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Spatial Patterns of Record-Setting Temperatures

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Abstract

We employ record-breaking statistics to study spatial correlations of record-setting terrestrial surface temperatures. To that end, a simple diagnostic tool is devised, reminiscent of a pair-correlation function. Data analysis reveals that while during the hottest years, record-breaking temperatures arrive in "heat waves", extending throughout almost the entire continental United States, this is not so for all years, not even recently. Recordbreaking temperatures generally exhibit spatial patterns and variability quite different from those of the mean temperatures.

Keywords: Record-setting, spatial correlation.

1. Introduction

Based on predictions of climate models, heat waves are widely expected to become more frequent and pronounced in the near future as climate change intensifies, e.g., Meehl and Tebaldi (2004). In the US, for example, last year was marked by extreme weather including droughts and record breaking heat in the middle west and Superstorm Sandy in the East. Thus, the perennial question on the public's mind is whether the anthropogenic climate change is responsible. This question of attribution is difficult as natural variability must be disentangled from the effects of a trend (i.e., global warming). Recent attempts to do so include work by Rahmstorf and Coumou (2011) who propose a statistical model that yields an increase of extreme events in the warming world. Most recently however, the mean global temperatures seem to have reached a plateau, e.g. see the recent perspective in Nature, entitled "The cause for the pause" (Held 2013). Perhaps one should pause for the cause instead. As a step toward disentangling natural variability from the possible trend, here we examine available evidence in terms of the "most extreme" variables, namely, record-breaking monthly temperatures within the continental United States. Recent years have seen many applications of record-breaking to statistical physics and meteorology e.g. Wergen (2013); Edery, Kostinski, Majumdar, and Berkowitz (2013). Here our emphasis is on surface temperatures and here too, record-breaking statistics have been used, e.g. Benestad (2004); Meehl, Tebaldi, Walton, Easterling, and McDaniel (2009); Anderson and Kostinksi (2010). However, while the spatial distribution of records has been explored, e.g. Elguindi, Rauscher, and Giorgi (2012); Meehl, Arblaster, and Branstator (2012), to the best of our knowledge, spatial correlations in record-setting have not been examined quantitatively.

For the reader's convenience we briefly summarize essential definitions and mathematical results. The *i*th entry in a time-series, x_i , is a record-breaking event (record) if it exceeds all prior values in the sequence, that is, x_i is a record high if

$$x_i > \max(x_1, x_2, \dots, x_{i-1})$$
 (1)

and is a record low if

$$x_i < \min(x_1, x_2, \dots x_{i-1})$$
 (2)

The first entry is always a record high and a record low by convention.

The crucial observation is the so-called reshuffling argument, namely, that for independent, identically distributed (i.i.d.), and continuous random variables, the *n*th trial has an equal chance of having the greatest value (denoted as $P_n(R)$) as all preceding trials, that is, 1/n.

$$P_n(R) = 1/n \tag{3}$$

The expected number of records in a time series is the sum over trial probabilities of being a record. Thus the expected number of records, E(R), for a time-series with n events is given by the harmonic series

$$E(R) = 1 + 1/2 + 1/3 \dots + 1/n \tag{4}$$

and, by Euler's formula for large n

$$E(R) = \ln(n) + \gamma \tag{5}$$

where $\gamma = 0.577...$, the Euler constant. These results are occasionally attributed to Rényi (1962) but in fact, originate with Foster and Stuart (1954). We stress the distribution independence of these results, i.e., they hold for any continuous probability densities. If the i.i.d. assumption is violated by a trend or by correlations, the number of records will deviate from the logarithmic dependence in Equation 5 and trends can, perhaps, be detected in a distribution-independent manner.

