The Limits of Social Choice Theory:
A Defense of the Voting Rights Act

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This Article presents a defense to the challenge that social choice theory presents to voting rights. Arrow's theorem, the crown jewel of social choice theory, holds that no voting procedure that meets some minimal conditions of democratic fairness can ensure transitive, meaningful outcomes. The theorem provides a powerful argument against the ability of any court to devise objective vote dilution standards. Because such standards are now a necessary element of claims under section 2 of the Voting Rights Act, Arrow's theorem may be viewed as a fundamental threat to the viability of all such claims. The defense of voting rights presented in this Article does not question the merits of the theorem (a difficult task indeed), but instead uses the theorem, some recent (and not-so-recent) work in social choice theory, and existing voting rights law to answer the fundamental challenge that Arrow's theorem poses to voting rights jurisprudence.

I. INTRODUCTION

At the end of next year, the Census Bureau will begin releasing the results of its decennial census of the United States. The new information will, no doubt, reveal many interesting things about the state of the union. More importantly, and more closely related to its central purpose, the census will provide the set of population data that will allow our legislatures to redistribute political power by creating new districts and redefining old ones. While the redistricting process will produce new political winners, it will also generate political losers, some of whom, such as racial and ethnic minorities, may need to be protected against that traditional status through legal action. The census and the resulting redistricting process will therefore set off a new round of voting rights litigation which, if the last ten years is any guide, will last well into the next decade.

The principal vehicle for bringing such districting claims is section 2 of the Voting Rights Act.[1] Section 2 allows individuals to bring claims for vote dilution, which occurs, in the words of the Act, when members of racial minorities "have less opportunity than other members of the electorate to participate in the political process and to elect representatives of their choice."[2] The guidelines for bringing such claims are still somewhat unsettled: the statutory language is opaque and the Supreme Court continues to struggle with basic questions that range from standing to appropriate remedies.[3] The Court has, however, made one point exceedingly clear: those asserting vote dilution claims must propose a standard by which to measure vote dilution.[4] In other words, voting rights plaintiffs must demonstrate the

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    1.   42 U.S.C. § 1973 (1994).
    2.   *Id.* § 1973(b).
    3.   For a recent Supreme Court case struggling with both standing and remedies, see *Shaw v. Hunt,* 517 U.S. 899, 904-18 (1996). For a recent case dealing with remedies, see *Abrams v. Johnson,* 521 U.S. 74, 79-90, 101 (1997).
    4.   *See, e.g.,* Reno v. Bossier Parish Sch. Bd., 520 U.S. 471, 480 (1997); Holder v. Hall, 512 U.S. 874, 880-81 (1994) (plurality opinion). For a more complete discussion, see *infra* notes 53-58 and accompanying text.

existence of an undiluted practice against which the fact of vote dilution may be measured.[5]

While the process of devising vote dilution standards is fraught with difficulties, a more serious challenge to the ability to devise vote dilution standards has arisen from the realm of social choice theory.[6] This challenge does not merely assert that one or another potential vote dilution standard is difficult to devise, but instead insists that *no* objective vote dilution standard may be devised, that vote dilution standards are theoretically doomed from the outset.[7] The source of the theoretical obstacle is none other than the crown jewel of social choice theory—Arrow's theorem.[8] And the conclusion drawn is that without an objective method of devising vote dilution standards, such claims should not be actionable under the Voting Rights Act.[9]

For those who believe that minority vote dilution remains a matter of serious concern, finding a solution to the challenge posed by Arrow's theorem is critical. The challenge goes to the very core of the concept of vote dilution and questions whether courts can ever recognize or remedy such dilution. It may also interject an additional degree of uncertainty into an area of law that the Supreme Court already views with a great deal of trepidation. In an earlier piece, I surveyed this problem and proposed a preliminary solution that called for the recognition of vote dilution claims only within districts that exhibited something loosely referred to as "spectrum agreement."[10] That solution, however, is far from complete: The only type of spectrum agreement that *guarantees* meaningful vote dilution standards almost never exists, even in districts that can demonstrate legally cognizable vote dilution claims. The purpose of this Article is to explain these problems and construct a more complete defense to this challenge posed by social choice theory.

The Article is divided into four principal Parts. Part II provides an overview of modern voting rights jurisprudence with special attention to the crucial role of vote dilution standards. Part III

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5.   A plaintiff may propose, for example, a proportional standard under which the number of minority-majority districts in a particular state is roughly proportional to the minority voting-age population in the state. But that type of proportional representation is clearly not guaranteed by the Voting Rights Act of 1965, 42 U.S.C. § 1973(b) (1994).

6.   *See* Grant M. Hayden, Comment, *Some Implications of Arrow's Theorem for Voting Rights*, 47 STAN. L. REV. 295, 295-96 (1995).

7.   *See id.* at 309.

8.   *See* KENNETH J. ARROW, SOCIAL CHOICE AND INDIVIDUAL VALUES 51-59 (2d ed. 1963). For a more thorough discussion of the theorem, see *infra* notes 64-81 and accompanying text.

9.   *See* Hayden, *supra* note 6, at 308-09.

10.   *See id.*

examines Arrow's theorem, its theoretical import, and its implications for voting rights. Part IV surveys one possible solution, spectrum agreement, and describes some of the shortcomings of the solution. Part V, comprising the bulk of the Article, explores recent advances in the field of social choice theory in an attempt to resolve those shortcomings and proposes a more complete solution to the challenge that social choice theory poses to voting rights.

II.   VOTE DILUTION JURISPRUDENCE

The modern battle for minority political participation and representation has taken place on two fronts with varying degrees of success. Initially, minority voters faced a battery of obstacles designed to keep them out of the voting booth altogether. Various screening devices, such as poll taxes and literacy tests, were administered in ways that effectively disenfranchised minorities, especially in the southern states.[11] The Voting Rights Act of 1965 was, by all accounts, very successful in eliminating those hurdles and thus securing the rights of minorities to register and vote.[12]

Even with individual access to the ballot secured, however, effective minority representation is often thwarted by district lines that are either drawn or, in many cases, preserved in the face of changing demographics, in ways that effectively "dilute" minority voting power. This vote dilution comes in two basic forms—quantitative and qualitative dilution.[13] Quantitative vote dilution occurs when votes are given unequal weight in determining the outcome, and thus the power of some votes is numerically diluted.[14] Qualitative vote dilution, on the other hand, occurs when a voter has less of an opportunity to elect a representative of her choice despite the fact that her vote is weighed

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11.   *See, e.g.*, BERNARD GROFMAN ET AL., MINORITY REPRESENTATION AND THE QUEST FOR VOTING EQUALITY 8-10 (1992); J. MORGAN KOUSSER, THE SHAPING OF SOUTHERN POLITICS: SUFFRAGE RESTRICTION AND THE ESTABLISHMENT OF THE ONE-PARTY SOUTH, 1880-1910 (1974); U.S. COMM'N ON CIVIL RIGHTS, POLITICAL PARTICIPATION: A STUDY OF THE PARTICIPATION BY NEGROES IN THE ELECTORAL AND POLITICAL PROCESSES IN 10 SOUTHERN STATES SINCE PASSAGE OF THE VOTING RIGHTS ACT OF 1965, at 19-132 (1968); C. VANN WOODWARD, ORIGINS OF THE NEW SOUTH, 1877-1913, at 342-49 (1951).

12.   Before the passage of the Voting Rights Act, the percentage of voting-age blacks registered in the southern states targeted by the Act was only 29.3% (compared with 73.4% for voting-age whites); less than two years after the passage of the Act, that number had risen to 52.1%. *See* GROFMAN ET AL., *supra* note 11, at 23 tbl.1; *see also* STEVEN F. LAWSON, BLACK BALLOTS: VOTING RIGHTS IN THE SOUTH, 1944-1969, at 330-39 (1976) (detailing the increase in black voting following the passage of the Voting Rights Act).

13.   *See* Pamela S. Karlan, *Maps and Misreadings: The Role of Geographic Compactness in Racial Vote Dilution Litigation*, 24 HARV. C.R.-C.L. L. REV. 173, 176 (1989).

14.   *See id.*

equally with all other votes cast.[15] While the principal focus of this
Article is qualitative, not quantitative, vote dilution, the latter describes
a less complex concept, and historical attempts to identify and remedy
such dilution provide an instructive contrast to the continuing struggle
over qualitative dilution.

A. Quantitative Vote Dilution

Quantitative vote dilution occurs when a vote is numerically
diluted, which happens whenever districts are drawn in ways that
deviate from an equiproportional or "one person, one vote" standard.[16]
Take, for example, a three-member governing body to be elected from
three single-member districts (Districts A, B, and C) in a county of
30,000 people. If the district lines are drawn such that District A has a
population of 5,000, District B 10,000, and District C 15,000, then
voters in District A have an obvious advantage over their neighbors in
influencing the outcome.[17] Indeed, they have double the voting power
of those in District B and triple the voting power of those in District
C.[18] Simply put, given the population disparities, a vote in District A
counts more than a vote in District B or District C.

An effective solution to quantitative vote dilution requires the
identification of a standard by which it can be measured and remedied.
The equiproportional standard for a single-member district may be
found very easily: one merely divides the total population by the
number of districts to reach the ideal number of people per district.[19]

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15.   *See id.*
16.   The phrase "one person, one vote" appears to have its origins in Justice William
Douglas's majority opinion in *Gray v. Sanders*, 372 U.S. 368, 381 (1963), where he noted:
"The conception of political equality from the Declaration of Independence, to Lincoln's
Gettysburg Address, to the Fifteenth, Seventeenth, and Nineteenth Amendments can mean
only one thing—one person, one vote."
17.   *See generally* Reynolds v. Sims, 377 U.S. 533, 562-64 (1964) (providing a
general discussion of district size and relative voting power).
18.   All voters in this example, of course, have the same number of votes—one each.
In a representative democracy, however, voting power must also be measured in a manner
that accounts for its republican form. That is, in order to measure an individual's voting
power, one must look at the relative indirect impact each voter has upon a particular piece of
legislation. The relative voting power of people in different-sized districts is most easily
illustrated at the extremes. If the members of a three-member governing body were elected
from three districts, and the populations of those districts were one person, one person, and
ten million people, the political advantage to living in one of the two less populated districts
is obvious. The same is true, in a less obvious manner but nonetheless significant degree, in
the example in the text.
19.   While the standard may be found relatively easily once the relevant population
data are known, there are various difficulties in arriving at the correct population figures.
Most districting is based on population counts from the decennial census which, according to
many commentators, systematically undercounts members of many minority groups. *See,*

In the example above, the ideal district size is 10,000 people per district (30,000 people divided by three districts). A comparison of the ideal district size with the district's actual size reveals the extent of any quantitative dilution, as deviations upward or downward from that ideal size result in a dilution or concentration of voting power. If this comparison reveals a significant deviation, certain adjustments, usually involving redrawing district lines, can remedy the situation.

Viewed almost three decades after *Baker v. Carr*[20] and its immediate progeny,[21] the twofold and threefold disparities in the example above are now seen as easily recognizable and manifestly unfair distributions of voting power. For many years, however, such disparities were relatively commonplace on the state and national political landscape. The districts at issue in *Baker*, for example, gave rise to differentials in voting power of approximately 20-to-1,[22] and those in *Reynolds v. Sims* up to an astonishing 41-to-1 ratio.[23] Such disparities provided a fairly strong justification for the Supreme Court's early forays into the political thicket of apportionment.

The Court, of course, first entered that thicket in *Baker*, where it found unequal apportionment to be a justiciable constitutional claim under the Equal Protection Clause.[24] The *Baker* Court did not, however, devise a substantive standard for such a claim.[25] A short time later, the Court devised an equiproportional or "one person, one vote" standard for state legislative bodies in *Reynolds*[26] and for

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e.g., Christine B. Hickman, *The Devil and the One Drop Rule: Racial Categories, African Americans, and the U.S. Census*, 95 MICH. L. REV. 1161, 1165 (1997); Samuel Issacharoff & Allan J. Lichtman, *The Census Undercount and Minority Representation: The Constitutional Obligation of the States to Guarantee Equal Representation*, 13 REV. LITIG. 1, 2-13 (1993). Indeed, the Census Bureau announced a plan to use statistical sampling in the 2000 Decennial Census to remedy the growing problem of undercounting some identifiable groups. That plan was challenged and held invalid under the Census Act, 13 U.S.C. §§ 1-401 (1994), in *Department of Commerce v. United States House of Representatives*, 119 S. Ct. 765 (1999). While such data problems may also lead to a dilution of minority voting strength, this Article focuses on other issues.

    20.   369 U.S. 186 (1962).

    21.   *See, e.g.*, Karcher v. Daggett, 462 U.S. 725 (1983); Kirkpatrick v. Preisler, 394 U.S. 526 (1969); Lucas v. Forty-Fourth Gen. Assembly, 377 U.S. 713 (1964); Reynolds v. Sims, 377 U.S. 533 (1964); Wesberry v. Sanders, 376 U.S. 1 (1964).

    22.   *See Baker*, 369 U.S. at 245 (Douglas, J., concurring).

    23.   377 U.S. at 545.

    24.   *See Baker*, 369 U.S. at 208-10.

    25.   *See id.*

    26.   *See Reynolds*, 377 U.S. at 568 (holding that the Equal Protection Clause requires each house of a bicameral state legislature to be apportioned by population). The equiproportional standard has become so firmly entrenched in subsequent decisions that it is one of the few claims that may be maintained under the Equal Protection Clause without a showing of discriminatory intent. *See* Tucker v. Department of Commerce, 958 F.2d 1411, 1414-15 (7th Cir. 1992).

