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DEVELOPMENT OF CHATTER ATTENUATION ROBUST CONTROL FOR AN AMB MACHINING SPINDLE

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ALEXANDER H. PESCH

ABSTRACT

This work develops a robust control strategy to avoid machining chatter in a highspeed machining spindle levitated on active magnetic bearings (AMBs). Chatter avoidance is an important aspect in machining because chatter limits the material removal rate, inhibiting production. The developed chatter avoiding method involves a controller designed for the AMBs through μ -synthesis based control. The μ -synthesis is performed by including a cutting force model with the spindle model. Such an approach causes the controller to stabilize the naturally unstable cutting process, avoiding chatter. Stability lobe diagrams are calculated for the new control strategy showing increased critical feed-The increase in critical feed-rate is confirmed through numerical machining rate. simulations. The developed method is implemented experimentally on the high-speed Impulse hammer testing is performed to measure the AMB machining spindle. controlled spindle's frequency response at the tooltip location. The frequency response is then used to evaluate the critical feed-rate for the experimental system. A 64% improvement in critical feed-rate is found relative to the current state-of-the-art.

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NOMENCLATURE

Acronyms

AC	Alternating Current
AMB	Active Magnetic Bearing
CA	Chatter Avoiding
CG	Center of Gravity
DS	Dynamic Stiffness
FE	Finite Element
FEM	Finite Element Method
LFT	Linear Fractional Transform
MIMO	Multiple-Input Multiple-Output
PID	Proportional Integral Derivative Controller
PWM	Pulse Width Modulation
SISO	Single-Input Single-Output
SLD	Stability Lobe Diagram
RPM	Revolutions per Minute
V13	Front Bearing Control Axis in V-plane
V24	Back Bearing Control Axis in V-plane
W13	Front Bearing Control Axis in W-plane
W24	Back Bearing Control Axis in W-plane

Chapter II

- D a matrix which commutes with Δ used to scale M in practical μ analysis
- *d* exogenous disturbances
- *d'* weighted exogenous disturbances
- *e* performance output signals
- *e'* weighted performance output signals
- F_L lower LFT
- F_U upper LFT
- *G* uncertain transfer function
- G_0 nominal transfer function
- *I* identity matrix
- *j* imaginary unit $(j = \sqrt{-1})$
- *K* controller
- *M* lower LFT of *P*' and *K* which is used for closed-loop stability analysis
- M_{11} partition of M which relates the first input vector to the first output vector
- M_{12} partition of M which relates the second input vector to the first output vector
- M_{21} partition of M which relates the first input vector to the second output vector
- M_{22} partition of M which relates the second input vector to the second output vector
- *P* plant including inputs and outputs for stability and performance
- P' plant weighted for performance (with $W_{\rm I}$ and $W_{\rm O}$)
- *r* an input vector
- r_1 upper partition of vector r
- r_2 lower partition of vector r

- *u* signals from controller to plant
- *v* an output vector
- v_1 upper partition of vector v
- v_2 lower partition of vector v
- *W* uncertainty weight transfer function
- *w* outputs for uncertainty loop
- $W_{\rm I}$ input performance weighting matrix transfer function
- $W_{\rm O}$ output performance weighting matrix transfer function
- *y* sensor signal used for control
- *z* disturbances due to uncertain perturbation
- Δ uncertain perturbation matrix
- Δ set of structured perturbation matrices
- $\Delta_{\rm P}$ fictitious perturbation matrix associated with performance
- $\Delta_{\rm U}$ perturbation matrix associated with uncertainties
- μ structured singular value
- $\sigma(\mathbf{A})$ singular value of \mathbf{A}
- $\bar{\sigma}(\mathbf{A})$ largest singular value of \mathbf{A}
- ω frequency

Chapter III

- A area of pole face $[\mu m^2]$
- A_{amp} state matrix of amplifier
- \mathbf{A}_{ff} state matrix of free-free rotor

B _{act}	input matrix for rotor at actuator (magnet) locations
B _{amp}	input matrix of amplifier
B _{load}	input matrix of rotor at external load locations
\mathbf{C}_{act}	output matrix of rotor at actuator (magnet) locations
\mathbf{C}_{amp}	output matrix of amplifier
\mathbf{C}_{perf}	output matrix of rotor at performance locations
$\mathbf{C}_{\text{sense}}$	output matrix of rotor at sensor locations
d	disturbance signal [A]
F	AMB force [N]
$G_{\rm OL}$	open-loop plant transfer function [µm/A]
G	MIMO plant matrix transfer function
g	nominal air gap in the AMB [µm]
G_{ff}	gyroscopic matrix of free-free rotor
I_b	bias current [A]
Ic	control current [A]
K	controller transfer function [A/ μ m]
K _a	diagonal matrix containing amplifier sensitivities
\mathbf{K}_{i}	diagonal matrix containing current stiffnesses
<i>k</i> _i	current stiffness [N/A]
Ks	diagonal matrix containing sensor sensitivities
K _x	diagonal matrix containing position stiffnesses
k_x	position stiffness [N/µm]
Ν	number of coil windings

- U matrix of all open-loop inputs
- *u* input to plant (control current plus disturbance) [A]
- u_1 first input [A]
- *u*₂ second input [A]
- *u*₃ third input [A]
- u_4 fourth input [A]
- x position of the rotor at the magnet location $[\mu m]$
- **x**_{amp} state vector of amplifier
- \mathbf{x}_{ff} state vector of free-free rotor
- Y matrix of all open-loop outputs
- y output of plant (position sensor signal) [µm]
- y_1 first output [µm]
- y_2 second output [µm]
- y_3 third output [µm]
- y_4 fourth output [µm]
- α pole angle [rad]
- ε derating factor [unitless]
- μ_0 permeability of free space [N/A²]
- Ω rotation speed [rad/s]
- ω_i i^{th} discrete identified frequency [Hz]

Chapter IV

 e_0 nominal cut depth [µm]

f_a	cutting	force	in	the	axial	direction	[N	

- f_c cutting force [N]
- f_r cutting force in the radial direction [N]
- f_t cutting force in the tangential direction [N]
- **K**_c matrix of linear cutting stiffnesses
- k_c cutting stiffness [N/ μ m²]
- k_d uncertain parameter which accounts for time delay in machining force function
- *l* uncut chip length [µm]
- t time [s]
- v(t) tool position for the current machining pass [µm]
- $v(t-\tau)$ tool position for the previous machining pass [µm]
- w uncut chip width [µm]
- τ amount of time to make one machining pass [s]

Chapter V

- E_0 nominal cut depth in the frequency domain [µm]
- F_e external force on the cutting tool [N]
- *n* integer number of full tool vibration cycles in one rotation
- T transfer function of the levitated spindle at the tool location $[\mu m/N]$
- V tool position for the current machining pass in the frequency domain [µm]
- v tool position for the current machining pass at the border of stability [µm]
- V' amplitude of tool vibrations, current machining pass, border of stability [µm]
- V_0 tool position for the previous machining pass in the frequency domain [µm]

- v_0 tool position for the previous machining pass at the border of stability [µm]
- V_0' amplitude of tool vibrations, previous machining pass, border of stability [µm]
- φ phase shift between tool vibration of current and previous machining passes [rad]

Chapter VI

- 1X synchronous component (1 times the running speed)
- ω_c chatter frequency [Hz]
- ω_{n1} first natural frequency of the spindle (first bending mode) [Hz]
- ω_{n2} second natural frequency of the spindle (second bending mode) [Hz]
- ω_T tooth passing frequency [Hz]

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CHAPTER I

INTRODUCTION

1.1 Background and Motivation

Machining is an important process in manufacturing in which an unfinished product (known as a workpiece) is formed to a specific geometry using some sort of powered machine such as a lathe, mill, saw, drill etc. Almost all manufactured metal products undergo some kind of machining. Types of machining include electrical discharge machining, chemical machining, drawing, and stamping, but the most common type of machining is the so-called "chip forming" type. In chip forming machining the workpiece material is physically cut to the desired geometry. The bits of material that are cut away are known as chips. Manufacturing companies are continuously seeking higher effectiveness in terms of production time and cost, but are burdened by the need to meet ever increasing tolerances for both modern materials and advanced geometries. This dissertation is motivated by the desire to increase effective material removal rates in chip forming machining processes. For the remainder of this work, chip forming machining is

simply referred to as machining.

There is a phenomenon in machining called chatter where, instead of getting a smooth cut, the tool starts to vibrate, and because of the conditions of the particular machining process, the vibrations grow over time, instead of dying out. Chatter is undesirable because it may create a rough surface finish, not hold part tolerance, and even break the cutting tool. There exists a critical feed-rate for each cutting speed below which chatter does not occur and above which chatter occurs. Therefore, chatter limits the material removal rate.

An industrial grade machining spindle, modified for experimental interface, is the platform for study in this work. The spindle is intended for boring operations. Chatter is of particular interest for rotating boring spindles and static boring bars because of the geometry involved. In boring, the cutting tool must reach inside the workpiece cavity. To accommodate the cavity, the tool holder must be relatively long and slender. The high length to diameter ratio of the tool holder causes low stiffness which is associated with the propensity for chatter vibrations.

Additionally, avoiding chatter is also especially problematic in so-called high-speed machining (HSM). Both cutting forces and the heat generated by metal machining tend to increase as the cutting speed increases. But, if cutting speed is raised high enough, there is an operating region where the cutting forces and heat generated decrease with cutting speed, and this is the region for HSM. There are several advantages to HSM:

- 1) Workpieces can be processed faster.
- Tool wear is reduced, leading to better part tolerances and less down time for replacing tools.

- More delicate workpieces can be machined because of the reduced forces and heat.
- In some applications, the need for coolant is eliminated, another cost saving measure.

Unfortunately the opportunity of HSM is often thwarted by the onset of chatter which requires the machinist to cut at a lower speed than the machinery is capable of providing. Actuators can be utilized to attenuate chatter vibrations so higher speeds can be attained. Active magnetic bearings are an example of such actuators.

Active magnetic bearings (AMBs) are electromechanical devices which use magnetic fields generated by electromagnets to levitate a rotor. The current in the magnetic coils is controlled by feedback of the rotor position using a stabilizing control law. Because there is no physical contact in the bearings, there are no losses due to friction. This attribute is especially beneficial for high-speed rotation. In addition to friction avoidance, the noncontact properties of AMBs make them useful for on-line health monitoring of rotordynamic systems. For example, in [1], a magnetic bearing was used as an actuator for identifying combinational resonances induced in laterally cracked shafts. Another example is [2] in which Wroblewski et al. were able to locate a rotor structural change through magnetic excitation.

Perhaps more important to HSM than noncontact is that the AMB's stators are static, i.e., there are no ball bearings outside the spindle which rotate; only the spindle rotates. The maximum rotational speed is limited by the element at maximum radius. Therefore, a spindle on AMBs may have a wider radius than a spindle on ball bearings. A wider radius gives the spindle a higher stiffness leading to less natural propensity to chatter. AMBs are inherently unstable because of the attractive nature of the electromagnets on the rotor, which is ferromagnetic. As a result, AMBs must be stabilized with active feedback control.

The AMB's controller will affect the chatter characteristics of the system as a whole. Therefore, proper design of the controller is necessary for maximizing the critical feed-rate across the operational speed range. This may be likened to selecting a traditional bearing based on its stiffness and damping properties in order to avoid chatter. However, the active control of the AMBs allows for the possibility of complicated bearing dynamics which are custom fit to the dynamics of the rotor and cutting process in order to counter the occurrence of chatter. In a real world application, prevention of chatter under all conditions is not possible. However, using modern control techniques a sophisticated controller for AMBs can be designed to achieve new levels of productivity. The modern control technique applied in the current work is μ -synthesis.

 μ -synthesis based control was developed in the late 1980's and early 1990's. Today there are commercially available software packages, such as MATLAB's Robust Control Toolbox for performing μ -synthesis for general applications. That is, the code is already available to execute the fundamental algorithm but it must be fitted by the engineer to a specific application. The μ -synthesis based control is a model-based approach, meaning the system to be controlled must be first modeled and that model is used to design the controller. Using the system model, a controller is derived by minimizing the system's response to disturbances. Then the closed-loop model is evaluated for robustness to uncertainties and the controller is re-derived if needed. Note that an accurate model of the system is required to perform μ -synthesis controller design. If the model does not accurately describe the system, there is no reasonable expectation that the resulting controller will be able to stabilize the system, let alone achieve desired performance.

The effort needed to create the needed system model might seem prohibitive. However, as Maslen and Sawicki [3] explain, the advantage of μ -synthesis is that it changes the design parameter space to one which is relatively easy to maintain and represents a durable investment from an engineering process view. Non-model-based control strategies might be thought of as easier to tune on a real system. However, μ synthesis based control is in fact easier to tune because the closed-loop behavior is dictated by performance weights which are cast as physically interpretable bounds. An example of non-model-based controller tuning is manually adjusting a PID's proportional gain iteratively which is straightforward to perform on a real system but the effect may be difficult to predict. A similar problem in a μ -synthesis framework is shifting a maximum deflection allowance, which has an exact performance implication. Also, the closedloop's maximum deflection can be specified and achieved in a single controller synthesis.

Another benefit of model-based control is that an engineer can specify performance requirements for unobserved points in the system. In the design process, the model is used to predict the behavior at any location and the controller is crafted accordingly. In implementation, actual behavior at the unobserved locations becomes what is required despite the fact that it is not monitored. This is significant for the current work because the AMB sensors are located at the bearings but the machining tolerance must be met at the tool tip. Additionally, the tooltip cannot be measured with sensors during machining with the workpiece and chips in the way.

To attain closed-loop robustness, the model used for controller design has to be

uncertain. Any parameters that will change during operation of the system, e.g., change of operating point or change of parameters over time, must be modeled as uncertain. For example, the spindle may be run at different speeds. Therefore, the running speed must be represented as an operating speed range. The uncertain parameters are accounted for with norm bounded perturbations on the nominal model. As a result, controller is designed which meets the performance requirements for any possible combination of uncertain parameter values inside the prescribed bounds. Uncertainties can also be used to compensate for modeling errors but this tactic should be avoided whenever possible as performance is usually sacrificed for robustness. To take full advantage of model-based control, a correct nominal model should be used and only parameters which are known to vary should be considered uncertain.

1.2 Goals of Research

The goal of the dissertation is to develop a chatter avoiding strategy which uses μ synthesis model-based robust control of AMBs. The developed strategy is to augment the system model, used for controller design, with a cutting force model such that the resulting controller stabilizes the cutting process, avoiding chatter. To achieve the goal, the proposed strategy is implemented on an industrial grade HSM spindle. The practical implementation of the developed strategy includes identification and modeling of the experimental system and selection of effective performance weights and uncertainties. The performance weights and uncertainties are for not only the AMB levitation but also for the newly included cutting force model. To demonstrate that the goal is achieved, the designed controller is compared to an AMB controller designed to maximize the cutting tool's dynamic stiffness. The dynamic stiffness controller represents the current state-ofthe-art in avoiding chatter. The controllers are compared through implementation on the spindle for machining simulations and experimental evaluation of the critical feed-rates.

The following section is a review of the body of literature relevant to the current work.

1.3 Literature Review

The current work involves machining, robust control, and active magnetic bearings, each of which is a field of study on its own. This literature review focuses on the past works of particular interest to the coming together of these fields. Keep in mind that the many of the topics brought up in the literature review, such as μ -synthesis, are discussed more thoroughly later in the dissertation as the proposed chatter avoiding strategy is explained.

There have been many works on the prevention of machining chatter. There have also been many works on the robust control of AMBs, even AMB spindles. There are only a few works on robust control of AMB spindles that address chatter, none of which employ the cutting model based method utilized in the current work.

1.3.1 The Chatter Phenomena

Chatter is unstable vibrations between the cutting tool and the workpiece. Chatter has been observed for essentially as long as there has been machining. Wiercigroch and Budak [4] conducted a critical review of the state-of-the-art of modeling and corresponding experimental investigations of chatter and identified four mechanisms which cause machining chatter: regeneration, mode coupling, variable friction, and thermo-mechanics of chip formation. The regenerative type is the most prevalent, the most costly to industry, and is therefore the focus of this work. For context, the types of chatter are briefly discussed.

Thermo-mechanics of chip formation is the thermodynamics of the cutting process influencing the motion during material removal through phenomena such as thermal expansion and the change of properties such as stiffness with temperature. An example of thermo-mechanics affecting chip formation is presented in [5], where Davies, Burns, and Evans used a simplified one dimensional thermo-mechanical model of a continuous homogenous material under shear to explain chip segmentation observed in experimental machining of electroplated nickel-phosphorous and hard bearing steel. This was also the first time that the mechanism of chip segmentation was successfully expressed mathematically. It is easy to see that with thermodynamics effecting the formation of the chip, there will be a corresponding effect on the force between the tool and workpiece. It is this link which allows for the possibility of thermo-mechanical chatter. The thermomechanical effect creates a force feedback of tool motion and under the right conditions instability will occur.

Mode coupling chatter can be associated with situations where vibration of the tool relative to the workpiece can exist simultaneously in at least two directions in the plane in which cutting takes place [6]. For example, assume cutting force magnitude is a function of cut depth, though cutting force direction may be in a different direction than cut depth. Furthermore, the cutting force excites two modes of vibration which are in two other directions. The vibration components can be decomposed in both the cut depth direction as well as perpendicular to the cut and are coupled by the cutting force. Both modes may be excited such that the tool moves in an elliptical pattern. If the elliptical motion

direction is opposite to that of the cutting force, energy is removed from the system. Conversely, if the motion is in the direction of cutting force, energy is added to the system. In this manner, vibrations may die off or grow in a self excited way depending on the particulars of the system such as cutting force angle, vibration modes angle, stiffness and damping, and cutting stiffness.

Frictional chatter arises due to the presence of the nonlinear (dry) friction component in the cutting force. Because the equation of motion becomes nonlinear with friction, it requires nonlinear methods to account for stability. In [7], Grabec studied this effect with a simplified two degree of freedom elastic structure model and found through numerical solution of the dynamical equations that the nonlinearity of the cutting force can lead to chaotic dynamics in intensive cutting situations. In [8], Wiercigroch and Krivtsov also studied this effect and included stochastic modeling of the cutting parameters. The authors used bifurcation diagrams and Poincaré maps to characterize the dynamic behavior and concluded that, in general, cutting force modulus was the most important factor in influencing the response of the system and that large viscous friction can always prevent this type of chatter. Wiercigroch and Krivtsov also concluded, through analysis of a simple one degree of freedom model, with such a nonlinear cutting force that if the optimum velocity for the dry friction force is smaller than the average relative velocity between the tool and workpiece, then frictional chatter will not occur.

Regenerative chatter is the most common type of chatter in turning operations [9] and therefore will be the focus of the developed chatter prevention strategy. Tobias and Fishwick [10], Tlusty and Polacek [11], and Merrit [12] developed the theory to explain regenerative chatter in the late 1950's and early 1960's. Simply put, in regenerative

chatter, the cutting force is a function of the uncut chip width. The chip width is the nominal chip width (set by the machinist) plus the vibration of the tool, minus the vibration of the previous pass of the tool. In the case of single point boring, the previous tool pass occurred at the previous rotation of the spindle. If the vibrations during one pass and the next are in phase, the chip will be of constant thickness. Therefore, the magnitude of the cutting force will be constant and stable cutting will occur. If the vibrations are out of phase, the chip thickness will vary and the magnitude of cutting force will also vary, causing vibrations to grow [13].

If the particular parameters, such as cutting stiffness, axial feed-rate, rotational speed (and therefore time delay), and tool dynamic stiffness, result in a system with eigenvalues on the right half of the real-imaginary plane, unstable chatter may occur. However, if the parameters result in a stable system, chatter will not occur and proper machining can take place. This behavior is often illustrated with the well known stability lobe diagram (SLD). The diagram plots either the limiting stable feed-rate or cutting stiffness against running speed. The SLD gets its name from a pattern of lobes, repeating as speed increases, between which stable machining is possible for relatively deep cuts. These speeds are known as "sweet spots." Regenerative chatter is a mature topic and some good texts with thorough discussion of the subject are [14], [15], [16], and [17].

For extreme cases of regenerative chatter, the tool can actually lose contact with the work piece. In such cases there is a discontinuity in the cutting force when contact is lost. As a result, the instantaneous chip width will be a function of the current tool vibration and the vibration of a previous tool pass which is not necessarily the immediately previous pass. This phenomenon is called multiple regenerative chatter and

has been studied by Kondo et al. [18], Tlusty and Ismail [19], and Shi and Tobias [20]. If vibrations grow to the level where the tool leaves the workpiece, chatter prevention can already be considered to have failed; therefore multiple regenerative chatter is not considered in this work.

