widehat{\phi} = U \widehat{Y}_x. \quad (2.30)$$

The solution of this equation by variation of parameters is

$$\widehat{\phi} = C_1 e^{kz} + C_2 e^{-kz} + \frac{U}{2k} \left[ e^{kz} \int \widehat{Y}_x e^{-kz} dz - e^{-kz} \int \widehat{Y}_x e^{kz} dz \right]. \quad (2.31)$$

Applying the boundary condition 2.23 gives

$$\widehat{\phi} = C e^{kz} + \frac{U}{2k} \left[ \int_{-\infty}^z \widehat{Y}_x e^{k(z-\zeta)} d\zeta - \int_{-\infty}^z \widehat{Y}_x e^{-k(z-\zeta)} d\zeta \right], \quad (2.32)$$

or simply

$$\widehat{\phi} = C e^{kz} + \frac{U}{k} \int_{-\infty}^z \widehat{Y}_x \sinh [k(z - \zeta)] d\zeta. \quad (2.33)$$

The boundary condition on  $z = 0$  is used to solve for the unknown constant. The Fourier transform of 2.24 is

$$k_0 \widehat{\phi}_z - \lambda^2 \widehat{\phi} = 0 \quad \text{on} \quad z = 0. \quad (2.34)$$

Substituting 2.33 into 2.34 and solving for C gives

$$C = -\frac{U}{k} \int_{-\infty}^0 \widehat{Y}_x \frac{k_0 k \cosh(k\zeta) + \lambda^2 \sinh(k\zeta)}{k_0 k - \lambda^2} d\zeta. \quad (2.35)$$

The solution to the ODE in Fourier space is then

$$\widehat{\phi} = \frac{U}{k} \left[ \int_{-\infty}^z \widehat{Y}_x \sinh [k(z - \zeta)] d\zeta - e^{kz} \int_{-\infty}^0 \widehat{Y}_x \frac{k_0 k \cosh(k\zeta) + \lambda^2 \sinh(k\zeta)}{k_0 k - \lambda^2} d\zeta \right]. \quad (2.36)$$

Fortunately, applying the inverse double transform (a quadruple integral) is not required to find the free wave spectrum  $A(\theta)$ . Equation 2.16 gives the linearized equation for the free surface elevation  $Z(x, y)$ . In Fourier space this equation becomes

$$\widehat{Z} = -\frac{i\lambda U}{g} \widehat{\phi} \quad \text{on} \quad z = 0. \quad (2.37)$$

Substituting 2.36 into 2.37 and using the identity  $\cosh(x) + \sinh(x) = e^x$  [3] gives the simple equation

$$\widehat{Z} = \frac{i\lambda}{k_0 k - \lambda^2} \int_{-\infty}^0 \widehat{Y}_x e^{k\zeta} d\zeta. \quad (2.38)$$

The inverse transform is then

$$Z = \frac{1}{\pi^2} \int_{-\infty}^{\infty} \int_0^{\infty} \widehat{Z} e^{-i\lambda x} \cos(\mu y) d\mu d\lambda. \quad (2.39)$$

Making a polar conversion  $\mu = k \sin \theta$ ,  $\lambda = k \cos \theta$  and using the Jacobian

$$J(k, \theta) = \begin{vmatrix} \frac{\partial \lambda}{\partial k} & \frac{\partial \lambda}{\partial \theta} \\ \frac{\partial \mu}{\partial k} & \frac{\partial \mu}{\partial \theta} \end{vmatrix} = k(\cos^2 \theta + \sin^2 \theta) = k$$



to make the change of integration variables gives

$$Z = \frac{2}{\pi^2} \int_0^{\pi/2} \int_0^\infty \widehat{Z} k e^{-ikx \cos(\theta)} \cos(ky \sin \theta) dk d\theta, \quad (2.40)$$

where

$$\widehat{Z} = \frac{i \cos \theta}{k_0 - k \cos^2 \theta} \int_{-\infty}^0 \int_{-\infty}^\infty Y_x(\xi, \zeta) e^{ik\xi \cos \theta} e^{k\zeta} d\xi d\zeta. \quad (2.41)$$

Re-writing 2.40 using the identity

$$2e^{-ikx \cos \theta} \cos(ky \sin \theta) = e^{-ik(x \cos \theta + y \sin \theta)} + e^{-ik(x \cos \theta - y \sin \theta)} \quad (2.42)$$

to factor an equation of the form of 2.1 gives

$$Z = \frac{2i}{\pi^2} \iiint Y_x(\xi, \zeta) e^{ik\xi \cos \theta} e^{k\zeta} \frac{k \sec \theta}{k_0 \sec^2 \theta - k} e^{-ik(x \cos \theta + y \sin \theta)} d\xi d\zeta d\theta dk$$

Integrating around the pole at  $k = k_0 \sec^2 \theta$  gives the triple integral

$$Z = -\frac{2}{\pi} \int_{-\pi/2}^{\pi/2} \int_{-\infty}^0 \int_{-\infty}^\infty Y_x(\xi, \zeta) e^{ik\xi \cos \theta} e^{k\zeta} e^{-ik(x \cos \theta + y \sin \theta)} k \sec \theta d\xi d\zeta d\theta. \quad (2.43)$$

Therefore the free-wave spectrum is

$$A(\theta) = -\frac{2}{\pi} k \sec \theta \int_{-\infty}^0 \int_{-\infty}^\infty Y_x(x, z) e^{ikx \cos \theta} e^{kz} dx dz. \quad (2.44)$$

This equation is then integrated by parts since working with the hull offsets  $Y(x, z)$  is generally preferred over working with the slope of the offsets  $Y_x(x, z)$ . If the hull offsets are zero at both the bow and stern, such a modification does not add any complexity to the equations. The equations must be able to handle a transom stern,

however. Integrating by parts assuming that only the bow offsets are zero gives

$$A(\theta) = \frac{2i}{\pi} k^2 \int_{-\infty}^0 \int_{-\infty}^{\infty} Y(x, z) e^{ikx \cos \theta} e^{kz} dx dz + \frac{2}{\pi} k \sec \theta \int_{-\infty}^0 Y(L, z) e^{ikx_s \cos \theta} e^{kz} dz, \quad (2.45)$$

where  $Y(L, z)$  represents the non-zero transom offsets. Substituting  $k = k_0 \sec^2 \theta$  gives the final equation

$$A(\theta) = \frac{2i}{\pi} k_0^2 \sec^4 \theta \int_{-\infty}^0 \int_{-\infty}^{\infty} Y(x, z) e^{ik_0 x \sec \theta} e^{k_0 z \sec^2 \theta} dx dz + \frac{2}{\pi} k_0 \sec^3 \theta \int_{-\infty}^0 Y(x_s, z) e^{ik_0 x_s \sec \theta} e^{k_0 z \sec^2 \theta} dz. \quad (2.46)$$

Substituting 2.46 into 2.2 will give an equation for the wave drag, but the full equation with non-zero transom offsets is unwieldy. The following sequence of single integrals simplifies the evaluation of the wave drag. The Z integral becomes

$$F = \int_{-\infty}^0 Y(x, z) e^{kz} dz, \quad (2.47)$$

with the separate case for  $x = L$

$$F_T = \int_{-\infty}^0 Y(L, z) e^{kz} dz. \quad (2.48)$$

The X integrals become

$$P = \int_{-\infty}^{\infty} F \cos(kx \cos \theta) dx \quad (2.49)$$

$$Q = \int_{-\infty}^{\infty} F \sin(kx \cos \theta) dx, \quad (2.50)$$

with the separate case

$$P_T = F_T \cos(kL \cos \theta) \quad (2.51)$$

$$Q_T = F_T \sin(kL \cos \theta). \quad (2.52)$$

such that the expression for the free wave spectrum reduces to

$$A(\theta) = \frac{2}{\pi} [ik^2(P + iQ) + k \sec \theta(P_T + iQ_T)] \quad (2.53)$$

and we can write  $|A(\theta)|^2$  in the relatively compact form

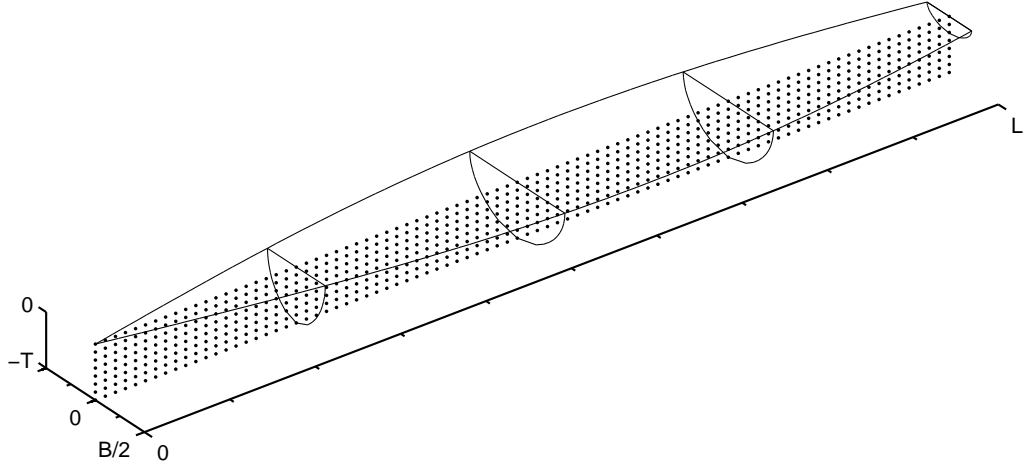
$$R_w = \frac{2}{\pi} \rho U^2 \int_{-\pi/2}^{\pi/2} [k^4(P^2 + Q^2) + 2k^3 \sec \theta(PQ_T - QP_T) + k^2 \sec^2 \theta(F_T^2)] \cos^3 \theta d\theta. \quad (2.54)$$

## 2.4 Numerical Method

The above derivation essentially leads to a centerplane source distribution with number and source strength described by the hull offsets as shown in Figure 2.2. The number of source points necessary to describe the hull depends upon hull shape and the efficiency of the numerical methods used. In order to evaluate the convergence properties of some numerical schemes, a canonical hull form with sinusoidal waterlines and stations can be evaluated analytically for the first two integrals. Such a hull form is shown in Figure 2.3a and has offsets  $Y$  described by

$$Y(x, z) = \frac{B}{2} \sin\left(\frac{\pi C x}{L}\right) \cos\left(\frac{\pi z}{2T}\right), \quad (2.55)$$

having length  $L$ , draft  $T$ , and beam  $B$ . The constant  $C$  determines the presence of a transom stern. Values  $1/2 \leq C < 1$  produce a range of transom size and  $C = 1$



**Figure 2.2.** Sketch of a hull described by a centerplane source distribution. Varying the source strength at each centerplane point approximates the hull shape shown in wireframe.

gives no transom. Making the substitution  $a = \sec \theta$  simplifies the presentation of the equations. Integrating from keel to waterline at each station gives

$$F(x, a) = \frac{B}{2} \sin \left( \frac{\pi C x}{L} \right) \int_{-T}^0 \cos \left( \frac{\pi z}{2T} \right) e^{k_0 z a^2} dz \quad (2.56)$$

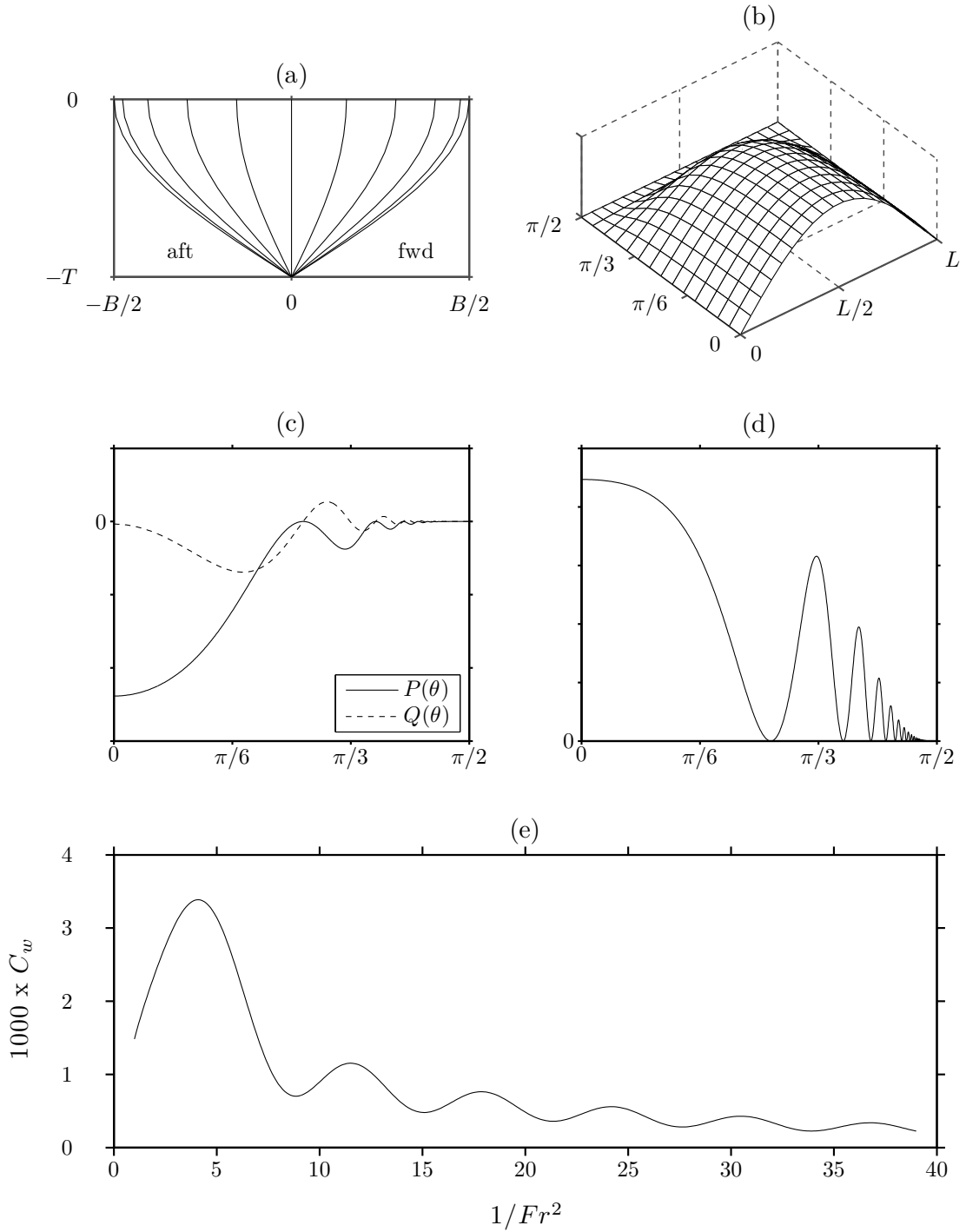
$$F(x, a) = \frac{B}{2} \sin \left( \frac{\pi C x}{L} \right) \frac{2T}{\pi^2 + 2Z^2} [\pi e^{-Z} + 2Z], \quad (2.57)$$

where  $Z = k_0 T a^2$ . The maximum section, at  $x = \frac{L}{2C}$  is used to test the numerical scheme for the  $z$  integral, specifically

$$F(L/2C, a) = \frac{B}{2} \frac{2T}{\pi^2 + 2Z^2} [\pi e^{-Z} + 2Z]. \quad (2.58)$$

The  $x$  cosine integral is then

$$P(a) = F(L/2C, a) \int_0^L \sin \left( \frac{\pi C x}{L} \right) \cos(k_0 x a) dx \quad (2.59)$$



**Figure 2.3.** Steps of the wave drag integration: (a) hull offsets (b)  $F(x, \theta)$  from  $z$  integration (c)  $P(\theta)$  and  $Q(\theta)$  from  $x$  integration (d) integrand of  $\theta$  integration (Eq. 2.54) (e) wave drag coefficient over a range of speeds. Each plot (b,c,d) is for a single speed.

$$P(a) = LBT \frac{\pi e^{-Z} + 2Z}{\pi^2 + 4Z^2} \left( \frac{\pi C - \pi C \cos \pi C \cos X - X \sin X \sin \pi C}{\pi^2 C^2 - X^2} \right), \quad (2.60)$$

and the x sine integral

$$Q(a) = LBT \frac{\pi e^{-Z} + 2Z}{\pi^2 + 4Z^2} \left( \frac{X \cos X \sin \pi C - \pi C \cos \pi C \sin X}{\pi^2 C^2 - X^2} \right), \quad (2.61)$$

where  $X = k_0 La$ . The  $\theta$  integral cannot be evaluated analytically, but the exact integrand can be used to test the numerical scheme for  $\theta$  integration. Since carrying the transom factor C becomes very tedious here, the  $C = 1$  case is used.

$$R_w = 8\pi\rho U^2 k_0^4 (LBT)^2 \int_0^{\pi/2} \frac{1 + \cos X}{(X - \pi)^2 (X + \pi)^2} \left[ \frac{\pi e^{-Z} + 2Z}{\pi^2 + (2Z)^2} \right]^2 a^5 d\theta \quad (2.62)$$

### 2.4.1 The z Integral

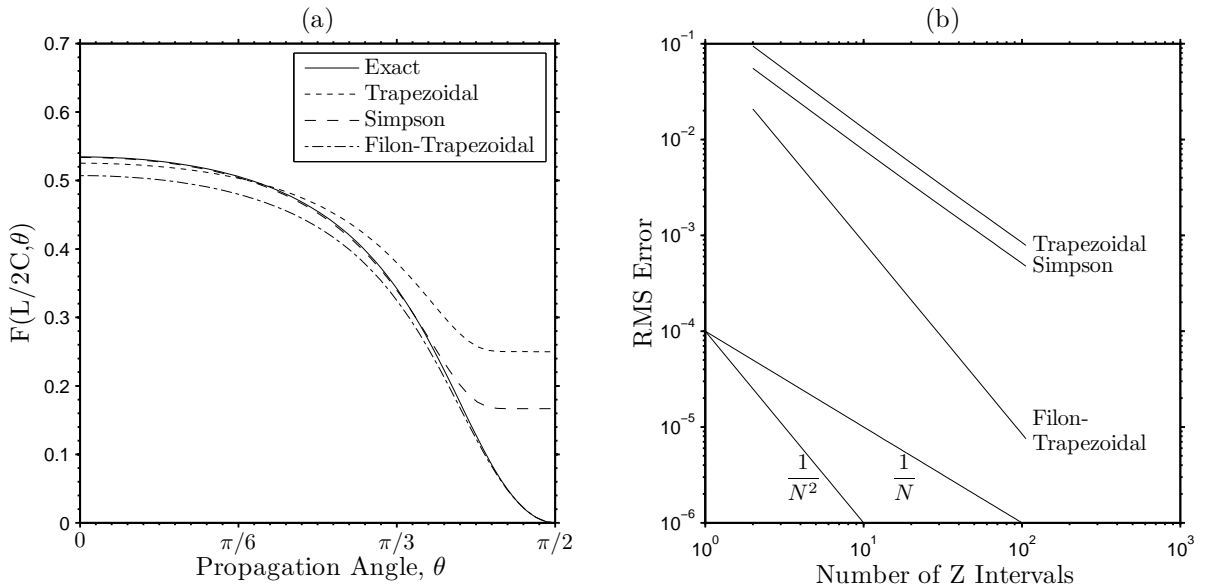
The numerical method for the z integral is Tuck's Filon-Trapezoidal rule [4, 29]. This method approximates equation 2.56 by

$$F(x, a) = \sum_{n=0}^{N_z} w_n Y(x, z_n) e^{kz_n} \delta_z, \quad (2.63)$$

with weights

$$w_n = \begin{cases} \frac{e^{k\delta_z} - 1 - k\delta_z}{(k\delta_z)^2}, & \text{if } n = 0 \\ \frac{e^{-k\delta_z} - 1 + k\delta_z}{(k\delta_z)^2}, & \text{if } n = N_z \\ \frac{e^{k\delta_z} + e^{-k\delta_z} + 2}{(k\delta_z)^2}, & \text{otherwise.} \end{cases} \quad (2.64)$$

Figure 2.4a shows the results of this scheme compared to the trapezoidal and Simpson rules. For small values of  $k$  the weights reduce to the standard trapezoidal



**Figure 2.4.** Performance of the  $z$  integration: (a) example for  $x = L/2C$  with a coarse  $z$  discretization and (b) convergence rate.

rule values, but for large  $k$  the effectiveness of this method is clear. The scheme used in the figure uses only a few  $z$  intervals in order to illustrate the point, but clearly the standard integration methods have difficulty near  $\theta = \pi/2$ .

Figure 2.4b shows the error quantified over a range of  $z$  intervals. The error is computed as the root mean square error

$$E = \sqrt{\frac{1}{N} \sum_{n=0}^N (F_n - F_{exact})^2}, \quad (2.65)$$

where  $F_{exact}$  is computed using equation 2.57. The Filon-Trapezoidal rule performs significantly better than the basic integration methods, achieving  $N^2$  convergence.

## 2.4.2 The $x$ Integral

The  $x$  integral follows in a similar manner but the form of the equation allows the use of the standard Filon integration method [4]. The Filon algorithm can be simplified by

assuming that the bow offsets are always zero. The method is substantially simplified if the transom offsets are also zero, but non-zero offsets are taken into consideration here. The method approximates P and Q as

$$P = [\alpha Q_T + \beta C_{2n} + \gamma C_{2n-1}] \delta_x \quad (2.66)$$

$$Q = [-\alpha P_T + \beta S_{2n} + \gamma S_{2n-1}] \delta_x, \quad (2.67)$$

with summation terms

$$C_{2n-1} = \sum_{n=1}^{N_x} F(x_{2n-1}) \cos(kx_{2n-1} \cos \theta) \quad (2.68)$$

$$C_{2n} = \sum_{n=0}^{N_x} F(x_{2n}) \cos(kx_{2n} \cos \theta) - \frac{1}{2} F_T \cos(kL \cos \theta) \quad (2.69)$$

$$S_{2n-1} = \sum_{n=1}^{N_x} F(x_{2n-1}) \sin(kx_{2n-1} \cos \theta) \quad (2.70)$$

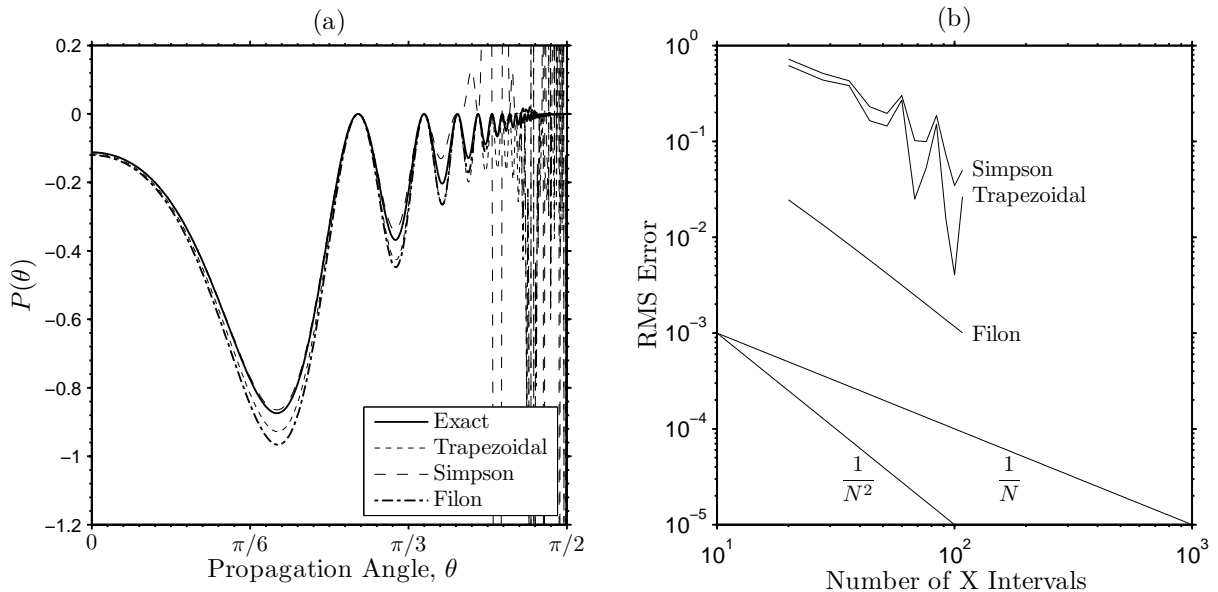
$$S_{2n} = \sum_{n=0}^{N_x} F(x_{2n}) \sin(kx_{2n} \cos \theta) - \frac{1}{2} F_T \sin(kL \cos \theta), \quad (2.71)$$

and weights

$$\begin{aligned} \alpha &= \frac{1}{\kappa^3} [\kappa^2 + \frac{1}{2} \kappa \sin 2\kappa + \cos 2\kappa - 1] \\ \beta &= \frac{1}{\kappa^3} [3\kappa + \kappa \cos 2\kappa - 2 \sin 2\kappa] \\ \gamma &= \frac{4}{\kappa^3} [\sin \kappa - \kappa \cos \kappa], \end{aligned} \quad (2.72)$$

where  $\kappa = k \cos \theta \delta_x$ . In this case the weights reduce to the Simpson rule values for low  $k$ . Again the standard trapezoidal and Simpson rules have trouble near  $\theta = \pi/2$ , while the Filon method is well behaved. Figure 2.5a shows the behavior of all three





**Figure 2.5.** Performance of the x integration: (a) example for  $P(\theta)$  with a coarse x discretization and (b) convergence rate.

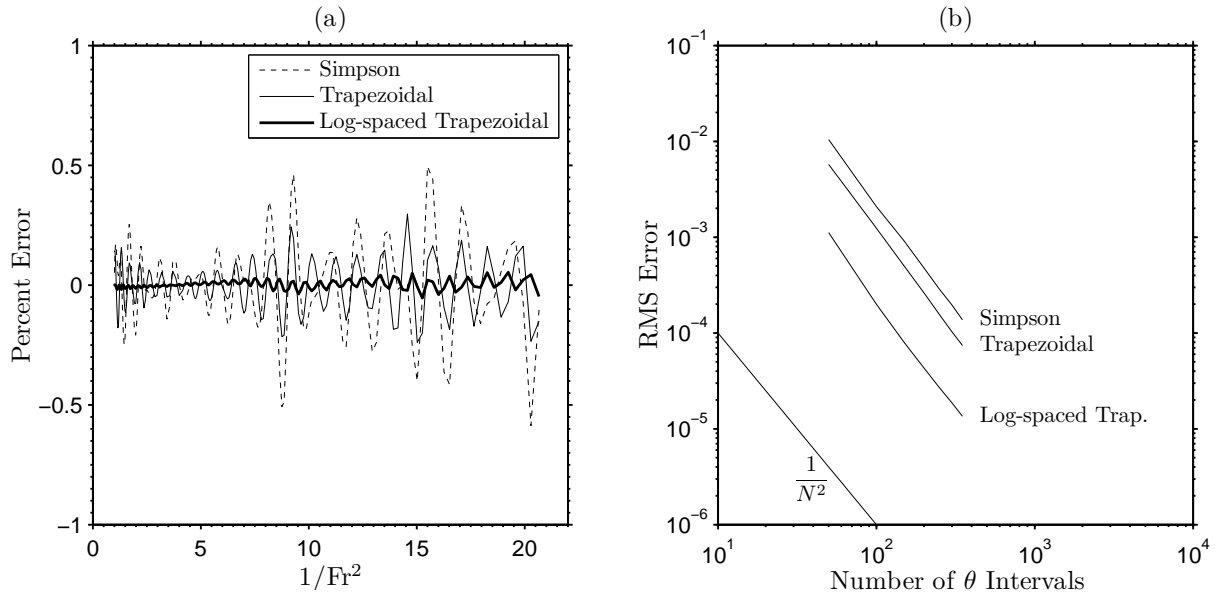
methods for a single speed. Though it is difficult to see in the plot, the Filon method does not “blow up” near  $\pi/2$  like the other methods.

Figure 2.5b shows the RMS error quantified over a range of x intervals, with  $F_{exact}$  taken from equation 2.60. The Filon method shows superior convergence and stability.

### 2.4.3 The $\theta$ Integral

The z and x integrals lead to a highly oscillatory but otherwise well behaved function of  $\theta$  (see Figure 2.3d). The exact analytical solution is not available, so the error is computed using the trapezoidal rule applied to Equation 2.62 with  $10^6$  points as the “exact” solution.

Since the frequency of oscillation increases near  $\pi/2$ , unequally spaced  $\theta$  intervals can provide some improvement in accuracy. An investigation of several spacing schemes showed that  $\log_{10}$  spacing performs best in the preferred range of 100 to 300

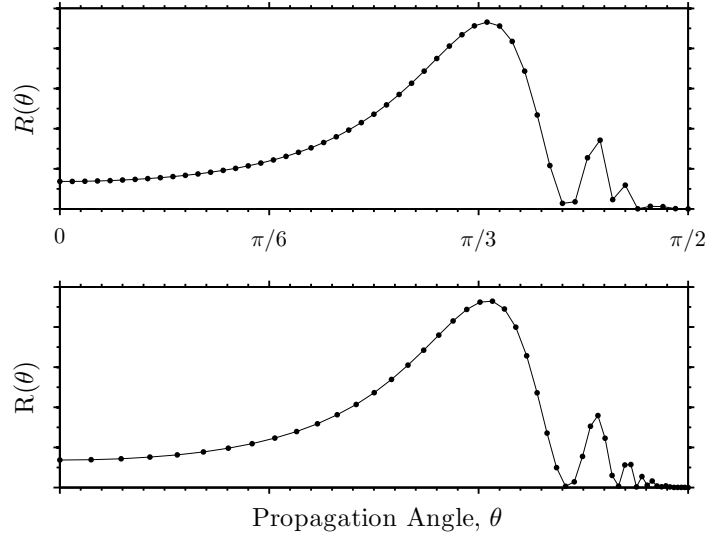


**Figure 2.6.** Performance of the  $\theta$  integration: (a) relative error over a range of speeds and (b) convergence rate.

$\theta$  intervals (Cosine spacing may be better when the number of  $\theta$  intervals is greater than 500). Figure 2.6a compares the performance of such spacing to equally spaced trapezoidal and Simpson schemes over a large range of speeds. This calculation is repeated for a range of  $\theta$  intervals in Figure 2.6b. While the rate of convergence does not improve for unequal spacing, the accuracy for a given number of intervals is superior. Figure 2.7 shows the difference in spacing for an arbitrary speed, illustrating the improvement in resolution as the oscillation frequency increases near  $\theta = \pi/2$ .

#### 2.4.4 Verification of Code

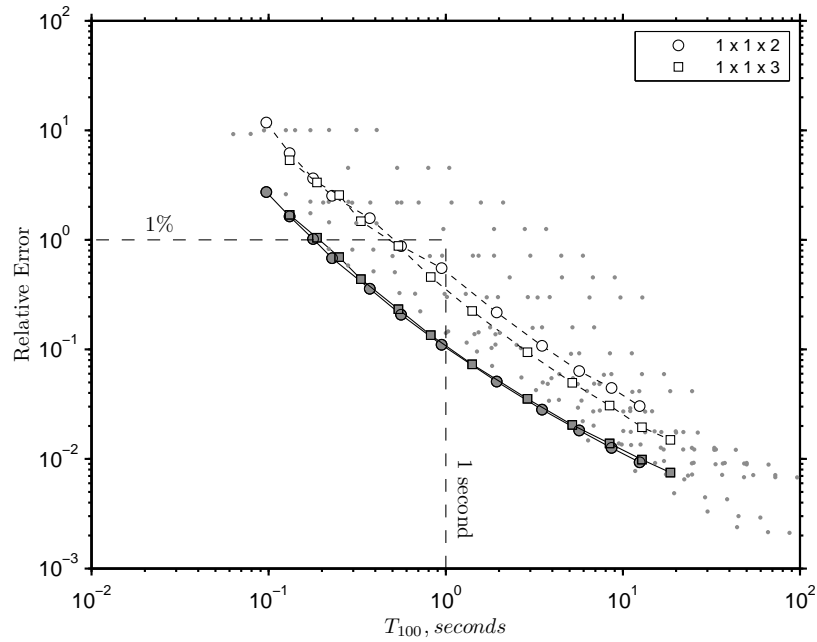
The previous sections have shown the performance of several numerical schemes on the exact solution of the Michell integral for a simple analytical hull shape. These tests show that the numerical scheme should be Filon-trapezoidal in  $z$ , Filon-Simpson in  $x$ , and log spaced trapezoidal in  $\theta$ . In the final test of the entire numerical method for all three integrals, the analytical hull offsets are computed at a discrete number of



**Figure 2.7.** Comparison of  $\theta$  discretization spacing. Top plot is equal spacing, bottom is log spacing.

points and passed to the algorithm. A MATLAB function to perform this integration is provided in Appendix A. Unlike the previous tests, the error from one integral is passed to the next integral. The results from the MATLAB script are again compared to the “exact” solution of numerically integrating Equation 2.62 with a very large number of points.

Since one of the advantages to using the linearized theory is speed, the numerical method was run with a range of discretization schemes to determine the best performance. Taking the number of  $z$  intervals as reference, the error and computation time were calculated for various numbers of  $x$  and  $\theta$  intervals. The  $z$  interval multipliers were  $1/2$ ,  $1$ ,  $2$ ,  $3$ , and  $4$  for both  $x$  and  $\theta$ . In other words a discretization denoted  $1 \times 2 \times 4$  with  $20$   $z$  intervals would have  $40$   $x$  intervals and  $80$   $\theta$  intervals. In total 25 of these discretization schemes were tested, as shown in Figure 2.8. The figure shows the average and maximum relative error versus the time to compute 100 speed evaluations. The speed range for each point is the same as that shown in Figure 2.3. Figure 2.6a illustrates the average and maximum error. The  $1 \times 1 \times 2$  and



**Figure 2.8.** Performance of the numerical scheme. Mean error (dark) and maximum error (light) are shown for the best interval ratios.  $T_{100}$  represents the time to compute the drag of the canonical example hull 100 times.

1x1x3 discretization schemes exhibited the best performance for the analytical hull (the remaining schemes are shown as the gray data points).

The combination of linear wave theory and careful selection of numerical algorithms shows the present method to be very efficient. The MATLAB function is written with basic speed increases in mind, and although it is not compiled, can compute 100 speed evaluations to 1% accuracy in 1 second. The computer processor is a 2.66 GHz dual core circa 2008, capable of approximately 1.5 billion floating point operations per second (GFLOPS).

# 3. Derivation and Behavior of Modified Linear Theory

The artificial neural networks developed in this work are based upon correcting the error between linear theory and the boundary element code. The networks accomplish this error correction in two different ways, as shown in Figure 1.4. The corrected linear theory approach uses the ratio of the two drag calculations directly, but this method must deal with the oscillations that occur as a function of Froude number. A second method, the modified linear theory, attempts to expand the Michell integral derivation in Chapter 2 to find a correction term that will work for all Froude numbers, reducing the number of training input variables. This modified linear theory is developed here. The analytical solution is re-developed with an additional boundary condition term, resulting in a waterline integral term that modifies the standard linear theory in the desired manner. The additional term contains unknown constants that are left unassigned to serve as variables in a constrained error minimization. The variable values that give minimum error to the SHIPFLOW data act as training targets for the modified linear theory networks.

### 3.1 Free Surface Boundary Condition

The extended derivation starts with the full free-surface boundary condition. In Chapter 2 all nonlinear terms are dropped, leaving the linearized boundary condition,

$$k_0\phi_z + \phi_{xx} = 0 \quad \text{on} \quad z = 0. \quad (3.1)$$

The full boundary condition derived from the Bernoulli equation and the Neumann condition on the free surface is

$$\begin{aligned} k_0\phi_z + \phi_{xx} + \frac{1}{U} [2\phi_x\phi_{xx} + \phi_z\phi_{xz} + 2\phi_y\phi_{xy}] + \frac{1}{U^2} [\phi_x^2\phi_{xx} + \phi_y^2\phi_{yy}] \\ + \frac{1}{U^2} [\phi_x\phi_z\phi_{xz} + 2\phi_x\phi_y\phi_{xy} + \phi_y\phi_z\phi_{yz}] = 0. \end{aligned} \quad (3.2)$$

The goal is to add a term that results in a solution with unknown constants, so that the value of the constants can be determined by minimizing the error with a set of boundary element code data. This objective is accomplished by adding a Michell-like assumption to the free surface boundary condition. Just as the Michell assumption for  $\phi_y$  is used to linearize the Neumann condition on the hull, the second and third order terms concerning a  $y$  velocity perturbation are investigated here. To that end the terms containing  $\phi_y\phi_{xy}$  and  $\phi_y^2\phi_{yy}$  were linearized by various assumptions in a trial and error investigation. Based on this study, only a  $\phi_{yy}$  term exhibits the mathematical behavior that will alter the wave drag in the desired manner. Adding this term leads to the boundary condition

$$k_0\phi_z + \phi_{xx} + \frac{1}{U^2}\phi_y^2\phi_{yy} = 0 \quad \text{on} \quad z = 0. \quad (3.3)$$

In the spirit of the Michell condition, the added term is linearized with an assumption concerning  $\phi_y$  such that

$$\phi_y^2 = f(C_p, C_{wp}, C_x, \dots). \quad (3.4)$$

This condition simply states that the square of the flow perturbation in the  $y$  direction is a function of the hull form coefficients and proportions. The network will determine this function as a correction factor based on the training data. To carry the assumption through the derivation,  $\frac{\phi_y^2}{U^2} = \Omega$  is used as a substitution.

$$k_0\phi_z + \phi_{xx} + \Omega\phi_{yy} = 0 \quad \text{on} \quad z = 0. \quad (3.5)$$

## 3.2 Alternative Derivation with Unknown Coefficients

The derivation of the linear theory can now proceed as in Chapter 2, starting from the step at which the free surface boundary condition is applied. Equation 2.33 is re-stated as the starting point.

$$\widehat{\phi} = Ce^{kz} + \frac{U}{k} \int_{-\infty}^z \widehat{Y}_x \sinh[k(z - \zeta)] d\zeta \quad (3.6)$$

The Fourier transform of the expanded boundary condition is

$$k_0\widehat{\phi}_z - \lambda^2\widehat{\phi} - \Omega\mu^2\widehat{\phi} - \Omega U\widehat{Y}_x = 0. \quad (3.7)$$

Following the procedure of Chapter 2 and substituting to solve for C gives

$$C = -\frac{U}{k} \int_{-\infty}^0 \widehat{Y}_x \frac{k_0 k \cosh(k\zeta) + [\lambda^2 + \Omega\mu^2] \sinh(k\zeta)}{k_0 k - \lambda^2 - \Omega\mu^2} d\zeta - \frac{\Omega U \widehat{Y}_x}{k_0 k - \lambda^2 - \Omega\mu^2}. \quad (3.8)$$

And the solution to  $\widehat{\phi}$  is then

$$\begin{aligned} \widehat{\phi} = & \frac{U}{k} \int_{-\infty}^z \widehat{Y}_x \sinh [k(z - \zeta)] d\zeta \\ & - \frac{U}{k} e^{kz} \int_{-\infty}^0 \widehat{Y}_x \frac{k_0 k \cosh(k\zeta) + [\lambda^2 + \Omega\mu^2] \sinh(k\zeta)}{k_0 k - \lambda^2 - \Omega\mu^2} d\zeta \\ & - e^{kz} \frac{\Omega U \widehat{Y}_x}{k_0 k - \lambda^2 - \Omega\mu^2}. \end{aligned} \quad (3.9)$$

Again, applying the inverse double transform is not required to find the free wave spectrum  $A(\theta)$ . Substituting Equation 3.9 into 2.16 and again using the hyperbolic function sum relation gives the equation for the free surface elevation in Fourier transformed space

$$\widehat{Z} = \frac{i\lambda}{k_0 k - \lambda^2 - \Omega\mu^2} \left[ \int_{-\infty}^0 \widehat{Y}_x e^{k\zeta} d\zeta - \frac{\Omega}{k_0} \widehat{Y}_x \right]. \quad (3.10)$$

The inverse transform is identical to Equation 2.40. Substituting the Fourier transform of the slope of the hull offsets  $\widehat{Y}_x$  and transforming the variables in Equation 3.10 gives

$$\begin{aligned} \widehat{Z} = & \frac{i \cos \theta}{k_0 - k \cos^2 \theta - k\Omega \sin^2 \theta} \\ & \times \left[ \int_{-\infty}^0 \int_{-\infty}^{\infty} Y_x(\xi, \zeta) e^{ik\xi \cos \theta} e^{k\zeta} d\xi d\zeta - \frac{\Omega}{k_0} \int_{-\infty}^{\infty} Y_x(\xi, 0) e^{ik\xi \cos \theta} d\xi \right]. \end{aligned} \quad (3.11)$$

Rearranging part of this equation

$$\frac{\cos \theta}{k_0 - k \cos^2 \theta - k\Omega \sin^2 \theta} \longrightarrow \frac{\sec \theta}{k_0 \sec^2 \theta - k - k\Omega \tan^2 \theta} \quad (3.12)$$



Setting the denominator equal to zero,

$$k = \frac{k_0}{\beta} \sec^2 \theta \quad (3.13)$$

where  $\beta = 1 + \Omega \tan^2 \theta$ . The equation for Z is then

$$Z = \frac{2i}{\pi} \iint \left[ \iiint Y_x(\xi, \zeta) e^{ik\xi \cos \theta} e^{k\zeta} d\xi d\zeta - \frac{\Omega}{k_0} \int Y_x(\xi, 0) e^{ik\xi \cos \theta} d\xi \right] \\ \times \frac{k \sec \theta}{k_0 \sec^2 \theta - k - k\Omega \tan^2 \theta} e^{-ik(x \cos \theta + y \sin \theta)} d\theta dk. \quad (3.14)$$

Now solving for the new pole at k and integrating over wave number gives a solution for  $A(\theta)$ . Substituting 3.11 into 2.40, and again using 2.42 to put the equation in the proper form, the free-wave spectrum is:

$$A(\theta) = \frac{2}{\pi} \frac{k \sec \theta}{\beta} \left[ \iint Y_x(x, z) e^{ikx \cos \theta} e^{kz} dx dz - \frac{\Omega}{k_0} \int Y_x(x, 0) e^{ikx \cos \theta} dx \right]. \quad (3.15)$$

Integrating by parts again

$$A(\theta) = \frac{2}{\pi} \frac{k \sec \theta}{\beta} \left[ \int Y(L, z) e^{ikL \cos \theta} e^{kz} dx dz - \frac{\Omega}{k_0} Y(L, 0) e^{ikL \cos \theta} \right] \\ - \frac{2i}{\pi} \frac{k^2}{\beta} \left[ \iint Y(x, z) e^{ikx \cos \theta} e^{kz} dx dz - \frac{\Omega}{k_0} \int Y(x, 0) e^{ikx \cos \theta} dx \right]. \quad (3.16)$$

Now assuming that the transom offsets are zero and substituting  $k = \frac{k_0}{\beta} \sec^2 \theta$  gives

$$A(\theta) = -\frac{2i}{\pi} \frac{k_0^2 \sec^4 \theta}{\beta^3} \left[ \iint Y(x, z) e^{i\frac{k_0}{\beta} x \sec \theta} e^{\frac{k_0}{\beta} z \sec^2 \theta} dx dz - \frac{\Omega}{k_0} \int Y(x, 0) e^{i\frac{k_0}{\beta} x \sec \theta} dx \right]. \quad (3.17)$$

Substituting 2.46 into 2.2 will give an equation for the wave drag, but the full equation with non-zero transom offsets is unwieldy due to the number of terms. The

following sequence of single integrals simplifies the evaluation of the wave drag. The Z integral becomes

$$F = \int_{-\infty}^0 Y(x, z) e^{\frac{k_0}{\beta} z \sec^2 \theta} dz. \quad (3.18)$$

The X integrals for the second term become

$$P_1 = \frac{1}{\beta^3} \int_{-\infty}^{\infty} F \cos\left(\frac{k_0}{\beta} x \cos \theta\right) dx \quad (3.19)$$

$$Q_1 = \frac{1}{\beta^3} \int_{-\infty}^{\infty} F \sin\left(\frac{k_0}{\beta} x \cos \theta\right) dx, \quad (3.20)$$

and the second term

$$P_2 = \frac{\Omega}{k_0 \beta^3} \int_{-\infty}^{\infty} Y(x, 0) \cos\left(\frac{k_0}{\beta} x \cos \theta\right) dx \quad (3.21)$$

$$Q_2 = \frac{\Omega}{k_0 \beta^3} \int_{-\infty}^{\infty} Y(x, 0) \sin\left(\frac{k_0}{\beta} x \cos \theta\right) dx, \quad (3.22)$$

such that the expression for the free wave spectrum reduces to

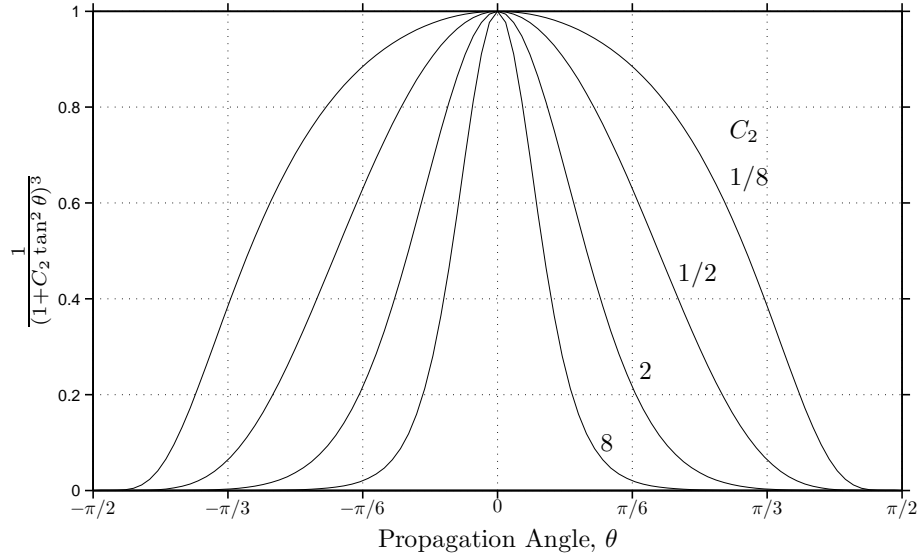
$$A(\theta) = \frac{2i}{\pi} k_0^2 \sec^4 \theta (P + iQ) \quad (3.23)$$

with  $P = P_1 - P_2$  and  $Q = Q_1 - Q_2$ . Again writing  $|A(\theta)|^2$  in a compact form

$$R_w = \frac{2}{\pi} \rho U^2 k_0^4 \int_{-\pi/2}^{\pi/2} [(P_1 - P_2)^2 + (Q_1 - Q_2)^2] \sec^5 \theta d\theta. \quad (3.24)$$

### 3.3 Behavior of Modified Theory

The new terms in the resistance integration (Equations 3.21 and 3.22) allow a selective alteration of the free wave spectrum. By selecting values for the unknown constants,



**Figure 3.1.** Behavior of the waterline integral coefficient. The constant in the denominator modulates the effect of the integral. Large values limit the effect to only the transverse wave energy.

we attempt to find a set of equations that can closely match the data from the training set described in the next chapter.

By examining part of the waterline integral coefficient, namely  $1/\beta^3$ , and picking different values for  $C_2$ , we can see which part of the wave spectrum is altered. The results for four values of  $C_2$  are shown in Figure 3.1. First, note that for any value of  $C_2$ , the coefficient goes rapidly to zero at  $\pi/2$ . Without the coefficient, the waterline integral term would not be well behaved near the singularity at that point. Second, we see how the term can affect the different components of the wave energy. Wave energy propagating between  $\pm 0.616$  radians comprises the transverse waves, while energy propagating at larger angles makes up the diverging waves. As shown in Figure 3.1, the  $1/\beta^3$  coefficient primarily alters the transverse wave energy, but can selectively affect the diverging waves. To examine the combined effect of the entire term, we examine Equation 2.54 and expand it into the original linear theory component and

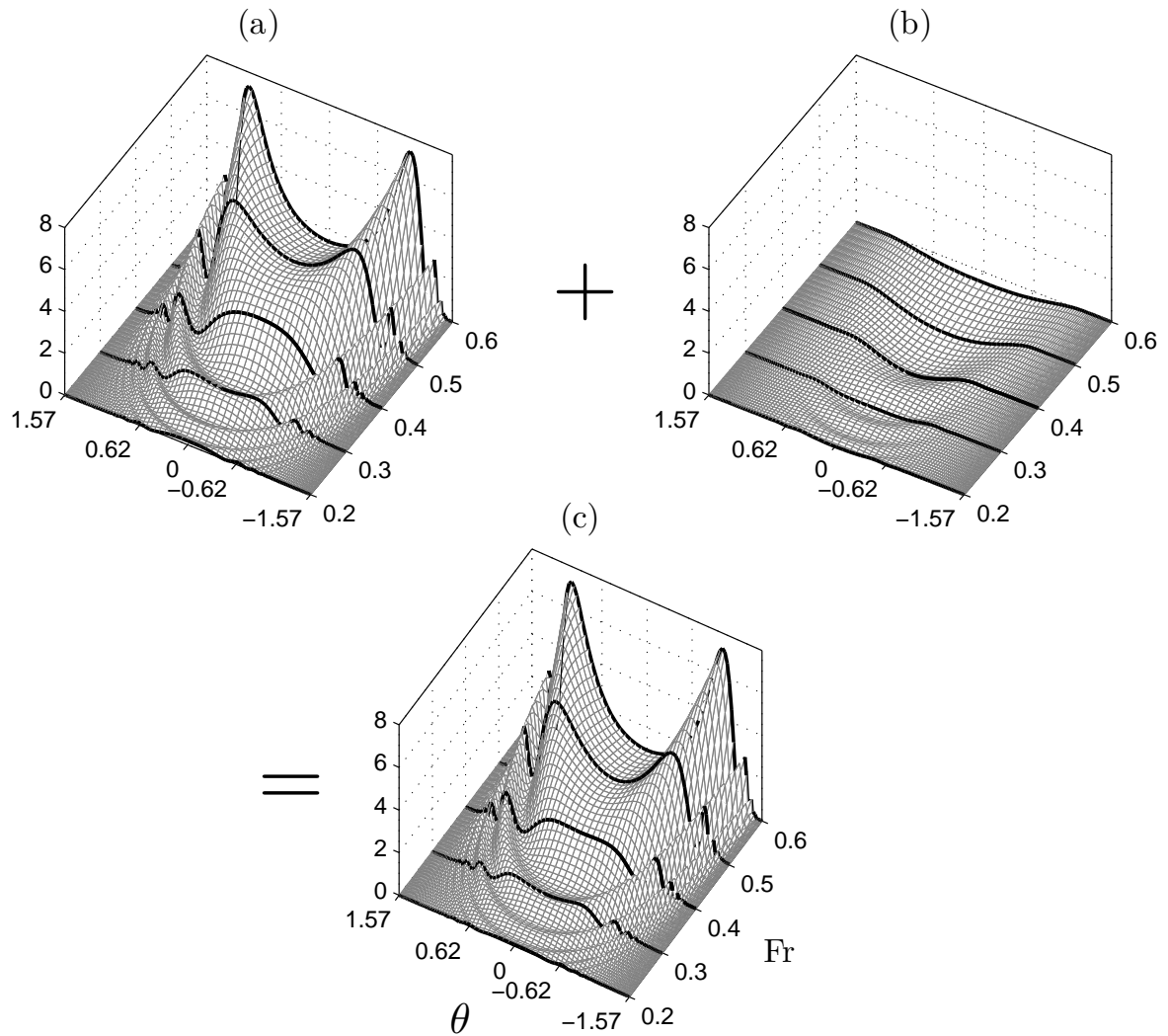
the new term component. (Details of the value of  $\Omega$  are discussed in the next Chapter.)

$$R_1(\theta, Fr) = \frac{2}{\pi} \rho U^2 k_0^4 (P_1^2 + Q_1^2) \sec^5 \theta. \quad (3.25)$$

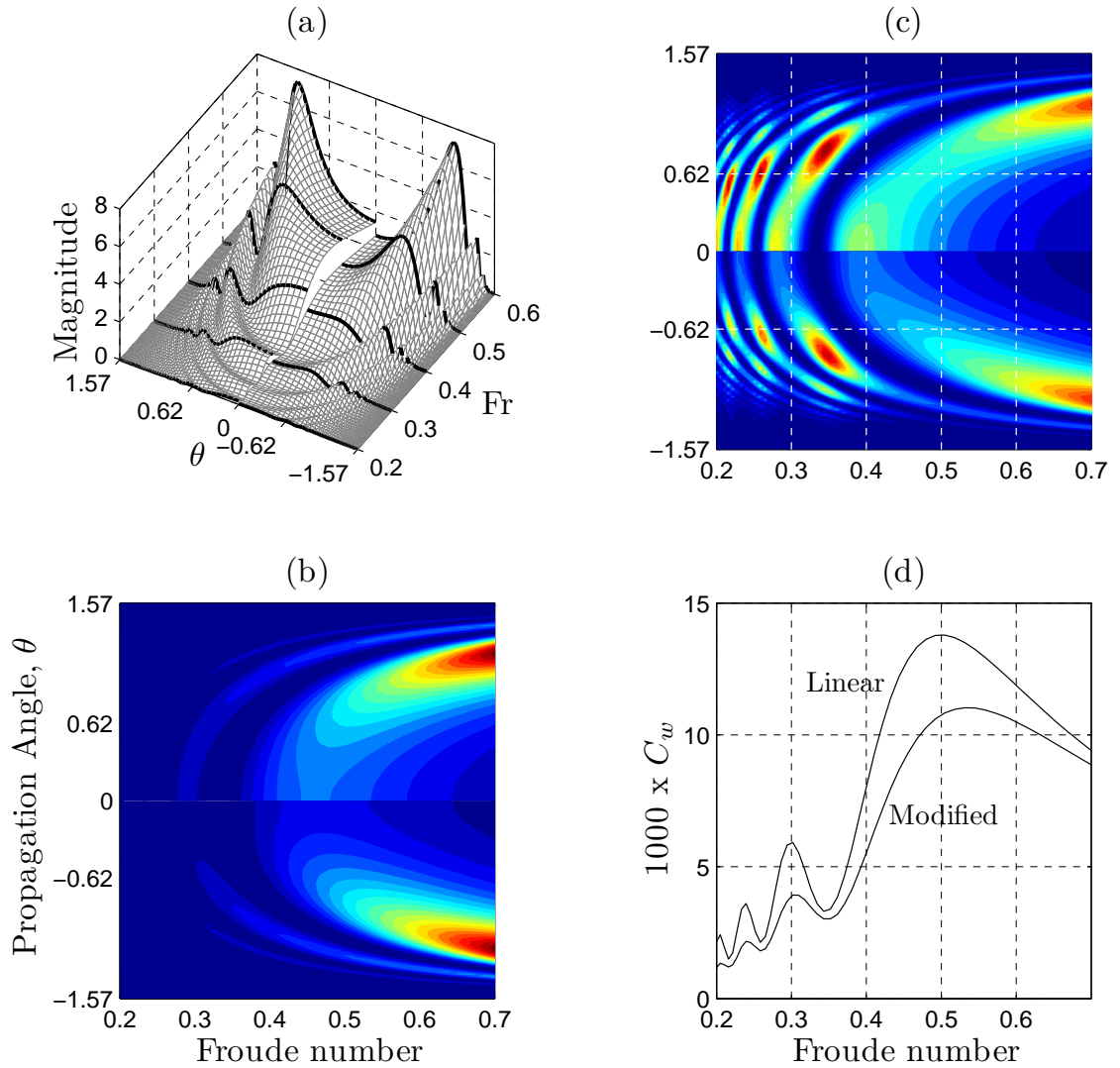
$$R_2(\theta, Fr) = \frac{2}{\pi} \rho U^2 k_0^4 (P_2^2 - 2P_1P_2 - 2Q_1Q_2 + Q_2^2) \sec^5 \theta \quad (3.26)$$

An example of the behavior of these two equations is shown in Figure 3.2. By choosing  $\beta = 1$  for the  $P_1$  and  $Q_1$  terms,  $R_1$  is the same as the unmodified linear theory. This surface is a function of  $\theta$  and Froude number and is shown in Figure 3.2a. Figure 3.2b represents the contribution of the new term  $R_2$  for  $C_2 = 2$  and a value of  $\Omega$  determined using the data set in Chapter 4. The effect of the new term is to subtract primarily transverse wave energy from the linear theory values. Note also that both the  $R_1$  and  $R_2$  surfaces exhibit similar shape characteristics. More specifically, both have minima and maxima near the same location along  $\theta = 0$ . This characteristic means that the new term can reduce the wave energy at peaks in the  $C_w$  curve while having little effect on the troughs. This effect can be seen in the resulting surface, shown in Figure 3.2c.

The changes due to the new term are shown in greater detail in Figure 3.3. The unmodified and modified linear theory surfaces are shown together in Figure 3.3a (unmodified theory on the positive  $\theta$  side). Note the cuts indicated by the heavy black lines, which show the effect for a single speed (the area under the line is the drag). In order to compare the two energy distributions, they are shown as two-dimensional contour plots. Such a plot is shown in Figure 3.3b, with the linear theory above and the modified theory below. Note that the  $\pm 0.62$  radian angle separating the transverse and diverging waves is shown in each plot. The reduction in transverse wave energy is shown in the contours between Froude numbers 0.40 and



**Figure 3.2.** Graphical depiction of the waterline integral. (a) Surface described by the function  $R_1$ . Compare dark lines at constant Froude number to 2.3d. (b) Surface described by the function  $R_2$  for  $C_2 = 2$ . (c) Total corrected wave energy. Note reduction in (primarily) transverse wave energy from (a) to (c).



**Figure 3.3.** Detailed comparison of altered wave energy distribution. (a) Side by side comparison of linear theory (left) and modified theory (right). See Figure 3.2a and 3.2c. (b) Contour plot of the same surface, with linear theory (top) and modified theory (bottom). (c) Wave energy normalized by drag at each speed to show behavior. (d) Corresponding change in wave resistance coefficient.

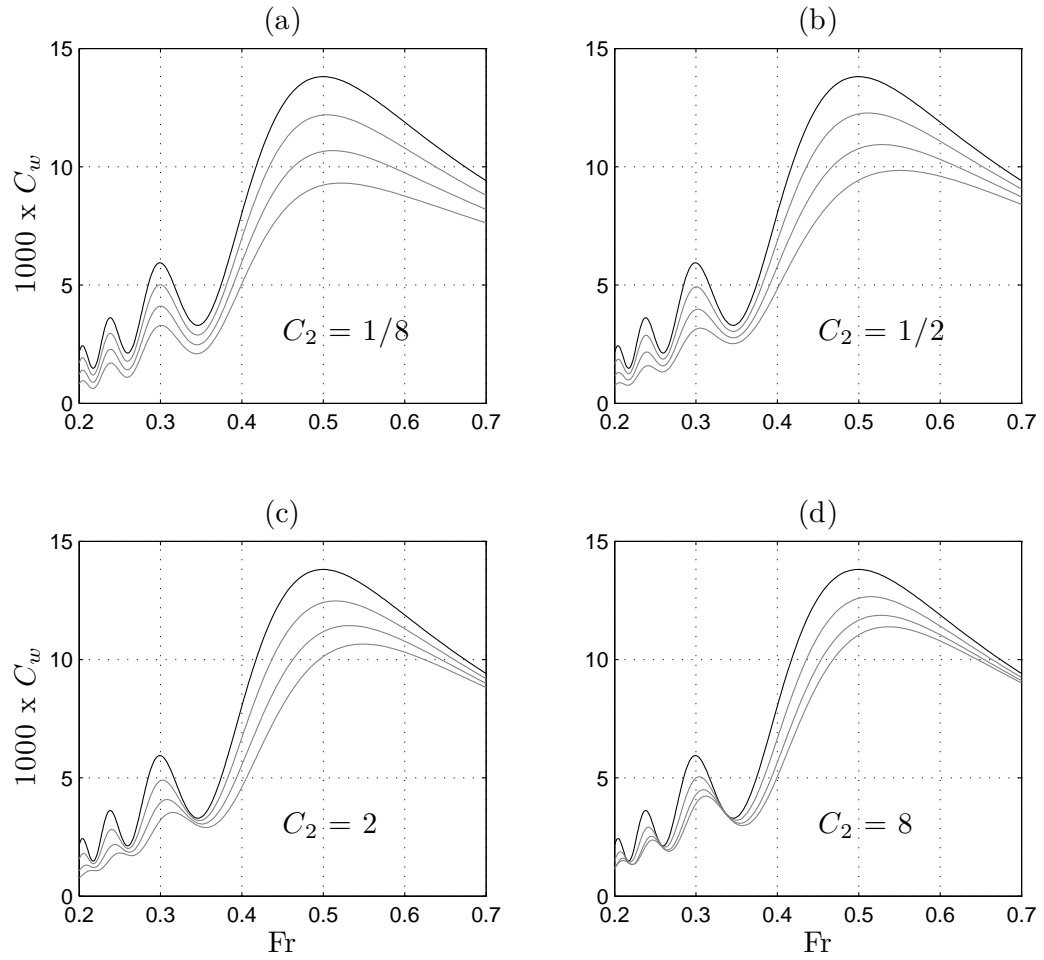
0.60. At lower speeds, however, the effect is difficult to discern. In order to see more clearly the effect of the new term, the surface is normalized by the linear theory wave drag at each Froude number.

$$R^*(\theta, Fr) = \frac{R(\theta, Fr)}{R_w(Fr)} \quad (3.27)$$

Here  $R(\theta, Fr)$  is either the linear or modified theory, but  $R_w(Fr)$  is the linear theory in either case. This “normalized” term is not non-dimensional, but serves to show the details of the surface over the entire speed range. The results of the calculation are shown in Figure 3.3c. As noted above, the effect is to reduce the peaks in the transverse waves without having a large impact on either the transverse wave trough or the diverging waves. (In fact reducing the diverging wave energy makes the exaggerations in the  $C_w$  curve even larger.) The result on the  $C_w$  curve is shown in in Figure 3.3d. The behavior is as desired, with large reductions in the peaks and small reductions in the troughs. Figures similar to in Figure 3.3c and 3.3d are used to show the results in subsequent chapters.

Different values of  $C_2$  are used in the error minimization used to match the modified theory to the training set that provides the target values for the neural network.

The effect of different values of  $C_2$  and  $\Omega$  is shown in Figure 3.4. The black line in each case is the unmodified linear theory, while the three gray lines represent different magnitudes of the  $\Omega$  coefficient. Each  $C_2$  value corresponds to the  $1/\beta^3$  plot shown in Figure 3.1. A low value such as  $C_2 = 1/8$  has the broadest effect on the wave energy, as shown in Figure 3.4a. As  $C_2$  nears zero, the effect of the waterline term becomes more like multiplying the standard linear theory by some fraction (although the two integrals are not exactly the same). For these low values of  $C_2$ , the effect on the trough in the  $C_w$  curve is more severe. For high values of  $C_2$  the new term



**Figure 3.4.** Possible behavior of altered  $C_W$  curves. Subplots (a) through (d) show result for four values of  $C_2$ . Multiple gray lines correspond to possible values of the unknown term  $\Omega$ .

impacts only the transverse waves, but the shape of the surface can be distorted. A slight shift in the position of the trough can also occur due to slight phase differences between the P and Q terms. The value of  $C_2$  that best matches the training data is determined by constrained error minimization in Chapter 5.



## 4. Artificial Neural Network

### Training Set

The basis for the wave drag estimate for both the corrected and modified linear theory methods is an artificial neural network. In order to train the networks over a set of inputs encompassing a range of practical ship forms, a large set of training data is required. This training set is comprised of wave resistance data from linear theory and a state-of-the-art panel code, in this case SHIPFLOW from Flowtech International AB [13, 14]. The hull geometry selected is that of the Taylor Standard Series [26, 24], modified slightly in order to have a simple keel shape in way of the rudder post. All geometry and resistance data for the systematic hull series are taken from Goertler's Reanalysis of the Taylor Standard Series data [5]. The geometrical parameter space covered by the work in this chapter is large, and includes hull proportions typical of small boats to ships to multihulls.