Congress in *Wesberry v. Sanders*.[27]　Since that time, the Court has tinkered with application of the equiproportional standard. It currently allows districts for state legislative bodies up to a 10% deviation[28] from the ideal district size without justification[29] and permits a greater deviation if there is some showing that the district lines promote a state objective such as the preservation of preexisting political boundaries.[30] Congressional districts, on the other hand, are increasingly held to zero deviation from the standard district size.[31]

The history of the equiproportional standard and its application in quantitative vote dilution is, of course, much more interesting and complex than the brief account presented above.[32] The most important aspect of that history for the purpose of this Article, however, is the

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27.　376 U.S. 1, 14, 17-18 (1964). The *Wesberry* Court applied Article 1, Section 2 of the Constitution to the election of a congressional delegation. *See id.* Article 1, Section 2 provides: "The House of Representatives shall be composed of Members chosen every second Year by the People of the several States, and the Electors in each State shall have the Qualifications requisite for Electors of the most numerous Branch of the State Legislature." U.S. CONST. art. I, § 2. The Court held that the prescription for election "by the People" requires that "one man's vote in a congressional election is to be worth as much as another's." *Wesberry*, 376 U.S. at 7-8. While the *Wesberry* decision was grounded on Article 1, Section 2, its holding and subsequent interpretation parallels jurisprudence under the Equal Protection Clause. *See infra* note 47.

28.　The percentage deviation is calculated by adding the percentage excess of the largest district over the ideal district size to the percentage deficit of the smallest district under the ideal district size. So, for example, if the ideal district size is 10,000 people per district, and the largest and smallest districts involved have populations of 11,500 and 8,000, the deviation is 35%.

29.　Indeed, these principles have become so firmly entrenched they now appear in standard reference materials. *See, e.g.*, 25 AM. JUR. 2D *Elections* § 25, at 820-21 (1996). The *American Jurisprudence* article provides as follows:

> By contrast, a maximum population deviation of more than 10[%] creates a prima facie case of invidious discrimination and requires justification by the state as based on legitimate considerations incident to the effectuation of a rational state policy. Population disparities greater than 10[%] can successfully be supported by evidence of a rational state policy, if it is shown that:
> (1)　there is an absence of any built-in bias in following the policy;
> (2)　the policy has been consistently followed without any taint of arbitrariness or discrimination; and
> (3)　population equality is the sole other criterion used in apportionments.

Id. § 25, at 821 (footnotes omitted).

30.　*See, e.g.*, Mahan v. Howell, 410 U.S. 315, 324-25 (upholding a Virginia state redistricting plan with a maximum percentage deviation of 16.4% on the basis of the State's interest in preserving the integrity of political subdivision boundary lines), *modified*, 411 U.S. 922 (1973).

31.　*See, e.g.*, Karcher v. Daggett, 462 U.S. 725, 731-40 (1983).

32.　*See, e.g.*, Issacharoff & Lichtman, *supra* note 19, at 19 (stating that "[t]here is no basis for concluding that mathematical fidelity to ideal population distributions according to the Census is required by the Constitution," and that, even within the one person, one vote rule, "states are given latitude to accommodate the need for representation of divergent political communities").

existence of a readily identifiable standard by which to measure such dilution. The equiproportional standard allows courts to easily recognize districting that results in quantitative vote dilution, quantify the degree of that dilution, and remedy the problem. There is, in other words, an objective, easily managed standard by which to measure and remedy quantitative vote dilution.[33]

B. *Qualitative Vote Dilution*

With the battle to increase minority access to the voting booth largely won, voting rights advocates turned to more invidious forms of discrimination that limited minority representation by effectively diluting their voting power.[34] In many cases, for example, minority representation was thwarted by racial gerrymanders in processes known as "cracking" and "packing."[35] As the Supreme Court noted, "[d]ilution of racial minority group voting strength may be caused [either] by the dispersal of blacks into districts in which they constitute an ineffective minority of voters or from the concentration of blacks into districts where they constitute an excessive majority."[36] In either case, a minority group may be denied the right to elect the representative of its choice despite the fact that its individual members cast votes that are weighed equally with those of majority voters.

The continuing problem of qualitative vote dilution gave rise to a series of judicial and legislative responses. The first wave of attack used section 5 of the Voting Rights Act.[37] Section 5 requires the

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33. Despite debates surrounding the desirability of the equiproportional standard, and the application of that standard, most agree that one of its principal benefits is its manageability. *See, e.g.*, Issacharoff & Lichtman, *supra* note 19, at 2. Indeed, one commentator noted that the equiproportional standard "is certainly administratable. In fact administrability is its long suit, and the more troublesome question is what else it has to recommend it." JOHN HART ELY, DEMOCRACY AND DISTRUST: A THEORY OF JUDICIAL REVIEW 121 (1980).

34. *See, e.g.*, GROFMAN ET AL., *supra* note 11, at 23-24 (describing more subtle schemes, such as at-large elections, anti-single-shot laws, reductions in the size of legislative bodies, racial gerrymandering, and exclusive slating for reducing minority voter participation).

35. *See* Frank R. Parker, *Racial Gerrymandering and Legislative Reapportionment*, *in* MINORITY VOTE DILUTION 89, 96 (Chandler Davidson ed., 1984). In the former practice, a politically cohesive minority group that is large enough to constitute a majority in a single-member district is "cracked," or divided among various districts such that it is a majority in none of those districts and prevented from electing a representative of its choice. *See id.* In the latter, a minority group with sufficient numbers to constitute a majority in three districts may be "packed" into two districts such that, while they make up a supermajority in those two districts, they are able to elect only two rather than three representatives of choice. *See id.*

36. Thornburg v. Gingles, 478 U.S. 30, 46 n.11 (1986).

37. 42 U.S.C. § 1973c (1994).

Attorney General or the United States District Court for the District of Columbia to "preclear" any proposed changes to "any voting qualification or prerequisite to voting, or standard, practice, or procedure with respect to voting" in certain jurisdictions.[38] In *Allen v. State Board of Elections*, the Court applied this section to changes that qualitatively diluted voting power as well as those that disenfranchised minority voters, and thus provided a potent weapon to attack the more subtle forms of discrimination.[39] The use of section 5 for such purposes, however, is limited in two respects. First, it does not apply to the entire United States, but instead to a limited number of targeted jurisdictions with a history of discrimination.[40] Second, and more importantly, section 5 only applies to *changes* in districting practices, and thus dilutive practices that existed prior to 1964, or before a jurisdiction's inclusion under section 5 coverage, are not subject to challenge.[41] These limitations meant that voting rights advocates often had to look elsewhere for a means of challenging districting that qualitatively diluted minority voting strength.

The Constitution provided the second avenue of attacking practices that qualitatively diluted minority voting power. Beginning

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38.   *Id.*

39.   393 U.S. 544, 569 (1969). The *Allen* Court stated:

The right to vote can be affected by a dilution of voting power as well as by an absolute prohibition on casting a ballot. Voters who are members of a racial minority might well be in the majority in one district, but in a decided minority in the county as a whole. [Switching from district to at-large elections] could therefore nullify their ability to elect the candidate of their choice just as would prohibiting some of them from voting.

Id. (citation omitted).

40.   Section 4(b) of the Voting Rights Act describes the covered jurisdictions as those meeting the following criteria: (1) the jurisdiction maintained a test or device as a precondition for registering or voting as of November 1, 1964, and (2) less than 50% of the voting-age population was registered to vote on November 1, 1964, or less than 50% of the voting-age population voted in the November 1964 presidential election.   42 U.S.C. §§ 1973b(b), 1973c (1994). The test was designed to target Southern states with a history of discrimination, and initially covered Alabama, Georgia, Louisiana, Mississippi, South Carolina, Virginia, and parts of North Carolina.   *See* Amy Snyder Weed, Note, *Getting Around the Voting Rights Act: The Supreme Court Sets the Limits of Racial Voting Discrimination in the South*, 10 B.C. THIRD WORLD L.J. 381, 381 (1990) (stating that "[s]ection 5 of the Act ... was intended to 'eradicat[e] the continuing effects of past discrimination' in the jurisdictions covered by the Act, and 'insure that old devices for disenfranchisement would not simply be replaced by new ones'" (quoting City of Lockhart v. United States, 460 U.S. 125, 141 (1983) (Marshall, J., dissenting))).

41.   *See* 42 U.S.C. § 1973c (1994); *see also* Beer v. United States, 425 U.S. 130, 138 (1976) (holding that "[t]he language of § 5 clearly provides that it applies only to proposed changes in voting procedures"). The at-large seats in *Beer* had been part of the city's electoral system since 1954 and were therefore not subject to review under section 5 because the plan was a continuation of an already existing practice and not a new practice. *See id.* at 138-39.

with *Reynolds v. Sims*, the Supreme Court generally recognized that vote dilution is an actionable wrong under the Equal Protection Clause.[42]   Though that case focused primarily upon quantitative dilution, the series of cases that followed, most of which focused upon the dilutive effect of multi-member districts, made clear that qualitative dilution claims are also actionable under the Constitution.[43] Dilution claims came to be routinely analyzed under a combination of considerations set forth by the Supreme Court in *White v. Regester*[44] and the Fifth Circuit in *Zimmer v. McKeithen*,[45] collectively known as the *White/Zimmer* factors. This changed, however, in 1981, when the Supreme Court decided *City of Mobile v. Bolden*.[46]

In a highly fractured opinion, the *Bolden* Court held that a party alleging qualitative vote dilution under the Fourteenth Amendment must demonstrate that the questioned practice or procedure was established or maintained with discriminatory intent.[47] The Court also applied the intent requirement to suits under the Fifteenth Amendment and, since section 2 of the Voting Rights Act was said to add nothing to the constitutional cause of action, to suits under section 2.[48] This requirement of proving discriminatory intent effectively put a halt to qualitative vote dilution claims.

The *Bolden* decision set off a storm of protest that culminated in passage of an amendment to section 2 of the Voting Rights Act.[49] The

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42.   377 U.S. 533, 565-66 (1964).

43.   *See, e.g.*, White v. Regester, 412 U.S. 755, 765-70 (1973) (holding that despite nearly equal population among the voting districts, the multi-member districts excluded minority participation and thus violated the Equal Protection Clause); *see also Allen*, 393 U.S. at 564-66 (holding that the Voting Rights Act should not only ensure that each citizen may cast a vote, but also that the state may pass no law, however minor, that affects that right).

44.   412 U.S. at 765-67.

45.   485 F.2d 1297, 1305-06 (5th Cir. 1973), *aff'd sub nom.* East Carroll Parish Sch. Bd. v. Marshall, 424 U.S. 636 (1976).

46.   446 U.S. 55, 65-68 (1980) (plurality opinion).

47.   *See id.* at 90 (Stevens, J., concurring). This intent requirement for voting rights cases came on the heels of the Court's similar requirement for actions brought under the Equal Protection Clause of the Fourteenth Amendment and the equal protection component of the Fifth Amendment as announced in *Washington v. Davis*, 426 U.S. 229, 238-42 (1976).

48.   *See Bolden*, 446 U.S. at 60-62 (plurality opinion) (stating that section 2 of the Voting Rights Act only elaborated on the language of the Fifteenth Amendment, thereby incorporating the requirement of discriminatory intent).

49.   Voting Rights Act Amendments of 1982, Pub. L. No. 97-205, 96 Stat. 131 (codified as amended at 42 U.S.C. § 1973aa-6 (1994)). A good history of the political and social events surrounding the 1982 amendments may be found in Peyton McCrary, *The Significance of the* City of Mobile v. Bolden, *in* MINORITY VOTE DILUTION, *supra* note 35, at 47-63.   A thorough description and analysis of the legislative history of the 1982 amendments are set out in Thomas M. Boyd & Stephen J. Markman, *The 1982 Amendments to the Voting Rights Act: A Legislative History*, 40 WASH. & LEE L. REV. 1347 (1983).

amendment effectively decoupled section 2 claims from constitutional claims of vote dilution, and specifically did not require proof of discriminatory intent as a prerequisite for a section 2 claim.[50] In *Thornburgh v. Gingles*, the Supreme Court affirmed the constitutionality of the statute, and set out a three-part test for vote dilution claims.[51] Section 2 has since become the weapon of choice in voting rights litigation.

While the concept of qualitative vote dilution now pervades section 2 voting rights litigation, the Supreme Court has yet to articulate an accepted definition of it. At a minimum, however, any such definition must involve a standard against which to measure dilution. In order to ascertain, in the words of the Voting Rights Act, when minorities "have less opportunity than other members of the electorate to participate in the political process and to elect representatives of their choice,"[52] one must first determine what the outcome should look like in the absence of dilution.

Although the Court has failed to provide anything but an operational definition of qualitative vote dilution,[53] it has recently made the need for such an underlying standard abundantly clear. The Court explained in 1994 that "[i]n a § 2 vote dilution suit, along with determining whether the *Gingles* preconditions are met and whether the totality of the circumstances supports a finding of liability, a court must find a reasonable alternative practice as a benchmark against which to measure the existing voting practice."[54] As the Court further

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50.   *See* 42 U.S.C. §§ 1971 to 1973bb-1 (1994).

51.   478 U.S. 30, 50-51 (1986). The three-part test for vote dilution requires the plaintiff to demonstrate (1) the minority group is sufficiently large and geographically compact to constitute a majority in a single-member district, (2) the minority group is politically cohesive, and (3) the majority votes sufficiently as a bloc to enable it to usually defeat the minority's preferred candidate. *See id.* The test is discussed in greater detail *infra* Parts IV.A and VI.

52.   42 U.S.C. § 1973(b) (1994).

