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A NEW PRE-LOADED BEAM GEOMETRIC STIFFNESS MATRIX WITH FULL RIGID BODY CAPABILITIES

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Abstract—Space structures, such as the Space Station solar arrays, must be extremely light-weight, flexible structures. Accurate prediction of the natural frequencies and mode shapes is essential for determining the structural adequacy of components, and designing a controls system. The tension pre-load in the 'blanket' of photovoltaic solar collectors, and the free/free boundary conditions of a structure in space, causes serious reservations on the use of standard finite element techniques of solution. In particular, a phenomenon known as 'grounding', or false stiffening, of the stiffness matrix occurs during rigid body rotation. The authors have previously shown that the grounding phenomenon is caused by a lack of rigid body rotational capability, and is typical in beam geometric stiffness matrices formulated by others, including those which contain higher order effects. The cause of the problem was identified as the force imbalance inherent in the formulations. In this paper, the authors develop a beam geometric stiffness matrix for a directed force problem, and show that the resultant global stiffness matrix contains complete rigid body mode capabilities, and performs very well in the diagonalization methodology customarily used in dynamic analysis.

NOTATION

A	area of beam cross-section, in ²
dR_i^{DF}/du_i	derivative of the load correction vector with respect to the nodal degrees of freedom
E	Young's modulus, psi
I	moment of inertia, in ⁴
$[K]$	the global stiffness matrix
$[K^{DFC}]$	directed force load correction matrix
$[K_e]$	elastic stiffness matrix for a Bernoulli beam
$[K_g]$	consistent geometric stiffness matrix for a Bernoulli beam
$[K_r]$	tangential stiffness matrix, $[K_e] + [K_g]$
L	length of beam
Λ	the square of the natural frequencies, rad ² /sec ²
m	mass per unit length, lb-sec ² /in ²
Ω	the natural frequencies of vibration, rad/sec
P	axial tension force
R^{DFC}	Argyris load correction vector for vertical force component
V_i	transverse displacement of node i
ϕ	angle of rotation as shown in Fig. 1
θ	rigid body rotation angle
$[\phi]$	the matrix of mode shapes (eigenvectors) used for diagonalization of the global stiffness matrix

INTRODUCTION

In order to be cost-effective, space structures must be extremely light-weight, and subsequently, very flexible structures. The power system for Space Station *Freedom* [1] is such a structure. Each array consists of a deployable truss mast and a split

'blanket' of photovoltaic solar collectors. The solar arrays are deployed in orbit, and the blanket is stretched into position as the mast is extended during deployment. Geometric stiffness due to the tension pre-load in the blanket make this an interesting non-linear problem.

A space station is subjected to various dynamic loads during shuttle docking, solar tracking, attitude adjustment, etc. Accurate prediction of the natural frequencies and mode shapes of the space station components, including the solar arrays, is critical for determining the structural adequacy of the components, and for designing a dynamic controls system.

This paper has the following objectives:

1. To examine the directed force (bow-string) problem for its potential as a basis for developing stiffness matrices which possess rigid body rotational capabilities.
2. To check the performance of any new matrix which possesses a complete set of rigid body motion capabilities in the diagonalization/partitioning methodology used in dynamic response.

GLOBAL FORMULATION OF BOW-STRING

The authors developed a geometric stiffness matrix for a bow-string [2] using the equations of motion

developed by Fertis and Lee [3], and found that the resultant matrix did possess all the rigid body modes. However, it was shown that an assemblage of such elements did not correctly model a force directed between the end nodes.

Examination of a two-element model of traditional beam elements indicated that the only fictitious forces that occurred during rigid body rotation were the end shears required for equilibrium [4]. The shear at the center node was zero. The corresponding row in the geometric stiffness matrix was full, indicating that there is a relationship between the stiffness terms at each degree of freedom, and the shear at the center node.

Examination of the first and fifth rows, however, indicate that there is not any relationship between the end nodes. This is inherent in the assembly process and is contrary to the basic supposition that we are considering a problem where the applied forces remain directed between the end nodes.

