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Heat Transfer in a Nanofluid Flow past a Permeable Continuous Moving Surface

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HEAT TRANSFER IN A NANOFLUID FLOW PAST A PERMEABLE

CONTINUOUS MOVING SURFACE

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DEDICATED TO

My family for their blessings, continued support and encouragement

throughout my career.

HEAT TRANSFER IN A NANOFLUID FLOW PAST A PERMEABLE CONTINUOUS MOVING SURFACE

SARVANG D SHAH

ABSTRACT

The main purpose of this paper is to introduce a boundary layer analysis for the fluid flow and heat transfer characteristics of an incompressible nanofluid flowing over a permeable isothermal surface moving continuously. The resulting system of non-linear ordinary differential equations is solved numerically using Runge-Kutta method with shooting techniques. Numerical results are obtained for the velocity, temperature and concentration distributions, as well as the friction factor, local Nusselt number and local Sherwood number for several values of the parameters, namely the velocity ratio parameter, suction/injection parameter and nanofluid parameters. The obtained results are presented graphically and in tabular form and the physical aspects of the problem are discussed.

Keywords: Suction/injection, moving surface, nanofluid

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NOMENCLATURE

D_B Brownian diffusion coefficient

D_T thermophoretic diffusion coefficient

f reduced stream function

g gravitational acceleration

k_m effective thermal conductivity of the porous material

K permeability of porous medium

Le Lewis number

N_b Brownian motion parameters

N_t thermophoresis parameters

Nu Nusselt number

p pressure

q wall heat flux

Re Reynolds number

T temperature

T_w wall temperature of the vertical plate

T_∞ ambient temperature

u_∞ freestream velocity

u, v Darcy velocity components

(x, y) Cartesian coordinates

Greek Symbols

α_m	thermal diffusivity of porous medium
η	dimensionless distance
θ	dimensionless temperature
μ	viscosity of fluid
ρ_f	fluid density
ρ_p	nano-particle mass density
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_m$	effective heat capacity of porous medium
$(\rho c)_p$	effective heat capacity of nano-particle material
τ	ratio between the effective heat capacity of the nano particle material and that of the fluid
ϕ	nano-particle volume fraction
ϕ_w	nano-particle volume fraction at the wall of the plate
ϕ_∞	ambient nano-particle volume fraction
ψ	stream function

Subscript

B	Blasius problem
S	Sakiadis problem
w	Refers to condition at wall
∞	Refers to condition far from the wall

CHAPTER I

INTRODUCTION

The study of convective heat transfer in nanofluids is gaining a lot of attention. The nanofluids have many applications in the industries since materials of nanometer size have unique physical and chemical properties. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically of 1-100 nm suspended in liquid. Nanofluids have attracted great interest recently because of reports of greatly enhanced thermal properties. For example, a small amount (< 1% volume fraction) of Cu nanoparticles or carbon nanotubes dispersed in ethylene glycol or oil is reported to increase the inherently poor thermal conductivity of the liquid by 40% and 150% respectively [1, 2]. Conventional particle-liquid suspensions require high concentration (>10%) of particles to achieve such enhancement. However, problems of rheology and stability are amplified at high concentration, precluding the widespread use

of conventional slurries as heat transfer fluids. In some cases, the observed enhancement in thermal conductivity of nanofluids is orders of magnitude larger than predicted by well-established theories. Other perplexing results in this rapidly evolving field include a surprisingly strong temperature dependence of the thermal conductivity [3] and a three-fold higher critical heat flux compared with the base fluids [4, 5]. These enhanced thermal properties are not merely of academic interest. If confirmed and found consistent, they would make nanofluids promising for application in thermal management. Furthermore, suspensions of metal nanoparticles are also being developed for other purposes, such as medical applications including cancer therapy. The interdisciplinary nature of nanofluid research presents a great opportunity for exploration and discovery at the frontiers of nanotechnology.

The characteristics of flow and heat transfer of a viscous and incompressible fluid over flexed or continuously moving flat surfaces in a moving or a quiescent fluid are well understood. These flows occur in many manufacturing processes in modern industry, such as hot rolling, hot extrusion, wire drawing and continuous casting. For example, in many metallurgical processes such as drawing of continuous filaments through quiescent fluids and annealing and tinning of copper wires, the properties of the end product depends greatly on the rare of cooling involved in these processes. Sakiadis [5] was the first one to analyze the boundary layer flow on continuous surfaces. Crane [6] obtained an exact solution the boundary layer flow of Newtonian fluid caused by the stretching of an elastic sheet moving in its own plane linearly. Tsou et al. [7] extended the research to the heat transfer phenomenon of the boundary layer flow on a continuous moving surface. Schowalter [8] applied the boundary layer theory into power law pseudoplastic

fluids and developed two and three dimensional boundary layer equations of the momentum transfer. Acrivos [9] analyzed the momentum and heat transfer of non-Newtonian fluid past arbitrary external surfaces. Howell [10] and Rao [11] investigated momentum and heat transfer phenomena on a continuous moving surface in power law fluids. Magyari and Keller [12, 13] have studied the thermal boundary layer of moving surfaces. Wang [14] studied free convection from a vertical stretching surface. Gorla and Sidawi [15] studied the characteristics of flow and heat transfer from a continuous surface with suction and blowing.

We present here a similarity analysis for the problem of steady boundary-layer flow of a nanofluid on a continuous moving permeable isothermal surface. The development of the velocity, temperature and concentration distributions have been illustrated for several values of nanofluid parameters, Prandtl number, Lewis number, velocity ratio and suction/injection parameters. Normally, for heat transfer application nanofluid works best for volume fraction of nano particle to fluid less than 10%. If concentration is more than 10% then a problem of stability may occur.

CHAPTER II

LITERATURE REVIEW

In recent time there are so many research work going on nanofluids about understanding their behavior so that they can be utilized where straight heat transfer enhancement is paramount as in many industrial applications. Described below are a few papers which show work done in the area of heat transfer in nanofluid. There are few papers also which describes earlier and recent work done for heat transfer in continuous moving surface for different types of fluid flow. Also, few papers are included here as they are somewhat relevant to the study area covered in this thesis report.

Kuznestov and Neild [16, 17] have studied the convective heat transfer in a nanofluid past a vertical plate. They have used a model in which Brownian motion and thermophoresis are accounted with the simplest possible boundary conditions, namely those in which both the temperature and the nanoparticle fraction are constant along the

wall. Using this they got solution which depends on five dimensionless parameters, namely a Prandtl number Pr, a Lewis number Le, a buoyancy-ratio parameter Nr, a Brownian motion parameter N_b, and a thermophoresis parameter N_t. They have explored the way in which the wall heat flux, represented by a Nusselt number Nu and then scaled in terms of $Ra_x^{1/4}$ - local Rayleigh number define in paper to produce a reduced Nusselt number, depends on these five parameters. They have also studied the Cheng–Minkowycz problem of natural convection past a vertical plate analytically in a porous medium saturated by a nanofluid. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis number. For the porous medium the Darcy model is used. In this study they have employed the Darcy model for the momentum equation and assumed boundary conditions in which both the temperature and the nanoparticle fraction are constant along the wall. This permits a solution which depends on four dimensionless parameters, namely a Lewis number Le, a buoyancy-ratio parameter N_r, a Brownian motion parameter N_b, and a thermophoresis parameter Nt.