#### 4.1 Parameter Space

The parameter space is based upon the original parameters varied in the Taylor series experiments. These tests varied B/T,  $C_P$ , and  $C_V$ . The choice of volumetric coefficient,  $C_V$  over length to beam ratio or slenderness is advantageous because the

$C_X$	$E$
0.650	0.8497
0.700	0.6267
0.750	0.4436
0.800	0.2910
0.850	0.1620
0.900	0.0516

**Table 4.1.** Exponents used to modify  $C_X$ .

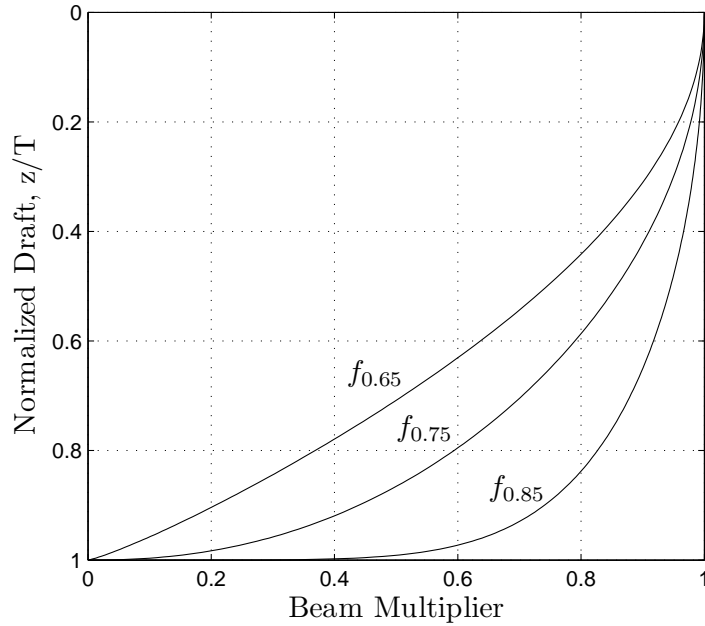
wave resistance coefficient varies linearly with  $C_V$ .

All Taylor series models had a midship section coefficient of 0.925. For the present study, the lines were modified to include three additional midship coefficients. The lines were altered by multiplying each station by a function

$$f_{C_X} = \cos\left(\frac{\pi z}{2T}\right)^E, \quad (4.1)$$

where the exponent  $E$  gives the desired  $C_X$ . Values of  $E$  for several midship coefficients are given in Table 4.1. The cosine function ensures that a knuckle is not formed at the waterline. Plots of the modifying function for the three chosen midship coefficients are shown in Figure 4.1.

Because the Taylor series hulls have different prismatic coefficient but identical midship coefficients, the method used to modify the midship coefficient results in a change to the prismatic coefficient as well. This change from the nominal selected prismatic coefficient is not detrimental, as the network is simply trained with the actual value of  $C_P$  for each hull.



**Figure 4.1.** Functions used to modify midship section. Multiplying each station by the function shown results in  $C_X$  given by subscript.

Once the geometric parameters were selected, a range and set of speeds was required. The goal was to use as few points as possible to reduce the computation time, but to have enough points to capture the features of the highly oscillatory wave resistance coefficient curve. The range of  $Fr = 0.22$  to  $0.60$  was selected to capture the last three peaks in the wave resistance coefficient. This range includes the majority of ship design speeds where wave resistance is important, although some fast multihulls operate above  $Fr = 0.60$ .

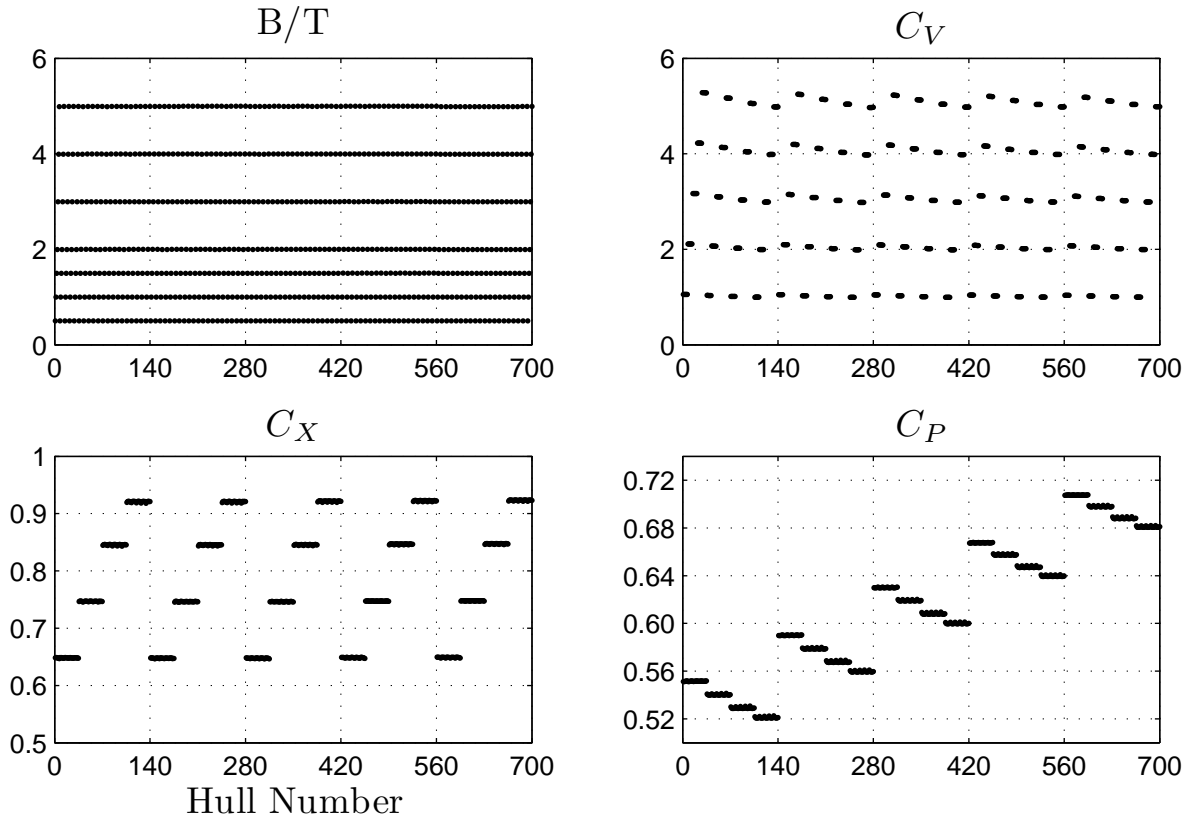
The final set of all parameters is shown in Table 4.2. The total number of Froude numbers for which the results are evaluated is 22, with spacing proceeding in steps of 0.01 from 0.22 to 0.36 and in steps of 0.05 from 0.40 to 0.60. A point at  $Fr = 0.38$  connects the lower and upper speed range. The total number of geometric parameters is 700, resulting from stretching length and beam of 20 unique lines plans. This combination of geometries and speeds required 15,400 SHIPFLOW evaluations.

Parameter	Values	Number
Froude number	0.22 0.23 0.24 0.25 0.26 0.27 0.28 0.29 0.30 0.31 0.32 0.33 0.34 0.35 0.36 0.38 0.40 0.45 0.50 0.55 0.60	22
Prismatic Coefficient	0.52 0.56 0.60 0.64 0.68	5
Midship Coefficient	0.650 0.750 0.850 0.925	4
Volumetric Coefficient	1 2 3 4 $5 \times 10^{-3}$	5
Beam to Draft Ratio	$\frac{1}{2}$ 1 $\frac{3}{2}$ 2 3 4 5	7
<b>Total</b>	700 hulls x 22 speeds	<b>15,400</b>

**Table 4.2.** Parameters used to generate the training set. Prismatic coefficient is nominal, see Figure 4.2 for actual values.

The range of primary input parameters is shown graphically in Figure 4.2. The actual prismatic coefficients due to the change in midship coefficient is evident in the figure. Additional parametric properties of the training set are presented in Figure 4.3. The ranges covered by these parameters encompass a wide range of ship types. A notable parameter to characterize the frictional part of total resistance is the surface to volume ratio. The lower values have less surface area for their enclosed volume, and are relatively more efficient with regard to frictional drag. The range for the ships in the data set is from just over 6 to about 14. A hemisphere has a surface to volume ratio of 3.84, representing the lowest value (it would pay a high price in viscous and wave drag however).

Finally, Figure 4.4 shows simple 10 station body plans for all 20 unique hulls. The effect of equation 4.1 is evident in the midship sections. To see the more subtle effect



**Figure 4.2.** Primary training set parameters. Note modifying  $C_X$  by the present method does not maintain  $C_P$ .

of prismatic coefficient, consider a single bow or stern station and track its centerline offset at the waterline as  $C_P$  increases.

## 4.2 Geometry

The body plan for one of the Taylor hulls is shown in Figure 4.5, with the modification to the stern shown in Figure 4.6. The shape of the Taylor series hulls is that of a pre-dreadnought cruiser, and the afterbody tapers to an integrated rudder post. Including the rudder post and rudder is possible in the panel code, but adds unnecessary complexity. To simplify the hull model and subsequent discretization,

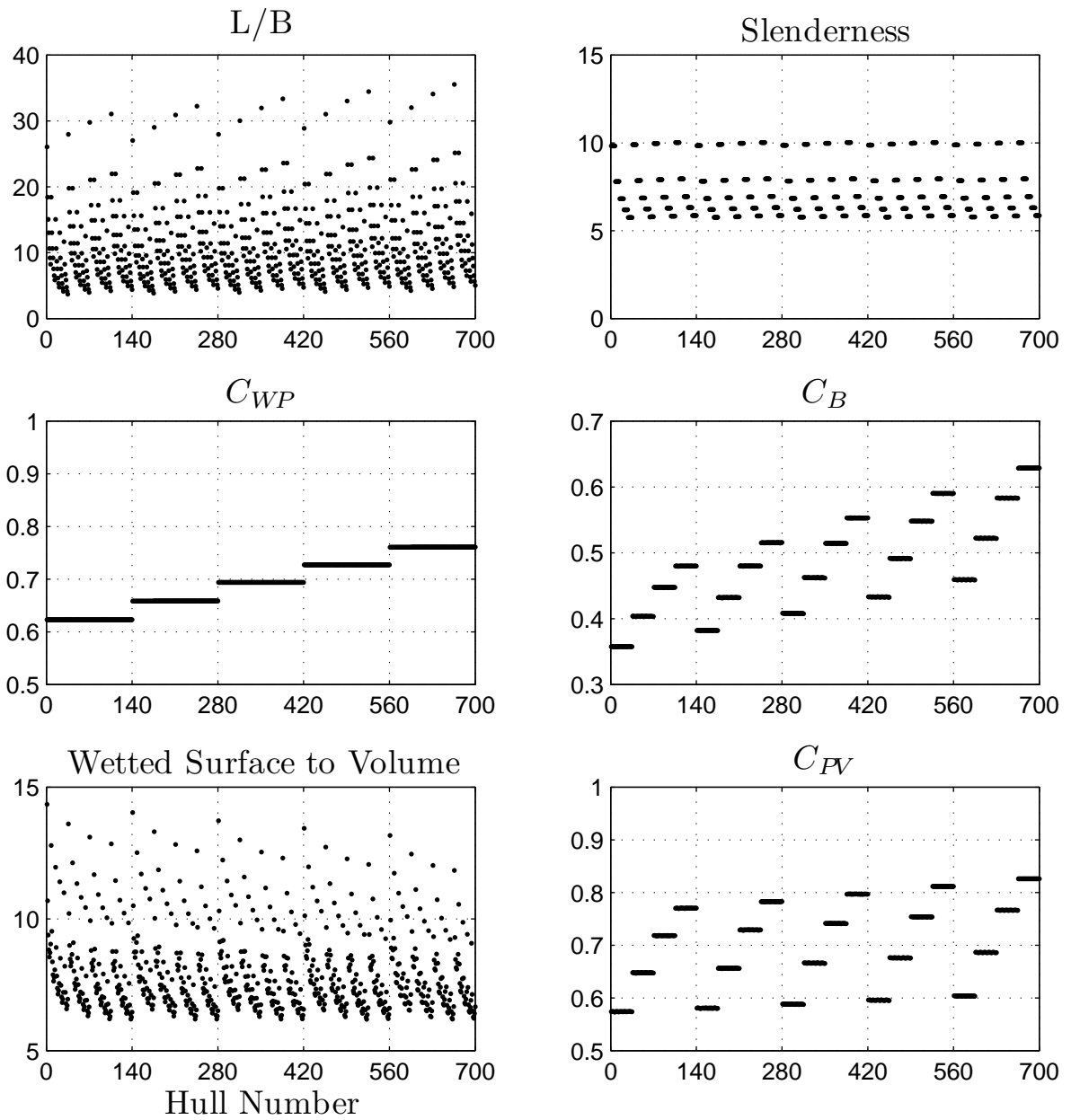
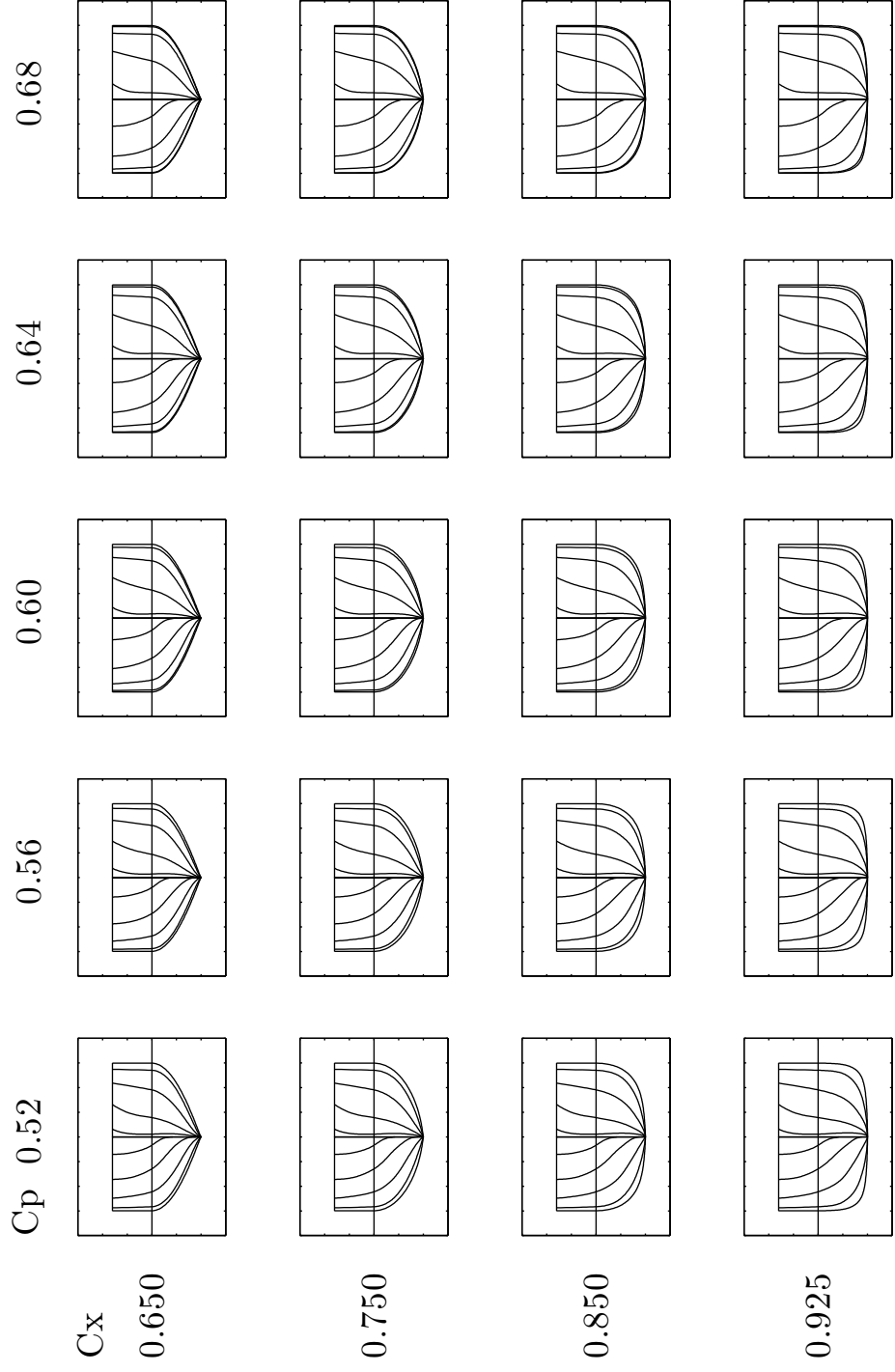
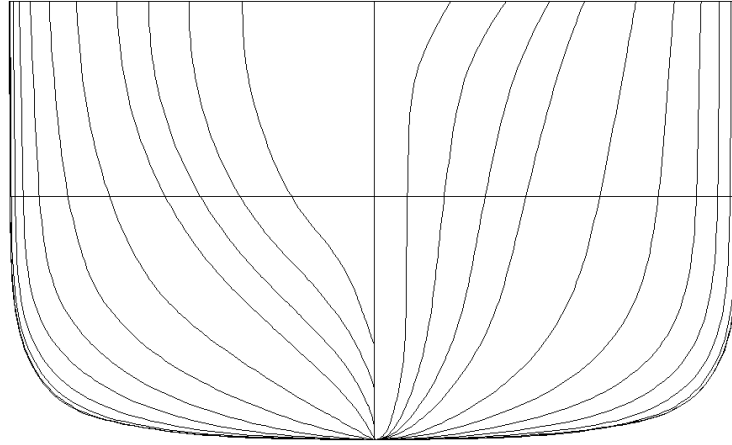


Figure 4.3. Additional training set parameters.



**Figure 4.4.** Body plans of the train sets hulls. Each of these 20 unique shapes is stretched into 35 hulls.  $C_p$  is nominal for  $C_x$  other than 0.925.



**Figure 4.5.** Body plan of a parent hull. For this example  $C_P = 0.68$ ,  $C_X = 0.925$ .

the keel was faired to the shape shown. This modification essentially adds a thin skeg to simplify the stern geometry. The change has negligible impact on the wave resistance and a small impact on the wetted surface.

For each of the 700 hulls, the draft  $T$  was fixed at 2 m and the following equation used to find the length.

$$L = \sqrt{\frac{C_P C_X B}{C_V T}} T^2 \quad (4.2)$$

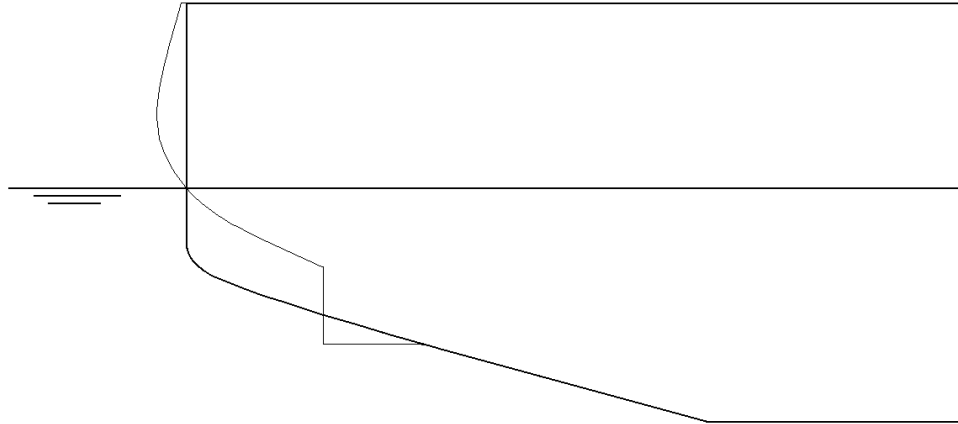
Beam is determined simply by the  $B/T$  ratio.

Three dimensional plots of three of the 700 hulls are shown in Figure 4.7 to better illustrate the geometry and range of shapes. The hull characteristics are given in the caption.

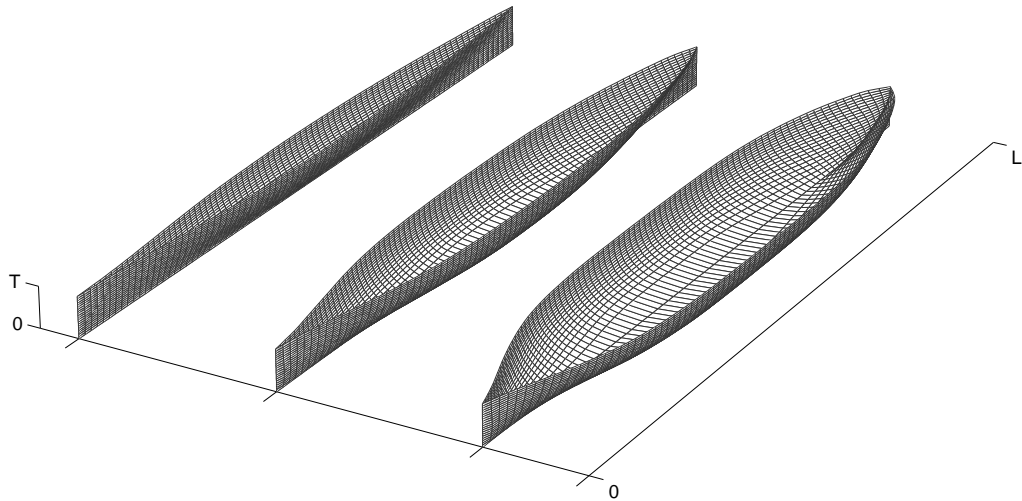
### 4.3 The SHIPFLOW Model

Boundary element potential flow codes such as SHIPFLOW determine the wave drag in a different manner than linear theory. The hull is modeled as distributed sources

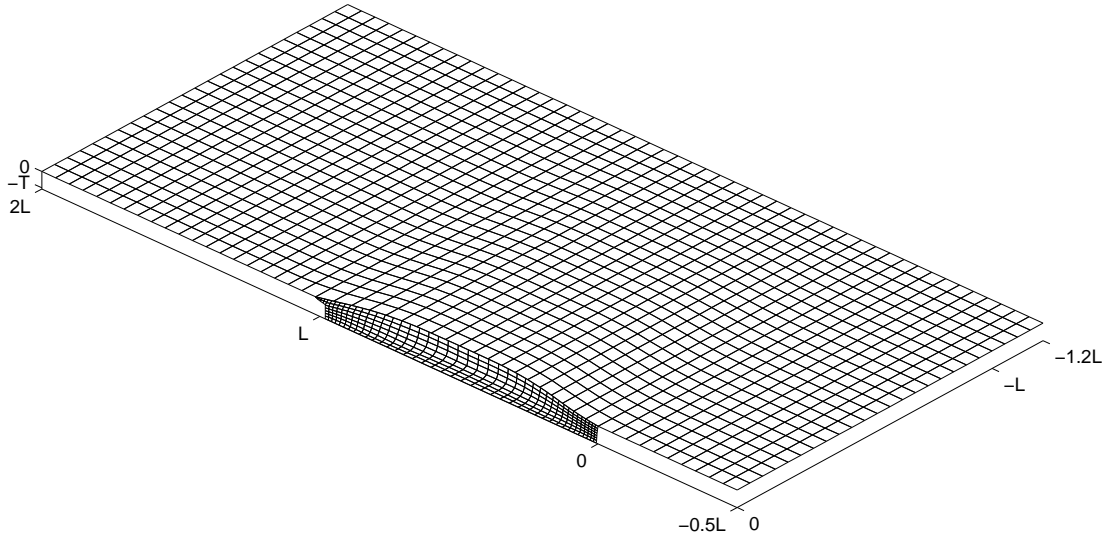




**Figure 4.6.** Simplified stern of SHIPFLOW model.



**Figure 4.7.** Example of hull shapes and proportions. Left to right:  
 $C_P = 0.52$ ,  $C_V = 1e-3$ ,  $C_M = 0.650$ ,  $B/T = 1$ .  
 $C_P = 0.60$ ,  $C_V = 3e-3$ ,  $C_M = 0.850$ ,  $B/T = 2$ .  
 $C_P = 0.68$ ,  $C_V = 5e-3$ ,  $C_M = 0.850$ ,  $B/T = 3$ .



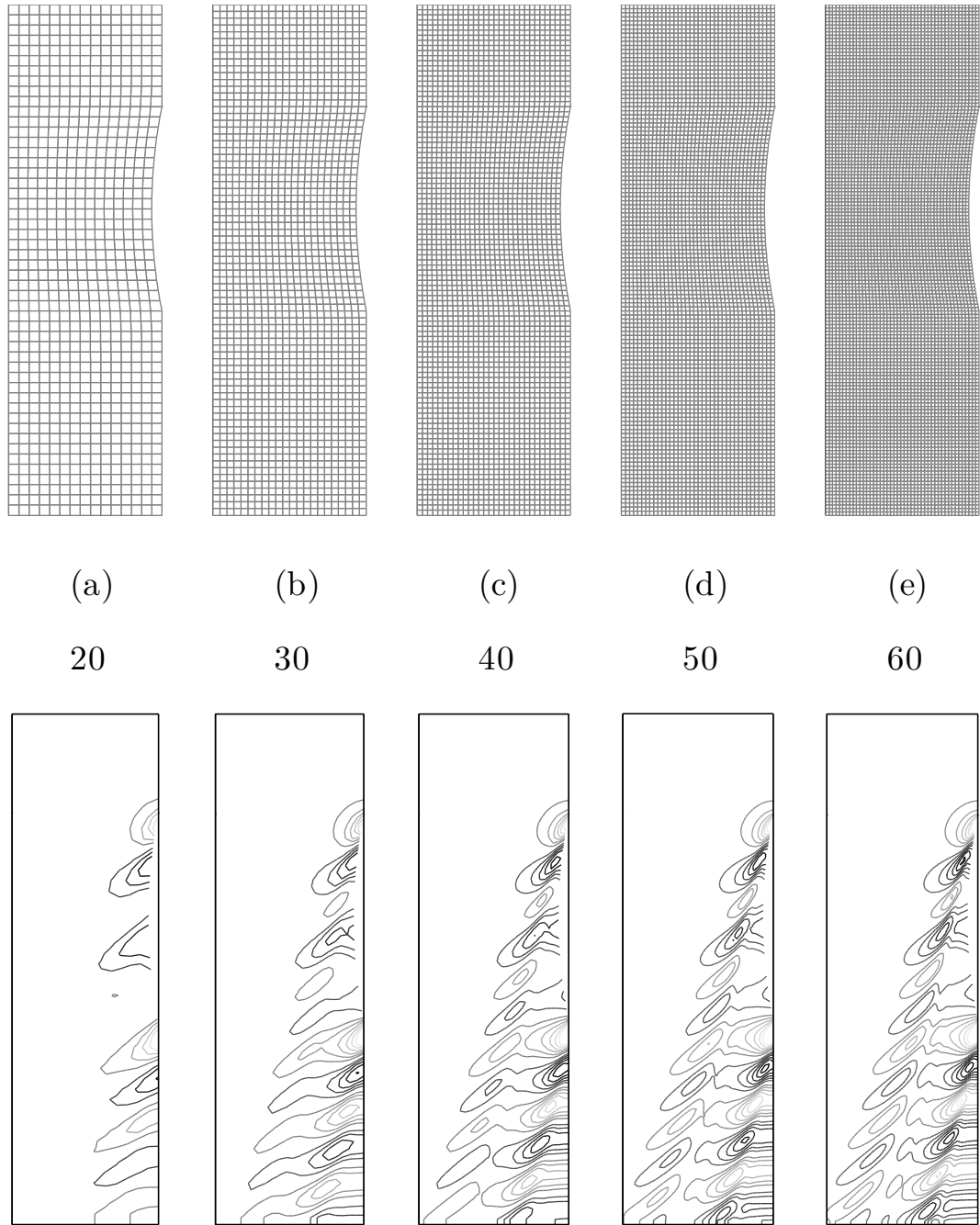
**Figure 4.8.** Typical hull and free surface discretization for SHIPFLOW.

on or near the hull surface instead of on the centerplane. The near field free surface is also discretized into panels, but the water volume is not (hence boundary element). A typical hull and free surface model is shown in Figure 4.8. The code then solves for the drag by integrating the pressure over the hull, taking into account calculated velocities and the near field waves generated by the ship. Since the free surface heights are initially unknown, the free surface boundary condition is applied at the undisturbed flat plane. Results from this calculation are referred to as the linear results. If the calculation is repeated with the boundary condition applied to the new free surface, the analysis can be run until it converges on a free surface shape. These results are called the nonlinear results. In the nonlinear case the free surface intersection with the hull is recomputed and the hull discretization repeated with the new wetted shape.

The discretization of the hull and free surface depend on the hull curvature and Froude number, respectively. Areas of high hull curvature must have a large number of panels since the pressure is integrated over each flat panel. In order to resolve a

free surface wave, the surface panel density must be high enough so that there are a certain number of panels per wavelength. Because the fundamental ship wavelength varies inversely with speed, low speeds require more panels to accurately resolve the wave field. An example of near field wave resolution with increasing panel density is shown in Figure 4.9. The Froude number is 0.23, which corresponds to a fundamental wavelength equal to three waves per hull length. For each case (a) through (e) the free surface panels are shown above with the linear free surface solution below. The numbers represent panels per ship length. For this speed, case (b) has 30 panels per ship length and 10 panels per fundamental wavelength. Case (e) has 60 panels per ship length and 20 panels per wavelength. Note the resolution of certain features moving from case (a) to (e). In case (a), less than 10 panels per wavelength, the shape of the near field waves is not well resolved. The diverging bow waves are not present at all, and the transverse waves cannot be discerned from the diverging stern waves. In case (b) both features start to appear. Note the transverse waves aft of the stern are now discernable from the diverging stern waves. Ten panels per fundamental wavelength is typically the minimum number of panels used in a SHIPFLOW model. As the resolution increases through case (c), (d), and (e), the three primary features become more clear. The bow and stern diverging waves and transverse waves are well resolved in cases (c) and (d). Note that at the highest resolution another smaller diverging wave pattern is visible between the bow and stern waves. This set of waves is due to the change in sectional area curve at midships. Many features of the wave field have a length scale less than the fundamental wavelength, so additional panels continue to resolve these features.

The free surface panels, and the corresponding resolution of the wave field, has a direct impact on the convergence of the wave resistance as well. Figure 4.10 shows the wave resistance coefficient for each case in Figure 4.9. The 20 and 30 panel per



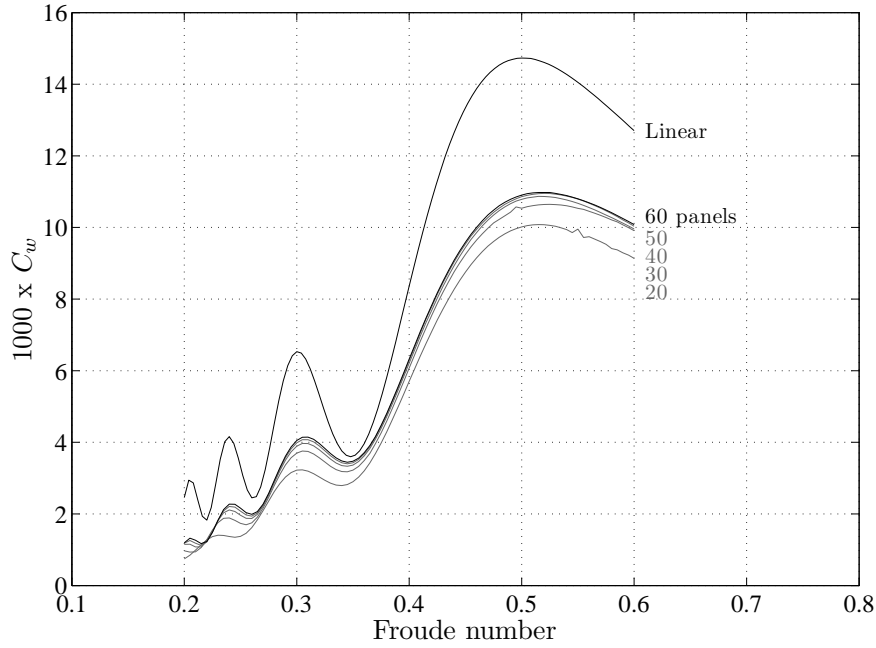
**Figure 4.9.** Convergence of SHIPFLOW wavefield. Each part (a) through (e) shows free surface discretization (above) vs. computed wavefield (below). The number of free surface panels per ship length is given in each case. Hull is simple shape with parabolic waterlines and stations similar to a Wigley hull. Froude number is 0.23.

<b>Fr</b>	<b>Waves per L</b>	<b>Panels per L</b>
0.23	3	59
0.25	2.5	56
0.28	2	53
0.33	1.5	49
0.40	1	46
0.56	0.5	43

**Table 4.3.** Number of fundamental waves per ship length.

ship length cases are not converged, particularly at low speed where the wavelength is small. For reference, the number of waves per ship length is given in Table 4.3. Note that at Froude number 0.40, the fundamental wave length is equal to the ship length. This point is known as hull speed, and generally represents the maximum practical speed for a displacement monohull ship.

The third column in Table 4.3 shows the actual number of panels per hull length used in the SHIPFLOW models. In order to reduce the computation time of SHIPFLOW without compromising the convergence, an adaptive panel density was used. The number of panels per ship length is approximately  $40 + 1/Fr^2$ . The number of panels near the maximum speed is always at least 40, rising to about 60 panels for the lowest speed. Compare the number of panels shown in Table 4.3 with Figure 4.10. Since computation time will be an important comparison, the adaptive panel density ensures that the SHIPFLOW model is not unnecessarily penalized.



**Figure 4.10.** Convergence of  $C_W$  for a SHIPFLOW model. Conditions are identical to case shown in Figure 4.9, with corresponding number of panels per ship length. Linear theory result shown for comparison

## 4.4 Results

The SHIPFLOW wave resistance coefficients for each of the 15,400 runs form the basis for the artificial neural network training set. An identical analysis using the linear theory provides the data to compute the drag ratio used for the corrected method. The computation time for the SHIPFLOW and Michell integral calculations are shown in Table 4.4. The difference is three orders of magnitude. The linear theory has a distinct speed advantage, but that advantage is useful only if it can be corrected to a better estimate of the drag.

Two examples, each representing 35 of the 700 hulls, are shown in Figures 4.11 and 4.12. Figure 4.11 shows all results for  $C_X = 0.650$  and (nominal)  $C_P = 0.52$ . Each subsurface represents the values for a single  $C_V$  over a range of B/T. Figure 4.12 shows all results for  $C_X = 0.925$  and  $C_P = 0.68$ . The Figures show how the

<b>Method</b>	<b>CPU Time</b>
Michell	3.36x10 <sup>3</sup> sec. (56 minutes)
SHIPFLOW	3.70x10 <sup>6</sup> sec. (6.2 weeks)

**Table 4.4.** Computation time.

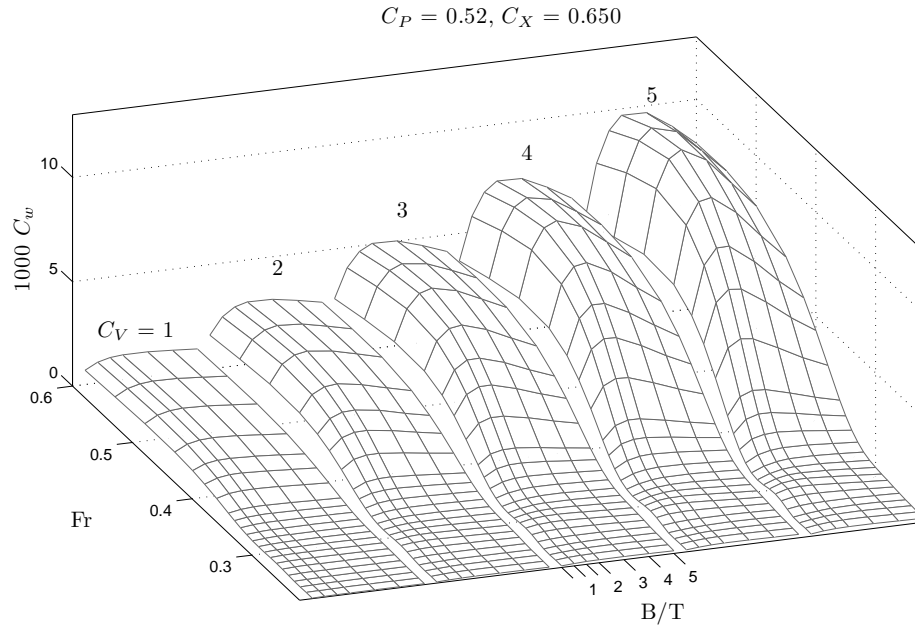
Froude number and B/T spacing resolve the surface in areas where  $C_W$  is sensitive to change in those parameters.

The SHIPFLOW data set is a useful repository of information regarding resistance trends for different hull shape and proportion parameters. Figure 1.1 was generated from the SHIPFLOW data. The entire data set is presented in Appendix B.

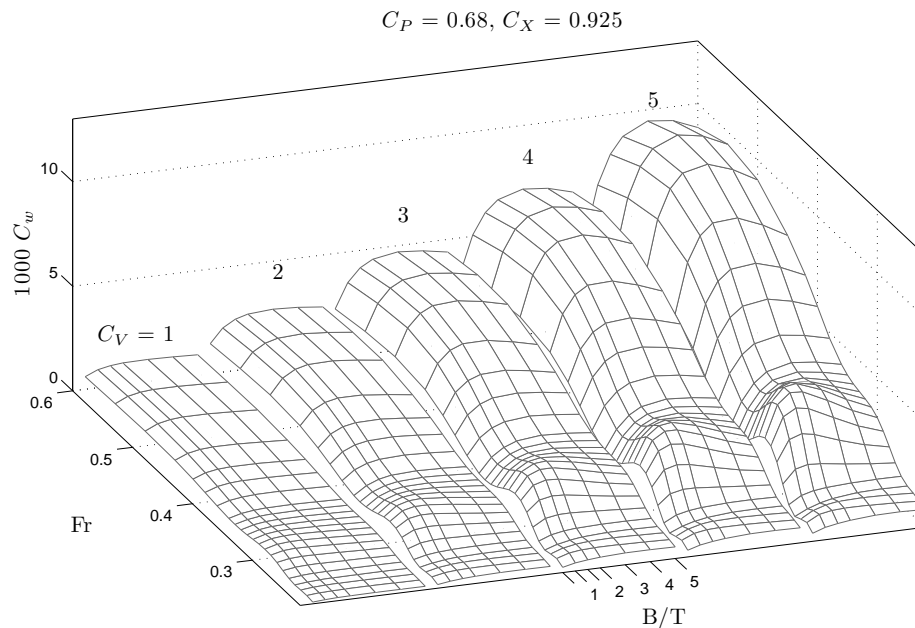
## 4.5 Comparison to Experiment

SHIPFLOW calculations were chosen primarily because an estimate of only the wave drag was desirable for comparison to the linear theory. Experiments must first distill the residual drag from the total drag, then the wave drag from the residual drag. In addition, experiments are almost always run with the ship free to sink and trim under the dynamic pressures on the hull. To that end, all SHIPFLOW calculations were restricted to the linear case, with the hull fixed.

Allowing SHIPFLOW to iterate the free surface in nonlinear mode is desirable, but would have increased the CPU time by an estimated factor of eight (increasing the run time to nearly a year). Fixing sinkage and trim is unrealistic in the speed range tested, but the hull in this position is essentially a slightly different submerged geometry. Once a sinkage and trim estimate is included, the present method will simply see the new geometry.



**Figure 4.11.** SHIPFLOW results for  $C_P = 0.52, C_X = 0.650$ . The five surfaces each show results for seven hulls, representing all 35 hulls produced from the unique lines plan.



**Figure 4.12.** SHIPFLOW results for  $C_P = 0.68, C_X = 0.925$ . Compare to trends in Figure 1.1 and note variable spacing used to resolve the surface.



Finally, one should note that the selection of SHIPFLOW data as the training targets is arbitrary. Another boundary element code or set of experiments could also serve as the training data without changing the method.

To understand the effects of using the fixed, linear SHIPFLOW model, three results are compared to the experiments. Figures 4.13 through 4.15 show the results of four SHIPFLOW calculations along with the Michell integral result and experimental values. The SHIPFLOW calculations include all four possible conditions: fixed linear, fixed nonlinear, free linear, and free nonlinear. The experimental data has been re-analyzed to provide upper and lower bounds to the wave drag.

The experimental curves given in Goertler [5] give coefficients of residual resistance,  $C_R$ , meaning that in addition to wave resistance they contain (primarily) the viscous pressure drag and additional friction due to hull curvature. These  $C_R$  curves came from subtracting the Schoenherr  $C_F$  from the total drag coefficient. Here the total drag has been recomputed and the residual resistance recalculated using the ITTC 1957 guidelines. The experimental  $C_W$  is then calculated using an estimate of form factor,  $k$ . The ITTC '57  $C_R$  and estimated  $C_W$  curves form upper and lower limits for wave drag, respectively.

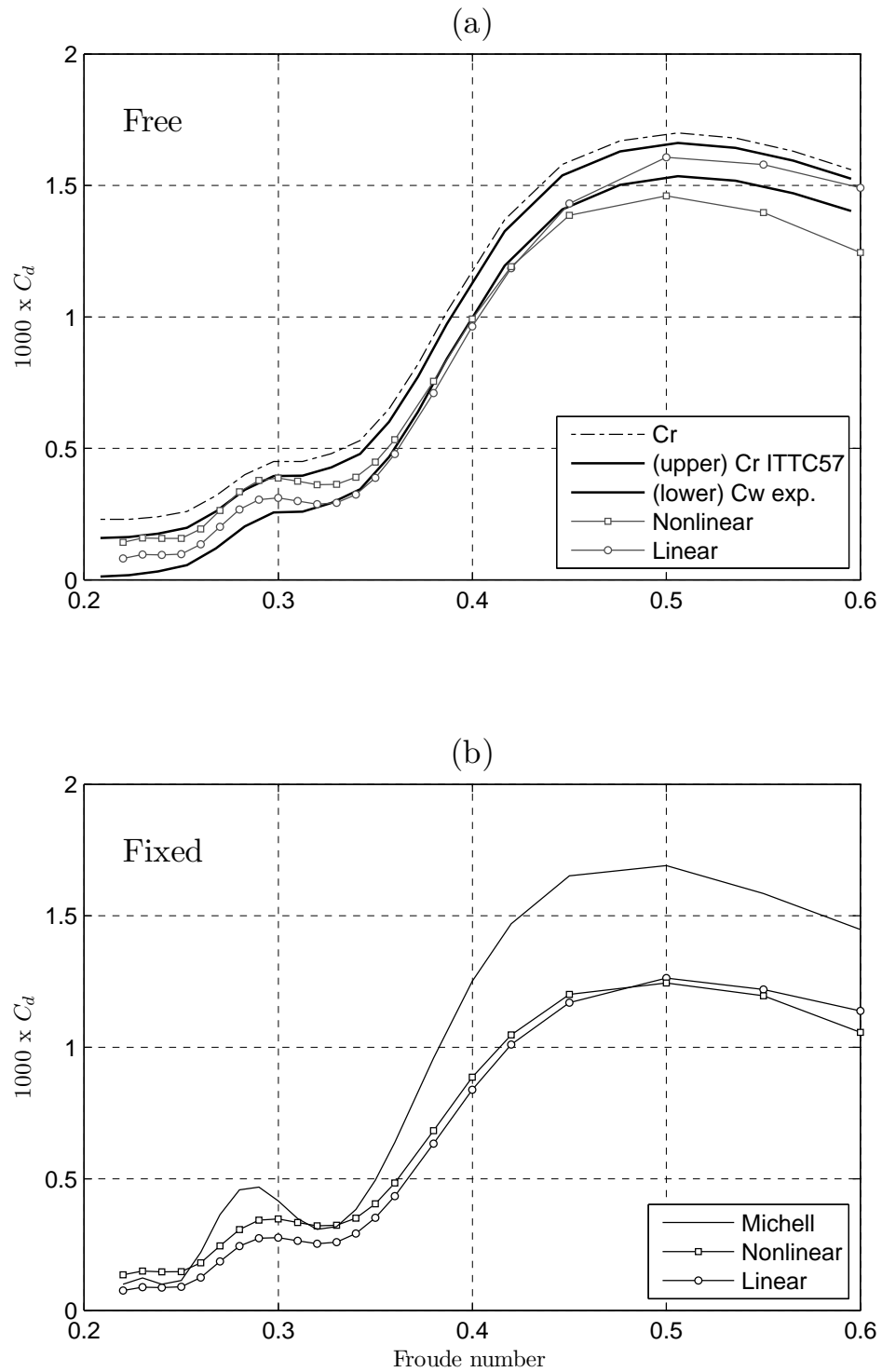
Each plot compares the free to sink and trim SHIPFLOW calculations with the experiment in the upper subplot, and the fixed SHIPFLOW calculations to the Michell integral in the lower subplot. (To avoid confusion between linear theory and linear mode SHIPFLOW, the term Michell is used for the linear theory in the figures.) Thus the accuracy of SHIPFLOW is judged in the upper plot and the suitability of the training data evaluated in the lower plot.

Figure 4.13 shows the comparison for the lowest value of volumetric coefficient. In part a) both the linear and nonlinear SHIPFLOW results compare favorably with the experimental data. The nonlinear  $C_W$  is higher for Froude number less than 0.40, as

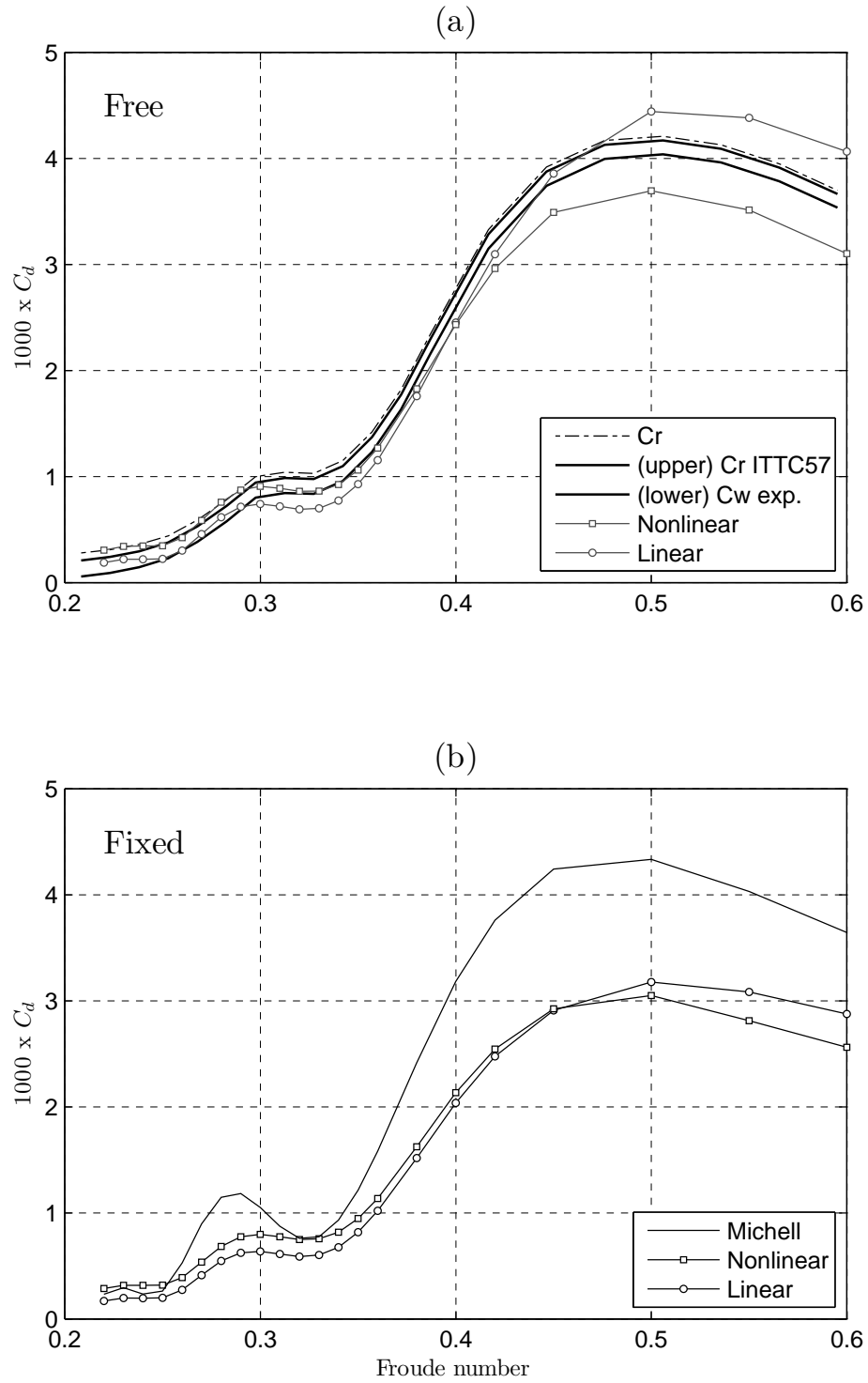
is typical. Above  $Fr = 0.40$  the linear result seems better, but we cannot compare the computed sinkage and trim since it is not provided in [5]. In part b) the hull is fixed but the trends remain the same. The nonlinear SHIPFLOW result is again higher in the low speed range, in this case coinciding with the Michell result from about  $Fr = 0.31$  to  $0.34$ . This case illustrates the Michell integral overestimating the wave drag only at the peaks. The network in the present work is trained with the lower, linear line however.

Figure 4.14 shows the results for a higher  $C_V$ . The trends are very similar to the previous case, except that now the nonlinear SHIPFLOW result is better in the low speed range. Both methods exhibit error at high speed, but again the sinkage and trim computed by SHIPFLOW may not match the experiment. If that is the case, then the submerged geometry SHIPFLOW is using does not match the test. The fixed case exhibits the same trends also, with the nonlinear computation offset higher than the linear case for much of the speed range. The relationship between the three curves in part b) is interesting when compared to the modified linear theory method described in Chapter 3, and will be discussed in more detail in Chapter 6.

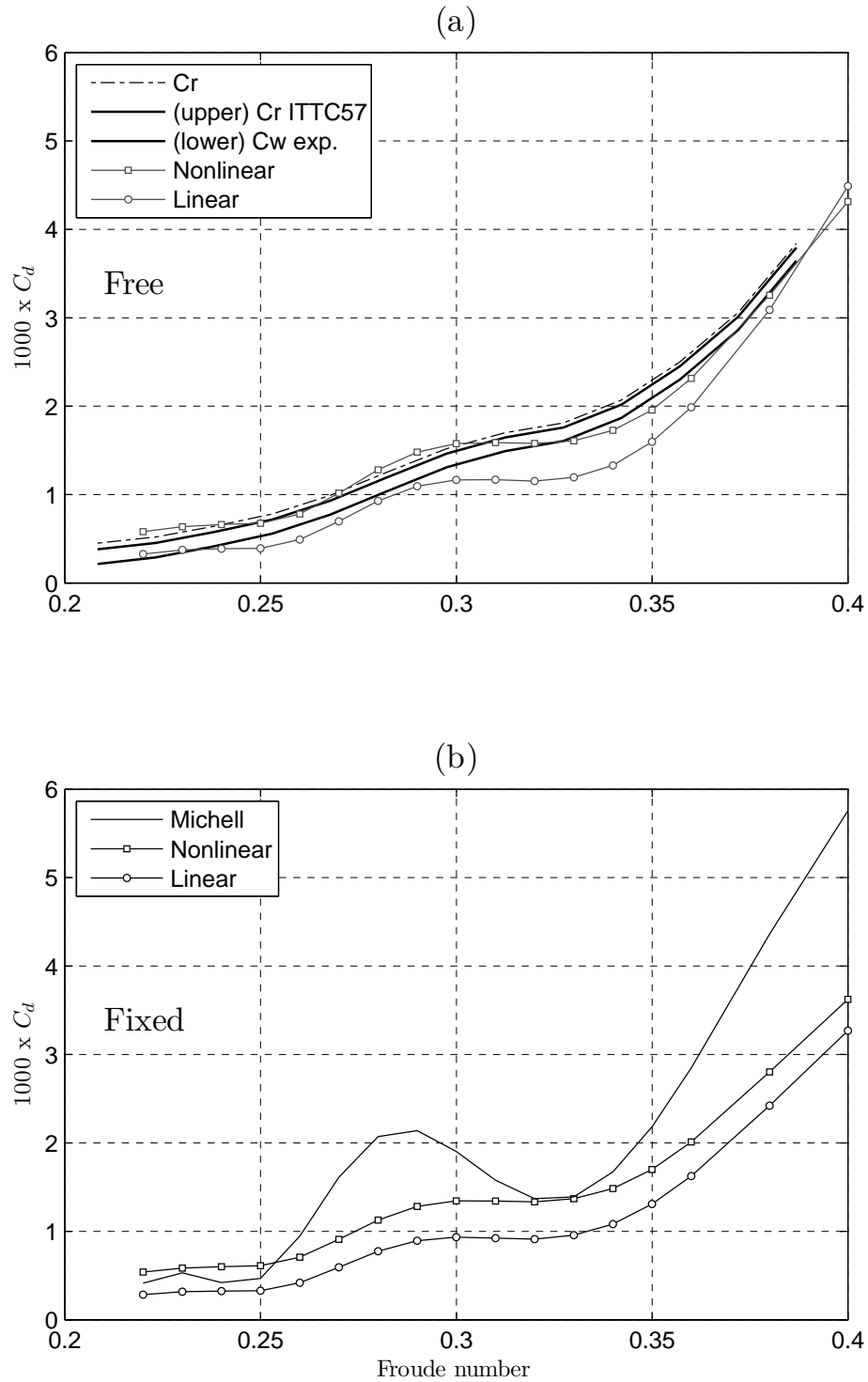
Figure 4.15 shows the final comparison, in this case the hull with the most realistic proportions for a monohull ship. The speed range only goes up to  $Fr = 0.40$  here, as experimental data is not provided above the point shown. This hull has a volumetric coefficient of 3 and  $B/T$  of 3, so the proportions of the ship are not as “thin” as the previous cases. Here the benefit of the nonlinear evaluation becomes more apparent. In the free to sink and trim comparison in part a), the nonlinear result is better than the linear result, especially in the important range from  $Fr = 0.30$  to  $0.40$ . In part b) though, the trend remains the same as for the more slender ships. The Michell integral still captures the physics of the problem as well as in the previous two cases.



**Figure 4.13.** Comparison of SHIPFLOW with experiment for  $C_V = 1$ ,  $B/T = 2.25$ . Part (a) shows free to sink and trim results while Part (b) shows fixed results.



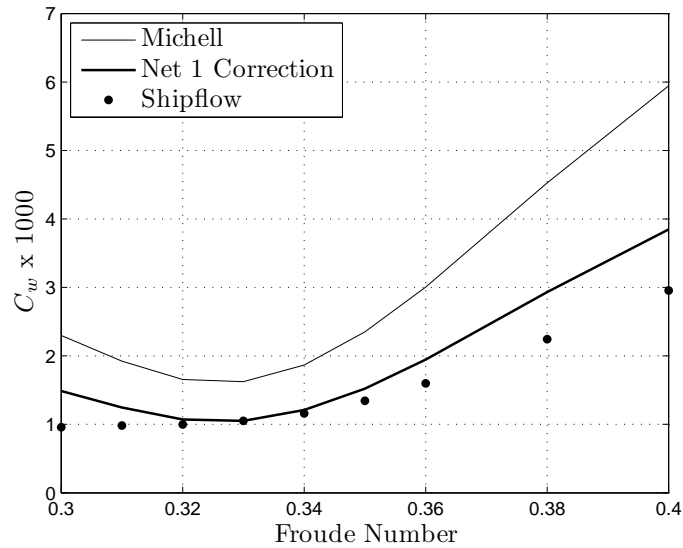
**Figure 4.14.** Comparison of SHIPFLOW with experiment for  $C_V = 2$ ,  $B/T = 2.25$ . Part (a) shows free to sink and trim results while Part (b) shows fixed results.



**Figure 4.15.** Comparison of SHIPFLOW with experiment for  $C_V = 3$ ,  $B/T = 3.00$ . Part (a) shows free to sink and trim results while Part (b) shows fixed results.

# 5. Artificial Neural Network Training and Implementation

Four artificial neural networks are implemented and evaluated for performance. Two networks are implemented for the corrected linear theory and two for the modified linear theory. The first corrected linear theory network includes only the four geometry variables of the parameter space  $(C_V, C_P, C_X, B/T)$ . This network computes the linear theory correction at a single Froude number in order to reduce the network complexity and provide an initial implementation of the method. The second network is identical to the first stage network, but the entire range of Froude numbers from the training set is included, increasing the number of inputs to five. The third network implements the modified linear theory, removing Froude number from the inputs, but increasing the number of outputs. Finally, based on the comparison to experiments in Chapter 4, the fourth network reduces the number of outputs back to one so that the complexity is similar to network 1. The modified theory networks also allow a rough estimation of the wave field instead of just the drag.



**Figure 5.1.** Single Froude number correction.

## 5.1 Network 1

Network 1 attempts to correct the linear theory at one Froude number, and then apply that simple correction over the entire range. By picking the single Froude number at the last trough in the wave resistance coefficient, the corrected data should behave as the example shown in Figure 5.1. In this case the method would provide improved performance in the range of Froude numbers from 0.30 to 0.40, a useful optimization range for many ships. The location of the  $C_W$  trough just before hull speed is dependent primarily on prismatic coefficient,  $C_P$ . Table 5.1 shows the target Froude number used for the nominal prismatic coefficients.

### 5.1.1 Training and Validation Set

The training targets for network 1 are the ratios of SHIPFLOW  $C_W$  to Michell integral  $C_W$ . The targets for a surface representing two of the four inputs are shown as squares connected by solid lines in Figure 5.2. The surface shows the expected relationship

$C_P$	Target Fr
0.52	0.31
0.56	0.32
0.60	0.33
0.64	0.34
0.68	0.35

**Table 5.1.** Froude number for network target.

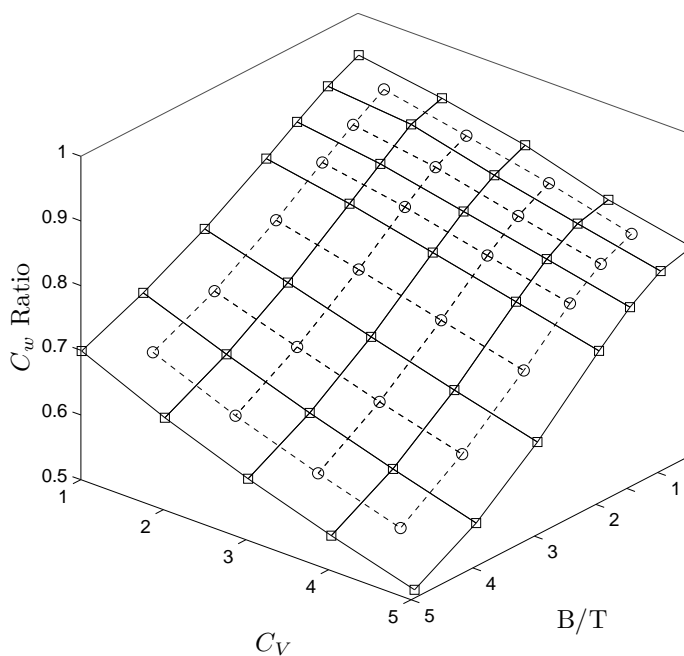
between SHIPFLOW and the linear theory, with low  $B/T$  and low  $C_V$  values needing the least correction. Note that there are 20 such surfaces to define the four dimensional problem.

Instead of segregating data points from the SHIPFLOW data set into training and validation points, the validation points are taken as the linearly interpolated midpoints of the training points. The validation points are shown as the circles and dotted lattice in Figure 5.2. The purpose of the validation points is to detect overfitting by the network, that is to prevent a network that goes exactly through all the training points but exhibits large error everywhere else. By using the midpoints shown, overfitting is detected without running more SHIPFLOW analysis or taking points away from the training set.

### 5.1.2 Network Architecture and Performance

The network 1 architecture is a simple feedforward backpropagation network, as shown in Figure 5.3. The network consists of the four geometric inputs, a single hidden layer with sigmoidal activation functions, and one output layer with a lin-

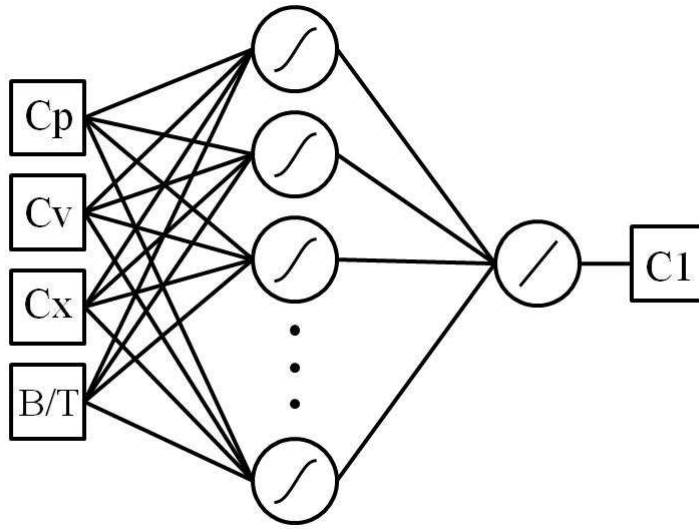




**Figure 5.2.** Target and validation values for network 1. Target values (dark lines) and validation values (dotted lines) are shown for two of four inputs.

ear activation function. Training is accomplished using the Levenberg-Marquardt algorithm [10]. Biases are not shown.

The next step is to determine the number of neurons to use in the hidden layer. For this simple network, the training time is short enough that a detailed analysis of the optimum number of neurons is practical. This calculation is done by training the network several times for each possible number of neurons and analyzing the error at both the training and validation points. The results of such an analysis are shown in Figure 5.4. The number of neurons varies from 6 to 30, with 10 networks trained for each number of neurons. (The weights are randomly initialized, leading to different results for each of the 10 networks.) The open circles represent the mean square error for the training set. Typically, the networks will have similar performance on the training set, and the error will continue to go down as more neurons are added. Such performance is seen here, with the error for the training points clustered along

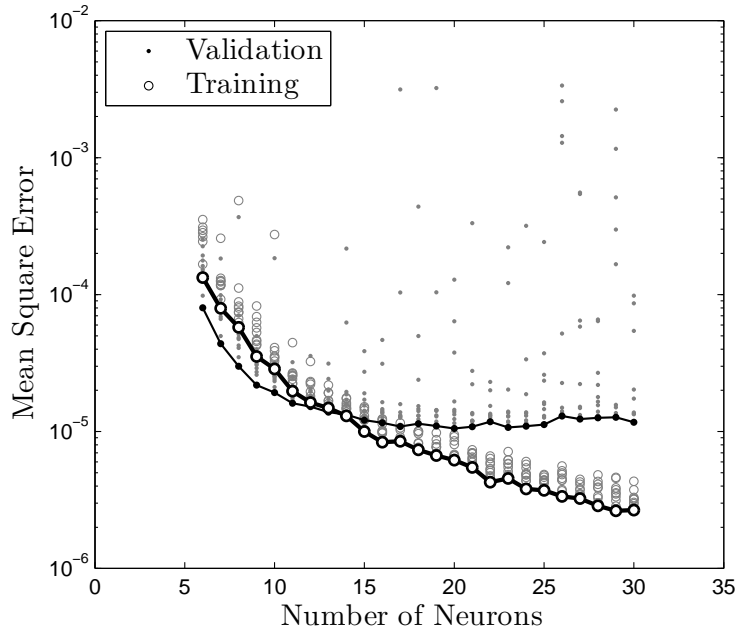


**Figure 5.3.** Network 1 architecture. The network has four inputs, a single layer of sigmoidal activation functions, one output.

the line of optimum performance. The key to picking the correct network lies in the validation data, represented by dots in the plot. The line of least error for the validation data does not continue to improve past a hidden layer size of about 15 neurons. In addition, the validation set errors do not lie near the line of least error, especially as the number of neurons becomes large. These validation error points far above the minimum error represent networks that are overfitting the data. Think of these networks as fitting a sine wave through three equally spaced points that should form a straight line.