53.   A complete definition of qualitative vote dilution would include identification of the standard against which dilution is measured. One commentator stated that the search for a definition of vote dilution is equivalent to the search for "the ideal against which vote dilution is identified and measured[.]" Larry Alexander, *Lost in the Political Thicket*, 41 FLA. L. REV. 563, 567 (1989). The Court's use of the three *Gingles* factors, while providing some indicia of qualitative dilution, do not actually define such dilution. Indeed, most of the recent controversies in this area revolve around the failure of the courts to come up with an adequate standard to measure dilution. *See, e.g.*, Abrams v. Johnson, 521 U.S. 74, 79-85 (1997) (recounting the extended history of the attempt by the Georgia legislature, the Justice Department, and federal courts to decide whether Georgia should have one, two, or three majority-minority districts).

54.   Holder v. Hall, 512 U.S. 874, 880 (1994) (plurality opinion) (footnote omitted). The *Holder* Court correctly cited Justice O'Connor's concurrence in *Gingles* for this proposition. *See id.* (plurality opinion). In *Gingles*, she noted, "[I]n order to decide whether

explained in 1997, "[b]ecause the very concept of vote dilution implies—and, indeed, necessitates—the existence of an 'undiluted' practice against which the fact of dilution may be measured, a § 2 plaintiff must also postulate a reasonable alternative voting practice to serve as the benchmark 'undiluted' voting practice."[55] The existence of such a benchmark is now viewed as a necessary component of section 2 dilution claims: "[W]here there is no objective and workable standard for choosing a reasonable benchmark by which to evaluate a challenged voting practice, it follows that the voting practice cannot be challenged as dilutive under § 2."[56]

In sum, like quantitative vote dilution, qualitative vote dilution requires, both conceptually and legally, the existence of some objective standard against which to measure dilution. It is conceptually required because, as Justice Scalia once noted in oral argument, "You don't know what watered beer is unless you know what beer is, right?"[57] It is legally required because the Supreme Court has made clear in the last few years that proof of such a standard is a necessary element of a section 2 case, and such cases are the principal vehicle for prosecuting qualitative vote dilution claims.[58] The future viability of qualitative vote dilution claims, then, turns on the existence and identification of these objective standards.

III. SOCIAL CHOICE THEORY'S CHALLENGE TO VOTE DILUTION JURISPRUDENCE

Given the pivotal importance of objective standards to the enterprise of identifying and remedying qualitative vote dilution, it should come as no surprise that the critics of section 2 focus upon the

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an electoral system has made it harder for minority voters to elect the candidates they prefer, a court must have an idea in mind of how hard it 'should' be for minority voters to elect their preferred candidates under an acceptable system." *Gingles*, 478 U.S. at 88 (O'Connor, J., concurring).

55.   Reno v. Bossier Parish Sch. Bd., 520 U.S. 471, 480 (1997). The *Bossier Parish* Court contrasted section 2 claims, which require an ideal, undiluted plan, with section 5 claims, which compare the challenged change in voting practice or procedure to the existing plan for signs of retrogression. *See id.*

56.   *Holder*, 512 U.S. at 881 (plurality opinion).

57.   Transcript of Oral Argument at 8, Chisom v. Roemer, 501 U.S. 380 (1991) (Nos. 90-757 & 90-1032).

58.   Commentators have proposed various standards by which to measure qualitative vote dilution. *See, e.g.*, Hayden, *supra* note 6, at 308. In the field of minority vote dilution, for example, some have proposed a standard of proportional representation wherein minorities constitute the same proportion of members in a legislative body as they do in the general population. *See id.* Under the Voting Rights Act, evidence of disproportionate representation, while not dispositive, may help substantiate a vote dilution claim.

difficulties in devising such standards.[59]   And, to be fair, those difficulties are by no measure inconsequential. Some arise out of a lack of complete information about voter preferences. For example, the use of secret ballots, the fact that polling data is usually only available for national elections, and the changing nature of voter preferences make it quite difficult to get a fix on those preferences in any particular jurisdiction.[60] Other difficulties arise when attempting to translate voter preferences into an ideal, standard outcome. When a hypothetical ten-district state has a 25% minority population and pure racial bloc voting, is it better to create two majority-minority districts or three majority-minority districts, or does the answer depend on the geographic distribution of the minority population?[61] These are the types of questions that courts have been grappling with since the inception of qualitative vote dilution jurisprudence, and their difficulty may help explain, at least in part, the Supreme Court's trepidation in the area.

While these difficulties continue to plague the courts, in the last few years a more serious challenge to the concept of qualitative vote dilution has been presented from the realm of social choice theory.[62] That criticism argues that Arrow's theorem renders the search for qualitative vote dilution standards not just difficult, but impossible—theoretically doomed from the outset.[63] Such an argument, if true, would effectively preempt the ongoing debate about the search for judicially manageable standards and, perhaps, explain some of the difficulties in developing such standards. That argument is the focus of this Part of the Article.

A.   *Arrow's Impossibility Theorem*

Arrow's impossibility theorem is the centerpiece of a broader enterprise known as social choice theory. Social choice theory seeks to describe, in some rigorous way, the translation of individual desires

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59.   *See* Larry Alexander, *Still Lost in the Political Thicket (or Why I Don't Understand the Concept of Vote Dilution)*, 50 VAND. L. REV. 327, 334 (1997).

60.   *See generally* GROFMAN ET AL., *supra* note 11, at 82-88 (explaining the difficulty in recognizing polarized voting).

61.   *See, e.g.,* Abrams v. Johnson, 521 U.S. 74, 77-90 (1997) (discussing the legislative, executive, and judicial deliberations over whether one, two, or three of Georgia's 11 congressional districts should be majority black districts given that blacks constitute 27% of Georgia's voting-age population).

62.   *See* Alexander, *supra* note 59, at 334.

63.   *See id.*

into group choices.[64]   More precisely, it examines social choice functions, the mechanisms by which we move from individual preference orders to social preference orders.[65] The ideal social choice function successfully amalgamates individual preference orders into social preference orders, translating individual desires into group choices. Democratic institutions have historically adopted some type of voting procedure to handle this task.[66] The adequacy of all social choice functions was called into question in 1951, however, with the publication of Arrow's theorem.[67]

Arrow demonstrated that no social choice function can simultaneously satisfy certain conditions of fairness and logic.[68] The theorem involves four fairness conditions (nondictatorship, Pareto efficiency, universal admissibility, and independence from irrelevant alternatives) and one logical condition (transitivity).[69] The conditions that a voting procedure must satisfy in order to appease Arrow's sense of democratic fairness and logic are fairly minimal.[70]

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64.   Social choice theory's entry into the legal literature has taken place largely under the guise of public choice theory. For two excellent summaries of the literature, see MAXWELL L. STEARNS, PUBLIC CHOICE AND PUBLIC LAW: READINGS AND COMMENTARY (1997), and DANIEL A. FARBER & PHILIP P. FRICKEY, LAW AND PUBLIC CHOICE: A CRITICAL INTRODUCTION (1991). Most of that literature, however, has focused more narrowly upon the decision making of relatively small groups—legislatures and courts—as opposed to that of the electorate at large.

65.   The following definitions, largely derived from WILLIAM H. RIKER, LIBERALISM AGAINST POPULISM: A CONFRONTATION BETWEEN THE THEORY OF DEMOCRACY AND THE THEORY OF SOCIAL CHOICE 18, 296-97 (1982), may prove useful to the reader unfamiliar with social choice theory. An "individual preference order" is a complete arrangement of alternatives in order of their desirability to an individual. *See id.* at 18, 296. The relationship between any two alternatives is either one of preference (P) or indifference (I). *See id.* Thus, if Chris's preference order is $xPyPzIw$, then Chris prefers x to y, prefers y to z, and is indifferent between z and w. *See id.* A "preference profile" is a "set of individual preference orders, one for each member of society." *Id.* at 296. A "social preference order" is a complete arrangement of alternatives in order of their attractiveness to society as a whole. *See id.* Finally, a "social choice function" is a rule that translates a preference profile into a social preference order. *See id.* at 18, 297.

66.   *See* NORMAN FROHLICH & JOE A. OPPENHEIMER, MODERN POLITICAL ECONOMY 16 (1978).

67.   *See* ARROW, *supra* note 8, at 51-58.

68.   *See id.* For a more concise outline of the proof, see PETER C. ORDESHOOK, GAME THEORY AND POLITICAL THEORY: AN INTRODUCTION 62-65 (1986).

69.   *See* RIKER, *supra* note 65, at 116-19. Riker actually refers to six fairness conditions. However, his additional criteria (monotonicity and citizens' sovereignty) comprise variations of one of the four conditions. *See id.* Subsuming these additional conditions into the primary four simplifies the analysis. *See id.*

70.   For a more complete discussion of these conditions, see Hayden, *supra* note 6, at 297-99.

The first condition, nondictatorship, stipulates that no single person's preferences dictate the social preference order.[71] One voter's desires, then, cannot determine the outcome of an election regardless of what others in society prefer. This condition comports with the basic democratic intuition that society, not a dictator, should make policy decisions.

The second condition, Pareto efficiency, is even weaker.[72] It ensures that if everyone in society prefers one alternative to another, then the outcome of the social choice function must reproduce that ordering.[73] If, for example, every single voter prefers candidate Gore to candidate Bush, the condition of Pareto efficiency condemns a voting procedure that declares Bush the winner. Like nondictatorship, Pareto efficiency comes out of basic democratic intuitions.

The third condition, universal admissibility, demands that a social choice function must be able to describe a social preference order for any possible preference profile.[74] In order to comply with this condition, a voting procedure must work with every permutation of voter preferences over a set of alternatives.[75] The converse of this condition, restricting individual preference orders, runs counter to a fundamental democratic principle: People should not be declared ineligible to vote because of their opinions.

The fourth condition, independence from irrelevant alternatives, requires that the introduction of new, "irrelevant" alternatives in a preference profile does not affect the relative orderings of the other alternatives.[76] The term "irrelevant" is not pejorative; it simply refers to an alternative outside the set of those from which a group must choose that does not substantively alter the desirability of the other alternatives relative to each other.

The final, logical condition of transitivity guarantees that a social choice function will produce a complete and transitive social preference order; that is, one in which if x is preferred to y, and y to z, then x is also preferred to z.[77] The alternative—an intransitive order in which x is preferred to y, y to z, and z to x—is referred to as a voting

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71.   *See id.* at 297.
72.   *See id.*
73.   *See id.*
74.   *See id.* at 298.
75.   That is, the voting procedure must work for both any individual ordering and any preference profile (combination of orderings). When choosing between x, y, and z, the procedure must work for any combination of the six possible individual orderings (excluding indifference): $xPyPz$, $xPzPy$, $yPxPz$, $yPzPx$, $zPxPy$, and $zPyPx$.
76.   *See id.*
77.   *See id.* at 299.

cycle.[78] The presence of such a cycle signals the inability of the voting procedure to declare a winner. This inability is a fundamental flaw given that the entire purpose of the procedure is to select an alternative from the range of alternatives presented to the voters as the social choice.

None of these conditions appears to impose outrageous demands on social choice functions. Even taken together, they do not seem to place a heavy burden on democratic voting procedures. According to Arrow's theorem, however, simultaneous fulfillment of all the conditions is impossible.[79] Simply put, no possible voting procedure can generate a result consistent with Arrow's five conditions. In an earlier piece, I surveyed four social choice procedures—the Condorcet method, amendment procedure, Borda count, and cumulative voting—to illustrate the inevitability of Arrovian problems.[80] So long as society preserves democratic institutions that embody the four fairness conditions, those institutions will inevitably produce some intransitive social preference orders. Some choices, in other words, will be always unordered and, hence, meaningless.[81]

B.   *The Legal Implications of Arrow's Theorem*

While Arrow's theorem has some obvious and profound implications for democratic theory, it also has some less obvious implications for our legal system. The impact of the theorem on some legal institutions has been well explored. Specifically, many commentators have examined our lawmaking institutions for structural devices that diminish the possibility of intransitive, cyclical outcomes. Some, for example, have explored legislative bodies for mechanisms that decrease the possibility of cycling.[82] Others have assessed judicial decision-making through the lens of social choice.[83] Strangely enough, however, little work has been done in the field of voting rights—a field of study that would naturally lend itself to examination

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78.   *See id.* at 306 n.57.

79.   *See* ARROW, *supra* note 8, at 51-60.

80.   *See* Hayden, *supra* note 6, at 299-304.

81.   *See* RIKER, *supra* note 65, at 136.

82.   *See, e.g.*, Saul Levmore, *Parliamentary Law, Majority Decisionmaking, and the Voting Paradox*, 75 VA. L. REV. 971, 997-1011 (1989); Abner J. Mikva, *Symposium on the Theory of Public Choice—Foreword*, 74 VA. L. REV. 167, 176-77 (1988). For an excellent overview of the literature, see William H. Panning, *Formal Models of Legislative Processes*, *in* HANDBOOK OF LEGISLATIVE RESEARCH 676-78, 681-82 (Gerhard Loewenberg et al. eds., 1985).

