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A Simple Demonstration of Mie Scattering Using an Overhead Projector

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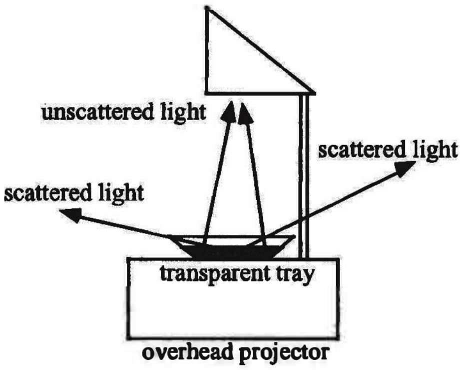

Fig. 1. Demonstration setup. In our demonstrations, the tray is a small, transparent plastic plate.

Light scattering by small particles is a complicated and sometimes counterintuitive subject. In particular, the dependence of the scattering cross section on wavelength of light can be quite different for different particle sizes. For example, Rayleigh scattering by particles that are small compared to the wavelength is responsible for the red sunset and blue sky, while scattering by larger particles can lead to the opposite effect: the so-called "blue moon" or "green sun" occasionally seen after forest fires or volcanic explosions.[1] Straightforward classroom demonstrations of light scattering exist: The Rayleigh scattering regime can be created by using a dilute suspension of milk in water, while demonstrations involving larger particles can be created using cigarette smoke.[1,2] However, the problem with both of these methods is that one cannot effectively control the size (or shape) of the scatterers, or even say much about the distribution of particle sizes.

A simple demonstration of the particle-size dependence of Mie scattering can be performed using an overhead projector and two suspensions of polystyrene microspheres: one with 500-nm-diam particles and the other with 1740-nm-diam particles. Suspensions of polystyrene microspheres are widely available; the particles are almost perfectly spherical, and the particle radii vary by no more than about ±1% in a typical suspension. The range of available sizes is also quite large; one can purchase suspensions with mean particle size from under 100 nm to greater than 100 μm. The refractive index of the particles is about 1.59 in the visible region of the spectrum, leading to a relative refractive index of 1.19 when the particles are suspended in water.

For the demonstration, an overhead projector is set up with a transparent tray on the platen. (A clear plastic plate, like the kind used for picnic lunches, is ideal.) The tray is filled with water to a depth of 2 or 3 mm, and the image of the tray is projected onto a screen, as shown in Fig. 1. The 500-nm-diam scatterers are now mixed in with the water, and the image on the screen turns a brownish-red! To make the demonstration more dramatic, the micropsheres should be dripped into the water directly from their original plastic squeeze bottle. The bottle can be passed around to the students before or after the demonstration to show that the particles are not colored in any way. After this, a new tray is placed on the overhead and the 1740-nm-diam particles are dripped into the water. This time, the image turns a vivid, dark blue!

Here's what's happening: Some of the light from the overhead lamp that passes through the suspension is scattered by the polystyrene microspheres. The scattered light misses the imaging optics of the overhead. (For a description of how an overhead projector works, see Goodman's wonderful little book.[2]) However, the scattering cross section of the particles is strongly wavelength dependent. For transparent spheres with relative index of refraction close to 1, the scattering cross section can be approximated by[3]

$$Q=2-\frac{4}{\rho}\sin\rho+\frac{4}{\rho^2}(1-\cos\rho), \tag{1}$$

where $Q=C_{\text{scat}}/\pi a^2$, C_{scat} is the scattering cross section, a is the particle radius (250 and 870 nm, respectively), and $\rho=4\pi(m-1)na/\lambda$. Here, $m=1.19$ is the relative index of refraction, $n=1.33$ is the index of the medium surrounding the particles, and λ is the wavelength of the incident light in air. In the words of van de Hulst, "this is one of the most useful formulae in the whole domain of Mie scattering, because it describes the salient features of the extinction curve not only for values of m close to 1, but even for values of m as large as 2."[3]

Figure 2 shows a graph of Q vs ρ for $n=1.33$ and $m=1.19$. We show both the exact Mie theory cross section and the approximation of Eq. (1). The chief differences between the two are that Eq. (1) underestimates Q by about 20% for the values of ρ examined here, and does not have the high-frequency ripple for large values of ρ. As is seen, Q is an oscillatory function of ρ. For fixed particle radius, increasing ρ implies decreasing wavelength, while for fixed wavelength, ρ increases as the particle size increases. For large ρ, Q approaches 2, meaning that for particles very large compared to the wavelength, the scattering cross section approaches $2\pi a^2$.[3] Several regions of the extinction curve are boxed in Fig. 2. The boxes enclose the region of the scattering curve for visible wavelengths of light (400–700 nm) for four different particle diameters. Region (a) represents the cross section for light scattering by particles with a fixed diameter of 500 nm, and region (b), a fixed diameter of 1740 nm. Note that for region (a), the center of the scattering region in Fig. 2 is approximately at the point where the slope of the extinction curve is greatest, implying the greatest difference in the scattering cross section for blue and red light. Because the cross section is greater for shorter wavelengths than for longer wavelengths, the image of the suspension will appear red, as more blue light is scattered out of the system than red. It should be noted that the wavelength dependence of the