Another avenue of research is the short regenerative effect, first proposed by Stépán [21], where the force on the tool is due in part to the cut chip before it loses contact with the tool rake face. As a result, the waviness of the chip causes the tool to vibrate which in turn causes the waviness to continue. In the short time between the material leaving the workpiece and the chip leaving the tool rake face, regeneration can occur. This phenomenon is a relatively new avenue of research. For example, Taylor, Turner and Sims recently studied the short regenerative effect as a possible explanation for experimentally observed stability at low cutting speeds in a case where traditional chatter models predict instability and found that the effect could not, on its own, account for the observed behavior [22].

1.3.2 High-Speed Machining

In typical machining, there is a recommended cutting speed according to the work piece material. Beyond that recommended speed machining becomes very difficult with problems such as tool breakage, tool wear, excess cutting force and heat, etc. The key to HSM is that if the speed is increased enough, the required cutting force and temperature decreases [6]. This leads to several advantages of HSM. For example, the workpiece tends to heat up less. The generated heat is handled in a better way because the chip production is faster so the heat leaves before it has a chance to dissipate to the workpiece. Because of the reduced heat generation and transmission, there tends to be a lower need for coolant, and in some cases the need for coolant is eliminated. In addition, lower cutting forces tend to cause less tool deflection which is required for high precision machining. Also because of lower cutting forces, the cutting tool tends to wear more slowly and there is less breakage due to shock. Likewise, the machining of thin walled components becomes feasible. With higher speeds, the same metal removal rate can be achieved using a smaller linear federate per turn resulting in a better surface finish. A disadvantage is the higher speeds usually invoke higher accelerations and decelerations which can cause increased bearing wear when traditional spindle bearings are used [23]. HSM also has the somewhat more intuitive benefit that it allows for production times to be decreased [24].

1.3.3 Active Control of Chatter

As previously stated, to avoid chatter the machinist sets the feed-rate slow enough to avoid instability. To achieve high productivity in the presence of this limiting requirement, the machinist seeks a rotational speed which allows for the fastest possible stable feed-rate. But, this is just working with the tool system as it is. Some systems are more prone to chatter than others. In general, high dynamic stiffness and high damping are desirable in avoiding chatter. This is intuitive because a very stiff tool is not free to vibrate and damping removes energy from vibrations, including chatter vibrations. Unfortunately, stiffness and damping are restricted to physically realizable levels. There is no such thing as a perfectly rigid spindle. As technology continues to advance, machining speeds are getting faster, workpiece materials are getting harder, and part geometries are getting more intricate. This makes increasing the dynamic stiffness and damping more and more difficult.

An alternative to the heuristic stiffness and damping solutions is to find a tool system with complicated dynamics that counteract the specific chatter vibration which is encountered. An example is the tunable boring bar which enables the machinist to set the natural frequency of the system such that a sweet spot will occur at the desired machining speed [25]. In practice, the system is tuned through prolonged trial and error but tunable boring bars demonstrate the possibility to significantly increase the stable feed-rate. To meet the ever increasing production demands, engineers are turning to control systems to provide the complicated dynamics needed for robustly stable machining.

The following is a literature review of research in the area of control of chatter. It is concerned with application of chatter control to many different machining systems, not just of the HSM AMB spindle type studied in this work. The intent is to give the reader the breadth of the field which will help put the literature review on robust control of AMB spindles (Subsection 1.2.4) in context of the focus of this work, i.e., chatter driven controller design for an AMB spindle.

Active chatter control efforts can be summarized as mainly active speed (and/or feedrate) control as in [26], [27], and [28] or active control of the cutting tool. Active speed control merely aids the machinist in hitting certain sweet spots which may be difficult to find manually, and is not the focus of this work. It is worth noting that active speed control is able to help with machining of flexible workpieces even if they are not modeled, but it has the disadvantages that it does not actually improve the SLD and chatter must occur before an update of speed can take place. Active control of the cutting tool, however, seeks to achieve better dynamic characteristics using an actuator added to the machine. The control schemes for the actuator can be optimal, robust, or classical with machine and actuator being the plant. Active control can prevent chatter by improving the characteristics of the machine beyond what can be achieved by passive design.

There have also been works where a cutting model is included in the plant. This is the most advanced area of active control for chatter prevention research. By using modern, model-based control techniques and including the cutting model, the system can be made to have the best possible combination of dynamic stiffness, natural frequency etc. to achieve maximum stable feed-rate over the entire speed range. It can also be used to achieve a sweet spot at a specific machining speed.

A feedback wave controller for chatter suppression was developed by Mei [29] for a non-rotating boring bar assumed as a continuous cantilever rod using an Euler-Bernoulli beam and the regenerative chatter model of [12]. The resulting controller was of proportional-derivative form. Similarly in [30], Mei et al. develop an optimal LQR controller for the same cutting model and a lumped mass model of the boring bar. Also, a hybrid time and frequency domain approach was developed for generating the SLD after the control was implemented. Both works include numerical simulations which show an increase in the minimum limiting depth of cut across all speeds because of the higher damping due to the controller.

A time-varying stiffness approach to chatter suppression was studied in [31] by Mei et al.. In the study, parameters of a harmonic signal to the magnetorheological fluid in a boring bar support were optimized in order to suppress chatter. Sine, triangle, and square waves were compared and the square wave was found to be superior. Simulated results are confirmed by an experiment. In [32] the previous findings were analyzed with a van der Pol-Mathieu-Duffing oscillator model with a time delay. The authors found it was possible to stabilize chatter with any parametric excitation of sufficient size regardless of waveform.

In [33], the possibility of tool wear was studied by Moradi et al. as a parametric uncertainty in simple orthogonal turning. A robust sliding mode controller intended for a piezoelectric actuator imbedded in a fixed tool holder was developed. It was also found that the existence of tool wear was not harmful from a chatter standpoint because it induces a damping like effect which raised the stability lobes. Numerical simulations were done using a 1-DOF mass-spring-damper model for the cutting tool and a simple regenerative cutting force model.

In several works, [34], [35], and [36], Tewani and his colleagues study a piezoelectric actuated mass inside a cavity inside a long boring bar. The device is patented by Rouch in [37]. This novel active dynamic absorber, which can also be used as an active mass damper, is controlled with model-based approaches such as LQR and H_{∞} , and shown experimentally with machining tests to significantly improve the chatter characteristics of the system. Although a model of the boring bar and actuator was used for control design, it was only able to achieve better dynamic stiffness than a long slender boring bar on its own, indirectly avoiding chatter. The system with active control was shown through calculation of the stability lobes to also be superior to the boring bar with a passive tuned mass. Experimental frequency responses of the system with different controllers and no control were measured at the tool tip to show the increase in dynamic stiffness due to

active control. This study is an important precursor to using a model of the system in conjunction with the actual cutting force model to design a controller specifically to prevent chatter. The H_{∞} controller was found to be superior to the LQR controller because the LQR often had unreasonably high gains. Also, the H_{∞} controller was experimentally found to be robust to changes in length to diameter ratio of the boring bar.

An early work in the area of robust control of machining, which made use of a cutting force model, is [38]. Kashani et al. designed a robust controller for a lathe tool holder which was modeled as a single degree of freedom lumped mass. An improvement in a simulated workpiece surface finish is achieved despite the delay term in the cutting model. Although the authors only considered robust surface finish optimization, they laid the foundation for robust chatter attenuation.

A key reference for this dissertation is Chen and Knospe [39] in which one axis of a single AMB is used as an actuator for control of a spring loaded non-rotating platform on which a lathe tool holder is supported. μ -synthesis is used for controller design with a plant model which included a simple regenerative cutting force. Feedback linearization of the AMB force is used. Separate controllers for different cutting cases, a specific speed, all speeds, and a specified speed range, are designed and compared. The time delay in the machining model is replaced with an uncertainty, with weighting based on the non-rational delay transfer function, developed in [40] for controller design. This handling of the delay is as opposed to the more common Padé approximation of delay. (Note that yet another alternative is to model the delay as an uncertainty with weighting based on the Padé approximation and modified to be conservative as in [41].) One of the developed controllers is tested experimentally for chatter limit while machining and
compared to results for PID control. All controllers were subject to impact testing to find tooltip dynamic stiffness. This work contains many of the elements of the current problem, i.e., control of unobserved tool tip, time delay regenerative chatter, parametric uncertainty of AMB and machining force, etc. This method is deficient for the current problem because it is for low speed machining and a SISO system which is not gyroscopic.

Similarly, in [42] and [43] Dijk et al. applied μ -synthesis to design controllers using a cutting force model. Unlike [39] which considered a lathe with non-rotating tool, the authors applied the technique to a model for a high-speed milling spindle. The rotor was modeled as a two degree of freedom spring-mass system and inherently stable as to mimic a spindle on traditional supports. Significant increases in the simulated critical cut depth are achieved for specific spindle speeds and small speed ranges. However, outside of the design speed range, it was found that the critical depth of cut might actually decrease when active control is applied. In [42] the control input was just a force applied to the rotor, while the uncertain delay model of [39] for small speed ranges is used. In [43] the work is extended to include a magnetic force actuator model and a lower order model for the nominal delay. The lower order delay model is found to be more likely to converge to a local minimum during the iterations which find the μ -synthesis controller.

One example of a spindle on traditional supports using active control to avoid chatter is Dohner et al. [44] in which electrostrictive actuators were incorporated in the housing of a milling spindle supported on ball bearings. With LQG control of the electrostrictive actuators to add damping, the experimental metal removal rate was significantly improved without the need to change speed to avoid chatter. Because the approach was active damping of the tool, the plant was only the spindle and actuators and no cutting model was included.

A more recent example of a spindle on traditional supports is presented by Späh et al. [45], in which a motor spindle for milling on ball bearings with an electromagnetic actuator and eddy current position probes outside the front bearing (near the milling head) is controlled with LQG for damping of chatter. Controllers are designed for both considering and neglecting gyroscopic cross coupling. Also, adaptive filtering is proposed which would remove frequency components going to the controller which occur at the rotational speed and tooth passing speed. As a result, only the chatter would be actively damped. Therefore, this method cannot address chatter which happens to coincides with the rotational speed or tooth passing frequency.

Ehmann et al. [46] conducted a study in which a milling machine was supported on piezoelectric actuators and active damping was designed through μ -synthesis control of the actuators. An increase in the minimum calculated stable cut depth was found. Although μ -synthesis is used as a control strategy for the actuators, active damping is still achieved by proper selection of inputs and outputs of the plant and the corresponding weighting function. Therefore, a feedback cutting model was not included.

Kern et al. [47] studied a similar spindle actuator configuration to that in [45]. A μ synthesis controller was developed to add active damping similar to [46] but instead of the common practice to achieve robustness by defining the operational speed range with an uncertain real parameter, which effects gyroscopics and ball bearing stiffness, two controllers are designed for different speed extremes. Then the study focuses on the experimental comparison of three gain-scheduling approaches: bumpless transfer using an observer like structure, fuzzy gain scheduling, and controllers' poles and zeros gain scheduling. The one switching and two continuous methods were all made to be effective for the entire speed range from zero to 22000 RPM. Although no proposed method could be said to be superior in general, it was determined that the switching method was more convenient when the speed was not continuously changing (assuming transients were not a problem). The continuous methods would handle continuously changing speed well, with the fuzzy gain scheduling easier to implement but requiring more computational power.

A state-of-the-art review of monitoring and control of machining processes was conducted in 2004 by Liang, Hecker, and Landers [48] which includes sections on chatter detection and chatter suppression. In 2011, a review of chatter in machining processes was conducted by Quintana and Ciurana [49] and includes sections on both active chatter elimination and passive chatter avoidance as well as recent works on modeling and recognition. It was followed in 2012 by Siddhpura and Paurobally [50] who conducted a similar review specifically for chatter in turning operations.

1.3.4 AMB Machining Spindles

The benefits of AMBs to HSM have been discussed. The following are some examples of successful application of AMBs for machining spindles. In most cases there is no special consideration of chatter. The active control serves merely to provide stable levitation so the AMBs can fill the role of a bearing support while the other advantages of the AMB, e.g., high speed, access to control current, etc., are taken advantage of. The next subsection will address robust control of spindles. An early study of AMBs to a machining spindle is Zivi et al. [51]. A machining spindle was supported on AMBs with PID control. A secondary outer control loop monitored the workpiece and acted to counter unwanted tool path trajectory errors with feed-forward compensation of the AMB clearance. The outer loop used a laser position sensor to monitor the workpiece table. The laser was affixed to the spindle tool holder such that relative position was monitored.

A spindle, fully levitated with PID control, was used for measuring cutting forces by Auchet et al. [52]. In this study, the transfer function of input force on the tool to output AMB control current was first measured. Next, the inverse transfer function was found such that the load on the tool could be calculated in real time using the control currents. The unbalance force and gyroscopic effect were neglected. Agreement was found between the force estimated from the developed inverse transfer function method and force measurement from a dynamometer.

Recently, Gourc, Seguy, and Arnaud [53] made a detailed model of a HSM spindle on AMBs and used it in conjunction with a cutting force model to predict the SLD. The spindle was levitated using industrial PID control and the cutting model was only used for stability predictions, not controller synthesis. The model was compared to the results of the experimental tool tip transfer function and SLD generated with cutting tests with mixed results. The authors also made a very interesting analysis of the modal contribution to the SLD. Depending on the shape of each flexible mode, it may have a larger or smaller effect on the SLD.

In [54], H_{∞} control was applied to on an experimental fully levitated spindle. The two block mixed sensitivity problem was used, taking into account unknown high-order

flexible modes and unknown plant disturbances. The resulting step and orbit responses were compared to those for PID control of the same experimental system and found to have similar performance. Speeds of up to 45,000 RPM were achieved.

A miniature milling spindle with shaft diameter of 1.2 cm was designed and built in [55] for a tool tip precision error of 1.5 μ m at 150,000 RPM. A PID controller with negative stiffness compensation was used. To simulate the cutting force disturbance on the spindle during micro milling, the model developed in [56] is used and no chatter is considered. Although the max speed of 150,000 RPM was achieved, the spindle error at 125,000 RPM was measured and found to have a standard deviation of 7.5 μ m. No experimental cutting was performed.

Kyung and Lee [57] performed a numerical analysis of a rigid, fully levitated spindle under local PD control including a milling cutting model. The authors show how changes in proportional and derivative gain of the controllers would affect the stability lobes of the system. These results demonstrate a key concept of the current study. With a limited case of a theoretical, rigid rotor, the possibility of chatter prevention was shown using only controller design. When advanced controller design is applied to a real-world flexible system, modern engineering solutions to the chatter problem can be realized. In the reference, the cutting force model was not used for controller design, but only to show the effect of the controller on stability lobes.

1.3.5 Robust Control Applied to AMB Spindles

There are also several examples of μ -synthesis control being applied to AMBs for fully levitated spindles. To summarize, there are novel differences in plant modeling

having to do with how the uncertainties are structured and how the performance weighting functions are made. Unfortunately, use of the cutting force model in the plant to take advantage of the robust control stabilizing property to prevent chatter has not been done.

An AMB system was designed by Stephens [58] for a HSM application with special considerations for power losses at high speeds, temperature distributions using finite element approximations, and advantageous "budgeting" of the space for radial bearing clearances. For control, μ -synthesis was utilized with uncertainties on the rotor force inputs and position outputs at the AMB locations and multiplicative uncertainties on the tool mass. Also, fictitious uncertainties for performance are used for sensor noise, control effort, and external tool load and displacement to achieve acceptable tool compliance for machining. The spindle with this modeling was further studied by Stephens and Knospe in [59] and by Knospe, Fittro and Stephens in [60] and it was concluded that the chatter tendency can be minimized by using performance weighting such that the minimum real part of the tool tip compliance is maximized.

Once again, μ -synthesis was utilized for the same spindle by Fittro [61] and Fittro and Knospe [62]. This time, uncertainty is placed on the linearized position stiffness and current stiffness of the AMB, PWM power amplifier, optical position sensor, elliptical low-pass filter, and control input to the system. Uncertainty was also used to account for eddy current effects and the effect of implementing the controller digitally, both of which are difficult to model reliably. Controllers were developed and compared for complex and mixed uncertainties with similar results. It was found that this approach yielded robust performance while not being overly conservative and experimental results agreed

with simulation results. This is another example of tooltip dynamic stiffness as a controller design performance requirement for an indirect approach to prevent the likelihood of chatter occurring. The thrust axis AMB for the spindle was given individual consideration in [63] and [64]. Finally, a summary of these efforts as well as the case for using AMBs in machining and chatter prevention is given by Knospe in [65].

Another example of robust control of an AMB milling spindle is [66], in which Nonami et al. develop a μ -synthesis controller using the descriptor form of the magnetic bearing system. Real additive uncertainty is placed on the elements of the mass matrix and on the linearized AMB current stiffness and position stiffness to account for real uncertainty in the nonlinear AMB force function calibration factor. Mixed sensitivity performance weights are placed on sensor noise input and control effort output. No machining model was used for controller synthesis. It was found that using this combination of descriptor form of a flexible rotor, mass uncertainty, and mixed sensitivity, that stabilization of the AMB could be achieved even with high stiffness. It was also found that the gyroscopic coupling could be neglected so that two independent controllers (one for each perpendicular plane) could be used.

References [67] and [68] are a two part series by Ahn and Han encompassing a complete robust controller design process for an AMB spindle. Part one is the rigorous modeling process required for any modern model-based control method including component measurement for uncertainty estimation and assembled open-loop plant model validation. Part two contains controller design and experimental implementation. Diagonal real uncertainties are placed on the poles of the plant which includes the rotor, AMB position stiffness feedback, sensor gain, and AMB current stiffness. Full matrix

complex multiplicative uncertainty is placed on the plant control input. Performance weights are put not only on sensitivity function and complementary sensitivity function but also on controller-sensitivity function and sensitivity function-plant for an augmented mixed sensitivity problem. This combination was found to be effective at preventing the synthesis of an unstable controller while still yielding the practical performance desired.

The spindle of interest in the present work was already studied in [69], in which μ synthesis control was performed for the fully levitated HSM spindle. It was shown through experiment and simulation that the μ -controller yielded a higher dynamic stiffness than the PID controller previously used in industrial applications. In [70], Pesch and Sawicki showed that an AMB controller can be designed to prevent machining chatter by including a cutting force model in the μ -synthesis process. Then in [71], Pesch and Sawicki applied an experimental method, based on Nyquist stability theory, to predict a SLD for the chatter avoiding μ -controller, showing an improvement over the current state-of-the-art.

1.4 Structure of This Work

Chapter II is an introduction to the μ -synthesis robust control design technique with the goal of explaining basic mechanisms such that the proposed chatter avoiding method can be understood. Chapter III is a description of the experimental research platform which includes modeling and identification of the AMB HSM spindle. The foundational model used for controller design is built and validated. Chapter IV presents the novel chatter avoiding controller design approach. A chatter avoiding controller is found for the AMB spindle considered and the controller is compared to a baseline controller made using the current state-of-the-art technique. Chapter V discusses criterion used to evaluate an arbitrary tool's suitability for machining. The criterion will be later applied to evaluate the spindle levitated with the different controllers. Chapter VI presents and discusses the results of simulations of the controlled system. The simulations include the chatter limiting criterion, SLDs, lift-off tests, and machining simulations. Chapter VII presents the results of the experimental implementation of the designed controllers including limiting chatter criterion and lift-off tests. Chapter VIII presents the concluding remarks for the dissertation.

CHAPTER II

PRIMER ON μ -SYNTHESIS

2.1 Overview

 μ -synthesis is a well established modern control technique. The basic concepts of μ synthesis are discussed in this chapter such that its application to AMB control and
machining can be understood.

 μ -synthesis is a linear, model-based robust controller design approach. The actual μ synthesis is performed by commercially available codes, e.g., the MATLAB Robust Control Toolbox. Therefore, successful implementation is contingent on the engineer correctly defining the system model. Specifically, a correct model includes a highly accurate description of the nominal plant, proper weighting of selected inputs and outputs to achieve desired performance, and the description of any possible change in operating conditions through the use of parametric uncertainties.