### 5.1.3 Results of Network 1

From Figure 5.4, a hidden layer size of 12 neurons was selected as a good compromise between complexity and accuracy. Candidate networks with low error on both the training and validation set were tested over the parameter space and plotted against the data set for all 20 surfaces like the one in Figure 5.2. From these networks, the

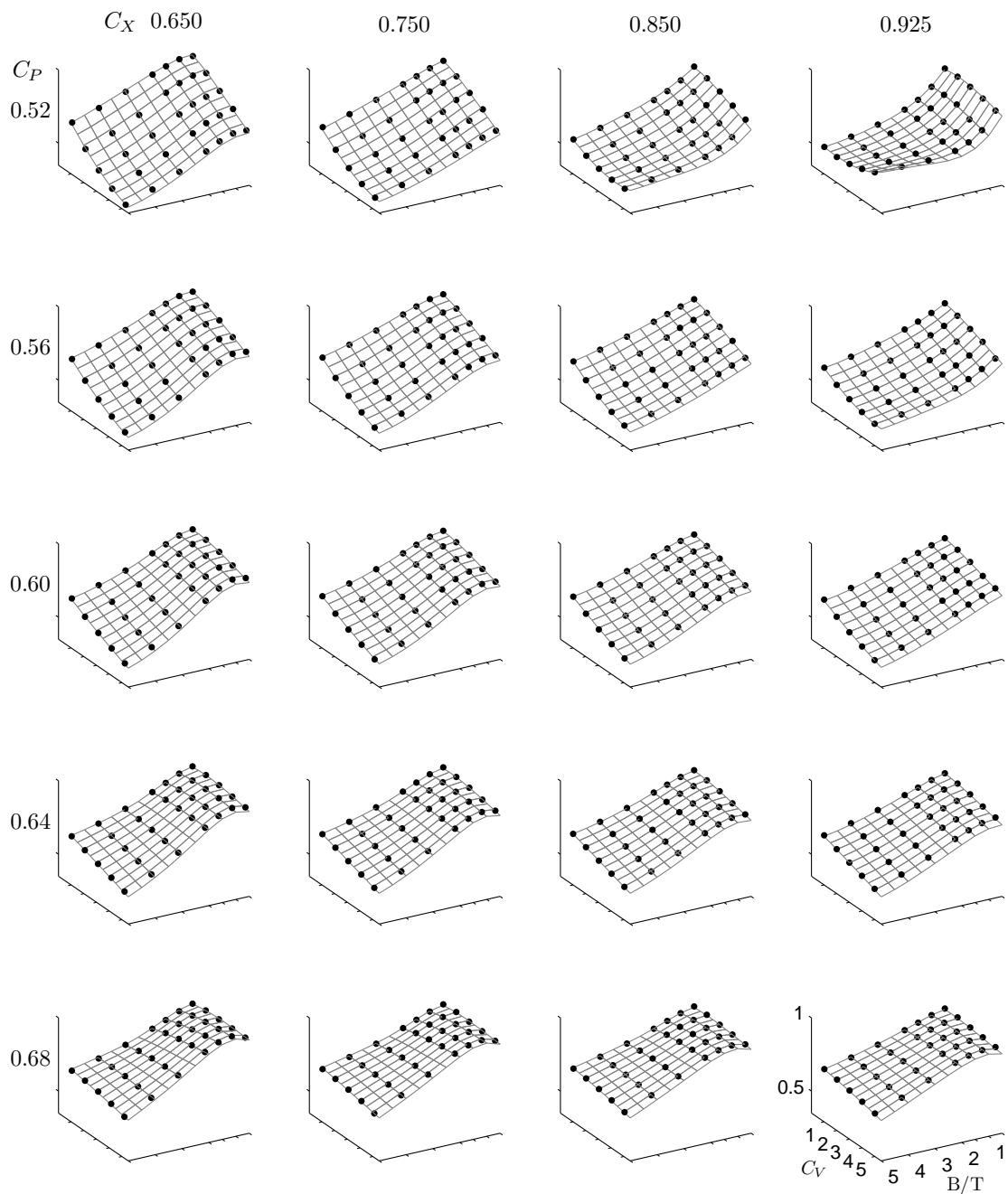


**Figure 5.4.** Network 1 size optimization. Dark markers represent best network for each number of neurons. Gray markers far from best networks caused by overfitting.

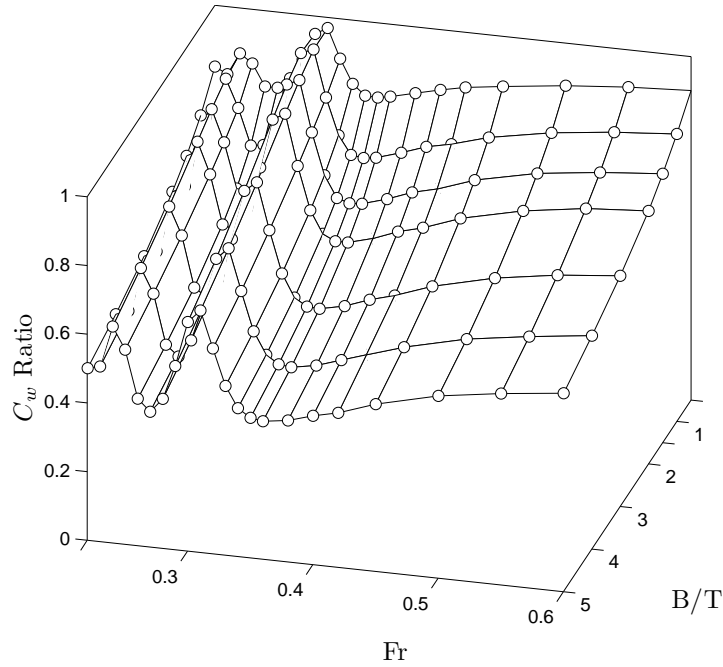
one with the best performance was selected based on a visual evaluation, as shown in Figure 5.5.

## 5.2 Network 2

Network 2 corrects the linear theory explicitly at every Froude number in the data set. Instead of using the ratio of SHIPFLOW  $C_W$  to linear theory  $C_W$  at a single Froude number, the network uses the ratio at each Froude number as a training target. Adding Froude number as a network input increases the complexity of the network not only by adding a dimension, but by the oscillatory nature of the  $C_W$  dependence on Froude number.



**Figure 5.5.** Network 1 results. Performance of chosen network 1 shown for all inputs. Black dots represent training data. Mesh calculated from simulating network over fine grid of input values.



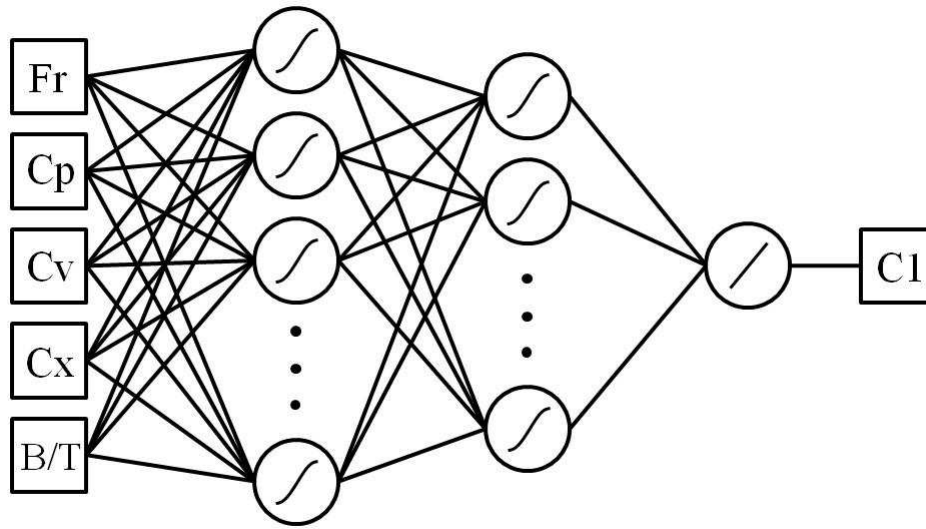
**Figure 5.6.** Target and validation values for network 2. Validation values omitted for clarity.

### 5.2.1 Training and Validation Set

The training set for network 2 uses all 15,400 points from the SHIPFLOW data set. A two dimensional portion of the five dimensional targets is shown in Figure 5.6. In this case there are 100 more surfaces such as this one in the training set. Again the trend in the  $C_W$  ratio is as expected, with low B/T cases and Froude numbers corresponding to troughs in the  $C_W$  curve having values nearest 1. The validation set is calculated as the center of the panels shown, just as with network 1. For clarity, the validation points are not shown on the figure.

### 5.2.2 Network Architecture and Performance

The network 2 architecture also consists of a feedforward backpropagation network trained with the Levenberg-Marquardt algorithm. The network consists of five inputs,



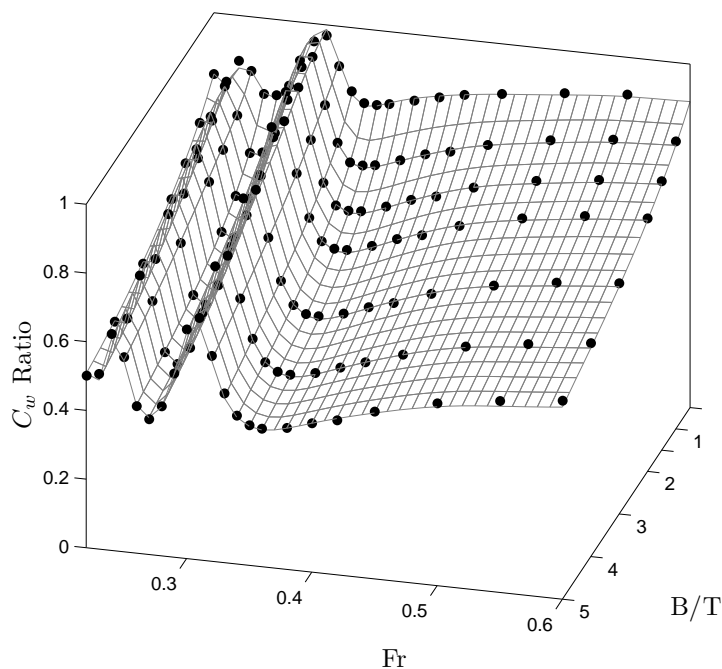
**Figure 5.7.** Network 2 architecture. The network has five inputs, two hidden layers with sigmoidal activation functions, one output.

two hidden layers with sigmoidal activation functions, and one output from a linear activation function. The network is shown in Figure 5.7.

The best number of hidden layers and neurons in each layer was determined by trial and error. Due to the increased complexity of the network, repeating the analysis used to demonstrate the optimization of network 1 would require significant computation time. Single hidden layer networks did not exhibit acceptable performance for network 2. Networks with two hidden layers were tested with varying numbers of neurons, with the validation error used to pick candidate networks.

### 5.2.3 Results of Network 2

The candidate networks were tested over a fine grid of input values and plotted much like Figure 5.5. Based on the trial and error process, a network with 20 neurons in the first hidden layer and 10 neurons in the second hidden layer was selected. The network was checked visually against all 100 surfaces like the one in Figure 5.6. A



**Figure 5.8.** Network 2 results. Performance of chosen network 2 shown for two of five inputs. Black dots represent training data. Mesh calculated from simulating network over fine grid of input values.

single example of the network output corresponding to the same surface is shown in Figure 5.8.

### 5.3 Network 3

Network 3 is used to determine the unknown coefficients  $C_1$  and  $C_2$  for the modified linear theory. Since these coefficients are the multipliers to the  $P_1$  and  $Q_1$  terms from linear theory and the  $P_2$  and  $Q_2$  waterline integrals from the new modified theory term, respectively, the target values for the network are not yet known. In addition, the coefficient in  $\beta$  must be determined, as well as the value of  $\Omega$ .

### 5.3.1 Determination of Network Targets

A constrained optimization routine is used to determine the unknown coefficients for different values of  $\beta$  and  $\Omega$  in the waterline integral coefficient ( $\beta$  values inside the integral are set to one). The MATLAB function `fmincon` is employed with the difference between the standard and modified linear theory as the function to be minimized. An experimental approach is undertaken to determine the best selection of  $\beta$  and  $\Omega$  that will allow the coefficients  $C_1$  and  $C_2$  to minimize the error over the entire range of Froude numbers. The values of  $\beta$  in this experimental approach correspond to the shapes in Figure 3.1. Various values of  $\Omega$  are tried such that  $\Omega$  is a constant or varies directly or inversely with  $Fr$  or  $Fr^2$ . Values of  $\beta$  that vary with Froude number were also included, but more work is necessary to evaluate their performance. These “experiments” judged not only the effect on the wave resistance coefficient, but on the wave energy distribution as well. Some results that matched the SHIPFLOW data exactly distorted the energy distribution too much and were discarded. The best fit is found to be

$$\beta = 1 + 2 \tan \theta^2 \quad (5.1)$$

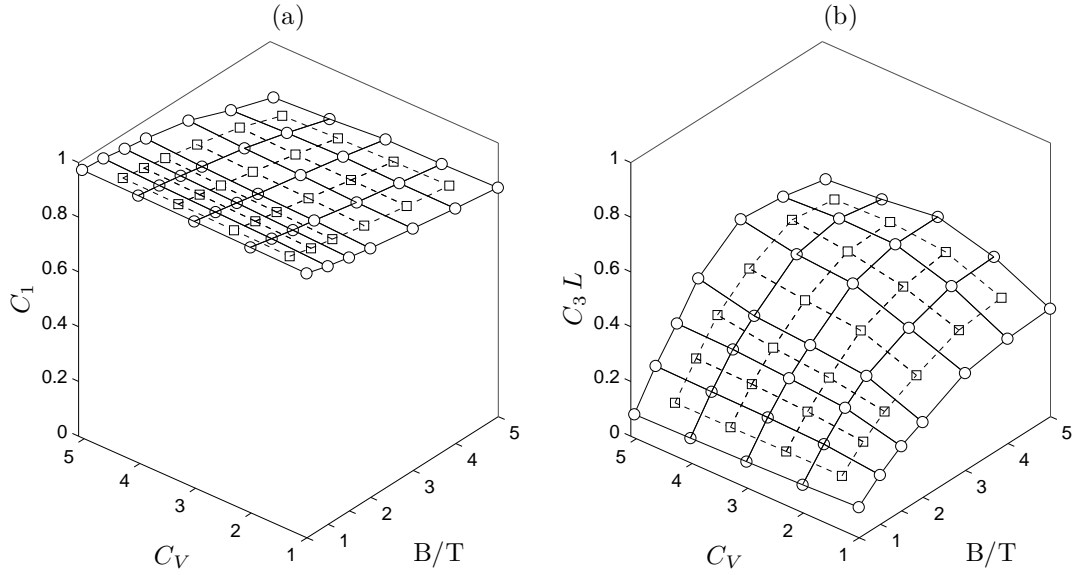
with

$$\frac{\Omega}{U^2} \propto \frac{1}{Fr^2}. \quad (5.2)$$

This relationship is achieved by setting  $\Omega = C_3 g L$  so that the numerator of the waterline term coefficient is  $C_3 k_0 L$ . The coefficient in front of the  $P_2$  and  $Q_2$  integrals (equations 2.51 and 2.52) becomes:

$$\frac{C_3 L}{(1 + 2 \tan \theta^2)^3} \quad (5.3)$$



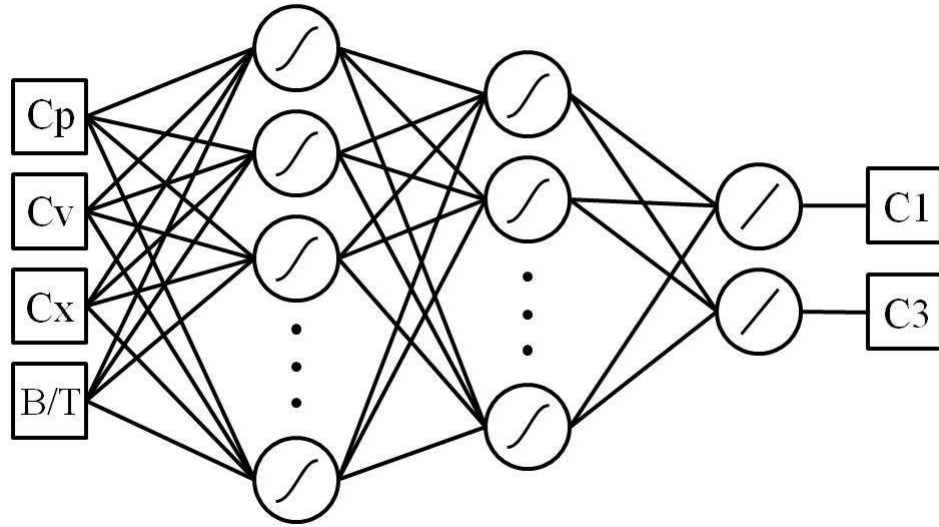


**Figure 5.9.** Target and validation values for network 3. Target values (dark lines) and validation values (dotted lines) for output values  $C_1$  (a), and  $C_3$  (b), plotted for two of four inputs.

The output of the constrained optimization (error minimization between linear theory and SHIPFLOW) then provides the two target values for network 3.

### 5.3.2 Training and Validation Set

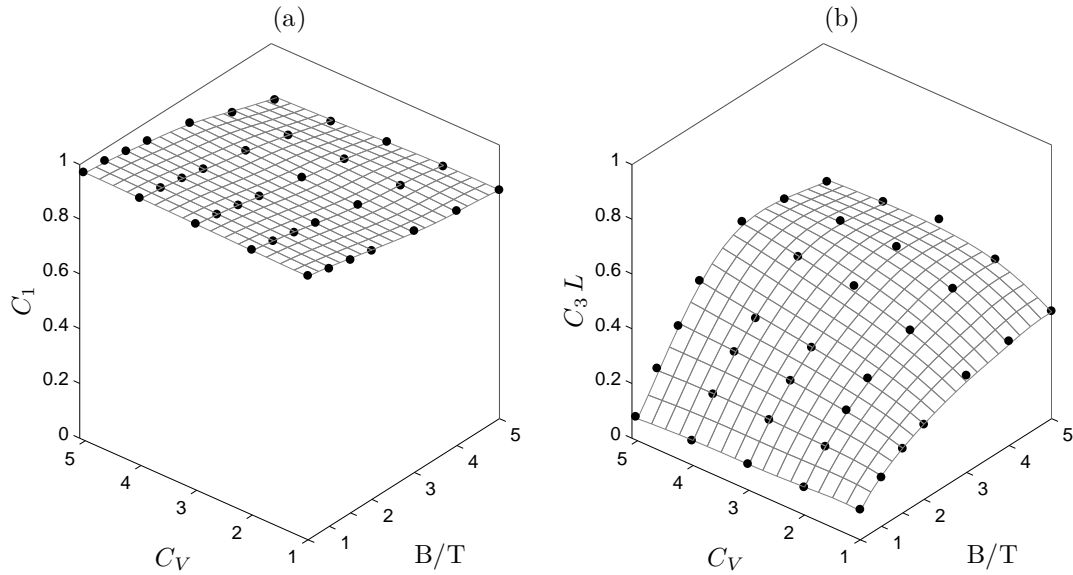
The training set for network 3 consists of the multiplier on the standard linear theory terms ( $C_1$ ) and the multiplier on the waterline integral terms ( $C_3$ ). These multipliers are known from the constrained optimization for each combination of geometric parameters in the training set. Since the modified theory provides a correction over the entire speed range, Froude number is not an input for network 3. One surface for each of the coefficients is shown in Figure 5.9, with  $C_3L$  plotted in part b) for convenience of scaling. Again these surfaces are each one of twenty that describe the output space. Validation of the network is performed as in the previous cases.



**Figure 5.10.** Network 3 architecture. The network has four inputs, two hidden layers with sigmoidal activation functions, two outputs.

### 5.3.3 Network Architecture and Performance

Network 3 is also a feedforward backpropagation network with Levenberg-Marquart training, with activation functions as shown in Figure 5.10. Though other architectures were tried, this type of network worked consistently for this study. The number of layers and neurons was determined in the same manner as that of network 2. The removal of Froude number from the inputs removes the oscillatory component of the targets and reduces the number of surfaces back to 20 from 100. With two outputs, however, a single hidden layer network again proved insufficient. Two hidden layers, with 14 neurons in the first and 10 in the second, provided good results without overfitting. The target surfaces for network 3 were much simpler than those of network 2, and fewer neurons were required.



**Figure 5.11.** Network 3 results. Performance of chosen network 3 for output values  $C_1$  (a), and  $C_3$  (b), plotted for two of four inputs. Black dots represent training data. Mesh calculated from simulating network over fine grid of input values.

### 5.3.4 Network 3 Results

The output of the network was again checked visually against the targets using a fine grid of points over the parameter space. Combined with the numerical error of the validation set, this visual check ensures the network is not overfitting the data. The result of the chosen network is shown in Figure 5.11 for the same case as Figure 5.9.

Because the targets for network 3 were determined by constrained error minimization in MATLAB, they are not as “smooth” as the targets formed from the ratio of SHIPFLOW and linear theory data. For this reason the network error for the training and validation sets was higher than in networks 1 and 2, but the network gives a smooth fit to the data.

Type	Name	Inputs	Neurons		Outputs
			Layer 1	Layer 2	
Corrected	Network 1	4	12		1
Linear Theory	Network 2	5	20	10	1
Modified	Network 3	4	14	10	2
Linear Theory	Network 4	4	12		1

**Table 5.2.** Summary of artificial neural networks.

## 5.4 Network 4

Looking at the comparison to experiment in Chapter 4, note that in the plots of fixed sinkage and trim, the SHIPFLOW nonlinear result is generally slightly higher than the linear result and coincident with the Michell integral at the trough in  $C_W$ . We know from Chapter 3 that the waterline integral of the modified theory can have an effect similar to this behavior. In fact, these plots look very much like the results that will be presented in Chapter 6, where the effects of the two outputs from network 3 are considered separately. The output  $C_1$  is a multiplier on the standard linear theory, much like network 1, while the output  $C_3$  is responsible for reducing the drag at the peaks in  $C_W$ .

For this reason, consider a network 4 that finds  $C_3$  only. There is no reason that two separate networks could not be used to find  $C_1$  and  $C_3$ , with architectures like the simple network 1. The details of these networks are not presented here, as the architecture is identical to network 1 and the inputs and results are identical to network 3. The advantage is that  $C_1$  or  $C_3$  can each be found with a single layer network consisting of only 12 neurons. Table 5.2 is a summary of the networks.

## 6. Results and Discussion

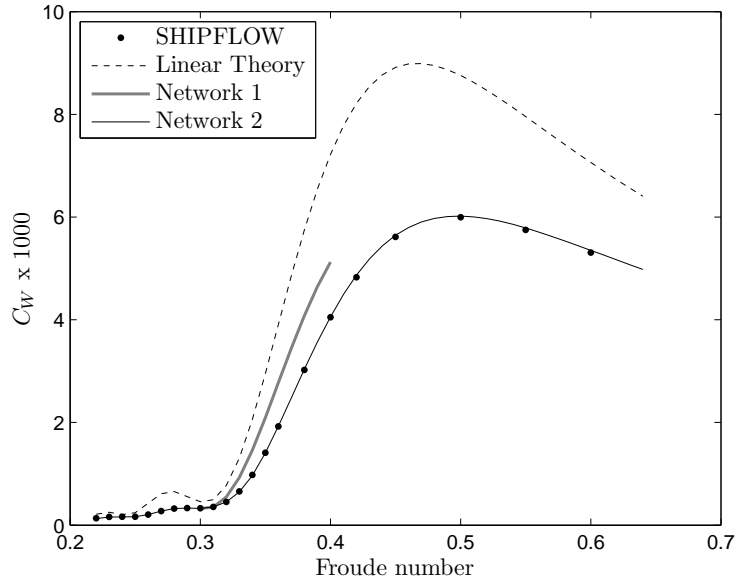
### 6.1 Results for Taylor Series Hulls

The artificial neural network results in Chapter 5 examine the networks' ability to match the target values. The targets are either ratios of wave resistance coefficients from different methods or sets of unknown coefficients determined by error minimization. None of the target values are actually the wave resistance, since both methods employ the standard linear theory to baseline the drag. Here we examine the results of the network output compared to the actual wave drag for the Taylor series hulls. These are the hulls used to train the network, so the predicted wave drag should closely match the SHIPFLOW values.

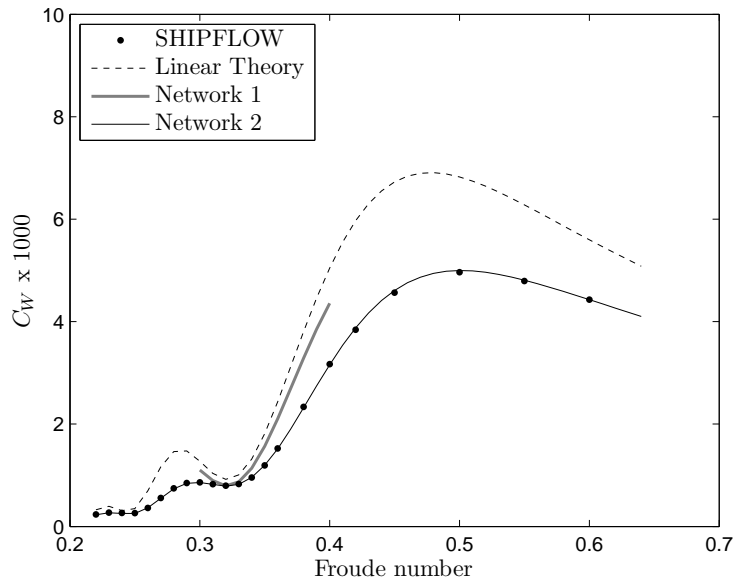
#### 6.1.1 Corrected Linear Theory Networks

The corrected theory networks use the network output as a multiplier for the standard linear theory wave resistance. Results for six of the 700 hulls are shown here in Figures 6.1 through 6.6. In each case the standard linear theory result is represented by a dashed line while the SHIPFLOW data are shown as black dots.

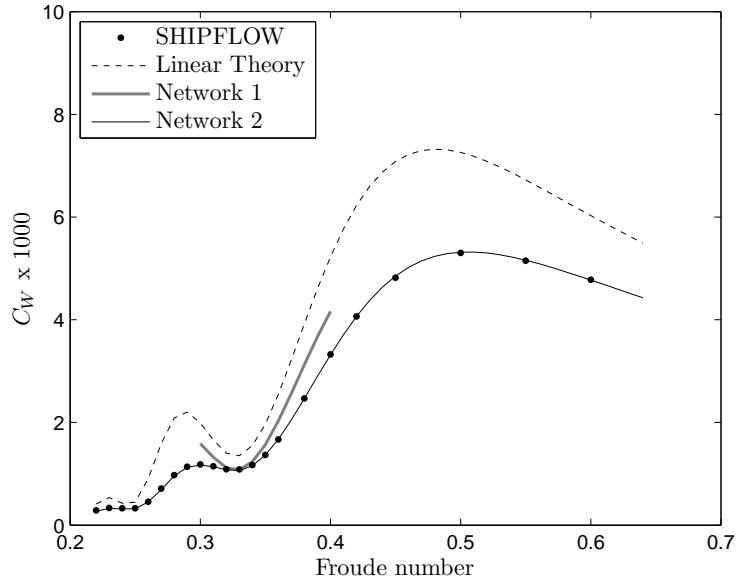
The network 1 result is shown as a heavy gray line. This network matches the SHIPFLOW data only at the trough in the wave resistance curve, but has been applied to the Froude number range 0.30 to 0.40 to see if it might be useful in that



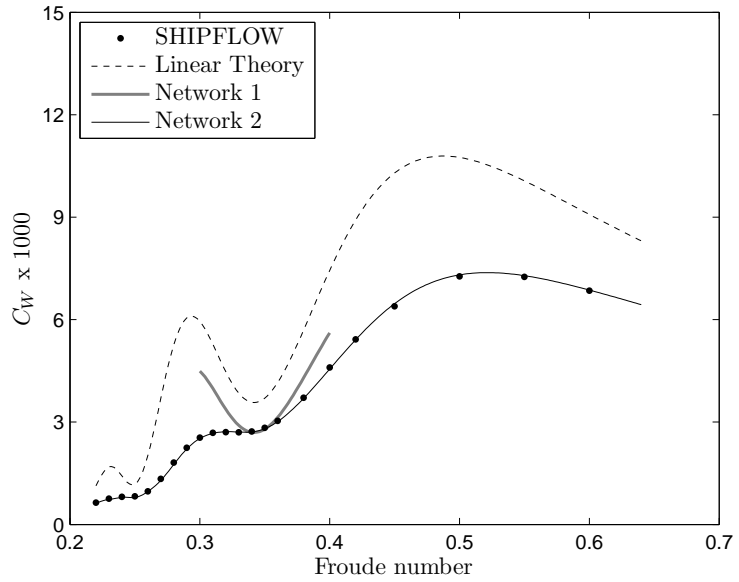
**Figure 6.1.** Corrected theory wave resistance coefficient for  $C_P = 0.52$ ,  $C_X = 0.750$ ,  $C_V = 3 \times 10^{-3}$ ,  $B/T = 2$ .



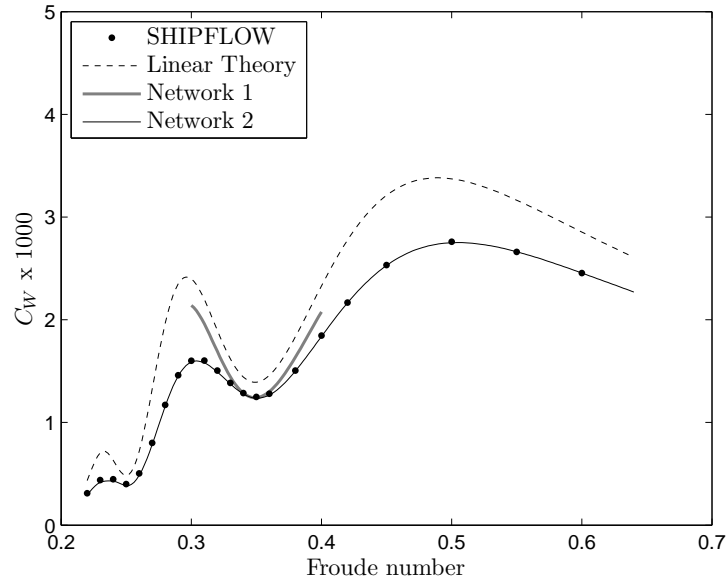
**Figure 6.2.** Corrected theory wave resistance coefficient for  $C_P = 0.56$ ,  $C_X = 0.650$ ,  $C_V = 3 \times 10^{-3}$ ,  $B/T = 1.5$ .



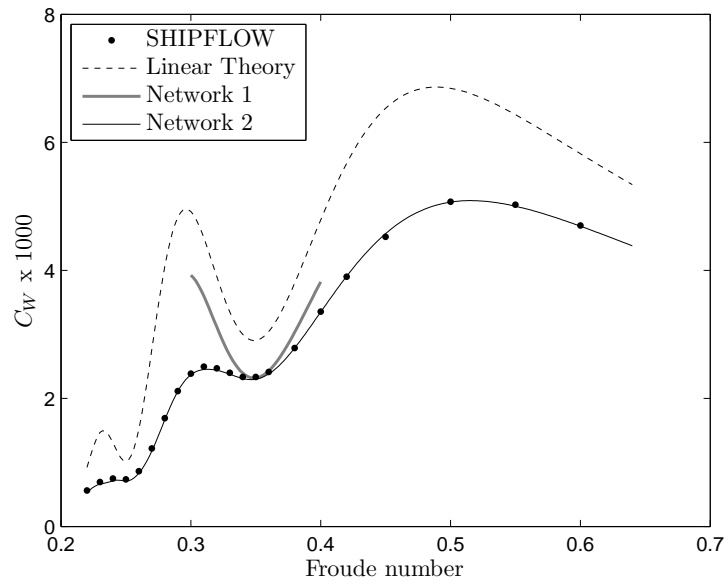
**Figure 6.3.** Corrected theory wave resistance coefficient for  $C_P = 0.60$ ,  $C_X = 0.850$ ,  $C_V = 3 \times 10^{-3}$ ,  $B/T = 2$ .



**Figure 6.4.** Corrected theory wave resistance coefficient for  $C_P = 0.64$ ,  $C_X = 0.750$ ,  $C_V = 4 \times 10^{-3}$ ,  $B/T = 3$ .



**Figure 6.5.** Corrected theory wave resistance coefficient for  $C_P = 0.68$ ,  $C_X = 0.925$ ,  $C_V = 2 \times 10^{-3}$ ,  $B/T = 1.5$ .



**Figure 6.6.** Corrected theory wave resistance coefficient for  $C_P = 0.68$ ,  $C_X = 0.925$ ,  $C_V = 3 \times 10^{-3}$ ,  $B/T = 3$ .

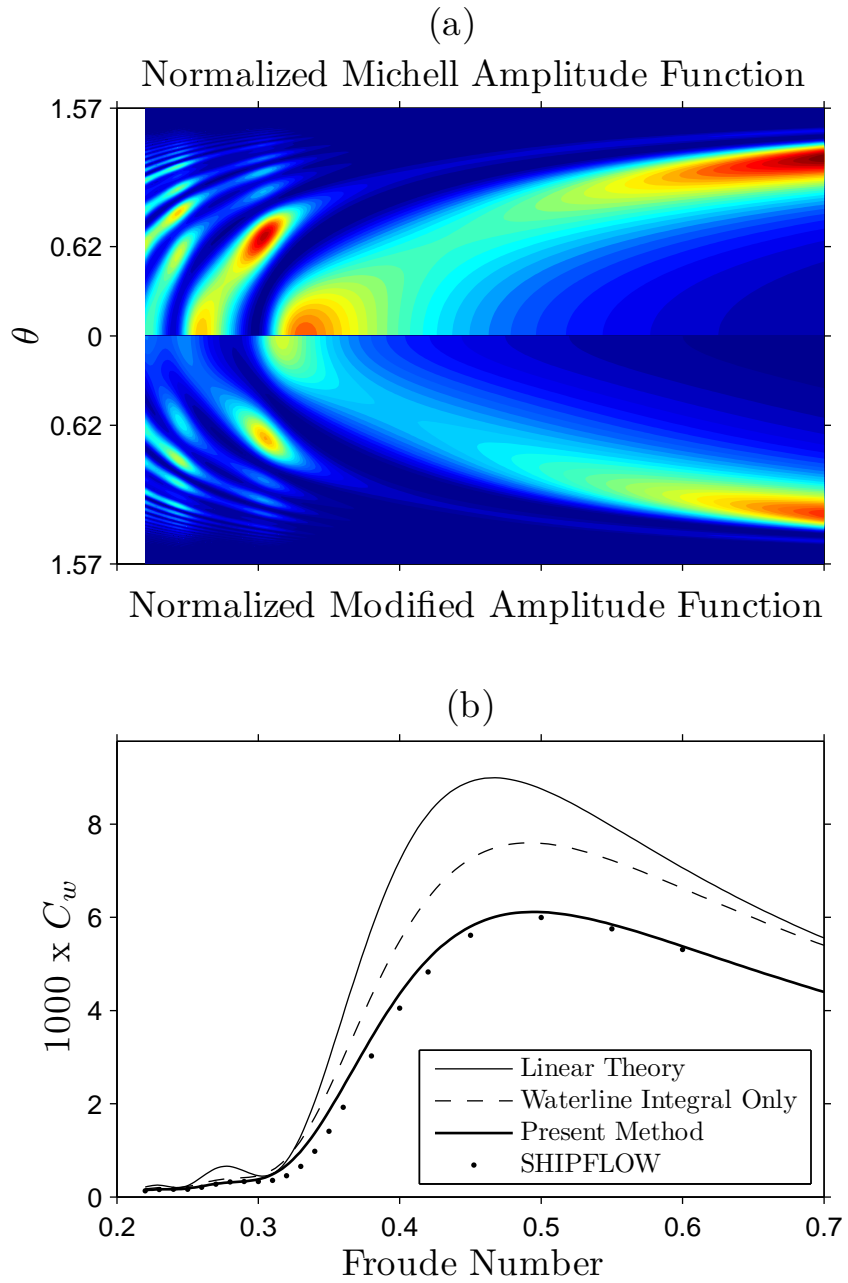


range. In cases where the hull was reasonably thin, the linear theory is very close to the SHIPFLOW value at the training point. In these cases (Figures 6.1 and 6.2) the network 1 result will not be significantly different from the linear theory since the network output is near one. In cases that need moderate correction, the network 1 result may be useful. Care must be exercised below the training point, however, since the exaggerated peak from the linear theory will not be accounted for (Figures 6.3 to 6.6). If the problem is carefully constrained, the network 1 output could be useful as a lower bound on the wave drag.

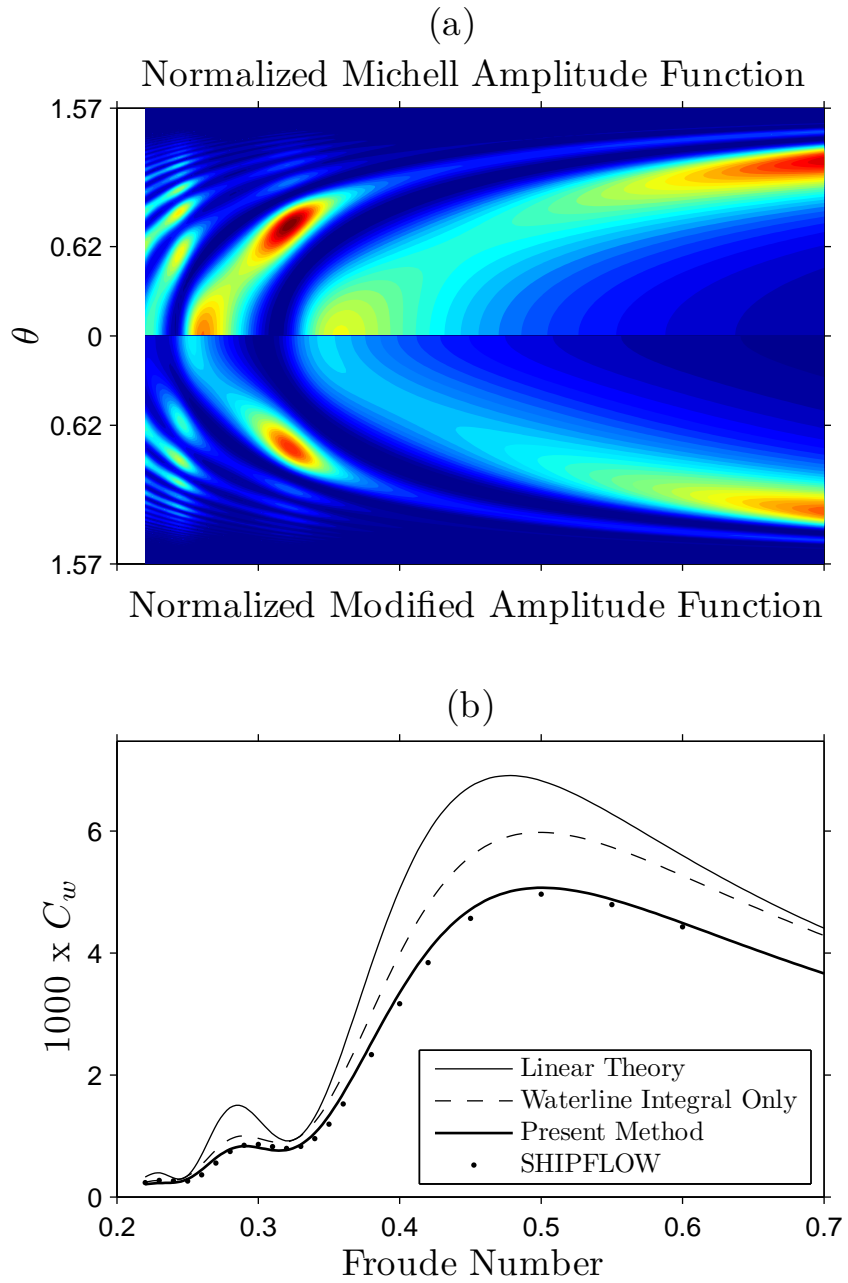
The network 2 result is shown as a solid black line. Since network 2 was trained with Froude number as input using these hulls, the results are simply curve fits of the SHIPFLOW data. In each case, the network 2 result approximates the training data well. Even if the artificial neural network is simply acting as a curve fit in this case, it is an elegant solution to a five parameter approximation. The method does still contain a link back to the physics of the problem through the linear theory, however. A more rigorous test is to see how network 2 performs on a hull outside the training set, and to compare that to what a Taylor hull approximation would give. Note that if network 2 is presented with the Taylor offsets, it automatically performs a Taylor resistance estimate for the physical parameters of any hull (extended to other midship coefficients).

### 6.1.2 Modified Linear Theory Networks

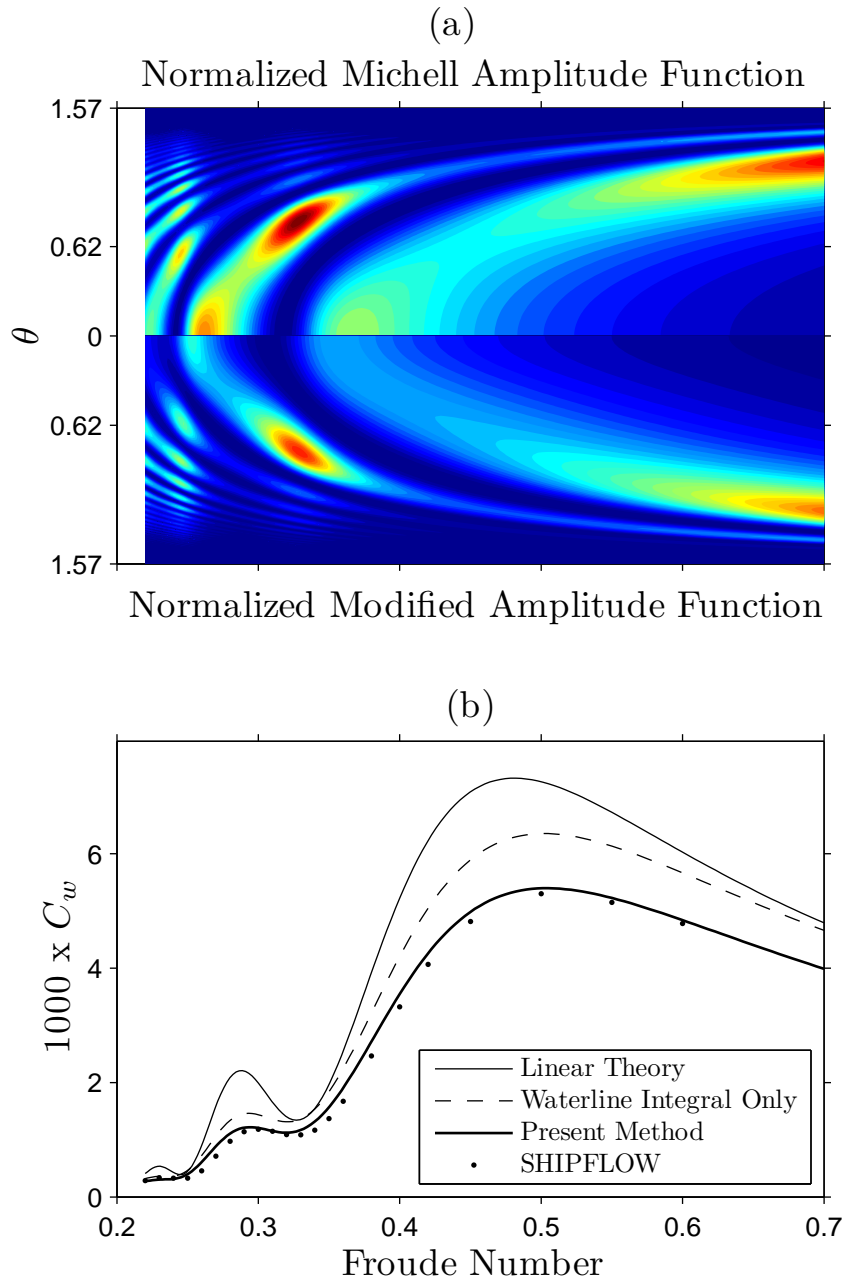
Modified theory networks alter the wave resistance one step earlier in the drag integration, by modifying the distribution of wave energy through a waterline integral. For this reason, the normalized amplitude function is plotted in addition to  $C_W$ . The results for the same hulls as in the previous section are shown in Figures 6.7 to 6.12.



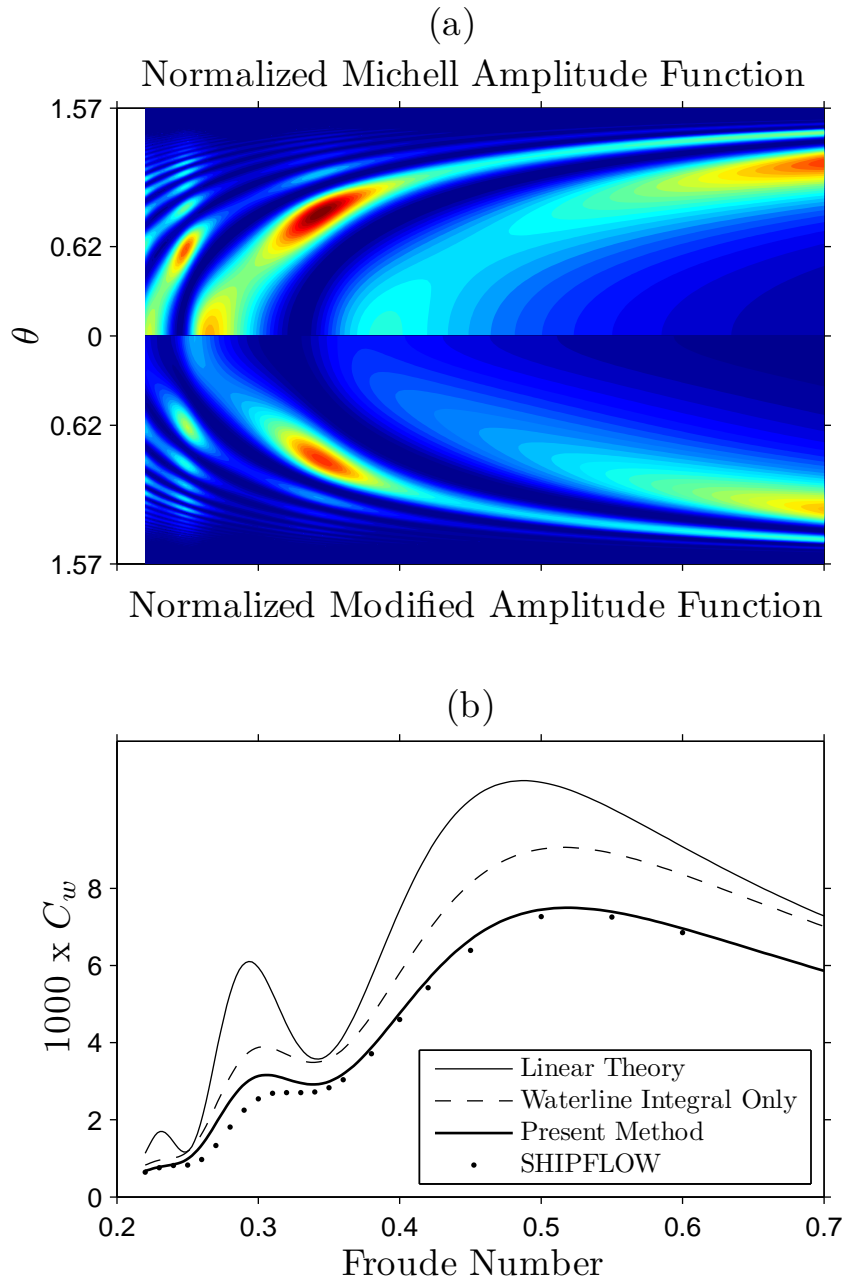
**Figure 6.7.** Modified theory energy distribution and wave resistance result for  $C_P = 0.52$ ,  $C_X = 0.750$ ,  $C_V = 3 \times 10^{-3}$ ,  $B/T = 2$ . Normalized wave energy (a) and wave resistance coefficient (b).



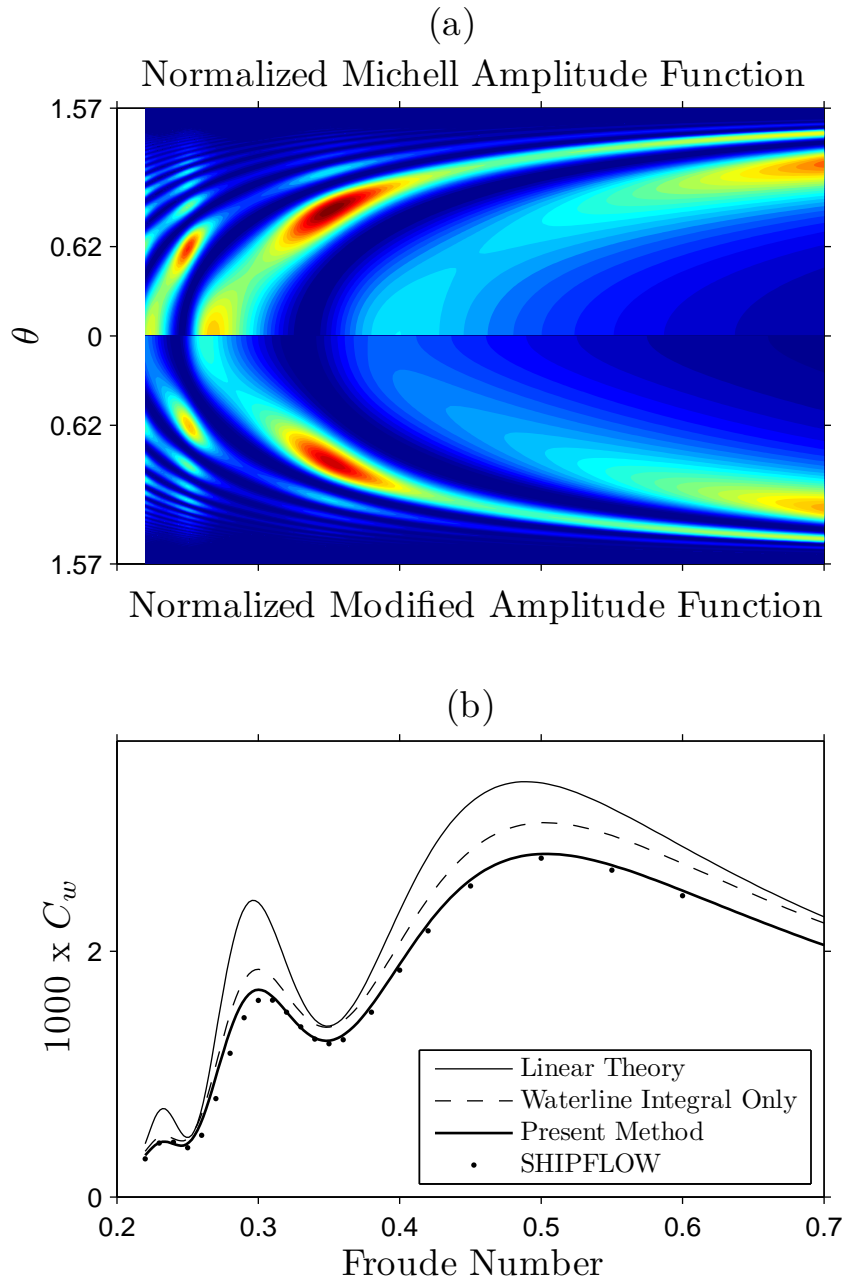
**Figure 6.8.** Modified theory energy distribution and wave resistance result for  $C_P = 0.56$ ,  $C_X = 0.650$ ,  $C_V = 3 \times 10^{-3}$ ,  $B/T = 1.5$ . Normalized wave energy (a) and wave resistance coefficient (b).



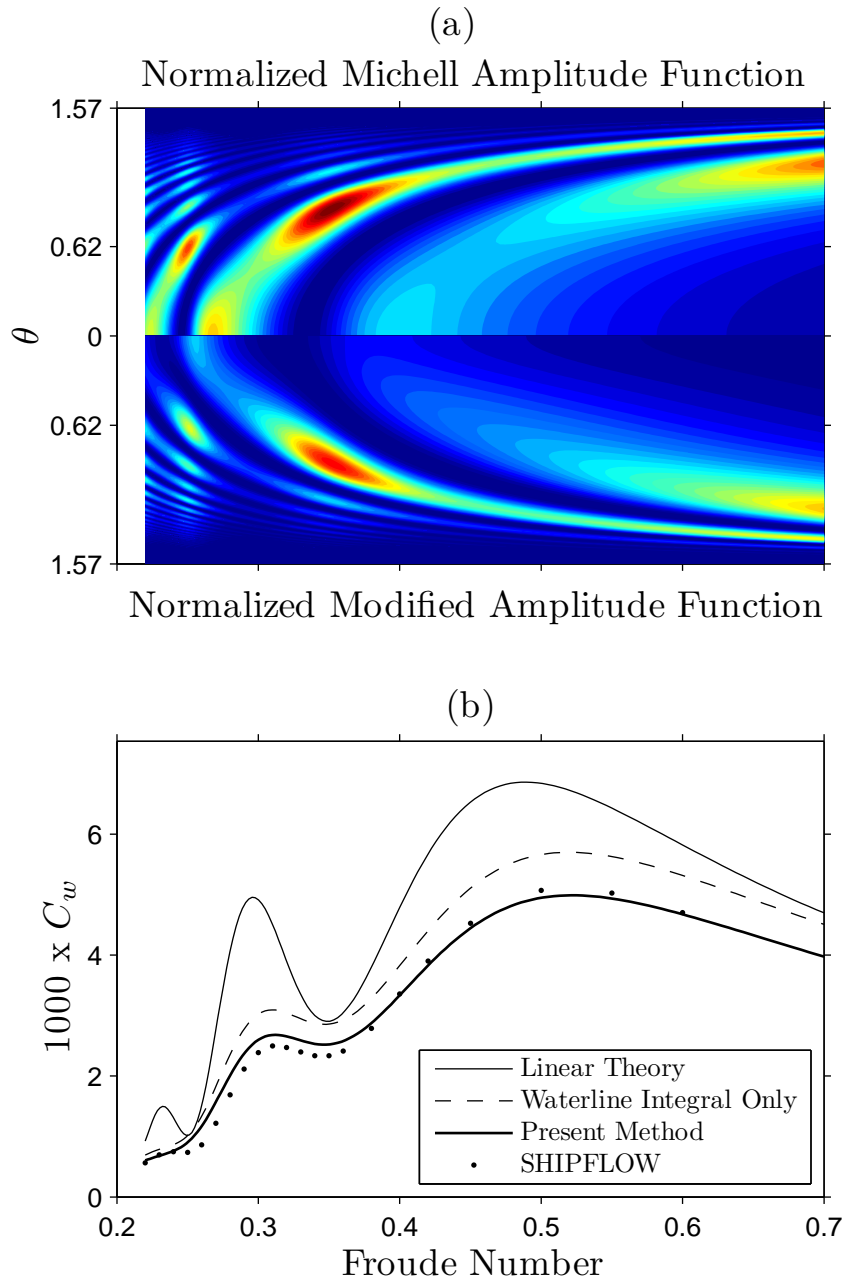
**Figure 6.9.** Modified theory energy distribution and wave resistance result for  $C_P = 0.60$ ,  $C_X = 0.850$ ,  $C_V = 3 \times 10^{-3}$ ,  $B/T = 2$ . Normalized wave energy (a) and wave resistance coefficient (b).



**Figure 6.10.** Modified theory energy distribution and wave resistance result for  $C_P = 0.64$ ,  $C_X = 0.750$ ,  $C_V = 4 \times 10^{-3}$ ,  $B/T = 3$ . Normalized wave energy (a) and wave resistance coefficient (b).



**Figure 6.11.** Modified theory energy distribution and wave resistance result for  $C_P = 0.68$ ,  $C_X = 0.925$ ,  $C_V = 2 \times 10^{-3}$ ,  $B/T = 1.5$ . Normalized wave energy (a) and wave resistance coefficient (b).



**Figure 6.12.** Modified theory energy distribution and wave resistance result for  $C_P = 0.68$ ,  $C_X = 0.925$ ,  $C_V = 3 \times 10^{-3}$ ,  $B/T = 3$ . Normalized wave energy (a) and wave resistance coefficient (b).

In these figures, the standard linear theory is shown as a thin black line while the SHIPFLOW data are again shown as black dots. The network 3 result, labeled present method in the graph, is plotted as a heavy black line. The waterline integral only line represents the network 4 output, shown as a dashed line. The normalized amplitude functions, with standard linear theory above and modified linear theory below, are included to ensure that the energy distribution is not altered too radically.

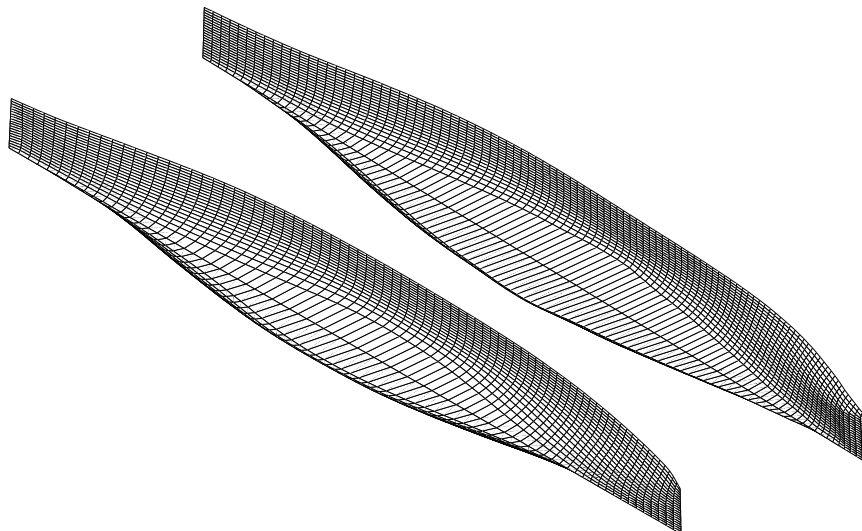
The network 3 result fits the SHIPFLOW data well in all cases, with the possible exception of the hull in Figure 6.10. For this hull, which has the highest  $C_V$  and  $B/T$  of the examples,  $C_W$  is too high near  $Fr = 0.30$ . Still, the correction is more realistic than the standard linear theory result. Network 3 does not fit the SHIPFLOW data as well as Network 2, but it does not have Froude number as an input. Taking this into account, the consistent performance over the large range of  $Fr$  from 0.22 to 0.60 is good.

The network 4 result is shown based on the performance of the nonlinear SHIPFLOW results in the comparison with the experiments. As can be seen in each case, the network 4 result always matches the linear theory  $C_W$  at the troughs in the curve, but significantly reduces the exaggerations at the peaks. It also gives consistent performance over the entire range of geometric parameters.

## 6.2 Results using Series 60 Hull

The true test of the present methods is to apply them to a hull not in the training data. Another systematic series hull was chosen, one of the Series 60 hulls. The geometric parameters are  $C_P = 0.60$ ,  $C_X = 0.97$ ,  $C_V = 4.1 \times 10^{-3}$ ,  $B/T = 2.5$ . This hull is commonly used for validation of computational methods. The Series 60 hull is shown in Figure 6.13 along with a Taylor series hull with identical form

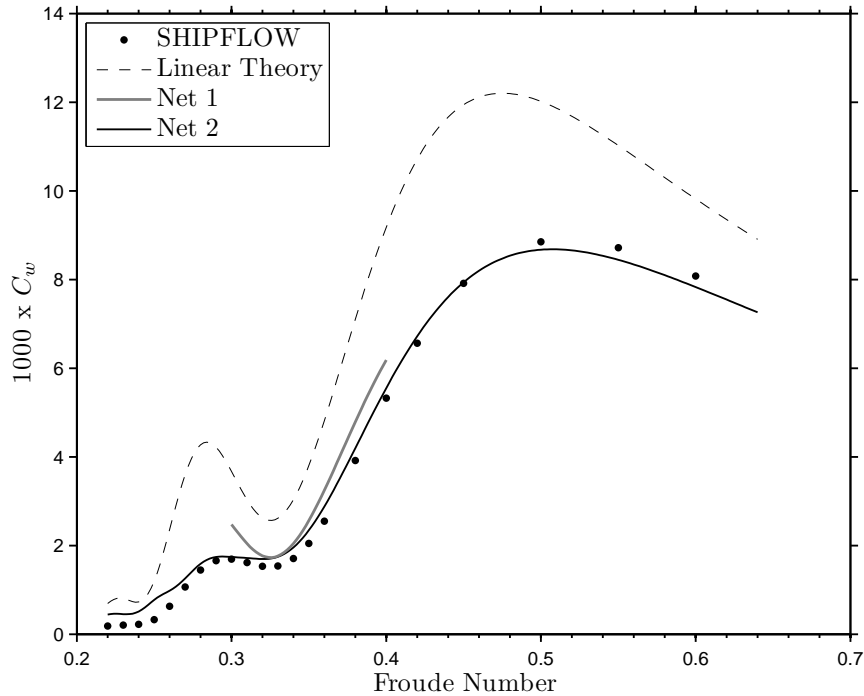




**Figure 6.13.** Comparison of Taylor and Series 60 hulls. Taylor hull (left) has identical shape coefficients and proportions as Series 60 hull (right).

coefficients and proportions (except for  $C_X$  which is slightly different). The hulls are subtly different, making a good test for the artificial neural network corrections. The Series 60 hull has a finer waterline forward but more U-shaped sections and a better defined flat-of-side. The waterline is full beam further aft on the Series 60, as one would expect from a cargo ship, but has a finer stern waterline than the Taylor series. The Series 60 stern has a more defined skeg shape to accommodate a single propeller, while the Taylor series hull was designed for the twin propeller struts and bossings of a warship.

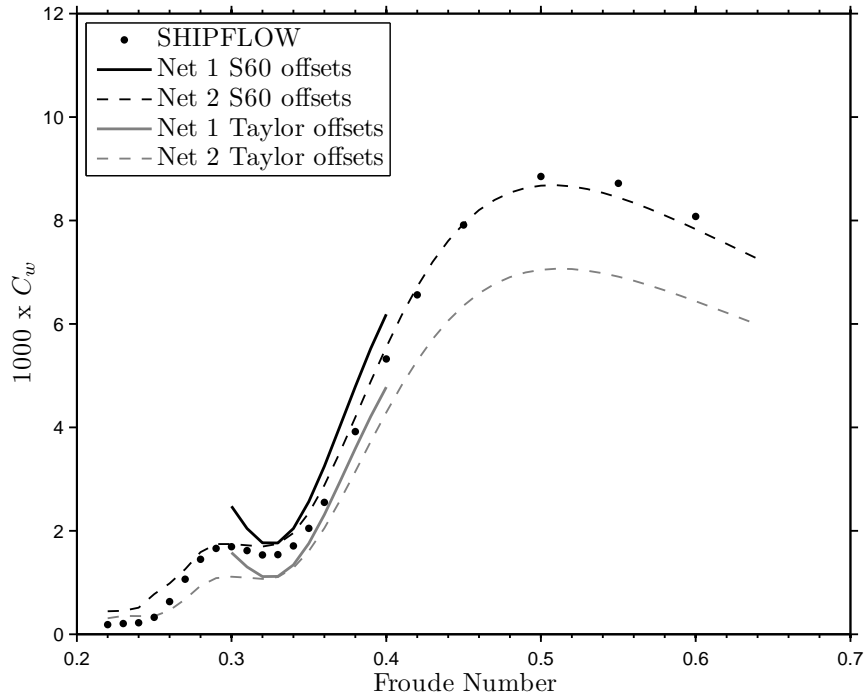
For comparison in the following plots, a SHIPFLOW model of the Series 60 hull was run under the same conditions as the Taylor series training set.