2. Problem Statement and Data Analysis

We shall now specialize the discussion, turning to terrestrial surface temperatures. Given the typical 100 years or so of temperature values, it becomes progressively less likely for any weather station to set a record high (or low), decaying as 1/n for stationary time series. In this sense, the classic lament by the elderly, e.g., "summers were hotter in my youth" is statistically valid insofar as record-breaking probabilities diminish as 1/n for stationary situations. Global warming, of course, being a trend, works against the 1/n decay and much work has been done recently (some of it by us, Anderson and Kostinksi (2010)) to extract trends this way e.g. Benestad (2004); Redner and Petersen (2006); Wergen and Krug (2010); Rowe and Derry (2012). A brief consideration of the extreme cases of steep and weak trend limits will provide some perspective. In the steep trend limit (regardless of whether the trend is linear), every value is a record high and the expected number of records then scales as $E(R) \sim n^1$ while in the stationary case $E(R) \sim \ln(n)$ so one can expect the general behaviour $E(R) \sim n^{\alpha}$ with $0 < \alpha < 1$ but with a generally n-dependent pre-factor (Edery, Kostinski, and Berkowitz 2011).

The simplest description of a global warming trend buried in noise can be incorporated via the temperature time series of the form (Ballerini and Resnick 1985)

$$T_k = T_{sk} + ck \tag{6}$$

where T_{sk} is the stationary (random) value, k is an integer and c represents the "drift", characterizing global warming. Of course, T_k and, therefore, T_{sk} being random variables, are associated with some standard deviation (natural variability) σ and one expects the parameter $C \equiv c/\sigma$ to figure prominently in the modified record-breaking statistics (see, for example, Newman, Malamud, and Turcotte (2010) for a straightforward approach to C). Indeed, for the linear Gaussian drift model, for example, in the $C \ll 1$ approximation, one obtains (Franke, Wergen, and Krug 2010)

$$P_n \approx \frac{1}{n} + \frac{c}{\sigma} \frac{2\sqrt{\pi}}{e^2} \sqrt{ln(\frac{n^2}{8\pi})}$$
(7)

Details of a specific model are not important for our purposes and the essential aspect is that while the drift c is viewed as "global", natural variability σ depends strongly on the location of a weather station as well as on the averaging period (e.g., daily vs. monthly temperatures). The latter is due to the range of temperature fluctuations e.g., being narrower near coasts, as well as by asymmetry of low and high temperatures, etc. A typical numerical value for the normalized drift c/σ , relevant to us below, is about 0.01 degrees per year (Wergen, Hense, and Krug 2013). However, as we noted earlier, the natural variability σ is not drawn from an ergodic ensemble because weather stations, say in coastal areas, differ greatly from those in the desert in Arizona. So the question is: given the the spatially heterogeneous natural variability, will the observed global drift alone suffice to set records in waves? Alternatively, can the normalized drift $C \equiv c/\sigma$ be regarded as a global parameter? If so, record-setting temperatures should arrive in bunches and wide-spread clusters on the entire globe. However, to the best of our knowledge, spatial correlations in record-setting have not been examined quantitatively.

A related question is: will record-breaking heat occur with spatial correlations similar to those of average monthly temperatures themselves? The answer is not obvious. Indeed, consider two nearby weather stations, say one in Chicago and another one in Milwaukee. Suppose that in 1921 (a relatively hot year), a heat wave reached and passed Chicago on the 17th of September but did not quite reach Milwaukee as the boundary of an associated front was sharper than the distance between the two cities. Hence, Chicago set a record high for that day but Milwaukee did not. Thus, the entire subsequent history of record-breaking in the two cities for all the 17 of September values were affected until such a time (if any) when a new record high occurred in both stations on Sept. 17. Clearly, although the underlying variables themselves (temperatures) are, likely, correlated because of the physical proximity of the two cities, it is not necessarily the case that the corresponding time-series of recordbreaking events are similarly correlated.

It is well known that temperatures at nearby weather stations are correlated, e.g., the twin cities, St. Paul and Minneapolis, will have coinciding hot and cold years. Will the neighbouring stations also set records in unison? Correlation radius for monthly temperatures has already been studied rather thoroughly, for example Hansen and Lededeff (1987), who found that temperatures tend to be correlated up to a distance of 1200 km in mid to high latitudes (the correlation coefficient threshold was was set at 0.5). If temperatures are correlated out to 1200 km, are record breaking events similarly correlated? In fact, reflecting on the question leads one to realize that the very notion of the correlation coefficient is inadequate for the task at hand as record-breaking events depend heavily on prior history and, therefore, represent non-stationary time series even if the parent variable is a stationary one. To that end, below we propose a tool for characterizing such situations.