The next section defines uncertainties and discusses how to model them. Section 2.3 covers linear fractional transformation (LFT), a linear operation that is employed in

modeling for μ -synthesis. Section 2.4 discusses the structured singular value, μ , from which μ -synthesis gets its name. Finally, Section 2.5 discusses the *D*-*K* iteration, the iterative process in which the bound on the μ -value is used to synthesis a robust controller.

2.2 Uncertainty

Robust control is control which achieves stability and performance for a plant which is uncertain. A plant is said to be uncertain when the parameters or the overall dynamics are not known. For example, the stiffness of a nonlinear spring might be found by linearizing the spring's force function about an operating point. If the operating point will be changing then the spring stiffness can be considered to vary in some range. Other examples of factors that can cause uncertainty include manufacturing tolerances and change in ambient conditions. There are three main types of uncertainty: additive, input multiplicative, and output multiplicative. These types of uncertainty are illustrated in Figure 2.1 [72].



Figure 2.1 Block structures for a) nominal system, b) additive uncertainty, c) input multiplicative uncertainty, and d) output multiplicative uncertainty.

The signals for the systems are described by:

- a) Nominal: b) Additive: c) Input Multiplicative: $y = G_0 u$ $y = (G_0 + \Delta)u$ $y = G_0(I + \Delta)u$
- d) Output Multiplicative: $y = (I + \Delta)G_0 u$

Therefore the transfer functions for the corresponding systems are:

a) Nominal: b) Additive: c) Input Multiplicative: d) Output Multiplicative: $G = G_0 + \Delta$ $G = G_0 (I + \Delta)$ $G = (I + \Delta)G_0$

 G_0 is the nominal open-loop plant to be controlled and Δ is the perturbation matrix which accounts for uncertainty in the system. *I* is the identity matrix of the appropriate dimensions.

If G_0 is a MIMO transfer function and Δ is a fully populated matrix (of the appropriate dimension) the uncertainty is called unstructured. Alternatively, if Δ is a block diagonal

matrix, it is called structured uncertainty. An uncertainty matrix which is structured is said to belong to Δ , which is the set of all uncertain matrices with the specified structure.

If G_0 is a SISO transfer function, Δ is of dimensions 1×1 and analyzing as structured or unstructured is equivalent. A common example is when G_0 is a term representing a single parameter in a larger system equation. This case is called parametric uncertainty.

Now consider if the open-loop plant to be controlled is the interconnections of several uncertain systems. Regardless of whether the uncertainty blocks are structured or unstructured for the individual subsystems, the overall uncertainty matrix will be structured. Specifically, parametric uncertainty within an interconnected plant will manifest itself as a purely diagonal portion of a block diagonal structured uncertainty matrix. Uncertainty structure of interconnected systems is discussed more thoroughly in Section 2.3, Linear Fractional Transformation.

The distinction between structured and unstructured uncertainty is significant because either structured or unstructured uncertainty representation can be used to guarantee robustness but maintaining the structure of the uncertainty allows for the design of a robust controller while not being overly conservative. This fact is discussed more thoroughly in Section 2.4, The Structured Singular Value.

It is common to weight the uncertain perturbation such that Δ is norm-bounded. For example, the multiplicative uncertainty shown below is given a weighting function W so that the uncertain plant covers the complete plant set while the infinity norm of the uncertain matrix Δ is less than or equal to 1.

$$y = G_0(I + W\Delta)u, \qquad \|\Delta\|_{\infty} \le 1$$

The infinity norm is defined as [72]:

$$\|\Delta\|_{\infty} \coloneqq \sup_{\omega} \overline{\sigma}(\Delta(j\omega)) \tag{2.1}$$

 ω is frequency and $\overline{\sigma}$ is the largest singular value, which is the maximum gain as the direction of the input is varied.

Consider the system $w = \Delta z$. If the system is SISO, its frequency dependent gain is

$$\frac{|z(\omega)|}{|w(\omega)|} = \frac{|\Delta(j\omega)w(\omega)|}{|w(\omega)|} = |\Delta(j\omega)|$$
(2.2)

which is independent of input magnitude. Alternatively, if the system is MIMO, the input w and output z are vectors. The size of the input and output vectors must be evaluated by a norm. In this case, the 2-norm is utilized. The frequency dependent gain of the MIMO system is therefore:

$$\frac{\left\|z(\omega)\right\|_{2}}{\left\|w(\omega)\right\|_{2}} = \frac{\left\|\Delta(j\omega)w(\omega)\right\|_{2}}{\left\|w(\omega)\right\|_{2}}$$
(2.3)

Again, the gain is independent of the magnitude of the input vector. The gain is, however, dependent on the direction of the input vector. The largest gain with respect to the direction of the input is the maximum singular value.

$$\max_{w\neq 0} \frac{\left\|\Delta(j\omega)w(\omega)\right\|_{2}}{\left\|w(\omega)\right\|_{2}} = \max_{\|w\|_{2}=1} \left\|\Delta(j\omega)w(\omega)\right\| = \overline{\sigma}(\Delta)$$
(2.4)

The weighting on the uncertainty, W, is a transfer function matrix of appropriate dimension. There may be left side weighting, right side weighting, or both. The weighting is a constant gain for the case of an unknown parameter with a range that is not affected by frequency. For robust controller synthesis, the uncertainty matrix is norm bounded in this way for interpretation of the μ -value. The corresponding plant model contains all weights to keep consistent with the norm-bounded uncertainty matrix, Δ .

2.3 Linear Fractional Transformation

Linear fractional transformation (LFT) is a procedure used to convert parametric uncertainties inside a system model to the usable representation for robustness analysis and subsequent controller design. The following is a brief summary of the concepts of LFT to provide the reader with a familiarity for the remainder of the dissertation. A more thorough discussion can be found in [73].

2.3.1 Basics of Linear Fractional Transformation

Consider a matrix M which relates input vector r to output vector v as shown in Figure 2.2 and Eq. (2.5),



Figure 2.2 Generalized matrix relation.

$$v = Mr \tag{2.5}$$

If *r* and *v* can be partitioned into upper parts and lower parts, i.e. $r = \{r_1, r_2\}^T$ and $v = \{v_1, v_2\}^T$, then *M* can be accordingly partitioned as shown in Figure 2.3 and Eq. (2.6).



Figure 2.3 Partitioned matrix relation.

$$\begin{cases} v_1 = M_{11}r_1 + M_{12}r_2 \\ v_2 = M_{21}r_1 + M_{22}r_2 \end{cases}$$
(2.6)

Note that the elements of M may be complex and could be the values of a MIMO transfer function evaluated at a particular frequency. The vector partitions r_1 , r_2 , v_1 and v_2 are themselves vectors. The matrix partitions M_{11} , M_{12} , M_{21} and M_{22} are themselves matrices. Finally, if part of the input can be related to part of the output, for example by the matrix Δ such that $r_2 = \Delta v_2$, then an LFT can be written which eliminates r_2 and v_2 and leaves the relationship from r_1 to v_1 . This is shown graphically in Figure 2.4 and analytically in Eq. (2.7).



Figure 2.4 Partitioned matrix relation with feedback, lower LFT.

$$v_1 = [M_{11} + M_{12}\Delta(I - M_{22}\Delta)^{-1}M_{21}]r_1$$
(2.7)

Therefore the matrix relating r_1 to v_1 is:

$$F_{L}(M,\Delta) := [M_{11} + M_{12}\Delta(I - M_{22}\Delta)^{-1}M_{21}]$$
(2.8)

For another case, set $r_1 = \Delta v_1$. The LFT can be written which eliminates r_1 and v_1 leaving the relationship from r_2 to v_2 . This is shown graphically in Figure 2.5 and analytically in Eq. (2.9).



Figure 2.5 Partitioned matrix relation with feedback, upper LFT.

$$v_2 = [M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}]r_2$$
(2.9)

Therefore the matrix relating r_2 to v_2 is:

$$F_{U}(M,\Delta) := [M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}]$$
(2.10)

The first case, where r_2 is related to v_2 , is called a lower LFT. The second case, where r_1 is related to v_1 , is called an upper LFT. The notation for the lower LFT is F_L and the notation for the upper LFT is F_U . Both are functions of the original matrix, M, and the matrix with which it is interconnected, Δ .

2.3.2 A Single Uncertainty Cast as a LFT

Uncertainties are defined and discussed in Section 2.2. LFTs are defined and discussed in Subsection 2.3.1. In the machinery of robust analysis the uncertainties are expressed with LFTs so the properties of LFTs can be exploited.

For an example of a parametric uncertainty cast as an LFT, consider a spring with

stiffness 10±2 N/m. This situation can be modeled as an input multiplicative uncertainty like the one shown in Figure 2.1c with $G_0 = 10$ and a pre-multiplied weight with magnitude 0.2 on the uncertainty Δ . To convert the original block structure to an LFT, designate the signals related by matrix Δ as w and z. Then, solve for both z and force in terms of w and deflection and put the system in matrix notation. Figure 2.6 shows the block diagram for the uncertainty example and the corresponding LFT.



Figure 2.6 Example of input multiplicative parametric uncertainty (left) and corresponding LFT (right).

2.3.3 LFTs for Robust Analysis

Linear combinations of LFTs are themselves LFTs. Also, the structure of the feedback is maintained when the LFT is performed. Because the structure is maintained, the LFT is useful in robust analysis. Specifically, the parametric uncertainties in a system are cast as a LFT so the μ -value can be evaluated using the structured uncertainty matrix. See Section 2.4 on the structured singular value, μ . As an example of maintaining structure, consider an upper LFT cascaded with a lower LFT as shown in Figure 2.7.



Figure 2.7 Upper LFT cascaded with lower LFT.

The resulting matrix M relates inputs u, z_2 , and z_1 to outputs y, w_2 , and w_1 .

$$\begin{cases} w_1 \\ w_2 \\ y \end{cases} = M \begin{cases} z_1 \\ z_2 \\ u \end{cases}$$
(2.11)

To find M, first solve for y and w_2 in terms of z_2 and the intermediate signal x. Then solve for w_1 and x in terms of z_1 and u. Finally substitute the expression for x in the solution for y and z_2 and put the system of equations in matrix notation. The result is Eq. (2.12).

$$\begin{cases} w_1 \\ w_2 \\ y \end{cases} = \begin{bmatrix} G_{11} & 0 & G_{12} \\ H_{21}G_{21} & H_{22} & H_{21}G_{22} \\ H_{11}G_{21} & H_{12} & H_{11}G_{22} \end{bmatrix} \begin{cases} z_1 \\ z_2 \\ u \end{cases}$$
(2.12)

The relations of z_1 to w_1 and z_2 to w_2 can also be put in matrix notation defining the matrix Δ .

$$\begin{cases} z_1 \\ z_2 \end{cases} = \begin{bmatrix} \Delta_1 & \mathbf{0} \\ \mathbf{0} & \Delta_2 \end{bmatrix} \begin{cases} w_1 \\ w_2 \end{cases}$$
(2.13)

With *M* and Δ defined, the system relating *y* to *u* can now be cast as the upper LFT $F_U(M,\Delta)$.



Figure 2.8 Upper LFT with structured uncertainty resulting from the interconnection of two LFTs.

The significant observation from this example is that the matrix Δ is not fully populated, rather it is diagonal. Note that Δ may be diagonal or block diagonal depending on the dimensions of Δ_1 and Δ_2 . If it is block diagonal, the zeros denote zeros matrices of the appropriate dimensions. The important conclusion is that the structure of the uncertainty is preserved throughout the LFT process. The set Δ is defined which is the set of all Δ matrices which have the structure dictated by the LFT. The structure is utilized in μ -analysis, discussed in the following Section 2.4.

2.4 The Structured Singular Value

The structured singular value, μ , is a measure of a system which provides for necessary and sufficient conditions for robust stability and robust performance. The structured singular value of matrix M, which is the closed-loop system evaluated at a particular frequency, is defined as [72]:

$$\mu(M) := \frac{1}{\inf_{\Delta} \{\overline{\sigma}(\Delta) \mid \det(I - M\Delta) = 0, \Delta \in \Delta\}}$$
(2.14)

unless no perturbation makes the determinant singular, in which case $\mu(M) := 0$. Notice that μ is a function of the uncertainty matrix Δ . The structured singular value is sometimes written $\mu_{\Delta}(M)$ to emphasize its dependence on Δ . There are four parts to the definition of μ :

1) The determinant of $(I - M\Delta)$ equals zero.

This condition means the uncertain perturbation causes the closed-loop system matrix to become singular. This condition provides for system stability when the μ -value is evaluated over all frequencies and is discussed later in this section.

2) Δ must belong to Δ .

The uncertainty matrix must belong to the set which has the specified structure and bound.

3) Maximum singular value of Δ .

This is a measure of the size of the uncertainty matrix. See the note on norms in Section 2.2.

4) Minimize with respect to Δ .

This seeks the smallest value of the perturbation which results in instability. The worst case scenario is the configuration of Δ that can destabilize the

system with the smallest size. Because M is typically created by normbounding Δ to 1, the desired μ -value is below 1.

The question of μ is: what is the smallest size of the uncertain perturbation matrix (in terms of the maximum singular value) which makes the closed-loop system matrix ($I - M\Delta$) singular? Because the perturbation matrix is norm bounded to 1, a μ -value below 1 means there is no combination of uncertainties within the specified ranges which makes the closed-loop system matrix singular. A μ -value above 1 means there is some combination of uncertainties within the specified ranges which makes the closed-loop system matrix singular. A μ -value above 1 means there is some combination of uncertainties within the specified ranges which makes the closed-loop system matrix singular. Therefore, systems are designed to keep the μ -value under 1.

Consider the generalized block diagram of an uncertain system shown in Figure 2.9.



Figure 2.9 Generalized block diagram of an uncertain system.

The uncertain system is described by the system of equations:

$$\begin{cases} w = Mz \\ z = \Delta w \end{cases}$$
(2.15)

Combining these equations to eliminate *z* yields:

$$w = M\Delta w \tag{2.16}$$

This can be rearranged as:

$$(I - M\Delta)w = 0 \tag{2.17}$$

To explain intuitively, albeit not rigorously, there are two possible solutions. The vector w can be zero which is likened to stability. Alternatively, the term $(I - M\Delta)$ can be singular allowing w (and also z) to take any value, however arbitrarily large. This can be likened to instability [73].

So far, the discussion of the structured singular value has been restricted to matrices which are from an uncertain system evaluated at a particular frequency. The μ -value has been shown to provide necessary and sufficient conditions for robust stability of the full system when evaluated over all frequencies and the worst case taken. Zhou and Doyle [74] showed that:

$$\sup_{s\in\overline{C}_{+}}\mu(G(s)) = \sup_{\omega\in\mathbf{R}}\mu(G(j\omega))$$
(2.18)

The proof follows: For any system G(s), $\sup_{s \in \overline{C}_+} \mu(G(s)) \ge \sup_{\omega \in \mathbb{R}} \mu(G(j\omega))$ because $j\omega$ is a subset of *s*. Also, $\sup_{s \in \overline{C}_+} \mu(G(s)) \le \sup_{\omega \in \mathbb{R}} \mu(G(j\omega))$ because, if the supremum with respect to *s* can be evaluated, there exists some frequency ω' with the same μ -value but requiring a

constant gain $0 < \alpha \le 1$ to maintain the singularity condition, $\det(I - G(j\omega')\alpha\Delta) = 0$. Therefore, the equality in Eq. (2.18) must be true. For the thorough proof, see the reference.

2.5 Model Augmentation for Controller Synthesis

How to model and analyze an uncertain system is discussed in the previous sections. This section clarifies a few details needed to use μ analysis for practical controller synthesis. First, consider Figure 2.10 where Δ_U is the structured uncertainty block and *P* is the nominal plant to be controlled but including weights to norm bound Δ_U as discussed in Section 2.2. *K* is a known controller for which the closed-loop system is analyzed. Now, input vector *d* and output vector *e* are added. The inputs and outputs not associated with the uncertainty or control are included in the system model at the discretion of the controls engineer for the purpose of achieving desired performance in the presence of anticipated disturbances.



Figure 2.10 Block diagram illustrating interconnection of nominal plant, uncertain perturbation, and controller.

For the nominal system ($\Delta_U = 0$), controller design can be carried out with the well known H_{∞} approach. In the H_{∞} approach, a controller K is found by solving two Riccati equations which minimize the infinity norm of the closed-loop transfer function from d to e [74]. To further improve closed-loop system performance weighting transfer functions are added to the inputs and outputs. The weighting functions are artificial components of the plant and are added at the discretion of the controls engineer. H_{∞} control synthesis is then performed on the weighted plant. Performance weighting is illustrated in Figure 2.11 where W_1 is the input weight, W_0 is the output weight and the infinity norm is minimized from d' to e'. It is helpful to think of the input weight as the maximum expected disturbance and the output weight as the inverse of the maximum allowable response. In the MIMO case, W_1 and W_0 are usually diagonal transfer function matrices.



Figure 2.11 Block diagram for robust control problem with performance weights.

An extension of H_{∞} control is robust H_{∞} control in which the infinity norm is minimized from $\{z \ d'\}^{T}$ to $\{w \ e'\}^{T}$. Robust H_{∞} is superior to nominal H_{∞} in that it can guarantee robust stability while still working to maximize robust performance. Unfortunately, Robust H_{∞} is overly conservative to interaction of uncertainties. To achieve higher levels of robust performance while still guaranteeing robust stability, the structured singular value is utilized in place of the infinity norm, which maintains information on the structure of the uncertain perturbation matrix. To evaluate μ for the system, including performance inputs and outputs, the uncertain perturbation matrix Δ_U is augmented with a fictitious performance matrix Δ_P . The augmented system is shown in Figure 2.12. In this way, robust performance is cast as a robust stability problem and can be addressed with the structured singular value.



Figure 2.12 Block diagram with augmented uncertainty matrix for robust stability and robust performance.

Comparing Figure 2.12 to Figure 2.9, it is clear that: 1) the perturbation block Δ is a combination of the uncertainty matrix Δ_U and the fictitious performance matrix Δ_P and 2) the system *M* is a lower LFT of the plant and a known controller with added weights for the performance inputs and outputs.

$$\Delta = \begin{bmatrix} \Delta_{\rm U} & \mathbf{0} \\ \mathbf{0} & \Delta_{\rm P} \end{bmatrix}$$
(2.19)

$$M = F_L(P', K) \tag{2.20}$$

Where P' is the nominal plant weighted for performance. Δ_U has block diagonal structure, originating from parametric uncertainties in the system factored out through the LFT process. Δ_P is fully populated therefore unstructured. Δ , being the diagonal assembly of Δ_U and Δ_P , is block diagonal and therefore is structured.

2.6 D-K Iteration

How to model uncertainty and analyze robustness is discussed in the previous sections. In this section, robust controller design using uncertainty modeling and robust analysis is discussed. The μ -synthesis robust controller design procedure is performed through *D-K* iteration in which μ is evaluated via an upper bound and a stabilizing controller is synthesized iteratively. The "*D*" step is the evaluation step and gets its name from the scaling used to upper bound μ . The "*K*" step is the controller synthesis step and gets its name from the common symbol for a controller.

2.6.1 D Scaling

The μ -value cannot be calculated directly except for in a few specific cases [75]. A practical method of approximating the μ -value for the general case has been developed which makes use of the upper bound on the μ -value and a scaling matrix D that

commutes with Δ .

Consider the extreme case for μ where Δ is a fully populated complex matrix. The structured singular value is then equivalent to the maximum singular value of M.

$$\mu(M) = \bar{\sigma}(M), \qquad \Delta \in \mathbb{C}^{n \times n} \tag{2.21}$$

The proof follows the small gain theorem [74]. Including any structure can only make the resulting μ -value smaller. Therefore, the maximum singular value of *M* represents an upper limit for the true structured singular value.

$$\mu(M) \le \overline{\sigma}(M), \qquad \Delta \in \Delta \tag{2.22}$$

Assume a matrix D which is included in the uncertain system, along with its inverse D^{-1} , such that the overall system is not changed. The situation is illustrated in Figure 2.13 which is the uncertain system shown in Figure 2.9 augmented by the additional matrices, the so-called D scaling.



Figure 2.13 D scaled uncertain system.