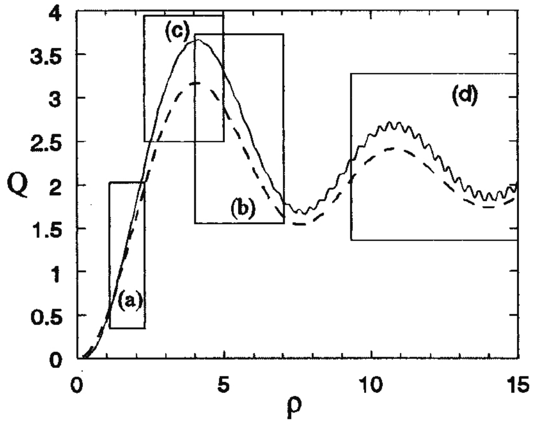

Fig. 2. Scattering cross section (Q) as a function of size parameter (ρ). The dashed line was calculated using Eq. (1), which is valid for small m, while the solid line was calculated using Mie theory. The boxes on the graph enclose the scattering curve for wavelengths in the visible region of the spectrum for the following particle diameters: (a) scattering cross section for 500-nm-diam particles as a function of ρ; (b) scattering cross section for 1740-nm-diam particles as a function of ρ; (c) scattering cross section for 1000-nm-diam particles as a function of ρ; (d) scattering cross section for 4000-nm-diam particles as a function of ρ. The entire scattering region for this particle size extends slightly beyond the end of the graph (to $\rho = 15.8$).

scattering cross section for the smaller particles cannot be explained by Rayleigh scattering.[3] Rayleigh scattering is only valid for particle radii small compared to the wavelength, which is not true here.

The situation is reversed for the 1740-nm-diam particles. Here, the center of region (b) in Fig. 2 lies where the slope of the extinction curve is near its minimum value (i.e., strongly negative). This means that the scattering cross section is greater for longer wavelengths, leading to the blue image. Note that the two particle sizes were chosen so that the *slope* of the scattering curve would be extremal, not the scattering curve itself.

The scattering region for particles with a diameter of 1000 nm for visible light is shown in box (c) of Fig. 2. Here, the center of the scattering curve lies near the maximum value of Q. This means that the image should be essentially uncolored, as the scattering cross section is almost the same for visible wavelengths. Doing the demonstration using these particles confirms this—no coloration of the image is seen.

One can perform similar demonstrations using larger particles. However, there is a complication: For larger particles, the range of ρ values corresponding to visible wavelengths increases as the particle diameter increases. This can be seen by comparing the two regions for the 500- and 1740-nm-diam particles: The range of ρ values is over three times as large for the larger particles as it is for the smaller ones. Could one use this to choose a particle size where both red and blue light were strongly scattered, leading to an image which was colored yellow or green? The range of ρ values for a 4000-nm-diam particle lies between about 9.1 and 15.8; this extends from just before one maximum in the scattering cross section to near the next maximum, implying that intermediate wavelength light (yellow and green) should be scattered least, and long- and short-wavelength light scattered most. Part of the scattering region is enclosed by box (d) in Fig. 2, although it extends beyond the end of the graph. We have performed this experiment: The resulting image looks dark grayish green. It may be possible to make the coloration more vivid by a better choice of particle size (4300-nm-diam particles would probably work better), but we have not tried this yet. If any readers make the attempt, we would be very interested in learning the results of their work.

The one drawback with this demonstration is the relatively high cost of polystyrene microspheres—roughly \$200 for a 15-ml bottle. One bottle is good for at least 20 demonstrations, however, which is probably good for a few years. (Our bottles are usually depleted because we do this demonstration quite often!) One can mix the suspensions ahead of time so that they can be used again, although this robs the demonstration of a good deal of drama. Finally, if any readers are interested in images we have made from the scattering demonstrations listed here, they can contact the authors.

[a] Electronic mail: cladler@osprey.smcm.edu.

[1] Craig Bohren, *Clouds in a Glass of Beer* (Wiley, New York, 1987), pp. 91–97.

[2] Douglas S. Goodman, *Optics Demonstrations on the Overhead Projector* (Optical Society of America, Washington, DC, 2000), p. S-6.

[3] H. C. van de Hulst, *Light Scattering by Small Particles* (Wiley, New York, 1957), pp. 176–178.