Bachok, Ishak and Pop [18] have studied the steady boundary-layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream. They assumed that the plate is moving in the same or opposite directions to the free stream to define resulting system of nonlinear ordinary differential equations. They solved governing equation using the Kellerbox method. The results are obtained for the skin-friction coefficient, the local Nusselt number and the local Sherwood number as well as the velocity, temperature and the nanoparticle volume fraction profiles for the governing parameters, namely, the plate velocity parameter, Prandtl number, Lewis number, the Brownian motion parameter and the thermophoresis parameter.

Khan and Pop [19] have studied the problem of laminar fluid flow which results from the stretching of a flat surface in a nanofluid and investigated it numerically. The model they used for the nanofluid incorporates the effects of Brownian motion and thermophoresis and found solution which depends on the Prandtl number Pr, Lewis number Le, Brownian motion number N_b and thermophoresis number N_t. They showed variation of the reduced Nusselt and reduced Sherwood numbers with N_b and N_t for various values of Pr and Le in tabular and graphical forms and conclude that reduced Nusselt number is a decreasing function while the reduced Sherwood number is and increasing function of each values of the parameters Pr, Le, Nb and Nt considered for study.

Ahmad and Pop [20] have studied steady mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with nanofluids. They used different types of nanoparticles as Cu (cuprom), Al_2O_3 (aluminium) and TiO_2 (titanium). The governing partial equations are reduced to an ordinary differential equation and solved numerically in matlab using shooting method for some values of the volume fraction and mixed convection parameters. The solution has positive or negative value for certain range of the parameters. The effects of these parameters on the velocity distribution are presented graphically. They also compare their work with the earlier work done by J. H. Merkin.

Polidori, Fohanno and Nguyen [21] have studied the problem of natural convection flow and heat transfer of Newtonian alumina–water nanofluids over a vertical semi-infinite plate from a theoretical viewpoint, for a range of nanoparticle volume fractions up to 4%. The analysis is based on a macroscopic modeling and under the

assumption of constant thermophysical nanofluid properties. They proposed Semi-analytical formulas of heat transfer parameters for both the uniform wall temperature (*UWT*) and uniform heat flux (*UHF*) surface thermal conditions and found that natural convection heat transfer is not solely characterized by the nanofluid effective thermal conductivity and that the sensitivity to the viscosity model used seems undeniable and plays a key role in the heat transfer behavior.

Koo and Kleinstreuer [22] have studied steady laminar liquid nanofluid flow in microchannels for conduction-convection heat transfer for two different base fluids water and ethylene glycol having copper oxide nanospheres at low volume concentrations. They conjugate heat transfer problem for microheat-sinks solved numerically. They employed new models for the effective thermal conductivity and dynamic viscosity of nanofluids in light of aspect ratio, viscous dissipation and enhanced temperature effects for computation of the impact of nanoparticle concentrations in these two mixture flows on the microchannel pressure gradients, temperature profiles and Nusselt numbers.

Xuan and Li [23] have experimentally investigated flow and convective heat transfer characteristics for Cu–water based nanofluids through a straight tube with a constant heat flux at wall. Results showed that the nanofluids give substantial enhancement of heat transfer rate compared to pure water.

Khan and Sanjayanand [24] have studied viscoelastic boundary layer flow and heat transfer over an exponential stretching continuous sheet. In this study, they focused on approximate analytical similarity solution of the highly non-linear momentum equation and confluent hypergeometric similarity solution of the heat transfer equation. They verified accuracy of the analytical solution for stream function using Runge–Kutta

fourth order method with shooting method in Matlab. These obtained solutions involve prescribed boundary temperature and prescribed boundary heat flux on the flow directional coordinate for an exponential dependent of stretching velocity. The effects of various physical parameters like viscoelastic parameter, Prandtl number, Reynolds number, Nusselt number and Eckert number on various momentum and heat transfer characteristics are also discussed.

Arnold, Asir, Somasundaram and Christopher [25] have studied the viscoelastic fluid flow and heat transfer characteristics over a stretching sheet with frictional heating and internal heat generation or absorption. The heat transfer analysis has been carried out for the cases of prescribed surface temperature (PST) and prescribed surface heat flux (PHF). The momentum equation is decoupled from the energy equation for the present incompressible boundary layer flow problem with constant physical parameters. Exact solution for the velocity field and the skin-friction are obtained while solutions for the temperature and heat transfer characteristics are obtained in terms of Kummer's function. The work due to deformation in energy equation, which is essential while formulating the viscoelastic boundary layer flow problems, is considered. In this paper they examines the effect of viscoelastic parameter, Eckert number, Prandtl number and non-uniform heat source/sink parameter on temperature distribution, wall temperature gradient for PST-case and wall temperature for PHF-case. In the result they found that the magnitude of the non-dimensional surface velocity gradient is found to increase with increasing the viscoelastic parameter (k_1) – a non-Newtonian parameter. They also found that magnitude of the non-dimensional surface temperature gradient increases with the Prandtl

number (Pr) and an increasing Pr causes reduction in the thickness of the thermal boundary layer.

Xu and Liao [26] have studied theoretical analysis of the laminar boundary-layer flow and heat transfer of power-law non-Newtonian fluids over a stretching sheet with the sheet velocity distribution of the form $U_w = Cx^m$ and the wall temperature distribution of the form $T_w = T_\infty + Ax^\gamma$ is presented, where x denotes the distance from the slit from which the surface emerges and C and A are constants, m and γ denote, the sheet velocity exponent and the temperature exponent, respectively. The nonlinear boundary layer momentum equation and the energy equation are reduced to a set of ordinary differential equations by them within the framework of the boundary layer approximations. They found that when the velocity exponent m = 1/3 or the power-law index n = 1, the similarity solutions are in existence for both the momentum equation and the energy equation. They formulate the global self-similarity equations for the flow problem and the accurate analytical approximations are then obtained with the help of the homotopy analysis method.

Yürüsoy [27] has studied Flow of a thin fluid film of a power-law caused by stretching of surface. He used a similarity transformation for reducing the unsteady boundary layer equations to a non-linear ordinary differential equation system. Numerical solutions of out coming nonlinear differential equations are found by using a combination a Runge–Kutta algorithm and shooting technique. He explored Boundary layer thickness numerically for different values of power-law index. In result he presented similarity solutions the shear-thinning and shear thickening power-law fluids and a comparison made with the Newtonian solutions.

Ishak, Nazar and Pop [28] have studied the analysis of the boundary-layer flow of a micropolar fluid on a fixed or continuously moving permeable surface. They considered both parallel and reverse moving surfaces to the free stream. The resulting system of non-linear ordinary differential equations is solved numerically using the Keller-box method and numerical results are obtained for the skin friction coefficient and the local Nusselt number for some values of the parameters, namely the velocity ratio parameter, suction/injection parameter and material parameter, while the Prandtl number is fixed to be unity. Their results indicate that dual solutions exist when the plate and the free stream move in opposite directions and they also observed that micropolar fluids show drag reduction characteristic compared to classical Newtonian fluids, and the boundary-layer separation is delayed for micropolar fluids or by introducing suction.

Ishak, Nazar and Pop [29] have also worked on steady boundary layer flow and heat transfer of a micropolar fluid on an isothermal continuously moving plane surface. They assumed that the microinertia density is variable and not constant and the viscous dissipation effect took into account. They used the Keller-box method for solving reduced to a system of nonlinear ordinary differential equations numerically to obtain result for the skin friction coefficient, local Nusselt number, as well as velocity, temperature and microrotation profiles. They also discussed effects of material parameter, Prandtl number and Eckert number (Ec) on the flow and heat transfer characteristics.

