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Reply to "Comment on 'Cooperation in an evolutionary prisoner's dilemma on networks with degree-degree correlations' "

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We respond to the comment of Zhu*et al.* [Phys. Rev. E **82**, 038101 (2010)] and show that the results in question are not misleading.

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Zhu *et al.* make several claims to which we respond below. It is worth pointing out however, that much of the content of their comment first appeared in [1].

The first point made by Zhu *et al.* is that the quantity $\nu_c - \nu_{st}$ used in [2] cannot account for degree correlations. However, we did not claim that $\nu_c - \nu_{st}$ accounted for network degree correlations, but rather, that it could be used to measure the added heterogeneity that is present in a network with degree-degree correlations, and which is undetectable from only the degree distribution.

Next, Zhu *et al.* claim that "The degree-degree correlation for each reshuffled network can be obtained by calculating the Pearson correlation coefficient r [3]." This is not true. A Barabási-Albert (BA) network is not uncorrelated: the probability of reaching a degree j vertex from a degree k vertex depends on k. This is the case despite the fact that the correlation coefficient r of a BA network is very close to zero. If one uses the degree distribution of a BA network to generate a configuration model (CM) analog (with the same degree distribution), then the CM network is indeed uncorrelated in the sense above. At the same time, both these networks have correlation coefficients $r \approx 0$. So, while r is certainly a tool that can be used to address correlations, it is misleading to assert that r is the measure of correlations.

Next, Zhu *et al.* use reshuffled BA networks to simulate an evolutionary prisoner's dilemma for a particular game parameter, and find that cooperation is highest with no reshuffling. From this, they conclude that "the presence of degree-degree correlation is harmful to the co-operation," and furthermore, "the presence of small amount of correlation always deteriorates cooperation."

There are three points to make here. First, note that cooperation thrives more on a BA network than on a CM network with the same degree distribution [4]. Even though both networks have $r \approx 0$, the BA network has degree-degree correlations that the CM network does not have. This runs counter to the claim in Zhu *et al.* that any correlations are detrimental to cooperation. For more examples that contradict this claim of Zhu *et al.*, see [5].

Second, since the CM gives a maximally random (uncorrelated) network with fixed degree distribution, it seems to us that the CM should serve as the natural baseline for comparison of cooperation phenomena—not a (correlated) BA network. That is the reason, in [2], that we considered networks that ranged from random CM, to BA, via an increasing percentage of vertices added by the growth and preferential attachment algorithm. For each network, we measured the added heterogeneity (beyond that coming from the degree distribution) arising from correlations introduced through the GPA algorithm (not the correlations themselves).

Finally, it is true that the reshuffled networks that Zhu *et al.* study, following [1], are not well behaved with respect to our measure of added heterogeneity, $\nu_c - \nu_{st}$. However, we would somewhat wary of making any firm conclusions about the specific role played by correlations in cooperation phenomena in these cases. First, they start with a baseline BA network that they consider uncorrelated, but as mentioned above, has correlations present to begin with. They then apply a nonrandom rewiring algorithm whereby larger vertices are more likely to be selected and rewired. The kinds of network that arise are difficult to categorize; for instance, they have highly non monotonic nearest neighbor functions. What does r mean when the nearest neighbor function is nonmonotonic (and nonlinear)? Further, one could reasonably ask if these reshuffled networks can be considered heterogeneous in the usual sense (see [5] for more discussion on this).

In conclusion, we have no argument with the results of Zhu *et al.* However, in the case of measuring added heterogeneity introduced by the GPA algorithm as in [2], there is no contradiction.

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