Consider Argyris' methodology [5] for the directed force problem (Fig. 1). Let

$$\phi \approx \frac{V_3 - V_1}{L}$$

If one neglects the change in the axial component of P that occurs during rotation, as is customarily done ($P \cos \phi \approx P$), we obtain the consistent geometric stiffness matrix,

Suppose we retain the vertical component, $P \sin \phi$, and use Argyris' approach to develop a load correction matrix. The load vector for this force becomes

$$R^{DFC} = [P \sin(V_3 - V_1)/L, 0, 0, 0, \\ -P \sin(V_3 - V_2)/L, 0]$$

The load correction matrix is generated using the equation

$$[K^{DFC}] = \left[\frac{dR_i^{DF}}{du_i} \right],$$

which yields

$$[K^{DFC}] = \begin{bmatrix} -\frac{P}{L} \cos \left[\frac{V_1 - V_3}{L} \right] & 0 & 0 & 0 & \frac{P}{L} \cos \left[\frac{V_1 - V_3}{L} \right] & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{P}{L} \cos \left[\frac{V_1 - V_3}{L} \right] & 0 & 0 & 0 & -\frac{P}{L} \cos \left[\frac{V_1 - V_3}{L} \right] & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

For small rotation, $\cos(V_1 - V_3)/L \approx 1$, and $[K^{DFC}]$ becomes

$$[K^{DFC}] = \begin{bmatrix} -\frac{P}{L} & 0 & 0 & 0 & \frac{P}{L} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{P}{L} & 0 & 0 & 0 & -\frac{P}{L} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At this point, combine $[K_g] + [K^{DFC}]$ and check rigid body rotation. Since $[K^{DFC}]$ contains non-zero terms in rows 1 and 5, and $[K_g]$ pseudo-forces occur only in the same two rows, only these two rows must be checked for rigid body rotation capability.

Row 1

$$P[12/5L - 1/L, 1/10, -12/5L, 1/10, 1/L, 0]$$

$$\begin{bmatrix} -L\theta/2 \\ \theta \\ 0 \\ \theta \\ L\theta/2 \\ \theta \end{bmatrix}$$

$$= P[-1.2\theta + 0.5\theta + 0.1\theta + 0.1\theta + 0.5\theta]$$

$$= 0.$$

Row 5

$$P[1/L, 0, -12/5L, -1/10, 12/5L - 1/L, -1/10]$$

$$\begin{bmatrix} -L\theta/2 \\ \theta \\ 0 \\ \theta \\ L\theta/2 \\ \theta \end{bmatrix}$$

$$= P[-0.5L - 0.1L + 1.2L - 0.5L - 0.1L]$$

$$= 0.$$

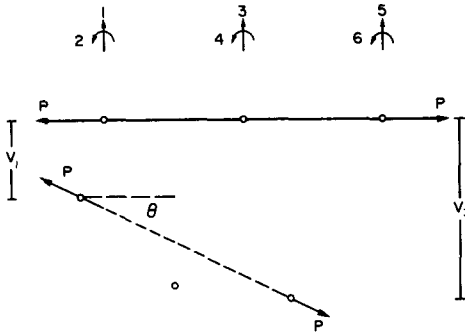


Fig. 1. Directed force two-element representation.

Therefore, $[K_g] + [K^{DFC}]$ does possess rigid body rotation capability. By inspection, it also possesses rigid body translation capabilities.

PERFORMANCE OF $[K_T] + [K]^{DFC}$

Consider a two-element model using using $[K_T] + [K]^{DFC}$, utilizing the computer program NLBO.FOR.† Table 1 compares the results from NLBO.FOR with the finite element solution using $[K_T]$ only (consistent $[K_e]$ and $[K_g]$ matrices). Note that the stiffness matrix generated by NLBO.FOR possesses the additional zero eigenvalue required for a complete set of rigid body modes. The other frequencies have extremely close correlation with the traditional finite element solution obtained using NLFINITE.FOR.† Most of them were identical. The largest difference was 2.8% for frequency No. 6.

When one considers the stiffness matrix $[K_T]$ generated by NLFINITE.FOR for this problem, the following is obtained:

$$[K_T] \times [\text{rigid body modes}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -6 \times 10^7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \times 10^7 \\ 0 & 0 & 0 \end{bmatrix}$$

As expected, large pseudo-forces occurred during rigid body rotation.