Hassanien [30] has studied the Boundary layer solutions to investigate the steady flow and heat transfer characteristics of a continuous flat surface moving in a parallel free stream of power-law fluid. He used similatity transformations for equations of motion to

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reduce to nonlinear ordinary differential equations. These equations are solved numerically with double precision, using a procedure based on finite difference approximations. The results are presented for the distribution of velocity and temperature profiles within the boundary layer. He also studied the effects of the power-law index of fluid on the shear stress at the wall and the rate of heat transfer. He stated in the paper that the solutions of the problems of flow past a moving continuous flat surface depend not only on the velocity difference but also on the velocity ratio. He shows numerical results for the fluid flow and heat transfer characteristics. He presented the missing wall values of the velocity and temperature function for a range of power-law index of the fluid. In the results he found that for the same values of the normalized velocity difference, power-law index, Re, and Pr, the case representing $u_w > u_\infty$ yields a larger surface friction factor and surface heat transfer rate when compared with the case where $u_w < u_\infty$.

Hassanien, Abdullah and Gorla [31] have studied the boundary layer analysis for the problem of flow and heat transfer from a power-law fluid to a continuous stretching sheet with variable wall temperature. They applied similarity transformation to reduce the Navier-Stokes and energy equations. The resulting system of nonlinear ordinary differential equations is by solved using the expansion of Chebyshev polynomials. They also performed parametric studies to investigate the effects of non-Newtonian flow index, generalized Prandtl number, power-law surface temperature and surface mass transfer rate. They have plotted velocity profiles for different power law index n, temperature profile for Prandtl number, velocity ratio and power law index keeping other parameter

fixed for skin friction coefficient and heat transfer rate and results exhibit that these both depends on fluid parameters.

Gorla and Reddy [32] have studied the boundary layer solution for the steady flow and heat transfer characteristics forma continuous flat surface moving in a parallel free stream of micropolar fluid. They presented numerical results for the distribution of velocity, mico-rotation and temperature profile within the boundary layer. They have used a similarity solution method to reduce the governing mass, energy and momentum equation into a non-linear ordinary differential equation. They have used fourth order Rugne-Kutta method of numerical integration to solve the equations. Missing wall values of velocity, angular velocity and temperature functions are tabulated for range of dimensionless groupings of material parameters of fluid. They founded that for the same values of normalized velocity difference, Re and Pr, the case representing $U_w > U_\infty$ yields larger surface friction factor and surface heat transfer rate compared to case when $U_w < U_\infty$.

Olajuwon [33] has studied the flow and convection heat transfer in a pseudoplastic power law fluid past a vertical plate with heat generation. In his study he used similarity transformation to transform governing non – linear partial differential equations which describes the flow and heat transfer problem into non –linear ordinary differential equation. The resulting problem after similarity transformation is solved numerically using Runge – Kutta shooting method. The power law exponents used by him for study is between 0 and 1. The analysis of results obtained showed that the heat generation parameter has significant influence on the flow and heat transfer and conclude that a pseudoplastic power law fluid with the power law exponent $0 < n < 1/2$ gives a

higher heat transfer coefficient than the pseudoplastic power law fluid with power law exponent $1/2 < n < 1$.

Ahmad, Siddiqui and Mishra [34] have studied the boundary layer flow of viscous incompressible fluid over a stretching plate for heat flow problem with variable thermal conductivity. They obtained velocity components using similarity transformation. The heat flow problem has been considered in two ways: (i) Prescribed surface temperature (PST), and (ii) Prescribed stretching plate heat flux (PHF) for variable thermal conductivity case. In their result they plot graphs for temperature profile for mean temperature and temperature profile induced due to variable thermal conductivity.

CHAPTER III

ANALYSIS

Consider a flat surface moving at a constant velocity u_w in a parallel direction to a free stream of a nanofluid of uniform velocity u_∞. Either the surface velocity or the free-stream velocity may be zero but not both at the same time. The physical properties of the fluid are assumed to be constant. Under such condition, the governing equations of the steady, laminar boundary-layer flow on the moving surface are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau\left\{D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right]\right\} \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

The boundary conditions are given by,

$y = 0$: $u = u_w$, $v = v_w(x)$, $T = T_w$, $c = C_w$

$y \to \infty$: $u = u_\infty$, $T = T_\infty$, $c = C_\infty$ (6)

The boundary condition of $u = u_w$ in (6) represents the case of a plane surface moving in parallel to the free stream.
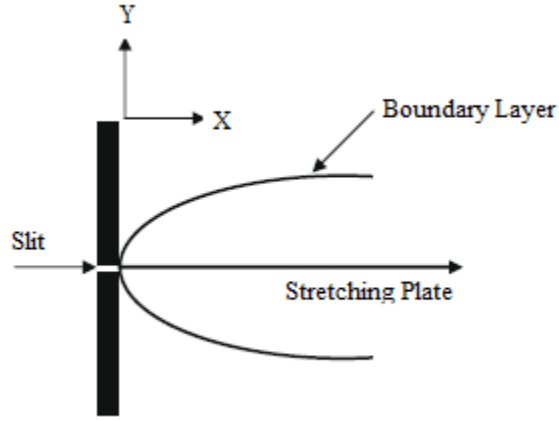

Figure 1 Physical Model and Co-Ordinates System

To analyze the effect of both the moving and the free stream on the boundary-layer flow, we propose a new similarly coordinate and a dimensionless stream function

$$\eta = \frac{y}{x}(Re_w + Re_\infty)^{\frac{1}{2}}, \qquad\qquad f = \frac{\psi}{v(Re_w + Re_\infty)^{\frac{1}{2}}} \qquad (7)$$

which are the combinations of the traditional ones:

$$\eta_B = \frac{y}{x}Re_\infty^{\frac{1}{2}}, \qquad f_B = \frac{\psi}{vRe_\infty^{\frac{1}{2}}} \qquad (8)$$

for the Blasius problem (stationary wall and uniform freestream velocity) and

$$\eta_S = \frac{y}{x} Re_w^{\frac{1}{2}}, \qquad f_S = \frac{\Psi}{\upsilon Re_w^{\frac{1}{2}}} \tag{9}$$

from the Sakiadis (uniformly moving wall with stagnant freestream) problem.

The Reynolds numbers are defined as:

$$Re_w = \frac{u_w x}{\upsilon}, \qquad Re_\infty = \frac{u_\infty x}{\upsilon} \tag{10}$$

A velocity ratio parameter γ is defined as

$$\gamma = \frac{u_w}{(u_w + u_\infty)} = \left(1 + \frac{u_\infty}{u_w}\right)^{-1} = \left(1 + \frac{Re_\infty}{Re_w}\right)^{-1}, \tag{11}$$

Note that form the Blasius problem, $u_w = 0$ therefore $\gamma = 0$. On the other hand, for the