If *D* is selected such that it commutes with Δ ,

$$D\Delta = \Delta D \tag{2.23}$$

it is straightforward to show that

$$\Delta = D\Delta D^{-1} \tag{2.24}$$

The structured singular value is unchanged by the *D* scaling because

$$(I - M\Delta) = (I - DMD^{-1}D\Delta D^{-1}) = (I - DMD^{-1}\Delta)$$

and the following equality can be written [73].

$$\mu(M) = \mu(DMD^{-1})$$
 (2.25)

With the inclusion of the *D* scaling one can make practical use of the upper bound.

$$\mu(M) = \mu(DMD^{-1}) \le \overline{\sigma}(DMD^{-1}), \qquad \Delta \in \Delta, \ D\Delta = \Delta D$$
(2.26)

Because there are many possible D matrices which commute with Δ the following statement can also be made:

$$\mu(M) \le \inf \overline{\sigma}(DMD^{-1}), \qquad D\Delta = \Delta D \qquad (2.27)$$

A very close, yet conservative, approximation of the μ -value can be found by minimizing the upper bound of μ with respect to the matrix *D*.

$$\mu(M) \approx \min_{D} \overline{\sigma}(DMD^{-1}), \qquad D\Delta = \Delta D \qquad (2.28)$$

The minimization is a convex numerical problem [75] and can therefore be driven to convergence within a desired tolerance. Furthermore, when the upper bound is minimized, it can be compared to the spectral radius, which is a lower bound for the μ -value.

$$\rho(M) \le \mu(M) \tag{2.29}$$

The comparison of the minimized upper bound and lower bound gives a narrow range for the actual μ -value. Similarly as for the upper bound, M can be scaled in such a way to affect the lower bounding spectral radius but not the structured singular value. A matrix U is used which belongs to the set of structured uncertainties Δ but is also a unitary matrix. Then, the lower bounding spectral radius can be maximized with respect to U.

$$\max_{U} \rho(UM) \le \mu(M) \tag{2.30}$$

Once the approximate μ -value (upper bound) is found, it is used to evaluate robust

stability and performance as described in Section 2.5. The D scaled system is also used to find a new controller as discussed in the next section.

2.6.2 Iterative Controller Synthesis

The following steps take place when performing a *D*-*K* iteration algorithm [76]:

- 0) Initialize with a starting controller K.
- 1) Find $M = F_L(P',K)$.
- 2) Evaluate the upper bound on μ using *D* scaling at all discrete frequencies of interest ω .
- 3) Curve fit the frequency-by-frequency $D(\omega)$ scale with rational transfer functions D(s).
- 4) Find the robust H_{∞} controller for the weighted and scaled system. That is:

$$\min_{K} \left\| D(s) F_{L}(P', K) D^{-1}(s) \right\|_{\infty}$$
(2.31)

The controller found in step 4) becomes the new controller in step 1). The process is repeated as *K* converges to the optimally robust controller. If the plant is correctly constructed, i.e., $\|\Delta\|_{\infty} \leq 1$, then a $\mu < 1$ indicates robust stability and performance using the synthesized controller, i.e. successful controller synthesis.

There is a know solution to the H_{∞} problem. Additionally, because *D* scaling is a convex numerical minimization, the bound on μ is guaranteed to converge to a solution. However, in *D*-*K* iteration, iterating between the *D* scaling and controller synthesis is not guaranteed to converge [72]. This issue can be mitigated somewhat by trying several different controllers as initial starting points for the *D*-*K* iteration.

CHAPTER III

IDENTIFICATION AND MODELING OF AMB SPINDLE

3.1 Overview

An industrial grade AMB HSM spindle is the platform for this research. In this chapter, the components of the spindle system are described, modeled, and experimentally identified. The components include AMB actuators, digital controller hardware, rotor, sensors, and power amplifiers. The open-loop system assembly, which constitutes the plant in the control problem, is discussed including the mathematical assembly of component models and the inputs and outputs which will be used for controller design. Finally, experimental identification of the open-loop plant is performed and compared to the model.

Figure 3.1 shows the AMB HSM spindle setup. The spindle's tool holder can be seen extended from the spindle housing. Opposing the spindle is a chuck which is on a movable track so that the workpiece can be fed over the cutting tool. The spindle and chuck are inside a protective cover which is open for the photograph.



Figure 3.1 AMB HSM spindle setup.

The spindle was developed for single point boring. The spindle is supported by two radial AMBs and one thrust AMB. The static load capacity is approximately 1400 N for the front bearing (nearest to the cutting tool), 600 N for the back bearing, and 500 N for the thrust bearing. An asynchronous AC electric motor is situated between the bearings, so that no additional motor bearings are needed. The motor can drive the spindle up to 50,000 RPM at 10 kW.

For research, the industrial spindle system is adapted with BNC access to the AMB integral position probe output signals and amplifier control current input signals. In this way, position data can be collected by an external data acquisition system while the machine operates with an industry standard PID control. This setup also allows for the implementation of the experimental controllers which are studied in the current work.

3.2 Components

3.2.1 Radial AMBs

The AMBs for the spindle are described in this section for the purpose of modeling for robust control of chatter. For a thorough discussion of the field of magnetic bearings see Schweitzer and Maslen [77].

In the AMBs, a magnetic field is generated by an electromagnet. The magnetic field creates an attractive force on a ferromagnetic rotor. To attain full range of control, with forces in both positive and negative directions, a second electromagnet is on the opposite side of the rotor. Each electromagnet has a corresponding position sensor and these components constitute a control axis.

The force from one of the electromagnet will always be attractive, no matter the direction of the current. A positive coil current and a negative coil current will induce a force in the same direction. Therefore, the coil current is constrained to be positive to avoid confusion in control. This is done by applying a positive bias current I_b to each coil and adding a control current I_c to it. The control current may be positive or negative but can never have a greater magnitude than the bias current.

Differential control is applied, i.e., in which a single control current signal is added to one coil and simultaneously subtracted from the opposing coil. For these particular AMBs, one position signal is obtained by adding one of the sensor signals to the opposing one such that the geometric center of the rotor is tracked. Note that the sensors are calibrated in this configuration. Figure 3.2 shows a conceptual diagram of one AMB axis with differential control where the nomenclature agrees with Eq. (3.1).



Figure 3.2 Diagram of one AMB axis with differential control.

The radial AMBs of the spindle are homopolar with eight poles, which is a common and successful AMB configuration. Two pairs of magnets are oriented around the rotor to comprise two perpendicular control axes. There are two axes per radial bearing so the two radial bearings define four axes of physical space. The other two axes of physical space are handled by the thrust bearing and the motor. Figure 3.3 shows the experimental spindle housing with back cover removed showing the rear AMB. The eight stator poles with wire coils that resemble four horseshoe electromagnets can be seen. The stator is actually a single piece of laminated material which is wired to behave as four separate magnets. The poles alternate between north and south, making the AMB homopolar. The control axes are at 45° from the vertical or horizontal directions. The 45° angle makes the gravity preload effect both axes equally. Therefore, the weight of the rotor is supported by both axes. As a result, the gravity preload does not cause anisotropy in the bearings, which may cause undesirable rotordynamic effects, such as backward whirl.


Figure 3.3 Photograph of spindle with back cover removed showing rear AMB coils and rotor.

The sections of the rotor inside the AMBs are made of laminated cobalt steel alloy shrunk fit on the shaft, which is made of AISI 4140 alloy steel. The cobalt steel alloy has a high magnetic saturation point and the laminations serve to break up eddy currents. Eddy currents are induced by the rapidly changing magnetic field of the AMBs and are a source of energy loss. Energy conservation in AMBs is studied by Barbaraci et al. in [78]. The rotor model including geometry and sensor/actuator non-collocation issues is directly addressed in Subsection 3.2.3.

The force from an AMB on the rotor in one axis is expressed with Eq. (3.1), which is a nonlinear function of control current and rotor position.

$$F = \frac{1}{4} \varepsilon \mu_0 N^2 A \cos(\alpha) \left[\frac{(I_b + I_c)^2}{(g - x)^2} - \frac{(I_b - I_c)^2}{(g + x)^2} \right]$$
(3.1)

where:

F = force of AMB on rotor [N] $x = \text{position of rotor } [\mu\text{m}]$ $g = \text{effective nominal air gap } [\mu\text{m}]$ $I_b = \text{bias current [A]}$ $I_c = \text{control current } [A]$ $\mu_0 = \text{permeability of free space } [\frac{N}{A^2}]$ N = number of coil windings $A = \text{area of pole face } [\mu\text{m}^2]$ $\alpha = \text{pole angle } [\text{rad}]$ $\varepsilon = \text{derating factor}$

The AMB parameter values are given in the appendix. In order to perform μ -synthesis, the force equation must be linearized about the operating point (in this case zero deflection and control current) to yield Eq. (3.2).

$$F = k_i I_c + k_x x \tag{3.2}$$

Here, k_i is current stiffness in $\frac{N}{A}$ and k_x is position stiffness in $\frac{N}{\mu m}$. The stiffnesses are defined as the partial derivative of AMB force with respect to each corresponding variable at the operating point, as follows:

$$k_{i} := \frac{\partial F}{\partial I_{c}} \bigg|_{x=x_{0}, I_{c}=I_{c}, x_{0}} \qquad \qquad k_{x} := \frac{\partial F}{\partial x} \bigg|_{x=x_{0}, I_{c}=I_{c}, x_{0}}$$
(3.3)

The operating point is the setpoint of the AMB controller x_0 and the resulting static

control current. In the present case, the setpoint is zero μ m and the resulting control current is zero Amps. Therefore:

$$k_{i} = \frac{1}{4} \varepsilon \mu_{0} N^{2} A \cos(\alpha) \left[(2)(1) \frac{(I_{b} + I_{c})}{(g - x)^{2}} - (2)(-1) \frac{(I_{b} - I_{c})}{(g + x)^{2}} \right]_{0,0}$$
(3.4)

$$k_{x} = \frac{1}{4} \varepsilon \mu_{0} N^{2} A \cos(\alpha) \left[(-2)(-1) \frac{(I_{b} + I_{c})^{2}}{(g - x)^{3}} - (-2)(1) \frac{(I_{b} - I_{c})^{2}}{(g + x)^{3}} \right]_{0,0}$$
(3.5)

yielding:

$$k_i = \frac{\varepsilon \mu_0 N^2 A \cos(\alpha) I_b}{g^2}, \qquad k_x = \frac{\varepsilon \mu_0 N^2 A \cos(\alpha) I_b^2}{g^3}$$
(3.6)

Given the linear force equation above, both position stiffness and current stiffness are positive. If the rotor is at zero position with zero control current and no external load, it will be at equilibrium with a net force of zero acting on it. If a small positive displacement is given to the rotor, a net positive force will result. It is easy to see that such a system will be unstable without active control. Also if starting at equilibrium and a small positive current disturbance is added, the net force will be positive as well. Therefore, in general, the control feedback must be negative such that a negative control current will result from a positive displacement.

The current stiffness and position stiffness are experimentally measured. The measurement procedure is as follows. Rotor levitation is achieved with a preliminary

PID controller. Incremental weights are applied to the spindle at the tool location. Transients are allowed to die out so the system is at steady state. The integrator terms in the localized PID controllers cause the rotor position to be zero at both bearings. The resulting control current is measured for each value of external load. The reaction forces at the bearings are readily calculated by solving the statics problem. The reaction force-control current relationship is found. The slope of the linear curve fit of the data is the current stiffness. Next, the external weight is removed and the controller's set point is incrementally changed. Transients are again allowed to die out. The steady state control current needed to achieve the new set point is recorded. The force component due to the control current for each displacement value is calculated using the current stiffness. The equilibrium force-position relationship is found. The slope of the linear curve fit of the data is position stiffness.

Figures 3.4 and 3.5 are the calibration for current stiffness and position stiffness, respectively. The current stiffness and position stiffness for the front radial AMB are found to be 196.15 $\frac{N}{A}$ and 1.16 $\frac{N}{\mu m}$, respectively. The stiffnesses for the back bearing are found to be 78.36 $\frac{N}{A}$ and 0.46 $\frac{N}{\mu m}$, respectively.

Note the following caveats:

- The loop shape in the current stiffness measurement is due to magnetic hysteresis. Hysteresis is residual magnetization of the bearing material when the current is changed and it causes slightly different forces when loading or unloading.
- Because the current stiffness value is used in the calculation of the position

stiffness, any measurement error in current stiffness will be reflected in position stiffness.

- The identified values of current stiffness and position stiffness are valid about the center of the AMBs.
- As seen in Eq. (3.6), the stiffnesses are functions of several parameters including bias current. For a given AMB, all the parameters are fixed except the bias current, which can be set by the controls engineer. Caution should be taken to ensure that the same bias current is used in the final controller design as was used in the identification process.



Figure 3.4 AMB current stiffness calibration.



Figure 3.5 AMB position stiffness calibration.

3.2.2 dSPACE Controller Hardware

The data acquisition and control implementation is implemented with a dSPACE system, specifically a DS1005 PPC processor board, two DS2001 analog-digital converter boards, and a DS2101 digital analog-converter board. The analog-digital converter board has five parallel channels and is capable of up to 16 bit resolution with a 5 μ s conversion time. The digital-analog converters have five parallel channels each, 12 bit resolution, and 4 μ s maximum settling time. This high performance hardware is necessary to ensure small phase lag in digital implementation of the controller. A sampling rate of 10 kHz is used for AMB control.

The dSPACE system is programmable via a PC with MATLAB Simulink and can be used as a data acquisition as well as a rapid control prototype. Figure 3.6 shows the dSPACE unit assembled in the housing rack. Both the BNC interface to the AMBs and the host PC used to program the dSPACE are housed in the same rack.



Figure 3.6 dSPACE rapid control prototyper and BNC interface with AMB sensors and amplifiers.

3.2.3 Spindle Rotor

The rotor was modeled using the finite element method by Wroblewski et al. in [79]. Then, in [80], the authors extended their work by studying the effects on mode shapes due to neglecting antiresonance information in the modeling procedure. The rotor modeling procedure used is summarized here. The finite element method is selected because of its ability to capture flexible dynamics for complicated geometry. First, the rotor is discretized into 73 beam elements. Next, the mass and stiffness matrices derived from the finite elements are transformed into modal coordinates. Then the system is reduced through modal truncation, retaining two rigid body modes and the first (lowest) three flexible modes. A state-space representation of the rotor is assembled from the transformed and truncated stiffness and mass matrices. Finally, a three-dimensional

model of the rotor is constructed by assuming the same dynamics in the two perpendicular axial planes, i.e., symmetric rotor, and coupling the planes with a skew symmetric gyroscopic matrix, which is also found with the finite element method.

A correct model is essential for model-based controller synthesis. Therefore, the model is updated with experimental data by numerically minimizing a cost function. The utilized cost function includes the error in resonance frequencies and error in anti-resonance frequencies at the front and back bearings. The use of experimental resonance frequencies in the cost function allows for not applying natural frequency uncertainty in the μ -synthesis. The use of anti-resonance data leads to a model with correct mode shapes. The mode shapes are of critical interest in the chatter control problem because chatter occurs at the tool location, away from the bearings and knowledge of how the rotor will bend at a particular frequency is needed to predict and attenuate chatter.

First, an initial rotor model is made using nominal parameter values. Then the numerical minimization corrects finite element modulus of elasticity values to minimize the cost function. In particular, the elements representing the shrink fit components of the AMB and motor and the tool holder coupling are corrected.

Model inputs and outputs are readily selected because the state-space input and output matrices are left in nodal coordinates corresponding to nodes of the finite element discritization. One may simply select the desired columns and rows of the input and output matrices, respectively. Inputs to the rotor model are taken as forces on the finite element nodes at the centers of the magnets and the tool. Outputs are taken as lateral positions of the finite element nodes at the centers of the magnets, tool, and AMB sensors. In this way, the finite element method allows for convenient modeling of sensor and actuator non-collocation. The utilization of the specific segments of the input and output matrices are explained in Section 3.3, Assembled Open-Loop System.

Figure 3.7 shows the rotor. A photograph of the rotor without the tool holder is shown on top. A schematic of the rotor with dimensions is shown in the middle. A plot of the rotor illustrating the finite element discretization and input and output nodes is shown on bottom.



Figure 3.7 Side view of rotor FE discretization with model inputs and outputs (top), photograph (middle), and dimensions (bottom) [81].

3.2.4 Eddy Current Position Sensors

The position sensors integral to the AMBs are eddy current type. Eddy current position sensors work by exciting a small wire coil with a harmonic current. This changing current induces eddy currents in a nearby metal object. Also, the eddy currents induce back voltage in the sensor. The amount of back voltage seen is related to the proximity of the metal object. The sensors in the AMBs operate with an excitation of 13.5 V at a frequency of 90 kHz. Specialized rings are built into the rotor at the sensor locations and serve as apt targets for the eddy current interactions. The target rings are made of solid Ti-6Al-4V. The bandwidth of the sensors is beyond the operational range of the AMB. As a result, the sensors are effectively modeled as constant gains. Figure 3.8 shows an inside view of the front radial AMB. One of the thrust axis eddy current position sensors can be seen. Also in the figure are the radial AMB pole faces and coils and the front touchdown bearing, which does not contact the rotor during normal operation.



Figure 3.8 Inside of AMB showing position sensors, magnetic poles and coils and touchdown bearing.

3.2.5 Power Amplifiers

The power amplifiers which supply the AMB coils with current are of the pulse width modulation (PWM) type. PWM amplifiers use rapid on-off switching of a constant voltage to create an output current which tracks a desired reference signal. The maximum supply voltage of the AMB amplifiers is 400 V and the switching frequency is 25 kHz. There are ten amplifiers, two for each radial AMB control axis and two for the thrust axis. Figure 3.9 shows a photograph of the amplifier chassis.



Figure 3.9 AMB amplifier chassis.

The nonlinear AMB force, Eq. (3.1), has an identical control current in the top and bottom coils. This consistency is required to make the linearized force expression, Eq. (3.2), which is used for robust controller design. To ensure the same dynamic control current in the top and bottom coils, the same type of amplifiers are used for all the coils.

The amplifiers, with AMB coils as a load, are experimentally identified by a sine sweep. The input is control current and the output is coil current. A model is found by

fitting the data with the prediction-error minimization method via the *pem* command of MATLAB's System Identification Toolbox. A model fit of 9th order (per axis) is selected, through trial and error, which is found to provide a good balance of accuracy and model size. Figure 3.10 shows the Bode plot for a front bearing top coil amplifier which is characteristic of all the amplifiers. The generated model is also shown.



Figure 3.10 Bode plot of AMB amplifier showing experimental identification and model.

Because the amplifier model is generated with the AMB coil as a load, the model includes dynamics of the AMB which are not captured by the force function Eq. (3.2) alone. Because the AMB coil acts as an inductor, and supply voltage is finite, it is not possible to change current at an arbitrarily high rate. This fact limits the high frequency load capacity of the bearing. The roll-off of load capacity with frequency is known as the

slew rate. The system is measured with the same 4 A bias current as is used for levitation. Also, the system is measured including the same digital hardware used for levitation. Therefore, the resulting amplifier model includes the additional phase lag associated with digital implementation.

3.3 Assembled Open-Loop System

The system which must be controlled consists of all the aforementioned components assembled into a single plant. This section discusses the interconnection of the component models into a single model. Then, that model is validated with experimental system identification.

3.3.1 Assembled Open-Loop Model

Figure 3.11 shows the block diagram of the open-loop assembly. All the signals are vectors with the units and dimensions as noted. The subscript "amp" denotes matrices and vectors pertaining to the amplifier model. The subscript "ff" denotes matrices and vectors pertaining to the finite element rotor with unconstrained boundary conditions on both sides, or free-free. The rotor can be thought of as the central component. Acting on the rotor are forces from the AMBs and external loads. The four radial AMB axes are expressed with the linear AMB force function, Eq. (3.2). The position stiffness is an unstable feedback of the magnet position. Current stiffness is multiplied by the coil current, which is the output of the amplifier model. Note that the coil current is not to be confused with control current, which is the input to the amplifier model. The force terms from the position and current stiffnesses are added together to become the AMB force

and are applied to the rotor at the AMB magnet center locations. Here, K_i and K_x are four by four diagonal matrices of the current stiffnesses and position stiffnesses respectively.



Figure 3.11 Open-loop plant assembly.

The external loads, which are inputs to the rotor model, become inputs to the assembled open-loop model. The external loads are applied to the rotor at the bearing locations and the tool location. The tool load input will be used in Chapter V to apply a feedback cutting force. The position outputs of the rotor become outputs of the assembled system and are used for performance specification in the controller design process. The rotor position outputs at the sensor locations are also used as the sensor signal control outputs. A noise input is added directly to the assembly at the sensor signal outputs. The amplifier state-space model includes four non-cross coupled

amplifiers, with the inputs being control current and the outputs being coil current. The inputs to the amplifier model are the control inputs for the assembled system.