83.   *See, e.g.*, Lewis A. Kornhauser & Lawrence G. Sager, *Unpacking the Court*, 96 YALE L.J. 82, 97-117 (1986); Frank H. Easterbrook, *Ways of Criticizing the Court*, 95 HARV. L. REV. 802, 815-32 (1982).

from a social choice perspective. Indeed, the dearth of commentary just a few years ago led one of the principal commentators in the field to note that he was not aware of *any* direct application of the insights of Arrow's theorem to voting rights law.[84]

This relative paucity of such commentary is even more surprising in light of the fact that Arrow's theorem, on its face, provides a powerful argument against the ability to sustain any claim of qualitative vote dilution. The argument goes as follows: A necessary element of any claim of qualitative vote dilution is the production of an objective standard against which to measure and remedy that dilution. Devising such a standard requires the use of some social choice mechanism to translate individual voter preferences into that ideal, standard outcome. Arrow's theorem, however, calls the viability of all such social choice mechanisms into question. More specifically, Arrow's theorem means that there is no objective way to move from individual desires to a meaningful group choice without sacrificing one of the theorem's very minimal conditions of democratic fairness. Arrow's theorem, in other words, proves that there is no objective method of devising qualitative vote dilution standards. Without an objective method of finding the "correct" social outcome, courts cannot even measure qualitative vote dilution, much less remedy it.

This argument was recently given a voice by Larry Alexander.[85] Alexander begins by differentiating a procedural conception of democracy from a substantive conception of democracy.[86] A procedural conception of democracy looks, unsurprisingly, to the procedures of democracy and relies on the principle of "one person, one vote" and majority rule in drawing district lines.[87] Such a conception is primarily concerned with quantitative vote dilution.[88] A substantive conception of democracy, on the other hand, "would have us draw district lines by reference to a set of policy outcomes that moral theory deems ideal."[89] This conception is more closely allied with qualitative vote dilution. According to Alexander, it would entail

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84. *See* Samuel Issacharoff, *Polarized Voting and the Political Process: The Transformation of Voting Rights Jurisprudence*, 90 MICH. L. REV. 1833, 1883 (1992).

85. *See* Alexander, *supra* note 53, at 570-79; Alexander, *supra* note 59, at 331-35. Samuel Issacharoff has also noted that "there is a stark disjuncture between a legal doctrine founded on the electoral preferences of racial and ethnic communities and [Arrow's theorem,] which disputes the ability to draw any conclusions about aggregate preferences from electoral results." Issacharoff, *supra* note 84, at 1883.

86. *See* Alexander, *supra* note 59, at 329-31.

87. *See id.* at 331.

88. *See id.*

89. *Id.* at 329.

determining ideal legislation and working back to district lines designed to achieve such legislation.[90]   The problem with such a conception, however, is that "[i]n order to structure voting, we must already know how the vote should turn out."[91] Using the terms of this Article, the problem with quantitative vote dilution is that we must be able to come up with an ideal standard outcome in order to draw district lines. Alexander views Arrow's theorem as an insurmountable obstacle to this enterprise.[92]

　　Arrow's theorem is a problem, from Alexander's point of view, because it means that we cannot determine, given the "real world" in which majorities shift depending on the issue or personality under discussion, a vote dilution standard.[93] More to the point, he believes that the theorem means that there is no "outcome blind" method for devising such standards—that all qualitative vote dilution cases are actually "vote maldistribution" claims, and such claims necessarily involve substantive political decisions that do not relate well to the constitutional text and are not well suited for the judiciary.[94] The decisions do not relate to the constitutional text because the Constitution gives no guidance on such substantive political decisions as what sort of legislation is ideal (and, if it did, it could have just as well included such legislation in its text).[95] Furthermore, such issues are ill-suited to the judiciary because they involve political questions.[96] Alexander's argument is, essentially, a version of the straightforward argument that Arrow's theorem eliminates the possibility of devising objective qualitative vote dilution standards, and thus dooms any claim of dilution that relies upon such standards. If Alexander is correct, voting rights advocates have no foolproof way to measure vote dilution.   Since finding an ideal outcome is usually impossible, qualitative dilution claims may ultimately lack foundation.[97]

　　90.　*See id.* at 330-31.
　　91.　*Id.* at 331.
　　92.　*See* Alexander, *supra* note 53 at 572-76; Alexander, *supra* note 59, at 335-36.
　　93.　*See* Alexander, *supra* note 59, at 332-35.
　　94.　*See id.* at 330-31.
　　95.　*See id.* at 330.
　　96.　*See* Alexander, *supra* note 53, at 578-79.
　　97.　Of course, this is not merely a problem attendant to vote dilution; there is no social choice function that guarantees meaningful outcomes in *any* election. Elections in districts with potential vote dilution claims are just like those in any other district. And if the outcomes of all elections are suspect, why does it matter that the standards for vote dilution are suspect as well? In one sense, it does not matter. At some theoretical level, Arrow's theorem dooms every democratic voting procedure's claim to credibility. In another sense, though, it does matter. We are, and should be, much less wary of allowing state legislative bodies to tinker with voting districts and procedures in the absence of objective standards than we are of allowing judges to do the same thing. Legislators, we feel, may be more

This lack of foundation is devastating for vote dilution claims from inception to conclusion. Initially, Arrow's theorem means that plaintiffs will be unable to assert standing in a qualitative vote dilution case. In order for a plaintiff to have standing, she must show some type of injury, which, in a qualitative vote dilution case, means being the member of a minority group that has less opportunity to elect a representative of its choice.[98] That burden begs the question, "Less opportunity than what?"—a question which, if Arrow's theorem is correct, cannot be given a meaningful answer. Plaintiffs, then, are left without any legitimate means of asserting standing. Or, to put it another way, if any person has standing, every other person who is a member of any group has an equal claim of standing to make a vote dilution claim and, indeed, has as many claims as there are groups in which he is a member.[99]

Assuming that a particular plaintiff gets past the standing stage, she is still left in the position of having to prove dilution which, again, entails production of an undiluted practice, or districting plan, against which to measure that dilution. The theoretical obstacle reappears once more at the remedy stage. Without a neutral procedural method of identifying qualitative vote dilution, any remedy devised by courts will actually be a substantive political decision.[100] Indeed, what is more political than being able to dictate the outcome of a popular election? In a sense, courts become dictators, usurping legislative authority and shaping agendas and districts to achieve the social preference orders they find most preferable. Arrovian obstacles plague the enterprise of qualitative vote dilution from start to finish. Thus, from the view point of social choice theorists generally and Larry Alexander in particular, Arrow's theorem means we should abandon the concept of qualitative vote dilution altogether.[101]

Arrow's theorem, then, provides the basis of a compelling argument against the ability of courts to devise objective standards against which to measure qualitative vote dilution. Such standards are an integral conceptual component of any qualitative vote dilution

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responsive and thus less likely to insulate their essentially dictatorial decisions from public scrutiny. The "messiness" of the legislative process may also contribute to greater stability. *See generally* Bernard Grofman, *Public Choice, Civic Republicanism, and American Politics: Perspectives of a "Reasonable Choice" Modeler,* 71 TEX. L. REV. 1541, 1551-53 (1993) (commenting that the cyclicality that characterizes majoritarian decision processes often favors the status quo).

98.   *See, e.g.,* Shaw v. Hunt, 517 U.S. 899, 904 (1996).
99.   *See* Alexander, *supra* note 53, at 576-77.
100.   *See id.* at 578.
101.   *See id.* at 577-79.

claim. Perhaps even more importantly, the Supreme Court recently held that such standards are a necessary element of any qualitative vote dilution claim under section 2 of the Voting Rights Act.[102] For voting rights advocates, these recent pronouncements of the Court and its general trepidation in voting rights cases make the search for a solution to the Arrovian problem even more pressing. And, as one of those advocates, it is that task to which I now turn.

IV. A PROPOSED SOLUTION AND ITS LIMITATIONS

There are no simple solutions to the challenge posed to voting rights by social choice theory. Some of the more obvious potential solutions can be readily dismissed. One potential solution or set of solutions goes to the heart of the issue and asks whether Arrow was wrong.[103] There may be, for example, some defect in the formal proof of the theorem that diminishes its force or, at a minimum, renders it unconnected to the search for qualitative vote dilution standards. But the conditions of fairness and logicality are minimal, the proof itself appears invulnerable,[104] and the theorem's connection to vote dilution claims is straightforward.

Other possible solutions involve the ability of courts to make an end run around the hazards posed by Arrow's theorem. That is, even if the theorem is formally correct, perhaps courts can nonetheless find some novel way to devise objective vote dilution standards that elude the strictures of the theorem. Unfortunately, any such standards must explicitly or implicitly equate inputs, voter preferences, with outputs, social choices. There simply is no secret method of amalgamating individual preferences to determine the "true" social choice. Any standard for evaluating social choices is vulnerable to the same violations of one of Arrow's five conditions that it is designed to test. Thus, solutions involving attempts to dodge the dictates of the theorem do not take us in a fruitful direction. We must instead acknowledge the power of Arrow's theorem and search for a solution that comes out of the theorem itself.

Such a solution will likely have several facets. I have argued elsewhere, for example, that limiting vote dilution claims to a few groups at a time and paying particular attention to control of the agenda may go a long way toward ensuring the presence of objective

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102. *See* Holder v. Hall, 512 U.S. 874, 875 (1994) (plurality opinion).
103. *See* Hayden, *supra* note 6, at 305.
104. *See* RIKER, *supra* note 65, at 129-36.

dilution standards.[105] A complete solution to the problem, however, must involve a cluster of concepts loosely referred to as "spectrum agreement."[106] The remainder of this Article, therefore, involves an examination of spectrum agreement, its possible limitations, and·its ultimate utility as a solution to the challenge to voting rights posed by social choice theory.

A. Spectrum Agreement

Spectrum agreement occurs when all individuals in society have a common spectrum upon which they array their preferences.[107] Such agreement on a spectrum of alternatives should not be confused with agreement on the ordering of those alternatives. Take, for example, a case of complete spectrum agreement where all voters are ideologues and array their preferences on a traditional right-left political spectrum. If there are three candidates running for office—a conservative (c), a liberal (l), and a moderate (m), voters will have one of four sets of preference orders.   Conservative voters will most prefer the conservative candidate and least prefer the liberal candidate, for a preference order of $cPmPl$. Liberal voter tastes will run in the opposite direction, and will thus give rise to a preference order of $lPmPc$. Moderate voters will rank the candidates either $mPcPl$ or $mPlPc$ depending on whether the voters are, respectively, right or left of center.   Although this assortment of political ideologues ranks the candidates in different orders, voter preferences can be aligned along the same right-left spectrum; no voter, for example, ranks the moderate candidate last, as agreement on the right-left spectrum precludes such an ordering.

Spectrum agreement is important for our purposes because it is a sufficient condition of transitivity.   When all voters arrange alternatives on a common spectrum, a simple majoritarian decision procedure guarantees a transitive outcome.[108] Spectrum agreement,

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105. *See* Hayden, *supra* note 6, at 310-13.

106. *See id.* at 310-17.

107. While the concept of spectrum *agreement* seems to imply some sort of express or implied prearranged understanding between voters, no such understanding is required. Instead, it is enough that voter preferences may be arrayed on a common continuum regardless of whether those voters made any sort of agreement or, indeed, even knew that their preferences could be arrayed on a particular spectrum.

108. *See* RIKER, *supra* note 65, at 123-28.   The seminal works on the subject of spectrum agreement are DUNCAN BLACK, THE THEORY OF COMMITTEES AND ELECTIONS (1958) [hereinafter BLACK, COMMITTEES AND ELECTIONS], and DUNCAN BLACK & R.A. NEWING, COMMITTEE DECISIONS WITH COMPLEMENTARY VALUATION (1951) [hereinafter BLACK & NEWING, COMMITTEE DECISIONS]. For a more concise discussion, see ORDESHOOK, *supra* note 68, at 160-66.   ·

therefore, solves the problem at hand. Simply put, courts can devise and apply neutral vote dilution standards in districts that display complete spectrum agreement.[109]   Given the dictates of Arrow's theorem, however, we know that such a guarantee must come at some cost.

Arrow's theorem, remember, holds that no voting procedure that meets minimal conditions of democratic fairness can guarantee a meaningful, transitive outcome. Thus, the one thing we know about spectrum agreement is that, because it guarantees a transitive outcome, we must be sacrificing one of those conditions. As it turns out, reliance upon spectrum agreement to generate a transitive vote dilution standard sacrifices the condition of universal admissibility. Spectrum agreement, by definition, implies the absence of certain individual preference orders.   In the example used above, for instance, no individual ranked the conservative, moderate, and liberal candidate in the order $cPlPm$ or $lPcPm$. The exclusion of individual preference orders rather directly violates universal admissibility. Thus, in order to insure a complete, transitive outcome, majoritarian social choice procedures must violate that condition.   Even naturally occurring spectrum agreement violates universal admissibility since that condition requires that a social choice function generates a complete transitive outcome for any *possible* preference profile.[110]

As discussed earlier, compromising universal admissibility jeopardizes a basic element of democratic fairness.[111]   Natural spectrum agreement, however, satisfies the fairness concerns embodied by the condition of universal admissibility. In cases of natural spectrum agreement, voters encounter no prior restraints on their preference orders. This means that natural spectrum agreement does not implicate the principal justification for universal admissibility: the immorality of denying the ballot to people with certain preference orders.   Instead, cases of natural spectrum agreement indicate situations in which individual preferences happen to align along a common spectrum. Thus, the condition of universal admissibility is not sacrificed by denying anyone the right to vote from the outset, but by determining when spectrum agreement naturally exists, thereby eliminating the possibility of intransitive results. "In

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109. *See* Hayden, *supra* note 6, at 311.
110. *See id.* at 311-12.
111. For a discussion of Arrow's theorem, see *supra* notes 64-81 and accompanying text.

short, to sacrifice universal admissibility in a district with spectrum agreement is to sacrifice very little."[112]

Legislatures and courts, therefore, should primarily focus on the qualitative vote dilution claims that arise within districts exhibiting spectrum agreement. Spectrum agreement eliminates the risk of intransitive outcomes and thereby permits courts to develop qualitative vote dilution standards that do not violate Arrow's basic conditions of fairness and logicality. "Therefore, qualitative vote dilution standards, and the legal cases built around them, are strongest where there is some agreement on the spectrum of alternatives."[113]

On a practical level, courts should focus their efforts on districts that exhibit some form of minority and majority bloc voting. Bloc voting makes spectrum agreement more likely; both majority and minority voters have preferences that align along some issue spectrum.[114] For example, racial bloc voting in a district with white-preferred candidates (w) and black-preferred candidates (b) means that while voters have preferences like $bPbPw$, $bPwPw$, $wPwPb$, and $wPbPb$, they do not have preferences like $bPwPb$ or $wPbPw$. Bloc voting, then, indicates the presence of a natural form of spectrum agreement that can eliminate the possibility of intransitivities without imposing prior restraints upon individual preferences.[115] The only difference is that here, unlike the example above, the spectrum is not a right-left political spectrum, but a spectrum of race.