Sakiadis problem, $u_\infty = 0$ and thus $\gamma = 1$. In addition, we also define dimensionless

temperature and concentration function as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{12}$$

$$\phi = \frac{c - c_\infty}{c_w - c_\infty} \tag{13}$$

Using the transformation variables defined in equations (7) – (13), the governing

transformed equations may be written as

$$f''' + \frac{ff''}{2} = 0 \tag{14}$$

$$\frac{\theta''}{Pr} + \frac{f\theta'}{2} + N_b \phi' \theta' + N_t (\theta')^2 = 0 \tag{15}$$

$$\phi'' + \frac{Le.f \, \phi'}{2} + \frac{N_t}{N_b} \theta'' = 0 \tag{16}$$

The transformed boundary conditions are given by

$$f(0) = f_w, \qquad f'(0) = \gamma, \qquad \theta(0) = 1, \qquad \phi(0) = 1$$

$$f'(\infty) = 1 - \gamma, \qquad \theta(\infty) = 0, \qquad \emptyset(\infty) = 0 \tag{17}$$

where prime denote differentiation with respect to η and the four parameters are defined by

$$Pr = \frac{\upsilon}{\alpha}, \qquad Le = \frac{\upsilon}{D_B}, \qquad N_b = \frac{(\rho c)_p D_B (\phi_w - \phi_\infty)}{(\rho c)_f \, \upsilon},$$

$$N_t = \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f \, T_\infty \upsilon} \tag{18}$$

Here, *Pr, Le, N_b* and *N_t* denote the Prandtl number, the Lewis number, the Brownian motion parameter and the thermophoresis parameter respectively. It is important to note that this boundary value problem reduces to the classical problem of flow and heat and mass transfer due to a stretching surface in a viscous fluid when N_b and N_t are zero.

The quantities of practical interest, in this study, are the Nusselt number Nu and the Sherwood number Sh which are defined as

Friction Factor,

$$C_\infty = \frac{\tau_w}{\left(\frac{\rho u_\infty{}^2}{2}\right)} = 2 \, Re_\infty{}^{-\frac{1}{2}} (1 - \gamma)^{-\frac{3}{4}} \left| f''(0) \right|$$

$$C_w = \frac{\tau_w}{\left(\frac{\rho u_w{}^2}{2}\right)} = 2 \, Re_w{}^{-\frac{1}{2}} \gamma^{-\frac{3}{4}} \left| f''(0) \right| \tag{19}$$

The local heat transfer rate (Local Nusselt number) is given by

$$Nu_x = \frac{q_w x}{R(T_w - T_\infty)} = -(Re_w - Re_\infty)^{\frac{1}{2}} \, \theta'(0) \tag{20}$$

Similarly the local Sherwood number is given by

$$Sh_x = \frac{q_m x}{D_B (C_w - C_\infty)} = -(Re_w - Re_\infty)^{\frac{1}{2}} \, \phi'(0) \tag{21}$$

where q_w and q_m are wall heat and mass flux rates, respectively.

CHAPTER IV

RESULTS AND DISCUSSIONS

The nonlinear ordinary differential equations (14)-(16), satisfying the boundary conditions (17) were integrated numerically by using the fourth-order Runge-Kutta scheme along with the shooting method for several values of the governing parameters, namely, Prandtl number (Pr), Lewis number (Le), Brownian motion parameter (N_b) and thermophoresis parameter (N_t). In order to assess the accuracy of the present results, we obtained results for the reduced Nusselt number $-\theta'(0)$ by ignoring the effects of N_b and N_t. These results are shown in Table 1. A comparison of our results with literature values indicates excellent agreement and therefore our results are highly accurate.

Table 1 Comparison of results for $-\theta'(0)$ for the Sakiadis problem ($N_t = N_b = 0; \gamma = 1$)

Pr	Present results	Wang [14]	Gorla and Sidawi [15]
0.07	0.0656	0.0656	0.0656
0.20	0.1691	0.1691	0.1691
0.70	0.4539	0.4539	0.4539
7.00	1.8907	1.8954	1.8905
20.00	3.3539	3.3539	3.3539
70.00	6.4622	6.4622	6.4622

Tables 2 and 3 display the resulting values of velocity gradient $f''(0)$, the sheet surface heat transfer rate $-\theta'(0)$ and the mass transfer rate $-\phi'(0)$, which are proportional to the friction factor, Nusselt number and Sherwood number respectively, for both the cases of the velocity ratio γ boundary layer flow over a stationary surface with a uniform free stream velocity ($\gamma = 0$) and uniformly moving plane surface moving in a stagnant free stream ($\gamma = 1$). The results in Tables 2 and 3 indicate that effect of increasing N_b or N_t is to decrease the heat and mass transfer rates from the surface.

Table 2 Variations of Nu and Sh with Nb and Nt for various values of Pr and γ at Le = 1

and $f_w = 0$

γ	Pr	Nt	$-\theta'(0)$			$-\phi'(0)$		
			Nb=.1	Nb=.2	Nb=.3	Nb=.1	Nb=.2	Nb=.3
0.0	0.7	0.1	0.9365	.9033	.8709	1.0648	1.0547	1.0510
		0.2	.9037	.8713	.8397	1.1835	1.1298	1.1113
		0.3	.8717	.8401	.8093	1.3651	1.2355	1.1915
	7.0	0.1	.5525	.3634	.2309	1.4468	1.3233	1.2634
		0.2	.3656	.2324	.1433	2.2543	1.7657	1.5737
		0.3	.2340	.1443	.0866	3.2695	2.2750	1.9117
	10.0	0.1	.4231	.2249	.1124	1.5755	1.3922	1.3028
		0.2	.2271	.1135	.0539	2.5300	1.8843	1.6334
		0.3	.1147	.0545	.0248	3.6266	2.4101	1.9744
1.0	0.7	0.1	.9724	.9377	.9039	1.0942	1.0932	1.0895
		0.2	.9387	.9048	.8719	1.1847	1.1574	1.1477
		0.3	.9058	.8728	.8406	1.3453	1.2534	1.2220
	7.0	0.1	.7537	.5104	.3332	1.3072	1.3070	1.2855
		0.2	.5162	.3370	.2126	2.0312	1.7286	1.5915
		0.3	.3408	.2150	.1316	3.0503	2.2497	1.9408
	10.0	0.1	.6664	.3757	.1974	1.3937	1.3749	1.3315
		0.2	.3820	.2007	.0992	2.3010	1.8673	1.6699
		0.3	.2040	.1008	.0473	3.4681	2.4275	2.0308

Table 3 Variations of Nu and Sh with Nb and Nt for various values of *Le* and γ at

Pr = 1 and f$_w$= 0

γ	*Le*	Nt	$-\theta'(0)$			$-\phi'(0)$		
			Nb=.1	Nb=.2	Nb=.3	Nb=.1	Nb=.2	Nb=.3
0.0	5.0	0.1	.9209	.8720	.8251	1.1816	1.1668	1.1612
		0.2	.8748	.8278	.7827	1.3489	1.2725	1.2459
		0.3	.8305	.7853	.7419	1.6012	1.4191	1.3566
	10.0	0.1	.9184	.8673	.8184	1.2819	1.2674	1.2618
		0.2	.8726	.8235	.7765	1.4471	1.3724	1.3462
		0.3	.8287	.7815	.7363	1.6946	1.5169	1.4556
	20.0	0.1	.9143	.8595	.8073	1.4659	1.4517	1.4462
		0.2	.8690	.8164	.7664	1.6263	1.5547	1.5293
		0.3	.8256	.7752	.7272	1.8638	1.6944	1.6356
1.0	5.0	0.1	.9557	.9023	.8513	1.3687	1.3676	1.3622
		0.2	.9086	.8573	.8081	1.5006	1.4618	1.4474
		0.3	.8633	.8139	.7666	1.7298	1.5995	1.5539
	10.0	0.1	.9501	.8919	.8365	1.7011	1.7001	1. 6944
		0.2	.9035	.8476	.7944	1.8351	1.7970	1.7826
		0.3	.8587	.8050	.7540	2.0657	1.9368	1.8914
	20.0	0.1	.9420	.8767	.8154	2. 3311	2.3304	2. 3249
		0.2	.8961	.8337	.7749	2.4640	2.4275	2.4130
		0.3	.8522	.7924	.7363	2.6889	2.5652	2.5207