The assembly can be expressed as a single state-space model as shown below. The state vector is a combination of the state vector of the rotor and that of the amplifier. The state matrices follow.

$$\begin{cases} \left\{ \dot{\mathbf{x}}_{\mathrm{ff}} \\ \left\{ \dot{\mathbf{x}}_{\mathrm{amp}} \right\} = \begin{bmatrix} \mathbf{A}_{\mathrm{ff}} + \mathbf{G}_{\mathrm{ff}} \Omega + \mathbf{B}_{\mathrm{act}} \mathbf{K}_{\mathrm{x}} \mathbf{C}_{\mathrm{act}} & \mathbf{B}_{\mathrm{act}} \mathbf{K}_{\mathrm{i}} \mathbf{C}_{\mathrm{amp}} \\ \mathbf{0} & \mathbf{A}_{\mathrm{amp}} \end{bmatrix} \left\{ \mathbf{x}_{\mathrm{ff}} \\ \mathbf{x}_{\mathrm{amp}} \right\} + \begin{bmatrix} \mathbf{B}_{\mathrm{load}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathrm{amp}} \end{bmatrix} \begin{bmatrix} \mathrm{Loads} \\ \mathrm{Noise} \\ \mathrm{Controls} \end{bmatrix} \\ \begin{cases} \mathrm{Rotor \ Locations} \\ \mathrm{Control \ Currents} \\ \mathrm{Sensor \ Signals} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{\mathrm{perf}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{C}_{\mathrm{sense}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\mathrm{ff}} \\ \mathbf{x}_{\mathrm{amp}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{\mathrm{a}} \\ \mathbf{0} & \mathbf{K}_{\mathrm{s}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathrm{Loads} \\ \mathrm{Noise} \\ \mathrm{Controls} \end{bmatrix}$$
(3.7)

What enables this expression to be succinct is the definition of input and output matrices for the rotor that select only the part of the total input or output matrix that is utilized. Specifically, **B**_{load} pertains to external loads, **B**_{act} pertains to AMB actuators, **C**_{perf} pertains to locations of the rotor where the system performance will be specified, i.e., sensors and tool, and **C**_{sense} pertains to the sensor locations. Also, the matrices **K**_a and **K**_s are diagonal matrices which allow for unit conversion of the noise input and control currents output if desired. For example, sensor signals can be reported as the original V or the calibrated μ m. This structure of noise and control effort feed-through is advantageous for avoiding the singular H_{∞} control problem [74]. Recall from Chapter II that H_{∞} is used for the controller step of the *D-K* iteration.

The matrix $G_{\rm ff}$ is the FE rotor gyroscopic matrix. This modifies the state matrix of the rotor based on rotation speed Ω to account for the gyroscopic effect. Note that $G_{\rm ff}\Omega$ is

not shown in the block diagram for space considerations.

3.3.2 Open-Loop System Identification

The assembled open-loop system model is confirmed experimentally through sine sweep testing. First, the rotor is levitated using an initial PID controller. Then a harmonic disturbance signal is superimposed on the AMB's control current. Even in the presence of a stabilizing controller, the open-loop dynamics can be extracted from the experimental data using the knowledge of the magnitude and phase of the total input current and the output position sensor signals. Because the system is linear, the position response and control current response will occur at the same frequency as the disturbance. Also, note that the transient response is allowed to die out before the frequency responses are recorded. Gähler [82] and Lösch [83] are good sources for a more thorough discussion of open-loop identification of an AMB system based on closed-loop experimental data. Figure 3.12 shows a closed-loop feedback control system with disturbance d(s). The transfer function $G_{OL}(s)$ is a SISO assembled AMB system corresponding to the MIMO one modeled in the previous section with control current input u(s) and position sensor signal output y(s). The transfer function K(s) can be any stabilizing controller.



Figure 3.12 Generic closed-loop feedback control system with disturbance.

For practical implementation of the controlled system in a working environment, closed-loop properties such as natural frequencies and stiffness must be known. For this reason, it is most common to take the transfer function from d(s) to y(s), the closed-loop dynamics. However, for controller design, the dynamics of the plant alone, u(s) to y(s), are needed. It is difficult to collect experimental data for an inherently unstable system like an AMB. Consequently, the system must be stabilized before input and output data can be taken. Therefore, the excitation signal alone cannot be used for system identification. However, having access to both u(s) and y(s), means that the transfer function of the open-loop plant can be calculated through simple arithmetic.

$$y(s) = G_{\rm OL}(s)u(s) \tag{3.8}$$

$$G_{\rm OL}(s) = \frac{y(s)}{u(s)} \tag{3.9}$$

Note that the disturbance signal d must be selected such that the input signal u does not

yield a singular result. Also, the linear dependence of u on noise in y must be negligible. See Balini et al [84] for a discussion on the linear dependence issue.

The actual spindle system is MIMO with four radial AMB axes, two for the front bearing and two for the back. The system identification method is applied with matrices:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \mathbf{G}_{OL}(s) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$
(3.10)

The method for obtaining the open-loop MIMO transfer function is explained by Sawicki and Maslen [85]. Each axis is excited individually and the responses of all four axes are measured. Sixteen frequency dependent vectors are collected as the input matrix $U(\omega_i)$ and output matrix $Y(\omega_i)$.

$$\mathbf{U}(\omega_{i}) = \begin{bmatrix} u_{1,1}(\omega_{i}) & u_{1,2}(\omega_{i}) & u_{1,3}(\omega_{i}) & u_{1,4}(\omega_{i}) \\ u_{2,1}(\omega_{i}) & u_{2,2}(\omega_{i}) & u_{2,3}(\omega_{i}) & u_{2,4}(\omega_{i}) \\ u_{3,1}(\omega_{i}) & u_{3,2}(\omega_{i}) & u_{3,3}(\omega_{i}) & u_{3,4}(\omega_{i}) \\ u_{4,1}(\omega_{i}) & u_{4,2}(\omega_{i}) & u_{4,3}(\omega_{i}) & u_{4,4}(\omega_{i}) \end{bmatrix}$$
(3.11)

$$\mathbf{Y}(\omega_{i}) = \begin{bmatrix} y_{1,1}(\omega_{i}) & y_{1,2}(\omega_{i}) & y_{1,3}(\omega_{i}) & y_{1,4}(\omega_{i}) \\ y_{2,1}(\omega_{i}) & y_{2,2}(\omega_{i}) & y_{2,3}(\omega_{i}) & y_{2,4}(\omega_{i}) \\ y_{3,1}(\omega_{i}) & y_{3,2}(\omega_{i}) & y_{3,3}(\omega_{i}) & y_{3,4}(\omega_{i}) \\ y_{4,1}(\omega_{i}) & y_{4,2}(\omega_{i}) & y_{4,3}(\omega_{i}) & y_{4,4}(\omega_{i}) \end{bmatrix}$$
(3.12)

Here the rows denote the output axes, the columns denote the axes which are excited, and

 ω_i is the frequency of excitation with the subscript denoting the counter of the discrete identification value. With input and output matrices, the MIMO transfer function can be identified for each frequency, in a method similar to the SISO case.

$$\mathbf{Y}(\omega_i) = \mathbf{G}_{OL}(j\omega_i)\mathbf{U}(\omega_i)$$
(3.13)

$$\mathbf{G}_{\mathrm{OL}}(j\omega_i) = \mathbf{Y}(\omega_i)\mathbf{U}^{-1}(\omega_i)$$
(3.14)

A series of frequencies is tested and the Bode plots shown in Figures 3.14 through 3.21 are created. The peaks in the Bode plots indicate the flexible modes of the spindle and occur at 1070 Hz, 1970 Hz, and 3250 Hz. Also shown in the figures is the model of the open-loop plant which is used for controller synthesis. A MIMO axis notation is established in Figure 3.13.



Figure 3.13 Schematic diagram of MIMO spindle showing open-loop input and output designations.

The V and W planes exist in physical space and are perpendicular with their intersection line being the center line of the rotor. The magnetic poles are aligned on each plane in the front and back AMBs. Each plane is 45° from the horizontal such that gravity preloading does not create anisotropy in the AMB control axes. The sensors for each AMB are in the same planes but are not collocated with the magnetic poles due to space restrictions. There is negligible coupling between perpendicular axes. Therefore, only input-output pairs which share a physical plane are shown.



Figure 3.14 Bode plot of the open-loop spindle from V13 axis control current input to V13 axis sensor position output, experimental and model.



Figure 3.15 Bode plot of the open-loop spindle from W13 axis control current input to W13 axis sensor position output, experimental and model.



Figure 3.16 Bode plot of the open-loop spindle from V24 axis control current input to V24 axis sensor position output, experimental and model.



Figure 3.17 Bode plot of the open-loop spindle from W24 axis control current input to W24 axis sensor position output, experimental and model.



Figure 3.18 Bode plot of the open-loop spindle from V13 axis control current input to V24 axis sensor position output, experimental and model.



Figure 3.19 Bode plot of the open-loop spindle from W13 axis control current input to W24 axis sensor position output, experimental and model.



Figure 3.20 Bode plot of the open-loop spindle from V24 axis control current input to V13 axis sensor position output, experimental and model.



Figure 3.21 Bode plot of the open-loop spindle from W24 axis control current input to W13 axis sensor position output, experimental and model.

CHAPTER IV

CONTROLLER DESIGN FOR CHATTER PREVENTION

4.1 Overview

This chapter discusses the design of a μ -synthesis controller for the HSM spindle's radial AMBs. The controller is designed such that machining chatter is least likely to occur by including a cutting force model in the synthesis process. For a comparison, a second controller is designed without the cutting force model by applying an external load performance specification to the spindle model at the tool location which results in the dynamic stiffness desired for machining. This second controller represents the current state-of-the-art in controller design for AMBs in machining applications and serves as a baseline for comparison of the performance of the new control method. The baseline controller is referred to as the *dynamic stiffness* μ -controller and the controller designed with the new method is referred to as the *chatter avoiding* μ -controller.

The following section explains the cutting force model and how it is used to augment the open-loop plant model used in controller synthesis. The uncertainties and weights in the cutting force model needed for robustness to chatter and practical implementation are also discussed. In Sections 4.3 and 4.4, the uncertainties and performance weights for the rest of the spindle system are described. An external tool load performance weight is also described in Section 4.4 which is only used to design the dynamic stiffness controller. All parameters not related to how the cutting force is addressed are held constant between the two controller designs and are presented in Section 4.5. Finally, the results of μ synthesis for the two cases are shown and discussed in Section 4.6.

4.2 Augmenting the Plant Model for Machining

4.2.1 Cutting Force Model

Figure 4.1 shows a photograph of the end of the spindle showing the cutting tool, counter weight, and an example workpiece held in approximate location.



Figure 4.1 End of the spindle with cutting tool, counter weight and an example workpiece.

The cutting force model of Tobias and Fishwick [10] is used in which the cutting force is proportional to uncut chip area. The proportionality constant which relates force to area is called the cutting stiffness k_c . The value of cutting stiffness is dependent on parameters such as workpiece material, temperature, cutting speed, and tool geometry, e.g., nose radius, rake angle and nominal cut depth region. A representative value of k_c for aluminum is about 1000 $\frac{N}{mm^2}$ [86]. The uncut chip area is the product of the axial feed-rate per revolution of the tool, or length *l*, and the radial depth of cut of the tool into the workpiece, or chip width *w*. Therefore, the cutting force can be expressed as:

$$f_{\rm c} = k_{\rm c} l w \tag{4.1}$$

Furthermore, the chip width is the sum of the nominal depth of cut e_0 which is set by the machinist, the vibration of the cutting tool v(t), and the vibration of the cutting tool made during the previous machining pass $v(t-\tau)$ [17].

$$f_{c} = k_{c} l(e_{0} + v(t) - v(t - \tau))$$
(4.2)

This relationship assumes a rigid workpiece which does not deflect under the machining force. If a workpiece cannot be assumed rigid, as in the present case, the vibrations v(t) and $v(t-\tau)$ would be the relative vibration between the tool and the workpiece.

The cutting force is decomposed into radial, tangential, and axial directions. For this particular machining application, a flexible boring spindle, the tangential cutting force is counteracted by the motor while the axial cutting force is counteracted by the thrust bearing. The dynamics in these directions are relatively simple and easy to control with little vibration. However, radial component of the cutting force excites the bending

modes of the flexible rotor. As a result, it is in this direction that machining chatter may occur. The cutting force equation is rewritten for the radial direction only for use in the chatter avoiding control problem. For convenience the same symbol for cutting stiffness is used for radial cutting stiffness. This is not problematic because the terms for tangential and axial cutting stiffness do not appear in the stability analysis.

$$f_{\rm r} = k_{\rm c} l(e_0 + v(t) - v(t - \tau)) \tag{4.3}$$

Figures 4.2 and 4.3 illustrate these parameters. Figure 4.2 is a three dimensional view showing the tool-workpiece interaction and Figure 4.3 is a two dimensional cross-sectional view of the same subject viewed from the direction of feed.

The cutting force equation includes a term for the tool vibration delayed by τ . This term is the value of tool vibration during the previous machining pass which created the overlaying surface. The time τ is how long it takes to make one pass and is related to the rotation speed by:

$$\tau = \frac{2\pi}{\Omega} \tag{4.4}$$

The presence of the delay term is what enables the cutting force feedback to cause system instability. When the running speed, and hence the delay time, is such that the current tool vibration and the previous tool vibration are in phase, the chip width does not oscillate. Therefore, the cutting force does not oscillate and the machining system remains stable. When the running speed, and hence the delay time, is such that the current tool vibration and the previous tool vibration are out of phase, the chip width oscillates. Therefore, the cutting force oscillates. The oscillating cutting force induces greater tool vibrations for the next machining pass because of the feedback nature of the cutting force. If the feedback gain is large enough for that given cutting speed, chatter will occur [13].

The mechanism in which the overlaying machining pass affects the force during the current machining pass is called the regenerative effect. Chatter caused by the regenerative effect is a self excited vibration because of the delayed feedback of the cutting force. The importance of vibration phase in the regenerative effect is seen in the results of the chatter avoiding controller design and is examined with the simulation results in Chapter VI.

The constant feed-rate per revolution, chip length l, and the cutting stiffness k_c can be multiplied to make a cutting force coefficient in units of force per distance. The linear cutting force coefficient is, in some literature, also referred to as cutting stiffness. The linear cutting stiffness is the cutting force feedback gain which must be below the critical value or chatter will occur. For two planes, V and W, this linear cutting stiffness is expressed as a diagonal matrix \mathbf{K}_c .

$$\mathbf{K}_{c} = (k_{c}l)I^{2\times 2} \tag{4.5}$$



Figure 4.2 Diagram of the cutting tool imbedded in the workpiece while machining (chip not shown).



Figure 4.3 Diagram showing tool cut depth and regenerative mechanism.

4.2.2 Open-Loop Plant with Cutting Included

Figure 4.4 shows the same block diagram of the open-loop system shown in Figure 3.11 with the addition of the machining force model Eq. (4.3). The block diagram components not discussed in this section are the same as in Figure 3.11 where no cutting is considered.



Figure 4.4 Open-loop plant assembly including machining force.

To realize the relationship of Eq. (4.3), the loads on the tool, which are an input in Chapter III, are here calculated as a function of tool position. The loads input here pertain only to the four external loads applied at the bearing locations and are used to achieve bearing stiffness. An input is included which accounts for the nominal cut depth, e_0 , for both V and W planes. The rotor locations output is the same as for no cutting force feedback and includes both tool and bearing locations for performance specification. As in Chapter III, the state-space representation is concise with the use of input and output matrices for the rotor which are subsets of those for the entire rotor, $\mathbf{B}_{\rm ff}$ and $\mathbf{C}_{\rm ff}$. The subscripts are the same as in Chapter III with the addition of "tool" which denotes inputs and outputs at the cutting tool location. The assembled state-space system is shown in Eq. (4.6).

$$\begin{cases} \left\{ \dot{\mathbf{x}}_{\mathrm{ff}}(t) \\ \dot{\mathbf{x}}_{\mathrm{amp}}(t) \right\} = \begin{bmatrix} \mathbf{A}_{\mathrm{ff}} + \mathbf{G}_{\mathrm{ff}} \Omega + \mathbf{B}_{\mathrm{act}} \mathbf{K}_{\mathrm{x}} \mathbf{C}_{\mathrm{act}} - \mathbf{B}_{\mathrm{tool}} \mathbf{K}_{\mathrm{c}} \mathbf{C}_{\mathrm{tool}} & \mathbf{B}_{\mathrm{act}} \mathbf{K}_{\mathrm{i}} \mathbf{C}_{\mathrm{amp}} \\ \mathbf{A}_{\mathrm{amp}}(t) \right\} \\ + \begin{bmatrix} \mathbf{B}_{\mathrm{tool}} \mathbf{K}_{\mathrm{c}} \mathbf{C}_{\mathrm{tool}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{cases} \mathbf{x}_{\mathrm{ff}}(t-\tau) \\ \mathbf{x}_{\mathrm{amp}}(t-\tau) \end{cases} + \begin{bmatrix} \mathbf{B}_{\mathrm{tool}} \mathbf{K}_{\mathrm{c}} & \mathbf{B}_{\mathrm{load}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathrm{amp}} \end{bmatrix} \begin{cases} e_{0} \\ \mathrm{Loads} \\ \mathrm{Noise} \\ \mathrm{Controls} \end{cases} \\ \begin{cases} \mathrm{Rotor \ Locations} \\ \mathrm{Sensor \ Signals} \end{cases} = \begin{bmatrix} \mathbf{C}_{\mathrm{perf}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{\mathrm{s}} \mathbf{C}_{\mathrm{sense}} & \mathbf{0} \end{bmatrix} \begin{cases} \mathbf{x}_{\mathrm{ff}} \\ \mathbf{x}_{\mathrm{amp}} \end{cases} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{\mathrm{a}} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{\mathrm{s}} & \mathbf{0} \end{bmatrix} \begin{cases} e_{0} \\ \mathrm{Loads} \\ \mathrm{Loads} \\ \mathrm{Noise} \\ \mathrm{Controls} \end{cases} \end{cases}$$
(4.6)

Here $\mathbf{x}_{\text{ff}}(t-\tau)$ and $\mathbf{x}_{\text{amp}}(t-\tau)$ are the delayed state vectors. Applying the Laplace Transform to the first equation yields:

$$\begin{cases} s\mathbf{X}_{\rm ff}(s) \\ s\mathbf{X}_{\rm amp}(s) \end{cases} = \begin{bmatrix} \mathbf{A}_{\rm ff} + \mathbf{G}_{\rm ff}\Omega + \mathbf{B}_{\rm act}\mathbf{K}_{\rm x}\mathbf{C}_{\rm act} - \mathbf{B}_{\rm tool}\mathbf{K}_{\rm c}\mathbf{C}_{\rm tool} & \mathbf{B}_{\rm act}\mathbf{K}_{\rm i}\mathbf{C}_{\rm amp} \\ \mathbf{0} & \mathbf{A}_{\rm amp} \end{bmatrix} \begin{cases} \mathbf{X}_{\rm ff}(s) \\ \mathbf{X}_{\rm amp}(s) \end{cases} + \begin{bmatrix} \mathbf{B}_{\rm tool}\mathbf{K}_{\rm c}\mathbf{C}_{\rm tool} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{cases} e^{-s\tau}\mathbf{X}_{\rm ff}(s) \\ e^{-s\tau}\mathbf{X}_{\rm amp}(s) \end{cases} + \begin{bmatrix} \mathbf{B}_{\rm tool}\mathbf{K}_{\rm c} & \mathbf{B}_{\rm load} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\rm amp} \end{bmatrix} \begin{cases} e_{0}(s) \\ {\rm Loads}(s) \\ {\rm Noise}(s) \\ {\rm Controls}(s) \end{cases} \end{cases}$$
(4.7)

The expression can be rearranged to bring the transcendental delay term into the state

matrix.

$$\begin{cases} s\mathbf{X}_{\rm ff}(s) \\ s\mathbf{X}_{\rm amp}(s) \end{cases} = \\ \begin{bmatrix} \mathbf{A}_{\rm ff} + \mathbf{G}_{\rm ff}\Omega + \mathbf{B}_{\rm act}\mathbf{K}_{\rm x}\mathbf{C}_{\rm act} - \mathbf{B}_{\rm tool}\mathbf{K}_{\rm c}\mathbf{C}_{\rm tool} + \mathbf{B}_{\rm tool}\mathbf{K}_{\rm c}\mathbf{C}_{\rm tool}e^{-s\tau} & \mathbf{B}_{\rm act}\mathbf{K}_{\rm i}\mathbf{C}_{\rm amp} \\ \mathbf{0} & \mathbf{A}_{\rm amp} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{\rm ff}(s) \\ \mathbf{X}_{\rm amp}(s) \end{bmatrix} (4.8) \\ + \begin{bmatrix} \mathbf{B}_{\rm tool}\mathbf{K}_{\rm c} & \mathbf{B}_{\rm load} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\rm amp} \end{bmatrix} \begin{bmatrix} e_{0}(s) \\ {\rm Loads}(s) \\ {\rm Noise}(s) \\ {\rm Controls}(s) \end{bmatrix}$$

Assume a term k_d such that the delay differential equation can be expressed as a linear state space:

$$\begin{cases} \dot{\mathbf{x}}_{\rm ff} \\ \dot{\mathbf{x}}_{\rm amp} \end{cases} = \\ \begin{bmatrix} \mathbf{A}_{\rm ff} + \mathbf{G}_{\rm ff} \mathbf{\Omega} + \mathbf{B}_{\rm act} \mathbf{K}_{\rm x} \mathbf{C}_{\rm act} - \mathbf{B}_{\rm tool} \mathbf{K}_{\rm c} \mathbf{C}_{\rm tool} + \mathbf{B}_{\rm tool} \mathbf{K}_{\rm c} \mathbf{C}_{\rm tool} k_{\rm d} & \mathbf{B}_{\rm act} \mathbf{K}_{\rm i} \mathbf{C}_{\rm amp} \\ \mathbf{0} & \mathbf{A}_{\rm amp} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\rm ff} \\ \mathbf{x}_{\rm amp} \end{bmatrix} \quad (4.9)$$
$$+ \begin{bmatrix} \mathbf{B}_{\rm tool} \mathbf{K}_{\rm c} & \mathbf{B}_{\rm load} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\rm amp} \end{bmatrix} \begin{bmatrix} e_{\rm 0} \\ \text{Loads} \\ \text{Noise} \\ \text{Controls} \end{bmatrix}$$

A robust control solution is found to the machining chatter problem by using the term k_d as an uncertainty perturbation, norm bounded to encompass any possible value for the delay term. The uncertainties and performance weights associated with the cutting force model are discussed in the next subsection.