Fortunately, section 2 of the Voting Rights Act generally accords with this prescription of Arrow's theorem. Several features of the act and its legislative history echo the call for spectrum agreement.[116] Further, the Supreme Court's initial interpretation of section 2 in *Thornburg v. Gingles* established spectrum agreement in the form of bloc voting as a prerequisite for vote dilution.[117] Justice Brennan, writing for the majority, devised a three-pronged test to identify the presence of vote dilution in multi-member districts:

> These circumstances are necessary preconditions for [a violation of section 2]. First, the minority group must be ... sufficiently large and

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112. Hayden, *supra* note 6, at 312.
113. *Id.* at 311.
114. Again, racial spectrum agreement should not be confused with agreement upon the candidates. With racial spectrum agreement, although white voters will prefer certain candidates and black voters will prefer other candidates, both groups agree that the candidates are arrayed on the same basic spectrum, with white-preferred on one side and black-preferred on the other.
115. *See* Hayden, *supra* note 6, at 312.
116. *See id.* at 314-16.
117. 478 U.S. 30, 48-51 (1986).

geographically compact to constitute a majority in a single-member district.... Second, the minority group must be ... politically cohesive.... Third, the minority must be able to demonstrate that the white majority votes sufficiently as a bloc to enable it ... usually to defeat the minority's preferred candidate.[118]

Taken together, the second and third conditions of minority and majority bloc voting require the presence of racially polarized voting as a precondition for any claim of vote dilution. In other words, the Supreme Court has installed a weak version of natural spectrum agreement as a necessary condition for proving qualitative vote dilution without sacrificing democratic fairness. The key question now becomes whether this version of spectrum agreement is sufficient to sidestep the implications of Arrow's theorem.

B. Further Problems

The Arrovian problems that lurk in vote dilution jurisprudence are not remedied by a simple trust that spectrum agreement will guarantee transitive outcomes and thus ensure the possibility of objective qualitative vote dilution standards. An initial problem with that solution is that absolute spectrum agreement in the form of pure racial bloc voting does not exist nor, of course, is it required by *Gingles*. Indeed, if the Supreme Court did have such a requirement, it would effectively eliminate legal cognizance of vote dilution claims since no district exhibits complete racial bloc voting. There will always be (and, perhaps, in increasing numbers) voters who cross racial lines. Given the absence of complete spectrum agreement—the only guarantee of transitive outcomes—the question becomes whether the presence of partial spectrum agreement has an effect upon the likelihood of transitive outcomes.

Even if a sufficient degree of spectrum agreement exists, there is the further problem of whether courts can successfully identify and measure it. That is, even if spectrum agreement exists, it may be difficult to verify its existence. Although the question of the existence of spectrum agreement logically precedes the question of the courts' ability to measure it, the two questions are intertwined on a practical level because it is impossible to discuss the existence of agreement without discussing the limitations upon our ability to verify its presence.[119] In any case, the principal difficulties with the solution of

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118. *Id.* at 50-51 (citations omitted) (footnotes omitted).

119. *See generally* Evelyn Elayne Shockley, Note, *Voting Rights Act Section 2: Racially Polarized Voting and the Minority Community's Representative of Choice*, 89 MICH.

spectrum agreement are whether sufficient agreement exists and, if so, whether we can identify it with an adequate degree of certainty.

While it is relatively easy to say that racial bloc voting is required for vote dilution claims, it is much more difficult to prove the existence of such voting patterns. One major difficulty is the lack of readily accessible data. Individual voting records are, of course, secret. Reliable exit poll data is often only available for presidential and statewide elections, which rarely involve minority candidates.[120] Voting rights proponents, then, must instead look to precinct-level, aggregate data to prove bloc voting. There are two methods of collecting precinct-level evidence of racial bloc voting, and both have limitations.

First, data can be gathered from racially homogeneous precincts to give accurate indications of racial bloc voting.[121] As Grofman notes, "[I]f the voters in a precinct with only black residents cast 80[%] of their ballots for a black candidate and 20[%] for a white candidate, it is obvious that that is how *black* voters in that precinct

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L. REV. 1038, 1042-44 (1991) (noting disagreement among the members of the *Gingles* Court over evidence necessary to show racial bloc voting). Shockley points out how only a plurality of the *Gingles* Court agreed on which considerations should be taken into account to determine whether racially polarized voting exists. *See id.* at 1042. For example, Justice Brennan argued that "the existence of racial bloc voting should be determined based on the facts of the case, in terms of how strong a correlation exists between the voters' race and the candidate they support, regardless of the candidate's race." *Id.* at 1043. However, Justice White disagreed with this approach arguing that "party politics and interest group politics can, in certain instances, produce electoral results that would look like racial bloc voting, as Justice Brennan defines it." *Id.* In part, the confusion surrounding racial bloc voting depends upon whether the courts focus on the race of the candidate. "The level of importance assigned to the candidate's race determines whether a court will consider all elections under the challenged districting scheme as relevant evidence of the presence or absence of racial bloc voting, or only those elections in which a minority candidate ran." *Id.* at 1044.

120. *See* GROFMAN ET AL., *supra* note 11, at 84, 146 n.6. Exit poll data, when available, is usually viewed as some of the best information regarding voters' race and preferences.

121. *See id.* at 85; *see also Gingles,* 478 U.S. at 52-54, 61 (approving the district court's reliance on two basic statistical techniques: homogeneous precinct analysis and bivariate ecological regression); Cuthair v. Montezuma-Cortez, 7 F. Supp. 2d 1152, 1168 (D. Colo. 1998) (stating that homogeneous precinct analysis is the "'standard in the literature for the analysis of racially polarized voting' and [is] expressly relied upon and approved by the Court in *Gingles*" (quoting *Gingles,* 478 U.S. at 53 n.20)); Barnett v. City of Chicago, 969 F. Supp. 1359, 1414 (N.D. Ill. 1997) (stating that "[w]hile lay testimony is instructive for the presentation of anecdotal evidence of minority cohesiveness or racial bloc voting, statistical evidence in the form of ecological regression analysis and extreme case analysis [such as homogeneous precinct analysis] ... is also typically offered"), *aff'd in part and vacated in part,* 141 F.3d 699 (7th Cir. 1998); Reed v. Town of Babylon, 914 F. Supp. 843, 852 (E.D.N.Y. 1996) (noting that homogeneous precinct analysis "assumes that the voting patterns or party affiliations of a particular racially homogeneous election district are likely to be a fair indicator of the ethnic group's voting pattern or party affiliation more generally").

voted."[122] There are, however, several drawbacks to this approach. Initially, absolutely racially homogenous precincts are rare.[123] Although data from predominantly homogenous precincts (say, 90% to 95%) is valuable, its usefulness is diluted to the extent of its lack of homogeneity.[124] Further, minorities in racially homogeneous precincts may vote differently from those in more heterogeneous precincts.[125] Given the limitations of data from racially homogeneous districts, parties in voting rights cases often turn to a second method of measuring bloc voting.

The second method of measuring the degree of racial bloc voting involves a statistical method known as "ecological regression."[126] Ecological regression compares the number of votes received by minority candidates in each precinct to the racial composition of the precinct.[127] Statistical correlations can then be drawn from the sum of the data from each precinct in the district at issue.[128] While this method does not require homogeneous precincts, it does have several drawbacks. First, different voter turnout rates for different races may

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122. GROFMAN ET AL., *supra* note 11, at 85.

123. *See id.* For one of the rare cases of abundant homogeneous precinct data, see *Cuthair*, 7 F. Supp. 2d at 1168 (stating that "[t]he homogeneous precinct analysis is particularly compelling and reliable in this case since virtually all the precincts from [the] elections were homogeneous").

124. *See generally Barnett*, 969 F. Supp. at 1415 (stating that "[w]hile conceptually simple, this analytic method is of limited applicability, because it is of questionable accuracy in districts which are less than 90% homogenous"); *Reed*, 914 F. Supp. at 852 (noting that "[o]nly election districts where the race or ethnicity of the voting age population is at least 90% homogenous should typically be subjected to extreme case analysis").

125. *See* GROFMAN ET AL., *supra* note 11, at 85. The differences in voter preferences, which may occur for a variety of political, economic, or social reasons, mean that the results from homogeneous precinct analysis are often compared to those from ecological regression analysis. *See, e.g., Barnett*, 969 F. Supp. at 1415 (noting that homogeneous precinct analysis is "frequently used as a cross checking device in conjunction with regression analysis by verifying how accurately the regression curves predict actual voter behavior in homogenous wards").

126. *See* GROFMAN ET AL., *supra* note 11, at 85-88; *see also Gingles*, 478 U.S. at 30, 52-54 (detailing the analysis used to determine racial bloc voting); *Reed*, 914 F. Supp. at 851 ("Ecological regression analysis is the standard technique used to infer voting behavior among distinct population groups.").

127. *See* GROFMAN ET AL., *supra* note 11, at 84-88.

128. For the exact methodology, see *id.* at 86-88. This methodology is used because of the obvious difficulties of interviewing every voter and then classifying them by race and preferred candidate. *See, e.g., Reed*, 914 F. Supp. at 851 ("Regression analysis allows the parties to surmount [the] proof-gathering burden by making reasonably accurate estimates of majority and minority voting behavior from demographic data and, depending on whether voting in a specific election or party affiliation is being estimated, election returns and party registration data, respectively.").

overestimate or underestimate the degree of bloc voting.[129] This is because voter-race data is only provided by census data, and precinct voting records provide no direct measure of the number of members of a particular race that votes (although advocates have tried counting Hispanic surnames in some precincts).[130] Second, it is often difficult to match census blocks (the minimum quantum of census data) to precincts.[131] Precincts can cut across census blocks, and even when a precinct and census block are coextensive, superior map–reading skills are often essential to parse out race ratios.[132] Third, a variety of potential statistical problems can plague the processing of the data.[133]

The second major difficulty in proving racial bloc voting is one of interpretation.[134] Once subject to heated debate, standard methods of interpreting the data are now set in the courtroom.[135] Indeed, the *Gingles* Court adopted a general notion of statistical significance of the ecological regressions, making it part of the case law.[136] But some

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129. *See* GROFMAN ET AL., *supra* note 11, at 93; *see also* Overton v. City of Austin, 871 F.2d 529, 539 & n.2 (5th Cir. 1989) (explaining that when computing regression analysis, it is necessary to estimate both the racial makeup of a district and the voter turnouts within that district).

130. *See* GROFMAN ET AL., *supra* note 11, at 93-94.

131. *See id.* at 94.

132. *See id.*

133. *See, e.g., id.* at 98-103; *see also* Barnett v. City of Chicago, 969 F. Supp. 1359, 1414 (N.D. Ill. 1997) (noting that "while ecological regression analysis has become a preferred tool to assist courts in divining voter behavior, it is hardly perfect and must not be accepted uncritically [since the methodology] is susceptible to subtle manipulation . . . [and] errors due to the 'ecological fallacy' of attributing the average behavior of voters in a given area to all voters in that area"), *aff'd in part and vacated in part*, 141 F.3d 699 (7th Cir. 1998).

134. *See* David D. O'Donnell, *Wading into the "Serbonian Bog" of Vote Dilution Claims Under Amended Section 2 of the Voting Rights Act: Making the Way Towards a Principled Approach to "Racially Polarized Voting"*, 65 MISS. L.J. 345, 364 (1995) ("The efforts of the lower courts to apply the *Gingles* evidentiary framework to minority vote dilution claims is marked largely by inconsistency in the evaluation of the expert statistical evidence presented in support of the . . . 'racial bloc voting' inquiries . . . .").

135. *See* GROFMAN ET AL., *supra* note 11, at 82, 84-85; *see also* O'Donnell, *supra* note 134, at 362 (noting that the *Gingles* Court approved a statistical methodology that "included a consideration of the existence and strength of any 'correlation' between the race of the voter and the selection of certain candidates, whether the revealed correlation was 'statistically significant,' and whether the differences in minority and majority electorate voting patterns [were] 'substantively significant'" (footnotes omitted)).