Figures 2 and 3 display results for the variation of temperature and concentration within the boundary layer. As N_t increases, the temperature increases whereas the concentration decreases. As N_b increases, the temperature decreases whereas the concentration increases. The thickness of the boundary layer for concentration is smaller than the thermal boundary layer thickness.
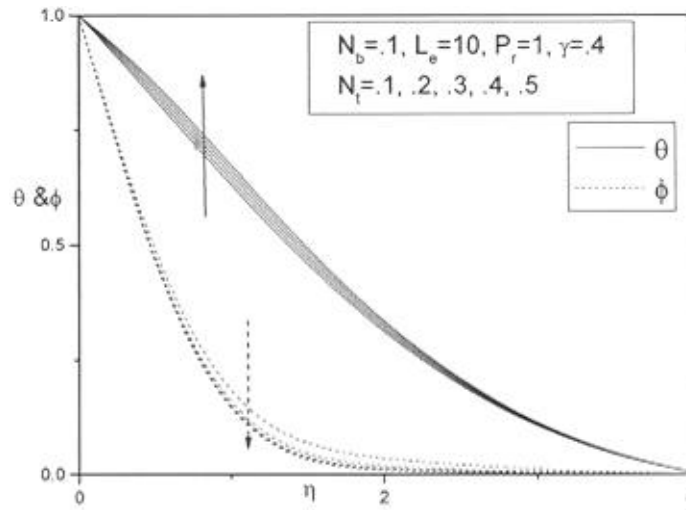

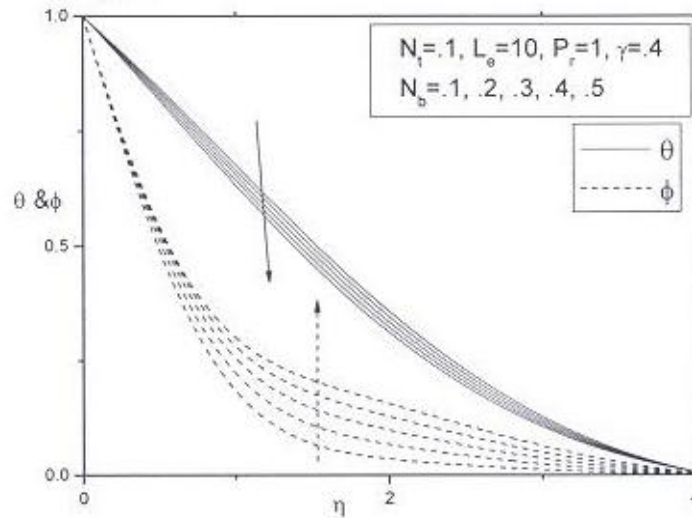Figure – 2 Temperature and Concentration Profiles

Figure – 3 Temperature and Concentration Profiles

Figure 4 shows the concentration distribution as the Lewis number increases. As Le increases, we observe that the concentration decreases and the concentration boundary layer thickness decreases. This in turn increases the surface mass transfer rates as Le increases.
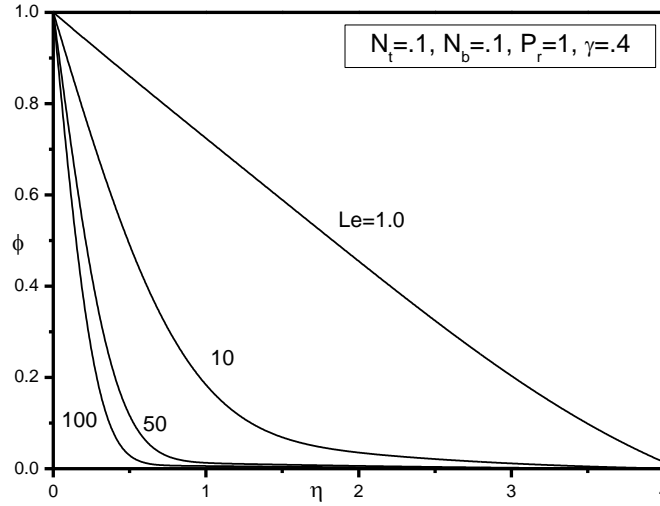
Figure – 4 Concentration Profiles

Figure 5 shows the temperature distribution as the Prandtl number increases. As Pr increases, we observe that the temperature decreases and the thermal boundary layer thickness decreases. This in turn increases the surface heat transfer rates as Pr increases.

Figure 6 shows the effect of surface mass transfer on temperature and concentration distributions. As the surface mass transfer parameter f_w increases, the temperature and concentration decrease and their surface gradients increase. Therefore, s surface mass transfer increases heat and mass transfer rates.
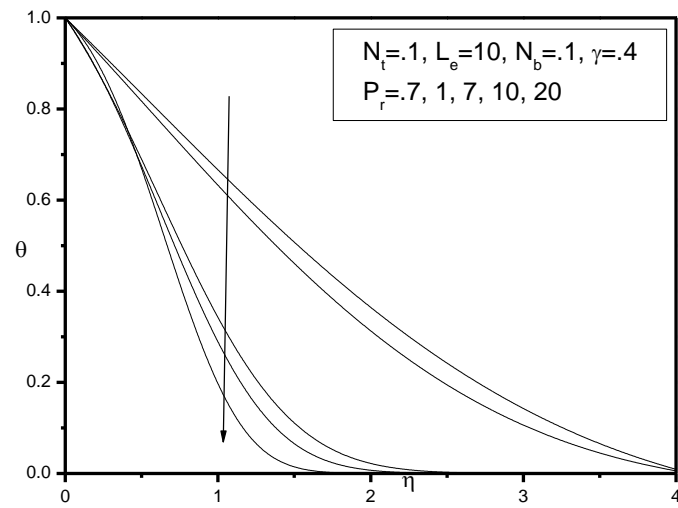
Figure – 5 Temperature Profiles
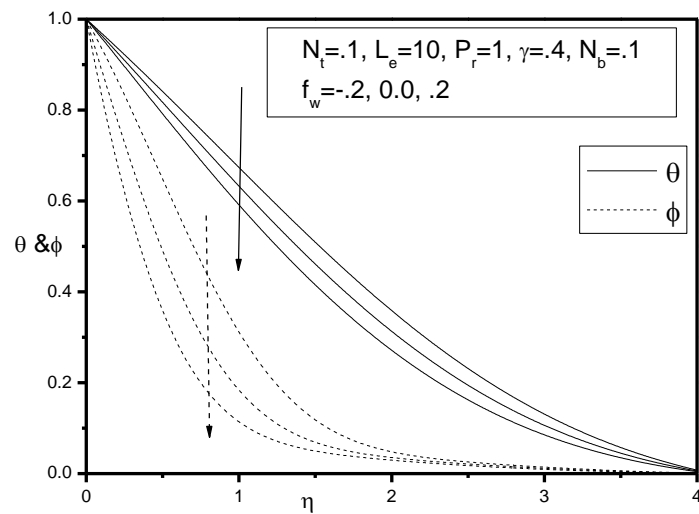
Figure – 6 Temperature and Concentration Profiles