4.2.3 Cutting Model Uncertainties and Weighting

Model uncertainties can be used to compensate for modeling errors for difficult to model or non-validated components. This however is not using the robust control technique to its fullest potential as it yields an overly conservative controller at the expense of performance. The most useful approach is to use uncertainties to only express values in the model which are known to change during the course of normal operation.

The cutting force model requires special uncertainties and performance weighting to apply robust control to the machining chatter problem. The first uncertainty, alluded to in the previous subsection, is the time delay. To be robustly stable, the system cannot chatter for any possible value of time delay. Specifically, delay is a phase shift between input and output. The angle of the phase shift is dependent on the delay time and frequency. However, it is straightforward to see that time delay term is norm bounded. The transfer function of a delay is $e^{-s\tau}$. Given a delay time τ and frequency ω , the complex response can be evaluated. For example, $e^{-j\omega\tau} = a + bj$. For any delay time and at any frequency, the magnitude of the complex response, |a + bj|, is unity. Consequently, delay lends itself to be robustly expresses as a complex uncertainty centered about zero.

In the case of single point boring, the delay time is the time for the tool to make one revolution on the workpiece inner diameter. To provide robust stability at any possible running speed, the delay is expressed as a complex uncertainty, k_d . The nominal value is zero and its radius in the real-imaginary plane is 1. This uncertainty captures any phase shift between zero and 2π . Therefore, it is a practical solution to stability in the presence of the regenerative effect.

The second uncertainty in the cutting force model which is necessary to design a controller for chatter is on the cutting stiffness. The uncertain cutting stiffness must range from zero to some maximum value. The reason for this is that the spindle must be stable even when no cutting is taking place. When the machining system is not in the chatter conditions, the negative feedback of the cutting stiffness has a stabilizing effect. The negative feedback cannot be relied on for support at the start of levitation before cutting begins. The cutting stiffness uncertainty also allows for the possibility of acceptable machining in the presence of variations in the cutting stiffness due to different cutting parameters. For example, if inaccuracies are made in the cutting stiffness measurements, if the cutting tool is changed for one with a different rake angle, or if a workpiece of a different material is used, etc. In controller design, the upper bound of the real uncertain cutting stiffness is increased incrementally until a μ -value just below 1 is obtained to maximize the chatter avoiding region regardless of cutting conditions.

The cutting force model, Eq. (4.3), has a nominal cut depth parameter e_0 which becomes an input to the open-loop model. For a rotating system such as the spindle, the nominal cut depth component will result in a harmonic load that occurs at the rotation frequency. Because the rotation frequency is uncertain, there is an opportunity to limit conservatism by using an uncertain weighting function such that the input only occurs at the same frequency as the running speed. Unfortunately, when such a scheme was attempted the *D-K* iteration could not converge to a controller solution. A conservative, but much simpler weighting function, shown in Figure 4.5, is used instead. A maximum nominal cut depth of 50 µm is specified for all frequencies.

The maximum nominal cut depth must be no less than twice the maximum allowable

tool vibrations or the tool may be allowed to jump off the workpiece during machining. The situation of the tool leaving contact with the workpiece leads to Eq. (4.3) predicting a negative chip width and therefore a negative cutting force. In other words, the workpiece will try to pull the tool back into itself. In reality when contact is lost the cutting force is zero. This mathematical inconsistency is avoided by limiting the allowable tool vibration to twice the nominal cut depth. Intuitively, vibrations large enough for the tool to bounces off the workpiece can be considered as a failure whether or not it is actually chattering.



Figure 4.5 Weighting function of maximum nominal cut depth input.
4.3 Selection of Spindle Structured Uncertainties

The operating speed range of the AMB spindle is zero to 50,000 RPM. It is required that the controller is able to levitate the spindle at any speed it may rotate. Therefore, the rotation speed, Ω , is considered as an uncertain parameter ranging from zero to 50,000 RPM. This ensures robustness to the gyroscopic effect. Uncertainty in AMB applications was studies by Pesch and Sawicki in [87].

4.4 Selection of Spindle Performance Weights

Performance weights are placed on all inputs and outputs of the assembled open-loop plant with the exception of control inputs (current) and control outputs (sensor signals). The weighting is applied by pre-multiplying and post-multiplying the plant by diagonal matrix transfer functions. Each transfer function is crafted specifically for the μ -synthesis process such that the maximum allowable disturbance and response is norm-bounded to unity at every frequency. It can be seen that the inputs and outputs of the plant discussed in Chapter III are selected in order to achieve the desired performance of the closed-loop system after model-based controller synthesis.

Load inputs are applied on the finite element rotor's nodes at the AMB force center locations. The load inputs are weighted to achieve bearing stiffnesses which allow for practical operation of the spindle. The front bearing and the back bearing are given a maximum static load of 300 N and 130 N, respectively. The static level is rolled off above 0.001 Hz. An intermediate level is weighted at 80 N and 50 N for the front and back bearings respectively. The intermediate range rolls off above 40 Hz.

The noise input, which is a disturbance that is added to the sensor signals, is taken as

0.9 μ m maximum and is weighted uniformly across all frequencies. The magnitude of position sensor noise is measured as 0.6 μ m from the variance of the sensor signal when the spindle is not in operation and the rotor is supported by the touchdown bearings. It is found that setting a more severe noise performance weight is helpful in obtaining μ -controllers that are open-loop stable. Interestingly, an open-loop unstable controller can achieve the design criteria if the closed-system is stable. However, it is advantageous for practical implementation to have an open-loop stable controller. For example, an unstable controller may lead to unacceptably large amplitude of control effort during initial transient response. The large response may cause the system to oscillate outside of the range where the system can be considered as linear and thus never reach the closed-loop stability predicted by the controller design.

The bearing deflections are limited to 1 μ m below 0.002 Hz and to 45 μ m above 0.002 Hz. The small allowable deflection at low frequency creates a high static gain in the controller and achieves high static stiffness. The 45 μ m allowable dynamic deflection of the bearings is more than the 30 μ m allowable dynamic deflections of the tool. This is due to the machining emphasis of the control problem. In contrast, for an AMB for only levitation, the limits on bearing deflection would be less. Regardless, 45 μ m allows for safe levitation and rotation while not being overly restrictive on the ability for tool location control. The deflection of the tool is limited to 30 μ m over the frequency range of machining for both chatter avoiding and dynamic stiffness controllers.

The control currents in differential control cannot be allowed to overcome the bias current. Therefore, a maximum weighting of 4 A is used. As to not saturate the amplifiers, the weighting rolls off at 1 kHz, which is below the bandwidth of the amplifier. Recall that the amplifiers are identified with the AMB coil as a load and using4 A bias current. Therefore, this weighting prevents saturation of the AMB slew rate.

The maximum rated current of the coil is below the bias current plus the maximum control current. Note that the magnetic saturation of the material occurs above the maximum current.

Figures 4.6 through 4.9 show Bode plots of the performance weighting functions. Note that the output weightings, which express maximum response, must be inverted for the norm-bounding as explained in Chapter II. All inputs to the system are multiplied component-wise by their maximum possible value and all outputs are divided component-wise by their maximum possible value such that the uncertain perturbation block has a magnitude of 1 for interpretation of the μ -value.



Figure 4.6 Weighting function of the maximum external force load input to rotor at bearing locations.



Figure 4.7 Weighting function of the maximum sensor noise input.



Figure 4.8 Weighting function of the maximum position outputs at tool and front and rear bearings.



Figure 4.9 Weighting function of the maximum control current outputs.

For the case of the dynamic stiffness controller, an external load is applied to the tool location and is weighted with 27 N. The weight is applied over the frequency range of machining, zero to 50,000 RPM, after which it is allowed to roll off. The tool load weight is shown in Figure 4.10. The magnitude of the weight is tuned manually to achieve a μ -value just below 1 while holding all other weights and uncertainties ranges constant. In this manner the dynamic stiffness at the tool is maximized.



Figure 4.10 Weighting function of the maximum external tool load input.

4.5 Summary of Performance Weights and Uncertainties

A summary of the performance weights for μ -synthesis is shown in Table I. The weights are the same for both controller design methods except for the tool load weight, which is only used in the dynamic stiffness method, and the nominal cut depth weight, which is used only in the chatter avoiding method.

Table I Performance weights summary

Weight	unit	at frequency:			
weight	unn	0.001 Hz	2 Hz	1000 Hz	
Max Front Bearing Load	Ν	300	80	3	
Max Back Bearing Load	Ν	130	50	2	
Max Bearing Deflection	μm	1	3	45	
Max Control Current	А	4	4	3	
Max Sensor Noise	μm 0.9		0.9	0.9	
Max Tool Deflection	μm	25	25	30	
Max Tool Load	N	27	27	4	
(for dynamic stiffness method)	IN	27	27		
Max nominal depth of cut		50	50	50	
(for chatter avoiding method)	μm	30	50	30	

Recall from Chapter II that output weights are inverted before multiplication with the nominal plant.

A summary of the uncertainties used for μ -synthesis is shown in Table II. Note that the cutting stiffness parameter and delay and hence their uncertainties only appear in the model used for the chatter avoiding controller design. All other uncertainties are the same for both dynamic stiffness and chatter avoiding controller designs.

Uncertainty Type		Form	Nominal Value	Range	
Running Speed	Real	Multiplicative	25,000 RPM	±100%	
Cutting Stiffness (for chatter avoiding method)	Real	Multiplicative	$7 \times 10^8 \frac{N}{m^2}$	±100%	
Delay (for chatter avoiding method)	Complex	Multiplicative	0	Radius = 1	

Table II Uncertainties summary

In the next section, the results of controller design using the two uncertain weighted models detailed in this section are presented and discussed.

4.6 Controller Design Results

The chatter avoiding μ -controller and the dynamic stiffness μ -controller are designed using the system with and without a cutting force model, respectively. For these two controllers to be a fair comparison of their respective methods, all the uncertainties and performance weights not associated with how the cutting force is handled are held constant between the two models. To maximize the chatter avoiding potential of each method, both models are tuned until the μ -value is just under 1. The μ -value below 1 indicates that the controller performance is acceptable for any possible combination of uncertain values. Additionally, forcing the μ -value to be close to 1 without going over maximizes the effective operation range. For the chatter avoiding μ -controller, the model is tuned by maximizing the uncertain cutting stiffness range. For the dynamic stiffness controller, the model is tuned by maximizing the tooltip external load weighting function. In this manner, the chatter avoiding controller is made to provide the largest possible chatter free range and the dynamic stiffness controller is made to have the largest possible dynamic stiffness. An interesting caveat: One should try several initial controllers as starting points for the *D*-K iterations for the reason discussed in Chapter II. To maximize the effective operation range, the model should be tuned such that about half of the starting points tried converge to final controllers with μ -values just under 1. This ensures that a thoroughly proven maximum level is achieved rather than just one for a given starting point. This is opposed to tuning the model to get a μ -value is just under 1 for an initial controller and then trying several starting points to achieve a better μ -value.

The μ -synthesis is performed using the MATLAB Robust Control Toolbox's command *dksyn* to carry out the *D*-*K* iterations. (See Chapter II for a discussion of μ -

synthesis theory.) The dynamic stiffness controller resulted with a μ -value of 0.9941 after 4 iterations. The chatter avoiding controller results with a μ -value of 0.9701 after 2 iterations.

Figures 4.11-4.14 show Bode plots for both chatter avoiding and dynamic stiffness controllers. Each MIMO controller has 4 inputs and 4 outputs for the 4 radial AMB control axes, i.e., two radial AMBs each with 2 perpendicular axes. Therefore, the controllers each have 16 transfer functions. Only those for the V plane are shown; refer to Fig. 3.13 for control axis orientation. The controller transfer functions are similar for the perpendicular planes of the spindle. Figures 4.11 and 4.12 show the direct transfer function for the front bearing control axis and back bearing control axis, respectively. Figures 4.13 and 4.14 show the cross coupling between the front and back bearing control axes.

Some initial observations: Overall, the two controllers show similar trends because they are designed for the same AMB-rotor system with the same performance requirements. The dynamic stiffness controller has higher gains at low frequencies. The chatter avoiding controller has sharp peaks at 754 Hz and 1840 Hz. The sharp peaks correspond to more abrupt phase shifts than are in the dynamic stiffness controller. As a result, the chatter avoiding controller holds the response at the most desirable phase in the region of the chatter frequency. Therefore, deliberate phase control is needed to attenuate the regenerative effect which causes the machining chatter when the current machining pass and previous machining pass are out of phase. The chatter frequency range is calculated in Chapter VI and is approximately 1 kHz to 1.5 kHz.

The first flexible mode shape of the spindle passes close to the position sensor of the

front AMB. See Wroblewski [80] for details on the spindle rotor's mode shapes. As a result of the geometry of the first flexible mode shape, that mode has relatively poor observability from the front sensor. This effect can be seen in the open-loop plant Bode plot shown in Fig. 3.14. The first natural frequency has relatively good observability at the back AMB sensor. The result is the MIMO μ -controllers using the cross coupled dynamics to control the first mode with the back AMB. This is especially true in the chatter avoiding controller which has a much larger 754 Hz peak in the back axis transfer function than the front. Note that the flexible modes of the rotor are 1070 Hz, 1970 Hz, and 3250 Hz.

The MIMO controllers are further summarized in Table III. Table III lists the number of poles and zeros in the transfer function for each control axis in the V plane. For example, "Front to Back" denotes the transfer function from the V13 bearing position sensor to the V24 bearing control current. The columns labeled "CA" are for the chatter avoiding controller and those labeled "DS" are for the dynamic stiffness controller. The steady-state gain and -3 dB bandwidth are also given in the table.

Parameter	Units	Front to Front		Front to Back		Back to Front		Back to Back	
		CA	DS	CA	DS	CA	DS	CA	DS
Number of Poles	NA	74	84	74	84	74	84	74	84
Number of Zeros	NA	73	83	73	83	73	83	73	83
Steady-State Gain	μm/A	-12.65	-18.51	-8.39	-10.46	-13.40	-31.61	-12.81	-27.17
-3 dB Bandwidth	Hz	0.025	0.028	0.034	0.045	0.025	0.026	0.034	0.038

Table III MIMO Controllers Summary and Comparison



Figure 4.11 Bode plot of the dynamic stiffness controller and the chatter avoiding controller for front AMB of HSM spindle.



Figure 4.12 Bode plot of the dynamic stiffness controller and the chatter avoiding controller for back AMB of HSM spindle.



Figure 4.13 Bode plot of the dynamic stiffness controller and the chatter avoiding controller for front AMB to back AMB of HSM spindle.



Figure 4.14 Bode plot of the dynamic stiffness controller and the chatter avoiding controller for back AMB to front AMB of HSM spindle.

CHAPTER V

MACHINING CHATTER LIMITING CRITERION

5.1 Overview

In the previous chapter, an AMB controller for the HSM spindle is developed with the intention of avoiding machining chatter. In order to evaluate the effectiveness of the controller at avoiding chatter, the criterion at which chatter occurs must be understood. With this criterion, the simulated and experimental tests of the levitated spindle in Chapters VI and VII, respectively, can be used to show the aptitude of the control design technique to chatter avoidance. In this chapter a chatter limiting criterion which is well established in the existing body of literature is derived and discussed.

5.2 Theory of Chatter Limiting Conditions

Tlusty [6] derived the limiting conditions for chatter. The key results are summarized here. Refer to Figure 4.3, the diagram of machining showing the cutting tool and workpiece interaction. The path v(t) occurs as the tool vibrates, while the path $v(t-\tau)$ was

generated by the tool vibration during the previous rotation of the tool inside the workpiece. For ease of notation in this chapter, v(t) is called v and $v(t-\tau)$ is called v_0 . Assume that the machining system is at the border of chatter stability such that the tool vibrations neither grow nor decay. The vibration can be described with the sine wave in Eq. (5.1).

$$v = V'\sin(\omega t + 2\pi n + \varphi) \tag{5.1}$$

The terms *n* and φ are the integer number of full cycles during one rotation and the phase shift between *v* and *v*₀, respectively. The prime denotes amplitude of vibration as to not be confused with capital letters used to denote frequency domain. Because *v*₀ is *v* delayed by one rotation, it will also be a constant amplitude vibration.

$$v_0 = V_0' \sin(\omega t) \tag{5.2}$$

Because of the relationship between the current pass and the previous pass, and the fact that the vibrations do not grow or decay, the magnitude of the vibration amplitude must be the same.

$$|V_0'| = |V'| \tag{5.3}$$

For the next step, the problem is cast in a controls framework so that classical stability analysis techniques can be applied. Figure 5.1 illustrates the block diagram of the machining process with the cutting force as a feedback on the levitated spindle's tooltip. With the machining force written as a feedback, the Nyquist stability criterion can be used.



Figure 5.1 Block diagram of the levitated spindle with regenerative machining force feedback in the frequency domain.

In Figure 5.1, $T(\omega)$ is the stable levitating spindle frequency response with input F_e , external force at the tool location, and output V, radial displacement at the tool location. The delay block $e^{-j\varphi}$ distinguishes V and V_0 , while the signal E_0 is the nominal cut depth. The gain $k_c \times l$ is the relation of radial cutting force to uncut chip width. The block as a whole is the linear cutting stiffness with k_c as the uncut chip area cutting stiffness. The units of k_c are force per area and the units of l are length.

The Nyquist stability criterion explained by Dorf and Bishop [88] can be stated as follows: A feedback system is stable if and only if the opened-loop frequency response in the complex plane does not encircle the (-1, 0j) point when the number of poles of the opened-loop transfer function in the right-hand *s*-plane is zero. Otherwise, a feedback control system is stable if and only if, for the opened-loop frequency response in the

complex plane, the number of counterclockwise encirclements of the (-1, 0j) point is equal to the number of poles of the opened-loop transfer function with positive real parts.

The case in point, a cutting force feedback, is opened-loop stable. Neither the cutting stiffness nor the delays on the outer modulus are marginally stable or unstable. The cutting tool transfer function is stable by itself. Note that this is a general criterion and can be used for machines with traditional supports, which are inherently stable, or for machines with AMB supports including a stabilizing controller. Because the system is opened-loop stable, at the border of stability the frequency response contour will touch -1 on the real axis but not encircle it. This situation is illustrated in the sketched Nyquist plot shown in Figure 5.2.