To address the above questions we use monthly mean temperatures from the United States Historical Climatology Network, version 2.5.0.20130501 (USHCN) (Menne, Williams, and Vose 2009). The US is chosen for this analysis because it is the densest region of stations extending back to 1900. Since our focus is on spatial correlation, a dense collection of stations is preferable. We use time series that have at least 90 years of data between the years 1900 and 2010 and expect 5.09 to 5.31 record-breaking events in a stationary and independent time series with 90 to 113 values. This results in 8290 time series (station-months) from about 690 stations (each month comprises its own time series); 38% of station-months were too short in duration and were excluded from the study. We use the adjusted data set, which accounts for irregularities in the raw data such as time of observation bias. Values that are estimates or are missing more than three entries in a monthly average are not used in this study, see Menne *et al.* (2009) and Menne, Williams, and Palecki (2010) for details regarding estimates and adjustments.

So, do record-setting temperatures arrive in spatial clusters and, if so, what is the cluster size? To answer these questions, we begin by examining the *fraction* of weather stations setting record highs and lows in the USHCN data. As a benchmark, we perform corresponding analyses for an independent, identically distributed (i.i.d.) Monte Carlo ensemble of the same dimensions as the used USHCN data, see Figure 1, panel (a). Immediately, the far greater variability of the fraction of stations with a record (y axis) in the USHCN data than the benchmark i.i.d. ensemble becomes evident. We see that the variability of both, record highs (orange) and record lows (blue) is much greater for USHCN than an i.i.d. Monte Carlo ensemble of the same dimensions. This contrast suggests that record-setting occurrences in USHCN time series are, indeed, correlated and, therefore cluster. Can the observed global warming trend (drift c) alone account for this? Panel (b) shows that the observed drift (mean trend, characterizing global warming) can not deliver such variability and, therefore, implicates spatial coherence of natural variability. The interplay is subtle, however, since the records certainly "know" about the mean trend: in panel (a) we see that between 1960 and

1980 there are several years with a deficiency of record highs and between 1998 and 2012 there is a deficiency of record lows. These are known cool and warm periods respectively, e.g. Shen, Lee, and Lawrimore (2012).

To test further, we mimic the mean trend in the USHCN data, e.g. Trenberth *et al.* (2007), and add a linear trend to an otherwise i.i.d. Monte Carlo ensemble (same dimensions are USHCN). We assume a normal distribution with an initial standard deviation equal to the average standard deviation for all series used in USHCN, 1.91°C. The results are also shown in Figure 1, see panel (b). We use two trends mimicking those reported in IPCC for land in the Northern hemisphere: $\Delta \mu_1 = 0.063^{\circ}$ C/decade, 1850-2005, and $\Delta \mu_2 = 0.344^{\circ}$ C/decade, 1979-2005 (Trenberth *et al.* 2007). We see that with the simulated IPCC trends, the fraction of stations with a record has less variability than observed in the USHCN data and the fraction values are less extreme. Furthermore, i.i.d. Monte Carlo ensembles with piecewise trends have the same result – much less variability of fraction of highs or lows. Even when we compute a (LOWESS) smoothed trend based on USHCN anomalies and substitute it for a trend, we do not achieve the variability observed in the data. So, it is unlikely that the correlation, evident in the variability of results of Figure 1) is the result of the temporal trend alone. We now proceed to examine the likely spatial coherence in more detail.

To that end, we examine the behavior of records on either side of the boundary for correlation of the mean monthly temperatures themselves. Hansen and Lededeff (1987) found monthly temperatures to be correlated for about 1200 km (taking the threshold as correlation coefficient > 0.5). Therefore, we examined neighbouring stations in two categories: (1) neighbours with strongly correlated mean monthly temperatures (0-1200 km) and (2) neighbours with weakly correlated mean monthly temperatures (beyond 1200 km). As a benchmark, the corresponding analyses for i.i.d. Monte Carlo ensembles of the same dimensions as the used USHCN data are also shown in Figure 2. The analysis is as follows: for a single year, say 1950, for each record-breaking station, we compute the fraction of its neighbours (first nearer than 1200 km, panel (a), then farther than 1200 km, panel (b)) that also set temperature records. Then we compute the average of these fractions, to obtain the y-axis value in Figure 2. One expects that the closer stations are, the more likely they are to set records in unison. Indeed, in panel (a) we see, almost exclusively, an excess of records – when a station sets a record, its neighbours are more likely to set a record. This is true for both record highs and lows. Meanwhile, in panel (b) the message changes – distant neighbours do not always set records in unison, but instead may be more likely not to set a record. In fact, they are somewhat anti-correlated.