† NLFINITE.FOR is based upon the finite element dynamics program FINITEL.FOR found in [6], with modifications added to solve the geometric non-linear problem. NLBO.FOR contains additional modifications for the directed force correction as well as capability for spring supports. The modifications will be provided by the authors upon request.

The eigenvalues and eigenvectors generated by NLFINITE were

$$\Lambda(1) = 0.0000$$

$$\Omega(1) = 0.0000 \text{ rad/sec.}$$

The associated eigenvector is

$$0.1000000000D + 01$$

$$0.0000000000D + 00$$

$$0.0000000000D + 00$$

$$0.1000000000D + 01$$

$$0.0000000000D + 00$$

$$0.0000000000D + 00$$

$$0.1000000000D + 01$$

$$0.0000000000D + 00$$

$$0.0000000000D + 00$$

$$\Lambda(2) = 0.0000$$

$$\Omega(2) = 0.0001 \text{ rad/sec.}$$

The associated eigenvector is

$$0.0000000000D + 00$$

$$0.1000000000D + 01$$

$$-0.1022543458D - 17$$

$$0.0000000000D + 00$$

$$0.1000000000D + 01$$

$$-0.1102256093D - 15$$

$$0.0000000000D + 00$$

$$0.1000000000D + 01$$

$$-0.5808321841D - 16$$

$$\Lambda(3) = 1930958.5265$$

$$\Omega(3) = 1389.5893 \text{ rad/sec.}$$

The associated eigenvector is

$$0.0000000000D + 00$$

$$-0.1000000000D + 01$$

$$0.1508421072D - 01$$

$$0.0000000000D + 00$$

$$0.1746464363D - 14$$

$$0.2418478835D - 01$$

$$0.0000000000D + 00$$

$$0.1000000000D + 01$$

$$0.1508421072D - 01$$

$$\Lambda(4) = 12477499.9590$$

$$\Omega(4) = 3532.3505 \text{ rad/sec.}$$

Table 1. Frequency comparison using NLFINITE.FOR and NLBO.FOR

				$A = 48 \text{ IN}^2$ $E = 30 \times 10^6 \text{ PSI}$ $I = 1000 \text{ IN}^4$ $m = 0.03525 \text{ LB-SEC}^2/\text{IN}^2$ $L = 100 \text{ IN}$ $P = 60,000,000 \text{ LBS}$		
Freq	NLFINITE.FOR		NLBO.FOR		% Diff	%Diff
	2 Elem.	4 Elem.	2 Elem.	4 Elem.		
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	1390	1385	0	0	0	0
4	3532	3524	3532	3524	0	0
5	7002	6514	7002	6524	0	0
6	7847	7163	7657	6969	2.4	2.7
7	14003	12565	14003	12565	0	0
8	17683	14003	17683	14003	0	0
9	27189	21782	27090	21715	0.4	0.3
10		22755		22755		0
11		28006		28006		0
12		33395		33395		0
13		51048		51083		0.07
14		84645		84645		0
15		93157		93200		0.05

The associated eigenvector is

0.0000000000D + 00
 0.1000000000D + 01
 -0.4125765357D - 01
 0.0000000000D + 00
 -0.6561862202D + 00
 -0.6954552347D - 17
 0.0000000000D + 00
 0.1000000000D + 01
 0.4125765357D - 01
 $\Lambda(5) = 49021276.5957$
 $\Omega(5) = 7001.5196 \text{ rad/sec.}$

The associated eigenvector is

0.1000000000D + 01
 0.0000000000D + 00
 0.0000000000D + 00
 0.2041110487D - 15
 0.0000000000D + 00
 0.0000000000D + 00
 -0.1000000000D + 01
 0.0000000000D + 00
 0.0000000000D + 00
 $\Lambda(6) = 61570298.8787$
 $\Omega(6) = 7847.6744 \text{ rad/sec.}$

The associated eigenvector is

$$\begin{array}{r}
 0.4120001744\text{D} + 00 \\
 -0.3619443243\text{D} - 18 \\
 0.0000000000\text{D} + 00 \\
 0.1000000000\text{D} + 01 \\
 -0.8622993042\text{D} - 01 \\
 0.0000000000\text{D} + 00 \\
 0.1067276729\text{D} - 15 \\
 0.7402023912\text{D} - 01 \\
 0.0000000000\text{D} + 00 \\
 -0.1000000000\text{D} + 01 \\
 -0.8622993042\text{D} - 01 \\
 A(7) = 196085106.3830 \\
 \Omega(7) = 14003.0392 \text{ rad/sec.}
 \end{array}$$