Figure 5.2 Typical Nyquist plot for a system on the border of stability.

Finding the critical feed-rate can be thought of as drawing the Nyquist contour and increasing the cutting stiffness gain until the (-1,0) point is reached. However, to consider it analytically gives additional insight. Begin by setting the system from Figure 5.1, opened-loop, equal to negative one.

$$k_{c} lT(\omega)(1 - e^{-j\varphi}) = -1$$
 (5.4)

Notice that length of chip l, cutting stiffness k_c , and -1 are real. Therefore:

$$T(1-e^{-j\varphi}) = T - Te^{-j\varphi}$$
(5.5)

is real. Furthermore, because the delay is a unit vector, the magnitude of these two terms must be the same.

$$\left|T\right| = \left|Te^{-j\varphi}\right| \tag{5.6}$$

With Eq. (5.5) and Eq. (5.6) the realization can be made that the difference between T and delayed T must be two times the real part of T.

$$T(1-e^{-j\varphi}) = 2\operatorname{Re}(T) \tag{5.7}$$

This can be seen in Figure 5.3 which graphically depicts T and delayed T while maintaining the equalities in Eq. (5.5) and (5.6). Note the conclusion of Eq. (5.7) must follow.



Figure 5.3 Tooltip frequency response and delayed frequency response at the border of stability.

Inserting Eq. (5.7) back into Eq. (5.4) and solving for $k_c l$ at the border of stability yields:

$$k_{\rm c}l = \frac{-1}{2\,{\rm Re}(T)}\tag{5.8}$$

The value of the response *T* is different depending upon frequency. The smallest value of $k_c l$ which will result in chatter occurs at the smallest possible value of the right side of the equation.

$$\left(k_{\rm c}l\right)_{\rm min} = \left(\frac{-1}{2\,{\rm Re}(T)}\right)_{\rm min} \tag{5.9}$$

The right side of the equation is minimized with the absolutely maximum negative real part of T with respect to frequency. This is only valid for negative values of the real part of T because cutting stiffness is always positive in negative feedback, and negative chip length would mean the tool is feeding away from the workpiece.

$$(k_{\rm c}l)_{\rm min} = \frac{-1}{2(-1)\max|({\rm Re}(T))|}, \quad {\rm Re}(T) < 0$$
 (5.10)

From a controls standpoint, $(k_c l)_{min}$ is a limiting feedback gain. In contrast, from a machining standpoint, this can be used to find either the maximum chatter free feed-rate l or the maximum chatter free area cutting stiffness k_c .

Because this conclusion is for the border of stability with a phase shift that is free to take any value, it is a conservative result for most cutting speeds. That is, it is the worst case scenario, corresponding to the troughs of the SLD, which is covered in Chapter VI. Because the speed information is lost in the preceding derivation, the criterion of Eq. (5.10) makes a limiting line across the bottom of the SLD.

5.3 Conclusions

The derivation in this chapter results in a chatter limiting criterion based on the largest negative real part of the frequency response at the tooltip. The chatter limiting criterion gives the critical value of feed-rate or cutting stiffness below which chatter will not occur, regardless of the running speed. This chatter limiting criterion has been used for many years to evaluate machining systems. It is applicable to any machining system that may encounter regenerative chatter, not just actively controlled systems.

In the current work, the criterion is used to evaluate and compare the chatter avoiding and the dynamic stiffness AMB controllers. In Chapter VI, the criterion is applied for the simulated systems after evaluating the tooltip frequency responses. Then, in Chapter VII the criterion is applied to the experimental systems using the measured tooltip frequency responses.

CHAPTER VI

SIMULATED CONTROLLER IMPLEMENTATION

6.1 Overview

In this chapter, the chatter avoiding μ -controller generated in Chapter IV is used in conjunction with models of the spindle and machining force to simulate and validate expected behavior of the closed-loop system. First, the controller and nominal plant model with no machining consideration are used to find the transfer function at the tool location of the stable levitating rotor. Next, these results are used to calculate the expected limiting feed-rate as is discussed in Chapter V. Then, using the cutting model with various values of feed-rate and spindle speed applied to the controlled spindle model, a SLD is plotted by calculating the eigenvalues at each combination of feed-rate and liftoff behavior and machining behavior of the system are found. All simulations are also performed using the baseline controller and results are compared.

6.2 Closed-Loop Transfer Function

Using the nominal spindle model (with no cutting model) and closing the loop with each controller, two closed-loop models are created. Figure 6.1 shows the frequency response of the two closed-loop systems at the tool location. The frequency response at the tool location using PID control is also included for additional comparison. The plot focuses on the first flexible mode of the spindle which is where the critical values of interest occur.



Figure 6.1 Modeled Bode plot and real and complex parts of frequency response of the spindle at the tooltip when using PID, and dynamic stiffness, and chatter avoiding μ-controllers.

The left two plots show the magnitude and phase of the frequency response of the tool. The minimum dynamic stiffness occurs at the peak of the magnitude plot at the first bending natural frequency. The dynamic stiffness μ -controller achieves significantly higher dynamic stiffness than the industry standard PID controller. However the chatter avoiding μ -controller achieves slightly greater dynamic stiffness than the dynamic stiffness controller. It can be concluded from these results that including the cutting force model in the total plant model can potentially yield a μ -controller with the high dynamic stiffness expected by historical machining experience. Anecdotally, when the performance weight on sensor noise was not artificially high (6 µm instead of 9 µm) the resulting μ -controllers were open-loop unstable but were projected to perform well in closed-loop. The system closed-loop with the chatter avoiding controller then had lower dynamic stiffness than with the dynamic stiffness controller. Yet, it had higher maximum negative real part.

The right two plots show the tool frequency response decomposed into real and imaginary parts for the same frequency span. The maximum negative real part occurs at slightly higher frequency than the first flexible mode for all cases. Recall from Chapter V that the maximum negative real part is the limiting criterion for chatter occurrence for an arbitrary cutting speed. The chatter avoiding μ -controller achieves a significantly better maximum negative real part over the dynamic stiffness controller. The maximum negative real part for the PID is far worse and not shown in the scale of the plot.

Using Eq. (5.10) and assuming a chip length of 100 μ m per revolution, the limiting stable cutting stiffness is calculated below.

Chatter Avoiding Controller:

$$k_{\rm c,lim} = \frac{-1}{2(-1)(0.494 \ \frac{\mu m}{N})(100 \ \mu m)} = 0.0101 \ \frac{N}{\mu m^2} = 10.01 \times 10^9 \ \frac{N}{m^2}$$

Dynamic Stiffness Controller:

$$k_{\rm c,lim} = \frac{-1}{2(-1)(1.258 \ \frac{\mu m}{N})(100 \ \mu m)} = 0.0040 \ \frac{N}{\mu m^2} = 4.00 \times 10^9 \ \frac{N}{m^2}$$

By using the new control approach, there is a 150% relative increase in the critical cutting stiffness. Note that the relative change is calculated as the difference between the test value and the control value, divided by the control value, times 100%. Below this critical cutting stiffness, chatter will not occur regardless of running speed..

Alternatively, by assuming the characteristic cutting stiffness of 0.001 $\frac{N}{\mu m^2}$, the critical chip length per revolution (critical feed-rate) is calculated as shown below.

Chatter Avoiding Controller:

$$l_{\text{lim}} = \frac{-1}{2(-1)(0.494 \ \frac{\mu\text{m}}{N})(0.001 \frac{N}{\mu\text{m}^2})} = 1012.15 \ \mu\text{m} \approx 1.0 \ \text{mm}$$

Dynamic Stiffness Controller:

$$l_{\rm lim} = \frac{-1}{2(-1)(1.258 \ \frac{\mu \rm m}{\rm N})(0.001 \ \frac{\rm N}{\mu \rm m^2})} = 397.456 \ \mu \rm m \approx 0.4 \ \rm mm$$

The relative increase in critical feed-rate is the same as the relative increase in critical cutting stiffness. The significant improvement in the critical feed-rate indicates that the proposed controller design strategy is effective at avoiding machining chatter.

6.3 Stability Lobe Diagram

The chatter characteristics of the rotating system are examined by calculating SLDs. First, a levitated spindle model is created using the nominal spindle model and an AMB controller. The levitated spindle model is then augmented with the cutting force model having a fixed value for feed-rate and delay. For the SLD generation, unlike the controller design, the delay term in the cutting force model is accounted for with a 30th order Padé approximation. The Padé approximation is not accurate for very low speeds so the SLD is only generated above 10 krpm. It can be assumed that the high-speed machining spindle will only be used for machining above this speed. Note that the uncertainty driven controller design model assures chatter robustness to any speed. The values of feed-rate and running speed are iteratively changed and stability checked by finding the system's eigenvalues. Figure 6.2 shows the SLD generation algorithm. This is similar to the techniques used by Nakano and Takahara [89]. The cutting stiffness is set to $1 \times 10^{-3} \frac{N}{um^2}$ which is a characteristic value for aluminum [90].

Figure 6.3 shows the resulting SLD's. Below the stability lobe line is stable machining conditions and above the stability lobe line chatter will occur. The stability lobe line is higher with the chatter avoiding controller than with the dynamic stiffness controller for almost all spindle speeds. This indicates that the chatter avoiding controller more is successful at avoiding machining chatter.



Figure 6.2 Stability lobe diagram generation algorithm.

The chatter frequency at the border of stability is readily identified because the SLD's are generated with the eigenvalue method. The chatter frequency is the frequency of the single marginally stable eigenvalue at the border of stability. Figure 6.4 plots the chatter frequency for both controllers against spindle speed. The speed range is the same as in the SLDs in Figure 6.3. The abrupt drops in chatter frequency correspond to moving from one stability lobe to the next. Regardless of spindle speed, the chatter frequency falls in a range of approximately 1100 Hz to 1500 Hz as shown by the shaded region on the plot. This is significant for interpretation of the differences between the chatter avoiding μ -controller and the dynamic stiffness μ -controller. Recall from Chapter IV that the chatter avoiding controller has two sharp peaks at 754 Hz and 1840 Hz. The

frequencies of the peaks in the chatter avoiding controller are indicated in Figure 6.4 by dashed lines. The peaks are a result of the presence of the cutting force model in the chatter avoiding control scheme which is not present in the dynamic stiffness control scheme. The first and second controller peaks occur at slightly lower frequencies than the first and second rotor flexible modes, respectively.

When comparing the chatter frequency region to the chatter avoiding μ -controller and dynamic stiffness μ -controller, the significance of the peaks in the chatter avoiding μ controller can be seen. Figure 6.5 shows the Bode plots of both controllers for the back AMB and the chatter frequency range is again indicated with the shaded region. The chatter frequency region falls after the first peak in the chatter avoiding controller. The peak corresponds to an abrupt 180° phase shift. The phase stays more or less flat until the second controller peak, where there is another abrupt drop. This frequency span includes the chatter range over which the chatter avoiding μ -controller frequency response stays between -35° and -46°. Over this same frequency span, the dynamic stiffness controller has a gradual shift in phase. The dynamic stiffness controller exhibits similar overall phase shift for the entire range of operation as does the chatter avoiding controller, but does not have the careful control of phase in the chatter frequency region. The deliberate control of phase in the frequency range of possible chatter is important in attenuating the regenerative effect which causes chatter. When the inner and outer cut modulations are out of phase, the resulting cutting force oscillates leading to greater vibrations which, in turn, affect the next machining pass [13]. In this manner, chatter is a self-excited instability. Careful control of the spindle's phase response works to avert regeneration and avoid chatter.



Figure 6.3 Stability Lobe Diagram generated with model eigenvalues and when using dynamic stiffness μ -controller and chatter avoiding μ -controller.



Figure 6.4 Chatter frequency at the border of stability vs. spindle speed for chatter avoiding μ controller and dynamic stiffness μ -controller.



Figure 6.5 Bode plot of dynamic stiffness μ -controller and chatter avoiding μ -controller for the back AMB shown with chatter frequency range.

To further investigate the results of the chatter avoiding controller design, two more SLDs are generated. For each SLD, the model is truncated such that only a single flexible mode of the rotor is present the model. This is an artificial representation which does not express the full dynamics of the system but is useful in gaining insight into the results of controller design. The flexible modes of the rotor are lightly damped and inherently stable. No amplifiers, AMBs, or stabilizing controller are present. Figure 6.6 shows the resulting stability lobes for the first and second flexible modes and the corresponding chatter frequencies at the border of stability. Also shown are dashed lines denoting the spindle's first and second natural frequencies, ω_{n1} and ω_{n2} , respectively, and dotted lines denoting the frequencies of the chatter avoiding controller peaks. The dotted lines are the same as those in Figure 6.4.



Figure 6.6 SLDs (top) and chatter frequencies (bottom) for the model including only the first or second flexible mode.

The significant observation from these SLDs is that the critical feed-rate is, at some speeds, lower for the first mode and, at other speeds, lower for the second mode. To maximize the minimum stable feed-rate, the AMB controller must act to mitigate both chatter associated with the first flexible mode and chatter associated with the second flexible mode. Over all, the first mode chatter behavior dominates the chatter behavior of the whole system. The chatter frequency for the second mode occurs above the second

controller peak frequency. This suggests the second controller peak is needed to prevent that mode of chatter from occurring. The higher frequency chatter is not seen in Figure 6.4 for the whole system because the chatter frequency is for the border of stability only. While both modes of chatter are possible, the chatter avoiding controller acts to attenuate both. The chatter associated with the first mode sets in at lower feed-rates and therefore is seen in Figure 6.4. The modally separated SLDs allow for examination of the system without the action of the stabilizing controllers. Note that the SLD for the third flexible mode is far above those for the first two and therefore is not shown in Figure 6.6. Also note from these partial models that the SLD for the full system including the AMB controller is higher overall than that for either mode.

6.4 Numerical Time Simulation

A time domain simulation is performed using MATLAB's *ode45* numerical integration and Simulink interface. The identified components discussed in Chapter III are used. Simulations are performed using the dynamic stiffness and chatter avoiding μ -controllers on the modeled spindle. First, a simulation is performed for the rotor lifting off of the touchdown bearings when the AMBs are initially activated. Position sensor signals and controller current response are plotted. Then simulations are performed for machining conditions to show the effectiveness of the chatter avoiding μ -controller.

6.4.1 Time Response during Initial Levitation

When lifting off of the touchdown bearings, the AMB system is at extreme values of both position and current. Therefore, nonlinear effects must be considered. Three nonlinear effects are found to be significant and are included in the simulation.

- The control current is limited by a maximum saturation value imposed on the simulation. The possibility of too high of a control current is present because the rotor begins well outside of the normal operation range. The μ-values below 1 resulting from each controller indicate that the AMB coil current limit of ±4 A will not be passed, but this is only guaranteed for the maximum allowable rotor position of ±50 µm. The addition of control current saturation limits is a practical solution to this issue. In the same way, saturation limits are imposed for experimental implementation as well. The sudden application of maximum allowable current causes a transient motion in the rotor which places the rotor in the proper operation range. After approximately 0.025 s, all control axes are within current limits and the rotor is close to the operating setpoint of 0 µm.
- 2) The transverse moment of inertia of the rotor causes one side of the rotor to tend to move downward as the other side is lifted up. This effect can be seen in the initial response of the rotor. The model-based MIMO control strategy is capable of handling this dynamic cross-coupling with either μ-controller. However, to achieve an accurate simulation, the touchdown bearing is included as a discontinuous stiffness via an if statement in the numerical integration. This limits the downward motion of one side of the rotor as the other side is lifted up off the touchdown bearings. The effect of the touchdown bearings is only a factor in the first few ms. After that, the rotor has lost contact with the touchdown bearings and they exert no force on the rotor.
- 3) The linear model for AMB force, Eq. (3.2), is a linearization of the nonlinear

AMB force function, Eq. (3.1), about the operating point of 0 deflection and no control current. At large values of deflection, the linear model of the AMB force is not accurate. For the liftoff simulation, the nonlinear AMB force model is used.

Figure 6.7 shows the simulated time response at the front bearing V13 axis. Figure 6.8 illustrates the simulated time response at the back bearing V24 axis. The position sensor signal is shown in the top plot of each figure and control current is shown in the bottom plot of each figure. The liftoff test starts with the rotor resting on the touchdown bearings. At a time of 0 s, the AMBs are turned on and bias current and control current are applied. With either controller, the main transients have died out by 0.03 seconds and the rotor is near the operating point.



Figure 6.7 Simulated position and control current responses in the V13 axis for the spindle controlled with the dynamic stiffness and the chatter avoiding μ -synthesis controllers.



Figure 6.8 Simulated position and control current responses in the V24 axis for the spindle controlled with the dynamic stiffness and the chatter avoiding μ -synthesis controllers.

6.4.2 Chatter Simulation

To further confirm the SLDs, a numerical simulation is conducted under machining conditions using each controller. The cutting model, Eq. (4.3), is used. The spindle speed is fixed at 40,000 RPM. The delay is created by a numerical hold of 0.0015 s, determined by the spindle speed according to Eq. (4.4). The simulated workpiece material is aluminum. The cutting stiffness is set to $1 \times 10^{-3} \frac{N}{\mu m^2}$ and is the same characteristic value for aluminum that is used for SLD generation. The nominal cut depth, e_0 , is set at 100 um. The machining feed-rate is increased with time.

Figure 6.9 shows the cutting tool vibrations and corresponding feed-rate vs. simulation time for using both the chatter avoiding and dynamic stiffness μ -controllers. As the feedrate is increased linearly in the stable region, the cutting forces and therefore tool vibration amplitude increase linearly. Machining chatter occurs at the critical feed-rate presenting with a sudden increase in cutting tool vibration amplitude. The onset of chatter occurs later and at a higher feed-rate when using the chatter avoiding controller than when using the dynamic stiffness controller. Chatter occurs at about 15 s and 0.75 mm/rev when using the dynamic stiffness controller and at about 23 s and 1.2 mm/rev when using the chatter avoiding controller.

Increasing the feed-rate in the time simulation can be likened to moving straight up on the SLD in Figure 6.3 at 40,000 RPM. The critical feed-rate appears to be about 0.2 mm/rev higher in the time simulation than the SLD for both controller cases. This small discrepancy can be attributed to the effect of ramping the feed-rate obscuring the initial onset of chatter.



Figure 6.9 Simulated machine tool vibrations for progressively severe feed-rates with feed-rate (top) and tool vibrations (bottom) for dynamic stiffness controller (left) and chatter avoiding controller (right).

The results confirm the findings from the static models and SLDs that the chatter avoiding controller is effective at improving the critical chatter limit.

The frequency components of the simulated time responses are analyzed. The frequency analysis is conducted using both controllers for time segments before and after the onset of chatter. Figure 6.10 and Figure 6.11 show the results for the dynamic stiffness controller and chatter avoiding controller, respectively. For both controllers, the pre-chatter time segment is taken at 5 s. The post-chatter time segment is at about 10.3 s for the dynamic stiffness controller and at 28.7 s for the chatter avoiding controller. All the time responses shown are the same data as in Figure 6.9 but with smaller time scales. The numerical simulation is conducted with a variable step size so for discrete frequency

analysis, the simulation is sampled at 10 kHz. The discrete frequency analysis is done with the fast Fourier transform (FFT) algorithm and is readily performed in MATLAB. 512 data points (2^9) are taken for each time segment which corresponds to about 0.05 s. Note that selecting a data vector of length 2^n is needed for computational efficiency.

In the pre-chatter time segment, only the 1X component is seen. This occurs at 40,000 RPM (~667 Hz). For single point boring, 1X is also the tooth passing frequency and is given the symbol ω_T . When chatter sets in, two additional frequencies are seen. These are combinational frequencies of the tooth passing frequency and the frequency of unstable chattering frequency, ω_c . Combinational frequencies can occur at harmonics of the tooth passing frequency around the chatter frequency ($\omega_c+i\omega_T$, $i = \pm 1, \pm 2, \pm 3$) [17]. For this case, the combinational frequencies are seen for *i* values of +1 and -1. The chatter frequency is 1168 Hz when using the dynamic stiffness controller and 1209 Hz when using the chatter avoiding controller. See Figure 6.4 at 40,000 RPM. This yields combinational frequencies at 501 Hz and 1835 Hz with the dynamic stiffness controller and combinational frequencies at 542 Hz and 1876 Hz with the chatter avoiding controller.


Figure 6.10 Simulated machine tool vibrations and corresponding frequency spectrum using the dynamic stiffness controller before chatter (left) and during chatter (right).