While Figure 2 suggests that 1200 km is a reasonable division between correlated and uncorrelated record-breaking, we want to look closer, considering the fractions of neighbours with records in concentric rings around a home station. Figure 3 demonstrates how we propose to do this. This is motivated by the notion of pair-correlation or radial distribution functions, often used in condensed matter physics. To find the fraction of records occurring as a function of distance from the base station at the origin, we consider concentric circles as depicted in panel (a). In panel (b), for each (base) station with a record-breaking high, we compute the fraction of its neighbours that also have a record high and then plot the average fraction. Note that for an extreme year like 2012 (the hottest in continental U.S. since 1900), the correlation

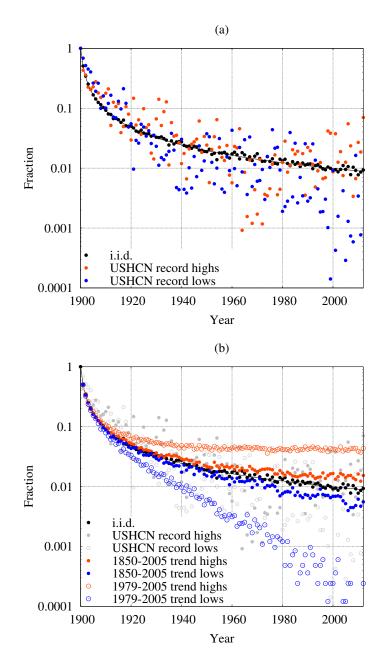


Figure 1: Fraction of stations setting records vs. the calendar year. Panel (a): the fraction of stations with a record high (orange) and low (blue) is shown per year for USHCN. Results for an i.i.d. Monte Carlo ensemble of the same dimensions as the used USHCN data is also shown (black). Panel (b): record breaking highs (orange) and lows (blue) for Monte Carlo ensembles of independent time series with two linear trends are shown that mimic trends reported by IPCC: 0.063°C/decade for 1850-2005 and 0.34°C/decade for 1979-2005 (Trenberth *et al.* 2007). For reference, results for USHCN record highs (gray filled circles) and lows (gray open circles) and an i.i.d. Monte Carlo ensemble (black) are included (same as in in panel (a)). The far greater variability of the fraction of stations with a record in the USHCN data versus the benchmark i.i.d. (identical and independently distributed) ensemble suggests that records in USHCN are correlated and, therefore, come in "bunches". Panel (b) shows that the observed mean trends alone do not account for the observed variability.

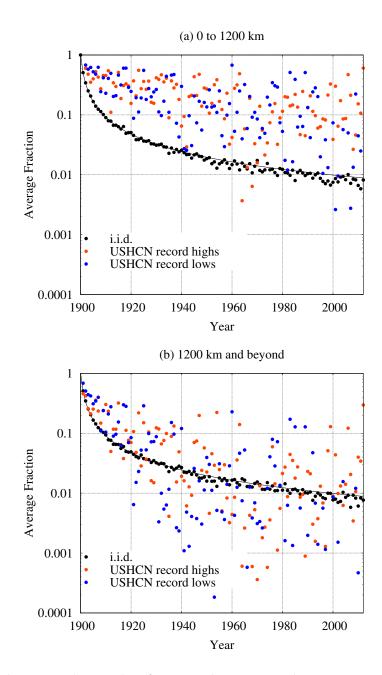


Figure 2: Same data as in the panel a of Figure 1 but separated into two parts: stations within 1200 km of each other (panel a) and the rest (figure b). The 1200 km boundary is chosen because it characterizes correlation radius for the monthly mean temperatures (see text). Record breaking highs and lows are spatially correlated out to at least 1200 km on average. The average fraction of neighbouring stations that set a record breaking high (orange) or low (blue) is shown for all stations that set a record breaking high. Neighbours in each panel are (panel a) those with correlated mean monthly temperatures (0-1200 km) and (panel b) stations with uncorrelated temperatures (1200 km and beyond); see text. Averages of the benchmark i.i.d. Monte Carlo ensembles are shown in black.