Figure 7 shows the velocity distribution within the boundary layer for several values of the velocity ratio parameter γ. A value of zero for the velocity parameter γ describes the Blasius problem whereas $\gamma = 1$ describes the Sakiadis problem.
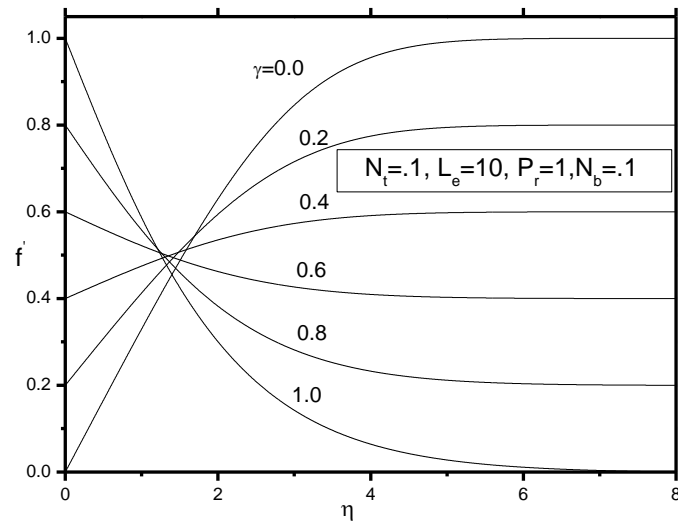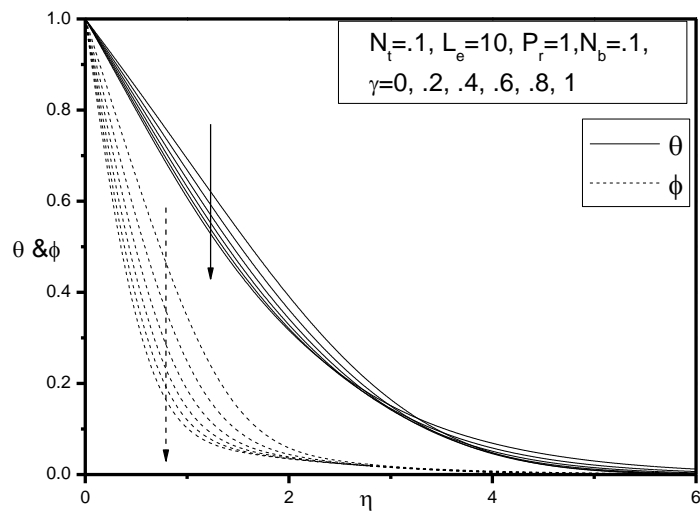

Figure – 7 Velocity Profiles

Figure – 8 Temperature and Concentration Profiles

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Figure 8 shows the temperature and concentration distribution within the boundary layer for several values of the velocity ratio parameter γ. As γ increases, the temperature and concentration values decrease whereas their surface gradients increase. This indicates that increasing values of γ will augment heat and mass transfer rates.

Figures 9 – 16 show the variation of heat and mass transfer rates versus N_b and N_t with Pr, Le and γ chosen as prescribable parameters. The heat transfer rates decrease as N_b or N_t increase. The mass transfer rates increase with N_t and decrease with N_b. The heat transfer rate increases as the Prandtl number Pr increases. At higher values of Pr, the thermal diffusivity decreases and therefore the heat transfer rate increases. Similarly, as Le increases, the surface mass transfer rates increase.
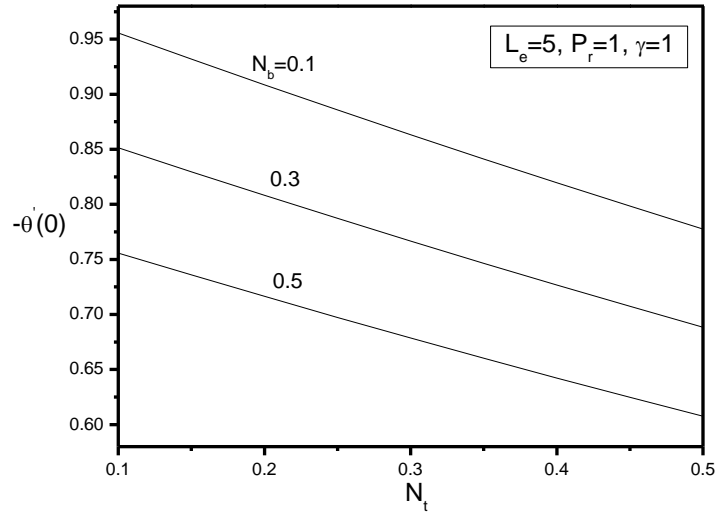

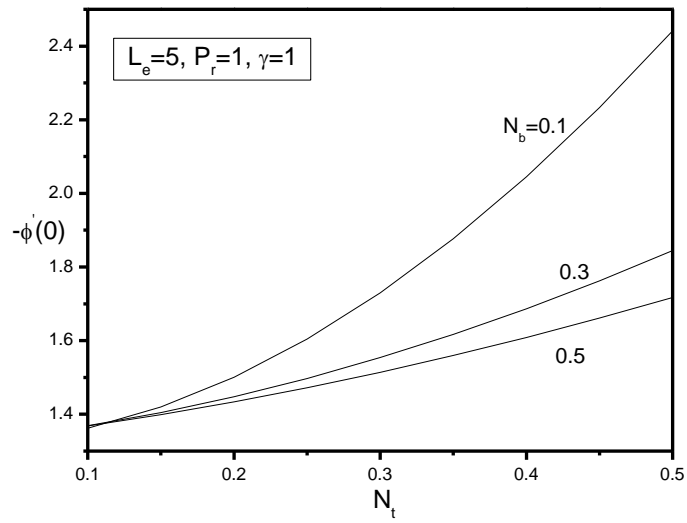Figure – 9 Reduced Local Nusselt Number Vs N_t

Figure – 10 Reduced Local Sherwood Number Vs N_t
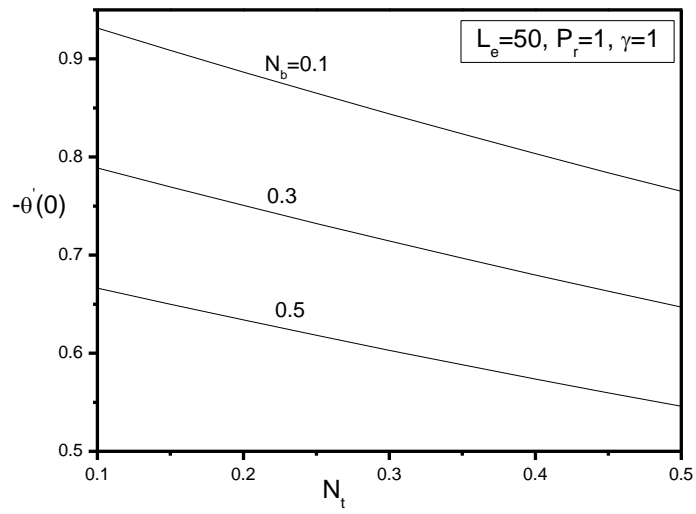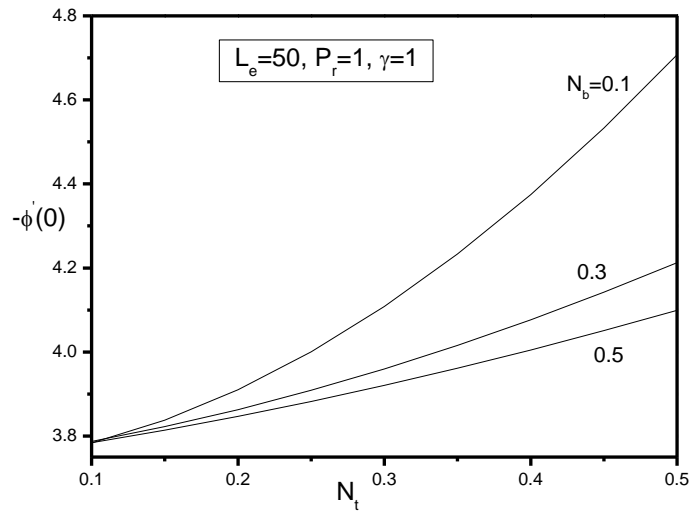