**Figure 6.14.** Corrected theory wave resistance coefficients for Series 60 hull.

### 6.2.1 Corrected Linear Theory Networks

The results of the Series 60 test for networks 1 and 2 are shown in Figure 6.14. The corrected theory networks exhibit similar performance to the test data. Network 1 matches the SHIPFLOW data for most of the speed range shown, while network 2 fits the data well except for a slight kink at low Froude number. Both networks provide improvement over the linear theory, especially in the context of an early stage drag estimate. The question remaining is whether the inclusion of the linear theory in the process provided any benefit. Ship hulls are often compared to the Taylor hulls by using the systematic series data to generate the resistance curve for a Taylor hull of the same proportions as the hull in question. If provided with the Taylor hull offsets, network 2 generates such a comparison curve. If the linear theory is capturing some of the physics differentiating the two hulls, then the corrected theory

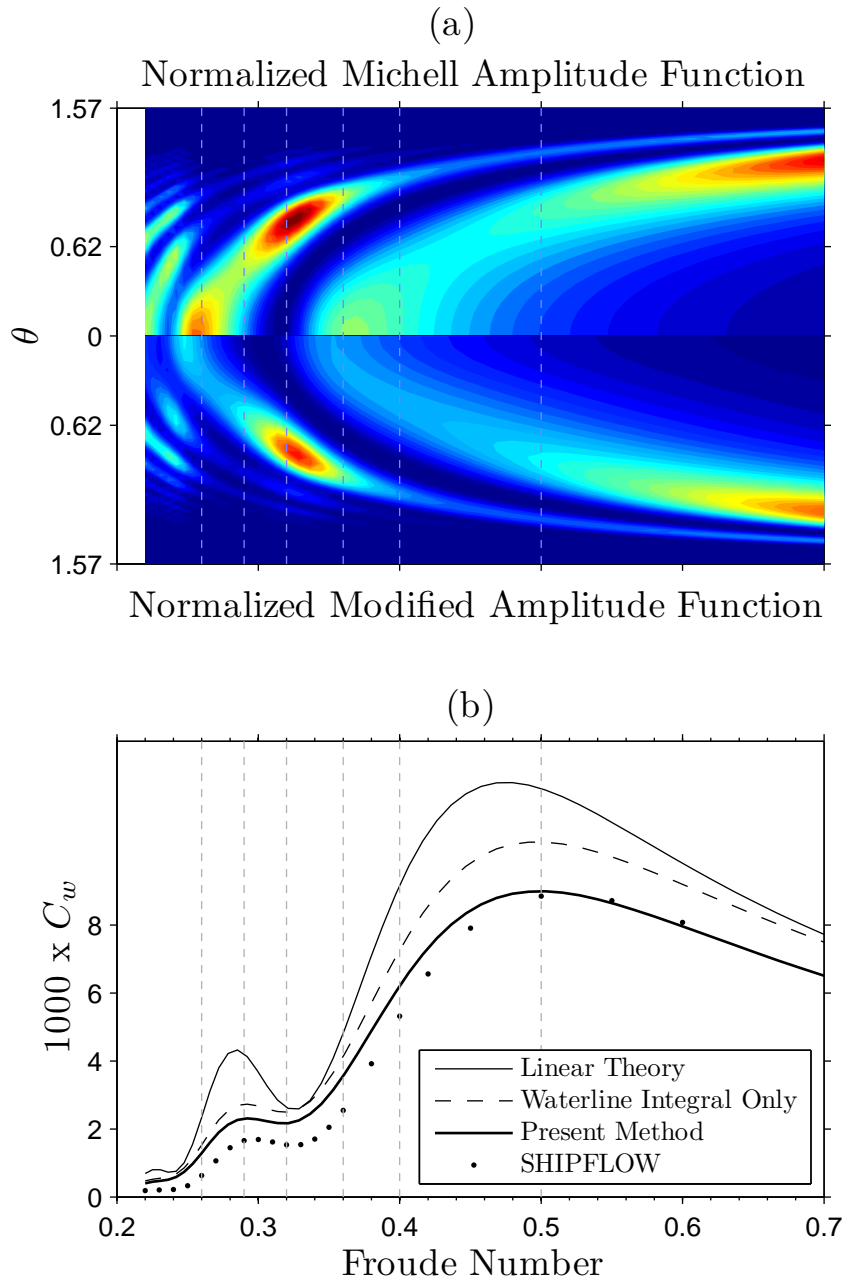


**Figure 6.15.** Comparison of corrected theory method to Taylor resistance.

networks should give a better estimate of the drag than the Taylor estimate. Figure 6.15 shows such a comparison. The SHIPFLOW analysis of the Series 60 hull is again shown as black dots. The neural network results for the Series 60 offsets are shown in black, with the Taylor series offsets plotted in gray. The corrected theory networks do match the SHIPFLOW Series 60 calculation better than the Taylor series estimate, suggesting that the combined parametric and numerical method can differentiate the performance of the two hull forms.

### 6.2.2 Modified Linear Theory Networks

The two modified linear theory networks also exhibit satisfactory behavior on the Series 60 hull, as shown in Figure 6.16. Performance is again similar to that seen on the training set, with the waterline integral (network 4) reducing the drag at the



**Figure 6.16.** Modified theory energy distribution and wave Resistance for Series 60 hull. Input parameters are  $C_P = 0.60$ ,  $C_X = 0.97$ ,  $C_V = 4.1 \times 10^{-3}$ ,  $B/T = 2.5$ . Normalized wave energy (a) and wave resistance coefficient (b).

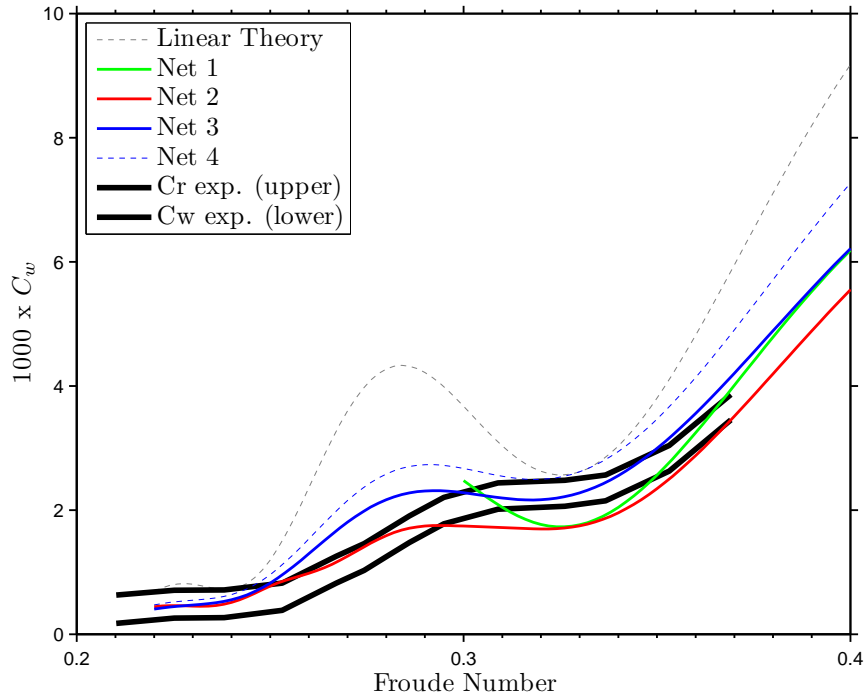
peaks in  $C_W$  and the standard linear theory multiplier (as part of network 3) further reducing the drag to approximate the SHIPFLOW data. Network 3 performance on the Series 60 hull is not quite as good as that of network 2. Network 3 overestimates the SHIPFLOW data over all but the highest speeds. The shape of the curve is captured well, however, and the performance would be acceptable as an early stage estimate. As stated earlier, the goal is to better balance the wave and friction drag in an optimization, which the modified theory network would certainly do in comparison to the standard linear theory.

### 6.2.3 Comparison to Experiment

Finally, all four networks are compared to the experimental data available for the Series 60 hull. It is important to note here that the experimental data is for a ship that is free to sink and trim, while the networks are trained with a fixed hull. The sinkage and trim will have a discernable effect on the wave drag over Froude number 0.30.

The results of the experimental comparison are shown in Figure 6.17. The upper heavy black line is the experimentally obtained residual resistance coefficient,  $C_R$ . The lower heavy black line is an estimate of  $C_W$  using a form factor of 0.12. The standard linear theory is plotted as a gray dotted line for reference. Again, since we have to compare the free to sink and trim experiment with the fixed training data, consider the black lines to be “a little high” above  $Fr = 0.30$ .

The corrected linear theory networks are shown in green (net1) and red (net2). These estimates appear to underestimate the wave resistance coefficient at the trough in the curve, but do better as the experimental curve starts to approach hull speed. The blue modified linear theory curves have the same shape as the network 2 curve,



**Figure 6.17.** Comparison of neural network methods to experiment for Series 60 hull.

but are offset towards the standard linear theory curve. Recalling that the sinkage and trim issue will cause the curves to have slightly different shapes, one could suggest that the corrected and modified linear theory curves could be used as lower and upper bounds to the wave resistance, respectively. In all cases, in the context of balancing drag components, these methods perform better than the standard linear theory.

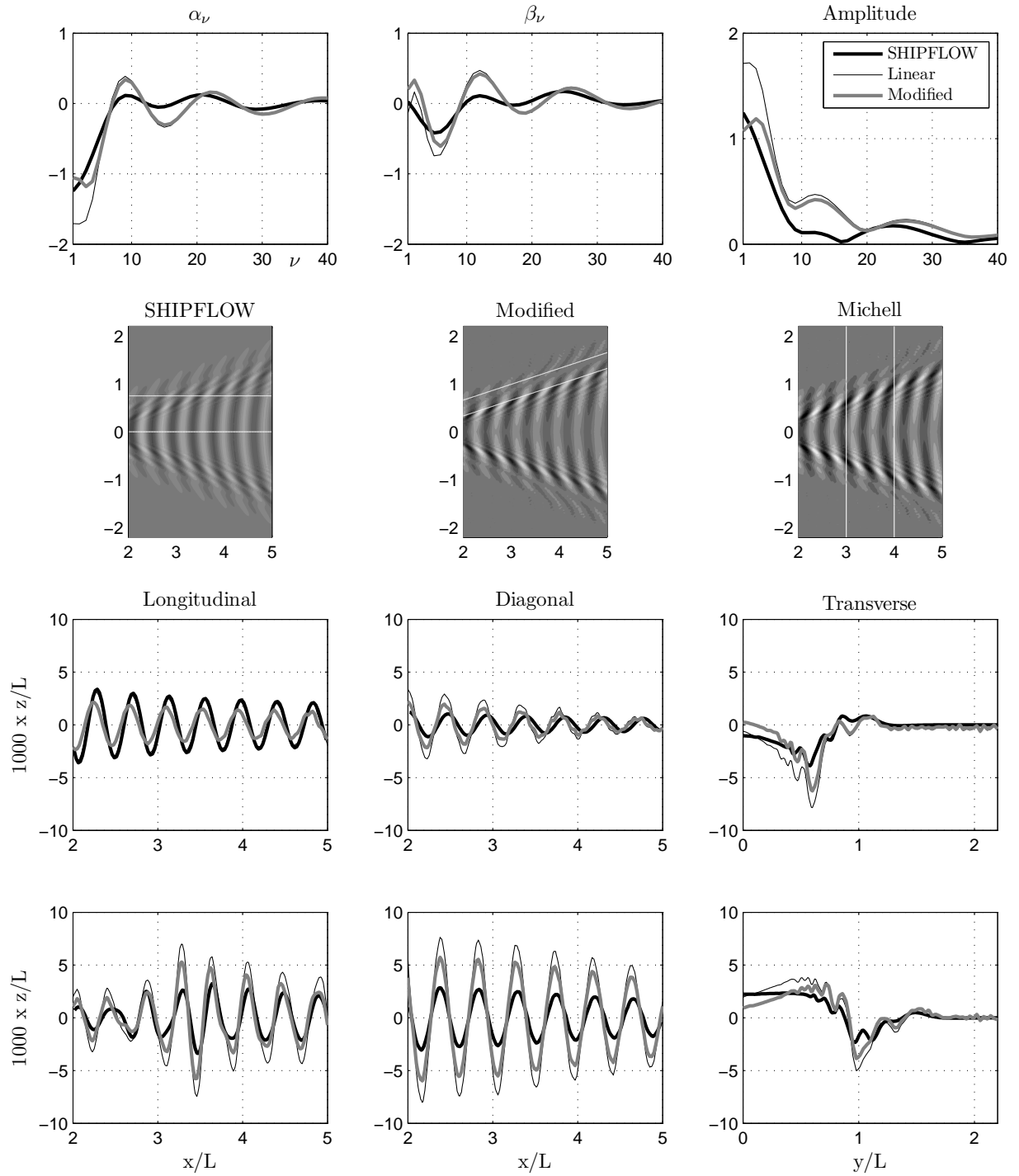
### 6.2.4 Wave Field Analysis

One of the benefits of the modified theory over the corrected linear theory is that the drag correction is accomplished by manipulating  $A(\theta)$ . Since this term can be used to construct the far field waves, the effect of the modified theory on the ship waves can be assessed. As stated in Chapter 5, solutions for the network 3 coefficients that unreasonably distorted the wave energy distribution were discarded. Analyzing the

wave field from the modified  $A(\theta)$  allows a further check on the performance of the method.

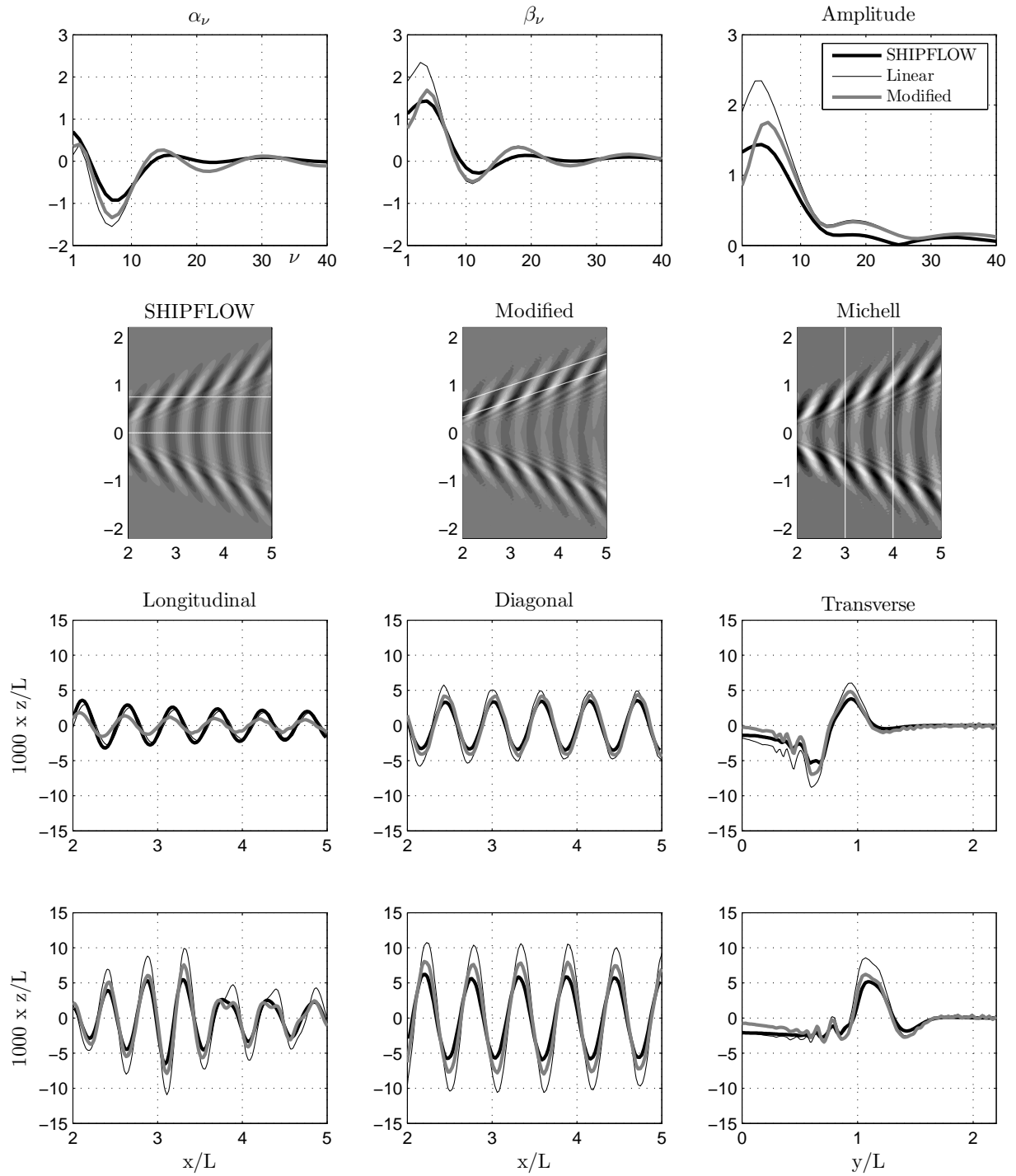
Figures 6.18 to 6.23 compare the wave fields of standard linear theory, SHIPFLOW, and the modified linear theory. Each figure shows the results for one of six Froude numbers, corresponding to the vertical dotted lines in Figure 6.16. In each plot, the top row illustrates a Fourier analysis of the wave field according to the method of Sharma [25]. The second row shows the computed far field waves for each method, all plotted on the same color scale. The bottom two rows show various 2-dimensional cuts through the wave fields. White lines on the wave fields represent the location of the wave cuts for each method. The white lines shown in each wave field represent the direction of the cuts shown directly below that field, but that cut is taken for each of the three methods.

In general, the wave field analysis shows that the modified linear theory method does not negatively alter the character of the ship waves. The waves of the modified method are typically closer in amplitude to the SHIPFLOW waves than the linear theory waves. The exception to this trend is the longitudinal wave cut on the center-line, where the linear theory and modified theory waves are both typically smaller in amplitude than the SHIPFLOW waves. A comparison to experimental data would be necessary to determine which is correct.

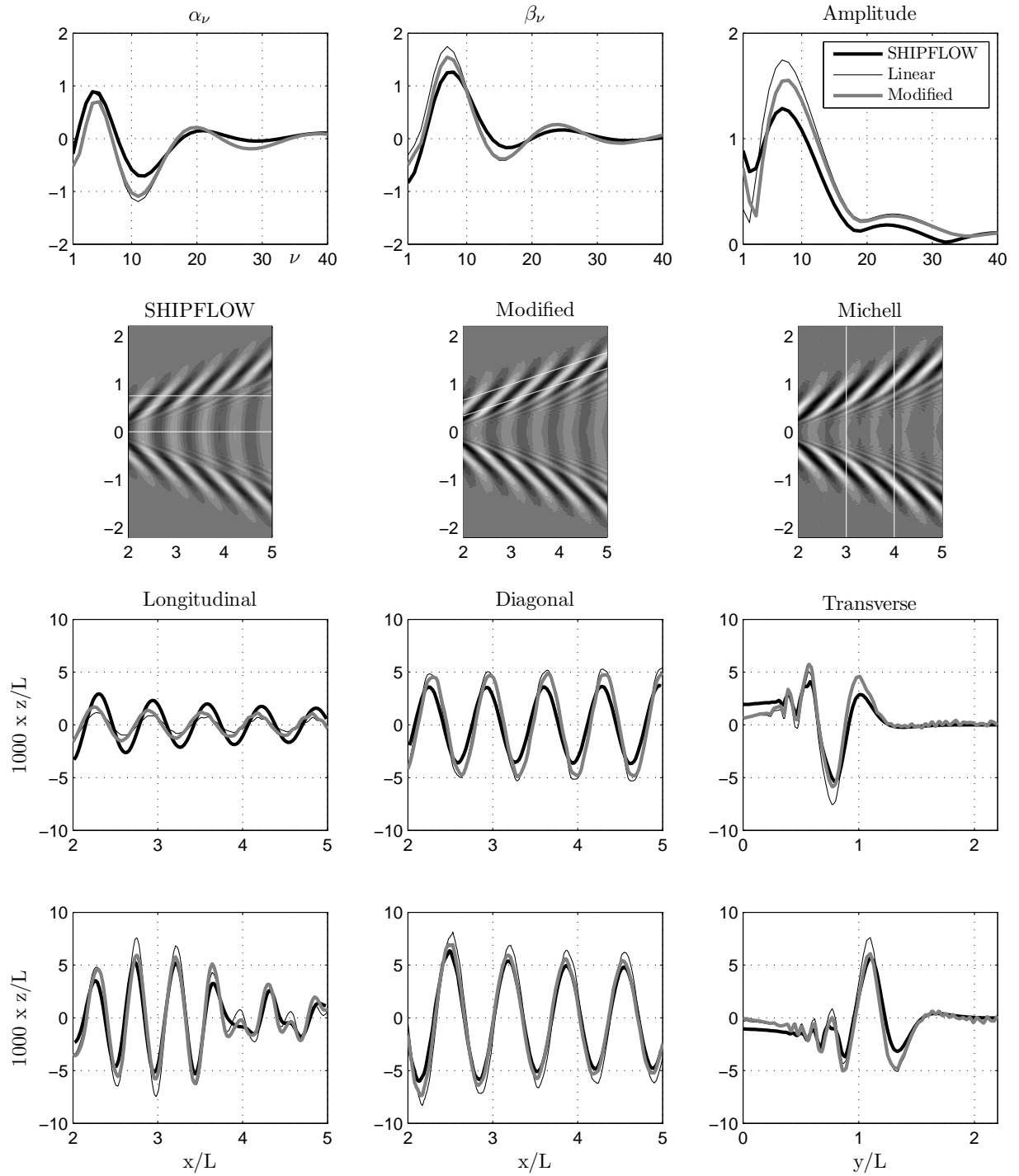


**Figure 6.18.** Wave field characteristics for  $Fr = 0.26$ . Fourier analysis (first row), far field waves (second row), wavecuts (third and fourth rows).

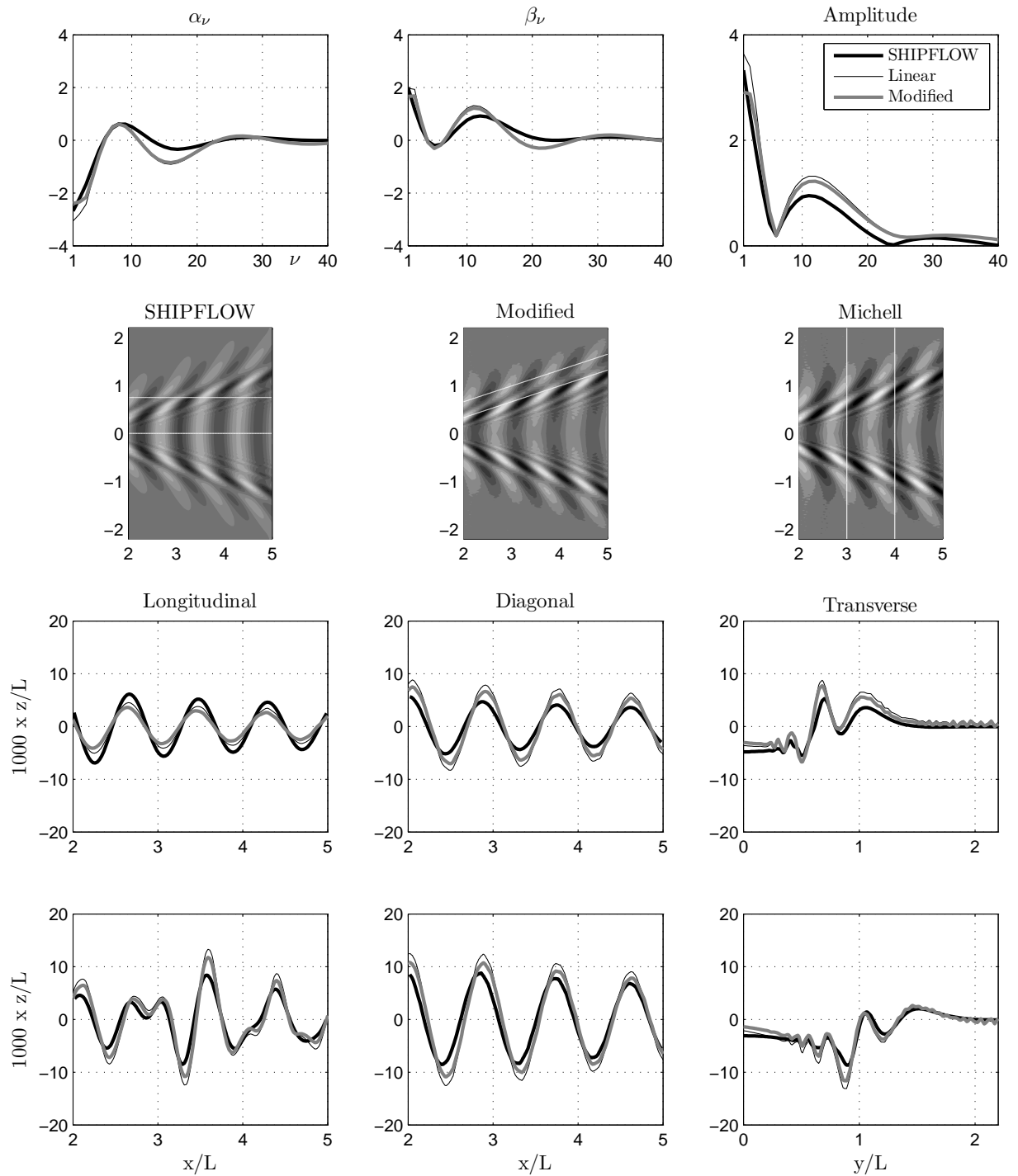




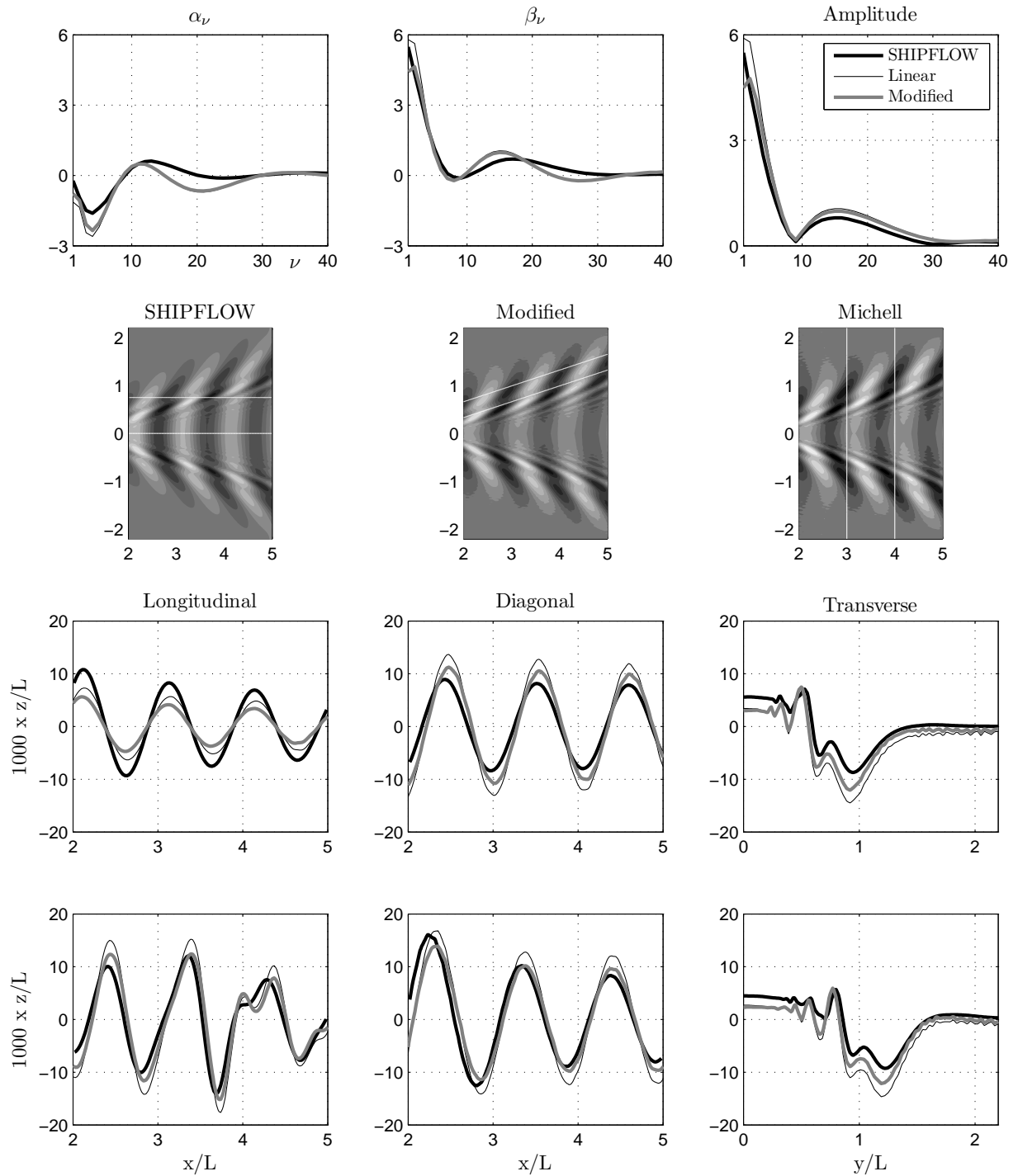
**Figure 6.19.** Wave field characteristics for  $Fr = 0.29$ . Fourier analysis (first row), far field waves (second row), wavecuts (third and fourth rows).



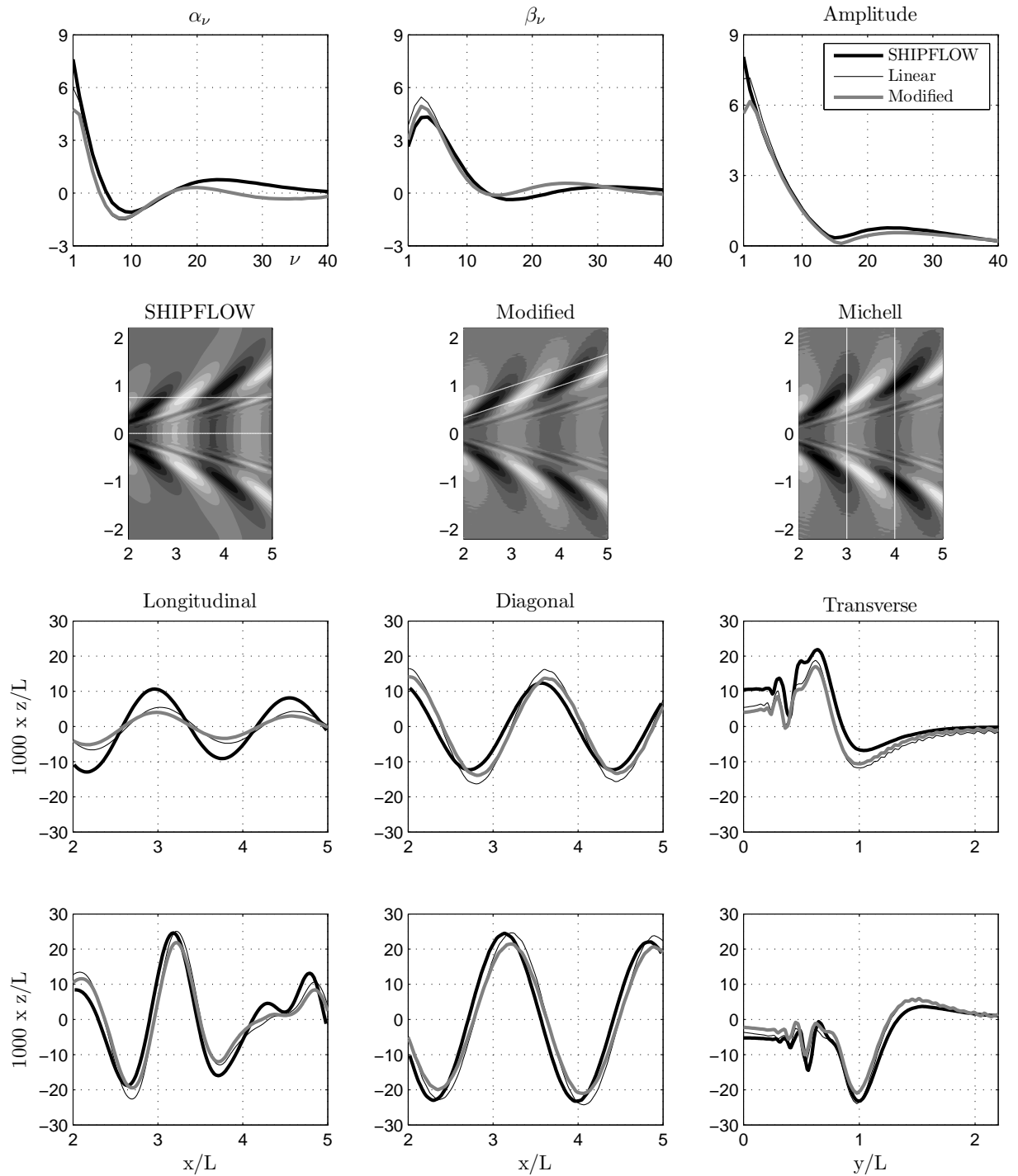
**Figure 6.20.** Wave field characteristics for  $Fr = 0.32$ . Fourier analysis (first row), far field waves (second row), wavecuts (third and fourth rows).



**Figure 6.21.** Wave field characteristics for  $Fr = 0.36$ . Fourier analysis (first row), far field waves (second row), wavecuts (third and fourth rows).



**Figure 6.22.** Wave field characteristics for  $Fr = 0.40$ . Fourier analysis (first row), far field waves (second row), wavecuts (third and fourth rows).



**Figure 6.23.** Wave field characteristics for  $Fr = 0.50$ . Fourier analysis (first row), far field waves (second row), wavecuts (third and fourth rows).

## 6.3 Discussion

The goal of this research was to find a ship wave resistance estimate with enough speed and accuracy to perform concept stage hull optimization using a multi-parameter optimization such as a genetic algorithm. The results of the hybrid numerical-parametric neural network methods shown here are encouraging in that regard. Both the corrected and modified linear theories will allow the wave and frictional resistance components to be better balanced by an optimization routine. Coupled with a way to parametrically alter hull shape, the methods described here should be able to act as reasonable fitness evaluations for such an algorithm. As the ship design matures, more rigorous resistance analysis and detailed optimization will further improve the hull shape.

## 6.4 Recommendations

Several recommendations can be made based on this work. The performance for the test hulls and Series 60 hull show improvement over linear theory and suggest that the hybrid method is accounting for different hull lines. Additional tests on hulls besides a single Series 60 example are necessary, however. The method should be extended to include not only waterline flow ships such as those used here, but also ships with transoms. Care must be taken with the waterline integral term, which was not presented in a form that will accept non-zero transom offsets. An example based on a destroyer hull was undertaken such that the transom offsets were zero but the stern waterline shape was transom-like. The waterline term did not behave as expected in this case, and the integration by parts that allows the waterline offsets to be used instead of the slope needs further investigation.

An estimate of sinkage and trim can be made from the linear theory, and the effects of this condition need to be included for most of the speed range presented here. The modified linear theory methods will also allow an investigation of how such a sinkage and trim estimate is effected by the change in wave energy distribution.

The data set generated by SHIPFLOW is by no means the only source of training target data. Other computational fluid dynamics software and experimental data could also be used. The range of hull shapes included here is very broad. If the type of hull is known, training with systematic series experimental data may be practical.

The data presented here also point out an interesting feature of the standard linear theory. In comparisons to both experiment and SHIPFLOW, the linear theory wave resistance coefficient at the last minima (trough) in the  $C_W$  curve appears to be an accurate estimate in many cases. Based on the results of this study, these cases extend into hull proportions for realistic monohull ships. If a hull optimization routine has control of prismatic coefficient and hull length, it may be able to put the trough in  $C_W$  at the correct Froude number for the linear theory to be used directly. For design  $Fr = 0.30$  to  $0.36$ , this method may be worth investigating.

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# Appendices

# A. Source Code

```
% MICHELL Michell integral wave drag computation
%
% MICHELL(Y,U,L,B,T,RHO,N) computes Rw for the non-dimensional
% hull offsets Y at speed U in water of density RHO. The offsets
% are scaled by length L, beam B, and draft T.
%
% The integration is carried out over Nz waterlines, Nz stations
% and N propagation angles.
%
% The matrix Y should be Nx by Nz ordered from bow to stern and
% keel to waterline. The maximum value of the offset matrix Y
% must be 1/2. Nx must be odd. All values must be specified
% in metric units.
%
% Doug Read          30.7.2008

function Rw = michell(Y,U,L,B,T,RHO,N)

Nx = size(Y,1);      % determine number of stations
Nz = size(Y,2);      % determine number of waterlines
YH = Y*B;           % scale non-dimensional offsets by the beam

if mod(Nx,2) == 0;
    warning('Nx must be odd.');
```

% required for x Filon algorithm

```
end

% ----- integration variables -----
dz = T/(Nz-1);  z = -T:dz:0;    z = z';
dx = L/(Nx-1);  x = 0:dx:L;     x = x';
%theta = linspace(0,pi/2,N);    theta = theta';
theta = michspace(N);          theta = theta';

g = 9.80665;
k0 = g/U^2;          % fundamental wave number
c = (4*RHO*U^2)/pi; % constant
a = sec(theta);     % convenient substitution
k = k0*a.^2;        % dispersion relation

%----- Z INTEGRAL -----

% --- variables for Filon trapezoidal algorithm ---
Kz = k0*dz*a.^2;
w0 = (exp(Kz)-1-Kz)./Kz.^2;
wn = (exp(Kz)+exp(-Kz)-2)./Kz.^2;
wN = (exp(-Kz)-1+Kz)./Kz.^2;

% --- preallocate ---
f = zeros(Nz,1);
F = zeros(Nx,N);
```

```

for j = 1:N;
  for m = 1:Nx;
    for n = 1:Nz;
      if n == 1;
        f(n) = w0(j)*YH(m,n)*exp(k0*z(n)*a(j)^2)*dz;
      elseif n == Nz;
        f(n) = wN(j)*YH(m,n)*exp(k0*z(n)*a(j)^2)*dz;
      else
        f(n) = wn(j)*YH(m,n)*exp(k0*z(n)*a(j)^2)*dz;
      end
    end
    F(m,j) = sum(f);
  end
end

%----- X INTEGRAL -----

% --- variables for Filon algorithm ---
Kx = k0*dx.*a;
alp = (Kx.^2+1/2*Kx.*sin(2*Kx)+cos(2*Kx)-1)./Kx.^3;
bet = (3*Kx+Kx.*cos(2*Kx)-2*sin(2*Kx))./Kx.^3;
gam = 4*(sin(Kx)-Kx.*cos(Kx))./Kx.^3;

Nev = (Nx+1)/2;    % even Filon index
Nod = (Nx-1)/2;    % odd Filon index

% --- preallocate ---
pev = zeros(Nev,1);   qev = zeros(Nev,1);
pod = zeros(Nod,1);   qod = zeros(Nod,1);
Pt = zeros(N,1);      Qt = zeros(N,1);
Pev = zeros(N,1);     Qev = zeros(N,1);
Pod = zeros(N,1);     Qod = zeros(N,1);
P = zeros(N,1);       Q = zeros(N,1);

for j = 1:N;
  for m = 1:Nev;
    pev(2*m-1) = F(2*m-1,j)*cos(k0*x(2*m-1)*a(j));
    qev(2*m-1) = F(2*m-1,j)*sin(k0*x(2*m-1)*a(j));
  end
  for m = 1:Nod;
    pod(2*m) = F(2*m,j)*cos(k0*x(2*m)*a(j));
    qod(2*m) = F(2*m,j)*sin(k0*x(2*m)*a(j));
  end

  Pt(j) = F(Nx,j)*cos(k0*L*a(j));
  Qt(j) = F(Nx,j)*sin(k0*L*a(j));

  Pev(j) = sum(pev)-1/2*Pt(j);
  Pod(j) = sum(pod);
  Qev(j) = sum(qev)-1/2*Qt(j);
  Qod(j) = sum(qod);

  P(j)=dx*( alp(j)*Qt(j)+bet(j)*Pev(j)+gam(j)*Pod(j));
  Q(j)=dx*(-alp(j)*Pt(j)+bet(j)*Qev(j)+gam(j)*Qod(j));
end

R = c*k.^2./a.^3.*...
    (k.^2.*( P.^2 + Q.^2 )+...
    2*k.*a.*( Q.*Pt - P.*Qt )+...
    a.^2.*( Pt.^2 + Qt.^2 ));
R(isnan(R)) = 0;

%----- THETA INTEGRAL -----

```

```

rw = zeros(N-1,1);

for k = 1:N-1;
    rw(k) = 1/2*(R(k)+R(k+1))*(theta(k+1)-theta(k));
end

Rw = sum(rw);

%----- END -----

function [xm]=michspace(N);

% MICHSPACE log spacing for Michell integral
%
% MICHSPACE(N) produces log base 10 spacing over N propagation
% angles between 0 and pi/2. Points are more closely spaced
% near pi/2.

xm = logspace(0,1,N)-1;
xm = xm*pi/18-pi/2;
xm = fliplr(-xm);

```

# B. Training Set Data

## B.1 Hull Characteristics

Dimensional values are in meters.

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	$1000 \times C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
1	26.00	1	2	1.296	16.172	100.59	18.570	0.5513	0.6484	1.057	0.5	14.344
2	36.77	2	2	2.591	45.742	149.89	52.520	0.5513	0.6487	1.056	1.0	10.688
3	45.03	3	2	3.886	84.034	197.39	96.478	0.5514	0.6486	1.056	1.5	9.383
4	52.00	4	2	5.180	129.378	247.51	148.524	0.5514	0.6484	1.056	2.0	8.825
5	63.69	6	2	7.765	237.683	359.68	272.793	0.5516	0.6481	1.056	3.0	8.551
6	73.54	8	2	10.348	365.935	488.59	419.840	0.5517	0.6475	1.056	4.0	8.714
7	82.22	10	2	12.928	511.409	633.38	586.585	0.5519	0.6470	1.055	5.0	9.039
8	18.38	1	2	1.295	11.435	71.16	13.131	0.5514	0.6487	2.113	0.5	12.785
9	26.00	2	2	2.591	32.345	106.08	37.138	0.5513	0.6483	2.113	1.0	9.530
10	31.84	3	2	3.885	59.421	139.73	68.220	0.5514	0.6486	2.113	1.5	8.369
11	36.77	4	2	5.179	91.484	175.25	105.022	0.5515	0.6485	2.113	2.0	7.873
12	45.03	6	2	7.765	168.066	254.73	192.893	0.5516	0.6481	2.112	3.0	7.630
13	52.00	8	2	10.348	258.753	346.04	296.870	0.5517	0.6478	2.111	4.0	7.776
14	58.14	10	2	12.928	361.617	448.56	414.780	0.5519	0.6474	2.111	5.0	8.065
15	15.01	1	2	1.295	9.337	58.13	10.722	0.5515	0.6486	3.170	0.5	11.956
16	21.23	2	2	2.591	26.409	86.70	30.323	0.5514	0.6487	3.169	1.0	8.916
17	26.00	3	2	3.885	48.517	114.23	55.702	0.5514	0.6482	3.169	1.5	7.832
18	30.02	4	2	5.179	74.696	143.29	85.750	0.5515	0.6485	3.169	2.0	7.369
19	36.77	6	2	7.765	137.224	208.31	157.494	0.5516	0.6482	3.168	3.0	7.143
20	42.46	8	2	10.348	211.270	282.98	242.388	0.5517	0.6478	3.167	4.0	7.279
21	47.47	10	2	12.928	295.256	366.80	338.671	0.5519	0.6475	3.166	5.0	7.549
22	13.00	1	2	1.295	8.086	50.37	9.285	0.5516	0.6483	4.226	0.5	11.402
23	18.38	2	2	2.590	22.871	75.14	26.260	0.5514	0.6486	4.226	1.0	8.505
24	22.52	3	2	3.885	42.016	99.05	48.239	0.5514	0.6485	4.226	1.5	7.474
25	26.00	4	2	5.179	64.688	124.26	74.261	0.5515	0.6481	4.225	2.0	7.033
26	31.84	6	2	7.765	118.839	180.67	136.392	0.5516	0.6482	4.224	3.0	6.819
27	36.77	8	2	10.348	182.962	245.44	209.908	0.5517	0.6478	4.222	4.0	6.949
28	41.11	10	2	12.928	255.696	318.13	293.299	0.5519	0.6475	4.222	5.0	7.207
29	11.63	1	2	1.295	7.232	45.07	8.305	0.5517	0.6483	5.283	0.5	10.991
30	16.44	2	2	2.590	20.456	67.28	23.488	0.5515	0.6484	5.282	1.0	8.203
31	20.14	3	2	3.885	37.580	88.69	43.146	0.5514	0.6486	5.282	1.5	7.210
32	23.26	4	2	5.179	57.859	111.30	66.421	0.5515	0.6484	5.281	2.0	6.786
33	28.48	6	2	7.765	106.292	161.84	121.990	0.5516	0.6482	5.280	3.0	6.580
34	32.89	8	2	10.348	163.645	219.87	187.743	0.5517	0.6478	5.278	4.0	6.706
35	36.77	10	2	12.929	228.698	284.97	262.333	0.5518	0.6475	5.277	5.0	6.954
36	27.93	1	2	1.491	17.375	108.48	22.523	0.5408	0.7465	1.034	0.5	13.602
37	39.50	2	2	2.984	49.143	162.65	63.682	0.5403	0.7471	1.034	1.0	10.199
38	48.37	3	2	4.477	90.282	215.03	116.967	0.5401	0.7473	1.033	1.5	8.991
39	55.86	4	2	5.969	138.998	270.09	180.053	0.5400	0.7472	1.033	2.0	8.470
40	68.41	6	2	8.952	255.356	392.46	330.692	0.5400	0.7469	1.033	3.0	8.207
41	78.99	8	2	11.931	393.145	532.29	508.996	0.5401	0.7463	1.033	4.0	8.350
42	88.32	10	2	14.907	549.436	688.92	711.300	0.5403	0.7458	1.033	5.0	8.646
43	19.75	1	2	1.491	12.286	76.75	15.926	0.5410	0.7465	2.068	0.5	12.125

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
44	27.93	2	2	2.984	34.750	115.12	45.030	0.5403	0.7470	2.067	1.0	9.095
45	34.21	3	2	4.477	63.839	152.23	82.708	0.5401	0.7472	2.067	1.5	8.020
46	39.50	4	2	5.969	98.287	191.26	127.318	0.5400	0.7472	2.066	2.0	7.557
47	48.37	6	2	8.952	180.564	277.97	233.835	0.5400	0.7471	2.066	3.0	7.324
48	55.86	8	2	11.931	277.994	377.03	359.910	0.5401	0.7467	2.065	4.0	7.451
49	62.45	10	2	14.907	388.508	487.94	502.968	0.5403	0.7464	2.065	5.0	7.715
50	16.12	1	2	1.490	10.031	62.70	13.004	0.5411	0.7462	3.102	0.5	11.339
51	22.80	2	2	2.984	28.373	94.09	36.767	0.5404	0.7470	3.101	1.0	8.509
52	27.93	3	2	4.477	52.124	124.45	67.531	0.5401	0.7471	3.100	1.5	7.505
53	32.25	4	2	5.969	80.251	156.38	103.954	0.5400	0.7472	3.099	2.0	7.073
54	39.50	6	2	8.952	147.429	227.33	190.924	0.5400	0.7471	3.099	3.0	6.856
55	45.61	8	2	11.931	226.981	308.35	293.861	0.5401	0.7468	3.098	4.0	6.976
56	50.99	10	2	14.907	317.214	399.05	410.678	0.5403	0.7465	3.098	5.0	7.223
57	13.96	1	2	1.490	8.687	54.33	11.262	0.5413	0.7461	4.136	0.5	10.813
58	19.75	2	2	2.983	24.572	81.55	31.841	0.5404	0.7470	4.134	1.0	8.118
59	24.19	3	2	4.476	45.141	107.92	58.483	0.5401	0.7467	4.133	1.5	7.163
60	27.93	4	2	5.969	69.499	135.63	90.026	0.5400	0.7471	4.133	2.0	6.752
61	34.21	6	2	8.951	127.676	197.19	165.341	0.5400	0.7471	4.131	3.0	6.546
62	39.50	8	2	11.931	196.570	267.48	254.486	0.5400	0.7468	4.130	4.0	6.661
63	44.16	10	2	14.907	274.713	346.15	355.658	0.5403	0.7465	4.130	5.0	6.896
64	12.49	1	2	1.489	7.770	48.62	10.073	0.5415	0.7458	5.170	0.5	10.424
65	17.66	2	2	2.983	21.977	73.01	28.480	0.5405	0.7468	5.168	1.0	7.829
66	21.63	3	2	4.476	40.375	96.64	52.309	0.5402	0.7471	5.167	1.5	6.910
67	24.98	4	2	5.969	62.161	121.48	80.521	0.5401	0.7467	5.166	2.0	6.515
68	30.59	6	2	8.951	114.197	176.66	147.884	0.5400	0.7471	5.164	3.0	6.317
69	35.33	8	2	11.931	175.817	239.63	227.616	0.5400	0.7468	5.163	4.0	6.428
70	39.50	10	2	14.907	245.710	310.10	318.110	0.5403	0.7465	5.163	5.0	6.655
71	29.73	1	2	1.687	18.499	116.70	26.581	0.5300	0.8445	1.011	0.5	13.102
72	42.05	2	2	3.379	52.323	176.84	75.170	0.5291	0.8458	1.011	1.0	9.928
73	51.50	3	2	5.070	96.124	235.16	138.084	0.5288	0.8462	1.011	1.5	8.802
74	59.46	4	2	6.761	147.992	296.10	212.578	0.5288	0.8462	1.011	2.0	8.313
75	72.83	6	2	10.140	271.879	430.20	390.471	0.5288	0.8458	1.011	3.0	8.053
76	84.10	8	2	13.516	418.584	582.01	601.172	0.5289	0.8453	1.011	4.0	8.171
77	94.02	10	2	16.888	584.988	751.08	840.211	0.5291	0.8449	1.011	5.0	8.435
78	21.02	1	2	1.686	13.081	82.57	18.796	0.5302	0.8443	2.023	0.5	11.680
79	29.73	2	2	3.378	36.998	125.16	53.153	0.5292	0.8458	2.022	1.0	8.854
80	36.41	3	2	5.070	67.970	166.49	97.641	0.5289	0.8461	2.022	1.5	7.852
81	42.05	4	2	6.761	104.646	209.69	150.316	0.5288	0.8462	2.022	2.0	7.417
82	51.50	6	2	10.140	192.247	304.73	276.104	0.5288	0.8461	2.022	3.0	7.187
83	59.46	8	2	13.516	295.983	412.27	425.093	0.5289	0.8459	2.022	4.0	7.292
84	66.48	10	2	16.889	413.649	532.03	594.120	0.5291	0.8455	2.022	5.0	7.528
85	17.17	1	2	1.686	10.680	67.45	15.347	0.5304	0.8436	3.034	0.5	10.922
86	24.28	2	2	3.378	30.209	102.30	43.399	0.5292	0.8451	3.033	1.0	8.284
87	29.73	3	2	5.070	55.497	136.12	79.723	0.5289	0.8461	3.033	1.5	7.348
88	34.33	4	2	6.760	85.443	171.46	122.732	0.5288	0.8462	3.033	2.0	6.943
89	42.05	6	2	10.140	156.969	249.23	225.436	0.5288	0.8462	3.032	3.0	6.729
90	48.55	8	2	13.516	241.669	337.21	347.088	0.5289	0.8459	3.033	4.0	6.828
91	54.28	10	2	16.889	337.743	435.16	485.101	0.5291	0.8456	3.033	5.0	7.048
92	14.87	1	2	1.685	9.249	58.45	13.291	0.5306	0.8437	4.045	0.5	10.417
93	21.02	2	2	3.378	26.162	88.68	37.585	0.5293	0.8456	4.045	1.0	7.904
94	25.75	3	2	5.069	48.062	118.03	69.042	0.5289	0.8455	4.044	1.5	7.013
95	29.73	4	2	6.760	73.996	148.71	106.289	0.5288	0.8462	4.044	2.0	6.628
96	36.41	6	2	10.140	135.939	216.20	195.230	0.5288	0.8462	4.043	3.0	6.424
97	42.05	8	2	13.516	209.291	292.54	300.589	0.5289	0.8459	4.043	4.0	6.519
98	47.01	10	2	16.889	292.493	377.51	420.110	0.5291	0.8456	4.044	5.0	6.730
99	13.30	1	2	1.684	8.273	52.30	11.888	0.5308	0.8430	5.057	0.5	10.042
100	18.80	2	2	3.377	23.399	79.39	33.617	0.5294	0.8454	5.056	1.0	7.622
101	23.03	3	2	5.069	42.988	105.71	61.753	0.5290	0.8460	5.055	1.5	6.766
102	26.59	4	2	6.760	66.184	133.21	95.067	0.5288	0.8458	5.055	2.0	6.395
103	32.57	6	2	10.139	121.587	193.70	174.617	0.5288	0.8462	5.054	3.0	6.200
104	37.61	8	2	13.516	187.195	262.11	268.854	0.5289	0.8459	5.054	4.0	6.292
105	42.05	10	2	16.888	261.612	338.23	375.755	0.5291	0.8456	5.055	5.0	6.495
106	31.02	1	2	1.836	19.299	123.26	29.722	0.5218	0.9194	0.996	0.5	12.846
107	43.86	2	2	3.679	54.587	188.63	84.082	0.5210	0.9210	0.996	1.0	9.828
108	53.72	3	2	5.521	100.282	252.17	154.472	0.5208	0.9214	0.996	1.5	8.759



hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
109	62.03	4	2	7.362	154.394	318.31	237.820	0.5207	0.9215	0.996	2.0	8.292
110	75.97	6	2	11.043	283.639	462.84	436.900	0.5208	0.9209	0.996	3.0	8.039
111	87.73	8	2	14.719	436.691	625.19	672.714	0.5210	0.9205	0.996	4.0	8.143
112	98.08	10	2	18.393	610.293	805.00	940.204	0.5212	0.9201	0.996	5.0	8.388
113	21.93	1	2	1.836	13.647	87.21	21.017	0.5220	0.9190	1.992	0.5	11.452
114	31.02	2	2	3.679	38.599	133.51	59.455	0.5211	0.9209	1.993	1.0	8.765
115	37.99	3	2	5.521	70.910	178.55	109.229	0.5208	0.9214	1.993	1.5	7.814
116	43.86	4	2	7.362	109.173	225.43	168.164	0.5207	0.9215	1.993	2.0	7.399
117	53.72	6	2	11.042	200.563	327.87	308.942	0.5208	0.9214	1.993	3.0	7.174
118	62.03	8	2	14.719	308.788	442.89	475.680	0.5210	0.9211	1.993	4.0	7.268
119	69.35	10	2	18.393	431.542	570.27	664.820	0.5212	0.9206	1.993	5.0	7.486
120	17.91	1	2	1.835	11.142	71.25	17.160	0.5222	0.9187	2.988	0.5	10.709
121	25.32	2	2	3.678	31.516	109.13	48.545	0.5211	0.9202	2.989	1.0	8.200
122	31.02	3	2	5.521	57.898	145.97	89.184	0.5208	0.9213	2.989	1.5	7.313
123	35.81	4	2	7.362	89.140	184.34	137.305	0.5208	0.9215	2.989	2.0	6.926
124	43.86	6	2	11.042	163.760	268.17	252.249	0.5208	0.9214	2.989	3.0	6.717
125	50.65	8	2	14.719	252.124	362.28	388.389	0.5210	0.9212	2.989	4.0	6.806
126	56.63	10	2	18.393	352.353	466.47	542.825	0.5212	0.9209	2.989	5.0	7.010
127	15.51	1	2	1.834	9.650	61.74	14.861	0.5224	0.9183	3.984	0.5	10.214
128	21.93	2	2	3.678	27.293	94.61	42.041	0.5212	0.9206	3.985	1.0	7.825
129	26.86	3	2	5.520	50.141	126.59	77.235	0.5209	0.9209	3.985	1.5	6.980
130	31.02	4	2	7.362	77.197	159.89	118.908	0.5208	0.9215	3.985	2.0	6.612
131	37.99	6	2	11.042	141.820	232.65	218.454	0.5208	0.9214	3.985	3.0	6.414
132	43.86	8	2	14.719	218.345	314.31	336.357	0.5210	0.9212	3.986	4.0	6.499
133	49.04	10	2	18.393	305.146	404.70	470.099	0.5212	0.9209	3.986	5.0	6.694
134	13.87	1	2	1.833	8.631	55.25	13.292	0.5227	0.9179	4.981	0.5	9.846
135	19.62	2	2	3.677	24.412	84.70	37.603	0.5213	0.9206	4.982	1.0	7.546
136	24.02	3	2	5.520	44.847	113.37	69.081	0.5209	0.9207	4.982	1.5	6.734
137	27.74	4	2	7.361	69.047	143.23	106.353	0.5208	0.9213	4.981	2.0	6.381
138	33.98	6	2	11.042	126.847	208.44	195.390	0.5208	0.9214	4.982	3.0	6.190
139	39.23	8	2	14.719	195.294	281.63	300.845	0.5210	0.9212	4.982	4.0	6.273
140	43.86	10	2	18.393	272.931	362.62	420.467	0.5212	0.9209	4.982	5.0	6.461
141	26.98	1	2	1.295	17.743	105.51	20.612	0.5898	0.6478	1.049	0.5	14.035
142	38.16	2	2	2.590	50.186	157.77	58.294	0.5898	0.6483	1.049	1.0	10.494
143	46.73	3	2	3.885	92.197	208.69	107.084	0.5898	0.6481	1.049	1.5	9.254
144	53.96	4	2	5.179	141.946	262.83	164.850	0.5899	0.6480	1.049	2.0	8.742
145	66.09	6	2	7.764	260.772	384.81	302.776	0.5901	0.6476	1.049	3.0	8.534
146	76.32	8	2	10.346	401.484	525.56	465.983	0.5902	0.6471	1.048	4.0	8.744
147	85.32	10	2	12.925	561.090	683.95	651.031	0.5903	0.6465	1.048	5.0	9.105
148	19.08	1	2	1.295	12.546	74.64	14.575	0.5899	0.6476	2.099	0.5	12.510
149	26.98	2	2	2.590	35.487	111.66	41.220	0.5898	0.6478	2.099	1.0	9.358
150	33.05	3	2	3.885	65.193	147.73	75.720	0.5899	0.6480	2.098	1.5	8.254
151	38.16	4	2	5.178	100.371	186.09	116.568	0.5899	0.6480	2.098	2.0	7.799
152	46.73	6	2	7.764	184.393	272.50	214.095	0.5901	0.6477	2.098	3.0	7.614
153	53.96	8	2	10.346	283.891	372.18	329.496	0.5902	0.6473	2.097	4.0	7.802
154	60.33	10	2	12.926	396.749	484.32	460.353	0.5903	0.6470	2.096	5.0	8.123
155	15.58	1	2	1.295	10.244	60.97	11.900	0.5899	0.6481	3.148	0.5	11.697
156	22.03	2	2	2.590	28.975	91.25	33.656	0.5898	0.6482	3.148	1.0	8.753
157	26.98	3	2	3.885	53.230	120.76	61.825	0.5899	0.6476	3.148	1.5	7.724
158	31.16	4	2	5.178	81.953	152.15	95.177	0.5899	0.6475	3.147	2.0	7.299
159	38.16	6	2	7.764	150.556	222.82	174.806	0.5901	0.6477	3.146	3.0	7.127
160	44.06	8	2	10.346	231.795	304.33	269.027	0.5902	0.6473	3.145	4.0	7.303
161	49.26	10	2	12.926	323.943	396.01	375.882	0.5903	0.6470	3.144	5.0	7.603
162	13.49	1	2	1.295	8.872	52.83	10.306	0.5900	0.6481	4.198	0.5	11.155
163	19.08	2	2	2.590	25.093	79.09	29.147	0.5899	0.6476	4.197	1.0	8.351
164	23.37	3	2	3.884	46.098	104.70	53.542	0.5899	0.6481	4.197	1.5	7.371
165	26.98	4	2	5.178	70.973	131.94	82.425	0.5899	0.6475	4.196	2.0	6.966
166	33.05	6	2	7.764	130.385	193.25	151.385	0.5901	0.6475	4.195	3.0	6.803
167	38.16	8	2	10.346	200.739	263.94	232.979	0.5901	0.6473	4.193	4.0	6.971
168	42.66	10	2	12.926	280.540	343.44	325.524	0.5903	0.6470	4.193	5.0	7.258
169	12.07	1	2	1.295	7.935	47.28	9.218	0.5901	0.6478	5.247	0.5	10.753
170	17.06	2	2	2.590	22.443	70.80	26.070	0.5899	0.6481	5.246	1.0	8.053
171	20.90	3	2	3.884	41.231	93.75	47.889	0.5899	0.6481	5.246	1.5	7.109
172	24.13	4	2	5.178	63.480	118.16	73.722	0.5899	0.6480	5.245	2.0	6.721
173	29.56	6	2	7.764	116.619	173.10	135.400	0.5901	0.6470	5.244	3.0	6.565

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
174	34.13	8	2	10.346	179.546	236.42	208.378	0.5901	0.6474	5.242	4.0	6.726
175	38.16	10	2	12.926	250.921	307.61	291.158	0.5903	0.6470	5.241	5.0	7.002
176	28.98	1	2	1.491	19.063	113.87	25.040	0.5795	0.7453	1.029	0.5	13.304
177	40.99	2	2	2.984	53.918	171.46	70.801	0.5789	0.7467	1.028	1.0	10.019
178	50.20	3	2	4.477	99.053	227.77	130.044	0.5786	0.7468	1.028	1.5	8.874
179	57.97	4	2	5.969	152.502	287.37	200.187	0.5786	0.7468	1.028	2.0	8.398
180	70.99	6	2	8.951	280.164	420.59	367.675	0.5786	0.7466	1.028	3.0	8.195
181	81.98	8	2	11.931	431.340	573.37	565.928	0.5786	0.7459	1.027	4.0	8.380
182	91.65	10	2	14.906	602.816	744.76	790.870	0.5789	0.7453	1.027	5.0	8.709
183	20.49	1	2	1.491	13.479	80.55	17.706	0.5796	0.7459	2.057	0.5	11.858
184	28.98	2	2	2.984	38.126	121.35	50.064	0.5789	0.7458	2.056	1.0	8.934
185	35.50	3	2	4.477	70.041	161.25	91.956	0.5787	0.7468	2.056	1.5	7.915
186	40.99	4	2	5.969	107.835	203.48	141.555	0.5786	0.7468	2.056	2.0	7.492
187	50.20	6	2	8.951	198.106	297.87	259.987	0.5786	0.7466	2.055	3.0	7.312
188	57.97	8	2	11.931	305.003	406.07	400.168	0.5786	0.7464	2.055	4.0	7.478
189	64.81	10	2	14.907	426.254	527.44	559.233	0.5789	0.7460	2.055	5.0	7.770
190	16.73	1	2	1.490	11.006	65.81	14.457	0.5797	0.7459	3.086	0.5	11.089
191	23.66	2	2	2.984	31.129	99.18	40.878	0.5789	0.7466	3.085	1.0	8.358
192	28.98	3	2	4.477	57.188	131.82	75.081	0.5787	0.7459	3.084	1.5	7.407
193	33.47	4	2	5.969	88.047	166.37	115.579	0.5786	0.7467	3.084	2.0	7.012
194	40.99	6	2	8.951	161.752	243.58	212.276	0.5786	0.7466	3.083	3.0	6.845
195	47.33	8	2	11.931	249.034	332.07	326.731	0.5786	0.7464	3.082	4.0	7.000
196	52.92	10	2	14.907	348.034	431.31	456.619	0.5789	0.7461	3.082	5.0	7.274
197	14.49	1	2	1.490	9.531	57.02	12.520	0.5798	0.7456	4.114	0.5	10.575
198	20.49	2	2	2.983	26.959	85.96	35.401	0.5790	0.7465	4.113	1.0	7.973
199	25.10	3	2	4.476	49.526	114.29	65.023	0.5787	0.7468	4.112	1.5	7.068
200	28.98	4	2	5.969	76.251	144.28	100.093	0.5786	0.7459	4.111	2.0	6.693
201	35.50	6	2	8.951	140.081	211.26	183.834	0.5786	0.7467	4.110	3.0	6.534
202	40.99	8	2	11.931	215.669	288.03	282.951	0.5786	0.7464	4.109	4.0	6.683
203	45.83	10	2	14.907	301.405	374.09	395.443	0.5789	0.7461	4.109	5.0	6.944
204	12.96	1	2	1.490	8.525	51.02	11.198	0.5800	0.7456	5.143	0.5	10.193
205	18.33	2	2	2.983	24.113	76.96	31.664	0.5790	0.7464	5.141	1.0	7.689
206	22.45	3	2	4.476	44.298	102.35	58.158	0.5787	0.7468	5.140	1.5	6.818
207	25.92	4	2	5.969	68.201	129.22	89.525	0.5786	0.7468	5.139	2.0	6.457
208	31.75	6	2	8.951	125.292	189.25	164.424	0.5786	0.7463	5.138	3.0	6.305
209	36.66	8	2	11.931	192.899	258.01	253.078	0.5786	0.7464	5.136	4.0	6.449
210	40.99	10	2	14.907	269.584	335.09	353.696	0.5789	0.7461	5.136	5.0	6.700
211	30.85	1	2	1.687	20.296	122.70	29.605	0.5687	0.8436	1.008	0.5	12.821
212	43.63	2	2	3.380	57.407	186.90	83.723	0.5677	0.8456	1.008	1.0	9.766
213	53.44	3	2	5.072	105.463	249.78	153.795	0.5674	0.8460	1.008	1.5	8.701
214	61.71	4	2	6.763	162.371	315.88	236.765	0.5674	0.8460	1.008	2.0	8.254
215	75.58	6	2	10.143	298.294	462.05	434.900	0.5673	0.8457	1.007	3.0	8.049
216	87.27	8	2	13.519	459.253	628.02	669.563	0.5675	0.8450	1.007	4.0	8.206
217	97.57	10	2	16.893	641.826	813.16	935.779	0.5678	0.8445	1.007	5.0	8.499
218	21.82	1	2	1.687	14.352	86.81	20.934	0.5688	0.8441	2.016	0.5	11.428
219	30.85	2	2	3.379	40.593	132.28	59.201	0.5678	0.8449	2.015	1.0	8.709
220	37.79	3	2	5.071	74.574	176.83	108.750	0.5675	0.8459	2.015	1.5	7.761
221	43.63	4	2	6.763	114.814	223.68	167.420	0.5674	0.8460	2.015	2.0	7.364
222	53.44	6	2	10.142	210.926	327.25	307.520	0.5673	0.8459	2.015	3.0	7.183
223	61.71	8	2	13.519	324.741	444.82	473.454	0.5675	0.8457	2.015	4.0	7.323
224	68.99	10	2	16.893	453.839	575.93	661.698	0.5677	0.8454	2.015	5.0	7.585
225	17.81	1	2	1.686	11.718	70.92	17.093	0.5690	0.8439	3.024	0.5	10.687
226	25.19	2	2	3.379	33.144	108.11	48.337	0.5678	0.8455	3.023	1.0	8.148
227	30.85	3	2	5.071	60.889	144.56	88.794	0.5675	0.8452	3.023	1.5	7.263
228	35.63	4	2	6.762	93.745	182.89	136.697	0.5674	0.8460	3.023	2.0	6.892
229	43.63	6	2	10.143	172.220	267.62	251.089	0.5673	0.8459	3.022	3.0	6.724
230	50.39	8	2	13.519	265.150	363.79	386.575	0.5675	0.8457	3.022	4.0	6.855
231	56.33	10	2	16.893	370.558	471.01	540.278	0.5677	0.8454	3.022	5.0	7.100
232	15.43	1	2	1.686	10.148	61.44	14.803	0.5691	0.8435	4.032	0.5	10.192
233	21.82	2	2	3.379	28.704	93.71	41.862	0.5679	0.8454	4.031	1.0	7.773
234	26.72	3	2	5.071	52.731	125.35	76.898	0.5675	0.8454	4.031	1.5	6.932
235	30.85	4	2	6.762	81.185	158.61	118.382	0.5674	0.8453	4.030	2.0	6.579
236	37.79	6	2	10.142	149.147	232.13	217.446	0.5673	0.8459	4.030	3.0	6.419
237	43.63	8	2	13.519	229.627	315.56	334.787	0.5675	0.8457	4.030	4.0	6.545
238	48.79	10	2	16.893	320.912	408.56	467.898	0.5677	0.8454	4.030	5.0	6.779