136. *See* Thornburg v. Gingles, 478 U.S. 30, 53 n.22; *see also* GROFMAN ET AL., *supra* note 11, at 84 (stating that the Court "accepted Grofman's judgment about the statistical significance of the correlation coefficients obtained in ecological regressions"); O'Donnell, *supra* note 134, at 364 ("Most lower court decisions engaged in the racial bloc voting inquiry have failed to include any analysis of the causes of white or majority voter behavior and instead, pointing to *Gingles*, focused their analysis exclusively on the results of the presented bivariate [ecological] regression and extreme case analysis statistics."). For examples of the use of bivariate regression and statistical significance, see *NAACP v. City of Niagara Falls*,

detractors remain, and lower courts have been known to accept less reliable methods.[137]

Because there is rarely, if ever, complete spectrum agreement, and because voter preference data is never completely accurate, the relationship between lesser degrees of spectrum agreement and transitive outcomes becomes pressing. For if a great deal of spectrum agreement is needed to drive down the probability of a voting cycle, then either a low degree of such agreement or a low degree of measurement precision in qualitative vote dilution cases may be sufficient to thwart attempts to devise meaningful vote dilution standards. If, on the other hand, very little spectrum agreement is needed to ensure transitive results, then high degrees of bloc voting and measurement precision are unnecessary. The second and third elements of the *Gingles* three-part test, after all, require evidence of racially polarized voting—a weak form of spectrum agreement.[138]

Unfortunately, the *Gingles* Court did not quantify the degree of racially polarized voting necessary to establish a section 2 claim.

65 F.3d 1002, 1008-09 (2d Cir. 1995); *Gomez v. City of Watsonville*, 863 F.2d 1407, 1409-11, 1416-17 (9th Cir. 1988); *Campos v. City of Baytown*, 840 F.2d 1240, 1244-49 (5th Cir. 1988).

137. *See* GROFMAN ET AL., *supra* note 11, at 103-04 (noting that the accuracy of the *Gingles* methodology has been questioned by both statisticians and social scientists acting as expert witnesses). The *Gingles* Court refused to articulate a "simple doctrinal test for the existence of legally significant racial bloc voting" because the degree of such block voting that is "cognizable as an element of a § 2 vote dilution claim will vary according to a variety of factual circumstances." *Gingles*, 478 U.S. at 57-58. Thus, as one commentator noted, "In establishing racial bloc voting as the key element ... but in failing to provide a precise definition of it or an evidentiary standard for proving its presence, ... Justice Brennan succeeded only in creating a new test for lower courts to follow, without providing definitive guidance for its implementation." Mary J. Kosterlitz, Note, Thornburg v. Gingles: *The Supreme Court's New Test for Analyzing Minority Vote Dilution*, 36 CATH. U. L. REV. 531, 562 (1987). As a result, lower courts have taken a number of different approaches in implementing the *Gingles* three-part test. *See, e.g.*, Sanchez v. Bond, 875 F.2d 1488, 1493-95 (10th Cir. 1989) (stating that the race of the candidate is relevant and holding that competent lay testimony can be used to establish polarized voting); *Campos*, 840 F.2d at 1245-49 (holding that a candidate's race must be considered in a racial bloc voting analysis); City of Carrollton Branch of the NAACP v. Stallings, 829 F.2d 1547, 1558-59 (11th Cir. 1987) (using the candidate's race in evaluating the level of racial bloc voting without acknowledging its use); Cuthair v. Montezuma-Cortez, 7 F. Supp. 2d 1152, 1169 (D. Colo. 1998) (stating that "[a]lthough an adequate statistical analysis was presented in this case ... a court should rely on other totality of the circumstances to determine if the electoral system has a discriminatory effect").

138. *See supra* notes 117-118 and accompanying text. Both the second and third prongs of the *Gingles* test require courts to consider the voting behavior of different races. The second prong differs from the third, however, in that the former merely asks whether voters of the same race vote alike and the latter evaluates the more complicated issue of whether a bloc-voting majority can routinely outvote the minority such that the minority's ability to elect a representative of its choice is impaired. *See generally* Johnson v. DeGrandy, 512 U.S. 997, 1007 (1994) (distinguishing between the second and third prongs of the *Gingles* test).

Instead, the Court explained that bloc voting must reach a certain level, one in which the ability of a minority group to elect representatives of its choice is impaired to a "legally significant" degree.[139] That factor, in turn, is measured on a sliding scale based on the district and a variety of other circumstances, and may emerge more distinctly over a period of time.[140] Given the number of variables at play in the bloc-voting determination, it is no wonder that, in the time since *Gingles*, neither the Supreme Court nor any of the circuit courts of appeals has really settled upon a threshold level of majority or minority bloc voting.

The failure of the courts to establish such a threshold level of polarized voting may be due to the fact that voting rights cases are fairly complicated factual matters analyzed under a flexible standard. Courts are often asked to look at multiple elections over several years involving different candidates with shifting voter loyalties.[141] And the complications involved in analyzing the extent of majority and minority bloc voting as part of the initial three-part test are only multiplied when courts engage in the second part of the inquiry—an examination whether, based on the totality of the circumstances, minorities have been denied an equal opportunity to participate and elect representatives of their choice.[142] Finally, because the presence of racially polarized voting patterns is reviewed only for clear error,[143] opinions of reviewing courts merely set relatively extreme upper and lower limits to judicially cognizable bloc voting.

That said, the Supreme Court has provided some guidance on the issue. In *Abrams v. Johnson*, for example, the Court upheld the district court finding of failure to prove legally cognizable racial bloc voting.[144] The district court had found that the average percentage of whites voting for black candidates ranged from 22% to 38%, and the average percentage of blacks voting for white candidates ranged from 20% to 23%.[145] In other words, it was not a clear error for the trial court to find that the second and third prongs of the *Gingles* test were not fulfilled by 62% to 78% majority bloc voting and 77% to 80% minority bloc voting. For the Supreme Court, these numbers were not

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139. *See Gingles*, 478 U.S. at 56.
140. *See id.* at 56-57.
141. *See, e.g.*, Abrams v. Johnson, 521 U.S. 74, 92-95 (1997).
142. *See, e.g., id.* at 91-95.
143. *See id.* at 95.
144. *See id.* at 91-95.
145. *See id.* at 92.

high enough; a higher percentage of racial bloc voting was necessary to overturn the lower court's holding as clearly erroneous.[146]

In sum, the formation of objective qualitative vote dilution standards is theoretically impossible in light of Arrow's theorem. The obstacles posed by the theorem, however, may be overcome when voter preferences can be arrayed on a common spectrum. The Supreme Court, in *Gingles*, mandated that section 2 plaintiffs prove a version of spectrum agreement in the form of racial bloc voting.[147] But in *Gingles* and, more recently, in *Abrams*, the Court did not and, indeed, could not, require 100% racial bloc voting, or pure spectrum agreement, as a prerequisite to qualitative vote dilution claims; instead, the Supreme Court and lower courts seem to require only about 70% to 80% bloc voting. This makes the identification of the exact degree of spectrum agreement necessary to guarantee transitive outcomes crucial to ensuring the presence of meaningful vote dilution standards. The remainder of this Article, therefore, concentrates on this problem, analyzing the precise relationship between spectrum agreement and transitive social preference orders.

V.   PARTIAL SPECTRUM AGREEMENT AND THE SEARCH FOR A MORE COMPLETE SOLUTION

As described above, the *theoretical* difficulties described by Arrow's theorem are unavoidable. Democratic decision procedures inevitably force a choice between universal admissibility and one of the other conditions of fairness or logicality. On a more practical level, however, faith in democratic choice procedures may not be wholly misplaced. The difficulties described by Arrow's theorem are alleviated if preference profiles that lead to intransitive social preference orders never occur in the real world. More to the point, the difficulties in finding qualitative vote dilution standards are resolved if such profiles do not occur in districts whose members have potential dilution claims. The practical impact of Arrow's theorem, then, depends upon how often preference profiles prone to intransitivities, or voting cycles, actually occur.

The probability that a given preference profile will result in an intransitive social preference order depends, of course, on the probability assumptions made about the composition of that profile.[148]

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146. *See id.* at 93.

147. *See* Thornburg v. Gingles, 478 U.S. 30, 51 (1986).

148. *See* Richard G. Niemi & Herbert F. Weisberg, *A Mathematical Solution for the Probability of the Paradox of Voting*, 13 BEHAV. SCI. 317, 321 (1968).

Most early researchers started from the neutral assumption that all individual preference orders were equally likely to occur in a profile.[149] Under this assumption, a substantial proportion of preference profiles result in cycles. In the set of all possible profiles with three voters and three alternatives, 5.56% produce voting cycles.[150] Both the number of voters and the number of alternatives positively affect the incidence of intransitivities.[151] For example, with three alternatives, as the number of voters increases, the incidence of cycling approaches a limit of 8.77%.[152] Intransitivities increase even more rapidly as a function of the number of alternatives.[153] With three voters, for example, as the number of alternatives increases, the incidence of cycling approaches 100%.[154] As a result, in a large election with as few as six alternatives, almost one-third of the possible preference profiles produce intransitive outcomes.[155] It would seem, therefore, that Arrow's theorem actually describes a significant practical problem in the search for meaningful vote dilution standards.

Empirical observations across a wide range of applications, however, have failed to discover the large number of predicted intransitivities.[156] The disparity could be the result of undetected cycles.[157] This is especially probable when we lack access to reliable data about preference orders, since in most democratic choice procedures, there is no way to work backward from result to preference profile. But even when reliable data about complete preference profiles is available, few real-world profiles produce intransitive results.[158] Thus, there is a real discrepancy between the actual and predicted incidence of cycling. In a world with elections involving millions of voters and hundreds of alternatives, the question

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149. *See id.*

150. *See id.* at 322 tbl.2.

151. *See id.* at 321-23.

152. *See id.* at 322 tbl.2.

153. *See id.* at 322-23.

154. *See* RIKER, *supra* note 65, at 122 tbl.5-1.

155. *See* Niemi & Weisberg, *supra* note 148, at 322 & tbl.2.

156. *See generally* Scott L. Feld & Bernard Grofman, *Partial Single-Peakedness: An Extension and Clarification,* 51 PUB. CHOICE 71, 71 (1986) (noting that "empirical observations of a wide variety of actual collective decisionmaking processes indicate that cyclical majorities are very rare"); Grofman, *supra* note 97, at 1553 (stating that "[c]ycles are much harder to find than early Social Choice models suggest they ought to be").

157. *See* Feld & Grofman, *supra* note 156, at 71-72; Grofman, *supra* note 97, at 1554, 1559-62.

158. And those that do are often suspected to be the result of strategic voting. *See* Grofman, *supra* note 97, at 1553 n.41.

becomes, as recently framed by Bernard Grofman: "Why so few observed cycles?"[159]

One answer is that factors beyond the number of voters and alternatives help minimize the frequency of cycling. The predicted number of cycles is based on an assumption of the equal likelihood of any preference ordering in a given profile.[160] Absent this assumption, of course, the predicted incidence of cycling varies tremendously. Certain types of preference profiles always produce transitive results. For example, a preference profile in which every individual has exactly the same preference order guarantees the absence of cycles, and the social preference order is identical to the orders in the profile.[161] The fact that a profile contains only identical preference orders, then, constitutes a sufficient condition of transitivity. Given the unlikelihood that a large number of people (or even two or three) completely agree on the ordering of multiple alternatives, the inquiry becomes whether there are other conditions that increase the probability of transitivity.

Several types of profiles that exhibit some type of spectrum agreement help ensure that a simple majority decision produces a transitive social preference order. This Part of the Article focuses upon three such categories: single-peaked profiles, value-restricted profiles, and socially homogeneous profiles. The example above in which every individual had identical orderings falls into all three categories: it is single-peaked, value-restricted, and perfectly socially homogeneous. Unfortunately, such an agreement only exists in the minds of utopian game theorists (and, perhaps, Utah), so it is of little use. Thus, we need to further explore the three types of profiles that increase the likelihood of acyclic outcomes.[162]

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159. *Id.* at 1553.

160. And, as discussed above, a social choice function must be able to produce transitive results with any possible preference profile to avoid violating the condition of universal admissibility.

161. *See* RIKER, *supra* note 65, at 124.

162. Much of the following analysis will make use of examples with three alternatives, or triples. This is no accident, nor is it opportunistic hand-waving. Amartya Sen proved that if, among a set of alternatives, every possible triple within that set yields a transitive result, then the same holds for the entire set of alternatives. *See* Amartya K. Sen, *A Possibility Theorem on Majority Decisions*, 34 ECONOMETRICA 491, 492 (1966). Thus the analysis concerning any number of triples can be expanded to encompass a greater number of alternatives.

A. Single-Peaked Preference Profiles

1. What Does It Mean to Be Single-Peaked?

Duncan Black was the first to recognize that if a certain profile was, in his terms, "single-peaked," it always produces a transitive outcome.[163] The description "single-peaked" comes from a graphic description of profiles (or the orderings in profiles). Consider first an individual preference order. An individual has a single-peaked preference order if he has a most desired alternative, and prefers other alternatives less as they are further from his ideal point.[164] Take, for example, Darren's individual preference order $yPxPz$. Placing his order on a graph, with desirability on the vertical axis and the alternatives ordered x, y, and z ($x\ y\ z$) on the horizontal axis, yields the following preference curve:

Figure 1
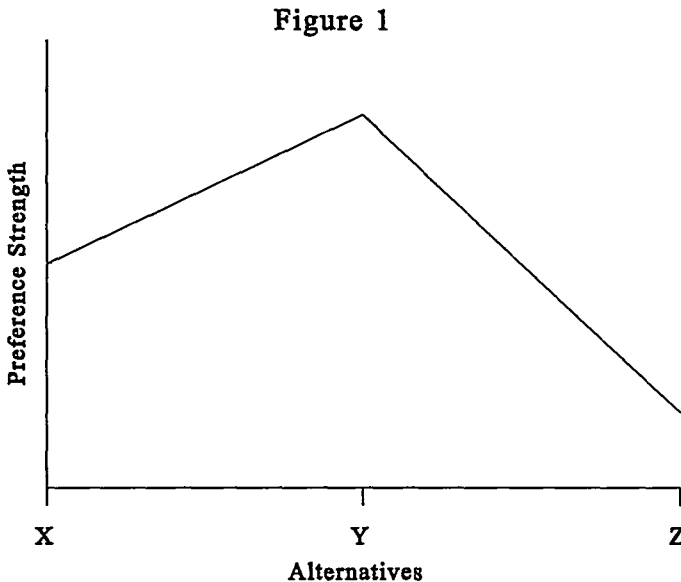
Darren's preferences with respect to this horizontal axis are single-peaked: He prefers y the most, and prefers x and z less as they are further from his ideal point y.[165]

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163. *See* BLACK, COMMITTEES AND ELECTIONS, *supra* note 108, at 19-25; BLACK & NEWING, COMMITTEE DECISIONS, *supra* note 108, at 19-28; Duncan Black, *On the Rationale of Group Decision-Making*, 56 J. POL. ECON. 23 (1948).