Figure 6.11 Simulated machine tool vibrations and corresponding frequency spectrum using the chatter avoiding controller before chatter (left) and during chatter (right).

CHAPTER VII

EXPERIMENTAL CONTROLLER IMPLEMENTATION

7.1 Overview

In this chapter, the two controllers designed in Chapter IV are experimentally implemented on the spindle. Controller reduction and discretization for digital implementation are discussed. Then, as an indication of machining suitability, the frequency response at the tool location is measured while using each controller. Limiting stable cutting stiffness for the experimental system is calculated from the data. Finally, the time responses of the system during initial levitation off the touchdown bearings are measured and compared.

7.2 Controller Reduction and Discretization

A natural consequence of μ -synthesis control of a MIMO flexible AMB spindle is a high-order controller. This type of model-based control is at least the order of the plant model used, and is usually higher order due to iterative and frequency curve fit *D* scaling.

In this case, the plant used for dynamic stiffness is 76^{th} order, i.e., 10 for the rotor $\times 2$ planes plus 9 for the amplifier \times 4 control axes, and 20 for performance weights. The chatter avoiding scheme uses the same physical system model and bearing performance weights as the dynamic stiffness scheme but does not have the tool load frequency dependent performance weight. Also note that using the complex uncertainty to bound any possible delay does not cause the cutting model to add degrees of freedom to the system. After the *D*-K iterations, the dynamic stiffness μ -controller is 84th order and the chatter avoiding μ -controller is 74th order. These are too large to implement in real time due to being too high order to calculate the controller response within a single time step. Therefore, the controllers are reduced prior to implementation. Balanced Hankel singular value based model reduction is performed on the controllers. This is readily done using the MATLAB *reduce* command. The resulting controllers are both of 44nd order. Both controllers are then discretized using a first order hold at 12.5 kHz. Figures 7.1 through 7.8 show the Bode plots of the V plane inputs and outputs of both full order and continuous controllers compared to those for the reduced and discretized controllers. The controller axes corresponding to the front bearing, back bearing and front-back cross coupling are shown for the V plane. Those for the W plane are similar. As shown, the dynamics of the controllers are maintained although there is some small loss of phase fidelity at high frequencies. The vertical line in the Bode plots is at 6.25 kHz and denotes half the sampling rate, or the Nyquist frequency, above which digital control cannot be implemented. The phase difference in some of the plots is 360° and does not create an inconsistency in implementation. These two discrete controllers are implemented experimentally via dSPACE as discussed in Chapter III.



Figure 7.1 Bode plot of full order continuous and reduced discretized dynamic stiffness μ -controllers, V13 input and V13 output.



Figure 7.2 Bode plot of full order continuous and reduced discretized dynamic stiffness μ -controllers, V24 input and V24 output.



Figure 7.3 Bode plot of full order continuous and reduced discretized dynamic stiffness μ -controllers, V13 input and V24 output.



Figure 7.4 Bode plots of full order continuous and reduced discretized dynamic stiffness μ -controllers, V24 input and V13 output.



Figure 7.5 Bode plot of full order continuous and reduced discretized chatter avoiding μ -controllers, V13 input and V13 output.



Figure 7.6 Bode plots of full order continuous and reduced discretized chatter avoiding μ -controllers, V24 input and V24 output.



Figure 7.7 Bode plot of full order continuous and reduced discretized chatter avoiding μ -controllers, V13 input and V24 output.



Figure 7.8 Bode plots of full order continuous and reduced discretized chatter avoiding μ -controllers, V24 input and V13 output.

7.3 Frequency Response Measurement at the Tool Location

To evaluate the aptitude of each controller to chatter-free machining, the spindle is levitated with each controller and an impulse hammer test is conducted to measure the frequency response at the tool location, *T* from Eq (5.10). The force transducer used is inside a PCB Piezotronics impulse hammer 086C03. The displacement transducer used is a Lion Precision capacitance probe C23-C with driver CPL290. The data is collected and processed via a HP dynamic signal analyzer 35670A. Figures 7.9 and 7.10 show photographs of the impulse test experimental setup with Figure 7.9 showing the whole setup and Figure 7.10 showing a close up of the spindle tool location, capacitance probe position and impulse hammer held in position.



Figure 7.9 Impulse hammer test experimental setup showing hammer, capacitance probe mount and driver, dynamic signal analyzer and spindle housing.



Figures 7.10 Impulse hammer experimental setup showing close up of impact area with hammer held in position.

The real part of *T* is found for each controller. The maximum negative real value occurs near the first bending mode for both controllers. Figure 7.11 shows the Bode plot of *T* and the real and imaginary parts at the first bending mode using both μ -controllers. The results for using an industry standard PID controller are also included for comparison. The values of $\max_{\omega} |\operatorname{Re}(T)|, T < 0$ are indicated for the μ -controllers while that for the PID exceeded the scale of the plot.

The first bending mode is at a natural frequency. As such, the Bode plot indicates the characteristic magnitude peak and phase shift of resonance. For all three controllers, the frequencies of the first bending mode remain in a very small range because the system dynamics are dominated by the dynamics of the flexible rotor. In keeping with classical theory, the height of the resonance peak and rate of phase shift are affected by the amount of damping. With all else being equal, the difference in damping must come from the different controllers. The PID controller results in a much higher resonance peak than either of the μ -controllers. Meanwhile, the Chatter avoiding μ -controller has a lower

peak than the dynamic stiffness μ -controller but only slightly so. It is at this frequency that the minimum dynamic stiffness occurs. These results are in agreement with the simulated results in Chapter VI.

Although heuristics tells of the need for dynamic stiffness, the limiting condition analysis depends on the real part of the response only. The real part of the frequency response is shown in the top right plot of Figure 7.11. The minimum dynamic stiffness occurs at resonance, where the real part of the response is zero due to the 90 degree phase shift. At this frequency, the response is limited by the damping. Despite this fact, the critical point occurs at a slightly higher frequency than resonance. The chatter avoiding μ -controller shows an improvement in the critical point over the μ -controller designed for only dynamic stiffness.

These measurements are projected to a value for limiting stable cutting stiffness. Using Eq. (5.10), and assuming a chip width of 100 μ m, the limiting stable cutting stiffness is calculated in the following equations. The value for the maximum negative real part of the frequency response using each controller is taken from Figure 7.11.



Figure 7.11 Experimental Bode plot and real and complex parts of frequency response of the spindle at the tooltip when using PID, and dynamic stiffness, and chatter avoiding μ -controllers.

Chatter Avoiding Controller:

$$K_{clim} = \frac{-1}{2(-1)(1.446 \ \frac{\mu m}{N})(100 \ \mu m)} = 0.0035 \ \frac{N}{\mu m^2} = 3.50 \times 10^9 \ \frac{N}{m^2}$$

Dynamic Stiffness Controller:

$$K_{clim} = \frac{-1}{2(-1)(2.372 \ \frac{\mu m}{N})(100 \ \mu m)} = 0.0021 \ \frac{N}{\mu m^2} = 2.10 \times 10^9 \ \frac{N}{m^2}$$

Alternatively, by assuming the characteristic cutting stiffness of 0.001 $\frac{N}{\mu m^2}$, the critical chip length per revolution (critical feed-rate) is calculated as shown below.

Chatter Avoiding Controller:

$$l_{\rm lim} = \frac{-1}{2(-1)(1.446 \ \frac{\mu m}{N})(0.001 \frac{N}{\mu m^2})} = 345.781 \ \mu m \approx 0.3 \ \rm mm$$

Dynamic Stiffness Controller:

$$l_{\rm lim} = \frac{-1}{2(-1)(2.372 \ \frac{\mu m}{N})(0.001 \ \frac{N}{\mu m^2})} = 210.793 \ \mu m \approx 0.2 \ \rm mm$$

By using the new control approach, there is a 64% relative increase in the critical cutting stiffness. The relative increase is calculated in the same way as in Chapter VI and the trend is in agreement with the predictions made in Chapter VI. However, the overall magnitude of improvement is less than predicted by the model. The discrepancy can be attributed to small mismatching between the spindle model and the system identification; In particular, the rotor's internal damping which was estimated manually. The increase in critical cutting stiffness confirms that the chatter avoiding μ -controller is more effective at avoiding chatter.

7.4 Time Response during Initial Levitation

To further validate the closed-loop spindle model which is used to simulate machining chatter, the liftoff behavior of the experimental system is measured. First, the rotor is at rest, supported by the touchdown bearings. Then the AMBs are activated and the rotor levitates. After a transient response, the rotor settles to the center of the AMBs. The AMB sensor signals and the resulting control currents are recorded. The liftoff procedure is performed for both the chatter avoiding μ -controller and the dynamic stiffness μ -controller. Recall that the touchdown bearings are traditional ball bearings with inner diameter larger than the outer diameter of the rotor such that they do not support the rotor during AMB levitation. Also recall that zero deflection corresponds to the center of the AMBs. This test is the same as the simulation in subsection 6.4.1.

Figures 7.12 and 7.13 show the time response for initial levitation. Figure 7.12 illustrate the front bearing V13 axis while Figure 7.13 shows for the back bearing V24 axis. The liftoff behavior of the perpendicular axes are similar. The top plot of each figure is for the position signal and the bottom plot is for the resulting control current. Both figures show the response when using the chatter avoiding μ -controller and the dynamic stiffness μ -controller.

In general, good agreement is found between the simulation and experimental results. However, there exists an unexpected low frequency component in the transient response which dies out before approximately 0.01 s. This is due to the digital implementation of control on the experimental test rig, whereas the numerical simulation is conducted using the original continuous controller. However, this low frequency discrepancy does not affect the chatter characteristics which occur in the neighborhood of 1-1.5 kHz.



Figure 7.12 Experimental position and control current response in the V13 axis for the spindle controlled with the dynamic stiffness and the chatter avoiding μ -synthesis controllers.



Figure 7.13 Experimental position and control current response in the V24 axis for the spindle controlled with the dynamic stiffness and the chatter avoiding *µ*-synthesis controllers.

CHAPTER VIII

CONCLUSIONS

8.1 Highlight of Contributions

A controller design strategy for a HSM spindle's AMBs has been proposed, in which μ -synthesis model-based robust control is used with a system model that has been augmented with a regenerative cutting force model. The resulting controller stabilizes not only the inherently unstable AMBs, but the instability associated with the machining process, or chatter. The proposed strategy was demonstrated on an industrial grade HSM AMB boring spindle. The practical implementation of the proposed strategy included identification and modeling of the experimental system and selection of effective performance weights and uncertainties. The performance weights and uncertainties were for not only the AMB levitation but also for the newly included cutting force model. A chatter avoiding robust controller was synthesized for the spindle. For performance evaluation, it was compared to a controller designed to maximize the tool's dynamic stiffness. The dynamic stiffness controller represents the current state-of-the-art in

avoiding chatter. The controllers were implemented on the spindle for machining simulations and experimental hammer tests to measure the chatter limiting criterion. A 64% relative increase in the critical feed-rate, which is the limit for the onset of chatter, was achieved.

8.2 Additional Insights

The plant augmentation is a natural extension of previous model-based chatter control attempts. In fact, if there was a full understanding of the machining system from the beginning of active chatter control research, the augmented plant would always have been used. Unfortunately, there is a learning curve in the coming together of controls expertise and machining expertise. In fact, it does not matter 1) what type of machining, 2) what type of actuator, 3) what your performance goals e.g., chatter, surface finish, tolerance, or even 4) what type of model-based controller is implemented. Using the augmented system is more than just superior; it is the only correct way to execute model-based controller synthesis. Model-based control requires an accurate model and the machining system has cutting dynamics with known form.

The augmented plant method can be applied to find the best machining control solution for any actuator. However, AMBs are perhaps the best type of actuator for this method because they are non-contacting supports. Compare an AMB spindle to a spindle on ball bearings with an actuator added somewhere in the system. Any chatter avoiding controller for the added actuator would have to work within the restrictions of the ball bearings. With AMBs, the spindle is free to move as needed to avoid chatter.

The method developed in this dissertation can be thought of as an optimal and robust

tuning method which also has the discretion of combining all possible chatter avoiding strategies. There are many chatter avoiding strategies in existence, e.g., active vibration absorber, active damping, etc., but for any given strategy one is presented with the problem of how best to tune it. Unfortunately, analytical solutions are usually not perfect for practical implementation because of the approximations of the system involved. For example, assume one wants to give the spindle stiffness so that it does not vibrate using the dynamic stiffness method. Also, one wants to give the spindle damping so that it attenuates chatter, using active damping. Additionally, one wants to allow the CG of the spindle to vibrate so the tool location is steady under machining loads, similar to an active vibration absorber. Finally, one wants to do all simultaneously to get the maximum chatter free region. A model-based optimization algorithm is needed to tune the controller values for each strategy given the complicated geometry of the spindle, cross-coupling of the inputs and outputs, and dynamics of the actuators. μ -synthesis has the advantage that the controller structure is not predefined. So μ -synthesis in essence selects the best possible combination of all the chatter avoiding strategies, given the specific geometry, and tunes the strategies at the same time. From this point of view, it can be seen that no other control strategy for chatter prevention can be more effective than the one proposed. The only limiting factor for the μ -synthesis is the geometry and actuators of the system.

What might not be immediately apparent is that this chatter avoiding method does not require information on what the cutting stiffness actually is. If a certain cutting stiffness value is used in the cutting model, and a successful controller designed, the resulting controller would guarantee stability to chatter but only at that specific cutting stiffness. That approach is not a practical solution because actual cutting stiffness is difficult to measure or calculate, and may change with different applications of the spindle. In contrast, the goal of the chatter avoiding method is to maximize the stable cutting stiffness. This is equivalent to maximizing the SLD for a given cutting stiffness. Because of the linear relationship of the cutting dynamics, regardless of what the cutting stiffness actually is, the resulting controller will provide the maximum possible feed-rate.

8.3 Possible Directions of Future Research

An exciting area of future research is the application of the chatter avoiding method developed in this work to systems with flexible workpieces. An example of flexible workpieces is thin-walled structures, which have received much interest in recent research [91]. Flexible workpieces can deflect under machining forces. Therefore, chatter, which is relative between the cutting tool and workpiece, may be attributed to vibration in the workpiece. This source of chatter is especially problematic because it cannot be addressed through traditional methods such as increasing tool stiffness.

The problem of a flexible workpiece is exacerbated by the nature of the machining process. As material is removed, the natural frequencies and mode shapes of the workpiece will change. As the tool is fed over the workpiece, the point of contact moves relative to the mode shape, causing an effective change in the cutting force feedback gain. These issues make finding a persistent sweet-spot on the SLD very difficult [92].

The problem of chatter with a flexible workpiece can only be addressed by a flexibility elsewhere in the system, be it a flexible tool or a flexible workpiece holder. The flexibility must be tuned or actively controlled to counteract the vibration in the

workpiece such that chatter is attenuated. The method developed in this work is the most apt solution to this controller design problem. The method can be extended to the flexible workpiece chatter by including a model of the workpiece dynamics in the plant for controller design. The changing workpiece natural frequency and mode shape can be accounted for with uncertainties in the model so the SLD will be optimized for the entire machining range.

This research can be also extended through further work with the cutting force model. The regenerative cutting force model itself has gained wide acceptance in describing the chatter phenomenon but the application of linear uncertainties to the regenerative cutting force model for robust control synthesis is fairly new.

The issue of the delay in the cutting force model is fully addressed in the current work in terms of robustness. However, there is the potential for significant improvements in the effectiveness of the proposed method if the delay were handled differently. Consider the running speed, which must be an uncertainty in order to make the rotor robust to the gyroscopic effect. The delay time is the difference between one machining pass and the next and is therefore a function of the running speed. Consequently, the delay need not introduce an additional uncertainty to the system on its own but can merely be expressed in terms of the running speed uncertainty. It is difficult to predict how much improvement is possible by eliminating one uncertainty from the system and it is impossible to predict for the general case. There is the potential for the controller using the knowledge of the gyroscopic stiffening at a particular speed to defeat chatter at that speed, but effectively eliminating the delay uncertainty is problematic due to the transcendental nature of the delay function, e^{-st} . An exact rational expression in terms of

the running speed cannot be written and approximate expressions for the delay of sufficient accuracy are too high order for a controller to be synthesized. Refer to the literature review section for more information on efforts in this area, but the only successes have come by using small speed ranges and have managed to shift sweet spots rather than increase the entire SLD over the entire operation range as was achieved in the current work. If the delay uncertainty is ever eliminated, the potential could be the entire SLD becoming a sweet spot.

Another way to improve the cutting force model is the inclusion of the high-speed effect in the cutting stiffness. In the current work, the cutting stiffness was assumed as a constant but uncertain gain and the maximum possible value of that gain was maximized so the chatter free region would be maximized. If the high-speed effect is taken into account, the cutting stiffness will be able to decrease as a function of the uncertain spindle speed. How much it will decrease is difficult to predict, which is why the conservative constant gain was assumed in the present work. If reliable high-speed cutting stiffness data was available, it can be incorporated into the uncertain model and a controller can be synthesized which is less conservative at high-speeds.

In the present work, the self-excited vibration of machining chatter is addressed through robust controller design by including a regenerative cutting force model in the plant. The abstract concept of augmenting the plant with dynamics to account for a selfexcited phenomenon can be applied to other engineering problems. Examples of other self-excited phenomenon of interest in engineering research are surge in compressors [93], blade flutter in gas turbines [94], and oil whip in hydrodynamic bearings [95].

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APPENDIX

ACTIVE MAGNETIC BEARING PARAMETERS

Parameter	Units	Front Radial	Back Radial	Thrust			
		Bearing	Bearing	Bearing			
Performance Specifications							
Saturation Current	A	11.25	11.10	10.25			
Static Capacity	N	1396.74	600.51	498.20			
Dynamic Capacity	N	1396.74	600.51				
at Frequency	Hz	800	1900				
Effective Gap	μm	575.25	576.27	939.86			
Calibration Factor	$\frac{N-\mu m^2}{A^2}$	13.71	5.85	17.02			
Bearing Geometry							
Poles per Quadrant	-	2	2	2			
Stator Stack Length	mm	44.5	19.0	20.00			
Stator OD	mm	136.0	136.0	135.99			
Stator ID	mm	65.0	57.0	65.00			
Rotor OD (at bearing)	mm	64.0	56.0	82.60			
Rotor Lamination ID (at bearing)	mm	38.0	30.0	NA			
Thrust Rotor Width	mm	NA	NA	6.50			
Nominal Gap	μm	508	508	762			
Pole Width	mm	12.0	12.0	NA			
Pole Height	mm	23.0	27.0	NA			
Slot Width (at ID)	mm	13.4	10.2	NA			
Pole Centerline Angle	degrees	22.5	22.5	NA			
Annular Area (thrust)	mm^2	NA	NA	903.2			

Parameter	Units	Front Radial Bearing	Back Radial Bearing	Thrust Bearing			
Material Properties							
Stator Material	-	Hiperco 50	Hiperco 50	AISI 4340 Ann			
Rotor Lamination Material	-	Hiperco 50HS	Hiperco 50HS	AISI 4340 Ann			
Saturation Flux Density	Т	2.04	2.04	1.21			
Relative Permeability	G Oe	1000	1000	500			
Stator Lamination Thickness	mm	0.36	0.36	Solid			
Rotor Lamination Thickness	mm	0.10	0.10	Solid			
Stator Material Resistivity	μΩ-mm	400	400	248			
Rotor Material Resistivity	μΩ-mm	420	420				
Density	$\frac{\text{kg}}{\text{m}^3}$	8110.21	8110.21	7888.71			
Lamination Insulation Material	-	Niox	Niox	Solid			
Lamination Stacking Factor	%	96	96	Solid			
Coil Specifications							
Wire Gage	AWG	17	17	17			
Wire Type	-	Phelps Dodge TZQS					
Coil Insulation Material	-	PD George 433-75VTC					
Coil Packing Factor	%	97	97	97			
Turns Per Coil	-	75	75	112			
Coil Extension (max)	mm	8.0	8.0	NA			
Quadrant Resistance	Ω	0.407	0.255	0.83			
Quadrant Inductance	mН	13.89	6.01	12.01			
Power Amplifier Specifications							
Max Continuous	А	12					
Dools Current	٨	12					
May DC Supply	A	12					
Waltage	V	400					
Voltage Min Doguired Voltage	V	200					
Min Lood Inductoria	v mU	200					
Switching Frequency		25					
Switching Frequency	КПΖ		23				