	Hottest Years		Coldest Years		Median Years	
Rank	Year	Relative Rank	Year	Relative Rank	Year	Relative Rank
1	2012	1 of 113	1917	18 of 18	1927	9 of 28
2	1998	1 of 99	1912	13 of 13	1942	18 of 43
3	2006	2 of 107	1924	23 of 25	1963	27 of 64
4	1931	1 of 32	1904	5 of 5	2008	53 of 109
5	1921	1 of 22	1978	75 of 79	2009	52 of 110

Table 1: In Figure 4 we examine records in three groups: 5 hot, 5 cold, and 5 typical years. The relative ranks for these years (hottest to coldest) are given here. The median years are listed chronologically rather in order of rank.

radius for record-setting temperatures far exceeds that of the monthly mean temperatures. Is this true for all years? Not so.

It turns out that the results of analysis illustrated in Figure 3 vary a great deal from year to year and this variation is summarized in Figure 4. We chose five hot, five cold and five unremarkable years to compute the correlation radius. These years are shown in Table 1 and Figure 4. The ranking is based on analysis of our USHCN subset (see above). Note, however, that other studies find slightly different rankings. For example, hottest years given by Shen et al. are 1998, 2006, 1934, 1921, 1999, etc. and the coldest years are 1917, 1895, 1912, 1924, 1903, etc., for the period 1985-2008, Shen *et al.* (2012).

It is evident from the panel (a) of Figure 4 that, during the hot years, spatial correlation extends well beyond 1200 km so that record-breaking heat arrives in huge waves. Somewhat surprisingly, in panel (b) we see similar pattern for record highs in one cold year (1904) but a much more rapid drop-off for other years. Overall, one expects fewer record highs in a cold year and this expectation is not in conflict with Figure 4. Observe also that if there is a record low set at the central station, its neighbours are likely to also set record lows. The correlation radius for these years appears to be well below the average for the five hottest years. Similarly, panel (c) shows that natural variability, as manifested by variation of the spatial correlation length from year to year, is remarkably pronounced. In passing, we note that Monte Carlo i.i.d. ensembles involve averaging over the number of stations with a record and with neighbours. The number of stations with records is constant along the x axis (approximately 1/n, where n is number of years in time series, dictated by year), but the number of neighbours is not. The latter begins to drop off around 2500 km, and therefore, variability increases there (approximately $1/\sqrt{N}$, where N is the number of neighbours.)

The data are very interesting and somewhat puzzling. Why should the remarkably hot (hottest year on record) 2012 be almost matched in the spatial coherence of record-setting temperatures by the seemingly unremarkable 2009 (Figure 4, panel c)? Should the record-setting be attributable mostly to the global warming trend (drift) C in an otherwise random and spatially incoherent random field, the correlation radius would presumably follow the "heat" rank of the year. This does not appear to be the case and the underlying spatial coherence of the natural variability σ plays an important role.