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
239	13.80	1	2	1.686	9.077	54.99	13.240	0.5693	0.8435	5.039	0.5	9.825
240	19.51	2	2	3.379	25.673	83.90	37.442	0.5679	0.8447	5.039	1.0	7.496
241	23.90	3	2	5.071	47.164	112.25	68.779	0.5675	0.8458	5.038	1.5	6.687
242	27.60	4	2	6.762	72.614	142.07	105.884	0.5674	0.8451	5.038	2.0	6.347
243	33.80	6	2	10.142	133.401	207.95	194.487	0.5673	0.8459	5.037	3.0	6.195
244	39.03	8	2	13.519	205.384	282.70	299.441	0.5675	0.8457	5.037	4.0	6.316
245	43.63	10	2	16.893	287.032	366.00	418.500	0.5677	0.8454	5.037	5.0	6.542
246	32.19	1	2	1.837	21.174	129.83	33.149	0.5605	0.9191	0.994	0.5	12.582
247	45.52	2	2	3.681	59.890	199.82	93.773	0.5596	0.9211	0.994	1.0	9.681
248	55.75	3	2	5.524	110.026	268.52	172.272	0.5594	0.9214	0.994	1.5	8.673
249	64.37	4	2	7.366	169.395	340.44	265.223	0.5593	0.9215	0.994	2.0	8.247
250	78.84	6	2	11.048	311.198	498.26	487.237	0.5594	0.9210	0.994	3.0	8.047
251	91.04	8	2	14.727	479.120	675.96	750.199	0.5595	0.9203	0.994	4.0	8.187
252	101.78	10	2	18.403	669.591	873.01	1048.500	0.5598	0.9198	0.994	5.0	8.459
253	22.76	1	2	1.837	14.973	91.86	23.440	0.5606	0.9193	1.988	0.5	11.215
254	32.19	2	2	3.681	42.349	141.43	66.307	0.5596	0.9206	1.988	1.0	8.633
255	39.42	3	2	5.524	77.800	190.11	121.815	0.5594	0.9214	1.988	1.5	7.736
256	45.52	4	2	7.366	119.781	241.08	187.542	0.5593	0.9215	1.988	2.0	7.358
257	55.75	6	2	11.048	220.051	352.91	344.537	0.5594	0.9214	1.988	3.0	7.181
258	64.37	8	2	14.727	338.790	478.80	530.476	0.5596	0.9212	1.989	4.0	7.307
259	71.97	10	2	18.403	473.473	618.36	741.401	0.5598	0.9209	1.989	5.0	7.549
260	18.58	1	2	1.837	12.225	75.05	19.139	0.5608	0.9185	2.982	0.5	10.488
261	26.28	2	2	3.681	34.578	115.59	54.140	0.5597	0.9208	2.983	1.0	8.077
262	32.19	3	2	5.524	63.523	155.42	99.461	0.5594	0.9210	2.983	1.5	7.240
263	37.17	4	2	7.366	97.800	197.12	153.126	0.5593	0.9215	2.983	2.0	6.887
264	45.52	6	2	11.048	179.671	288.62	281.311	0.5594	0.9214	2.983	3.0	6.723
265	52.56	8	2	14.727	276.620	391.60	433.131	0.5595	0.9212	2.983	4.0	6.841
266	58.77	10	2	18.403	386.588	505.74	605.350	0.5598	0.9209	2.983	5.0	7.067
267	16.09	1	2	1.836	10.587	65.02	16.575	0.5609	0.9188	3.977	0.5	10.002
268	22.76	2	2	3.680	29.945	100.20	46.887	0.5597	0.9208	3.977	1.0	7.706
269	27.87	3	2	5.524	55.013	134.77	86.136	0.5594	0.9203	3.977	1.5	6.910
270	32.19	4	2	7.366	84.697	170.95	132.610	0.5593	0.9211	3.977	2.0	6.574
271	39.42	6	2	11.048	155.599	250.35	243.622	0.5594	0.9214	3.977	3.0	6.418
272	45.52	8	2	14.727	239.560	339.70	375.104	0.5595	0.9212	3.977	4.0	6.531
273	50.89	10	2	18.403	334.796	438.71	524.253	0.5598	0.9209	3.977	5.0	6.748
274	14.39	1	2	1.836	9.470	58.19	14.825	0.5611	0.9185	4.971	0.5	9.643
275	20.36	2	2	3.680	26.784	89.70	41.937	0.5598	0.9205	4.971	1.0	7.432
276	24.93	3	2	5.523	49.205	120.69	77.042	0.5594	0.9213	4.971	1.5	6.666
277	28.79	4	2	7.366	75.756	153.13	118.609	0.5593	0.9204	4.971	2.0	6.343
278	35.26	6	2	11.048	139.172	224.28	217.903	0.5594	0.9214	4.971	3.0	6.194
279	40.71	8	2	14.727	214.269	304.34	335.504	0.5596	0.9212	4.971	4.0	6.303
280	45.52	10	2	18.403	299.450	393.04	468.903	0.5598	0.9209	4.972	5.0	6.512
281	27.93	1	2	1.295	19.358	110.25	22.776	0.6298	0.6482	1.046	0.5	13.721
282	39.50	2	2	2.589	54.752	165.48	64.416	0.6298	0.6478	1.045	1.0	10.298
283	48.37	3	2	3.883	100.585	219.88	118.327	0.6299	0.6478	1.045	1.5	9.123
284	55.86	4	2	5.177	154.861	278.16	182.156	0.6300	0.6478	1.045	2.0	8.656
285	68.41	6	2	7.761	284.497	410.23	334.558	0.6301	0.6475	1.045	3.0	8.512
286	78.99	8	2	10.342	438.011	563.21	514.888	0.6302	0.6471	1.045	4.0	8.767
287	88.32	10	2	12.921	612.138	735.63	719.356	0.6304	0.6468	1.044	5.0	9.163
288	19.75	1	2	1.295	13.688	77.99	16.106	0.6298	0.6482	2.091	0.5	12.230
289	27.93	2	2	2.589	38.715	117.11	45.549	0.6298	0.6481	2.091	1.0	9.182
290	34.21	3	2	3.883	71.124	155.65	83.670	0.6299	0.6480	2.091	1.5	8.137
291	39.50	4	2	5.177	109.503	196.95	128.805	0.6300	0.6475	2.090	2.0	7.722
292	48.37	6	2	7.761	201.170	290.50	236.570	0.6301	0.6473	2.090	3.0	7.595
293	55.86	8	2	10.343	309.720	398.83	364.077	0.6302	0.6471	2.089	4.0	7.822
294	62.45	10	2	12.922	432.846	520.90	508.666	0.6304	0.6468	2.089	5.0	8.175
295	16.12	1	2	1.295	11.176	63.71	13.150	0.6299	0.6481	3.137	0.5	11.436
296	22.80	2	2	2.589	31.611	95.71	37.191	0.6298	0.6478	3.136	1.0	8.590
297	27.93	3	2	3.883	58.073	127.24	68.316	0.6299	0.6480	3.136	1.5	7.614
298	32.25	4	2	5.177	89.409	161.01	105.169	0.6300	0.6478	3.136	2.0	7.227
299	39.50	6	2	7.761	164.254	237.54	193.156	0.6301	0.6472	3.135	3.0	7.109
300	45.61	8	2	10.343	252.885	326.12	297.265	0.6302	0.6469	3.134	4.0	7.322
301	50.99	10	2	12.922	353.417	425.91	415.332	0.6304	0.6467	3.133	5.0	7.651
302	13.96	1	2	1.295	9.679	55.20	11.389	0.6299	0.6480	4.182	0.5	10.906
303	19.75	2	2	2.589	27.376	82.95	32.208	0.6299	0.6481	4.182	1.0	8.195

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
304	24.19	3	2	3.883	50.292	110.31	59.163	0.6299	0.6476	4.181	1.5	7.265
305	27.93	4	2	5.177	77.430	139.62	91.078	0.6300	0.6478	4.181	2.0	6.897
306	34.21	6	2	7.761	142.248	206.00	167.275	0.6301	0.6475	4.180	3.0	6.786
307	39.50	8	2	10.343	219.005	282.83	257.433	0.6302	0.6468	4.178	4.0	6.989
308	44.16	10	2	12.922	306.068	369.36	359.690	0.6304	0.6466	4.177	5.0	7.303
309	12.49	1	2	1.295	8.657	49.40	10.186	0.6299	0.6479	5.228	0.5	10.512
310	17.66	2	2	2.589	24.486	74.26	28.808	0.6299	0.6477	5.227	1.0	7.902
311	21.63	3	2	3.883	44.983	98.78	52.918	0.6299	0.6480	5.227	1.5	7.008
312	24.98	4	2	5.177	69.255	125.04	81.462	0.6300	0.6476	5.226	2.0	6.654
313	30.59	6	2	7.761	127.230	184.52	149.615	0.6301	0.6475	5.225	3.0	6.547
314	35.33	8	2	10.343	195.884	253.33	230.251	0.6302	0.6471	5.223	4.0	6.743
315	39.50	10	2	12.922	273.756	330.82	321.720	0.6304	0.6465	5.221	5.0	7.046
316	30.00	1	2	1.491	20.796	119.07	27.721	0.6199	0.7460	1.027	0.5	12.999
317	42.43	2	2	2.983	58.819	180.14	78.380	0.6192	0.7463	1.026	1.0	9.836
318	51.96	3	2	4.476	108.057	240.49	143.964	0.6190	0.7466	1.026	1.5	8.755
319	60.00	4	2	5.968	166.364	304.79	221.614	0.6189	0.7468	1.026	2.0	8.323
320	73.48	6	2	8.950	305.629	449.22	407.032	0.6189	0.7466	1.026	3.0	8.179
321	84.85	8	2	11.929	470.546	615.36	626.499	0.6190	0.7463	1.025	4.0	8.405
322	94.87	10	2	14.904	657.608	801.98	875.528	0.6192	0.7459	1.025	5.0	8.763
323	21.21	1	2	1.490	14.705	84.23	19.602	0.6200	0.7460	2.053	0.5	11.587
324	30.00	2	2	2.983	41.591	127.49	55.424	0.6192	0.7466	2.053	1.0	8.770
325	36.74	3	2	4.476	76.408	170.25	101.799	0.6190	0.7465	2.052	1.5	7.809
326	42.43	4	2	5.968	117.637	215.81	156.706	0.6189	0.7464	2.052	2.0	7.425
327	51.96	6	2	8.950	216.113	318.14	287.816	0.6189	0.7465	2.051	3.0	7.298
328	60.00	8	2	11.929	332.727	435.80	443.000	0.6190	0.7463	2.051	4.0	7.499
329	67.08	10	2	14.904	464.999	567.94	619.099	0.6192	0.7460	2.051	5.0	7.819
330	17.32	1	2	1.490	12.006	68.81	16.005	0.6200	0.7456	3.080	0.5	10.835
331	24.49	2	2	2.983	33.959	104.19	45.253	0.6193	0.7462	3.079	1.0	8.205
332	30.00	3	2	4.476	62.386	139.17	83.118	0.6190	0.7468	3.078	1.5	7.307
333	34.64	4	2	5.968	96.050	176.45	127.950	0.6189	0.7468	3.078	2.0	6.949
334	42.43	6	2	8.950	176.455	260.15	234.999	0.6189	0.7462	3.077	3.0	6.831
335	48.99	8	2	11.929	271.669	356.37	361.703	0.6189	0.7461	3.076	4.0	7.020
336	54.77	10	2	14.905	379.668	464.41	505.498	0.6192	0.7459	3.076	5.0	7.318
337	15.00	1	2	1.490	10.398	59.62	13.861	0.6201	0.7458	4.107	0.5	10.333
338	21.21	2	2	2.983	29.409	90.31	39.191	0.6193	0.7466	4.105	1.0	7.827
339	25.98	3	2	4.476	54.028	120.67	71.982	0.6190	0.7466	4.105	1.5	6.973
340	30.00	4	2	5.968	83.182	153.01	110.807	0.6189	0.7468	4.104	2.0	6.633
341	36.74	6	2	8.950	152.815	225.63	203.513	0.6189	0.7464	4.103	3.0	6.521
342	42.43	8	2	11.929	235.272	309.09	313.241	0.6189	0.7460	4.102	4.0	6.701
343	47.43	10	2	14.905	328.802	402.78	437.776	0.6192	0.7458	4.102	5.0	6.986
344	13.42	1	2	1.490	9.300	53.35	12.398	0.6202	0.7458	5.134	0.5	9.960
345	18.97	2	2	2.983	26.304	80.85	35.053	0.6193	0.7465	5.132	1.0	7.548
346	23.24	3	2	4.476	48.324	108.05	64.383	0.6190	0.7464	5.131	1.5	6.727
347	26.83	4	2	5.968	74.400	137.04	99.109	0.6189	0.7468	5.130	2.0	6.399
348	32.86	6	2	8.950	136.681	202.11	182.024	0.6188	0.7466	5.129	3.0	6.293
349	37.95	8	2	11.929	210.434	276.87	280.172	0.6189	0.7460	5.127	4.0	6.466
350	42.43	10	2	14.905	294.089	360.78	391.562	0.6192	0.7457	5.127	5.0	6.741
351	31.94	1	2	1.688	22.140	128.51	32.833	0.6092	0.8445	1.008	0.5	12.533
352	45.17	2	2	3.381	62.621	196.90	92.852	0.6081	0.8456	1.008	1.0	9.603
353	55.32	3	2	5.073	115.043	264.55	170.565	0.6078	0.8462	1.008	1.5	8.602
354	63.87	4	2	6.764	177.120	336.05	262.583	0.6078	0.8463	1.008	2.0	8.195
355	78.23	6	2	10.145	325.390	494.77	482.327	0.6077	0.8462	1.007	3.0	8.045
356	90.33	8	2	13.522	500.970	675.39	742.577	0.6079	0.8459	1.007	4.0	8.236
357	101.00	10	2	16.896	700.127	877.01	1037.842	0.6082	0.8456	1.007	5.0	8.556
358	22.58	1	2	1.687	15.655	90.92	23.217	0.6093	0.8443	2.016	0.5	11.172
359	31.94	2	2	3.380	44.280	139.36	65.657	0.6081	0.8459	2.015	1.0	8.563
360	39.12	3	2	5.073	81.348	187.29	120.609	0.6078	0.8458	2.015	1.5	7.672
361	45.17	4	2	6.764	125.243	237.95	185.676	0.6077	0.8460	2.015	2.0	7.311
362	55.32	6	2	10.145	230.085	350.41	341.056	0.6077	0.8461	2.015	3.0	7.178
363	63.87	8	2	13.522	354.239	478.34	525.091	0.6079	0.8459	2.015	4.0	7.349
364	71.41	10	2	16.897	495.063	621.12	733.868	0.6082	0.8456	2.015	5.0	7.634
365	18.44	1	2	1.687	12.783	74.27	18.957	0.6093	0.8441	3.024	0.5	10.447
366	26.08	2	2	3.380	36.155	113.89	53.609	0.6082	0.8457	3.023	1.0	8.011
367	31.94	3	2	5.073	66.420	153.10	98.477	0.6079	0.8462	3.023	1.5	7.180
368	36.88	4	2	6.764	102.260	194.56	151.603	0.6078	0.8460	3.023	2.0	6.843

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
369	45.17	6	2	10.145	187.863	286.55	278.469	0.6077	0.8458	3.022	3.0	6.720
370	52.15	8	2	13.523	289.235	391.18	428.737	0.6079	0.8458	3.022	4.0	6.880
371	58.31	10	2	16.897	404.217	507.93	599.203	0.6082	0.8456	3.022	5.0	7.146
372	15.97	1	2	1.687	11.070	64.36	16.417	0.6094	0.8443	4.032	0.5	9.963
373	22.58	2	2	3.380	31.311	98.72	46.426	0.6082	0.8456	4.031	1.0	7.643
374	27.66	3	2	5.073	57.522	132.75	85.284	0.6079	0.8462	4.031	1.5	6.852
375	31.94	4	2	6.764	88.560	168.72	131.292	0.6078	0.8463	4.030	2.0	6.531
376	39.12	6	2	10.145	162.695	248.54	241.160	0.6077	0.8458	4.030	3.0	6.415
377	45.17	8	2	13.523	250.484	339.30	371.297	0.6079	0.8456	4.030	4.0	6.568
378	50.50	10	2	16.897	350.062	440.56	518.930	0.6082	0.8455	4.030	5.0	6.822
379	14.28	1	2	1.687	9.901	57.59	14.684	0.6095	0.8437	5.040	0.5	9.604
380	20.20	2	2	3.380	28.005	88.38	41.525	0.6082	0.8458	5.039	1.0	7.370
381	24.74	3	2	5.072	51.449	118.88	76.280	0.6079	0.8458	5.038	1.5	6.609
382	28.57	4	2	6.764	79.210	151.11	117.431	0.6077	0.8463	5.038	2.0	6.302
383	34.99	6	2	10.145	145.518	222.64	215.697	0.6077	0.8462	5.037	3.0	6.190
384	40.40	8	2	13.523	224.040	303.95	332.102	0.6079	0.8455	5.037	4.0	6.338
385	45.17	10	2	16.897	313.104	394.64	464.144	0.6082	0.8453	5.037	5.0	6.583
386	33.32	1	2	1.839	23.097	136.23	36.813	0.6009	0.9203	0.995	0.5	12.310
387	47.12	2	2	3.684	65.328	211.06	104.136	0.6000	0.9215	0.996	1.0	9.535
388	57.71	3	2	5.528	120.016	285.21	191.308	0.5997	0.9221	0.996	1.5	8.590
389	66.63	4	2	7.371	184.777	363.22	294.532	0.5997	0.9222	0.996	2.0	8.205
390	81.61	6	2	11.055	339.455	534.97	541.076	0.5997	0.9221	0.996	3.0	8.057
391	94.23	8	2	14.736	522.624	728.67	833.088	0.5999	0.9218	0.996	4.0	8.230
392	105.36	10	2	18.414	730.391	943.55	1164.341	0.6002	0.9215	0.996	5.0	8.525
393	23.56	1	2	1.839	16.332	96.38	26.032	0.6009	0.9197	1.991	0.5	10.973
394	33.32	2	2	3.684	46.194	149.38	73.635	0.6000	0.9217	1.991	1.0	8.503
395	40.80	3	2	5.528	84.864	201.92	135.277	0.5997	0.9216	1.991	1.5	7.662
396	47.12	4	2	7.371	130.657	257.20	208.267	0.5997	0.9219	1.991	2.0	7.320
397	57.71	6	2	11.055	240.031	378.89	382.606	0.5997	0.9221	1.991	3.0	7.189
398	66.63	8	2	14.736	369.553	516.10	589.090	0.5999	0.9218	1.991	4.0	7.344
399	74.50	10	2	18.414	516.464	668.28	823.312	0.6002	0.9215	1.991	5.0	7.608
400	19.24	1	2	1.839	13.335	78.73	21.255	0.6010	0.9201	2.986	0.5	10.261
401	27.20	2	2	3.684	37.718	122.08	60.124	0.6000	0.9217	2.987	1.0	7.955
402	33.32	3	2	5.528	69.291	165.07	110.452	0.5997	0.9221	2.987	1.5	7.170
403	38.47	4	2	7.371	106.681	210.30	170.048	0.5997	0.9218	2.987	2.0	6.851
404	47.12	6	2	11.055	195.984	309.85	312.396	0.5997	0.9218	2.987	3.0	6.730
405	54.41	8	2	14.737	301.737	422.08	480.989	0.5999	0.9217	2.987	4.0	6.875
406	60.83	10	2	18.415	421.691	546.52	672.235	0.6001	0.9216	2.987	5.0	7.122
407	16.66	1	2	1.838	11.549	68.22	18.407	0.6011	0.9199	3.982	0.5	9.786
408	23.56	2	2	3.683	32.664	105.83	52.069	0.6000	0.9212	3.982	1.0	7.589
409	28.85	3	2	5.528	60.008	143.12	95.654	0.5998	0.9221	3.982	1.5	6.843
410	33.32	4	2	7.371	92.388	182.37	147.265	0.5997	0.9222	3.982	2.0	6.540
411	40.80	6	2	11.056	169.728	268.76	270.543	0.5997	0.9216	3.982	3.0	6.425
412	47.12	8	2	14.737	261.312	366.11	416.550	0.5999	0.9215	3.982	4.0	6.564
413	52.68	10	2	18.415	365.194	474.05	582.174	0.6001	0.9214	3.983	5.0	6.799
414	14.90	1	2	1.838	10.329	61.05	16.464	0.6012	0.9196	4.977	0.5	9.434
415	21.07	2	2	3.683	29.216	94.74	46.571	0.6001	0.9216	4.978	1.0	7.319
416	25.81	3	2	5.528	53.673	128.17	85.556	0.5998	0.9219	4.978	1.5	6.601
417	29.80	4	2	7.371	82.635	163.34	131.718	0.5997	0.9222	4.978	2.0	6.310
418	36.50	6	2	11.056	151.809	240.75	241.981	0.5997	0.9219	4.978	3.0	6.200
419	42.14	8	2	14.737	233.724	327.98	372.576	0.5999	0.9214	4.978	4.0	6.334
420	47.12	10	2	18.415	326.638	424.67	520.711	0.6001	0.9213	4.978	5.0	6.561
421	28.84	1	2	1.298	20.956	114.81	24.979	0.6673	0.6492	1.041	0.5	13.436
422	40.79	2	2	2.595	59.272	172.95	70.644	0.6673	0.6487	1.041	1.0	10.121
423	49.96	3	2	3.892	108.890	230.81	129.767	0.6674	0.6490	1.041	1.5	9.005
424	57.69	4	2	5.188	167.647	293.20	199.767	0.6675	0.6489	1.041	2.0	8.580
425	70.65	6	2	7.778	307.986	435.31	366.898	0.6677	0.6485	1.040	3.0	8.494
426	81.58	8	2	10.365	474.174	600.41	564.651	0.6678	0.6482	1.040	4.0	8.789
427	91.21	10	2	12.949	662.678	786.71	788.839	0.6679	0.6478	1.039	5.0	9.215
428	20.40	1	2	1.298	14.818	81.22	17.663	0.6674	0.6492	2.082	0.5	11.975
429	28.84	2	2	2.595	41.912	122.41	49.953	0.6674	0.6491	2.082	1.0	9.025
430	35.33	3	2	3.892	76.997	163.40	91.760	0.6674	0.6488	2.081	1.5	8.032
431	40.79	4	2	5.188	118.543	207.60	141.257	0.6675	0.6484	2.081	2.0	7.654
432	49.96	6	2	7.778	217.778	308.26	259.436	0.6676	0.6485	2.080	3.0	7.578
433	57.69	8	2	10.365	335.291	425.17	399.264	0.6677	0.6482	2.080	4.0	7.841

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
434	64.50	10	2	12.949	468.581	557.07	557.798	0.6679	0.6478	2.079	5.0	8.221
435	16.65	1	2	1.298	12.099	66.35	14.422	0.6674	0.6492	3.123	0.5	11.198
436	23.55	2	2	2.595	34.220	100.03	40.787	0.6674	0.6491	3.122	1.0	8.442
437	28.84	3	2	3.892	62.867	133.57	74.921	0.6674	0.6490	3.122	1.5	7.516
438	33.31	4	2	5.188	96.790	169.73	115.336	0.6675	0.6489	3.122	2.0	7.163
439	40.79	6	2	7.778	177.814	252.06	211.827	0.6676	0.6481	3.121	3.0	7.093
440	47.10	8	2	10.365	273.762	347.65	325.994	0.6677	0.6482	3.119	4.0	7.340
441	52.66	10	2	12.949	382.594	455.48	455.450	0.6679	0.6478	3.118	5.0	7.694
442	14.42	1	2	1.297	10.478	57.49	12.490	0.6675	0.6491	4.163	0.5	10.679
443	20.40	2	2	2.595	29.636	86.70	35.322	0.6674	0.6491	4.163	1.0	8.054
444	24.98	3	2	3.892	54.444	115.80	64.884	0.6675	0.6486	4.163	1.5	7.172
445	28.84	4	2	5.188	83.823	147.18	99.883	0.6675	0.6488	4.162	2.0	6.837
446	35.33	6	2	7.778	153.992	218.60	183.446	0.6676	0.6483	4.161	3.0	6.771
447	40.79	8	2	10.365	237.084	301.51	282.311	0.6677	0.6477	4.159	4.0	7.006
448	45.61	10	2	12.949	331.336	395.00	394.433	0.6679	0.6477	4.158	5.0	7.344
449	12.90	1	2	1.297	9.372	51.44	11.171	0.6675	0.6491	5.204	0.5	10.294
450	18.24	2	2	2.595	26.507	77.62	31.593	0.6674	0.6491	5.204	1.0	7.767
451	22.34	3	2	3.891	48.696	103.70	58.034	0.6675	0.6490	5.203	1.5	6.918
452	25.80	4	2	5.188	74.973	131.81	89.338	0.6675	0.6486	5.203	2.0	6.596
453	31.60	6	2	7.778	137.734	195.80	164.075	0.6676	0.6485	5.201	3.0	6.533
454	36.49	8	2	10.365	212.055	270.06	252.502	0.6677	0.6477	5.199	4.0	6.760
455	40.79	10	2	12.949	296.355	353.78	352.793	0.6679	0.6474	5.197	5.0	7.086
456	30.98	1	2	1.493	22.512	124.09	30.458	0.6583	0.7471	1.024	0.5	12.723
457	43.82	2	2	2.989	63.675	188.60	86.121	0.6576	0.7474	1.024	1.0	9.671
458	53.67	3	2	4.484	116.978	252.99	158.183	0.6573	0.7478	1.023	1.5	8.649
459	61.97	4	2	5.979	180.100	321.97	243.503	0.6572	0.7478	1.023	2.0	8.257
460	75.89	6	2	8.966	330.863	477.58	447.236	0.6572	0.7476	1.023	3.0	8.166
461	87.64	8	2	11.950	509.397	657.02	688.390	0.6573	0.7473	1.023	4.0	8.427
462	97.98	10	2	14.931	711.903	858.76	962.002	0.6576	0.7470	1.023	5.0	8.812
463	21.91	1	2	1.493	15.919	87.79	21.538	0.6583	0.7471	2.048	0.5	11.341
464	30.98	2	2	2.989	45.025	133.48	60.897	0.6576	0.7476	2.047	1.0	8.624
465	37.95	3	2	4.484	82.716	179.10	111.853	0.6573	0.7472	2.047	1.5	7.715
466	43.82	4	2	5.979	127.350	227.98	172.184	0.6572	0.7476	2.047	2.0	7.366
467	53.67	6	2	8.967	233.956	338.21	316.246	0.6572	0.7476	2.046	3.0	7.286
468	61.97	8	2	11.951	360.198	465.29	486.761	0.6573	0.7473	2.046	4.0	7.519
469	69.28	10	2	14.932	503.391	608.14	680.247	0.6576	0.7470	2.046	5.0	7.863
470	17.89	1	2	1.493	12.998	71.72	17.586	0.6584	0.7470	3.072	0.5	10.605
471	25.30	2	2	2.989	36.763	109.08	49.723	0.6576	0.7473	3.071	1.0	8.067
472	30.98	3	2	4.484	67.537	146.41	91.327	0.6573	0.7478	3.070	1.5	7.219
473	35.78	4	2	5.979	103.980	186.40	140.587	0.6572	0.7474	3.070	2.0	6.894
474	43.82	6	2	8.967	191.024	276.56	258.212	0.6572	0.7474	3.069	3.0	6.820
475	50.60	8	2	11.951	294.100	380.48	397.435	0.6573	0.7473	3.068	4.0	7.039
476	56.57	10	2	14.932	411.016	497.28	555.423	0.6576	0.7470	3.068	5.0	7.360
477	15.49	1	2	1.493	11.256	62.14	15.230	0.6584	0.7468	4.096	0.5	10.113
478	21.91	2	2	2.989	31.837	94.56	43.061	0.6576	0.7476	4.095	1.0	7.697
479	26.83	3	2	4.484	58.489	126.94	79.092	0.6573	0.7478	4.094	1.5	6.889
480	30.98	4	2	5.979	90.049	161.64	121.751	0.6572	0.7478	4.093	2.0	6.580
481	37.95	6	2	8.967	165.431	239.87	223.615	0.6572	0.7470	4.092	3.0	6.511
482	43.82	8	2	11.951	254.697	330.00	344.181	0.6573	0.7471	4.091	4.0	6.719
483	48.99	10	2	14.932	355.949	431.28	481.016	0.6576	0.7470	4.091	5.0	7.025
484	13.86	1	2	1.493	10.068	55.60	13.622	0.6585	0.7469	5.120	0.5	9.749
485	19.60	2	2	2.989	28.476	84.65	38.515	0.6576	0.7475	5.118	1.0	7.422
486	24.00	3	2	4.484	52.314	113.67	70.742	0.6573	0.7475	5.117	1.5	6.646
487	27.71	4	2	5.979	80.543	144.76	108.898	0.6572	0.7478	5.117	2.0	6.348
488	33.94	6	2	8.967	147.966	214.86	200.005	0.6572	0.7476	5.115	3.0	6.282
489	39.19	8	2	11.951	227.808	295.60	307.844	0.6572	0.7467	5.114	4.0	6.484
490	43.82	10	2	14.932	318.370	386.30	430.237	0.6576	0.7468	5.114	5.0	6.778
491	32.98	1	2	1.690	23.968	134.18	36.144	0.6483	0.8455	1.007	0.5	12.274
492	46.65	2	2	3.386	67.792	206.74	102.215	0.6472	0.8469	1.007	1.0	9.457
493	57.13	3	2	5.080	124.542	279.15	187.766	0.6469	0.8472	1.007	1.5	8.513
494	65.97	4	2	6.774	191.744	356.05	289.064	0.6468	0.8472	1.007	2.0	8.144
495	80.80	6	2	10.160	352.256	527.30	530.976	0.6468	0.8471	1.007	3.0	8.042
496	93.30	8	2	13.542	542.333	722.53	817.451	0.6470	0.8468	1.007	4.0	8.265
497	104.31	10	2	16.921	757.933	940.65	1142.473	0.6473	0.8465	1.007	5.0	8.607
498	23.32	1	2	1.690	16.948	94.93	25.558	0.6484	0.8454	2.014	0.5	10.941

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
499	32.98	2	2	3.386	47.936	146.33	72.278	0.6472	0.8468	2.014	1.0	8.433
500	40.40	3	2	5.080	88.064	197.63	132.772	0.6469	0.8466	2.014	1.5	7.594
501	46.65	4	2	6.774	135.584	252.11	204.401	0.6468	0.8472	2.014	2.0	7.266
502	57.13	6	2	10.160	249.083	373.44	375.456	0.6468	0.8471	2.013	3.0	7.175
503	65.97	8	2	13.542	383.487	511.72	578.036	0.6470	0.8468	2.013	4.0	7.374
504	73.76	10	2	16.921	535.938	666.17	807.852	0.6473	0.8465	2.013	5.0	7.680
505	19.04	1	2	1.690	13.838	77.55	20.868	0.6484	0.8451	3.021	0.5	10.231
506	26.93	2	2	3.386	39.140	119.58	59.015	0.6472	0.8468	3.021	1.0	7.889
507	32.98	3	2	5.080	71.904	161.56	108.408	0.6469	0.8472	3.021	1.5	7.106
508	38.09	4	2	6.774	110.704	206.13	166.893	0.6468	0.8465	3.021	2.0	6.800
509	46.65	6	2	10.160	203.375	305.38	306.558	0.6468	0.8471	3.020	3.0	6.717
510	53.86	8	2	13.543	313.116	418.47	471.968	0.6470	0.8468	3.020	4.0	6.903
511	60.22	10	2	16.922	437.591	544.76	659.613	0.6473	0.8465	3.020	5.0	7.189
512	16.49	1	2	1.690	11.984	67.19	18.072	0.6485	0.8453	4.029	0.5	9.757
513	23.32	2	2	3.385	33.896	103.66	51.108	0.6472	0.8468	4.028	1.0	7.527
514	28.57	3	2	5.080	62.271	140.08	93.884	0.6469	0.8471	4.028	1.5	6.781
515	32.98	4	2	6.774	95.872	178.76	144.533	0.6468	0.8472	4.027	2.0	6.491
516	40.40	6	2	10.160	176.128	264.87	265.484	0.6468	0.8465	4.027	3.0	6.412
517	46.65	8	2	13.543	271.165	362.96	408.736	0.6470	0.8468	4.027	4.0	6.590
518	52.15	10	2	16.922	378.964	472.49	571.247	0.6473	0.8465	4.027	5.0	6.863
519	14.75	1	2	1.690	10.719	60.13	16.164	0.6485	0.8452	5.036	0.5	9.406
520	20.86	2	2	3.385	30.318	92.80	45.713	0.6473	0.8468	5.035	1.0	7.258
521	25.55	3	2	5.080	55.696	125.43	83.971	0.6469	0.8468	5.035	1.5	6.541
522	29.50	4	2	6.774	85.751	160.10	129.274	0.6468	0.8472	5.034	2.0	6.262
523	36.13	6	2	10.160	157.533	237.26	237.452	0.6468	0.8466	5.033	3.0	6.187
524	41.72	8	2	13.543	242.537	325.14	365.587	0.6470	0.8464	5.033	4.0	6.359
525	46.65	10	2	16.922	338.956	423.24	510.940	0.6473	0.8465	5.034	5.0	6.622
526	34.41	1	2	1.841	25.004	142.50	40.583	0.6406	0.9210	0.996	0.5	12.067
527	48.66	2	2	3.688	70.722	222.18	114.795	0.6396	0.9225	0.996	1.0	9.406
528	59.60	3	2	5.534	129.925	301.80	210.890	0.6394	0.9229	0.996	1.5	8.518
529	68.82	4	2	7.380	200.033	385.91	324.679	0.6393	0.9229	0.996	2.0	8.169
530	84.29	6	2	11.068	367.483	571.59	596.441	0.6393	0.9228	0.996	3.0	8.067
531	97.32	8	2	14.753	565.777	781.27	918.323	0.6396	0.9225	0.996	4.0	8.269
532	108.81	10	2	18.436	790.696	1014.02	1283.454	0.6398	0.9222	0.996	5.0	8.586
533	24.33	1	2	1.841	17.681	100.82	28.697	0.6406	0.9205	1.992	0.5	10.756
534	34.41	2	2	3.688	50.008	157.25	81.173	0.6396	0.9225	1.992	1.0	8.388
535	42.14	3	2	5.534	91.871	213.66	149.124	0.6394	0.9224	1.992	1.5	7.598
536	48.66	4	2	7.380	141.445	273.26	229.584	0.6393	0.9229	1.992	2.0	7.288
537	59.60	6	2	11.069	259.849	404.81	421.755	0.6393	0.9228	1.992	3.0	7.198
538	68.82	8	2	14.754	400.065	553.34	649.360	0.6396	0.9225	1.992	4.0	7.379
539	76.94	10	2	18.436	559.108	718.17	907.542	0.6398	0.9222	1.992	5.0	7.662
540	19.87	1	2	1.841	14.436	82.36	23.431	0.6407	0.9208	2.988	0.5	10.058
541	28.10	2	2	3.688	40.832	128.51	66.277	0.6396	0.9225	2.989	1.0	7.847
542	34.41	3	2	5.534	75.012	174.67	121.759	0.6394	0.9229	2.989	1.5	7.110
543	39.73	4	2	7.380	115.489	223.42	187.455	0.6393	0.9222	2.989	2.0	6.821
544	48.66	6	2	11.069	212.166	331.03	344.361	0.6393	0.9228	2.988	3.0	6.738
545	56.19	8	2	14.754	326.651	452.52	530.200	0.6396	0.9225	2.989	4.0	6.908
546	62.82	10	2	18.436	456.508	587.30	741.005	0.6398	0.9222	2.989	5.0	7.172
547	17.20	1	2	1.841	12.502	71.37	20.292	0.6408	0.9208	3.985	0.5	9.593
548	24.33	2	2	3.688	35.361	111.40	57.398	0.6396	0.9220	3.985	1.0	7.487
549	29.80	3	2	5.534	64.963	151.45	105.446	0.6394	0.9228	3.985	1.5	6.785
550	34.41	4	2	7.380	100.016	193.76	162.339	0.6393	0.9229	3.985	2.0	6.511
551	42.14	6	2	11.069	183.741	287.12	298.227	0.6393	0.9224	3.985	3.0	6.432
552	48.66	8	2	14.754	282.888	392.51	459.169	0.6396	0.9225	3.985	4.0	6.595
553	54.41	10	2	18.436	395.347	509.41	641.734	0.6398	0.9222	3.985	5.0	6.847
554	15.39	1	2	1.841	11.182	63.86	18.150	0.6408	0.9207	4.981	0.5	9.247
555	21.76	2	2	3.688	31.628	99.73	51.338	0.6397	0.9224	4.981	1.0	7.220
556	26.65	3	2	5.534	58.104	135.62	94.314	0.6394	0.9228	4.981	1.5	6.545
557	30.78	4	2	7.380	89.457	173.54	145.200	0.6393	0.9229	4.981	2.0	6.282
558	37.69	6	2	11.069	164.343	257.20	266.741	0.6393	0.9221	4.981	3.0	6.207
559	43.52	8	2	14.754	253.022	351.61	410.695	0.6396	0.9222	4.981	4.0	6.364
560	48.66	10	2	18.436	353.608	456.33	573.984	0.6398	0.9222	4.981	5.0	6.607
561	29.73	1	2	1.296	22.567	119.22	27.257	0.7073	0.6494	1.037	0.5	13.163
562	42.05	2	2	2.592	63.828	180.27	77.085	0.7073	0.6493	1.037	1.0	9.953
563	51.50	3	2	3.887	117.259	241.64	141.599	0.7073	0.6492	1.037	1.5	8.895

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
564	59.46	4	2	5.182	180.532	308.23	217.983	0.7074	0.6490	1.037	2.0	8.510
565	72.83	6	2	7.769	331.657	460.59	400.353	0.7076	0.6487	1.036	3.0	8.479
566	84.10	8	2	10.353	510.620	638.04	616.121	0.7077	0.6483	1.036	4.0	8.812
567	94.02	10	2	12.933	713.614	838.47	860.675	0.7078	0.6480	1.036	5.0	9.267
568	21.02	1	2	1.296	15.957	84.34	19.274	0.7073	0.6494	2.074	0.5	11.733
569	29.73	2	2	2.592	45.133	127.59	54.508	0.7073	0.6493	2.074	1.0	8.875
570	36.41	3	2	3.887	82.914	171.08	100.126	0.7073	0.6492	2.074	1.5	7.934
571	42.05	4	2	5.182	127.655	218.25	154.138	0.7074	0.6490	2.073	2.0	7.592
572	51.50	6	2	7.769	234.518	326.18	283.091	0.7076	0.6487	2.073	3.0	7.565
573	59.46	8	2	10.353	361.066	451.85	435.659	0.7077	0.6483	2.072	4.0	7.862
574	66.48	10	2	12.934	504.608	593.76	608.598	0.7078	0.6480	2.071	5.0	8.268
575	17.17	1	2	1.296	13.029	68.90	15.737	0.7073	0.6494	3.111	0.5	10.972
576	24.28	2	2	2.592	36.851	104.27	44.506	0.7073	0.6493	3.111	1.0	8.303
577	29.73	3	2	3.887	67.700	139.85	81.753	0.7074	0.6492	3.110	1.5	7.424
578	34.33	4	2	5.182	104.231	178.45	125.853	0.7074	0.6490	3.110	2.0	7.106
579	42.05	6	2	7.769	191.485	266.72	231.141	0.7076	0.6487	3.109	3.0	7.082
580	48.55	8	2	10.353	294.812	369.48	355.708	0.7077	0.6484	3.108	4.0	7.360
581	54.28	10	2	12.934	412.014	485.49	496.928	0.7078	0.6480	3.107	5.0	7.738
582	14.87	1	2	1.296	11.283	59.70	13.629	0.7073	0.6493	4.148	0.5	10.463
583	21.02	2	2	2.592	31.914	90.39	38.543	0.7073	0.6493	4.148	1.0	7.921
584	25.75	3	2	3.887	58.630	121.26	70.800	0.7073	0.6492	4.147	1.5	7.085
585	29.73	4	2	5.182	90.267	154.74	108.992	0.7074	0.6490	4.147	2.0	6.782
586	36.41	6	2	7.769	165.832	231.33	200.172	0.7076	0.6487	4.146	3.0	6.760
587	42.05	8	2	10.353	255.315	320.45	308.046	0.7077	0.6484	4.144	4.0	7.026
588	47.01	10	2	12.934	356.813	421.04	430.356	0.7078	0.6480	4.142	5.0	7.387
589	13.30	1	2	1.296	10.092	53.42	12.190	0.7073	0.6494	5.185	0.5	10.086
590	18.80	2	2	2.592	28.545	80.92	34.474	0.7073	0.6493	5.185	1.0	7.639
591	23.03	3	2	3.887	52.440	108.58	63.326	0.7074	0.6492	5.184	1.5	6.834
592	26.59	4	2	5.182	80.738	138.59	97.485	0.7074	0.6491	5.184	2.0	6.543
593	32.57	6	2	7.769	148.325	207.21	179.037	0.7076	0.6487	5.182	3.0	6.523
594	37.61	8	2	10.353	228.360	287.04	275.518	0.7076	0.6484	5.180	4.0	6.779
595	42.05	10	2	12.934	319.145	377.12	384.924	0.7078	0.6480	5.178	5.0	7.127
596	31.94	1	2	1.492	24.247	128.97	33.298	0.6988	0.7475	1.022	0.5	12.460
597	45.17	2	2	2.986	68.580	196.96	94.144	0.6980	0.7480	1.022	1.0	9.517
598	55.32	3	2	4.480	125.989	265.48	172.917	0.6977	0.7482	1.022	1.5	8.553
599	63.87	4	2	5.974	193.973	339.29	266.188	0.6976	0.7482	1.021	2.0	8.199
600	78.23	6	2	8.958	356.351	506.39	488.908	0.6977	0.7480	1.021	3.0	8.160
601	90.33	8	2	11.939	548.637	699.44	752.535	0.6978	0.7477	1.021	4.0	8.454
602	101.00	10	2	14.917	766.744	916.63	1051.591	0.6980	0.7473	1.021	5.0	8.864
603	22.58	1	2	1.492	17.145	91.24	23.545	0.6988	0.7475	2.044	0.5	11.107
604	31.94	2	2	2.986	48.493	139.40	66.571	0.6980	0.7480	2.044	1.0	8.487
605	39.12	3	2	4.480	89.088	187.96	122.273	0.6977	0.7482	2.043	1.5	7.630
606	45.17	4	2	5.974	137.159	240.25	188.224	0.6976	0.7482	2.043	2.0	7.315
607	55.32	6	2	8.958	251.978	358.63	345.708	0.6977	0.7480	2.042	3.0	7.281
608	63.87	8	2	11.939	387.946	495.36	532.121	0.6978	0.7477	2.042	4.0	7.544
609	71.41	10	2	14.917	542.172	649.15	743.597	0.6980	0.7474	2.042	5.0	7.909
610	18.44	1	2	1.492	13.999	74.54	19.225	0.6989	0.7474	3.066	0.5	10.386
611	26.08	2	2	2.986	39.595	113.93	54.355	0.6980	0.7480	3.065	1.0	7.940
612	31.94	3	2	4.480	72.740	153.65	99.836	0.6978	0.7482	3.065	1.5	7.140
613	36.88	4	2	5.974	111.990	196.44	153.685	0.6976	0.7482	3.064	2.0	6.847
614	45.17	6	2	8.958	205.740	293.27	282.267	0.6976	0.7480	3.063	3.0	6.815
615	52.15	8	2	11.939	316.758	405.08	434.469	0.6977	0.7477	3.063	4.0	7.062
616	58.31	10	2	14.917	442.684	530.82	607.151	0.6980	0.7474	3.063	5.0	7.403
617	15.97	1	2	1.492	12.123	64.59	16.649	0.6989	0.7474	4.089	0.5	9.905
618	22.58	2	2	2.986	34.290	98.75	47.073	0.6980	0.7480	4.087	1.0	7.575
619	27.66	3	2	4.480	62.995	133.23	86.460	0.6977	0.7482	4.086	1.5	6.814
620	31.94	4	2	5.974	96.987	170.35	133.095	0.6976	0.7482	4.086	2.0	6.535
621	39.12	6	2	8.958	178.177	254.37	244.449	0.6976	0.7480	4.085	3.0	6.506
622	45.17	8	2	11.939	274.320	351.34	376.253	0.6977	0.7477	4.084	4.0	6.741
623	50.50	10	2	14.918	383.374	460.39	525.815	0.6980	0.7474	4.083	5.0	7.067
624	14.28	1	2	1.492	10.843	57.80	14.891	0.6989	0.7474	5.111	0.5	9.549
625	20.20	2	2	2.986	30.670	88.41	42.103	0.6980	0.7480	5.109	1.0	7.305
626	24.74	3	2	4.480	56.344	119.30	77.332	0.6977	0.7482	5.108	1.5	6.573
627	28.57	4	2	5.974	86.748	152.57	119.043	0.6976	0.7482	5.107	2.0	6.305
628	34.99	6	2	8.958	159.366	227.85	218.640	0.6976	0.7480	5.106	3.0	6.278



hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	1000x $C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
629	40.40	8	2	11.939	245.360	314.73	336.525	0.6977	0.7477	5.104	4.0	6.505
630	45.17	10	2	14.918	342.899	412.38	470.304	0.6980	0.7474	5.104	5.0	6.819
631	34.00	1	2	1.689	25.817	139.70	39.579	0.6893	0.8461	1.007	0.5	12.029
632	48.08	2	2	3.383	73.022	216.54	111.930	0.6881	0.8474	1.007	1.0	9.324
633	58.89	3	2	5.076	134.150	293.90	205.614	0.6878	0.8477	1.007	1.5	8.437
634	68.00	4	2	6.769	206.537	376.41	316.545	0.6877	0.8478	1.007	2.0	8.104
635	83.28	6	2	10.152	379.432	560.65	581.458	0.6877	0.8477	1.007	3.0	8.048
636	96.17	8	2	13.531	584.174	770.97	895.145	0.6879	0.8474	1.007	4.0	8.301
637	107.52	10	2	16.908	816.408	1006.03	1251.044	0.6882	0.8471	1.007	5.0	8.665
638	24.04	1	2	1.689	18.256	98.84	27.987	0.6893	0.8461	2.014	0.5	10.722
639	34.00	2	2	3.383	51.634	153.27	79.147	0.6881	0.8474	2.014	1.0	8.315
640	41.64	3	2	5.076	94.859	208.08	145.392	0.6878	0.8477	2.014	1.5	7.525
641	48.08	4	2	6.769	146.044	266.53	223.832	0.6877	0.8478	2.013	2.0	7.230
642	58.89	6	2	10.152	268.300	397.07	411.154	0.6877	0.8477	2.013	3.0	7.181
643	68.00	8	2	13.531	413.074	546.04	632.976	0.6879	0.8474	2.013	4.0	7.407
644	76.03	10	2	16.908	577.288	712.50	884.626	0.6882	0.8471	2.013	5.0	7.732
645	19.63	1	2	1.689	14.906	80.75	22.851	0.6893	0.8460	3.021	0.5	10.027
646	27.76	2	2	3.383	42.159	125.26	64.623	0.6881	0.8474	3.021	1.0	7.778
647	34.00	3	2	5.076	77.452	170.10	118.712	0.6878	0.8477	3.020	1.5	7.042
648	39.26	4	2	6.769	119.244	217.93	182.758	0.6877	0.8478	3.020	2.0	6.767
649	48.08	6	2	10.152	219.065	324.71	335.702	0.6877	0.8477	3.020	3.0	6.722
650	55.52	8	2	13.531	337.273	446.54	516.824	0.6879	0.8474	3.020	4.0	6.934
651	62.08	10	2	16.908	471.354	582.65	722.298	0.6882	0.8471	3.020	5.0	7.238
652	17.00	1	2	1.689	12.909	69.96	19.790	0.6894	0.8460	4.028	0.5	9.563
653	24.04	2	2	3.383	36.511	108.58	55.966	0.6881	0.8474	4.027	1.0	7.421
654	29.44	3	2	5.076	67.075	147.49	102.807	0.6878	0.8477	4.027	1.5	6.721
655	34.00	4	2	6.769	103.269	189.00	158.273	0.6877	0.8478	4.027	2.0	6.459
656	41.64	6	2	10.152	189.717	281.64	290.725	0.6877	0.8477	4.026	3.0	6.418
657	48.08	8	2	13.532	292.087	387.32	447.583	0.6879	0.8474	4.026	4.0	6.619
658	53.76	10	2	16.908	408.204	505.37	625.534	0.6882	0.8471	4.026	5.0	6.909
659	15.21	1	2	1.689	11.546	62.61	17.700	0.6894	0.8460	5.035	0.5	9.218
660	21.50	2	2	3.383	32.656	97.21	50.057	0.6881	0.8473	5.034	1.0	7.157
661	26.34	3	2	5.076	59.994	132.08	91.954	0.6878	0.8477	5.034	1.5	6.483
662	30.41	4	2	6.769	92.366	169.27	141.563	0.6877	0.8478	5.034	2.0	6.232
663	37.25	6	2	10.152	169.688	252.29	260.030	0.6877	0.8477	5.033	3.0	6.193
664	43.01	8	2	13.532	261.250	346.97	400.334	0.6879	0.8474	5.033	4.0	6.388
665	48.08	10	2	16.908	365.107	452.69	559.494	0.6882	0.8471	5.033	5.0	6.667
666	35.47	1	2	1.840	26.935	148.64	44.493	0.6818	0.9218	0.997	0.5	11.838
667	50.16	2	2	3.686	76.184	233.36	125.859	0.6808	0.9233	0.997	1.0	9.292
668	61.43	3	2	5.531	139.958	318.75	231.220	0.6805	0.9236	0.997	1.5	8.461
669	70.94	4	2	7.375	215.479	409.29	355.981	0.6805	0.9237	0.997	2.0	8.149
670	86.88	6	2	11.060	395.860	609.57	653.942	0.6805	0.9235	0.997	3.0	8.091
671	100.32	8	2	14.743	609.466	835.93	1006.827	0.6807	0.9233	0.997	4.0	8.321
672	112.16	10	2	18.422	851.753	1087.19	1407.123	0.6810	0.9229	0.997	5.0	8.658
673	25.08	1	2	1.840	19.046	105.16	31.462	0.6818	0.9217	1.994	0.5	10.552
674	35.47	2	2	3.686	53.870	165.18	88.996	0.6808	0.9232	1.995	1.0	8.286
675	43.44	3	2	5.531	98.965	225.67	163.498	0.6805	0.9236	1.995	1.5	7.547
676	50.16	4	2	7.375	152.367	289.82	251.718	0.6805	0.9237	1.995	2.0	7.270
677	61.43	6	2	11.061	279.915	431.71	462.411	0.6805	0.9236	1.994	3.0	7.220
678	70.94	8	2	14.743	430.957	592.06	711.948	0.6808	0.9233	1.995	4.0	7.426
679	79.31	10	2	18.422	602.280	770.01	994.996	0.6810	0.9229	1.995	5.0	7.726
680	20.48	1	2	1.840	15.551	85.91	25.689	0.6819	0.9217	2.992	0.5	9.868
681	28.96	2	2	3.686	43.985	134.99	72.665	0.6808	0.9232	2.992	1.0	7.752
682	35.47	3	2	5.531	80.805	184.49	133.496	0.6805	0.9236	2.992	1.5	7.063
683	40.96	4	2	7.375	124.407	236.97	205.525	0.6805	0.9237	2.992	2.0	6.804
684	50.16	6	2	11.061	228.550	353.04	377.557	0.6805	0.9236	2.992	3.0	6.758
685	57.92	8	2	14.743	351.874	484.19	581.303	0.6808	0.9233	2.992	4.0	6.952
686	64.76	10	2	18.422	491.758	629.70	812.410	0.6810	0.9230	2.992	5.0	7.232
687	17.73	1	2	1.840	13.468	74.44	22.247	0.6819	0.9216	3.989	0.5	9.411
688	25.08	2	2	3.686	38.092	117.02	62.930	0.6808	0.9232	3.989	1.0	7.396
689	30.72	3	2	5.531	69.979	159.96	115.611	0.6806	0.9236	3.989	1.5	6.740
690	35.47	4	2	7.375	107.739	205.51	177.990	0.6805	0.9237	3.989	2.0	6.495
691	43.44	6	2	11.061	197.929	306.21	326.973	0.6805	0.9236	3.989	3.0	6.452
692	50.16	8	2	14.743	304.732	419.98	503.425	0.6807	0.9233	3.989	4.0	6.637
693	56.08	10	2	18.423	425.875	546.19	703.573	0.6810	0.9230	3.989	5.0	6.905

hull	L	B	T	$A_X$	$A_{WP}$	S	$\nabla$	$C_P$	$C_X$	$1000 \times C_V$	B/T	$\frac{S}{\nabla^{2/3}}$
694	15.86	1	2	1.840	12.046	66.62	19.898	0.6819	0.9216	4.986	0.5	9.072
695	22.43	2	2	3.686	34.070	104.76	56.286	0.6808	0.9232	4.986	1.0	7.133
696	27.47	3	2	5.530	62.591	143.25	103.405	0.6806	0.9236	4.986	1.5	6.502
697	31.72	4	2	7.375	96.365	184.06	159.198	0.6805	0.9237	4.986	2.0	6.266
698	38.85	6	2	11.061	177.033	274.31	292.453	0.6805	0.9236	4.986	3.0	6.226
699	44.86	8	2	14.743	272.560	376.23	450.279	0.6808	0.9233	4.986	4.0	6.404
700	50.16	10	2	18.423	380.914	489.28	629.299	0.6810	0.9230	4.986	5.0	6.663

## B.2 Low Froude Numbers

Value given is  $1000 \times C_w$ .