164. *See* Black, *supra* note 163, at 24; Scott L. Feld & Bernard Grofman, *Ideological Consistency as a Collective Phenomenon*, 82 AM. POL. SCI. REV. 773, 775 (1988).

165. There are two caveats to this analysis. First, within such a representation, no significance attaches to the distance between any two points: Single-peakedness is a property

One can imagine a profile with respect to the horizontal axis ordering (*x y z*) that is not single-peaked. Take, for example, Joanna's preference order *xPzPy*:

Figure 2
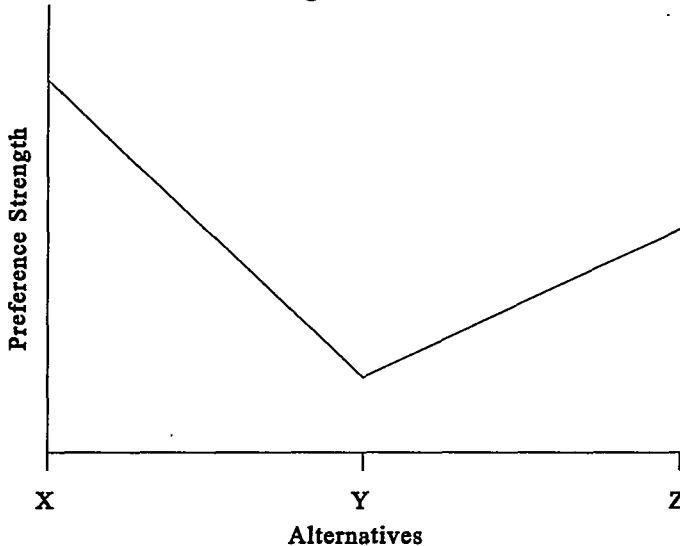
Here, Joanna most prefers *x*, but does not prefer the other alternatives less as they are further from *x*; instead, her preference curve goes down to *y* and then back up to *z*. Of course, on other possible horizontal axes, such as (*x z y*), Joanna's preferences are single-peaked. And, indeed, any individual preference order is single-peaked on a graph whose horizontal axis places alternatives in the order of desirability to that individual. Why, then, is not every social preference profile single-peaked and thus capable of producing transitive outcomes?

The catch is that for a preference profile to be single-peaked, it is insufficient that there be any arbitrary way for each individual order to be single-peaked.[166] Instead, there must exist *one* horizontal ordering such that every one of the individual orders in the profile is single-peaked.[167] Preference orders, in other words, must all align upon a

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of a set of *orderings*, not utility functions. Second, this analysis can also apply to ordering with indifference. In that case, the highest point on the preference curve would not be peaked, but truncated, or plateaued. But single-peaked or single-plateaued, the consequences for ensuring transitive outcomes is the same. *See* AMARTYA K. SEN, COLLECTIVE CHOICE AND SOCIAL WELFARE 168 (1970).
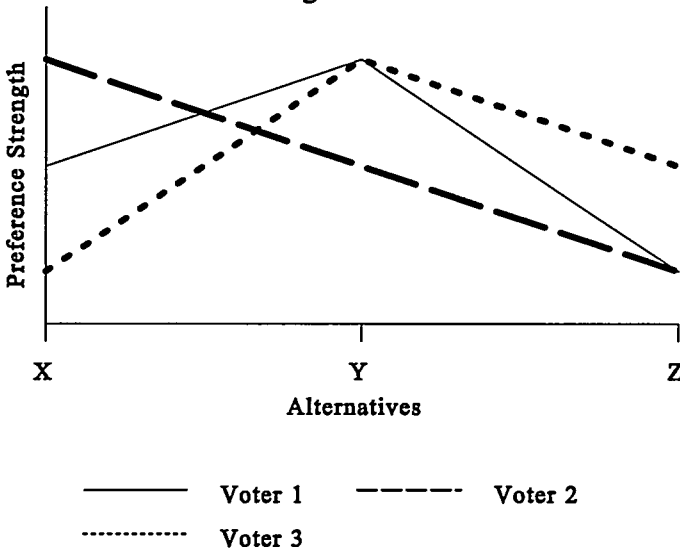
166. *See id.* at 167.
167. *See id.*

single dimension. Take, for example, the following profile with three
voters (V) and three alternatives:

$$P_1$$
$$V_1: \ y\,x\,z$$
$$V_2: \ x\,y\,z$$
$$V_3: \ y\,z\,x$$

This profile is single-peaked because it can be arrayed on a horizontal
axis (such as ($x\ y\ z$), among others) such that each of the individual
orders gives rise to a single-peaked preference curve:

Figure 3

When each individual's ordering is single-peaked along some ordering
on the horizontal axis, it may be said that they agree on the spectrum
of alternatives. Voters may array candidates for office, for example,
along a traditional left-right political spectrum; although all voters do
not agree on which candidate is the best, the fact that their preferences
are arrayed on a common spectrum is important. And determining the
existence of a common spectrum on which every preference order is
single-peaked has significant implications for the difficulties described
by Arrow's theorem.

2.   What Does Single-Peakedness Imply?

When a profile is single-peaked, the outcome of a simple majority vote is guaranteed to be transitive.[168] Preference profiles that are, analogously, single-caved or polarized are also guaranteed to yield transitive results.[169] The winner of a simple majority decision procedure with a single-peaked profile will inevitably be the ideal point of the median voter, and the same is analogously true if all the individuals in a given profile have single-caved or polarized preference orders.[170]

Profiles that result in cycles are not single-peaked. Take, for example, profile P_2:

$$P_2$$
$$V_1:\ x\,y\,z$$
$$V_2:\ y\,z\,x$$
$$V_3:\ z\,x\,y$$

This profile, when arrayed along horizontal ordering $(x\,y\,z)$, yields the following graph:

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168. Duncan Black was the first to demonstrate this result. *See* Black, *supra* note 163, at 30. A good modern summary can be found in RIKER, *supra* note 65, at 123-28.

169. *See* SEN, *supra* note 165, at 168; Feld & Grofman, *supra* note 164, at 776. Since the same conditions and results hold true for single-caved and polarized preferences as for single-peaked preferences, this discussion focuses on the latter.

170. *See* BLACK, COMMITTEES AND ELECTIONS, *supra* note 108, at 126-29; *see also* Feld & Grofman, *supra* note 164, at 776 (concluding that "[w]hen all individuals have single-peaked preference orderings . . . the majority choice is consistent with all of the requirements of an ideal decision rule as set forth by Arrow").

Figure 4
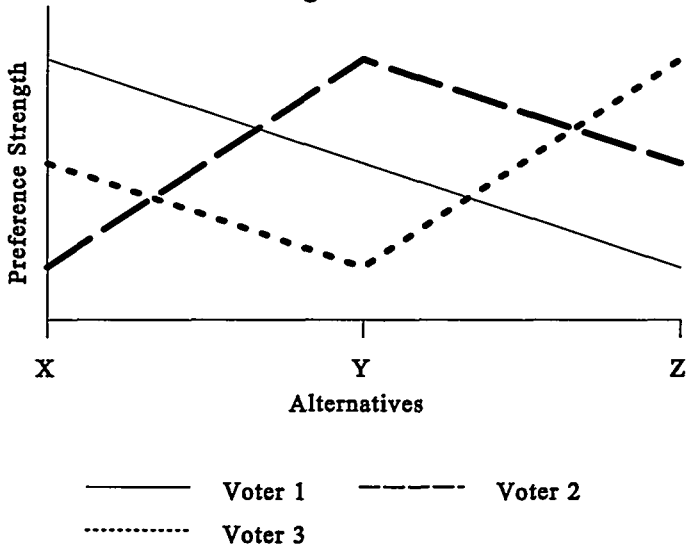
Profile P_2, as can be seen, is not single-peaked with reference to dimension $(x\ y\ z)$. But, more importantly, there is *no* dimension or spectrum upon which the individual orders of P_2 can be arrayed such that each is single-peaked (or single-caved or polarized).[171] The same can be said for any profile that produces intransitivities. Single-peakedness, it turns out, is a sufficient condition of transitivity.

B. *Value-Restricted Preference Profiles*

1. What Does It Mean to Be Value-Restricted?

A second, related sufficient condition of transitivity is that a profile be value-restricted. A triple of alternatives is value-restricted if there is at least one alternative that is either not first, not middle, or not last in every individual's ordering of the triple.[172] As discussed above, if this property holds for every triple among a set of alternatives, then it holds for the entire set of alternatives.[173] And, if a preference profile is value-restricted in this way, it will always produce a transitive social preference order.[174]

Consider, for example, the situation discussed earlier in this Article of an election with three candidates: a conservative (c), a

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171. *See* Black, *supra* note 163, at 32-33.
172. *See* Sen, *supra* note 162, at 492-93.
173. *See id.* at 493-94.
174. *See id.*

moderate (*m*), and a liberal (*l*).[175] Although voters may not support the same candidate, the profile may be value-restricted in that there is one candidate, *m*, that all voters agree is not the worst. Conservative voters would have a preference order of *cPmPl*, liberal voters *lPmPc*, and moderates either *mPlPc* or *mPcPl*. In no case is candidate *m* the least-preferred alternative.   The social preference order is, therefore, guaranteed to be transitive.

So far, the concept of value restrictiveness seems to add little to the analysis since, as in the simple example above, many profiles that are value-restricted are also single-peaked. Indeed, whenever a triple is value-restricted such that one alternative is not worst, that triple is also single-peaked. Similarly, when one alternative is not best, the triple is single-caved, and when one is not in the middle, the triple is polarized. So what does the concept of value restrictiveness add to the single-peaked analysis?

Value restrictiveness provides an additional sufficient condition of transitivity in that certain profiles that are not entirely single-peaked, single-caved, or polarized, may still be value-restricted and produce a transitive outcome. For example, there are profiles that are value-restricted even though not all of the triples are single-peaked, single-caved, or polarized. Consider the following profile with four voters and five alternatives in which *I* represents indifference between two alternatives:[176]

$$P_3$$
$$V_1: w\,I\,x\,P\,y\,P\,z$$
$$V_2: x\,I\,w\,P\,z\,P\,y$$
$$V_3: z\,I\,x\,P\,y\,P\,w$$
$$V_4: z\,P\,y\,I\,x\,P\,w$$
$$V_5: z\,P\,y\,P\,x\,P\,w$$

Each of the four possible triples are value-restricted: (*w*, *x*, *y*) is single-peaked since *x* is not the worse; (*x*, *y*, *z*) is single-caved since *y* is not the best; (*w*, *x*, *z*) is single-peaked since *x* is not the worse; and (*w*, *y*, *z*) is both polarized since *w* is not in the middle and single-caved since *y* is not the best.[177] Thus, even though the profile as a whole is not single-peaked (or single-caved or polarized), the fact that each of the triples within the profile is value-restricted in some way guarantees a

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175. This example was discussed in Hayden, *supra* note 6, at 306.
176. This example was discussed in Sen, *supra* note 162, at 498.
177. *See id.*

transitive outcome. Here, majority decision procedures yield the transitive social preference order $x I z P y P w$.[178]

The concept of value restrictiveness, then, both expands upon and encompasses the idea of single-peakedness. Every single-peaked profile is value-restricted (though not all value-restricted profiles, as seen above, are single-peaked). The import of this concept, however, is that it describes rather large groups of profiles that guarantee transitive outcomes. If real-world preferences regularly fell into such patterns, social preference orders would be acyclic, and there would be little to worry about.

2. The Likelihood of Value-Restricted Profiles

Political and sociological conditions suggest that some degree of spectrum agreement exists in most societies. All democracies require a degree of consensus as a precondition to formation; absent some agreement, no social contract would exist.[179] Moreover, common socialization may shape individual perceptions of the spectrum of alternatives, producing the type of value restrictiveness that prevents cycles.[180]

Unfortunately, the condition that a profile be value-restricted is, itself, extremely restrictive. In order to guarantee a transitive outcome, *every* individual's preference order must be value-restricted.[181] If even one voter does not agree on the spectrum, there can be a cycle. To demonstrate this, consider the following preference order with seventeen voters and three alternatives:[182]

$$P_4$$
8 Voters: $x\ y\ z$
5 Voters: $y\ z\ x$
4 Voters: $z\ y\ x$

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178. *See id.*
179. *See* FROHLICH & OPPENHEIMER, *supra* note 66, at 19-20.
180. *See id.* at 20; *see also* BENJAMIN I. PAGE & ROBERT Y. SHAPIRO, THE RATIONAL PUBLIC: FIFTY YEARS OF TRENDS IN AMERICANS' POLICY PREFERENCES 14 (1992) (arguing that the collective policy preferences of the American public are coherent and stable).
181. *See generally* Feld & Grofman, *supra* note 156, at 72-73 (noting that *"if even one* individual has non-single-peaked preferences then there can be a paradox of cyclical majorities"); Richard G. Niemi, *Majority Decision-Making with Partial Unidimensionality*, 63 AM. POL. SCI. REV. 488, 488 (1969) (stating that "the preference ordering of *every* individual must be single-peaked" for "majority voting [to] yield a transitive social ordering of the alternatives").
182. This example was discussed in Feld & Grofman, *supra* note 164, at 775-76.