Figure – 11 Reduced Local Nusselt Number Vs N_t

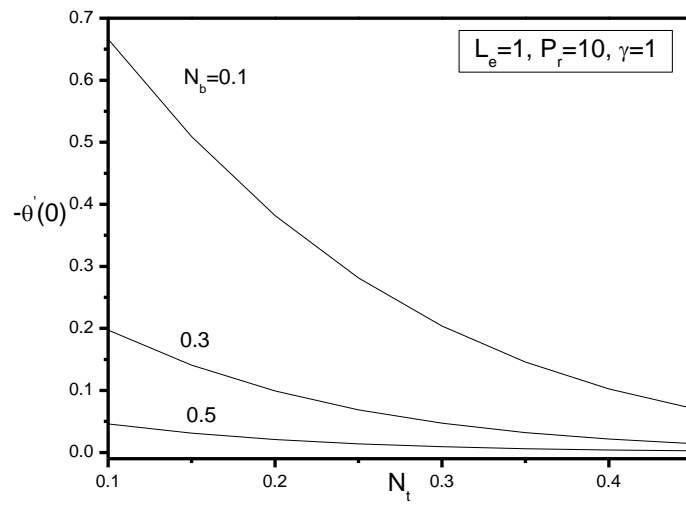Figure – 12 Reduced Local Sherwood Number Vs N_t

Figure – 13 Reduced Local Nusselt Number Vs N_t
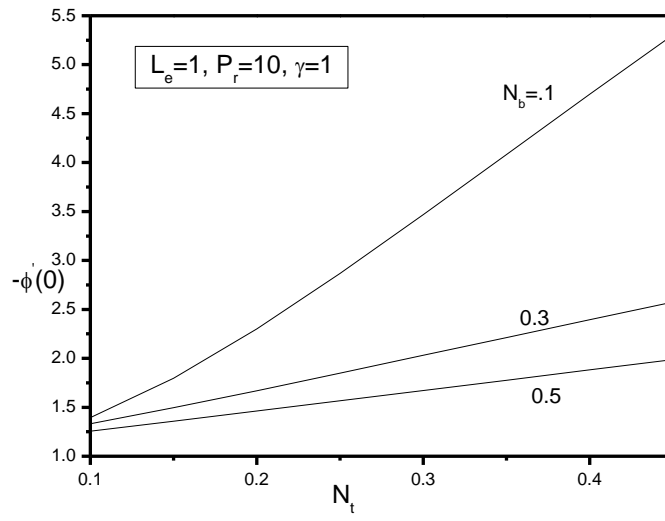
28

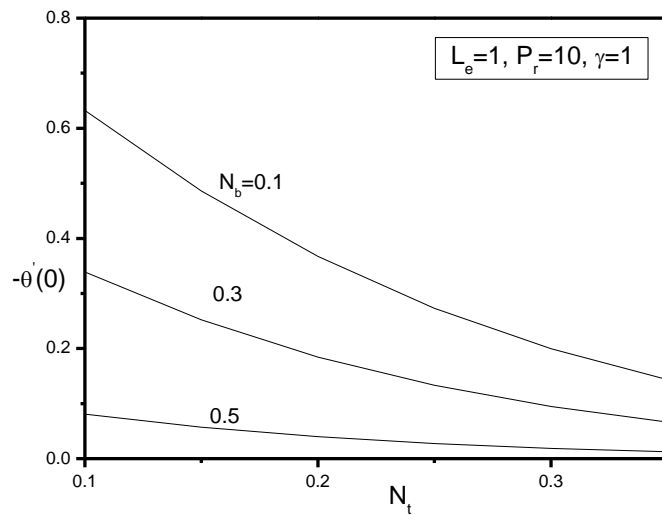Figure – 14 Reduced Local Sherwood Number Vs N_t

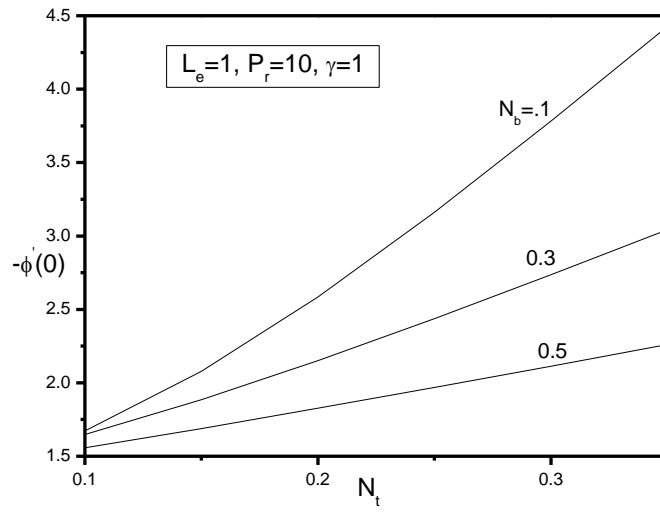Figure – 15 Reduced Local Nusselt Number Vs N_t

Figure – 16 Reduced Local Sherwood Number Vs N_t

CHAPTER V

CONCLUDING REMARKS

In this work, we have studied the problem of the steady boundary-layer flow of a nanofluid on a permeable continuous moving isothermal surface moving in parallel to a free stream. The governing boundary layer equations are solved numerically using the fourth-order Runge-Kutta scheme along with the shooting method. The development of the Nusselt number and Sherwood number as well as the temperature, concentration and velocity distributions for various values of the velocity ratio, suction/injection and nanofluid parameters has been discussed and illustrated in tabular forms and graphs. The results indicate that the suction/injection parameter is found to reduce the Nusselt number, friction factor and mass transfer. The effect of the nanofluid parameters on the temperature and concentration distributions as well as the friction factor and heat and

mass transfer depends on the ratio of the velocity of the plate and the free stream fluid velocity.

REFERENCES

1. Eastman, J. A., Choi, S.U.S. Li, S., Yu, W. and Thompson, L.J., "Anomalously Increased Effective Thermal Conductivities Containing Copper Nanoparticles," Applied Physics Letters, Vol. 78, 2001, pp. 718 – 720.

2. Choi, S.U.S., Zhang, Z.G., Yu, W., Lockwood, F.E., and Grulke, E.A., "Anomalous Thermal Conductivity Enhancement on Nanotube Suspension," Applied Physics Letters, Vol. 79, 2001, pp.2252 – 2254.

3. Patel, H. E., Das, S. K., Sundararajan, T., Sreekumaran, A., George B. and Pradeep, T., "Thermal Conductivities of Naked and Monolayer Protected Metal Nanoparticle Based Nanofluids: Manifestation of Anomalous Enhancement and Chemical Effects," Applied Physics Letters, Vol. 83, 2003, pp. 2931 – 2933.