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
1	0.0228	0.0261	0.0227	0.0270	0.0462	0.0688	0.0795	0.0757	0.0660	0.0621	0.0736
2	0.0346	0.0407	0.0381	0.0422	0.0645	0.0940	0.1116	0.1118	0.1034	0.1003	0.1148
3	0.0410	0.0486	0.0474	0.0514	0.0732	0.1031	0.1243	0.1283	0.1229	0.1218	0.1378
4	0.0442	0.0524	0.0525	0.0565	0.0767	0.1055	0.1274	0.1337	0.1312	0.1321	0.1490
5	0.0455	0.0537	0.0553	0.0594	0.0764	0.1009	0.1210	0.1292	0.1306	0.1347	0.1524
6	0.0442	0.0516	0.0539	0.0578	0.0721	0.0925	0.1097	0.1176	0.1212	0.1274	0.1452
7	0.0421	0.0486	0.0510	0.0547	0.0668	0.0839	0.0981	0.1058	0.1105	0.1176	0.1352
8	0.0494	0.0562	0.0479	0.0561	0.0985	0.1493	0.1742	0.1657	0.1429	0.1319	0.1554
9	0.0779	0.0914	0.0849	0.0915	0.1399	0.2059	0.2493	0.2502	0.2317	0.2235	0.2550
10	0.0936	0.1107	0.1086	0.1143	0.1591	0.2254	0.2740	0.2864	0.2773	0.2757	0.3113
11	0.1010	0.1197	0.1214	0.1276	0.1673	0.2285	0.2769	0.2950	0.2940	0.2993	0.3381
12	0.0980	0.1161	0.1223	0.1297	0.1612	0.2096	0.2506	0.2715	0.2801	0.2949	0.3379
13	0.0938	0.1099	0.1173	0.1253	0.1507	0.1886	0.2213	0.2402	0.2528	0.2725	0.3159
14	0.0880	0.1019	0.1090	0.1167	0.1377	0.1676	0.1933	0.2096	0.2239	0.2462	0.2896
15	0.0754	0.0849	0.0711	0.0821	0.1473	0.2273	0.2642	0.2499	0.2125	0.1941	0.2302
16	0.1135	0.1336	0.1222	0.1296	0.2025	0.3050	0.3708	0.3741	0.3436	0.3289	0.3779
17	0.1371	0.1634	0.1599	0.1650	0.2297	0.3304	0.4061	0.4264	0.4125	0.4096	0.4651
18	0.1479	0.1767	0.1805	0.1867	0.2417	0.3315	0.4055	0.4348	0.4351	0.4445	0.5057
19	0.1494	0.1777	0.1898	0.2000	0.2410	0.3075	0.3665	0.3986	0.4149	0.4411	0.5104
20	0.1409	0.1658	0.1799	0.1918	0.2235	0.2736	0.3177	0.3456	0.3679	0.4026	0.4748
21	0.1301	0.1513	0.1642	0.1759	0.2011	0.2388	0.2717	0.2952	0.3195	0.3580	0.4313
22	0.0997	0.1117	0.0924	0.1055	0.1905	0.2961	0.3461	0.3250	0.2730	0.2463	0.2964
23	0.1504	0.1764	0.1601	0.1662	0.2591	0.3972	0.4848	0.4876	0.4442	0.4224	0.4907
24	0.1823	0.2173	0.2130	0.2151	0.2936	0.4267	0.5269	0.5539	0.5345	0.5292	0.6060
25	0.1963	0.2352	0.2423	0.2466	0.3101	0.4251	0.5213	0.5604	0.5618	0.5745	0.6586
26	0.1962	0.2343	0.2544	0.2666	0.3123	0.3925	0.4649	0.5065	0.5298	0.5673	0.6620
27	0.1817	0.2154	0.2377	0.2543	0.2900	0.3466	0.3984	0.4336	0.4647	0.5134	0.6127
28	0.1646	0.1929	0.2128	0.2287	0.2566	0.2977	0.3346	0.3643	0.3984	0.4526	0.5537
29	0.1112	0.1248	0.1004	0.1148	0.2182	0.3502	0.4079	0.3794	0.3129	0.2789	0.3435
30	0.1847	0.2158	0.1949	0.1974	0.3087	0.4777	0.5844	0.5853	0.5285	0.5001	0.5892
31	0.2244	0.2675	0.2630	0.2603	0.3496	0.5089	0.6306	0.6625	0.6367	0.6291	0.7275
32	0.2410	0.2893	0.3010	0.3019	0.3712	0.5050	0.6193	0.6667	0.6678	0.6831	0.7902
33	0.2368	0.2847	0.3145	0.3301	0.3784	0.4664	0.5484	0.5976	0.6271	0.6740	0.7926
34	0.2149	0.2571	0.2892	0.3118	0.3508	0.4111	0.4671	0.5083	0.5476	0.6093	0.7335
35	0.1909	0.2262	0.2535	0.2749	0.3054	0.3478	0.3866	0.4219	0.4659	0.5359	0.6630
36	0.0199	0.0231	0.0199	0.0230	0.0385	0.0558	0.0622	0.0570	0.0497	0.0516	0.0713
37	0.0303	0.0363	0.0336	0.0360	0.0534	0.0755	0.0865	0.0836	0.0771	0.0808	0.1063
38	0.0360	0.0436	0.0421	0.0441	0.0607	0.0830	0.0967	0.0965	0.0921	0.0972	0.1241
39	0.0392	0.0472	0.0470	0.0489	0.0641	0.0854	0.0997	0.1014	0.0990	0.1054	0.1324
40	0.0413	0.0492	0.0504	0.0525	0.0650	0.0830	0.0964	0.1001	0.1008	0.1088	0.1348
41	0.0407	0.0479	0.0497	0.0519	0.0623	0.0772	0.0890	0.0932	0.0955	0.1044	0.1289
42	0.0393	0.0457	0.0476	0.0497	0.0585	0.0709	0.0807	0.0851	0.0885	0.0978	0.1207
43	0.0438	0.0502	0.0426	0.0488	0.0837	0.1230	0.1382	0.1264	0.1084	0.1100	0.1516
44	0.0689	0.0819	0.0754	0.0790	0.1175	0.1676	0.1956	0.1885	0.1733	0.1801	0.2365
45	0.0832	0.0999	0.0973	0.0994	0.1340	0.1838	0.2154	0.2172	0.2086	0.2202	0.2807
46	0.0909	0.1091	0.1101	0.1123	0.1422	0.1878	0.2198	0.2266	0.2242	0.2401	0.3014
47	0.0894	0.1073	0.1126	0.1163	0.1391	0.1747	0.2021	0.2128	0.2186	0.2404	0.3005
48	0.0873	0.1033	0.1102	0.1147	0.1331	0.1604	0.1822	0.1931	0.2030	0.2278	0.2852

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
49	0.0832	0.0974	0.1043	0.1091	0.1240	0.1454	0.1624	0.1724	0.1843	0.2106	0.2644
50	0.0673	0.0765	0.0637	0.0722	0.1266	0.1896	0.2116	0.1920	0.1620	0.1624	0.2272
51	0.1079	0.1277	0.1165	0.1200	0.1782	0.2564	0.2977	0.2880	0.2625	0.2715	0.3613
52	0.1222	0.1481	0.1444	0.1444	0.1943	0.2700	0.3189	0.3219	0.3081	0.3255	0.4217
53	0.1333	0.1618	0.1651	0.1655	0.2067	0.2734	0.3219	0.3330	0.3301	0.3558	0.4531
54	0.1381	0.1662	0.1778	0.1819	0.2119	0.2603	0.2992	0.3160	0.3275	0.3638	0.4596
55	0.1327	0.1582	0.1718	0.1794	0.2030	0.2379	0.2670	0.2834	0.3017	0.3437	0.4360
56	0.1247	0.1471	0.1604	0.1688	0.1866	0.2136	0.2351	0.2500	0.2715	0.3162	0.4051
57	0.0902	0.1015	0.0838	0.0939	0.1657	0.2485	0.2790	0.2510	0.2090	0.2081	0.2961
58	0.1336	0.1595	0.1439	0.1457	0.2211	0.3250	0.3795	0.3643	0.3278	0.3398	0.4650
59	0.1635	0.1985	0.1937	0.1904	0.2502	0.3505	0.4146	0.4172	0.3977	0.4216	0.5556
60	0.1783	0.2172	0.2234	0.2217	0.2698	0.3536	0.4156	0.4300	0.4265	0.4616	0.5967
61	0.1827	0.2213	0.2409	0.2472	0.2806	0.3373	0.3846	0.4063	0.4233	0.4743	0.6056
62	0.1729	0.2080	0.2308	0.2430	0.2694	0.3087	0.3424	0.3638	0.3902	0.4494	0.5761
63	0.1597	0.1905	0.2121	0.2258	0.2461	0.2751	0.2992	0.3194	0.3508	0.4136	0.5362
64	0.0992	0.1125	0.0899	0.1012	0.1880	0.2908	0.3272	0.2905	0.2365	0.2349	0.3464
65	0.1650	0.1964	0.1767	0.1753	0.2632	0.3921	0.4573	0.4358	0.3882	0.4037	0.5670
66	0.2026	0.2460	0.2415	0.2333	0.3007	0.4203	0.4966	0.4979	0.4722	0.5028	0.6766
67	0.2204	0.2692	0.2807	0.2749	0.3250	0.4240	0.4966	0.5124	0.5077	0.5523	0.7258
68	0.2226	0.2716	0.3018	0.3114	0.3447	0.4080	0.4609	0.4862	0.5081	0.5728	0.7391
69	0.2068	0.2512	0.2852	0.3045	0.3327	0.3757	0.4123	0.4383	0.4736	0.5487	0.7080
70	0.1875	0.2264	0.2576	0.2786	0.3023	0.3332	0.3597	0.3852	0.4280	0.5092	0.6631
71	0.0178	0.0215	0.0184	0.0196	0.0306	0.0426	0.0454	0.0403	0.0368	0.0455	0.0739
72	0.0269	0.0338	0.0311	0.0311	0.0426	0.0570	0.0621	0.0581	0.0558	0.0683	0.1055
73	0.0320	0.0404	0.0391	0.0386	0.0490	0.0635	0.0699	0.0674	0.0664	0.0802	0.1195
74	0.0348	0.0438	0.0438	0.0433	0.0525	0.0656	0.0728	0.0717	0.0719	0.0861	0.1251
75	0.0371	0.0461	0.0475	0.0475	0.0548	0.0656	0.0723	0.0728	0.0748	0.0890	0.1252
76	0.0371	0.0453	0.0475	0.0479	0.0538	0.0625	0.0685	0.0695	0.0726	0.0863	0.1193
77	0.0363	0.0438	0.0462	0.0469	0.0519	0.0588	0.0637	0.0652	0.0689	0.0821	0.1122
78	0.0393	0.0465	0.0393	0.0421	0.0681	0.0960	0.1043	0.0919	0.0813	0.0972	0.1588
79	0.0613	0.0762	0.0703	0.0692	0.0950	0.1280	0.1423	0.1319	0.1259	0.1526	0.2370
80	0.0737	0.0930	0.0912	0.0885	0.1101	0.1416	0.1571	0.1528	0.1510	0.1823	0.2721
81	0.0808	0.1019	0.1040	0.1016	0.1192	0.1470	0.1627	0.1620	0.1643	0.1976	0.2867
82	0.0800	0.1010	0.1078	0.1079	0.1204	0.1407	0.1540	0.1569	0.1648	0.1996	0.2825
83	0.0792	0.0986	0.1073	0.1093	0.1191	0.1340	0.1442	0.1484	0.1592	0.1940	0.2702
84	0.0769	0.0944	0.1035	0.1066	0.1146	0.1258	0.1333	0.1377	0.1504	0.1849	0.2537
85	0.0608	0.0710	0.0591	0.0631	0.1037	0.1491	0.1604	0.1418	0.1219	0.1454	0.2396
86	0.0968	0.1196	0.1097	0.1070	0.1463	0.1978	0.2176	0.2023	0.1911	0.2329	0.3685
87	0.1081	0.1384	0.1361	0.1302	0.1609	0.2081	0.2317	0.2245	0.2218	0.2713	0.4163
88	0.1180	0.1519	0.1577	0.1517	0.1755	0.2152	0.2383	0.2372	0.2419	0.2956	0.4394
89	0.1235	0.1577	0.1731	0.1732	0.1891	0.2154	0.2331	0.2382	0.2526	0.3099	0.4430
90	0.1208	0.1524	0.1710	0.1760	0.1887	0.2069	0.2194	0.2266	0.2468	0.3047	0.4250
91	0.1157	0.1443	0.1628	0.1706	0.1813	0.1945	0.2035	0.2116	0.2349	0.2927	0.4050
92	0.0819	0.0946	0.0782	0.0831	0.1376	0.2012	0.2137	0.1846	0.1584	0.1879	0.3156
93	0.1193	0.1492	0.1358	0.1299	0.1809	0.2494	0.2747	0.2522	0.2359	0.2937	0.4833
94	0.1451	0.1867	0.1853	0.1749	0.2117	0.2722	0.3019	0.2910	0.2873	0.3570	0.5608
95	0.1577	0.2052	0.2163	0.2076	0.2342	0.2825	0.3108	0.3085	0.3158	0.3913	0.5926
96	0.1637	0.2116	0.2381	0.2416	0.2587	0.2882	0.3084	0.3149	0.3372	0.4176	0.6020
97	0.1574	0.2018	0.2333	0.2454	0.2609	0.2807	0.2949	0.3055	0.3362	0.4182	0.5861
98	0.1489	0.1890	0.2199	0.2358	0.2494	0.2649	0.2757	0.2890	0.3249	0.4067	0.5622
99	0.1020	0.1170	0.0963	0.1016	0.1689	0.2478	0.2622	0.2244	0.1901	0.2259	0.3866
100	0.1471	0.1846	0.1683	0.1587	0.2192	0.3022	0.3316	0.3012	0.2800	0.3553	0.6016
101	0.1794	0.2326	0.2338	0.2173	0.2563	0.3295	0.3631	0.3478	0.3433	0.4348	0.7004
102	0.1948	0.2558	0.2751	0.2641	0.2887	0.3450	0.3760	0.3719	0.3819	0.4800	0.7415
103	0.1987	0.2609	0.3026	0.3120	0.3286	0.3615	0.3829	0.3909	0.4208	0.5245	0.7608
104	0.1884	0.2457	0.2929	0.3162	0.3349	0.3584	0.3745	0.3897	0.4322	0.5379	0.7513
105	0.1751	0.2269	0.2721	0.3003	0.3210	0.3397	0.3539	0.3752	0.4256	0.5325	0.7294
106	0.0167	0.0210	0.0180	0.0176	0.0252	0.0334	0.0342	0.0299	0.0301	0.0445	0.0797
107	0.0251	0.0330	0.0306	0.0286	0.0356	0.0447	0.0464	0.0428	0.0449	0.0650	0.1115
108	0.0298	0.0397	0.0387	0.0362	0.0419	0.0505	0.0528	0.0502	0.0533	0.0747	0.1240
109	0.0324	0.0432	0.0436	0.0412	0.0458	0.0530	0.0559	0.0541	0.0579	0.0795	0.1283
110	0.0348	0.0459	0.0482	0.0466	0.0497	0.0550	0.0575	0.0568	0.0614	0.0818	0.1268
111	0.0350	0.0456	0.0489	0.0481	0.0505	0.0543	0.0562	0.0561	0.0612	0.0801	0.1206
112	0.0345	0.0443	0.0481	0.0480	0.0498	0.0527	0.0540	0.0544	0.0596	0.0771	0.1138
113	0.0367	0.0451	0.0382	0.0381	0.0570	0.0768	0.0803	0.0694	0.0665	0.0951	0.1710

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
114	0.0571	0.0745	0.0693	0.0644	0.0804	0.1013	0.1061	0.0976	0.1016	0.1460	0.2527
115	0.0685	0.0916	0.0910	0.0845	0.0957	0.1138	0.1196	0.1144	0.1222	0.1717	0.2857
116	0.0749	0.1010	0.1050	0.0991	0.1067	0.1211	0.1267	0.1240	0.1344	0.1850	0.2981
117	0.0797	0.1071	0.1175	0.1152	0.1192	0.1279	0.1316	0.1319	0.1455	0.1947	0.2986
118	0.0738	0.1001	0.1133	0.1143	0.1176	0.1227	0.1249	0.1266	0.1419	0.1885	0.2823
119	0.0724	0.0970	0.1112	0.1141	0.1175	0.1208	0.1219	0.1244	0.1407	0.1848	0.2691
120	0.0569	0.0688	0.0574	0.0580	0.0876	0.1208	0.1249	0.1083	0.1002	0.1429	0.2603
121	0.0903	0.1171	0.1092	0.1005	0.1249	0.1575	0.1646	0.1503	0.1553	0.2260	0.3982
122	0.0995	0.1370	0.1381	0.1262	0.1410	0.1675	0.1756	0.1672	0.1801	0.2603	0.4463
123	0.1081	0.1515	0.1619	0.1515	0.1603	0.1795	0.1867	0.1827	0.2010	0.2839	0.4680
124	0.1131	0.1595	0.1827	0.1815	0.1859	0.1950	0.1987	0.2000	0.2246	0.3059	0.4743
125	0.1118	0.1563	0.1847	0.1913	0.1963	0.2014	0.2033	0.2077	0.2360	0.3136	0.4648
126	0.1087	0.1503	0.1798	0.1912	0.1977	0.2015	0.2026	0.2094	0.2399	0.3141	0.4515
127	0.0767	0.0916	0.0761	0.0761	0.1175	0.1652	0.1682	0.1418	0.1311	0.1862	0.3447
128	0.1099	0.1465	0.1357	0.1232	0.1545	0.1975	0.2058	0.1850	0.1916	0.2902	0.5317
129	0.1331	0.1857	0.1895	0.1719	0.1872	0.2217	0.2306	0.2186	0.2375	0.3513	0.6142
130	0.1440	0.2060	0.2254	0.2126	0.2205	0.2416	0.2486	0.2432	0.2703	0.3881	0.6485
131	0.1487	0.2156	0.2560	0.2610	0.2639	0.2739	0.2765	0.2793	0.3167	0.4320	0.6680
132	0.1453	0.2092	0.2575	0.2762	0.2854	0.2911	0.2934	0.3023	0.3457	0.4562	0.6668
133	0.1399	0.1998	0.2491	0.2748	0.2875	0.2950	0.2988	0.3132	0.3610	0.4674	0.6577
134	0.0958	0.1133	0.0940	0.0937	0.1455	0.2053	0.2081	0.1737	0.1583	0.2256	0.4244
135	0.1359	0.1818	0.1695	0.1524	0.1866	0.2407	0.2489	0.2216	0.2303	0.3584	0.6720
136	0.1644	0.2325	0.2417	0.2193	0.2337	0.2724	0.2807	0.2647	0.2906	0.4408	0.7840
137	0.1761	0.2582	0.2908	0.2772	0.2804	0.3038	0.3091	0.3020	0.3390	0.4936	0.8338
138	0.1797	0.2677	0.3310	0.3469	0.3518	0.3615	0.3631	0.3682	0.4190	0.5704	0.8736
139	0.1735	0.2571	0.3304	0.3672	0.3841	0.3951	0.3996	0.4160	0.4766	0.6211	0.8911
140	0.1661	0.2440	0.3165	0.3631	0.3895	0.4044	0.4151	0.4414	0.5094	0.6500	0.8934
141	0.0360	0.0421	0.0358	0.0394	0.0697	0.1142	0.1468	0.1544	0.1433	0.1254	0.1145
142	0.0564	0.0661	0.0613	0.0644	0.0988	0.1535	0.1999	0.2177	0.2116	0.1952	0.1842
143	0.0681	0.0794	0.0771	0.0803	0.1133	0.1682	0.2184	0.2419	0.2417	0.2299	0.2219
144	0.0742	0.0859	0.0857	0.0895	0.1200	0.1718	0.2212	0.2470	0.2509	0.2437	0.2382
145	0.0774	0.0885	0.0907	0.0956	0.1215	0.1651	0.2083	0.2339	0.2421	0.2409	0.2416
146	0.0753	0.0852	0.0883	0.0939	0.1161	0.1526	0.1892	0.2123	0.2219	0.2242	0.2287
147	0.0714	0.0801	0.0835	0.0893	0.1088	0.1397	0.1707	0.1910	0.2009	0.2052	0.2119
148	0.0774	0.0902	0.0747	0.0802	0.1479	0.2485	0.3255	0.3440	0.3181	0.2753	0.2463
149	0.1265	0.1478	0.1357	0.1385	0.2138	0.3401	0.4495	0.4968	0.4858	0.4474	0.4187
150	0.1512	0.1761	0.1710	0.1738	0.2427	0.3663	0.4829	0.5454	0.5516	0.5281	0.5095
151	0.1660	0.1917	0.1926	0.1975	0.2590	0.3713	0.4828	0.5499	0.5676	0.5577	0.5507
152	0.1728	0.1968	0.2044	0.2140	0.2642	0.3533	0.4448	0.5067	0.5339	0.5412	0.5517
153	0.1658	0.1869	0.1967	0.2091	0.2514	0.3235	0.3966	0.4489	0.4776	0.4927	0.5132
154	0.1543	0.1724	0.1825	0.1962	0.2340	0.2928	0.3521	0.3957	0.4234	0.4424	0.4685
155	0.1173	0.1357	0.1098	0.1164	0.2206	0.3798	0.4990	0.5295	0.4870	0.4165	0.3678
156	0.1887	0.2205	0.1985	0.1979	0.3146	0.5161	0.6927	0.7704	0.7533	0.6897	0.6397
157	0.2349	0.2726	0.2629	0.2613	0.3634	0.5568	0.7444	0.8484	0.8626	0.8265	0.7953
158	0.2592	0.2982	0.2996	0.3017	0.3879	0.5599	0.7354	0.8464	0.8810	0.8702	0.8607
159	0.2681	0.3047	0.3187	0.3314	0.3987	0.5278	0.6647	0.7628	0.8120	0.8313	0.8544
160	0.2528	0.2852	0.3034	0.3222	0.3797	0.4795	0.5841	0.6631	0.7123	0.7445	0.7851
161	0.2293	0.2573	0.2757	0.2973	0.3480	0.4289	0.5108	0.5750	0.6214	0.6592	0.7096
162	0.1471	0.1696	0.1327	0.1397	0.2799	0.4958	0.6569	0.6969	0.6366	0.5373	0.4686
163	0.2516	0.2918	0.2584	0.2520	0.4063	0.6840	0.9280	1.0357	1.0107	0.9195	0.8472
164	0.3158	0.3644	0.3489	0.3383	0.4677	0.7302	0.9883	1.1352	1.1574	1.1074	1.0627
165	0.3489	0.3999	0.4015	0.3966	0.5020	0.7277	0.9656	1.1209	1.1738	1.1623	1.1511
166	0.3571	0.4065	0.4283	0.4414	0.5201	0.6805	0.8569	0.9891	1.0614	1.0946	1.1321
167	0.3293	0.3736	0.4019	0.4267	0.4943	0.6143	0.7437	0.8462	0.9160	0.9664	1.0296
168	0.2903	0.3281	0.3565	0.3859	0.4465	0.5424	0.6409	0.7230	0.7881	0.8460	0.9240
169	0.1794	0.2045	0.1565	0.1635	0.3378	0.6079	0.8087	0.8567	0.7778	0.6499	0.5619
170	0.3112	0.3577	0.3117	0.2972	0.4871	0.8374	1.1472	1.2836	1.2497	1.1297	1.0350
171	0.3931	0.4507	0.4283	0.4050	0.5571	0.8832	1.2102	1.3988	1.4294	1.3651	1.3068
172	0.4341	0.4960	0.4977	0.4823	0.5993	0.8720	1.1685	1.3670	1.4390	1.4272	1.4138
173	0.4376	0.4998	0.5310	0.5440	0.6275	0.8102	1.0199	1.1832	1.2777	1.3255	1.3779
174	0.3932	0.4500	0.4904	0.5218	0.5953	0.7283	0.8749	0.9979	1.0877	1.1560	1.2426
175	0.3355	0.3834	0.4224	0.4606	0.5287	0.6341	0.7441	0.8421	0.9258	1.0039	1.1101
176	0.0310	0.0358	0.0302	0.0344	0.0628	0.1020	0.1281	0.1310	0.1184	0.1028	0.0976
177	0.0484	0.0563	0.0515	0.0550	0.0872	0.1355	0.1733	0.1837	0.1743	0.1589	0.1546
178	0.0588	0.0680	0.0650	0.0682	0.0987	0.1474	0.1887	0.2041	0.1993	0.1880	0.1856

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
179	0.0647	0.0741	0.0729	0.0761	0.1040	0.1500	0.1910	0.2089	0.2079	0.2003	0.2006
180	0.0688	0.0777	0.0785	0.0822	0.1055	0.1442	0.1806	0.1994	0.2030	0.2005	0.2045
181	0.0679	0.0759	0.0776	0.0818	0.1015	0.1338	0.1649	0.1824	0.1880	0.1890	0.1958
182	0.0653	0.0723	0.0745	0.0786	0.0957	0.1231	0.1495	0.1653	0.1718	0.1749	0.1833
183	0.0674	0.0775	0.0636	0.0713	0.1350	0.2244	0.2856	0.2941	0.2643	0.2260	0.2095
184	0.1098	0.1269	0.1145	0.1191	0.1910	0.3038	0.3934	0.4225	0.4016	0.3648	0.3498
185	0.1360	0.1561	0.1491	0.1521	0.2175	0.3285	0.4257	0.4684	0.4616	0.4364	0.4289
186	0.1462	0.1669	0.1648	0.1684	0.2257	0.3270	0.4210	0.4694	0.4736	0.4599	0.4612
187	0.1557	0.1751	0.1791	0.1857	0.2308	0.3107	0.3890	0.4364	0.4519	0.4545	0.4696
188	0.1526	0.1697	0.1762	0.1849	0.2225	0.2860	0.3489	0.3902	0.4095	0.4207	0.4441
189	0.1444	0.1595	0.1668	0.1765	0.2091	0.2610	0.3118	0.3468	0.3673	0.3830	0.4113
190	0.1032	0.1176	0.0940	0.1041	0.2037	0.3450	0.4417	0.4543	0.4055	0.3419	0.3126
191	0.1645	0.1897	0.1674	0.1710	0.2830	0.4637	0.6086	0.6558	0.6211	0.5590	0.5309
192	0.2067	0.2367	0.2237	0.2231	0.3211	0.4964	0.6531	0.7243	0.7155	0.6744	0.6597
193	0.2311	0.2621	0.2584	0.2585	0.3410	0.4965	0.6456	0.7260	0.7371	0.7176	0.7192
194	0.2456	0.2751	0.2830	0.2902	0.3518	0.4674	0.5854	0.6611	0.6912	0.7012	0.7294
195	0.2372	0.2639	0.2767	0.2891	0.3399	0.4281	0.5183	0.5816	0.6166	0.6408	0.6843
196	0.2203	0.2439	0.2579	0.2734	0.3163	0.3874	0.4578	0.5099	0.5454	0.5774	0.6298
197	0.1377	0.1552	0.1216	0.1336	0.2675	0.4594	0.5903	0.6060	0.5369	0.4472	0.4050
198	0.2211	0.2525	0.2189	0.2184	0.3694	0.6177	0.8173	0.8817	0.8314	0.7417	0.6999
199	0.2806	0.3187	0.2985	0.2903	0.4174	0.6549	0.8706	0.9702	0.9585	0.8998	0.8774
200	0.3149	0.3548	0.3491	0.3414	0.4419	0.6490	0.8512	0.9640	0.9823	0.9564	0.9591
201	0.3325	0.3719	0.3852	0.3904	0.4605	0.6063	0.7594	0.8622	0.9077	0.9258	0.9677
202	0.3153	0.3523	0.3733	0.3891	0.4464	0.5534	0.6653	0.7483	0.7987	0.8383	0.9032
203	0.2859	0.3189	0.3414	0.3628	0.4132	0.4970	0.5820	0.6489	0.7000	0.7502	0.8290
204	0.1582	0.1777	0.1339	0.1471	0.3128	0.5542	0.7172	0.7350	0.6454	0.5296	0.4747
205	0.2755	0.3111	0.2653	0.2591	0.4438	0.7592	1.0119	1.0922	1.0249	0.9065	0.8510
206	0.3527	0.3970	0.3685	0.3488	0.4981	0.7962	1.0690	1.1964	1.1813	1.1042	1.0741
207	0.3965	0.4442	0.4359	0.4174	0.5302	0.7819	1.0345	1.1783	1.2040	1.1715	1.1747
208	0.4140	0.4639	0.4836	0.4856	0.5590	0.7262	0.9088	1.0366	1.0975	1.1247	1.1812
209	0.3847	0.4327	0.4635	0.4833	0.5442	0.6624	0.7906	0.8902	0.9565	1.0109	1.0988
210	0.3393	0.3824	0.4152	0.4431	0.4988	0.5903	0.6850	0.7653	0.8323	0.9012	1.0076
211	0.0268	0.0311	0.0261	0.0299	0.0545	0.0870	0.1064	0.1058	0.0931	0.0813	0.0831
212	0.0416	0.0486	0.0441	0.0470	0.0743	0.1138	0.1419	0.1457	0.1348	0.1233	0.1277
213	0.0506	0.0586	0.0556	0.0578	0.0837	0.1234	0.1543	0.1619	0.1544	0.1455	0.1518
214	0.0559	0.0641	0.0625	0.0645	0.0880	0.1254	0.1563	0.1663	0.1619	0.1562	0.1637
215	0.0604	0.0681	0.0683	0.0704	0.0896	0.1210	0.1488	0.1603	0.1602	0.1584	0.1677
216	0.0605	0.0675	0.0684	0.0709	0.0869	0.1130	0.1368	0.1481	0.1502	0.1514	0.1622
217	0.0591	0.0652	0.0667	0.0692	0.0828	0.1048	0.1251	0.1356	0.1390	0.1419	0.1537
218	0.0589	0.0678	0.0554	0.0628	0.1190	0.1942	0.2418	0.2399	0.2097	0.1799	0.1788
219	0.0950	0.1101	0.0986	0.1027	0.1650	0.2584	0.3272	0.3379	0.3120	0.2834	0.2879
220	0.1180	0.1356	0.1284	0.1300	0.1862	0.2778	0.3510	0.3741	0.3587	0.3384	0.3489
221	0.1272	0.1453	0.1423	0.1437	0.1921	0.2753	0.3466	0.3753	0.3692	0.3578	0.3741
222	0.1380	0.1549	0.1574	0.1606	0.1973	0.2623	0.3220	0.3525	0.3577	0.3597	0.3851
223	0.1377	0.1528	0.1577	0.1624	0.1924	0.2434	0.2913	0.3189	0.3293	0.3392	0.3701
224	0.1328	0.1464	0.1522	0.1583	0.1834	0.2245	0.2631	0.2870	0.2999	0.3144	0.3483
225	0.0910	0.1035	0.0824	0.0927	0.1806	0.3012	0.3750	0.3733	0.3235	0.2728	0.2676
226	0.1427	0.1648	0.1440	0.1477	0.2457	0.3961	0.5045	0.5243	0.4808	0.4314	0.4353
227	0.1798	0.2062	0.1933	0.1911	0.2762	0.4212	0.5392	0.5775	0.5533	0.5191	0.5335
228	0.2026	0.2299	0.2247	0.2222	0.2921	0.4201	0.5332	0.5810	0.5736	0.5561	0.5816
229	0.2199	0.2459	0.2513	0.2531	0.3031	0.3968	0.4867	0.5352	0.5479	0.5555	0.5987
230	0.2169	0.2406	0.2510	0.2580	0.2976	0.3672	0.4355	0.4780	0.4988	0.5200	0.5738
231	0.2059	0.2274	0.2395	0.2494	0.2819	0.3374	0.3900	0.4261	0.4505	0.4796	0.5399
232	0.1224	0.1374	0.1075	0.1200	0.2391	0.4036	0.5039	0.5002	0.4298	0.3575	0.3479
233	0.1929	0.2203	0.1894	0.1906	0.3212	0.5297	0.6788	0.7046	0.6416	0.5697	0.5732
234	0.2457	0.2791	0.2590	0.2504	0.3607	0.5578	0.7196	0.7727	0.7382	0.6886	0.7075
235	0.2782	0.3133	0.3057	0.2952	0.3820	0.5515	0.7044	0.7709	0.7620	0.7377	0.7731
236	0.3006	0.3352	0.3453	0.3442	0.3995	0.5177	0.6336	0.7000	0.7208	0.7340	0.7961
237	0.2922	0.3249	0.3430	0.3518	0.3948	0.4790	0.5635	0.6192	0.6512	0.6857	0.7643
238	0.2718	0.3021	0.3222	0.3368	0.3741	0.4384	0.5015	0.5485	0.5855	0.6319	0.7208
239	0.1399	0.1566	0.1175	0.1318	0.2806	0.4872	0.6125	0.6063	0.5151	0.4211	0.4070
240	0.2417	0.2727	0.2307	0.2267	0.3896	0.6529	0.8412	0.8718	0.7883	0.6932	0.6967
241	0.3108	0.3495	0.3217	0.3034	0.4327	0.6803	0.8842	0.9510	0.9054	0.8400	0.8641
242	0.3528	0.3943	0.3843	0.3635	0.4590	0.6669	0.8573	0.9413	0.9308	0.8996	0.9455
243	0.3781	0.4213	0.4371	0.4320	0.4889	0.6242	0.7624	0.8448	0.8732	0.8934	0.9752

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
244	0.3608	0.4033	0.4306	0.4419	0.4867	0.5794	0.6755	0.7436	0.7871	0.8355	0.9399
245	0.3284	0.3680	0.3976	0.4180	0.4587	0.5287	0.5995	0.6569	0.7077	0.7728	0.8919
246	0.0241	0.0284	0.0238	0.0268	0.0477	0.0749	0.0895	0.0866	0.0749	0.0671	0.0748
247	0.0372	0.0440	0.0398	0.0417	0.0647	0.0971	0.1180	0.1179	0.1072	0.1001	0.1119
248	0.0450	0.0529	0.0501	0.0511	0.0725	0.1047	0.1278	0.1310	0.1225	0.1173	0.1311
249	0.0498	0.0580	0.0564	0.0571	0.0761	0.1063	0.1294	0.1345	0.1286	0.1255	0.1403
250	0.0543	0.0621	0.0622	0.0630	0.0779	0.1028	0.1235	0.1299	0.1281	0.1283	0.1432
251	0.0550	0.0621	0.0632	0.0642	0.0764	0.0967	0.1143	0.1210	0.1214	0.1238	0.1391
252	0.0542	0.0607	0.0622	0.0634	0.0737	0.0905	0.1050	0.1117	0.1133	0.1172	0.1326
253	0.0535	0.0621	0.0508	0.0568	0.1058	0.1694	0.2062	0.1989	0.1706	0.1492	0.1615
254	0.0855	0.1002	0.0895	0.0919	0.1449	0.2221	0.2723	0.2746	0.2485	0.2297	0.2519
255	0.1057	0.1233	0.1166	0.1161	0.1625	0.2371	0.2917	0.3021	0.2842	0.2721	0.3000
256	0.1132	0.1318	0.1292	0.1279	0.1667	0.2336	0.2866	0.3019	0.2917	0.2866	0.3186
257	0.1237	0.1415	0.1447	0.1447	0.1727	0.2233	0.2670	0.2850	0.2851	0.2908	0.3283
258	0.1249	0.1412	0.1468	0.1489	0.1708	0.2093	0.2437	0.2607	0.2662	0.2784	0.3187
259	0.1221	0.1368	0.1435	0.1471	0.1653	0.1956	0.2227	0.2379	0.2464	0.2628	0.3035
260	0.0830	0.0950	0.0758	0.0844	0.1627	0.2648	0.3242	0.3118	0.2646	0.2270	0.2425
261	0.1282	0.1499	0.1308	0.1324	0.2161	0.3404	0.4218	0.4247	0.3807	0.3472	0.3803
262	0.1605	0.1875	0.1755	0.1709	0.2409	0.3588	0.4465	0.4637	0.4346	0.4137	0.4574
263	0.1805	0.2093	0.2054	0.1994	0.2549	0.3570	0.4403	0.4657	0.4508	0.4432	0.4951
264	0.1972	0.2252	0.2323	0.2304	0.2674	0.3392	0.4039	0.4329	0.4364	0.4493	0.5124
265	0.1970	0.2230	0.2351	0.2388	0.2670	0.3185	0.3666	0.3928	0.4059	0.4308	0.5000
266	0.1898	0.2135	0.2276	0.2348	0.2585	0.2977	0.3340	0.3573	0.3757	0.4078	0.4801
267	0.1122	0.1267	0.0993	0.1101	0.2166	0.3568	0.4349	0.4199	0.3530	0.2983	0.3166
268	0.1737	0.2010	0.1729	0.1716	0.2849	0.4559	0.5675	0.5699	0.5062	0.4570	0.5020
269	0.2196	0.2546	0.2370	0.2253	0.3162	0.4754	0.5950	0.6180	0.5766	0.5463	0.6077
270	0.2480	0.2860	0.2810	0.2668	0.3354	0.4692	0.5809	0.6158	0.5960	0.5860	0.6598
271	0.2696	0.3074	0.3207	0.3160	0.3568	0.4452	0.5277	0.5670	0.5752	0.5961	0.6870
272	0.2657	0.3014	0.3228	0.3287	0.3587	0.4204	0.4790	0.5139	0.5363	0.5759	0.6765
273	0.2514	0.2844	0.3083	0.3206	0.3471	0.3935	0.4370	0.4687	0.4987	0.5499	0.6564
274	0.1277	0.1438	0.1081	0.1206	0.2529	0.4312	0.5293	0.5089	0.4221	0.3499	0.3705
275	0.2181	0.2496	0.2116	0.2051	0.3441	0.5626	0.7030	0.7037	0.6196	0.5544	0.6127
276	0.2780	0.3195	0.2958	0.2740	0.3823	0.5802	0.7298	0.7577	0.7034	0.6637	0.7444
277	0.3145	0.3604	0.3548	0.3309	0.4046	0.5688	0.7066	0.7500	0.7252	0.7132	0.8104
278	0.3391	0.3862	0.4076	0.4001	0.4395	0.5412	0.6387	0.6872	0.7006	0.7306	0.8502
279	0.3285	0.3740	0.4068	0.4167	0.4480	0.5157	0.5827	0.6268	0.6594	0.7148	0.8477
280	0.3046	0.3471	0.3822	0.4023	0.4330	0.4840	0.5342	0.5752	0.6196	0.6922	0.8337
281	0.0541	0.0696	0.0608	0.0589	0.0977	0.1673	0.2307	0.2617	0.2596	0.2363	0.2076
282	0.0847	0.1066	0.1019	0.0998	0.1427	0.2258	0.3101	0.3593	0.3690	0.3507	0.3219
283	0.1022	0.1257	0.1256	0.1253	0.1663	0.2483	0.3360	0.3923	0.4106	0.4002	0.3773
284	0.1112	0.1343	0.1373	0.1393	0.1776	0.2541	0.3384	0.3957	0.4187	0.4148	0.3983
285	0.1155	0.1359	0.1420	0.1475	0.1809	0.2450	0.3173	0.3697	0.3950	0.3989	0.3924
286	0.1117	0.1292	0.1362	0.1435	0.1733	0.2271	0.2881	0.3334	0.3576	0.3649	0.3645
287	0.1054	0.1204	0.1275	0.1357	0.1626	0.2086	0.2596	0.2994	0.3216	0.3304	0.3336
288	0.1130	0.1469	0.1249	0.1180	0.2051	0.3644	0.5120	0.5867	0.5827	0.5272	0.4569
289	0.1837	0.2334	0.2212	0.2114	0.3069	0.5012	0.7018	0.8266	0.8562	0.8165	0.7469
290	0.2216	0.2743	0.2751	0.2696	0.3575	0.5445	0.7507	0.8942	0.9491	0.9345	0.8847
291	0.2439	0.2943	0.3037	0.3053	0.3850	0.5545	0.7477	0.8916	0.9588	0.9634	0.9339
292	0.2545	0.2971	0.3141	0.3268	0.3959	0.5309	0.6876	0.8124	0.8829	0.9071	0.9060
293	0.2444	0.2799	0.2986	0.3171	0.3791	0.4892	0.6150	0.7183	0.7815	0.8120	0.8260
294	0.2272	0.2572	0.2757	0.2972	0.3531	0.4465	0.5493	0.6348	0.6912	0.7227	0.7443
295	0.1678	0.2182	0.1811	0.1684	0.3054	0.5600	0.7952	0.9137	0.9057	0.8135	0.6968
296	0.2708	0.3470	0.3240	0.3029	0.4527	0.7696	1.0987	1.3060	1.3573	1.2915	1.1752
297	0.3364	0.4172	0.4167	0.4016	0.5349	0.8344	1.1715	1.4129	1.5114	1.4931	1.4142
298	0.3722	0.4485	0.4639	0.4610	0.5770	0.8427	1.1533	1.3934	1.5144	1.5324	1.4923
299	0.3887	0.4516	0.4809	0.4991	0.5965	0.7989	1.0404	1.2415	1.3645	1.4166	1.4277
300	0.3689	0.4210	0.4532	0.4831	0.5710	0.7319	0.9179	1.0775	1.1853	1.2457	1.2824
301	0.3359	0.3801	0.4118	0.4477	0.5292	0.6631	0.8112	0.9387	1.0314	1.0933	1.1428
302	0.2107	0.2749	0.2213	0.2029	0.3913	0.7408	1.0633	1.2241	1.2100	1.0786	0.9141
303	0.3535	0.4536	0.4169	0.3817	0.5880	1.0301	1.4928	1.7855	1.8585	1.7636	1.5953
304	0.4417	0.5482	0.5441	0.5143	0.6885	1.1037	1.5775	1.9224	2.0688	2.0476	1.9365
305	0.4905	0.5902	0.6104	0.5979	0.7444	1.1027	1.5334	1.8751	2.0561	2.0916	2.0411
306	0.5105	0.5921	0.6337	0.6549	0.7736	1.0344	1.3546	1.6319	1.8121	1.8982	1.9266
307	0.4771	0.5454	0.5917	0.6329	0.7407	0.9422	1.1796	1.3912	1.5440	1.6396	1.7044
308	0.4228	0.4809	0.5276	0.5788	0.6810	0.8472	1.0314	1.1978	1.3268	1.4208	1.5039

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
309	0.2537	0.3292	0.2587	0.2350	0.4738	0.9172	1.3248	1.5261	1.5043	1.3327	1.1189
310	0.4288	0.5491	0.4963	0.4447	0.7049	1.2749	1.8729	2.2525	2.3469	2.2207	1.9972
311	0.5378	0.6667	0.6569	0.6076	0.8186	1.3467	1.9590	2.4116	2.6089	2.5854	2.4407
312	0.5980	0.7179	0.7418	0.7149	0.8835	1.3288	1.8780	2.3237	2.5698	2.6261	2.5665
313	0.6179	0.7167	0.7722	0.7939	0.9247	1.2323	1.6230	1.9737	2.2135	2.3374	2.3875
314	0.5651	0.6494	0.7124	0.7643	0.8858	1.1168	1.3957	1.6536	1.8505	1.9821	2.0796
315	0.4847	0.5576	0.6207	0.6870	0.8058	0.9965	1.2086	1.4088	1.5722	1.6981	1.8180
316	0.0468	0.0592	0.0505	0.0516	0.0921	0.1585	0.2161	0.2399	0.2329	0.2080	0.1816
317	0.0727	0.0906	0.0845	0.0850	0.1304	0.2115	0.2885	0.3283	0.3303	0.3081	0.2806
318	0.0878	0.1071	0.1047	0.1060	0.1491	0.2295	0.3105	0.3575	0.3674	0.3521	0.3294
319	0.0962	0.1150	0.1154	0.1179	0.1577	0.2329	0.3115	0.3604	0.3753	0.3662	0.3490
320	0.1013	0.1178	0.1213	0.1261	0.1600	0.2230	0.2913	0.3373	0.3560	0.3552	0.3473
321	0.0993	0.1134	0.1180	0.1242	0.1537	0.2066	0.2645	0.3051	0.3241	0.3275	0.3257
322	0.0947	0.1069	0.1118	0.1187	0.1449	0.1901	0.2392	0.2748	0.2929	0.2984	0.3004
323	0.0988	0.1259	0.1042	0.1041	0.1956	0.3491	0.4822	0.5401	0.5237	0.4637	0.3979
324	0.1592	0.1994	0.1836	0.1802	0.2835	0.4748	0.6591	0.7608	0.7698	0.7181	0.6491
325	0.1918	0.2347	0.2292	0.2271	0.3218	0.5083	0.7010	0.8225	0.8552	0.8252	0.7718
326	0.2125	0.2536	0.2560	0.2583	0.3427	0.5122	0.6951	0.8201	0.8667	0.8556	0.8201
327	0.2261	0.2606	0.2708	0.2811	0.3507	0.4857	0.6361	0.7484	0.8033	0.8145	0.8067
328	0.2209	0.2498	0.2628	0.2779	0.3383	0.4473	0.5687	0.6631	0.7156	0.7359	0.7444
329	0.2087	0.2332	0.2466	0.2645	0.3188	0.4096	0.5089	0.5878	0.6358	0.6596	0.6772
330	0.1478	0.1881	0.1516	0.1499	0.2941	0.5387	0.7505	0.8419	0.8137	0.7139	0.6044
331	0.2354	0.2966	0.2676	0.2579	0.4229	0.7348	1.0370	1.2050	1.2199	1.1323	1.0143
332	0.2934	0.3589	0.3476	0.3382	0.4849	0.7859	1.1022	1.3063	1.3652	1.3180	1.2298
333	0.3277	0.3897	0.3927	0.3900	0.5163	0.7842	1.0805	1.2901	1.3749	1.3634	1.3095
334	0.3501	0.4012	0.4190	0.4319	0.5310	0.7352	0.9690	1.1516	1.2494	1.2783	1.2753
335	0.3397	0.3825	0.4052	0.4283	0.5137	0.6736	0.8550	1.0024	1.0931	1.1365	1.1617
336	0.3160	0.3525	0.3766	0.4054	0.4826	0.6139	0.7578	0.8771	0.9576	1.0061	1.0458
337	0.1854	0.2366	0.1843	0.1809	0.3778	0.7136	1.0032	1.1267	1.0845	0.9430	0.7886
338	0.3095	0.3890	0.3441	0.3260	0.5537	0.9902	1.4142	1.6494	1.6686	1.5409	1.3703
339	0.3889	0.4743	0.4549	0.4334	0.6284	1.0483	1.4933	1.7837	1.8706	1.8042	1.6770
340	0.4365	0.5170	0.5194	0.5065	0.6679	1.0342	1.4469	1.7450	1.8722	1.8615	1.7873
341	0.4666	0.5330	0.5592	0.5711	0.6920	0.9577	1.2711	1.5242	1.6688	1.7194	1.7242
342	0.4479	0.5046	0.5385	0.5684	0.6726	0.8738	1.1073	1.3046	1.4344	1.5050	1.5513
343	0.4084	0.4575	0.4940	0.5345	0.6298	0.7926	0.9725	1.1282	1.2406	1.3169	1.3853
344	0.2243	0.2842	0.2158	0.2108	0.4592	0.8846	1.2499	1.4034	1.3464	1.1621	0.9617
345	0.3779	0.4725	0.4098	0.3813	0.6691	1.2327	1.7790	2.0816	2.1037	1.9328	1.7058
346	0.4780	0.5801	0.5502	0.5125	0.7532	1.2894	1.8645	2.2438	2.3596	2.2731	2.1038
347	0.5389	0.6349	0.6350	0.6071	0.7972	1.2562	1.7843	2.1733	2.3458	2.3370	2.2418
348	0.5744	0.6555	0.6901	0.6982	0.8318	1.1485	1.5347	1.8567	2.0501	2.1253	2.1401
349	0.5427	0.6135	0.6615	0.6977	0.8130	1.0446	1.3207	1.5636	1.7327	1.8319	1.9022
350	0.4823	0.5455	0.5966	0.6496	0.7582	0.9432	1.1514	1.3387	1.4828	1.5869	1.6863
351	0.0405	0.0507	0.0425	0.0455	0.0848	0.1457	0.1951	0.2118	0.2009	0.1762	0.1539
352	0.0622	0.0766	0.0701	0.0726	0.1172	0.1917	0.2579	0.2871	0.2822	0.2584	0.2349
353	0.0750	0.0904	0.0868	0.0893	0.1318	0.2062	0.2765	0.3122	0.3140	0.2956	0.2756
354	0.0825	0.0974	0.0961	0.0991	0.1381	0.2080	0.2768	0.3149	0.3214	0.3085	0.2930
355	0.0881	0.1012	0.1025	0.1068	0.1393	0.1981	0.2585	0.2957	0.3071	0.3022	0.2946
356	0.0876	0.0989	0.1014	0.1066	0.1343	0.1836	0.2351	0.2687	0.2815	0.2810	0.2792
357	0.0849	0.0946	0.0976	0.1032	0.1276	0.1694	0.2130	0.2432	0.2561	0.2583	0.2600
358	0.0862	0.1083	0.0878	0.0927	0.1827	0.3241	0.4388	0.4800	0.4539	0.3938	0.3370
359	0.1372	0.1693	0.1524	0.1546	0.2579	0.4355	0.5947	0.6698	0.6601	0.6023	0.5411
360	0.1647	0.1990	0.1899	0.1916	0.2867	0.4614	0.6299	0.7233	0.7335	0.6927	0.6429
361	0.1836	0.2163	0.2141	0.2172	0.3014	0.4610	0.6228	0.7216	0.7457	0.7217	0.6865
362	0.1987	0.2261	0.2314	0.2395	0.3068	0.4335	0.5682	0.6602	0.6962	0.6945	0.6842
363	0.1980	0.2209	0.2292	0.2408	0.2974	0.3991	0.5083	0.5872	0.6247	0.6344	0.6393
364	0.1906	0.2102	0.2196	0.2335	0.2830	0.3669	0.4562	0.5230	0.5589	0.5743	0.5884
365	0.1300	0.1626	0.1284	0.1345	0.2761	0.5028	0.6857	0.7504	0.7066	0.6064	0.5111
366	0.2034	0.2519	0.2212	0.2216	0.3884	0.6784	0.9394	1.0625	1.0451	0.9458	0.8399
367	0.2539	0.3058	0.2885	0.2855	0.4354	0.7190	0.9958	1.1519	1.1710	1.1029	1.0175
368	0.2859	0.3349	0.3297	0.3287	0.4574	0.7111	0.9738	1.1392	1.1841	1.1476	1.0899
369	0.3118	0.3522	0.3616	0.3708	0.4679	0.6598	0.8699	1.0201	1.0854	1.0906	1.0791
370	0.3094	0.3436	0.3589	0.3759	0.4563	0.6043	0.7676	0.8915	0.9576	0.9816	0.9980
371	0.2944	0.3240	0.3416	0.3639	0.4332	0.5535	0.6827	0.7842	0.8452	0.8784	0.9111
372	0.1628	0.2041	0.1555	0.1628	0.3565	0.6675	0.9179	1.0046	0.9413	0.7992	0.6643
373	0.2691	0.3316	0.2850	0.2816	0.5111	0.9192	1.2843	1.4553	1.4271	1.2819	1.1280

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
374	0.3394	0.4062	0.3785	0.3673	0.5704	0.9654	1.3543	1.5752	1.6021	1.5033	1.3796
375	0.3849	0.4478	0.4387	0.4288	0.5947	0.9441	1.3099	1.5447	1.6117	1.5617	1.4803
376	0.4216	0.4740	0.4879	0.4938	0.6113	0.8637	1.1463	1.3546	1.4522	1.4660	1.4555
377	0.4150	0.4605	0.4846	0.5053	0.6004	0.7877	0.9987	1.1650	1.2601	1.3024	1.3337
378	0.3886	0.4292	0.4570	0.4875	0.5717	0.7199	0.8816	1.0137	1.1006	1.1548	1.2105
379	0.1978	0.2459	0.1825	0.1909	0.4357	0.8291	1.1447	1.2522	1.1684	0.9838	0.8084
380	0.3305	0.4040	0.3401	0.3316	0.6243	1.1489	1.6183	1.8360	1.7955	1.6015	1.3964
381	0.4208	0.4998	0.4597	0.4362	0.6871	1.1945	1.6958	1.9818	2.0168	1.8853	1.7197
382	0.4801	0.5548	0.5397	0.5162	0.7145	1.1539	1.6221	1.9267	2.0174	1.9537	1.8466
383	0.5267	0.5906	0.6094	0.6083	0.7386	1.0414	1.3911	1.6557	1.7866	1.8110	1.8023
384	0.5122	0.5705	0.6047	0.6279	0.7315	0.9478	1.1978	1.4024	1.5279	1.5891	1.6377
385	0.4698	0.5233	0.5641	0.6025	0.6967	0.8641	1.0503	1.2093	1.3220	1.3989	1.4806
386	0.0363	0.0451	0.0374	0.0413	0.0787	0.1342	0.1768	0.1883	0.1752	0.1518	0.1337
387	0.0553	0.0674	0.0609	0.0644	0.1072	0.1751	0.2323	0.2536	0.2445	0.2210	0.2017
388	0.0665	0.0795	0.0754	0.0784	0.1190	0.1869	0.2478	0.2749	0.2713	0.2521	0.2356
389	0.0732	0.0860	0.0838	0.0868	0.1236	0.1874	0.2472	0.2767	0.2775	0.2631	0.2503
390	0.0790	0.0902	0.0905	0.0940	0.1242	0.1775	0.2301	0.2596	0.2655	0.2584	0.2524
391	0.0794	0.0891	0.0906	0.0947	0.1199	0.1644	0.2093	0.2363	0.2442	0.2417	0.2394
392	0.0778	0.0861	0.0882	0.0926	0.1144	0.1520	0.1898	0.2144	0.2230	0.2230	0.2244
393	0.0778	0.0968	0.0777	0.0850	0.1710	0.3009	0.4022	0.4294	0.3980	0.3400	0.2926
394	0.1226	0.1498	0.1328	0.1382	0.2379	0.4008	0.5384	0.5936	0.5723	0.5137	0.4622
395	0.1465	0.1757	0.1652	0.1686	0.2603	0.4206	0.5666	0.6376	0.6328	0.5878	0.5451
396	0.1638	0.1917	0.1873	0.1907	0.2708	0.4172	0.5577	0.6346	0.6426	0.6122	0.5816
397	0.1793	0.2029	0.2058	0.2118	0.2738	0.3890	0.5062	0.5795	0.6005	0.5911	0.5820
398	0.1811	0.2009	0.2069	0.2157	0.2664	0.3577	0.4523	0.5155	0.5404	0.5429	0.5478
399	0.1766	0.1936	0.2007	0.2117	0.2552	0.3295	0.4064	0.4603	0.4855	0.4945	0.5082
400	0.1179	0.1459	0.1141	0.1241	0.2609	0.4688	0.6282	0.6734	0.6209	0.5243	0.4436
401	0.1820	0.2228	0.1926	0.1986	0.3600	0.6262	0.8512	0.9409	0.9035	0.8023	0.7125
402	0.2270	0.2710	0.2517	0.2520	0.3975	0.6578	0.8971	1.0148	1.0071	0.9304	0.8565
403	0.2564	0.2984	0.2899	0.2895	0.4128	0.6456	0.8732	1.0009	1.0168	0.9676	0.9163
404	0.2836	0.3185	0.3238	0.3293	0.4189	0.5932	0.7755	0.8940	0.9335	0.9236	0.9122
405	0.2858	0.3154	0.3269	0.3392	0.4094	0.5423	0.6830	0.7819	0.8263	0.8372	0.8516
406	0.2760	0.3019	0.3163	0.3332	0.3928	0.4982	0.6082	0.6895	0.7335	0.7554	0.7857
407	0.1475	0.1830	0.1378	0.1504	0.3377	0.6234	0.8418	0.9025	0.8273	0.6902	0.5751
408	0.2416	0.2940	0.2487	0.2534	0.4761	0.8504	1.1645	1.2876	1.2305	1.0826	0.9518
409	0.3048	0.3614	0.3314	0.3256	0.5232	0.8858	1.2206	1.3855	1.3727	1.2607	1.1532
410	0.3473	0.4011	0.3872	0.3790	0.5388	0.8594	1.1753	1.3549	1.3787	1.3086	1.2356
411	0.3866	0.4315	0.4399	0.4413	0.5492	0.7782	1.0223	1.1861	1.2451	1.2357	1.2238
412	0.3870	0.4264	0.4455	0.4592	0.5424	0.7086	0.8892	1.0215	1.0867	1.1081	1.1354
413	0.3687	0.4045	0.4277	0.4504	0.5216	0.6497	0.7864	0.8921	0.9556	0.9933	1.0444
414	0.1797	0.2209	0.1621	0.1773	0.4127	0.7756	1.0512	1.1261	1.0276	0.8494	0.6993
415	0.2979	0.3593	0.2975	0.3000	0.5839	1.0646	1.4673	1.6226	1.5440	1.3466	1.1725
416	0.3799	0.4464	0.4037	0.3885	0.6330	1.0983	1.5285	1.7397	1.7214	1.5719	1.4282
417	0.4361	0.4994	0.4788	0.4582	0.6499	1.0528	1.4559	1.6871	1.7191	1.6276	1.5311
418	0.4868	0.5412	0.5528	0.5463	0.6654	0.9401	1.2413	1.4486	1.5279	1.5203	1.5082
419	0.4828	0.5328	0.5601	0.5740	0.6631	0.8539	1.0679	1.2299	1.3167	1.3509	1.3927
420	0.4517	0.4985	0.5333	0.5611	0.6388	0.7825	0.9391	1.0660	1.1490	1.2064	1.2817
421	0.0761	0.1092	0.1008	0.0872	0.1239	0.2112	0.3061	0.3666	0.3832	0.3642	0.3263
422	0.1190	0.1625	0.1630	0.1500	0.1894	0.2909	0.4108	0.4968	0.5327	0.5245	0.4890
423	0.1442	0.1883	0.1959	0.1879	0.2258	0.3241	0.4456	0.5386	0.5847	0.5873	0.5606
424	0.1579	0.1994	0.2108	0.2077	0.2440	0.3350	0.4500	0.5414	0.5911	0.6010	0.5829
425	0.1661	0.2005	0.2141	0.2179	0.2514	0.3275	0.4245	0.5047	0.5526	0.5693	0.5633
426	0.1611	0.1896	0.2029	0.2099	0.2415	0.3055	0.3865	0.4548	0.4975	0.5155	0.5162
427	0.1532	0.1773	0.1897	0.1986	0.2282	0.2834	0.3518	0.4100	0.4476	0.4651	0.4693
428	0.1560	0.2294	0.2076	0.1736	0.2569	0.4599	0.6810	0.8260	0.8663	0.8215	0.7293
429	0.2485	0.3490	0.3497	0.3144	0.4021	0.6420	0.9294	1.1458	1.2417	1.2293	1.1458
430	0.3069	0.4072	0.4277	0.4061	0.4878	0.7152	1.0029	1.2375	1.3635	1.3852	1.3311
431	0.3410	0.4321	0.4626	0.4556	0.5331	0.7385	1.0045	1.2318	1.3671	1.4104	1.3830
432	0.3643	0.4353	0.4702	0.4824	0.5559	0.7209	0.9347	1.1260	1.2523	1.3111	1.3173
433	0.3567	0.4135	0.4462	0.4678	0.5383	0.6749	0.8465	1.0022	1.1104	1.1692	1.1902
434	0.3363	0.3837	0.4140	0.4406	0.5074	0.6255	0.7661	0.8936	0.9859	1.0405	1.0686
435	0.2287	0.3401	0.3011	0.2456	0.3798	0.7081	1.0639	1.2967	1.3599	1.2835	1.1305
436	0.3668	0.5250	0.5213	0.4583	0.5988	0.9951	1.4727	1.8351	1.9990	1.9802	1.8411
437	0.4562	0.6135	0.6457	0.6046	0.7291	1.1006	1.5777	1.9748	2.1969	2.2439	2.1618
438	0.5106	0.6503	0.7010	0.6858	0.8009	1.1291	1.5637	1.9463	2.1864	2.2752	2.2446



hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
439	0.5503	0.6540	0.7116	0.7318	0.8401	1.0949	1.4307	1.7427	1.9629	2.0786	2.1097
440	0.5369	0.6181	0.6714	0.7095	0.8149	1.0222	1.2829	1.5274	1.7106	1.8230	1.8779
441	0.4977	0.5661	0.6167	0.6650	0.7680	0.9449	1.1539	1.3482	1.5002	1.6027	1.6678
442	0.2869	0.4323	0.3735	0.2970	0.4854	0.9424	1.4354	1.7554	1.8396	1.7281	1.5094
443	0.4705	0.6838	0.6715	0.5769	0.7729	1.3372	2.0178	2.5360	2.7718	2.7443	2.5419
444	0.5859	0.7977	0.8391	0.7727	0.9391	1.4618	2.1424	2.7171	3.0467	3.1236	3.0106
445	0.6587	0.8429	0.9127	0.8845	1.0321	1.4841	2.0969	2.6491	3.0077	3.1523	3.1223
446	0.7140	0.8448	0.9250	0.9515	1.0873	1.4241	1.8802	2.3178	2.6413	2.8268	2.8922
447	0.6902	0.7921	0.8672	0.9224	1.0568	1.3246	1.6662	1.9979	2.2594	2.4339	2.5344
448	0.6247	0.7129	0.7855	0.8575	0.9928	1.2202	1.4890	1.7473	1.9588	2.1121	2.2255
449	0.3427	0.5181	0.4378	0.3416	0.5864	1.1728	1.8021	2.2084	2.3114	2.1626	1.8757
450	0.5598	0.8237	0.7994	0.6704	0.9240	1.6622	2.5534	3.2346	3.5452	3.5076	3.2369
451	0.6960	0.9572	1.0056	0.9085	1.1132	1.7899	2.6830	3.4470	3.8934	4.0052	3.8607
452	0.7847	1.0071	1.0950	1.0483	1.2204	1.7935	2.5886	3.3202	3.8097	4.0187	3.9941
453	0.8516	1.0048	1.1073	1.1376	1.2912	1.6984	2.2672	2.8304	3.2651	3.5286	3.6390
454	0.8114	0.9321	1.0296	1.1015	1.2585	1.5741	1.9847	2.3972	2.7372	2.9769	3.1313
455	0.7118	0.8186	0.9155	1.0125	1.1768	1.4460	1.7633	2.0777	2.3452	2.5500	2.7168
456	0.0672	0.0952	0.0848	0.0755	0.1180	0.2071	0.2983	0.3517	0.3612	0.3374	0.2983
457	0.1036	0.1410	0.1373	0.1273	0.1739	0.2796	0.3968	0.4748	0.5012	0.4853	0.4465
458	0.1249	0.1631	0.1656	0.1590	0.2031	0.3063	0.4265	0.5125	0.5492	0.5436	0.5125
459	0.1369	0.1727	0.1791	0.1762	0.2175	0.3132	0.4279	0.5137	0.5552	0.5573	0.5344
460	0.1452	0.1749	0.1842	0.1869	0.2234	0.3033	0.4010	0.4780	0.5201	0.5306	0.5202
461	0.1421	0.1666	0.1763	0.1821	0.2151	0.2822	0.3644	0.4308	0.4695	0.4829	0.4799
462	0.1365	0.1573	0.1666	0.1741	0.2045	0.2619	0.3315	0.3887	0.4233	0.4375	0.4387
463	0.1391	0.2010	0.1747	0.1504	0.2473	0.4547	0.6668	0.7944	0.8170	0.7597	0.6638
464	0.2175	0.3037	0.2928	0.2646	0.3713	0.6238	0.9065	1.1030	1.1733	1.1392	1.0442
465	0.2666	0.3539	0.3607	0.3407	0.4388	0.6814	0.9695	1.1885	1.2904	1.2882	1.2186
466	0.2964	0.3760	0.3934	0.3846	0.4743	0.6939	0.9633	1.1799	1.2950	1.3165	1.2728
467	0.3203	0.3823	0.4066	0.4145	0.4935	0.6688	0.8880	1.0748	1.1885	1.2319	1.2243
468	0.3186	0.3676	0.3920	0.4089	0.4817	0.6248	0.8012	0.9556	1.0559	1.1038	1.1146
469	0.3053	0.3458	0.3692	0.3910	0.4581	0.5802	0.7249	0.8526	0.9395	0.9862	1.0058
470	0.2052	0.2989	0.2535	0.2135	0.3693	0.7030	1.0431	1.2468	1.2805	1.1835	1.0241
471	0.3226	0.4577	0.4348	0.3839	0.5578	0.9761	1.4455	1.7728	1.8912	1.8325	1.6710
472	0.3982	0.5351	0.5435	0.5046	0.6592	1.0583	1.5382	1.9087	2.0868	2.0891	1.9756
473	0.4463	0.5688	0.5968	0.5771	0.7130	1.0685	1.5124	1.8782	2.0826	2.1303	2.0672
474	0.4884	0.5792	0.6197	0.6311	0.7471	1.0201	1.3684	1.6762	1.8760	1.9641	1.9683
475	0.4864	0.5563	0.5973	0.6259	0.7329	0.9509	1.2215	1.4671	1.6384	1.7319	1.7674
476	0.4615	0.5198	0.5594	0.5987	0.7003	0.8827	1.0993	1.2957	1.4403	1.5291	1.5790
477	0.2572	0.3793	0.3127	0.2578	0.4742	0.9372	1.4064	1.6851	1.7274	1.5873	1.3602
478	0.4160	0.5971	0.5578	0.4818	0.7262	1.3224	1.9895	2.4544	2.6212	2.5330	2.2964
479	0.5148	0.6986	0.7053	0.6421	0.8525	1.4190	2.1045	2.6384	2.9003	2.9073	2.7441
480	0.5800	0.7420	0.7791	0.7425	0.9215	1.4163	2.0453	2.5732	2.8776	2.9576	2.8738
481	0.6406	0.7559	0.8127	0.8239	0.9704	1.3350	1.8126	2.2469	2.5425	2.6844	2.7067
482	0.6360	0.7244	0.7828	0.8227	0.9580	1.2409	1.5993	1.9342	2.1818	2.3287	2.3973
483	0.5938	0.6691	0.7275	0.7852	0.9169	1.1513	1.4312	1.6923	1.8955	2.0319	2.1203
484	0.3083	0.4550	0.3662	0.2975	0.5760	1.1680	1.7648	2.1164	2.1653	1.9799	1.6832
485	0.4979	0.7206	0.6618	0.5590	0.8775	1.6563	2.5273	3.1339	3.3496	3.2281	2.9100
486	0.6161	0.8425	0.8446	0.7525	1.0179	1.7550	2.6542	3.3605	3.7121	3.7248	3.5075
487	0.6969	0.8937	0.9377	0.8792	1.0945	1.7277	2.5471	3.2449	3.6591	3.7768	3.6730
488	0.7746	0.9104	0.9841	0.9917	1.1578	1.6041	2.2056	2.7683	3.1658	3.3706	3.4174
489	0.7631	0.8685	0.9456	0.9966	1.1519	1.4875	1.9224	2.3424	2.6665	2.8715	2.9813
490	0.6972	0.7900	0.8696	0.9477	1.1041	1.3800	1.7119	2.0321	2.2907	2.4750	2.6091
491	0.0590	0.0822	0.0710	0.0656	0.1112	0.1982	0.2822	0.3269	0.3295	0.3024	0.2642
492	0.0895	0.1201	0.1133	0.1073	0.1584	0.2631	0.3720	0.4384	0.4543	0.4321	0.3924
493	0.1072	0.1385	0.1369	0.1326	0.1812	0.2843	0.3973	0.4720	0.4976	0.4842	0.4508
494	0.1175	0.1469	0.1488	0.1470	0.1918	0.2879	0.3966	0.4724	0.5034	0.4976	0.4715
495	0.1258	0.1501	0.1554	0.1576	0.1960	0.2763	0.3699	0.4397	0.4735	0.4770	0.4629
496	0.1246	0.1447	0.1508	0.1555	0.1893	0.2566	0.3359	0.3969	0.4291	0.4369	0.4305
497	0.1213	0.1382	0.1444	0.1506	0.1810	0.2384	0.3059	0.3589	0.3884	0.3980	0.3965
498	0.1230	0.1743	0.1463	0.1313	0.2361	0.4389	0.6346	0.7416	0.7473	0.6811	0.5864
499	0.1885	0.2591	0.2405	0.2217	0.3418	0.5939	0.8579	1.0252	1.0679	1.0148	0.9147
500	0.2296	0.3014	0.2971	0.2824	0.3934	0.6392	0.9127	1.1038	1.1756	1.1503	1.0703
501	0.2555	0.3212	0.3269	0.3193	0.4190	0.6429	0.9017	1.0949	1.1823	1.1802	1.1236
502	0.2795	0.3304	0.3446	0.3498	0.4329	0.6115	0.8249	0.9960	1.0893	1.1131	1.0924
503	0.2826	0.3226	0.3387	0.3517	0.4247	0.5694	0.7421	0.8856	0.9713	1.0037	1.0038

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
504	0.2753	0.3082	0.3246	0.3422	0.4088	0.5298	0.6713	0.7912	0.8667	0.9018	0.9126
505	0.1825	0.2600	0.2126	0.1877	0.3557	0.6817	0.9952	1.1652	1.1711	1.0597	0.9016
506	0.2810	0.3911	0.3559	0.3217	0.5181	0.9377	1.3755	1.6525	1.7214	1.6287	1.4570
507	0.3449	0.4572	0.4468	0.4170	0.5957	1.0027	1.4587	1.7810	1.9049	1.8636	1.7285
508	0.3872	0.4885	0.4964	0.4782	0.6327	0.9988	1.4272	1.7533	1.9076	1.9107	1.8203
509	0.4301	0.5050	0.5289	0.5344	0.6578	0.9385	1.2798	1.5631	1.7275	1.7794	1.7566
510	0.4370	0.4943	0.5222	0.5433	0.6508	0.8712	1.1377	1.3678	1.5147	1.5809	1.5945
511	0.4233	0.4708	0.4998	0.5307	0.6296	0.8110	1.0234	1.2091	1.3352	1.4037	1.4359
512	0.2288	0.3296	0.2612	0.2269	0.4590	0.9108	1.3425	1.5740	1.5778	1.4179	1.1939
513	0.3641	0.5108	0.4552	0.4043	0.6831	1.2795	1.8999	2.2901	2.3835	2.2445	1.9919
514	0.4488	0.5990	0.5790	0.5300	0.7773	1.3571	2.0080	2.4695	2.6487	2.5877	2.3893
515	0.5073	0.6411	0.6491	0.6152	0.8229	1.3362	1.9442	2.4132	2.6413	2.6508	2.5220
516	0.5701	0.6660	0.7000	0.7016	0.8584	1.2366	1.7079	2.1084	2.3513	2.4368	2.4141
517	0.5796	0.6528	0.6939	0.7220	0.8571	1.1445	1.4996	1.8146	2.0271	2.1329	2.1657
518	0.5553	0.6173	0.6623	0.7075	0.8335	1.0666	1.3419	1.5891	1.7676	1.8740	1.9341
519	0.2750	0.3960	0.3059	0.2632	0.5605	1.1364	1.6843	1.9756	1.9753	1.7652	1.4730
520	0.4381	0.6175	0.5388	0.4704	0.8342	1.6121	2.4192	2.9249	3.0410	2.8508	2.5112
521	0.5412	0.7255	0.6928	0.6211	0.9377	1.6937	2.5458	3.1527	3.3890	3.3062	3.0388
522	0.6153	0.7778	0.7839	0.7288	0.9853	1.6459	2.4387	3.0566	3.3636	3.3810	3.2115
523	0.6984	0.8124	0.8565	0.8500	1.0305	1.4979	2.0950	2.6143	2.9405	3.0656	3.0463
524	0.7074	0.7959	0.8530	0.8864	1.0401	1.3825	1.8163	2.2124	2.4924	2.6426	2.6996
525	0.6662	0.7454	0.8090	0.8698	1.0172	1.2907	1.6181	1.9211	2.1503	2.2962	2.3901
526	0.0532	0.0728	0.0617	0.0591	0.1058	0.1894	0.2667	0.3044	0.3020	0.2732	0.2368
527	0.0798	0.1054	0.0972	0.0942	0.1475	0.2490	0.3497	0.4063	0.4146	0.3886	0.3496
528	0.0952	0.1214	0.1175	0.1153	0.1658	0.2662	0.3712	0.4358	0.4529	0.4345	0.4007
529	0.1044	0.1290	0.1283	0.1276	0.1738	0.2675	0.3689	0.4352	0.4578	0.4465	0.4192
530	0.1125	0.1329	0.1354	0.1376	0.1763	0.2545	0.3423	0.4043	0.4308	0.4290	0.4129
531	0.1136	0.1303	0.1341	0.1382	0.1715	0.2368	0.3108	0.3661	0.3922	0.3953	0.3868
532	0.1104	0.1244	0.1286	0.1338	0.1634	0.2189	0.2824	0.3302	0.3546	0.3601	0.3566
533	0.1116	0.1552	0.1273	0.1190	0.2268	0.4223	0.6025	0.6929	0.6865	0.6159	0.5248
534	0.1721	0.2312	0.2091	0.1983	0.3247	0.5704	0.8146	0.9570	0.9789	0.9144	0.8145
535	0.2045	0.2648	0.2545	0.2450	0.3623	0.6036	0.8584	1.0236	1.0718	1.0307	0.9467
536	0.2277	0.2830	0.2816	0.2765	0.3809	0.6014	0.8441	1.0132	1.0772	1.0579	0.9946
537	0.2512	0.2938	0.3016	0.3059	0.3898	0.5651	0.7666	0.9191	0.9926	1.0005	0.9712
538	0.2569	0.2901	0.3008	0.3115	0.3834	0.5240	0.6871	0.8162	0.8858	0.9050	0.8970
539	0.2533	0.2805	0.2921	0.3067	0.3710	0.4874	0.6208	0.7291	0.7918	0.8155	0.8195
540	0.1662	0.2324	0.1855	0.1710	0.3437	0.6578	0.9465	1.0897	1.0763	0.9577	0.8053
541	0.2521	0.3441	0.3035	0.2828	0.4922	0.8999	1.3039	1.5382	1.5706	1.4574	1.2842
542	0.3084	0.4026	0.3818	0.3613	0.5527	0.9533	1.3777	1.6545	1.7350	1.6642	1.5202
543	0.3469	0.4320	0.4279	0.4137	0.5783	0.9406	1.3425	1.6265	1.7377	1.7079	1.6028
544	0.3894	0.4521	0.4653	0.4683	0.5946	0.8715	1.1945	1.4462	1.5753	1.5963	1.5550
545	0.4009	0.4489	0.4681	0.4843	0.5898	0.8046	1.0566	1.2631	1.3824	1.4237	1.4202
546	0.3939	0.4334	0.4553	0.4806	0.5749	0.7491	0.9488	1.1161	1.2206	1.2682	1.2859
547	0.2082	0.2944	0.2273	0.2073	0.4450	0.8800	1.2772	1.4719	1.4492	1.2796	1.0638
548	0.3278	0.4499	0.3876	0.3568	0.6523	1.2328	1.8037	2.1311	2.1708	2.0015	1.7480
549	0.4033	0.5286	0.4945	0.4600	0.7270	1.2980	1.9020	2.2952	2.4084	2.3020	2.0891
550	0.4573	0.5694	0.5603	0.5330	0.7573	1.2666	1.8362	2.2418	2.4037	2.3611	2.2078
551	0.5206	0.6011	0.6201	0.6177	0.7799	1.1545	1.6005	1.9553	2.1445	2.1816	2.1282
552	0.5378	0.5993	0.6289	0.6493	0.7814	1.0616	1.3976	1.6796	1.8517	1.9188	1.9233
553	0.5237	0.5765	0.6118	0.6485	0.7669	0.9897	1.2479	1.4698	1.6166	1.6928	1.7294
554	0.2509	0.3544	0.2665	0.2417	0.5452	1.0990	1.6027	1.8473	1.8136	1.5915	1.3104
555	0.3958	0.5446	0.4585	0.4170	0.8023	1.5581	2.2986	2.7197	2.7640	2.5336	2.1934
556	0.4890	0.6421	0.5918	0.5405	0.8842	1.6284	2.4168	2.9300	3.0755	2.9298	2.6424
557	0.5587	0.6944	0.6786	0.6329	0.9131	1.5698	2.3107	2.8424	3.0571	3.0013	2.7964
558	0.6446	0.7407	0.7652	0.7528	0.9411	1.4061	1.9714	2.4301	2.6825	2.7392	2.6746
559	0.6644	0.7403	0.7831	0.8051	0.9541	1.2888	1.6990	2.0532	2.2789	2.3758	2.3922
560	0.6379	0.7068	0.7595	0.8077	0.9440	1.2041	1.5104	1.7809	1.9710	2.0762	2.1367
561	0.1128	0.1759	0.1768	0.1447	0.1643	0.2578	0.3813	0.4769	0.5216	0.5166	0.4770
562	0.1797	0.2572	0.2751	0.2467	0.2664	0.3712	0.5211	0.6476	0.7189	0.7316	0.6986
563	0.2196	0.2951	0.3225	0.3048	0.3255	0.4256	0.5740	0.7054	0.7867	0.8121	0.7907
564	0.2412	0.3104	0.3413	0.3323	0.3556	0.4482	0.5867	0.7128	0.7950	0.8272	0.8156
565	0.2513	0.3078	0.3381	0.3412	0.3679	0.4463	0.5622	0.6700	0.7443	0.7799	0.7807
566	0.2438	0.2907	0.3180	0.3268	0.3557	0.4231	0.5191	0.6111	0.6751	0.7086	0.7152
567	0.2303	0.2702	0.2947	0.3061	0.3360	0.3955	0.4769	0.5552	0.6105	0.6408	0.6500
568	0.2282	0.3711	0.3690	0.2908	0.3347	0.5556	0.8465	1.0769	1.1853	1.1744	1.0783