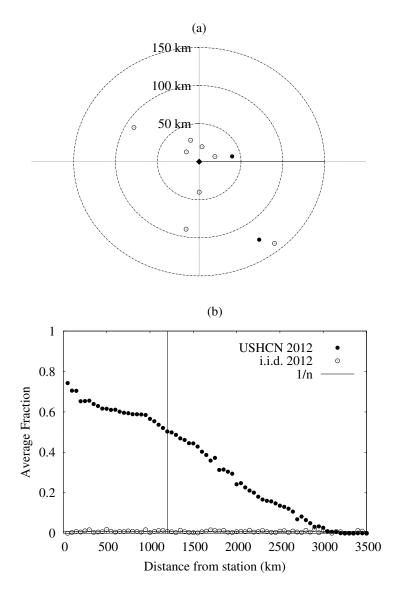


Figure 3: Average fraction of record-setting neighbours versus the distance from a given record-setting station. Panel (a): Computing the fraction of records occurring as a function of distance from a station at the origin, by counting within the concentric circles as depicted. A record-breaking station (black diamond) is at the origin and its neighbours are examined. Small circles represent stations, filled circles are stations with records. As illustrated in this figure, the fraction of record-breaking stations nearest to the station at the origin (within 0-50 km) is 1/6; 0/2 stations are between 50-100 km have a record and 1/2 stations between 100-150 km have a record. Panel (b): For each station with a record-breaking high, the fraction of its neighbours that also have a record high, is computed. The average fraction is depicted. A vertical line is shown at 1200 km, the point at which monthly temperatures drop below correlation coefficient of 0.5) (Hansen and Lededeff 1987). For an extreme year like 2012 (the hottest in continental U.S. since 1900), the correlation radius for records is considerably longer than anticipated by studies of the mean monthly temperatures.

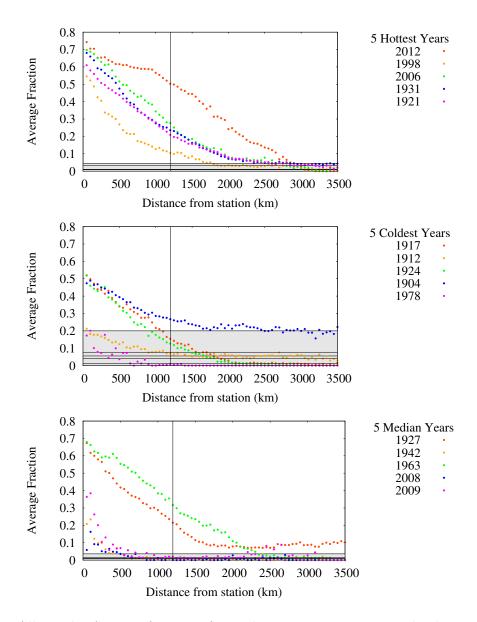


Figure 4: All panels: Average fraction of record-setting stations versus the distance from the base station. Panel (a): 5 hottest years. Panel (b); 5 coldest years. Panel (c): 5 typical years. 1900-2012. The average fraction of neighbouring stations that have a record breaking high is shown for all stations that have a record breaking high. A vertical line is shown at 1200 km, the point at which station temperature variables are considered to be uncorrelated (correlation defined as coefficient > 0.5) (Hansen and Lededeff 1987). Note that the correlation radius varies a great deal from year to year, indicating pronounced natural variability. I.i.d. Monte Carlo ensembles are included for reference, see the text for details. The grey shaded regions demonstrate the expected average fraction of neighbours with simultaneous record breaking. The upper limit corresponds to the shortest time series displayed, 22 years for (a), 5 years for (b), and 28 years for (c). Black horizontal lines below the upper limit correspond to expectations for longer time series corresponding again to the longer time periods shown.

3. Concluding Remarks

Viewing the attribution problem in terms of the competition between the global trend cand natural variability σ , the following question was posed here: given the the spatially heterogeneous natural variability, will the observed global drift alone suffice to set records in waves as observed? Alternatively, can the normalized drift $C \equiv c/\sigma$ be regarded as a global parameter? If so, record-setting temperatures should arrive in bunches and widespread clusters on the entire globe. However, spatial correlations in record-setting have not, to the best of our knowledge, been previously examined. Therefore, we used record-breaking statistics to examine spatial coherence of record-setting terrestrial surface temperatures, with the eye toward disentangling effects of global warming (mean drift or trend) from natural variability.

Devising an analogue of a pair-correlation function we have been able to examine recordsetting temperatures by quantifying the size of underlying spatial clusters. We find that record-breaking of average monthly temperature time series exhibit spatial correlations quite different from those of the temperatures themselves. In particular, the variability of the correlation radius is much more pronounced for the record-breaking events. Furthermore, there is no clear correspondence between the extent of spatial correlations and the heat rank of the year, thereby indicating that the spatial coherence of natural variability plays an important role in setting up "waves" of extreme events.

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