The associated eigenvector is

$$\begin{array}{r}
 0.1000000000\text{D} + 01 \\
 0.0000000000\text{D} + 00 \\
 0.0000000000\text{D} + 00 \\
 -0.1000000000\text{D} + 01 \\
 0.0000000000\text{D} + 00 \\
 0.0000000000\text{D} + 00 \\
 0.1000000000\text{D} + 01 \\
 0.0000000000\text{D} + 00 \\
 0.0000000000\text{D} + 00
 \end{array}$$

0	0	0	0	0	0	0	0	0	0
0	1	1.92	-0.66	0	-1.98	0	0.41	-1.36	
0	1	2.54×10^6	0	0	109	0	-1	41	
0	-2	1	1.19×10^7	0	-3	0	220	-2	
0	0	0	0	5.6×10^7	0	-2	0	0	
0	-1	113	-1	0	4.9×10^7	0	-1	-909	
0	0	0	0	-1	0	2.30×10^8	0	0	
0	2	1	219	0	-1	0	1.33×10^8	-1	
0	1	43	-1	0	-908	0	0	3.30×10^8	

$$\begin{array}{l}
 A(8) = 312696968.0165 \\
 \Omega(8) = 17683.2397 \text{ rad/sec.}
 \end{array}$$

The associated eigenvector is

$$\begin{array}{r}
 0.0000000000\text{D} + 00 \\
 0.1000000000\text{D} + 01 \\
 -0.1694400209\text{D} + 00 \\
 0.0000000000\text{D} + 00
 \end{array}$$

The associated eigenvector is

$$\begin{array}{r}
 0.0000000000\text{D} + 00 \\
 0.1000000000\text{D} + 01 \\
 -0.2124292332\text{D} + 00 \\
 0.0000000000\text{D} + 00 \\
 0.3363477755\text{D} - 15 \\
 -0.1091964722\text{D} + 00 \\
 0.0000000000\text{D} + 00 \\
 -0.1000000000\text{D} + 01 \\
 -0.2124292332\text{D} + 00.
 \end{array}$$

Let $[\phi]$ be the matrix of mode shapes (eigenvectors). Then $[\phi]^T [K] [\phi]$ would yield a diagonalized stiffness matrix if all of the rigid body modes were present. Performing that matrix multiplication yields

It should be noted that small errors occur during the computations (initial data errors, roundoff errors, truncation errors, relative errors, etc.). The 2.54×10^6 term in the 3,3 position is due to the lack of rigid body rotation capability. The other non-diagonal terms should also be zero, but may be attributed to the above mentioned errors. The largest of these, ± 909 , is still relatively insignificant compared to the magnitude of the diagonal terms.

If one neglects the relatively small terms due to arithmetic errors, the following diagonal matrix is obtained

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.54 \times 10^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.19 \times 10^7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.76 \times 10^7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.91 \times 10^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.30 \times 10^8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.33 \times 10^8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.30 \times 10^8 & 0 \end{bmatrix}$$

Now consider the modified finite element solution from NLBO.FOR, which utilized the directed force correction matrix. The two-element stiffness matrix generated is

$$\begin{bmatrix} 0.288E8 & 0 & 0 & -0.288E8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.372E7 & 0.78E8 & 0 & -0.432E7 & 0.78E8 & 0 & 0 & 0 & 0 \\ 0 & 0.78E8 & 0.28E10 & 0 & -0.78E8 & 0.11E10 & 0 & 0 & 0 & 0 \\ -0.288E8 & 0 & 0 & 0.576E8 & 0 & 0 & -0.288E8 & 0 & 0 & 0 \\ 0 & -0.432E7 & -0.78E8 & 0 & 0.864E7 & 0 & 0 & -0.432E7 & 0.78E8 & 0 \\ 0 & 0.78E8 & 0.11E10 & 0 & 0 & 0.56E10 & 0 & -0.78E8 & 0.11E10 & 0 \\ 0 & 0 & 0 & -0.288E8 & 0 & 0 & 0.288E8 & 0 & 0 & 0 \\ 0 & 0.6E6 & 0 & 0 & -0.432E7 & -0.78E8 & 0 & 0.372E7 & -0.78E8 & 0 \\ 0 & 0 & 0 & 0 & 0.78E8 & 0.11E10 & 0 & -0.78E8 & 0.28E10 & 0 \end{bmatrix}$$