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
569	0.3717	0.5553	0.5988	0.5269	0.5671	0.8172	1.1796	1.4988	1.6854	1.7281	1.6546
570	0.4632	0.6392	0.7108	0.6690	0.7116	0.9454	1.2996	1.6318	1.8488	1.9319	1.8969
571	0.5156	0.6722	0.7532	0.7383	0.7878	1.0008	1.3248	1.6395	1.8589	1.9622	1.9582
572	0.5466	0.6679	0.7449	0.7621	0.8262	1.0039	1.2646	1.5247	1.7183	1.8279	1.8575
573	0.5293	0.6269	0.6944	0.7260	0.7982	0.9521	1.1633	1.3748	1.5376	1.6373	1.6781
574	0.4948	0.5768	0.6368	0.6757	0.7511	0.8880	1.0637	1.2370	1.3740	1.4630	1.5074
575	0.3276	0.5496	0.5379	0.4092	0.4830	0.8461	1.3204	1.6951	1.8704	1.8489	1.6876
576	0.5425	0.8409	0.9063	0.7795	0.8422	1.2631	1.8741	2.4159	2.7374	2.8143	2.6918
577	0.6788	0.9640	1.0831	1.0088	1.0704	1.4588	2.0554	2.6247	3.0088	3.1664	3.1202
578	0.7597	1.0073	1.1448	1.1206	1.1944	1.5407	2.0794	2.6165	3.0076	3.2065	3.2217
579	0.8084	0.9906	1.1193	1.1547	1.2561	1.5383	1.9576	2.3904	2.7312	2.9421	3.0213
580	0.7766	0.9197	1.0306	1.0917	1.2091	1.4514	1.7823	2.1228	2.4028	2.5917	2.6906
581	0.7132	0.8346	0.9324	1.0069	1.1313	1.3470	1.6176	1.8902	2.1191	2.2843	2.3872
582	0.4179	0.7148	0.6880	0.5080	0.6176	1.1310	1.7963	2.3191	2.5606	2.5247	2.2913
583	0.6880	1.1024	1.1846	0.9949	1.0841	1.6936	2.5781	3.3647	3.8343	3.9478	3.7685
584	0.8574	1.2527	1.4195	1.3047	1.3836	1.9401	2.8055	3.6422	4.2185	4.4649	4.4090
585	0.9599	1.2954	1.4922	1.4545	1.5470	2.0333	2.8071	3.5951	4.1863	4.5035	4.5494
586	1.0198	1.2543	1.4365	1.4903	1.6233	2.0071	2.5896	3.2093	3.7209	4.0559	4.2057
587	0.9665	1.1476	1.3009	1.3941	1.5538	1.8788	2.3246	2.7976	3.2048	3.4991	3.6768
588	0.8663	1.0210	1.1577	1.2719	1.4452	1.7332	2.0905	2.4598	2.7824	3.0299	3.2089
589	0.4980	0.8644	0.8185	0.5879	0.7380	1.4070	2.2667	2.9387	3.2459	3.1925	2.8823
590	0.8086	1.3357	1.4297	1.1716	1.2906	2.1026	3.2787	4.3276	4.9563	5.1090	4.8673
591	0.9972	1.4986	1.7122	1.5511	1.6433	2.3769	3.5313	4.6606	5.4511	5.8007	5.7394
592	1.1134	1.5293	1.7858	1.7296	1.8335	2.4617	3.4835	4.5452	5.3627	5.8200	5.9105
593	1.1758	1.4519	1.6846	1.7568	1.9130	2.3892	3.1312	3.9456	4.6431	5.1243	5.3665
594	1.0933	1.3032	1.4966	1.6241	1.8196	2.2146	2.7623	3.3635	3.9013	4.3087	4.5826
595	0.9493	1.1318	1.3066	1.4625	1.6800	2.0298	2.4597	2.9163	3.3300	3.6591	3.9231
596	0.1019	0.1586	0.1538	0.1250	0.1540	0.2554	0.3801	0.4708	0.5079	0.4958	0.4519
597	0.1585	0.2295	0.2388	0.2113	0.2412	0.3576	0.5119	0.6347	0.6975	0.7005	0.6605
598	0.1916	0.2612	0.2800	0.2609	0.2901	0.4019	0.5564	0.6860	0.7601	0.7762	0.7471
599	0.2100	0.2739	0.2968	0.2858	0.3153	0.4184	0.5637	0.6896	0.7665	0.7904	0.7715
600	0.2200	0.2720	0.2962	0.2962	0.3266	0.4131	0.5353	0.6451	0.7166	0.7467	0.7417
601	0.2158	0.2587	0.2811	0.2870	0.3176	0.3914	0.4931	0.5873	0.6503	0.6803	0.6825
602	0.2061	0.2425	0.2628	0.2718	0.3024	0.3666	0.4530	0.5337	0.5886	0.6165	0.6225
603	0.2080	0.3359	0.3204	0.2498	0.3161	0.5548	0.8478	1.0655	1.1544	1.1250	1.0171
604	0.3290	0.4979	0.5181	0.4459	0.5126	0.7935	1.1694	1.4799	1.6432	1.6585	1.5628
605	0.4042	0.5687	0.6168	0.5677	0.6305	0.8960	1.2699	1.6004	1.7991	1.8558	1.7969
606	0.4489	0.5956	0.6561	0.6310	0.6958	0.9348	1.2801	1.5985	1.8054	1.8869	1.8605
607	0.4812	0.5936	0.6558	0.6625	0.7323	0.9279	1.2070	1.4753	1.6650	1.7614	1.7739
608	0.4736	0.5629	0.6189	0.6414	0.7145	0.8804	1.1067	1.3267	1.4888	1.5807	1.6108
609	0.4503	0.5247	0.5750	0.6057	0.6804	0.8255	1.0126	1.1946	1.3323	1.4150	1.4511
610	0.2995	0.4975	0.4651	0.3495	0.4587	0.8488	1.3244	1.6759	1.8177	1.7648	1.5839
611	0.4828	0.7563	0.7811	0.6536	0.7624	1.2368	1.8704	2.3949	2.6729	2.6989	2.5356
612	0.5944	0.8618	0.9387	0.8493	0.9460	1.3910	2.0239	2.5903	2.9398	3.0471	2.9535
613	0.6635	0.8974	0.9989	0.9531	1.0486	1.4442	2.0231	2.5686	2.9372	3.0945	3.0649
614	0.7172	0.8872	0.9916	1.0055	1.1120	1.4249	1.8784	2.3286	2.6641	2.8506	2.8972
615	0.7052	0.8360	0.9287	0.9721	1.0880	1.3480	1.7045	2.0624	2.3432	2.5178	2.5940
616	0.6633	0.7721	0.8554	0.9143	1.0350	1.2615	1.5505	1.8378	2.0702	2.2262	2.3113
617	0.3835	0.6471	0.5933	0.4325	0.5896	1.1382	1.8014	2.2889	2.4816	2.4010	2.1400
618	0.6163	0.9941	1.0165	0.8273	0.9853	1.6721	2.5866	3.3433	3.7442	3.7783	3.5360
619	0.7547	1.1256	1.2286	1.0903	1.2221	1.8641	2.7836	3.6137	4.1333	4.2988	4.1662
620	0.8428	1.1621	1.3050	1.2320	1.3557	1.9174	2.7526	3.5531	4.1081	4.3577	4.3296
621	0.9146	1.1350	1.2835	1.3029	1.4398	1.8681	2.5038	3.1518	3.6559	3.9524	4.0488
622	0.8942	1.0601	1.1901	1.2558	1.4090	1.7578	2.2420	2.7430	3.1540	3.4290	3.5688
623	0.8279	0.9669	1.0840	1.1748	1.3395	1.6407	2.0250	2.4167	2.7477	2.9842	3.1353
624	0.4587	0.7826	0.7038	0.4993	0.7092	1.4189	2.2713	2.8945	3.1367	3.0249	2.6800
625	0.7293	1.2073	1.2222	0.9670	1.1794	2.0925	3.3045	4.3070	4.8371	4.8778	4.5477
626	0.8840	1.3545	1.4809	1.2867	1.4529	2.3041	3.5306	4.6467	5.3535	5.5845	5.4106
627	0.9846	1.3833	1.5668	1.4606	1.6063	2.3387	3.4471	4.5252	5.2890	5.6455	5.6253
628	1.0691	1.3316	1.5231	1.5461	1.7034	2.2403	3.0579	3.9144	4.6026	5.0306	5.1921
629	1.0364	1.2293	1.3955	1.4848	1.6685	2.0934	2.6949	3.3363	3.8844	4.2692	4.4886
630	0.9385	1.1025	1.2550	1.3804	1.5851	1.9500	2.4155	2.9036	3.3304	3.6501	3.8776
631	0.0912	0.1404	0.1315	0.1073	0.1439	0.2487	0.3698	0.4523	0.4806	0.4621	0.4157
632	0.1381	0.1998	0.2014	0.1771	0.2167	0.3394	0.4913	0.6047	0.6556	0.6489	0.6037
633	0.1651	0.2259	0.2359	0.2177	0.2555	0.3745	0.5282	0.6500	0.7127	0.7182	0.6825

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
634	0.1805	0.2364	0.2510	0.2392	0.2754	0.3855	0.5312	0.6511	0.7180	0.7321	0.7063
635	0.1902	0.2357	0.2529	0.2508	0.2849	0.3769	0.5007	0.6072	0.6720	0.6944	0.6831
636	0.1889	0.2261	0.2429	0.2463	0.2789	0.3568	0.4612	0.5528	0.6112	0.6354	0.6326
637	0.1825	0.2139	0.2296	0.2360	0.2675	0.3349	0.4239	0.5028	0.5545	0.5781	0.5801
638	0.1875	0.2988	0.2739	0.2139	0.2985	0.5451	0.8292	1.0271	1.0942	1.0483	0.9333
639	0.2881	0.4353	0.4350	0.3698	0.4622	0.7618	1.1337	1.4205	1.5519	1.5391	1.4263
640	0.3490	0.4939	0.5183	0.4688	0.5542	0.8417	1.2184	1.5302	1.6985	1.7244	1.6426
641	0.3865	0.5162	0.5543	0.5240	0.6044	0.8649	1.2168	1.5225	1.7042	1.7572	1.7068
642	0.4184	0.5170	0.5618	0.5600	0.6360	0.8470	1.1347	1.3981	1.5721	1.6479	1.6409
643	0.4184	0.4958	0.5381	0.5523	0.6276	0.8028	1.0362	1.2550	1.4073	1.4846	1.4991
644	0.4040	0.4677	0.5068	0.5300	0.6047	0.7553	0.9483	1.1303	1.2613	1.3332	1.3576
645	0.2709	0.4429	0.3964	0.2984	0.4368	0.8382	1.2980	1.6159	1.7211	1.6408	1.4481
646	0.4253	0.6628	0.6529	0.5380	0.6922	1.1991	1.8255	2.3073	2.5277	2.5016	2.3061
647	0.5157	0.7511	0.7862	0.6955	0.8327	1.3190	1.9583	2.4917	2.7847	2.8331	2.6959
648	0.5736	0.7814	0.8432	0.7862	0.9107	1.3458	1.9396	2.4639	2.7859	2.8882	2.8124
649	0.6277	0.7780	0.8536	0.8500	0.9667	1.3060	1.7774	2.2222	2.5308	2.6782	2.6859
650	0.6303	0.7437	0.8147	0.8420	0.9592	1.2345	1.6052	1.9636	2.2286	2.3767	2.4226
651	0.6052	0.6982	0.7642	0.8089	0.9274	1.1618	1.4608	1.7502	1.9722	2.1084	2.1703
652	0.3481	0.5765	0.5048	0.3692	0.5653	1.1272	1.7664	2.2054	2.3464	2.2270	1.9500
653	0.5460	0.8722	0.8458	0.6768	0.9017	1.6352	2.5365	3.2268	3.5395	3.4940	3.2024
654	0.6586	0.9847	1.0258	0.8858	1.0798	1.7851	2.7139	3.4917	3.9223	3.9938	3.7907
655	0.7329	1.0178	1.1018	1.1013	1.1787	1.8018	2.6619	3.4301	3.9111	4.0715	3.9666
656	0.8082	1.0049	1.1119	1.1035	1.2539	1.7232	2.3892	3.0316	3.4931	3.7287	3.7586
657	0.8114	0.9556	1.0575	1.0979	1.2506	1.6209	2.1280	2.6318	3.0210	3.2554	3.3451
658	0.7710	0.8906	0.9859	1.0556	1.2139	1.5255	1.9247	2.3208	2.6390	2.8479	2.9605
659	0.4177	0.6975	0.5980	0.4263	0.6837	1.4079	2.2265	2.7854	2.9600	2.7987	2.4341
660	0.6501	1.0608	1.0125	0.7871	1.0896	2.0624	3.2521	4.1606	4.5683	4.4984	4.1008
661	0.7771	1.1902	1.2331	1.0383	1.2916	2.2291	3.4658	4.5059	5.0851	5.1815	4.9051
662	0.8634	1.2204	1.3243	1.1920	1.4005	2.2199	3.3630	4.3945	5.0510	5.2777	5.1437
663	0.9565	1.1932	1.3327	1.3150	1.4892	2.0835	2.9467	3.7972	4.4287	4.7660	4.8280
664	0.9571	1.1280	1.2614	1.3155	1.4948	1.9486	2.5840	3.2331	3.7542	4.0828	4.2286
665	0.8964	1.0391	1.1667	1.2650	1.4584	1.8348	2.3220	2.8178	3.2309	3.5159	3.6892
666	0.0831	0.1263	0.1152	0.0951	0.1367	0.2420	0.3583	0.4334	0.4548	0.4320	0.3846
667	0.1237	0.1778	0.1745	0.1537	0.2000	0.3250	0.4720	0.5761	0.6174	0.6036	0.5556
668	0.1468	0.2001	0.2042	0.1877	0.2316	0.3536	0.5032	0.6162	0.6688	0.6663	0.6267
669	0.1599	0.2090	0.2177	0.2064	0.2473	0.3603	0.5030	0.6150	0.6727	0.6788	0.6485
670	0.1692	0.2088	0.2210	0.2179	0.2544	0.3487	0.4707	0.5714	0.6290	0.6444	0.6286
671	0.1692	0.2015	0.2139	0.2159	0.2497	0.3292	0.4316	0.5197	0.5725	0.5910	0.5841
672	0.1650	0.1920	0.2039	0.2088	0.2407	0.3092	0.3965	0.4728	0.5200	0.5390	0.5375
673	0.1721	0.2701	0.2402	0.1897	0.2862	0.5343	0.8072	0.9868	1.0368	0.9797	0.8618
674	0.2593	0.3881	0.3751	0.3185	0.4292	0.7364	1.0973	1.3598	1.4651	1.4314	1.3090
675	0.3109	0.4384	0.4466	0.4008	0.5031	0.8013	1.1706	1.4598	1.6005	1.6018	1.5056
676	0.3431	0.4575	0.4794	0.4486	0.5419	0.8135	1.1613	1.4483	1.6043	1.6329	1.5662
677	0.3733	0.4595	0.4909	0.4850	0.5682	0.7859	1.0723	1.3236	1.4785	1.5339	1.5107
678	0.3774	0.4441	0.4757	0.4848	0.5625	0.7420	0.9748	1.1850	1.3233	1.3846	1.3854
679	0.3686	0.4229	0.4531	0.4714	0.5463	0.6989	0.8910	1.0664	1.1865	1.2461	1.2592
680	0.2492	0.4008	0.3473	0.2646	0.4216	0.8242	1.2649	1.5530	1.6299	1.5312	1.3337
681	0.3843	0.5915	0.5606	0.4615	0.6477	1.1678	1.7745	2.2131	2.3858	2.3220	2.1083
682	0.4611	0.6678	0.6747	0.5907	0.7595	1.2663	1.8935	2.3859	2.6268	2.6285	2.4624
683	0.5110	0.6945	0.7277	0.6692	0.8185	1.2754	1.8642	2.3550	2.6287	2.6835	2.5736
684	0.5634	0.6951	0.7479	0.7356	0.8631	1.2181	1.6906	2.1149	2.3883	2.4962	2.4704
685	0.5733	0.6714	0.7254	0.7424	0.8628	1.1462	1.5179	1.8633	2.1020	2.2207	2.2386
686	0.5581	0.6378	0.6902	0.7260	0.8437	1.0809	1.3794	1.6589	1.8620	1.9744	2.0134
687	0.3211	0.5224	0.4422	0.3278	0.5484	1.1105	1.7217	2.1185	2.2199	2.0751	1.7922
688	0.4952	0.7786	0.7233	0.5788	0.8507	1.6019	2.4722	3.0970	3.3374	3.2348	2.9156
689	0.5916	0.8771	0.8772	0.7486	0.9915	1.7277	2.6373	3.3513	3.7002	3.6980	3.4485
690	0.6564	0.9077	0.9493	0.8560	1.0635	1.7219	2.5747	3.2908	3.6952	3.7787	3.6178
691	0.7307	0.9043	0.9787	0.9557	1.1230	1.6178	2.2880	2.9005	3.3079	3.4785	3.4516
692	0.7458	0.8720	0.9510	0.9759	1.1320	1.5147	2.0253	2.5108	2.8631	3.0482	3.0909
693	0.7206	0.8244	0.9034	0.9594	1.1149	1.4298	1.8287	2.2112	2.5020	2.6753	2.7497
694	0.3860	0.6325	0.5238	0.3792	0.6659	1.3881	2.1694	2.6733	2.7971	2.6037	2.2322
695	0.5918	0.9472	0.8628	0.6716	1.0363	2.0304	3.1753	3.9929	4.3010	4.1534	3.7186
696	0.7017	1.0621	1.0512	0.8737	1.1949	2.1742	3.3826	4.3316	4.7950	4.7862	4.4439
697	0.7784	1.0934	1.1405	1.0067	1.2712	2.1401	3.2725	4.2294	4.7758	4.8916	4.6744
698	0.8731	1.0839	1.1813	1.1427	1.3402	1.9716	2.8426	3.6527	4.2061	4.4498	4.4282

hull	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
699	0.8909	1.0432	1.1504	1.1822	1.3637	1.8350	2.4772	3.1030	3.5729	3.8340	3.9104
700	0.8511	0.9786	1.0887	1.1680	1.3559	1.7360	2.2229	2.7019	3.0803	3.3168	3.4348

## B.3 High Froude Numbers

Value given is  $1000 \times C_w$ .

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
1	0.1069	0.1607	0.2322	0.3140	0.4825	0.6280	0.7347	0.8176	0.8281	0.7652	0.6846
2	0.1569	0.2235	0.3166	0.4229	0.6459	0.8449	0.9859	1.1199	1.1564	1.0889	0.9944
3	0.1820	0.2517	0.3503	0.4636	0.7050	0.9249	1.0825	1.2443	1.3028	1.2372	1.1422
4	0.1931	0.2629	0.3594	0.4726	0.7154	0.9403	1.1039	1.2765	1.3483	1.2950	1.1972
5	0.1940	0.2584	0.3468	0.4514	0.6774	0.8908	1.0507	1.2227	1.3089	1.2722	1.1949
6	0.1838	0.2419	0.3210	0.4142	0.6179	0.8110	0.9653	1.1217	1.2099	1.1815	1.1256
7	0.1708	0.2235	0.2943	0.3775	0.5592	0.7329	0.8624	1.0139	1.1037	1.0834	1.0350
8	0.2287	0.3518	0.5199	0.7157	1.1262	1.4841	1.7416	1.9525	1.9875	1.8238	1.6191
9	0.3473	0.5043	0.7222	0.9800	1.5331	2.0330	2.4022	2.7431	2.8509	2.6780	2.4224
10	0.4075	0.5700	0.7979	1.0705	1.6661	2.2183	2.6334	3.0488	3.2241	3.0723	2.8263
11	0.4336	0.5922	0.8149	1.0859	1.6758	2.2355	2.6639	3.1097	3.3342	3.2143	2.9895
12	0.4283	0.5721	0.7718	1.0147	1.5510	2.0691	2.5027	2.9231	3.1948	3.1358	2.9666
13	0.4018	0.5305	0.7065	0.9184	1.3894	1.8474	2.2131	2.6240	2.9034	2.8792	2.7644
14	0.3693	0.4862	0.6420	0.8278	1.2395	1.6402	1.9642	2.3311	2.6016	2.6023	2.5270
15	0.3462	0.5451	0.8192	1.1407	1.8198	2.4155	2.8543	3.2078	3.2616	2.9822	2.6275
16	0.5270	0.7845	1.1452	1.5754	2.5073	3.3549	3.9870	4.5719	4.7644	4.4675	4.0278
17	0.6197	0.8847	1.2601	1.7134	2.7164	3.6561	4.3868	5.0925	5.4209	5.1702	4.7392
18	0.6582	0.9145	1.2782	1.7237	2.7116	3.6612	4.4000	5.1808	5.6047	5.4233	5.0420
19	0.6534	0.8823	1.2021	1.5935	2.4749	3.3410	4.0479	4.8155	5.3351	5.2704	5.0155
20	0.6077	0.8112	1.0886	1.4251	2.1831	2.9335	3.5563	4.2595	4.7874	4.8094	4.6518
21	0.5551	0.7383	0.9809	1.2710	1.9230	2.5657	3.1067	3.7232	4.2293	4.2982	4.2157
22	0.4575	0.7356	1.1188	1.5750	2.5389	3.3873	4.0159	4.5176	4.5918	4.1937	3.6855
23	0.7008	1.0664	1.5817	2.1993	3.5438	4.7724	5.7057	6.5495	6.8355	6.3972	5.7436
24	0.8224	1.1972	1.7324	2.3869	3.8317	5.1992	6.2841	7.3169	7.8180	7.4605	6.8228
25	0.8702	1.2298	1.7440	2.3774	3.8005	5.1799	6.2807	7.4254	8.0907	7.8469	7.2879
26	0.8575	1.1729	1.6169	2.1637	3.4135	4.6568	5.6921	6.8326	7.6556	7.6134	7.2622
27	0.7927	1.0695	1.4482	1.9086	2.9649	4.0235	4.9503	5.9520	6.7879	6.8849	6.7035
28	0.7211	0.9685	1.2957	1.6874	2.5805	3.4707	4.2496	5.1284	5.9126	6.0913	6.0196
29	0.5510	0.9106	1.4121	2.0014	3.2606	4.3719	5.1815	5.8560	5.9547	5.4304	4.7411
30	0.8638	1.3436	2.0221	2.8382	4.6222	6.2582	7.5027	8.6318	9.0179	8.4318	7.5426
31	1.0084	1.4988	2.2025	3.0619	4.9888	6.8184	8.2658	9.6716	10.3697	9.8997	9.0291
32	1.0615	1.5278	2.1998	3.0279	4.9188	6.7612	8.2779	9.8041	10.7437	10.4395	9.6916
33	1.0387	1.4395	2.0083	2.7122	4.3468	5.9905	7.4032	8.9297	10.1040	10.1136	9.6688
34	0.9569	1.3034	1.7797	2.3631	3.7196	5.0968	6.3044	7.6721	8.8576	9.0694	8.8795
35	0.8701	1.1766	1.5830	2.0725	3.2007	4.3371	5.3489	6.5068	7.6057	7.9380	7.8986
36	0.1143	0.1778	0.2586	0.3481	0.5278	0.6794	0.7813	0.8684	0.8625	0.8011	0.7160
37	0.1616	0.2417	0.3491	0.4674	0.7088	0.9181	1.0607	1.1961	1.2239	1.1415	1.0406
38	0.1825	0.2673	0.3817	0.5092	0.7730	1.0066	1.1690	1.3316	1.3806	1.3054	1.2003
39	0.1898	0.2745	0.3884	0.5167	0.7838	1.0241	1.1946	1.3696	1.4347	1.3689	1.2694
40	0.1884	0.2665	0.3708	0.4901	0.7419	0.9729	1.1578	1.3215	1.4050	1.3548	1.2709
41	0.1775	0.2482	0.3412	0.4484	0.6765	0.8883	1.0442	1.2169	1.3056	1.2717	1.2008
42	0.1650	0.2285	0.3116	0.4075	0.6124	0.8040	0.9807	1.1088	1.1968	1.1718	1.1150
43	0.2465	0.3918	0.5810	0.7951	1.2313	1.6019	1.8637	2.0703	2.0804	1.8984	1.6744
44	0.3603	0.5513	0.8031	1.0915	1.6911	2.2167	2.5873	2.9305	3.0101	2.8074	2.5393
45	0.4101	0.6105	0.8775	1.1868	1.8411	2.4293	2.8839	3.2737	3.4230	3.2421	2.9642
46	0.4286	0.6249	0.8880	1.1951	1.8528	2.4557	2.9289	3.3585	3.5596	3.4117	3.1564
47	0.4172	0.5940	0.8311	1.1121	1.7157	2.2847	2.7356	3.1872	3.4501	3.3587	3.1619
48	0.3910	0.5478	0.7563	1.0026	1.5374	2.0467	2.4445	2.8842	3.1610	3.1174	2.9754
49	0.3616	0.5016	0.6854	0.9015	1.3717	1.8216	2.1771	2.5772	2.8606	2.8397	2.7377
50	0.3767	0.6102	0.9173	1.2676	1.9871	2.6015	3.0390	3.3816	3.3957	3.0884	2.7055
51	0.5621	0.8759	1.2930	1.7743	2.7831	3.6730	4.3212	4.8913	5.0277	4.6797	4.1923
52	0.6314	0.9600	1.4019	1.9180	3.0213	4.0222	4.7601	5.4826	5.7587	5.4395	4.9596
53	0.6579	0.9774	1.4096	1.9229	3.0233	4.0484	4.8296	5.6149	5.9965	5.7500	5.3076

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
54	0.6449	0.9279	1.3106	1.7629	2.7665	3.7224	4.4789	5.2856	5.7823	5.6642	5.3472
55	0.6022	0.8502	1.1811	1.5752	2.4443	3.2859	3.9717	4.7227	5.2526	5.2246	5.0158
56	0.5557	0.7752	1.0635	1.4031	2.1555	2.8856	3.4895	4.1632	4.6833	4.7173	4.5866
57	0.5022	0.8270	1.2569	1.7496	2.7673	3.6399	4.2583	4.7516	4.7722	4.3281	3.7735
58	0.7482	1.1936	1.7890	2.4788	3.9327	5.2191	6.1687	6.9873	7.1904	6.6756	5.9505
59	0.8510	1.3175	1.9484	2.6895	4.2845	5.7400	6.8308	7.8812	8.2950	7.8426	7.1190
60	0.8830	1.3337	1.9477	2.6753	4.2682	5.7580	6.9223	8.0686	8.6561	8.3156	7.6559
61	0.8606	1.2536	1.7888	2.4281	3.8555	5.2327	6.3429	7.5364	8.3222	8.1897	7.7376
62	0.8025	1.1416	1.5970	2.1408	3.3623	4.5582	5.5539	6.6550	7.4909	7.5121	7.2389
63	0.7402	1.0379	1.4287	1.8932	2.9327	3.9529	4.8342	5.7974	6.6053	6.7269	6.5817
64	0.6099	1.0275	1.5823	2.2221	3.5475	4.6866	5.4952	6.1445	6.1713	5.5854	4.8491
65	0.9364	1.5200	2.3028	3.2131	5.1381	6.8456	8.1360	9.1976	9.4733	8.7721	7.7950
66	1.0615	1.6729	2.5039	3.4838	5.6035	7.5468	9.0052	10.4207	10.9921	10.3860	9.4027
67	1.0965	1.6840	2.4888	3.4489	5.5634	7.5519	9.1385	10.6688	11.4929	11.0444	10.1593
68	1.0634	1.5672	2.2580	3.0892	4.9638	6.7896	8.2996	9.9000	11.0177	10.8789	10.2900
69	0.9919	1.4202	1.9990	2.6942	4.2734	5.8392	7.1736	8.6477	9.8292	9.9243	9.5997
70	0.9177	1.2896	1.7807	2.3652	3.6894	5.0062	6.1526	7.4407	8.5737	8.8128	8.6721
71	0.1263	0.1984	0.2871	0.3831	0.5707	0.7249	0.8261	0.9093	0.9048	0.8266	0.7406
72	0.1736	0.2679	0.3865	0.5134	0.7686	0.9833	1.1227	1.2548	1.2734	1.1878	1.0776
73	0.1917	0.2921	0.4199	0.5589	0.8384	1.0794	1.2598	1.3994	1.4367	1.3350	1.2404
74	0.1968	0.2966	0.4248	0.5654	0.8504	1.1004	1.2729	1.4453	1.4990	1.4234	1.3124
75	0.1918	0.2831	0.4026	0.5347	0.8065	1.0500	1.2407	1.4033	1.4786	1.4197	1.3284
76	0.1793	0.2626	0.3688	0.4884	0.7370	0.9627	1.1268	1.3019	1.3857	1.3419	1.2660
77	0.1662	0.2411	0.3363	0.4438	0.6682	0.8745	1.0242	1.1913	1.2762	1.2469	1.1817
78	0.2740	0.4395	0.6468	0.8755	1.3300	1.7063	1.9608	2.1614	2.1469	1.9508	1.7105
79	0.3926	0.6145	0.8969	1.2100	1.8420	2.3799	2.7508	3.0759	3.1222	2.8937	2.5969
80	0.4368	0.6738	0.9759	1.3154	2.0121	2.6200	3.0765	3.4536	3.5649	3.3547	3.0533
81	0.4489	0.6826	0.9828	1.3231	2.0300	2.6589	3.1151	3.5603	3.7309	3.5460	3.2646
82	0.4296	0.6410	0.9143	1.2300	1.8885	2.4934	2.9438	3.4142	3.6542	3.5335	3.3100
83	0.4012	0.5881	0.8293	1.1089	1.6980	2.2476	2.6816	3.1185	3.3849	3.3194	3.1299
84	0.3722	0.5380	0.7507	0.9961	1.5193	2.0097	2.3886	2.8103	3.0821	3.0506	2.9265
85	0.4216	0.6866	1.0222	1.3953	2.1429	2.7652	3.1910	3.5211	3.4996	3.1591	2.7540
86	0.6217	0.9884	1.4549	1.9771	3.0389	3.9481	4.5914	5.1334	5.2104	4.8083	4.2862
87	0.6850	1.0754	1.5773	2.1455	3.3219	4.3556	5.1076	5.7904	5.9948	5.6264	5.0948
88	0.7026	1.0858	1.5823	2.1490	3.3404	4.4105	5.2070	5.9703	6.2868	5.9797	5.4860
89	0.6786	1.0204	1.4651	1.9819	3.0799	4.0997	4.8853	5.6960	6.1535	5.9737	5.6009
90	0.6345	0.9323	1.3188	1.7680	2.7364	3.6504	4.3715	5.1508	5.6583	5.5851	5.3202
91	0.5901	0.8514	1.1880	1.5780	2.4230	3.2270	3.8724	4.5884	5.1115	5.0998	4.9240
92	0.5653	0.9323	1.4004	1.9241	2.9788	3.8602	4.4859	4.9330	4.8997	4.4122	3.8295
93	0.8407	1.3608	2.0257	2.7735	4.3021	5.6129	6.5438	7.3239	7.4367	6.8423	6.0677
94	0.9410	1.4967	2.2145	3.0311	4.7308	6.2311	7.3172	8.3275	8.6344	8.0905	7.2967
95	0.9636	1.5072	2.2145	3.0277	4.7476	6.3029	7.4868	8.5930	9.0772	8.6374	7.8984
96	0.9300	1.4079	2.0343	2.7619	4.3371	5.8107	6.9625	8.1603	8.8713	8.6420	8.0959
97	0.8718	1.2828	1.8190	2.4459	3.8144	5.1195	6.1676	7.3144	8.1119	8.0479	7.6842
98	0.8145	1.1714	1.6326	2.1691	3.3471	4.4804	5.4286	6.4530	7.2548	7.3033	7.0822
99	0.7042	1.1743	1.7769	2.4535	3.8243	4.9734	5.7681	6.3755	6.3335	5.6977	4.9218
100	1.0674	1.7480	2.6210	3.6057	5.6261	7.3625	8.6307	9.6310	9.7755	8.9745	7.9287
101	1.1973	1.9267	2.8724	3.9522	6.2099	8.2101	9.7099	11.0124	11.4285	10.6967	9.6158
102	1.2245	1.9359	2.8664	3.9399	6.2233	8.2975	9.8997	11.3819	12.0495	11.4603	10.4606
103	1.1819	1.7993	2.6132	3.5642	5.6396	7.5971	9.1783	10.7625	11.7616	11.4858	10.7591
104	1.1135	1.6371	2.3246	3.1311	4.9124	6.6278	8.0544	9.5684	10.6913	10.6550	10.1938
105	1.0469	1.4970	2.0806	2.7630	4.2749	5.7448	6.9920	8.3654	9.4773	9.6116	9.3545
106	0.1390	0.2177	0.3116	0.4116	0.6037	0.7588	0.8674	0.9381	0.9259	0.8431	0.7473
107	0.1893	0.2927	0.4192	0.5531	0.8141	1.0299	1.1682	1.2973	1.3051	1.2121	1.0906
108	0.2068	0.3174	0.4542	0.5988	0.8880	1.1311	1.2903	1.4450	1.4709	1.3806	1.2604
109	0.2106	0.3206	0.4588	0.6068	0.9017	1.1550	1.3465	1.4936	1.5379	1.4551	1.3392
110	0.2028	0.3053	0.4342	0.5746	0.8579	1.1069	1.2801	1.4594	1.5292	1.4584	1.3587
111	0.1891	0.2822	0.3981	0.5260	0.7869	1.0197	1.1848	1.3600	1.4374	1.3899	1.3071
112	0.1759	0.2597	0.3634	0.4788	0.7165	0.9301	1.0849	1.2517	1.3349	1.2989	1.2315
113	0.3026	0.4825	0.7022	0.9404	1.4052	1.7829	2.0373	2.2227	2.1983	1.9765	1.7310
114	0.4321	0.6767	0.9787	1.3072	1.9561	2.4971	2.8604	3.1762	3.1917	2.9433	2.6313
115	0.4772	0.7403	1.0659	1.4244	2.1439	2.7581	3.1843	3.5684	3.6535	3.4175	3.0994
116	0.4875	0.7486	1.0744	1.4356	2.1702	2.8088	3.2616	3.6932	3.8302	3.6480	3.3223
117	0.4709	0.7090	1.0081	1.3438	2.0390	2.6602	3.1116	3.5783	3.7932	3.6491	3.4100
118	0.4355	0.6463	0.9120	1.2140	1.8393	2.4101	2.8534	3.2913	3.5431	3.4490	3.2674

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
119	0.4065	0.5936	0.8286	1.0955	1.6541	2.1692	2.5579	2.9870	3.2508	3.2036	3.0639
120	0.4667	0.7544	1.1092	1.4971	2.2607	2.8841	3.3095	3.6140	3.5603	3.1991	2.7795
121	0.6910	1.0951	1.5942	2.1420	3.2321	4.1446	4.7724	5.2888	5.3069	4.8770	4.3308
122	0.7604	1.1956	1.7377	2.3386	3.5543	4.5977	5.3323	5.9909	6.1377	5.7235	5.1627
123	0.7781	1.2088	1.7495	2.3535	3.5936	4.6811	5.4685	6.2088	6.4688	6.1101	5.5826
124	0.7531	1.1407	1.6310	2.1856	3.3473	4.3989	5.1855	5.9860	6.3952	6.1732	5.7551
125	0.7100	1.0488	1.4779	1.9678	3.0004	3.9547	4.6954	5.4725	5.9512	5.8373	5.5319
126	0.6684	0.9655	1.3400	1.7661	2.6766	3.5252	4.1943	4.9249	5.4229	5.3818	5.1696
127	0.6270	1.0244	1.5184	2.0621	3.1376	4.0191	4.6217	5.0540	4.9797	4.4601	3.8568
128	0.9449	1.5178	2.2289	3.0128	4.5797	5.8935	6.8026	7.5402	7.5741	6.9277	6.1181
129	1.0602	1.6806	2.4568	3.3210	5.0768	6.5884	7.6762	8.6162	8.8316	8.2214	7.3834
130	1.0877	1.7007	2.4735	3.3416	5.1330	6.7128	7.8708	8.9473	9.3360	8.8191	8.0303
131	1.0586	1.6042	2.2983	3.0875	4.7538	6.2741	7.4464	8.6083	9.2375	8.9327	8.3204
132	1.0065	1.4771	2.0762	2.7628	4.2295	5.5968	6.6635	7.8163	8.5599	8.4230	7.9932
133	0.9557	1.3641	1.8806	2.4725	3.7468	4.9488	5.9306	6.9775	7.7523	7.7384	7.4532
134	0.7824	1.2899	1.9246	2.6262	4.0219	5.1701	5.9502	6.5217	6.4227	5.7459	4.9480
135	1.2100	1.9589	2.8920	3.9229	5.9925	7.7300	8.9274	9.9049	9.9466	9.0754	7.9814
136	1.3682	2.1833	3.2068	4.3490	6.6810	8.6931	10.1508	11.3973	11.6841	10.8597	9.7134
137	1.4082	2.2124	3.2305	4.3779	6.7575	8.8645	10.4372	11.8565	12.3890	11.6956	10.6217
138	1.3796	2.0882	2.9939	4.0284	6.2274	8.2477	9.8256	11.3848	12.2646	11.8778	11.0523
139	1.3248	1.9285	2.7001	3.5876	5.5042	7.3039	8.7300	10.2749	11.3173	11.1728	10.6072
140	1.2706	1.7885	2.4466	3.2034	4.8470	6.4124	7.6810	9.1036	10.1767	10.2065	9.8493
141	0.1198	0.1465	0.1932	0.2546	0.3959	0.5305	0.6304	0.7257	0.7539	0.7062	0.6373
142	0.1915	0.2250	0.2838	0.3618	0.5443	0.7231	0.8580	0.9965	1.0503	0.9993	0.9173
143	0.2313	0.2672	0.3290	0.4115	0.6064	0.8013	0.9497	1.1091	1.1824	1.1335	1.0522
144	0.2506	0.2872	0.3489	0.4309	0.6254	0.8220	0.9739	1.1394	1.2255	1.1830	1.1063
145	0.2570	0.2929	0.3505	0.4264	0.6073	0.7909	0.9335	1.0982	1.1908	1.1619	1.0999
146	0.2457	0.2802	0.3329	0.4013	0.5637	0.7297	0.8669	1.0103	1.1082	1.0856	1.0368
147	0.2298	0.2627	0.3110	0.3725	0.5177	0.6667	0.7827	0.9230	1.0129	1.0059	0.9603
148	0.2562	0.3149	0.4242	0.5727	0.9181	1.2511	1.5108	1.7460	1.8205	1.6970	1.5155
149	0.4327	0.5066	0.6443	0.8351	1.2861	1.7366	2.0887	2.4452	2.6023	2.4721	2.2586
150	0.5304	0.6091	0.7528	0.9520	1.4306	1.9193	2.3100	2.7210	2.9372	2.8243	2.6156
151	0.5798	0.6598	0.8020	0.9981	1.4710	1.9605	2.3549	2.7882	3.0456	2.9573	2.7623
152	0.5917	0.6730	0.8057	0.9841	1.4134	1.8626	2.2321	2.6494	2.9388	2.8938	2.7684
153	0.5596	0.6395	0.7614	0.9203	1.2996	1.6964	2.0444	2.4048	2.6918	2.6758	2.5820
154	0.5173	0.5953	0.7078	0.8506	1.1848	1.5327	1.8191	2.1610	2.4298	2.4360	2.3741
155	0.3817	0.4773	0.6566	0.9008	1.4782	2.0364	2.4700	2.8711	3.0002	2.7875	2.4791
156	0.6612	0.7805	1.0088	1.3278	2.0923	2.8609	3.4763	4.0848	4.3668	4.1465	3.7634
157	0.8282	0.9555	1.1926	1.5248	2.3338	3.1689	3.8416	4.5663	4.9630	4.7915	4.4290
158	0.9072	1.0387	1.2722	1.5961	2.3910	3.2248	3.9252	4.6685	5.1502	5.0261	4.7002
159	0.9223	1.0568	1.2733	1.5617	2.2748	3.0294	3.6904	4.3937	4.9455	4.9089	4.6893
160	0.8644	0.9977	1.1964	1.4532	2.0728	2.7277	3.2891	3.9396	4.4813	4.5044	4.3783
161	0.7941	0.9239	1.1070	1.3366	1.8761	2.4397	2.9305	3.4984	3.9970	4.0630	4.0062
162	0.4877	0.6226	0.8768	1.2247	2.0495	2.8488	3.4605	4.0518	4.2421	3.9354	3.4799
163	0.8748	1.0460	1.3740	1.8330	2.9449	4.0661	4.9919	5.8652	6.2867	5.9632	5.3944
164	1.1080	1.2881	1.6275	2.1007	3.2818	4.5041	5.5140	6.5746	7.1914	6.9432	6.3915
165	1.2167	1.4019	1.7336	2.1934	3.3480	4.5662	5.5993	6.7142	7.4643	7.3078	6.8252
166	1.2297	1.4192	1.7245	2.1327	3.1521	4.2414	5.1904	6.2666	7.1335	7.1317	6.8292
167	1.1433	1.3316	1.6092	1.9695	2.8428	3.7735	4.5918	5.5513	6.3979	6.5024	6.3550
168	1.0451	1.2289	1.4832	1.8012	2.5532	3.3417	4.0479	4.8705	5.6373	5.8099	5.7763
169	0.5871	0.7637	1.0964	1.5522	2.6369	3.6885	4.5162	5.2762	5.5309	5.1245	4.5156
170	1.0705	1.2968	1.7317	2.3371	3.8274	5.3282	6.5501	7.7478	8.3250	7.8933	7.1159
171	1.3641	1.6008	2.0495	2.6786	4.2586	5.9033	7.2871	8.7130	9.5744	9.2521	8.4938
172	1.4982	1.7400	2.1768	2.7853	4.3250	5.9613	7.3403	8.8877	9.9545	9.7669	9.1162
173	1.5052	1.7509	2.1471	2.6808	4.0221	5.4717	6.7591	8.2256	9.4596	9.5273	9.1401
174	1.3907	1.6334	1.9902	2.4521	3.5881	4.8063	5.9185	7.2013	8.3973	8.6131	8.4737
175	1.2681	1.5045	1.8288	2.2336	3.1994	4.2137	5.1341	6.2339	7.3048	7.6166	7.6397
176	0.1108	0.1463	0.2027	0.2725	0.4272	0.5700	0.6724	0.7708	0.7931	0.7383	0.6645
177	0.1720	0.2178	0.2895	0.3798	0.5826	0.7747	0.9148	1.0566	1.1093	1.0489	0.9590
178	0.2051	0.2537	0.3297	0.4258	0.6442	0.8551	1.0120	1.1761	1.2469	1.1910	1.1016
179	0.2212	0.2700	0.3457	0.4417	0.6610	0.8754	1.0369	1.2127	1.2946	1.2463	1.1609
180	0.2273	0.2739	0.3441	0.4332	0.6379	0.8410	0.9974	1.1705	1.2656	1.2323	1.1573
181	0.2188	0.2622	0.3261	0.4063	0.5913	0.7761	0.9181	1.0818	1.1787	1.1569	1.0940
182	0.2061	0.2466	0.3044	0.3767	0.5432	0.7094	0.8554	0.9883	1.0852	1.0685	1.0234
183	0.2370	0.3170	0.4478	0.6164	0.9934	1.3453	1.6074	1.8464	1.9090	1.7668	1.5737

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
184	0.3875	0.4902	0.6599	0.8815	1.3847	1.8691	2.2374	2.6001	2.7403	2.5932	2.3512
185	0.4717	0.5808	0.7592	0.9942	1.5333	2.0647	2.4976	2.9044	3.1033	2.9749	2.7399
186	0.5099	0.6180	0.7940	1.0260	1.5630	2.1007	2.5452	2.9779	3.2259	3.1211	2.8984
187	0.5241	0.6280	0.7897	1.0008	1.4922	1.9925	2.4072	2.8423	3.1332	3.0724	2.9106
188	0.5010	0.5992	0.7453	0.9328	1.3682	1.8144	2.2006	2.5877	2.8877	2.8633	2.7442
189	0.4685	0.5614	0.6941	0.8607	1.2460	1.6401	1.9839	2.3352	2.6214	2.6216	2.5367
190	0.3552	0.4835	0.6971	0.9722	1.6009	2.1881	2.6324	3.0309	3.1375	2.8946	2.5590
191	0.5902	0.7583	1.0403	1.4115	2.2633	3.0882	3.7240	4.3470	4.6004	4.3288	3.9120
192	0.7298	0.9057	1.2012	1.5934	2.5079	3.4164	4.1708	4.8741	5.2495	5.0265	4.6030
193	0.7964	0.9737	1.2638	1.6488	2.5548	3.4726	4.2272	5.0004	5.4634	5.2965	4.9212
194	0.8179	0.9882	1.2522	1.5972	2.4156	3.2600	3.9532	4.7374	5.2880	5.2224	4.9522
195	0.7789	0.9397	1.1770	1.4800	2.1952	2.9365	3.5857	4.2694	4.8242	4.8333	4.6589
196	0.7262	0.8781	1.0929	1.3610	1.9855	2.6297	3.1821	3.8093	4.3505	4.3877	4.2854
197	0.4629	0.6430	0.9441	1.3361	2.2283	3.0659	3.7251	4.2727	4.4275	4.0760	3.5892
198	0.7819	1.0222	1.4268	1.9611	3.1967	4.3971	5.3366	6.2367	6.6089	6.2133	5.5854
199	0.9732	1.2255	1.6495	2.2094	3.5445	4.8745	5.9350	7.0258	7.5941	7.2679	6.6502
200	1.0667	1.3177	1.7320	2.2791	3.5987	4.9404	6.0528	7.2055	7.9204	7.6938	7.1345
201	1.0942	1.3328	1.7059	2.1916	3.3691	4.5933	5.6253	6.7800	7.6468	7.5860	7.2014
202	1.0376	1.2632	1.5955	2.0195	3.0339	4.0936	5.0112	6.0560	6.9310	6.9900	6.7651
203	0.9655	1.1788	1.4776	1.8500	2.7243	3.6323	4.4271	5.3452	6.1666	6.2994	6.1943
204	0.5492	0.7827	1.1742	1.6828	2.8538	3.9533	4.7965	5.5448	5.7472	5.2836	4.6323
205	0.9574	1.2753	1.8108	2.5143	4.1644	5.7692	7.0137	8.2337	8.7291	8.2070	7.3521
206	1.1986	1.5301	2.0915	2.8362	4.6209	6.4062	7.8454	9.3153	10.0968	9.6735	8.8194
207	1.3136	1.6426	2.1885	2.9133	4.6758	6.4771	7.9828	9.5531	10.5575	10.2695	9.5029
208	1.3437	1.6536	2.1394	2.7763	4.3325	5.9652	7.3829	8.9379	10.1652	10.1375	9.6302
209	1.2723	1.5624	1.9901	2.5380	3.8627	5.2592	6.5120	7.9043	9.1452	9.2917	9.0214
210	1.1847	1.4583	1.8399	2.3147	3.4448	4.6232	5.6602	6.9047	8.0500	8.2979	8.2151
211	0.1052	0.1499	0.2148	0.2922	0.4577	0.6059	0.7104	0.8066	0.8237	0.7650	0.6841
212	0.1573	0.2139	0.2992	0.4005	0.6195	0.8205	0.9626	1.1054	1.1492	1.0797	0.9849
213	0.1831	0.2439	0.3344	0.4428	0.6801	0.9018	1.0629	1.2278	1.2905	1.2296	1.1299
214	0.1954	0.2574	0.3466	0.4551	0.6947	0.9213	1.1042	1.2647	1.3409	1.2868	1.1954
215	0.1997	0.2579	0.3410	0.4423	0.6678	0.8853	1.0492	1.2269	1.3194	1.2785	1.2027
216	0.1929	0.2462	0.3213	0.4129	0.6184	0.8179	0.9861	1.1396	1.2375	1.2045	1.1580
217	0.1828	0.2318	0.2997	0.3823	0.5677	0.7492	0.8867	1.0472	1.1420	1.1223	1.0723
218	0.2260	0.3269	0.4781	0.6646	1.0666	1.4312	1.6966	1.9295	1.9722	1.8170	1.6115
219	0.3531	0.4852	0.6872	0.9368	1.4814	1.9883	2.3608	2.7225	2.8373	2.6663	2.4101
220	0.4199	0.5612	0.7739	1.0412	1.6305	2.1916	2.6396	3.0415	3.2236	3.0689	2.8088
221	0.4475	0.5877	0.7982	1.0633	1.6552	2.2276	2.6909	3.1248	3.3523	3.2266	2.9917
222	0.4584	0.5899	0.7826	1.0255	1.5728	2.1140	2.5529	3.0013	3.2850	3.2058	3.0201
223	0.4421	0.5628	0.7357	0.9516	1.4404	1.9282	2.3435	2.7517	3.0528	3.0107	2.8738
224	0.4182	0.5294	0.6850	0.8769	1.3113	1.7460	2.0919	2.4943	2.7841	2.7761	2.6757
225	0.3407	0.5026	0.7483	1.0517	1.7198	2.3259	2.7702	3.1612	3.2349	2.9695	2.6114
226	0.5395	0.7593	1.0929	1.5072	2.4325	3.2943	3.9401	4.5517	4.7557	4.4496	3.9991
227	0.6481	0.8809	1.2351	1.6830	2.6842	3.6443	4.3876	5.1170	5.4456	5.1828	4.7257
228	0.7005	0.9307	1.2799	1.7233	2.7251	3.7039	4.4717	5.2670	5.6892	5.4809	5.0646
229	0.7191	0.9331	1.2494	1.6492	2.5665	3.4838	4.2600	5.0302	5.5637	5.4573	5.1449
230	0.6929	0.8888	1.1703	1.5214	2.3300	3.1465	3.8126	4.5690	5.1314	5.1002	4.8850
231	0.6554	0.8364	1.0881	1.3980	2.1073	2.8243	3.4171	4.1040	4.6364	4.6734	4.5376
232	0.4477	0.6728	1.0175	1.4471	2.3934	3.2562	3.8997	4.4486	4.5570	4.1705	3.6583
233	0.7198	1.0329	1.5117	2.1082	3.4475	4.6985	5.6471	6.5290	6.8238	6.3756	5.6987
234	0.8690	1.2019	1.7113	2.3569	3.8140	5.2160	6.2971	7.3825	7.8740	7.4826	6.7987
235	0.9409	1.2690	1.7696	2.4017	3.8635	5.2953	6.4704	7.6048	8.2531	7.9538	7.3295
236	0.9653	1.2686	1.7178	2.2837	3.6086	4.9425	6.0351	7.2316	8.0604	7.9410	7.4811
237	0.9314	1.2073	1.6029	2.0958	3.2497	4.4234	5.4116	6.5171	7.3932	7.3994	7.1166
238	0.8841	1.1374	1.4893	1.9206	2.9203	3.9378	4.8084	5.8000	6.6483	6.7316	6.5620
239	0.5339	0.8237	1.2699	1.8261	3.0629	4.1913	5.0358	5.7635	5.9020	5.3921	4.6976
240	0.8887	1.3010	1.9333	2.7241	4.5049	6.1704	7.4443	8.6101	9.0068	8.3969	7.4789
241	1.0750	1.5148	2.1903	3.0423	4.9963	6.8745	8.3517	9.7915	10.4641	9.9234	8.9984
242	1.1633	1.5965	2.2584	3.0987	5.0508	6.9723	8.5189	10.0972	10.9992	10.6036	9.7476
243	1.1954	1.5897	2.1782	2.9225	4.6801	6.4626	7.9667	9.5638	10.7318	10.6068	9.9860
244	1.1561	1.5126	2.0251	2.6668	4.1793	5.7325	7.0451	8.5570	9.7896	9.8504	9.4678
245	1.1029	1.4288	1.8803	2.4360	3.7326	5.0624	6.2253	7.5539	8.7309	8.9053	8.7178
246	0.1042	0.1554	0.2269	0.3096	0.4822	0.6334	0.7371	0.8321	0.8453	0.7792	0.6969
247	0.1507	0.2175	0.3106	0.4191	0.6482	0.8536	1.0113	1.1367	1.1724	1.1010	1.0019
248	0.1726	0.2421	0.3428	0.4595	0.7081	0.9353	1.0961	1.2601	1.3152	1.2500	1.1507



hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
249	0.1813	0.2518	0.3520	0.4692	0.7212	0.9544	1.1396	1.2978	1.3678	1.3085	1.2143
250	0.1834	0.2505	0.3428	0.4527	0.6916	0.9172	1.1001	1.2624	1.3498	1.3061	1.2260
251	0.1766	0.2380	0.3217	0.4216	0.6402	0.8490	1.0205	1.1781	1.2721	1.2399	1.1740
252	0.1677	0.2238	0.2996	0.3900	0.5883	0.7790	0.9225	1.0851	1.1812	1.1578	1.1010
253	0.2245	0.3415	0.5075	0.7063	1.1250	1.4960	1.7586	1.9897	2.0178	1.8511	1.6323
254	0.3396	0.4952	0.7188	0.9847	1.5575	2.0751	2.4720	2.8015	2.8967	2.7117	2.4409
255	0.3945	0.5612	0.7989	1.0847	1.7088	2.2851	2.7370	3.1307	3.2920	3.0857	2.8521
256	0.4144	0.5800	0.8163	1.1051	1.7315	2.3223	2.7936	3.2212	3.4310	3.2865	3.0364
257	0.4211	0.5746	0.7918	1.0579	1.6432	2.2079	2.6719	3.1113	3.3783	3.2877	3.0906
258	0.4065	0.5465	0.7412	0.9791	1.5054	2.0194	2.4524	2.8673	3.1583	3.1095	2.9582
259	0.3871	0.5152	0.6900	0.9014	1.3719	1.8336	2.1975	2.6125	2.9054	2.8854	2.7746
260	0.3405	0.5276	0.7969	1.1198	1.8137	2.4294	2.8729	3.2504	3.2994	3.0151	2.6401
261	0.5218	0.7805	1.1518	1.5977	2.5660	3.4447	4.0939	4.6839	4.8492	4.5166	4.0516
262	0.6123	0.8885	1.2860	1.7678	2.8284	3.8134	4.5714	5.2746	5.5580	5.2560	4.7778
263	0.6514	0.9265	1.3207	1.8004	2.8693	3.8811	4.6554	5.4440	5.8254	5.5823	5.1382
264	0.6644	0.9175	1.2764	1.7176	2.7032	3.6643	4.4207	5.2356	5.7373	5.6052	5.2618
265	0.6442	0.8737	1.1923	1.5819	2.4581	3.3232	4.0238	4.7907	5.3404	5.2867	5.0413
266	0.6169	0.8259	1.1103	1.4534	2.2271	2.9943	3.6293	4.3290	4.8768	4.8804	4.7147
267	0.4500	0.7093	1.0858	1.5418	2.5224	3.3972	4.0350	4.5704	4.6376	4.2219	3.6812
268	0.7017	1.0709	1.6029	2.2449	3.6449	4.9186	5.8589	6.7164	6.9560	6.4518	5.7453
269	0.8283	1.2241	1.7972	2.4941	4.0358	5.4729	6.5669	7.6155	8.0377	7.5913	6.8701
270	0.8831	1.2763	1.8438	2.5374	4.0922	5.5714	6.7247	7.8744	8.4557	8.1047	7.4273
271	0.9024	1.2627	1.7751	2.4031	3.8315	5.2315	6.3499	7.5527	8.3340	8.1539	7.6478
272	0.8802	1.2041	1.6549	2.2044	3.4604	4.7094	5.7301	6.8697	7.7226	7.6810	7.3291
273	0.8497	1.1433	1.5420	2.0232	3.1194	4.2134	5.1349	6.1640	7.0114	7.0616	6.8355
274	0.5393	0.8717	1.3576	1.9469	3.2264	4.3678	5.2274	5.9025	5.9982	5.4463	4.7314
275	0.8745	1.3596	2.0615	2.9118	4.7702	6.4630	7.7149	8.8552	9.1640	8.4909	7.5288
276	1.0337	1.5584	2.3191	3.2465	5.3057	7.2296	8.7166	10.1044	10.6758	10.0711	9.0792
277	1.1045	1.6237	2.3765	3.2997	5.3771	7.3612	8.9242	10.4682	11.2686	10.7943	9.8669
278	1.1319	1.6040	2.2787	3.1066	5.0066	6.8784	8.4251	10.0184	11.1116	10.9062	10.2095
279	1.1133	1.5341	2.1204	2.8388	4.4926	6.1480	7.5361	9.0609	10.2554	10.2431	9.7755
280	1.0852	1.4641	1.9788	2.6023	4.0298	5.4649	6.6713	8.0749	9.2514	9.3680	9.0837
281	0.1881	0.1878	0.2084	0.2472	0.3564	0.4738	0.5679	0.6642	0.7080	0.6642	0.6022
282	0.3011	0.3018	0.3274	0.3756	0.5134	0.6652	0.7893	0.9213	0.9846	0.9427	0.8695
283	0.3604	0.3637	0.3916	0.4427	0.5884	0.7514	0.8942	1.0346	1.1135	1.0712	0.9962
284	0.3865	0.3928	0.4219	0.4732	0.6186	0.7815	0.9254	1.0698	1.1584	1.1226	1.0497
285	0.3892	0.3992	0.4305	0.4792	0.6143	0.7670	0.8914	1.0408	1.1373	1.1060	1.0480
286	0.3685	0.3808	0.4115	0.4566	0.5791	0.7173	0.8415	0.9662	1.0605	1.0384	0.9925
287	0.3413	0.3558	0.3858	0.4276	0.5381	0.6625	0.7683	0.8848	0.9771	0.9588	0.9257
288	0.4081	0.4027	0.4495	0.5437	0.8116	1.1057	1.3473	1.5931	1.6992	1.6002	1.4426
289	0.6956	0.6898	0.7467	0.8635	1.2020	1.5875	1.9236	2.2582	2.4375	2.3331	2.1393
290	0.8476	0.8493	0.9121	1.0342	1.3892	1.8007	2.1638	2.5390	2.7712	2.6774	2.4859
291	0.9139	0.9261	0.9937	1.1166	1.4671	1.8768	2.2290	2.6262	2.8896	2.8148	2.6408
292	0.9134	0.9431	1.0175	1.1354	1.4581	1.8351	2.1635	2.5330	2.8228	2.7795	2.6449
293	0.8494	0.8913	0.9679	1.0785	1.3689	1.7044	2.0057	2.3274	2.6152	2.5961	2.4987
294	0.7763	0.8263	0.9022	1.0062	1.2666	1.5632	1.8222	2.1141	2.3849	2.3800	2.3210
295	0.6151	0.6054	0.6829	0.8382	1.2916	1.7889	2.2002	2.6230	2.8095	2.6408	2.3662
296	1.0855	1.0733	1.1669	1.3623	1.9395	2.6016	3.1644	3.7703	4.0994	3.9291	3.5834
297	1.3511	1.3530	1.4560	1.6566	2.2624	2.9694	3.5824	4.2652	4.6960	4.5532	4.2140
298	1.4624	1.4841	1.5972	1.7982	2.3940	3.0956	3.7190	4.4111	4.9075	4.8069	4.4944
299	1.4503	1.5093	1.6357	1.8298	2.3740	3.0128	3.5827	4.2331	4.7845	4.7541	4.5333
300	1.3320	1.4137	1.5465	1.7327	2.2172	2.7777	3.2776	3.8569	4.3966	4.4159	4.2765
301	1.2063	1.2987	1.4332	1.6093	2.0425	2.5295	2.9659	3.4742	3.9678	4.0271	3.9537
302	0.7977	0.7855	0.8970	1.1206	1.7751	2.4932	3.1076	3.7034	3.9792	3.7443	3.3352
303	1.4641	1.4454	1.5806	1.8607	2.7102	3.6831	4.5286	5.4149	5.9128	5.6622	5.1505
304	1.8452	1.8462	1.9953	2.2866	3.1728	4.2149	5.1401	6.1499	6.8151	6.6225	6.1148
305	2.0018	2.0359	2.1987	2.4897	3.3581	4.3910	5.3158	6.3610	7.1409	7.0199	6.5617
306	1.9670	2.0585	2.2449	2.5268	3.3142	4.2451	5.1066	6.0712	6.9414	6.9506	6.6392
307	1.7848	1.9098	2.1063	2.3746	3.0741	3.8786	4.6135	5.4815	6.3236	6.4212	6.2531
308	1.6041	1.7440	1.9407	2.1935	2.8144	3.5045	4.1373	4.8884	5.6470	5.7979	5.7509
309	0.9686	0.9546	1.1039	1.4009	2.2692	3.2216	4.0235	4.8313	5.1977	4.8858	4.3405
310	1.8224	1.7967	1.9797	2.3562	3.5005	4.8130	5.9608	7.1615	7.8476	7.5185	6.8141
311	2.3195	2.3210	2.5195	2.9099	4.1054	5.5167	6.7724	8.1653	9.0961	8.8589	8.1631
312	2.5176	2.5655	2.7823	3.1719	4.3404	5.7381	7.0214	8.4471	9.5496	9.4278	8.8023
313	2.4481	2.5742	2.8248	3.2015	4.2526	5.5025	6.6580	8.0141	9.2561	9.3413	8.9408