4. You, S. M., Kim, J. H. and Kim, K. H., "Effects of Nanoparticles on Critical Heat Flux of Water in Pool Boiling Heat Transfer," Applied Physics Letters, Vol. 83, 2003, pp. 3374 – 3376.

5. Sakiadis, B. C., "Boundary-layer behavior on a continuous solid surface: II-The boundary layer on a continuous flat surface," AIChE Journal, Vol. 7, 1961, pp. 221 – 225.

6. Crane, L. J., "Flow Past a Stretching Plate," ZAMP, Vol. 21, 1970, pp. 645 – 647.

7. Tsou, F. K., Sparrow, E. M. and Goldstein, R. J., "Flow and Heat Transfer in The Boundary Layer in the Continuous Moving Surfaces," Int. J. Heat Mass Transfer, Vol. 10, 1967, pp. 219 – 235.

8. Schowalter, W. R., "The Application of Boundary-Layer Theory to Power-Law Pseudoplastic Fluid," AIChE Journal, Vol. 6, 1960, pp. 24 – 28.

9. Acrivos, A., Shah, M. J. and Peterson, E. E., "Momentum and Heat Transfer in Laminar Boundary-Layer Flows of Non-Newtonian Fluids Past External Surfaces," AIChE Journal, Vol. 6, 1960, pp. 312 – 317.

10. Howell, T. G., Jeng, D. R. and De Witt, K. J., "Momentum and Heat Transfer on a Continuous Moving Surface in a Power Law Fluid," Int. J. Heat Mass Transfer, Vol. 40, 1997, pp. 1853 – 1861.

11. Rao, J. H., Jeng, D. R. and De Witt, K. J., "Momentum and Heat Transfer in a Power-law Fluid with Arbitrary Injection/Suction at a Moving Wall," Int. J. Heat Mass Transfer, Vol. 42, 1999, pp. 2837 – 2847.

12. Magyari, E. and Keller, B., "Heat and Mass Transfer in the Boundary Layers on an Exponentially Stretching Continuous Surface," J. Phys. D: Appl. Phys, Vol. 32, 1999, pp. 577 – 585.

13. Magyari, E. and Keller, B., "Heat Transfer Characteristics of the Separation Boundary Flow Induced by a Continuous Stretching Surface," J. Phys. D: Appl. Phys, Vol. 32, 1999, pp. 2876 – 2881.

14. Wang, C.Y., "Free Convection on a Vertical Stretching Surface," ZAMM, Vol. 69, 1989, pp. 418 – 420.

15. Gorla, R.S.R. and Sidawi, I., "Free Convection on a Vertical Stretching Surface with Suction and Blowing," Applied Scientific Research, Vol. 52, 1994, pp. 247 – 257.

16. Kuznetsov, A.V., and Nield, D.A., "Natural Convective Boundary-layer Flow of a Nanofluid Past a Vertical Plate," Int. J. of Thermal Sciences, Vol. 49, 2010, pp. 243 – 247.

17. Kuznetsov, A.V., and Nield, D.A., "The Cheng–Minkowycz Problem for Natural Convective Boundary-layer Flow in a Porous Medium Saturated by a Nanofluid," In. J. of Heat and Mass Transfer, Vol. 52, 2009, pp. 5792 – 5795.

18. Bachok, N., Ishak, A. and Pop, I., "Boundary-layer Flow of Nanofluids over a Moving Surface in a Flowing Fluid," Int. J. of Thermal Sciences, VOl. 49, 2010, pp. 1663 – 1668.

19. Khan, W.A. and Pop, I., "Boundary-Layer Flow of a Nanofluid Past a Stretching Sheet," International Journal of Heat and Mass Transfer, Vol. 53, 2010, pp. 2477 – 2483.

20. Ahmad, S. and Pop, I., "Mixed Convection Boundary Layer Flow from a Vertical Flat Plate Embedded in a Porous Medium Filled with Nanofluids," Int. Communications in Heat and Mass Transfer, Vol. 37, 2010, pp. 987 – 991.

21. Polidori, G., Fohanno, S. and Nguyen, C.T., "A Note on Heat Transfer Modeling of Newtonian Nanofluids in Laminar Free Convection," International Journal of Thermal Sciences, Vol. 46, 2007, pp. 739 – 744.

22. Koo, J. and Kleinstreuer, C., "Laminar Nanofluid Flow in Microheat-Sinks," International Journal of Heat and Mass Transfer, Vol. 48, 2005, pp. 2652 – 2661.

23. Xuan, Y. and Li, Q., "Investigation on Convective Heat Transfer and Flow Features of Nanofluids," Journal of Heat Transfer, Vol. 125, 2003, pp. 151 – 155.

24. Khan, S. and Sanjayanand, E., "Viscoelastic Boundary Layer Flow and Heat Transfer over an Exponential Stretching Sheet," Int. J. of Heat and Mass Transfer, Vol. 48, 2005, pp. 1534 – 1542.

25. Arnold, J. C., Asir, A. A., Somasundaram, S. and Christopher, T., "Heat Transfer in a Viscoelastic Boundary Layer Flow over a Stretching Sheet," Int. J. of Heat and Mass Transfer, Vol. 53, 2010, pp. 1112 – 1118.

26. Xu, H. and Liao, S., "Laminar Flow and Heat Transfer in the Boundary-Layer of Non-Newtonian Fluids over a Stretching Flat Sheet," Computers and Mathematics with Applications, Vol. 57, 2009, pp. 1425 – 1431.

27. Yürüsoy, M., "Unsteady Boundary Layer Flow of Power-Law Fluid on Stretching Sheet Surface," International Journal of Engineering Science, Vol. 44, 2006, pp. 325 – 332.

28. Ishak, A., Nazar, R. and Pop, I., "Boundary-layer flow of a Micropolar Fluid on a Continuously Moving or Fixed Permeable Surface," International Journal of Heat and Mass Transfer, Vol. 50, 2007, pp. 4743 – 4748.

29. Ishak, A., Nazar, R. and Pop, I., "Flow of a Micropolar Fluid on a Continuous Moving Surface," Arch. Mech., Vol. 58, 2006, pp. 529 – 541.

30. Hassanien, I. A., "Flow and Heat Transfer on a Continuous Flat Surface Moving in a Parallel Free Stream of Power-Law Fluid," Appl. Math. Modelling, Vol. 20, 1996, pp. 779 – 784.

31. Hassanien, I. A., Abdullah, A. and Gorla, R. S. R., "Flow and Heat Transfer in a Power-Law Fluid over a Non-isothermal Stretching Sheet," Mathl. Comput. Modelling, Vol. 28, 1998, pp. 105 – 116.

32. Gorla, R. S. R. and Reddy, Pottapu V, " Flow and Heat Transfer from a Continuous Surface in a Parallel Free Stream of Micropolar Fluid," Int. J. Engg. Sci., Vol. 25, 1987, pp. 1243 – 1249.

33. Olajuwon, B.I., "Flow and Natural Convection Heat Transfer in a Power Law Fluid Past a Vertical Plate with Heat Generation," Int. J. of Nonlinear Science, Vol.7, 2009, pp. 50 – 56.

34. Ahmad, Naseem, Siddiqui, Z. U. and Mishra, M. K., "Boundary Layer Flow and Heat Transfer Past a Stretching Plate with Variable Thermal Conductivity," Int. J. of Non-Linear Mechanics, Vol. 45, 2010, pp. 306 – 309.