The rigid body rotation matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -50 & 1 & 0 & 0 & 1 & 0 & 50 & 1 \end{bmatrix}^T$$

$$[K_T] \times [\text{rigid body modes}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The associated eigenvector is

$$\begin{aligned} &0.0000000000D + 00 \\ &-0.9999999660D - 01 \\ &0.0000000000D + 00 \\ &0.1701762335D - 07 \\ &0.1999999966D - 01 \\ &0.0000000000D + 00 \\ &0.0000000000D + 00 \\ &0.1000000000D + 01 \\ &0.1999999971D - 01 \\ &A(2) = 0.0000 \\ &\Omega(2) = 0.0000 \text{ rad/sec.} \end{aligned}$$

The large, erroneous term ($\pm 6 \times 10^7$) due to lack of rigid body rotation capability has been eliminated.

The eigenvalues and eigenvectors generated by NLBO, corresponding to all rigid body and elastic modes and frequencies, were

$$A(1) = -0.0122$$

$$\Omega(1) = 0.000 \text{ rad/sec.}$$

The associated eigenvector is

$$\begin{aligned} &0.0000000000D + 00 \\ &0.1000000000D + 01 \\ &-0.1021057361D - 08 \\ &0.0000000000D + 00 \end{aligned}$$

$$\begin{aligned}
&0.9999999489D + 00 \\
&-0.1021057341D - 08 \\
&0.0000000000D + 00 \\
&0.9999998979D + 00 \\
&-0.1021057345D - 08 \\
&\Lambda(3) = 0.0000 \\
&\Omega(3) = 0.0001 \text{ rad/sec.}
\end{aligned}$$

The associated eigenvector is

$$\begin{aligned}
&0.1000000000D + 01 \\
&0.0000000000D + 00 \\
&0.0000000000D + 00 \\
&0.1000000000D + 01 \\
&0.0000000000D + 00 \\
&0.0000000000D + 00 \\
&0.1000000000D + 01 \\
&0.0000000000D + 00 \\
&0.0000000000D + 00 \\
&\Lambda(4) = 12477499.9590 \\
&\Omega(4) = 3532.3505 \text{ rad/sec.}
\end{aligned}$$

The associated eigenvector is

$$\begin{aligned}
&0.0000000000D + 00 \\
&0.1000000000D + 01 \\
&-0.4125765357D - 01 \\
&0.0000000000D + 00 \\
&-0.6561862202D + 00 \\
&-0.1373447533D - 16 \\
&0.0000000000D + 00 \\
&0.1000000000D + 01 \\
&0.4125765357D - 01 \\
&\Lambda(5) = 49021276.5957 \\
&\Omega(5) = 7001.5196 \text{ rad/sec.}
\end{aligned}$$

The associated eigenvector is

$$\begin{aligned}
&0.1000000000D + 01 \\
&0.0000000000D + 00 \\
&0.0000000000D + 00 \\
&0.8376574057D - 16 \\
&0.0000000000D + 00 \\
&0.0000000000D + 00 \\
&-0.1000000000D + 01
\end{aligned}$$

$$\begin{aligned}
&0.0000000000D + 00 \\
&0.0000000000D + 00 \\
&\Lambda(6) = 58630070.5158 \\
&\Omega(6) = 7657.0275
\end{aligned}$$

The associated eigenvector is

$$\begin{aligned}
&0.0000000000D + 00 \\
&0.1000000000D + 01 \\
&-0.8951411454D - 01 \\
&0.0000000000D + 00 \\
&0.1259126794D - 15 \\
&0.7572882821D - 01 \\
&0.0000000000D + 00 \\
&-0.1000000000D + 01 \\
&-0.8951411454D - 01 \\
&\Lambda(7) = 196085106.3830 \\
&\Omega(7) = 14003.0392 \text{ rad/sec.}
\end{aligned}$$