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
314	2.1936	2.3645	2.6275	2.9836	3.9101	4.9757	5.9655	7.1602	8.3519	8.5635	8.3902
315	1.9568	2.1474	2.4086	2.7413	3.5569	4.4549	5.3008	6.3067	7.3598	7.6423	7.6519
316	0.1674	0.1747	0.2037	0.2509	0.3741	0.5021	0.6013	0.7033	0.7382	0.6917	0.6277
317	0.2656	0.2756	0.3122	0.3720	0.5295	0.6972	0.8391	0.9695	1.0317	0.9865	0.9019
318	0.3174	0.3298	0.3691	0.4324	0.6008	0.7810	0.9362	1.0840	1.1627	1.1173	1.0363
319	0.3411	0.3556	0.3957	0.4591	0.6274	0.8093	0.9664	1.1207	1.2113	1.1711	1.0933
320	0.3466	0.3644	0.4036	0.4631	0.6204	0.7911	0.9313	1.0893	1.1890	1.1600	1.0957
321	0.3301	0.3489	0.3872	0.4418	0.5841	0.7394	0.8767	1.0132	1.1147	1.0918	1.0410
322	0.3082	0.3281	0.3645	0.4145	0.5432	0.6835	0.8027	0.9313	1.0302	1.0148	0.9725
323	0.3614	0.3732	0.4400	0.5547	0.8571	1.1751	1.4284	1.6819	1.7806	1.6694	1.4965
324	0.6101	0.6255	0.7093	0.8549	1.2464	1.6712	2.0317	2.3806	2.5559	2.4360	2.2234
325	0.7433	0.7649	0.8546	1.0079	1.4212	1.8791	2.2624	2.6714	2.9011	2.7954	2.5837
326	0.8058	0.8347	0.9267	1.0792	1.4887	1.9478	2.3354	2.7608	3.0274	2.9409	2.7509
327	0.8161	0.8582	0.9509	1.0936	1.4705	1.8956	2.2625	2.6651	2.9670	2.9218	2.7605
328	0.7685	0.8193	0.9102	1.0411	1.3791	1.7598	2.0888	2.4547	2.7578	2.7353	2.6093
329	0.7095	0.7654	0.8538	0.9745	1.2772	1.6149	1.9165	2.2343	2.5223	2.5211	2.4421
330	0.5422	0.5606	0.6704	0.8605	1.3677	1.9035	2.3362	2.7648	2.9412	2.7431	2.4468
331	0.9447	0.9678	1.1069	1.3514	2.0190	2.7478	3.3521	3.9816	4.2926	4.0919	3.7150
332	1.1785	1.2119	1.3597	1.6122	2.3203	3.1082	3.7854	4.4943	4.9167	4.7477	4.3717
333	1.2851	1.3318	1.4850	1.7349	2.4338	3.2214	3.8945	4.6452	5.1432	5.0188	4.6741
334	1.2970	1.3709	1.5271	1.7608	2.3963	3.1200	3.7568	4.4660	5.0357	4.9962	4.7328
335	1.2096	1.3016	1.4565	1.6740	2.2367	2.8761	3.4538	4.0897	4.6520	4.6670	4.4864
336	1.1088	1.2092	1.3612	1.5628	2.0630	2.6217	3.1131	3.6849	4.2222	4.2689	4.1614
337	0.6995	0.7257	0.8825	1.1512	1.8833	2.6542	3.2842	3.8980	4.1490	3.8768	3.4371
338	1.2662	1.2985	1.4996	1.8504	2.8306	3.8993	4.7934	5.7164	6.1878	5.8903	5.3278
339	1.6006	1.6473	1.8603	2.2267	3.2638	4.4244	5.4411	6.4852	7.1354	6.8923	6.3314
340	1.7524	1.8204	2.0401	2.4019	3.4214	4.5811	5.6003	6.7070	7.4815	7.3214	6.8041
341	1.7602	1.8700	2.0965	2.4332	3.3537	4.4110	5.3551	6.4214	7.3305	7.2938	6.9204
342	1.6277	1.7643	1.9897	2.3009	3.1114	4.0333	4.8618	5.8229	6.7163	6.7908	6.5549
343	1.4822	1.6314	1.8519	2.1397	2.8558	3.6505	4.3697	5.2137	6.0406	6.1707	6.0538
344	0.8466	0.8825	1.0890	1.4432	2.4099	3.4269	4.2602	5.0764	5.4132	5.0455	4.4586
345	1.5668	1.6103	1.8791	2.3493	3.6667	5.1041	6.3226	7.5572	8.2003	7.7994	7.0318
346	2.0008	2.0618	2.3465	2.8381	4.2351	5.8053	7.1471	8.6159	9.5151	9.2036	8.4309
347	2.1957	2.2864	2.5786	3.0630	4.4345	6.0046	7.3957	8.9156	10.0060	9.8105	9.1100
348	2.1918	2.3412	2.6409	3.0897	4.3179	5.7401	7.0422	8.5004	9.7707	9.8067	9.3052
349	2.0083	2.1926	2.4914	2.9027	3.9766	5.2021	6.3044	7.6395	8.9020	9.0788	8.7969
350	1.8198	2.0199	2.3112	2.6894	3.6298	4.6721	5.6423	6.7728	7.9256	8.1757	8.0745
351	0.1467	0.1623	0.2001	0.2555	0.3915	0.5273	0.6293	0.7317	0.7650	0.7188	0.6471
352	0.2280	0.2487	0.2971	0.3678	0.5437	0.7229	0.8715	1.0045	1.0630	1.0058	0.9259
353	0.2711	0.2934	0.3455	0.4206	0.6083	0.8034	0.9660	1.1185	1.1947	1.1431	1.0597
354	0.2915	0.3159	0.3676	0.4426	0.6315	0.8279	0.9953	1.1524	1.2395	1.1975	1.1167
355	0.2983	0.3241	0.3735	0.4438	0.6205	0.8068	0.9562	1.1241	1.2233	1.1913	1.1234
356	0.2868	0.3129	0.3590	0.4232	0.5843	0.7549	0.9040	1.0478	1.1507	1.1278	1.0725
357	0.2701	0.2964	0.3392	0.3979	0.5436	0.6984	0.8357	0.9670	1.0663	1.0500	1.0079
358	0.3161	0.3474	0.4347	0.5685	0.9018	1.2384	1.4989	1.7556	1.8418	1.7182	1.5360
359	0.5208	0.5621	0.6749	0.8495	1.2877	1.7426	2.1212	2.4732	2.6337	2.4992	2.2743
360	0.6305	0.6771	0.7968	0.9811	1.4479	1.9425	2.3605	2.7650	2.9857	2.8700	2.6428
361	0.6843	0.7357	0.8558	1.0388	1.5032	2.0022	2.4275	2.8543	3.1140	3.0205	2.8091
362	0.7004	0.7589	0.8746	1.0438	1.4729	1.9398	2.3358	2.7622	3.0650	3.0068	2.8433
363	0.6679	0.7308	0.8403	0.9943	1.3793	1.8005	2.1592	2.5531	2.8651	2.8362	2.7142
364	0.6242	0.6894	0.7929	0.9328	1.2779	1.6543	1.9656	2.3327	2.6371	2.6248	2.5406
365	0.4741	0.5235	0.6664	0.8874	1.4441	2.0085	2.4527	2.8811	3.0305	2.8180	2.4986
366	0.8018	0.8678	1.0560	1.3500	2.0971	2.8766	3.5284	4.1387	4.4262	4.1942	3.7909
367	0.9918	1.0681	1.2673	1.5777	2.3753	3.2273	3.9508	4.6637	5.0633	4.8653	4.4619
368	1.0846	1.1688	1.3690	1.6709	2.4669	3.3254	4.0654	4.8167	5.3003	5.1490	4.7795
369	1.1093	1.2089	1.4019	1.6801	2.4073	3.2055	3.8946	4.6469	5.2167	5.1509	4.8669
370	1.0511	1.1603	1.3446	1.5993	2.2433	2.9552	3.5592	4.2648	4.8523	4.8509	4.6440
371	0.9776	1.0915	1.2667	1.4986	2.0705	2.6980	3.2377	3.8665	4.4337	4.4717	4.3353
372	0.6097	0.6790	0.8814	1.1928	1.9921	2.8011	3.4436	4.0570	4.2728	3.9668	3.5000
373	1.0685	1.1638	1.4361	1.8568	2.9534	4.0938	5.0195	5.9435	6.3757	6.0252	5.4217
374	1.3395	1.4475	1.7358	2.1805	3.3559	4.6098	5.6518	6.7408	7.3445	7.0567	6.4493
375	1.4702	1.5910	1.8799	2.3185	3.4833	4.7484	5.8497	6.9761	7.7159	7.5070	6.9474
376	1.5007	1.6456	1.9245	2.3266	3.3820	4.5532	5.5746	6.7036	7.5930	7.5444	7.1130
377	1.4143	1.5744	1.8401	2.2031	3.1332	4.1657	5.0790	6.1106	7.0274	7.0814	6.7877
378	1.3109	1.4782	1.7303	2.0608	2.8798	3.7792	4.5739	5.5018	6.3800	6.4915	6.3139

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
379	0.7374	0.8277	1.0921	1.5004	2.5512	3.6152	4.4844	5.2730	5.5651	5.1534	4.5323
380	1.3165	1.4436	1.8070	2.3695	3.8394	5.3685	6.6517	7.8621	8.4385	7.9635	7.1462
381	1.6647	1.8083	2.1939	2.7909	4.3741	6.0673	7.4776	8.9609	9.7933	9.4060	8.5675
382	1.8321	1.9925	2.3784	2.9666	4.5346	6.2474	7.7220	9.2858	10.3170	10.0590	9.2816
383	1.8641	2.0580	2.4281	2.9634	4.3762	5.9537	7.3261	8.8998	10.1545	10.1279	9.5499
384	1.7476	1.9614	2.3128	2.7934	4.0270	5.4054	6.6309	8.0558	9.3548	9.4805	9.1106
385	1.6163	1.8391	2.1712	2.6057	3.6835	4.8700	5.9503	7.1947	8.4210	8.6333	8.4317
386	0.1324	0.1548	0.1994	0.2607	0.4057	0.5465	0.6498	0.7540	0.7815	0.7326	0.6572
387	0.2021	0.2305	0.2884	0.3672	0.5548	0.7410	0.8933	1.0251	1.0813	1.0235	0.9366
388	0.2380	0.2689	0.3302	0.4136	0.6146	0.8172	0.9839	1.1355	1.2078	1.1562	1.0678
389	0.2549	0.2867	0.3480	0.4316	0.6332	0.8390	1.0104	1.1694	1.2549	1.2102	1.1269
390	0.2608	0.2934	0.3507	0.4288	0.6194	0.8145	0.9804	1.1397	1.2420	1.2052	1.1351
391	0.2517	0.2835	0.3365	0.4082	0.5819	0.7621	0.9152	1.0666	1.1699	1.1464	1.0903
392	0.2384	0.2693	0.3183	0.3835	0.5413	0.7060	0.8482	0.9863	1.0887	1.0731	1.0272
393	0.2854	0.3326	0.4358	0.5841	0.9386	1.2866	1.5498	1.8078	1.8825	1.7478	1.5558
394	0.4578	0.5216	0.6569	0.8518	1.3219	1.7947	2.1635	2.5306	2.6809	2.5351	2.2970
395	0.5483	0.6170	0.7604	0.9675	1.4706	1.9872	2.4162	2.8209	3.0309	2.8950	2.6665
396	0.5931	0.6643	0.8076	1.0131	1.5161	2.0393	2.4804	2.9089	3.1600	3.0405	2.8351
397	0.6074	0.6818	0.8173	1.0077	1.4746	1.9689	2.3953	2.8184	3.1187	3.0561	2.8834
398	0.5829	0.6582	0.7843	0.9569	1.3776	1.8263	2.2045	2.6137	2.9275	2.9000	2.7668
399	0.5490	0.6243	0.7418	0.8982	1.2760	1.6795	2.0085	2.3966	2.7063	2.6967	2.6042
400	0.4283	0.5031	0.6715	0.9155	1.5060	2.0882	2.5359	2.9610	3.0910	2.8631	2.5290
401	0.7034	0.8057	1.0320	1.3605	2.1628	2.9719	3.6076	4.2403	4.5028	4.2423	3.8238
402	0.8588	0.9718	1.2124	1.5629	2.4252	3.3155	4.0444	4.7691	5.1462	4.9232	4.5051
403	0.9322	1.0522	1.2926	1.6351	2.5005	3.4041	4.1725	4.9276	5.3874	5.2115	4.8298
404	0.9567	1.0827	1.3098	1.6251	2.4214	3.2700	3.9917	4.7618	5.3264	5.2416	4.9395
405	0.9143	1.0438	1.2557	1.5402	2.2499	3.0140	3.6838	4.3883	4.9721	4.9702	4.7449
406	0.8592	0.9895	1.1875	1.4463	2.0769	2.7548	3.3360	3.9939	4.5779	4.6051	4.4567
407	0.5508	0.6549	0.8919	1.2377	2.0800	2.9126	3.5578	4.1637	4.3577	4.0276	3.5335
408	0.9354	1.0826	1.4103	1.8822	3.0567	4.2383	5.2022	6.0939	6.4749	6.0939	5.4629
409	1.1537	1.3164	1.6665	2.1709	3.4424	4.7520	5.8090	6.9032	7.4672	7.1329	6.4946
410	1.2583	1.4303	1.7798	2.2797	3.5484	4.8805	6.0113	7.1487	7.8628	7.6019	7.0093
411	1.2888	1.4721	1.8015	2.2593	3.4203	4.6677	5.7272	6.8942	7.7635	7.6742	7.2218
412	1.2288	1.4180	1.7238	2.1345	3.1612	4.2741	5.2241	6.3166	7.2400	7.2680	6.9413
413	1.1547	1.3449	1.6305	2.0005	2.9061	3.8846	4.7334	5.7171	6.6159	6.7184	6.4992
414	0.6665	0.8007	1.1096	1.5572	2.6652	3.7574	4.6196	5.4060	5.6581	5.2151	4.5686
415	1.1491	1.3462	1.7830	2.4138	3.9856	5.5679	6.8184	8.0541	8.5669	8.0438	7.1812
416	1.4273	1.6458	2.1150	2.7929	4.5056	6.2720	7.7289	9.1821	9.9500	9.5010	8.6195
417	1.5610	1.7911	2.2594	2.9313	4.6419	6.4431	7.9293	9.5294	10.5004	10.1766	9.3586
418	1.5960	1.8411	2.2814	2.8933	4.4499	6.1344	7.5916	9.1783	10.3986	10.3315	9.6931
419	1.5196	1.7716	2.1777	2.7224	4.0879	5.5776	6.8941	8.3645	9.6574	9.7541	9.3165
420	1.4303	1.6832	2.0598	2.5479	3.7442	5.0397	6.2009	7.5136	8.7719	8.9599	8.6923
421	0.2889	0.2649	0.2612	0.2778	0.3527	0.4492	0.5333	0.6265	0.6697	0.6356	0.5779
422	0.4501	0.4232	0.4195	0.4398	0.5318	0.6539	0.7674	0.8848	0.9465	0.9083	0.8368
423	0.5285	0.5040	0.5044	0.5271	0.6243	0.7536	0.8745	1.0030	1.0781	1.0384	0.9662
424	0.5588	0.5398	0.5435	0.5675	0.6653	0.7949	0.9116	1.0460	1.1366	1.0910	1.0199
425	0.5524	0.5432	0.5529	0.5784	0.6720	0.7940	0.9067	1.0238	1.1191	1.0851	1.0264
426	0.5138	0.5123	0.5258	0.5514	0.6384	0.7504	0.8485	0.9597	1.0523	1.0244	0.9763
427	0.4725	0.4761	0.4918	0.5174	0.5970	0.6993	0.7913	0.8868	0.9768	0.9522	0.9149
428	0.6364	0.5752	0.5636	0.6037	0.7882	1.0334	1.2531	1.4915	1.6154	1.5339	1.3847
429	1.0496	0.9799	0.9654	1.0125	1.2363	1.5473	1.8338	2.1528	2.3378	2.2508	2.0622
430	1.2584	1.2021	1.1964	1.2488	1.4828	1.8117	2.1212	2.4627	2.6885	2.6008	2.4137
431	1.3367	1.2996	1.3062	1.3638	1.5992	1.9272	2.2318	2.5763	2.8308	2.7517	2.5752
432	1.3108	1.3070	1.3356	1.4016	1.6302	1.9353	2.2132	2.5329	2.8112	2.7519	2.6107
433	1.2058	1.2252	1.2669	1.3371	1.5500	1.8287	2.0840	2.3610	2.6408	2.6085	2.4901
434	1.0953	1.1272	1.1785	1.2502	1.4471	1.6977	1.9142	2.1724	2.4382	2.4159	2.3336
435	0.9744	0.8724	0.8536	0.9217	1.2377	1.6572	2.0355	2.4491	2.6670	2.5343	2.2813
436	1.6761	1.5553	1.5285	1.6041	1.9909	2.5300	3.0372	3.5951	3.9340	3.7865	3.4649
437	2.0410	1.9464	1.9356	2.0197	2.4217	2.9900	3.5304	4.1404	4.5617	4.4287	4.1012
438	2.1752	2.1199	2.1332	2.2288	2.6314	3.1978	3.7317	4.3464	4.8264	4.7214	4.4088
439	2.1157	2.1272	2.1845	2.3001	2.6895	3.2175	3.7068	4.2727	4.8045	4.7579	4.5089
440	1.9220	1.9726	2.0585	2.1876	2.5535	3.0279	3.4615	3.9654	4.4938	4.4969	4.3062
441	1.7292	1.7992	1.8998	2.0335	2.3759	2.7955	3.1797	3.6256	4.1167	4.1567	4.0247
442	1.2874	1.1437	1.1190	1.2184	1.6836	2.2955	2.8589	3.4563	3.7834	3.5936	3.2229
443	2.3005	2.1248	2.0849	2.1966	2.7693	3.5680	4.3341	5.1605	5.6817	5.4745	4.9967

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
444	2.8382	2.7003	2.6852	2.8088	3.4019	4.2461	5.0555	5.9787	6.6326	6.4655	5.9727
445	3.0316	2.9576	2.9804	3.1223	3.7149	4.5555	5.3597	6.2905	7.0450	6.9287	6.4645
446	2.9195	2.9507	3.0468	3.2234	3.7989	4.5785	5.3095	6.1805	7.0208	7.0220	6.6516
447	2.6151	2.7053	2.8445	3.0418	3.5871	4.2802	4.9297	5.6991	6.5277	6.6091	6.3500
448	2.3292	2.4450	2.6051	2.8090	3.3193	3.9244	4.4942	5.1669	5.9285	6.0503	5.9108
449	1.5859	1.4001	1.3724	1.5088	2.1345	2.9535	3.6979	4.5119	4.9548	4.7086	4.2113
450	2.9120	2.6766	2.6273	2.7809	3.5622	4.6494	5.6806	6.8293	7.5552	7.2835	6.6288
451	3.6314	3.4503	3.4308	3.5990	4.4071	5.5611	6.6654	7.9528	8.8809	8.6772	8.0009
452	3.8819	3.7904	3.8259	4.0207	4.8273	5.9760	7.1018	8.3908	9.4602	9.3549	8.7156
453	3.6925	3.7502	3.8893	4.1379	4.9228	5.9821	6.9919	8.2198	9.4262	9.5074	9.0165
454	3.2553	3.3921	3.5912	3.8663	4.6121	5.5434	6.4342	7.5143	8.6985	8.8927	8.5889
455	2.8690	3.0393	3.2617	3.5406	4.2310	5.0321	5.7988	6.7288	7.8065	8.0444	7.9242
456	0.2632	0.2443	0.2474	0.2716	0.3604	0.4678	0.5589	0.6573	0.7008	0.6643	0.6020
457	0.4089	0.3875	0.3920	0.4219	0.5328	0.6706	0.7964	0.9169	0.9858	0.9441	0.8681
458	0.4805	0.4621	0.4690	0.5012	0.6190	0.7656	0.8999	1.0373	1.1183	1.0754	0.9994
459	0.5095	0.4939	0.5052	0.5384	0.6564	0.8033	0.9379	1.0751	1.1684	1.1304	1.0568
460	0.5078	0.5011	0.5164	0.5499	0.6615	0.8001	0.9206	1.0606	1.1573	1.1418	1.0632
461	0.4760	0.4759	0.4938	0.5263	0.6293	0.7562	0.8675	0.9901	1.0923	1.0637	1.0140
462	0.4403	0.4450	0.4643	0.4956	0.5898	0.7057	0.8119	0.9195	1.0150	0.9949	0.9534
463	0.5757	0.5262	0.5306	0.5891	0.8079	1.0797	1.3155	1.5690	1.6866	1.5947	1.4377
464	0.9485	0.8895	0.8947	0.9655	1.2386	1.5906	1.9119	2.2449	2.4331	2.3313	2.1343
465	1.1413	1.0927	1.1044	1.1805	1.4659	1.8404	2.1773	2.5548	2.7874	2.6920	2.4915
466	1.2193	1.1873	1.2075	1.2871	1.5714	1.9453	2.2900	2.6665	2.9284	2.8468	2.6576
467	1.2097	1.2054	1.2445	1.3273	1.5961	1.9457	2.2604	2.6178	2.9108	2.8574	2.7033
468	1.1236	1.1398	1.1903	1.2735	1.5226	1.8390	2.1210	2.4424	2.7416	2.7027	2.5806
469	1.0276	1.0592	1.1143	1.1964	1.4245	1.7098	1.9700	2.2479	2.5357	2.5235	2.4268
470	0.8761	0.7933	0.8013	0.8976	1.2722	1.7348	2.1467	2.5711	2.7830	2.6325	2.3591
471	1.5043	1.4020	1.4076	1.5236	1.9955	2.6047	3.1695	3.7532	4.0951	3.9234	3.5808
472	1.8435	1.7599	1.7754	1.8986	2.3906	3.0387	3.6430	4.2972	4.7300	4.5849	4.2275
473	1.9809	1.9278	1.9622	2.0915	2.5793	3.2256	3.8240	4.5020	4.9914	4.8824	4.5462
474	1.9561	1.9630	2.0320	2.1711	2.6304	3.2312	3.7826	4.4174	4.9729	4.9259	4.6517
475	1.7974	1.8433	1.9356	2.0798	2.5036	3.0440	3.5293	4.1026	4.6654	4.6700	4.4577
476	1.6296	1.6972	1.8028	1.9486	2.3381	2.8177	3.2515	3.7551	4.2917	4.3301	4.1748
477	1.1497	1.0333	1.0470	1.1883	1.7334	2.4057	3.0012	3.6237	3.9399	3.7228	3.3250
478	2.0511	1.9018	1.9095	2.0805	2.7782	3.6784	4.4947	5.3906	5.9090	5.6675	5.1493
479	2.5503	2.4257	2.4488	2.6294	3.3556	4.3184	5.2278	6.2085	6.8762	6.6805	6.1457
480	2.7525	2.6779	2.7282	2.9177	3.6360	4.5944	5.4935	6.5144	7.2845	7.1471	6.6473
481	2.7017	2.7223	2.8299	3.0371	3.7105	4.5984	5.4213	6.3893	7.2667	7.2537	6.8493
482	2.4549	2.5349	2.6798	2.8971	3.5189	4.3092	5.0417	5.9052	6.7881	6.8591	6.5643
483	2.2078	2.3169	2.4775	2.7000	3.2727	3.9651	4.6110	5.3677	6.1968	6.3187	6.1216
484	1.4091	1.2596	1.2816	1.4724	2.2006	3.0957	3.8935	4.7246	5.1501	4.8609	4.3283
485	2.5809	2.3812	2.3950	2.6287	3.5770	4.7987	5.9318	7.1295	7.8462	7.5287	6.8192
486	3.2471	3.0814	3.1115	3.3571	4.3463	5.6590	6.8881	8.2602	9.1931	8.9445	8.2144
487	3.5133	3.4171	3.4866	3.7447	4.7207	6.0309	7.2706	8.6859	9.7786	9.6290	8.9385
488	3.4232	3.4611	3.6123	3.8964	4.8089	6.0153	7.1687	8.5050	9.7610	9.8175	9.2643
489	3.0719	3.1904	3.3937	3.6922	4.5342	5.5968	6.5931	7.8051	9.0632	9.2514	8.8647
490	2.7372	2.8946	3.1197	3.4204	4.1926	5.1125	5.9770	7.0256	8.1973	8.4350	8.2125
491	0.2334	0.2208	0.2321	0.2640	0.3664	0.4832	0.5789	0.6809	0.7228	0.6826	0.6177
492	0.3588	0.3448	0.3595	0.3995	0.5288	0.6792	0.8132	0.9425	1.0084	0.9621	0.8844
493	0.4214	0.4101	0.4265	0.4692	0.6058	0.7669	0.9113	1.0549	1.1362	1.0921	1.0129
494	0.4481	0.4404	0.4584	0.5016	0.6384	0.8005	0.9463	1.0938	1.1794	1.1464	1.0704
495	0.4506	0.4482	0.4702	0.5121	0.6409	0.7943	0.9317	1.0738	1.1765	1.1436	1.0805
496	0.4261	0.4290	0.4523	0.4920	0.6108	0.7514	0.8690	1.0102	1.1127	1.0878	1.0344
497	0.3973	0.4041	0.4280	0.4654	0.5738	0.7027	0.8142	0.9382	1.0400	1.0204	0.9762
498	0.5080	0.4741	0.4972	0.5747	0.8262	1.1205	1.3794	1.6259	1.7401	1.6383	1.4745
499	0.8268	0.7863	0.8150	0.9122	1.2324	1.6192	1.9640	2.3086	2.4946	2.3817	2.1730
500	0.9962	0.9624	0.9963	1.0987	1.4354	1.8491	2.2108	2.6075	2.8370	2.7412	2.5313
501	1.0691	1.0478	1.0873	1.1913	1.5255	1.9399	2.3045	2.7114	2.9759	2.8963	2.6944
502	1.0730	1.0759	1.1266	1.2286	1.5414	1.9298	2.2758	2.6627	2.9639	2.9124	2.7497
503	1.0076	1.0268	1.0865	1.1852	1.4715	1.8248	2.1378	2.4893	2.7969	2.7773	2.6448
504	0.9301	0.9634	1.0253	1.1200	1.3815	1.7004	1.9730	2.2973	2.6037	2.5956	2.4918
505	0.7700	0.7126	0.7509	0.8781	1.3062	1.8052	2.2265	2.6668	2.8676	2.6955	2.4110
506	1.3024	1.2311	1.2769	1.4375	1.9927	2.6614	3.2496	3.8670	4.1965	4.0070	3.6414
507	1.5982	1.5374	1.5917	1.7602	2.3427	3.0608	3.6949	4.3978	4.8263	4.6585	4.2897
508	1.7277	1.6900	1.7557	1.9262	2.5025	3.2210	3.8709	4.5869	5.0828	4.9578	4.6067

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
509	1.7310	1.7429	1.8312	2.0010	2.5361	3.2063	3.8135	4.4998	5.0712	5.0201	4.7340
510	1.6118	1.6579	1.7629	1.9321	2.4169	3.0220	3.5683	4.1905	4.7785	4.7798	4.5613
511	1.4771	1.5432	1.6588	1.8218	2.2650	2.8043	3.2823	3.8458	4.4166	4.4603	4.2892
512	1.0057	0.9251	0.9812	1.1655	1.7850	2.5065	3.1254	3.7630	4.0561	3.8095	3.3865
513	1.7641	1.6592	1.7269	1.9637	2.7826	3.7690	4.6295	5.5541	6.0514	5.7751	5.2258
514	2.1961	2.1052	2.1844	2.4320	3.2935	4.3603	5.3087	6.3603	7.0157	6.7873	6.2227
515	2.3871	2.3298	2.4279	2.6779	3.5288	4.5969	5.5725	6.6525	7.4245	7.2567	6.7286
516	2.3852	2.4066	2.5407	2.7910	3.5759	4.5684	5.4961	6.5205	7.4175	7.3936	6.9591
517	2.2020	2.2789	2.4381	2.6855	3.3973	4.2854	5.1069	6.0452	6.9688	7.0403	6.7099
518	2.0049	2.1111	2.2852	2.5276	3.1742	3.9566	4.6817	5.5143	6.3991	6.5263	6.2900
519	1.2284	1.1258	1.2019	1.4479	2.2707	3.2272	4.0641	4.8942	5.2903	4.9612	4.3983
520	2.2059	2.0663	2.1609	2.4825	3.5926	4.9266	6.0955	7.3482	8.0290	7.6594	6.9063
521	2.7777	2.6557	2.7637	3.1006	4.2736	5.7258	7.0424	8.4674	9.3786	9.0763	8.2976
522	3.0309	2.9580	3.0886	3.4285	4.5858	6.0442	7.3814	8.8804	9.9567	9.6049	8.0309
523	3.0151	3.0500	3.2338	3.5743	4.6379	5.9886	7.2468	8.6958	9.9662	9.9965	9.4052
524	2.7570	2.8678	3.0879	3.4247	4.3844	5.5825	6.7054	8.0142	9.3240	9.5065	9.0644
525	2.4930	2.6434	2.8824	3.2110	4.0798	5.1231	6.1016	7.2544	8.5042	8.7499	8.4543
526	0.2096	0.2034	0.2210	0.2591	0.3713	0.4945	0.5928	0.6966	0.7373	0.6934	0.6284
527	0.3193	0.3131	0.3355	0.3832	0.5250	0.6841	0.8224	0.9530	1.0184	0.9704	0.8913
528	0.3748	0.3694	0.3939	0.4443	0.5944	0.7648	0.9143	1.0606	1.1323	1.0957	1.0154
529	0.3984	0.3961	0.4214	0.4719	0.6220	0.7935	0.9459	1.0963	1.1873	1.1456	1.0676
530	0.4017	0.4037	0.4313	0.4797	0.6212	0.7838	0.9284	1.0771	1.1779	1.1525	1.0794
531	0.3828	0.3890	0.4170	0.4626	0.5924	0.7426	0.8752	1.0141	1.1177	1.0966	1.0427
532	0.3575	0.3668	0.3947	0.4373	0.5562	0.6939	0.8124	0.9423	1.0444	1.0297	0.9868
533	0.4563	0.4359	0.4742	0.5664	0.8422	1.1517	1.4184	1.6675	1.7746	1.6652	1.4957
534	0.7363	0.7123	0.7608	0.8782	1.2323	1.6409	1.9852	2.3471	2.5246	2.4095	2.1926
535	0.8791	0.8605	0.9136	1.0371	1.4105	1.8498	2.2244	2.6315	2.8614	2.7054	2.5381
536	0.9435	0.9347	0.9914	1.1146	1.4863	1.9277	2.3202	2.7284	2.9953	2.9113	2.7057
537	0.9510	0.9610	1.0251	1.1440	1.4905	1.9059	2.2732	2.6748	2.9780	2.9316	2.7607
538	0.8985	0.9239	0.9918	1.1044	1.4207	1.7997	2.1333	2.5048	2.8222	2.8007	2.6688
539	0.8346	0.8703	0.9405	1.0465	1.3348	1.6785	1.9797	2.3160	2.6295	2.6271	2.5253
540	0.6897	0.6546	0.7176	0.8717	1.3362	1.8591	2.3106	2.7355	2.9228	2.7389	2.4377
541	1.1471	1.1034	1.1842	1.3797	1.9941	2.7006	3.2985	3.9330	4.2487	4.0440	3.6632
542	1.4000	1.3654	1.4533	1.6588	2.3086	3.0726	3.7319	4.4474	4.8667	4.6934	4.3060
543	1.5135	1.4962	1.5920	1.7974	2.4418	3.2102	3.9062	4.6289	5.1195	4.9862	4.6244
544	1.5255	1.5461	1.6570	1.8584	2.4521	3.1727	3.8156	4.5364	5.1102	5.0606	4.7644
545	1.4311	1.4816	1.6024	1.7931	2.3323	2.9857	3.5720	4.2306	4.8316	4.8383	4.6119
546	1.3214	1.3909	1.5173	1.6991	2.1878	2.7730	3.2915	3.8915	4.4866	4.5297	4.3540
547	0.8981	0.8485	0.9393	1.1569	1.8303	2.5842	3.2328	3.8508	4.1266	3.8562	3.4187
548	1.5448	1.4817	1.6004	1.8883	2.7942	3.8357	4.7215	5.6526	6.1271	5.8210	5.2514
549	1.9105	1.8575	1.9884	2.2918	3.2550	4.3896	5.3730	6.4466	7.0845	6.8253	6.2406
550	2.0764	2.0520	2.1924	2.4955	3.4506	4.5947	5.6212	6.7266	7.4800	7.2990	6.7491
551	2.0897	2.1254	2.2896	2.5822	3.4617	4.5325	5.4848	6.5909	7.4901	7.4543	7.0092
552	1.9474	2.0279	2.2102	2.4899	3.2818	4.2451	5.1145	6.1215	7.0642	7.1380	6.7903
553	1.7901	1.8995	2.0890	2.3567	3.0702	3.9255	4.7051	5.5998	6.5246	6.6548	6.3992
554	1.0948	1.0319	1.1528	1.4412	2.3323	3.3286	4.1852	5.0101	5.3747	5.0242	4.4337
555	1.9214	1.8390	2.0021	2.3927	3.6185	5.0250	6.2226	7.4811	8.1239	7.7094	6.9274
556	2.4007	2.3329	2.5100	2.9236	4.2353	5.7809	7.1226	8.5921	9.4653	9.1219	8.3099
557	2.6195	2.5854	2.7796	3.1936	4.4949	6.0570	7.4621	8.9916	10.0423	9.8088	9.0500
558	2.6278	2.6795	2.9054	3.3035	4.4983	5.9585	7.2839	8.8096	10.0756	10.0789	9.4636
559	2.4313	2.5462	2.7956	3.1754	4.2447	5.5489	6.7728	8.1413	9.4713	9.6433	9.1729
560	2.2238	2.3775	2.6363	2.9981	3.9574	5.1032	6.1659	7.3998	8.6980	8.9448	8.6124
561	0.4259	0.3803	0.3521	0.3450	0.3804	0.4509	0.5222	0.6042	0.6487	0.6159	0.5595
562	0.6471	0.5966	0.5643	0.5555	0.5966	0.6828	0.7726	0.8757	0.9346	0.8925	0.8281
563	0.7489	0.7041	0.6753	0.6685	0.7130	0.8040	0.8976	1.0075	1.0748	1.0292	0.9541
564	0.7845	0.7478	0.7247	0.7210	0.7679	0.8599	0.9525	1.0604	1.1373	1.0893	1.0132
565	0.7664	0.7448	0.7322	0.7346	0.7844	0.8735	0.9573	1.0572	1.1407	1.0963	1.0233
566	0.7116	0.7009	0.6964	0.7030	0.7527	0.8373	0.9122	0.9991	1.0880	1.0478	0.9849
567	0.6527	0.6497	0.6508	0.6604	0.7080	0.7882	0.8561	0.9312	1.0178	0.9841	0.9312
568	0.9517	0.8386	0.7671	0.7485	0.8363	1.0190	1.2111	1.4248	1.5470	1.4770	1.3422
569	1.5287	1.4026	1.3177	1.2902	1.3871	1.6090	1.8418	2.1178	2.2890	2.1964	2.0136
570	1.8045	1.6997	1.6277	1.6070	1.7102	1.9433	2.1851	2.4736	2.6768	2.5789	2.3845
571	1.9005	1.8244	1.7729	1.7652	1.8754	2.1114	2.3481	2.6295	2.8577	2.7623	2.5654
572	1.8503	1.8231	1.8093	1.8244	1.9495	2.1811	2.3975	2.6495	2.9075	2.8312	2.6428
573	1.6979	1.7024	1.7149	1.7465	1.8763	2.0984	2.3000	2.5151	2.7825	2.7309	2.5546

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
574	1.5399	1.5626	1.5924	1.6356	1.7663	1.9728	2.1552	2.3466	2.6023	2.5843	2.4250
575	1.4735	1.2840	1.1651	1.1347	1.2921	1.6098	1.9468	2.3213	2.5493	2.4421	2.2031
576	2.4761	2.2595	2.1117	2.0637	2.2313	2.6180	3.0327	3.5236	3.8455	3.7029	3.3865
577	2.9701	2.7951	2.6726	2.6363	2.8120	3.2176	3.6468	4.1619	4.5443	4.4030	4.0561
578	3.1409	3.0241	2.9436	2.9321	3.1229	3.5342	3.9561	4.4621	4.8893	4.7649	4.4115
579	3.0365	3.0167	3.0146	3.0532	3.2771	3.6854	4.0769	4.5303	5.0191	4.9533	4.6045
580	2.7505	2.7877	2.8364	2.9116	3.1541	3.5441	3.9056	4.3038	4.8078	4.8036	4.4833
581	2.4646	2.5301	2.6089	2.7052	2.9567	3.3144	3.6422	4.0009	4.4862	4.5243	4.2561
582	1.9835	1.7127	1.5450	1.5062	1.7457	2.2165	2.7134	3.2731	3.6232	3.4679	3.1262
583	3.4508	3.1318	2.9149	2.8458	3.1000	3.6796	4.3067	5.0515	5.5520	5.3574	4.8925
584	4.1945	3.9414	3.7626	3.7106	3.9729	4.5811	5.2395	6.0231	6.6198	6.4418	5.9243
585	4.4484	4.2896	4.1804	4.1691	4.4562	5.0736	5.7274	6.4969	7.1717	7.0361	6.5007
586	4.2576	4.2567	4.2778	4.3536	4.6998	5.3166	5.9287	6.6306	7.4069	7.3945	6.8652
587	3.7926	3.8771	3.9781	4.1130	4.5036	5.0878	5.6508	6.2784	7.0799	7.1723	6.7048
588	3.3470	3.4692	3.6080	3.7748	4.1837	4.7150	5.2076	5.7824	6.5630	6.7025	6.3336
589	2.4766	2.1215	1.9051	1.8610	2.1944	2.8332	3.4997	4.2657	4.7461	4.5486	4.0903
590	4.4371	4.0066	3.7153	3.6259	3.9832	4.7852	5.6427	6.6820	7.3867	7.1465	6.5061
591	5.4555	5.1169	4.8776	4.8103	5.1752	6.0157	6.9300	8.0338	8.8884	8.6728	7.9609
592	5.7922	5.5923	5.4544	5.4471	5.8483	6.7031	7.6116	8.7140	9.6822	9.5533	8.8168
593	5.4686	5.4983	5.5524	5.6777	6.1765	7.0322	7.8997	8.9160	10.0374	10.1254	9.4067
594	4.7682	4.9145	5.0810	5.2904	5.8601	6.6665	7.4478	8.3731	9.5473	9.7801	9.1744
595	4.1308	4.3225	4.5305	4.7756	5.3659	6.0887	6.7740	7.5998	8.7499	9.0166	8.5827
596	0.3997	0.3564	0.3329	0.3318	0.3799	0.4612	0.5402	0.6293	0.6743	0.6403	0.5817
597	0.6060	0.5570	0.5297	0.5281	0.5856	0.6868	0.7874	0.9005	0.9636	0.9201	0.8452
598	0.7012	0.6569	0.6325	0.6326	0.6940	0.8009	0.9083	1.0288	1.1029	1.0563	0.9782
599	0.7358	0.6989	0.6793	0.6821	0.7452	0.8524	0.9582	1.0808	1.1620	1.1160	1.0387
600	0.7229	0.7000	0.6899	0.6974	0.7607	0.8639	0.9609	1.0754	1.1642	1.1242	1.0532
601	0.6748	0.6610	0.6598	0.6705	0.7328	0.8291	0.9162	1.0176	1.1114	1.0756	1.0144
602	0.6219	0.6159	0.6195	0.6324	0.6906	0.7814	0.8614	0.9498	1.0440	1.0133	0.9601
603	0.8872	0.7792	0.7185	0.7150	0.8342	1.0443	1.2577	1.4839	1.6097	1.5362	1.3911
604	1.4263	1.3008	1.2262	1.2176	1.3549	1.6162	1.8781	2.1805	2.3625	2.2659	2.0748
605	1.6885	1.5798	1.5148	1.5108	1.6542	1.9287	2.2057	2.5273	2.7417	2.6438	2.4416
606	1.7855	1.7026	1.6547	1.6582	1.8075	2.0831	2.3554	2.6754	2.9193	2.8278	2.6229
607	1.7519	1.7160	1.7021	1.7253	1.8818	2.1454	2.3969	2.6886	2.9682	2.8912	2.7024
608	1.6182	1.6145	1.6255	1.6627	1.8169	2.0673	2.2988	2.5520	2.8367	2.7878	2.6136
609	1.4745	1.4913	1.5187	1.5644	1.7153	1.9480	2.1551	2.3823	2.6602	2.6320	2.4824
610	1.3649	1.1836	1.0831	1.0773	1.2885	1.6517	2.0079	2.4157	2.6527	2.5286	2.2793
611	2.2984	2.0799	1.9494	1.9303	2.1720	2.6282	3.0997	3.6304	3.9691	3.8155	3.4834
612	2.7701	2.5840	2.4700	2.4577	2.7056	3.1840	3.6711	4.2490	4.6548	4.5117	4.1502
613	2.9475	2.8126	2.7332	2.7390	2.9935	3.4734	3.9585	4.5315	4.9849	4.8600	4.5006
614	2.8810	2.8401	2.8312	2.8769	3.1450	3.6106	4.0624	4.5830	5.0980	5.0344	4.6939
615	2.6318	2.6510	2.6916	2.7678	3.0437	3.4812	3.8904	4.3520	4.8845	4.8834	4.5668
616	2.3732	2.4251	2.4960	2.5916	2.8667	3.2698	3.6420	4.0497	4.5716	4.6167	4.3342
617	1.8257	1.5680	1.4276	1.4244	1.7410	2.2766	2.8192	3.4089	3.7663	3.5903	3.2235
618	3.1827	2.8622	2.6703	2.6447	3.0091	3.6911	4.3990	5.2007	5.7239	5.5155	5.0205
619	3.8966	3.6221	3.4533	3.4356	3.8056	4.5227	5.2669	6.1434	6.7755	6.5787	6.0467
620	4.1662	3.9740	3.8609	3.8713	4.2505	4.9697	5.7038	6.5861	7.2977	7.1466	6.6218
621	4.0476	4.0081	4.0115	4.0908	4.4959	5.1935	5.8854	6.6910	7.4913	7.4861	6.9658
622	3.6482	3.7011	3.7831	3.9149	4.3401	4.9934	5.6255	6.3399	7.1905	7.2773	6.8062
623	3.2474	3.3466	3.4715	3.6334	4.0653	4.6615	5.2260	5.8610	6.6905	6.8323	6.4316
624	2.2677	1.9310	1.7515	1.7543	2.1883	2.9101	3.6308	4.4390	4.9205	4.6963	4.2060
625	4.0704	3.6360	3.3800	3.3508	3.8582	4.7961	5.7558	6.8805	7.6113	7.3358	6.6589
626	5.0461	4.6743	4.4464	4.4251	4.9383	5.9265	6.9617	8.1832	9.0765	8.8422	8.1070
627	5.4138	5.1601	5.0116	5.0313	5.5554	6.5471	7.5928	8.8168	9.8256	9.6864	8.9377
628	5.2150	5.1830	5.2053	5.3279	5.8951	6.8576	7.8333	8.9813	10.1513	10.2214	9.5053
629	4.6203	4.7191	4.8533	5.0524	5.6567	6.5542	7.4448	8.4686	9.6912	9.9256	9.2886
630	4.0500	4.2076	4.3968	4.6347	5.2490	6.0562	6.8226	7.7458	8.9446	9.2393	8.7197
631	0.3648	0.3260	0.3086	0.3144	0.3758	0.4673	0.5563	0.6469	0.6921	0.6587	0.5959
632	0.5489	0.5043	0.4842	0.4912	0.5659	0.6804	0.7925	0.9109	0.9760	0.9324	0.8561
633	0.6349	0.5936	0.5760	0.5845	0.6631	0.7847	0.9030	1.0314	1.1091	1.0651	0.9844
634	0.6678	0.6313	0.6190	0.6294	0.7088	0.8306	0.9485	1.0782	1.1642	1.1219	1.0444
635	0.6608	0.6367	0.6324	0.6458	0.7233	0.8396	0.9494	1.0730	1.1676	1.1301	1.0614
636	0.6214	0.6074	0.6089	0.6244	0.6986	0.8072	0.9062	1.0199	1.1171	1.0847	1.0287
637	0.5763	0.5696	0.5753	0.5922	0.6635	0.7630	0.8533	0.9554	1.0541	1.0290	0.9778
638	0.8057	0.7080	0.6621	0.6755	0.8277	1.0629	1.2847	1.5313	1.6596	1.5753	1.4197

hull	0.33	0.34	0.35	0.36	0.37	0.38	0.40	0.45	0.50	0.55	0.60
639	1.2858	1.1687	1.1113	1.1224	1.3066	1.6040	1.8897	2.2126	2.3966	2.2953	2.1017
640	1.5251	1.4192	1.3688	1.3849	1.5733	1.8868	2.1902	2.5399	2.7634	2.6669	2.4593
641	1.6197	1.5364	1.4980	1.5201	1.7095	2.0231	2.3259	2.6756	2.9279	2.8411	2.6378
642	1.6046	1.5640	1.5539	1.5887	1.7784	2.0760	2.3586	2.6854	2.9674	2.9072	2.7189
643	1.4947	1.4850	1.4970	1.5419	1.7241	2.0037	2.2615	2.5508	2.8481	2.8109	2.6452
644	1.3716	1.3823	1.4095	1.4610	1.6354	1.8943	2.1248	2.3863	2.6818	2.6645	2.5419
645	1.2325	1.0688	0.9932	1.0157	1.2815	1.6860	2.0748	2.4952	2.7272	2.5896	2.3247
646	2.0604	1.8556	1.7541	1.7725	2.0933	2.6134	3.1233	3.6927	4.0332	3.8723	3.5239
647	2.4902	2.3074	2.2149	2.2367	2.5649	3.1120	3.6514	4.2755	4.6922	4.5426	4.1752
648	2.6653	2.5239	2.4576	2.4906	2.8189	3.3664	3.9077	4.5348	5.0046	4.8825	4.5198
649	2.6380	2.5813	2.5736	2.6346	2.9585	3.4821	3.9947	4.5700	5.1095	5.0551	4.7154
650	2.4340	2.4375	2.4745	2.5587	2.8771	3.3645	3.8234	4.3465	4.9167	4.9089	4.6071
651	2.2128	2.2515	2.3170	2.4178	2.7257	3.1745	3.5894	4.0486	4.6113	4.6465	4.3851
652	1.6426	1.4093	1.3047	1.3418	1.7352	2.3282	2.9028	3.5171	3.8609	3.6661	3.2796
653	2.8369	2.5349	2.3871	2.4172	2.9000	3.6753	4.4474	5.2952	5.8168	5.5829	5.0688
654	3.4855	3.2113	3.0741	3.1069	3.5995	4.4217	5.2409	6.1858	6.8300	6.6247	6.0734
655	3.7523	3.5453	3.4482	3.4982	3.9878	4.8094	5.6322	6.5878	7.3187	7.1700	6.6272
656	3.7031	3.6333	3.6319	3.7291	4.2137	4.9976	5.7703	6.6684	7.5124	7.4970	6.9792
657	3.3809	3.4048	3.4753	3.6135	4.0916	4.8205	5.5223	6.3292	7.2154	7.3112	6.8481
658	3.0404	3.1167	3.2317	3.3939	3.8632	4.5272	5.1642	5.8719	6.7460	6.9026	6.4989
659	2.0308	1.7277	1.5955	1.6512	2.1844	2.9797	3.7484	4.5735	5.0468	4.7874	4.2722
660	3.6050	3.1994	3.0035	3.0504	3.7185	4.7816	5.8179	7.0023	7.7272	7.4202	6.7116
661	4.4869	4.1157	3.9307	3.9791	4.6616	5.7953	6.9322	8.2403	9.1459	8.8838	8.1278
662	4.8576	4.5769	4.4476	4.5192	5.1960	6.3309	7.4646	8.8190	9.8552	9.6894	8.9351
663	4.7680	4.6872	4.6979	4.8393	5.5087	6.5899	7.6679	8.9501	10.1525	10.2122	9.5065
664	4.2959	4.3479	4.4607	4.6627	5.3299	6.3279	7.3127	8.4614	9.7194	9.9711	9.3375
665	3.8143	3.9338	4.1079	4.3436	5.0004	5.9100	6.7801	7.7838	9.0395	9.3628	8.8149
666	0.3357	0.3010	0.2893	0.3008	0.3713	0.4711	0.5648	0.6580	0.7012	0.6662	0.6036
667	0.5021	0.4618	0.4480	0.4619	0.5492	0.6728	0.7874	0.9139	0.9788	0.9375	0.8597
668	0.5792	0.5417	0.5297	0.5447	0.6363	0.7671	0.8925	1.0269	1.1032	1.0621	0.9820
669	0.6092	0.5769	0.5681	0.5846	0.6764	0.8076	0.9326	1.0684	1.1548	1.1176	1.0396
670	0.6044	0.5817	0.5811	0.5995	0.6878	0.8130	0.9306	1.0599	1.1560	1.1234	1.0578
671	0.5708	0.5574	0.5616	0.5815	0.6647	0.7816	0.8857	1.0083	1.1086	1.0816	1.0253
672	0.5316	0.5249	0.5328	0.5532	0.6331	0.7400	0.8362	0.9470	1.0476	1.0259	0.9775
673	0.7388	0.6505	0.6184	0.6464	0.8232	1.0762	1.3113	1.5589	1.6870	1.5974	1.4374
674	1.1701	1.0626	1.0207	1.0497	1.2679	1.5900	1.8905	2.2223	2.4130	2.3080	2.1075
675	1.3847	1.2857	1.2488	1.2799	1.5058	1.8464	2.1672	2.5323	2.7592	2.6610	2.4542
676	1.4719	1.3909	1.3644	1.4019	1.6245	1.9653	2.2869	2.6564	2.9117	2.8257	2.6258
677	1.4646	1.4220	1.4187	1.4646	1.6798	2.0040	2.3083	2.6551	2.9457	2.8889	2.7105
678	1.3716	1.3577	1.3735	1.4265	1.6330	1.9334	2.2102	2.5246	2.8311	2.8005	2.6508
679	1.2646	1.2717	1.3003	1.3578	1.5521	1.8309	2.0913	2.3640	2.6733	2.6641	2.5209
680	1.1265	0.9788	0.9254	0.9712	1.2786	1.7121	2.1168	2.5461	2.7706	2.6232	2.3488
681	1.8635	1.6760	1.6021	1.6520	2.0333	2.5974	3.1383	3.7185	4.0633	3.8860	3.5319
682	2.2490	2.0752	2.0066	2.0589	2.4518	3.0491	3.6254	4.2720	4.6851	4.5482	4.1670
683	2.4109	2.2704	2.2224	2.2818	2.6714	3.2700	3.8476	4.5080	4.9838	4.8627	4.5020
684	2.4002	2.3353	2.3356	2.4140	2.7879	3.3568	3.9007	4.5257	5.0724	5.0256	4.7019
685	2.2291	2.2214	2.2607	2.3557	2.7123	3.2406	3.7325	4.3029	4.8821	4.8962	4.6085
686	2.0387	2.0680	2.1315	2.2394	2.5782	3.0636	3.5098	4.0195	4.6011	4.6567	4.3981
687	1.4961	1.2874	1.2137	1.2837	1.7356	2.3680	2.9674	3.5881	3.9222	3.7110	3.3095
688	2.5519	2.2763	2.1699	2.2474	2.8207	3.6616	4.4540	5.3402	5.8537	5.6027	5.0731
689	3.1281	2.8690	2.7678	2.8476	3.4393	4.3384	5.2086	6.1866	6.8319	6.6155	6.0583
690	3.3760	3.1681	3.0978	3.1879	3.7727	4.6734	5.5532	6.5590	7.2993	7.1445	6.5988
691	3.3575	3.2724	3.2784	3.4000	3.9595	4.8141	5.6601	6.6099	7.4708	7.4603	6.9611
692	3.0908	3.0957	3.1650	3.3153	3.8492	4.6400	5.4086	6.2748	7.1875	7.2982	6.8455
693	2.8000	2.8589	2.9685	3.1385	3.6485	4.3703	5.0636	5.8316	6.7444	6.9279	6.5234
694	1.8452	1.5738	1.4825	1.5807	2.1893	3.0343	3.8347	4.6651	5.1172	4.8396	4.3021
695	3.2264	2.8581	2.7194	2.8312	3.6219	4.7717	5.8838	7.0677	7.7754	7.4372	6.7122
696	4.0051	3.6525	3.5193	3.6337	4.4548	5.6928	6.8945	8.2515	9.1507	8.8678	8.0960
697	4.3437	4.0650	3.9714	4.0994	4.9110	6.1563	7.3838	8.7854	9.8266	9.6465	8.8795
698	4.3093	4.2022	4.2176	4.3930	5.1669	6.3487	7.5223	8.8798	10.1041	10.1550	9.4687
699	3.9238	3.9436	4.0507	4.2679	5.0075	6.0935	7.1739	8.4001	9.7021	9.9528	9.3368
700	3.5164	3.6106	3.7737	4.0166	4.7242	5.7050	6.6530	7.7536	9.0539	9.4071	8.8604

## C. Mathematically Defined Hulls

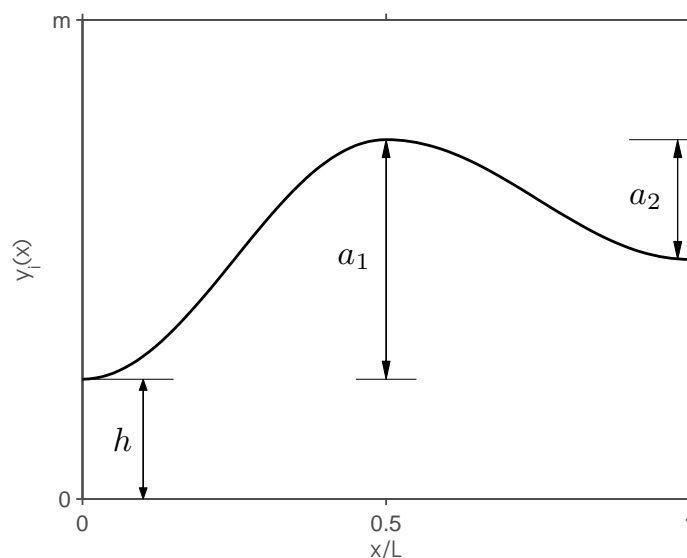
The following equations define a 12 parameter mathematical hull shape. The form of the equation starts with a three parameter series as shown in Equation C.1 [15]

$$Y(x, z) = \begin{cases} 0, & \text{if } X(x) = 0 \\ 0, & \text{if } X(x)^{f_2(x)} \leq z^{f_3(x)} \\ \frac{1}{2}X(x)^{f_0} \left[ 1 - \frac{z^2}{X(x)^{2f_2}} \right]^{f_1}, & \text{otherwise} \end{cases} \quad (\text{C.1})$$

where  $X(x) = 4x(1-x)$ . Another exponent shape function is added and each is made a function of  $x$ , as shown in Equation C.2. Each one of those functions is made up of two cosine functions defined by three parameters according to Equations C.3 and C.4. An example is shown in Figure C.1.

$$Y(x, z) = \begin{cases} 0, & \text{if } X(x) = 0 \\ 0, & \text{if } X(x)^{f_2(x)} \leq z^{f_3(x)} \\ \frac{1}{2}X(x)^{f_0(x)} \left[ 1 - \frac{z^{2f_3(x)}}{X(x)^{2f_2(x)}} \right]^{f_1(x)}, & \text{otherwise} \end{cases} \quad (\text{C.2})$$





**Figure C.1.** Typical three parameter shape function

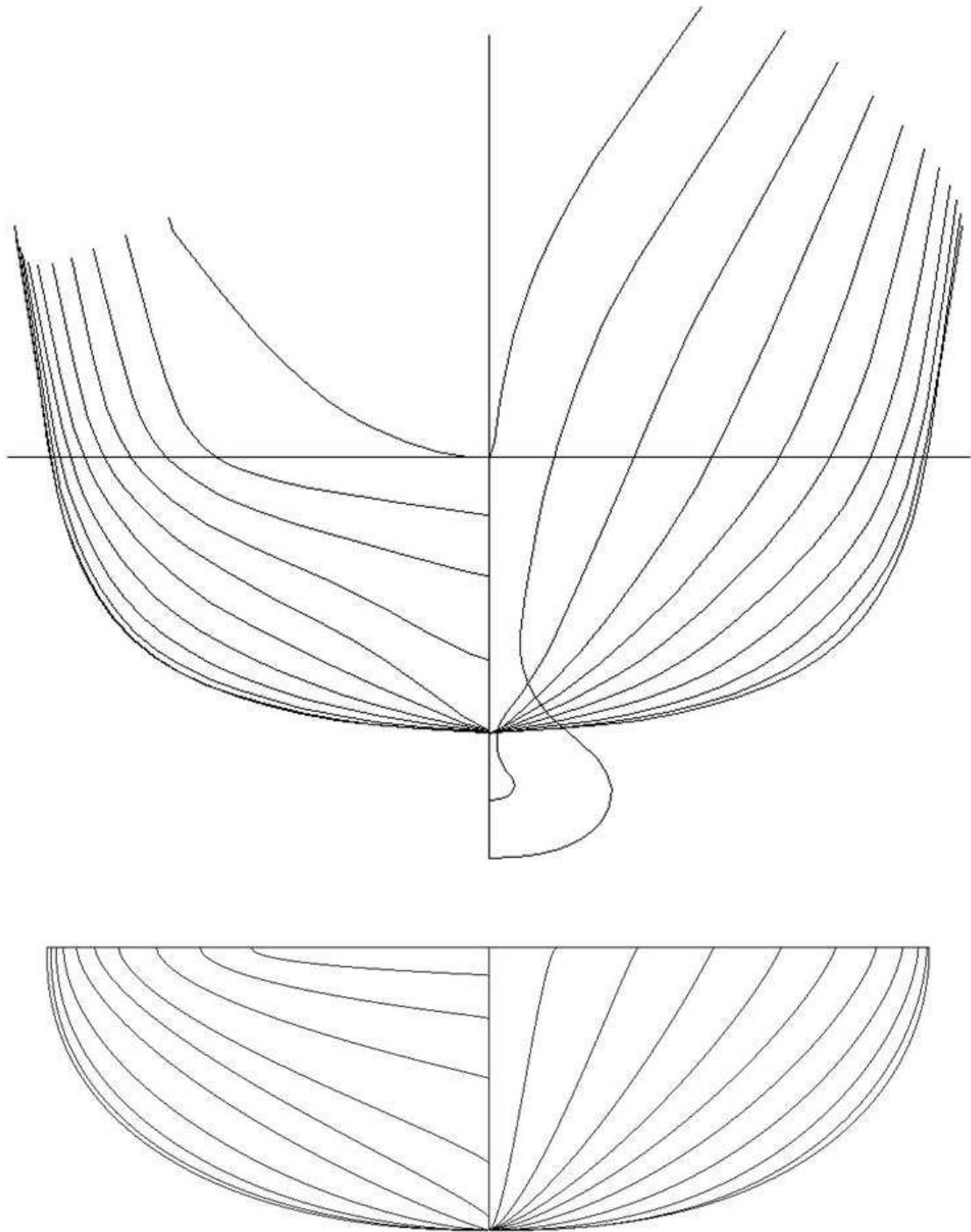
$$f_i(x) = \begin{cases} h_i - \frac{a_{1i}}{2} [\cos(2\pi x) - 1], & \text{if } x < \frac{1}{2} \\ h_i + \frac{a_{2i}}{2} [\cos(2\pi x) + 1] + a_{1i}, & \text{if } x \geq \frac{1}{2} \end{cases} \quad (\text{C.3})$$

$$0 \leq h_i \leq m_i \quad (\text{C.4})$$

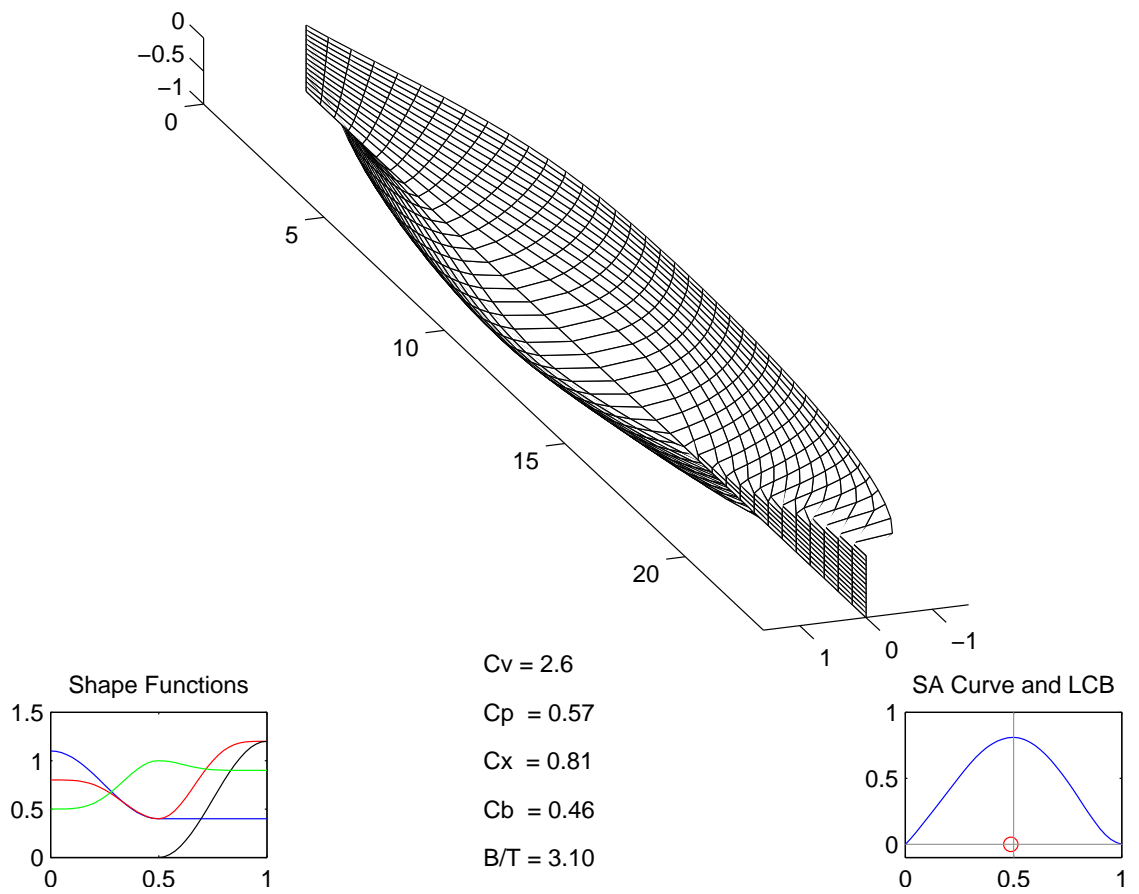
$$-h_i \leq a_{1i} \leq m_i - h_i$$

$$-(h_i + a_{1i}) \leq a_{2i} \leq m_i - (h_i + a_{1i}),$$

This equation can approximate realistic ship forms. If the cosine terms in Equation C.3 are each raised to a power, the equations form a 20 parameter series and can nearly match actual ship lines. Figure C.2 shows an approximation to the David Taylor Model Basin hull number 5415 using the 20 parameter series. Figure C.3 shows an isometric view of the mathematically defined hull, along with the shape functions, shape parameters, and sectional area curve.



**Figure C.2.** Math form compared to DTMB 5415. Math form is below, destroyer hull above.



**Figure C.3.** Isometric of mathematical destroyer hull

## BIOGRAPHY OF THE AUTHOR

Douglas Read was born in Bedford, Indiana on March 3, 1975. He was raised in Shoals, Indiana and graduated from Shoals Community High School in 1993. He attended Webb Institute and graduated in 1997 with a Bachelor of Science in Naval Architecture and Marine Engineering. He spent his sophomore sea term aboard the steamship *Arco Prudhoe Bay*. In 2001 he received a Master of Science degree in Naval Architecture and Marine Engineering from Massachusetts Institute of Technology. Doug worked at Bath Iron Works in Bath, Maine from 1999 to 2004, primarily on the DD21/DDX destroyer and littoral combat ship programs. In 2004 he entered the Ocean Engineering Interdisciplinary Ph.D. program at The University of Maine.

Doug published his M.S. research in *Journal of Fluids and Structures*. He is a member of the Society of Naval Architects and Marine Engineers (SNAME) and presented his Ph.D. work to the New England section in 2009. He is also a member of the American Society of Naval Engineers (ASNE) and the honor society Phi Kappa Phi.

After receiving his degree, Doug will be joining the faculty at Maine Maritime Academy, where he has been teaching as an adjunct since the fall of 2007. Doug is a candidate for the Doctor of Philosophy degree Interdisciplinary in Ocean Engineering from The University of Maine in August 2009.