A quick glance at this profile tells us that it is value-restricted in several ways. The profile is single-peaked, for example, since y is never last. Thus, a transitive outcome is guaranteed. Of the seventeen voters, a majority of thirteen prefer *y* over *z*; a majority of nine prefer *y* over *x*; and a majority of nine prefer *z* over *x*. The outcome of majoritarian decision procedures is the transitive social order *yPzPx*.

Suppose, however, that one of the voters with preference order *zPyPx* changed his order to *zPxPy*. In that case, there would still be majorities that preferred *y* to *z* and *z* to *x*, but there would now also be a majority of nine that preferred *x* to *y*. The outcome of a majority vote is now the cycle *yPzPxPy*. Thus, even though sixteen of the seventeen voters have single-peaked preferences, a majority vote still produces an intransitive outcome.

Thus, while some preference profiles may be value-restricted, such a condition is still extremely limiting.[183] As Gerald H. Kramer explained, "[T]he various equilibrium conditions for majority rule are incompatible with even a very modest degree of heterogeneity of tastes, and for most purposes are probably not significantly less restrictive than the extreme condition of complete unanimity of individual preferences."[184]

There are several reasons to believe that such unanimity on the spectrum does not exist in the real world. In some cases, voters may prefer either of two extremes to a more middling position. Voters disappointed in an inert centrist government may, for example, prefer both conservative and liberal platforms over moderate proposals. And indeed, empirical data suggest that most voters in the United States do not make choices along a standard left-right ideological spectrum.[185]

The likelihood of achieving spectrum agreement further declines when the array of alternatives is comprised of candidates instead of discrete issues. Since candidates take positions on many different issues, single-issue voters will make spectrum agreement unlikely as voters exhibit different profiles according to their particular issue preference.[186] Thus, the fact that we make decisions on multiple dimensions renders complete spectrum agreement unlikely as well.

There are some additional, more pedestrian reasons that make complete spectrum agreement unlikely. There may be a lack of

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183. *See* Joseph Greenberg, *Consistent Majority Rules over Compact Sets of Alternatives*, 47 ECONOMETRICA 627, 627 (1979).

184. Gerald H. Kramer, *On a Class of Equilibrium Conditions for Majority Rule*, 41 ECONOMETRICA 285, 285 (1973).

185. *See* Feld & Grofman, *supra* note 164, at 773.

186. *See* Niemi, *supra* note 181, at 488.

information or misperceptions about the alternatives.[187]   Some
alternatives may be very similar.[188]   Finally, voter error may preclude
the occurrence of complete spectrum agreement.[189]

In practice, then, social choice procedures will encounter
preference profiles that contain groups of voters who lack agreement
on the spectrum of alternatives.   Without complete spectrum
agreement, the validity of social preference orders remains uncertain.
The degree of that uncertainty, however, varies with several other
factors, a topic taken up in the next two subparts.

C.   Social Homogeneity

Fortunately, the likelihood of transitive outcomes does not wholly
depend upon assurances of complete spectrum agreement.   Lesser
degrees of voter homogeneity may, in certain cases, be sufficient.
Ascertaining the degree of social homogeneity and its effect on
outcomes has been explored in two principal ways: (1) measuring
how the proportion of single-peaked individual preference orders in a
given profile affects the likelihood of transitive outcomes and
(2) measuring how other indices of the similarity or dissimilarity of
preference orders affect the likelihood of such outcomes.[190]

1.   The Proportion of Single-Peaked Preferences

If, as seen above, complete spectrum agreement in the form of
value-restricted preference profiles absolutely guarantees a transitive
outcome, then, intuitively, one would predict that a large proportion of
single-peaked or otherwise value-restricted preference orders would
increase the probability of an acyclic result, and vice versa.   And, as
Richard Niemi proved in 1969, this is precisely the case.[191]

Niemi made two findings.   First, he found that the incidence of
acyclic results increases with the *proportion* of individuals in the
profile with single-peaked preferences.[192]   Such a result makes
intuitive sense given what is known about complete spectrum
agreement.   Second, and more surprisingly, he proved that with some
constant degree of spectrum agreement, the probability of transitive
results increases with the *number* of individuals in the profile.[193]   As

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187. *See id.*
188. *See id.*
189. *See id.*
190. *See id.*
191. *See id.*
192. *See id.* at 493.
193. *See id.* at 493-94.

Niemi explained, "This means, for example, that if about 70% of the preference orderings satisfy unidimensional criteria, the occurrence of the paradox . . . is relatively likely if the population is small, but is very unlikely if the population is large."[194]

This second result is surprising in light of what we saw before— the likelihood of transitive outcomes *decreased* with increasing numbers of individuals within a neutrally chosen profile. This incongruity is the result of the two different determinants of the incidence of transitivity.[195] First, there is the low probability of achieving some level of agreement in a neutrally determined profile.[196] Second, there is the effect of that degree of spectrum agreement on the incidence of transitive outcomes.[197] Within any random group of voters, the probability against getting a large level of spectrum agreement by chance is so great that it overwhelms the second factor, so the total probability of intransitivity in a neutrally chosen profile increases as the number of individuals increases.[198]

But within a profile that is not randomly chosen, where there is some degree of spectrum agreement, Niemi's result is not without meaning. In a large group with only 70% to 75% spectrum agreement, the probability of a cycle is rare indeed. Thus "[t]he [voting] paradox can be very satisfactorily avoided if common frames of reference are widespread but far less than unanimous."[199] This general result was confirmed by later studies of the effect of social homogeneity on the probability of transitive outcomes.[200]

2. Other Measures of Social Homogeneity

There are at least two other indices of social homogeneity. Dean Jamison and Edward Luce have devised a definition of a coefficient, *sigma* (or, more clearly, 1/*sigma*), indicating the degree of social homogeneity.[201] And, like Niemi, they calculate that as 1/*sigma* increases (and the degree of social homogeneity increases), so too does the likelihood of a simple majority winner.[202] They also provide a mathematical method to move from an observation of the probability

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194. *Id.* at 493.
195. *See id.* at 493-94.
196. *See id.*
197. *See id.*
198. *See id.*
199. *Id.* at 494.
200. *See infra* notes 201-206 and accompanying text.
201. *See* Dean Jamison & Edward Luce, *Social Homogeneity and the Probability of Intransitive Majority Rule*, 5 J. ECON. THEORY 79, 86 (1972).
202. *See id.* at 84-86.

of intransitivity on a number of issues by a number of different-sized groups within society to an estimated value of *sigma*.[203]

Peter Fishburn provides evidence that another measure of social homogeneity, the Kendall-Smith coefficient of concordance, also provides an accurate prediction of the likelihood of transitive social preference orderings.[204] As to be expected at this point, when the coefficient of concordance increases, so does the likelihood of acyclic outcomes.[205] Thus, by any measure, the greater the degree of social homogeneity, the greater the probability of a transitive outcome.

The conditions spelled out by Niemi, Luce and Edwards, and Fishburn, however, are still somewhat restrictive. After all, they require up to 70% agreement on a spectrum of alternatives. The occurrence of such agreement is unlikely in modern representative democracies; the morass of conflicting issue and candidate preferences in such democracies frustrates any attempt to discern a dimension of agreement. The number of possible voter-relevant dimensions is quite large, as one commentator noted:

> As voters we are Democrats and Republicans, blacks and whites, males and females. But we are also hawks and doves, redistributionists and laissez-faire advocates. We are atheist, agnostic, Catholic, Protestant, Jewish, Muslim, and Buddhist, all of various stripes. We are trade unionists and managers, Main Streeters and cosmopoles. Some of us prefer hot, charismatic candidates; others prefer cooler types. Some of us prefer the well-educated or the well-bred. Others prefer regular Joes and Joans. The list of our voting-relevant divisions is virtually endless.[206]

The natural occurrence of a high degree of spectrum agreement is unlikely given this multiplicity of personal preferences and social influences. Voters can and do cross race and party lines for a variety of reasons. The resultant lack of spectrum agreement increases the likelihood of intransitive outcomes.

Such diversity, however, is not completely devastating to the search for transitivity. Remember, we need only find one dimension that about 70% of the voters agree on in order to drive the probability of an intransitive result down to near zero. Moreover, each of the three methods of measuring partial social homogeneity examined thus far— those of Niemi, Jamison and Luce, and Fishburn—analyzes profiles as

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203. *See id.* at 81-84.

204. *See* Peter C. Fishburn, *Voter Concordance, Simple Majorities, and Group Decision Methods,* 18 BEHAV. SCI. 364 (1973).

205. *See id.* at 371-72.

206. Alexander, *supra* note 53, at 575.

a sum of their individual preference orders. Might there be another way to look at preference profiles that lends additional insight to the problem?

D. *Looking at Preference Profiles as a Whole*

Several researchers have recently investigated the effects of looking at preference profiles as a whole in their quest for acyclic results. Scott Feld and Bernard Grofman, for example, put forth the idea of "net preferences."[207] They showed that intransitive outcomes were unlikely if one subgroup of individuals in a profile had single-peaked preferences and the remaining preference orders were randomly distributed.[208] The random orders, in effect, cancel each other out, leaving the single-peaked subgroup to dominate both the likelihood of a cycle and the social preference order.[209] Feld and Grofman explained, that in the presence of a small subgroup with value-restricted preferences, "the impartial culture assumption for the rest of the society, rather than producing generic instability, *on the contrary*, guarantees that the relatively coherent minority will prevail and impose its net preference ordering on the rest of society."[210] There will not be a cycle since that subgroup has value-restricted preferences, and the social ordering will be identical to the majority preference ordering within the subgroup.[211]

Further, the social preference order will be transitive if one continuum exists such that the probability that any randomly chosen individual would align the alternatives along that continuum is greater than 50%.[212] Thus, at least for voters of a large electorate choosing among relatively few alternatives, if more than half of them agree on a spectrum, then (because of the canceling out of random preferences), society's majority choices will be made *as if* all the members of society viewed the range of alternatives along the same continuum. Thus, society may be ideologically consistent even if individuals are not.[213]

These results were confirmed by examining results from the 1980 presidential election.[214] In that election, only a bare majority of voters

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207. *See* Feld & Grofman, *supra* note 156, at 73-79.
208. *See id.* at 75.
209. *See id.*
210. *Id.* at 78.
211. *See id.* at 75.
212. *See id.* at 75-76.
213. *See id.* at 78.
214. *See* Feld & Grofman, *supra* note 164, at 780-86.

had single-peaked preferences, yet 80% of the subsets, and society as a whole, had single-peaked preferences.[215] From that study, Feld and Grofman concluded:

> The fact that the ideological nature of groups is not a simple function of the proportion of individuals who have single-peaked preferences suggests that the sources of ideological orientation must largely be attributed to tendencies toward ideological perceptions which may be only dimly realized in any single individual but which may cumulate across individuals (almost none of whom are perfectly ideological) so as to consistently provide an ideological cast to the decisions of the society as a whole and to virtually all of its subgroups.[216]

Thus society may be more ideological than the individuals that compose it, and agreement at the aggregate level on a spectrum does not require a high level of agreement on an individual level.[217] Our inquiry into the conditions that guarantee or increase the likelihood of transitive social preference orders has been completed.

VI. CONCLUSION

The difficulties posed by Arrow's theorem to voting rights advocates are not insurmountable. Recent work in the social choice theory has shown that absolute spectrum agreement is not required to elude Arrovian difficulties. Instead, increasing degrees of social homogeneity make transitive outcomes more likely, and there are some qualities of large groups that make such outcomes almost certain with relatively low levels of spectrum agreement. In more precise terms, 70% to 75% agreement all but ensures a transitive outcome, and the concept of net preferences lowers the bar even further.

When translated back into the voting rights arena, these recent advances in social choice theory mean that relatively low levels of racial bloc voting are necessary to ensure the possibility of a transitive outcome. Indeed, *Gingles* and its progeny require a level of bloc voting that is more than sufficient to the task, and thus provide the

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215. *See id.* at 785.

216. *Id.* at 786.

217. One should not be suspicious of this line of argument; it does not posit some sort of "emergent" group property possessed by none of the group's members. It is instead dictated by the mathematics of the situation. An example of such an "emergent" property is that larger groups are more likely to make correct decisions than its members if its members are more likely than not to be correct. Consider what happens, for example, if individuals make correct decisions 80% of the time. A three-person society composed of such individuals using a majority vote procedure will make the correct decision almost 90% of the time, more often than any single individual. Thus, the fact that a society may be more likely to make meaningful, transitive decisions should not come as a complete surprise.

degree of spectrum agreement necessary to solve the challenge posed by Arrow's theorem. They do so, however, without questioning the viability of the theorem and, because the racial bloc voting in section 2 cases is a form of natural spectrum agreement, without sacrificing the concerns embodied by Arrow's conditions of democratic fairness.

Such results are important to voting rights advocates in at least two ways. First, they mean that high levels of spectrum agreement in the form of bloc voting are not required to devise meaningful vote dilution standards. Second, the results dispense with the notion that high degrees of measurement precision are necessary in making claims of bloc voting. Because the standard has been lowered, relatively large degrees of imprecision may not doom bloc voting measurements. In sum, these advances in social choice theory mean that courts can be assured of finding meaningful qualitative vote dilution standards, and society can be more confident that courts go about that task in a neutral manner.