The associated eigenvector is

$$\begin{aligned}
&0.1000000000D + 01 \\
&0.0000000000D + 00 \\
&0.0000000000D + 00 \\
&-0.1000000000D + 01 \\
&0.0000000000D + 00 \\
&0.0000000000D + 00 \\
&0.1000000000D + 01 \\
&0.0000000000D + 00 \\
&0.0000000000D + 00 \\
&\Lambda(8) = 3126968.0165 \\
&\Omega(8) = 17683.2397 \text{ rad/sec.}
\end{aligned}$$

The associated eigenvector is

$$\begin{aligned}
&0.0000000000D + 00 \\
&0.1000000000D + 01 \\
&-0.1694400209D + 00 \\
&0.0000000000D + 00 \\
&0.4120001744D + 00 \\
&0.3280451486D - 16 \\
&0.0000000000D + 00 \\
&0.1000000000D + 01 \\
&0.1694400209D + 00 \\
&\Lambda(9) = 733880567.5204 \\
&\Omega(9) = 27090.2301 \text{ rad/sec.}
\end{aligned}$$

The associated eigenvector is

0.0000000000D + 00
 0.1000000000D + 01
 -0.2134858855D + 00
 0.0000000000D + 00
 0.6206811895D - 15
 -0.1102288283D + 00
 0.0000000000D + 00
 -0.1000000000D + 01
 -0.2134858855D + 00.

Let $[\phi]$ be the matrix of mode shapes (eigenvectors). Then $[\phi]^T[K][\phi]$ should yield a diagonalized stiffness matrix if all of the rigid body modes were present. Performing that matrix multiplication yields

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.08 & 0 & 0 & 0 & 0.34 & 0 & 0 \\ 0 & 1.71 & 1 & -0.55 & 0 & -1.97 & 0 & 0.33 & 0.84 & 0 \\ 0 & 1 & -1.99 & 1.19E7 & 0 & -3 & 0 & -39 & -1 & 0 \\ 0 & 0 & 0 & 0 & 5.76E7 & 0 & -2 & 0 & 0 & 0 \\ 0 & -1 & -1.91 & -2 & 0 & 4.91E7 & 0 & -1 & -81 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 2.31E8 & 0 & 0 & 0 \\ 0 & 0 & 0.56 & -39 & 0 & 1 & 0 & 1.33E8 & -1 & 0 \\ 0 & 0 & 1.65 & -1 & 0 & -81 & 0 & -2 & 3.32E8 & 0 \end{bmatrix}$$

It should be noted that minor errors still occur during the computations (initial data errors, roundoff errors, truncation errors, relative errors. etc.). The examination of these errors is beyond the scope of this dissertation. It can be readily seen, however, that the largest of these error has been reduced an order of magnitude (from ± 909 to 81).

Neglecting the relatively small terms due to arithmetic errors, the following diagonal matrix is obtained

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.19E7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.76E7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.91E7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.31E8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.33E8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.32E8 & 0 \end{bmatrix}$$

Most importantly, the large erroneous term in the 3,3 position of the matrix obtained using the conventional finite element formulation is now identically zero, and the matrix has been properly diagonalized. Thus, adding $[K]^{DFC}$, as developed in this dissertation, corrected the lack of rigid body rotation capability of the pre-loaded beam element, as well as provided the correct diagonalized stiffness matrix in the diagonalization/partitioning methodology used in finite element dynamic analysis.

SUMMARY

Based upon this investigation, the following conclusions have been developed:

1. Grounding is due to the development of pseudo-forces at the element level required to counteract a force-imbalance inherent in the development. This

causes a lack of rigid body rotational capability of the geometric stiffness matrix.

2. The directed force (i.e., bow-string) problem was examined, since the force unbalance inherent in the other developments does not occur in this situation.

3. By considering the directed force problem at the global level, using traditional development of $[K_g]$

from the horizontal component of the directed force, and Argyris' load correction method for the vertical component, a load correction matrix $[K^{DFC}]$ was developed, which, when combined with $[K_g]$, provided a complete set of rigid body modes. This combined matrix performs properly in the diagonalization/partitioning methodology used in dynamic response.

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